

Section 1.2

C01S02.001: The slope of L is $m = (3 - 0)/(2 - 0) = \frac{3}{2}$, so L has equation

$$y - 0 = \frac{3}{2}(x - 0); \quad \text{that is,} \quad 2y = 3x.$$

C01S02.002: Because L is vertical and $(7, 0)$ lies on L , every point on L has Cartesian coordinates $(7, y)$ for some number y (and every such point lies on L). Hence an equation of L is $x = 7$.

C01S02.003: Because L is horizontal, it has slope zero. Hence an equation of L is

$$y - (-5) = 0 \cdot (x - 3); \quad \text{that is,} \quad y = -5.$$

C01S02.004: Because $(2, 0)$ and $(0, -3)$ lie on L , it has slope $(0 + 3)/(2 - 0) = \frac{3}{2}$. Hence an equation of L is

$$y - 0 = \frac{3}{2}(x - 2); \quad \text{that is,} \quad y = \frac{3}{2}x - 3.$$

C01S02.005: The slope of L is $(3 - (-3))/(5 - 2) = 2$, so an equation of L is

$$y - 3 = 2(x - 5); \quad \text{that is,} \quad y = 2x - 7.$$

C01S02.006: An equation of L is $y - (-4) = \frac{1}{2}(x - (-1))$; that is, $2y + 7 = x$.

C01S02.007: The slope of L is $\tan(135^\circ) = -1$, so L has equation

$$y - 2 = -1 \cdot (x - 4); \quad \text{that is,} \quad x + y = 6.$$

C01S02.008: Equation: $y - 7 = 6(x - 0)$; that is, $y = 6x + 7$.

C01S02.009: The second line's equation can be written in the form $y = -2x + 10$ to show that it has slope -2 . Because L is parallel to the second line, L also has slope -2 and thus equation $y - 5 = -2(x - 1)$.

C01S02.010: The equation of the second line can be rewritten as $y = -\frac{1}{2}x + \frac{17}{2}$ to show that it has slope $-\frac{1}{2}$. Because L is perpendicular to the second line, L has slope 2 and thus equation $y - 4 = 2(x + 2)$.

C01S02.011: $x^2 - 4x + 4 + y^2 = 4$: $(x - 2)^2 + (y - 0)^2 = 2^2$. Center $(2, 0)$, radius 2 .

C01S02.012: $x^2 + y^2 + 6y + 9 = 9$: $(x - 0)^2 + (y + 3)^2 = 3^2$. Center $(0, -3)$, radius 3 .

C01S02.013: $x^2 + 2x + 1 + y^2 + 2y + 1 = 4$: $(x + 1)^2 + (y + 1)^2 = 2^2$. Center $(-1, -1)$, radius 2 .

C01S02.014: $x^2 + 10x + 25 + y^2 - 20y + 100 = 25$: $(x + 5)^2 + (y - 10)^2 = 5^2$. Center $(-5, 10)$, radius 5 .

C01S02.015: $x^2 + y^2 + x - y = \frac{1}{2}$: $x^2 + x + \frac{1}{4} + y^2 - y + \frac{1}{4} = 1$; $(x + \frac{1}{2})^2 + (y - \frac{1}{2})^2 = 1$. Center: $(-\frac{1}{2}, \frac{1}{2})$, radius 1 .

C01S02.016: $x^2 + y^2 - \frac{2}{3}x - \frac{4}{3}y = \frac{11}{9}$: $x^2 - \frac{2}{3}x + \frac{1}{9} + y^2 - \frac{4}{3}y + \frac{4}{9} = \frac{16}{9}$; $(x - \frac{1}{3})^2 + (y - \frac{2}{3})^2 = (\frac{4}{3})^2$. Center $(\frac{1}{3}, \frac{2}{3})$, radius $\frac{4}{3}$.

C01S02.017: $y = (x - 3)^2$: Opens upward, vertex at $(3, 0)$.

C01S02.018: $y - 16 = -x^2$: Opens downward, vertex at $(0, 16)$.

C01S02.019: $y - 3 = (x + 1)^2$: Opens upward, vertex at $(-1, 3)$.

C01S02.020: $2y = x^2 - 4x + 4 + 4$: $y - 2 = \frac{1}{2}(x - 2)^2$. Opens upward, vertex at $(2, 2)$.

C01S02.021: $y = 5(x^2 + 4x + 4) + 3 = 5(x + 2)^2 + 3$: Opens upward, vertex at $(-2, 3)$.

C01S02.022: $y = -(x^2 - x) = -(x^2 - x + \frac{1}{4}) + \frac{1}{4}$: $y - \frac{1}{4} = -(x - \frac{1}{2})^2$. Opens downward, vertex at $(\frac{1}{2}, \frac{1}{4})$.

C01S02.023: $x^2 - 6x + 9 + y^2 + 8y + 16 = 25$: $(x - 3)^2 + (y + 4)^2 = 5^2$. Circle, center $(3, -4)$, radius 5.

C01S02.024: $(x - 1)^2 + (y + 1)^2 = 0$: The graph consists of the single point $(1, -1)$.

C01S02.025: $(x + 1)^2 + (y + 3)^2 = -10$: There are no points on the graph.

C01S02.026: $x^2 + y^2 - x + 3y + 2.5 = 0$: $x^2 - x + 0.25 + y^2 + 3y + 2.25 = 0$: $(x - 0.5)^2 + (y + 1.5)^2 = 0$. The graph consists of the single point $(0.5, -1.5)$.

C01S02.027: The graph is the straight line segment connecting the two points $(-1, 7)$ and $(1, -3)$ (including those two points).

C01S02.028: The graph is the straight line segment connecting the two points $(0, 2)$ and $(2, -8)$, including the first of these two points but not the second.

C01S02.029: The graph is the parabola that opens downward, symmetric around the y -axis, with vertex at $(0, 10)$ and x -intercepts $\pm\sqrt{10}$.

C01S02.030: The graph of $y = 1 + 2x^2$ is a parabola that opens upwards, is symmetric around the y -axis, and has vertex at $(0, 1)$.

C01S02.031: The graph of $y = x^3$ can be visualized by modifying the familiar graph of the parabola with equation $y = x^2$: The former may be obtained by multiplying the y -coordinate of the latter's point (x, x^2) by x . Thus both have flat spots at the origin. For $0 < x < 1$, the graph of $y = x^3$ is below that of $y = x^2$. They cross at $(1, 1)$, and for $x > 1$ the graph of $y = x^2$ is below that of $y = x^3$, with the difference becoming arbitrarily large as x increases without bound. If the graph of $y = x^3$ for $x \geq 0$ is rotated 180° around the point $(0, 0)$, the graph of $y = x^3$ for $x < 0$ is the result.

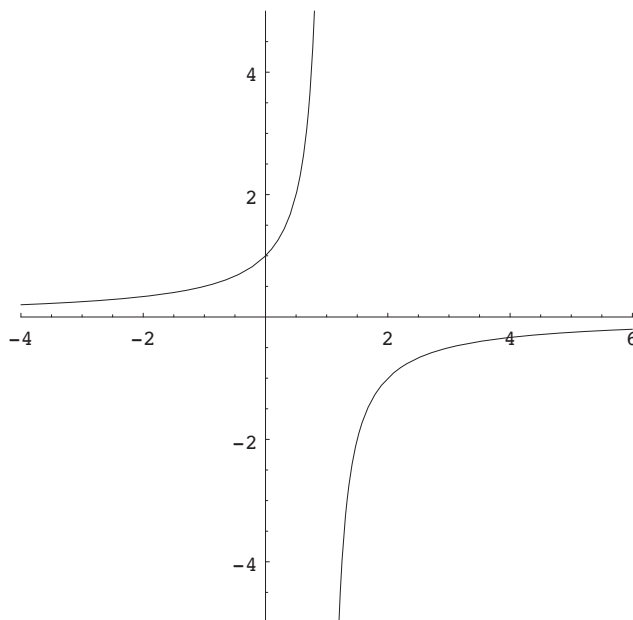
C01S02.032: The graph of $f(x) = x^4$ can be visualized by first visualizing the graph of $y = x^2$. If the y -coordinate of each point on this graph is replaced with its square (x^4), the result is the graph of f . The effect on the graph of $y = x^2$ is to multiply the y -coordinate by x^2 , which is between 0 and 1 for $0 < |x| < 1$ and which is larger than 1 for $|x| > 1$. Thus the graph of f superficially resembles that of $y = x^2$, but is much closer to the x -axis for $|x| < 1$ and much farther away for $|x| > 1$. The two graphs cross at $(0, 0)$ (where each has a flat spot) and at $(\pm 1, 1)$, but the graph of f is much steeper at the latter two points.

C01S02.033: To graph $y = f(x) = \sqrt{4 - x^2}$, note that $y \geq 0$ and that $y^2 = 4 - x^2$; that is, $x^2 + y^2 = 4$. Hence the graph of f is the *upper half* of the circle with center $(0, 0)$ and radius 2.

C01S02.034: To graph $y = f(x) = -\sqrt{9 - x^2}$, note that $y \leq 0$ and that $y^2 = 9 - x^2$; that is, that $x^2 + y^2 = 9$. Hence the graph of f is the *lower half* of the circle with center $(0, 0)$ and radius 3.

C01S02.035: To graph $f(x) = \sqrt{x^2 - 9}$, note that there is no graph for $-3 < x < 3$, that $f(\pm 3) = 0$, and that $f(x) > 0$ for $x < -3$ and for $x > 3$. If x is large positive, then $\sqrt{x^2 - 9} \approx \sqrt{x^2} = x$, so the graph of f has x -intercept $(3, 0)$ and rises as x increases, nearly coinciding with the graph of $y = x$ for x large positive. The case $x < -3$ is trickier. In this case, if x is a large negative number, then $f(x) = \sqrt{x^2 - 9} \approx \sqrt{x^2} = -x$ (Note the minus sign!). So for $x \leq -3$, the graph of f has x -intercept $(-3, 0)$ and, for x large negative, almost coincides with the graph of $y = -x$. Later we will see that the graph of f becomes arbitrarily steep as x gets closer and closer to ± 3 .

C01S02.036: As x increases without bound—either positively or negatively— $f(x)$ gets arbitrarily close to zero. Moreover, if x is large positive then $f(x)$ is negative and close to zero, so the graph of f lies just below the x -axis for such x . Similarly, the graph of f lies just above the x -axis for x large negative. If x is slightly less than 1 but very close to 1, then $f(x)$ is the reciprocal of a tiny positive number, hence is a large positive number. So the graph of f just to the left of the vertical line $x = 1$ almost coincides with the top half of that line. Similarly, just to the right of the line $x = 1$, then graph of f almost coincides with the bottom half of that line. There is no graph where $x = 1$, so the graph resembles the one in the next figure. The only intercept is the y -intercept $(0, 1)$. The graph correctly shows that the graph of f is increasing for $x < 1$ and for $x > 1$.



C01S02.037: Note that $f(x)$ is positive and close to zero for x large positive, so that the graph of f is just above the x -axis—and nearly coincides with it—for such x . Similarly, the graph of f is just below the x -axis and nearly coincides with it for x large negative. There is no graph where $x = -2$, but if x is slightly greater than -2 then $f(x)$ is the reciprocal of a very small positive number, so $f(x)$ is large and nearly coincides with the upper half of the vertical line $x = -2$. Similarly, if x is slightly less than -2 , then the graph of $f(x)$ is large negative and nearly coincides with the lower half of the line $x = -2$. The graph of f is decreasing for $x < -2$ and for $x > -2$ and its only intercept is the y -intercept $(0, \frac{1}{2})$.

C01S02.038: Note that $f(x)$ is very small but positive if x is either large positive or large negative. There is no graph for $x = 0$, but if x is very close to zero, then $f(x)$ is the reciprocal of a very small positive number, and hence is large positive. So the graph of f is just above the x -axis and almost coincides with it if $|x|$ is large, whereas the graph of f almost coincides with the positive y -axis for x near zero. There are no intercepts; the graph of f is increasing for $x < 0$ and is decreasing for $x > 0$.

C01S02.039: Note that $f(x) > 0$ for all x other than $x = 1$, where f is not defined. If $|x|$ is large, then $f(x)$ is near zero, so the graph of f almost coincides with the x -axis for such x . If x is very close to 1, then $f(x)$ is the reciprocal of a very small positive number, hence $f(x)$ is large positive. So for such x , the graph of $f(x)$ almost coincides with the upper half of the vertical line $x = 1$. The only intercept is $(0, 1)$.

C01S02.040: Note first that $f(x)$ is undefined at $x = 0$. To handle the absolute value symbol, we look at two cases: If $x > 0$, then $f(x) = 1$; if $x < 0$, then $f(x) = -1$. So the graph of f consists of the part of the horizontal line $y = 1$ for which $x > 0$, together with the part of the horizontal line $y = -1$ for which $x < 0$.

C01S02.041: Note that $f(x)$ is undefined when $2x + 3 = 0$; that is, when $x = -\frac{3}{2}$. If x is large positive, then $f(x)$ is positive and close to zero, so the graph of f is slightly above the x -axis and almost coincides with the x -axis. If x is large negative, then $f(x)$ is negative and close to zero, so the graph of f is slightly below the x -axis and almost coincides with the x -axis. If x is slightly greater than $-\frac{3}{2}$ then $f(x)$ is very large positive, so the graph of f almost coincides with the upper half of the vertical line $x = -\frac{3}{2}$. If x is slightly less than $-\frac{3}{2}$ then $f(x)$ is very large negative, so the graph of f almost coincides with the lower half of that vertical line. The graph of f is decreasing for $x < -\frac{3}{2}$ and also decreasing for $x > -\frac{3}{2}$. The only intercept is at $(0, \frac{1}{3})$.

C01S02.042: Note that $f(x)$ is undefined when $2x + 3 = 0$; that is, when $x = -\frac{3}{2}$. If x is large positive or large negative, then $f(x)$ is positive and close to zero, so the graph of f is slightly above the x -axis and almost coincides with the x -axis for $|x|$ large. If x is close to $-\frac{3}{2}$ then $f(x)$ is very large positive, so the graph of f almost coincides with the upper half of the vertical line $x = -\frac{3}{2}$. The graph of f is increasing for $x < -\frac{3}{2}$ and decreasing for $x > -\frac{3}{2}$. The only intercept is at $(0, \frac{1}{9})$.

C01S02.043: Given $y = f(x) = \sqrt{1-x}$, note that $y \geq 0$ and that $y^2 = 1-x$; that is, $x = 1-y^2$. So the graph is the part of the parabola $x = 1-y^2$ for which $y \geq 0$. This parabola has horizontal axis of symmetry the y -axis, opens to the left (because the coefficient of y^2 is negative), and has vertex $(1, 0)$. Therefore the graph of f is the upper half of this parabola.

C01S02.044: Note that the interval $x < 1$ is the domain of f , so there is no graph for $x \geq 1$. If x is a large negative number, then the denominator is large positive, so that its reciprocal $f(x)$ is very small positive. As x gets closer and closer to 1 (while $x < 1$), the denominator approaches zero, so its reciprocal $f(x)$ takes on arbitrarily large positive values. So the graph of f is slightly above the x -axis and almost coincides with that axis for x large negative; the graph of f almost coincides with the upper half of the vertical line $x = 1$ for x near (and less than) 1. The graph of f is increasing for all $x < 1$ and $(0, 1)$ is the only intercept.

C01S02.045: Note that $f(x)$ is defined only if $2x + 3 > 0$; that is, if $x > -\frac{3}{2}$. Note also that $f(x) > 0$ for all such x . If x is large positive, then $f(x)$ is positive but near zero, so the graph of f is just above the x -axis and almost coincides with it. If x is very close to $-\frac{3}{2}$ (but larger), then the denominator in $f(x)$ is very tiny positive, so the graph of f almost coincides with the upper half of the vertical line $x = -\frac{3}{2}$ for such x . The graph of f is decreasing for all $x > -\frac{3}{2}$.

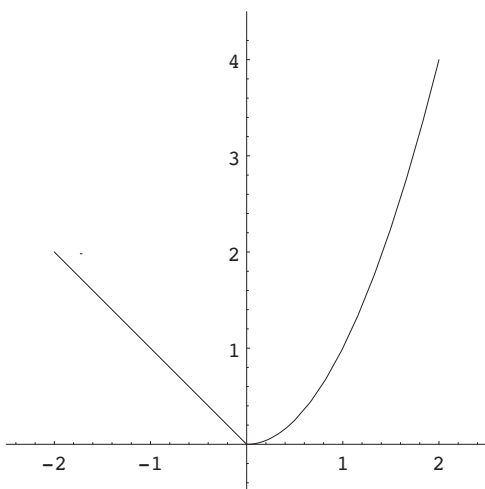
C01S02.046: Given: $f(x) = |2x - 2|$. Case 1: $x \geq 1$. Then $2x - 2 \geq 0$, so that $f(x) = 2x - 2$. Because $f(1) = 0$, the graph of f for $x \geq 1$ consists of the part of the straight line through $(1, 0)$ with slope 2. Case 2: $x < 1$. Then $2x - 2 < 0$, so that $f(x) = -2x + 2$. The line $y = -2x + 2$ passes through $(1, 0)$, so the graph of f for $x < 1$ consists of the part of the straight line through $(1, 0)$ with slope -2 .

C01S02.047: Given: $f(x) = |x| + x$. If $x \geq 0$ then $f(x) = x + x = 2x$, so if $x \geq 0$ then the graph of f is the part of the straight line through $(0, 0)$ with slope 2 for which $x \geq 0$. If $x < 0$ then $f(x) = -x + x = 0$, so the rest of the graph of f coincides with the negative x -axis.

C01S02.048: Given: $f(x) = |x - 3|$. If $x \geq 0$ then $f(x) = x - 3$, so the graph of f consists of the straight line through $(3, 0)$ with slope 1 for $x \geq 3$. If $x < 0$ then $f(x) = -x + 3$, so the graph of f consists of that part of the straight line with slope -3 and y -intercept $(0, 3)$. These two line segments fit together perfectly at the point $(3, 0)$; there is no break or gap or discontinuity in the graph of f .

C01S02.049: Given: $f(x) = |2x + 5|$. The two cases are determined by the point where $2x + 5$ changes sign, which is where $x = -\frac{5}{2}$. If $x \geq -\frac{5}{2}$, then $f(x) = 2x + 5$, so the graph of f consists of the part of the line with slope 2 and y -intercept 5 for which $x \geq -\frac{5}{2}$. If $x < -\frac{5}{2}$, then the graph of f is the part of the straight line $y = -2x - 5$ for which $x < -\frac{5}{2}$.

C01S02.050: The graph consists of the part of the line $y = -x$ for which $x < 0$ together with the part of the parabola $y = x^2$ for which $x \geq 0$. The two graphs fit together perfectly at the point $(0, 0)$; there is no break, gap, jump, or discontinuity there. The graph is shown next.



C01S02.051: The graph consists of the horizontal line $y = 0$ for $x < 0$ together with the horizontal line $y = 1$ for $x \geq 0$. As x moves from left to right through the value zero, there is an abrupt and unavoidable “jump” in the value of f from 0 to 1. That is, f is discontinuous at $x = 0$. To see part of the graph of f , enter the *Mathematica* commands

```
f[x_] := If[x < 0, 0, 1]
Plot[f[x], {x, -3.5, 3.5}, AspectRatio -> Automatic, PlotRange -> {{-3.5, 3.5}, {-1.5, 2.5}}];
```

C01S02.052: The graph of f consists of the open intervals $\dots, (-2, -1), (-1, 0), (0, 1), (1, 2), (2, 3), \dots$ on the x -axis together with the isolated points $\dots, (-1, 1), (0, 1), (1, 1), (2, 1), (3, 1), \dots$. There is a discontinuity at every integral value of x . A *Mathematica* plot of

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f[x_] := If[IntegerQ[x], 1, 0]
```

will produce a graph that’s completely different because *Mathematica*, like most plotting programs, “connects the dots,” in effect assuming that every function is continuous at every point in its domain.

C01S02.053: Because the graph of the greatest integer function changes at each integral value of x , the graph of $f(x) = \llbracket 2x \rrbracket$ changes twice as often—at each integral multiple of $\frac{1}{2}$. So as x moves from left to right through such points, the graph jumps upward one unit. Thus there is a discontinuity at each integral multiple of $\frac{1}{2}$. Because f is constant otherwise, these are the only discontinuities. To see something like the graph of f , enter the *Mathematica* commands

`f[x_] := Floor[2*x];`

`Plot[f[x], {x, -3.5, 3.5}, AspectRatio -> Automatic, PlotRange -> {{-3.5, 3.5}, {-4.5, 4.5}}];`

Mathematica will draw vertical lines connecting points that it shouldn't, making the graph look like treads and risers of a staircase, whereas only the treads are on the graph.

C01S02.054: The function f is undefined at $x = 1$. The graph consists of the horizontal line $y = 1$ for $x > 1$ together with the horizontal line $y = -1$ for $x < 1$. There is a discontinuity at $x = 1$.

C01S02.055: Given: $f(x) = \llbracket x \rrbracket$. If n is an integer and $n \leq x < n + 1$, then express x as $x = n + ((x))$ where $((x)) = x - \llbracket x \rrbracket$ is the *fractional part* of x . Then $f(x) = n - x = n - [n + ((x))] = -((x))$. So $f(x)$ is the negative of the fractional part of x . So as x ranges from n up to (but not including) $n + 1$, $f(x)$ begins at 0 and drops linearly down not quite to -1 . That is, on the interval $(n, n + 1)$, the graph of f is the straight line segment connecting the two points $(n, 0)$ and $(n + 1, -1)$ with the first of these points included and the second excluded. There is a discontinuity at each integral value of x .

C01S02.056: Given: $f(x) = \llbracket x \rrbracket + \llbracket -x \rrbracket + 1$. If x is an integer, then $f(x) = x + (-x) + 1 = 1$. If x is not an integer, then choose the integer n such that $n < x < n + 1$. Then $-(n + 1) < -x < -n$, so

$$f(x) = \llbracket x \rrbracket + \llbracket -x \rrbracket + 1 = n - (n + 1) + 1 = 0.$$

So f is the same function as the one defined in Problem 52 and has the same discontinuities: one at each integral value of x .

C01S02.057: Because $y = 2x^2 - 6x + 7 = 2(x^2 - 3x + 3.5) = 2(x^2 - 3x + 2.25 + 1.25) = 2(x - 1.5)^2 + 2.5$, the vertex of the parabola is at $(1.5, 2.5)$.

C01S02.058: Because $y = 2x^2 - 10x + 11 = 2(x^2 - 5x + 5.5) = 2(x^2 - 5x + 6.25 - 0.75) = 2(x - 2.5)^2 - 1.5$, the vertex of the parabola is at $(2.5, -1.5)$.

C01S02.059: Because $y = 4x^2 - 18x + 22 = 4(x^2 - (4.5)x + 5.5) = 4(x^2 - (4.5)x + 5.0625 + 0.4375) = 4(x - 2.25)^2 + 1.75$, the vertex of the parabola is at $(2.25, 1.75)$.

C01S02.060: Because $y = 5x^2 - 32x + 49 = 5(x^2 - (6.4)x + 9.8) = 5(x^2 - (6.4)x + 10.24 - 0.44) = 5(x - 3.2)^2 - 2.2$, the vertex of the parabola is at $(3.2, -2.2)$.

C01S02.061: Because $y = -8x^2 + 36x - 32 = -8(x^2 - (4.5)x + 4) = -8(x^2 - (4.5)x + 5.0625 - 1.0625) = -8(x - 2.25)^2 + 8.5$, the vertex of the parabola is at $(2.25, 8.5)$.

C01S02.062: Because $y = -5x^2 - 34x - 53 = -5(x^2 + (6.8)x + 10.6) = -5(x^2 + (6.8)x + 11.56 - 0.96) = -5(x + 3.4)^2 + 4.8$, the vertex of the parabola is at $(-3.4, 4.8)$.

C01S02.063: Because $y = -3x^2 - 8x + 3 = -3(x^2 + \frac{8}{3}x - 1) = -3(x^2 + \frac{8}{3}x + \frac{16}{9} - \frac{25}{9}) = -3(x + \frac{4}{3})^2 + \frac{25}{3}$, the vertex of the parabola is at $(-\frac{4}{3}, \frac{25}{3})$.

C01S02.064: Because $y = -9x^2 + 34x - 28 = -9(x^2 - \frac{34}{9}x + \frac{28}{9}) = -9(x^2 - \frac{34}{9}x + \frac{289}{81} - \frac{37}{81}) = -9(x - \frac{17}{9})^2 + \frac{37}{9}$, the vertex of the parabola is at $(\frac{17}{9}, \frac{37}{9})$.

C01S02.065: To find the maximum height $y = -16t^2 + 96t$ of the ball, we find the vertex of the parabola: $y = -16(t^2 - 6t) = -16(t^2 - 6t + 9 - 9) = -16(t - 3)^2 + 144$. The vertex of the parabola is at $(3, 144)$ and therefore the maximum height of the ball is 144 ft.

C01S02.066: Recall that the area of the rectangle is given by $y = A(x) = x(50 - x)$. To maximize $A(x)$ we find the vertex of the parabola: $y = 50x - x^2 = -(x^2 - 50x) = -(x^2 - 50x + 625 - 625) = -(x - 25)^2 + 625$. Because the vertex of the parabola is at $(25, 625)$ and $x = 25$ is in the domain of the function A , the maximum value of $A(x)$ occurs at $x = 25$ and is $A(25) = 625$ (ft²).

C01S02.067: If two positive numbers x and y have sum 50, then $y = 50 - x$ and $x < 50$ (because $y > 0$). To maximize their product $p(x)$ we find the vertex of the parabola

$$\begin{aligned} y = p(x) &= x(50 - x) = -(x^2 - 50x) \\ &= -(x^2 - 50x + 625 - 625) = -(x - 25)^2 + 625, \end{aligned}$$

which is at $(25, 625)$. Because $0 < 25 < 50$, $x = 25$ is in the domain of the product function $p(x) = x(50 - x)$, and hence the maximum value of the product of x and y is $p(25) = 625$.

C01S02.068: Recall that if x new wells are drilled, then the resulting total production p is given by $p(x) = 4000 + 100x - 5x^2$. To maximize $p(x)$ we find the vertex of the parabola

$$\begin{aligned} y = p(x) &= -5x^2 + 100x + 4000 = -5(x^2 - 20x - 800) \\ &= -5(x^2 - 20x + 100 - 900) = -5(x - 10)^2 + 4500. \end{aligned}$$

The vertex of the parabola $y = p(x)$ is therefore at $(10, 4500)$. Because $x = 10$ is in the domain of p (it is an integer between 0 and 40) and because the parabola opens downward (the coefficient of x^2 is negative), $x = 10$ indeed maximizes $p(x)$.

C01S02.069: The graph looks like the graph of $y = |x|$ because the slope of the left-hand part is -1 and that of the right-hand part is 1 ; but the vertex is shifted to $(-1, 0)$, so—using the translation principle—the graph in Fig. 1.2.29 must be the graph of $f(x) = |x + 1|$, $-2 \leq x \leq 2$.

C01S02.070: Because the graph in Fig. 1.2.30 is composed of three straight-line segments, it can be described most easily using a “three-part” function:

$$f(x) = \begin{cases} 2x + 6 & \text{if } -3 \leq x < -2; \\ 2 & \text{if } -2 \leq x < 2; \\ \frac{1}{3}(10 - 2x) & \text{if } 2 \leq x \leq 5. \end{cases}$$

C01S02.071: The graph in Fig. 1.2.31 is much like the graph of the greatest integer function—it takes on only integral values—but the “jumps” occur twice as often, so this must be very like—indeed, it is exactly—the graph of $f(x) = \llbracket 2x \rrbracket$, $-1 \leq x < 2$.

C01S02.072: The graph in Fig. 1.2.32 resembles the graph of the greatest integer function in that it takes on all integral values and only those, but it is decreasing rather than increasing and the “jumps” occur only at the even integers. Thus it must be the graph of something similar to $f(x) = -\llbracket \frac{1}{2}x \rrbracket$, $-4 \leq x < 4$. Comparing values of f at $x = -4, -3, -2.1, -2, -1, -0.1, 0, 1, 1.9, 2, 3$, and 3.9 with points on the graph is sufficient evidence that the graph of f is indeed that shown in the figure.

C01S02.073: Clearly $x(t) = 45t$ for the first hour; that is, for $0 \leq t \leq 1$. In the second hour the graph of $x(t)$ must be a straight line (because of constant speed) of slope 75, thus with equation $x(t) = 75t + C$ for some constant C . The constant C is determined by the fact that $45t$ and $75t + C$ must be equal at time

$t = 1$, as the automobile cannot suddenly jump from one position to a completely different position in an instant. Hence $45 = 75 + C$, so that $C = -30$. Therefore

$$x(t) = \begin{cases} 45t & \text{if } 0 \leq t \leq 1; \\ 75t - 30 & \text{if } 1 < t \leq 2. \end{cases}$$

To see the graph of $x(t)$, plot in *Mathematica*

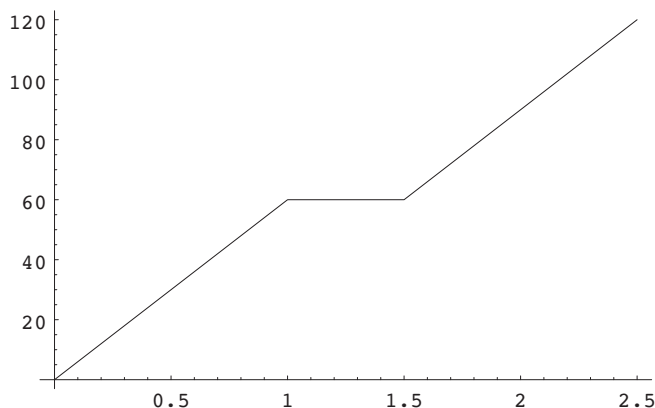
```
x[t_] := If[t < 1, 45*t, 75*t - 30 ]
```

on the interval $0 \leq t \leq 2$.

C01S02.074: The graph of $x(t)$ will consist of three straight-line segments (because of the constant speeds), the first of slope 60 for $0 \leq t \leq 1$, the second of slope zero for $1 \leq t \leq 1.5$, and the third of slope 60 for $1.5 \leq t \leq 2.5$. The first pair must coincide when $t = 1$ and the second pair must coincide when $t = 1.5$ because the graph of $x(t)$ can have no discontinuities. So if we write $x(t) = 60t$ for $0 \leq t \leq 1$, we must have $x(t) = 60$ for $1 \leq t \leq 1.5$. Finally, $x(t) = 60t + C$ for some constant C if $1.5 \leq t \leq 2.5$, but the latter must equal 60 when $t = 1.5$, so that $C = -30$. Hence

$$x(t) = \begin{cases} 60t & \text{if } 0 \leq t \leq 1, \\ 60 & \text{if } 1 < t \leq 1.5, \\ 60t - 30 & \text{if } 1.5 < t \leq 2.5. \end{cases}$$

The graph of $x(t)$ is shown next.



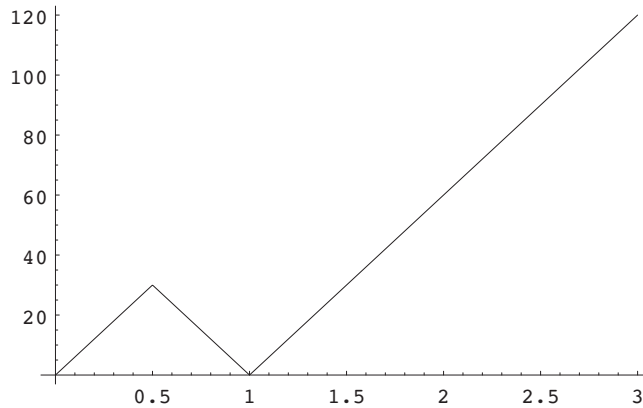
C01S02.075: The graph must consist of two straight-line segments (because of the constant speeds). The first must have slope 60, so we have $x(t) = 60t$ for $0 \leq t \leq 1$. The second must have slope -30 , negative because you're driving in the reverse direction, so $x(t) = -30t + C$ for some constant C if $1 \leq t \leq 3$. The two segments must coincide when $t = 1$, so that $60 = -30 + C$. Thus $C = 90$ and thus a formula for $x(t)$ is

$$x(t) = \begin{cases} 60t & \text{if } 0 \leq t \leq 1, \\ 90 - 30t & \text{if } 1 < t \leq 3. \end{cases}$$

C01S02.076: We need three straight line segments, the first of slope 60 for $0 \leq t \leq 0.5$, the second of slope -60 for $0.5 \leq t \leq 1$, and the third of slope 60 for $1 \leq t \leq 3$. Clearly the first must be $x(t) = 60t$ for $0 \leq t \leq 0.5$. The second must have the form $x(t) = -60t + C$ for some constant C , and the first and second must coincide when $t = 0.5$, so that $30 = -30 + C$, and thus $C = 60$. The third segment must have the form $x(t) = 60t + K$ for some constant K , and the second and third must coincide when $t = 1$, so that $0 = 60 + K$, and so $K = -60$. Therefore a formula for $x(t)$ is

$$x(t) = \begin{cases} 60t & \text{if } 0 \leq t \leq 0.5, \\ 60 - 60t & \text{if } 0.5 < t \leq 1, \\ 60t - 60 & \text{if } 1 < t \leq 3. \end{cases}$$

The graph of $x(t)$ is shown next.



C01S02.077: Initially we work in units of pages and cents (to avoid decimals and fractions). The graph of C , as a function of p , must be a straight line segment, and its slope is (by information given)

$$\frac{C(79) - C(34)}{79 - 34} = \frac{305 - 170}{79 - 34} = \frac{135}{45} = 3.$$

Thus $C(p) = 3p + K$ for some constant K . So $3 \cdot 34 + K = 170$, and it follows that $K = 68$. So $C(p) = 3p + 68$, $1 \leq p \leq 100$, if C is to be expressed in cents. If C is to be expressed in dollars, we have

$$C(p) = (0.03)p + 0.68, \quad 1 \leq p \leq 100.$$

The “fixed cost” is incurred regardless of the number of pamphlets printed; it is \$0.68. The “marginal cost” of printing each additional page of the pamphlet is the coefficient \$0.03 of p .

C01S02.078: We are given $C(x) = a + bx$ where a and b are constants; we are also given

$$99.45 = C(207) = a + 207b \quad \text{and}$$

$$79.15 = C(149) = a + 149b.$$

Subtraction of the second equation from the first yields $20.3 = 58b$, so that $b = 0.35$. Substitution of this datum in the first of the preceding equations then yields

$$99.45 = a + 207 \cdot 0.35 = a + 72.45, \quad \text{so that} \quad a = 27.$$

Therefore $C(x) = 27 + (0.35)x$, $0 \leq x < +\infty$. Thus if you drive 175 miles on the third day, the cost for that day will be $C(175) = 88.25$ (in dollars). The slope $b = 0.35$ represents a cost of \$0.35 per mile. The C -intercept $a = 27$ represents the daily base cost of renting the car. In civil engineering and in some branches of applied mathematics, the intercept $a = 27$ is sometimes called the *offset*, representing the vertical amount by which $C(0)$ is “offset” from zero.

C01S02.079: Suppose that the letter weighs x ounces, $0 < x \leq 16$. If $x \leq 8$, then the cost is simply 8 (dollars). If $8 < x \leq 9$, add \$0.80; if $9 < x \leq 10$, add \$1.60, and so on. Very roughly, one adds \$0.80 if

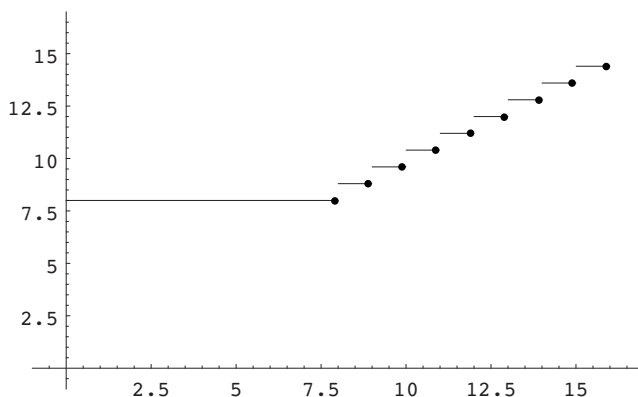
$\lfloor x - 8 \rfloor = 1$, \$1.60 if $\lfloor x - 8 \rfloor = 2$, and so on. But this isn't quite right—we are using the FLOOR function of Section 1.1, whereas we should really be using the CEILING function. By the result of Problem 51 of that section, we see that instead of cost

$$C(x) = 8 + (0.8)\lfloor x - 8 \rfloor$$

for $8 < x \leq 16$, we should instead write

$$C(x) = \begin{cases} 8 & \text{if } 0 < x \leq 8, \\ 8 - (0.8)\lceil -(x - 8) \rceil & \text{if } 8 < x \leq 16. \end{cases}$$

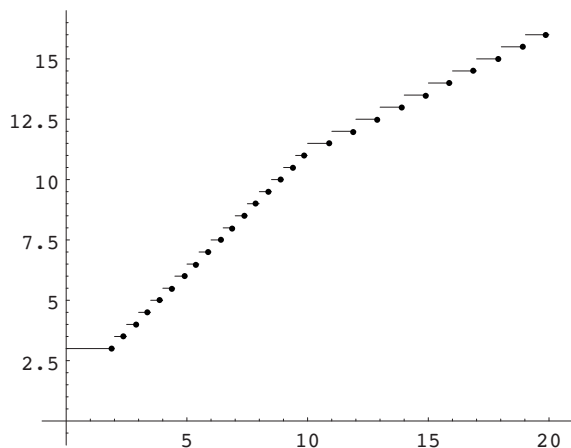
The graph of the cost function is shown next.



C01S02.080: Solve this problem like Problem 79 (but it is more complicated). Result:

$$C(x) = \begin{cases} 3 & \text{if } 0 < x \leq 2; \\ 3 - 0.5\lceil -2(x - 2) \rceil & \text{if } 2 < x \leq 10; \\ 11 - 0.5\lceil -(x - 10) \rceil & \text{if } 10 < x \leq 20. \end{cases}$$

The graph of C is shown below.



C01S02.081: Boyle's law states that under conditions of constant temperature, the product of the pressure p and the volume V of a fixed mass of gas remains constant. If we assume that $pV = c$, a constant, for the given data, we find that the given five data points yield the values $c = 1.68, 1.68, 1.675, 1.68$, and 1.62 . The

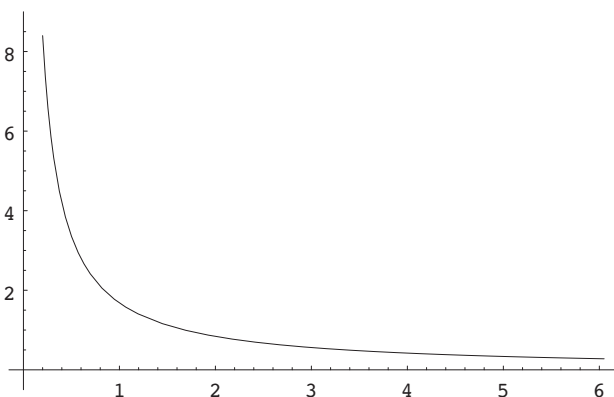
average of these is 1.65 (to two places) and should be a good estimate of the true value of c . Alternatively, you can use a computer algebra program to find c ; in *Mathematica*, for example, the command `Fit` will fit given data points to a sum of constant multiples of functions you specify. We used the commands

```
data = {{0.25, 6.72}, {1.0, 1.68}, {2.5, 0.67}, {4.0, 0.42}, {6.0, 0.27}};
Fit[data, {1/p}, p]
```

to find that

$$V(p) = \frac{1.67986}{p}$$

yields the best *least-squares* fit of the given data to a function of the form $V(p) = c/p$. We rounded the numerator to 1.68 to find the estimates $V(0.5) \approx 3.36$ and $V(5) \approx 0.336$ (L). The graph of $V(p)$ is shown next.



C01S02.082: It seems reasonable to assume that the maximum average temperature occurs on July 15 and the minimum on January 15, so that a multiple of a cosine function should fit the given data if we take $t = 0$ on July 15. So we assume a solution of the form

$$T(t) = c_1 + c_2 \cos\left(\frac{2\pi t}{365}\right).$$

Also assuming that the average year-round daily temperature is the average of the minimum and the maximum, we find that $c_1 = 61.25$, so we could find c_2 by the averaging method of Problem 81. Alternatively, we could use the `Fit` command in *Mathematica* to find both c_1 and c_2 simultaneously as follows:

```
data = {{0, 79.1}, {62, 70.2}, {123, 52.3}, {184, 43.4}, {224, 52.2}, {285, 70.1}};
Fit[data, {1, Cos[2*Pi*t/365]}, t]
```

The result is the formula

$$T(t) = 62.9602 + (17.437) \cos\left(\frac{2\pi t}{365}\right).$$

The values predicted by this function at the six dates in question are [approximately] 80.4, 71.4, 53.9, 45.5, 49.8, and 66.3. Not bad, considering we are dealing with weather, a most unpredictable phenomenon. The graph of $T(t)$ is shown next. Units on the horizontal axis are days, measured from July 15. Units on

the vertical axis are degrees Fahrenheit. Remember that these are *average* daily temperatures; it is not uncommon for a winter low in Athens to be below 28°F and for a summer high to be as much as 92°F.

