13–1.
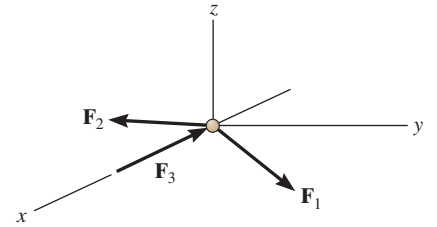
The 6-lb particle is subjected to the action of its weight and forces $\mathbf{F}_1 = \{2\mathbf{i} + 6\mathbf{j} - 2t\mathbf{k}\}$ lb, $\mathbf{F}_2 = \{t^2\mathbf{i} - 4t\mathbf{j} - 1\mathbf{k}\}$ lb, and $\mathbf{F}_3 = \{-2t\mathbf{i}\}$ lb, where t is in seconds. Determine the distance the ball is from the origin 2 s after being released from rest.

SOLUTION

$\Sigma \mathbf{F} = m\mathbf{a};\quad (2\mathbf{i} + 6\mathbf{j} - 2t\mathbf{k}) + (t^2\mathbf{i} - 4t\mathbf{j} - 1\mathbf{k}) - 2t\mathbf{i} - 6\mathbf{k} = \left(\dfrac{6}{32.2}\right)(a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k})$

Equating components:

$\left(\dfrac{6}{32.2}\right)a_x = t^2 - 2t + 2 \qquad \left(\dfrac{6}{32.2}\right)a_y = -4t + 6 \qquad \left(\dfrac{6}{32.2}\right)a_z = -2t - 7$

Since $dv = a\,dt$, integrating from $v = 0, t = 0$, yields

$\left(\dfrac{6}{32.2}\right)v_x = \dfrac{t^3}{3} - t^2 + 2t \qquad \left(\dfrac{6}{32.2}\right)v_y = -2t^2 + 6t \qquad \left(\dfrac{6}{32.2}\right)v_z = -t^2 - 7t$

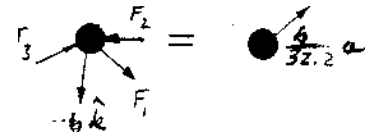Since $ds = v\,dt$, integrating from $s = 0, t = 0$ yields

$\left(\dfrac{6}{32.2}\right)s_x = \dfrac{t^4}{12} - \dfrac{t^3}{3} + t^2 \qquad \left(\dfrac{6}{32.2}\right)s_y = -\dfrac{2t^3}{3} + 3t^2 \qquad \left(\dfrac{6}{32.2}\right)s_z = -\dfrac{t^3}{3} - \dfrac{7t^2}{2}$

When $t = 2$ s then, $\quad s_x = 14.31$ ft, $\qquad s_y = 35.78$ ft $\qquad s_z = -89.44$ ft

Thus,

$s = \sqrt{(14.31)^2 + (35.78)^2 + (-89.44)^2} = 97.4$ ft $\hspace{3cm}$ **Ans.**

245

13–2.

The two boxcars A and B have a weight of 20 000 lb and 30 000 lb, respectively. If they are freely coasting down the incline when the brakes are applied to all the wheels of car A, determine the force in the coupling C between the two cars. The coefficient of kinetic friction between the wheels of A and the tracks is $\mu_k = 0.5$. The wheels of car B are free to roll. Neglect their mass in the calculation. *Suggestion:* Solve the problem by representing single resultant normal forces acting on A and B, respectively.
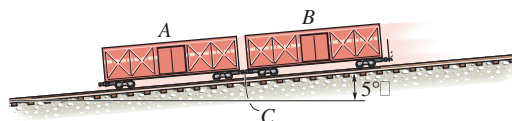
SOLUTION

Car A:

$$+\nwarrow\Sigma F_y = 0; \qquad N_A - 20\,000 \cos 5° = 0 \qquad N_A = 19\,923.89 \text{ lb}$$

$$+\nearrow\Sigma F_x = ma_x; \qquad 0.5(19\,923.89) - T - 20\,000 \sin 5° = \left(\frac{20\,000}{32.2}\right)a \qquad \textbf{(1)}$$
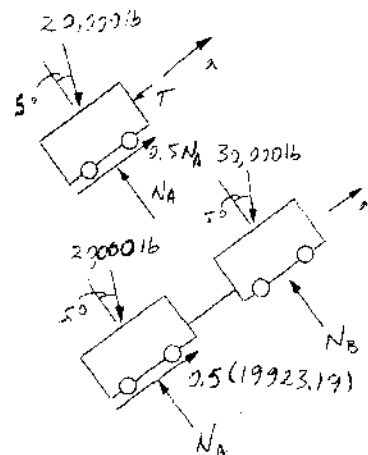
Both cars:

$$+\nearrow\Sigma F_x = ma_x; \qquad 0.5(19\,923.89) - 50\,000 \sin 5° = \left(\frac{50\,000}{32.2}\right)a$$

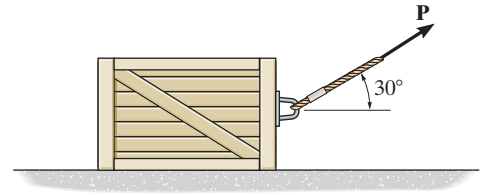Solving,

$$a = 3.61 \text{ ft/s}^2$$

$$T = 5.98 \text{ kip} \qquad\qquad \textbf{Ans.}$$

246

13–3.

If the coefficient of kinetic friction between the 50-kg crate and the ground is $\mu_k = 0.3$, determine the distance the crate travels and its velocity when $t = 3$ s. The crate starts from rest, and $P = 200$ N.

SOLUTION

Free-Body Diagram: The kinetic friction $F_f = \mu_k N$ is directed to the left to oppose the motion of the crate which is to the right, Fig. *a*.

Equations of Motion: Here, $a_y = 0$. Thus,

$+\uparrow \Sigma F_y = 0;$ $N - 50(9.81) + 200 \sin 30° = 0$

$N = 390.5$ N

$\xrightarrow{+} \Sigma F_x = ma_x;$ $200 \cos 30° - 0.3(390.5) = 50a$

$a = 1.121$ m/s^2

Kinematics: Since the acceleration **a** of the crate is constant,

$(\xrightarrow{+})$ $v = v_0 + a_c t$
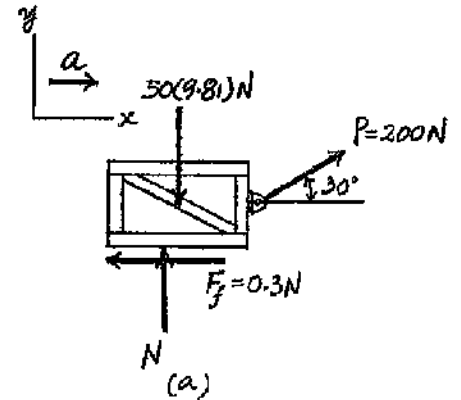
$v = 0 + 1.121(3) = 3.36$ m/s   **Ans.**

and

$(\xrightarrow{+})$ $s = s_0 + v_0 t + \dfrac{1}{2} a_c t^2$

$s = 0 + 0 + \dfrac{1}{2}(1.121)(3^2) = 5.04$ m   **Ans.**
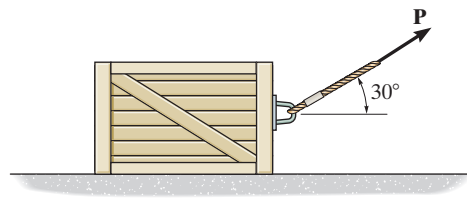
Ans:
$v = 3.36$ m/s
$s = 5.04$ m

*13–4.

If the 50-kg crate starts from rest and achieves a velocity of $v = 4$ m/s when it travels a distance of 5 m to the right, determine the magnitude of force **P** acting on the crate. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.3$.
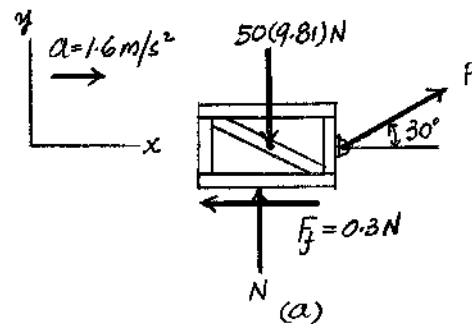
SOLUTION

Kinematics: The acceleration **a** of the crate will be determined first since its motion is known.

$(\xrightarrow{+})$ $v^2 = v_0{}^2 + 2a_c(s - s_0)$

         $4^2 = 0^2 + 2a(5 - 0)$

         $a = 1.60$ m/s$^2 \rightarrow$

Free-Body Diagram: Here, the kinetic friction $F_f = \mu_k N = 0.3N$ is required to be directed to the left to oppose the motion of the crate which is to the right, Fig. *a*.

Equations of Motion:

$+\uparrow \Sigma F_y = ma_y;$ $N + P \sin 30° - 50(9.81) = 50(0)$

                 $N = 490.5 - 0.5P$

Using the results of **N** and **a**,

$\xrightarrow{+} \Sigma F_x = ma_x;$ $P \cos 30° - 0.3(490.5 - 0.5P) = 50(1.60)$

              $P = 224$ N   **Ans.**

Ans:
$P = 224$ N

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13–5.

If blocks A and B of mass 10 kg and 6 kg, respectively, are placed on the inclined plane and released, determine the force developed in the link. The coefficients of kinetic friction between the blocks and the inclined plane are $\mu_A = 0.1$ and $\mu_B = 0.3$. Neglect the mass of the link.
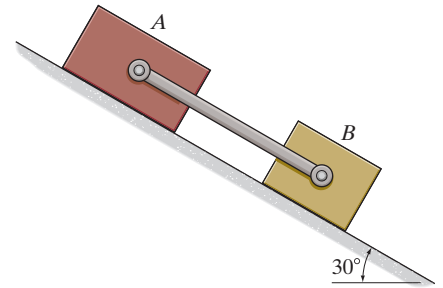
SOLUTION

Free-Body Diagram: Here, the kinetic friction $(F_f)_A = \mu_A N_A = 0.1 N_A$ and $(F_f)_B = \mu_B N_B = 0.3 N_B$ are required to act up the plane to oppose the motion of the blocks which are down the plane. Since the blocks are connected, they have a common acceleration $\mathbf{a}$.

Equations of Motion: By referring to Figs. (a) and (b),

$$+\nearrow\Sigma F_{y'} = ma_{y'}; \qquad N_A - 10(9.81)\cos 30° = 10(0)$$

$$N_A = 84.96 \text{ N}$$

$$\searrow+\Sigma F_{x'} = ma_{x'}; \qquad 10(9.81)\sin 30° - 0.1(84.96) - F = 10a$$

$$40.55 - F = 10a \qquad (1)$$

and

$$+\nearrow\Sigma F_{y'} = ma_{y'}; \qquad N_B - 6(9.81)\cos 30° = 6(0)$$
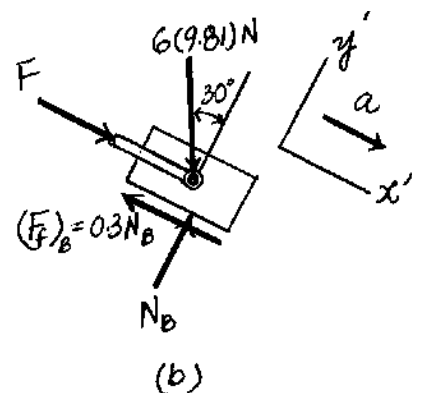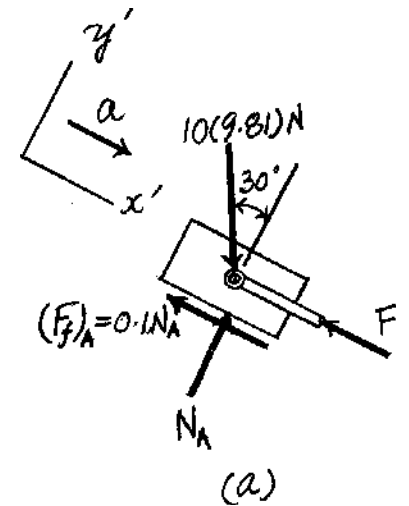
$$N_B = 50.97 \text{ N}$$

$$\searrow+\Sigma F_{x'} = ma_{x'}; \qquad F + 6(9.81)\sin 30° - 0.3(50.97) = 6a$$

$$F + 14.14 = 6a \qquad (2)$$

Solving Eqs. (1) and (2) yields
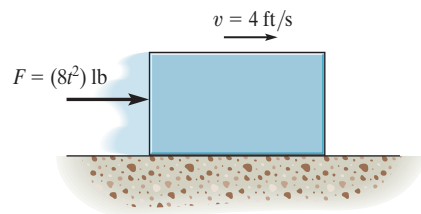
$$a = 3.42 \text{ m/s}^2$$

$$F = 6.37 \text{ N} \qquad \text{Ans.}$$

(a)

(b)

Ans:
$F = 6.37$ N

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13–6.

The 10-lb block has a speed of 4 ft/s when the force of $F = (8t^2)$ lb is applied. Determine the velocity of the block when $t = 2$ s. The coefficient of kinetic friction at the surface is $\mu_k = 0.2$.

$v = 4$ ft/s

$F = (8t^2)$ lb

SOLUTION

Equations of Motion. Here the friction is $F_f = \mu_k N = 0.2N$. Referring to the FBD of the block shown in Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 10 = \frac{10}{32.2}(0) \qquad N = 10 \text{ lb}$$
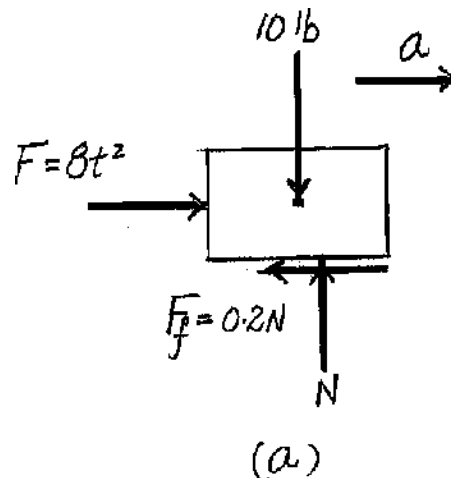
$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 8t^2 - 0.2(10) = \frac{10}{32.2}a$$

$$a = 3.22(8t^2 - 2) \text{ ft/s}^2$$

Kinematics. The velocity of the block as a function of t can be determined by integrating $dv = a \, dt$ using the initial condition $v = 4$ ft/s at $t = 0$.

$$\int_{4 \text{ ft/s}}^{v} dv = \int_{0}^{t} 3.22 \, (8t^2 - 2)dt$$

$$v - 4 = 3.22\left(\frac{8}{3}t^3 - 2t\right)$$

$$v = \{8.5867t^3 - 6.44t + 4\} \text{ ft/s}$$

When $t = 2$ s,

$$v = 8.5867(2^3) - 6.44(2) + 4$$

$$= 59.81 \text{ ft/s}$$

$$= 59.8 \text{ ft/s} \qquad\qquad \textbf{Ans.}$$
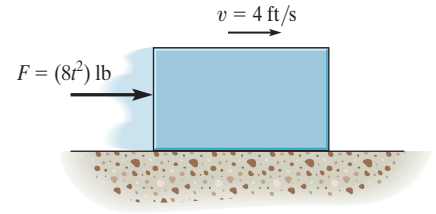
$10\,lb$

a

$F = 8t^2$

$F_f = 0.2N$

N

(a)

Ans:
$v = 59.8$ ft/s

13–7.

The 10-lb block has a speed of 4 ft/s when the force of $F = (8t^2)$ lb is applied. Determine the velocity of the block when it moves $s = 30$ ft. The coefficient of kinetic friction at the surface is $\mu_s = 0.2$.

SOLUTION

Equations of Motion. Here the friction is $F_f = \mu_k N = 0.2N$. Referring to the FBD of the block shown in Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \quad N - 10 = \frac{10}{32.2}(0) \quad N = 10 \text{ lb}$$

$$\underrightarrow{+} \Sigma F_x = ma_x; \quad 8t^2 - 0.2(10) = \frac{10}{32.2}a$$

$$a = 3.22(8t^2 - 2) \text{ ft/s}^2$$

Kinematics. The velocity of the block as a function of t can be determined by integrating $dv = adt$ using the initial condition $v = 4$ ft/s at $t = 0$.

$$\int_{4 \text{ ft/s}}^{v} dv = \int_{0}^{t} 3.22(8t^2 - 2)dt$$

$$v - 4 = 3.22\left(\frac{8}{3}t^3 - 2t\right)$$

$$v = \{8.5867t^3 - 6.44t + 4\} \text{ ft/s}$$

The displacement as a function of t can be determined by integrating $ds = vdt$ using the initial condition $s = 0$ at $t = 0$

$$\int_{0}^{s} ds = \int_{0}^{t} (8.5867t^3 - 6.44t + 4)dt$$

$$s = \{2.1467t^4 - 3.22t^2 + 4t\} \text{ ft}$$

At $s = 30$ ft,
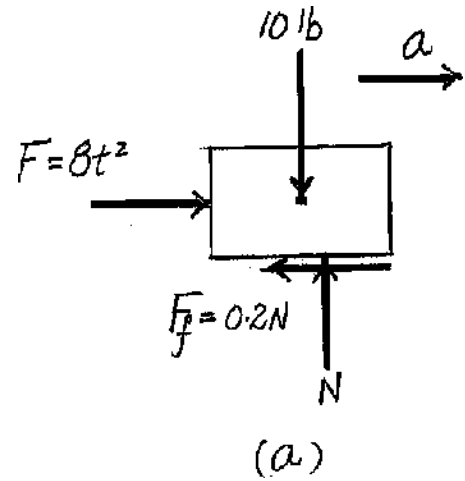
$$30 = 2.1467t^4 - 3.22t^2 + 4t$$

Solved by numerically,

$$t = 2.0089 \text{ s}$$

Thus, at $s = 30$ ft,

$$v = 8.5867(2.0089^3) - 6.44(2.0089) + 4$$

$$= 60.67 \text{ ft/s}$$
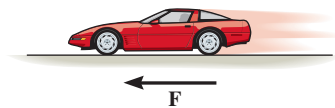
$$= 60.7 \text{ ft/s} \quad \textbf{Ans.}$$

Ans:
$v = 60.7$ ft/s

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*13–8.

The speed of the 3500-lb sports car is plotted over the 30-s time period. Plot the variation of the traction force **F** needed to cause the motion.

SOLUTION

Kinematics: For $0 \leq t < 10$ s. $v = \dfrac{60}{10}t = \{6t\}$ ft/s. Applying equation $a = \dfrac{dv}{dt}$, we have

$$a = \frac{dv}{dt} = 6 \text{ ft/s}^2$$

For $10 < t \leq 30$ s, $\dfrac{v - 60}{t - 10} = \dfrac{80 - 60}{30 - 10}$, $v = \{t + 50\}$ ft/s. Applying equation $a = \dfrac{dv}{dt}$, we have

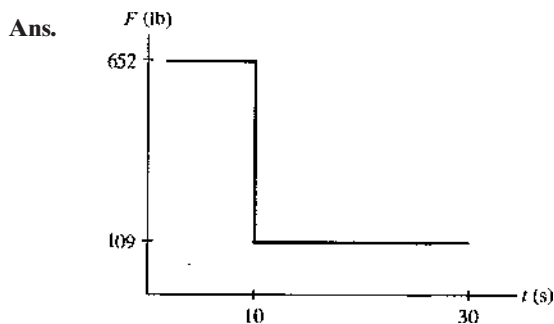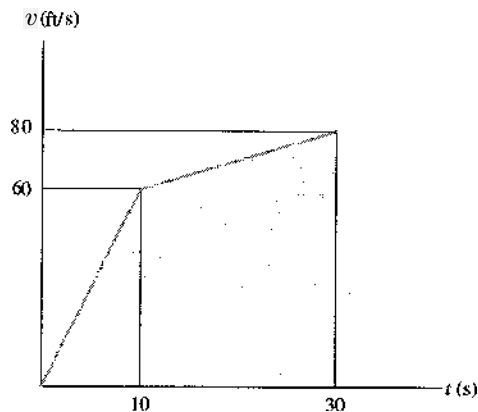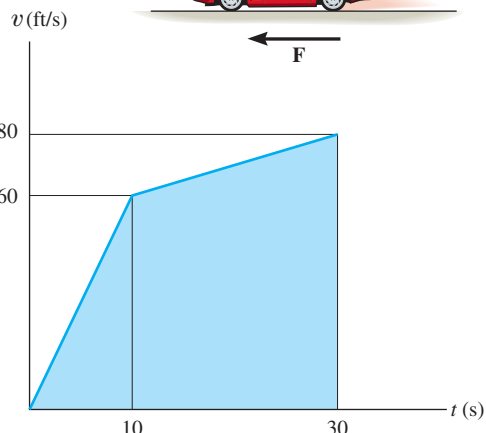$$a = \frac{dv}{dt} = 1 \text{ ft/s}^2$$

Equation of Motion:

For $0 \leq t < 10$ s

$$\xleftarrow{+} \Sigma F_x = ma_x; \quad F = \left(\frac{3500}{32.2}\right)(6) = 652 \text{ lb} \qquad \textbf{Ans.}$$

For $10 < t \leq 30$ s

$$\xleftarrow{+} \Sigma F_x = ma_x; \quad F = \left(\frac{3500}{32.2}\right)(1) = 109 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
$$\xleftarrow{+} \Sigma F_x = ma_x; F = 652 \text{ lb}$$
$$\xleftarrow{+} \Sigma F_x = ma_x; F = 109 \text{ lb}$$

13–9.

The conveyor belt is moving at 4 m/s. If the coefficient of static friction between the conveyor and the 10-kg package B is $\mu_s = 0.2$, determine the shortest time the belt can stop so that the package does not slide on the belt.
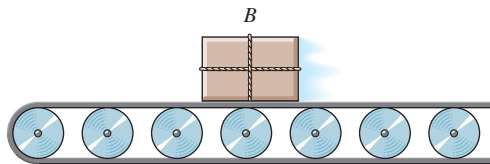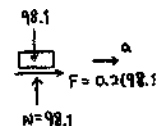
SOLUTION

$\stackrel{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 0.2(98.1) = 10\,a$

$\qquad\qquad\qquad a = 1.962 \text{ m/s}^2$

$(\stackrel{+}{\rightarrow})v = v_0 + a_c t$

$\qquad 4 = 0 + 1.962\,t$

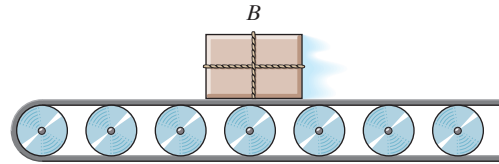$\qquad t = 2.04 \text{ s} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ **Ans.**

Ans:
$t = 2.04$ s

13–10.

The conveyor belt is designed to transport packages of various weights. Each 10-kg package has a coefficient of kinetic friction $\mu_k = 0.15$. If the speed of the conveyor is 5 m/s, and then it suddenly stops, determine the distance the package will slide on the belt before coming to rest.

SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 0.15\, m(9.81) = ma$

$\qquad\qquad a = 1.4715 \text{ m/s}^2$

$(\xrightarrow{+})\ v^2 = v_0^2 + 2a_c(s - s_0)$

$\qquad 0 = (5)^2 + 2(-1.4715)(s - 0)$

$\qquad s = 8.49 \text{ m}$ \qquad\qquad\qquad\qquad **Ans.**

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13–11.

Determine the time needed to pull the cord at B down 4 ft starting from rest when a force of 10 lb is applied to the cord. Block A weighs 20 lb. Neglect the mass of the pulleys and cords.

SOLUTION

$$+\uparrow \Sigma F_y = ma_y; \qquad 40 - 20 = \frac{20}{32.2} a_A \qquad \textbf{Ans.}$$

$$a_A = 32.2 \text{ ft/s}^2$$

$$s_B + 2s_C = l; \qquad a_B = -2a_C$$

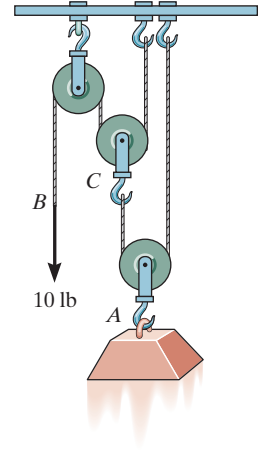$$2s_A - s_C = l'; \qquad 2a_A = a_C$$

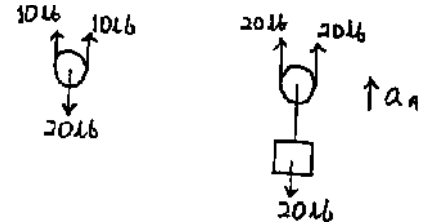$$a_B = -4a_A$$

$$a_B = 128.8 \text{ ft/s}^2$$

$$(+\downarrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$
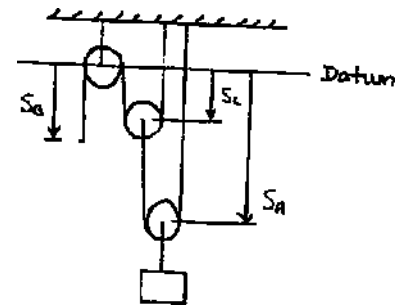
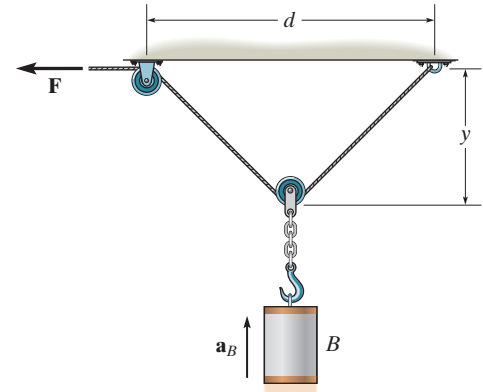$$4 = 0 + 0 + \frac{1}{2}(128.8) t^2$$

$$t = 0.249 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$t = 0.249$ s

*13–12.

Cylinder B has a mass m and is hoisted using the cord and pulley system shown. Determine the magnitude of force $\mathbf{F}$ as a function of the block's vertical position y so that when $\mathbf{F}$ is applied the block rises with a constant acceleration $\mathbf{a}_B$. Neglect the mass of the cord and pulleys.

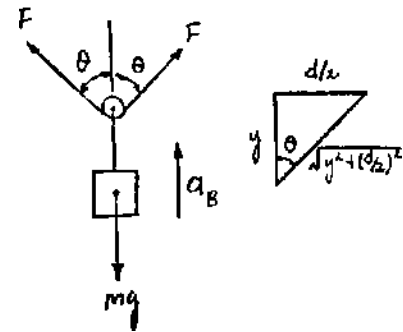## SOLUTION

$+\uparrow \Sigma F_y = ma_y;$ $2F\cos\theta - mg = ma_B$ where $\cos\theta = \dfrac{y}{\sqrt{y^2 + \left(\frac{d}{2}\right)^2}}$

$$2F\left(\frac{y}{\sqrt{y^2 + \left(\frac{d}{2}\right)^2}}\right) - mg = ma_B$$

$$F = \frac{m(a_B + g)\sqrt{4y^2 + d^2}}{4y}$$ **Ans.**
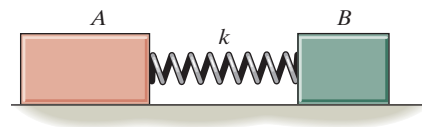
Ans:
$$F = \frac{m(a_B + g)\sqrt{4y^2 + d^2}}{4y}$$

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13–13.

Block A has a weight of 8 lb and block B has a weight of 6 lb. They rest on a surface for which the coefficient of kinetic friction is $\mu_k = 0.2$. If the spring has a stiffness of $k = 20$ lb/ft, and it is compressed 0.2 ft, determine the acceleration of each block just after they are released.

SOLUTION

Block A:

$$\xleftarrow{+} \Sigma F_x = ma_x; \qquad 4 - 1.6 = \frac{8}{32.2} a_A$$

$$a_A = 9.66 \text{ ft/s}^2 \leftarrow \qquad\qquad \textbf{Ans.}$$

Block B:

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 4 - 12 = \frac{6}{32.2} a_B$$

$$a_B = 15.0 \text{ ft/s}^2 \rightarrow \qquad\qquad \textbf{Ans.}$$
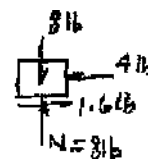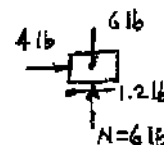
Ans:
$a_A = 9.66 \text{ ft/s}^2 \leftarrow$
$a_B = 15.0 \text{ ft/s}^2 \rightarrow$

13–14.

The 2-Mg truck is traveling at 15 m/s when the brakes on all its wheels are applied, causing it to skid for a distance of 10 m before coming to rest. Determine the constant horizontal force developed in the coupling C, and the frictional force developed between the tires of the truck and the road during this time. The total mass of the boat and trailer is 1 Mg.

SOLUTION

Kinematics: Since the motion of the truck and trailer is known, their common acceleration **a** will be determined first.

$$\left(\xrightarrow{+} \right) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = 15^2 + 2a(10 - 0)$$

$$a = -11.25 \text{ m/s}^2 = 11.25 \text{ m/s}^2 \leftarrow$$

Free-Body Diagram: The free-body diagram of the truck and trailer are shown in Figs. (a) and (b), respectively. Here, **F** representes the frictional force developed when the truck skids, while the force developed in coupling C is represented by **T**.

Equations of Motion: Using the result of **a** and referrning to Fig. (a),

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -T = 1000(-11.25)$$

$$T = 11\,250 \text{ N} = 11.25 \text{ kN} \qquad\qquad \textbf{Ans.}$$

Using the results of **a** and **T** and referring to Fig. (b),

$$+\uparrow \Sigma F_x = ma_x; \qquad 11\,250 - F = 2000(-11.25)$$

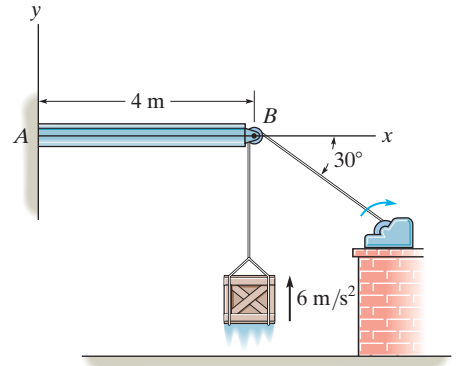$$F = 33\,750 \text{ N} = 33.75 \text{ kN} \qquad\qquad \textbf{Ans.}$$

(a)

(b)
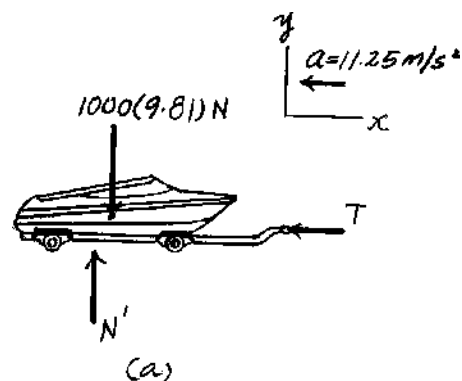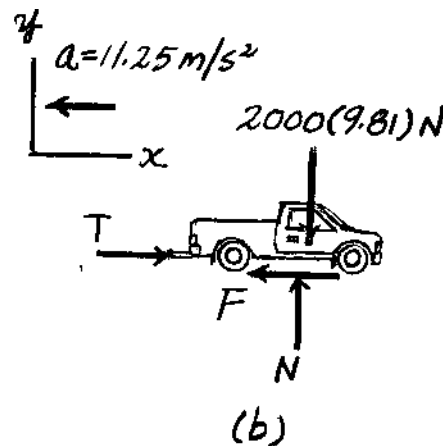
Ans:
$T = 11.25$ kN
$F = 33.75$ kN

13–15.

The motor lifts the 50-kg crate with an acceleration of 6 m/s^2. Determine the components of force reaction and the couple moment at the fixed support A.

SOLUTION

Equation of Motion. Referring to the FBD of the crate shown in Fig. *a*,

$+\uparrow \Sigma F_y = ma_y;$ $T - 50(9.81) = 50(6)$ $T = 790.5 \text{ N}$

Equations of Equilibrium. Since the pulley is smooth, the tension is constant throughout entire cable. Referring to the FBD of the pulley shown in Fig. *b*,

$\xrightarrow{+} \Sigma F_x = 0;$ $790.5 \cos 30° - B_x = 0$ $B_x = 684.59 \text{ N}$

$+\uparrow \Sigma F_y = 0;$ $B_y - 790.5 - 790.5 \sin 30° = 0$ $B_y = 1185.75 \text{ N}$

Consider the FBD of the cantilever beam shown in Fig. *c*,

$\xrightarrow{+} \Sigma F_x = 0;$ $684.59 - A_x = 0$ $A_x = 684.59 \text{ N} = 685 \text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $A_y - 1185.75 = 0$ $A_y = 1185.75 \text{ N} = 1.19 \text{ kN}$ **Ans.**

$\zeta + \Sigma M_A = 0;$ $M_A - 1185.75(4) = 0$ $M_A = 4743 \text{ N} \cdot \text{m} = 4.74 \text{ kN} \cdot \text{m}$ **Ans.**
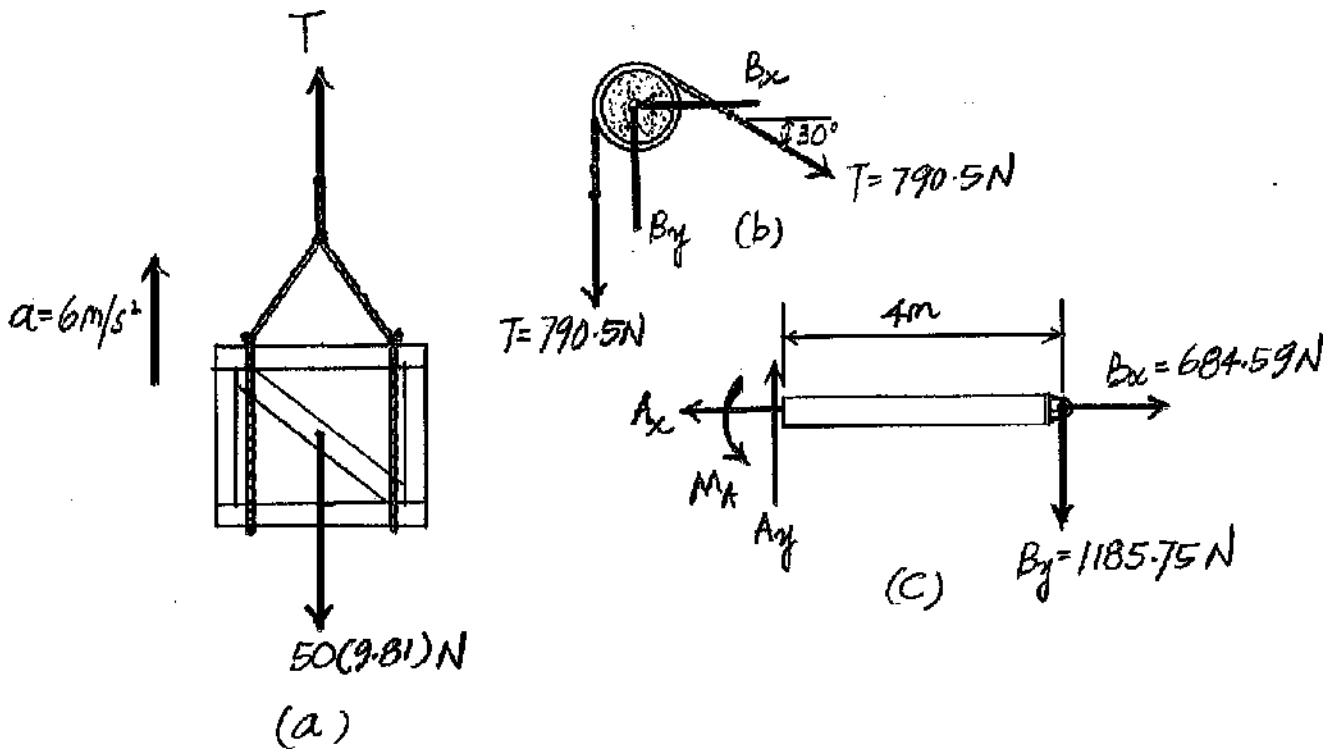
Ans:
$A_x = 685 \text{ N}$
$A_y = 1.19 \text{ kN}$
$M_A = 4.74 \text{ kN} \cdot \text{m}$

*13–16.

The 75-kg man pushes on the 150-kg crate with a horizontal force **F**. If the coefficients of static and kinetic friction between the crate and the surface are $\mu_s = 0.3$ and $\mu_k = 0.2$, and the coefficient of static friction between the man's shoes and the surface is $\mu_s = 0.8$, show that the man is able to move the crate. What is the greatest acceleration the man can give the crate?

SOLUTION

Equation of Equilibrium. Assuming that the crate is on the verge of sliding $(F_f)_C = \mu_s N_C = 0.3N_C$. Referring to the FBD of the crate shown in Fig. *a*,

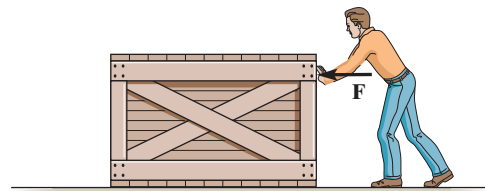$+\uparrow \Sigma F_y = 0; \qquad N_C - 150(9.81) = 0 \qquad N_C = 1471.5 \text{ N}$

$\overset{+}{\underset{\rightarrow}{}} \Sigma F_x = 0; \qquad 0.3(1471.5) - F = 0 \qquad F = 441.45 \text{ N}$

Referring to the FBD of the man, Fig. *b*,

$+\uparrow \Sigma F_y = 0; \qquad N_m - 75(9.81) = 0 \qquad N_B = 735.75 \text{ N}$

$\overset{+}{\underset{\rightarrow}{}} \Sigma F_x = 0; \qquad 441.45 - (F_f)_m = 0 \qquad (F_f)_m = 441.45 \text{ N}$

Since $(F_f)_m < \mu'_s N_m = 0.8(735.75) = 588.6 \text{ N}$, **the man is able to move the crate.**

Equation of Motion. The greatest acceleration of the crate can be produced when the man is on the verge of slipping. Thus, $(F_f)_m = \mu'_s N_m = 0.8(735.75) = 588.6 \text{ N}$.

$\overset{+}{\underset{\rightarrow}{}} \Sigma F_x = 0; \qquad F - 588.6 = 0 \qquad F = 588.6 \text{ N}$

Since the crate slides, $(F_f)_C = \mu_k N_C = 0.2(1471.5) = 294.3 \text{ N}$. Thus,

$\overset{+}{\underset{\leftarrow}{}} \Sigma F_x = ma_x; \qquad 588.6 - 294.3 = 150 a$

$\qquad\qquad a = 1.962 \text{ m/s}^2 = 1.96 \text{ m/s}^2 \qquad\qquad$ **Ans.**

(a)   (b)
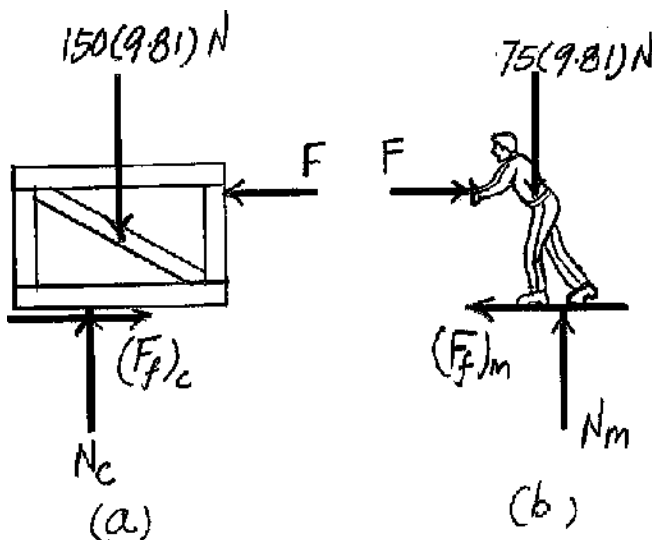
Ans:
$a = 1.96 \text{ m/s}^2$

13–17.

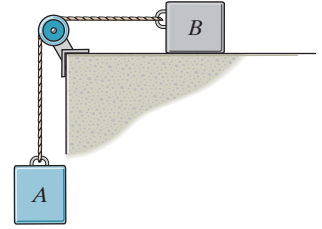Determine the acceleration of the blocks when the system is released. The coefficient of kinetic friction is μ_k, and the mass of each block is m. Neglect the mass of the pulleys and cord.

SOLUTION

Free Body Diagram. Since the pulley is smooth, the tension is constant throughout the entire cord. Since block B is required to slide, $F_f = \mu_k N$. Also, blocks A and B are attached together with inextensible cord, so $a_A = a_B = a$. The FBDs of blocks A and B are shown in Figs. a and b, respectively.

Equations of Motion. For block A, Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad T - mg = m(-a) \tag{1}$$
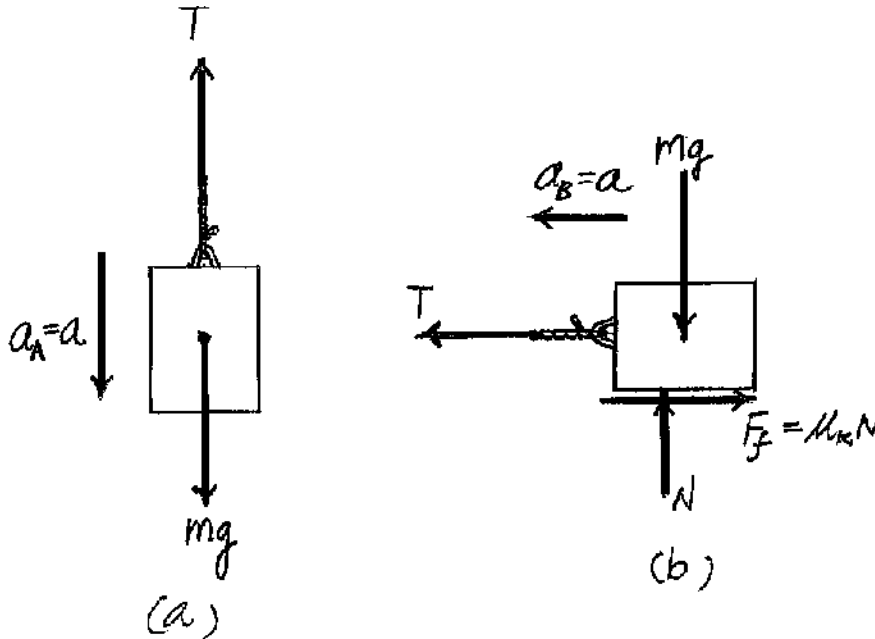
For block B, Fig. b,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - mg = m(0) \qquad N = mg$$

$$(\pm)\Sigma F_x = ma_x; \qquad T - \mu_k mg = ma \tag{2}$$

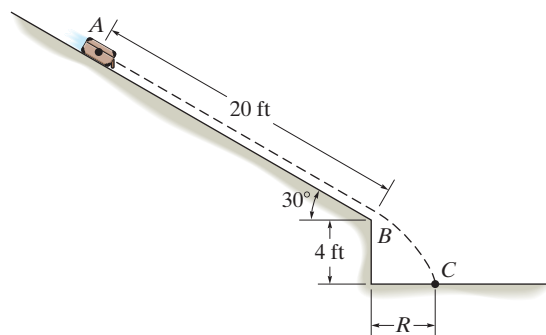Solving Eqs. (1) and (2)

$$a = \frac{1}{2}(1 - \mu_k)\,g \qquad\qquad \textbf{Ans.}$$

$$T = \frac{1}{2}(1 + \mu_k)\,mg$$

(a)

(b)

Ans:
$$a = \frac{1}{2}(1 - \mu_k)\,g$$

13–18.

A 40-lb suitcase slides from rest 20 ft down the smooth ramp. Determine the point where it strikes the ground at C. How long does it take to go from A to C?

SOLUTION

$+\searrow \Sigma F_x = m\, a_x; \qquad 40 \sin 30° = \dfrac{40}{32.2}a$

$a = 16.1 \text{ ft/s}^2$

$(+\searrow)v^2 = v_0^2 + 2\, a_c(s - s_0);$

$v_B^2 = 0 + 2(16.1)(20)$

$v_B = 25.38 \text{ ft/s}$

$(+\searrow)\, v = v_0 + a_c t;$

$25.38 = 0 + 16.1\, t_{AB}$

$t_{AB} = 1.576 \text{ s}$

$(\xrightarrow{+})s_x = (s_x)_0 + (v_x)_0\, t$

$\qquad R = 0 + 25.38 \cos 30°(t_{BC})$

$(+\downarrow)\, s_y = (s_y)_0 + (v_y)_0\, t + \dfrac{1}{2}a_c t^2$

$\qquad 4 = 0 + 25.38 \sin 30°\, t_{BC} + \dfrac{1}{2}(32.2)(t_{BC})^2$

$t_{BC} = 0.2413 \text{ s}$

$R = 5.30 \text{ ft}$ **Ans.**

Total time $= t_{AB} + t_{BC} = 1.82 \text{ s}$ **Ans.**

13–19.

Solve Prob. 13–18 if the suitcase has an initial velocity down the ramp of $v_A = 10$ ft/s and the coefficient of kinetic friction along AB is $\mu_k = 0.2$.

SOLUTION

$+\searrow \Sigma F_x = ma_x; \qquad 40 \sin 30° - 6.928 = \dfrac{40}{32.2} a$

$\qquad\qquad\qquad a = 10.52 \text{ ft/s}^2$

$(+\searrow)\ v^2 = v_0^2 + 2\, a_c (s - s_0);$

$v_B^2 = (10)^2 + 2(10.52)(20)$

$v_B = 22.82 \text{ ft/s}$

$(+\searrow)\ v = v_0 + a_c t;$

$22.82 = 10 + 10.52\, t_{AB}$

$t_{AB} = 1.219 \text{ s}$

$(\xrightarrow{+})\ s_x = (s_x)_0 + (v_x)_0\, t$
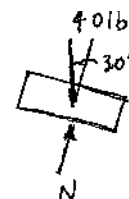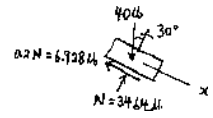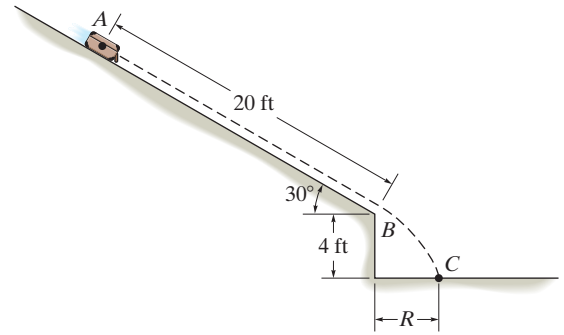
$R = 0 + 22.82 \cos 30° (t_{BC})$

$(+\downarrow)\ s_y = (s_y)_0 + (v_y)_0\, t + \dfrac{1}{2} a_c\, t^2$

$4 = 0 + 22.82 \sin 30°\, t_{BC} + \dfrac{1}{2}(32.2)(t_{BC})^2$

$t_{BC} = 0.2572 \text{ s}$

$R = 5.08 \text{ ft}$ **Ans.**

Total time $= t_{AB} + t_{BC} = 1.48 \text{ s}$ **Ans.**

*13–20.

The conveyor belt delivers each 12-kg crate to the ramp at A such that the crate's speed is $v_A = 2.5$ m/s, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is $\mu_k = 0.3$, determine the speed at which each crate slides off the ramp at B. Assume that no tipping occurs. Take $\theta = 30°$.
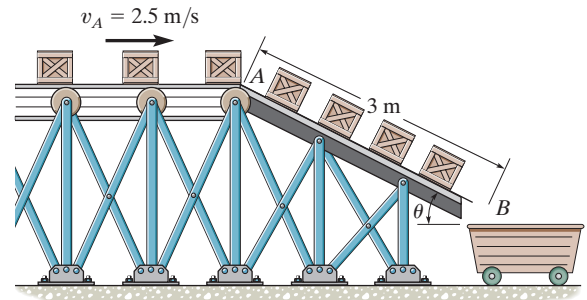
SOLUTION

$\nearrow + \Sigma F_y = ma_y; \quad N_C - 12(9.81)\cos 30° = 0$

$N_C = 101.95$ N

$+ \searrow \Sigma F_x = ma_x; \quad 12(9.81)\sin 30° - 0.3(101.95) = 12\, a_C$

$a_C = 2.356$ m/s^2

$(+\searrow) \quad v_B^2 = v_A^2 + 2\, a_C(s_B - s_A)$

$v_B^2 = (2.5)^2 + 2(2.356)(3 - 0)$

$v_B = 4.5152 = 4.52$ m/s   **Ans.**
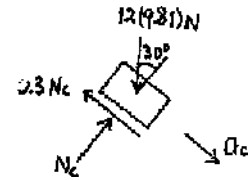

Ans:
$v_B = 4.52$ m/s

13–21.

The conveyor belt delivers each 12-kg crate to the ramp at A such that the crate's speed is $v_A = 2.5$ m/s, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is $\mu_k = 0.3$, determine the smallest incline θ of the ramp so that the crates will slide off and fall into the cart.

$v_A = 2.5$ m/s

3 m

B

θ
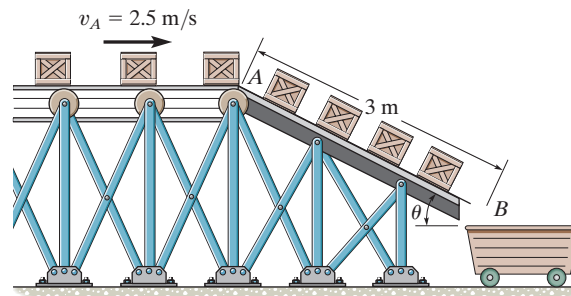
SOLUTION

$(+\searrow) \quad v_B^2 = v_A^2 + 2a_C(s_B - s_A)$

$\qquad 0 = (2.5)^2 + 2(a_C)(3 - 0)$

$\qquad a_C = 1.0417$

$\nearrow + \Sigma F_y = ma_y; \qquad N_C - 12(9.81)\cos\theta = 0$

$\qquad\qquad\qquad\quad N_C = 117.72\cos\theta$

$+\searrow \Sigma F_x = ma_x; \qquad 12(9.81)\sin\theta - 0.3(N_C) = 12\,(1.0417)$

$\qquad\qquad\qquad\quad 117.72\sin\theta - 35.316\cos\theta - 12.5 = 0$

Solving,

$\theta = 22.6°$ **Ans.**

$12(9.81)N$

$30°$

$N_C \quad 0.3N_C$
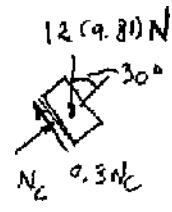
13–22.

The 50-kg block A is released from rest. Determine the velocity of the 15-kg block B in 2 s.

SOLUTION

Kinematics. As shown in Fig. a, the position of block B and point A are specified by s_B and s_A respectively. Here the pulley system has only one cable which gives

$$s_A + s_B + 2(s_B - a) = l$$

$$s_A + 3s_B = l + 2a \tag{1}$$

Taking the time derivative of Eq. (1) twice,

$$a_A + 3a_B = 0 \tag{2}$$

Equations of Motion. The FBD of blocks B and A are shown in Fig. b and c. To be consistent to those in Eq. (2), $\mathbf{a}_A$ and $\mathbf{a}_B$ are assumed to be directed towards the positive sense of their respective position coordinates s_A and s_B. For block B,

$$+\uparrow \Sigma F_y = ma_y; \qquad 3T - 15(9.81) = 15(-a_B) \tag{3}$$

For block A,

$$+\uparrow \Sigma F_y = ma_y; \qquad T - 50(9.81) = 50(-a_A) \tag{4}$$

Solving Eqs. (2), (3) and (4),

$$a_B = -2.848 \text{ m/s}^2 = 2.848 \text{ m/s}^2 \uparrow \qquad a_A = 8.554 \text{ m/s}^2 \qquad T = 63.29 \text{ N}$$

The negative sign indicates that $\mathbf{a}_B$ acts in the sense opposite to that shown in FBD. The velocity of block B can be determined using

$$+\uparrow \quad v_B = (v_A)_0 + a_B t; \qquad v_B = 0 + 2.848(2)$$

$$v_B = 5.696 \text{ m/s} = 5.70 \text{ m/s} \uparrow \qquad \qquad \textbf{Ans.}$$

(a)

15(9.81)N

(b)

50(9.81)N

(c)
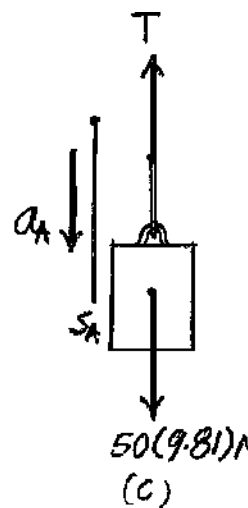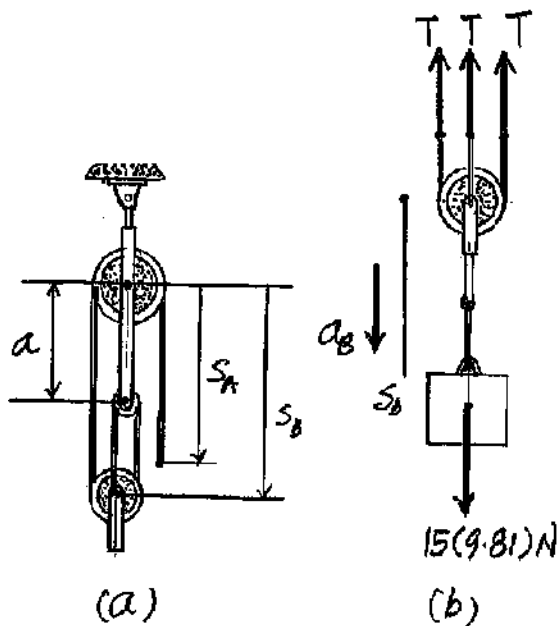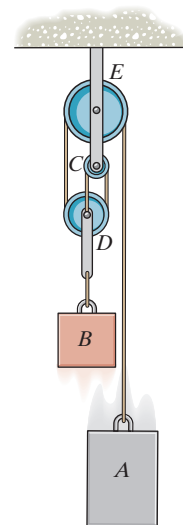
Ans:
$v_B = 5.70$ m/s $\uparrow$

13–23.

If the supplied force $F = 150$ N, determine the velocity of the 50-kg block A when it has risen 3 m, starting from rest.
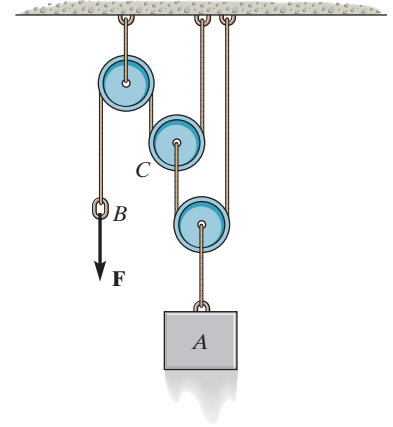
SOLUTION

Equations of Motion. Since the pulleys are smooth, the tension is constant throughout each entire cable. Referring to the FBD of pulley C, Fig. a, of which its mass is negligible.

$+\uparrow \Sigma F_y = 0;$ $150 + 150 - T = 0$ $T = 300$ N

Subsequently, considered the FBD of block A shown in Fig. b,

$+\uparrow \Sigma F_y = ma_y;$ $300 + 300 - 50(9.81) = 50a$

$$a = 2.19 \text{ m/s}^2 \uparrow$$

Kinematics. Using the result of **a**,

$(+\uparrow)$ $v^2 = v_0^2 + 2a_c s;$

$$v^2 = 0^2 + 2(2.19)(3)$$

$$v = 3.6249 \text{ m/s} = 3.62 \text{ m/s} \qquad \textbf{Ans.}$$

150N 150N

T

(a)

T=300N T=300N

a

50(9.81)N

(b)

Ans:
$v = 3.62$ m/s $\uparrow$

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*13–24.

A 60-kg suitcase slides from rest 5 m down the smooth ramp. Determine the distance R where it strikes the ground at B. How long does it take to go from A to B?
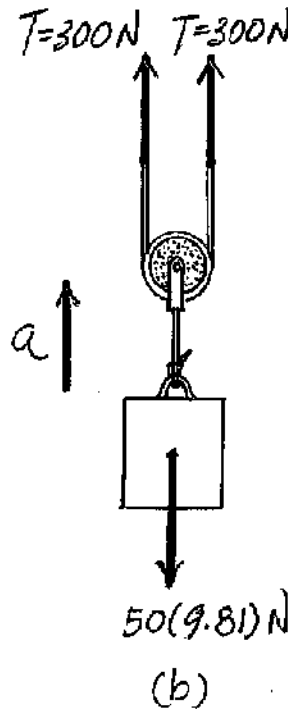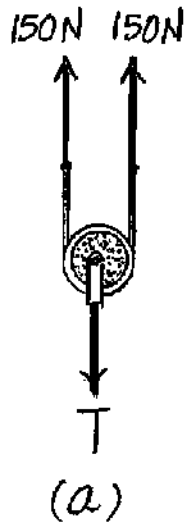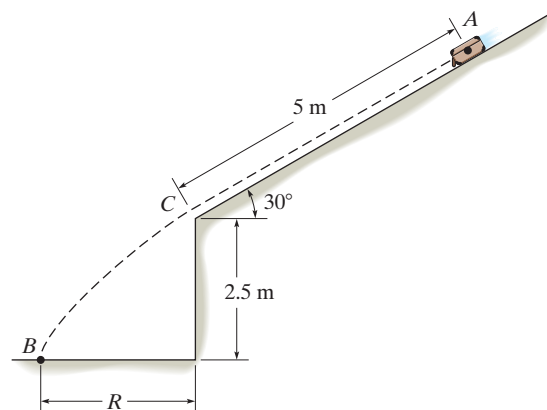
SOLUTION

Equation of Motion. Referring to the FBD of the suitcase shown in Fig. a

$+\swarrow\Sigma F_{x'} = ma_{x'};\qquad 60(9.81)\sin 30° = 60a\qquad a = 4.905 \text{ m/s}^2$

Kinematics. From A to C, the suitcase moves along the inclined plane (straight line).

$(+\swarrow)\ v^2 = v_0^2 + 2a_c s;\qquad v^2 = 0^2 + 2(4.905)(5)$

$$v = 7.0036 \text{ m/s}$$

$(+\swarrow)\ s = s_0 + v_0 t + \dfrac{1}{2}a_c t^2;\qquad 5 = 0 + 0 + \dfrac{1}{2}(4.905)t_{AC}^2$

$$t_{AC} = 1.4278 \text{ s}$$

From C to B, the suitcase undergoes projectile motion. Referring to x–y coordinate system with origin at C, Fig. b, the vertical motion gives

$(+\downarrow)\quad s_y = (s_0)_y + v_y t + \dfrac{1}{2}a_y t^2;$

$$2.5 = 0 + 7.0036\sin 30°\, t_{CB} + \dfrac{1}{2}(9.81)t_{CB}^2$$

$$4.905\, t_{CB}^2 + 3.5018\, t_{CB} - 2.5 = 0$$

Solve for positive root,

$$t_{CB} = 0.4412 \text{ s}$$

Then, the horizontal motion gives

$(\underset{\leftarrow}{\pm})\ s_x = (s_0)_x + v_x t;$

$$R = 0 + 7.0036\cos 30° (0.4412)$$

$$= 2.676 \text{ m} = 2.68 \text{ m}\qquad\qquad \textbf{Ans.}$$

The time taken from A to B is

$$t_{AB} = t_{AC} + t_{CB} = 1.4278 + 0.4412 = 1.869 \text{ s} = 1.87 \text{ s}\qquad \textbf{Ans.}$$

(a)

(b)
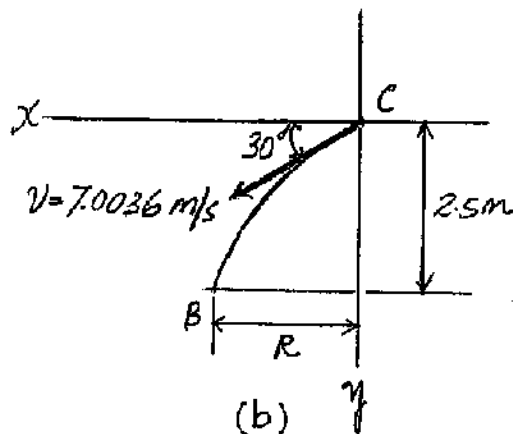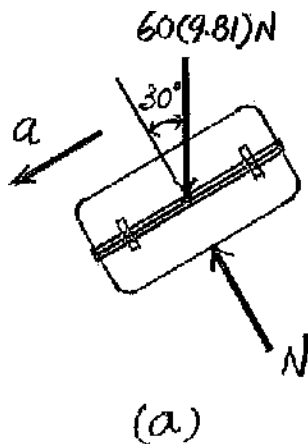
Ans:
$(\underset{\leftarrow}{\pm})\ s_x = 2.68$ m
$t_{AB} = 1.87$ s

13–25.

Solve Prob. 13–24 if the suitcase has an initial velocity down the ramp of $v_A = 2$ m/s, and the coefficient of kinetic friction along AC is $\mu_k = 0.2$.

SOLUTION

Equations of Motion. The friction is $F_f = \mu_k N = 0.2N$. Referring to the FBD of the suitcase shown in Fig. *a*

$$\nwarrow^+ \Sigma F_{y'} = ma_{y'}; \quad N - 60(9.81) \cos 30° = 60(0)$$

$$N = 509.74 \text{ N}$$

$$+\swarrow \Sigma F_{x'} = ma_{x'}; \quad 60(9.81) \sin 30° - 0.2(509.74) = 60 \, a$$

$$a = 3.2059 \text{ m/s}^2 \checkmark$$

Kinematics. From A to C, the suitcase moves along the inclined plane (straight line).

$$(+\swarrow) \quad v^2 = v_0^2 + 2a_c s; \quad v^2 = 2^2 + 2(3.2059)(5)$$

$$v = 6.0049 \text{ m/s} \checkmark$$

$$(+\swarrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2; \quad 5 = 0 + 2t_{AC} + \frac{1}{2}(3.2059)t_{AC}^2$$

$$1.6029 \, t_{AC}^2 + 2t_{AC} - 5 = 0$$

Solve for positive root,

$$t_{AC} = 1.2492 \text{ s}$$

From C to B, the suitcase undergoes projectile motion. Referring to x–y coordinate system with origin at C, Fig. *b*, the vertical motion gives

$$(+\downarrow) \quad s_y = (s_0)_y + v_y t + \frac{1}{2}a_y t^2;$$

$$2.5 = 0 + 6.0049 \sin 30° \, t_{CB} + \frac{1}{2}(9.81)t_{CB}^2$$

$$4.905 \, t_{CB}^2 + 3.0024 \, t_{CB} - 2.5 = 0$$

Solve for positive root,

$$t_{CB} = 0.4707 \text{ s}$$

Then, the horizontal motion gives

$$(\underset{\pm}{\rightleftarrows}) \quad s_x = (s_0)_x + v_x t;$$

$$R = 0 + 6.0049 \cos 30° \, (0.4707)$$

$$= 2.448 \text{ m} = 2.45 \text{ m} \qquad \textbf{Ans.}$$

The time taken from A to B is

$$t_{AB} = t_{AC} + t_{CB} = 1.2492 + 0.4707 = 1.7199 \text{ s} = 1.72 \text{ s} \qquad \textbf{Ans.}$$

(a)

(b)
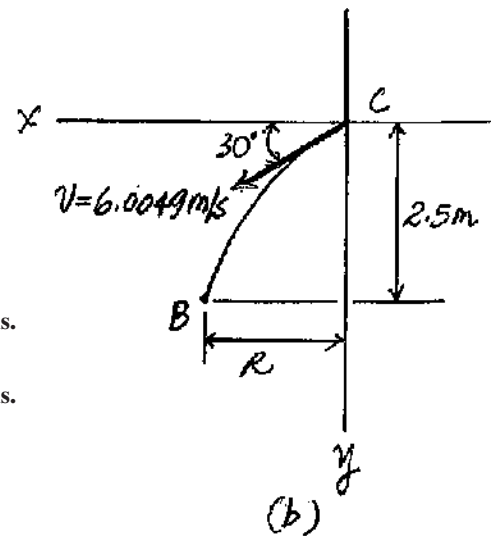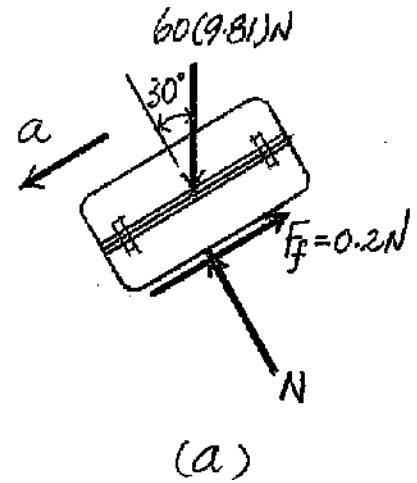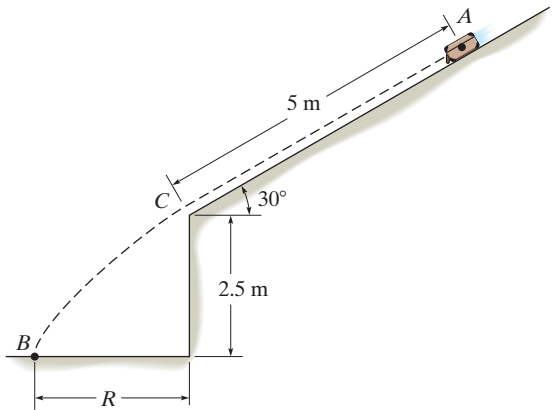
Ans:
$R = 2.45$ m
$t_{AB} = 1.72$ s

13–26.

The 1.5 Mg sports car has a tractive force of $F = 4.5$ kN. If it produces the velocity described by v-t graph shown, plot the air resistance R versus t for this time period.
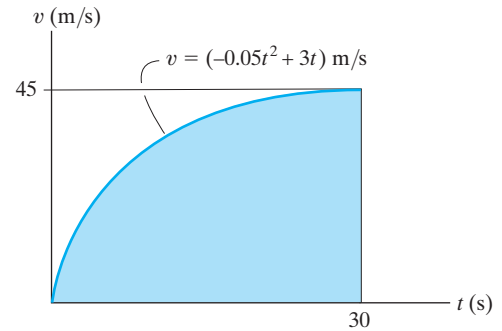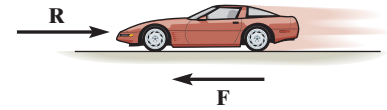
SOLUTION

Kinematic. For the v–t graph, the acceleration of the car as a function of t is

$$a = \frac{dv}{dt} = \{-0.1t + 3\}\,\text{m/s}^2$$

Equation of Motion. Referring to the FBD of the car shown in Fig. *a*,

$$(\stackrel{+}{\leftarrow})\Sigma F_x = ma_x; \quad 4500 - R = 1500(-0.1t + 3)$$

$$R = \{150t\}\,\text{N}$$

The plot of R vs t is shown in Fig. *b*
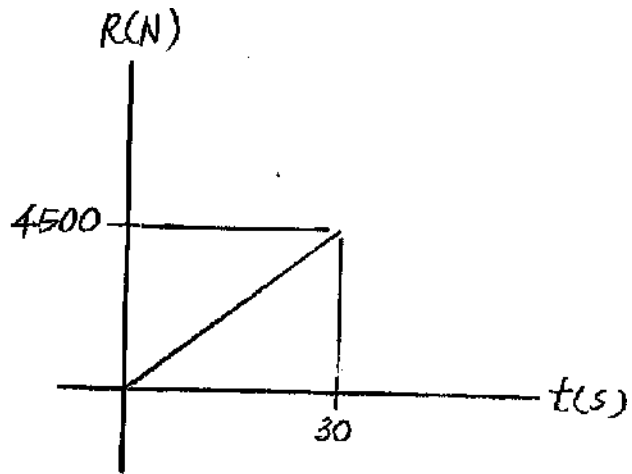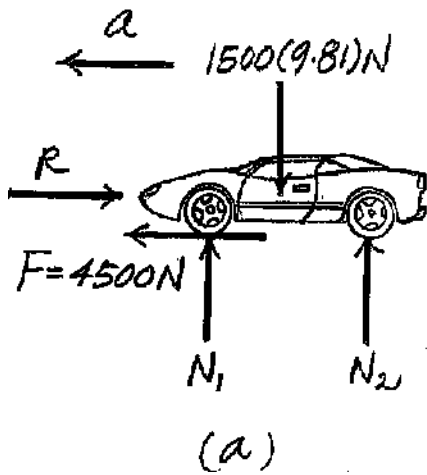
Ans:
$R = \{150t\}$ N

13–27.

The conveyor belt is moving downward at 4 m/s. If the coefficient of static friction between the conveyor and the 15-kg package B is $\mu_s = 0.8$, determine the shortest time the belt can stop so that the package does not slide on the belt.

4 m/s

B

30°
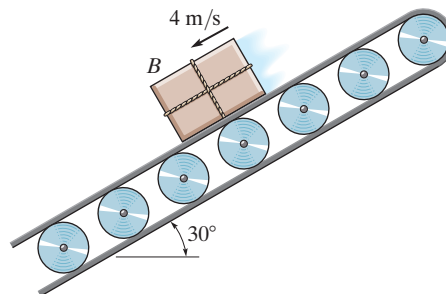
SOLUTION

Equations of Motion. It is required that the package is on the verge to slide. Thus, $F_f = \mu_s N = 0.8N$. Referring to the FBD of the package shown in Fig. a,

$+\nwarrow \Sigma F_{y'} = ma_{y'};$ $N - 15(9.81)\cos 30° = 15(0)$ $N = 127.44$ N

$+\nearrow \Sigma F_{x'} = ma_{x'};$ $0.8(127.44) - 15(9.81)\sin 30° = 15\,a$

$$a = 1.8916 \text{ m/s}^2 \nearrow$$

Kinematic. Since the package is required to stop, $v = 0$. Here $v_0 = 4$ m/s.

$(+\swarrow)$ $v = v_0 + a_0 t;$

$0 = 4 + (-1.8916)\,t$

$t = 2.1146$ s $= 2.11$ s   **Ans.**

$15(9.81)N$

30°

a

$F_f = 0.8N$

N

(a)

Ans:
$t = 2.11$ s

*13–28.

At the instant shown the 100-lb block A is moving down the plane at 5 ft/s while being attached to the 50-lb block B. If the coefficient of kinetic friction between the block and the incline is $\mu_k = 0.2$, determine the acceleration of A and the distance A slides before it stops. Neglect the mass of the pulleys and cables.
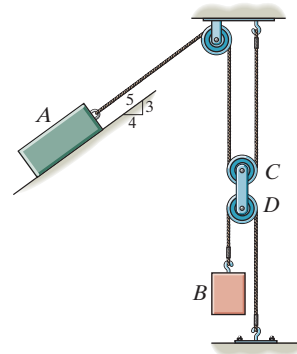
SOLUTION

Block A:

$$+\swarrow \Sigma F_x = ma_x; \qquad -T_A - 0.2N_A + 100\left(\frac{3}{5}\right) = \left(\frac{100}{32.2}\right)a_A$$
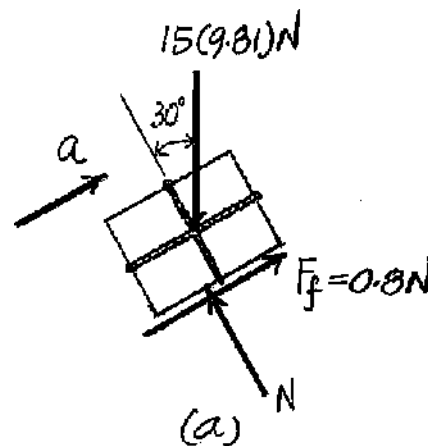
$$+\nwarrow \Sigma F_y = ma_y; \qquad N_A - 100\left(\frac{4}{5}\right) = 0$$

Thus,

$$T_A - 44 = -3.1056a_A \qquad\qquad (1)$$

Block B:

$$+\uparrow \Sigma F_y = ma_y; \qquad T_B - 50 = \left(\frac{50}{32.2}\right)a_B$$

$$T_B - 50 = 1.553a_B \qquad\qquad (2)$$

Pulleys at C and D:

$$+\uparrow \Sigma F_y = 0; \qquad 2T_A - 2T_B = 0$$

$$T_A = T_B \qquad\qquad (3)$$

Kinematics:

$$s_A + 2s_C = l$$

$$s_D + (s_D - s_B) = l'$$

$$s_C + d + s_D = d'$$

Thus,

$$a_A = -2a_C$$

$$2a_D = a_B$$

$$a_C = -a_{D'}$$

so that $a_A = a_B$ $\qquad\qquad (4)$

Solving Eqs. (1)–(4):

$$a_A = a_B = -1.288 \text{ ft/s}^2$$

$$T_A = T_B = 48.0 \text{ lb}$$

Thus,

$$a_A = 1.29 \text{ ft/s}^2 \qquad\qquad \textbf{Ans.}$$

$$(+\swarrow) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = (5)^2 + 2(-1.288)(s - 0)$$
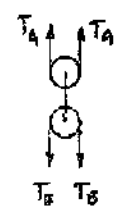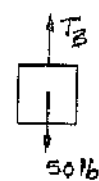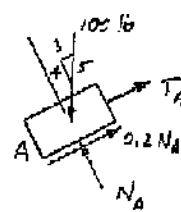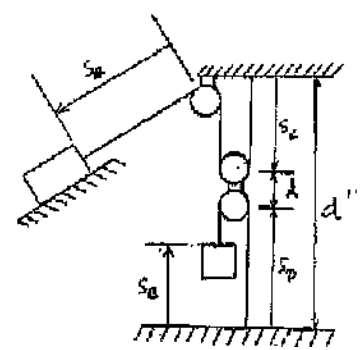
$$s = 9.70 \text{ ft} \qquad\qquad \textbf{Ans.}$$

Ans:
$a_A = 1.29 \text{ ft/s}^2$
$s = 9.70 \text{ ft}$

272

13–29.

The force exerted by the motor on the cable is shown in the graph. Determine the velocity of the 200-lb crate when $t = 2.5$ s.
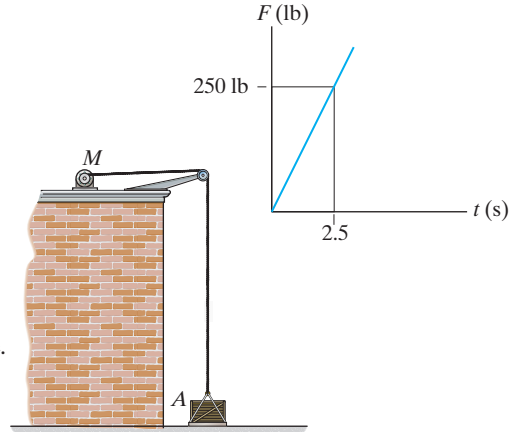
SOLUTION

Free-Body Diagram: The free-body diagram of the crate is shown in Fig. *a*.

Equilibrium: For the crate to move, force **F** must overcome the weight of the crate. Thus, the time required to move the crate is given by

$+\uparrow\Sigma F_y = 0;$ $100t - 200 = 0$ $t = 2$ s

Equation of Motion: For 2 s $< t <$ 2.5 s, $F = \dfrac{250}{2.5}t = (100t)$ lb. By referring to Fig. *a*,

$+\uparrow\Sigma F_y = ma_y;$ $100t - 200 = \dfrac{200}{32.2}a$

$a = (16.1t - 32.2)$ ft/s²

Kinematics: The velocity of the crate can be obtained by integrating the kinematic equation, $dv = adt$. For 2 s $\le t <$ 2.5 s, $v = 0$ at $t = 2$ s will be used as the lower integration limit. Thus,

$(+\uparrow)$ $\displaystyle\int dv = \int adt$

$\displaystyle\int_0^v dv = \int_{2\,s}^t (16.1t - 32.2)dt$

$v = \left(8.05t^2 - 32.2t\right)\Big|_{2\,s}^t$

$= \left(8.05t^2 - 32.2t + 32.2\right)$ft/s

When $t = 2.5$ s,

$v = 8.05(2.5^2) - 32.2(2.5) + 32.2 = 2.01$ ft/s **Ans.**

Ans:
$v = 2.01$ ft/s

273
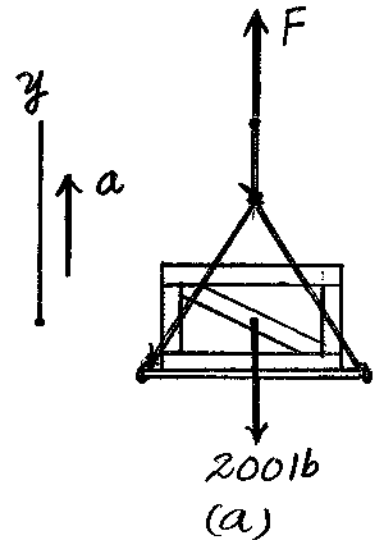
13–30.

The force of the motor M on the cable is shown in the graph. Determine the velocity of the 400-kg crate A when $t = 2$ s.
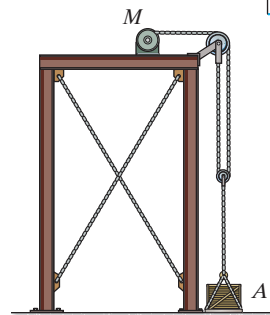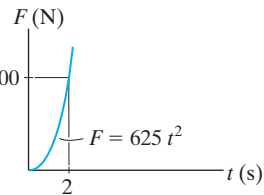
SOLUTION

Free-Body Diagram: The free-body diagram of the crate is shown in Fig. *a*.

Equilibrium: For the crate to move, force 2**F** must overcome its weight. Thus, the time required to move the crate is given by

$$+\uparrow \Sigma F_y = 0; \qquad 2(625t^2) - 400(9.81) = 0$$

$$t = 1.772 \text{ s}$$

Equations of Motion: $F = (625t^2)$ N. By referring to Fig. *a*,

$$+\uparrow \Sigma F_y = ma_y; \qquad 2(625t^2) - 400(9.81) = 400a$$

$$a = (3.125t^2 - 9.81) \text{ m/s}^2$$

Kinematics: The velocity of the crate can be obtained by integrating the kinematic equation, $dv = a\,dt$. For 1.772 s $\leq t < 2$ s, $v = 0$ at $t = 1.772$ s will be used as the lower integration limit. Thus,

$$(+\uparrow) \qquad \int dv = \int a\,dt$$

$$\int_0^v dv = \int_{1.772 \text{ s}}^t (3.125t^2 - 9.81)dt$$

$$v = \left.(1.0417t^3 - 9.81t)\right|_{1.772 \text{ s}}^t$$

$$= (1.0417t^3 - 9.81t + 11.587) \text{ m/s}$$

When $t = 2$ s,

$$v = 1.0417(2^3) - 9.81(2) + 11.587 = 0.301 \text{ m/s} \qquad \textbf{Ans.}$$

(a)
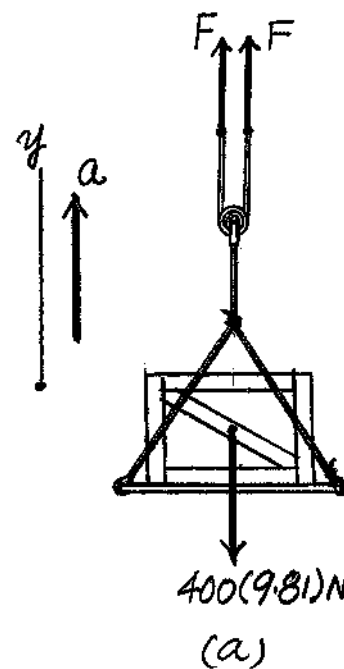
Ans:
$v = 0.301$ m/s

13–31.

The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right at a constant speed of 4 m/s, determine the tension in the rope when $s_A = 5$ m. When $s_A = 0$, $s_B = 0$.
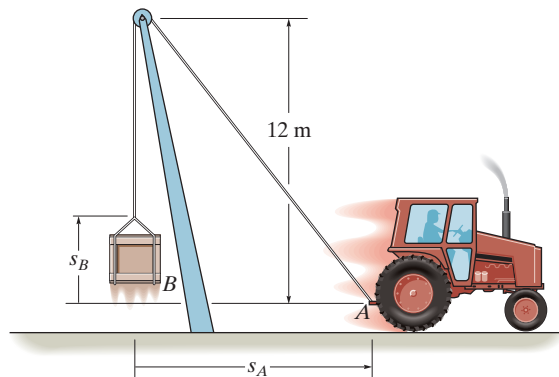
SOLUTION

$$12 - s_B + \sqrt{s_A^2 + (12)^2} = 24$$

$$-s_B + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \dot{s}_A\right) = 0$$

$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A \dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \ddot{s}_A\right) = 0$$

$$\ddot{s}_B = -\left[\frac{s_A^2 \dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A \ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$

$$a_B = -\left[\frac{(5)^2(4)^2}{\left((5)^2 + 144\right)^{\frac{3}{2}}} - \frac{(4)^2 + 0}{\left((5)^2 + 144\right)^{\frac{1}{2}}}\right] = 1.0487 \text{ m/s}^2$$

$$+\uparrow \Sigma F_y = ma_y; \qquad T - 150(9.81) = 150(1.0487)$$

$$T = 1.63 \text{ kN} \qquad\qquad\qquad\qquad\qquad \textbf{Ans.}$$

T

a_B
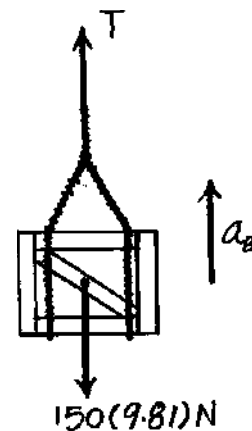
$150(9.81)N$

*13–32.

The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right with an acceleration of 3 m/s² and has a velocity of 4 m/s at the instant $s_A = 5$ m, determine the tension in the rope at this instant. When $s_A = 0$, $s_B = 0$.

12 m

s_B

B

A

s_A
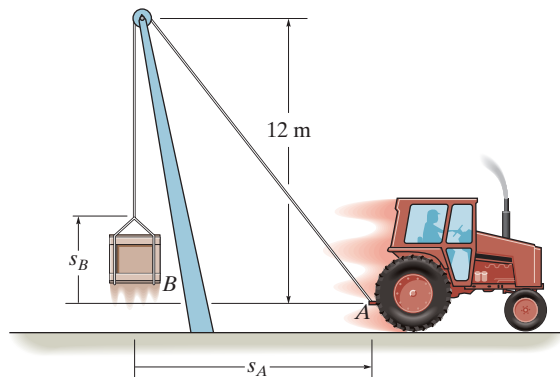
SOLUTION

$12 = s_B + \sqrt{s_A^2 + (12)^2} = 24$

$-\dot{s}_B + \dfrac{1}{2}\left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(2s_A\dot{s}_A\right) = 0$

$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A\dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A\ddot{s}_A\right) = 0$

$\ddot{s}_B = -\left[\dfrac{s_A^2\,\dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \dfrac{\dot{s}_A^2 + s_A\ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$

$a_B = -\left[\dfrac{(5)^2(4)^2}{\left((5)^2 + 144\right)^{\frac{3}{2}}} - \dfrac{(4)^2 + (5)(3)}{\left((5)^2 + 144\right)^{\frac{1}{2}}}\right] = 2.2025 \text{ m/s}^2$

$+\uparrow \Sigma F_y = ma_y; \qquad T - 150(9.81) = 150(2.2025)$

$\qquad\qquad\qquad T = 1.80 \text{ kN}$ **Ans.**

T

a_B
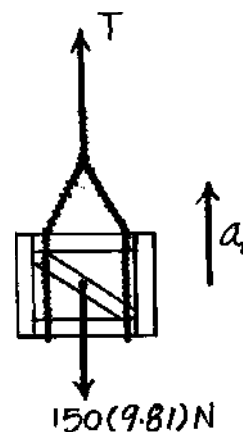
$150(9.81)N$

13–33.

Block A and B each have a mass m. Determine the largest horizontal force **P** which can be applied to B so that it will not slide on A. Also, what is the corresponding acceleration? The coefficient of static friction between A and B is μ_s. Neglect any friction between A and the horizontal surface.
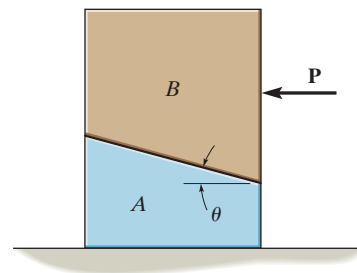
SOLUTION

Equations of Motion. Since block B is required to be on the verge to slide on A, $F_f = \mu_s N_B$. Referring to the FBD of block B shown in Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad N_B \cos \theta - \mu_s N_B \sin \theta - mg = m(0)$$

$$N_B = \frac{mg}{\cos \theta - \mu_s \sin \theta} \qquad (1)$$

$$\xleftarrow{+} \Sigma F_x = ma_x; \qquad P - N_B \sin \theta - \mu_s N_B \cos \theta = ma$$

$$P - N_B(\sin \theta + \mu_s \cos \theta) = ma \qquad (2)$$

Substitute Eq. (1) into (2),

$$P - \left(\frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta} \right) mg = ma \qquad (3)$$

Referring to the FBD of blocks A and B shown in Fig. b

$$\xleftarrow{+} \Sigma F_x = ma_x; \qquad P = 2ma \qquad (4)$$

Solving Eqs. (2) into (3),

$$P = 2mg \left(\frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta} \right) \qquad \textbf{Ans.}$$

$$a = \left(\frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta} \right) g \qquad \textbf{Ans.}$$

(a)

(b)
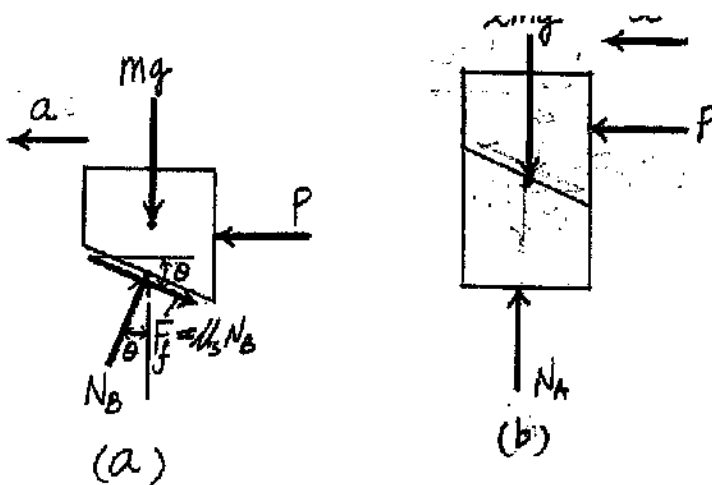
Ans:

$$P = 2mg \left(\frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta} \right)$$

$$a = \left(\frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta} \right) g$$

13–34.

The 4-kg smooth cylinder is supported by the spring having a stiffness of $k_{AB} = 120$ N/m. Determine the velocity of the cylinder when it moves downward $s = 0.2$ m from its equilibrium position, which is caused by the application of the force $F = 60$ N.

SOLUTION

Equation of Motion. At the equilibrium position, realizing that $F_{sp} = kx_0 = 120x_0$ the compression of the spring can be determined from

$+\uparrow \Sigma F_y = 0;$ $120x_0 - 4(9.81) = 0$ $x_0 = 0.327$ m

Thus, when 60 N force is applied, the compression of the spring is $x = s + x_0 = s + 0.327$. Thus, $F_{sp} = kx = 120(s + 0.327)$. Then, referring to the FBD of the collar shown in Fig. a,

$+\uparrow \Sigma F_y = ma_y;$ $120(s + 0.327) - 60 - 4(9.81) = 4(-a)$

$$a = \{15 - 30\,s\} \text{ m/s}^2$$

Kinematics. Using the result of **a** and integrate $\int v\,dv = a\,ds$ with the initial condition $v = 0$ at $s = 0$,

$$\int_0^v v\,dv = \int_0^s (15 - 30\,s)ds$$

$$\frac{v^2}{2} = 15\,s - 15\,s^2$$

$$v = \left\{ \sqrt{30(s - s^2)} \right\} \text{ m/s}$$

At $s = 0.2$ m,

$$v = \sqrt{30(0.2 - 0.2^2)} = 2.191 \text{ m/s} = 2.19 \text{ m/s} \qquad \textbf{Ans.}$$

(a)
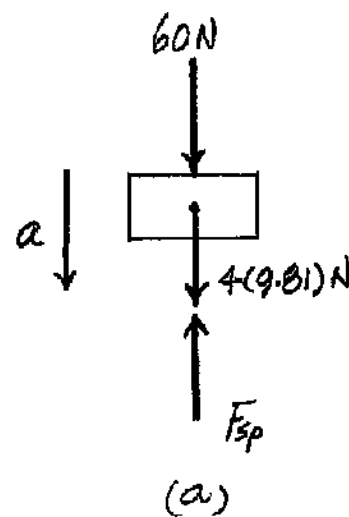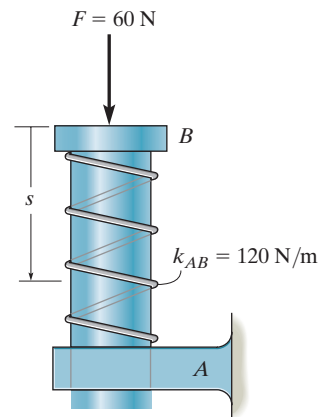
13–35.

The coefficient of static friction between the 200-kg crate and the flat bed of the truck is $\mu_s = 0.3$. Determine the shortest time for the truck to reach a speed of 60 km/h, starting from rest with constant acceleration, so that the crate does not slip.
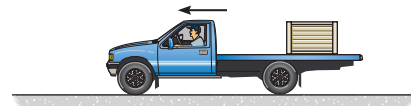
SOLUTION

Free-Body Diagram: When the crate accelerates with the truck, the frictional force F_f develops. Since the crate is required to be on the verge of slipping, $F_f = \mu_s N = 0.3N$.

Equations of Motion: Here, $a_y = 0$. By referring to Fig. a,

$+\uparrow \Sigma F_y = ma_y;$ $N - 200(9.81) = 200(0)$

$N = 1962$ N

$\xrightarrow{+} \Sigma F_x = ma_x;$ $-0.3(1962) = 200(-a)$

$a = 2.943$ m/s^2 $\leftarrow$

Kinematics: The final velocity of the truck is $v = \left(60\dfrac{\text{km}}{\text{h}}\right)\left(\dfrac{1000\ \text{m}}{1\ \text{km}}\right)\left(\dfrac{1\ \text{h}}{3600\ \text{s}}\right) = $ 16.67 m/s. Since the acceleration of the truck is constant,

$(\xrightarrow{+})$ $v = v_0 + a_c t$

$16.67 = 0 + 2.943t$
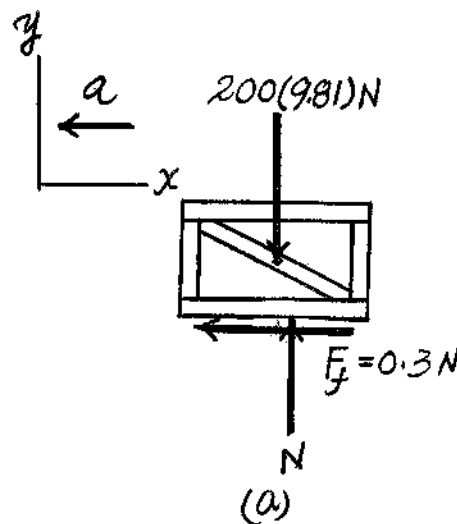
$t = 5.66$ s   **Ans.**

*13–36.

The 2-lb collar C fits loosely on the smooth shaft. If the spring is unstretched when $s = 0$ and the collar is given a velocity of 15 ft/s, determine the velocity of the collar when $s = 1$ ft.

SOLUTION

$F_s = kx;$ $\qquad F_s = 4\left(\sqrt{1 + s^2} - 1\right)$

$\xrightarrow{+} \Sigma F_x = ma_x;$ $\qquad -4\left(\sqrt{1 + s^2} - 1\right)\left(\dfrac{s}{\sqrt{1 + s^2}}\right) = \left(\dfrac{2}{32.2}\right)\left(v\dfrac{dv}{ds}\right)$

$-\displaystyle\int_0^1 \left(4s\,ds - \dfrac{4s\,ds}{\sqrt{1 + s^2}}\right) = \int_{15}^v \left(\dfrac{2}{32.2}\right)v\,dv$

$-\left[2s^2 - 4\sqrt{1 + s^2}\right]_0^1 = \dfrac{1}{32.2}\left(v^2 - 15^2\right)$
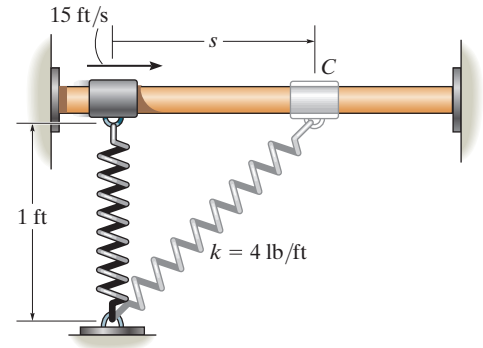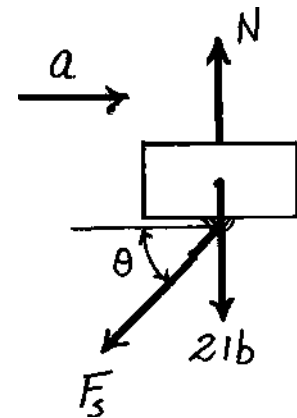
$v = 14.6$ ft/s $\qquad\qquad\qquad\qquad\qquad\qquad$ **Ans.**

13–37.

The 10-kg block A rests on the 50-kg plate B in the position shown. Neglecting the mass of the rope and pulley, and using the coefficients of kinetic friction indicated, determine the time needed for block A to slide 0.5 m *on the plate* when the system is released from rest.

SOLUTION

Block A:

$+\nwarrow \Sigma F_y = ma_y;$ $N_A - 10(9.81) \cos 30° = 0$ $N_A = 84.96$ N

$+\swarrow \Sigma F_x = ma_x;$ $-T + 0.2(84.96) + 10(9.81) \sin 30° = 10a_A$

$T - 66.04 = -10a_A$　　　　　　　　　　　　　　　　(1)

Block B:

$+\nwarrow \Sigma F_y = ma_y;$ $N_B - 84.96 - 50(9.81) \cos 30° = 0$

$N_B = 509.7$ N

$+\swarrow \Sigma F_x = ma_x;$ $-0.2(84.96) - 0.1(509.7) - T + 50(9.81 \sin 30°) = 50a_B$

$177.28 - T = 50a_B$　　　　　　　　　　　　　　　　(2)

$s_A + s_B = l$

$\Delta s_A = -\Delta s_B$

$a_A = -a_B$　　　　　　　　　　　　　　　　　　　(3)

Solving Eqs. (1) – (3):

$a_B = 1.854$ m/s^2

$a_A = -1.854$ m/s^2 $T = 84.58$ N

In order to slide 0.5 m along the plate the block must move 0.25 m. Thus,

$(+\swarrow)$ $s_B = s_A + s_{B/A}$

$-\Delta s_A = \Delta s_A + 0.5$

$\Delta s_A = -0.25$ m

$(+\swarrow)$ $s_A = s_0 + v_0 t + \dfrac{1}{2} a_A t^2$

$-0.25 = 0 + 0 + \dfrac{1}{2}(-1.854)t^2$
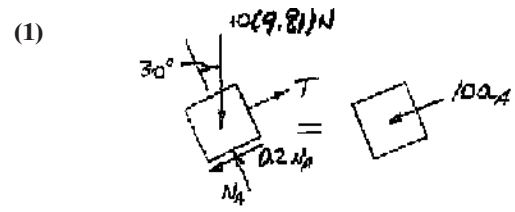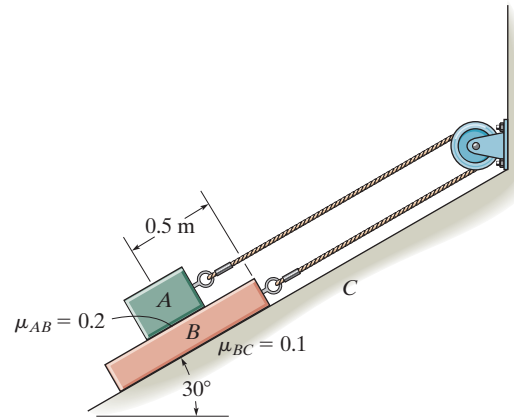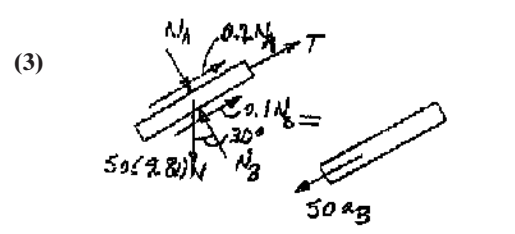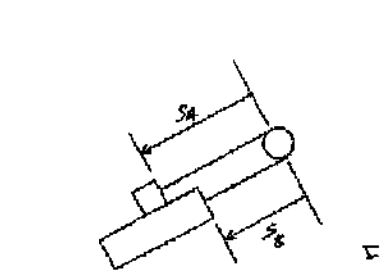
$t = 0.519$ s　　　　　　　　　　　　　　　　　**Ans.**

Ans:
$t = 0.519$ s

13–38.

The 300-kg bar B, originally at rest, is being towed over a series of small rollers. Determine the force in the cable when $t = 5$ s, if the motor M is drawing in the cable for a short time at a rate of $v = (0.4t^2)$ m/s, where t is in seconds $(0 \le t \le 6$ s$)$. How far does the bar move in 5 s? Neglect the mass of the cable, pulley, and the rollers.

SOLUTION

$\pm \Sigma F_x = ma_x; \qquad T = 300a$

$v = 0.4t^2$

$a = \dfrac{dv}{dt} = 0.8t$

When $t = 5$ s, $a = 4$ m/s^2

$T = 300(4) = 1200$ N $= 1.20$ kN   **Ans.**

$ds = v\, dt$

$\displaystyle \int_0^s ds = \int_0^5 0.4t^2\, ds$

$s = \left(\dfrac{0.4}{3}\right)(5)^3 = 16.7$ m   **Ans.**
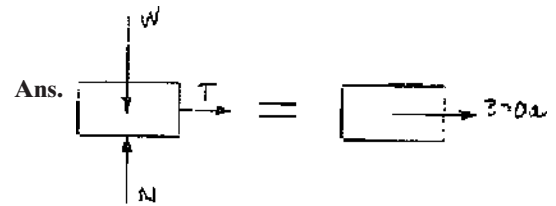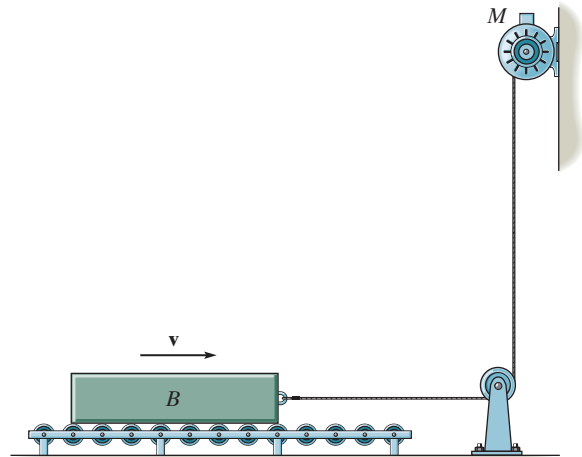
13–39.

An electron of mass m is discharged with an initial horizontal velocity of $\mathbf{v}_0$. If it is subjected to two fields of force for which $F_x = F_0$ and $F_y = 0.3F_0$, where F_0 is constant, determine the equation of the path, and the speed of the electron at any time t.

SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_0 = ma_x$

$+\uparrow \Sigma F_y = ma_y; \qquad 0.3\,F_0 = ma_y$

Thus,

$\displaystyle\int_{v_0}^{v_x} dv_x = \int_0^t \frac{F_0}{m}\,dt$

$v_x = \dfrac{F_0}{m}t + v_0$

$\displaystyle\int_0^{v_y} dv_y = \int_0^t \frac{0.3F_0}{m}\,dt \qquad v_y = \dfrac{0.3F_0}{m}t$

$v = \sqrt{\left(\dfrac{F_0}{m}t + v_0\right)^2 + \left(\dfrac{0.3F_0}{m}t\right)^2}$

$\qquad = \dfrac{1}{m}\sqrt{1.09F_0^2t^2 + 2F_0tmv_0 + m^2v_0^2}$　　**Ans.**

$\displaystyle\int_0^x dx = \int_0^t \left(\frac{F_0}{m}t + v_0\right) dt$

$x = \dfrac{F_0 t^2}{2m} + v_0\,t$

$\displaystyle\int_0^y dy = \int_0^t \frac{0.3F_0}{m}t\,dt$

$y = \dfrac{0.3F_0 t^2}{2m}$

$t = \left(\sqrt{\dfrac{2m}{0.3F_0}}\right) y^{\frac{1}{2}}$

$x = \dfrac{F_0}{2m}\left(\dfrac{2m}{0.3F_0}\right)y + v_0\left(\sqrt{\dfrac{2m}{0.3F_0}}\right)y^{\frac{1}{2}}$
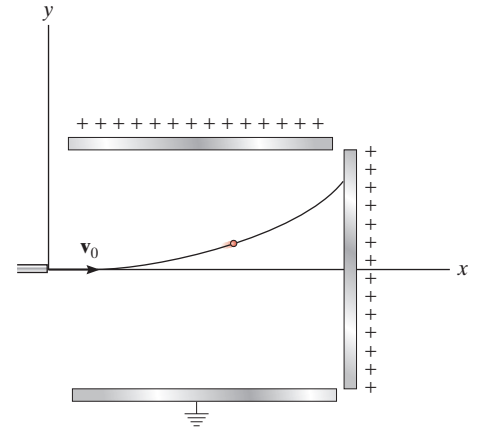
$x = \dfrac{y}{0.3} + v_0\left(\sqrt{\dfrac{2m}{0.3F_0}}\right)y^{\frac{1}{2}}$　　**Ans.**

Ans:

$v = \dfrac{1}{m}\sqrt{1.09F_0^2t^2 + 2F_0tmv_0 + m^2v_0^2}$

$x = \dfrac{y}{0.3} + v_0\left(\sqrt{\dfrac{2m}{0.3F_0}}\right)y^{1/2}$

*13–40.

The 400-lb cylinder at A is hoisted using the motor and the pulley system shown. If the speed of point B on the cable is increased at a constant rate from zero to $v_B = 10$ ft/s in $t = 5$ s, determine the tension in the cable at B to cause the motion.

SOLUTION

$2s_A + s_B = l$

$2a_A = -a_B$

$(\overset{+}{\rightarrow}) \quad v = v_0 + a_B t$

$10 = 0 + a_B(5)$

$a_B = 2$ ft/s^2

$a_A = -1$ ft/s^2

$+\downarrow \Sigma F_y = ma_y; \qquad 400 - 2T = \left(\dfrac{400}{32.2}\right)(-1)$

Thus, $\quad T = 206$ lb **Ans.**

Ans:
$T = 206$ lb

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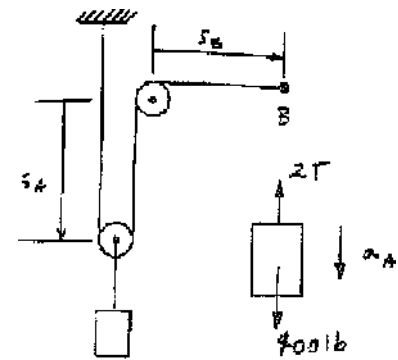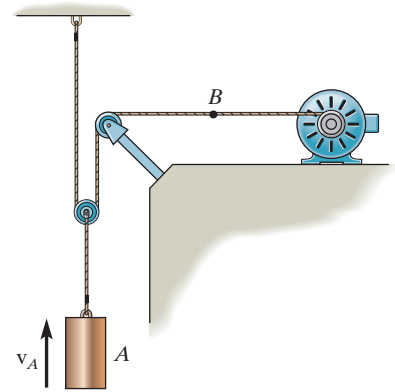
13–41.

Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, determine the distance both blocks slide on the smooth surface before they begin to separate. What is their velocity at this instant?
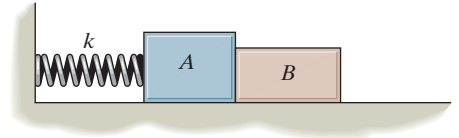
SOLUTION

Block A:

$\xrightarrow{+} \Sigma F_x = ma_x;$ $-k(x - d) - N = m_A a_A$

Block B:

$\xrightarrow{+} \Sigma F_x = ma_x;$ $N = m_B a_B$

Since $a_A = a_B = a$,

$-k(x - d) - m_B a = m_A a$

$a = \dfrac{k(d - x)}{(m_A + m_B)}$ $N = \dfrac{km_B (d - x)}{(m_A + m_B)}$

$N = 0$ when $d - x = 0,$ or $x = d$ **Ans.**

$v \, dv = a \, dx$

$\displaystyle\int_0^v v \, dv = \int_0^d \dfrac{k(d - x)}{(m_A + m_B)} dx$

$\dfrac{1}{2} v^2 = \dfrac{k}{(m_A + m_B)} \left[(d)x - \dfrac{1}{2}x^2 \right]_0^d = \dfrac{1}{2}\dfrac{kd^2}{(m_A + m_B)}$
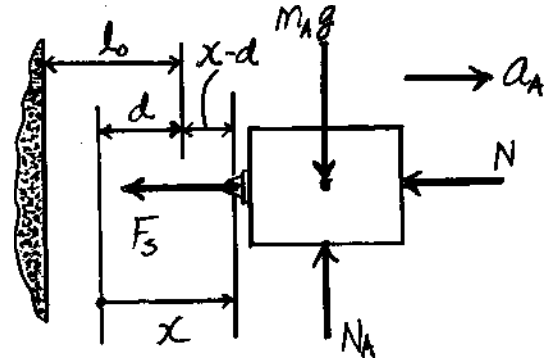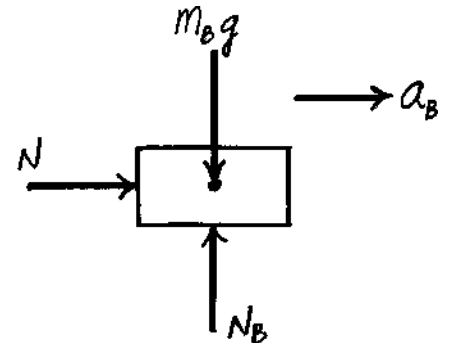
$v = \sqrt{\dfrac{kd^2}{(m_A + m_B)}}$ **Ans.**

Ans:
$x = d$
$v = \sqrt{\dfrac{kd^2}{m_A + m_B}}$

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13–42.

Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, show that for separation to occur it is necessary that $d > 2\mu_k g(m_A + m_B)/k$, where μ_k is the coefficient of kinetic friction between the blocks and the ground. Also, what is the distance the blocks slide on the surface before they separate?
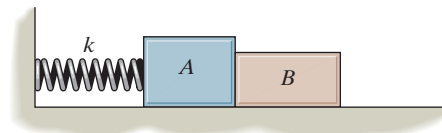
SOLUTION

Block A:

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -k(x - d) - N - \mu_k m_A g = m_A a_A$

Block B:

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad N - \mu_k m_B g = m_B a_B$

Since $a_A = a_B = a$,

$a = \dfrac{k(d - x) - \mu_k g(m_A + m_B)}{(m_A + m_B)} = \dfrac{k(d - x)}{(m_A + m_B)} - \mu_k g$

$N = \dfrac{km_B(d - x)}{(m_A + m_B)}$

$N = 0$, then $x = d$ for separation.　　　　　　　　　　**Ans.**

At the moment of separation:

$v\, dv = a\, dx$

$\displaystyle \int_0^v v\, dv = \int_0^d \left[\dfrac{k(d - x)}{(m_A + m_B)} - \mu_k g \right] dx$

$\dfrac{1}{2} v^2 = \dfrac{k}{(m_A + m_B)} \left[(d)x - \dfrac{1}{2} x^2 - \mu_k g\, x \right]_0^d$

$v = \sqrt{\dfrac{kd^2 - 2\mu_k g(m_A + m_B)d}{(m_A + m_B)}}$

Require $v > 0$, so that

$kd^2 - 2\mu_k g(m_A + m_B)d > 0$

Thus,

$kd > 2\mu_k g(m_A + m_B)$
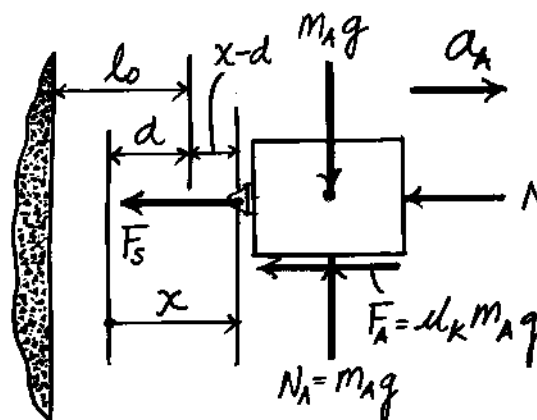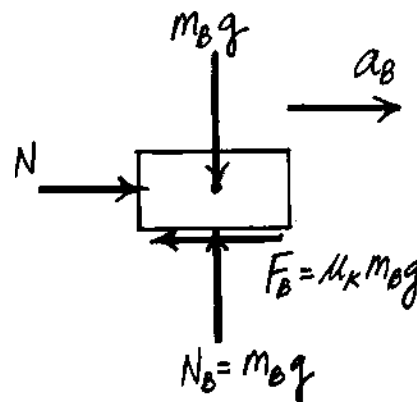
$d > \dfrac{2\mu_k g}{k}(m_A + m_B)$　　　　　　　　**Q.E.D.**

Ans:
$x = d$ for separation.

13–43.

A parachutist having a mass m opens his parachute from an at-rest position at a very high altitude. If the atmospheric drag resistance is $F_D = kv^2$, where k is a constant, determine his velocity when he has fallen for a time t. What is his velocity when he lands on the ground? This velocity is referred to as the *terminal velocity*, which is found by letting the time of fall $t \rightarrow \infty$.
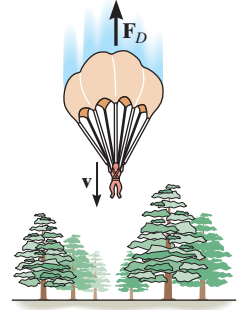
SOLUTION

$+\downarrow \Sigma F_z = ma_z;$ $\qquad mg - kv^2 = m\dfrac{dv}{dt}$

$$m\int_0^v \frac{m\,dv}{(mg - kv^2)} = \int_0^t dt$$

$$\frac{m}{k}\int_0^v \frac{dv}{\frac{mg}{k} - v^2} = t$$

$$\frac{m}{k}\left(\frac{1}{2\sqrt{\frac{mg}{k}}}\right)\ln\left[\frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}\right]_0^v = t$$

$$\frac{k}{m}t\left(2\sqrt{\frac{mg}{k}}\right) = \ln\frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}$$

$$e^{2t\sqrt{\frac{mg}{k}}} = \frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}$$

$$\sqrt{\frac{mg}{k}}e^{2t\sqrt{\frac{mg}{k}}} - ve^{2t\sqrt{\frac{mg}{k}}} = \sqrt{\frac{mg}{k}} + v$$

$$v = \sqrt{\frac{mg}{k}}\left[\frac{e^{2t\sqrt{\frac{mg}{k}}} - 1}{e^{2t\sqrt{\frac{mg}{k}}} + 1}\right] \qquad \textbf{Ans.}$$

$$\text{When } t \rightarrow \infty \qquad v_t = \sqrt{\frac{mg}{k}} \qquad \textbf{Ans.}$$

Ans:
$$v = \sqrt{\frac{mg}{k}}\left[\frac{e^{2t\sqrt{mg/k}} - 1}{e^{2t\sqrt{mg/k}} + 1}\right]$$
$$v_t = \sqrt{\frac{mg}{k}}$$

*13–44.

If the motor draws in the cable with an acceleration of $3\,\text{m/s}^2$, determine the reactions at the supports A and B. The beam has a uniform mass of 30 kg/m, and the crate has a mass of 200 kg. Neglect the mass of the motor and pulleys.
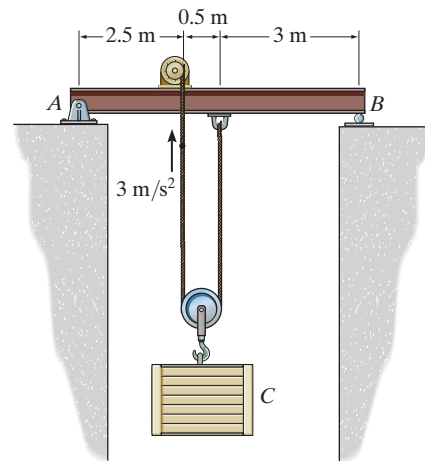
SOLUTION

$S_c + (S_c - S_p)$

$2v_c = v_p$

$2a_c = a_p$

$2a_c = 3\,\text{m/s}^2$

$a_c = 1.5\,\text{m/s}^2$

$+\uparrow \Sigma F_y = ma_y \qquad 2T - 1962 = 200(1.5)$

$\qquad\qquad\qquad\qquad T = 1{,}131\,\text{N}$

$\zeta + \Sigma M_A = 0; \qquad B_y(6) - (1765.8 + 1{,}131)3 - (1{,}131)(2.5) = 0$

$\qquad\qquad\qquad\qquad B_y = 1{,}919.65\,\text{N} = 1.92\,\text{kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad A_y - 1765.8 - (2)(1{,}131) + 1919.65 = 0$

$\qquad\qquad\qquad\qquad A_y = 2108.15\,\text{N} = 2.11\,\text{kN}$ **Ans.**
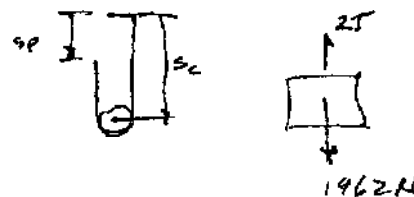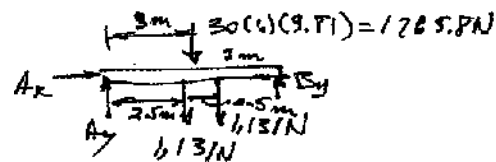
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**

13–45.

If the force exerted on cable AB by the motor is $F = (100t^{3/2})$ N, where t is in seconds, determine the 50-kg crate's velocity when $t = 5$ s. The coefficients of static and kinetic friction between the crate and the ground are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively. Initially the crate is at rest.

SOLUTION

Free-Body Diagram: The frictional force $\mathbf{F}_f$ is required to act to the left to oppose the motion of the crate which is to the right.

Equations of Motion: Here, $a_y = 0$. Thus,

$+\uparrow \Sigma F_y = ma_y;$ $N - 50(9.81) = 50(0)$

$\qquad\qquad N = 490.5$ N

Realizing that $F_f = \mu_k N = 0.3(490.5) = 147.15$ N,

$+\uparrow \Sigma F_x = ma_x;$ $100t^{3/2} - 147.15 = 50a$

$\qquad\qquad a = (2t^{3/2} - 2.943)$ m/s

Equilibrium: For the crate to move, force $\mathbf{F}$ must overcome the static friction of $F_f = \mu_s N = 0.4(490.5) = 196.2$ N. Thus, the time required to cause the crate to be on the verge of moving can be obtained from.

$\xrightarrow{+} \Sigma F_x = 0;$ $100t^{3/2} - 196.2 = 0$

$\qquad\qquad t = 1.567$ s

Kinematics: Using the result of **a** and integrating the kinematic equation $dv = a\,dt$ with the initial condition $v = 0$ at $t = 1.567$ as the lower integration limit,

$(\xrightarrow{+})$ $\int dv = \int a\,dt$

$\qquad \int_0^v dv = \int_{1.567\,\text{s}}^t (2t^{3/2} - 2.943)dt$

$\qquad v = (0.8t^{5/2} - 2.943t)\Big|_{1.567\,\text{s}}^t$

$\qquad v = (0.8t^{5/2} - 2.943t + 2.152)$ m/s

When $t = 5$ s,

$\qquad v = 0.8(5)^{5/2} - 2.943(5) + 2.152 = 32.16$ ft/s $= 32.2$ ft/s   **Ans.**

(a)
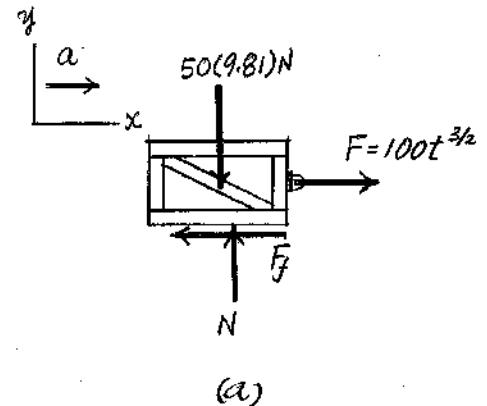
Ans:
$v = 32.2$ ft/s

13–46.

Blocks A and B each have a mass m. Determine the largest horizontal force **P** which can be applied to B so that A will not move relative to B. All surfaces are smooth.
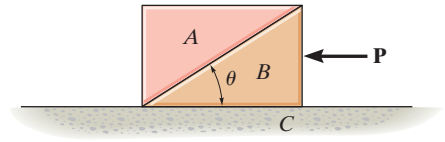
SOLUTION

Require

$$a_A = a_B = a$$

Block A:

$$+\uparrow \Sigma F_y = 0; \qquad N\cos\theta - mg = 0$$

$$\xleftarrow{+} \Sigma F_x = ma_x; \qquad N\sin\theta = ma$$

$$a = g\tan\theta$$

Block B:

$$\xleftarrow{+} \Sigma F_x = ma_x; \qquad P - N\sin\theta = ma$$

$$P - mg\tan\theta = mg\tan\theta$$

$$P = 2mg\tan\theta \qquad\qquad \textbf{Ans.}$$

Ans:
$$P = 2mg\tan\theta$$

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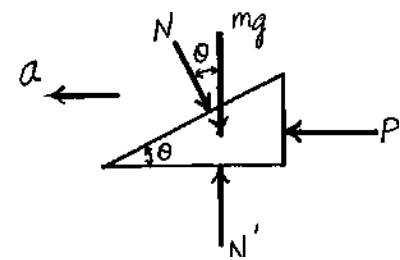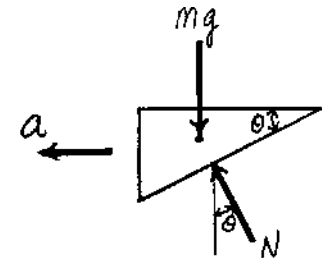
13–47.

Blocks A and B each have a mass m. Determine the largest horizontal force **P** which can be applied to B so that A will not slip on B. The coefficient of static friction between A and B is μ_s. Neglect any friction between B and C.
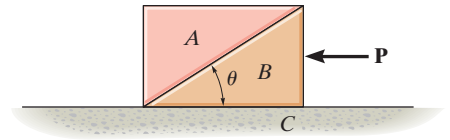
SOLUTION

Require

$$a_A = a_B = a$$

Block A:

$$+\uparrow \Sigma F_y = 0; \qquad N \cos\theta - \mu_s N \sin\theta - mg = 0$$

$$\xleftarrow{+} \Sigma F_x = ma_x; \qquad N \sin\theta + \mu_s N \cos\theta = ma$$

$$N = \frac{mg}{\cos\theta - \mu_s \sin\theta}$$

$$a = g\left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right)$$

Block B:

$$\xleftarrow{+} \Sigma F_x = ma_x; \qquad P - \mu_s N \cos\theta - N \sin\theta = ma$$

$$P - mg\left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right) = mg\left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right)$$

$$P = 2mg\left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right) \qquad \textbf{Ans.}$$
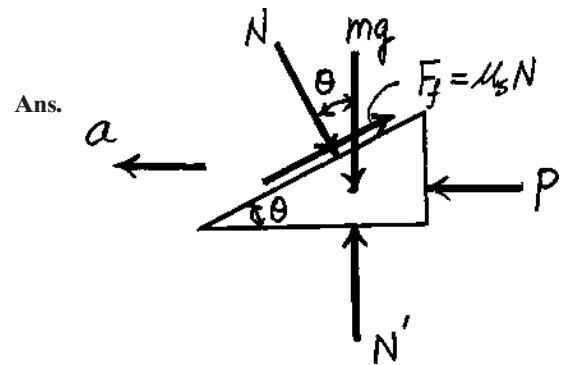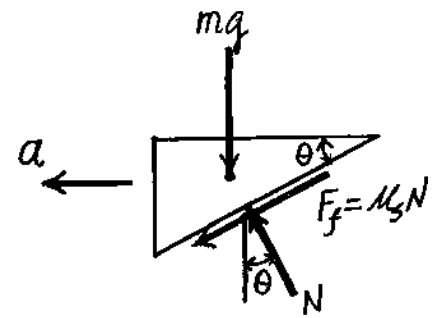
Ans:

$$P = 2mg\left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right)$$

*13–48.

The smooth block B of negligible size has a mass m and rests on the horizontal plane. If the board AC pushes on the block at an angle θ with a constant acceleration $\mathbf{a}_0$, determine the velocity of the block along the board and the distance s the block moves along the board as a function of time t. The block starts from rest when $s = 0$, $t = 0$.
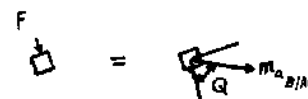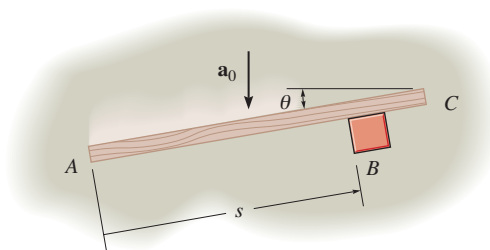
SOLUTION

$\nearrow + \Sigma F_x = m\, a_x; \qquad 0 = m\, a_B \sin \phi$

$\qquad \mathbf{a}_B = \mathbf{a}_{AC} + \mathbf{a}_{B/AC}$

$\qquad \mathbf{a}_B = \mathbf{a_0} + \mathbf{a}_{B/AC}$

$\nearrow + \qquad a_B \sin \phi = -a_0 \sin \theta + a_{B/AC}$

Thus,

$\qquad 0 = m(-a_0 \sin \theta + a_{B/AC})$

$\qquad a_{B/AC} = a_0 \sin \theta$

$\qquad \displaystyle\int_0^{v_{B/AC}} dv_{B/AC} = \int_0^t a_0 \sin \theta\, dt$

$\qquad v_{B/AC} = a_0 \sin \theta\, t$ **Ans.**

$\qquad s_{B/AC} = s = \displaystyle\int_0^t a_0 \sin \theta\, t\, dt$

$\qquad s = \dfrac{1}{2} a_0 \sin \theta\, t^2$ **Ans.**

Ans:

$v_{B/AC} = a_0 \sin \theta\, t$

$s = \dfrac{1}{2} a_0 \sin \theta\, t^2$

13–49.

If a horizontal force $P = 12$ lb is applied to block A determine the acceleration of block B. Neglect friction.
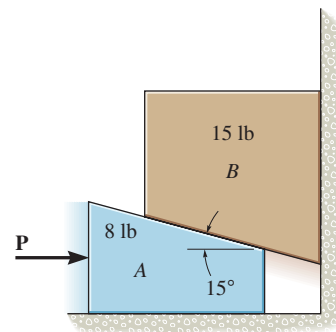
SOLUTION

Block A:

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 12 - N_B \sin 15° = \left(\frac{8}{32.2}\right) a_A \tag{1}$$

Block B:

$$+\uparrow \Sigma F_y = ma_y; \qquad N_B \cos 15° - 15 = \left(\frac{15}{32.2}\right) a_B \tag{2}$$

$$s_B = s_A \tan 15°$$

$$a_B = a_A \tan 15° \tag{3}$$

Solving Eqs. (1)–(3)

$$a_A = 28.3 \text{ ft/s}^2 \qquad N_B = 19.2 \text{ lb}$$

$$a_B = 7.59 \text{ ft/s}^2 \qquad \textbf{Ans.}$$
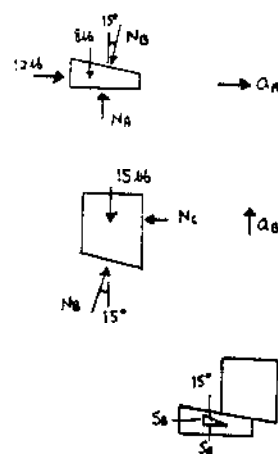
Ans:
$a_B = 7.59$ ft/s^2

13–50.

A freight elevator, including its load, has a mass of 1 Mg. It is prevented from rotating due to the track and wheels mounted along its sides. If the motor M develops a constant tension $T = 4$ kN in its attached cable, determine the velocity of the elevator when it has moved upward 6 m starting from rest. Neglect the mass of the pulleys and cables.
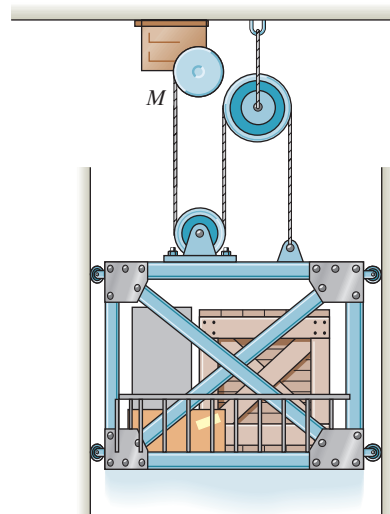
SOLUTION

Equation of Motion. Referring to the FBD of the freight elevator shown in Fig. a,

$+\uparrow \Sigma F_y = ma_y;$ $3(4000) - 1000(9.81) = 1000a$

$$a = 2.19 \text{ m/s}^2 \uparrow$$

Kinematics. Using the result of a,

$(+\uparrow)$ $v^2 = v_0^2 + 2as;$ $v^2 = 0^2 + 2(2.19)(6)$

$$v = 5.126 \text{ m/s} = 5.13 \text{ m/s} \qquad\qquad \textbf{Ans.}$$

4000 N

4000 N (**4000 N**

a

N_1

N_2

$1000(9.81) N$

(a)
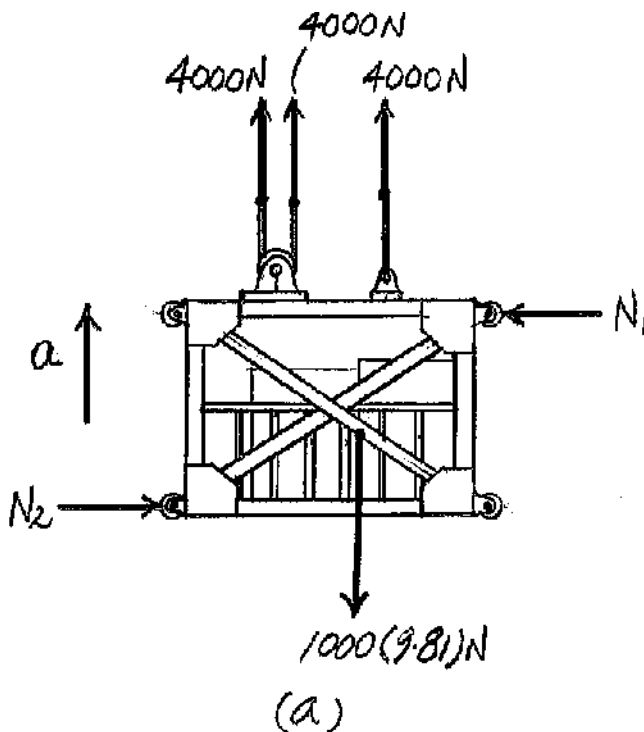
Ans:
$v = 5.13$ m/s

13–51.

The block A has a mass m_A and rests on the pan B, which has a mass m_B. Both are supported by a spring having a stiffness k that is attached to the bottom of the pan and to the ground. Determine the distance d the pan should be pushed down from the equilibrium position and then released from rest so that separation of the block will take place from the surface of the pan at the instant the spring becomes unstretched.
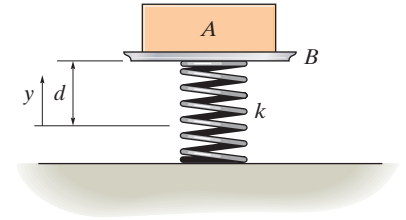
SOLUTION

For Equilibrium

$+\uparrow \Sigma F_y = ma_y;$ $F_s = (m_A + m_B)g$

$$y_{eq} = \frac{F_s}{k} = \frac{(m_A + m_B)g}{k}$$

Block:

$+\uparrow \Sigma F_y = ma_y;$ $-m_A g + N = m_A a$

Block and pan

$+\uparrow \Sigma F_y = ma_y;$ $-(m_A + m_B)g + k(y_{eq} + y) = (m_A + m_B)a$

Thus,

$$-(m_A + m_B)g + k\left[\left(\frac{m_A + m_B}{k}\right)g + y\right] = (m_A + m_B)\left(\frac{-m_A g + N}{m_A}\right)$$

Require $y = d, N = 0$

$$kd = -(m_A + m_B)g$$

Since d is downward,
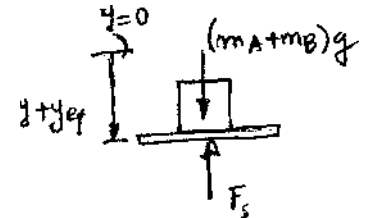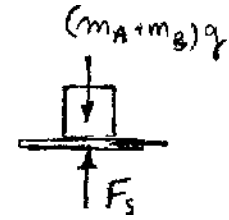
$$d = \frac{(m_A + m_B)g}{k}$$ **Ans.**

Ans:

$$d = \frac{(m_A + m_B)g}{k}$$

*13–52.

A girl, having a mass of 15 kg, sits motionless relative to the surface of a horizontal platform at a distance of $r = 5$ m from the platform's center. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which the girl will have before she begins to slip off the platform. The coefficient of static friction between the girl and the platform is $\mu = 0.2$.

SOLUTION

Equation of Motion: Since the girl is on the verge of slipping, $F_f = \mu_s N = 0.2N$. Applying Eq. 13–8, we have

$$\Sigma F_b = 0; \qquad N - 15(9.81) = 0 \qquad N = 147.15 \text{ N}$$

$$\Sigma F_n = ma_n; \qquad 0.2(147.15) = 15\left(\frac{v^2}{5}\right)$$

$$v = 3.13 \text{ m/s} \qquad\qquad\qquad \textbf{Ans.}$$
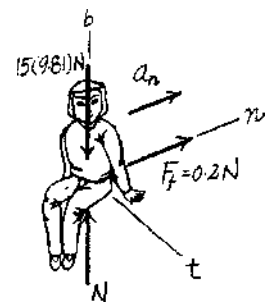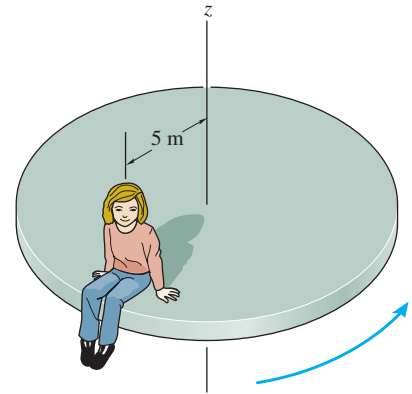
Ans:
$v = 3.13$ m/s

13–53.

The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block is given a speed of $v = 10\,\text{m/s}$, determine the radius r of the circular path along which it travels.
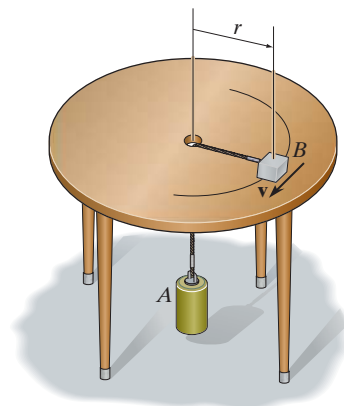
SOLUTION

Free-Body Diagram: The free-body diagram of block B is shown in Fig. (a). The tension in the cord is equal to the weight of cylinder A, i.e., $T = 15(9.81)\,\text{N} = 147.15\,\text{N}$. Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{10^2}{r}$ and referring to Fig. (a),

$$\Sigma F_n = ma_n; \qquad 147.15 = 2\left(\frac{10^2}{r}\right)$$

$$r = 1.36\,\text{m} \qquad\qquad\qquad \textbf{Ans.}$$
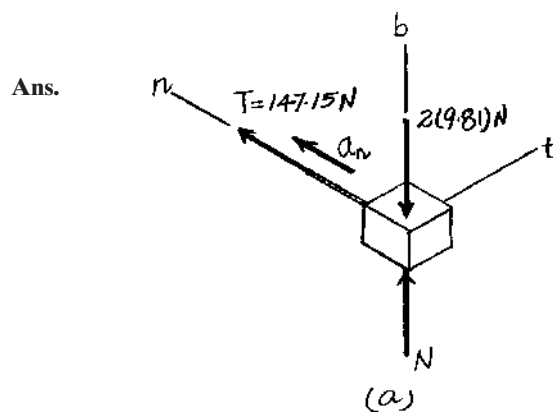
13–54.

The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block travels along a circular path of radius $r = 1.5\,\text{m}$, determine the speed of the block.
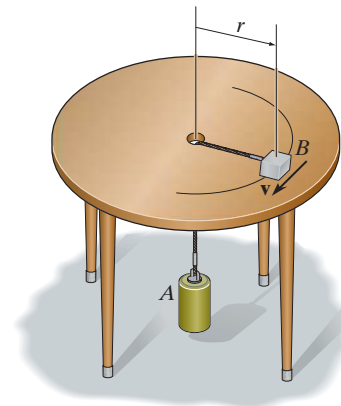
SOLUTION

Free-Body Diagram: The free-body diagram of block B is shown in Fig. (a). The tension in the cord is equal to the weight of cylinder A, i.e., $T = 15(9.81)\,\text{N} = 147.15\,\text{N}$. Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: Realizing that $a_n = \dfrac{v^2}{r} = \dfrac{v^2}{1.5}$ and referring to Fig. (a),

$$\Sigma F_n = ma_n; \qquad 147.15 = 2\left(\frac{v^2}{1.5}\right)$$

$$v = 10.5\ \text{m/s} \qquad\qquad\qquad \textbf{Ans.}$$

(a)
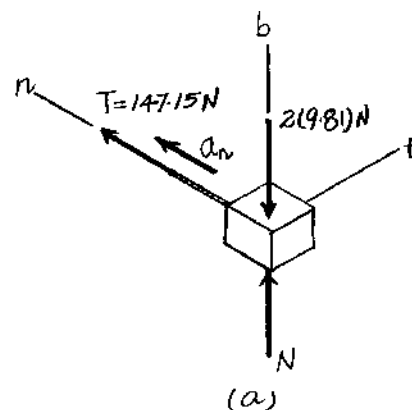
Ans:
$v = 10.5\,\text{m/s}$

13–55.

Determine the maximum constant speed at which the pilot can travel around the vertical curve having a radius of curvature $\rho = 800$ m, so that he experiences a maximum acceleration $a_n = 8g = 78.5$ m/s². If he has a mass of 70 kg, determine the normal force he exerts on the seat of the airplane when the plane is traveling at this speed and is at its lowest point.

$\rho = 800$ m

SOLUTION

$$a_n = \frac{v^2}{\rho}; \qquad 78.5 = \frac{v^2}{800}$$

$v = 251$ m/s   **Ans.**

$+\uparrow \Sigma F_n = ma_n; \qquad N - 70(9.81) = 70(78.5)$

$\qquad\qquad N = 6.18$ kN   **Ans.**

$70(9.81)$N

a_n
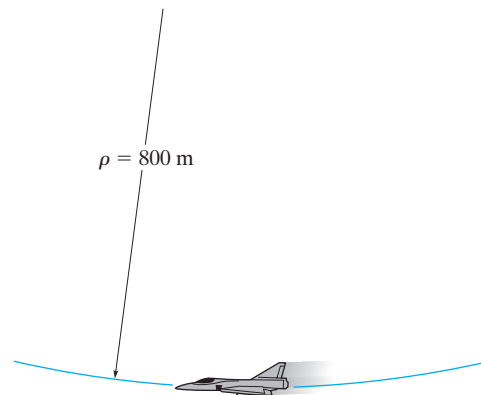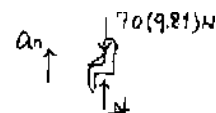
Ans:
$N = 6.18$ kN

*13–56.

Cartons having a mass of 5 kg are required to move along the assembly line at a constant speed of 8 m/s. Determine the smallest radius of curvature, ρ, for the conveyor so the cartons do not slip. The coefficients of static and kinetic friction between a carton and the conveyor are $\mu_s = 0.7$ and $\mu_k = 0.5$, respectively.
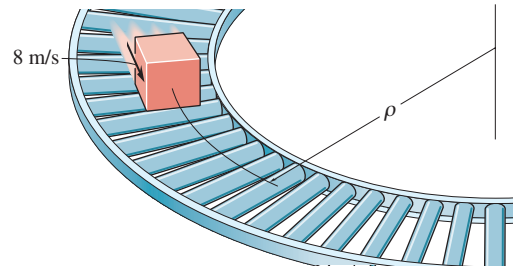
SOLUTION

$+\uparrow \Sigma F_b = m\,a_b; \qquad N - W = 0$

$$N = W$$

$$F_x = 0.7W$$

$\xleftarrow{+} \Sigma F_n = m\,a_n; \qquad 0.7W = \dfrac{W}{9.81}\left(\dfrac{8^2}{\rho}\right)$

$$\rho = 9.32 \text{ m} \qquad\qquad\qquad\qquad\qquad \textbf{Ans.}$$

13–57.

The collar A, having a mass of 0.75 kg, is attached to a spring having a stiffness of $k = 200$ N/m. When rod BC rotates about the vertical axis, the collar slides outward along the smooth rod DE. If the spring is unstretched when $s = 0$, determine the constant speed of the collar in order that $s = 100$ mm. Also, what is the normal force of the rod on the collar? Neglect the size of the collar.
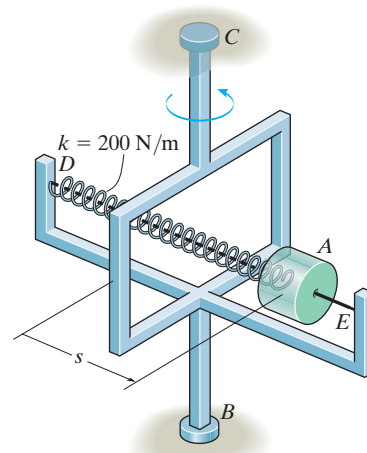
SOLUTION

$\Sigma F_b = 0;$ $N_b - 0.75(9.81) = 0$ $N_b = 7.36$

$\Sigma F_n = ma_n;$ $200(0.1) = 0.75\left(\dfrac{v^2}{0.10}\right)$

$\Sigma F_t = ma_t;$ $N_t = 0$

$\qquad\qquad v = 1.63$ m/s   **Ans.**

$\qquad\qquad N = \sqrt{(7.36)^2 + (0)} = 7.36$ N   **Ans.**

$0.75(9.8)N$

N_t

$F_s = 200(0.1)$

N_b

$0.75\left(\dfrac{v^2}{0.10}\right)$
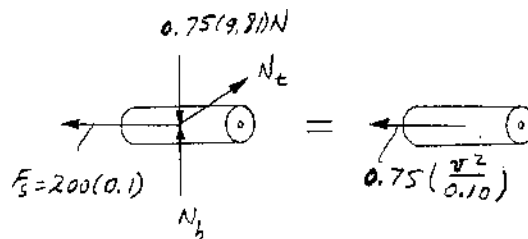
Ans:
$v = 1.63$ m/s
$N = 7.36$ N

13–58.

The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the minimum constant speed the spool can have so that it does not slip down the rod.
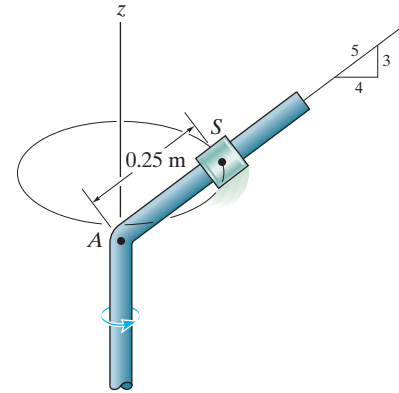
SOLUTION

$$\rho = 0.25\left(\frac{4}{5}\right) = 0.2 \text{ m}$$

$\underset{\leftarrow}{+} \Sigma F_n = m\, a_n;$ $\quad N_s\left(\dfrac{3}{5}\right) - 0.2 N_s\left(\dfrac{4}{5}\right) = 2\left(\dfrac{v^2}{0.2}\right)$

$+\uparrow \Sigma F_b = m\, a_b;$ $\quad N_s\left(\dfrac{4}{5}\right) + 0.2 N_s\left(\dfrac{3}{5}\right) - 2(9.81) = 0$

$\qquad N_s = 21.3 \text{ N}$

$\qquad v = 0.969 \text{ m/s}$ $\qquad$ **Ans.**

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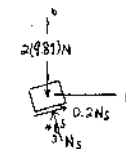
13–59.

The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the maximum constant speed the spool can have so that it does not slip up the rod.
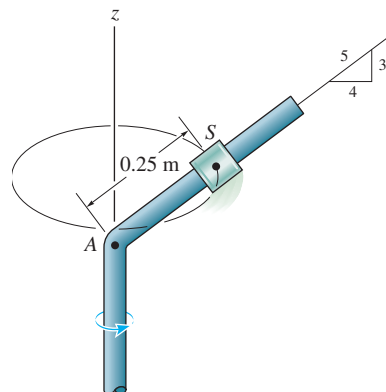
SOLUTION

$$\rho = 0.25(\frac{4}{5}) = 0.2 \text{ m}$$

$\xleftrightarrow{} \Sigma F_n = m\,a_n;$ $N_s(\frac{3}{5}) + 0.2N_s(\frac{4}{5}) = 2(\frac{v^2}{0.2})$

$+\uparrow \Sigma F_b = m\,a_b;$ $N_s(\frac{4}{5}) - 0.2N_s(\frac{3}{5}) - 2(9.81) = 0$

$\qquad N_s = 28.85 \text{ N}$
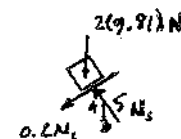
$\qquad v = 1.48 \text{ m/s}$ **Ans.**

Ans:
$v = 1.48$ m/s

*13–60.

At the instant $\theta = 60°$, the boy's center of mass G has a downward speed $v_G = 15$ ft/s. Determine the rate of increase in his speed and the tension in each of the two supporting cords of the swing at this instant. The boy has a weight of 60 lb. Neglect his size and the mass of the seat and cords.

SOLUTION

$$+\searrow \Sigma F_t = ma_t; \qquad 60 \cos 60° = \frac{60}{32.2} a_t \qquad a_t = 16.1 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$\nearrow + \Sigma F_n = ma_n; \qquad 2T - 60 \sin 60° = \frac{60}{32.2}\left(\frac{15^2}{10}\right) \qquad T = 46.9 \text{ lb} \qquad \textbf{Ans.}$$
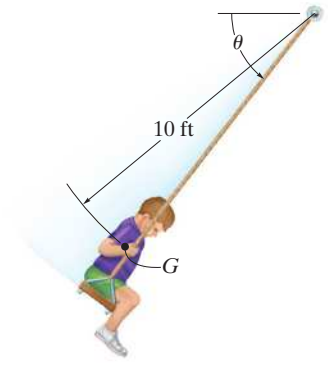
Ans:
$a_t = 16.1 \text{ ft/s}^2$
$T = 46.9 \text{ lb}$

13–61.

At the instant $\theta = 60°$, the boy's center of mass G is momentarily at rest. Determine his speed and the tension in each of the two supporting cords of the swing when $\theta = 90°$. The boy has a weight of 60 lb. Neglect his size and the mass of the seat and cords.

10 ft
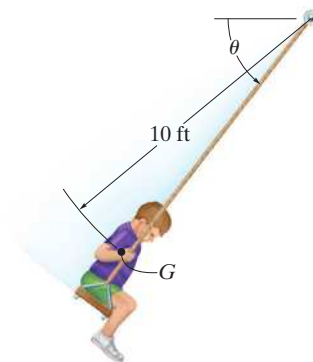
SOLUTION

$+\searrow \Sigma_t = ma_t;$ $\quad 60 \cos \theta = \dfrac{60}{32.2} a_t \quad a_t = 32.2 \cos \theta$

$\nearrow + \Sigma F_n = ma_n;$ $\quad 2T - 60 \sin \theta = \dfrac{60}{32.2}\left(\dfrac{v^2}{10}\right)$ $\qquad$ **(1)**

$\quad v \, dv = a \, ds \qquad$ however $ds = 10 d\theta$

$\displaystyle\int_0^v v \, dv = \int_{60°}^{90°} 322 \cos \theta \, d\theta$

$\quad v = 9.289 \text{ ft/s}$ $\hspace{4cm}$ **Ans.**

From Eq. (1)

$2T - 60 \sin 90° = \dfrac{60}{32.2}\left(\dfrac{9.289^2}{10}\right) \qquad T = 38.0 \text{ lb}$ $\hspace{2cm}$ **Ans.**

60 lb

2T
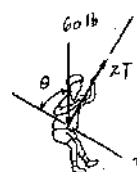
Ans:
$v = 9.29 \text{ ft/s}$
$T = 38.0 \text{ lb}$

13–62.

A girl having a mass of 25 kg sits at the edge of the merry-go-round so her center of mass G is at a distance of 1.5 m from the axis of rotation. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which she can have before she begins to slip off the merry-go-round. The coefficient of static friction between the girl and the merry-go-round is $\mu_s = 0.3$.

SOLUTION

$$\xrightarrow{+} \Sigma F_n = ma_n; \qquad 0.3(245.25) = 25\left(\frac{v^2}{1.5}\right)$$

$$v = 2.10 \text{ m/s} \qquad\qquad \textbf{Ans.}$$

245.25 N

0.3N

$N = 245.25$ N
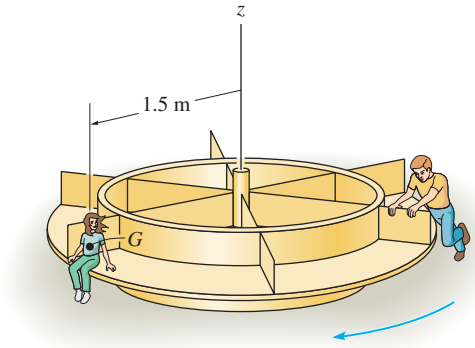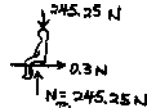
Ans:

$v = 2.10 \text{ m/s}$

13–63.

The pendulum bob B has a weight of 5 lb and is released from rest in the position shown, $\theta = 0°$. Determine the tension in string BC just after the bob is released, $\theta = 0°$, and also at the instant the bob reaches point D, $\theta = 45°$. Take $r = 3$ ft.
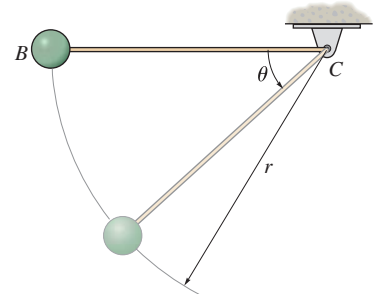
SOLUTION

Equation of Motion: Since the bob is just being release, $v = 0$. Applying Eq. 13–8 to FBD(a), we have

$$\Sigma F_n = ma_n; \qquad T = \frac{5}{32.2}\left(\frac{0^2}{3}\right) = 0 \qquad \text{**Ans.**}$$

Applying Eq. 13–8 to FBD(b), we have

$$\Sigma F_t = ma_t; \qquad 5\cos\theta = \frac{5}{32.2}a_t \qquad a_t = 32.2\cos\theta$$

$$\Sigma F_n = ma_n; \qquad T - 5\sin\theta = \frac{5}{32.2}\left(\frac{v^2}{3}\right) \qquad \text{[1]}$$

Kinematics: The speed of the bob at the instant when $\theta = 45°$ can be determined using $v\,dv = a_t\,ds$. However, $ds = 3d\theta$, then $v\,dv = 3a_t\,d\theta$.

$$\int_0^v v\,dv = 3(32.2)\int_0^{45°}\cos\theta\,d\theta$$

$$v^2 = 136.61 \text{ ft}^2/\text{s}^2$$

Substitute $\theta = 45°$ and $v^2 = 136.61$ ft^2/s^2 into Eq. [1] yields

$$T - 5\sin 45° = \frac{5}{32.2}\left(\frac{136.61}{3}\right)$$

$$T = 10.6 \text{ lb} \qquad \text{**Ans.**}$$

(a)

(b)
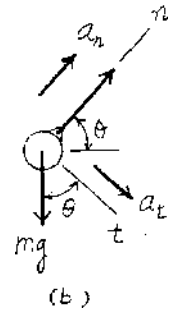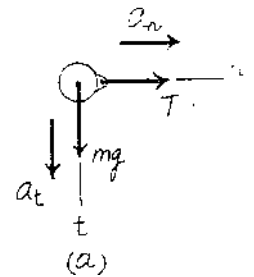
Ans:
$T = 0$
$T = 10.6$ lb

*13–64.

The pendulum bob B has a mass m and is released from rest when $\theta = 0°$. Determine the tension in string BC immediately afterwards, and also at the instant the bob reaches the arbitrary position θ.
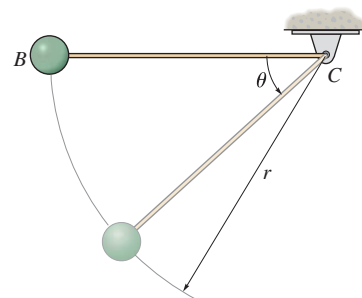
SOLUTION

Equation of Motion: Since the bob is just being release, $v = 0$. Applying Eq. 13–8 to FBD(a), we have

$$\Sigma F_n = ma_n; \qquad T = m\left(\frac{0^2}{r}\right) = 0 \qquad\qquad \textbf{Ans.}$$

Applying Eq. 13–8 to FBD(b), we have

$$\Sigma F_t = ma_t; \qquad mg\cos\theta = ma_t \qquad a_t = g\cos\theta$$

$$\Sigma F_n = ma_n; \qquad T - mg\sin\theta = m\left(\frac{v^2}{r}\right) \qquad\qquad \textbf{[1]}$$

Kinematics: The speed of the bob at the arbitrary position θ can be detemined using $vdv = a_t\, ds$. However, $ds = rd\theta$, then $vdv = a_t\, rd\theta$.

$$\int_0^v vdv = gr\int_0^\theta \cos\theta d\theta$$

$$v^2 = 2gr\sin\theta$$

Substitute $v^2 = 2gr\sin\theta$ into Eq. [1] yields

$$T - mg\sin\theta = m\left(\frac{2gr\sin\theta}{r}\right)$$

$$T = 3mg\sin\theta \qquad\qquad \textbf{Ans.}$$

Ans:
$T = 0$
$T = 3mg\sin\theta$

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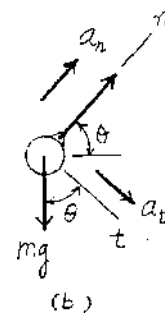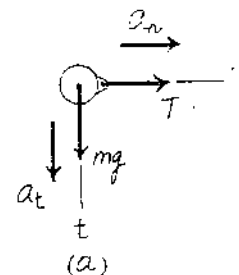
13–65.

Determine the constant speed of the passengers on the amusement-park ride if it is observed that the supporting cables are directed at $\theta = 30°$ from the vertical. Each chair including its passenger has a mass of 80 kg. Also, what are the components of force in the n, t, and b directions which the chair exerts on a 50-kg passenger during the motion?

SOLUTION

$\xleftrightarrow{} \Sigma F_n = m\, a_n;$ $T \sin 30° = 80(\dfrac{v^2}{4 + 6 \sin 30°})$

$+\uparrow \Sigma F_b = 0;$ $T \cos 30° - 80(9.81) = 0$

$T = 906.2$ N

$v = 6.30$ m/s **Ans.**

$\Sigma F_n = m\, a_n;$ $F_n = 50(\dfrac{(6.30)^2}{7}) = 283$ N **Ans.**

$\Sigma F_t = m\, a_t;$ $F_t = 0$ **Ans.**

$\Sigma F_b = m\, a_b;$ $F_b - 490.5 = 0$
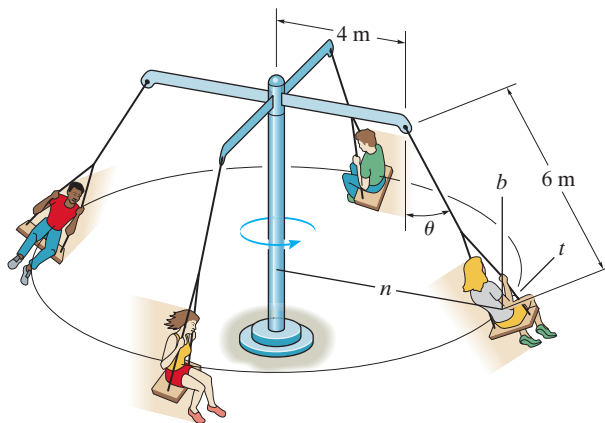
$F_b = 490$ N **Ans.**
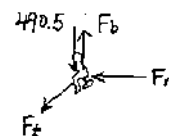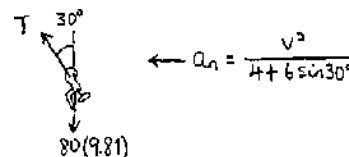
Ans:
$v = 6.30$ m/s
$F_n = 283$ N
$F_t = 0$
$F_b = 490$ N

13–66.

A motorcyclist in a circus rides his motorcycle within the confines of the hollow sphere. If the coefficient of static friction between the wheels of the motorcycle and the sphere is $\mu_s = 0.4$, determine the minimum speed at which he must travel if he is to ride along the wall when $\theta = 90°$. The mass of the motorcycle and rider is 250 kg, and the radius of curvature to the center of gravity is $\rho = 20$ ft. Neglect the size of the motorcycle for the calculation.

SOLUTION

$\xrightarrow{+} \Sigma F_n = ma_n;$ $N = 250\left(\dfrac{v^2}{20}\right)$

$+\uparrow \Sigma F_b = ma_b;$ $0.4\,N - 250(9.81) = 0$

Solving,
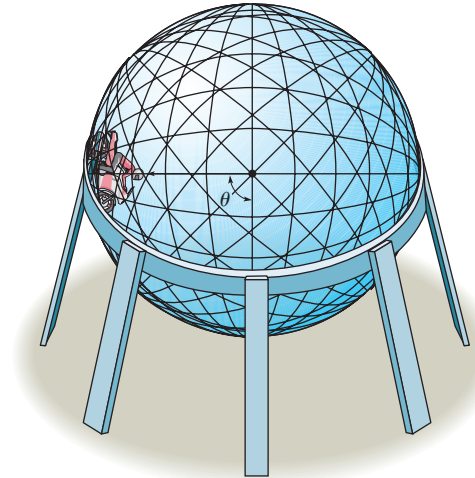
$$v = 22.1 \text{ m/s} \qquad \text{Ans.}$$
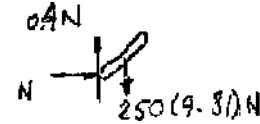
Ans:
$v = 22.1$ m/s

13–67.

The vehicle is designed to combine the feel of a motorcycle with the comfort and safety of an automobile. If the vehicle is traveling at a constant speed of 80 km/h along a circular curved road of radius 100 m, determine the tilt angle θ of the vehicle so that only a normal force from the seat acts on the driver. Neglect the size of the driver.
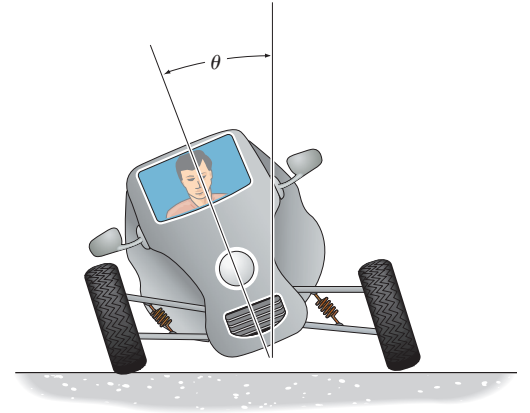
SOLUTION

Free-Body Diagram: The free-body diagram of the passenger is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: The speed of the passenger is $v = \left(80\,\dfrac{\text{km}}{\text{h}} \right)\left(\dfrac{1000\,\text{m}}{1\,\text{km}} \right)\left(\dfrac{1\,\text{h}}{3600\,\text{s}} \right)$

$= 22.22$ m/s. Thus, the normal component of the passenger's acceleration is given by

$a_n = \dfrac{v^2}{\rho} = \dfrac{22.22^2}{100} = 4.938$ m/s². By referring to Fig. (a),

$+\uparrow \Sigma F_b = 0; \qquad N\cos\theta - m(9.81) = 0 \qquad\qquad N = \dfrac{9.81m}{\cos\theta}$

$\pm\, \Sigma F_n = ma_n; \qquad \dfrac{9.81m}{\cos\theta}\sin\theta = m(4.938)$

$\qquad\qquad\qquad \theta = 26.7° \qquad\qquad\qquad\qquad\qquad$ **Ans.**

(a)
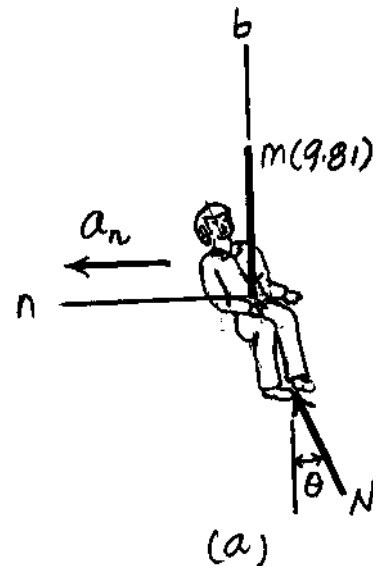
Ans:
$\theta = 26.7°$

*13–68.

The 0.8-Mg car travels over the hill having the shape of a parabola. If the driver maintains a constant speed of 9 m/s, determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at the instant it reaches point A. Neglect the size of the car.
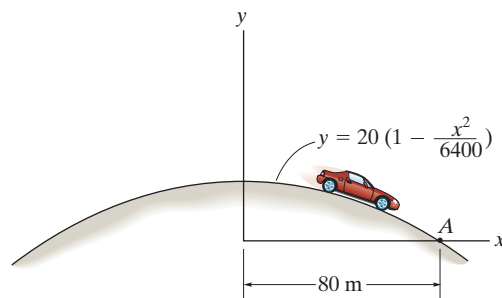
$y = 20 \left(1 - \dfrac{x^2}{6400}\right)$

80 m

SOLUTION

Geometry: Here, $\dfrac{dy}{dx} = -0.00625x$ and $\dfrac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan \theta = \dfrac{dy}{dx}\bigg|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57°$$

and the radius of curvature at point A is

$$\rho = \dfrac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \dfrac{[1 + (-0.00625x)^2]^{3/2}}{|-0.00625|}\bigg|_{x=80\text{ m}} = 223.61\text{ m}$$

Equations of Motion: Here, $a_t = 0$. Applying Eq. 13–8 with $\theta = 26.57°$ and $\rho = 223.61$ m, we have

$$\Sigma F_t = ma_t; \qquad 800(9.81)\sin 26.57° - F_f = 800(0)$$

$$F_f = 3509.73\text{ N} = 3.51\text{ kN} \qquad\qquad \textbf{Ans.}$$

$$\Sigma F_n = ma_n; \qquad 800(9.81)\cos 26.57° - N = 800\left(\dfrac{9^2}{223.61}\right)$$

$$N = 6729.67\text{ N} = 6.73\text{ kN} \qquad\qquad \textbf{Ans.}$$
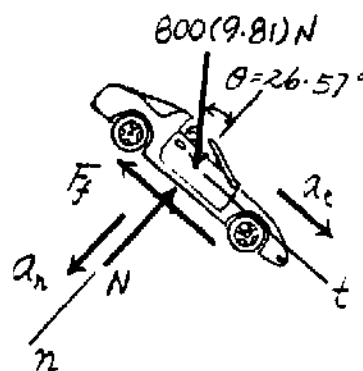
Ans:
$F_f = 3.51$ kN
$N = 6.73$ kN

312

13–69.

The 0.8-Mg car travels over the hill having the shape of a parabola. When the car is at point A, it is traveling at 9 m/s and increasing its speed at 3 m/s². Determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at this instant. Neglect the size of the car.
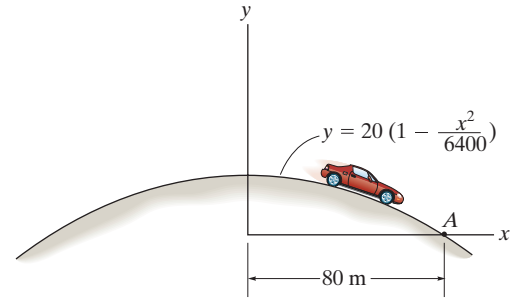
SOLUTION

Geometry: Here, $\dfrac{dy}{dx} = -0.00625x$ and $\dfrac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan \theta = \left.\frac{dy}{dx}\right|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57°$$

and the radius of curvature at point A is

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{[1 + (-0.00625x)^2]^{3/2}}{|-0.00625|}\right|_{x=80\text{ m}} = 223.61 \text{ m}$$

Equation of Motion: Applying Eq. 13–8 with $\theta = 26.57°$ and $\rho = 223.61$ m, we have

$$\Sigma F_t = ma_t; \qquad 800(9.81) \sin 26.57° - F_f = 800(3)$$

$$F_f = 1109.73 \text{ N} = 1.11 \text{ kN} \qquad\qquad \textbf{Ans.}$$

$$\Sigma F_n = ma_n; \qquad 800(9.81) \cos 26.57° - N = 800\left(\frac{9^2}{223.61}\right)$$

$$N = 6729.67 \text{ N} = 6.73 \text{ kN} \qquad\qquad \textbf{Ans.}$$

$y = 20\left(1 - \dfrac{x^2}{6400}\right)$

80 m

800(9.81) N

$\theta = 26.57°$
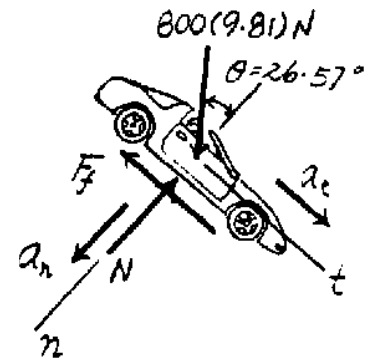
Ans:
$F_f = 1.11$ kN
$N = 6.73$ kN

313

13–70.

The package has a weight of 5 lb and slides down the chute. When it reaches the curved portion AB, it is traveling at 8 ft/s ($\theta = 0°$). If the chute is smooth, determine the speed of the package when it reaches the intermediate point C ($\theta = 30°$) and when it reaches the horizontal plane ($\theta = 45°$). Also, find the normal force on the package at C.
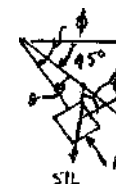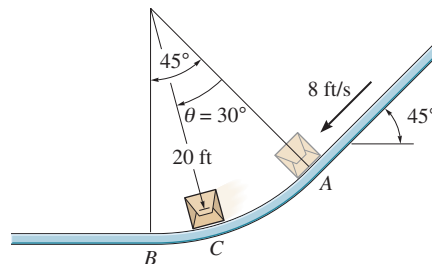
SOLUTION

$+\swarrow \Sigma F_t = ma_t;$ $\qquad 5 \cos \phi = \dfrac{5}{32.2} a_t$

$\qquad\qquad\qquad a_t = 32.2 \cos \phi$

$+\nwarrow \Sigma F_n = ma_n;$ $\qquad N - 5 \sin \phi = \dfrac{5}{32.2} \left(\dfrac{v^2}{20}\right)$

$\qquad\qquad\qquad v \, dv = a_t \, ds$

$$\int_g^v v \, dv = \int_{45°}^{\phi} 32.2 \cos \phi \, (20 \, d\phi)$$

$$\frac{1}{2}v^2 - \frac{1}{2}(8)^2 = 644 \, (\sin \phi - \sin 45°)$$

At $\phi = 45° + 30° = 75°$,

$\qquad\qquad\qquad v_C = 19.933 \text{ ft/s} = 19.9 \text{ ft/s}$ $\qquad\qquad$ **Ans.**

$\qquad\qquad\qquad N_C = 7.91 \text{ lb}$ $\qquad\qquad$ **Ans.**

At $\phi = 45° + 45° = 90°$

$\qquad\qquad\qquad v_B = 21.0 \text{ ft/s}$ $\qquad\qquad$ **Ans.**

Ans:
$v_C = 19.9 \text{ ft/s}$
$N_C = 7.91 \text{ lb}$
$v_B = 21.0 \text{ ft/s}$

13–71.

The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. Determine the resultant normal and frictional forces the cushion exerts on him if, due to rotation about the z axis, he has a constant speed $v = 20$ ft/s. Neglect the size of the man. Take $\theta = 60°$.

SOLUTION

$+\nwarrow\sum F_y = m(a_n)_y;$ $N - 150\cos 60° = \dfrac{150}{32.2}\left(\dfrac{20^2}{8}\right)\sin\ 60°$

$N = 277$ lb   **Ans.**

$+\swarrow\sum F_x = m(a_n)_x;$ $-F + 150\sin\ 60° = \dfrac{150}{32.2}\left(\dfrac{20^2}{8}\right)\cos 60°$

$F = 13.4$ lb   **Ans.**

Note: No slipping occurs

Since $\mu_s N = 138.4$ lb > 13.4 lb
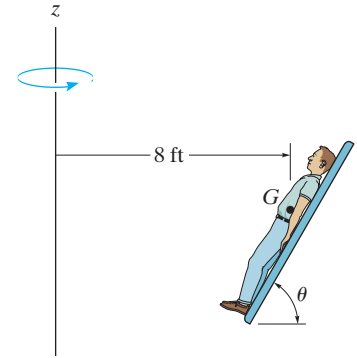
Ans:
$N = 277$ lb
$F = 13.4$ lb

*13–72.

The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. If he rotates about the z axis with a constant speed $v = 30$ ft/s, determine the smallest angle θ of the cushion at which he will begin to slip off.

SOLUTION

$\xleftarrow{+} \Sigma F_n = ma_n;$ $0.5N \cos \theta + N \sin \theta = \dfrac{150}{32.2}\left(\dfrac{(30)^2}{8}\right)$

$+\uparrow \Sigma F_b = 0;$ $-150 + N \cos \theta - 0.5 N \sin \theta = 0$

$\qquad\qquad N = \dfrac{150}{\cos \theta - 0.5 \sin \theta}$

$\qquad\qquad \dfrac{(0.5 \cos \theta + \sin \theta)150}{(\cos \theta - 0.5 \sin \theta)} = \dfrac{150}{32.2}\left(\dfrac{(30)^2}{8}\right)$

$\qquad\qquad 0.5 \cos \theta + \sin \theta = 3.49378 \cos \theta - 1.74689 \sin \theta$

$\qquad\qquad \theta = 47.5°$ **Ans.**
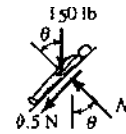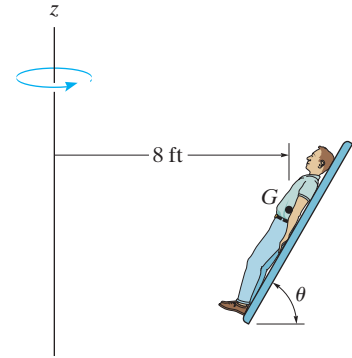
13–73.

Determine the maximum speed at which the car with mass m can pass over the top point A of the vertical curved road and still maintain contact with the road. If the car maintains this speed, what is the normal reaction the road exerts on the car when it passes the lowest point B on the road?
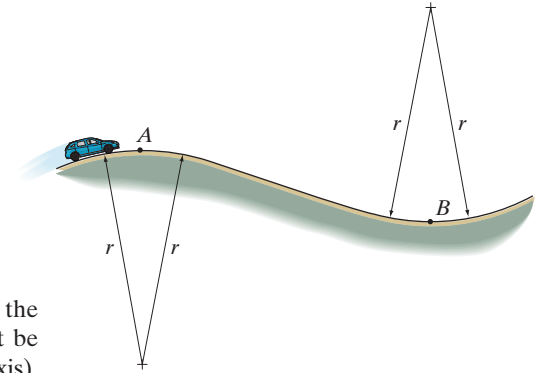
SOLUTION

Free-Body Diagram: The free-body diagram of the car at the top and bottom of the vertical curved road are shown in Figs. (a) and (b), respectively. Here, $\mathbf{a}_n$ must be directed towards the center of curvature of the vertical curved road (positive n axis).

Equations of Motion: When the car is on top of the vertical curved road, it is required that its tires are about to lose contact with the road surface. Thus, $N = 0$. Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{v^2}{r}$ and referring to Fig. (a),

$$+\downarrow \Sigma F_n = ma_n; \qquad mg = m\left(\frac{v^2}{r}\right) \qquad\qquad v = \sqrt{gr} \qquad\qquad \text{Ans.}$$

Using the result of v, the normal component of car acceleration is $a_n = \dfrac{v^2}{\rho} = \dfrac{gr}{r} = g$ when it is at the lowest point on the road. By referring to Fig. (b),

$$+\uparrow \Sigma F_n = ma_n; \qquad N - mg = mg$$

$$N = 2mg \qquad\qquad \text{Ans.}$$

(a)

(b)

Ans:
$v = \sqrt{gr}$
$N = 2mg$

317
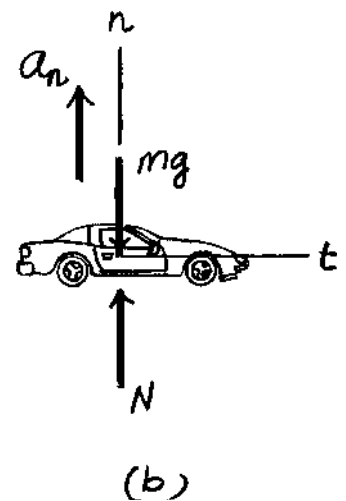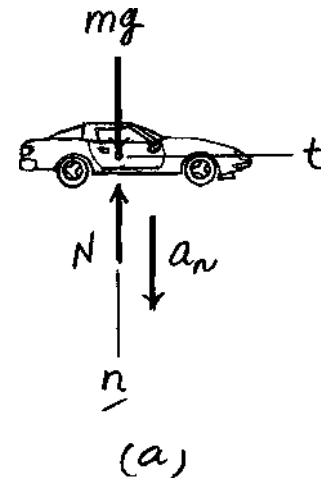
13–74.

Determine the maximum constant speed at which the 2-Mg car can travel over the crest of the hill at A without leaving the surface of the road. Neglect the size of the car in the calculation.

$$y = 20\left(1 - \frac{x^2}{10\,000}\right)$$

A

100 m
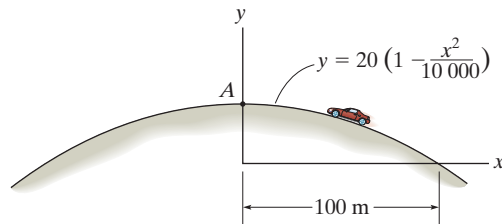
SOLUTION

Geometry. The radius of curvature of the road at A must be determined first. Here

$$\frac{dy}{dx} = 20\left(-\frac{2x}{10000}\right) = -0.004x$$

$$\frac{d^2y}{dx^2} = -0.004$$

At point A, $x = 0$. Thus, $\left.\frac{dy}{dx}\right|_{x=0} = 0$. Then

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{(1 + 0^2)^{3/2}}{0.004} = 250 \text{ m}$$

Equation of Motion. Since the car is required to be on the verge to leave the road surface, $N = 0$.

$$\Sigma F_n = ma_n; \qquad 2000(9.81) = 2000\left(\frac{v^2}{250}\right)$$

$$v = 49.52 \text{ m/s} = 49.5 \text{ m/s} \qquad\qquad \textbf{Ans.}$$

$2000(9.81)\text{N}$

t

$N \downarrow a_n$

n

(a)

Ans:
$v = 49.5$ m/s

318
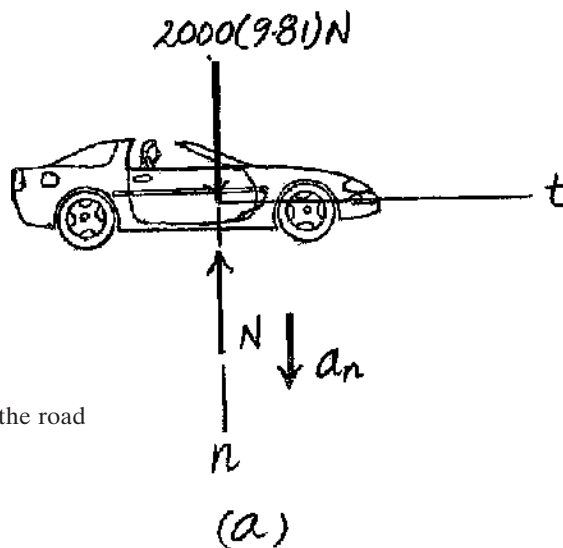
13–75.

The box has a mass m and slides down the smooth chute having the shape of a parabola. If it has an initial velocity of v_0 at the origin, determine its velocity as a function of x. Also, what is the normal force on the box, and the tangential acceleration as a function of x?

SOLUTION

$$x = -\frac{1}{2}x^2$$

$$\frac{dy}{dx} = -x$$

$$\frac{d^2y}{dx^2} = -1$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + x^2\right]^{\frac{3}{2}}}{|-1|} = \left(1 + x^2\right)^{\frac{3}{2}}$$

$$+\nearrow\Sigma F_n = ma_n; \qquad mg\left(\frac{1}{\sqrt{1 + x^2}}\right) - N = m\left(\frac{v^2}{(1 + x^2)^{\frac{3}{2}}}\right) \qquad \textbf{(1)}$$

$$+\searrow\Sigma F_t = ma_t; \qquad mg\left(\frac{x}{\sqrt{1 + x^2}}\right) = ma_t$$

$$a_t = g\left(\frac{x}{\sqrt{1 + x^2}}\right) \qquad \textbf{Ans.}$$

$$v\, dv = a_t\, ds = g\left(\frac{x}{\sqrt{1 + x^2}}\right)ds$$

$$ds = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}} dx = \left(1 + x^2\right)^{\frac{1}{2}} dx$$

$$\int_{v_0}^{v} v\, dv = \int_0^x gx\, dx$$

$$\frac{1}{2}v^2 - \frac{1}{2}v_0^2 = g\left(\frac{x^2}{2}\right)$$

$$v = \sqrt{v_0^2 + gx^2} \qquad \textbf{Ans.}$$

From Eq. (1):
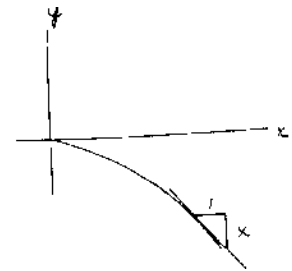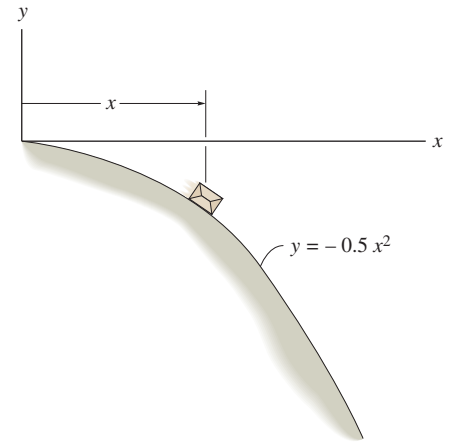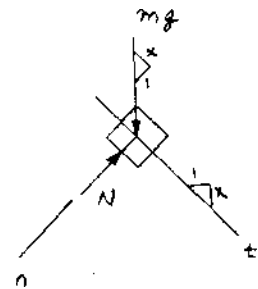
$$N = \frac{m}{\sqrt{1 + x^2}}\left[g - \frac{(v_0^2 + gx^2)}{(1 + x^2)}\right] \qquad \textbf{Ans.}$$

Ans:

$$a_t = g\left(\frac{x}{\sqrt{1 + x^2}}\right)$$

$$v = \sqrt{v_0^2 + gx^2}$$

$$N = \frac{m}{\sqrt{1 + x^2}}\left[g - \frac{v_0^2 + gx^2}{1 + x^2}\right]$$

*13–76.

Prove that if the block is released from rest at point B of a smooth path of *arbitrary shape*, the speed it attains when it reaches point A is equal to the speed it attains when it falls freely through a distance h; i.e., $v = \sqrt{2gh}$.
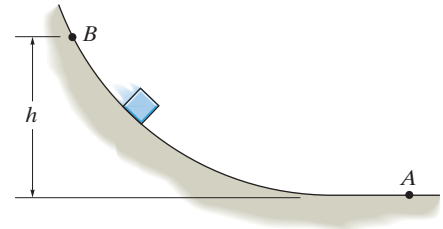
SOLUTION

$+\searrow\Sigma F_t = ma_t;$ $mg \sin\theta = ma_t$ $a_t = g \sin\theta$

$v\,dv = a_t\,ds = g\sin\theta\,ds$ However $dy = ds\sin\theta$

$$\int_0^v v\,dv = \int_0^h g\,dy$$

$$\frac{v^2}{2} = gh$$

$$v = \sqrt{2gh}$$ **Q.E.D.**
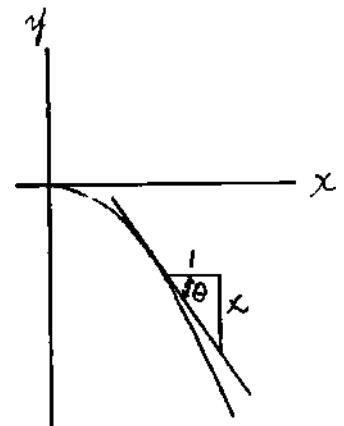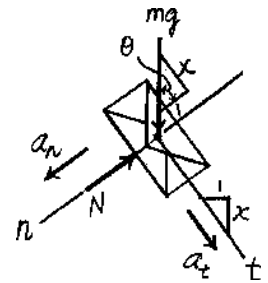
Ans:
$v = \sqrt{2gh}$

12–77.

The cylindrical plug has a weight of 2 lb and it is free to move within the confines of the smooth pipe. The spring has a stiffness $k = 14$ lb/ft and when no motion occurs the distance $d = 0.5$ ft. Determine the force of the spring on the plug when the plug is at rest with respect to the pipe. The plug is traveling with a constant speed of 15 ft/s, which is caused by the rotation of the pipe about the vertical axis.
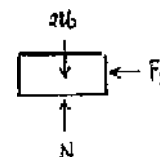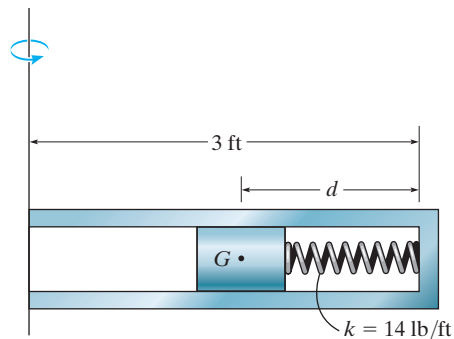
SOLUTION

$$\xleftarrow{+} \Sigma F_n = ma_n; \qquad F_s = \frac{2}{32.2}\left[\frac{(15)^2}{3 - d}\right]$$

$$F_s = ks; \qquad F_s = 14(0.5 - d)$$

Thus,

$$14(0.5 - d) = \frac{2}{32.2}\left[\frac{(15)^2}{3 - d}\right]$$

$$(0.5 - d)(3 - d) = 0.9982$$

$$1.5 - 3.5d + d^2 = 0.9982$$

$$d^2 - 3.5d + 0.5018 = 0$$

Choosing the root < 0.5 ft

$$d = 0.1498 \text{ ft}$$

$$F_s = 14(0.5 - 0.1498) = 4.90 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
$F_s = 4.90$ lb

13–78.

When crossing an intersection, a motorcyclist encounters the slight bump or crown caused by the intersecting road. If the crest of the bump has a radius of curvature $\rho = 50$ ft, determine the maximum constant speed at which he can travel without leaving the surface of the road. Neglect the size of the motorcycle and rider in the calculation. The rider and his motorcycle have a total weight of 450 lb.
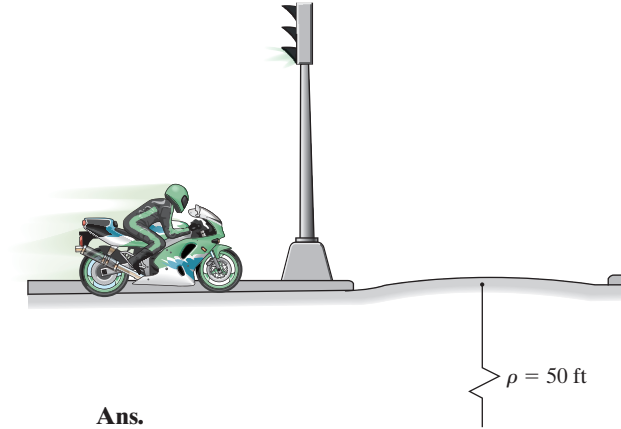
SOLUTION

$+\downarrow \Sigma F_n = ma_n;$ $\qquad 450 - 0 = \dfrac{450}{32.2}\left(\dfrac{v^2}{50}\right)$

$\qquad\qquad v = 40.1$ ft/s **Ans.**

$\rho = 50$ ft
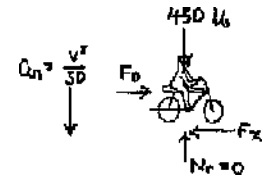
Ans:
$v = 40.1$ ft/s

13–79.

The airplane, traveling at a constant speed of 50 m/s, is executing a horizontal turn. If the plane is banked at $\theta = 15°$, when the pilot experiences only a normal force on the seat of the plane, determine the radius of curvature ρ of the turn. Also, what is the normal force of the seat on the pilot if he has a mass of 70 kg.
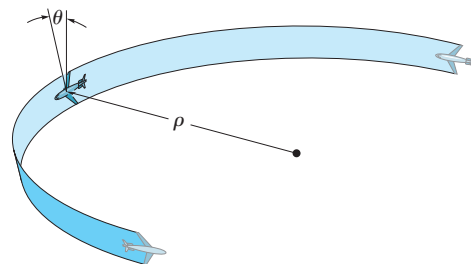
SOLUTION

$+\uparrow \sum F_b = ma_b;$ $N_P \sin 15° - 70(9.81) = 0$

$N_P = 2.65$ kN   **Ans.**

$\xleftarrow{+} \sum F_n = ma_n;$ $N_P \cos 15° = 70\left(\dfrac{50^2}{\rho}\right)$

$\rho = 68.3$ m   **Ans.**

70(9.81) N

N_P
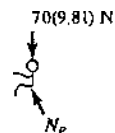
Ans:
$N_P = 2.65$ kN
$\rho = 68.3$ m

*13–80.

The 2-kg pendulum bob moves in the vertical plane with a velocity of 8 m/s when $\theta = 0°$. Determine the initial tension in the cord and also at the instant the bob reaches $\theta = 30°$. Neglect the size of the bob.

2 m
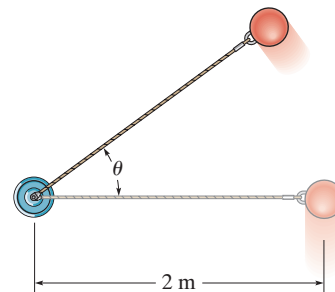
SOLUTION

Equations of Motion. Referring to the FBD of the bob at position $\theta = 0°$, Fig. a,

$$\Sigma F_n = ma_n; \quad T = 2\left(\frac{8^2}{2}\right) = 64.0 \text{ N} \qquad \textbf{Ans.}$$

For the bob at an arbitrary position θ, the FBD is shown in Fig. b.

$$\Sigma F_t = ma_t; \quad -2(9.81)\cos\theta = 2a_t$$

$$a_t = -9.81\cos\theta$$

$$\Sigma F_n = ma_n; \quad T + 2(9.81)\sin\theta = 2\left(\frac{v^2}{2}\right)$$

$$T = v^2 - 19.62\sin\theta \qquad (1)$$

Kinematics. The velocity of the bob at the position $\theta = 30°$ can be determined by integrating $v\,dv = a_t\,ds$. However, $ds = r\,d\theta = 2\,d\theta$.
Then,

$$\int_{8 \text{ m/s}}^{v} v\,dv = \int_{0°}^{30°} -9.81\cos\theta(z\,d\theta)$$

$$\left.\frac{v^2}{2}\right|_{8 \text{ m/s}}^{v} = -19.62\sin\theta\Big|_{0°}^{30°}$$

$$\frac{v^2}{2} - \frac{8^2}{2} = -19.62(\sin 30° - 0)$$

$$v^2 = 44.38 \text{ m}^2/\text{s}^2$$

Substitute this result and $\theta = 30°$ into Eq. (1),

$$T = 44.38 - 19.62\sin 30°$$

$$= 34.57 \text{ N} = 34.6 \text{ N} \qquad \textbf{Ans.}$$
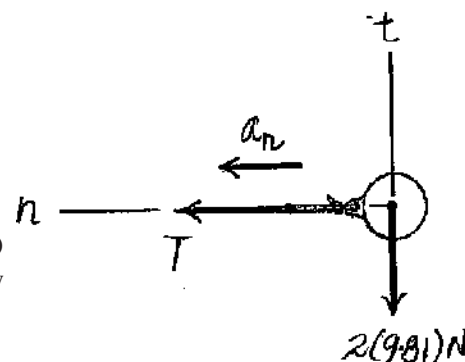
(a)
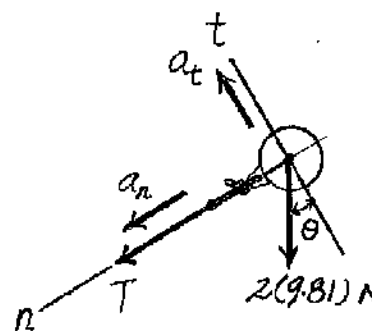
(b)

Ans:
$T = 64.0$ N
$T = 34.6$ N

13–81.

The 2-kg pendulum bob moves in the vertical plane with a velocity of 6 m/s when $\theta = 0°$. Determine the angle θ where the tension in the cord becomes zero.

2 m
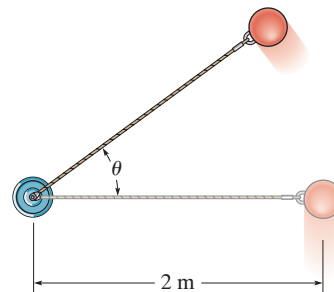
SOLUTION

Equation of Motion. The FBD of the bob at an arbitrary position θ is shown in Fig. *a*. Here, it is required that $T = 0$.

$$\Sigma F_t = ma_t; \qquad -2(9.81) \cos \theta = 2a_t$$
$$a_t = -9.81 \cos \theta$$

$$\Sigma F_n = ma_n; \qquad 2(9.81) \sin \theta = 2\left(\frac{v^2}{2}\right)$$
$$v^2 = 19.62 \sin \theta \qquad \qquad (1)$$

Kinematics. The velocity of the bob at an arbitrary position θ can be determined by integrating $v\,dv = a_t\,ds$. However, $ds = r\,d\theta = 2\,d\theta$.

Then

$$\int_{6\,\text{m/s}}^{v} v\,dv = \int_{0°}^{\theta} -9.81 \cos \theta(2\,d\theta)$$

$$\left.\frac{v^2}{2}\right|_{6\,\text{m/s}}^{v} = \left. -19.62 \sin \theta \right|_{0°}^{\theta}$$

$$v^2 = 36 - 39.24 \sin \theta \qquad \qquad (2)$$

Equating Eqs. (1) and (2)

$$19.62 \sin \theta = 36 - 39.24 \sin \theta$$
$$58.86 \sin \theta = 36$$
$$\theta = 37.71° = 37.7° \qquad \qquad \textbf{Ans.}$$

(a)

Ans:
$\theta = 37.7°$

325
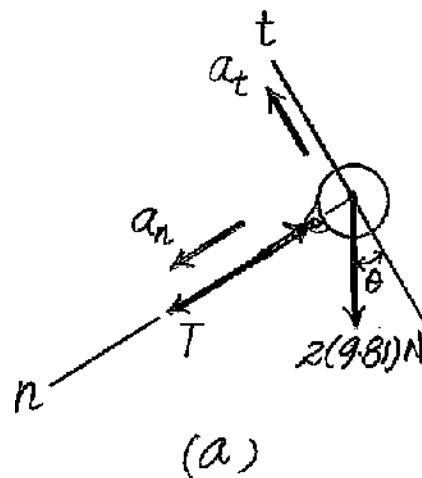
13–82.

The 8-kg sack slides down the smooth ramp. If it has a speed of 1.5 m/s when $y = 0.2$ m, determine the normal reaction the ramp exerts on the sack and the rate of increase in the speed of the sack at this instant.
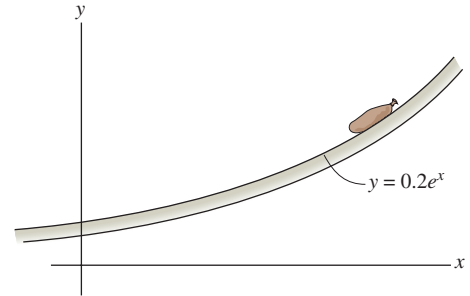
SOLUTION

$y = 0.2 \qquad x = 0$

$y = 0.2e^x$

$\dfrac{dy}{dx} = 0.2e^x \Big|_{x=0} = 0.2$

$\dfrac{d^2y}{dx^2} = 0.2e^x \Big|_{x=0} = 0.2$

$\rho = \dfrac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\dfrac{d^2y}{dx^2}\right|} = \dfrac{\left[1 + (0.2)^2\right]^{\frac{3}{2}}}{|0.2|} = 5.303$

$\theta = \tan^{-1}(0.2) = 11.31°$

$+\nwarrow \Sigma F_n = ma_n; \qquad N_B - 8(9.81)\cos 11.31° = 8\left(\dfrac{(1.5)^2}{5.303}\right)$

$\qquad\qquad\qquad\qquad N_B = 80.4 \text{ N}$ **Ans.**

$+\swarrow \Sigma F_t = ma_t; \qquad 8(9.81)\sin 11.31° = 8a_t$

$\qquad\qquad\qquad\qquad a_t = 1.92 \text{ m/s}^2$ **Ans.**
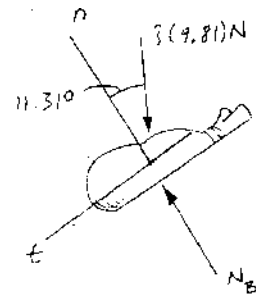
Ans:
$N_B = 80.4 \text{ N}$
$a_t = 1.92 \text{ m/s}^2$

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13–83.

The ball has a mass m and is attached to the cord of length l. The cord is tied at the top to a swivel and the ball is given a velocity $\mathbf{v}_0$. Show that the angle θ which the cord makes with the vertical as the ball travels around the circular path must satisfy the equation $\tan \theta \sin \theta = v_0^2/gl$. Neglect air resistance and the size of the ball.
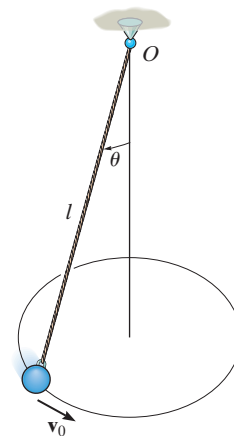
SOLUTION

$$\xrightarrow{+} \Sigma F_n = ma_n; \qquad T \sin \theta = m\left(\frac{v_0^2}{r}\right)$$

$$+\uparrow \Sigma F_b = 0; \qquad T \cos \theta - mg = 0$$

Since $r = l \sin \theta \qquad T = \dfrac{mv_0^2}{l \sin^2 \theta}$

$$\left(\frac{mv_0^2}{l}\right)\left(\frac{\cos \theta}{\sin^2 \theta}\right) = mg$$

$$\tan \theta \sin \theta = \frac{v_0^2}{gl} \qquad\qquad \textbf{Q.E.D.}\ .$$
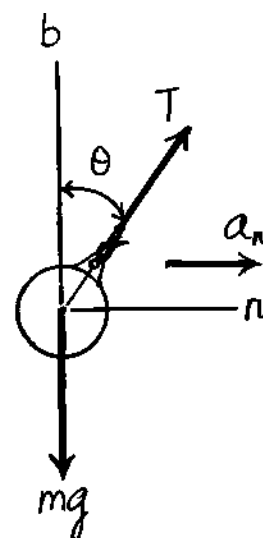
Ans:

$$\tan \theta \sin \theta = \frac{v_0^2}{gl}$$

*13–84.

The 2-lb block is released from rest at A and slides down along the smooth cylindrical surface. If the attached spring has a stiffness $k = 2$ lb/ft, determine its unstretched length so that it does not allow the block to leave the surface until $\theta = 60°$.
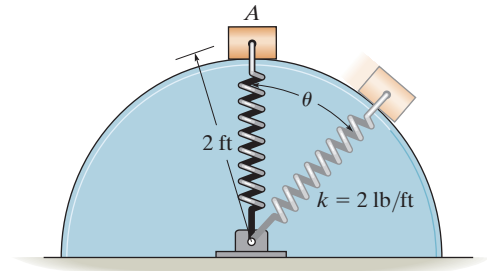
SOLUTION

$$+\swarrow\Sigma F_n = ma_n; \quad F_s + 2\cos\theta = \frac{2}{32.2}\left(\frac{v^2}{2}\right) \qquad (1)$$

$$+\searrow\Sigma F_t = ma_t; \quad 2\sin\theta = \frac{2}{32.2}a_t$$

$$a_t = 32.2\sin\theta$$

$$v\,dv = a_t\,ds; \quad \int_0^v v\,dv = \int_0^\theta 32.2(\sin\theta)2d\theta$$

$$\frac{1}{2}v^2 = 64.4(-\cos\theta + 1)$$

When $\theta = 60°$

$v^2 = 64.4$

From Eq. (1)

$$F_s + 2\cos 60° = \frac{2}{32.2}\left(\frac{64.4}{2}\right)$$

$F_s = 1$ lb

$F_s = ks; \quad 1 = 2s; \quad s = 0.5$ ft

$l_0 = l - s = 2 - 0.5 = 1.5$ ft   **Ans.**
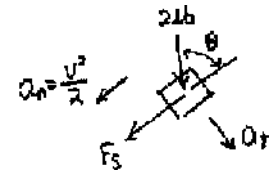
Ans:
$l_0 = 1.5$ ft

13–85.

The spring-held follower AB has a weight of 0.75 lb and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.2$ ft and $z = (0.1 \sin \theta)$ ft. If the cam is rotating at a constant rate of 6 rad/s, determine the force at the end A of the follower when $\theta = 90°$. In this position the spring is compressed 0.4 ft. Neglect friction at the bearing C.
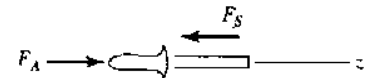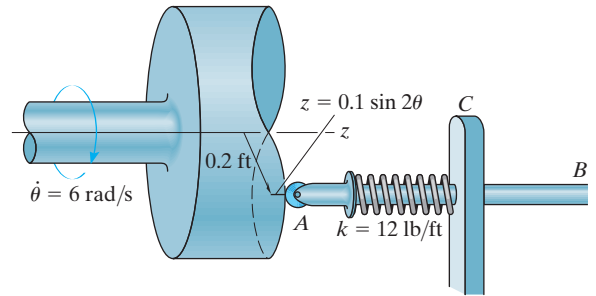
SOLUTION

$z = 0.1 \sin 2\theta$

$\dot{z} = 0.2 \cos 2\theta \dot{\theta}$

$\ddot{z} = -0.4 \sin 2\theta \dot{\theta}^2 + 0.2 \cos 2\theta \ddot{\theta}$

$\dot{\theta} = 6$ rad/s

$\ddot{\theta} = 0$

$\ddot{z} = -14.4 \sin 2\theta$

$\sum F_z = ma_z;$ $F_A - 12(z + 0.3) = m\ddot{z}$

$F_A - 12(0.1 \sin 2\theta + 0.3) = \dfrac{0.75}{32.2}(-14.4 \sin 2\theta)$

For $\theta = 45°$,

$F_A - 12(0.4) = \dfrac{0.75}{32.2}(-14.4)$

$F_A = 4.46$ lb   **Ans.**

Ans:
$F_A = 4.46$ lb

13–86.

Determine the magnitude of the resultant force acting on a 5-kg particle at the instant $t = 2$ s, if the particle is moving along a horizontal path defined by the equations $r = (2t + 10)$ m and $\theta = (1.5t^2 - 6t)$ rad, where t is in seconds.

SOLUTION

$r = 2t + 10|_{t=2\,s} = 14$

$\dot{r} = 2$

$\ddot{r} = 0$

$\theta = 1.5t^2 - 6t$

$\dot{\theta} = 3t - 6|_{t=2\,s} = 0$

$\ddot{\theta} = 3$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0 = 0$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 14(3) + 0 = 42$

Hence,

$\Sigma F_r = ma_r; \qquad F_r = 5(0) = 0$

$\Sigma F_\theta = ma_\theta; \qquad F_\theta = 5(42) = 210$ N

$F = \sqrt{(F_r)^2 + (F_\theta)^2} = 210$ N   **Ans.**
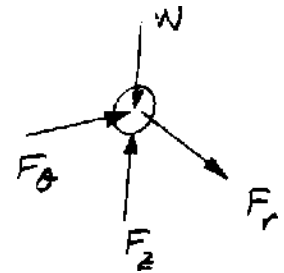
Ans:
$F = 210$ N

13–87.

The path of motion of a 5-lb particle in the horizontal plane is described in terms of polar coordinates as $r = (2t + 1)$ ft and $\theta = (0.5t^2 - t)$ rad, where t is in seconds. Determine the magnitude of the unbalanced force acting on the particle when $t = 2$ s.

SOLUTION

$r = 2t + 1|_{t=2\,s} = 5$ ft $\qquad \dot{r} = 2$ ft/s $\qquad \ddot{r} = 0$

$\theta = 0.5t^2 - t|_{t=2\,s} = 0$ rad $\qquad \dot{\theta} = t - 1|_{t=2\,s} = 1$ rad/s $\qquad \ddot{\theta} = 1$ rad/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 5(1)^2 = -5$ ft/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 5(1) + 2(2)(1) = 9$ ft/s^2

$\Sigma F_r = ma_r; \qquad F_r = \dfrac{5}{32.2}(-5) = -0.7764$ lb

$\Sigma F_\theta = ma_\theta; \qquad F_\theta = \dfrac{5}{32.2}(9) = 1.398$ lb

$F = \sqrt{F_r^2 + F_\theta^2} = \sqrt{(-0.7764)^2 + (1.398)^2} = 1.60$ lb $\qquad\qquad$ **Ans.**

Ans:
$F = 1.60$ lb

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*13–88.

Rod OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 5$ rad/s. The double collar B is pin-connected together such that one collar slides over the rotating rod and the other slides over the *horizontal* curved rod, of which the shape is described by the equation $r = 1.5(2 - \cos\theta)$ ft. If both collars weigh 0.75 lb, determine the normal force which the curved rod exerts on one collar at the instant $\theta = 120°$. Neglect friction.

SOLUTION

Kinematic: Here, $\dot{\theta} = 5$ rad/s and $\ddot{\theta} = 0$. Taking the required time derivatives at $\theta = 120°$, we have

$r = 1.5(2 - \cos\theta)|_{\theta=120°} = 3.75$ ft

$\dot{r} = 1.5\sin\theta\dot{\theta}|_{\theta=120°} = 6.495$ ft/s

$\ddot{r} = 1.5(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2)|_{\theta=120°} = -18.75$ ft/s²

Applying Eqs. 12–29, we have

$a_r = \ddot{r} - r\dot{\theta}^2 = -18.75 - 3.75(5^2) = -112.5$ ft/s²

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 3.75(0) + 2(6.495)(5) = 64.952$ ft/s²

Equation of Motion: The angle ψ must be obtained first.

$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{1.5(2 - \cos\theta)}{1.5\sin\theta}\bigg|_{\theta=120°} = 2.8867 \qquad \psi = 70.89°$

Applying Eq. 13–9, we have

$\sum F_r = ma_r; \qquad -N\cos 19.11° = \dfrac{0.75}{32.2}(-112.5)$

$\qquad\qquad N = 2.773$ lb $= 2.77$ lb   **Ans.**

$\sum F_\theta = ma_\theta; \qquad F_{OA} + 2.773\sin 19.11° = \dfrac{0.75}{32.2}(64.952)$

$\qquad\qquad F_{OA} = 0.605$ lb
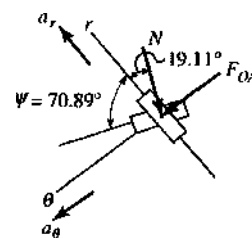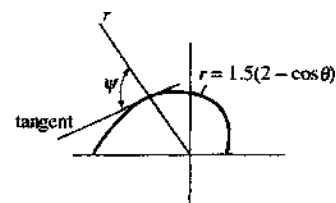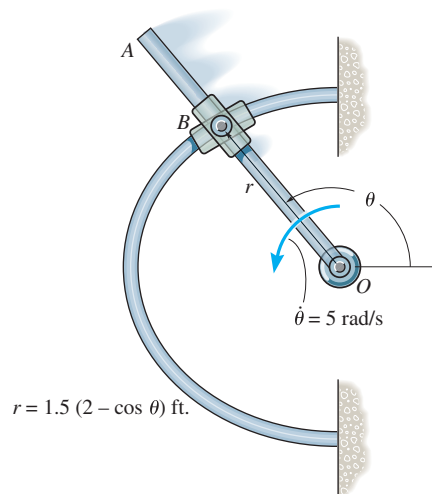
Ans:
$N = 2.77$ lb

13–89.

The boy of mass 40 kg is sliding down the spiral slide at a constant speed such that his position, measured from the top of the chute, has components $r = 1.5$ m, $\theta = (0.7t)$ rad, and $z = (-0.5t)$ m, where t is in seconds. Determine the components of force $\mathbf{F}_r$, $\mathbf{F}_\theta$, and $\mathbf{F}_z$ which the slide exerts on him at the instant $t = 2$ s. Neglect the size of the boy.
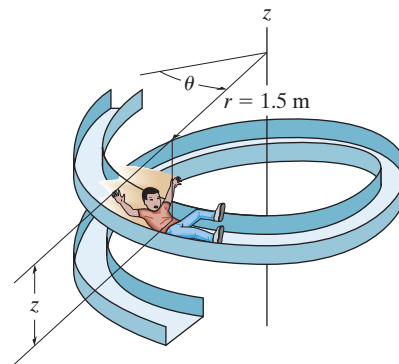
SOLUTION

$r = 1.5$ 　　　$\theta = 0.7t$ 　　　$z = -0.5t$

$\dot{r} = \ddot{r} = 0$ 　　　$\dot{\theta} = 0.7$ 　　　$\dot{z} = -0.5$

　　　　　　　$\ddot{\theta} = 0$ 　　　$\ddot{z} = 0$

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 1.5(0.7)^2 = -0.735$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$a_z = \ddot{z} = 0$

$\Sigma F_r = ma_r;$ 　　　$F_r = 40(-0.735) = -29.4$ N 　　　　　**Ans.**

$\Sigma F_\theta = ma_\theta;$ 　　　$F_\theta = 0$ 　　　　　　　　　　　　**Ans.**

$\Sigma F_z = ma_z;$ 　　　$F_z - 40(9.81) = 0$

　　　　　　　　　$F_z = 392$ N 　　　　　　　　　　　　**Ans.**
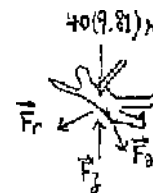
Ans:
$F_r = -29.4$ N
$F_\theta = 0$
$F_z = 392$ N

13–90.

The 40-kg boy is sliding down the smooth spiral slide such that $z = -2$ m/s and his speed is 2 m/s. Determine the r, θ, z components of force the slide exerts on him at this instant. Neglect the size of the boy.
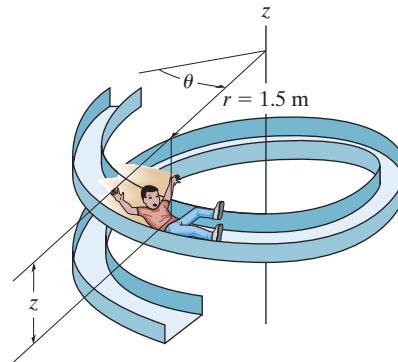
SOLUTION

$r = 1.5$ m

$\dot{r} = 0$

$\ddot{r} = 0$

$v_\theta = 2 \cos 11.98° = 1.9564$ m/s

$v_z = -2 \sin 11.98° = -0.41517$ m/s

$v_\theta = r\dot{\theta}; \quad 1.9564 = 1.5\,\dot{\theta}$

$\dot{\theta} = 1.3043$ rad/s

$\Sigma F_r = ma_r; \qquad -F_r = 40(0 - 1.5(1.3043)^2)$

$\qquad\qquad\qquad F_r = 102$ N **Ans.**

$\Sigma F_\theta = ma_\theta; \qquad N_b \sin 11.98° = 40(a_\theta)$

$\Sigma F_z = ma_z; \qquad -N_b \cos 11.98° + 40(9.81) = 40a_z$

Require $\tan 11.98° = \dfrac{a_z}{a_\theta}, \qquad a_\theta = 4.7123a_z$

Thus,

$a_z = 0.423$ m/s²

$a_\theta = 1.99$ m/s²

$N_b = 383.85$ N

$N_z = 383.85 \cos 11.98° = 375$ N **Ans.**
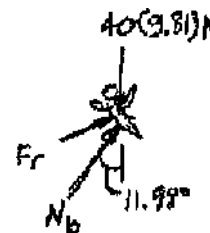
$N_\theta = 383.85 \sin 11.98° = 79.7$ N **Ans.**
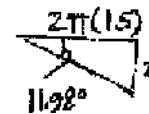
Ans:
$F_r = 102$ N
$F_z = 375$ N
$F_\theta = 79.7$ N

13–91.

Using a forked rod, a 0.5-kg smooth peg P is forced to move along the *vertical slotted* path $r = (0.5\theta)$ m, where θ is in radians. If the angular position of the arm is $\theta = (\frac{\pi}{8}t^2)$ rad, where t is in seconds, determine the force of the rod on the peg and the normal force of the slot on the peg at the instant $t = 2$ s. The peg is in contact with only *one edge* of the rod and slot at any instant.
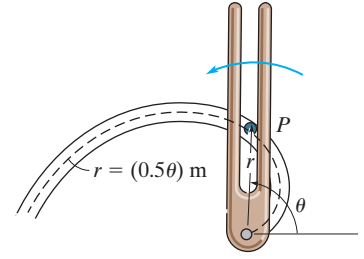
SOLUTION

Equation of Motion. Here, $r = 0.5\theta$. Then $\dfrac{dr}{d\theta} = 0.5$. The angle ψ between the extended radial line and the tangent can be determined from

$$\tan\psi = \frac{r}{dr/d\theta} = \frac{0.5\theta}{0.5} = \theta$$

At the instant $t = 25$, $\theta = \dfrac{\pi}{8}(2^2) = \dfrac{\pi}{2}$ rad

$$\tan\psi = \frac{\pi}{2} \qquad \psi = 57.52°$$

The positive sign indicates that ψ is measured from extended radial line in positive sense of θ (counter clockwise) to the tangent. Then the FBD of the peg shown in Fig. *a* can be drawn.

$$\Sigma F_r = ma_r; \quad N\sin 57.52° - 0.5(9.81) = 0.5a_r \qquad (1)$$
$$\Sigma F_\theta = ma_\theta; \quad F - N\cos 57.52° = 0.5a_\theta \qquad (2)$$

Kinematics. Using the chain rule, the first and second derivatives of r and θ with respect to t are

$$r = 0.5\theta = 0.5\left(\frac{\pi}{8}t^2\right) = \frac{\pi}{16}t^2 \qquad \theta = \frac{\pi}{8}t^2$$

$$\dot{r} = \frac{\pi}{8}t \qquad\qquad \dot{\theta} = \frac{\pi}{4}t$$

$$\ddot{r} = \frac{\pi}{8} \qquad\qquad \ddot{\theta} = \frac{\pi}{4}$$

When $t = 2$ s,

$$r = \frac{\pi}{16}(2^2) = \frac{\pi}{4}\text{ m} \qquad \theta = \frac{\pi}{8}(2^2) = \frac{\pi}{2}\text{ rad}$$

$$\dot{r} = \frac{\pi}{8}(2) = \frac{\pi}{4}\text{ m/s} \qquad \dot{\theta} = \frac{\pi}{4}(2) = \frac{\pi}{2}\text{ rad/s}$$

$$\ddot{r} = \frac{\pi}{8}\text{ m/s}^2 \qquad \ddot{\theta} = \frac{\pi}{4}\text{ rad/s}^2$$

Thus,

$$a_r = \ddot{r} - r\dot{\theta}^2 = \frac{\pi}{8} - \frac{\pi}{4}\left(\frac{\pi}{2}\right)^2 = -1.5452\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{\pi}{4}\left(\frac{\pi}{4}\right) + 2\left(\frac{\pi}{4}\right)\left(\frac{\pi}{2}\right) = 3.0843\text{ m/s}^2$$

Substitute these results in Eqs. (1) and (2)

$$N = 4.8987\text{ N} = 4.90\text{ N} \qquad\qquad \textbf{Ans.}$$
$$F = 4.173\text{ N} = 4.17\text{ N} \qquad\qquad \textbf{Ans.}$$

Ans:
$N = 4.90$ N
$F = 4.17$ N
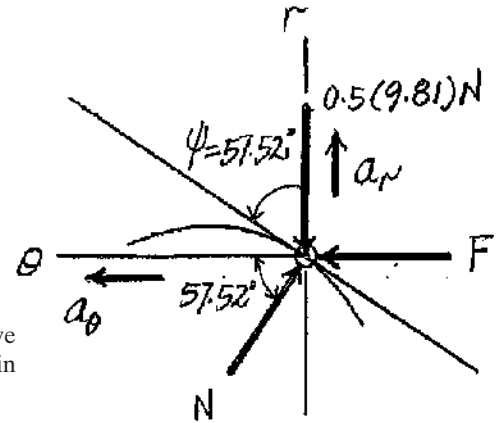
*13–92.

The arm is rotating at a rate of $\dot{\theta} = 4$ rad/s when $\ddot{\theta} = 3$ rad/s² and $\theta = 180°$. Determine the force it must exert on the 0.5-kg smooth cylinder if it is confined to move along the slotted path. Motion occurs in the horizontal plane.

$\dot{\theta} = 4$ rad/s, $\ddot{\theta} = 3$ rad/s²

$\theta = 180°$

$r = \left(\dfrac{2}{\theta}\right)$ m
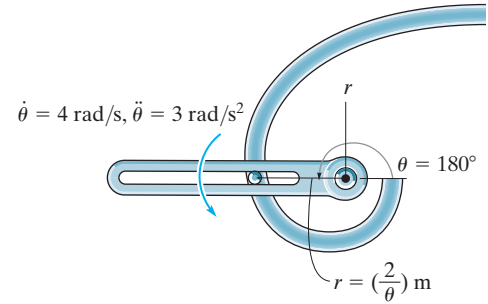
SOLUTION

Equation of Motion. Here, $r = \dfrac{2}{\theta}$. Then $\dfrac{dr}{d\theta} = -\dfrac{2}{\theta^2}$. The angle ψ between the extended radial line and the tangent can be determined from

$$\tan \psi = \frac{r}{dr/d\theta} = \frac{2/\theta}{-2/\theta^2} = -\theta$$

At $\theta = 180° = \pi$ rad,

$$\tan \psi = -\pi \qquad \psi = -72.34°$$

The negative sign indicates that ψ is measured from extended radial line in the negative sense of θ (clockwise) to the tangent. Then, the FBD of the peg shown in Fig. *a* can be drawn.

$$\Sigma F_r = ma_r; \qquad -N \sin 72.34° = 0.5a_r \qquad (1)$$
$$\Sigma F_\theta = ma_\theta; \qquad F - N \cos 72.34° = 0.5a_\theta \qquad (2)$$

Kinematics. Using the chain rule, the first and second time derivatives of r are

$$r = 2\theta^{-1}$$

$$\dot{r} = -2\theta^{-2}\dot{\theta} = -\left(\frac{2}{\theta^2}\right)\dot{\theta}$$

$$\ddot{r} = -2(-2\theta^{-3}\dot{\theta}^2 + \theta^{-2}\ddot{\theta}) = \frac{2}{\theta^3}(2\dot{\theta}^2 - \theta\ddot{\theta})$$

When $\theta = 180° = \pi$ rad, $\dot{\theta} = 4$ rad/s and $\ddot{\theta} = 3$ rad/s². Thus

$$r = \frac{2}{\pi} \text{ m} = 0.6366 \text{ m}$$

$$\dot{r} = -\left(\frac{2}{\pi^2}\right)(4) = -0.8106 \text{ m/s}$$

$$\ddot{r} = \frac{2}{\pi^3}\left[2(4^2) - \pi(3)\right] = 1.4562 \text{ m/s}^2$$

Thus,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 1.4562 - 0.6366(4^2) = -8.7297 \text{ m/s}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6366(3) + 2(-0.8106)(4) = -4.5747 \text{ m/s}^2$$

Substitute these result into Eqs. (1) and (2),

$$N = 4.5807 \text{ N}$$
$$F = -0.8980 \text{ N} = -0.898 \text{ N} \qquad\qquad \textbf{Ans.}$$

The negative sign indicates that **F** acts in the sense opposite to that shown in the FBD.

(a)
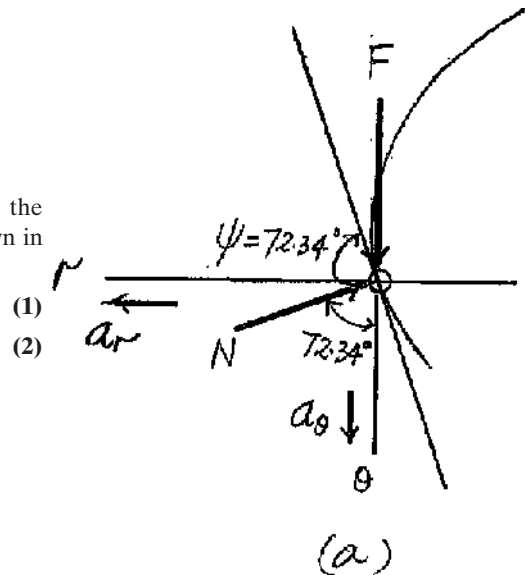
13–93.

If arm OA rotates with a constant clockwise angular velocity of $\dot{\theta} = 1.5$ rad/s. determine the force arm OA exerts on the smooth 4-lb cylinder B when $\theta = 45°$.
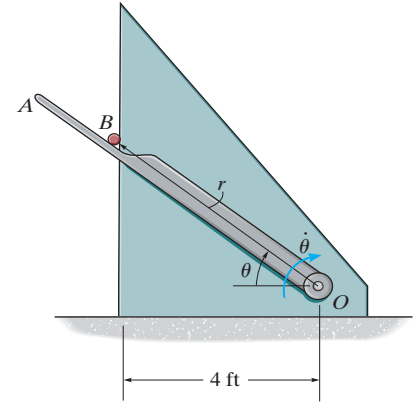
SOLUTION

Kinematics: Since the motion of cylinder B is known, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ will be determined first. Here, $\dfrac{4}{r} = \cos\theta$ or $r = 4 \sec\theta$ ft. The value of r and its time derivatives at the instant $\theta = 45°$ are

$$r = 4\sec\theta|_{\theta=45°} = 4\sec 45° = 5.657 \text{ ft}$$

$$\dot{r} = 4\sec\theta(\tan\theta)\dot{\theta}|_{\theta=45°} = 4\sec 45°\tan 45°(1.5) = 8.485 \text{ ft/s}$$

$$\ddot{r} = 4\left[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}\left(\sec\theta\,\sec^2\theta\dot{\theta} + \tan\theta\sec\theta\tan\theta\dot{\theta}\right)\right]$$

$$= 4\left[\sec\theta(\tan\theta)\ddot{\theta} + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan^2\theta\dot{\theta}^2\right]\Big|_{\theta=45°}$$

$$= 4\left[\sec 45°\tan 45°(0) + \sec^3 45°(1.5)^2 + \sec 45°\tan^2 45°(1.5)^2\right]$$

$$= 38.18 \text{ ft/s}^2$$

Using the above time derivatives,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 38.18 - 5.657(1.5^2) = 25.46 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} - 2\dot{r}\dot{\theta} = 5.657(0) + 2(8.485)(1.5) = 25.46 \text{ ft/s}^2$$

Equations of Motion: By referring to the free-body diagram of the cylinder shown in Fig. a,

$$\Sigma F_r = ma_r; \qquad N\cos 45° - 4\cos 45° = \frac{4}{32.2}(25.46)$$

$$N = 8.472 \text{ lb}$$

$$\Sigma F_\theta = ma_\theta; \qquad F_{OA} - 8.472\sin 45° - 4\sin 45° = \frac{4}{32.2}(25.46)$$

$$F_{OA} = 12.0 \text{ lb} \qquad\qquad\qquad\qquad \textbf{Ans.}$$

(a)
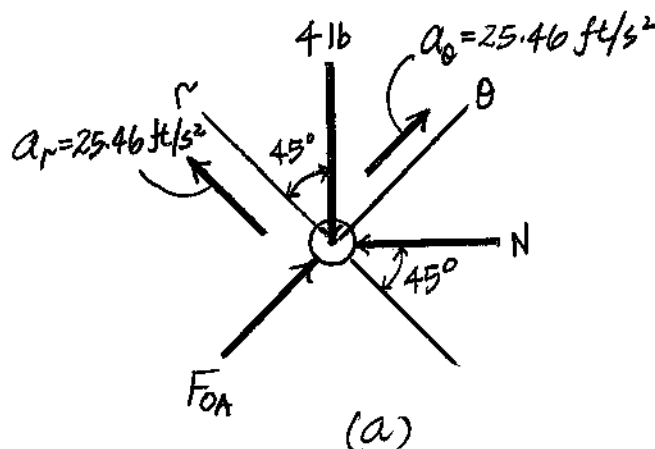
Ans:
$F_{OA} = 12.0$ lb

13–94.

Determine the normal and frictional driving forces that the partial spiral track exerts on the 200-kg motorcycle at the instant $\theta = \frac{5}{3}\pi$ rad, $\dot{\theta} = 0.4$ rad/s, and $\ddot{\theta} = 0.8$ rad/s². Neglect the size of the motorcycle.

$r = (5\theta)$ m
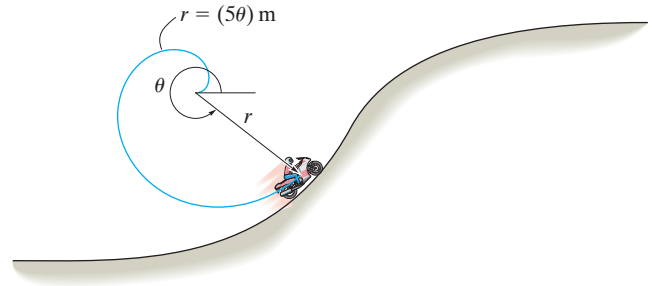
SOLUTION

$\theta = \left(\frac{5}{3}\pi\right) = 300°$ $\dot{\theta} = 0.4$ $\ddot{\theta} = 0.8$

$r = 50 = 5\left(\frac{5}{3}\pi\right) = 26.18$

$\dot{r} = 5\dot{\theta} = 5(0.4) = 2$

$\ddot{r} = 5\ddot{\theta} = 5(0.8) = 4$

$a_r = \ddot{r} - r\dot{\theta}^2 = 4 - 26.18(0.4)^2 = -0.1888$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 26.18(0.8) + 2(2)(0.4) = 22.54$

$\tan \psi = \dfrac{r}{dr/d\theta} = \dfrac{5\left(\frac{5}{3}\pi\right)}{5} = 5.236$ $\psi = 79.19°$

$+\searrow \Sigma F_r = ma_r;$ $F \sin 10.81° - N \cos 10.81° + 200(9.81) \cos 30° = 200(-0.1888)$

$+\nearrow \Sigma F_\theta = ma_\theta;$ $F \cos 10.81° - 200(9.81) \sin 30° + N \sin 10.81° = 200(22.54)$

$F = 5.07$ kN   **Ans.**

$N = 2.74$ kN   **Ans.**
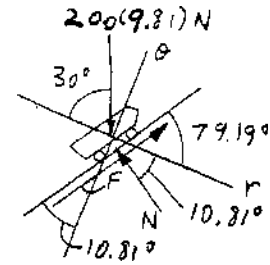
13–95.

A smooth can C, having a mass of 3 kg, is lifted from a feed at A to a ramp at B by a rotating rod. If the rod maintains a constant angular velocity of $\dot\theta = 0.5$ rad/s, determine the force which the rod exerts on the can at the instant $\theta = 30°$. Neglect the effects of friction in the calculation and the size of the can so that $r = (1.2 \cos \theta)$ m. The ramp from A to B is circular, having a radius of 600 mm.
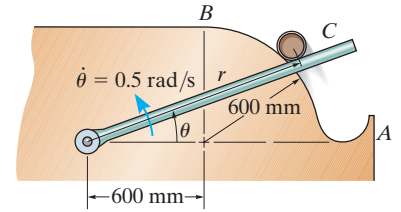
SOLUTION

$r = 2(0.6 \cos \theta) = 1.2 \; \cos \theta$

$\dot r = -1.2 \sin \theta \dot\theta$

$\ddot r = -1.2 \cos \theta \dot\theta^2 - 1.2 \sin \theta \ddot\theta$

At $\theta = 30°, \dot\theta = 0.5$ rad/s and $\ddot\theta = 0$

$r = 1.2 \cos 30° = 1.0392$ m

$\dot r = -1.2 \sin 30°(0.5) = -0.3$ m/s

$\ddot r = -1.2 \cos 30°(0.5)^2 - 1.2 \sin 30°(0) = -0.2598$ m/s^2

$a_r = \ddot r - r\dot\theta^2 = -0.2598 - 1.0392(0.5)^2 = -0.5196$ m/s^2

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 1.0392(0) + 2(-0.3)(0.5) = -0.3$ m/s^2

$+\nearrow \Sigma F_r = ma_r; \qquad N \cos 30° - 3(9.81) \sin 30° = 3(-0.5196) \qquad N = 15.19$ N

$+\nwarrow \Sigma F_\theta = ma_\theta; \qquad F + 15.19 \sin 30° - 3(9.81) \cos 30° = 3(-0.3)$

$$F = 17.0 \text{ N} \qquad\qquad \textbf{Ans.}$$
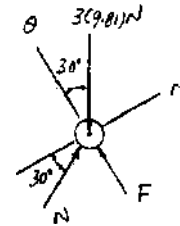
*13–96.

The spring-held follower AB has a mass of 0.5 kg and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.15$ m and $z = (0.02 \cos 2\theta)$ m. If the cam is rotating at a constant rate of 30 rad/s, determine the force component F_z at the end A of the follower when $\theta = 30°$. The spring is uncompressed when $\theta = 90°$. Neglect friction at the bearing C.

$z = (0.02 \cos 2\theta)$ m

z

0.15 m

$\dot{\theta} = 30$ rad/s

C

A

B

$k = 1000$ N/m

SOLUTION

Kinematics. Using the chain rule, the first and second time derivatives of z are

$$z = (0.02 \cos 2\theta) \text{ m}$$

$$\dot{z} = 0.02[-\sin 2\theta(2\dot{\theta})] = [-0.04(\sin 2\theta)\dot{\theta}] \text{ m/s}$$

$$\ddot{z} = -0.04[\cos 2\theta(2\dot{\theta})\dot{\theta} + (\sin 2\theta)\ddot{\theta}] = [-0.04(2 \cos 2\theta(\dot{\theta})^2 + \sin 2\theta(\ddot{\theta}))] \text{ m/s}^2$$

Here, $\dot{\theta} = 30$ rad/s and $\ddot{\theta} = 0$. Then

$$\ddot{z} = -0.04[2 \cos 2\theta(30^2) + \sin 2\theta(0)] = (-72 \cos 2\theta) \text{ m/s}^2$$

N

F_{sp}

z

a_z

(a)

Equation of Motion. When $\theta = 30°$, the spring compresses $x = 0.02 + 0.02 \cos 2(30°) = 0.03$ m. Thus, $F_{sp} = kx = 1000(0.03) = 30$ N. Also, at this position $a_z = \ddot{z} = -72 \cos 2(30°) = -36.0$ m/s². Referring to the FBD of the follower, Fig. a,
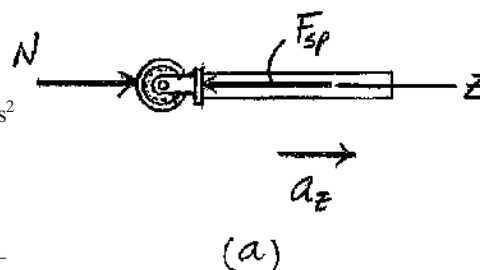
$$\Sigma F_z = ma_z; \quad N - 30 = 0.5(-36.0)$$

$$N = 12.0 \text{ N} \quad\quad\quad \textbf{Ans.}$$

Ans:
$N = 12.0$ N

13–97.

The spring-held follower AB has a mass of 0.5 kg and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.15$ m and $z = (0.02 \cos 2\theta)$ m. If the cam is rotating at a constant rate of 30 rad/s, determine the maximum and minimum force components F_z the follower exerts on the cam if the spring is uncompressed when $\theta = 90°$.
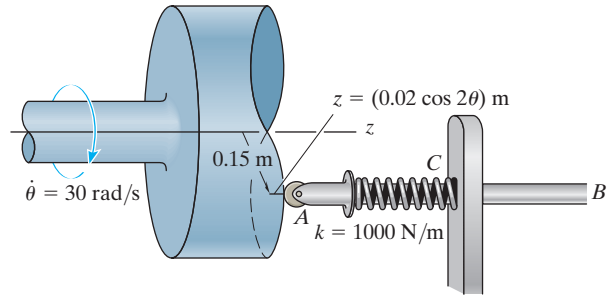
SOLUTION

Kinematics. Using the chain rule, the first and second time derivatives of z are

$z = (0.02 \cos 2\theta)$ m

$\dot{z} = 0.02[-\sin 2\theta(2\dot{\theta})] = (-0.04 \sin 2\theta \dot{\theta})$ m/s

$\ddot{z} = -0.04[\cos 2\theta(2\dot{\theta})\dot{\theta} + \sin 2\theta\ddot{\theta}] = [-0.04(2 \cos 2\theta(\dot{\theta})^2 + \sin 2\theta(\ddot{\theta}))]$ m/s²

Here $\dot{\theta} = 30$ rad/s and $\ddot{\theta} = 0$. Then,

$\ddot{z} = -0.04[2 \cos 2\theta(30^2) + \sin 2\theta(0)] = (-72 \cos 2\theta)$ m/s²

Equation of Motion. At any arbitrary θ, the spring compresses $x = 0.02(1 + \cos 2\theta)$. Thus, $F_{sp} = kx = 1000[0.02(1 + \cos 2\theta)] = 20(1 + \cos 2\theta)$. Referring to the FBD of the follower, Fig. *a*,
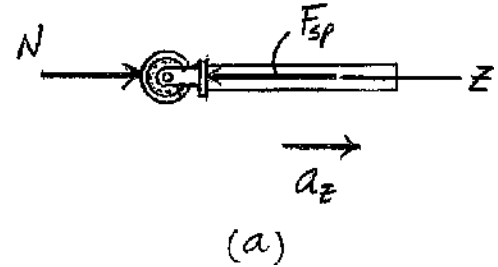
$\Sigma F_z = ma_z;\quad N - 20(1 + \cos 2\theta) = 0.5(-72 \cos 2\theta)$

$N = (20 - 16 \cos 2\theta)$ N

N is maximum when $\cos 2\theta = -1$. Then

$(N)_{max} = 36.0$ N **Ans.**

N is minimum when $\cos 2\theta = 1$. Then

$(N)_{min} = 4.00$ N **Ans.**

Ans:
$(N)_{max} = 36.0$ N
$(N)_{min} = 4.00$ N

13–98.

The particle has a mass of 0.5 kg and is confined to move along the smooth vertical slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = 30°$. The rod is rotating with a constant angular velocity $\dot\theta = 2$ rad/s. Assume the particle contacts only one side of the slot at any instant.
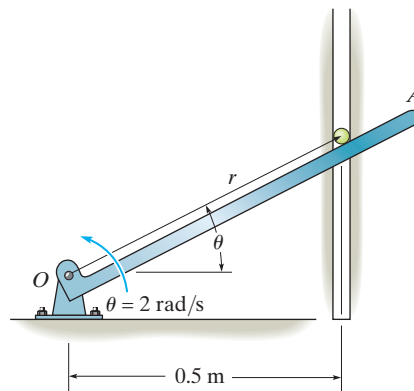
SOLUTION

$r = \dfrac{0.5}{\cos\theta} = 0.5\sec\theta, \qquad \dot r = 0.5\sec\theta\tan\theta\,\dot\theta$

$\ddot r = 0.5\sec\theta\tan\theta\,\ddot\theta + 0.5\sec^3\theta\,\dot\theta^2 + 0.5\sec\theta\tan^2\theta\,\dot\theta^2$

At $\theta = 30°$.

$\dot\theta = 2$ rad/s

$\ddot\theta = 0$

$r = 0.5774$ m

$\dot r = 0.6667$ m/s

$\ddot r = 3.8490$ m/s^2

$a_r = \ddot r - r\dot\theta^2 = 3.8490 - 0.5774(2)^2 = 1.5396$ m/s^2

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 + 2(0.6667)(2) = 2.667$ m/s^2

$+\nearrow\Sigma F_r = ma_r; \qquad N_P\cos 30° - 0.5(9.81)\sin 30° = 0.5(1.5396)$

$\qquad\qquad N_P = 3.7208 = 3.72$ N   **Ans.**

$+\nwarrow\Sigma F_\theta = ma_\theta; \qquad F - 3.7208\sin 30° - 0.5(9.81)\cos 30° = 0.5(2.667)$

$\qquad\qquad F = 7.44$ N   **Ans.**
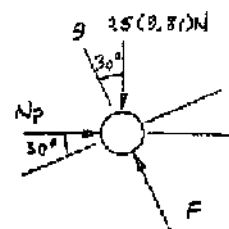
Ans:
$N_s = 3.72$ N
$F_r = 7.44$ N

13–99.

A car of a roller coaster travels along a track which for a short distance is defined by a conical spiral, $r = \frac{3}{4}z$, $\theta = -1.5z$, where r and z are in meters and θ in radians. If the angular motion $\dot{\theta} = 1$ rad/s is always maintained, determine the r, θ, z components of reaction exerted on the car by the track at the instant $z = 6$ m. The car and passengers have a total mass of 200 kg.
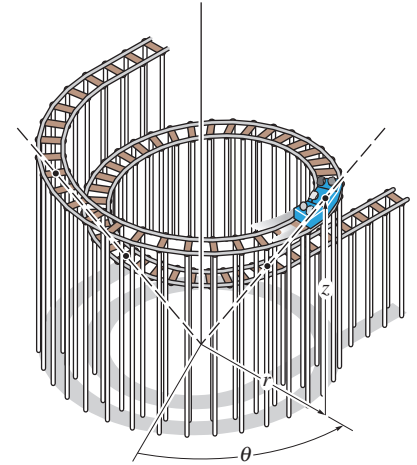
SOLUTION

$$r = 0.75z \qquad \dot{r} = 0.75\dot{z} \qquad \ddot{r} = 0.75\ddot{z}$$

$$\theta = -1.5z \qquad \dot{\theta} = -1.5\dot{z} \qquad \ddot{\theta} = -1.5\ddot{z}$$

$$\dot{\theta} = 1 = -1.5\dot{z} \qquad \dot{z} = -0.6667 \text{ m/s} \qquad \ddot{z} = 0$$

At $z = 6$ m,

$r = 0.75(6) = 4.5$ m $\dot{r} = 0.75(-0.6667) = -0.5$ m/s $\ddot{r} = 0.75(0) = 0$ $\ddot{\theta} = 0$

$a_t = \ddot{r} - r\dot{\theta}^2 = 0 - 4.5(1)^2 = -4.5$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.5(0) + 2(-0.5)(1) = -1$ m/s^2

$a_z = \ddot{z} = 0$

$\Sigma F_t = ma_r;$ $F_r = 200(-4.5)$ $F_r = -900$ N   **Ans.**

$\Sigma F_\theta = ma_\theta;$ $F_\theta = 200(-1)$ $F_\theta = -200$N   **Ans.**

$\Sigma F_z = ma_z;$ $F_z - 200(9.81) = 0$ $F_z = 1962$ N $= 1.96$ kN   **Ans.**
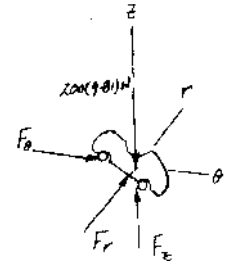
Ans:
$F_r = -900$ N
$F_\theta = -200$ N
$F_z = 1.96$ kN

*13–100.

The 0.5-lb ball is guided along the vertical circular path $r = 2r_c \cos \theta$ using the arm OA. If the arm has an angular velocity $\dot{\theta} = 0.4$ rad/s and an angular acceleration $\ddot{\theta} = 0.8$ rad/s^2 at the instant $\theta = 30°$, determine the force of the arm on the ball. Neglect friction and the size of the ball. Set $r_c = 0.4$ ft.
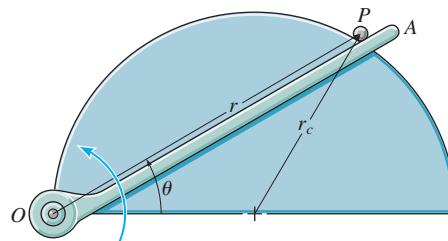
SOLUTION

$r = 2(0.4) \cos \theta = 0.8 \cos \theta$

$\dot{r} = -0.8 \sin \theta \dot{\theta}$

$\ddot{r} = -0.8 \cos \theta \dot{\theta}^2 - 0.8 \sin \theta \ddot{\theta}$

At $\theta = 30°, \dot{\theta} = 0.4$ rad/s, and $\ddot{\theta} = 0.8$ rad/s^2

$r = 0.8 \cos 30° = 0.6928$ ft

$\quad \dot{r} = -0.8 \sin 30°(0.4) = -0.16$ ft/s

$\quad \ddot{r} = -0.8 \cos 30°(0.4)^2 - 0.8 \sin 30°(0.8) = -0.4309$ ft/s^2

$\quad a_r = \ddot{r} - r\dot{\theta}^2 = -0.4309 - 0.6928(0.4)^2 = -0.5417$ ft/s^2

$\quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6928(0.8) + 2(-0.16)(0.4) = 0.4263$ ft/s^2

$+\nearrow \Sigma F_r = ma_r; \quad N \cos 30° - 0.5 \sin 30° = \dfrac{0.5}{32.2}(-0.5417) \quad N = 0.2790$ lb

$\nwarrow + \Sigma F_\theta = ma_\theta; \quad F_{OA} + 0.2790 \sin 30° - 0.5 \cos 30° = \dfrac{0.5}{32.2}(0.4263)$

$\quad\quad\quad F_{OA} = 0.300$ lb **Ans.**
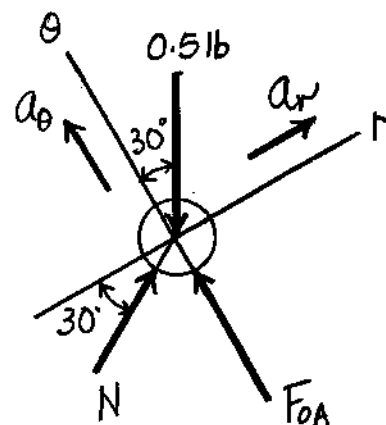
Ans:
$F_{OA} = 0.300$ lb

13–101.

The ball of mass m is guided along the vertical circular path $r = 2r_c \cos \theta$ using the arm OA. If the arm has a constant angular velocity $\dot{\theta}_0$, determine the angle $\theta \leq 45°$ at which the ball starts to leave the surface of the semicylinder. Neglect friction and the size of the ball.
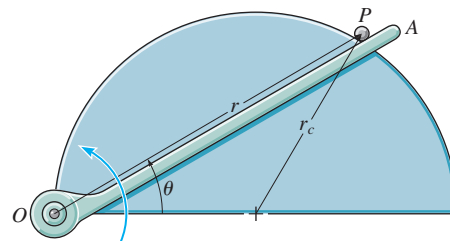
SOLUTION

$r = 2r_c \cos \theta$

$\dot{r} = -2r_c \sin \theta \dot{\theta}$

$\ddot{r} = -2r_c \cos \theta \dot{\theta}^2 - 2r_c \sin \theta \ddot{\theta}$

Since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$.

$a_r = \ddot{r} - r\dot{\theta}^2 = -2r_c \cos \theta \dot{\theta}_0^2 - 2r_c \cos \theta \dot{\theta}_0^2 = -4r_c \cos \theta \dot{\theta}_0^2$

$+\nearrow \Sigma F_r = ma_r; \qquad -mg \sin \theta = m(-4r_c \cos \theta \dot{\theta}_0^2)$

$$\tan \theta = \frac{4r_c \dot{\theta}_0^2}{g} \qquad \theta = \tan^{-1}\left(\frac{4r_c \dot{\theta}_0^2}{g}\right) \qquad \textbf{Ans.}$$
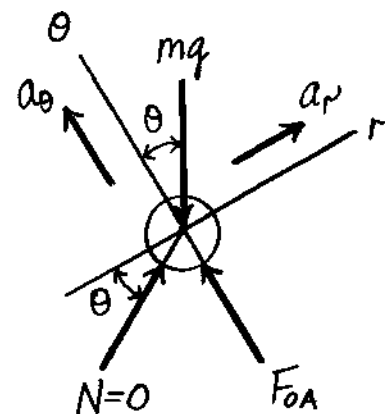
Ans:
$$\theta = \tan^{-1}\left(\frac{4r_c \dot{\theta}_0^2}{g}\right)$$

13–102.

Using a forked rod, a smooth cylinder P, having a mass of 0.4 kg, is forced to move along the *vertical slotted* path $r = (0.6\theta)$ m, where θ is in radians. If the cylinder has a constant speed of $v_C = 2$ m/s, determine the force of the rod and the normal force of the slot on the cylinder at the instant $\theta = \pi$ rad. Assume the cylinder is in contact with only *one edge* of the rod and slot at any instant. *Hint:* To obtain the time derivatives necessary to compute the cylinder's acceleration components a_r and a_θ, take the first and second time derivatives of $r = 0.6\theta$. Then, for further information, use Eq. 12–26 to determine $\dot\theta$. Also, take the time derivative of Eq. 12–26, noting that $\dot v_C = 0$, to determine $\ddot\theta$.
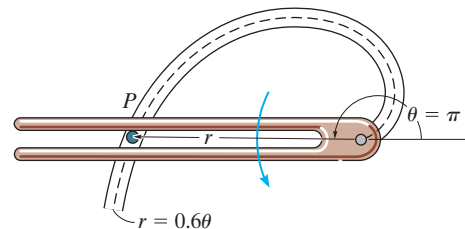
SOLUTION

$r = 0.6\theta \qquad \dot r = 0.6\dot\theta \qquad \ddot r = 0.6\ddot\theta$

$v_r = \dot r = 0.6\dot\theta \qquad v_\theta = r\dot\theta = 0.6\theta\dot\theta$

$v^2 = \dot r^2 + \left(r\dot\theta\right)^2$

$2^2 = \left(0.6\dot\theta\right)^2 + \left(0.6\theta\dot\theta\right)^2 \qquad \dot\theta = \dfrac{2}{0.6\sqrt{1 + \theta^2}}$

$0 = 0.72\dot\theta\ddot\theta + 0.36\left(2\theta\dot\theta^3 + 2\theta^2\dot\theta\ddot\theta\right) \qquad \ddot\theta = -\dfrac{\theta\dot\theta^2}{1 + \theta^2}$

At $\theta = \pi$ rad, $\quad \dot\theta = \dfrac{2}{0.6\sqrt{1 + \pi^2}} = 1.011$ rad/s

$\ddot\theta = -\dfrac{(\pi)(1.011)^2}{1 + \pi^2} = -0.2954$ rad/s^2

$r = 0.6(\pi) = 0.6\,\pi$ m $\qquad \dot r = 0.6(1.011) = 0.6066$ m/s

$\ddot r = 0.6(-0.2954) = -0.1772$ m/s^2

$a_r = \ddot r - r\dot\theta^2 = -0.1772 - 0.6\,\pi(1.011)^2 = -2.104$ m/s^2

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0.6\pi(-0.2954) + 2(0.6066)(1.011) = 0.6698$ m/s^2

$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{0.6\theta}{0.6} = \theta = \pi \qquad \psi = 72.34°$

$\xleftrightarrow{} \Sigma F_r = ma_r; \qquad -N\cos 17.66° = 0.4(-2.104) \qquad N = 0.883$ N $\qquad$ **Ans.**

$+\downarrow \Sigma F_\theta = ma_\theta; \qquad -F + 0.4(9.81) + 0.883\sin 17.66° = 0.4(0.6698)$

$\qquad\qquad\qquad F = 3.92$ N $\qquad$ **Ans.**
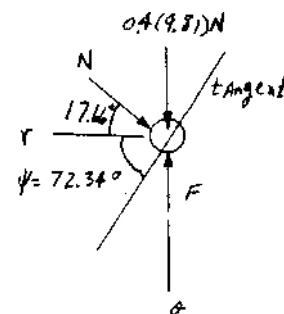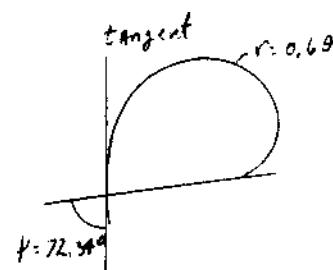
Ans:
$N = 0.883$ N
$F = 3.92$ N

13–103.

The pilot of the airplane executes a vertical loop which in part follows the path of a cardioid, $r = 200(1 + \cos\theta)$ m, where θ is in radians. If his speed at A is a constant $v_p = 85$ m/s, determine the vertical reaction the seat of the plane exerts on the pilot when the plane is at A. He has a mass of 80 kg. *Hint:* To determine the time derivatives necessary to calculate the acceleration components a_r and a_θ, take the first and second time derivatives of $r = 200(1 + \cos\theta)$. Then, for further information, use Eq. 12–26 to determine $\dot{\theta}$.
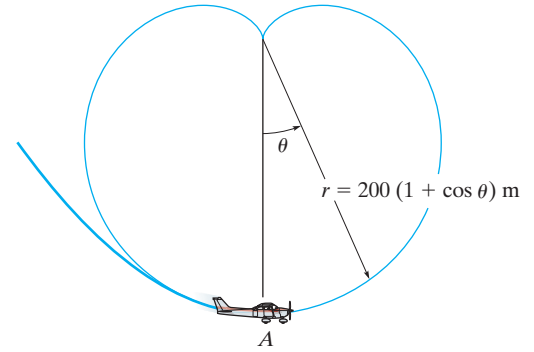
$r = 200 (1 + \cos \theta)$ m

SOLUTION

Kinematic. Using the chain rule, the first and second time derivatives of r are

$$r = 200(1 + \cos\theta)$$

$$\dot{r} = 200(-\sin\theta)(\dot{\theta}) = -200(\sin\theta)\dot{\theta}$$

$$\ddot{r} = -200[(\cos\theta)(\dot{\theta})^2 + (\sin\theta)(\ddot{\theta})]$$

When $\theta = 0°$,

$$r = 200(1 + \cos 0°) = 400 \text{ m}$$

$$\dot{r} = -200(\sin 0°)\,\dot{\theta} = 0$$

$$\ddot{r} = -200\big[(\cos 0°)(\dot{\theta})^2 + (\sin 0°)(\ddot{\theta})\big] = -200\dot{\theta}^2$$

Using Eq. 12–26

$$v = \sqrt{\dot{r}^2 + (r\dot{\theta})^2}$$

$$v^2 = \dot{r}^2 + (r\dot{\theta})^2$$

$$85^2 = 0^2 + (400\dot{\theta})^2$$

$$\dot{\theta} = 0.2125 \text{ rad/s}$$

Thus,

$$a_r = \ddot{r} - r\dot{\theta}^2 = -200(0.2125^2) - 400(0.2125^2) = -27.09 \text{ m/s}^2$$

Equation of Motion. Referring to the FBD of the pilot, Fig. *a*,

$$\downarrow + \Sigma F_r = ma_r; \qquad 80(9.81) - N = 80(-27.09)$$

$$N = 2952.3 \text{ N} = 2.95 \text{ kN} \qquad\qquad \textbf{Ans.}$$

$80(9.81)N$

$a_r \downarrow \quad \uparrow N$

(a)
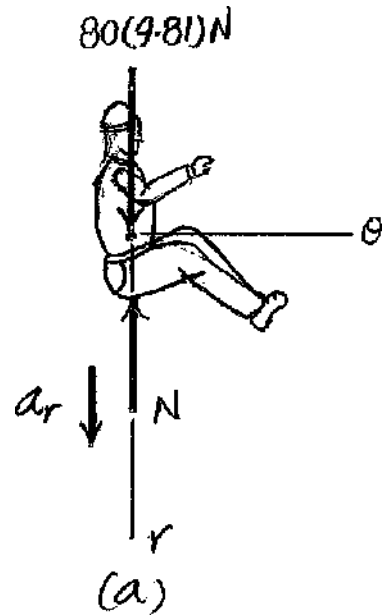
*13–104.

The collar has a mass of 2 kg and travels along the smooth horizontal rod defined by the equiangular spiral $r = (e^\theta)$ m, where θ is in radians. Determine the tangential force F and the normal force N acting on the collar when $\theta = 45°$, if the force F maintains a constant angular motion $\dot\theta = 2$ rad/s.
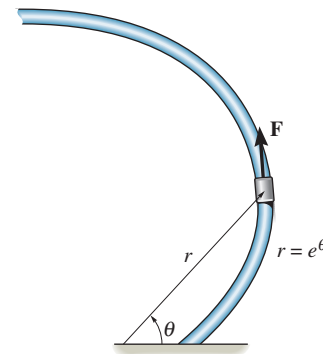
SOLUTION

$r = e^\theta$

$\dot r = e^\theta \dot\theta$

$\ddot r = e^\theta(\dot\theta)^2 + e^\theta\ddot\theta$

At $\theta = 45°$

$\dot\theta = 2$ rad/s

$\ddot\theta = 0$

$r = 2.1933$

$\dot r = 4.38656$

$\ddot r = 8.7731$

$a_r = \ddot r - r(\dot\theta)^2 = 8.7731 - 2.1933(2)^2 = 0$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 + 2(4.38656)(2) = 17.5462 \text{ m/s}^2$

$\tan\psi = \dfrac{r}{\left(\dfrac{dr}{d\theta}\right)} = e^\theta/e^\theta = 1$

$\psi = \theta = 45°$

$\nearrow + \sum F_r = ma_r;\qquad -N_C\cos 45° + F\cos 45° = 2(0)$

$+\nwarrow \sum F_\theta = ma_\theta;\qquad F\sin 45° + N_C\sin 45° = 2(17.5462)$

$N = 24.8 \text{ N}$ **Ans.**

$F = 24.8 \text{ N}$ **Ans.**
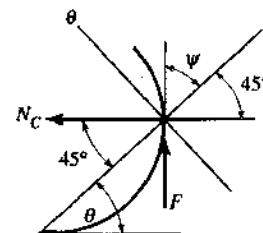
Ans:
$N = 24.8$ N
$F = 24.8$ N

13–105.

The particle has a mass of 0.5 kg and is confined to move along the smooth horizontal slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = 30°$. The rod is rotating with a constant angular velocity $\dot{\theta} = 2$ rad/s. Assume the particle contacts only one side of the slot at any instant.

SOLUTION

$r = \dfrac{0.5}{\cos \theta} = 0.5 \sec \theta$

$\dot{r} = 0.5 \sec \theta \tan \theta \dot{\theta}$
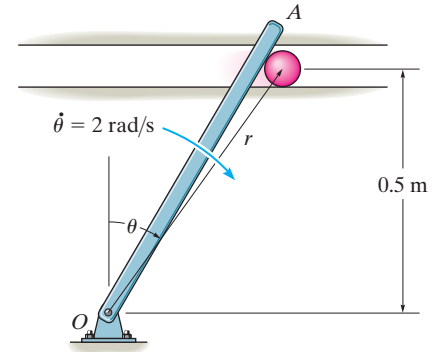
$\ddot{r} = 0.5 \left\{ \left[(\sec \theta \tan \theta \dot{\theta})\tan \theta + \sec \theta(\sec^2 \theta \dot{\theta}) \right]\dot{\theta} + \sec \theta \tan \theta \ddot{\theta} \right\}$

$\quad = 0.5[\sec \theta \tan^2 \theta \dot{\theta}^2 + \sec^3 \theta \dot{\theta}^2 + \sec \theta \tan \theta \ddot{\theta}]$

When $\theta = 30°, \dot{\theta} = 2$ rad/s and $\ddot{\theta} = 0$

$r = 0.5 \sec 30° = 0.5774$ m

$\dot{r} = 0.5 \sec 30° \tan 30°(2) = 0.6667$ m/s

$\ddot{r} = 0.5\left[\sec 30° \tan^2 30°(2)^2 + \sec^3 30°(2)^2 + \sec 30° \tan 30°(0) \right]$

$\quad = 3.849$ m/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = 3.849 - 0.5774(2)^2 = 1.540$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(0) + 2(0.6667)(2) = 2.667$ m/s^2

$\nearrow + \Sigma F_r = ma_r;$ $N \cos 30° - 0.5(9.81) \cos 30° = 0.5(1.540)$

$\qquad\qquad\qquad\qquad N = 5.79$ N   **Ans.**

$+\searrow \Sigma F_\theta = ma_\theta;$ $F + 0.5(9.81)\sin 30° - 5.79 \sin 30° = 0.5(2.667)$

$\qquad\qquad\qquad\qquad F = 1.78$ N   **Ans.**
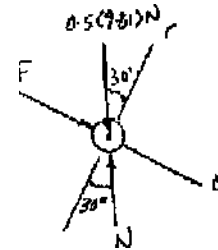
Ans:
$F_r = 1.78$ N
$N_s = 5.79$ N

13–106.

Solve Prob. 13–105 if the arm has an angular acceleration of $\ddot{\theta} = 3$ rad/s^2 when $\dot{\theta} = 2$ rad/s at $\theta = 30°$.

SOLUTION

$r = \dfrac{0.5}{\cos \theta} = 0.5 \sec \theta$

$\dot{r} = 0.5 \sec \theta \tan \theta \dot{\theta}$
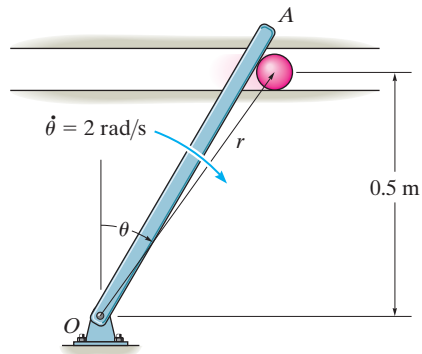
$\ddot{r} = 0.5 \left\{ \left[(\sec \theta \tan \theta \dot{\theta}) \tan \theta + \sec \theta (\sec^2 \theta \dot{\theta}) \right] \dot{\theta} + \sec \theta \tan \theta \ddot{\theta} \right\}$

$\quad = 0.5 \left[\sec \theta \tan^2 \theta \dot{\theta}^2 + \sec^3 \theta \dot{\theta}^2 + \sec \theta \tan \theta \ddot{\theta} \right]$

When $\theta = 30°$, $\dot{\theta} = 2$ rad/s and $\ddot{\theta} = 3$ rad/s^2

$r = 0.5 \sec 30° = 0.5774$ m

$\dot{r} = 0.5 \sec 30° \tan 30°(2) = 0.6667$ m/s

$\ddot{r} = 0.5 \left[\sec 30° \tan^2 30°(2)^2 + \sec^3 30°(2)^2 + \sec 30° \tan 30°(3) \right]$

$\quad = 4.849$ m/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = 4.849 - 0.5774(2)^2 = 2.5396$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(3) + 2(0.6667)(2) = 4.3987$ m/s^2

$\nearrow + \Sigma F_r = ma_r;$ $N \cos 30° - 0.5(9.81) \cos 30° = 0.5(2.5396)$

$\qquad\qquad\qquad N = 6.3712 = 6.37$ N   **Ans.**

$+ \searrow \Sigma F_\theta = ma_\theta;$ $F + 0.5(9.81) \sin 30° - 6.3712 \sin 30° = 0.5(4.3987)$

$\qquad\qquad\qquad F = 2.93$ N   **Ans.**
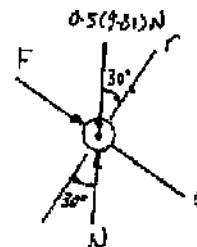
Ans:
$F_r = 2.93$ N
$N_s = 6.37$ N

13–107.

The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos \theta)$ ft. If $\theta = (0.5t^2)$ rad, where t is in seconds, determine the force which the rod exerts on the particle at the instant $t = 1$ s. The fork and path contact the particle on only one side.
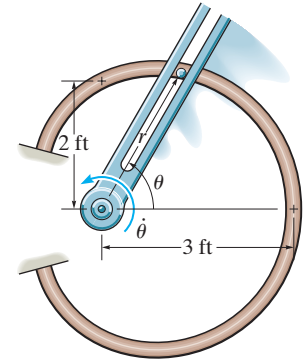
SOLUTION

$r = 2 + \cos \theta \qquad\qquad \theta = 0.5t^2$

$\dot{r} = -\sin \theta \dot{\theta} \qquad\qquad \dot{\theta} = t$

$\ddot{r} = -\cos \theta \dot{\theta}^2 - \sin \theta \ddot{\theta} \qquad \ddot{\theta} = 1 \text{ rad/s}^2$

At $t = 1$ s, $\theta = 0.5$ rad, $\dot{\theta} = 1$ rad/s, and $\ddot{\theta} = 1$ rad/s^2

$r = 2 + \cos 0.5 = 2.8776$ ft

$\dot{r} = -\sin 0.5(1) = -0.4974$ ft/s^2

$\ddot{r} = -\cos 0.5(1)^2 - \sin 0.5(1) = -1.357$ ft/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = -1.375 - 2.8776(1)^2 = -4.2346$ ft/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.8776(1) + 2(-0.4794)(1) = 1.9187$ ft/s^2

$\tan \psi = \dfrac{r}{dr/d\theta} = \dfrac{2 + \cos \theta}{-\sin \theta}\bigg|_{\theta=0.5 \text{ rad}} = -6.002 \qquad \psi = -80.54°$

$+\nearrow \Sigma F_r = ma_r; \qquad -N \cos 9.46° = \dfrac{2}{32.2}(-4.2346) \qquad N = 0.2666$ lb

$+\nwarrow \Sigma F_\theta = ma_\theta; \qquad F - 0.2666 \sin 9.46° = \dfrac{2}{32.2}(1.9187)$

$\qquad\qquad\qquad\qquad F = 0.163$ lb $\qquad\qquad$ **Ans.**
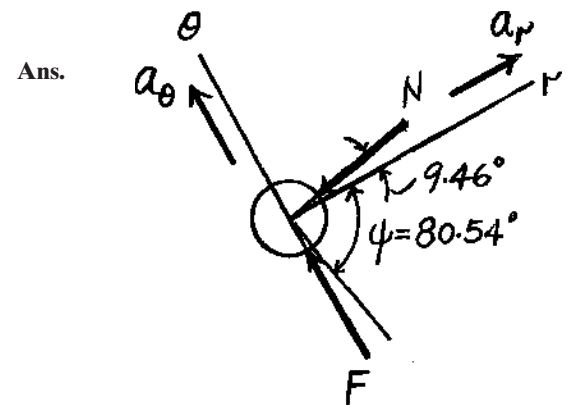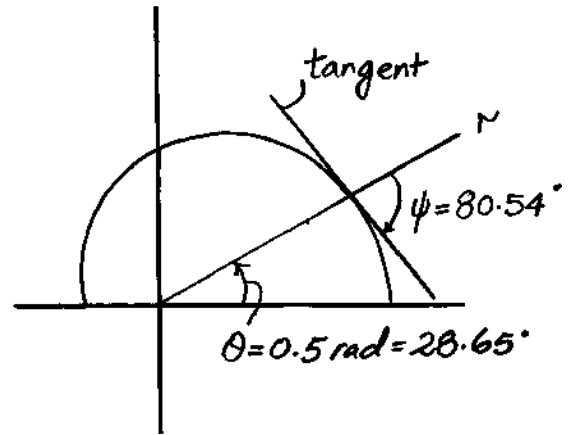
*13–108.

The collar, which has a weight of 3 lb, slides along the smooth rod lying in the *horizontal plane* and having the shape of a parabola $r = 4/(1 - \cos\theta)$, where θ is in radians and r is in feet. If the collar's angular rate is constant and equals $\dot\theta = 4$ rad/s, determine the tangential retarding force P needed to cause the motion and the normal force that the collar exerts on the rod at the instant $\theta = 90°$.
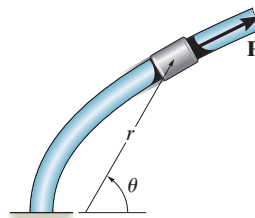
SOLUTION

$$r = \frac{4}{1 - \cos\theta}$$

$$\dot r = \frac{-4\sin\theta\,\dot\theta}{(1 - \cos\theta)^2}$$

$$\ddot r = \frac{-4\sin\theta\,\ddot\theta}{(1 - \cos\theta)^2} + \frac{-4\cos\theta\,(\dot\theta)^2}{(1 - \cos\theta)^2} + \frac{8\sin^2\theta\,\dot\theta^2}{(1 - \cos\theta)^3}$$

At $\theta = 90°$, $\qquad \dot\theta = 4$, $\qquad \ddot\theta = 0$

$$r = 4$$

$$\dot r = -16$$

$$\ddot r = 128$$

$$a_r = \ddot r - r(\dot\theta)^2 = 128 - 4(4)^2 = 64$$

$$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 + 2(-16)(4) = -128$$

$$r = \frac{4}{1 - \cos\theta}$$

$$\frac{dr}{d\theta} = \frac{-4\sin\theta}{(1 - \cos\theta)^2}$$

$$\tan\psi = \frac{r}{\left(\frac{dr}{d\theta}\right)} = \left.\frac{\frac{4}{1 - \cos\theta}}{\frac{-4\sin\theta}{(1 - \cos\theta)^2}}\right|_{\theta=90°} = \frac{4}{-4} = -1$$

$$\psi = -45° = 135°$$

$$+\uparrow \Sigma F_r = m\,a_r ; \qquad P\sin 45° - N\cos 45° = \frac{3}{32.2}(64)$$

$$\overset{+}{\leftarrow} \Sigma F_\theta = ma_\theta ; \qquad -P\cos 45° - N\sin 45° = \frac{3}{32.2}(-128)$$

Solving,

$$P = 12.6 \text{ lb} \qquad\qquad \textbf{Ans.}$$

$$N = 4.22 \text{ lb} \qquad\qquad \textbf{Ans.}$$

Ans:
$P = 12.6$ lb
$N = 4.22$ lb

13–109.

Rod *OA* rotates counterclockwise at a constant angular rate $\dot{\theta} = 4$ rad/s. The double collar *B* is pin-connected together such that one collar slides over the rotating rod and the other collar slides over the circular rod described by the equation $r = (1.6 \cos \theta)$ m. If *both* collars have a mass of 0.5 kg, determine the force which the circular rod exerts on one of the collars and the force that *OA* exerts on the other collar at the instant $\theta = 45°$. Motion is in the horizontal plane.

SOLUTION

$r = 1.6 \cos \theta$

$\dot{r} = -1.6 \sin \theta \dot{\theta}$

$\ddot{r} = -1.6 \cos \theta \dot{\theta}^2 - 1.6 \sin \theta \ddot{\theta}$

At $\theta = 45°, \dot{\theta} = 4$ rad/s and $\ddot{\theta} = 0$

$r = 1.6 \cos 45° = 1.1314$ m

$\dot{r} = -1.6 \sin 45°(4) = -4.5255$ m/s

$\ddot{r} = -1.6 \cos 45°(4)^2 - 1.6 \sin 45°(0) = -18.1019$ m/s²

$a_r = \ddot{r} - r\dot{\theta}^2 = -18.1019 - 1.1314(4)^2 = -36.20$ m/s²

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.1314(0) + 2(-4.5255)(4) = -36.20$ m/s²

$\nearrow + \Sigma F_r = ma_r;$ $-N_C \cos 45° = 0.5(-36.20)$ $N_C = 25.6$ N   **Ans.**

$+ \nwarrow \Sigma F_\theta = ma_\theta;$ $F_{OA} - 25.6 \sin 45° = 0.5(-36.20)$ $F_{OA} = 0$ **Ans.**
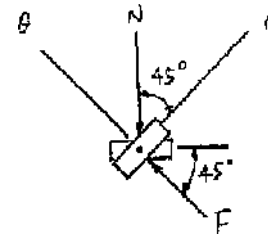
Ans:
$F_r = 25.6$ N
$F_{OA} = 0$

13–110.

Solve Prob. 13–109 if motion is in the vertical plane.

$r = 1.6 \cos \theta$

$\dot{\theta} = 4 \text{ rad/s}$

0.8 m

SOLUTION

$r = 1.6 \cos \theta$

$\dot{r} = -1.6 \sin \theta \dot{\theta}$

$\ddot{r} = -1.6 \cos \theta \dot{\theta}^2 - 1.6 \sin \theta \ddot{\theta}$

At $\theta = 45°$, $\dot{\theta} = 4 \text{ rad/s}$ and $\ddot{\theta} = 0$

$r = 1.6 \cos 45° = 1.1314 \text{ m}$

$\dot{r} = -1.6 \sin 45°(4) = -4.5255 \text{ m/s}$

$\ddot{r} = -1.6 \cos 45°(4)^2 - 1.6 \sin 45°(0) = -18.1019 \text{ m/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -18.1019 - 1.1314(4)^2 = -36.20 \text{ m/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.1314(0) + 2(-4.5255)(4) = -36.20 \text{ m/s}^2$

$+\swarrow \Sigma F_r = ma_r;\quad -N_C \cos 45° - 4.905 \cos 45° = 0.5(-36.204)$

$+\nwarrow \Sigma F_\theta = ma_\theta;\quad F_{OA} - N_C \sin 45° - 4.905 \sin 45° = 0.5(-36.204)$

$N_C = 20.7 \text{ N}$ **Ans.**
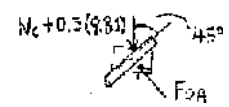
$F_{OA} = 0$ **Ans.**

Ans:
$F_r = 20.7 \text{ N}$
$F_{OA} = 0$

13–111.

A 0.2-kg spool slides down along a smooth rod. If the rod has a constant angular rate of rotation $\dot{\theta} = 2$ rad/s in the vertical plane, show that the equations of motion for the spool are $\ddot{r} - 4r - 9.81 \sin \theta = 0$ and $0.8\dot{r} + N_s - 1.962 \cos \theta = 0$, where N_s is the magnitude of the normal force of the rod on the spool. Using the methods of differential equations, it can be shown that the solution of the first of these equations is $r = C_1 e^{-2t} + C_2 e^{2t} - (9.81/8) \sin 2t$. If r, $\dot{r}$, and θ are zero when $t = 0$, evaluate the constants C_1 and C_2 to determine r at the instant $\theta = \pi/4$ rad.
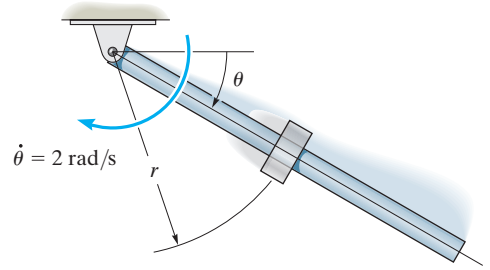
SOLUTION

Kinematic: Here, $\dot{\theta}. = 2$ rad/s and $\ddot{\theta} = 0$. Applying Eqs. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = \ddot{r} - r(2^2) = \ddot{r} - 4r$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = r(0) + 2\dot{r}(2) = 4\dot{r}$$

Equation of Motion: Applying Eq. 13–9, we have

$$\Sigma F_r = ma_r; \qquad 1.962 \sin \theta = 0.2(\ddot{r} - 4r)$$

$$\ddot{r} - 4r - 9.81 \sin \theta = 0 \qquad \textbf{(Q.E.D.)} \qquad \textbf{(1)}$$

$$\Sigma F_\theta = ma_\theta; \qquad 1.962 \cos \theta - N_s = 0.2(4\dot{r})$$

$$0.8\dot{r} + N_s - 1.962 \cos \theta = 0 \qquad \textbf{(Q.E.D.)} \qquad \textbf{(2)}$$
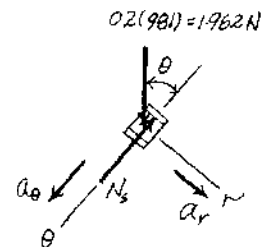
Since $\dot{\theta}. = 2$ rad/s, then $\int_0^\theta \dot{\theta} = \int_0^1 2dt$, $\theta = 2t$. The solution of the differential equation (Eq.(1)) is given by

$$r = C_1\, e^{-2t} + C_2\, e^{2t} - \frac{9.81}{8} \sin 2t \qquad \textbf{(3)}$$

Thus,

$$\dot{r} = -2\,C_1 e^{-2t} + 2C_2\, e^{2t} - \frac{9.81}{4} \cos 2t \qquad \textbf{(4)}$$

At $t = 0$, $r = 0$. From Eq.(3) $0 = C_1(1) + C_2(1) - 0 \qquad \textbf{(5)}$

At $t = 0$, $\dot{r} = 0$. From Eq.(4) $0 = -2\,C_1(1) + 2C_2(1) - \dfrac{9.81}{4} \qquad \textbf{(6)}$

Solving Eqs. (5) and (6) yields

$$C_1 = -\frac{9.81}{16} \qquad C_2 = \frac{9.81}{16}$$

Thus,

$$r = -\frac{9.81}{16} e^{-2t} + \frac{9.81}{16} e^{2t} - \frac{9.81}{8} \sin 2t$$

$$= \frac{9.81}{8} \left(\frac{-e^{-2t} + e^{2t}}{2} - \sin 2t \right)$$

$$= \frac{9.81}{8} (\sin h\, 2t - \sin 2t)$$

At $\theta = 2t = \dfrac{\pi}{4}$, $\quad r = \dfrac{9.81}{8} \left(\sin h\dfrac{\pi}{4} - \sin \dfrac{\pi}{4}3 \right) = 0.198$ m $\qquad$ **Ans.**

0.2(9.81) = 1.962 N

Ans:
$r = 0.198$ m

*13–112.

The pilot of an airplane executes a vertical loop which in part follows the path of a "four-leaved rose," $r = (-600 \cos 2\theta)$ ft, where θ is in radians. If his speed at A is a constant $v_P = 80$ ft/s, determine the vertical reaction the seat of the plane exerts on the pilot when the plane is at A. He weighs 130 lb. *Hint:* To determine the time derivatives necessary to compute the acceleration components a_r and a_θ, take the first and second time derivatives of $r = 400(1 + \cos \theta)$. Then, for further information, use Eq. 12–26 to determine $\dot\theta$. Also, take the time derivative of Eq. 12–26, noting that $\dot v_C = 0$, to determine $\ddot\theta$.

80 ft/s ← A

$r = -600 \cos 2\theta$

r

θ
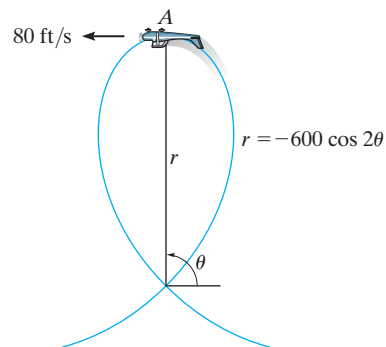
SOLUTION

$r = -600 \cos 2\theta \qquad \dot r = 1200 \sin 2\theta \dot\theta \qquad \ddot r = 1200(2 \cos 2\theta \dot\theta^2 + \sin 2\theta \ddot\theta)$

At $\theta = 90°$

$r = -600 \cos 180° = 600$ ft $\qquad \dot r = 1200 \sin 180° \dot\theta = 0$

$\ddot r = 1200(2 \cos 180° \dot\theta^2 + \sin 180° \ddot\theta) = -2400\dot\theta^2$

$v_r = \dot r = 0 \qquad v_\theta = r\dot\theta = 600\dot\theta$

$v_p^2 = v_r^2 + v_\theta^2$

$80^2 = 0^2 + (600\dot\theta)^2 \qquad \dot\theta = 0.1333$ rad/s

$\ddot r = -2400(0.1333)^2 = -42.67$ ft/s^2

$a_r = \ddot r - r\dot\theta^2 = -42.67 - 600(0.1333)^2 = -53.33$ ft/s^2

$+\uparrow \Sigma F_r = ma_r; \qquad -N - 130 = \dfrac{130}{32.2}(-53.33) \qquad N = 85.3$ lb $\qquad$ **Ans.**
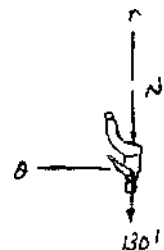
13–113.

The earth has an orbit with eccentricity $e = 0.0167$ around the sun. Knowing that the earth's minimum distance from the sun is $146(10^6)$ km, find the speed at which the earth travels when it is at this distance. Determine the equation in polar coordinates which describes the earth's orbit about the sun.

SOLUTION

$e = \dfrac{Ch^2}{GM_S}$ where $C = \dfrac{1}{r_0}\left(1 - \dfrac{GM_S}{r_0 v_0^2}\right)$ and $h = r_0 v_0$

$e = \dfrac{1}{GM_S r_0}\left(1 - \dfrac{GM_S}{r_0 v_0^2}\right)(r_0 v_0)^2 \qquad e = \left(\dfrac{r_0 v_0^2}{GM_S} - 1\right) \qquad \dfrac{r_0 v_0^2}{GM_S} = e + 1$

$v_0 = \sqrt{\dfrac{GM_S(e+1)}{r_0}}$

$\quad = \sqrt{\dfrac{66.73(10^{-12})(1.99)(10^{30})(0.0167 + 1)}{146(10^9)}} = 30409 \text{ m/s} = 30.4 \text{ km/s}$ **Ans.**

$\dfrac{1}{r} = \dfrac{1}{r_0}\left(1 - \dfrac{GM_S}{r_0 v_0^2}\right)\cos\theta + \dfrac{GM_S}{r_0^2 v_0^2}$

$\dfrac{1}{r} = \dfrac{1}{146(10^9)}\left(1 - \dfrac{66.73(10^{-12})(1.99)(10^{30})}{151.3(10^9)(30409)^2}\right)\cos\theta + \dfrac{66.73(10^{-12})(1.99)(10^{30})}{\left[146(10^9)\right]^2(30409)^2}$

$\dfrac{1}{r} = 0.348(10^{-12})\cos\theta + 6.74(10^{-12})$ **Ans.**

Ans:
$v_o = 30.4 \text{ km/s}$
$\dfrac{1}{r} = 0.348\left(10^{-12}\right)\cos\theta + 6.74\left(10^{-12}\right)$

13–114.

A communications satellite is in a circular orbit above the earth such that it always remains directly over a point on the earth's surface. As a result, the period of the satellite must equal the rotation of the earth, which is approximately 24 hours. Determine the satellite's altitude h above the earth's surface and its orbital speed.

SOLUTION

The period of the satellite around the circular orbit of radius $r_0 = h + r_e = \left[h + 6.378(10^6)\right]$ m is given by

$$T = \frac{2\pi r_0}{v_s}$$

$$24(3600) = \frac{2\pi\left[h + 6.378(10^6)\right]}{v_s}$$

$$v_s = \frac{2\pi\left[h + 6.378(10^6)\right]}{86.4(10^3)} \qquad (1)$$

The velocity of the satellite orbiting around the circular orbit of radius $r_0 = h + r_e = \left[h + 6.378(10^6)\right]$ m is given by

$$v_S = \sqrt{\frac{GM_e}{r_0}}$$

$$v_S = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{h + 6.378(10^6)}} \qquad (2)$$

Solving Eqs.(1) and (2),

$$h = 35.87(10^6) \text{ m} = 35.9 \text{ Mm} \qquad v_S = 3072.32 \text{ m/s} = 3.07 \text{ km/s} \quad \textbf{Ans.}$$

Ans:
$h = 35.9 \text{ mm}$
$v_s = 3.07 \text{ km/s}$

13–115.

The speed of a satellite launched into a circular orbit about the earth is given by Eq. 13–25. Determine the speed of a satellite launched parallel to the surface of the earth so that it travels in a circular orbit 800 km from the earth's surface.

SOLUTION

For a 800-km orbit

$$v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{(800 + 6378)(10^3)}}$$

$$= 7453.6 \text{ m/s} = 7.45 \text{ km/s} \qquad\qquad \textbf{Ans.}$$

Ans:
$v_0 = 7.45\,\text{km/s}$

*13–116.

The rocket is in circular orbit about the earth at an altitude of 20 Mm. Determine the minimum increment in speed it must have in order to escape the earth's gravitational field.

20 Mm
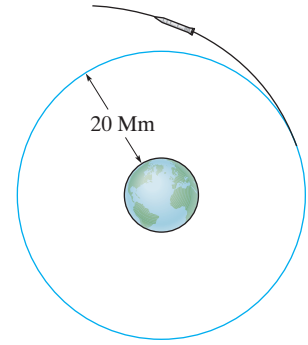
SOLUTION

The speed of the rocket in circular orbit is

$$v_c = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})\left[5.976(10^{24})\right]}{20(10^6) + 6378(10^3)}} = 3888.17 \text{ m/s}$$

To escape the earth's gravitational field, the rocket must enter the parabolic trajectory, which require its speed to be

$$v_e = \sqrt{\frac{2GM_e}{r_0}} = \sqrt{\frac{2\left[66.73(10^{-12})\right]\left[5.976(10^{24})\right]}{20(10^6) + 6378(10^3)}} = 5498.70 \text{ m/s}$$

The required increment in speed is

$$\Delta v = v_e - v_c = 5498.70 - 3888.17$$

$$= 1610.53 \text{ m/s}$$

$$= 1.61(10^3) \text{ m/s} \qquad\qquad \textbf{Ans.}$$

Ans:
$\Delta v = 1.61(10^3) \text{ m/s}$

13–117.

Prove Kepler's third law of motion. *Hint:* Use Eqs. 13–19, 13–28, 13–29, and 13–31.

SOLUTION

From Eq. 13–19,

$$\frac{1}{r} = C \cos \theta + \frac{GM_s}{h^2}$$

For $\theta = 0°$ and $\theta = 180°$,

$$\frac{1}{r_p} = C + \frac{GM_s}{h^2}$$

$$\frac{1}{r_a} = -C + \frac{GM_s}{h^2}$$

Eliminating C, from Eqs. 13–28 and 13–29,

$$\frac{2a}{b^2} = \frac{2GM_s}{h^2}$$

From Eq. 13–31,

$$T = \frac{\pi}{h}(2a)(b)$$

Thus,

$$b^2 = \frac{T^2 h^2}{4\pi^2 a^2}$$

$$\frac{4\pi^2 a^3}{T^2 h^2} = \frac{GM_s}{h^2}$$

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right)a^3 \qquad\qquad \textbf{Q.E.D.}$$

Ans:

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right)a^3$$

13–118.

The satellite is moving in an elliptical orbit with an eccentricity $e = 0.25$. Determine its speed when it is at its maximum distance A and minimum distance B from the earth.
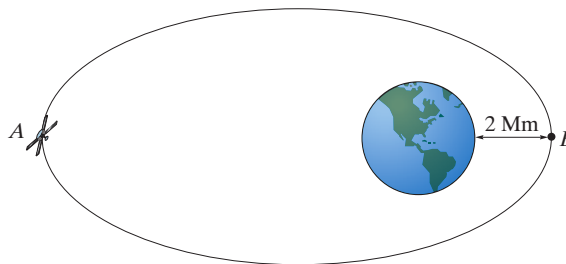
SOLUTION

$$e = \frac{Ch^2}{GM_e}$$

where $C = \dfrac{1}{r_0}\left(1 - \dfrac{GM_e}{r_0 v_0^2}\right)$ and $h = r_0 v_0$.

$$e = \frac{1}{GM_e r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)(r_0 v_0)^2$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1 \qquad v_0 = \sqrt{\frac{GM_e(e+1)}{r_0}}$$

where $r_0 = r_p = 2(10^6) + 6378(10^3) = 8.378(10^6)$ m.

$$v_B = v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})(0.25+1)}{8.378(10^6)}} = 7713 \text{ m/s} = 7.71 \text{ km/s} \quad \textbf{Ans.}$$

$$r_a = \frac{r_0}{\dfrac{2GM_e}{r_0 v_0} - 1} = \frac{8.378(10^6)}{\dfrac{2(66.73)(10^{-12})(5.976)(10^{24})}{8.378(10^6)(7713)^2} - 1} = 13.96(10^6) \text{ m}$$

$$v_A = \frac{r_p}{r_a}v_B = \frac{8.378(10^6)}{13.96(10^6)}(7713) = 4628 \text{ m/s} = 4.63 \text{ km/s} \qquad \textbf{Ans.}$$

Ans:
$v_B = 7.71 \text{ km/s}$
$v_A = 4.63 \text{ km/s}$

362

13–119.

The rocket is traveling in free flight along the elliptical orbit. The planet has no atmosphere, and its mass is 0.60 times that of the earth. If the rocket has the orbit shown, determine the rocket's speed when it is at A and at B.
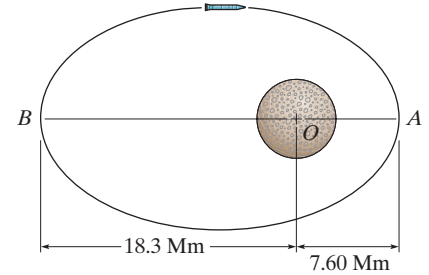
SOLUTION

Applying Eq. 13–27,

$$r_a = \frac{r_p}{(2GM/r_p v_p^2) - 1}$$

$$\frac{2GM}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM\, r_a}{r_p(r_p + r_a)}}$$

The elliptical orbit has $r_p = 7.60(10^6)$ m, $r_a = 18.3(10^6)$ m and $v_p = v_A$. Then

$$v_A = \sqrt{\frac{2\left[66.73(10^{-12})\right]\left[0.6(5.976)(10^{24})\right]\left[18.3(10^6)\right]}{7.60(10^6)\left[7.60(10^6) + 18.3(10^6)\right]}}$$

$$= 6669.99 \text{ m/s} = 6.67(10^3) \text{ m/s} \qquad\qquad \textbf{Ans.}$$

In this case,

$$h = r_p v_A = r_a v_B$$

$$7.60(10^6)(6669.99) = 18.3(10^6)v_B$$

$$v_B = 2770.05 \text{ m/s} = 2.77(10^3) \text{ m/s} \qquad\qquad \textbf{Ans.}$$

Ans:
$v_A = 6.67(10^3)$ m/s
$v_B = 2.77(10^3)$ m/s

*13–120.

Determine the constant speed of satellite S so that it circles the earth with an orbit of radius $r = 15$ Mm. *Hint: Use Eq. 13–1.*

$r = 15$ Mm

S
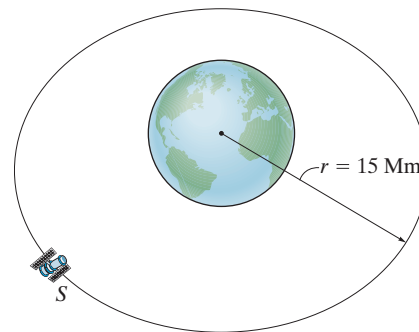
SOLUTION

$$F = G\frac{m_s\, m_e}{r^2} \qquad \text{Also} \qquad F = m_s\left(\frac{v_s^2}{r}\right) \qquad \text{Hence}$$

$$m_s\left(\frac{v_0^2}{r}\right) = G\frac{m_s\, m_e}{r^2}$$

$$v = \sqrt{G\frac{m_e}{r}} = \sqrt{66.73(10^{-12})\left(\frac{5.976(10^{24})}{15(10^6)}\right)} = 5156 \text{ m/s} = 5.16 \text{ km/s} \qquad \textbf{Ans.}$$

Ans:
$v = 5.16$ km/s

364

13–121.

The rocket is in free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.70 times that of the earth. If the rocket has an apoapsis and periapsis as shown in the figure, determine the speed of the rocket when it is at point A.

r = 3 Mm

A   *B*   *O*   *A'*

6 Mm   9 Mm
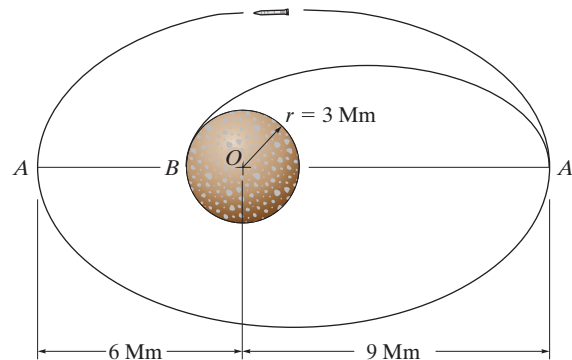
SOLUTION

Central-Force Motion: Use $r_a = \dfrac{r_0}{(2\,GM/r_0\,v_0^2)-1}$, with $r_0 = r_p = 6\left(10^6\right)$ m and $M = 0.70 M_e$, we have

$$9\left(10^6\right) = \frac{6(10)^6}{\left(\dfrac{2(66.73)\,(10^{-12})\,(0.7)\,[5.976(10^{24})]}{6(10^6)v_P^2}\right) - 1}$$

$$v_A = 7471.89 \text{ m/s} = 7.47 \text{ km/s} \qquad\qquad \textbf{Ans.}$$

Ans:
$v_A = 7.47$ km/s

365

13–122.

The Viking Explorer approaches the planet Mars on a parabolic trajectory as shown. When it reaches point A its velocity is 10 Mm/h. Determine r_0 and the required velocity at A so that it can then maintain a circular orbit as shown. The mass of Mars is 0.1074 times the mass of the earth.
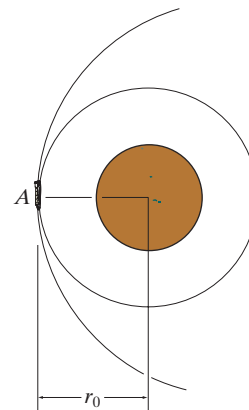
SOLUTION

When the Viking explorer approaches point A on a parabolic trajectory, its velocity at point A is given by

$$v_A = \sqrt{\frac{2GM_M}{r_0}}$$

$$\left[10(10^6)\,\frac{m}{h}\right]\left(\frac{1\ h}{3600\ s}\right) = \sqrt{\frac{2(66.73)(10^{-12})\left[0.1074(5.976)(10^{24})\right]}{r_0}}$$

$$r_0 = 11.101(10^6)\ m = 11.1\ Mm \qquad\qquad \textbf{Ans.}$$

When the explorer travels along a circular orbit of $r_0 = 11.101(10^6)$ m, its velocity is

$$v_{A'} = \sqrt{\frac{GM_r}{r_0}} = \sqrt{\frac{66.73(10^{-12})\left[0.1074(5.976)(10^{24})\right]}{11.101(10^6)}}$$

$$= 1964.19\ m/s$$

Thus, the required sudden decrease in the explorer's velocity is

$$\Delta v_A = v_A - v_{A'}$$

$$= 10(10^6)\left(\frac{1}{3600}\right) - 1964.19$$

$$= 814\ m/s \qquad\qquad \textbf{Ans.}$$

Ans:
$r_0 = 11.1$ Mm
$\Delta v_A = 814$ m/s

13–123.

The rocket is initially in free-flight circular orbit around the earth. Determine the speed of the rocket at A. What change in the speed at A is required so that it can move in an elliptical orbit to reach point A'?
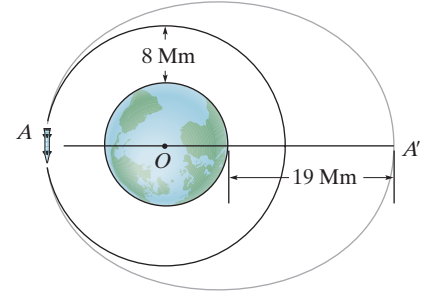
SOLUTION

The required speed to remain in circular orbit containing point A of which $r_0 = 8(10^6) + 6378(10^3) = 14.378(10^6)$ m can be determined from

$$(v_A)_C = \sqrt{\frac{GM_e}{r_0}}$$

$$= \sqrt{\frac{[66.73(10^{-12})][5.976(10^{24})]}{14.378(10^6)}}$$

$$= 5266.43 \text{ m/s} = 5.27(10^3) \text{ m/s} \qquad \textbf{Ans.}$$

To more from A to A', the rocket has to follow the elliptical orbit with $r_p = 8(10^6) + 6378(10^3) = 14.378(10^6)$ m and $r_a = 19(10^6) + 6378(10^3) = 25.378(10^6)$ m. The required speed at A to do so can be determined using Eq. 13–27.

$$r_a = \frac{r_p}{(2GM_e/r_p v_p^2) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM_e r_a}{r_p(r_p + r_a)}}$$

Here, $v_p = (v_A)_e$. Then

$$(v_A)_e = \sqrt{\frac{2[66.73(10^{-12})][5.976(10^{24})][25.378(10^6)]}{14.378(10^6)[14.378(10^6) + 25.378(10^6)]}}$$

$$= 5950.58 \text{ m/s}$$

Thus, the required change in speed is

$$\Delta v = (v_A)_e - (v_A)_c = 5950.58 - 5266.43 = 684.14 \text{ m/s} = 684 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$$(v_A)_C = 5.27(10^3) \text{ m/s}$$
$$\Delta v = 684 \text{ m/s}$$

*13–124.

The rocket is in free-flight circular orbit around the earth. Determine the time needed for the rocket to travel from the inner orbit at A to the outer orbit at A'.
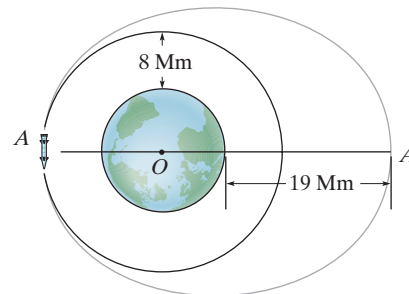
SOLUTION

To move from A to A', the rocket has to follow the elliptical orbit with $r_p = 8(10^6) + 6378(10^3) = 14.378(10^6)$ m   and $r_a = 19(10^6) + 6378(10^3)$ $= 25.378(10^6)$ m. The required speed at A to do so can be determined using Eq. 13–27.

$$r_a = \frac{r_p}{(2GM_e/r_p v_p^2) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM_e r_a}{r_p(r_p + r_a)}}$$

Here, $v_p = v_A$. Then

$$v_A = \sqrt{\frac{2\left[66.73(10^{-12})\right]\left[5.976(10^{24})\right]\left[25.378(10^6)\right]}{14.378(10^6)\left[14.378(10^6) + 25.378(10^6)\right]}} = 5950.58 \text{ m/s}$$

Then

$$h = v_A r_p = 5950.58\left[14.378(10^6)\right] = 85.5573(10^9) \text{ m}^2/\text{s}$$

The period of this elliptical orbit can be determined using Eq. 13–31.

$$T = \frac{\pi}{h}(r_p + r_a)\sqrt{r_p r_a}$$

$$= \frac{\pi}{85.5573(10^9)}\left[14.378(10^6) + 25.378(10^6)\right]\sqrt{\left[14.378(10^6)\right]\left[25.378(10^6)\right]}$$

$$= 27.885(10^3) \text{ s}$$

Thus, the time required to travel from A to A' is

$$t = \frac{T}{2} = \frac{27.885(10^3)}{2} = 13.94(10^3) \text{ s} = 3.87 \text{ h} \qquad \textbf{Ans.}$$

Ans:
$t = 3.87$ h

13–125.

A satellite is launched with an initial velocity $v_0 = 2500$ mi/h parallel to the surface of the earth. Determine the required altitude (or range of altitudes) above the earth's surface for launching if the free-flight trajectory is to be (a) circular, (b) parabolic, (c) elliptical, and (d) hyperbolic. Take $G = 34.4(10^{-9})(\text{lb} \cdot \text{ft}^2)/\text{slug}^2$, $M_e = 409(10^{21})$ slug, the earth's radius $r_e = 3960$ mi, and 1 mi $= 5280$ ft.

SOLUTION

$$v_0 = 2500 \text{ mi/h} = 3.67(10^3) \text{ ft/s}$$

(a) $e = \dfrac{C^2 h}{GM_e} = 0$ or $C = 0$

$1 = \dfrac{GM_e}{r_0 v_0^2}$

$GM_e = 34.4(10^{-9})(409)(10^{21})$

$\quad\;\; = 14.07(10^{15})$

$r_0 = \dfrac{GM_e}{v_0^2} = \dfrac{14.07(10^{15})}{[3.67(10^{13})]^2} = 1.046(10^9) \text{ ft}$

$r = \dfrac{1.047(10^9)}{5280} - 3960 = 194(10^3) \text{ mi}$ **Ans.**

(b) $e = \dfrac{C^2 h}{GM_e} = 1$

$\dfrac{1}{GM_e}(r_0^2 v_0^2)\left(\dfrac{1}{r_0}\right)\left(1 - \dfrac{GM_e}{r_0 v_0^2}\right) = 1$

$r_0 = \dfrac{2GM_e}{v_0^2} = \dfrac{2(14.07)(10^{15})}{[3.67(10^3)]^2} = 2.09(10^9) \text{ ft} = 396(10^3) \text{ mi}$

$r = 396(10^3) - 3960 = 392(10^3) \text{ mi}$ **Ans.**

(c) $e < 1$

$194(10^3) \text{ mi} < r < 392(10^3) \text{ mi}$ **Ans.**

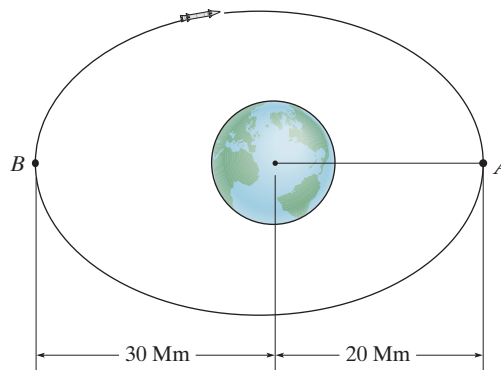(d) $e > 1$

$r > 392(10^3) \text{ mi}$ **Ans.**

Ans:
(a) $r = 194 \left(10^3\right)$ mi
(b) $r = 392 \left(10^3\right)$ mi
(c) $194 \left(10^3\right)$ mi $< r < 392 \left(10^3\right)$ mi
(d) $r > 392 \left(10^3\right)$ mi

13–126.

The rocket is traveling around the earth in free flight along the elliptical orbit. If the rocket has the orbit shown, determine the speed of the rocket when it is at A and at B.

SOLUTION

Here $r_p = 20(10^6)$ m and $r_a = 30(10^6)$ m. Applying Eq. 13–27,

$$r_a = \frac{r_p}{(2GM_e/r_p v_p^2) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_P = \sqrt{\frac{2GM_e r_a}{r_p(r_p + r_a)}}$$

Here $v_p = v_A$. Then

$$v_A = \sqrt{\frac{2\left[66.73(10^{-12})\right]\left[5.976(10^{24})\right]\left[30(10^6)\right]}{20(10^6)\left[20(10^6) + 30(10^6)\right]}}$$

$$= 4891.49 \text{ m/s} = 4.89(10^3) \text{ m/s} \qquad\qquad \textbf{Ans.}$$

For the same orbit h is constant. Thus,

$$h = r_p v_p = r_a v_a$$

$$\left[20(10^6)\right](4891.49) = \left[30(10^6)\right]v_B$$

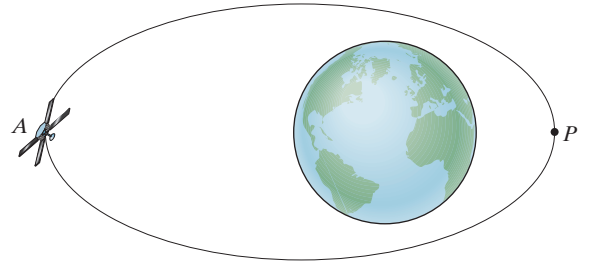$$v_B = 3261.00 \text{ m/s} = 3.26(10^3) \text{ m/s} \qquad\qquad \textbf{Ans.}$$

Ans:
$v_A = 4.89(10^3)$ m/s
$v_B = 3.26(10^3)$ m/s

370

13–127.

An elliptical path of a satellite has an eccentricity $e = 0.130$. If it has a speed of 15 Mm/h when it is at perigee, P, determine its speed when it arrives at apogee, A. Also, how far is it from the earth's surface when it is at A?

SOLUTION

$e = 0.130$

$v_p = v_0 = 15 \text{ Mm/h} = 4.167 \text{ km/s}$

$e = \dfrac{Ch^2}{GM_e} = \dfrac{1}{r_0}\left(1 - \dfrac{GM_e}{r_0 v_0^2}\right)\left(\dfrac{r_0^2 v_0^2}{GM_e}\right)$

$e = \left(\dfrac{r_0 v_0^2}{GM_e} - 1\right)$

$\dfrac{r_0 v_0^2}{GM_e} = e + 1$

$r_0 = \dfrac{(e + 1)GM_e}{v_0^2}$

$= \dfrac{1.130(66.73)(10^{-12})(5.976)(10^{24})}{\left[4.167(10^3)\right]^2}$

$= 25.96 \text{ Mm}$

$\dfrac{GM_e}{r_0 v_0^2} = \dfrac{1}{e + 1}$

$r_A = \dfrac{r_0}{\frac{2GM_e}{r_0 v_0^2} - 1} = \dfrac{r_0}{\left(\frac{2}{e + 1}\right) - 1}$

$r_A = \dfrac{r_0(e + 1)}{1 - e}$

$= \dfrac{25.96(10^6)(1.130)}{0.870}$

$= 33.71(10^6) \text{ m} = 33.7 \text{ Mm}$

$v_A = \dfrac{v_0 r_0}{r_A}$

$= \dfrac{15(25.96)(10^6)}{33.71(10^6)}$

$= 11.5 \text{ Mm/h}$ **Ans.**

$d = 33.71(10^6) - 6.378(10^6)$

$= 27.3 \text{ Mm}$ **Ans.**

Ans:
$v_A = 11.5 \text{ Mm/h}$
$d = 27.3 \text{ Mm}$

*13–128.

A rocket is in free-flight elliptical orbit around the planet Venus. Knowing that the periapsis and apoapsis of the orbit are 8 Mm and 26 Mm, respectively, determine (a) the speed of the rocket at point A', (b) the required speed it must attain at A just after braking so that it undergoes an 8-Mm free-flight circular orbit around Venus, and (c) the periods of both the circular and elliptical orbits. The mass of Venus is 0.816 times the mass of the earth.

SOLUTION

a)
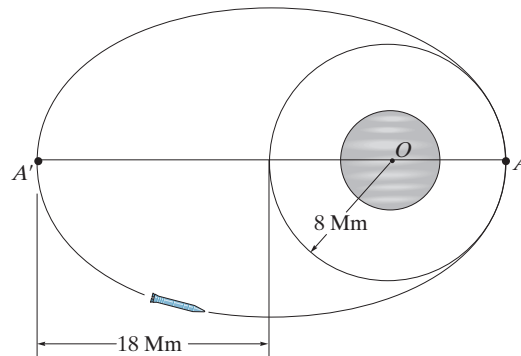
$$M_\nu = 0.816(5.976(10^{24})) = 4.876(10^{24})$$

$$OA' = \frac{OA}{\left(\dfrac{2GM\nu}{OA\, v_A^2} - 1\right)}$$

$$26(10)^6 = \frac{8(10^6)}{\left(\dfrac{2(66.73)(10^{-12})4.876(10^{24})}{8(10^6)v_A^2} - 1\right)}$$

$$\frac{81.35(10^6)}{v_A^2} = 1.307$$

$$v_A = 7887.3 \text{ m/s} = 7.89 \text{ km/s}$$

$$v_A = \frac{OA\, v_A}{OA'} = \frac{8(10^6)(7887.3)}{26(10^6)} = 2426.9 \text{ m/s} = 2.43 \text{ m/s} \qquad \textbf{Ans.}$$

b)

$$v_{A''} = \sqrt{\frac{GM_\nu}{OA'}} = \sqrt{\frac{66.73(10^{-12})4.876(10^{24})}{8(10^6)}}$$

$$v_{A''} = 6377.7 \text{ m/s} = 6.38 \text{ km/s} \qquad \textbf{Ans.}$$

c)

Circular orbit:

$$T_c = \frac{2\pi OA}{v_{A''}} = \frac{2\pi 8(10^6)}{6377.7} = 7881.41 \text{ s} = 2.19 \text{ h} \qquad \textbf{Ans.}$$

Elliptic orbit:

$$T_e = \frac{\pi}{OA v_A}(OA + OA')\sqrt{(OA)(OA')} = \frac{\pi}{8(10^6)(7886.8)}(8 + 26)(10^6)(\sqrt{(8)(26)})(10^6)$$

$$T_e = 24414.2 \text{ s} = 6.78 \text{ h} \qquad \textbf{Ans.}$$
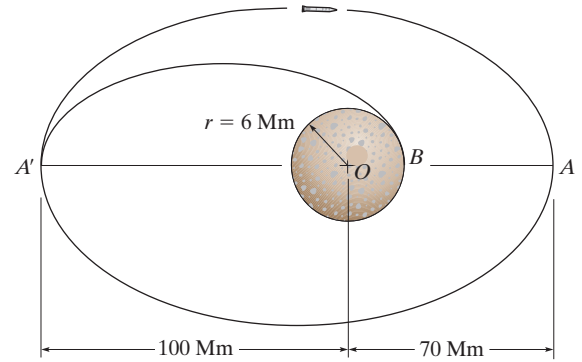
Ans:
$v_A = 2.43$ m/s
$v_{A''} = 6.38$ km/s
$T_c = 2.19$ h
$T_e = 6.78$ h

13–129.

The rocket is traveling in a free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.60 times that of the earth. If the rocket has the orbit shown, determine the rocket's velocity when it is at point A.

SOLUTION

Applying Eq. 13–27,

$$r_a = \frac{r_p}{(2GM/r_p v_p^2) - 1}$$

$$\frac{2GM}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GMr_a}{r_p(r_p + r_a)}}$$

The rocket is traveling around the elliptical orbit with $r_p = 70(10^6)$ m, $r_a = 100(10^6)$ m and $v_p = v_A$. Then

$$v_A = \sqrt{\frac{2[66.73(10^{-12})][0.6(5.976)(10^{24})][100(10^6)]}{70(10^6)[70(10^6) + 100(10^6)]}}$$
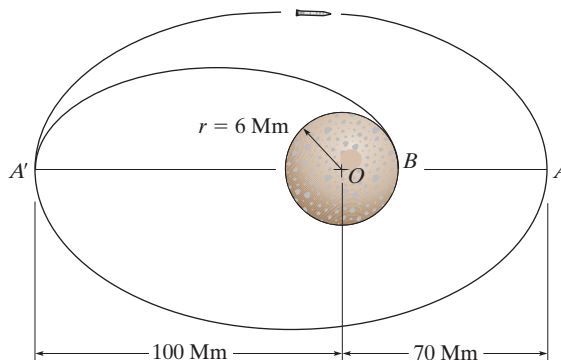
$$= 2005.32 \text{ m/s} = 2.01(10^3) \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 2.01(10^3)$ m/s

13–130.

If the rocket is to land on the surface of the planet, determine the required free-flight speed it must have at A' so that the landing occurs at B. How long does it take for the rocket to land, going from A' to B? The planet has no atmosphere, and its mass is 0.6 times that of the earth.

SOLUTION

Applying Eq. 13–27,

$$r_a = \frac{r_p}{(2GM/r_p v_p^2) - 1}$$

$$\frac{2GM}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GMr_a}{r_p(r_p + r_a)}}$$

To land on B, the rocket has to follow the elliptical orbit $A'B$ with $r_p = 6(10^6)$, $r_a = 100(10^6)$ m and $v_p = v_B$.

$$v_B = \sqrt{\frac{2\left[66.73(10^{-12})\right]\left[0.6(5.976)(10^{24})\right]\left[100(10^6)\right]}{6(10^6)\left[6(10^6) + 100(10^6)\right]}} = 8674.17 \text{ m/s}$$

In this case

$$h = r_p v_B = r_a v_{A'}$$

$$6(10^6)(8674.17) = 100(10^6) v_{A'}$$

$$v_{A'} = 520.45 \text{ m/s} = 521 \text{ m/s} \qquad \textbf{Ans.}$$

The period of the elliptical orbit can be determined using Eq. 13–31.

$$T = \frac{\pi}{h}(r_p + r_a)\sqrt{r_p r_a}$$

$$= \frac{\pi}{6(10^6)(8674.17)}\left[6(10^6) + 100(10^6)\right]\sqrt{\left[6(10^6)\right]\left[100(10^6)\right]}$$

$$= 156.73(10^3) \text{ s}$$

Thus, the time required to travel from A' to B is

$$t = \frac{T}{2} = 78.365(10^3) \text{ s} = 21.8 \text{ h} \qquad \textbf{Ans.}$$
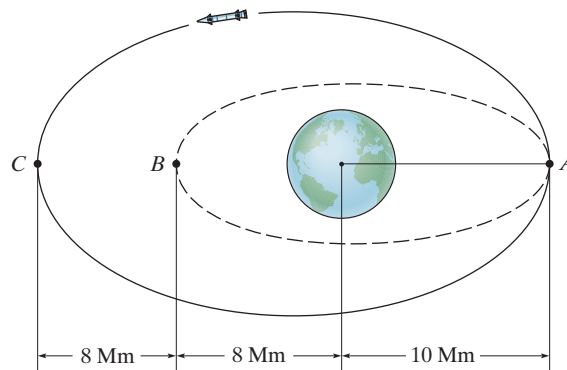
Ans:
$v_{A'} = 521 \text{ m/s}$
$t = 21.8 \text{ h}$

13–131.

The rocket is traveling around the earth in free flight along an elliptical orbit AC. If the rocket has the orbit shown, determine the rocket's velocity when it is at point A.

8 Mm · 8 Mm · 10 Mm

SOLUTION

For orbit AC, $r_p = 10(10^6)$ m and $r_a = 16(10^6)$ m. Applying Eq. 13–27

$$r_a = \frac{r_p}{(2GM_e/r_p v_p^2) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM_e r_a}{r_p(r_p + r_a)}}$$
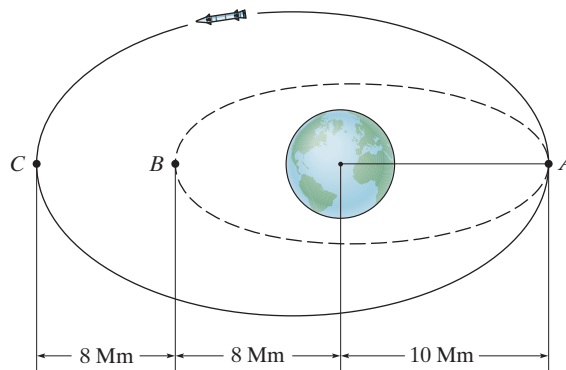
Here $v_p = v_A$. Then

$$v_A = \sqrt{\frac{2\left[66.73(10^{-12})\right]\left[5.976(10^{24})\right]\left[16(10^6)\right]}{10(10^6)\left[10(10^6) + 16(10^6)\right]}}$$

$$= 7005.74 \text{ m/s} = 7.01(10^3) \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 7.01(10^3)$ m/s

*13–132.

The rocket is traveling around the earth in free flight along the elliptical orbit AC. Determine its change in speed when it reaches A so that it travels along the elliptical orbit AB.

SOLUTION

Applying Eq. 13–27,

$$r_a = \frac{r_p}{(2GM_e/r_p v_p^2) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_a v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM_e r_a}{r_p(r_p + r_a)}}$$

For orbit AC, $r_p = 10(10^6)$ m, $r_a = 16(10^6)$ m and $v_p = (v_A)_{AC}$. Then

$$(v_A)_{AC} = \sqrt{\frac{2[66.73(10^{-12})][5.976(10^{24})][16(10^6)]}{10(10^6)[10(10^6) + 16(10^6)]}} = 7005.74 \text{ m/s}$$

For orbit AB, $r_p = 8(10^6)$ m, $r_a = 10(10^6)$ m and $v_p = v_B$. Then

$$v_B = \sqrt{\frac{2[66.73(10^{-12})][5.976(10^{24})][10(10^6)]}{8(10^6)[8(10^6) + 10(10^6)]}} = 7442.17 \text{ m/s}$$

Since h is constant at any position of the orbit,

$$h = r_p v_p = r_a v_a$$

$$8(10^6)(7442.17) = 10(10^6)(v_A)_{AB}$$

$$(v_A)_{AB} = 5953.74 \text{ m/s}$$

Thus, the required change in speed is

$$\Delta v = (v_A)_{AB} - (v_A)_{AC} = 5953.74 - 7005.74$$

$$= -1052.01 \text{ m/s} = -1.05 \text{ km/s} \qquad \textbf{Ans.}$$

The negative sign indicates that the speed must be decreased.

Ans:
$\Delta v = -1.05$ km/s