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## Solutions Manual for

**Machine Elements in Mechanical Design, 5<sup>th</sup> ed.**

**By: Robert L. Mott**

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**MDESIGN Software - Its application to**  
***Machine Elements in Mechanical Design, 5th edition***  
**By: Robert L. Mott**  
**Published by: Pearson Education - Prentice-Hall**

### **General Description of the Software**

A powerful computer-aided calculation software package called MDESIGN is included with each purchase of this book. A total of 66 modules divided among 15 categories make up the complete package, outlined in the Introduction to the software. The software is an updated version of one that first appeared in the 4th edition of *Machine Elements in Mechanical Design*.

The software was created by TEDATA, GMBH, a German company that has a long history of producing such software for professional use throughout Europe and many other parts of the world. The version included with this book has 32 modules that have general applicability or that were produced specially for the book, following the analysis and design methods presented in the book, most of which are patterned on methods and standards commonly used in the United States. The other 34 modules were developed primarily for use by professionals and conform to common practice in Europe as represented by DIN standards, VDI publications, and the popular reference book on machine elements often called Roloff/Matek Machine Elements, written by Herbert Wittel, Dieter Muhs, Dieter Jannasch, and Joachim Vobiek and published by Vieweg+Teubner, Wiesbaden, Germany, 2009.

#### ***Identification of the two types of MDESIGN modules listed in the Introduction to the software:***

- The list identifies the 32 modules most closely aligned with this book by the symbol ⊕. The chapter and section of the book most relevant to each module is indicated. The text includes several sections where a special icon appears to indicate that the use of MDESIGN is pertinent to that topic.
- The other 34 modules are denoted by the symbol ⊗ and are more closely aligned to European standards. They may use terminology, notations, and symbols unfamiliar to those experienced primarily in U.S.-based practices.

It is important to note that the inclusion of this extensive and diverse set of modules can be useful to users of this book throughout the world as a means of expanding the breadth of knowledge of design practices in different regions. Furthermore, many users of the book are likely to engage in projects with industrial companies, design services, consultants, and university faculty members from many parts of the world and having these modules available can aid in communicating across traditional geographic boundaries and between different technical cultures.

#### **Advice on Use of the MDESIGN Software**

The following comments are directed primarily to those using this book as a learning tool either in college and university degree programs or in professional self-study.

The author's approach to the inclusion of calculation aids within initial learning of technical subject matter is:

- *Users of computer software and calculation aids must have solid understanding of the relevant principles of design and stress analysis to ensure that design decisions are based on reliable foundations.*
- *Software should be used only after mastering a given design methodology by careful study and practicing manual techniques.*
- *Then, data with known results can be applied to the software as a check on the understanding of the program's input data requirements, symbols and notation used, limits on the range of acceptable data, and analysis methods.*
- *Only then should users rely on implementation of design decisions based on output results from the software.*

### **General HELP for Running MDESIGN**

An extensive 123-page help file can be accessed from the main menu ribbon. Particular attention should be paid to the Graphical User Interface section on pages 33-39 for those few modules that permit graphical data input.

### **Recommended Primary Uses for MDESIGN Software with this Book**

Upon launching the MDESIGN software package, reading the Introduction, and opening the software, the left side of the initial screen will include the list of 15 categories of modules. Each category name is preceded by a plus-sign (+) that, when selected, yields the list of modules in that category. Double-clicking will open any selected module. Alternatively, you can right click and select Open.

The 32 modules most closely aligned with the presentations in the book, identified by the symbol ⊕ in the Introduction, are obviously those that should be considered first for incorporation into courses and individual study. Pertinent sections of the book for which these modules may be useful are indicated by the graphic symbol in the left margin.

Particularly for design projects and where multiple trials for design decisions are to be expected and where the large catalogs of data in MDESIGN can be accessed, the following modules enable learners to try many options in a short amount of time after learning the basic fundamentals. The following modules serve well these purposes.

Beam Calculations	Column Analysis	ISO Fit System
Statically determinate beams	Column Design	Parallel Keys
Statically indeterminate beams	Ball and Roller Bearings	V-belts
Helical Compression Springs	Plain Surface/Journal Bearings	Synchronous Belts
Helical Extension Springs	Clutches and Brakes (5 modules)	Roller Chains
Helical Torsion Springs	Combined Stresses/Mohr's Circle	Shafts-U.S. Standards

Certainly, in an academic learning situation, instructors must enforce expectations on when and where use of the MDESIGN software is accepted, expected, or prohibited.

The process for using any module should be as follows:

1. Open the relevant module and read the General Text Help screen in the lower left part of the opening page. This outlines the basic functions of the module, shows the technical bases for the analyses performed, and identifies relevant references, terms, and symbols. Be aware that for some modules, the text help has been translated from the original German language and the result may not be in adequate standard English.
2. Use the pull-down menu on that screen to peruse what other textual aids are included. These often elaborate on design approaches by,
  - a. Explaining unfamiliar terms
  - b. Stating typical units for input data or results
  - c. Setting acceptable limits on values of certain variables
  - d. Providing tables of data from which some input data must be selected by the user
3. Observe the graphic aids in the lower right of the opening page, again scrolling among all available topics. Some of these can be accessed directly from the Input page.
4. Peruse the data required for the input screen. Open any available help icons for text, data, or “choice” options to determine requirements or available options.
5. Under the Tools tab on the main menu ribbon, select the pull down menu on the Measures System icon and select U.S. System, Metric System, or All Systems. These choices set the primary units in which data are to be entered and for which output results will be shown. In any case, you have the option to change units for any item by passing the cursor over the unit and pulling down the local menu.
  - a. Pay special attention to the precision of results data shown on the Output pages. At times, only one or two significant figures of accuracy are displayed and that may not be adequate for your use. You may be able to select a smaller unit that will show higher precision. For example if a length of diameter measurement shows 6.0 in, selecting the *mil* unit (0.001 – thousandths) may show 6075 mils, indicating 6.075 in.
  - b. Note that the standard European Metric system uses the comma rather than the decimal point for separating digits in floating point calculations. For example, In the U.S. system a number may be 12.456; in Metric it will appear 12,456.
6. When all data are entered, select the Calculation tab on the main menu or, simply, press the F10 key on the keyboard to initiate the module’s calculations. Note the following:
  - a. In some modules, intermediate data entry screens pop up for which some initial calculations have generated data on which subsequent design decisions are based. You are asked then to make the final decision before the complete results are found.



- b. After a short time for completing the calculations (typically only a few seconds), select the Output Page option at the upper left of the data page to see results.
  - c. Carefully evaluate results for reasonableness and check that proper units have been selected.
  - d. Some modules include internal checks on output to assess acceptability, with unacceptable results shown in RED. If that occurs, you must return to the Input Page and change design decisions, recalculate results, and re-evaluate their acceptability.
  - e. Consider the degree of optimization of any particular result and, where possible, make adjustments to hone into a more optimum design. *It is typical for mechanical design analyses to require proposing and analyzing several alternative solutions to achieve the most efficient and effective design.*
7. When the final result has been found, use the Print command to print out both the Input Page and the Output Page. It is essential for an instructor or a client to see complete records of the data used along with the results.

### Descriptions of Selected Modules

The following sections describe certain topics from this book for which the use of MDESIGN is particularly pertinent. Suggestions for applying the modules are also given, but practice with known data is a good way to gain skill at entering data and seeking optimum results. Use of data taken from Example Problems from the text is highly recommended. However, there may be slight differences between results in book problems and those from MDESIGN because of rounding of numbers and slightly modified ways of making calculations.

#### ***Combined Stress and Mohr's Circle – Chapter 4***

**Module group: Shafts, Axles, and Beams.** This module solves problems of the type featured in Sections 4-4 and 4-5 of the text in which data for applied normal and shear stresses in one plane are known and the program computes the maximum principal stress, the minimum principal stress, and the maximum shear stress. The complete Mohr's circle and the pertinent stress elements are also developed and included in the output.

#### ***Columns – Chapter 6 – Two modules: Column Analysis and Column Design***

**Module group: Shafts, Axles, and Beams.** These modules follow closely the methodology used in the text for applying the Euler formula for long columns and the J. B. Johnson formula for short columns to either analysis or design problems. Loading can be either central or eccentric

and both straight and crooked columns can be analyzed. Problems of the types shown in the text in Example Problems 6-1 to 6-6

***Belt Drives and Chain Drives – Chapter 7 – Three modules: V-Belts, U.S. Standards, Synchronous Belts, and Roller Chains ISO 10823***

**Module Group: Belt-, Chain Drives:** These modules are pertinent to Sections 7-4 to 7-7. Each module contains large databases of commercially available products that can be selected for designs of power transmission drives. It is recommended that the use of this module be combined with student use of actual online catalogs of belts, sheaves, chains, and sprockets to specify part numbers and model numbers that can be specified for purchase.

- **Comments: V-Belts, U.S. Standards:** This module is pertinent to Section 7-4 in the text. Data entry and calculations are modeled after the method demonstrated in the book in Example Problem 7-1. Users are given options for selecting the belt size (3V, 5V, or 8V) and Figure 7-9 is used by the program to suggest a choice, which may be overridden by selecting another size. Selections for ‘Driver’ and ‘Driven machine’ types are identical to those used in Table 7-1 in the text. A design value for center distance is selected by the user after being given nominal minimum and maximum values. Then the user is presented with a set of optional combination of sheave sizes from which one must be chosen. That design is then evaluated and output data show the results. Iterations can be done easily by restarting the calculation and making modified selections.
- **Comments: Synchronous Belts:** This module approximates the methodology described in Section 7-5 of the text. Belts of the styles shown in Figure 7-18, both metric and U.S. sizes, are selected and analyzed by the program. Most data are shown in metric units for either style, although pull-down menus permit some features to be shown in U.S. units. The program is quite powerful, allowing multiple pulleys to be driven by one belt in serpentine arrangement. Most applications in this book will include two and only two pulleys. Data are input in tabular style and some practice may be required to become familiar with the details. It is recommended that the center of the driver pulley (No. 1) be positioned at  $x = 0$ ,  $y = 0$  on the coordinate system shown in the graphic aids. Then position the center of the driven pulley (No. 2; called a ‘jockey pulley’ in the module) at  $x =$  desired center distance and  $y = 0$ . Entering  $0^\circ$  for the ‘Displacement angle’ will place the driven pulley to the right of the driver pulley. Enter 0 values for the ‘max effective  $\phi$ ’ for each pulley and select ‘within’ for the ‘Location’ because the pulleys are positioned within the belt. For loads, it is normal to specify the power input to pulley 1 and the power output from pulley 2 to be equal and positive numbers. For U.S. data, select ‘hp US’ as the unit for power. Then leave the ‘Torque’ and ‘Tangential force’ entries as zero. The ‘Load Factor’ should be obtained from Table 7-1 (the same table as used for V-Belt drives). Then start the calculation. You will be presented screens offering choices for belt style, belt length, and belt width and you would normally select the nominal value offered for initial trials. Other values can be tried for subsequent trials until a satisfactory design is achieved.

- Comments: Chain Drives – Roller Chains ISO 10823:** This module is pertinent to Section 7-7 of the text. It uses an ISO standard approach to the selection of chain drives that produces recommend chain sizes from ISO 606 as shown in Table 7-6 of the text. These sizes are identical to standard U.S. sizes for chain pitch as shown in the table. The performance analysis may differ slightly from the methods shown in the text. When using U.S. units for input data, ensure that Power is in ‘hp US’ and that units for other pertinent input and output data are expressed in the desired units, typically rpm for rotational speed and inches for dimensions. Selecting first the input page option: ‘Selection and calculation of one chain’ will result in a selection table being presented with options for different chain pitches and number of strands. Each design will list the rated power of the drive and the ‘Utilization’ (the value of the required corrected power to the rated power of the design, expressed as a percentage). The ‘Application factor to allow for the operating conditions,  $f_1$ , is similar to the ‘Service factors’ shown in the book in Table 7-10. It is recommended that users select the option ‘Allow adjustment of factors  $f_1$  and  $f_2$ ’ (Yes). Then manually enter the service factor for  $f_1$  and set  $f_2 = 0$ . This will match most closely to the methods used in the text. Common U.S. practice generally does not use a ‘Factor for number of teeth on drive sprocket’ ( $f_2$ ).

### ***Keys and Keyseats – Chapter 11***

**Module group: Shafts-Hub Connections:** This module performs the calculations as described in Section 11-4 in the text, similar to Example Problem 11-1. Also included are calculations for the dimensioning variables  $Y$ ,  $S$ , and  $T$  from Figure 11-2. Users enter the shaft diameter and either the torque of the combination of power and rotational speed. The yield strength of the key, shaft, and hub are entered or can be selected from a list of possible materials. After specifying the design factor to be used, the module produces the calculated results.

### ***Rolling Contact Bearings – Chapter 14***

**Module group: Roller Bearings - Ball and Roller Bearings:** The primary features of this module and its use are described in Section 14-11 of the text. The module provides access to a prominent manufacturer’s entire catalog for many types of bearings.

### ***Plain Surface Bearings – Chapter 16***

**Module group: Journals – Plain Surface/Journals, US Standards:** This module is patterned after Section 16-5 of the text – Design of Boundary-Lubricated Bearings. Problems of the types shown in Example Problems 16-1 and 16-2 are solved using this module. After entering data for radial load on the bearing, the rotational speed of the shaft, the selected trial value for shaft diameter, and a trial value of the L/D ratio, the program calculates the actual length,  $L$ , the bearing pressure,  $p$ , the surface speed,  $V$ , and the  $pV$  value, and the design value of  $pV$

(23calculated  $pV$ ). It then searches a modest table of possible materials similar to that shown in Table 16-1 of the text for one that has a rated  $pV$  value closest to but more than the design  $pV$  value. The nominal diametral clearance for the bearing is also computed, using data shown in Figure 16-4 of the text.

- ***Please note that the label for the Diameter value on the Input page of this module uses the term, 'Nominal minimum diameter of the journal,  $D_{min}$ '. That is not the same as the minimum acceptable shaft diameter (based on shaft stress analysis). It should be the actual trial diameter for the shaft that is selected by the user and that must be greater than the minimum acceptable value based on strength.***
- A recommended use for this module is as an aid in selecting commercially available sizes and materials for plain surface bearings from vendors such as those listed in the Internet sites for Chapter 16, particularly sites: 2 – Thomson Engineering & Polymers; 3 – Saint-Gobain Performance Plastics; 4 – GGB Bearing Technology; 5 – Graphite Metallizing Corporation; 6 – Beemer Precision, Inc.; and 7 – Bunting Bearings Corporation. These sites offer catalog data for their products and they list the design  $pV$  values for the various materials from which the bearings are made.
- For use of catalog data, users should select a preliminary value for the internal diameter and length for a particular bearing and make note of the design  $pV$  value for the selected material. Then compute the actual  $L/D$  ratio. Then enter the dimensions into the MDESIGN module ( $D$  and  $L/D$ ), along with other given data for bearing load and rotational speed. The computed required "Design value of  $pV$  factor" should then be compared with the catalog-listed value for the selected material. The suggested material given on the module output page should be ignored. Iterations are easily and quickly done by trying other sizes until an optimum design is identified.

**MACHINE ELEMENTS IN MECHANICAL DESIGN**  
**Fifth Edition**

**Robert L. Mott**

**Prentice-Hall Publishing Company**

**Description of Spreadsheets Included with the Instructors Manual**

### **Introduction**

The Instructors Manual for this book contains a set of 26 computational aids that are keyed to the book. The files are written as Microsoft Excel spreadsheets.

Many of the spreadsheets appear in the text. Others were prepared to produce solutions for the Solutions Manual. The given spreadsheets include data and results from certain figures in the text, from certain example problems, or for certain problems from the end of chapters containing the analysis and design procedures featured in the programs.

The following sections give brief descriptions of each spreadsheet. Many are discussed in the text in more extensive detail. It is expected that you will verify all of the elements of each spreadsheet before using them for solutions to specific problems.

### **Using the Spreadsheets:**

- *It is recommended that you maintain the given spreadsheets as they initially appear on the disk, considering them to be master copies.*
- *To use a program for solving other problems, call it up in Excel and use the "Save as" command to give it a different name.*
- *For instance, the original program called Column Analysis should be considered the master. Use "Save as" and call it, for example, Column Analysis – Working. Then use that version for general problem solving.*

You should study the concepts and the solution techniques for each type of problem before using the spreadsheets. You should work sample problems by hand first. Then enter the appropriate data into the spreadsheet to verify the solution. In most spreadsheets in the text, the data that need to be entered are identified by gray-shaded areas and by italic type.

### **Descriptions of Spreadsheets**

The descriptions are given here in the order that the subjects for the spreadsheets are covered in the text.

**Column Analysis:** Chapter 6. Analyzes straight columns of uniform cross section to determine the critical buckling load and the allowable load. The spreadsheet shows results for Example Problem 6-1. U.S. Customary units are used. A description is given in Section 6-8. The process is essentially the same as that shown in the flow chart of Figure 6-4. Note that a short macro program in Visual Basic is used to decide whether the column is *long* (Euler) or *short* (J. B. Johnson) and to complete the calculation of the critical buckling load. Be sure that your Excel program enables macros.

**Column Analysis SI:** Chapter 6. Same as **Column Analysis:** except SI units are used. The solution to Example Problem 6-2 is given as an example for data entry.

**Circular Column Analysis:** Chapter 6. Special version of **Column Analysis** in which the geometric properties of a column with a solid circular cross section are computed when the diameter is input. The spreadsheet can be used as an iterative design tool to determine the required diameter of a column with a circular cross section to carry a given load. See Figure 6-14.



**Crooked Column Analysis:** Chapter 6. Section 6-11. Analyzes the allowable load on a column of constant cross section with a given amount of crookedness. Data from Example Problem 6-4 are used as shown in Figure 6-16 on page 252.

**Eccentric Column Analysis:** Chapter 6. Section 6-12. Computes the required yield strength of the material and the resulting maximum deflection of the middle of a column that is loaded eccentrically. Data from Example Problem 6-6 are used as shown in Figure 6-18.

**Chain Drive Design:** Chapter 7. Design of roller chain drives as described in Section 7-6. User must obtain rated power data from Tables 7-7, 7-8, or 7-9 to specify a suitable chain number and number of teeth in the smaller sprocket. A service factor must be selected from Table 7-10 in the text. Data from Example Problem 7-4 are shown in the master spreadsheet.

**Gear Geometry:** Chapter 8. Computes the geometric features of spur and helical gears using the relationships in Sections 8-4 and 8-6. Can be used for Problems 1-9 and 41-44.

**Contact Ratio-Spur Gears:** Chapter 8. Computes the contact ratio for spur gears using the procedure shown in Section 8-4.

**Bevel Gear Geometry:** Chapter 8. Computes the geometric features of straight bevel gears using the formulas listed in Table 8-8 in Section 8-7 and illustrated in Example Problem 8-3. Two identical programs are shown side-by-side. One shows the results of Example Problem 8-3 and the other can be used to solve any given problem.

**Wormgearing Geometry, C, VR:** Chapter 8. Computes essential geometric features of a worm and wormgear, the center distance ( $C$ ) between their shafts, and the velocity ratio,  $VR$ . Uses procedure from Section 8-9 as illustrated in Example Problem 8-4. The spreadsheet was used to complete Problems 52-57 at the end of the chapter.

**Gears VR Design:** Chapter 8. Aids in the specification of the number of teeth in a pinion and gear to produce a specified velocity ratio. Uses a procedure similar to that shown in Section 8-11 and illustrated in Table 8-10. An integer is entered for the number of teeth in the pinion. The program computes the required approximate number of teeth in the gear to produce the given velocity ratio. The user then enters an integer for the actual number of gear teeth. The program identifies the combination of numbers of teeth that produces the minimum differential between the desired ratio and the actual ratio. The spreadsheet was used to complete Problems 62-65 at the end of the chapter.

**Spur Gear Forces:** Chapter 9. Computes the tangential, radial, and normal forces on spur gear teeth of a given design transmitting a given power at a given pinion speed. It uses the method of Section 9-3. The spreadsheet was used to complete Problems 1-6 at the end of Chapter 9. The results for Problems 1 and 2 are shown in the master.

**Spur Gears-Design-U.S.:** Chapter 9. Performs a complete design analysis for a pair of spur gears, including the essential geometry, tangential force, required bending stress number, and required contact stress number. All modifying factors for stress calculations as described in Sections 9-8 to 9-11 are included. The data from Example Problem 9-4 are shown in the given spreadsheet as illustrated in Figure 9-25. An extensive discussion of the spreadsheet is given in Section 9-13. A feature of the spreadsheet is the computation of the required hardness (HB) for through-hardened Grade 1 steel using the equations in Figures 9-11 and 9-12. The user can then specify suitable materials and list them at the bottom of the spreadsheet.

**Geometry Factor-I-Pitting:** Chapter 9. Computes the value of the geometry factor,  $I$ , used in the calculation of contact stress for spur gears in Equation 9-23. Program uses the algorithm from Appendix A18.

**Spur Gears-Design-U.S.-With I:** Chapter 9. Same as **Spur Gears-Design** except the geometry factor,  $I$ , is computed within the program instead of being input by the user. The program **Geometry Factor-I-Pitting** is integrated within **Spur Gears-Design**. One additional input value is needed for the pressure angle,  $\phi$ .

**Spur Gears-Design-SI:** Chapter 9. Similar to *Spur Gears-Design:* except SI metric data are used as described in Section 9-13 and illustrated in Example Problem 9-5. Data from Example Problem 9-5 are used in the given spreadsheet.

**Spur Gears-Capacity-U.S.:** Chapter 9. Section 9-15. Determines the power transmitting capacity of a given set of spur gears considering both bending strength and pitting resistance. The user must input the allowable bending stress and allowable contact stress based on the material specified for the pinion and the gear using Figures 9-11 and 9-12 and Table 9-5. The spreadsheet includes the computation of the required bending stress number,  $s_{ab}$ , and contact stress number,  $s_{ac}$ , based on user-entered hardness (HB) for through-hardened Grade 1 steel using the equations in Figures 9-11 and 9-12. The user must transcribe these values into the spreadsheet if, in fact, this kind of material is specified.

**Plastic Gears- Design:** Chapter 9. Completes the design of plastic gears using the procedure from Section 9-16. Data are shown for Example Problem 9-7.

**Helical Gears-Design:** Chapter 10. Computes the forces on helical gear teeth as described in Section 10-2 and illustrated in Example Problem 10-1. Completes the design analysis for a pair of helical gears as described in Sections 10-3 to 10-5 and illustrated in Example Problem 10-3. Used for the solutions to Problems 1-11 at the end of Chapter 10.

**Helical Gears-Capacity:** Chapter 10. Similar to *Spur Gears-Capacity:* with modifications for the special geometry of helical gear teeth. Used for the solutions to Problems 12 and 13 at the end of Chapter 10. The user must input the allowable bending stress and allowable contact stress based on the material specified for the pinion and the gear using Figures 9-11 and 9-12 and Table 9-5. The spreadsheet includes the computation of the required bending stress number,  $s_{ab}$ , and contact stress number,  $s_{ac}$ , based on user-entered hardness (HB) for through-hardened Grade 1 steel using the equations in Figures 9-11 and 9-12. The user must transcribe these values into the spreadsheet if, in fact, this kind of material is specified.

**Bevel Gears – Design:** Chapter 10. Computes forces and stresses on bevel gears using the methods shown in Section 10-9.

**Wormgearing – Design:** Computes worm and wormgear geometry values, forces, and stresses for wormgearing, using methods and data from Chapter 8 (Section 8-9) and Chapter 10 (Sections 10-10, 10-11, and 10-12). The master spreadsheet uses data from Example Problems 8-4, 10-9, and 10-10.

**Keyseat Data:** Chapter 11. Computes the data required to dimension keyseats and keyways on shaft drawings according to the information in Figure 11-2.

**Shaft Design:** Chapter 12. Computes the minimum acceptable diameter for shafts using Equation 12-24 when both bending and torsion are present and Equation 12-16 when only vertical shearing stress is present. Requires prior analysis for torques, forces, bending moments, pertinent material strengths, modifying factors on material strength, and stress concentration factor. The program is typically applied at several selected sections of the shaft as illustrated in Design Example 12-1 in Section 12-6. If the location being analyzed has a retaining ring installed, the computed minimum shaft diameter is considered to be for the base of the ring groove. The spreadsheet computes the nominal full shaft diameter by applying a factor of 1.06 as described at the end of Section 12-4. The data used in the master spreadsheet are for one location on the shaft in Design Example 12-1 as illustrated in Figure 12-19 in Section 12-9 where the spreadsheet and its use are described.

**Force Fits:** Chapter 13, Section 13-8. Stresses for Force Fits. Computes the pressure at the interface between mating members assembled with an interference fit (See Section 13-6.) Also computes the resulting stresses and deformations for the mating members using the procedure in Section 13-8. Data from Example Problem 13-2 are shown in the example.

**Spring Design-Method 1:** Chapter 18, Section 18-6. The given spreadsheet uses data and the method from Example Problem 18-2 to design a safe helical compression spring for a given loading and to fit given geometrical limitations. See Figure 18-16 and the accompanying discussion.

**Spring Design-Method 2:** Chapter 19. Similar to *Spring Design-Method 1* without the restriction of designing to a set of geometrical limitations. See Example Problem 18-3, Figure 18-17, and the accompanying discussion.

## CHAPTER 1 THE NATURE OF MECHANICAL DESIGN

Problems 1 - 14 require the specification of functions and design requirements for design projects and have no unique solution.

15.  $D = 1.75 \text{ in.} \times 25.4 \text{ mm/in} = \underline{44.5 \text{ mm}}$

16.  $L = 46 \text{ FT} \times 0.3048 \text{ m/FT} = \underline{14.0 \text{ m}}$

17.  $T = 12\,550 \text{ LB}\cdot\text{IN} \times 0.1130 \text{ N}\cdot\text{m/LB}\cdot\text{IN} = \underline{1418 \text{ N}\cdot\text{m}}$

18.  $A = 4.12 \text{ IN}^2 \times 645.2 \text{ mm}^2/\text{IN}^2 = \underline{2658 \text{ mm}^2}$

19.  $Z = 14.8 \text{ IN}^3 \times 1.639 \times 10^4 \text{ mm}^3/\text{IN}^3 = \underline{2.43 \times 10^5 \text{ mm}^3}$

20.  $I = 88.0 \text{ IN}^4 \times 4.162 \times 10^5 \text{ mm}^4/\text{IN}^4 = \underline{3.66 \times 10^7 \text{ mm}^4}$

21. GIVEN  $A_{\text{MIN}} = 750 \text{ mm}^2$ ; IN U.S. UNITS:  $A_{\text{MIN}} = 1.162 \text{ IN}^2$   
 APP. 15-1:  $L = 2 \times 2 \times \frac{3}{8}$ ,  $A = 1.36 \text{ IN}^2 = 890 \text{ mm}^2$   
 APP 15-3: ANGLES  $50 \times 100 \times 6$  AND  $75 \times 75 \times 5$  HAVE  $A = 864 \text{ mm}^2$

22.  $P = 7.5 \text{ hp} \times 745.7 \text{ W/hp} = 5.59 \times 10^3 \text{ W} = \underline{5.59 \text{ kW}}$

23.  $S_m = 127 \text{ ksi} \times 6.895 \text{ MPa/ksi} = \underline{876 \text{ MPa}}$

24. LET  $D = 0.035 \text{ m}$ ;  $L = 0.675 \text{ m}$ ; VOLUME  $= V = A \times L = \left(\frac{\pi D^2}{4}\right) \times L$   
 $V = \frac{\pi (0.035 \text{ m})^2}{4} \times 0.675 \text{ m} = 6.49 \times 10^{-4} \text{ m}^3$

MASS  $= \text{DENSITY} \times V = 7680 \text{ kg/m}^3 \times 6.49 \times 10^{-4} \text{ m}^3 = 4.98 \text{ kg}$

WEIGHT  $= m \times g = 4.98 \text{ kg} \times 9.81 \text{ m/s}^2 = 48.9 \text{ kg}\cdot\text{m/s}^2 = \underline{48.9 \text{ N}}$

$$25. \quad T = 180 \text{ LB}\cdot\text{IN} \times 0.1130 \text{ N}\cdot\text{m}/\text{LB}\cdot\text{IN} = \underline{20.3 \text{ N}\cdot\text{m}}$$

$$\theta = 35^\circ \times \pi \text{ RAD}/180^\circ = \underline{0.611 \text{ RAD.}}$$

$$\text{SCALE} = T/\theta = 180 \text{ LB}\cdot\text{IN}/35^\circ = \underline{5.14 \text{ LB}\cdot\text{IN}/\text{DEGREE}}$$

$$\text{SCALE} = T/\theta = 20.3 \text{ N}\cdot\text{m}/0.611 \text{ RAD.} = \underline{33.3 \text{ N}\cdot\text{m}/\text{RAD.}}$$

$$26. \quad \text{ENERGY} = \text{POWER} \times \text{TIME}$$

$$E = 12.5 \text{ hp} \times \frac{16 \text{ h}}{\text{DAY}} \times \frac{5 \text{ DAYS}}{\text{WEEK}} \times \frac{52 \text{ WKS}}{\text{YEAR}} \times \frac{550 \text{ FT}\cdot\text{LB}}{\text{s}\cdot\text{hp}} \times \frac{3600 \text{ s}}{\text{h}}$$

$$E = \underline{1.03 \times 10^8 \text{ FT}\cdot\text{LB}/\text{YEAR}}$$

$$E = 1.03 \times 10^8 \frac{\text{FT}\cdot\text{LB}}{\text{YEAR}} \times \frac{1.356 \text{ J}}{\text{FT}\cdot\text{LB}} \times \frac{1.0 \text{ N}\cdot\text{m}}{\text{J}} \times \frac{1.0 \text{ W}}{\text{N}\cdot\text{m}/\text{s}} \times \frac{1 \text{ h}}{3600 \text{ s}}$$

$$E = 38.8 \times 10^6 \text{ W}\cdot\text{h}/\text{YEAR} = \underline{38.8 \text{ MW}\cdot\text{h}/\text{YEAR}}$$

$$27. \quad \text{VISCOSITY} = \mu = 3.75 \text{ REYN} \times \frac{1.0 \text{ LB}\cdot\text{s}}{\text{IN}^2\cdot\text{REYN}} \times \frac{144 \text{ IN}^2}{\text{FT}^2} = \underline{540 \frac{\text{LB}\cdot\text{s}}{\text{FT}^2}}$$

$$\mu = 3.75 \frac{\text{LB}\cdot\text{s}}{\text{IN}^2} \times \frac{4.448 \text{ N}}{\text{LB}} \times \frac{1.0 \text{ IN}^2}{645.2 \text{ mm}^2} \times \frac{10^6 \text{ mm}^2}{\text{m}^2} = \underline{25.9 \times 10^3 \frac{\text{N}\cdot\text{s}}{\text{m}^2}}$$

$$28. \quad \text{LIFE} = \frac{1750 \text{ REV}}{\text{MIN}} \times \frac{24 \text{ h}}{\text{DAY}} \times \frac{60 \text{ MIN.}}{\text{h}} \times \frac{365 \text{ DAYS}}{\text{YEAR}} \times 5 \text{ YEARS}$$

$$\text{LIFE} = \underline{4.60 \times 10^9 \text{ REVOLUTIONS}}$$

## CHAPTER 2

### MATERIALS IN MECHANICAL DESIGN

1. Ultimate tensile strength is the apparent stress at the peak of the stress-strain curve.
2. Yield point is the value of the apparent stress from the stress-strain curve at which there is a large increase in strain with no increase in stress. It is the point where the stress-strain curve exhibits a horizontal slope.
3. Yield strength is the apparent stress from the stress-strain curve at which there is a large increase in strain with little increase in stress for materials that do not exhibit a yield point. The offset method is used by drawing a line parallel to the straight part of the stress-strain curve through a value of 0.2% on the strain axis.
4. Many low alloy steels exhibit a yield point.
5. The proportional limit is the apparent stress on the stress-strain curve at which the curve deviates from a straight line. At this value, the material is usually still elastic. The elastic limit is the apparent stress at which the material is deformed plastically and will not return to its original size and shape.
6. Hooke's law applies to that portion of the stress-strain curve that is a straight line for which stress is proportional to strain.
7. The modulus of elasticity is a measure of the stiffness of a material.
8. The percent elongation is a measure of the ductility of a material.
9. The material is not ductile. Materials having a percent elongation greater than 5% are considered to be ductile.
10. Poisson's ratio is the ratio of the lateral strain in a material to the axial strain when subjected to a tensile load.
11. From Eq. 2-5  $G = E/[2(1+\nu)] = (114 \text{ GPa})/[2(1+0.33)]$   
 $G = 42.9 \text{ GPa}$



12. Hardness = 52.8 HRC (Approximate; Appendix 17)
13. Tensile strength = 235 ksi (Approximate; Appendix 17)

14.-17. Errors in given statements:

14. A hardness of HB 750 is extremely hard, characteristic of the hardest steels in the as-quenched or surface hardened condition. Appendix 3 shows annealed steels to have hardness values in the approximate range of HB 120 to 230.
15. Hardness on the HRB scale is normally limited to HRB 100.
16. Hardness on the HRC scale is normally no lower than HRC 20.
17. The relationship between hardness and tensile strength is only valid for steels.
18. Charpy and Izod tests measure impact strength.
19. Iron and carbon. Other elements are often present.
20. In addition to iron and carbon SAE 4340 steel contains nickel, chromium, and molybdenum. (Table 2-8)
21. Approximately 0.40% carbon in SAE 4340 steel.
22. Low-carbon: Less than 0.3%  
Medium-carbon: 0.30% to 0.50%  
High-carbon: 0.50% to 0.95%
23. Typically a bearing steel contains 1.0% carbon.
24. Lead is added to SAE 12L13 steel to improve machinability.
25. Shafts are often made from SAE 1040, 4140, 4640, 5150, 6150, and 8650 steels. (Table 2-9)
26. Gears are often made from SAE 1040, 4140, 4340, 4640, 5150, 6150, and 8650 steels. (Table 2-9)
27. The blades of a post hole digger should have good wear resistance, high strength, and good ductility. SAE 1080 steel is a reasonable choice.
28. SAE 5160 OQT 1000 is a high-carbon, chromium steel, containing approximately 0.60% carbon and 0.80% chromium. It was heat treated by heating above its upper critical temperature, quenched in oil, and then tempered at 1000 degrees Fahrenheit. It has fairly high strength ( $s_y = 151$  ksi or 1040 MPa) and good ductility (14% elongation).

29. In general, a high hardness with good ductility are desirable for machine parts and tools subjected to impact loads as seen by a shovel. A hardness of HRC 40 corresponds to approximately HB 375 and is considered moderately hard. While this is a good level, even a higher value up to HRC 50 (HB 475) would be better, provided ductility is fairly high, say about 15% elongation. Appendix 3 shows some forms of oil-quenched SAE 1040 and none listed have sufficiently high hardness. Appendix 4-1 shows the same material quenched in water and tempered. SAE 1040 WQT 700 has a hardness of HB 401 (HRC 43) with approximately 20% elongation and a yield point of 92 ksi.
30. Through hardening involves heating the entire part followed by quenching to achieve the hardened condition. Except for some variation in thick sections, the part is hardened throughout. But no chemical composition changes occur. In carburizing, the chemical composition of the surface is changed by the infusion of carbon. Thus, carburizing results in a hard surface while the core is softer.
31. Induction hardening is a heat treating process in which the area to be hardened is subjected to a high-frequency electric current created by a coil, inducing current flow near the surface of the part and causing local heating. After sufficient time to bring the surface to a temperature above the upper critical temperature of the material, the part is quenched to harden the surface.
32. Some carburizing grades of steels are SAE 1015, 1020, 1022, 1117, 1118, 4118, 4320, 4620, 4820, 8620 and 9310. The carbon content ranges from 0.10% to 0.20%. App. A-5.
33. The AISI 200 and 300 series of stainless steels are nonmagnetic.
34. Chromium gives stainless steels good corrosion resistance.
35. ASTM A992 structural steel is used for most wide-flange beams.
36. HSLA structural steels are high-strength, low-alloy steels having yield strengths in the range of 42 - 100 ksi (290 - 700 MPa).
37. Three types of cast iron are gray iron, ductile iron, and malleable iron.
38. ASTM A48 , Grade 30 is a gray iron with a tensile strength of 30 ksi (207 MPa); no yield strength; less than 1% elongation (brittle); modulus of elasticity (stiffness) of  $169 \times 10^6$  psi (117 GPa).

Problem 38. (continued)

ASTM A536 Grade 100-70-03 is a ductile iron with a tensile strength of 100 ksi (689 MPa); a yield strength of 70 ksi (483 MPa); 3% elongation (brittle); modulus of elasticity (stiffness) of  $24 \times 10^6$  psi (165 GPa).

ASTM A47, Grade 32510 is a malleable iron with a tensile strength of 50 ksi (345 MPa); a yield strength of 32.5 ksi (224 MPa); 10% elongation (ductile); modulus of elasticity (stiffness) of  $25 \times 10^6$  psi (172 GPa).

ASTM A220, Grade 70003 is a malleable iron with a tensile strength of 85 ksi (586 MPa); a yield strength of 70 ksi (483 MPa); 3% elongation (brittle); modulus of elasticity (stiffness) of  $26 \times 10^6$  psi (179 GPa).

39. Powdered metals are preformed in a die under high pressure and sintered at a high temperature to fuse the particles. Re-pressing after sintering is sometimes used.
40. Parts made from Zamak 3 zinc casting alloy typically have good dimensional accuracy and smooth surfaces, a tensile strength of approximately 41 Ksi (283 MPa), a yield strength of 32 Ksi (221 MPa), 10% elongation, and a modulus of elasticity of  $12.4 \times 10^6$  psi (85 GPa). (Appendix 10)
41. Type D tool steels are typically used for stamping dies, punches, and gages. (Table 2-11)
42. The suffix O on aluminum 6061-O indicates the annealed condition.
43. The suffix H on aluminum 3003-H14 indicates that it was strain hardened.
44. The suffix T on aluminum 6061-T6 indicates that it was heat treated.
45. Aluminum 7075-T6 has the highest strength; tensile strength = 88 ksi (607 MPa); yield strength = 78 ksi (538 MPa).
46. Aluminum alloy 6061 is one of the most versatile.
47. Three typical uses of titanium alloys are aerospace structures, chemical processing equipment, and marine hardware.
48. Bronze is an alloy of copper with tin, aluminum, lead, phosphorus, nickel, zinc, manganese, or silicon.

49. Bronze C86200 is a manganese bronze casting alloy with a tensile strength of 95 ksi (655MPa); yield strength of 48 ksi (331 MPa); 20% elongation (ductile); modulus of elasticity of  $15 \times 10^6$  psi (103 GPa).
50. Bronze is used for gears and bearings.
51. Thermosetting plastics undergo a chemical change during forming resulting in a structure of cross-linked molecules. The process cannot be reversed or repeated. Thermoplastic materials can be formed repeatedly by reheating because the molecular structure is essentially unchanged during processing.
52. a) Gears: Nylon, polycarbonate, acetal, <sup>PET,</sup> polyurethane elastomer, phenolic. b) Helmets: ABS and polycarbonates. c) Transparent shield: Acrylic. d) Structural housing: <sup>PET,</sup> ABS, polycarbonate, acrylic, PVC, phenolic, polyester/glass composite. e) Pipe: ABS, PVC. f) Wheels: Polyurethane elastomer. g) Switch parts: polyimide, phenolic, <sup>PET.</sup>
53. Designers of parts to be made from composite materials can control 1) base resin, 2) reinforcing fibers, 3) amount of fibers, 4) orientation of fibers, 5) number of layers, 6) overall thickness, 7) orientation of layers, 8) combinations of types of materials.
54. Composite materials are comprised of two or more different materials, typically a resin reinforced by fibers.
55. Resins used for composites include polyesters, epoxies, polyimides, <sup>PHENOLICS (ALL THERMOSETS) + THERMO PLASTICS: PE, PA,</sup> PEEK, PPS, PVC.
56. Reinforcing fibers used for composites are glass, boron, aramid, and carbon/graphite.
57. Sporting equipment is made from glass/epoxy, boron/epoxy, and graphite/epoxy composites.
58. Aerospace structures are made from glass/epoxy, boron/epoxy, graphite/epoxy, and aramid/epoxy composites.
59. Sheet molding compound is typically a glass/polyester composite.
60. SMC's are used for auto and truck body panels and large housings.
61. Reinforcing fibers are produced as continuous filaments, chopped fibers, roving, fabric, yarn, and mats.

62. Wet processing of composites involves the layup of fabric reinforcing sheets on a form, saturation of the sheets with the resin, and curing under heat and pressure.
63. Preimpregnated composite materials are produced with the resin already on the fibers in a convenient form, called a prepreg. The prepreg is layered onto the form and cured.
64. SMC's are preimpregnated fabric sheets formed in a mold and cured simultaneously under heat and pressure.
65. Pultrusion is a process of coating the fiber reinforcement as it is pulled through a heated die to produce a continuous form such as tubing, structural shapes, rod, and hat sections used to stiffen aircraft structures.
66. In the filament winding process, continuous filaments are placed around a mandrel in a controlled pattern and then cured. The process is used for pipe, pressure vessels, rocket motor cases, containers and enclosures.
67. Specific strength is the ratio of the strength of a material to its specific weight.
68. Specific stiffness is the ratio of the modulus of elasticity of a material to its specific weight.
69. Many composites have significantly higher values of specific strength and specific stiffness than metals.

70 - 73 refer to Figure 2-23 and Table 2-17.

General conclusions from Questions 70 - 73: The specific strengths of the metals listed range from  $0.194 \times 10^6$  to  $1.00 \times 10^6$  in, approximately a factor of 5.0. The specific stiffnesses are very nearly equal for all metals listed, approximately  $1.0 \times 10^8$  in. The specific strengths of the composites listed range 1.87 to  $4.88 \times 10^6$  in, much higher than any of the metals. Glass/epoxy has a specific stiffness about 2/3 that of the metals. The other composites listed range from 2.2 to 8.3 times as stiff as the metals.

See Section 2-17 for answers to Questions 74 to 100.



### ***Supplementary Problems – Chapter 2***

1. Poisson's ratio: a) Carbon steel – 0.29; c) Lead – 0.43; e) Concrete – 0.10 to 0.25
2. See Section 2-2, subsection: Flexural Strength and Modulus, and Figure 2-5.
3. Erosive, abrasive, adhesive, fretting, surface fatigue
4. From Table 2-6: 14 alloys listed, Examples: ASTM A36, SAE 1018 HR or CD, SAE 1045 HR or CD, SAE 8620 CD.
5. From Table 2-6: SAE 304 and SAE 316
6. From Table 2-6: Six alloys listed, Examples: 2024-T4, 3003-H14, 6061-T6, 6063-T6
7. From Section 2-3: ASTM International, AISI, SAE
8. From Section 2-3: Aluminum Association
9. From Table 2-7: a) DIN 42CrMo4 or W-1.7225; b) BS 708A42; c) EN 42CrMo4; d) GB ML42CrMo4; e) JIS SCM 440H
10. From Table 2-7: a) DIN C45 or W-1.0503; b) BS 060A47; c) EN C45; d) GB 699-45; e) JIS S45C
11. From Table 2-7: a) DIN X6Cr17 or W-1.4016; b) BS 430S17; c) EN X6Cr17; d) GB ML1Cr17; e) JIS SUS430
12. From Table 2-7: a) DIN AlZnMgCu1.5 or W-3.4365; b) BS L.95, L.96; c) EN AlZn6MgCu
13. Water, brine, mineral oil, water-soluble polyalkylene glycol (PAG)
14. From Section 2-6: Fine steel or cast iron shot is projected at high velocity on critical surfaces to produce residual compressive stress that tends to improve the fatigue strength.
15. From Table 2-10: ASTM A27/A27M; A915/A915M; A128/A128M; A148/A148M
16. From Table 2-10: ASTM A757; ASTM A351; ASTM A216; ASTM A389
17. Carbide austempered ductile iron – used for: railroad rolling stock, earthmoving equipment, agricultural machinery, crushers
18. From Section 2-10: White iron is made by rapidly chilling a casting of gray iron or ductile iron during solidification. ASTM Standard A532 describes the process. Used to improve wear resistance for ball mills, crushers, mixing equipment, and material handling devices.
19. From Section 2-11: Powders are pressed to their basic form and then heated to sinter the powder particles into a strong solid.
20. From Section 2-11: Powders are compressed by a flexible membrane in a hermetic chamber to produce a high density; may be done cold or at elevated temperatures.
21. From Section 2-11: Metal powders are fed into an injection molding machine to form a green part that is then sintered to complete the solidification and bonding processes.
22. From Section 2-11: Metal powders are first pressed and sintered, then forged in a closed-die press to achieve final form and properties.
23. From Table 2-12: Carbon steel F-0008-HT,  $s_u = 85$  ksi (590 MPa);

- Low-alloy steel FL-4405-HT,  $s_u = 160$  ksi (1100 MPa);  
 Diffusion-alloyed steel FD-0205-HT,  $s_u = 130$  ksi (900 MPa);  
 Sinter-hardened steel FLC-4608-HT,  $s_u = 100$  ksi (690 MPa)
24. From Table 2-12: a) Nickel silver – CNZ-1818;  $s_u = 20$  ksi (118 MPa)  
 b) Bronze – CTG-1001; (no strength listed; used for bearings)  
 c) Copper – C-0000; No strength listed; used for electrical applications  
 d) Aluminum -  $s_u = 32$  ksi (221 MPa)
  25. From Section 2-11: Projected surface area less than 50 in<sup>2</sup> (32 000 mm<sup>2</sup>)
  26. From Section 2-12: Aluminum casting alloys: 202, 222, 319, 360, 413, 444, 512, 535, 712, 771, 850, 852. Others available.
  27. From Section 2-12: Aluminum 2014, 2024, 6061
  28. From Section 2-13: Zinc *alloy No. 3* or *Zamak 3*.
  29. From Section 2-13 and Appendix A10-1: Zinc ZA-8,  $s_u = 54$  ksi (374 MPa)  
 ZA-12,  $s_u = 59$  ksi (404 MPa); ZA-27,  $s_u = 61$  ksi (421 MPa)
  30. From Section 2-14: Nickel-based alloys have good corrosion resistance and retain good levels of strength at high temperatures.
  31. From Section 2-15: a) Bearing bronze C93200l; b) Phosphor bronze C54400;  
 c) Muntz metal C37000; d) Manganese bronze C86200;  
 e) Copper-nickel-zinc alloy C96200; f) Manganese bronze C67500
  32. From Section 2-15: H-numbers indicate the degree of hardening by strain hardening methods; a) H04 – Full hard; b) H02 – ½ hard; c) H01 – 1/8 hard; d)H08 – Spring hard
  33. From Section 2-15: TD temper indicates – solution heat treated and cold worked
  34. From Section 2-15 and Figure 2-18: As the percent cold reduction increases, tensile and yield strengths increase and ductility as indicated by percent elongation decreases.  
 10% cold work:  $s_u = 133$  ksi (917 MPa),  $s_y = 121$  ksi (834 MPa), 17% elongation  
 40% cold work:  $s_u = 154$  ksi (1062 MPa),  $s_y = 142$  ksi (979 MPa), 1% elongation
  35. From Section 2-18: Metals, polymers, ceramics, glasses, elastomers, hybrids
  36. From Section 2-18: Foams, sandwich structures, honeycomb structures
  37. From Fig. 2-31: d) Metals, b) ceramics, g) composites, c) polymers, a) wood,  
 h) rubbers/elastomers, f) foams
  38. From Fig. 2-32: b) ceramics, d) metals, g) composites, a) wood, c) polymers, f) foams,  
 h) elastomers
  39. From Figure 3-32: (lightest to heaviest) e) foams, a) wood, h) elastomers, g) composites,  
 d and b) Metals and ceramics (about equal)

## CHAPTER 3 STRESS AND DEFORMATION ANALYSIS

### Direct Tension and Compression

1.  $\sigma = F/A$ ;  $A = \pi (18^2 - 12^2)/4 = 141.4 \text{ mm}^2$   
 $\sigma = 4500 \text{ N} / 141.4 \text{ mm}^2 = 31.8 \text{ N/mm}^2 = \underline{31.8 \text{ MPa}}$   
 $\delta = \frac{PL}{EA} = \frac{(4500 \text{ N})(750 \text{ mm})}{(207 \times 10^9 \text{ N/m}^2)(141.4 \text{ mm}^2)} \times \frac{10^6 \text{ mm}^2}{\text{m}^2} = \underline{0.12 \text{ mm}}$

2.  $\sigma = F/A = 3500 \text{ N} / (\pi (10)^2 / 4) \text{ mm}^2 = \underline{44.6 \text{ MPa}}$

3.  $\sigma = F/A = 20 \times 10^3 \text{ N} / (0.30) \text{ mm}^2 = \underline{66.7 \text{ MPa}}$

4.  $\sigma = F/A = 860 \text{ LB} / (0.40 \text{ in})^2 = \underline{5375 \text{ psi}}$

5.  $\sigma = F/A = 1900 \text{ LB} / \pi (0.375 \text{ in})^2 / 4 = \underline{17,200 \text{ psi}}$

6.  $\sigma = P/A$ ;  $A = (12 \text{ mm})^2 = 144 \text{ mm}^2$ ;  $\sigma = 5000 \text{ N} / 144 \text{ mm}^2 = \underline{34.7 \text{ MPa}}$  (ALL)  
 $\delta = \frac{PL}{EA} = \frac{(5000 \text{ N})(1650 \text{ mm})}{(E \text{ N/mm}^2)(144 \text{ mm}^2)} = \frac{57292}{E} \text{ mm}$

a) AISI 1020;  $E = 207 \text{ GPa} = 207 \times 10^3 \text{ N/mm}^2$ ;  $\delta = \underline{0.277 \text{ mm}}$

b) AISI 8650;  $E = 207 \text{ GPa}$ ;  $\delta = \underline{0.277 \text{ mm}}$

c) DUCTILE IRON;  $E = 165 \text{ GPa}$ ;  $\delta = \underline{0.347 \text{ mm}}$

d) ALUMINUM 6061-T6;  $E = 69 \text{ GPa}$ ;  $\delta = \underline{0.830 \text{ mm}}$

e) TITANIUM Ti-6Al-4V;  $E = 114 \text{ GPa}$ ;  $\delta = \underline{0.503 \text{ mm}}$

f) PVC;  $E = 2410 \text{ MPa} = 2070 \text{ N/mm}^2$ ;  $\delta = \underline{27.7 \text{ mm}}$

g) PHENOLIC;  $E = 7580 \text{ MPa}$ ;  $\delta = \underline{7.56 \text{ mm}}$

NOTE: STRESS IS CLOSE TO THE ULTIMATE FOR f AND g.

7.  $\delta = PL/EA$ ;  $P = \frac{\delta EA}{L}$

$A = (2.25^2 - 2.01^2) \text{ in}^2 = 1.02 \text{ in}^2$

$P = \frac{(0.004 \text{ in})(10 \times 10^6 \text{ lb/in}^2)(1.02 \text{ in}^2)}{16.0 \text{ in.}} = 2556 \text{ LB}$

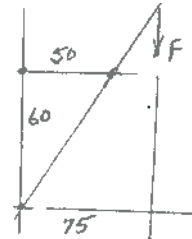


$\sigma = P/A = 2556 \text{ LB} / 1.02 \text{ in}^2 = 2506 \text{ PSI}$

8.  $\sum M_B = 0 = 2500(75) - F_c(60)$

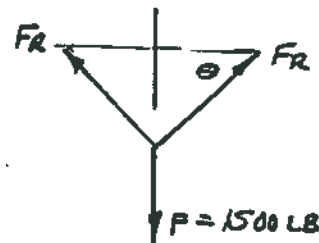
$F_c = 2500(75/60) = 3125 \text{ LB} = \text{TENSILE FORCE IN AC}$

$\sigma = \frac{P}{A} = \frac{3125 \text{ LB}}{(1.50)(3.50) \text{ in}^2} = 595 \text{ PSI}$



9.  $2 F_R \sin \theta = 1500 \text{ LB}$

$F_R = 1500 / (2 \sin(45)) = 1061 \text{ LB}$



10.  $\sigma = P/A$ ; REQ'D.  $A = P/\sigma_{\text{ALLOW}}$

$A = \frac{1061 \text{ LB}}{18000 \text{ LB/in}^2} = 0.0589 \text{ in}^2 = \pi D^2/4$

$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(0.0589)}{\pi}} = 0.274 \text{ in. MINIMUM}$

11.  $\theta = 15^\circ$ ;  $F_R = \frac{1500 \text{ LB}}{2 \sin \theta} = \frac{1500 \text{ LB}}{2 \sin(15)} = 2898 \text{ LB}$

$A = 2898 / 18000 = 0.161 \text{ in}^2$ ;  $D = \sqrt{\frac{4(161)}{\pi}} = 0.453 \text{ in}$

12.

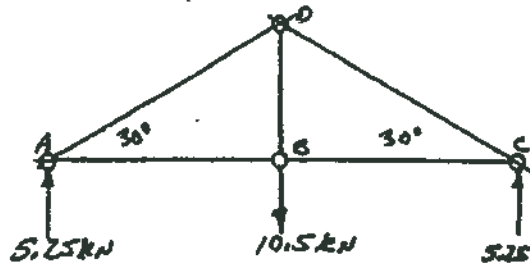
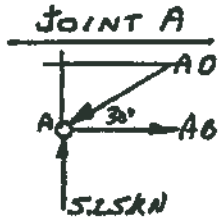
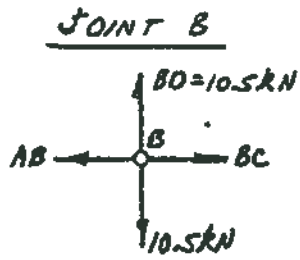


FIG. P2-12

$$AD \sin 30^\circ = 5.25 \text{ kN}$$

$$AO = 10.5 \text{ kN} = CO$$

$$AB = AO \cos 30^\circ = 9.09 \text{ kN} = BC$$

STRESSES:

$$AB, BC: \sigma_{AB} = \sigma_{BC} = \frac{9.09 \times 10^3 \text{ N}}{(2 \times 30) \text{ mm}^2} = \underline{25.3 \text{ MPa TENSION}}$$

$$BD: \sigma_{BD} = \frac{10.5 \times 10^3 \text{ N}}{(2 \times 10)(30) \text{ mm}^2} = \underline{17.5 \text{ MPa TENSION}}$$

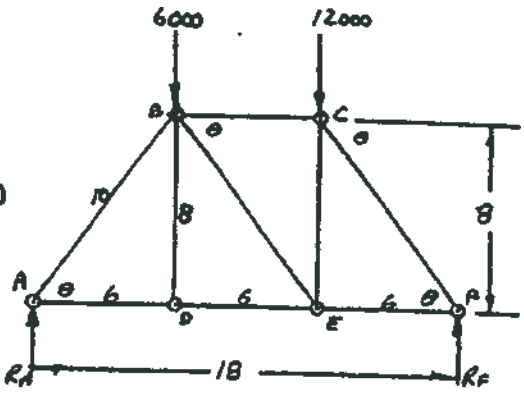
$$AO, CO: A = (30)^2 - (20)^2 = 500 \text{ mm}^2$$

$$\sigma_{AO} = \sigma_{CO} = \frac{-10.5 \times 10^3 \text{ N}}{500 \text{ mm}^2} = \underline{-21.0 \text{ MPa COMPRESSION}}$$



13.

FIGURE P8-26

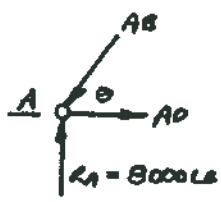


$$\sum M_A = 0 = 6000(6) + 12000(12) - R_F(18)$$

$$R_F = 10000 \text{ LB}$$

$$\sum M_F = 0 = 12000(6) + 6000(12) - R_A(18)$$

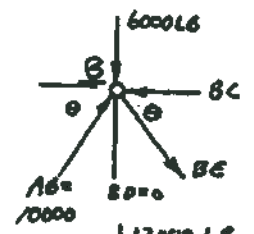
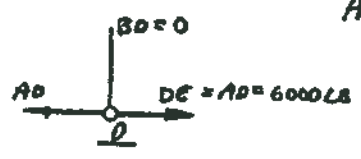
$$R_A = 8000 \text{ LB}$$



$$R_A = AB \sin \theta = AB(0.8)$$

$$AB = R_A / 0.8 = 8000 / 0.8 = 10000 \text{ LB COMP.}$$

$$AD = AB \cos \theta = 10000(0.6) = 6000 \text{ LB TENS.}$$



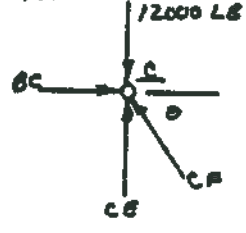
$$BD = 0$$

$$DE = AD = 6000 \text{ LB}$$

$$DE \sin \theta + 6000 - AB \sin \theta = 0$$

$$BE = \frac{AB \sin \theta - 6000}{\sin \theta} = \frac{10000(0.8) - 6000}{0.8} = 2500 \text{ LB TENS.}$$

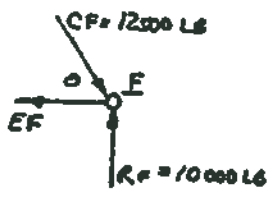
$$BC = AB \cos \theta + BE \cos \theta = 10000(0.6) + 2500(0.6) = 7500 \text{ LB COMP.}$$



$$BC = CF \cos \theta$$

$$CF = BC / \cos \theta = 7500 / 0.6 = 12500 \text{ LB COMP.}$$

$$CE = 12000 - CF \sin \theta = 12000 - 12500(0.8) = 2000 \text{ LB } \perp$$



$$EF = CF \cos \theta = 12500(0.6) = 7500 \text{ LB TENS.}$$

AREAS OF MEMBERS: (APP. A5, A6)

- AD, DE, EF - 2(0.484) = 0.968 in<sup>2</sup>
- BD, BE, CE - 0.484 in<sup>2</sup>
- AB, BC, CF - 2(1.21) = 2.42 in<sup>2</sup>

NOTE: COMPRESSION MEMBERS MUST BE CHECKED FOR COLUMN BUCKLING

STRESSES:

$$\sigma_{AD} = \sigma_{DE} = 6000 / 0.968 = 6199 \text{ psi}$$

$$\sigma_{EF} = 7500 / 0.968 = 7748 \text{ psi}$$

$$\sigma_{BD} = 0$$

$$\sigma_{BE} = 2500 / 0.484 = 5165 \text{ psi}$$

$$\sigma_{CE} = 2000 / 0.484 = 4132 \text{ psi}$$

$$\sigma_{AB} = -10000 / 2.42 = -4132 \text{ psi}$$

$$\sigma_{BC} = -7500 / 2.42 = -3099 \text{ psi}$$

$$\sigma_{CF} = -12500 / 2.42 = -5165 \text{ psi}$$

14.

$$A = (2.65)(1.40) + 2[(0.40)(0.5)(t)] = 4.41 \text{ in}^2$$

$$\sigma = F/A = (52000 \text{ LB} / 4.41 \text{ in}^2) = 11791 \text{ psi}$$

15.

$$A = (80)(40) + \pi(40)^2/4 = 4457 \text{ mm}^2$$

$$\sigma = F/A = 640 \times 10^3 \text{ N} / 4457 \text{ mm}^2 = \underline{143.6 \text{ MPa}}$$

### Direct Shear Stress

16.

PIN DIA = 0.50 IN.; DOUBLE SHEAR

$$A_s = 2(\pi d^2/4) = 2\pi(0.50)^2/4 = 0.3927 \text{ IN}^2$$

$$\tau = F_s/A_s$$

MEMBER BC:

$$\sum M_B = 0 = (2500)(75) - F_{AC}(60)$$

$$F_{AC} = 2500(75/60) = 3125 \text{ LB}$$

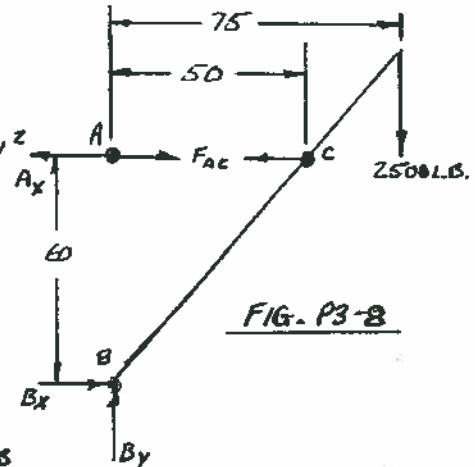
$$\text{AND } B_x = F_{AC} = 3125 \text{ LB}$$

$$B_y = 2500 \text{ LB}$$

$$\text{RESULTANT AT B: } B = \sqrt{3125^2 + 2500^2} = 4002 \text{ LB}$$

$$\text{PINS A AND C: } \tau = F_s/A_s = 3125 \text{ LB} / 0.3927 \text{ IN}^2 = \underline{7958 \text{ PSI}}$$

$$\text{PIN B: } \tau = F_s/A_s = 4002 \text{ LB} / 0.3927 \text{ IN}^2 = \underline{10,190 \text{ PSI}}$$



17.

FROM PROB. 9: FORCE IN EACH ROD =  $F_R = 1500 \text{ LB} / 2 \sin \theta$

FOR  $\theta = 40^\circ$ ;  $F_R = 1167 \text{ LB} = \text{SHEAR FORCE ON UPPER PINS}$

ASSUME DOUBLE SHEAR:  $A_s = 2\pi d^2/4 = 2\pi(0.75)^2/4 = 0.8836 \text{ IN}^2$

$$\tau = F_s/A_s = 1167 \text{ LB} / 0.8836 \text{ IN}^2 = \underline{1321 \text{ PSI}}$$

LOWER PIN:  $F_s = 1500 \text{ LB}$

$$\tau = F_s/A_s = 1500 \text{ LB} / 0.8836 \text{ IN}^2 = \underline{1698 \text{ PSI}}$$

18.

ANALYSIS FROM PROBLEMS 9 AND 17. LET  $\theta = 15^\circ$

$$F_R = 1500 \text{ LB} / 2 \sin \theta = 1500 \text{ LB} / 2 \sin 15^\circ = 2898 \text{ LB}$$

$$\tau = F_s/A_s = 2898 \text{ LB} / 0.8836 \text{ IN}^2 = \underline{3280 \text{ PSI}} \text{ IN ALL PINS}$$

19.

FIGURE 3-7 KEY IN SHEAR.  $A_s = b \cdot L = (12)(45) = 540 \text{ mm}^2$

$$F_s = \text{TORQUE} / \text{RADIUS} = 1600 \text{ N}\cdot\text{m} / 30 \text{ mm} \times \frac{10 \text{ mm}}{\text{m}} = 53333 \text{ N}$$

$$\tau = F_s/A_s = \frac{53333 \text{ N}}{540 \text{ mm}^2} = 98.8 \text{ N/mm}^2 = \underline{98.8 \text{ MPa}}$$

20. PUNCH-FIG P3-20  $A_s = (\text{PERIM.}) t = [2.50 + 2.00 + 1.50 + \sqrt{0.5^2 + 2.5^2}] (0.060)$   
 $A_s = (8.55 \text{ IN}) (0.060 \text{ IN}) = 0.513 \text{ IN}^2$   
 $\tau = F_s / A_s = 52,000 \text{ LB} / 0.513 \text{ IN}^2 = \underline{101,400 \text{ PSI}}$

21. PUNCH-FIG. P3-21.  $\text{PERIM} = 60 + 2(30) + 2(7.5) + 3 \left[ \frac{\pi(15)}{2} \right] = 205.7 \text{ mm}$   
 $A_s = (\text{PERIM.}) t = (205.7)(2.0) = 411.4 \text{ mm}^2$   
 $\tau = F_s / A_s = \frac{225,000 \text{ N}}{411.4 \text{ mm}^2} = 547 \text{ N/mm}^2 = \underline{547 \text{ MPa}}$

Torsion

22.  $\tau = \frac{T}{Z_p} = \frac{T}{\pi D^3 / 16} = \frac{800 \text{ N}\cdot\text{m}}{\pi (50)^3 / 16 \text{ mm}^3} \times \frac{10^3 \text{ mm}}{1 \text{ m}} = 32.6 \frac{\text{N}}{\text{mm}^2} = \underline{32.6 \text{ MPa}}$

23.  $\theta = \frac{TL}{GJ}$  ;  $T = 800 \text{ N}\cdot\text{m} = 800 \times 10^3 \text{ N}\cdot\text{mm}$  ;  $G = 80 \text{ GPa} = 80 \times 10^3 \text{ N/mm}^2$   
 $J = \frac{\pi D^4}{32} = \frac{\pi (50)^4}{32} \text{ mm}^4 = 6.14 \times 10^5 \text{ mm}^4$   
 $\theta = \frac{(800 \times 10^3 \text{ N}\cdot\text{mm})(850 \text{ mm})}{(80 \times 10^3 \text{ N/mm}^2)(6.14 \times 10^5 \text{ mm}^4)} = 0.0138 \text{ RAD} \times \frac{180 \text{ DEG}}{\pi \text{ RAD}} = \underline{0.79 \text{ DEG.}}$

24.  $\tau = \frac{T}{Z_p} = \frac{T}{\pi D^3 / 16} = \frac{88.0 \text{ LB}\cdot\text{IN.}}{\pi (0.40 \text{ IN})^3 / 16} = \underline{7003 \text{ PSI}}$

25.  $T = 63,000 \text{ (P) / m} = 63,000 (110 \text{ HP}) / 560 \text{ RPM} = 12,375 \text{ LB}\cdot\text{IN.}$   
 $\tau = \frac{T}{Z_p} = \frac{12,375 \text{ LB}\cdot\text{IN.}}{\pi (1.25 \text{ IN})^3 / 16} = \underline{32,270 \text{ PSI}}$

26.  $T = P / \omega = 28 \times 10^3 \text{ N}\cdot\text{m} / \text{s} / 45 \text{ RAD/s} = 622 \text{ N}\cdot\text{m}$   
 $Z_p = \frac{\pi (D^4 - d^4)}{16 D} = \frac{\pi [40^4 - 30^4] \text{ mm}^4}{16 (40 \text{ mm})} = 8590 \text{ mm}^3$   
 $\tau = \frac{T}{Z_p} = \frac{622 \text{ N}\cdot\text{m}}{8590 \text{ mm}^3} \times \frac{10^3 \text{ mm}}{\text{m}} = 72.4 \text{ N/mm}^2 = \underline{72.4 \text{ MPa}}$

27.  $\theta = \frac{TL}{GJ}$  ;  $J = \frac{\pi (D^4 - d^4)}{32} = \frac{\pi [40^4 - 30^4]}{32} \text{ mm}^4 = 1.718 \times 10^5 \text{ mm}^4$   
 $\theta = \frac{(622 \text{ N}\cdot\text{m})(400 \text{ mm})}{(80 \times 10^3 \text{ N/mm}^2)(1.718 \times 10^5 \text{ mm}^4)} \times \frac{10^3 \text{ mm}}{\text{m}} = 0.018 \text{ RAD} \times \frac{180}{\pi} = \underline{1.04^\circ}$

## Noncircular Members in Torsion

28.

$$\text{SQUARE: } \left. \begin{aligned} a &= 25 \text{ mm}; Q = 0.208 a^3 = 3250 \text{ mm}^3 \\ k &= 0.141 a^4 = 5.51 \times 10^4 \text{ mm}^4 \end{aligned} \right\} \text{FIG. 3-6}$$

$$\tau = T/Q = \left( \frac{230 \text{ N}\cdot\text{m}}{3250 \text{ mm}^3} \right) \frac{10^3 \text{ mm}}{\text{m}} = 70.8 \text{ N/mm}^2 = \underline{70.8 \text{ MPa}}$$

$$\theta = \frac{TL}{GK} = \frac{(230 \times 10^3 \text{ N}\cdot\text{mm})(650 \text{ mm})}{(80 \times 10^3 \text{ N/mm}^2)(5.51 \times 10^4 \text{ mm}^4)} = 0.0339 \text{ RAD} \times \frac{180^\circ}{\pi \text{ RAD}} = \underline{1.94^\circ}$$

29.

$$h/n = 0.60/1.50 = 0.40 \quad (\text{FIG. P3-29})$$

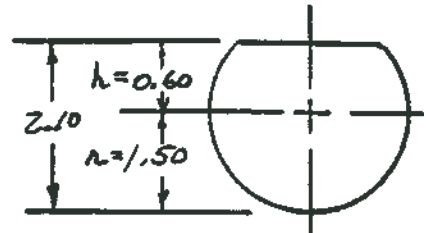
$$C_1 = 0.78; C_2 = 0.70$$

$$K = C_1 n^4 = 0.78 (1.50)^4 = 3.95 \text{ in}^4$$

$$Q = C_2 n^3 = 0.70 (1.50)^3 = 2.36 \text{ in}^3$$

$$\tau = \frac{T}{Q} = \frac{10,600 \text{ LB}\cdot\text{in.}}{2.36 \text{ in}^3} = \underline{4487 \text{ PSI}}$$

$$\theta = \frac{TL}{GK} = \frac{(10,600 \text{ LB}\cdot\text{in.})(44.0 \text{ in.})}{(11.5 \times 10^6 \text{ LB/in}^2)(3.95 \text{ in}^4)} = 0.0103 \text{ RAD} \times \frac{180^\circ}{\pi \text{ RAD}} = \underline{0.59^\circ}$$



30.

$$a = 2.0 \text{ in}; b = 4.0 \text{ in}; t = 0.109 \text{ in}; (a-t) = 1.891 \text{ in}; (b-t) = 3.891 \text{ in}; L = 6.5 \text{ FT}$$

$$K = \frac{2t(a-t)^2(b-t)^2}{a+b-2t} = \frac{2(0.109)(1.891)^2(3.891)^2}{[2.0+4.0-2(0.109)]} = 2.041 \text{ in}^4$$

$$Q = 2t(a-t)(b-t) = 2(0.109)(1.891)(3.891) = 1.604 \text{ in}^3$$

$$\tau = T/Q = (6000 \text{ LB/in}^2)(1.604 \text{ in}^3) = \underline{9624 \text{ LB}\cdot\text{in}}$$

$$\theta = \frac{TL}{GK} = \frac{(9624 \text{ LB}\cdot\text{in.})(78 \text{ in.})}{(11.5 \times 10^6 \text{ LB/in}^2)(2.041 \text{ in}^4)} = 0.032 \text{ RAD} \times \frac{180^\circ}{\pi \text{ RAD}} = \underline{1.83^\circ}$$

Beams

31.

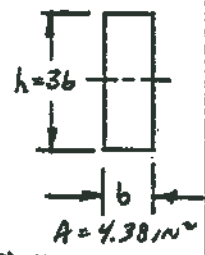
$\sigma = M/S$  : REQ'D.  $S = M/\sigma_{ALLOW}$

$S = \frac{3600 \text{ LB}\cdot\text{FT}}{18000 \text{ LB}/\text{IN}^2} \times \frac{12 \text{ IN}}{\text{FT}} = 2.40 \text{ IN}^3$   $A = 6.25 \text{ IN}^2$

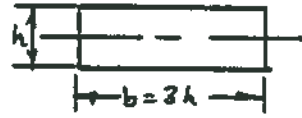
a) SQUARE;  $S = a^3/6$   
 $a = \sqrt[3]{6S} = 2.43 \text{ IN}$   
 USE  $a = 2.50 \text{ IN}$



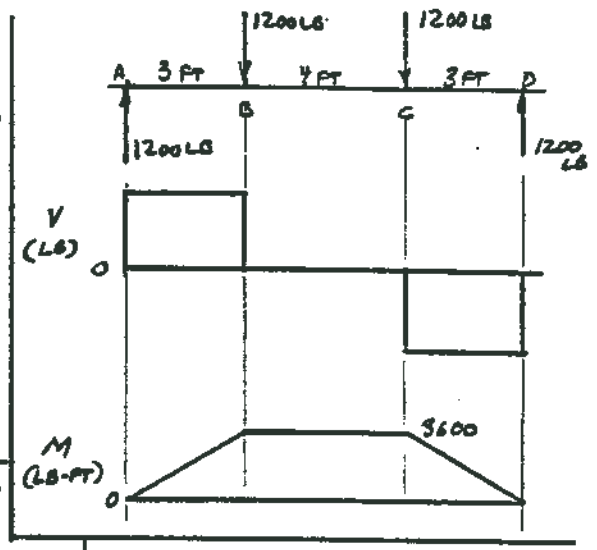
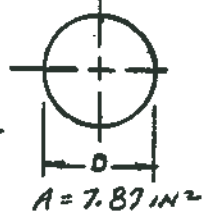
b) RECTANGLE  
 $S = \frac{bh^2}{6} = \frac{b(3b)^2}{6} = 1.5b^3$   
 $b = \sqrt[3]{S/1.5} = 1.17 \text{ IN}$   
 $h = 3b = 3.50 \text{ IN}$   
 USE  $b = 1.25 \text{ IN}, h = 3.50 \text{ IN}$



c) RECTANGLE  
 $S = \frac{bh^2}{6} = \frac{3A(h)^2}{6} = \frac{Ah^2}{2}$   
 $h = \sqrt{2S/A} = 1.69 \text{ IN}; b = 5.06 \text{ IN}$   
 USE  $h = 1.75 \text{ IN}; b = 5.00 \text{ IN}$   $A = 8.75 \text{ IN}^2$



d) CIRCLE  
 $S = \pi D^3/32$   
 $D = \sqrt[3]{\frac{32S}{\pi}} = 2.90 \text{ IN}$   
 USE  $D = 3.00 \text{ IN}$   $A = 7.87 \text{ IN}^2$



e)  $S_{4 \times 7.7} = 3.04 \text{ IN}^3$   $A = 2.26 \text{ IN}^2$

f)  $C_{15 \times 40} S_y = 3.37 \text{ IN}^3$   $A = 11.8 \text{ IN}^2$

g) 4-IN SCHEDULE 40 PIPE  $S = 3.215 \text{ IN}^3$   $A = 3.17 \text{ IN}^2$

32.

a) VOLUME =  $A \times L = (2.5 \text{ IN})^2 \times 120 \text{ IN} = 750 \text{ IN}^3$ ;  $W = (0.283 \text{ LB}/\text{IN}^3)(750 \text{ IN}^3)$

b)  $V = A \times L = (1.25)(3.50)(120) = 525 \text{ IN}^3$ ;  $W = (0.283)(525) = 149 \text{ LB}$

c)  $V = (1.75)(5.00)(120) = 1050 \text{ IN}^3$ ;  $W = (0.283)(1050) = 297 \text{ LB}$

d)  $V = \frac{\pi D^2}{4} \times L = \frac{\pi (3.00 \text{ IN})^2}{4} \times 120 \text{ IN} = 848 \text{ IN}^3$ ;  $W = (0.283)(848) = 240 \text{ LB}$

e)  $7.7 \text{ LB}/\text{FT} (10 \text{ FT}) = 77.0 \text{ LB}$  (f)  $40 \text{ LB}/\text{FT} (10 \text{ FT}) = 400 \text{ LB}$

g)  $10.78 \text{ LB}/\text{FT} (10 \text{ FT}) = 107.8 \text{ LB}$   $V = A \cdot L = (3.17 \text{ IN}^2)(120 \text{ IN}) = 38.09 \text{ IN}^3$  FOR 1.0 FT  
 $W = 0.283(38.09) = 10.78 \text{ LB}/\text{FT}$

33.

FROM CASE C; APPENDIX 14 :  $a = 36 \text{ IN}$ ;  $L = 120 \text{ IN}$ ;  $P = 1200 \text{ LB}$ ;  $E = 30 \times 10^6 \text{ PSI}$

$M_{MAX} = \frac{Pa}{24EI} (3L^2 - 4a^2) = \frac{1200(36)[3(120)^2 - 4(36)^2]}{24(30 \times 10^6) I} = \frac{2.281}{I} \text{ IN}$

$M_{B} = M_C = \frac{Pa^2}{6EI} (3L - 4a) = \frac{1200(36)^2 [3(120) - 4(36)]}{6(30 \times 10^6) I} = \frac{1.866}{I} \text{ IN}$

(SEE NEXT PAGE)

33. (CONT)

a)  $I = a^4/12 = (2.50)^4/12 = 3.26 \text{ IN}^4$ ;  $\gamma_{MAX} = \frac{2.281}{3.26} = 0.701 \text{ IN}$

$\gamma_B = \gamma_C = \frac{1.866}{3.26} = 0.572 \text{ IN}$

b)  $I = bh^3/12 = (1.25 \times 3.50)^3/12 = 4.47 \text{ IN}^4$ ;  $\gamma_{MAX} = 0.511 \text{ IN}$ ;  $\gamma_B = 0.418 \text{ IN}$

c)  $I = bh^3/12 = 5.00(1.75)^3/12 = 2.23 \text{ IN}^4$ ;  $\gamma_{MAX} = 1.021 \text{ IN}$ ;  $\gamma_B = 0.836 \text{ IN}$

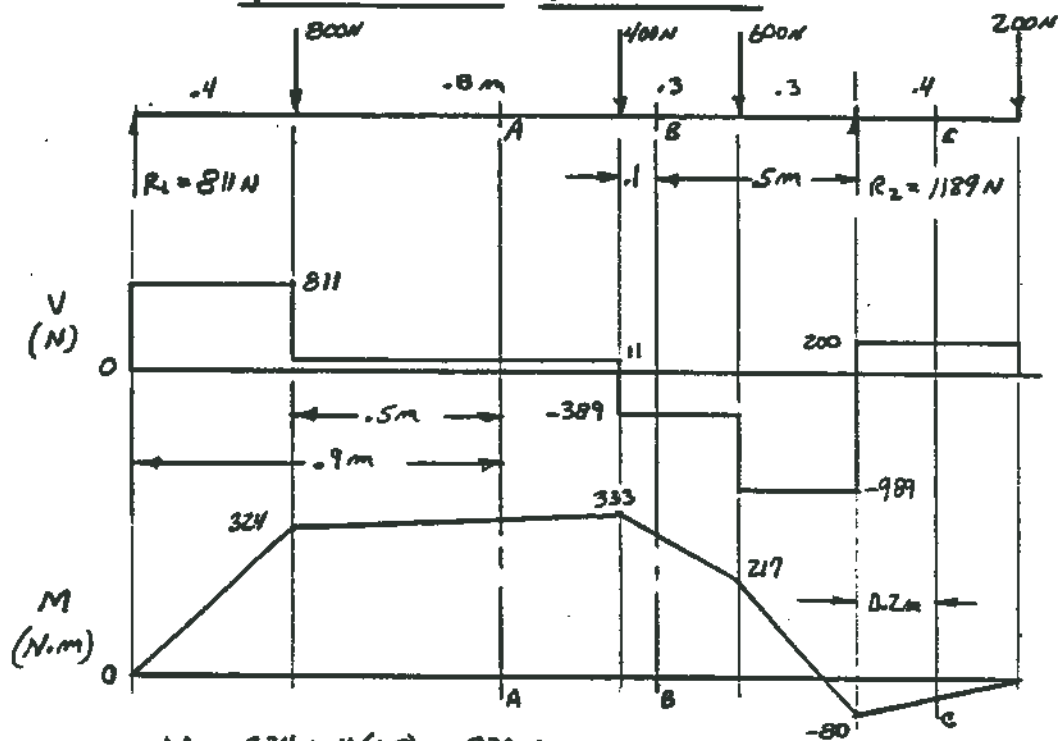
d)  $I = \frac{\pi D^4}{64} = \frac{\pi (3.00)^4}{64} = 3.98 \text{ IN}^4$ ;  $\gamma_{MAX} = 0.574 \text{ IN}$ ;  $\gamma_B = 0.469 \text{ IN}$

e)  $I = 6.08 \text{ IN}^4$ ;  $\gamma_{MAX} = 0.375 \text{ IN}$ ;  $\gamma_B = 0.307 \text{ IN}$

f)  $I_y = 9.23 \text{ IN}^4$ ;  $\gamma_{MAX} = 0.247 \text{ IN}$ ;  $\gamma_B = 0.202 \text{ IN}$

g)  $I = 7.23 \text{ IN}^4$ ;  $\gamma_{MAX} = 0.315 \text{ IN}$ ;  $\gamma_B = 0.258 \text{ IN}$

34.



$M_A = 324 + 11(0.5) = 330 \text{ N}\cdot\text{m}$

$M_B = 333 - 389(0.1) = 294 \text{ N}\cdot\text{m}$

$M_C = -80 + 200(0.2) = -40 \text{ N}\cdot\text{m}$

35.

FOR STRENGTH:

REQ'D  $S = \frac{M_{MAX}}{\sigma_{ALLOW}} = \frac{333 \text{ N}\cdot\text{m}}{100 \text{ N/mm}^2} \times \frac{10^3 \text{ mm}}{\text{m}} = 3330 \text{ mm}^3$  OR  $0.203 \text{ IN}^3$

METRIC SHAPE WITH SMALLEST  $A$  AND  $S \geq 3330 \text{ mm}^3$ : APP. 16-19

RES.  $\gamma$ : MECH. TUBING - ROUND:  $D_o = 45 \text{ mm}$ ,  $D_i = 40 \text{ mm}$ ,  $A = 333.8 \text{ mm}^2$

$S = 3361 \text{ mm}^3$



36. a) SIMPLE CANTILEVER - CASE 6 - APPENDIX 14-2  $I = 7.23 \text{ IN}^4$  APP. 15-17

$$M_{YA} = \frac{-P \cdot x^2}{6EI} (3a - x) = \frac{-800(48)^2}{6(30 \times 10^6)(7.23)} [3(72) - 48] = -0.238 \text{ IN} = M_{YA}$$

$$x_1 = 4 \text{ ft} = 48 \text{ IN}; a = 6 \text{ ft} = 72 \text{ IN}; x_2 = 8 \text{ ft} = 96 \text{ IN}$$

$$M_{YB} = \frac{-P a^2}{6EI} (3x_2 - a) = \frac{-800(72)^2}{6(30 \times 10^6)(7.23)} [3(96) - 72] = 0.688 \text{ IN} = M_{YB}$$

b) SUPPORTED CANTILEVER - CASE 6 - APPENDIX 14-3

$$M_{YA} = \frac{-P \cdot x_1^2 \cdot b}{12 E I L^3} (3 C_1 - C_2 x)$$

$$x_1 = 4 \text{ ft} = 48 \text{ IN}; a = 6 \text{ ft} = 72 \text{ IN}; b = 4 \text{ ft} = 48 \text{ IN}; L = 10 \text{ ft} = 120 \text{ IN}; N = 2 \text{ ft} = 24 \text{ IN}$$

$$C_1 = a L (L + b) = 72(120)(168) = 1.452 \times 10^6 \text{ IN}^3$$

$$C_2 = (L + a)(L + b) + a L = (192)(168) + 72(120) = 4.090 \times 10^4 \text{ IN}^2$$

$$M_{YA} = \frac{-(800)(48)^2(48)}{12(30 \times 10^6)(7.23)(120)^3} [3(1.452 \times 10^6) - 4.09 \times 10^4(48)] = 0.047 \text{ IN} = M_{YA}$$

$$M_{YB} = \frac{-P a^2 N}{12 E I L^3} [3 L^2 b - N^2 (3L - a)]$$

$$M_{YB} = \frac{-(800)(72)^2(24)}{12(30 \times 10^6)(7.23)(120)^3} [3(120)^2(48) - (24)^2(3(120) - 72)] = 0.042 \text{ IN} = M_{YB}$$

37.  $\sigma = M/S$ ;  $S = M/\sigma_{\text{ALLOW}}$

$$S = \frac{8000 \text{ LB-IN}}{12000 \text{ LB/IN}^2} = 0.667 \text{ IN}^3$$

SMALLEST BEAM OK.

$$3 \text{ I} \times 1.637; S = 1.49 \text{ IN}^3; I = 2.24 \text{ IN}^4$$

$$M_{YA1} = \frac{+P a b c}{6 E I L} (L + a)$$

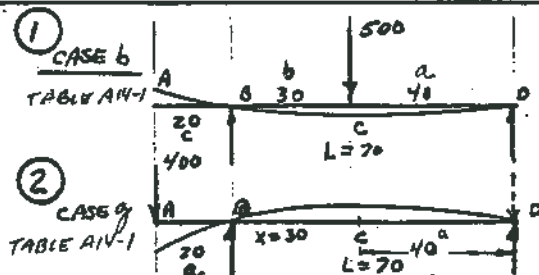
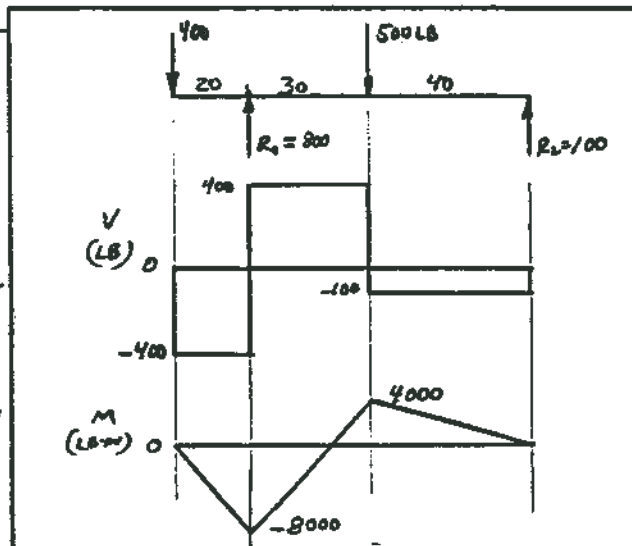
$$= \frac{(500)(30)(40)(20)(70+40)}{6(10 \times 10^6)(2.24)(70)} = 0.140 \text{ IN UP}$$

$$M_{YA2} = \frac{-P a^2}{3 E I} (a + L)$$

$$= \frac{400(20)^2(20+70)}{3(10 \times 10^6)(2.24)} = -0.214 \text{ IN DOWN}$$

$$M_{YA} = M_{YA1} + M_{YA2} = +.140 - .214 = -0.074 \text{ IN DOWN}$$

SEE NEXT PAGE FOR  $M_{YC}$



37. (CONT.)  $\eta_{c1} = \frac{-Pa^2b^2}{3EI} = \frac{-(500)(30)^2(40)^2}{3(10 \times 10^6)(2.24)(70)} = -0.153 \text{ IN DOWN}$

$\eta_{c2} = \frac{PaL^2}{EI} (0.06415) = \frac{(400)(20)(70)^2}{(10 \times 10^6)(2.24)} (0.06415) = 0.112 \text{ IN UP}$

NOTE:  $a/L = 40/70 = 0.571$ . THEN POINT C IS CLOSE TO  $\eta_{MAX}$  IN CASE OF.

$\eta_c = \eta_{c1} + \eta_{c2} = -0.153 + 0.112 = -0.041 \text{ IN DOWN}$

38.  $M_{MAX} = 3810 \text{ LB-FT (12 IN/FT)}$

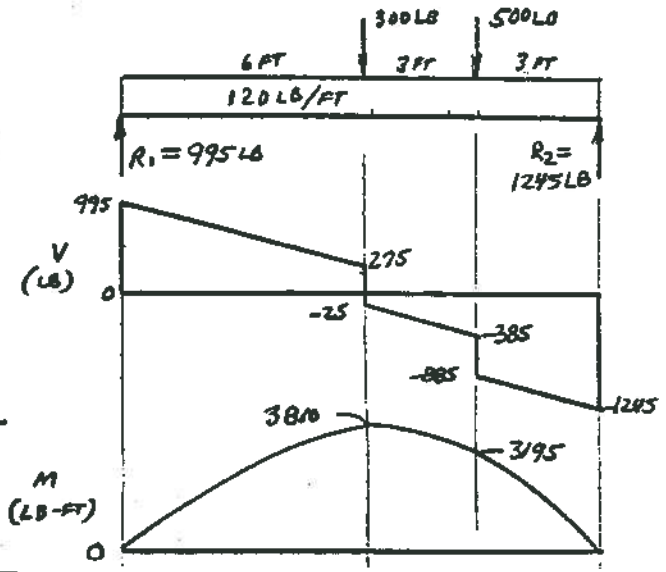
$= 45720 \text{ LB-IN}$

$I = \frac{bh^3}{12} = \frac{(150)(7.25)^3}{12} = 47.6 \text{ IN}^4$

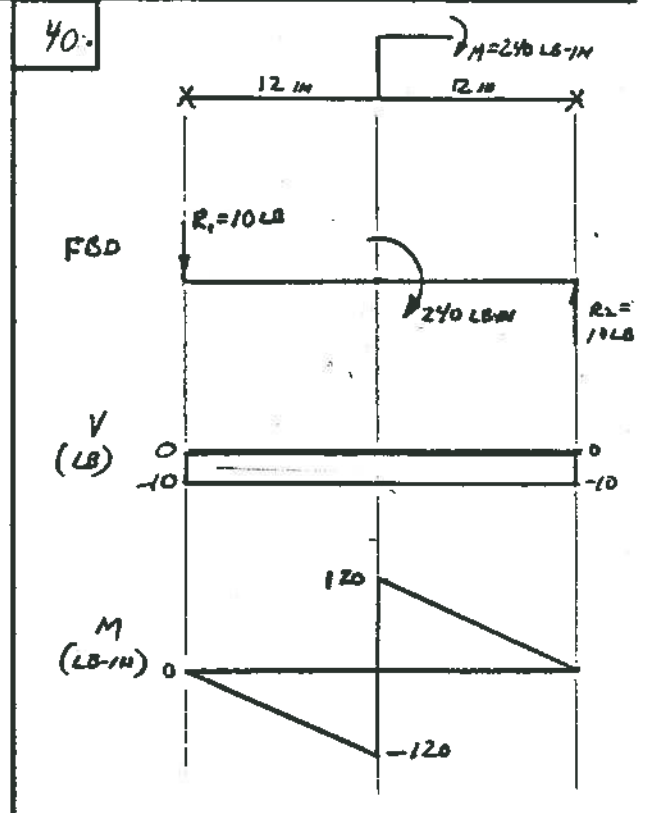
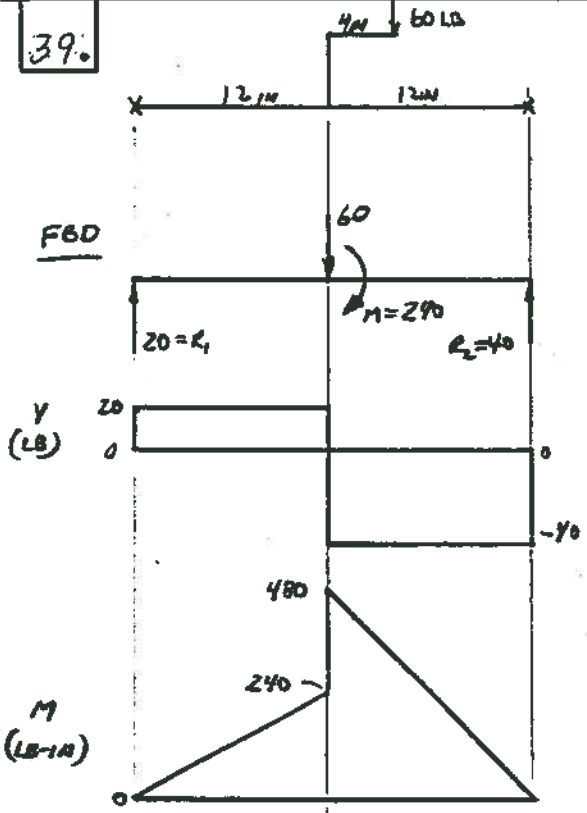
$S = \frac{bh^2}{6} = \frac{(150)(7.25)^2}{6} = 13.1 \text{ IN}^3$

$\sigma = \frac{M}{S} = \frac{45720}{13.1} = 3480 \text{ PSI}$

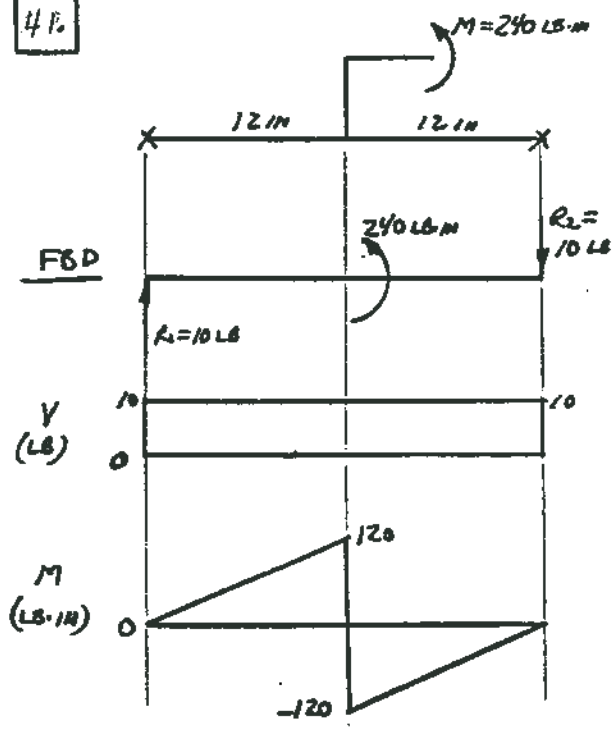
$\tau = \frac{3V}{2A} = \frac{3(1245)}{2(150)(7.25)} = 172 \text{ PSI}$



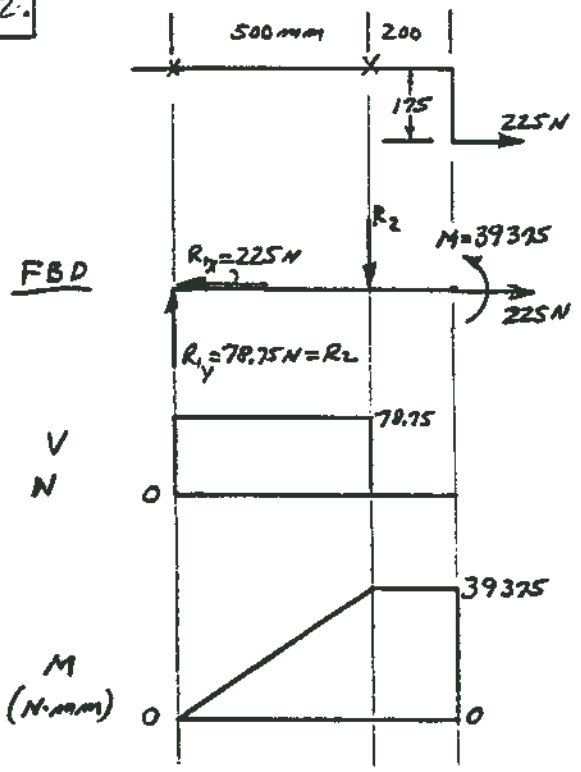
**Beams with Concentrated Bending Moments**



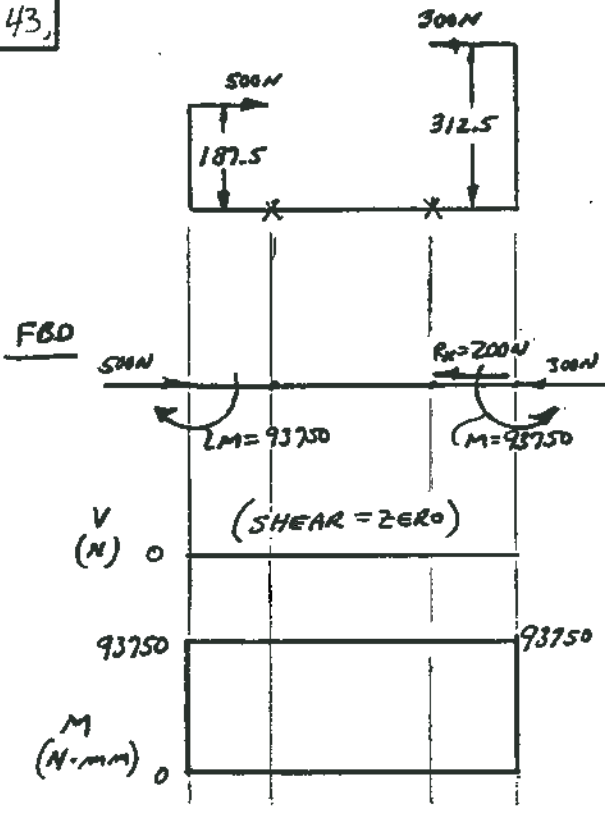
41.



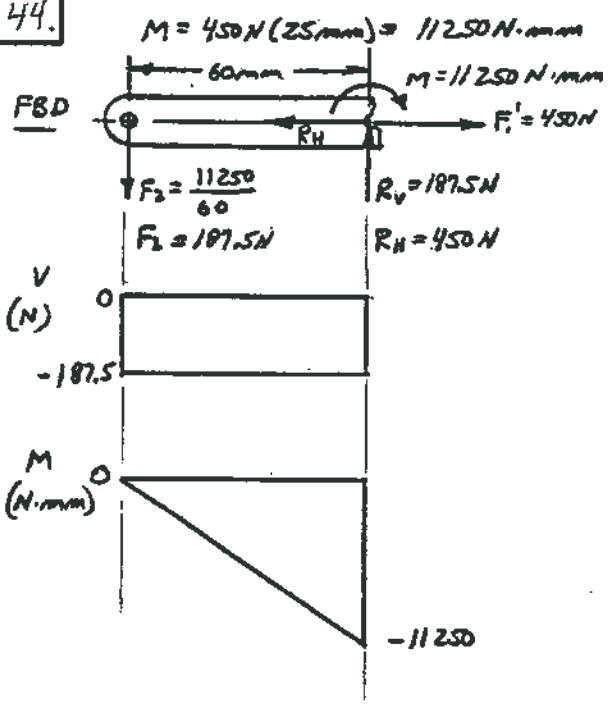
42.



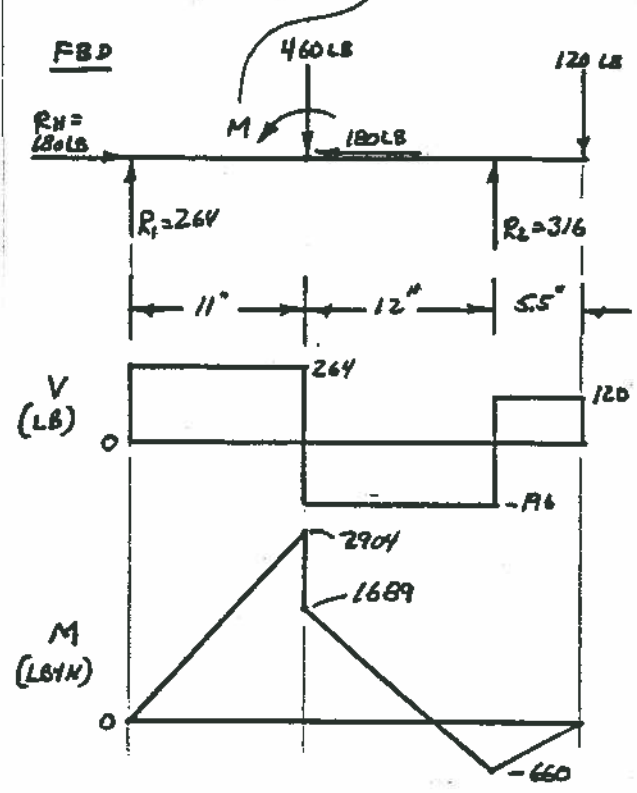
43.



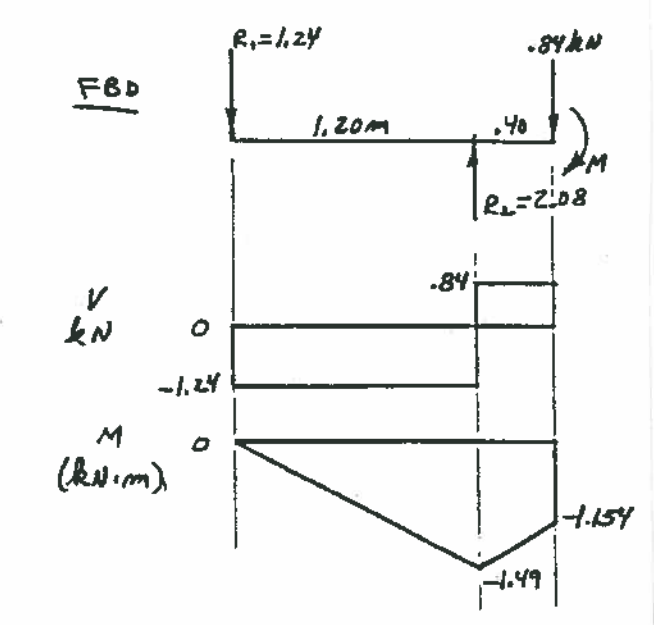
44.



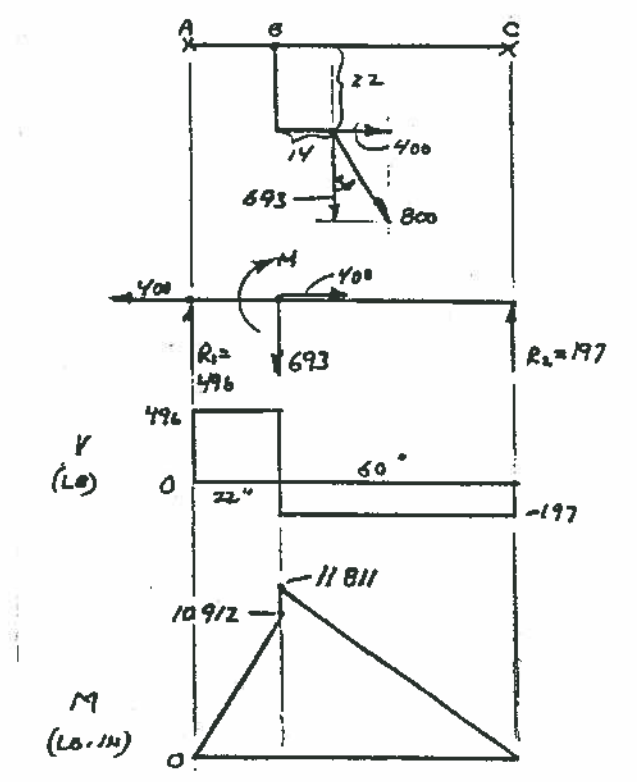
45  $M = 180(6.75) = 1215 \text{ LB}\cdot\text{IN}$



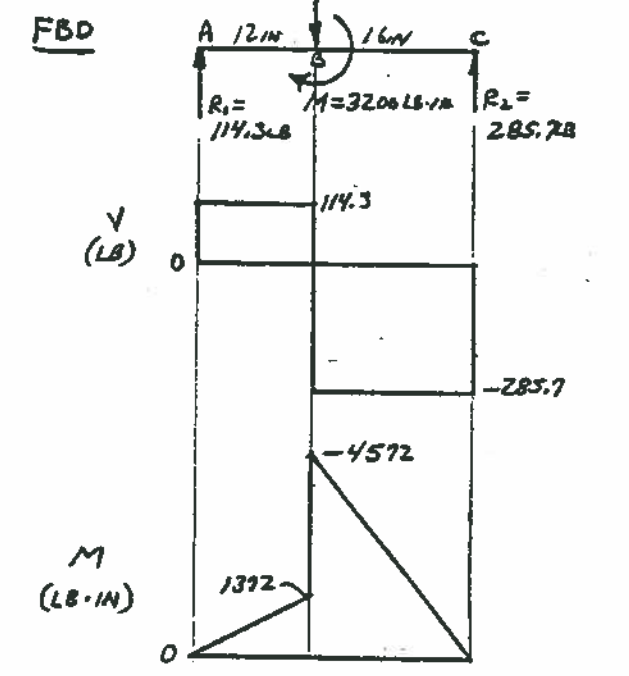
46 AT RIGHT END:  
 NET VERTICAL FORCE =  $2.24 - 1.80 = 0.84 \text{ kN}$   
 $M = 2.64(2.60) + 1.80(2.60) = 1.154 \text{ kN}\cdot\text{m}$



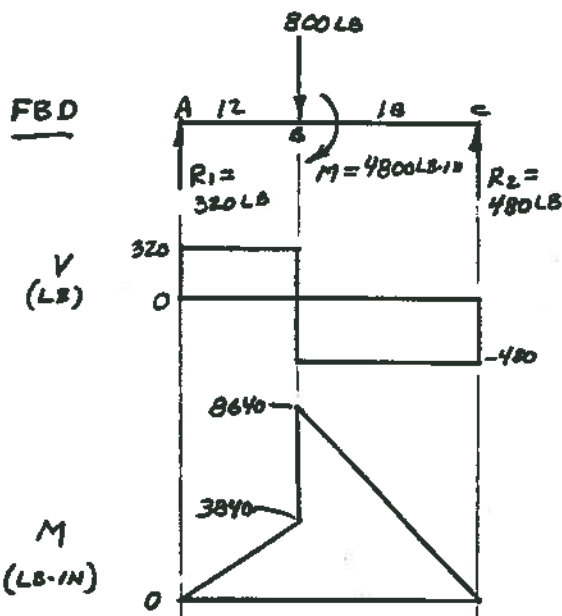
47  $M = 693(14) - 400(22) = 899 \text{ LB}\cdot\text{IN}$



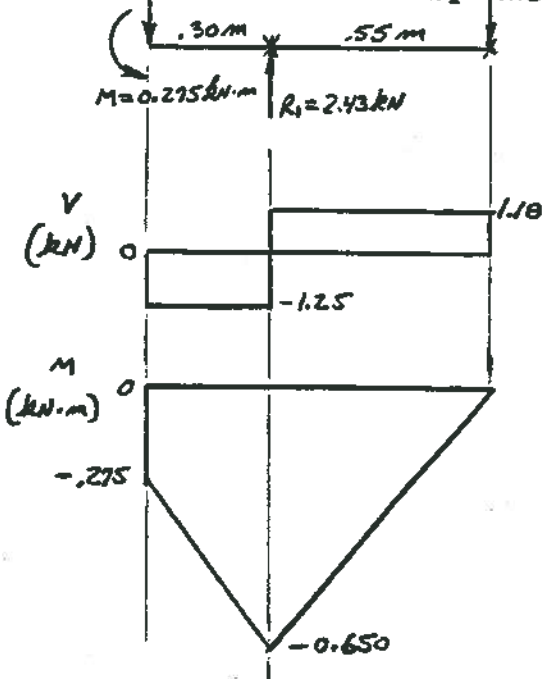
48.  $M = 200(1) + 600(4) = 3200 \text{ LB}\cdot\text{IN}$   
 TORQUE =  $T = 200(8) + 600(8) = 6400 \text{ LB}\cdot\text{IN}$   
 (NOT SHOWN)  
 CW - VIEWED FROM C.



49  $M = 600(6) + 200(6) = 4800 \text{ LB}\cdot\text{ft}$   
 NET TORQUE =  $600(4) - 200(10) = 400 \text{ LB}\cdot\text{ft}$   
 NOT SHOWN

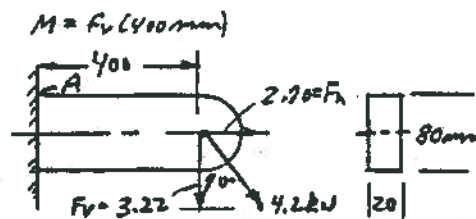


50  $M = 1.25 \text{ kN} \cdot (2.2 \text{ m}) = 0.275 \text{ kN}\cdot\text{m}$   
 $T = 1.25 \text{ kN} \cdot (3.0 \text{ m}) = 0.375 \text{ kN}\cdot\text{m}$   
 NOT SHOWN  
 $R_2 = 1.18 \text{ kN}$

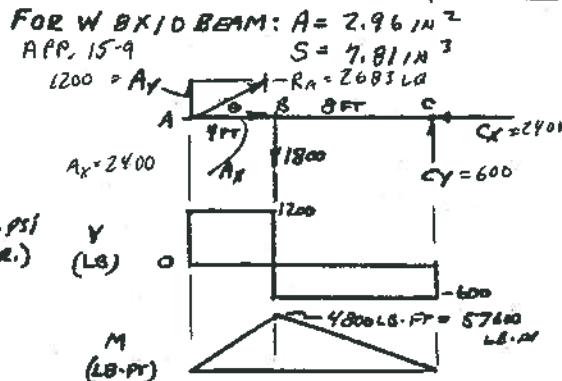


Combined Normal Stresses

51.  $A = 20(80) = 1600 \text{ mm}^2$   
 $S = bh^2/6 = 20(80)^2/6 = 21333 \text{ mm}^3$   
 $\sigma_A = \frac{F_h}{A} + \frac{F_v(400)}{S} = \frac{2700 \text{ N}}{1600 \text{ mm}^2} + \frac{(3220 \text{ N})400 \text{ mm}}{21333 \text{ mm}^3}$   
 $\sigma_A = 1.69 + 60.38 = 62.07 \text{ MPa}$



52.  $\tan \theta = 4/12 = 0.5; \theta = 26.6^\circ$   
 $\sum M_C = 0 = 1800(8) - A_y(12)$   
 $A_y = 1200 \text{ LB}; C_y = 1800 - A_y = 600 \text{ LB}$   
 $A_x = 2A_y = 2400 \text{ LB} = C_x$   
 AT B ON TOP OF BEAM  
 $\sigma = \frac{-A_x}{A} - \frac{M}{S} = \frac{-2400}{2.96} - \frac{57600}{7.81} = -8186 \text{ PSI}$   
 AT B ON BOTTOM OF BEAM  
 $\sigma = \frac{-A_x}{A} + \frac{M}{S} = \frac{-2400}{2.96} + \frac{57600}{7.81} = +6564 \text{ PSI}$   
 (TENSILE)



53.

AT A (TOP)

$$\sigma = \frac{F_h}{A} + \frac{M}{S} = \frac{30 \text{ N}}{50(20) \text{ mm}^2} + \frac{(430 \text{ N})(400 \text{ mm})}{(20)(50)^2/6 \text{ mm}^3}$$

$$\sigma = 0.301 + 20.64 = 20.94 \text{ MPa TENSION}$$

AT B,  $M = 430(200) = 86000 \text{ N}\cdot\text{mm}$

ASSUME AXIAL STRESS IS SMALL:  $\sigma \approx M/S$

$$\text{REQ'D } S = \frac{M}{\sigma} = \frac{86000 \text{ N}\cdot\text{mm}}{20.94 \text{ N/mm}^2} = 4107 \text{ mm}^3 = \frac{t h^3}{6}$$

$$h = \sqrt{\frac{6S}{t}} = \sqrt{\frac{6(4107)}{20}} = 35.1 \text{ mm} \quad \text{LET } h = 36 \text{ mm}; S = \frac{t h^3}{6} = \frac{20(36)^3}{6} = 4320 \text{ mm}^3$$

$$A = t h = 720 \text{ mm}^2$$

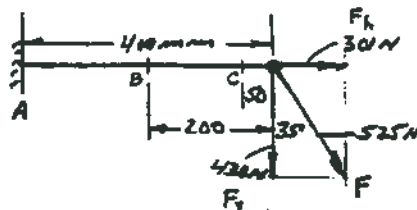
$$\sigma_B = \frac{30 \text{ N}}{720 \text{ mm}^2} + \frac{86000 \text{ N}\cdot\text{mm}}{4320 \text{ mm}^3} = 20.33 \text{ MPa OK}$$

AT C,  $M = 430(50) = 21500 \text{ N}\cdot\text{mm}$

$$S = \frac{M}{\sigma} = \frac{21500}{20.94} = 1027 \text{ mm}^3; h = \sqrt{\frac{6S}{t}} = \sqrt{\frac{6(1027)}{20}} = 17.6 \text{ mm}$$

$$\text{LET } h = 18 \text{ mm}; t = 20 \text{ mm}; S = \frac{20(18)^3}{6} = 1080 \text{ mm}^3; A = 18(20) = 360 \text{ mm}^2$$

$$\sigma_C = \frac{30 \text{ N}}{360 \text{ mm}^2} + \frac{21500 \text{ N}\cdot\text{mm}}{1080 \text{ mm}^3} = 20.74 \text{ MPa OK}$$



54.

6x2 x 1/4 HOLLOW RECT. TUBE; APP. 15-14  
FOR CROSS SECTION OF BEAM

$$A = 3.59 \text{ IN}^2$$

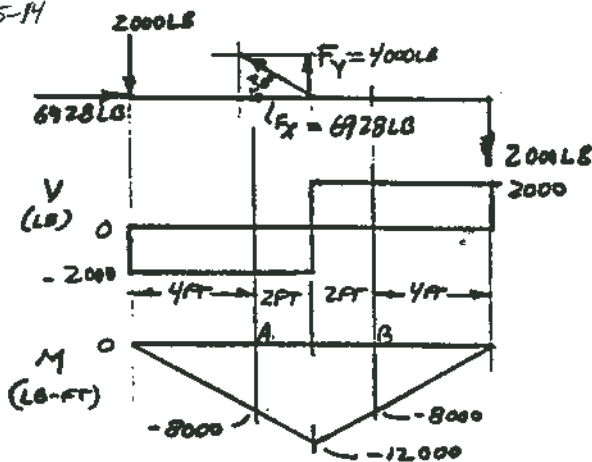
$$S = 4.60 \text{ IN}^3$$

AT A:  $\sigma = \frac{-F_y}{A} + \frac{M}{S}$  TENSION ON TOP SURFACE

$$\sigma_a = \frac{-6928 \text{ LB}}{3.59 \text{ IN}^2} + \frac{8000(12) \text{ LB}\cdot\text{IN}}{4.60 \text{ IN}^3}$$

$$\sigma_a = -1930 \text{ PSI} + 20870 \text{ PSI} = 18940 \text{ PSI}$$

AT B:  $\sigma = \frac{M}{S} = 20870 \text{ PSI}$



55.

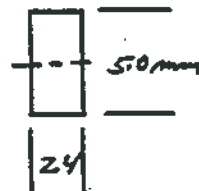
FROM FIG 3-22;  $M = 875 \text{ N}\cdot\text{mm}$ ;  $F_x = 35.0 \text{ N}$

$$A = 5.0(24) = 120 \text{ mm}^2$$

$$S = 24(5.0)^2/6 = 100 \text{ mm}^3$$

$$\sigma = \frac{-F}{A} + \frac{M}{S} = \frac{-35.0 \text{ N}}{120 \text{ mm}^2} + \frac{875 \text{ N}\cdot\text{mm}}{100 \text{ mm}^3} = 84.58 \text{ MPa TENSION}$$

ON BOTTOM OF SECTION





56.

$$BC = \sqrt{50^2 + 60^2} = 78.1 \text{ in}$$

$$CF = 25 \text{ in} / \sin \theta = 39.05 \text{ in}$$

$$BF = 117.15$$

$$\sum M_B = 0 = -C_y(78.1) + F_y(117.15)$$

$$C_y = F_y(117.15/78.1) = 1728 \text{ LB}$$

$$C_x = C_y / \tan \beta = 1439 \text{ LB}$$

$$\sum M_C = 0 = 1152(39.05) - B_y(78.1)$$

$$B_y = 576 \text{ LB}$$

$$B_x = C_x + F_x = 1439 + 1383 = 2822 \text{ LB}$$

FOR 6x4 x 1/4 TUBE: (APP. 15-14)

$$A = 4.59 \text{ in}^2; S_x = 7.36 \text{ in}^3$$

JUST ABOVE C:

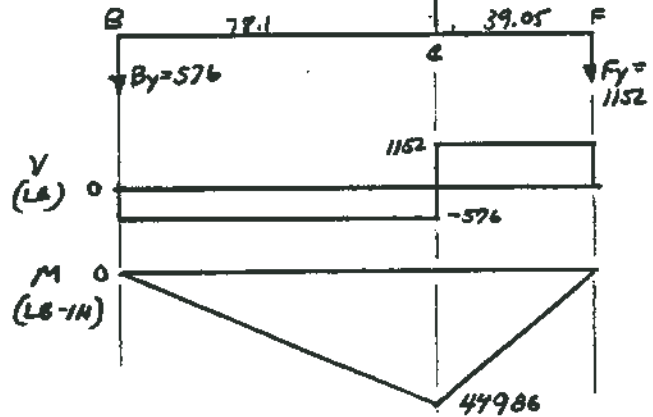
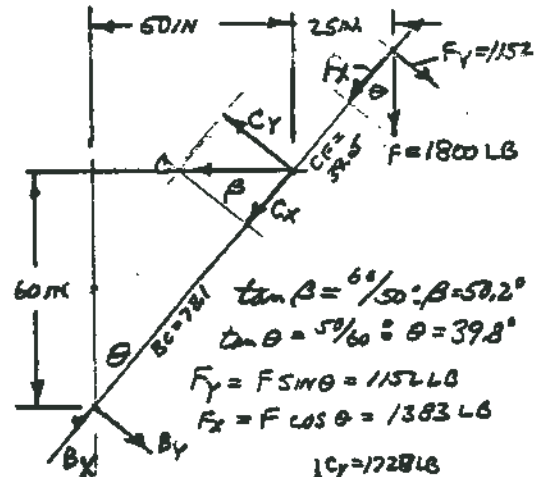
$$\sigma = \frac{-F_x}{A} + \frac{M}{S} = \frac{-1383}{4.59} + \frac{44986}{7.36}$$

$$\sigma = 5811 \text{ PSI TENSION}$$

JUST BELOW C:

$$\sigma = \frac{-B_x}{A} - \frac{M}{S} = \frac{-2822}{4.59} - \frac{44986}{7.36}$$

$$\sigma = -6727 \text{ PSI COMPRESSION}$$



57.

$$A = b^2; S = b^3/6$$

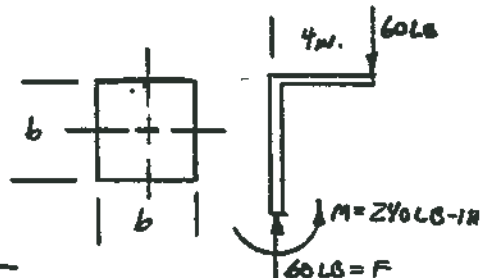
$$\sigma = -\frac{F}{A} - \frac{M}{S} = -\frac{60}{b^2} - \frac{240}{b^3/6}$$

$$\sigma = -\frac{60}{b^2} - \frac{1440}{b^3} \quad \text{BUT ASSUME } F/A \text{ IS SMALL}$$

$$\sigma \approx -1440/b^3; b = \sqrt[3]{\frac{-1440}{\sigma_{\text{ALLOW}}}} = \sqrt[3]{\frac{1440 \text{ LB} \cdot \text{in}}{12000 \text{ LB}/\text{in}^2}} = 0.493 \text{ in}$$

$$\text{TRY } b = 0.500 \text{ in} = 1/2 \text{ in}; A = 0.25 \text{ in}^2; S = b^3/6 = 0.0208 \text{ in}^3$$

$$\sigma = \frac{-60 \text{ LB}}{0.25 \text{ in}^2} - \frac{240 \text{ LB} \cdot \text{in}}{0.0208 \text{ in}^3} = -240 - 11520 = -11760 \text{ PSI OK}$$



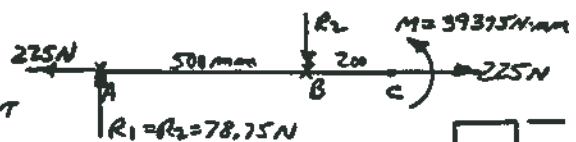
58.

FBD FROM PROBLEM 3-42:

PART FROM B-C SEES 39375 N/mm MOMENT

$$\sigma_{\text{max}} = \frac{F}{A} + \frac{M}{S} = \frac{225 \text{ N}}{2500 \text{ mm}^2} + \frac{39375 \text{ N} \cdot \text{mm}}{20833 \text{ mm}^3}$$

$$\sigma_{\text{max}} = 0.09 + 1.89 = 1.98 \text{ MPa TENSION ON BOTTOM SURFACE BETWEEN B AND C.}$$



$$A = 50^2 = 2500 \text{ mm}^2$$

$$S = 50^3/6 = 20833 \text{ mm}^3$$



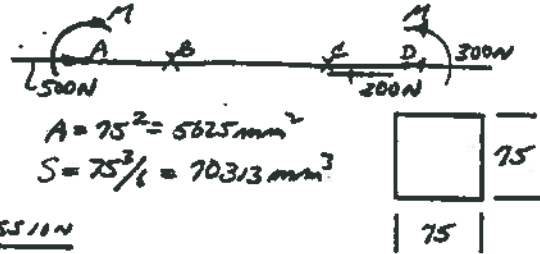
69. FROM PROB. 3-43:  $M = \text{CONSTANT} = 93750 \text{ N}\cdot\text{mm}$   
 $F_x = 500 \text{ N}$  FROM A TO C

$$\sigma_{\text{MAX}} = \frac{-F_x}{A} - \frac{M}{S}$$

$$= \frac{-500 \text{ N}}{5625 \text{ mm}^2} - \frac{93750 \text{ N}\cdot\text{mm}}{70313 \text{ mm}^3}$$

$$\sigma_{\text{MAX}} = -0.089 - 1.333 = \underline{7.422 \text{ MPa COMPRESSION}}$$

ON TOP SURFACE BETWEEN A AND C



60.

FROM PROB 3-47: POINT B IS WHERE BRACKET ATTACHES

TO RIGHT OF B;  $M = 11811 \text{ LB}\cdot\text{IN}$ ;  $\sigma = M/S$ ;  $R = 2500 \text{ S} = \frac{M}{\sigma} = \frac{11811 \text{ LB}\cdot\text{IN}}{25000 \text{ LB/IN}^2} = 0.472 \text{ IN}^3$

$$S = \pi D^3/32; D = \sqrt[3]{32 R / \pi} = \sqrt[3]{32 (0.472) / \pi} = 1.69 \text{ IN}; \text{ USE } D = 1.75 \text{ IN}$$

CHECK TO LEFT OF B:  $M = 10912 \text{ LB}\cdot\text{IN}$ ;  $F_x = 400 \text{ LB}$

$$\sigma = \frac{F_x}{A} + \frac{M}{S} = \frac{400}{2.41} + \frac{10912}{0.526} = 166 + 20739 = 20900 \text{ PSI}$$

OK

Stress Concentrations:  $K_t$  factors obtained from the website:

[www.efatigue.com/constantamplitude/stress-concentration/](http://www.efatigue.com/constantamplitude/stress-concentration/) [Note for a flat plate with a circular hole in bending: For  $d/W < 0.50$ , use  $K_t = 1.0$ . The eFatigue site does not show this.]

61.  $D/d = 9/6 = 1.50$   
 $N/d = 0.5/6 = 0.083$   
 $K_t = 2.03$   
 $\sigma_{\text{MAX}} = \frac{K_t F}{A} = \frac{(2.03)(1250 \text{ N})}{\pi (6)^2 / 4} = 89.7 \text{ MPa}$

62.  $\sigma_{\text{MAX}} = K_t F/A$   
 BOTTOM:  $D/d = 2.0/0.5 = 4.0$   
 $N/d = 0.08/0.5 = 0.16$   
 $K_t = 1.86$   
 $\sigma_{\text{MAX}} = \frac{(1.86)(1200)}{\pi (0.5)^2 / 4} = 11,367 \text{ PSI MAX}$

MIDDLE:  $D/d = 2.0/0.75 = 2.67$   
 $N/d = 0.08/0.75 = 0.11$   
 $K_t = 2.08$   
 $\sigma_{\text{MAX}} = \frac{(2.08)(2400)}{\pi (0.75)^2 / 4} = 11,300 \text{ PSI}$

TOP:  $D/d = 2.0/1.0 = 2.0$   
 $N/d = 0.08/1.0 = 0.08$   
 $K_t = 2.22$   
 $\sigma_{\text{MAX}} = \frac{(2.22)(3600)}{\pi (1.0)^2 / 4} = 10,176 \text{ PSI}$

63. LEFT HOLE:  $d/W = 1.7/1.40 = 0.57$   
 $K_t = 2.15$   
 $\sigma_{\text{MAX}} = \frac{(2.15)(6200)}{(1.40 - 0.72)(0.5)} = 39,206 \text{ PSI MAX}$   
 MIDDLE:  $d/W = 0.40/1.40 = 0.29 \rightarrow K_t = 2.38$   
 $\sigma_{\text{MAX}} = \frac{(2.38)(6200)}{(1.40 - 0.40)(0.5)} = 29,572 \text{ PSI}$   
 RIGHT:  $d/W = 0.50/1.40 = 0.36 \rightarrow K_t = 2.29$   
 $\sigma_{\text{MAX}} = \frac{(2.29)(6200)}{(1.40 - 0.50)(0.5)} = 31,551 \text{ PSI}$

64.  $H/h = 1.50/0.80 = 1.875$   
 $N/h = 0.12/0.80 = 0.15$   
 $K_t = 2.25$   
 $\sigma_{\text{MAX}} = \frac{2.25(1625)}{(25)(0.80)} = 18,281 \text{ PSI}$

65.  $D/d = 42/30 = 1.40$   
 $N/d = 1.50/30 = 0.05$   
 $K_t = 2.30$   
 $\sigma_{\text{MAX}} = \frac{(2.30)(30300 \text{ N})}{\pi (30)^2 / 4} = 98.6 \text{ MPa}$

66.  $D/d = 2.00/1.25 = 1.60$   
 $N/d = 0.10/1.25 = 0.08$   
 $K_t = 1.43$   
 $T_{\text{MAX}} = \frac{(1.43)(2200 \text{ LB}\cdot\text{IN})}{\pi (1.25)^2 / 16 \text{ IN}^2} = 8203 \text{ PSI}$

67.

$$D/d = 2.00/1.25 = 1.60 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} K_t = 2.23 \quad \sigma_m = \frac{2.23 (2800) \text{ LB/IN}}{\pi (1.25)^3 / 32 \text{ IN}^3} = 32564 \text{ PSI}$$

$$r/d = 0.06/1.25 = 0.048$$

68.

$$d/W = 1.38/2.00 = 0.69 \rightarrow K_t = 1.38$$

$$\sigma_m = K_t \sigma_{nom} = \frac{K_t 6MW}{(W^3 - d^3)t} = \frac{(1.38)(6)(12000)(2.00)}{[(2.00)^3 - (1.38)^3]0.75} = 49323 \text{ PSI}$$

### Problems of a General Nature

69.

$$\sum M_C = 0 = 12.5 \text{ kN}(4.0 \text{ m}) - R_B(2.5 \text{ m})$$

$$R_B = (2.5)(4.0)/2.5 = 20.0 \text{ kN} \uparrow$$

$$R_C = 20 \text{ kN} - 12.5 \text{ kN} = 7.5 \text{ kN} \downarrow$$

$$M_{MAX} = 18.75 \text{ kN} \cdot \text{m} \times \frac{10^3 \text{ N}}{\text{kN}} \cdot \frac{10^3 \text{ mm}}{\text{m}}$$

$$M_{MAX} = 18.75 \times 10^6 \text{ N} \cdot \text{mm}$$

$$\text{SECTION MODULUS} = S = \frac{a^3}{6} = \frac{(20.0 \text{ mm})^3}{6}$$

$$S = 1333 \text{ mm}^3$$

$$\text{AREA OF AB} = (20 \text{ mm})^2 = 400 \text{ mm}^2$$

SHEAR AREA OF PIN - DOUBLE SHEAR

$$A_s = \frac{2\pi D^2}{4} = \frac{\pi D^2}{2} = \frac{\pi (8.0 \text{ mm})^2}{2} = 100.5 \text{ mm}^2$$

$$\text{TENSION IN AB: } \sigma = \frac{R_B}{A} = \frac{20000 \text{ N}}{400 \text{ mm}^2} = 50 \text{ MPa}$$

$$\text{SHEAR IN PIN: } \tau = \frac{R_B}{A_s} = \frac{20000 \text{ N}}{100.5 \text{ mm}^2} = 199 \text{ MPa}$$

$$\text{BENDING IN CD; AT B: } \sigma_B = M/S = \frac{18.75 \times 10^6 \text{ N} \cdot \text{mm}}{1333 \text{ mm}^3} = 14063 \text{ MPa}$$

VERY HIGH

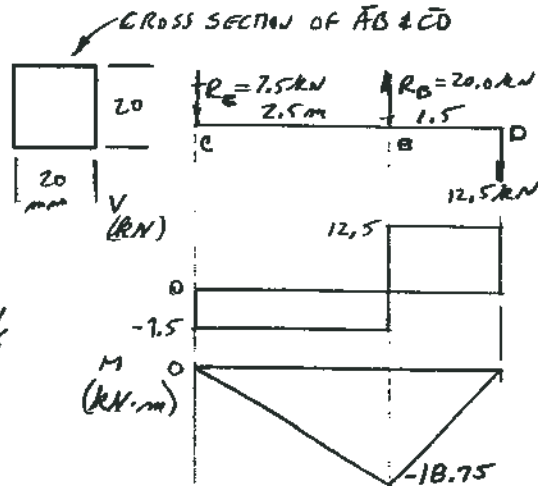
SUGGEST CHANGE IN CROSS SECTION OF CD TO MAKE  $\sigma_B < 400 \text{ MPa}$

$$\text{REQ'D } S = M/\sigma = \frac{18.75 \times 10^6 \text{ N} \cdot \text{mm}}{400 \text{ N/mm}^2} = 46875 \text{ mm}^3$$

$$S = (20)(h)^2/6 : \text{REQ'D } h = \sqrt{6S/20} = \sqrt{6(46875 \text{ mm}^3)/20} = 118 \text{ mm}$$

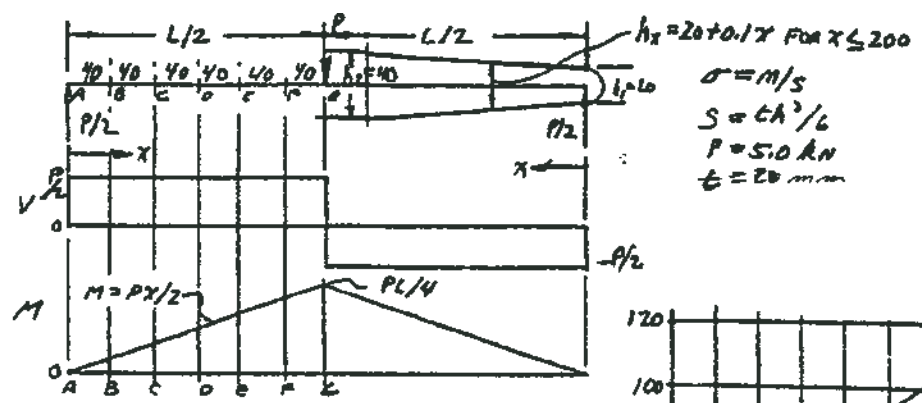
$$\text{LET } h = 120 \text{ mm: THEN } S = \frac{(20)(120)^2 \text{ mm}^3}{6} = 48000 \text{ mm}^3$$

$$\sigma = \frac{M}{S} = \frac{18.75 \times 10^6 \text{ N} \cdot \text{mm}}{48000 \text{ mm}^3} = 391 \text{ MPa OK}$$



PROPOSED CROSS SECTION OF CD

70.



X (m)	M (kN.m)	h (mm)	S (mm <sup>3</sup> )	$\sigma$ (MPa)
A	0	20	1333	0
B	0.040 $0.20P = 0.10$	24	1920	52.1
C	0.080 $0.40P = 0.20$	28	2613	76.5
D	0.120 $0.60P = 0.30$	32	3413	87.9
E	0.160 $0.80P = 0.40$	36	4320	92.6
F	0.200 $1.00P = 0.50$	40	5333	93.8
G	0.240 $1.20P = 0.60$	40	5333	112.5

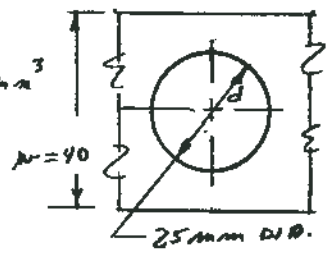
71.

FROM PROB. 70,  $M = 0.60 \text{ kN}\cdot\text{m}$  UNDER LOAD.

$$\sigma = \frac{K_t M}{S_{NET}} ; S_{NET} = \frac{(w^3 - d^3) t}{6w} = \frac{(40^3 - 25^3)(20)}{6(40)} = 4031 \text{ mm}^3$$

$$d/w = 25/40 = 0.625 ; K_t = 1.25$$

$$\sigma = \frac{(1.25)(0.60 \text{ kN}\cdot\text{m})}{4031 \text{ mm}^3} \cdot \frac{10^3 \text{ N}}{\text{kN}} \cdot \frac{10^3 \text{ mm}^3}{\text{m}^3} = 186 \text{ MPa}$$



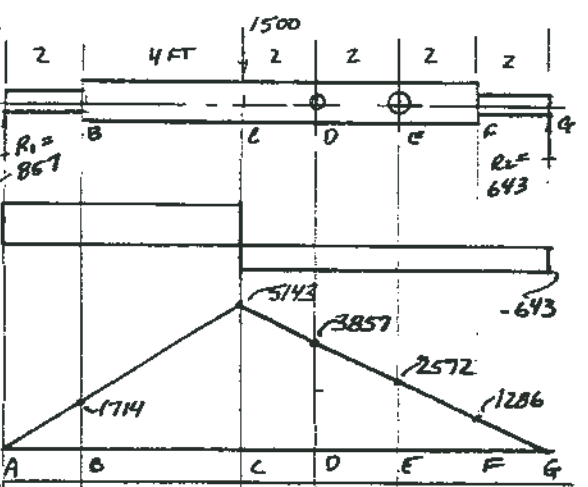
72.

$\sigma = \frac{K_t M}{S}$  ASSUME LATERAL BRACING

(a) AT C AT LOAD;  $M_C = 5143 \text{ LB}\cdot\text{FT}$   
 $K_t = 1.0$ ;  $S = \frac{t h^3}{6} = \frac{(1.20)(4.0)^3}{6} = 3.20 \text{ in}^3$   
 $\sigma_c = \frac{(1.0)(5143 \text{ LB}\cdot\text{FT})(12 \text{ IN/FT})}{3.20 \text{ in}^3} = 19286 \text{ PSI}$

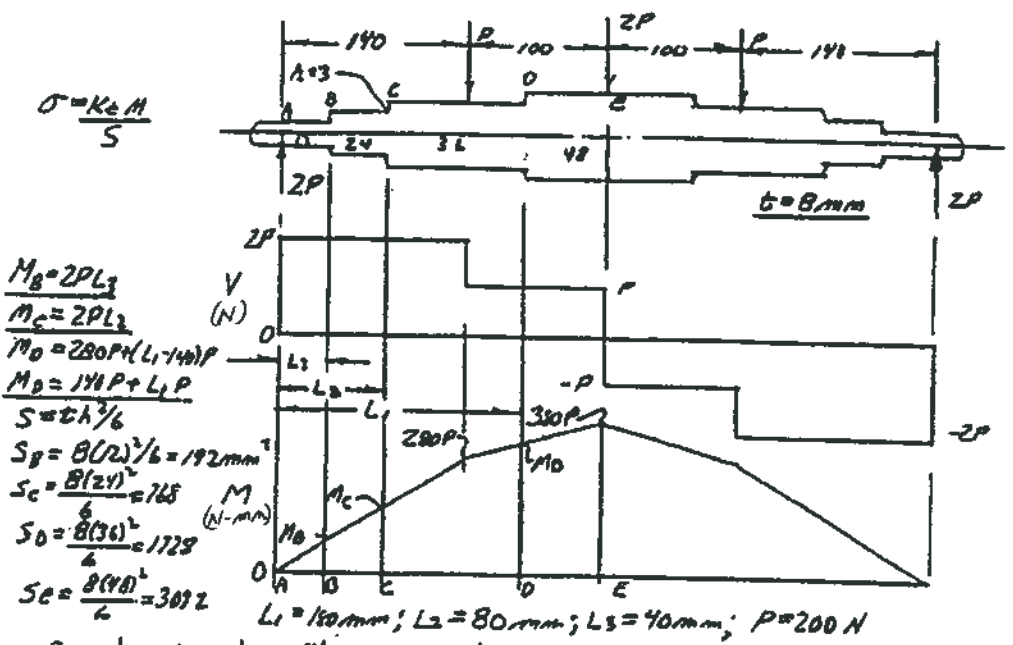
(b) AT D AT 1.50 IN DIA. HOLE  
 $M_D = 3857 \text{ LB}\cdot\text{FT}$ ;  $d/w = \frac{1.50}{4.00} = 0.375$   
 $K_t = 1.00$   
 $S_{NET} = \frac{(w^3 - d^3) t}{6w} = \frac{(4.0^3 - 1.50^3)(1.20)}{6(4.0)} = 3.03 \text{ in}^3$   
 $\sigma_D = \frac{(1.0)(3857)(12)}{3.03} = 15270 \text{ PSI}$

(c) AT E AT 2.50 IN DIA. HOLE  
 $M_E = 2572 \text{ LB}\cdot\text{FT}$ ;  $d/w = \frac{2.50}{4.00} = 0.625$   
 $K_t = 1.25$   
 $S_{NET} = \frac{(4.0^3 - 2.50^3)(1.20)}{6(4.00)} = 2.419 \text{ in}^3$   
 $\sigma_E = \frac{(1.25)(2572)(12)}{(2.419)} = 15948 \text{ PSI}$



(d) AT B AT STEP;  $H = 4.00$ ,  $h = 2.80$ ,  $t = 0.15$   
 $d/h = 1.43$ ;  $d/w = 0.054$ ;  $K_t = 2.21$   
 $M_B = 1714 \text{ LB}\cdot\text{FT}$ ;  $S = \frac{t h^3}{6} = \frac{(0.15)(2.80)^3}{6} = 1.57 \text{ in}^3$   
 $\sigma_B = \frac{(2.21)(1714)(12)}{1.57} = 29952 \text{ PSI}$   
 MAXIMUM 1.57

73.



$$\sigma = \frac{K_t M}{S}$$

$$M_B = 2PL_1$$

$$M_C = 2PL_2$$

$$M_D = 280P(L_1 + L_2)P$$

$$M_E = 140P + L_3 P$$

$$S = \frac{bh^3}{6}$$

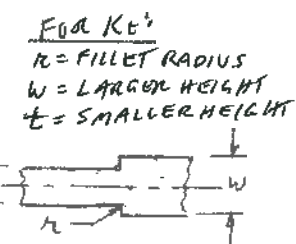
$$S_g = \frac{8(2)^3}{6} = 192 \text{ mm}^3$$

$$S_c = \frac{8(2)^3}{6} = 768$$

$$S_D = \frac{8(4)^3}{6} = 1728$$

$$S_e = \frac{8(4)^3}{6} = 3072$$

POINT	M (N-mm)	S (mm <sup>3</sup> )	t/w	K <sub>t</sub>	σ (MPa)	
B	16000	192	0.25	0.50	1.46	121.7 MAXIMUM
C	32000	768	0.125	0.67	1.72	71.67
D	64000	1728	0.083	0.75	1.88	69.63
E	76000	3072	-	-	1.0	24.7



74.

$$M = F(S_2 + 2S_1/2) = (2500 \text{ N})(64.5 \text{ mm}) = 161250 \text{ N}\cdot\text{mm} \text{ ALONG UPPER PART}$$

$$\sigma = \frac{K_t M}{S_{NET}} ; S_{NET} = \frac{(w^3 - d^3)(t)}{6w}$$

AT B-B:  $d/w = 15/25 = 0.6 \rightarrow K_t = 1.20$

$$\sigma = \frac{(1.20)(6)(161250)(25)}{(25^3 - 15^3)(16)} = 148.1 \text{ MPa}$$

NOTE: For K<sub>t</sub> at hole in flat plate in bending: if d/W < 0.50, use K<sub>t</sub> = 1.0

75.

SEE ALSO PROBLEM 74.

AT B-B:  $d/w = 12/25 = 0.48 \rightarrow K_t = 1.0$

$$\sigma = \frac{K_t M}{S} = \frac{(1.0)(6)(161250)(25)}{(25^3 - 12^3)(16)} = 108.8 \text{ MPa}$$

76.

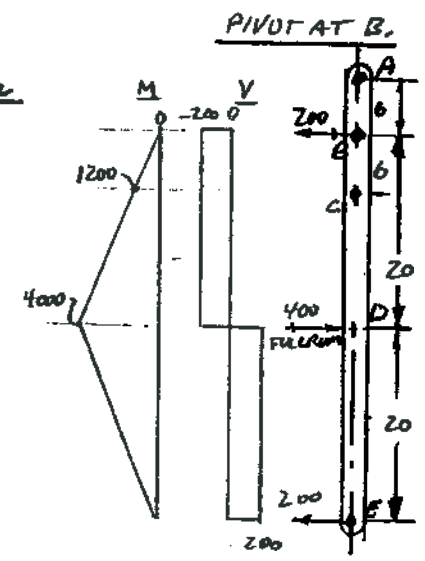
AT FULLRUM:  $S = \frac{th^3}{6} = \frac{(0.75)(2.0)^3}{6} = 0.50 \text{ IN}^3$

$$\sigma_b = \frac{M}{S} = \frac{4000 \text{ LB}\cdot\text{IN}}{0.50 \text{ IN}^3} = 8000 \text{ PSI}$$

AT HOLE: APP 15-2:  $d/w = 1.25/2.0 = 0.625, K_t = 1.25$

$$S_{NET} = \frac{(w^3 - d^3)t}{6w} = \frac{(2.00^3 - 1.25^3)(0.75)}{6(2.00)} = 0.378 \text{ IN}^3$$

$$\sigma_c = \frac{K_t M_c}{S} = \frac{1.25(1200)}{0.378} = 3969 \text{ PSI}$$



77. PIVOT AT A.  
AT FULCRUM:  $S = \frac{bh^2}{6} = \frac{(0.75)(2.00)^2}{6} = 0.50 \text{ IN}^3$

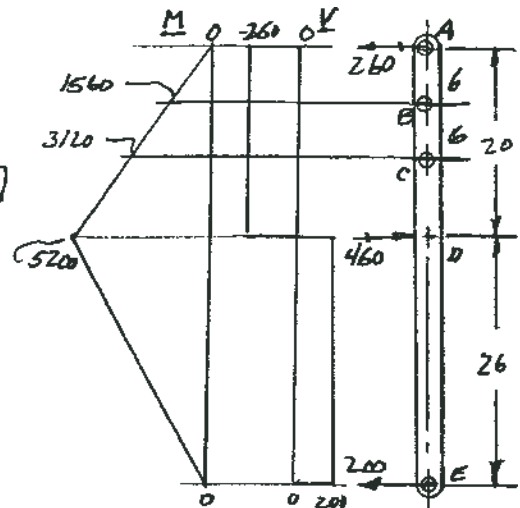
$\sigma_D = \frac{M}{S} = \frac{5200 \text{ LB}\cdot\text{IN}}{0.50 \text{ IN}^3} = 10,400 \text{ PSI}$

AT B:  $K_t = 1.25$ ;  $S_{NET} = 0.378 \text{ IN}^3$  [PROB 76.]

$\sigma_B = \frac{K_t M}{S} = \frac{1.25(1560)}{0.378} = 5160 \text{ PSI}$

AT C:

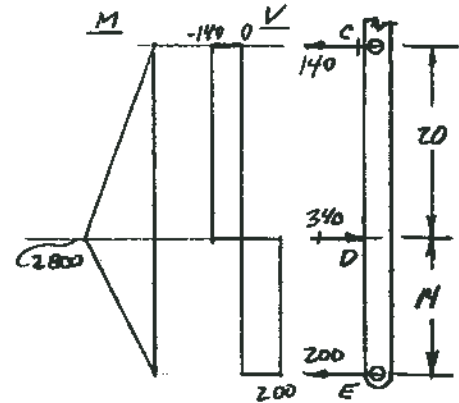
$\sigma_C = \frac{K_t M}{S} = \frac{1.25(3120)}{0.378} = 10320 \text{ PSI}$



PIVOT AT L.

AT FULCRUM:  $S = 0.50 \text{ IN}^3$

$\sigma_D = \frac{M_D}{S} = \frac{2800 \text{ LB}\cdot\text{IN}}{0.50 \text{ IN}^3} = 5600 \text{ PSI}$



78.

$M = 0$  AT A, E

POINT B IS CRITICAL

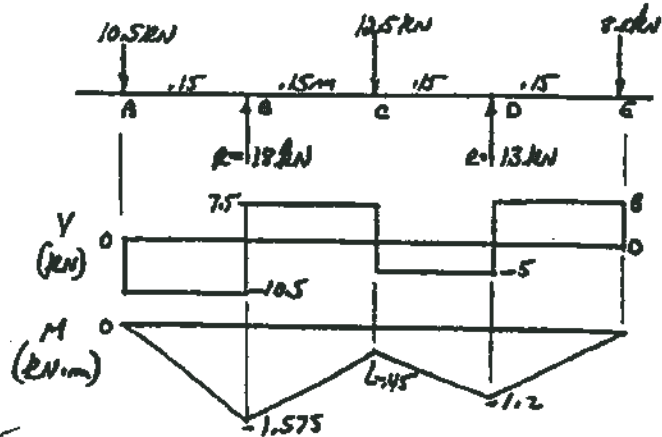
$\sigma = \frac{MK_x}{S}$

$S = \frac{\pi(YS)^3}{32} = 8946 \text{ mm}^3$

FOR FILLET

$\frac{r}{d} = \frac{2}{45} = 0.044$   
 $\frac{D}{d} = \frac{55}{45} = 1.22$   
 $K_x = 2.05$

$\sigma = \frac{MK_x}{S} = \frac{1.575 \times 10^3 \text{ N}\cdot\text{m} (2.05)}{8946 \text{ mm}^3} = 360.9 \text{ MPa}$





79.

ONE POSSIBLE DESIGN

LUG JOINT: SEE FIG. 3-69;  $d = 8.0 \text{ mm}$ ,  $w = 20.0 \text{ mm}$   
 FIND: THICKNESS  $t$ , MATERIALS FOR LUG AND PIN.  $N = 5$ .  
 FROM SOLUTION FOR PROB. 3-69,  $F = 20,000 \text{ N}$ .

$$\text{LET } t = 0.5d = 0.5(8.0 \text{ mm}) = 4.0 \text{ mm}$$

$$\text{LET } w = d/0.40 = 8.0 \text{ mm}/0.40 = 20.0 \text{ mm}$$

$$\text{LET END DISTANCE} = h = w = 20.0 \text{ mm}$$

$$\text{HOLE DIAMETER} = d_{\text{hole}} = d_{\text{pin}}(1.002) = 8.016 \text{ mm}$$

$$\sigma_{\text{MAX}} = K_t \sigma_{\text{NOM}}; \sigma_{\text{NOM}} = \frac{F}{(w-d)t} = \frac{20000 \text{ N}}{(20-8)(4.0 \text{ mm})} = 416.7 \text{ MPa}$$

$$\text{IN FIG. 3-29: } d/w = 8/20 = 0.40; K_t = 3.00$$

$$\sigma_{\text{MAX}} = K_t \sigma_{\text{NOM}} = (3.00)(416.7 \text{ MPa}) = 1250 \text{ MPa}$$

$$\text{LET } \sigma_{\text{MAX}} = \sigma_B = \frac{S_u}{N}; \text{REQ. } S_u = N \sigma_{\text{MAX}} = 5(1250) = 6250 \text{ MPa}$$

TOO HIGH FOR TYPICAL STEELS IN APP. 3.

RE-DESIGN FOR USE OF SAE 4340 OQT 800,  $S_u = 1450 \text{ MPa}$

$$\sigma_B = \frac{S_u}{N} = \frac{1450 \text{ MPa}}{5} = 290 \text{ MPa}$$

$$\text{LET } \sigma_B = \sigma_{\text{MAX}} = K_t \sigma_{\text{NOM}} = 3.0 \sigma_{\text{NOM}} \text{ FOR } d/w = 0.40$$

$$\sigma_{\text{NOM}} = \frac{\sigma_B}{3.0} = \frac{290 \text{ MPa}}{3} = 96.67 \text{ MPa} = \frac{F}{A_{\text{NET}}} = \frac{F}{(w-d)t}$$

$$\text{REQ'D } A_{\text{NET}} = \frac{F}{96.67 \text{ MPa}} = \frac{20000 \text{ N}}{96.67 \text{ N/mm}^2} = 206.9 \text{ mm}^2$$

$$\text{FOR } w = \frac{d}{0.40} = 2.5d; t = 0.5d;$$

$$A_{\text{NET}} = (w-d)t = (2.5d-d)(0.5d) = 0.75d^2 = 206.9 \text{ mm}^2$$

$$d = \sqrt{206.9 \text{ mm}^2 / 0.75} = 16.6 \text{ mm.}$$

SPECIFY PREFERRED SIZE:  $d = 18 \text{ mm}$  (APP. 2)

$$\text{THEN: } w = \frac{d}{0.4} = \frac{18}{0.4} = 45.0 \text{ mm}; t = 0.5d = 9.0 \text{ mm}$$

$$\sigma_{\text{NOM}} = \frac{F}{(w-d)t} = \frac{20000 \text{ N}}{(45-18)(9)} = \frac{20000 \text{ N}}{243 \text{ mm}^2} = 82.3 \text{ MPa}$$

(GIVEN DESIGN PARAMETERS WERE NOT FEASIBLE.)

$$\text{CHECK SHEAR STRESS IN PIN: } \tau = \frac{F}{A_s} = \frac{F}{2A} = \frac{F}{2\pi r^2} = \frac{F}{\pi d^2/2}$$

$$\tau = \frac{20000 \text{ N}}{\pi(18 \text{ mm})^2/2} = 39.3 \text{ MPa}$$

$$\text{CHECK } \tau_B = \frac{S_{su}}{N} = \frac{0.75 S_u}{5} = \frac{0.75(1450 \text{ MPa})}{5} = 217.5 \text{ MPa} > 39.3 \text{ MPa}$$

OK FOR PIN SHEAR

$$\text{HOLE DIA.} = d_{\text{HOLE}} = d_{\text{PIN}}(1.002) = 18.0 \text{ mm}(1.002) = 18.036 \text{ mm}$$

81

CURVED BEAMS: FIND F FOR YIELDING OF STEEL

ASTM A36 STEEL  $S_y = 36 \text{ ksi} = 248 \text{ MPa}$

$$\sigma_o = \frac{M(R - r_o)}{A r_o (r_c - R)} \quad \sigma_i = \frac{M(R - r_i)}{A r_i (r_c - R)}$$

$$R = A / ASF; \quad A = b^2 = 10^2 = 100 \text{ mm}^2$$

GIVEN:  $r_i = 150 \text{ mm}; \quad r_o = r_i + 10 = 160 \text{ mm}$

$$r_c = r_i + b/2 = 150 + 5 = 155 \text{ mm}$$

$$ASF = b \ln(r_o/r_i) = 10 \ln(160/150) = 0.64539 \text{ mm}$$

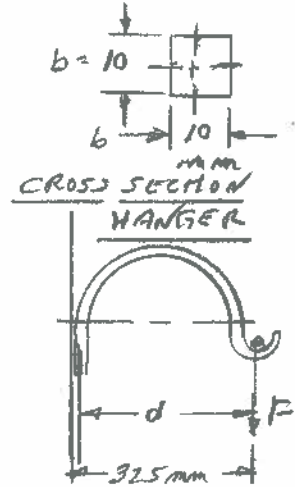
$$R = A / ASF = 100 / 0.64539 = 154.946 \text{ mm}$$

$$M = F \cdot d = F(325 - b/2) = 320F \text{ (NEGATIVE)}$$

$$\sigma_o = \frac{(-320F)(154.946 - 160)}{(100)(160)(155 - 154.946)} = \frac{1.87948F}{\text{mm}^2}$$

$$\sigma_i = \frac{(-320F)(154.946 - 150)}{(100)(150)(155 - 154.946)} = -1.962F \text{ MAXIMUM AT INSIDE SURFACE}$$

LET  $\sigma_{\text{MAX}} = \frac{-1.962F}{\text{mm}^2} = -248 \text{ N/mm}^2$ ;  $F = \frac{-248 \text{ N}}{-1.962} = 126.4 \text{ N}$  ANSWER



82

CURVED BEAM: COPING SAW FIND N FOR TENSION IN BLADE

SAE 1020 CD STEEL;  $S_y = 352 \text{ MPa}$

$$M = F \cdot d = (120 \text{ N})(145 \text{ mm}) = -17400 \text{ N}\cdot\text{mm (NEG.)}$$

$$r_i = 22 \text{ mm GIVEN}; \quad r_o = 22 + 10 = 32 \text{ mm}$$

$$r_c = 22 + 5 = 27 \text{ mm}$$

$$ASF = b \ln(r_o/r_i) = 4 \ln(32/22) = 1.4987 \text{ mm}$$

$$R = A / ASF = 40 / 1.4987 = 26.688 \text{ mm}$$

SEE PROB. 81 FOR EQUATIONS.

$$(R - r_i) = 26.688 - 22 = 4.688 \text{ mm}$$

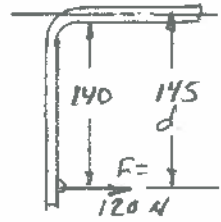
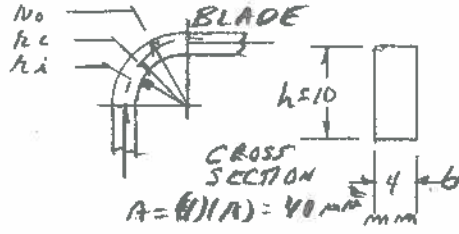
$$(r_c - R) = 27.0 - 26.688 = 0.3115 \text{ mm}$$

$$(R - r_o) = 26.688 - 32.0 = -5.3115 \text{ mm}$$

$$\sigma_i = \frac{(-17400)(4.688)}{(40)(22)(0.3115)} = -297.6 \text{ MPa COMPRESSION ON INSIDE SURFACE}$$

$$\sigma_o = \frac{(-17400)(-5.3115)}{(40)(32)(0.3115)} = 231.8 \text{ MPa TENSION ON OUTSIDE SURFACE}$$

$$\left. \begin{aligned} N_i &= \frac{S_y}{\sigma_i} = \frac{-352 \text{ MPa}}{-297.6 \text{ MPa}} = 1.18 \text{ MINIMUM} \\ N_o &= \frac{S_y}{\sigma_o} = \frac{352 \text{ MPa}}{231.8 \text{ MPa}} = 1.52 \end{aligned} \right\} \text{ BOTH LOW}$$



83

## CURVED BEAM HACK SAW FIG. P3-83

GIVEN:  $F = 480\text{ N}$ ; FIND  $N$ .SAE 120 CD;  $S_y = 352\text{ MPa}$ 

CONSIDER TUBE TO BE A COMPOSITE SECTION: ① - ②

$$A_1 = \pi(10)^2/4 = 78.54\text{ mm}^2; A_2 = \pi(6)^2/4 = 28.27\text{ mm}^2$$

$$A = A_1 - A_2 = 50.265\text{ mm}^2$$

$$ASF = ASF_1 - ASF_2$$

$$ASF_1 = 2\pi \left[ r_c - (r_c^2 - D_1^2/4)^{1/2} \right] = 3,990.3\text{ mm}$$

$$ASF_2 = 2\pi \left[ r_c - (r_c^2 - D_2^2/4)^{1/2} \right] = 1,421.8\text{ mm}$$

$$ASF = 3,990.3 - 1,421.8 = 2,568.5\text{ mm}$$

$$R = \frac{A}{ASF} = \frac{50.265\text{ mm}^2}{2,568.5\text{ mm}} = 19.569\text{ mm}$$

SEE PROBLEM B1 FOR EQUATIONS:

$$(R - r_o) = (19.569 - 25) = -5.4307\text{ mm}$$

$$(r_c - R) = (20 - 19.569) = 0.4307\text{ mm}$$

$$(R - r_i) = (19.569 - 15) = 4.569\text{ mm}$$

$$M = F \cdot d = 480\text{ N}(80\text{ mm}) = 38,400\text{ N}\cdot\text{mm}$$

$$\sigma_i = \frac{(-38,400)(4.569)}{(50.265)(15)(0.4307)} = -540.3\text{ MPa COMPRESSION ON INSIDE SURFACE}$$

$$\sigma_o = \frac{(-38,400)(-5.4307)}{(50.265)(25)(0.4307)} = 385.3\text{ MPa TENSION ON OUTSIDE SURFACE}$$

$$\left. \begin{aligned} N_i &= \frac{S_y}{\sigma_i} = \frac{-352\text{ MPa}}{-540.3\text{ MPa}} = 0.651 \\ N_o &= \frac{S_y}{\sigma_o} = \frac{352\text{ MPa}}{385.3\text{ MPa}} = 0.913 \end{aligned} \right\} \text{ BOTH INDICATE FAILURE}$$

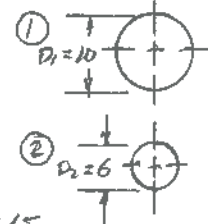
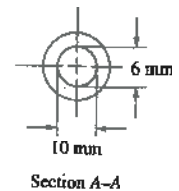
ADDITION TO PROBLEM: FIND  $F_{\text{ALL}}$  TO ACHIEVE  $N \geq 2.0$ STRESSES ARE PROPORTIONAL TO MOMENT AND TO  $F$ .

$$\text{STRESS REDUCTION REQD: } \sigma_{\text{ALL}} = \frac{S_y}{2} = \frac{352\text{ MPa}}{2} = 176\text{ MPa}$$

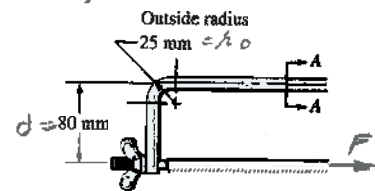
$$\frac{\sigma_{\text{ALL}}}{\sigma_i} = \frac{-176\text{ MPa}}{-540.3\text{ MPa}} = 0.326 = \frac{M_{\text{ALL}}}{M_1}$$

$$M_{\text{ALL}} = 0.326 M_1 = 0.326(-38,400) = -12,509\text{ N}\cdot\text{mm} = F_{\text{ALL}} \cdot d$$

$$F_{\text{ALL}} = \frac{12,509\text{ N}\cdot\text{mm}}{80\text{ mm}} = 156.4\text{ N}$$



$$r_c = 20; r_i = 15$$



84

CURVED BEAM GARDEN TOOL

FIND F FOR YIELDING.

CAST ALUMINUM: 356.0-T6;  $S_y = 207 \text{ MPa}$

$$A_{SF} = 2\pi \left[ r_c - (r_c^2 - D^2/4)^{1/2} \right]$$

$$A_{SF} = 2\pi \left[ 12 - (12^2 - 8^2/4)^{1/2} \right] = 4.312 \text{ mm}$$

$$R = \frac{A}{A_{SF}} = \frac{50.265 \text{ mm}^2}{4.312 \text{ mm}} = 11.6568 \text{ mm}$$

$$(r_c - R) = (12 - 11.6568) = 0.3431 \text{ mm}$$

$$(R - r_o) = (11.6568 - 16) = -4.343 \text{ mm}$$

$$(R - r_i) = (11.6568 - 8) = 3.6568 \text{ mm}$$

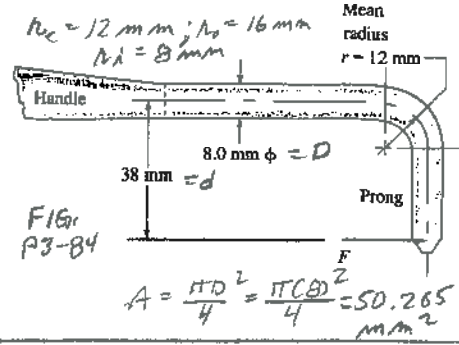
LET  $M = F \cdot d$ ,  $\sigma = S_y = 207 \text{ MPa}$  (POS.)

$$F_{O_{AIL}} = \frac{(-207 \text{ N/mm}^2)(50.265 \text{ mm}^2)(16 \text{ mm})(0.3431 \text{ mm})}{(38 \text{ mm})(-4.343 \text{ mm})} = 346.1 \text{ N}$$

SIMILARLY:

$$F_{I_{AIL}} = \frac{S_y A r_i (r_c - R)}{d (R - r_i)} = \frac{(207)(50.265)(8)(0.3431)}{(38)(3.6568)} = 205.6 \text{ N}$$

GOVERNS



$$\sigma_o = S_y = \frac{(E_o \cdot d)(R - r_o)}{A r_o (r_c - R)}$$

$$F_{O_{AIL}} = \frac{S_y (A) r_o (r_c - R)}{d (R - r_o)}$$

LET  $S_y = -207 \text{ MPa}$  COMPRESSION

85

CURVED BEAM HOOP SUPPORT FIG. P3-85:  $r_i = r_o - D$

STEEL: ASTM A53-GR B  $S_y = 35 \text{ KSI}$

$$M = F \cdot d = (230 \text{ lb})(48 \text{ in}) = -11040 \text{ lb-in (NEG.)}$$

ANALYSIS AS IN PROB 3-83

STRESS EQNS. IN PROB 3-81

$$A_{SF} = A_{SF1} - A_{SF2} = 0.61748 - 0.45484 = 0.16264 \text{ IN}$$

$$R = \frac{A}{A_{SF}} = \frac{1.704 \text{ IN}^2}{0.16264 \text{ IN}} = 10.4769 \text{ IN}$$

$$(R - r_o) = -1.5231 \text{ IN}; (R - r_i) = 1.35185 \text{ IN}; (r_c - R) = 0.08564 \text{ IN}$$

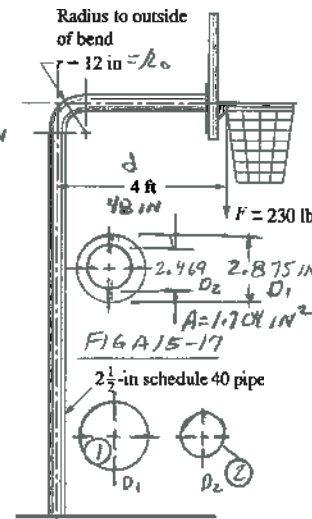
$$\sigma_i = \frac{(-11040 \text{ lb-in})(1.35185 \text{ IN})}{(1.704 \text{ IN}^2)(9.125 \text{ IN})(0.08564 \text{ IN})} = -11208 \text{ PSI COMP.}$$

$$\sigma_o = \frac{(-11040 \text{ lb-in})(-1.5231)}{(1.704 \text{ IN}^2)(12)(0.08564 \text{ IN})} = 9602.4 \text{ PSI TENSION}$$

$$N_i = \frac{S_y}{\sigma_i} = \frac{-35000 \text{ PSI}}{-11208 \text{ PSI}} = 3.123 \text{ LOWEST - GOVERNS}$$

$$N_o = \frac{S_y}{\sigma_o} = \frac{35000 \text{ PSI}}{9602.4 \text{ PSI}} = 3.645$$

SATISFACTORY N



86

CURVED BEAM C-C LAMP FIG. P3-86

CAST ZINC ZA-12;  $S_{UT} = 401 \text{ MPa}$   
 $S_{UC} = 269 \text{ MPa}$

FIND:  $F$  FOR  $N=3$ 

$$\bar{y} = \frac{1}{A} \left[ (b_1 y_1) \left( \frac{r_1}{2} \right) + (b_2 y_2) \left( \frac{r_1 + r_2}{2} \right) \right]$$

$$\bar{y} = \frac{1}{57} \left[ (8 \cdot 3) (1.5) + (3 \cdot 11) (3 + 5.5) \right] = 5.553 \text{ mm}$$

$$ASF = b_1 \ln \left( \frac{r_1}{r_2} \right) + b_2 \ln \left( \frac{r_1}{r_2} \right); r_1 = r_i + r_i = 5 + 3 = 8 \text{ mm}$$

$$ASF = 8 \ln \left( \frac{8}{5} \right) + 3 \ln \left( \frac{19}{8} \right) = 6.355 \text{ mm}$$

$$R = A / ASF = 57 / 6.355 = 8.969 \text{ mm}$$

$$M = F (26 + \bar{y}) = F (26 + 5.553) = 31.553 F \text{ (Pos)}$$

$$\sigma_i = \frac{M (R - r_i)}{A r_i (r_c - R)} = \frac{31.553 F (3.969)}{(57)(5)(1.583)} = 0.2776 F_i \text{ TENSION}$$

$$\text{LET } \sigma_i = \frac{S_{UT}}{3} = \frac{401 \text{ MPa}}{3} = 134.67 \text{ MPa}$$

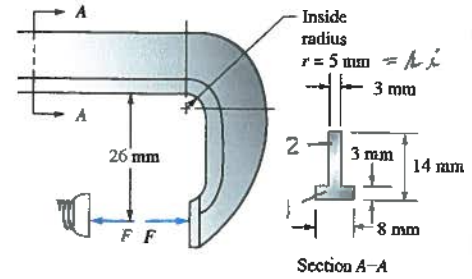
$$F_i = \frac{134.67 \text{ N/mm}^2}{0.2776 \text{ mm}^2} = 485.1 \text{ N}$$

$$\sigma_o = \frac{M (R - r_o)}{A r_o (r_c - R)} = \frac{31.553 F (-10.0307)}{(57)(19)(1.583)} = -0.1846 F_o \text{ COMPRESSION}$$

$$\text{LET } \sigma_o = \frac{S_{UC}}{3} = \frac{-269 \text{ MPa}}{3} = -89.67 \text{ N/mm}^2$$

$$F_o = \frac{-89.67 \text{ N/mm}^2}{-0.1846 \text{ mm}^2} = 485.7 \text{ N}$$

BECAUSE  $F_o \approx F_i$ , THE DESIGN OF THE INVERTED T-SECTION USES THE MATERIAL VERY EFFICIENTLY.

PART 1:  $A_1 = 3 \cdot 8 = 24 \text{ mm}^2$ PART 2:  $A_2 = 3 \cdot 11 = 33 \text{ mm}^2$  $A = A_1 + A_2 = 57 \text{ mm}^2$  $r_o = r_i + 14 = 5 + 14 = 19 \text{ mm}$  $r_c = r_i + \bar{y} = 5 + 5.553$  $r_c = 10.553 \text{ mm}$  $(R - r_i) = 8.969 - 5$  $= 3.969 \text{ mm}$  $(R - r_o) = 8.969 - 19$  $= -10.0307 \text{ mm}$  $(r_c - R) = 10.553 - 8.969$  $= 1.583 \text{ mm}$

## CHAPTER 4 COMBINED STRESSES AND MOHR'S CIRCLE

**NOTE:** The solutions to Chapter 4 problems 1 – 30 are shown on the following pages as images of the output from the MDESIGN – MOTT software that is included in the text. Each problem produces a solution in line with the procedure shown for manual solution in Section 4-4 in Chapter 4 of the text and as shown in the four Example Problems in Section 4-5 of the text. Problem 4-1 is shown worked out in manual form below and the MDESIGN – MOTT solution is shown on the following page. Solutions for all other problems are shown only as the results from the MDESIGN – MOTT solutions. Note that in the MDESIGN-MOTT output, the graphic view of Mohr's circle and the stress elements show the stress values only in psi.

1

$$\sigma_x = 20 \text{ ksi}; \sigma_y = 0; \tau_{xy} = 10 \text{ ksi}; \text{ X-AXIS IN 1ST QUADRANT}$$

$$\sigma_{AVG} = (\sigma_x + \sigma_y)/2 = (20 + 0)/2 = 10.0 \text{ ksi}$$

$$a = \sigma_x - \sigma_{AVG} = 20 - 10 = 10 \text{ ksi}; b = \tau_{xy} = 10 \text{ ksi}$$

$$R = \sqrt{a^2 + b^2} = \sqrt{10^2 + 10^2} = 14.14 \text{ ksi} = \tau_{MAX}$$

$$\alpha = \tan^{-1}(b/a) = \tan^{-1}(10/10) = 45^\circ = 2\phi_\sigma \text{ CW FROM X-AXIS TO } \sigma_1$$

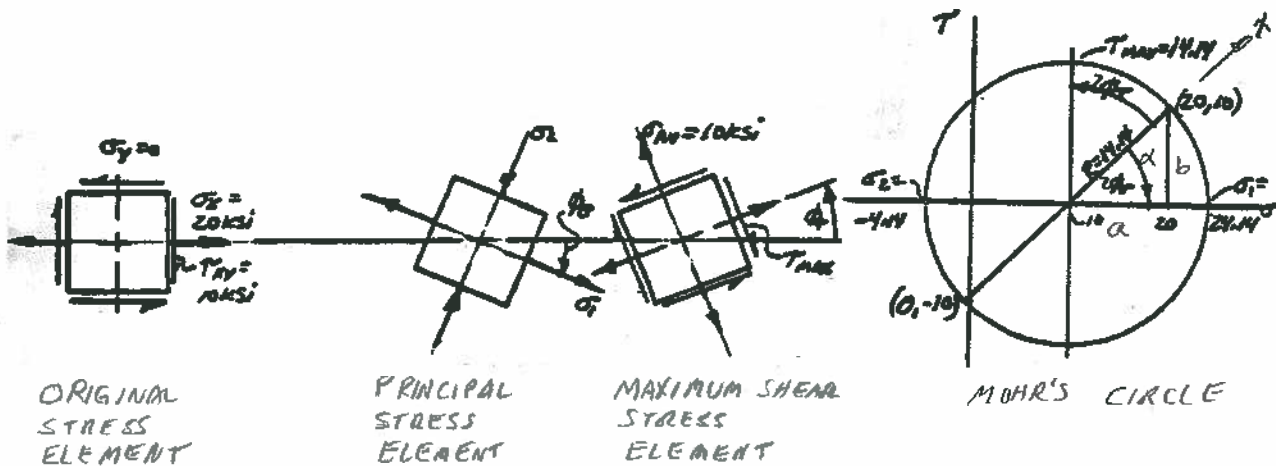
$$\phi_\sigma = 2\phi_\sigma / 2 = 45.0^\circ / 2 = 22.5^\circ$$

$$2\phi_\tau = 90^\circ - \alpha = 90 - 45 = 45^\circ \text{ CCW FROM X-AXIS TO } \tau_{MAX}$$

$$\phi_\tau = 2\phi_\tau / 2 = 45^\circ / 2 = 22.5^\circ$$

$$\sigma_1 = \sigma_{AVG} + R = 10 + 14.14 = 24.14 \text{ ksi}$$

$$\sigma_2 = \sigma_{AVG} - R = 10 - 14.14 = -4.14 \text{ ksi}$$



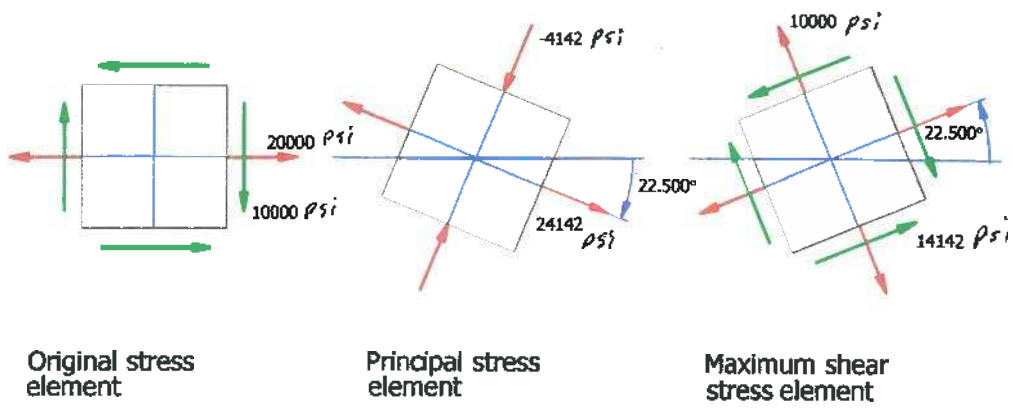
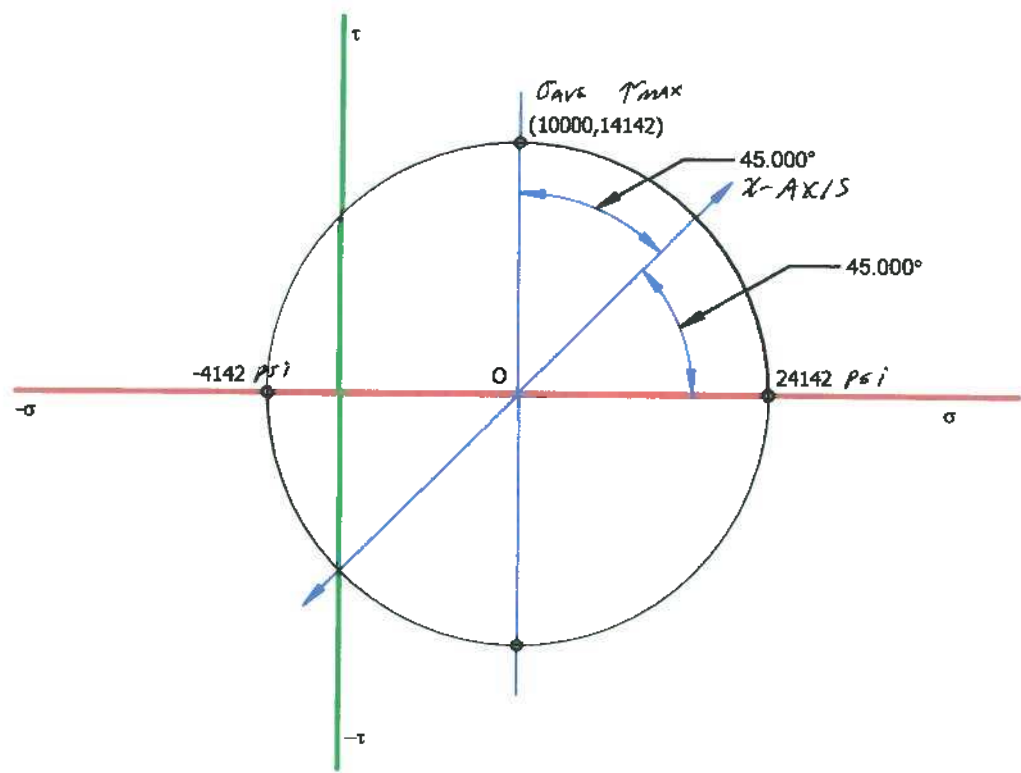




$\sigma_x = 20$  ksi  
 $\sigma_y = 0$  ksi  
 $\tau_{xy} = 10$  ksi

**Results:**

Maximum principal stress	$\sigma_1 =$	24.142	ksi
Minimum principal stress	$\sigma_2 =$	-4.142	ksi
Maximum shear stress	$\tau_{max} =$	14.142	ksi
Average normal stress	$\sigma_{avg} =$	10.000	ksi
Principal planes	$\phi_\sigma =$	22.500	°
Angle of maximum shear stress	$\phi_\tau =$	22.500	°
			CW
			CCW

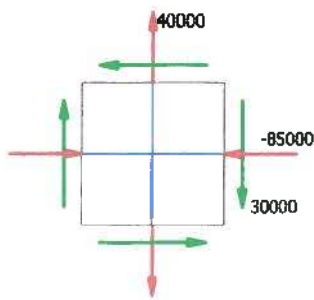
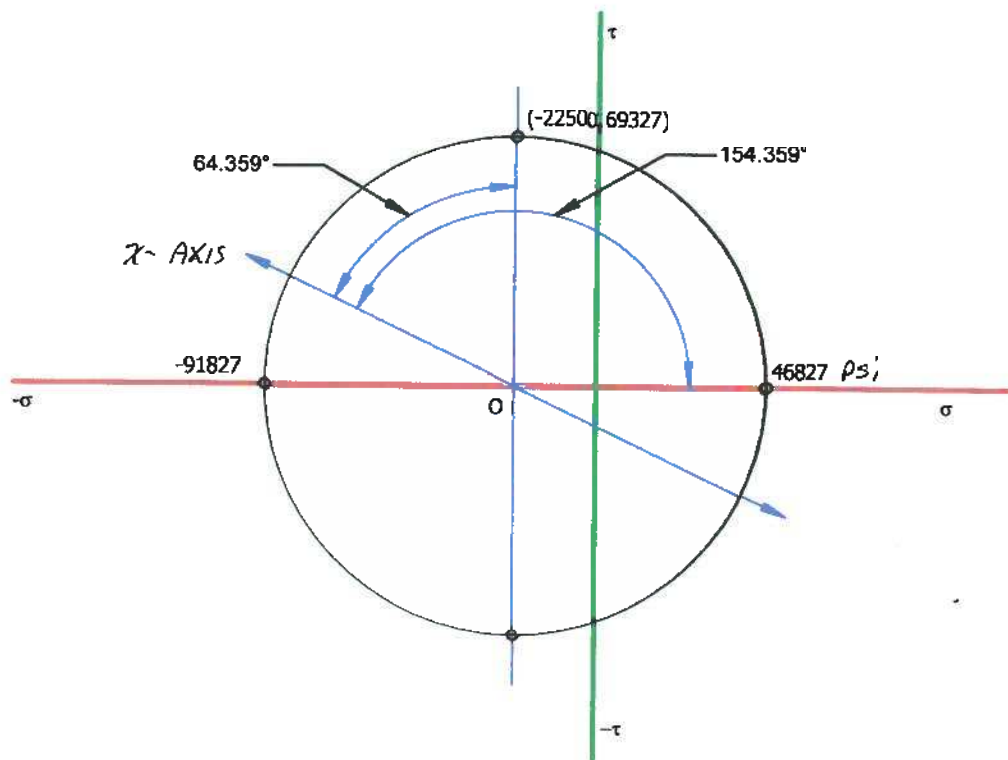


2

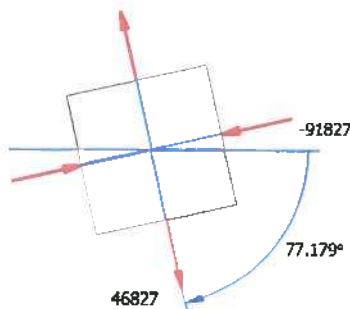
$\sigma_x = -85000$  psi  
 $\sigma_y = 40000$  psi  
 $\tau_{xy} = 30000$  psi

**Results:**

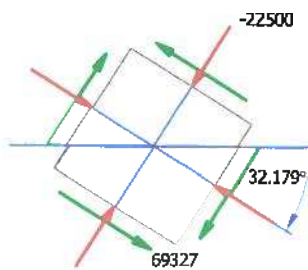
Maximum principal stress  $\sigma_1 = 46827.123$  psi  
 Minimum principal stress  $\sigma_2 = -91827.123$  psi  
 Maximum shear stress  $\tau_{max} = 69327.123$  psi  
 Average normal stress  $\sigma_{avg} = -22500.000$  psi  
 Principal planes  $\phi_\sigma = 77.179^\circ$  CW  
 Angle of maximum shear stress  $\phi_\tau = 32.179^\circ$  CW



Original stress element



Principal stress element



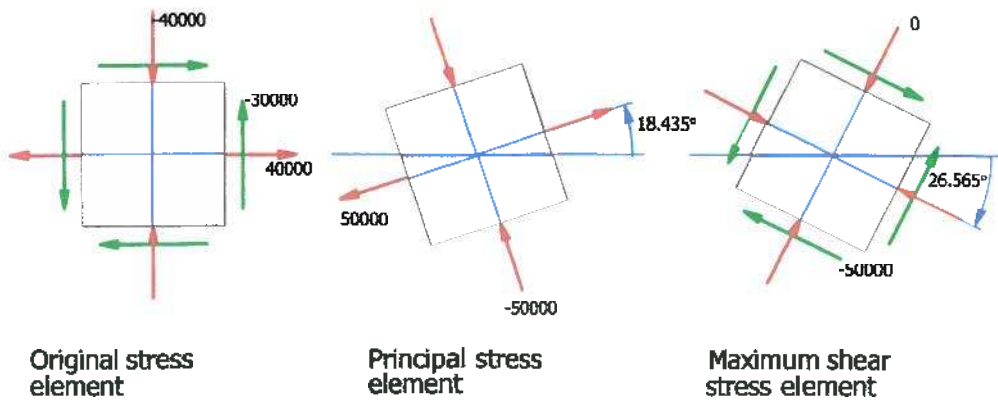
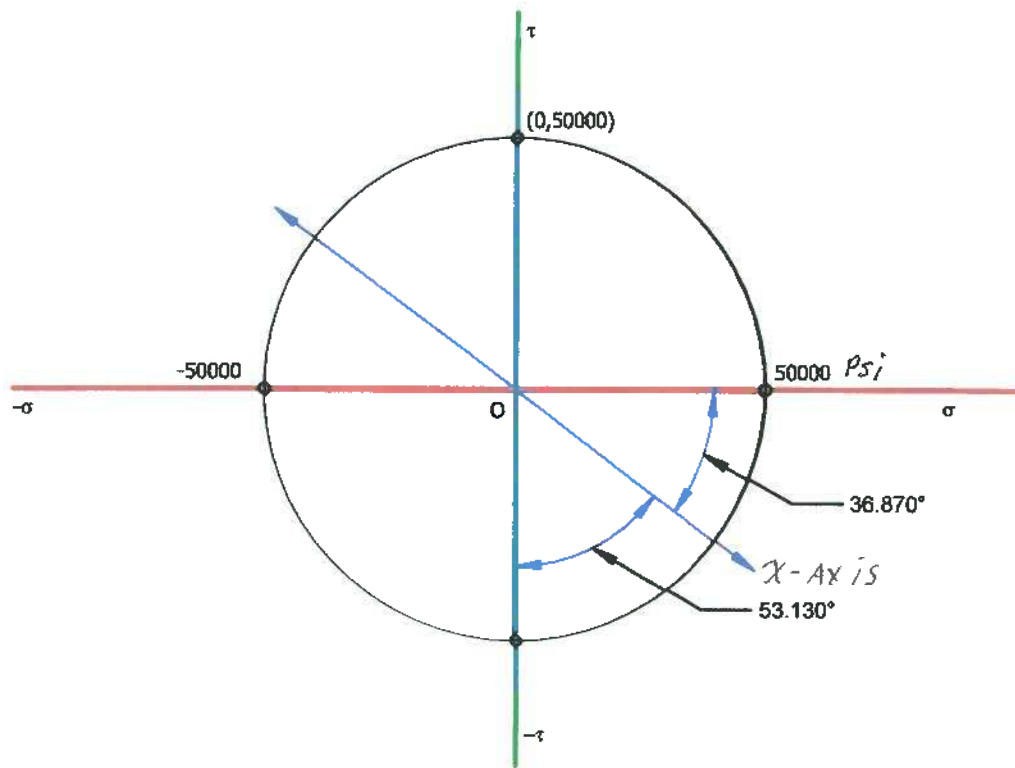
Maximum shear stress element

3

$\sigma_x = 40$  ksi  
 $\sigma_y = -40$  ksi  
 $\tau_{xy} = -30$  ksi

**Results:**

Maximum principal stress	$\sigma_1 =$	50.000	ksi
Minimum principal stress	$\sigma_2 =$	-50.000	ksi
Maximum shear stress	$\tau_{max} =$	50.000	ksi
Average normal stress	$\sigma_{avg} =$	0.000	ksi
Principal planes	$\phi_\sigma =$	18.435	°
			CCW
Angle of maximum shear stress	$\phi_\tau =$	26.565	°
			CW to $-\tau_{max}$



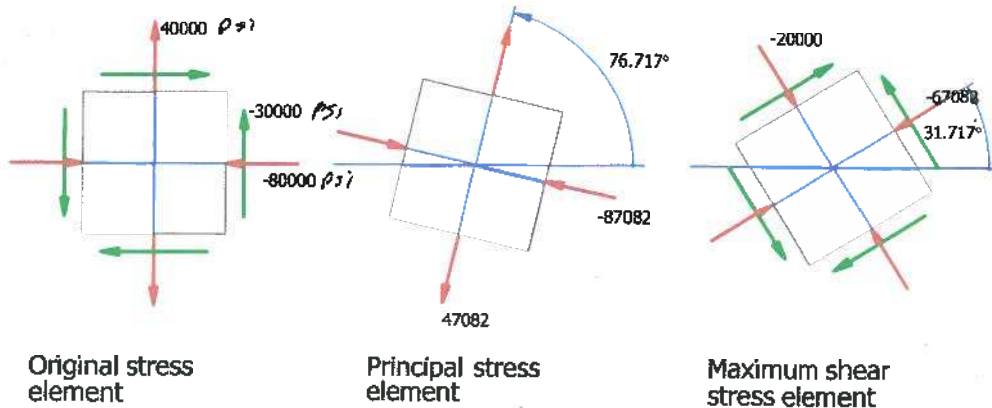
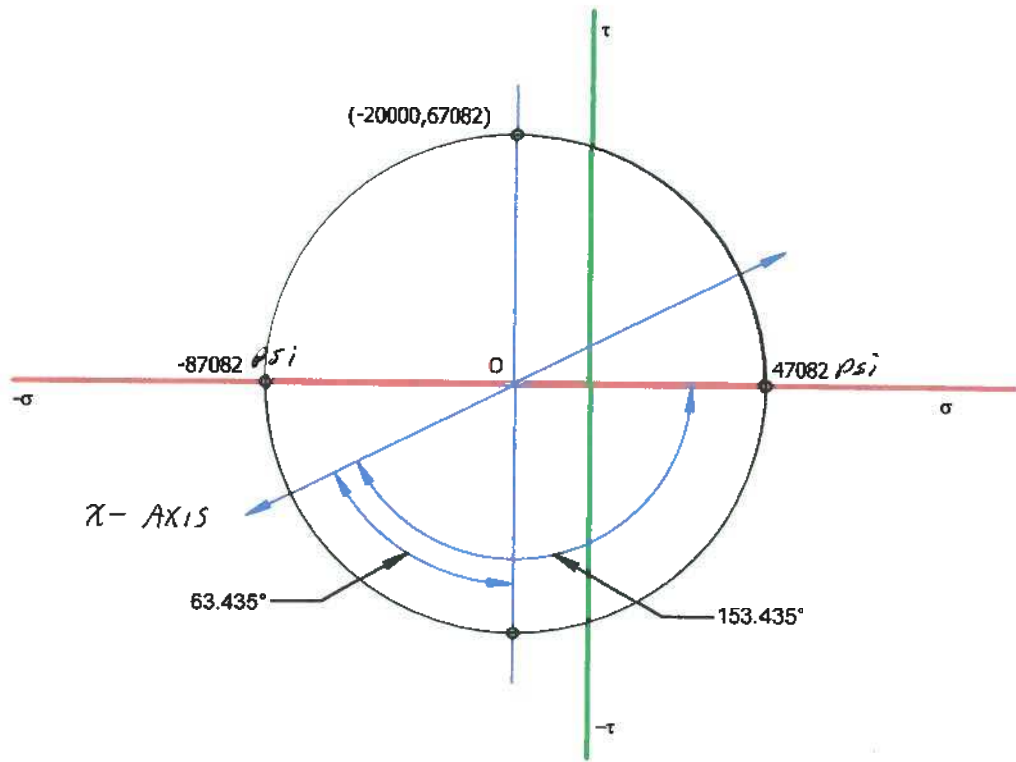
4

$\sigma_x = -80$  ksi  
 $\sigma_y = 40$  ksi  
 $\tau_{xy} = -30$  ksi

**Results:**

Maximum principal stress  $\sigma_1 = 47.082$  ksi  
 Minimum principal stress  $\sigma_2 = -87.082$  ksi  
 Maximum shear stress  $\tau_{max} = 67.082$  ksi  
 Average normal stress  $\sigma_{avg} = -20.000$  ksi  
 Principal planes  $\phi_\sigma = 76.717^\circ$  °  
 Angle of maximum shear stress  $\phi_\tau = 31.717^\circ$  °

CCW  
 CCW to  $-\tau_{max}$



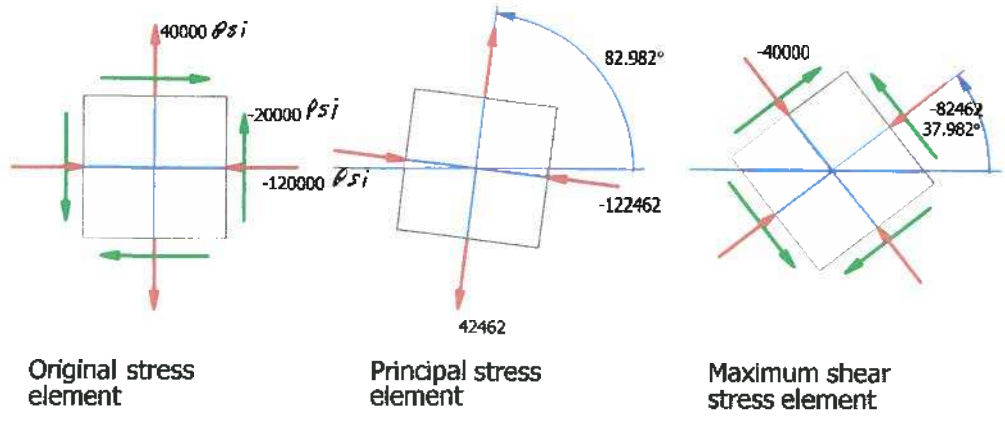
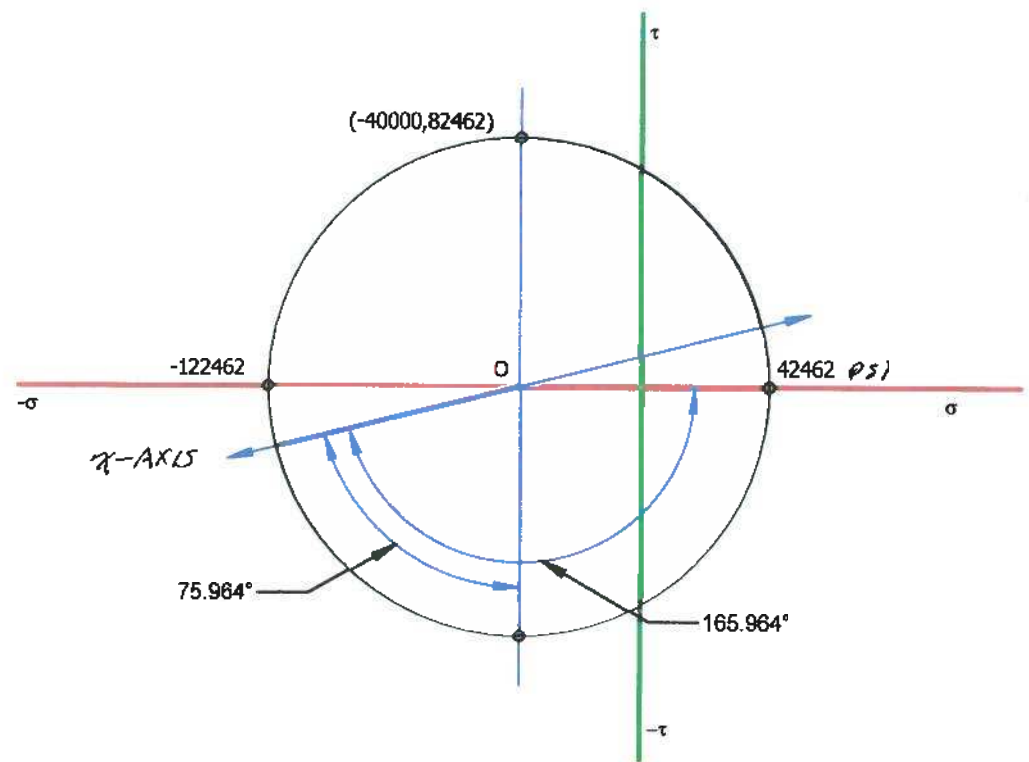
5

$\sigma_x = -120000$  psi  
 $\sigma_y = 40000$  psi  
 $\tau_{xy} = -20000$  psi

**Results:**

Maximum principal stress	$\sigma_1 = 42462.113$	psi
Minimum principal stress	$\sigma_2 = -122462.113$	psi
Maximum shear stress	$\tau_{max} = 82462.113$	psi
Average normal stress	$\sigma_{avg} = -40000.000$	psi
Principal planes	$\phi_\sigma = 82.982$	°
Angle of maximum shear stress	$\phi_\tau = 37.982$	°

CCW  
 CCW to  $-\tau_{max}$



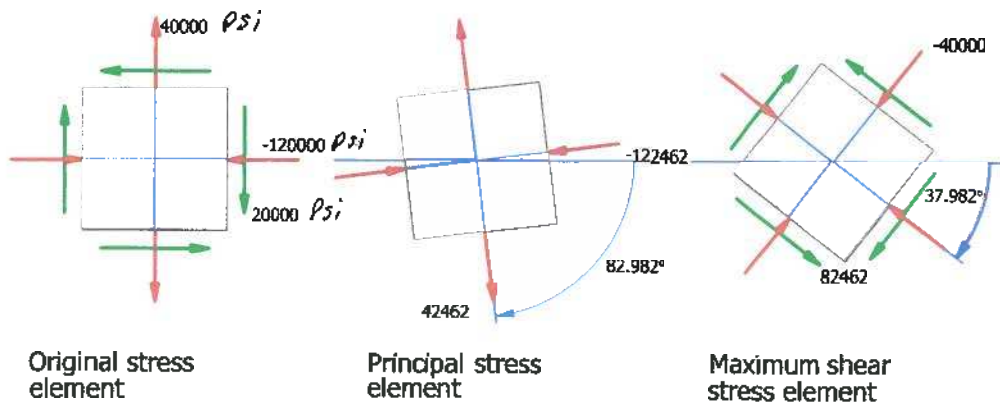
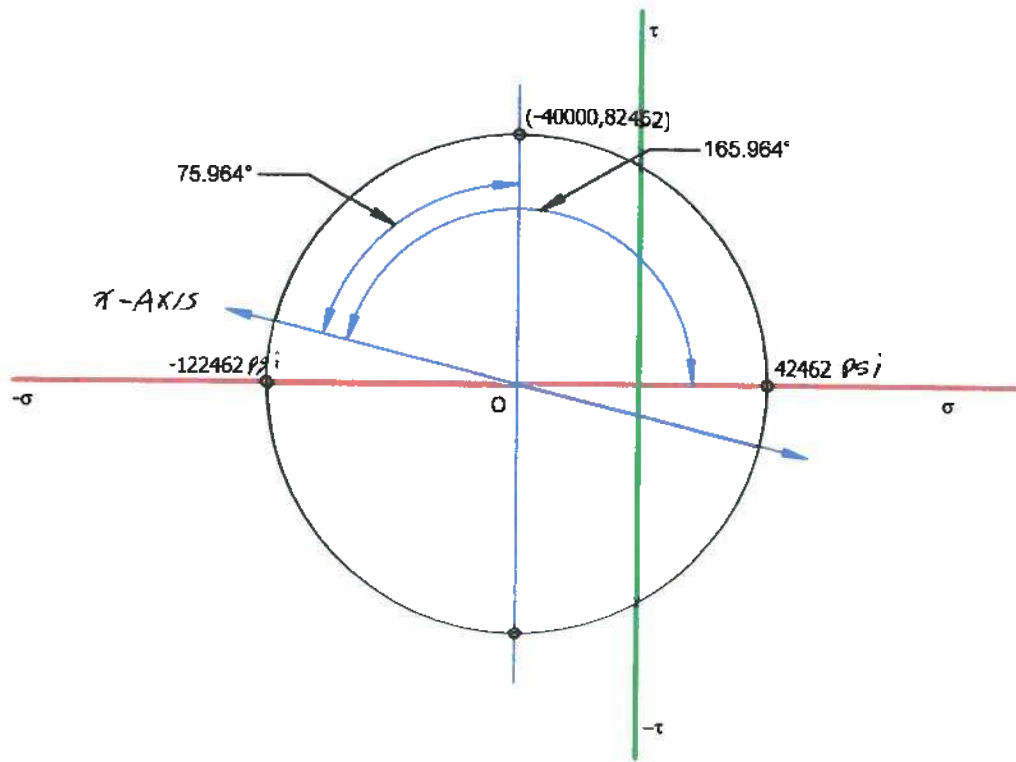
6

$\sigma_x = -120$  ksi  
 $\sigma_y = 40$  ksi  
 $\tau_{xy} = 20$  ksi

**Results:**

Maximum principal stress	$\sigma_1 =$	42.462	ksi
Minimum principal stress	$\sigma_2 =$	-122.462	ksi
Maximum shear stress	$\tau_{max} =$	82.462	ksi
Average normal stress	$\sigma_{avg} =$	-40.000	ksi
Principal planes	$\phi_\sigma =$	82.982	°
Angle of maximum shear stress	$\phi_\tau =$	37.982	°

CW  
CW

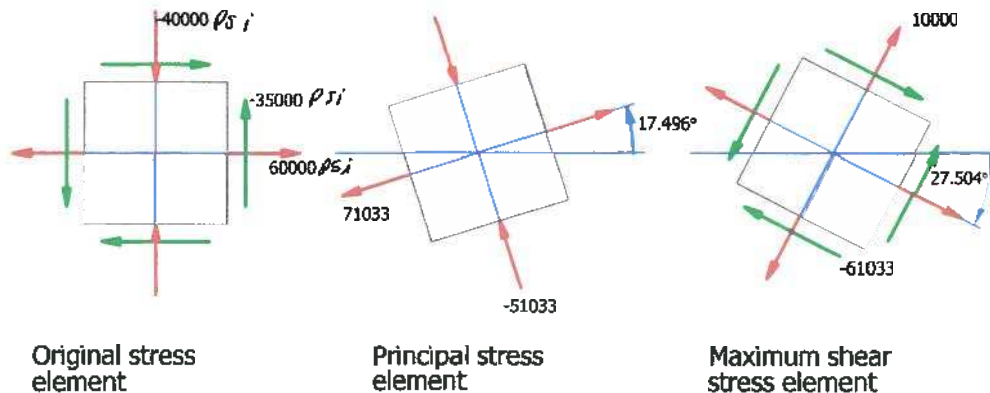
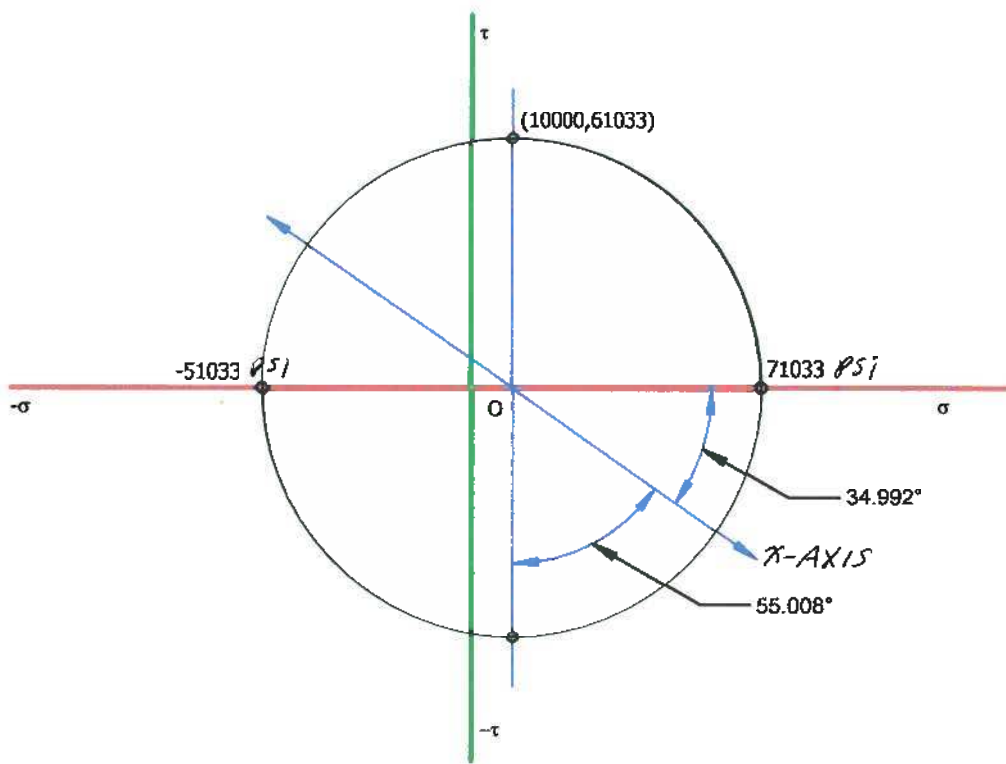


7

$\sigma_x = 60000$  psi  
 $\sigma_y = -40000$  psi  
 $\tau_{xy} = -35000$  psi

**Results:**

Maximum principal stress	$\sigma_1 = 71032.778$	psi
Minimum principal stress	$\sigma_2 = -51032.778$	psi
Maximum shear stress	$\tau_{max} = 61032.778$	psi
Average normal stress	$\sigma_{avg} = 10000.000$	psi
Principal planes	$\phi_\sigma = 17.496$	°
		CCW
Angle of maximum shear stress	$\phi_\tau = 27.504$	°
		CW to $-\tau_{max}$





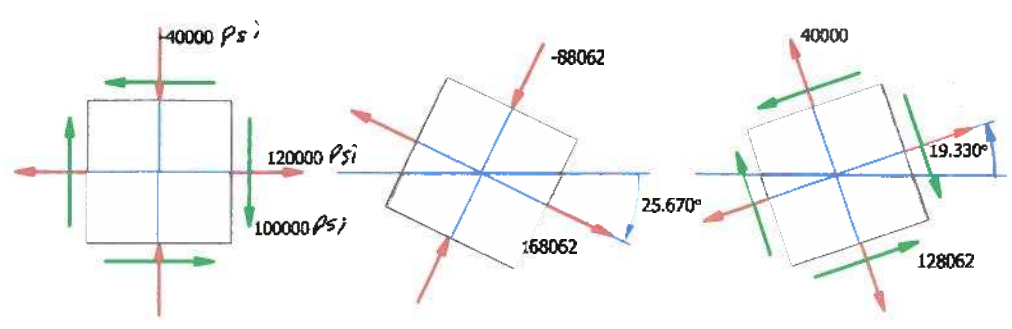
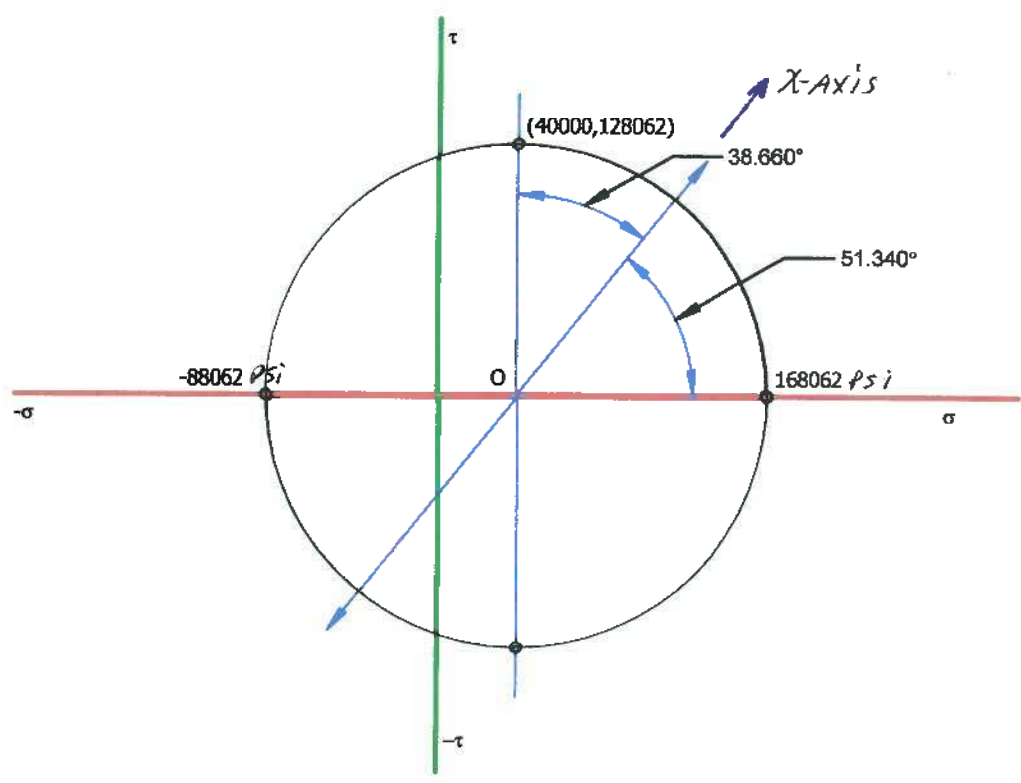
8

$\sigma_x = 120$  ksi  
 $\sigma_y = -40$  ksi  
 $\tau_{xy} = 100$  ksi

**Results:**

Maximum principal stress	$\sigma_1 = 168.062$	ksi
Minimum principal stress	$\sigma_2 = -88.062$	ksi
Maximum shear stress	$\tau_{max} = 128.062$	ksi
Average normal stress	$\sigma_{avg} = 40.000$	ksi
Principal planes	$\phi_\sigma = 25.670$	°
Angle of maximum shear stress	$\phi_\tau = 19.330$	°

CW  
 CCW



Original stress element

Principal stress element

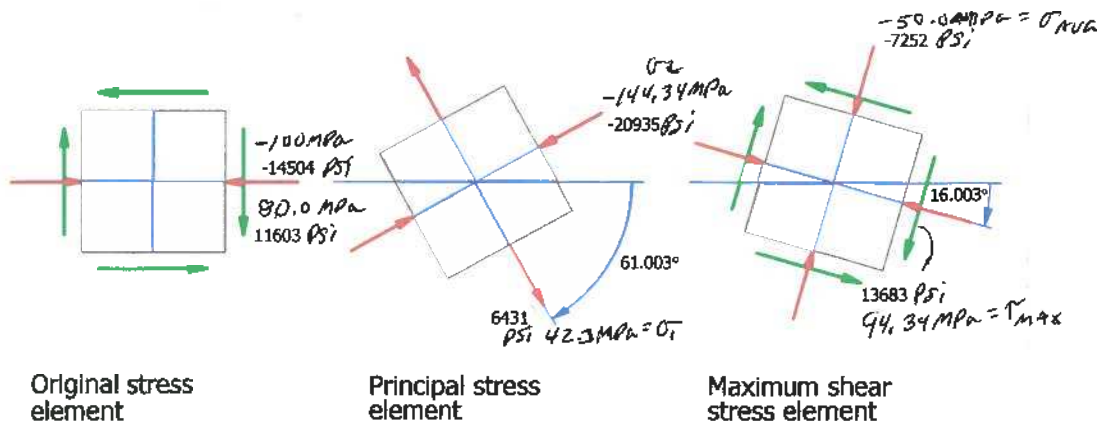
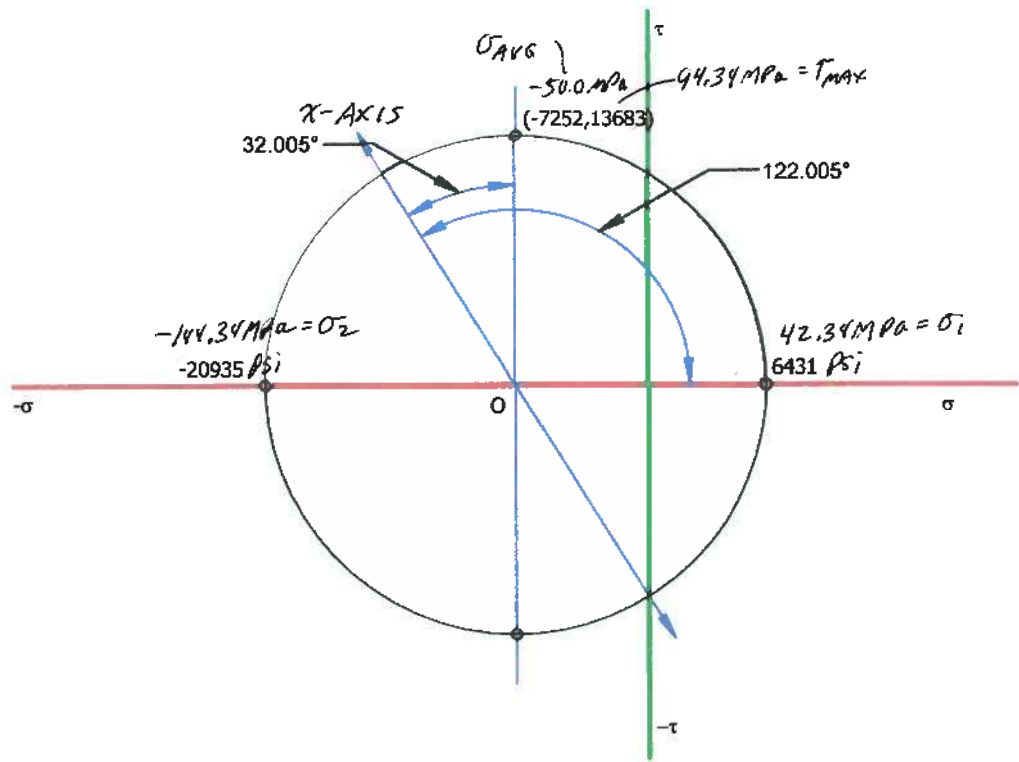
Maximum shear stress element

9

$\sigma_x = -100$  MPa  
 $\sigma_y = 0$  MPa  
 $\tau_{xy} = 80$  MPa

**Results:**

Maximum principal stress	$\sigma_1 =$	44.340	MPa
Minimum principal stress	$\sigma_2 =$	-144.340	MPa
Maximum shear stress	$\tau_{max} =$	94.340	MPa
Average normal stress	$\sigma_{avg} =$	-50.000	MPa
Principal planes	$\phi_\sigma =$	61.003	°
			CW
Angle of maximum shear stress	$\phi_\tau =$	16.003	°
			CW

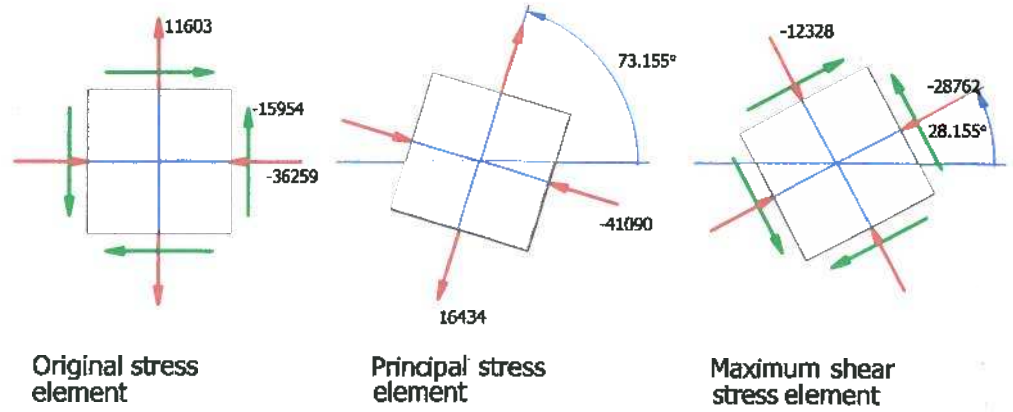
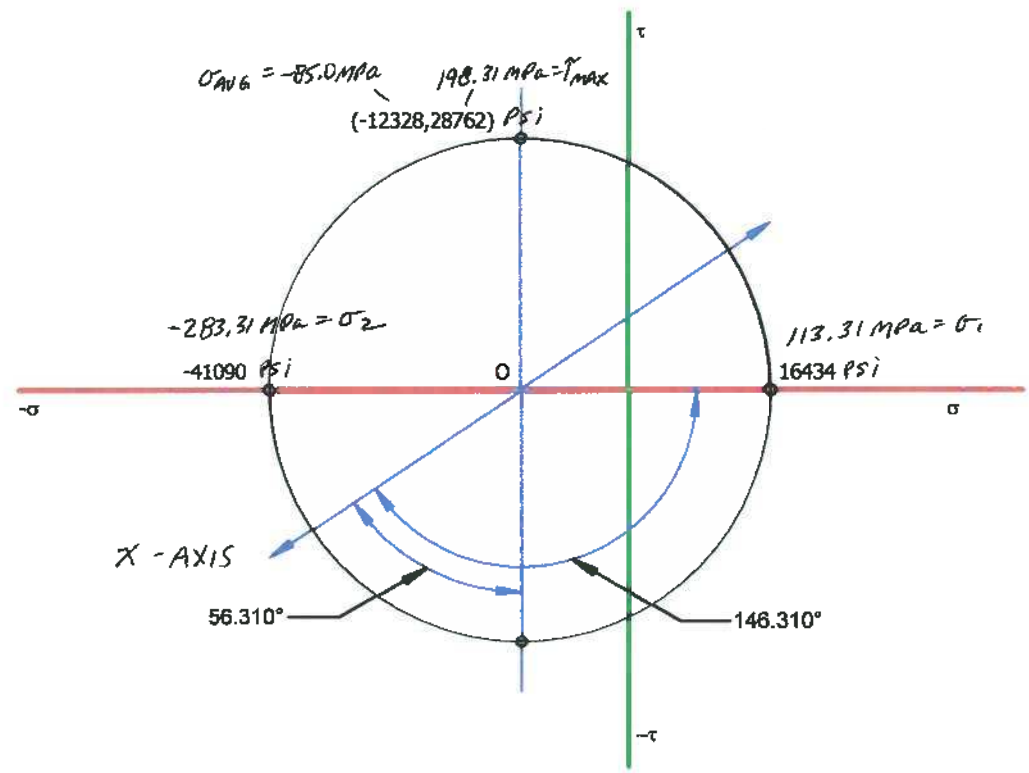


10

$\sigma_x = -250$  MPa  
 $\sigma_y = 80$  MPa  
 $\tau_{xy} = -110$  MPa

**Results:**

Maximum principal stress	$\sigma_1 = 113.305$ MPa
Minimum principal stress	$\sigma_2 = -283.305$ MPa
Maximum shear stress	$\tau_{max} = 198.305$ MPa
Average normal stress	$\sigma_{avg} = -85.000$ MPa
Principal planes	$\phi_\sigma = 73.155^\circ$
	ccw
Angle of maximum shear stress	$\phi_\tau = 28.155^\circ$
	ccw to $-\tau_{max}$



//

$\sigma_x = 50$  MPa  
 $\sigma_y = -80$  MPa  
 $\tau_{xy} = 40$  MPa

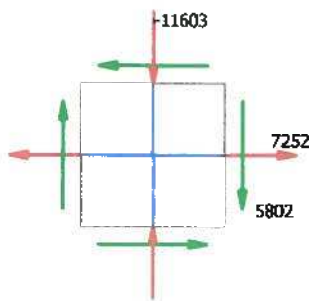
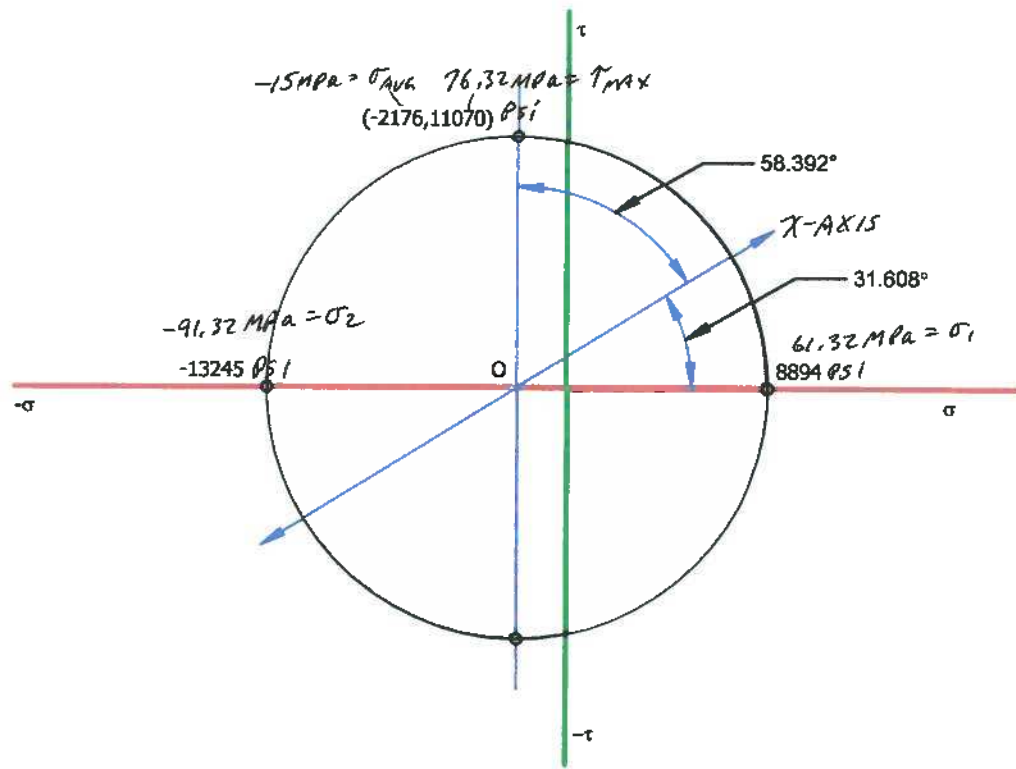
**Results:**

Maximum principal stress  $\sigma_1 = 61.322$  MPa  
 Minimum principal stress  $\sigma_2 = -91.322$  MPa  
 Maximum shear stress  $\tau_{max} = 76.322$  MPa  
 Average normal stress  $\sigma_{avg} = -15.000$  MPa  
 Principal planes  $\phi_\sigma = 15.804^\circ$

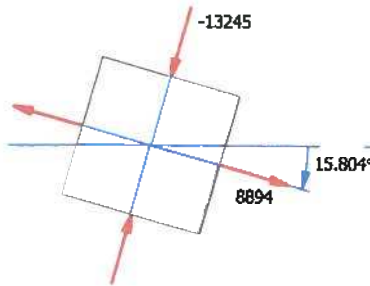
Angle of maximum shear stress  $\phi_\tau = 29.196^\circ$

CW

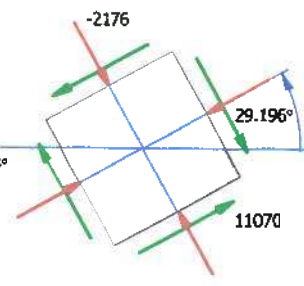
CCW



Original stress element



Principal stress element



Maximum shear stress element

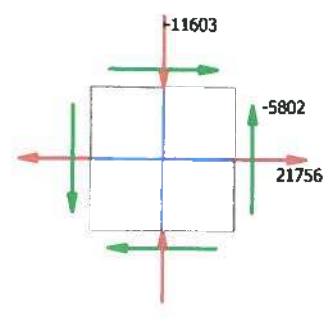
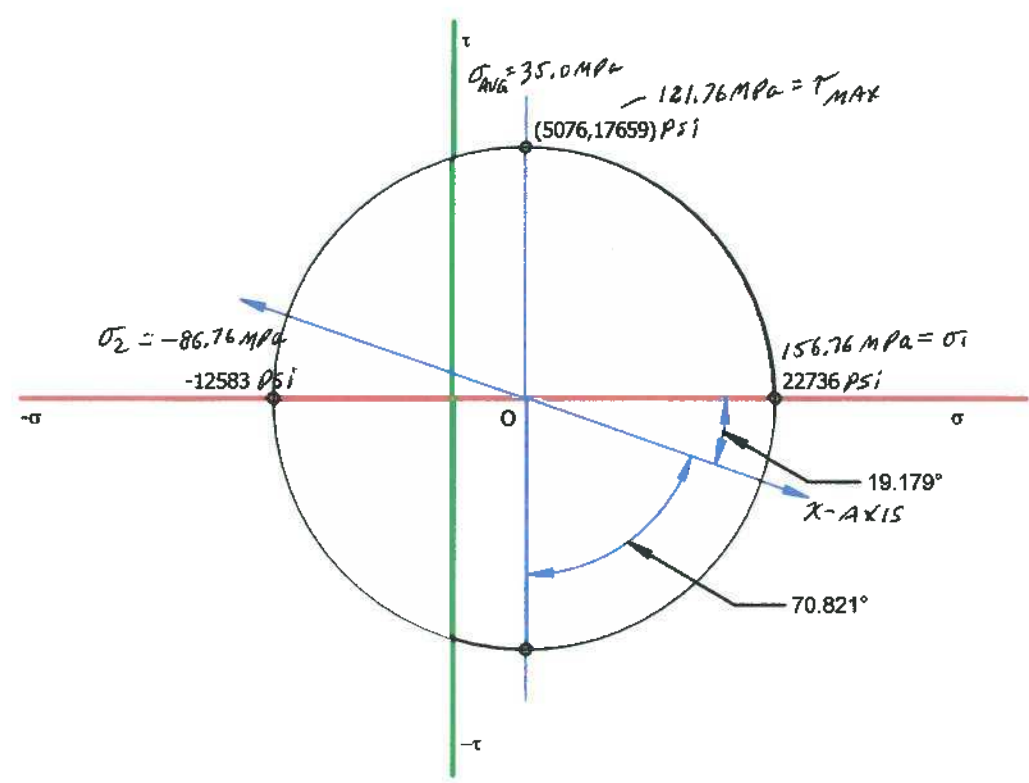
12

$\sigma_x = 150$  MPa  
 $\sigma_y = -80$  MPa  
 $\tau_{xy} = -40$  MPa

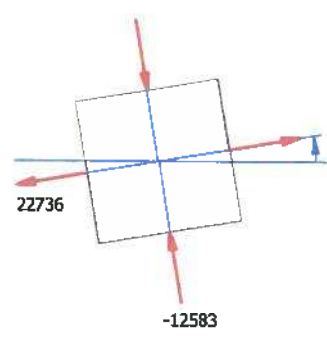
**Results:**

Maximum principal stress  $\sigma_1 = 156.758$  MPa  
 Minimum principal stress  $\sigma_2 = -86.758$  MPa  
 Maximum shear stress  $\tau_{max} = 121.758$  MPa  
 Average normal stress  $\sigma_{avg} = 35.000$  MPa  
 Principal planes  $\phi_\sigma = 9.590^\circ$

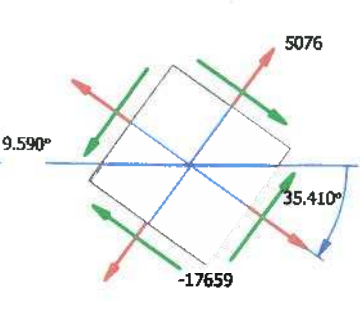
Angle of maximum shear stress  $\phi_\tau = 35.410^\circ$   
 CCW  
 CW to  $-\tau_{max}$



Original stress element



Principal stress element



Maximum shear stress element

13

$\sigma_x = -150$  MPa

$\sigma_y = 80$  MPa

**Results:**  $\tau_{xy} = -40$  MPa

Maximum principal stress  $\sigma_1 = 86.758$  MPa

Minimum principal stress  $\sigma_2 = -156.758$  MPa

Maximum shear stress  $\tau_{max} = 121.758$  MPa

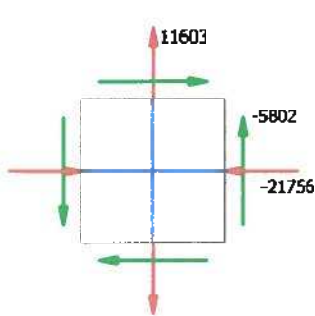
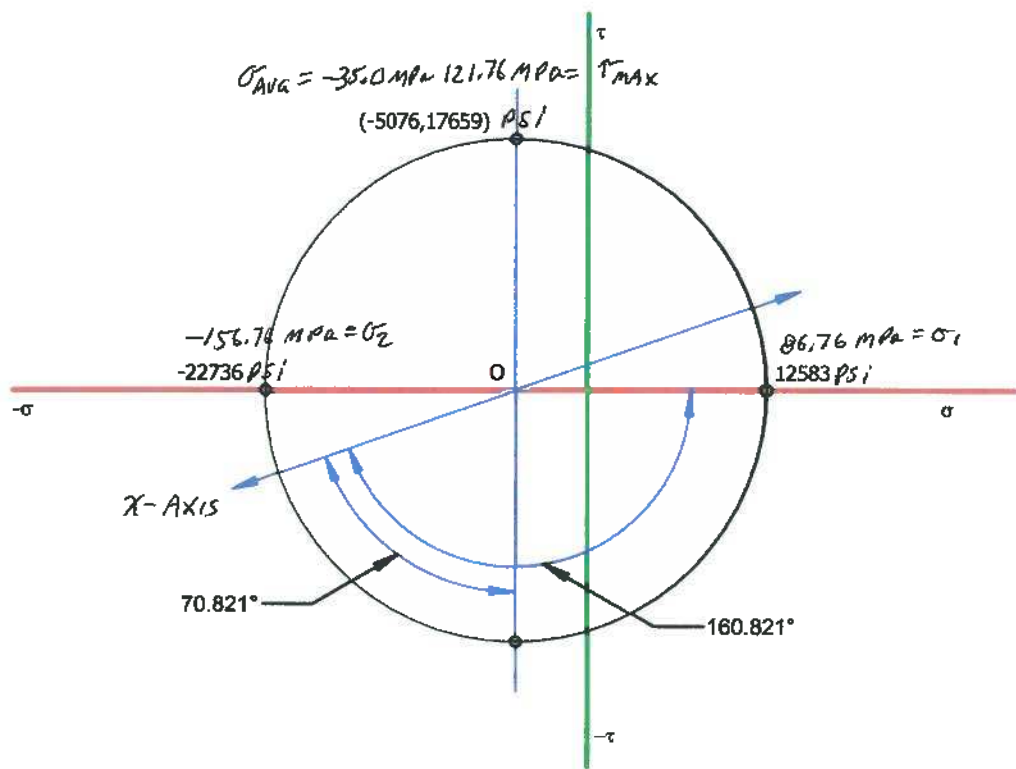
Average normal stress  $\sigma_{avg} = -35.000$  MPa

Principal planes  $\phi_\sigma = 80.410^\circ$

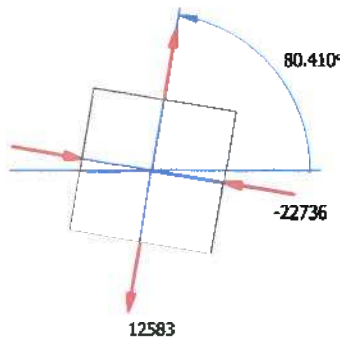
Angle of maximum shear stress  $\phi_\tau = 35.410^\circ$

CCW

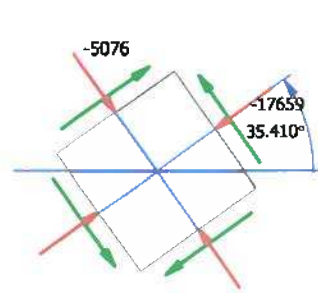
CCW to  $-\tau_{max}$



Original stress element

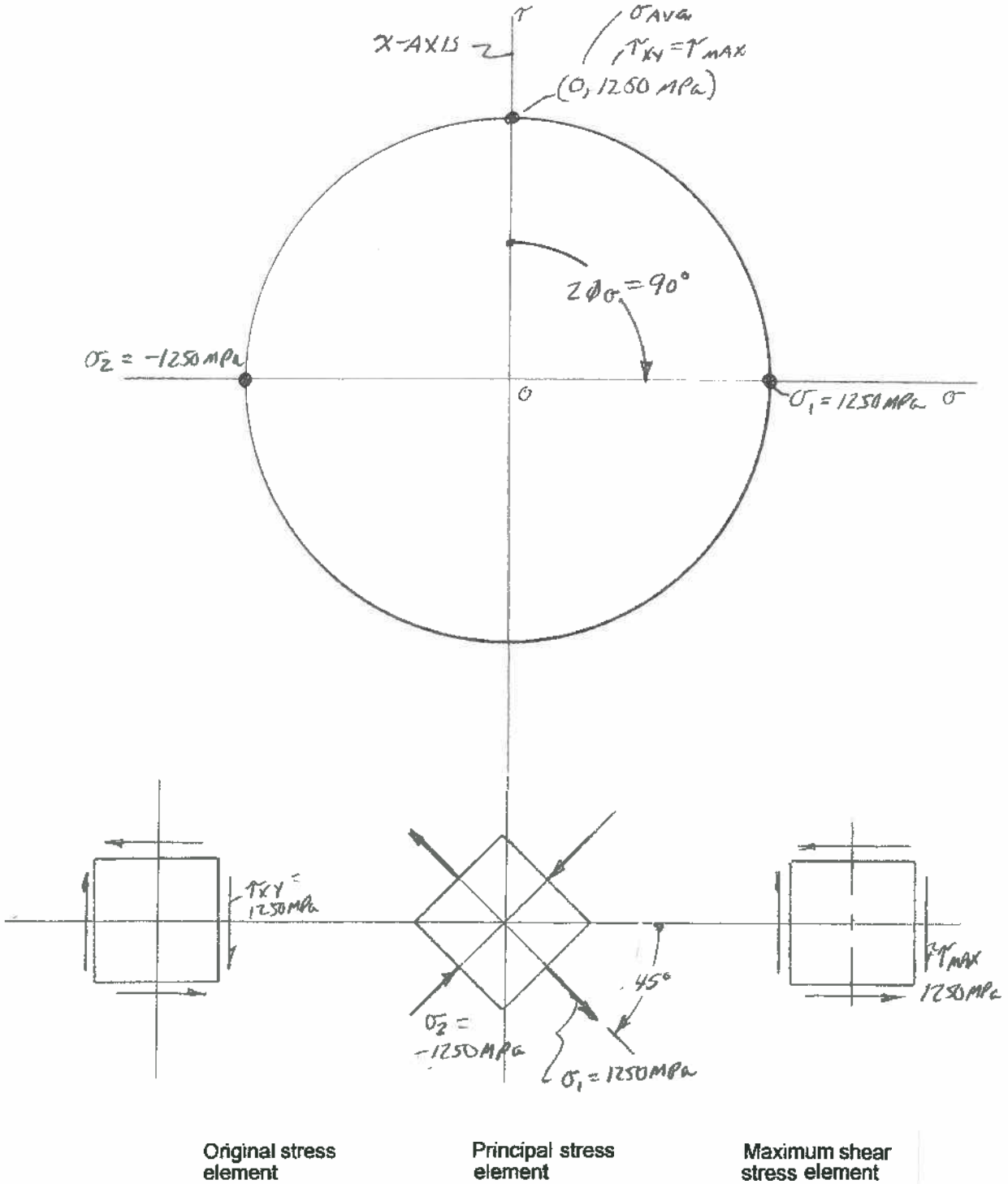


Principal stress element



Maximum shear stress element

14.  $\sigma_x = 0, \sigma_y = 0, \tau_{xy} = 1250 \text{ MPa}$



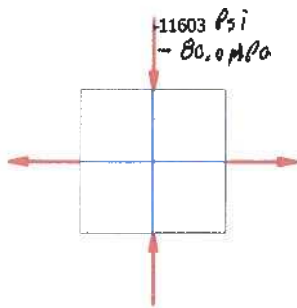
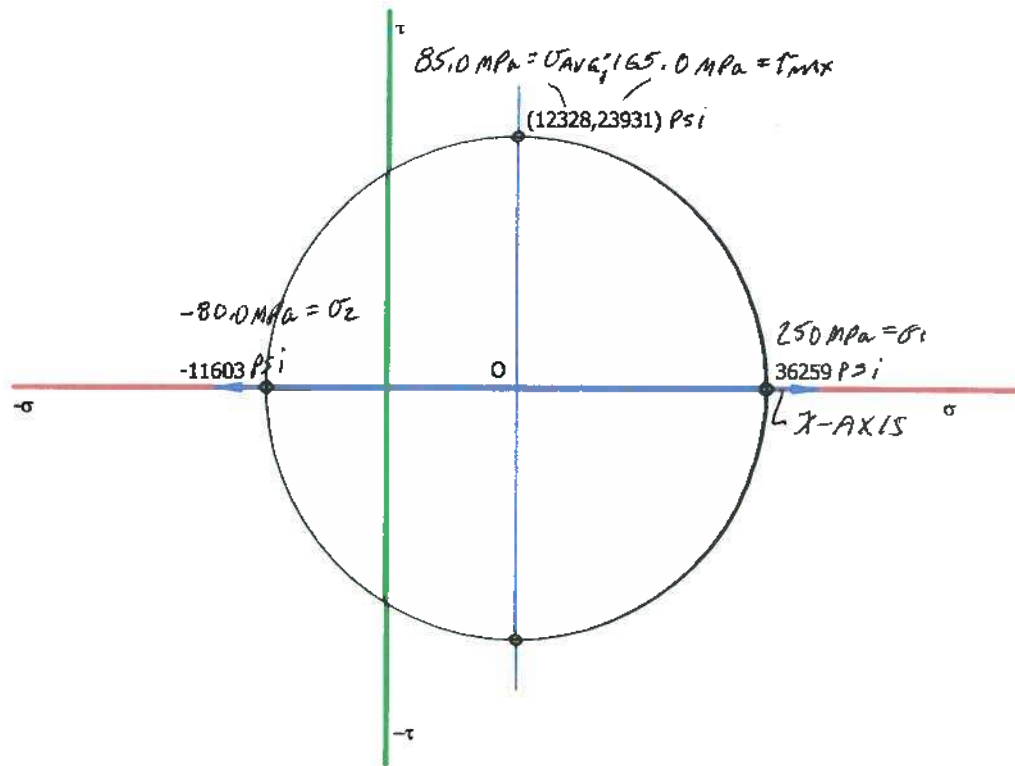


15

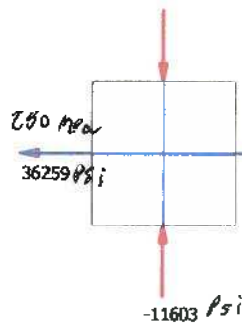
$\sigma_x = 250$  MPa  
 $\sigma_y = -80$  MPa  
 $\tau_{xy} = 0$  MPa

**Results:**

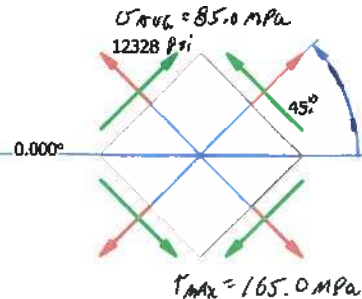
Maximum principal stress	$\sigma_1 =$	250.000	MPa
Minimum principal stress	$\sigma_2 =$	-80.000	MPa
Maximum shear stress	$\tau_{max} =$	165.000	MPa
Average normal stress	$\sigma_{avg} =$	85.000	MPa
Principal planes	$\phi_\sigma =$	0.000	°
			ccw
Angle of maximum shear stress	$\phi_\tau =$	45.000	°
			ccw to $-\tau_{max}$



Original stress element



Principal stress element



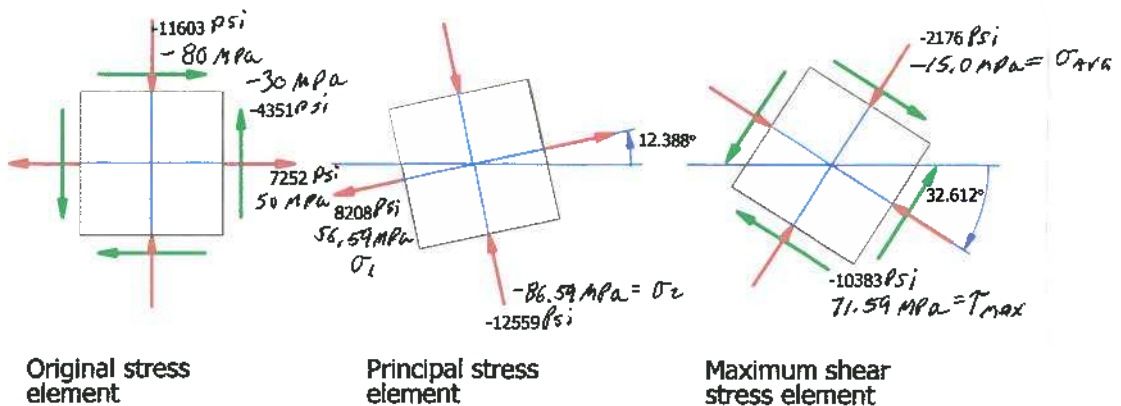
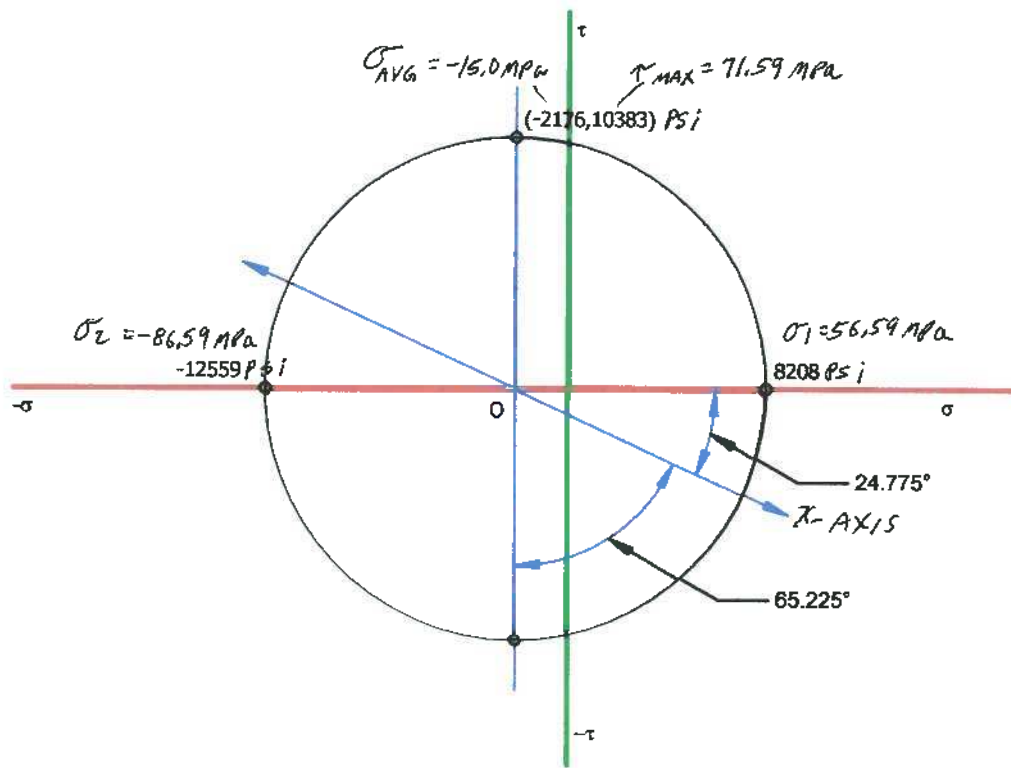
Maximum shear stress element

16

$\sigma_x = 50$  MPa  
 $\sigma_y = -80$  MPa  
 $\tau_{xy} = -30$  MPa

**Results:**

Maximum principal stress	$\sigma_1 =$	56.589	MPa
Minimum principal stress	$\sigma_2 =$	-86.589	MPa
Maximum shear stress	$\tau_{max} =$	71.589	MPa
Average normal stress	$\sigma_{avg} =$	-15.000	MPa
Principal planes	$\phi_\sigma =$	12.388	°
			CCW
Angle of maximum shear stress	$\phi_\tau =$	32.612	°
			CW to $-\tau_{max}$

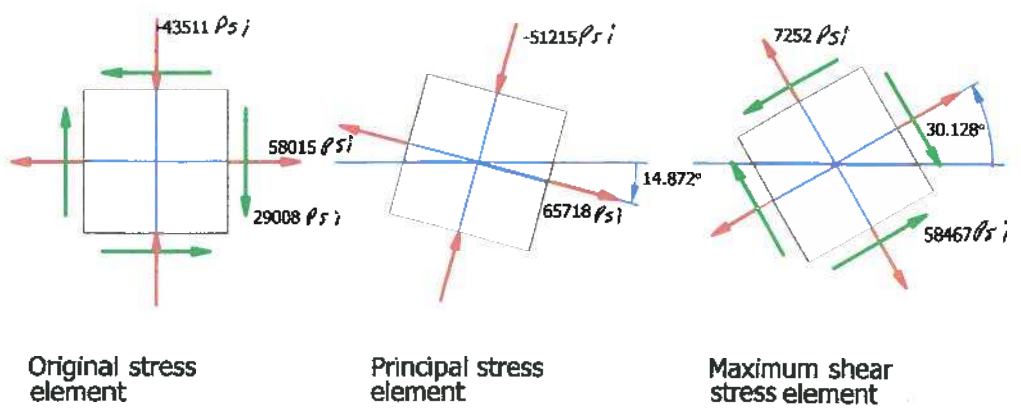
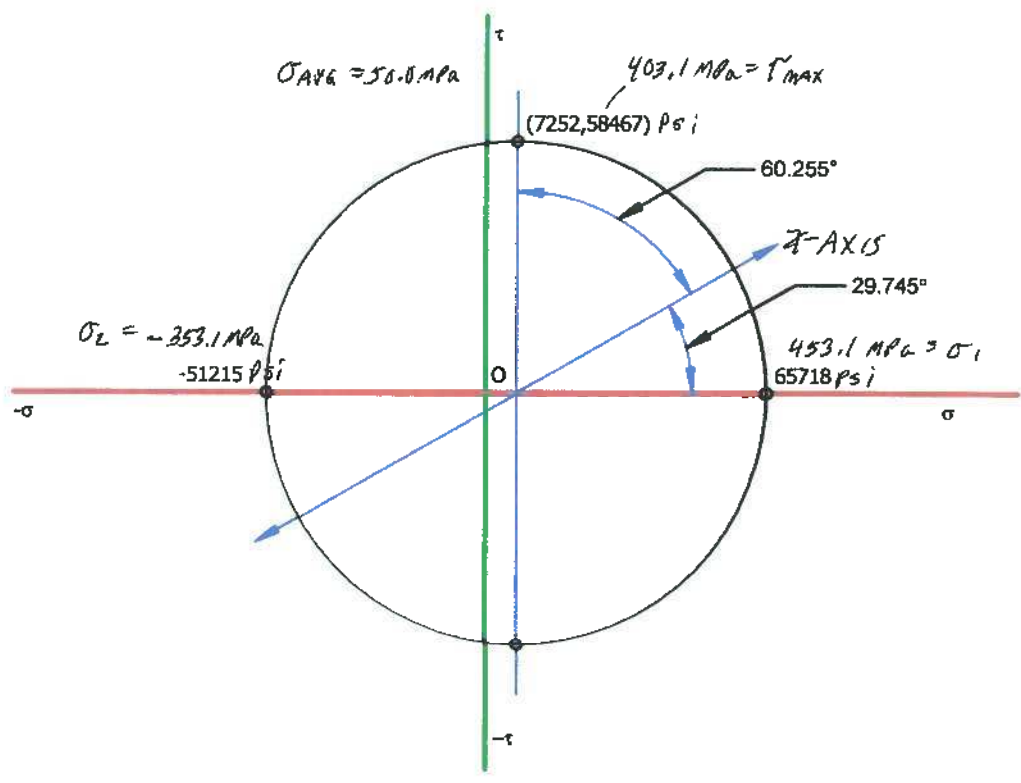


17

$\sigma_x = 400$  MPa  
 $\sigma_y = -300$  MPa  
 $\tau_{xy} = 200$  MPa

**Results:**

Maximum principal stress  $\sigma_1 = 453.113$  MPa  
 Minimum principal stress  $\sigma_2 = -353.113$  MPa  
 Maximum shear stress  $\tau_{max} = 403.113$  MPa  
 Average normal stress  $\sigma_{avg} = 50.000$  MPa  
 Principal planes  $\phi_\sigma = 14.872^\circ$   
 Angle of maximum shear stress  $\phi_\tau = 30.128^\circ$

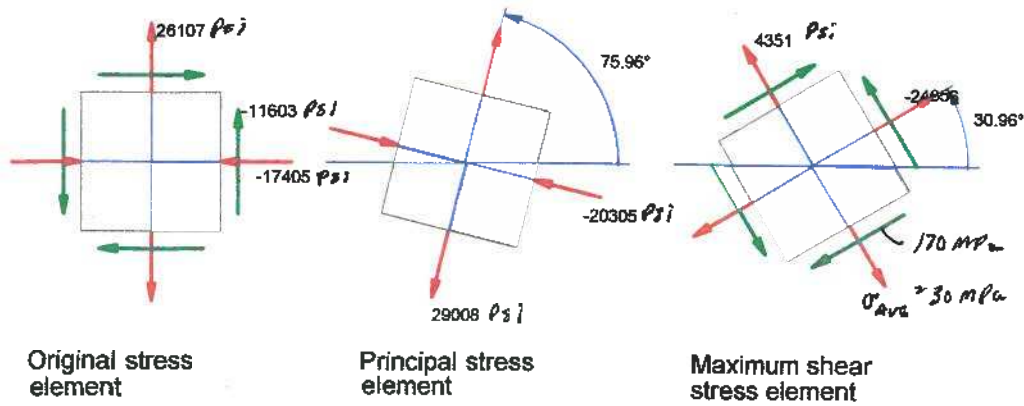
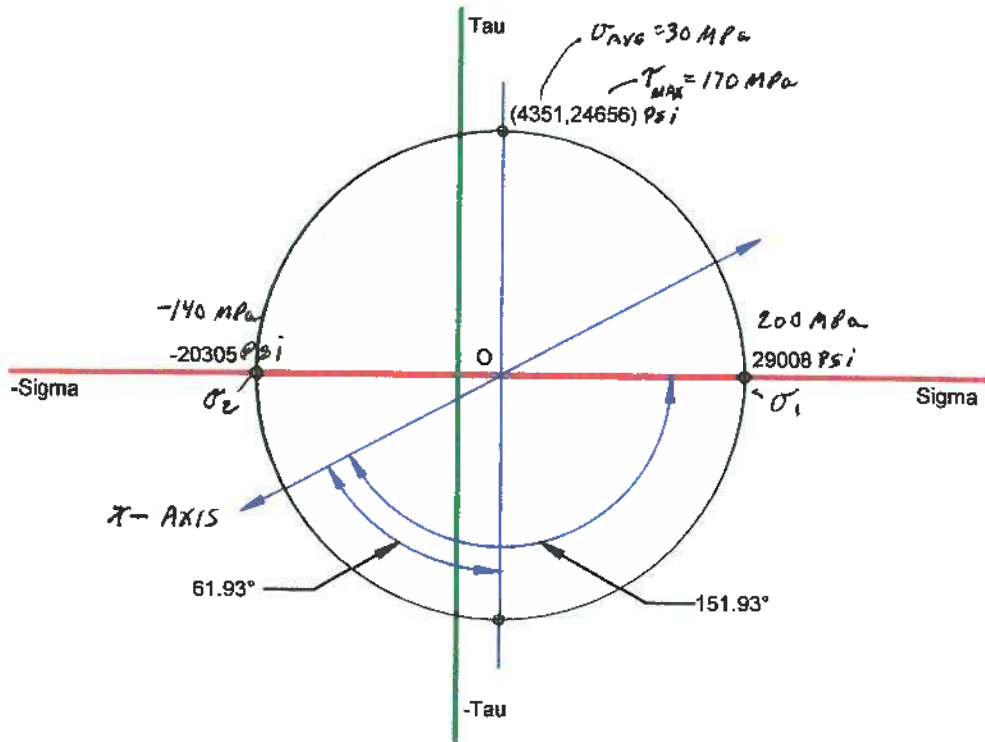


18.

$\sigma_x = -120$  MPa  
 $\sigma_y = 180$  MPa  
 $\tau_{xy} = -80$  MPa

**Results**

Maximum principal stress  $\sigma_1 = 200.000$  MPa  
 Minimum principal stress  $\sigma_2 = -140.000$  MPa  
 Maximum shear stress  $\tau_{max} = 170.000$  MPa  
 Average normal stress  $\sigma_{avg} = 30.000$  MPa  
 Principal planes  $\phi_\sigma = 75.964^\circ$  CCW  
 Angle of maximum shear stress  $\phi_\tau = 30.964^\circ$  CCW to  $-\tau_{max}$

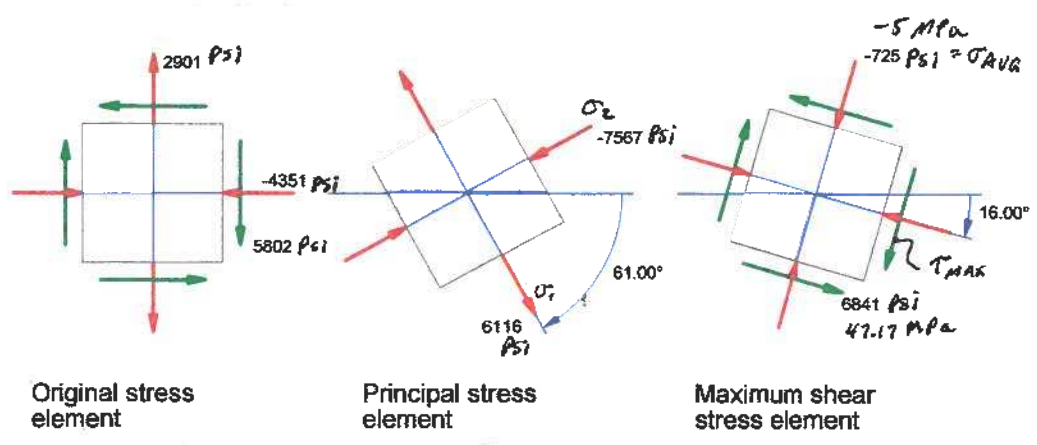
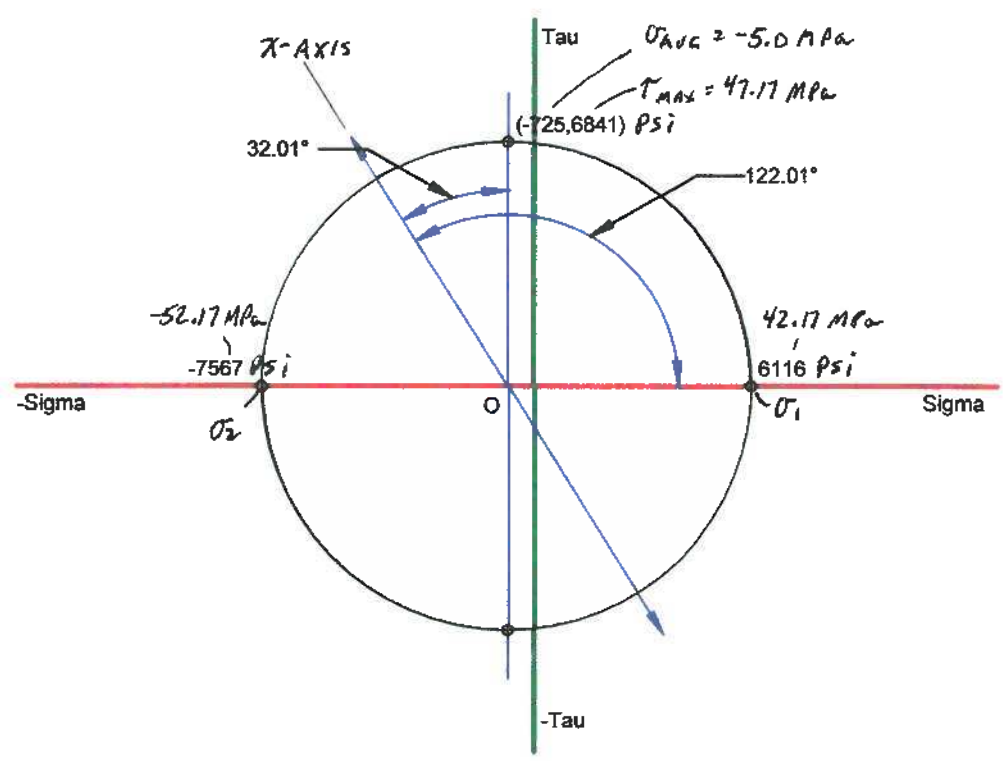


19,

$\sigma_x = -30$  MPa  
 $\sigma_y = 20$  MPa  
 $\tau_{xy} = 40$  MPa

**Results**

Maximum principal stress	$\sigma_1$	=	42.170MPa
Minimum principal stress	$\sigma_2$	=	-52.170MPa
Maximum shear stress	$\tau_{max}$	=	47.170MPa
Average normal stress	$\sigma_{avg}$	=	-5.000MPa
Principal planes	$\phi_\sigma$	=	61.003° CW
Angle of maximum shear stress	$\phi_\tau$	=	16.003° CW

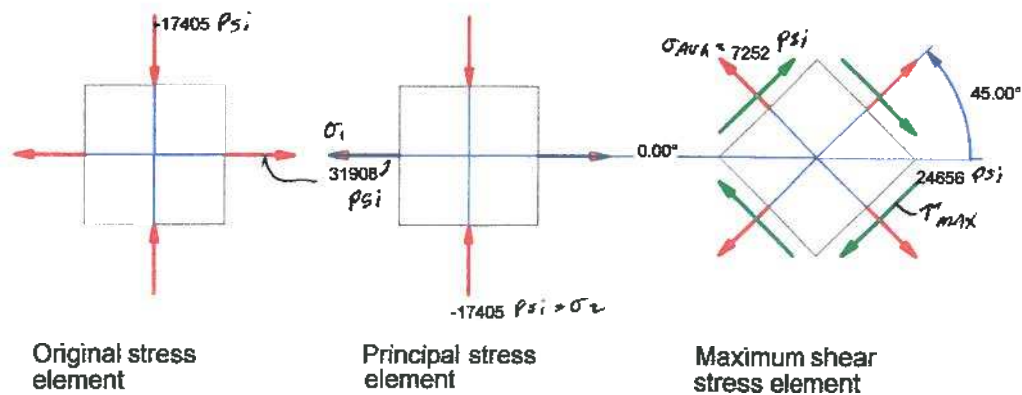
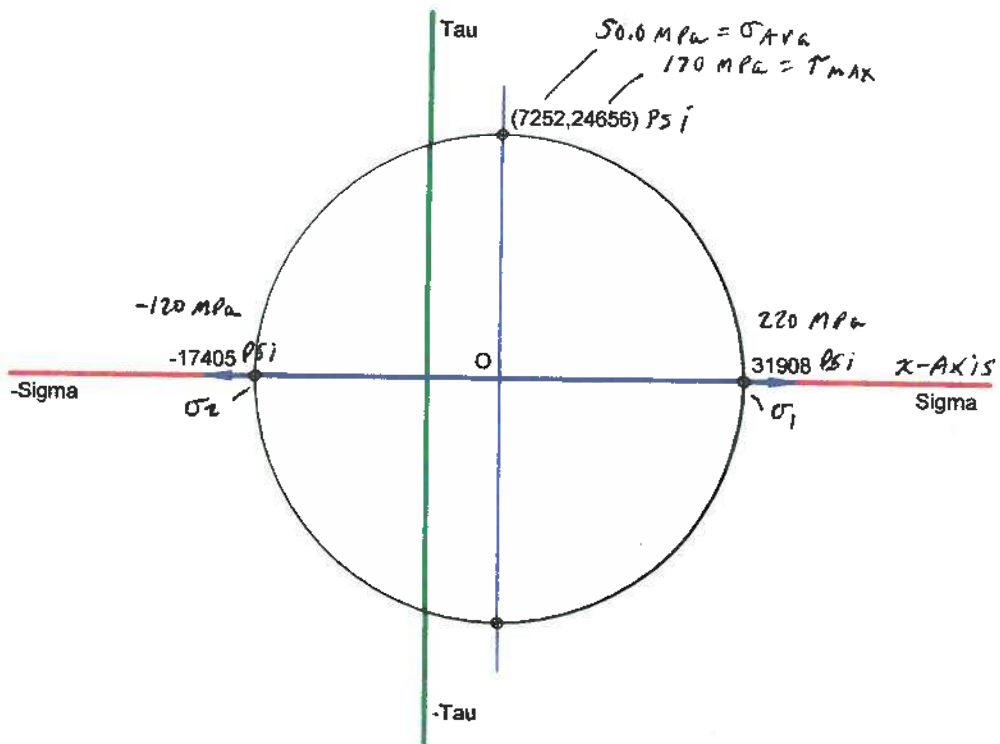


20.

$\sigma_x = 220$  MPa  
 $\sigma_y = -120$  MPa  
 $\tau_{xy} = 0$  MPa

**Results**

Maximum principal stress	$\sigma_1$	=	220.000 MPa
Minimum principal stress	$\sigma_2$	=	-120.000 MPa
Maximum shear stress	$\tau_{max}$	=	170.000 MPa
Average normal stress	$\sigma_{avg}$	=	50.000 MPa
Principal planes	$\phi_\sigma$	=	0.000°
Angle of maximum shear stress	$\phi_\tau$	=	45.000°
			ccw to $-\tau_{max}$

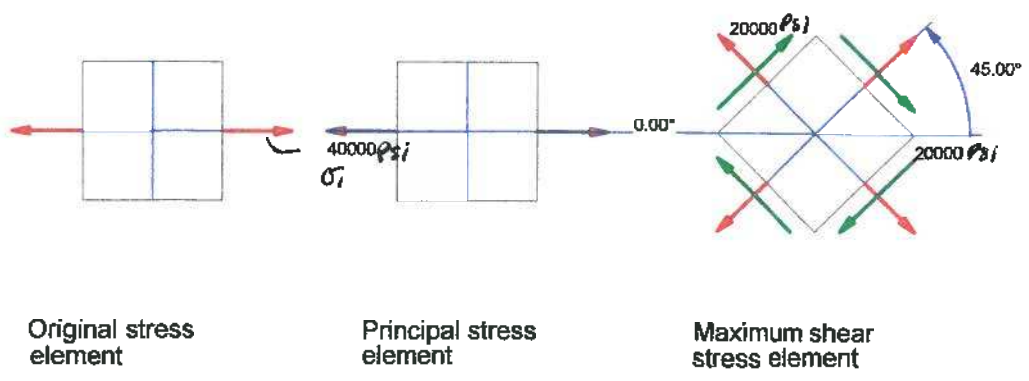
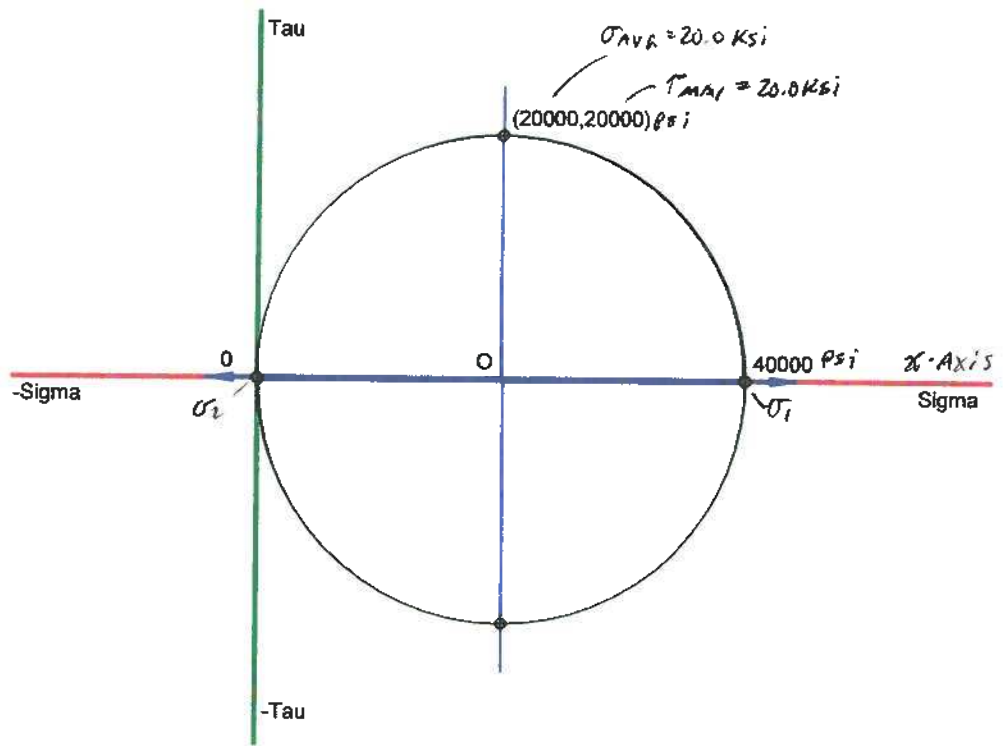


21.

$\sigma_x = 40$  Ksi  
 $\sigma_y = 0$  Ksi  
 $\tau_{xy} = 0$  Ksi

**Results**

Maximum principal stress	$\sigma_1$	=	40.000 Ksi
Minimum principal stress	$\sigma_2$	=	0.000 Ksi
Maximum shear stress	$\tau_{max}$	=	20.000 Ksi
Average normal stress	$\sigma_{avg}$	=	20.000 Ksi
Principal planes	$\phi_\sigma$	=	0.000°
Angle of maximum shear stress	$\phi_\tau$	=	45.000°
			ccw to $-\tau_{max}$



Original stress element

Principal stress element

Maximum shear stress element

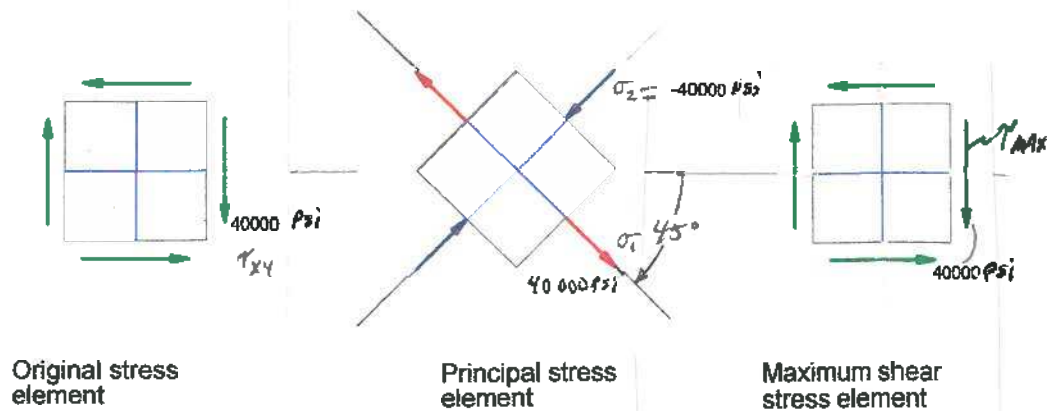
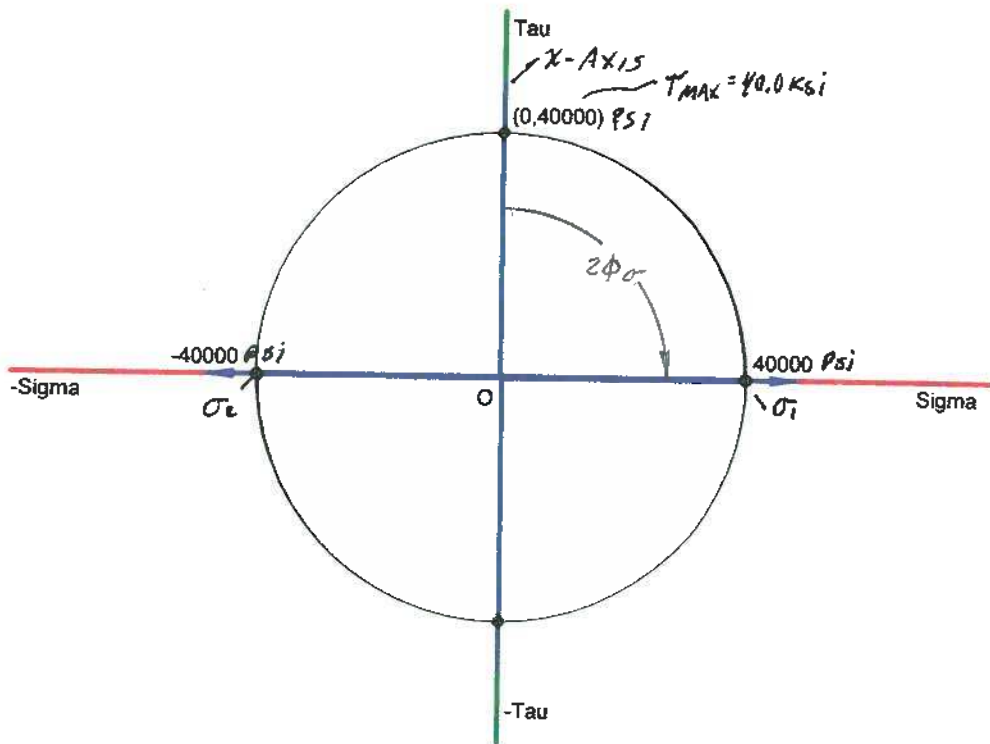


22.

$\sigma_x = 0$                       Ksi  
 $\sigma_y = 0$                       Ksi  
 $\tau_{xy} = 40$                     Ksi

**Results**

Maximum principal stress	$\sigma_1$	=	40.000 Ksi
Minimum principal stress	$\sigma_2$	=	-40.000 Ksi
Maximum shear stress	$\tau_{max}$	=	40.000 Ksi
Average normal stress	$\sigma_{avg}$	=	0.000 psi
Principal planes	$\phi_\sigma$	=	45.000°
Angle of maximum shear stress	$\phi_\tau$	=	0.000°
			CW CCW

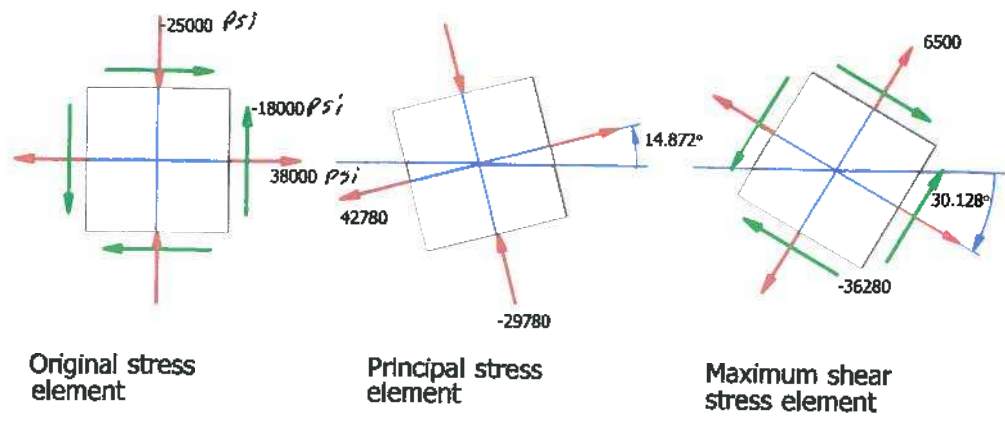
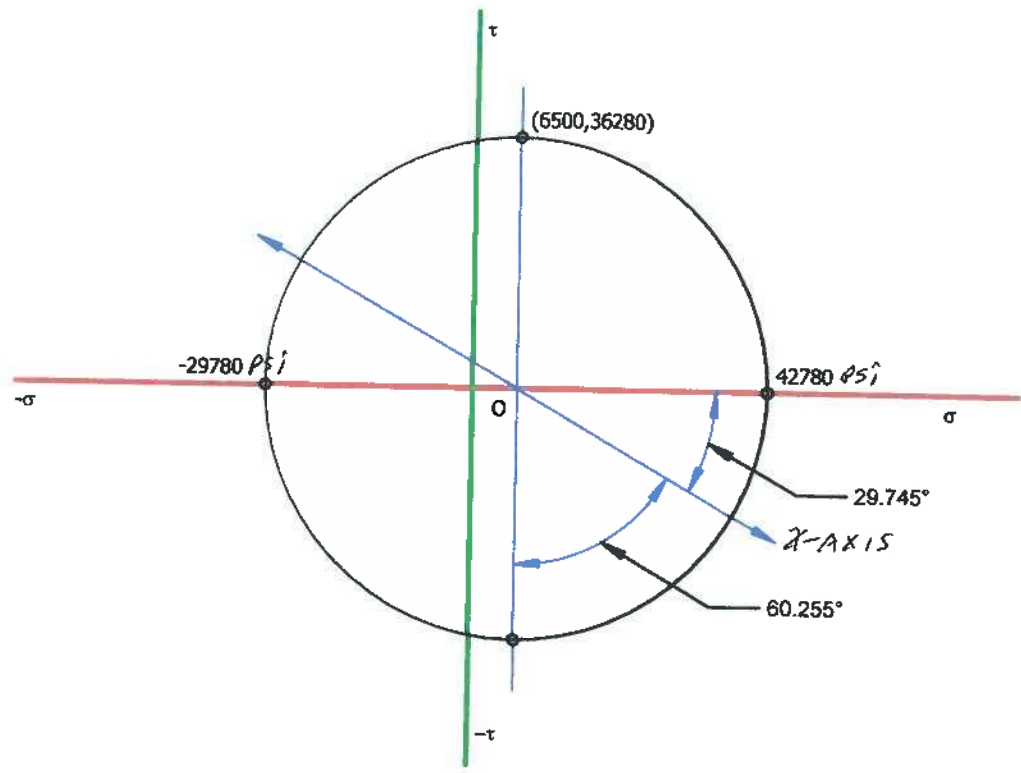


23

$\sigma_x = 38$  ksi  
 $\sigma_y = -25$  ksi  
 $\tau_{xy} = -18$  ksi

**Results:**

Maximum principal stress	$\sigma_1 =$	42.780	ksi
Minimum principal stress	$\sigma_2 =$	-29.780	ksi
Maximum shear stress	$\tau_{max} =$	36.280	ksi
Average normal stress	$\sigma_{avg} =$	6.500	ksi
Principal planes	$\phi_\sigma =$	14.872	°
			CCW
Angle of maximum shear stress	$\phi_\tau =$	30.128	°
			CW to $-\tau_{max}$

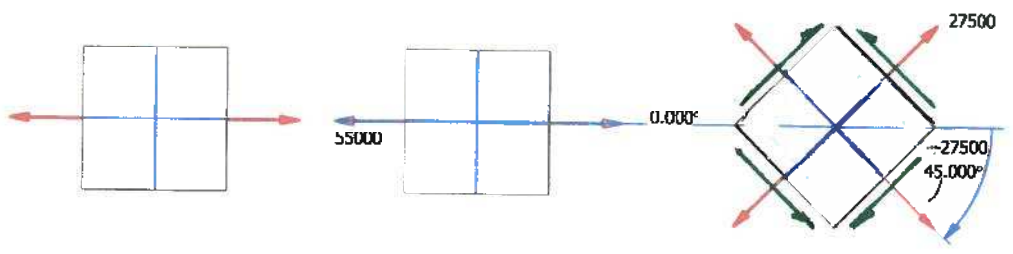
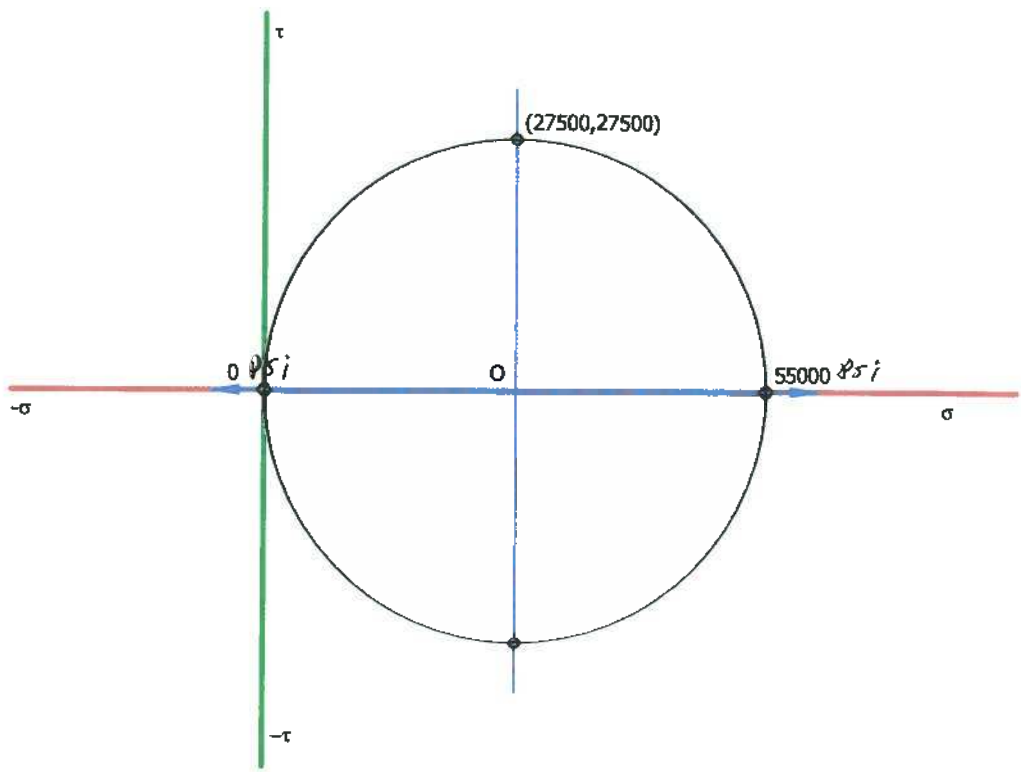


24

$\sigma_x = 55$       ksi  
 $\sigma_y = 0$         ksi  
 $\tau_{xy} = 0$         ksi

**Results:**

Maximum principal stress	$\sigma_1 =$	55.000	ksi
Minimum principal stress	$\sigma_2 =$	0.000	ksi
Maximum shear stress	$\tau_{max} =$	27.500	ksi
Average normal stress	$\sigma_{avg} =$	27.500	ksi
Principal planes	$\phi_\sigma =$	0.000	°
			ccw
Angle of maximum shear stress	$\phi_\tau =$	45.000	°
			cw to $-\tau_{max}$



Original stress element

Principal stress element

Maximum shear stress element

25

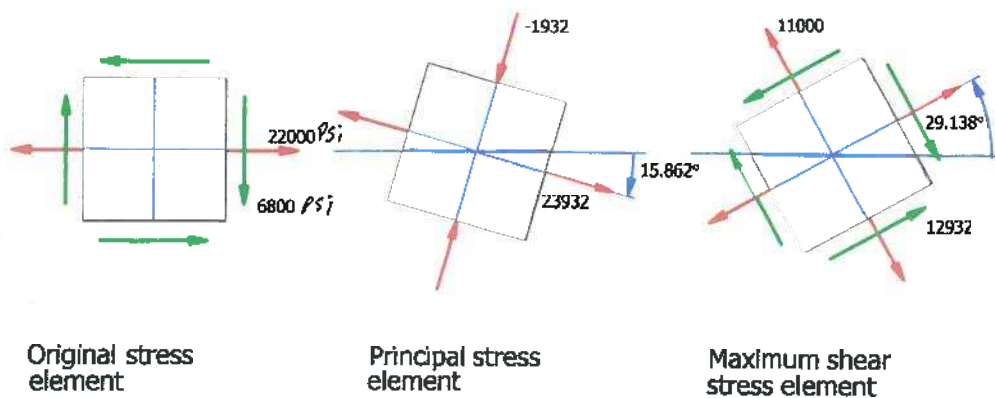
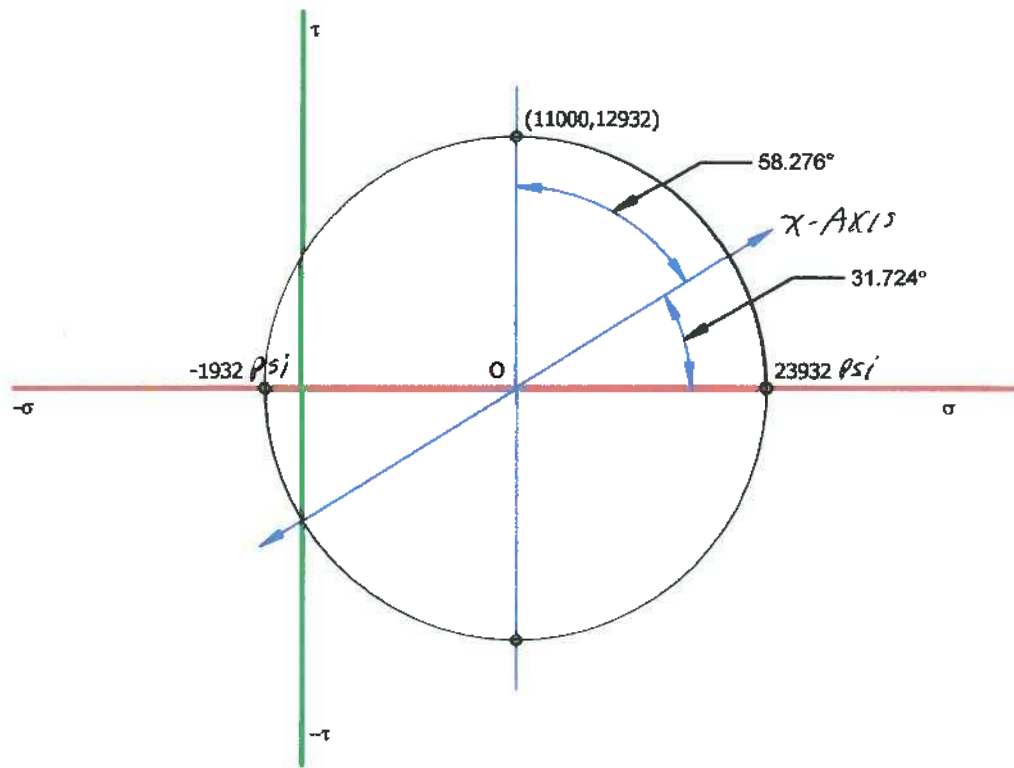
$\sigma_x = 22$  ksi  
 $\sigma_y = 0$  ksi  
 $\tau_{xy} = 6.8$  ksi

**Results:**

Maximum principal stress  $\sigma_1 = 23.932$  ksi  
 Minimum principal stress  $\sigma_2 = -1.932$  ksi  
 Maximum shear stress  $\tau_{max} = 12.932$  ksi  
 Average normal stress  $\sigma_{avg} = 11.000$  ksi  
 Principal planes  $\phi_\sigma = 15.862^\circ$

Angle of maximum shear stress  $\phi_\tau = 29.138^\circ$

CW  
 CCW

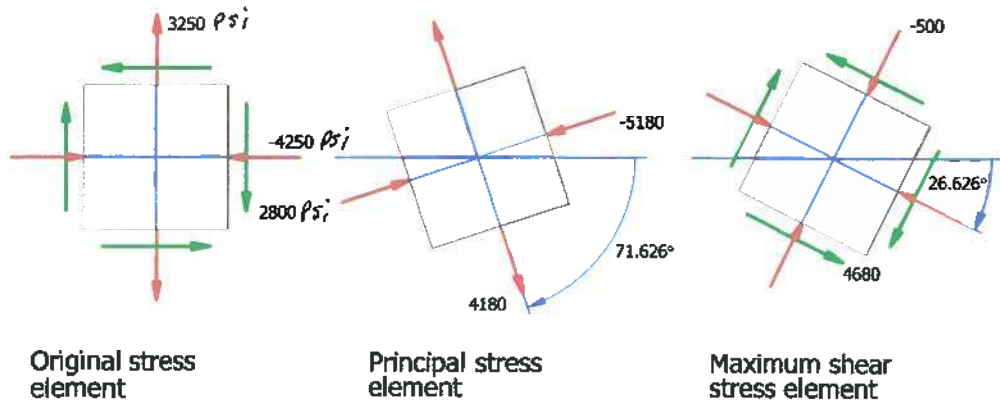
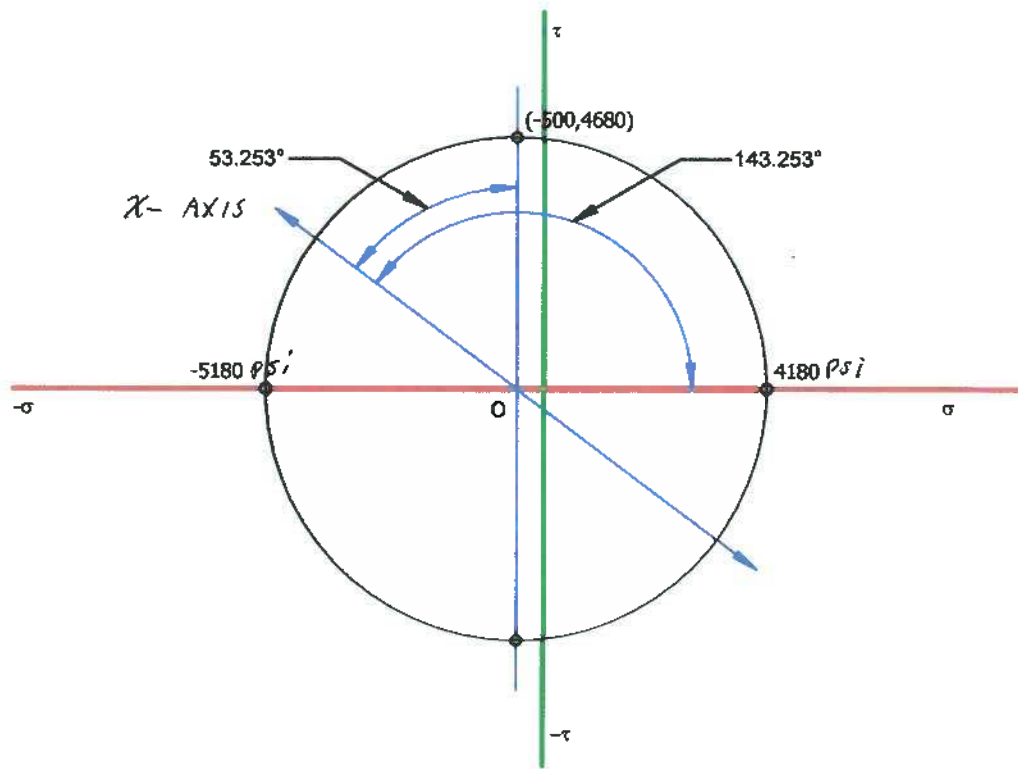


26

$\sigma_x = -4250$  psi  
 $\sigma_y = 3250$  psi  
 $\tau_{xy} = 2800$  psi

**Results:**

Maximum principal stress  $\sigma_1 = 4180.011$  psi  
 Minimum principal stress  $\sigma_2 = -5180.011$  psi  
 Maximum shear stress  $\tau_{max} = 4680.011$  psi  
 Average normal stress  $\sigma_{avg} = -500.000$  psi  
 Principal planes  $\phi_\sigma = 71.626^\circ$   
 Angle of maximum shear stress  $\phi_\tau = 26.626^\circ$  CW



27

BOTH PRINCIPAL STRESSES ARE TENSILE - SAME SIGN

**Input data:****Combined Stresses and Mohr's Circle**

Normal stress acting along x-axis	$\sigma_x = 300$	MPa
Normal stress acting along y-axis	$\sigma_y = 100$	MPa
Shear stress	$\tau_{xy} = 80$	MPa

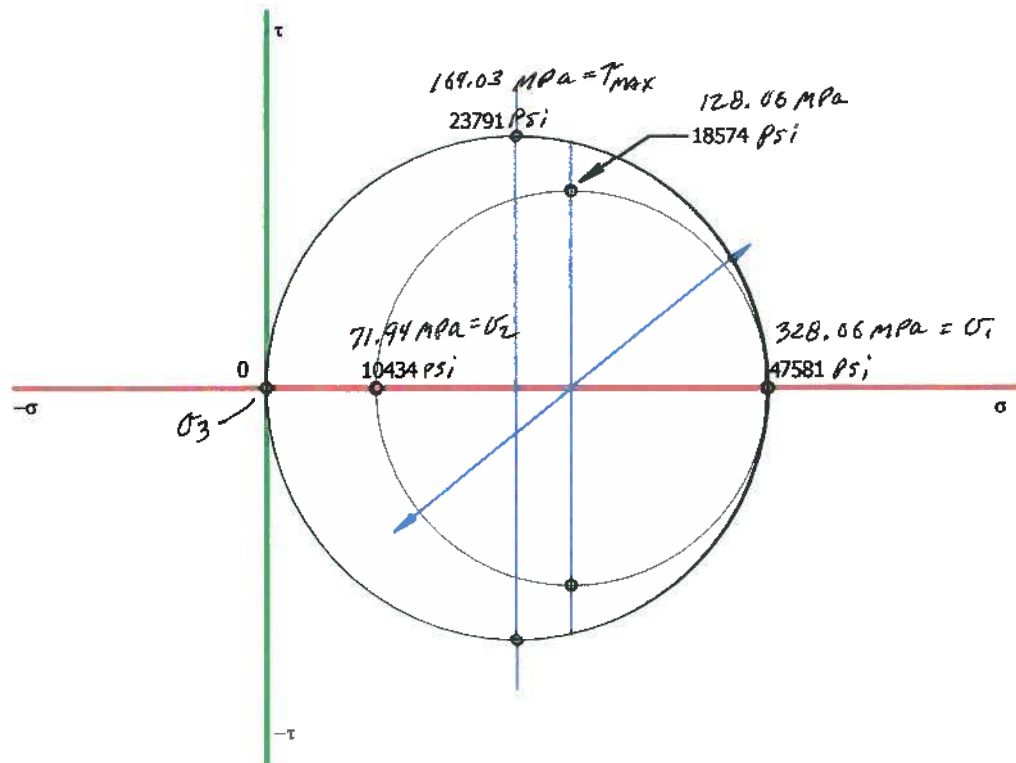
**Results:**

Maximum principal stress	$\sigma_1 =$	328.062	MPa
Minimum principal stress	$\sigma_2 =$	71.938	MPa
Minimum principal stress	$\sigma_3 =$	0.000	MPa
Maximum shear stress	$\tau_{max} =$	164.031	MPa
Shear stress	$\tau =$	128.062	MPa

Note: Both principle stresses have the same sign (both are tensile or both are compressive).

User must consider the resulting three-dimensional case.

Due to compound angles, elements as calculated are not applicable.



28

BOTH PRINCIPAL STRESSES ARE TENSILE - SAME SIGN.

**Input data:**

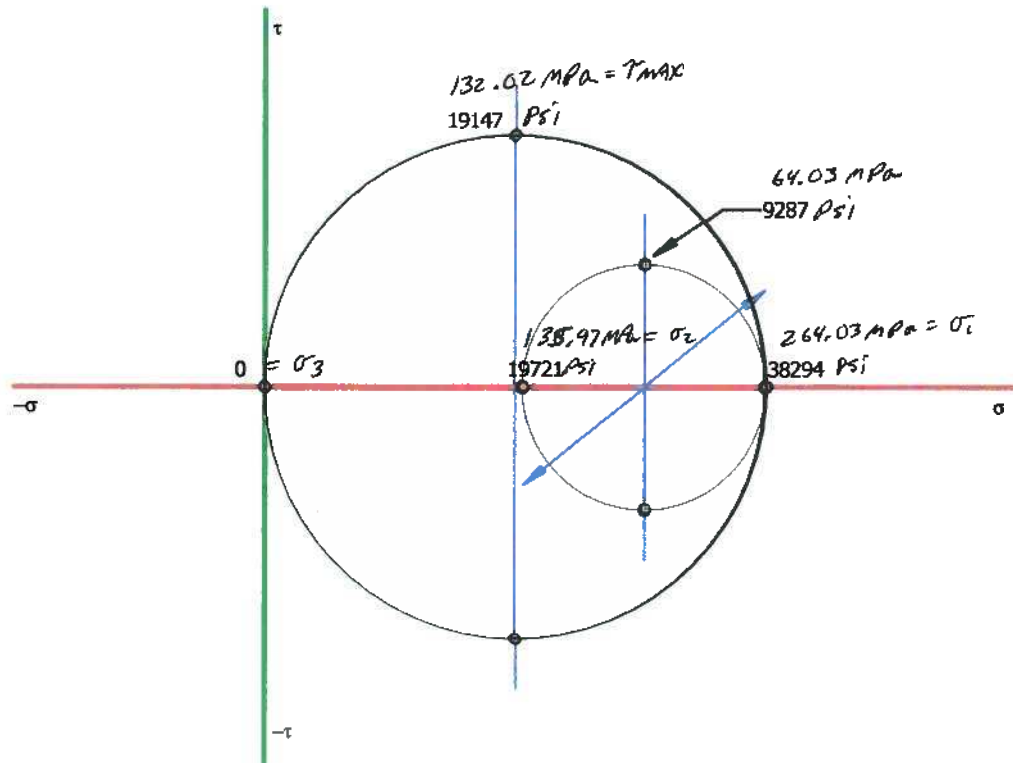
**Combined Stresses and Mohr's Circle**

Normal stress acting along x-axis	$\sigma_x = 250$	MPa
Normal stress acting along y-axis	$\sigma_y = 150$	MPa
Shear stress	$\tau_{xy} = 40$	MPa

**Results:**

Maximum principal stress	$\sigma_1 =$	264.031	MPa
Minimum principal stress	$\sigma_2 =$	135.969	MPa
Minimum principal stress	$\sigma_3 =$	0.000	MPa
Maximum shear stress	$\tau_{max} =$	132.016	MPa
Shear stress	$\tau =$	64.031	MPa

Note: Both principle stresses have the same sign (both are tensile or both are compressive).  
 User must consider the resulting three-dimensional case.  
 Due to compound angles, elements as calculated are not applicable.





29

BOTH PRINCIPAL STRESSES ARE COMPRESSIVE - SAME SIGN

**Input data:**

**Combined Stresses and Mohr's Circle**

Normal stress acting along x-axis	$\sigma_x = -840$	kPa
Normal stress acting along y-axis	$\sigma_y = -335$	kPa
Shear stress	$\tau_{xy} = -120$	kPa

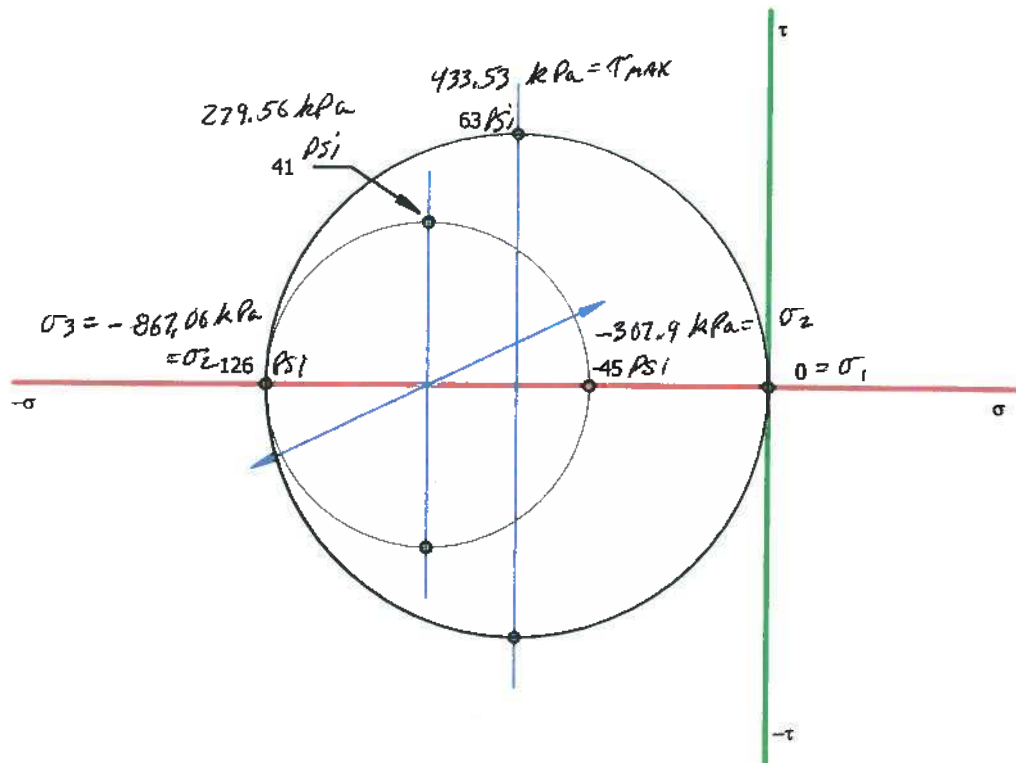
**Results:**

Maximum principal stress	$\sigma_1 = 0.000$	kPa
Minimum principal stress	$\sigma_2 = -307.936$	kPa
Minimum principal stress	$\sigma_3 = -867.064$	kPa
Maximum shear stress	$\tau_{max} = 433.532$	kPa
Shear stress	$\tau = 279.564$	kPa

Note: Both principle stresses have the same sign (both are tensile or both are compressive).

User must consider the resulting three-dimensional case.

Due to compound angles, elements as calculated are not applicable.



30

BOTH PRINCIPAL STRESSES ARE COMPRESSIVE - SAME SIGN

**Input data:**

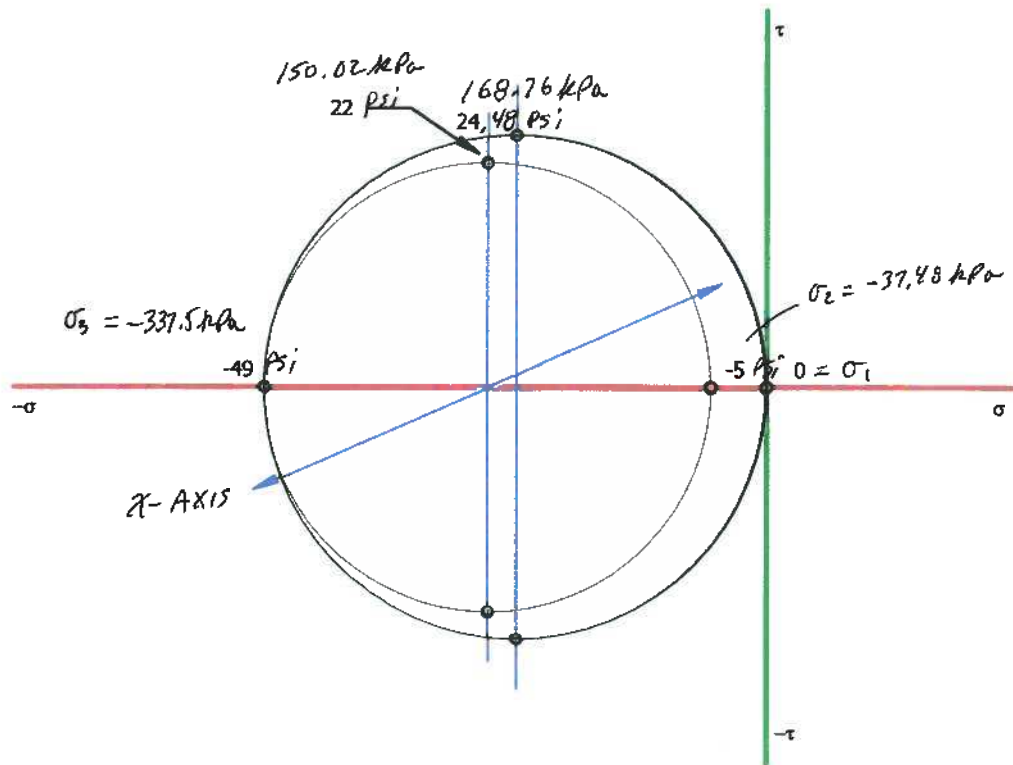
**Combined Stresses and Mohr's Circle**

Normal stress acting along x-axis	$\sigma_x = -325$	kPa
Normal stress acting along y-axis	$\sigma_y = -50$	kPa
Shear stress	$\tau_{xy} = -60$	kPa

**Results:**

Maximum principal stress	$\sigma_1 =$	0.000	kPa
Minimum principal stress	$\sigma_2 =$	-37.479	kPa
Minimum principal stress	$\sigma_3 =$	-337.521	kPa
Maximum shear stress	$\tau_{max} =$	168.760	kPa
Shear stress	$\tau =$	150.021	kPa

Note: Both principle stresses have the same sign (both are tensile or both are compressive).  
 User must consider the resulting three-dimensional case.  
 Due to compound angles, elements as calculated are not applicable.



31.

USE  $D = 0.500$  IN FOR SHAFT ABC. STRESS ELEMENT ON BOTTOM. FROM FIG. 3-23:  $M_B = 252$  LB·IN;  $T_B = 300$  LB·IN.

$$Z = \pi D^3 / 32 = \pi (0.500 \text{ in})^3 / 32 = 0.01227 \text{ in}^3$$

$$\sigma_B = \frac{M_B}{Z} = \frac{252 \text{ LB·IN}}{0.01227 \text{ in}^3} = 20538 \text{ psi} = \sigma_x$$

$$Z_p = \pi D^3 / 16 = 2Z = 0.02454 \text{ in}^3$$

$$T_B = \frac{T_B}{Z_p} = \frac{300 \text{ LB·IN}}{0.02454 \text{ in}^3} = 12225 \text{ psi}$$

$$R = T_{MAX} = \sqrt{10269^2 + 12225^2} = 15966 \text{ psi}$$

$$\sigma_1 = 10269 + 15966 = 26235 \text{ psi}$$

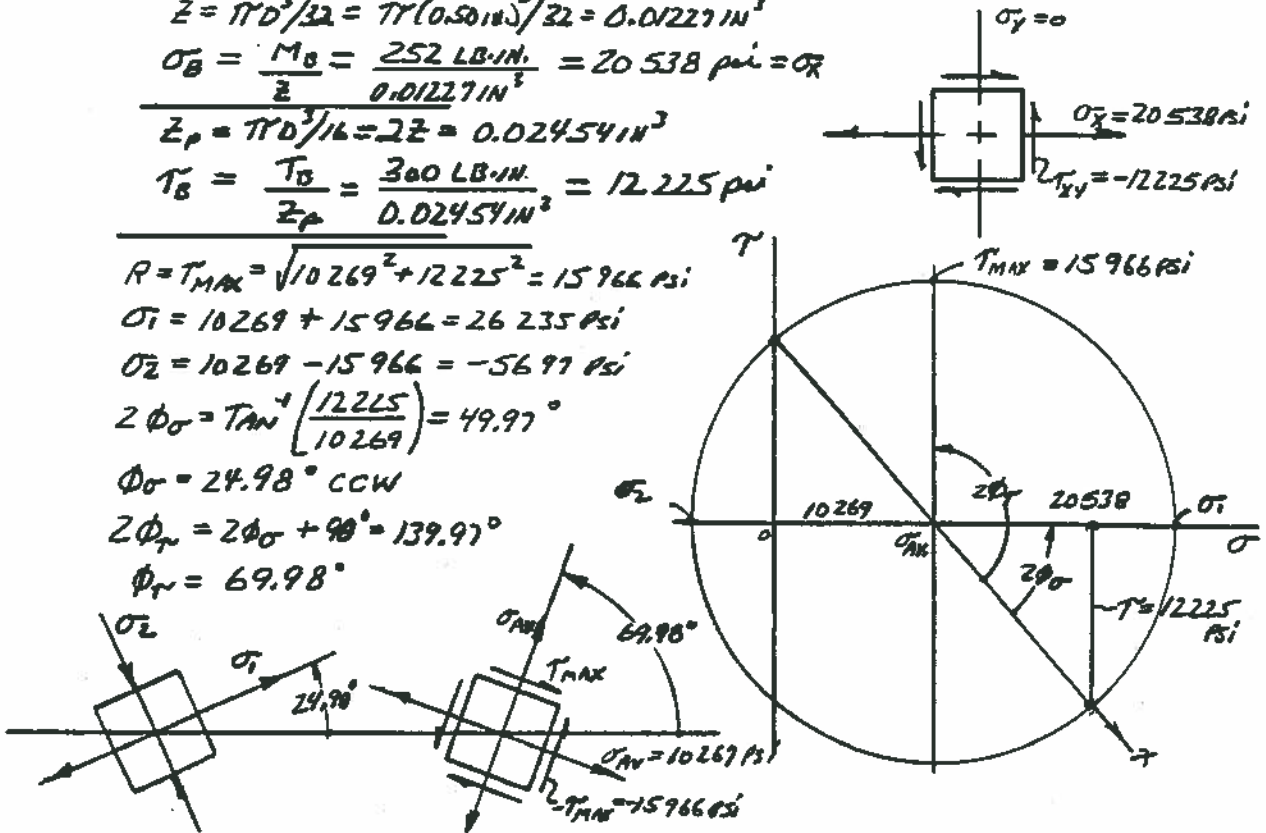
$$\sigma_2 = 10269 - 15966 = -5697 \text{ psi}$$

$$2\phi_\sigma = \tan^{-1} \left( \frac{12225}{10269} \right) = 49.97^\circ$$

$$\phi_\sigma = 24.98^\circ \text{ CCW}$$

$$2\phi_r = 2\phi_\sigma + 90^\circ = 139.97^\circ$$

$$\phi_r = 69.98^\circ$$



32.

USE  $D = 1.500$  IN FOR SHAFT ABC. STRESS ELEMENT ON BOTTOM.

FROM SOLUTION FOR PROBLEM 3-40,

$$M_B = 4572 \text{ LB·IN}; T_B = 6400 \text{ LB·IN}$$

$$Z = \pi D^3 / 32 = 0.3313 \text{ in}^3; \sigma_B = M_B / Z = 4572 \text{ LB·IN} / 0.3313 \text{ in}^3 = 13800 \text{ psi} = \sigma_x$$

$$Z_p = \pi D^3 / 16 = 2Z = 0.6627 \text{ in}^3; T_B = T_B / Z_p = 6400 / 0.6627 = 9658 \text{ psi}$$

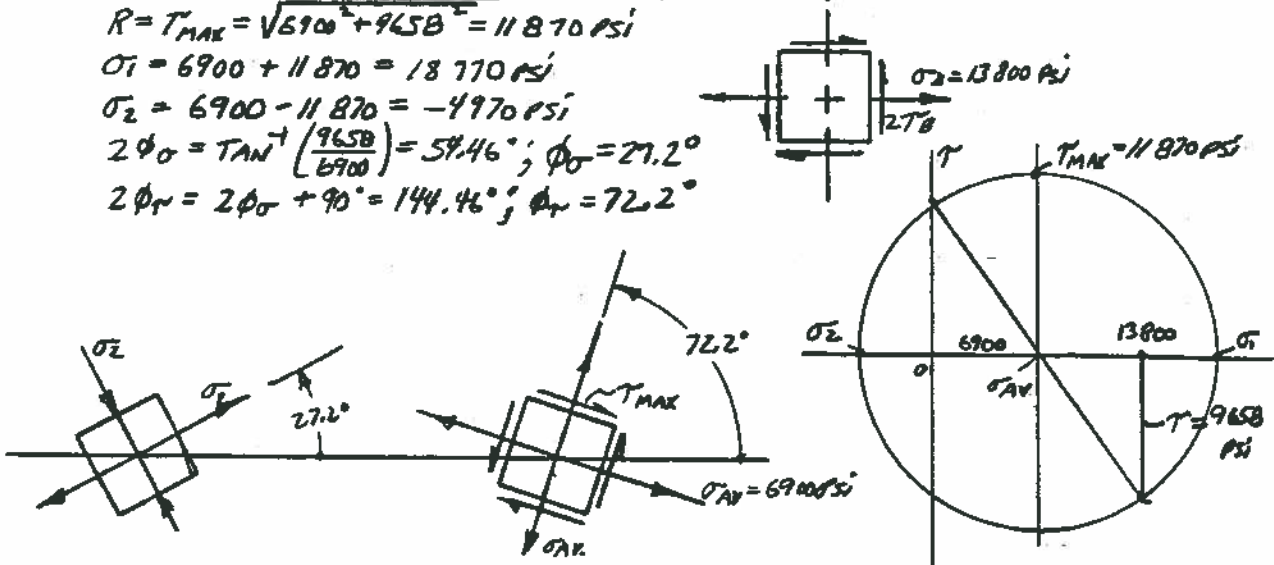
$$R = T_{MAX} = \sqrt{6900^2 + 9658^2} = 11870 \text{ psi}$$

$$\sigma_1 = 6900 + 11870 = 18770 \text{ psi}$$

$$\sigma_2 = 6900 - 11870 = -4970 \text{ psi}$$

$$2\phi_\sigma = \tan^{-1} \left( \frac{9658}{6900} \right) = 54.46^\circ; \phi_\sigma = 27.2^\circ$$

$$2\phi_r = 2\phi_\sigma + 90^\circ = 144.46^\circ; \phi_r = 72.2^\circ$$



33.

USE  $D = 2.25 \text{ IN.}$  FOR SHAFT ABC.: STRESS ELEMENT ON BOTTOM.

FROM SOLUTION FOR PROBLEM 3-49,

$M_B = 8640 \text{ LB}\cdot\text{IN.}; T_B = 400 \text{ LB}\cdot\text{IN.}$

$Z = \pi D^3/32 = 1.118 \text{ IN}^3; \sigma_B = M/z = 8640 \text{ LB}\cdot\text{IN.}/1.118 \text{ IN}^3 = 7726 \text{ PSI TENSION}$

$Z_P = \pi D^3/16 = 2Z = 2.237 \text{ IN}^3; T_B = T_B/z_P = 400 \text{ LB}\cdot\text{IN.}/2.237 \text{ IN}^3 = 179 \text{ PSI}$

$R = T_{MAX} = \sqrt{179^2 + 3863^2} = 3867 \text{ psi}$

$\sigma_1 = 3863 + 3867 = 7730 \text{ psi}$

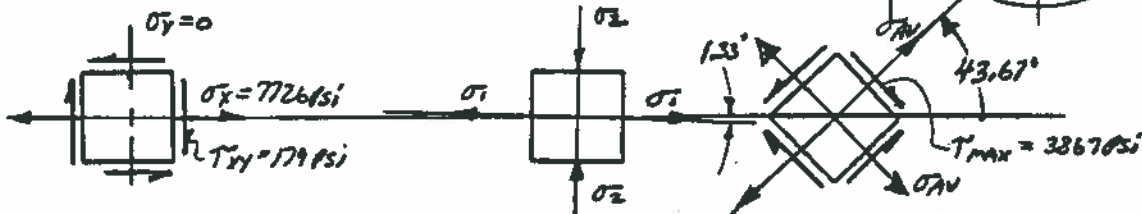
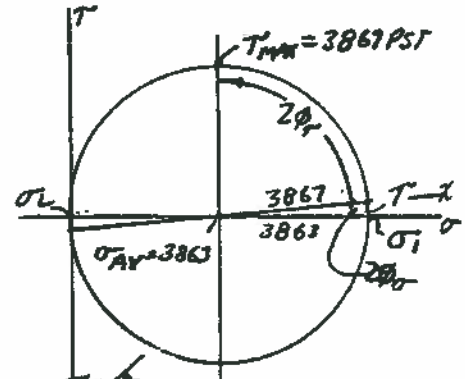
$\sigma_2 = 3863 - 3867 = -4 \text{ psi}$

$2\phi_\sigma = \tan^{-1}(179/3863) = 2.65^\circ$

$\phi_\sigma = 1.33^\circ \text{ CCW}$

$2\phi_r = 90^\circ - 2\phi_\sigma = 87.35^\circ$

$\phi_r = 43.67^\circ \text{ CCW}$



34.

USE  $D = 50.0 \text{ mm}$  FOR SHAFT AB: STRESS ELEMENT ON BOTTOM.

FROM SOLUTION FOR PROBLEM 3-50,

$M_A = -0.650 \text{ kN}\cdot\text{m.}; T_A = 0.375 \text{ kN}\cdot\text{m.}$

$Z = \pi D^3/32 = 12.272 \text{ mm}^3; \sigma_A = M/z = \frac{650 \text{ N}\cdot\text{m}}{12.272 \text{ mm}^3} \cdot \frac{10^3 \text{ mm}^3}{\text{m}} = -52.91 \text{ MPa COMPRESSION.}$

$Z_P = \pi D^3/16 = 2Z = 24.544 \text{ mm}^3; T_A = T/z_P = \frac{375 \text{ N}\cdot\text{m}}{24.544 \text{ mm}^3} \cdot \frac{10^3 \text{ mm}^3}{\text{m}} = 15.28 \text{ MPa}$

$R = T_{MAX} = \sqrt{15.28^2 + 26.48^2} = 30.57 \text{ MPa}$

$\sigma_1 = -26.48 + 30.57 = 4.09 \text{ MPa}$

$\sigma_2 = -26.48 - 30.57 = -57.05 \text{ MPa}$

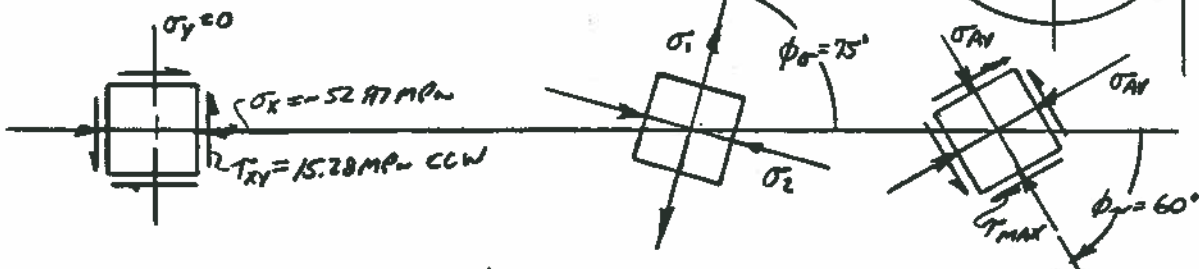
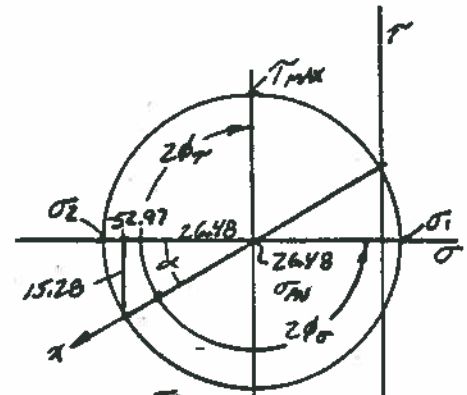
$\alpha = \tan^{-1}(15.28/26.48) = 30.0^\circ$

$2\phi_\sigma = 180^\circ - \alpha = 150^\circ$

$\phi_\sigma = 75^\circ \text{ CCW}$

$2\phi_r = 90^\circ + \alpha = 120^\circ$

$\phi_r = 60^\circ \text{ CW}$



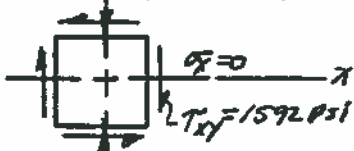
35.

$D = 4.00 \text{ IN} ; A = \pi D^2/4 = 12.57 \text{ IN}^2 ; Z_p = \pi D^3/16 = 12.57 \text{ IN}^3$

$\sigma = \frac{-F}{A} = \frac{-75000 \text{ LB}}{12.57 \text{ IN}^2} = -5968 \text{ PSI}$

$T = \frac{T}{Z_p} = \frac{20000 \text{ LB}\cdot\text{IN}}{12.57 \text{ IN}^3} = 1592 \text{ PSI}$

$\sigma_y = -5968 \text{ PSI}$

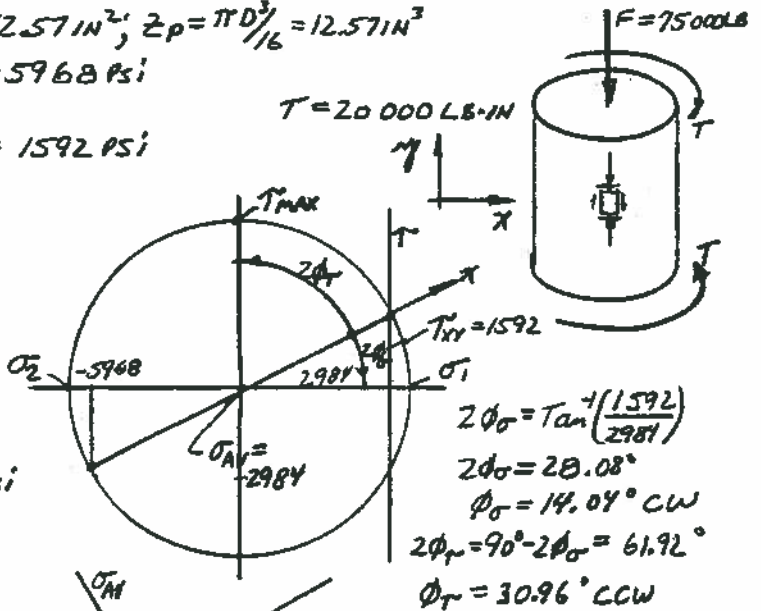
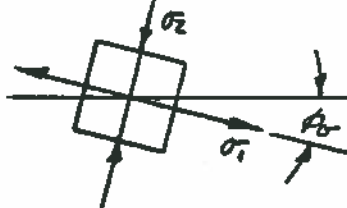


SURFACE ELEMENT

$R = T_{MAX} = \sqrt{1592^2 + 2984^2} = 3382 \text{ PSI}$

$\sigma_1 = -2984 + 3382 = 398 \text{ PSI}$

$\sigma_2 = -2984 - 3382 = -6366 \text{ PSI}$



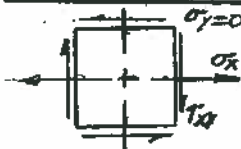
36.

$D = 20 \text{ mm} ; A = \frac{\pi D^2}{4} = 314 \text{ mm}^2$

$\sigma_x = \frac{F}{A} = \frac{36000 \text{ N}}{314 \text{ mm}^2} = 114.6 \text{ MPa TENSION}$

$Z_p = \frac{\pi D^3}{16} = 1571 \text{ mm}^3$

$T_{xy} = \frac{T}{Z_p} = \frac{450 \text{ N}\cdot\text{m}}{1571 \text{ mm}^3} \cdot \frac{10 \text{ mm}}{1 \text{ m}} = 286.48 \text{ MPa}$



$\sigma_{AVG} = \frac{114.6 + 0}{2} = 57.3 \text{ MPa}$

$R = T_{MAX} = \sqrt{57.3^2 + 286.5^2}$

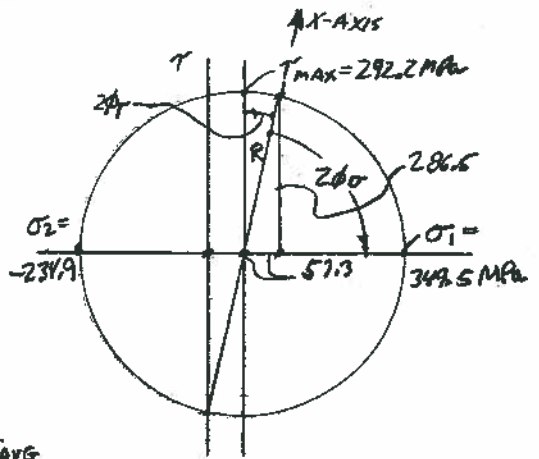
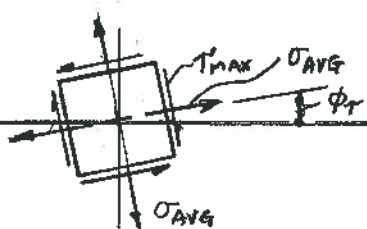
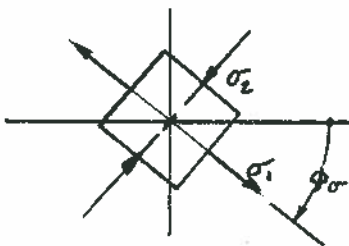
$T_{MAX} = 292.2 \text{ MPa}$

$\sigma_1 = 57.3 + 292.2 = 349.5 \text{ MPa}$

$\sigma_2 = 57.3 - 292.2 = -234.9 \text{ MPa}$

$2\phi_\sigma = \tan^{-1}(286.5/57.3) = 78.7^\circ ; \phi_\sigma = 39.35^\circ$

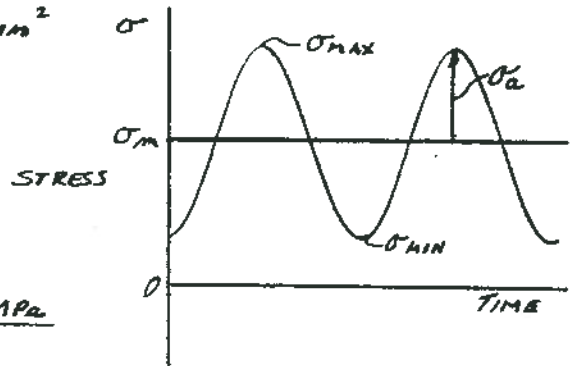
$2\phi_\tau = 90 - 2\phi_\sigma = 11.3^\circ ; \phi_\tau = 5.65^\circ$



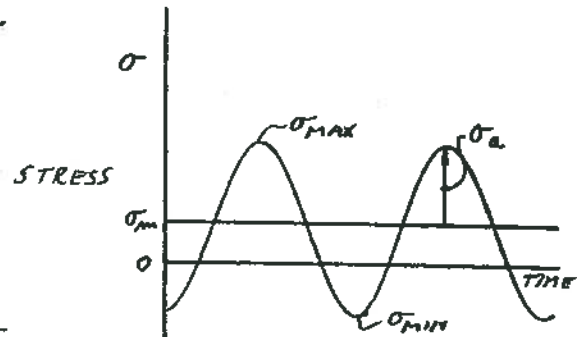
## CHAPTER 5 DESIGN FOR DIFFERENT TYPES OF LOADING

### Stress Ratio

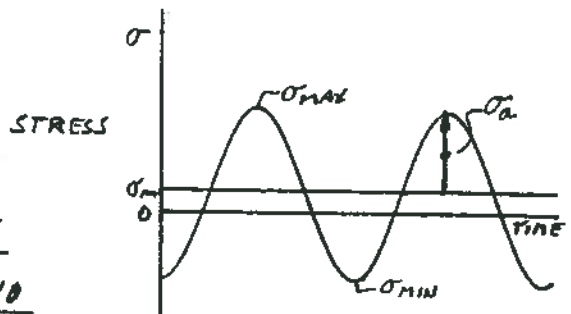
1.  $\sigma = F/A$ ;  $A = \frac{\pi(10\text{mm})^2}{4} = 78.54\text{mm}^2$   
 $\sigma_{\text{MAX}} = 3500\text{N}/78.54\text{mm}^2 = 44.6\text{MPa}$   
 $\sigma_{\text{MIN}} = 500\text{N}/A = 6.37\text{MPa}$   
 $F_m = (3500 + 500)/2 = 2000\text{N}$   
 $\sigma_m = 2000\text{N}/A = 25.5\text{MPa}$   
 $\sigma_a = \sigma_{\text{MAX}} - \sigma_m = 44.6 - 25.5 = 19.1\text{MPa}$   
 $R = \sigma_{\text{MIN}}/\sigma_{\text{MAX}} = 6.37/44.6 = 0.143$



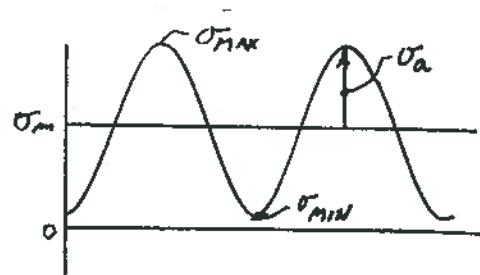
2.  $\sigma = F/A$ ;  $A = (10\text{mm})(30\text{mm}) = 300\text{mm}^2$   
 $\sigma_{\text{MAX}} = 20 \times 10^3\text{N}/300\text{mm}^2 = 66.7\text{MPa}$   
 $\sigma_{\text{MIN}} = -8.0 \times 10^3\text{N}/A = -26.7\text{MPa}$   
 $F_m = (20 - 8)/2 = 6\text{kN}$   
 $\sigma_m = 6 \times 10^3\text{N}/A = 20.0\text{MPa}$   
 $\sigma_a = \sigma_{\text{MAX}} - \sigma_m = 66.7 - 20.0 = 46.7\text{MPa}$   
 $R = \sigma_{\text{MIN}}/\sigma_{\text{MAX}} = -26.7/66.7 = -0.40$



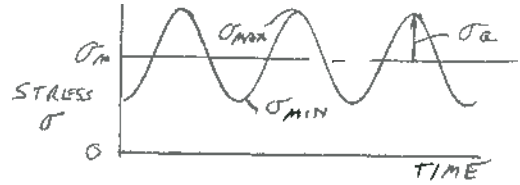
3.  $\sigma = F/A$ ;  $A = (0.40\text{in})^2 = 0.16\text{in}^2$   
 $\sigma_{\text{MAX}} = 860\text{lb}/0.16\text{in}^2 = 5375\text{psi}$   
 $\sigma_{\text{MIN}} = -120\text{lb}/0.16\text{in}^2 = -750\text{psi}$   
 $F_m = (860 - 120)/2 = 370\text{lb}$   
 $\sigma_m = 370\text{lb}/A = 2313\text{psi}$   
 $\sigma_a = \sigma_{\text{MAX}} - \sigma_m = 5375 - 2313 = 3062\text{psi}$   
 $R = \sigma_{\text{MIN}}/\sigma_{\text{MAX}} = -750/5375 = -0.140$



4.  $\sigma = F/A$ ;  $A = \pi D^2/4 = \pi(0.375)^2/4 = 0.1104\text{in}^2$   
 $\sigma_{\text{MAX}} = 1800\text{lb}/0.1104\text{in}^2 = 16297\text{psi}$   
 $\sigma_{\text{MIN}} = 150\text{lb}/0.1104\text{in}^2 = 1358\text{psi}$   
 $F_m = (1800 + 150)/2 = 975\text{lb}$   
 $\sigma_m = 975\text{lb}/A = 8828\text{psi}$   
 $\sigma_a = \sigma_{\text{MAX}} - \sigma_m = 16297 - 8828 = 7470\text{psi}$   
 $R = \sigma_{\text{MIN}}/\sigma_{\text{MAX}} = 1358/16297 = 0.083$

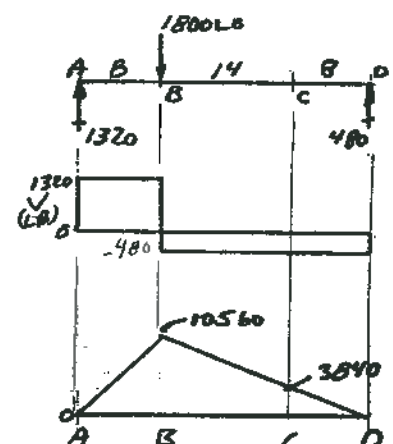
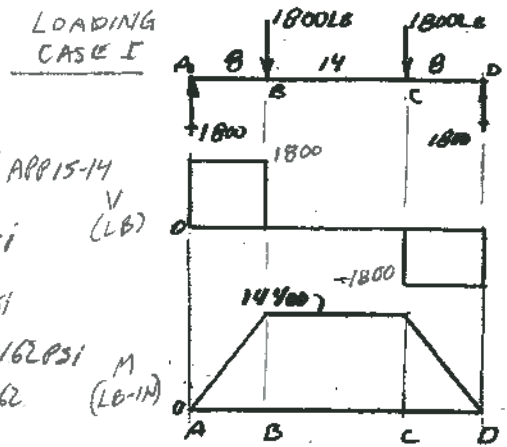


5.  $\sigma = F/A$ ;  $A = \frac{\pi d^2}{4} = \frac{\pi (3.0 \text{ mm})^2}{4} = 7.069 \text{ mm}^2$   
 $\sigma_{\text{MAX}} = 780 \text{ N} / 7.069 \text{ mm}^2 = 110.3 \text{ MPa}$   
 $\sigma_{\text{MIN}} = 360 \text{ N} / A = 50.9 \text{ MPa}$   
 $F_m = (180 + 360) / 2 = 570 \text{ N}$   
 $\sigma_m = 570 \text{ N} / A = 80.6 \text{ MPa}$   
 $\sigma_a = 110.3 - 80.6 = 29.7 \text{ MPa}$   
 $R = 50.9 / 110.3 = 0.462$



6.  $\sigma = M/S$ ;  $S_y = 1.48 \text{ in}^3$  FOR  $4 \times 2 \times 1/4$  TUBE  $\sigma = Y$  APP 15-14  
 AT B AT C

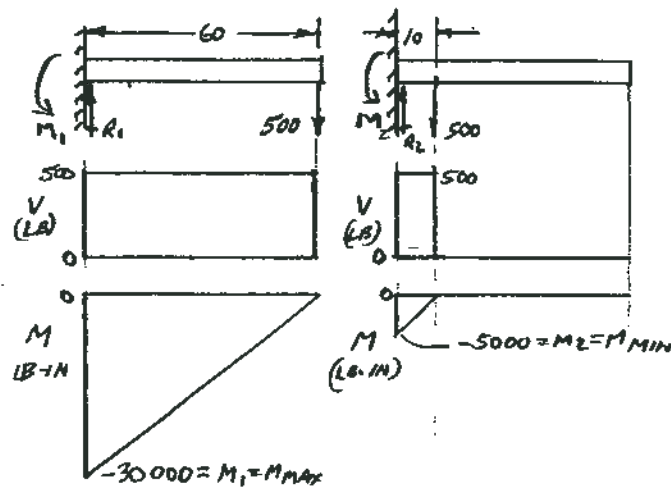
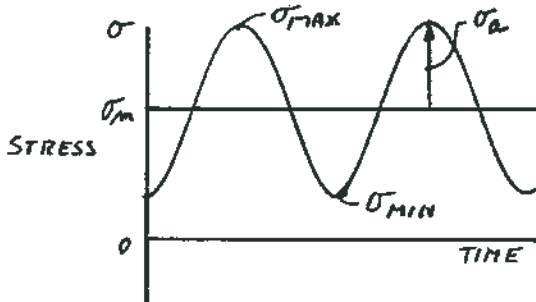
$\sigma_{\text{MAX}} = \frac{14400}{1.48} = 9730 \text{ psi}$ $\sigma_{\text{MIN}} = \frac{10560}{1.48} = 7135 \text{ psi}$ $\sigma_m = (9730 + 7135) / 2 = 8432 \text{ psi}$ $\sigma_a = \sigma_{\text{MAX}} - \sigma_m = 9730 - 8432 = 1298 \text{ psi}$ $\sigma_a = 2595 \text{ psi}$ $R = \frac{\sigma_{\text{MIN}}}{\sigma_{\text{MAX}}} = \frac{7135}{9730} = 0.733$ $R = 0.233$	$\sigma_{\text{MAX}} = \frac{14400}{1.48} = 9730 \text{ psi}$ $\sigma_{\text{MIN}} = \frac{3840}{1.48} = 2595 \text{ psi}$ $\sigma_m = (9730 + 2595) / 2 = 6162 \text{ psi}$ $\sigma_a = \sigma_{\text{MAX}} - \sigma_m = 9730 - 6162 = 3568 \text{ psi}$ $\sigma_a = 3568 \text{ psi}$ $R = \frac{\sigma_{\text{MIN}}}{\sigma_{\text{MAX}}} = \frac{2595}{9730} = 0.267$ $R = 0.267$
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STRESS VS. TIME DIAGRAM - SAME AS FOR PROB. 5.

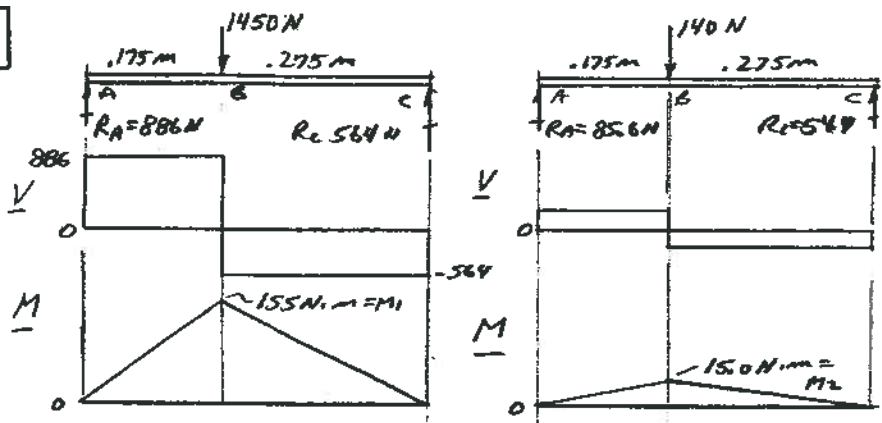
7. BEAM:  $\sigma = \frac{M}{S}$ ;  $S = 3.04 \text{ in}^3$  FOR  $5 \times 7.7$

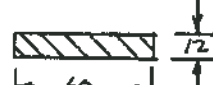
$M_{\text{MAX}} = (500 \text{ LB})(60 \text{ IN}) = 30000 \text{ LB}\cdot\text{IN} = M_1$   
 $M_{\text{MIN}} = (500 \text{ LB})(10 \text{ IN}) = 5000 \text{ LB}\cdot\text{IN} = M_2$   
 $M_m = (30000 + 5000) / 2 = 17500 \text{ LB}\cdot\text{IN}$   
 $M_a = 30000 - 17500 = 12500 \text{ LB}\cdot\text{IN}$   
 $\sigma_{\text{MAX}} = \frac{30000 \text{ LB}\cdot\text{IN}}{3.04 \text{ in}^3} = 9868 \text{ psi}$   
 $\sigma_{\text{MIN}} = 5000 / 3.04 = 1645 \text{ psi}$   
 $\sigma_m = 17500 / 3.04 = 5757 \text{ psi}$   
 $\sigma_a = 12500 / 3.04 = 4112 \text{ psi}$   
 $R = \frac{\sigma_{\text{MIN}}}{\sigma_{\text{MAX}}} = \frac{1645}{9868} = 0.167$





8.



BEAM CROSS SECTION  
  
 $S = \frac{BH^2}{6} = \frac{(60)12^2}{6}$   
 $S = 1440 \text{ mm}^3$   
 $\sigma = \frac{M}{S}$  BENDING

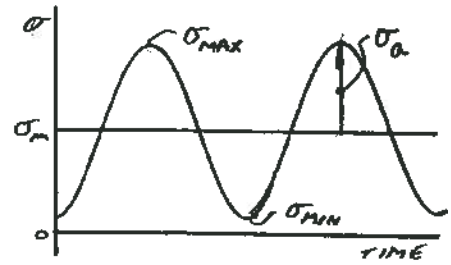
$$\sigma_{MAX} = \frac{M_1}{S} = \frac{155 \text{ N}\cdot\text{m}}{1440 \text{ mm}^3} \cdot \frac{10^3 \text{ mm}^3}{\text{m}} = 107.7 \text{ MPa}$$

$$\sigma_{MIN} = \frac{M_2}{S} = \frac{15.0 \times 10^3 \text{ N}\cdot\text{mm}}{1440 \text{ mm}^3} = 10.4 \text{ MPa}$$

$$\sigma_m = \frac{(107.7 + 10.4)}{2} = 59.1 \text{ MPa}$$

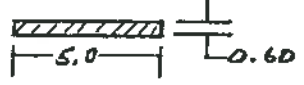
$$\sigma_a = \sigma_{MAX} - \sigma_m = 107.7 - 59.1 = 48.6 \text{ MPa}$$

$$R = \frac{\sigma_{MIN}}{\sigma_{MAX}} = \frac{10.4}{107.7} = 0.097$$



9.

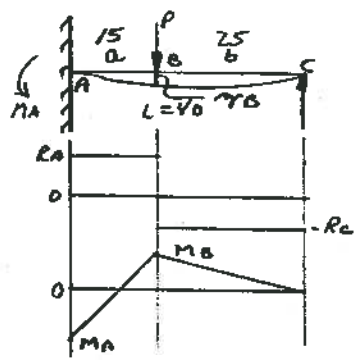
SPRING IS A SUPPORTED CANTILEVER - CASE (b) APP A14-3.  
 DEFLECTION PROPORTIONAL TO FORCE. BENDING MOMENT PROPORTIONAL TO FORCE.  
 DEFLECTION AT LOAD B:

$$M_B = \frac{-Pa^3b^2}{12EIL^3}(3L+b)$$


$$I = \frac{(5)(.6)^3}{12} = 0.090 \text{ mm}^4$$

$$S = \frac{5(.6)^2}{6} = 0.30 \text{ mm}^3$$

$$E = 207 \times 10^3 \text{ N/mm}^2$$



SOLVE FOR P:

$$P = \frac{12EIL^3 M_B}{a^3 b^2 (3L+b)} = \frac{(12)(207 \times 10^3)(0.090)(40)^3 M_B}{(0.5)^3 (0.25)^2 [3(40) + 0.25]} \text{ N}$$

$$P = 46.78 M_B$$

FOR  $M_1 = 0.25 \text{ mm}$ ;  $P_1 = 46.78(0.25) = 11.7 \text{ N}$ ; FOR  $M_2 = 0.40 \text{ mm}$ ,  $P_2 = 18.7 \text{ N}$   
 MOMENTS:

$$M_A = \frac{-Pa^2b}{2L^2}(b+L) = \frac{-P(0.5)(0.25)}{2(40)^2}(0.25+40) = 7.617 P \text{ MAXIMUM}$$

$$M_B = \frac{Pa^2b}{2L^3}(b+2L) = \frac{P(0.5)^2(0.25)}{2(40)^3}(0.25+2(40)) = 4.614 P$$

SUMMARY

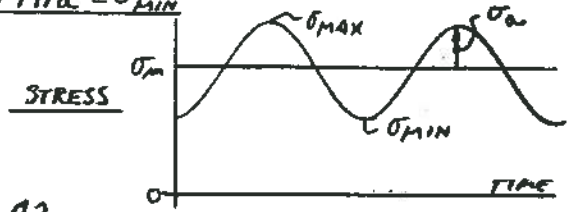
$$P_2 = 18.7 \text{ N}; M_{A2} = 7.617(18.7) = 142.5 \text{ N}\cdot\text{mm}; \sigma_{A2} = \frac{M}{S} = \frac{142.5 \text{ N}\cdot\text{mm}}{0.30 \text{ mm}^3} = 475 \text{ MPa} = \sigma_{MAX}$$

$$P_1 = 11.7 \text{ N}; M_{A1} = 89.1 \text{ N}\cdot\text{mm}; \sigma_{A1} = 297 \text{ MPa} = \sigma_{MIN}$$

$$\sigma_m = \frac{(475 + 297)}{2} = 386 \text{ MPa}$$

$$\sigma_a = 475 - 386 = 89.0 \text{ MPa}$$

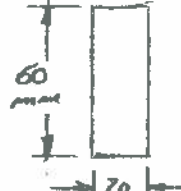
$$R = \frac{\sigma_{MIN}}{\sigma_{MAX}} = \frac{297}{475} = 0.625$$



10. FIND  $S_m'$ ; SAE 1040 CD;  $S_u = 80 \text{ ksi}$ ;  $S_m = 31 \text{ ksi}$  FOR CD CURVE  
 0.75 IN DIA.  $C_s = 0.90$  FIG. 5-9 FIG. 5-8.  
 $C_m = 1.0$  WROUGHT STEEL;  $C_{ST} = 1.0$  REV. BENDING;  $C_R = 0.81$  ( $R = 0.99$ )  
 $S_m' = S_m C_s C_m C_{ST} C_R = 31 \text{ ksi} (0.90)(1.0)(1.0)(0.81) = 22.6 \text{ ksi}$


11. FIND  $S_m'$ ; SAE 5160 OQT 1300;  $S_u = 715 \text{ MPa}$ ;  $S_m = 300 \text{ MPa}$  FIG. 5-8.  
 20.0 mm DIA.  $C_s = 0.90$  FIG. 5-9;  $C_m = 1.0$  WROUGHT STEEL; MACHINED.  
 $C_{ST} = 1.0$  REV. BENDING;  $C_R = 0.81$  ( $R = 0.99$ )  
 $S_m' = S_m C_s C_m C_{ST} C_R = 300 \text{ MPa} (0.90)(1.0)(1.0)(0.81) = 219 \text{ MPa}$

12. FIND  $S_m'$ ; SAE 4130 WQT 1300;  $S_u = 676 \text{ MPa}$ ;  $S_m = 260 \text{ MPa}$  FIG. 5-8  
 EQ. (5-8);  $D_e = 0.808 \sqrt{hb} = 0.808 \sqrt{(60)(20)} = 28.0 \text{ mm}$   
 $C_s = 0.87$  FIG. 5-9;  $C_m = 1.0$  WROUGHT STEEL  
 $C_{ST} = 1.0$  REV. BENDING;  $C_R = 0.81$  ( $R = 0.99$ )  
 $S_m' = S_m C_s C_m C_{ST} C_R = 260 \text{ MPa} (0.87)(1.0)(1.0)(0.81) = 183 \text{ MPa}$



13. FIND  $S_m'$ ; SAE 301 ST. ST. 1/2 HARD;  $S_u = 150 \text{ ksi}$ ;  $S_m = 52 \text{ ksi}$  FIG. 5-8.  
 $C_s = 1.0$  FOR AXIAL TENSILE STRESS.  $C_m = 1.0$  WROUGHT STEEL  
 $C_{ST} = 0.80$  AXIAL TENSILE STRESS.  $C_R = 0.75$  FOR  $R = 0.999$ .  
 $S_m' = S_m C_s C_m C_{ST} C_R = 52 \text{ ksi} (1.0)(1.0)(0.80)(0.75) = 31.2 \text{ ksi}$

14. FIND  $S_m'$ ; ASTM A242;  $S_u = 70 \text{ ksi}$ ;  $S_m = 27.0 \text{ ksi}$ , FIG. 5-8  
 EQ. (5-8);  $D_e = 0.808 \sqrt{hb} = 0.808 \sqrt{(3.5)(0.375)} = 0.926$ ;  $C_s = 0.883$   
 $C_m = 1.0$  WROUGHT STEEL;  $C_{ST} = 1.0$  REV. BENDING;  $C_R = 0.81$  ( $R = 0.99$ )  
 $S_m' = S_m C_s C_m C_{ST} C_R = (27.0 \text{ ksi})(0.883)(1.0)(1.0)(0.81) = 19.3 \text{ ksi}$



### Design and Analysis

Problems 15 - 18 are open-ended design problems for which there is no unique answer. The General Design Procedure from Section 5-9 should be used. The loading and support conditions should be compared with the cases described in Section 5-8 to determine the appropriate design stress. A design factor should be specified using the guidelines in Section 5-7. When needed, the endurance strength should be computed from Equation 5-6 in Section 5-4.

15. The link is subjected to a fluctuating normal stress. Use Case 5 from Section 5-8. See also the solution for Problem 1.
16. The rod is subjected to a fluctuating normal stress. Use Case 5 from Section 5-8. See also the solution for Problem 4.
17. The strut is subjected to a fluctuating normal stress. Use Case 5 from Section 5-8. See also the solution for Problem 2.
18. The latch part is subjected to a fluctuating normal stress. Use Case 5 from Section 5-8. See also the solution for Problem 5.

19.

FIG. P5-8. SEE ALSO PROBLEM 8 SOLUTION. FLUCTUATING LOAD

$$\frac{1}{N} = \frac{\sigma_m + K_t \sigma_a}{S_u} \quad \text{CASE 5} \quad \sigma_m = 59.1 \text{ MPa}; \sigma_a = 48.6 \text{ MPa}$$

FIND  $N$ . SAE 1020 HR;  $S_y = 207 \text{ MPa}$ ;  $S_u = 379 \text{ MPa}$ ;  $S_m = 140 \text{ MPa}$

$C_s$ : RECTANGLE:  $D_e = 0.808 \sqrt{12(60)} = 26.7 \text{ mm}$ ;  $C_s = 0.89$  FIG. 5-8 HOT ROL.

$C_m = 1.0$  (WELDED HT STEEL),  $C_{st} = 1.0$  (REV. BENDING); LET  $R = 0.99 - C_R = 0.81$

$$S_m' = (0.89)(1.0)(1.0)(0.81)(140) = 101 \text{ MPa}$$

$$\frac{1}{N} = \frac{59.1}{379} + \frac{(1.0)(48.6)}{101} = 0.637; \quad N = 1.57$$

LOW

20.

DESIGN PROBLEM - NO UNIQUE SOLUTION.

SUGGESTIONS: KEEP 60 mm WIDTH FOR SEAT APPLICATION.

CONSIDER HIGHER STRENGTH MATERIAL; THINNER STOCK FORMED INTO CHANNEL SHAPE . CONSIDER TAPERING CROSS SECTION

DEPTH - DEEPER AT LOAD - LESS DEEP NEAR SUPPORTS WHERE MOMENT IS SMALLER. CONSIDER REF. R - APP. 15-7 OR REF. 9 - APP. 15-B. ALUMINUM.

21.

DATA ARE SAME AS IN PROB. 7 WHERE AN S4X7.7 STEEL BEAM WAS PROPOSED. FLUCTUATING STRESS. USE CASE 5.  $A_t = 2.26 \text{ in}^2$

$$\frac{1}{N} = \frac{\sigma_m + K_t \sigma_a}{S_u} \quad \text{SPECIFY ASTM A36 STEEL } S_y = 36 \text{ ksi}; S_u = 58 \text{ ksi}$$

$$C_s: A_{95} = 0.05 A_t = 0.05(2.26 \text{ in}^2) = 0.113 \text{ in}^2 = 0.0766 D_e^2$$

$$S_m = 20 \text{ ksi (FIG. 5-8)} \quad D_e = \sqrt{0.113 / 0.0766} = 1.215 \Rightarrow C_s = 0.86$$

$C_m = 1.0$ ;  $C_{st} = 1.0$ ; LET  $R = 0.99 - C_R = 0.81$ .

$S_m' = (86)(0.81)(20) = 13.9 \text{ ksi}$ ;  $\sigma_m = 5757 \text{ psi}$ ;  $\sigma_a = 4112 \text{ psi}$  - PROB. 7.

$$\frac{1}{N} = \frac{5757 \text{ psi}}{58000 \text{ psi}} + \frac{1.0(4112 \text{ psi})}{13900 \text{ psi}} = 0.395; \quad N = 2.53$$

CHECK YIELD:

$$N = \frac{S_y}{K_t(\sigma_a + \sigma_m)} = \frac{36000}{1(4112 + 5757)} = 3.65$$

$N = 2.53$  IS SATISFACTORY IF NO UNUSUAL CONDITIONS OR UNCERTAINTY OF DATA EXIST.

22.

DATA SAME AS PROBLEM 9, FLUCTUATING NORMAL STRESS - CASE 5.

$\sigma_m = 386 \text{ MPa}$ ;  $\sigma_a = 89 \text{ MPa}$ . FROM APP. 3 AISI 4140 QT 408 HAS THE HIGHEST  $S_m$  WITH  $> 10\%$  ELONGATION FOR 6000 DUCTILITY.

$S_u = 2000 \text{ MPa}$ ;  $S_y = 1730 \text{ MPa}$ ;  $S_m = 450 \text{ MPa}$  (FIG. 5-8); LET  $R = 99\% - C_R = 0.81$

$S_m' = (0.81)(450 \text{ MPa}) = 364 \text{ MPa}$  | FOR REF.:  $D_e = 0.808 \sqrt{12(5)} = 0.808 \sqrt{60} = 1.40 \text{ mm}$

$$\frac{1}{N} = \frac{\sigma_m + K_t \sigma_a}{S_u} = \frac{386}{2000} + \frac{(1.0)(89)}{364} = 0.438; \quad N = 2.29$$

SUGGEST TRYING TO FIND AN EVEN STRONGER MATERIAL FROM OTHER REFERENCES. MAY ADJUST WIDTH OR THICKNESS OF SPRING STOCK. USE ANALYSIS FROM PROB. 9 TO COMPUTE FORCE VS. DEFLECTION FOR SPRING.

CONSIDER MOVING LATCH PIN FARTHER FROM FIXED END OF SPRING. THIS IS A GOOD SPREAD SHEET PROBLEM.

23.

DATA SAME AS PROB. 6. FLUCTUATING NORMAL STRESS, CASE 5.

AT B:  $\sigma_m = 8432 \text{ psi}$ ,  $\sigma_a = 2595 \text{ psi}$ . FOR ASTM A500 GRADE B:  $S_u = 58 \text{ ksi}$ ,  $S_y = 46 \text{ ksi}$

AT C:  $\sigma_m = 6162 \text{ psi}$ ,  $\sigma_a = 3568 \text{ psi}$ .  $S_u = 20 \text{ ksi}$ ;  $R = 0.99 \rightarrow C_R = 0.81$ ;  $C_S$  FOR 4x4x1/4 TUBE

$A_T = 3.59 \text{ in}^2$ ;  $A_{RS} = 0.05(3.59) = 0.180 \text{ in}^2 = 0.0716 \text{ in}^2$ ;  $D_o = 1.53 \text{ in} \rightarrow C_S = 0.84$ ;  $S'_m = (0.84)(0.81)(20) = 13.6 \text{ ksi}$

$$\text{AT C: } \frac{1}{N} = \frac{6162}{58000} + \frac{(60)(3568)}{13600} = 0.369; \quad \boxed{N = 2.71} \quad \begin{array}{l} \text{CHECK} \\ \text{YIELD: } N = \frac{46000}{1(3429 + 5922)} = 4.92 \text{ OK} \end{array}$$

AT B:  $N = 2.97 > 2.71 \text{ OK}$ . 4x4x1/4 TUBING WEIGHS 8.78 LB/FT. APP 15-14. A LIGHTER BEAM CAN BE DESIGNED BY PLACING 4 INSIDE VERTICAL AND USING A THINNER WALL. IN APP 15-15, THE SOURCE-JORGENSEN OFFERS 4x2x0.134; 5.223 LB/FT. WITH 4.0 INSIDE VERTICAL,  $S_x = 1.58 \text{ in}^3$ . ORIGINAL  $S_x = 1.48$  SO BEAM IS SAFE. WT IS REDUCED BY  $\approx 40\%$ .

24.

PISTON ROD. F16 P5-24. DIA = 0.60 IN.  $A = \pi D^2/4 = 0.283 \text{ in}^2$ FLUCTUATING LOAD.  $F_{\text{MAX}} = 500 \text{ LB TENS.}$ ;  $F_{\text{MIN}} = -400 \text{ LB COMP.}$  CASE 5.

$$F_m = (500 - 400)/2 = 55 \text{ LB}; \quad F_a = 500 - 55 = 445 \text{ LB}$$

$$\sigma_m = F_m/A = 55 \text{ LB}/0.283 \text{ in}^2 = 1943 \text{ psi}; \quad \sigma_a = F_a/A = 445 \text{ LB}/0.283 \text{ in}^2 = 1572 \text{ psi}$$

SAE 4130 WOT 1300;  $S_u = 98 \text{ ksi}$ ;  $S_y = 89 \text{ ksi}$ ;  $S_m = 37 \text{ ksi}$  (FIG. 5-8) $C_S = 0.93$ ;  $C_m = 1.0$ ;  $C_{ST} = 0.80$  (ANIAL);  $R = 99\% - C_R = 0.81$ 

$$S'_m = (0.93)(1.0)(0.80)(0.81) 37 \text{ ksi} = 22.3 \text{ ksi}$$

$$\frac{1}{N} = \frac{\sigma_m}{S'_m} + \frac{K_t(\sigma_a)}{S'_m} = \frac{1943 \text{ psi}}{22300 \text{ psi}} + \frac{(1.0)(1572 \text{ psi})}{22300 \text{ psi}} = 0.170; \quad N = 5.87$$

SAFE BUT HIGH. SHOULD ALSO CHECK FOR  $K_t$  IN FINAL DESIGN.IF ROD DIA. IS REDUCED TO 0.50 IN.  $A = 0.196 \text{ in}^2$ 

$$\sigma_m = 2801 \text{ psi}; \quad \sigma_a = 4838 \text{ psi}; \quad N = 4.15 \text{ BETTER. USE } D = 0.50 \text{ IN.}$$

25.

BRITTLE MATERIAL - STATIC LOAD - CASE 1:  $N = S_{uc}/\sigma_{\text{MAX}}$ 

$$\sigma_{\text{MAX}} = \frac{K_t F}{A} = \frac{(1.99)(75000 \text{ LB})}{\pi (4.00 \text{ in.})^2/4} = 11877 \text{ psi COMPRESSION}$$

$$r/d = 0.25 \text{ in}/4.00 \text{ in} = 0.0625; \quad D/b = 5.00 \text{ in}/4.00 \text{ in} = 1.25; \quad \text{THEN } K_t = 1.99$$

$$N = \frac{S_{uc}}{\sigma_{\text{MAX}}} = \frac{140000 \text{ psi}}{11877 \text{ psi}} = 11.8$$

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26.

BRITTLE MATERIAL - STATIC LOAD - CASE 1:  $N = S_{uc}/\sigma_{\text{MAX}}$ 

$$\sigma_{\text{MAX}} = \frac{K_t F}{A} = \frac{(1.99)(12000 \text{ LB})}{\pi (4.00 \text{ in.})^2/4} = 1900 \text{ psi}; \quad [K_t \text{ SAME AS PROB. 25}]$$

$$N = \frac{S_{uc}}{\sigma_{\text{MAX}}} = \frac{40000 \text{ psi}}{1900 \text{ psi}} = 21.0$$

27.

BRITTLE MATERIAL - BIAXIAL STRESS - SECTION 5-11.1

USE MODIFIED MOHR METHOD.

 $\sigma_1, \sigma_2$  FOUND FROM MOHR CIRCLE  
STRESS ELEMENT IN FILLET AREAAXIAL COMPRESSIVE STRESS FROM PROB.:  $\sigma_{\text{MAX}} = -11877 \text{ psi} = \sigma_y$ 

CONTINUED - NEXT PAGE.

27. CONTINUED

TORSION:  $T = \frac{K_e T}{Z_p}$  ; FOR  $r/d = 0.0625$  ;  $D/d = 1.25$  ,  $K_e = 1.48$   
SEE PROB. 25.

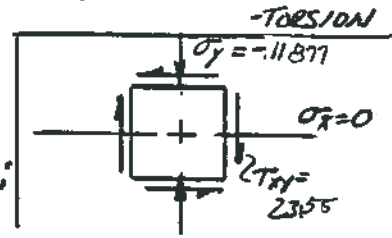
$$Z_p = \frac{\pi D^3}{16} = \frac{\pi (4.00 \text{ IN})^3}{16} = 12.57 \text{ IN}^3$$

$$T = (1.48 \times 20,000 \text{ LB}\cdot\text{IN}) / 12.57 \text{ IN}^3 = 2355 \text{ PSI}$$

FROM MOHR CIRCLE:  $R = \sqrt{2355^2 + 5939^2} = 6389 \text{ PSI}$

$$\sigma_1 = \sigma_{AV} + R = -5939 + 6389 = 450 \text{ PSI TENSION}$$

$$\sigma_2 = \sigma_{AV} - R = -5939 - 6389 = -12328 \text{ PSI COMP.$$



GRAPHICAL SOLUTION  
4TH QUADRANT

PT. A AT  $\sigma_1 = 450 \text{ PSI}$ ,  $\sigma_2 = -12328 \text{ PSI}$

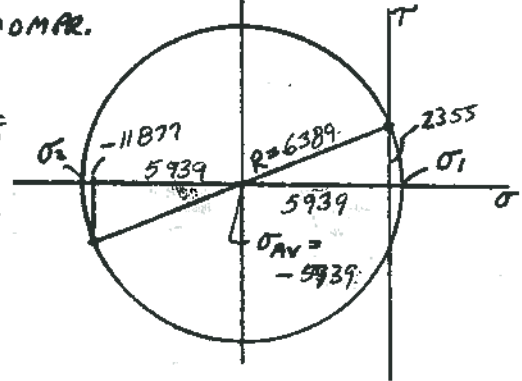
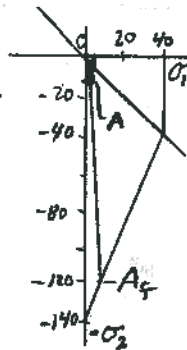
LINE  $OA = 12336 \text{ PSI}$

PT  $A_f$  IS FAILURE POINT

LINE  $OA_f = 120300 \text{ PSI (SCALED)}$

$$N = \frac{OA_f}{OA} = 9.75$$

VERY SAFE



28.

DUCTILE MATERIAL - STATIC LOAD - CASE 2:

SAE 1137 CD ;  $S_y = 565 \text{ MPa}$

$$\sigma_B = S_y / N = 565 \text{ MPa} / 3 = 188 \text{ MPa}$$

IN MIDDLE OF SHAFT,  $M = 337.5 \text{ kN}\cdot\text{mm}$

$$\text{REQ'D } S = \frac{M}{\sigma_B} = \frac{337.5 \times 10^3 \text{ N}\cdot\text{mm}}{188 \text{ N}/\text{mm}^2} = 1795 \text{ mm}^3$$

$$S = \pi d^3 / 32 ; d = \sqrt[3]{32S / \pi} = 26.3 \text{ mm}$$

USE PREFERRED VALUE  $d = 28 \text{ mm}$   
(TABLE A2-1)

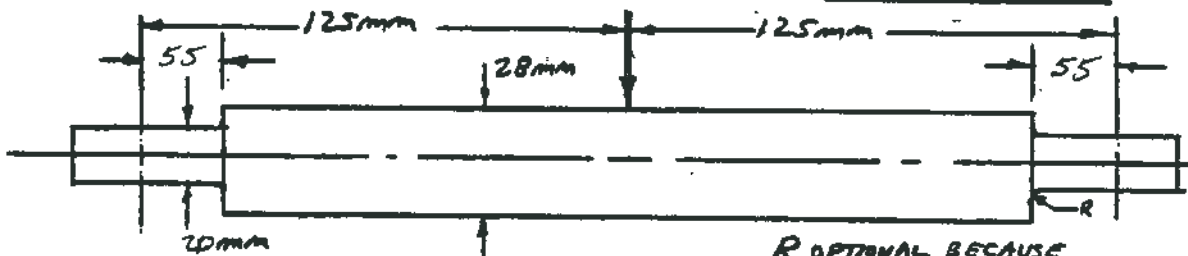
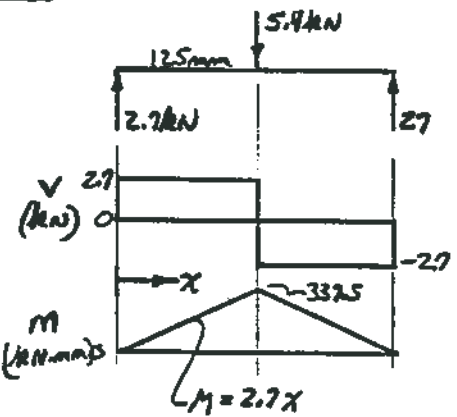
FIND  $x$  WHICH WOULD BE SAFE FOR  $d = 20 \text{ mm}$

$$S = \pi d^3 / 32 = \pi (20)^3 / 32 = 785.4 \text{ mm}^3$$

$$M = \sigma_B S = (188 \text{ N}/\text{mm}^2) (785.4 \text{ mm}^3) = 147,650 \text{ N}\cdot\text{mm} = 147.7 \text{ kN}\cdot\text{mm}$$

$$\text{BUT } M = 2.7x ; x = \frac{M}{2.7} = \frac{147.7 \text{ kN}\cdot\text{mm}}{2.7} = 54.7 \text{ mm (MAX)}$$

USE  $a = 55 \text{ mm}$



R OPTIONAL BECAUSE  
OF STATIC STRESS

29.

FIGURE 5-28. SEE ALSO PROB. 28.

CASE 4; REPEATED REVERSED NORMAL STRESS;  $\sigma_a = S_m' / N$

SAE 1137 CD  $S_u = 676 \text{ MPa}$ ;  $S_m = 250 \text{ MPa}$  (F165-8); FOR  $d = 20 \text{ mm}$ ,  
 $C_s = 0.90$ ,  $C_m = 1.0$ ,  $C_{cr} = 1.0$ ,  $R = 99\%$  -  $C_R = 0.81$ ;  $S_m' = (0.90)(0.81)(250) = 182 \text{ MPa}$

$\sigma_a = S_m' / N = 182 / 3 = 60.7 \text{ MPa}$ ;  $M_{max} = 337.5 \times 10^3 \text{ N}\cdot\text{mm}$  AT LOAD (PROB. 28)

REQ'D  $S = \frac{M}{\sigma_a} = \frac{337.5 \times 10^3 \text{ N}\cdot\text{mm}}{60.7 \text{ N/mm}^2} = 5563 \text{ mm}^3$ ; BUT  $S = \pi D^3 / 32$

REQ'D  $D = \sqrt[3]{32 S / \pi} = 38.4 \text{ mm}$ ; USE  $D = 40.0 \text{ mm}$  NEW  $C_s = 0.87$   
RECOMPUTE  $D_{min} = 38.4 \text{ mm}$   
 $D = 40.0 \text{ mm}$  OK

ALLOWABLE DISTANCE "a". LET  $R = 2.0 \text{ mm}$ ,  $R/d = 2.0/20 = 0.100$  }  $K_t = 1.80$

$\sigma = \frac{K_t M}{S}$ ;  $M_{max} = \frac{\sigma_a S}{K_t}$ ;  $S = 785.4 \text{ mm}^3$  (PROB. 28);  $D/d = 40/20 = 2.00$

$M_{max} = \frac{60.7 \text{ N/mm}^2 \cdot 785.4 \text{ mm}^3}{1.80} = 26485 \text{ N}\cdot\text{mm} = 26.5 \text{ kN}\cdot\text{mm} = 2.7 \text{ X}$  (PROB. 28)

$X_{max} = M_{max} / 2.7 = 26.5 \text{ kN}\cdot\text{mm} / 2.7 \text{ kN} = 9.81 \text{ mm}$ ; USE  $a = 9.0 \text{ mm}$  (SMALL)

30.

SEE FIG. 5-28 AND PROB. 28 AND 29.  $K_t = 2.0$  FOR KEYSEAT NON-CIRCLE FOR ALTERNATING STRESS

EQU. 5-22:  $\frac{1}{N} = \frac{K_t (\sigma_a)_{max}}{S_s S_m'} + \frac{(\tau_m)_{max}}{S_s S_m}$   $(\tau_m)_{max} = T/2$   
 $(\sigma_a)_{max} = \sigma_a/2$

$\sigma_a = \frac{M_a}{S} = \frac{M_a}{Z_P/2} = \frac{2 M_a}{Z_P}$ ; THEN  $(\sigma_a)_{max} = \frac{M_a}{Z_P}$

$\frac{1}{N} = \frac{K_t M_a}{Z_P S_s S_m'} + \frac{T}{Z_P S_s S_m}$ ; REQ'D  $Z_P = N \left[ \frac{K_t M_a}{S_s S_m'} + \frac{T}{S_s S_m} \right]$

SAE 1137 CD:  $S_u = 676 \text{ MPa}$ ,  $S_m = 250 \text{ MPa}$   
 ASSUME  $D = 50 \text{ mm}$ ,  $C_s = 0.81$ ,  $C_R = 0.81$   
 $S_s S_m' = 0.577 S_m' = (0.50)(0.81)(0.81)(250) = 820 \text{ MPa}$   
 $S_s S_m = 0.75 S_m = 0.75(676) = 507 \text{ MPa}$

REQ'D  $Z_P = 3 \left[ \frac{(2.0)(337.5 \times 10^3)}{820} + \frac{150 \times 10^3}{507} \right] = 25579 \text{ mm}^3 = \pi D^3 / 16$

$D_{min} = \sqrt[3]{16 Z_P / \pi} = \sqrt[3]{16(25579) / \pi} = 50.69 \text{ mm}$ ; SPECIFY  $D = 52.0 \text{ mm}$

LOCATION OF STEP FROM 20 mm TO 50 mm:

$Z_P = \pi D^3 / 16 = \pi (20)^3 / 16 = 1571 \text{ mm}^3$ ; SOLVE EQ (I) FOR  $M_a$

$M_a = \frac{S_s S_m'}{K_t} \left[ \frac{Z_P}{N} - \frac{T}{S_s S_m} \right]$

FOR  $d = 20 \text{ mm}$ ;  $C_s = 0.90$ ;  $S_s S_m' = (0.90)(0.81)(250)(0.577) = 105 \text{ MPa}$

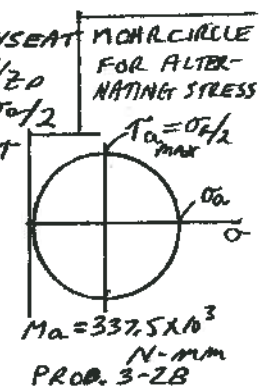
$M_a = \frac{105 \text{ N/mm}^2}{1.82} \left[ \frac{1571 \text{ mm}^3}{3} - \frac{150 \times 10^3 \text{ N}\cdot\text{mm}}{507 \text{ N/mm}^2} \right] = 13143 \text{ N}\cdot\text{mm}$

FROM PROB. 3-28,  $M = (2700 \text{ N}) X$

$X_{max} = M_a / 2700 = \frac{13143 \text{ N}\cdot\text{mm}}{2700 \text{ N}} = 4.86 \text{ mm}$  VERY SMALL

X IS FROM MIDDLE OF BEARING TO STEP.

REDESIGN IS REQUIRED, CONSIDER LARGER  $d$  OR MATERIAL WITH HIGHER STRENGTH.



FOR  $K_t$ : BENDING  
 $D = 52 \text{ mm}$   
 $d = 20 \text{ mm}$   
 $r = 2.0 \text{ mm}$   
 $K_t = 1.82$



31.

$S_y = 30 \text{ ksi} ; T_d = 0.5 S_y / 4 = 0.5(30) / 3 = 5 \text{ ksi} = 5000 \text{ psi}$

CASE 3 : COMBINED STRESS-STATIC LOAD, MAX SHEAR STRESS METHOD.

ASSUME AXIAL COMPRESSION IS SMALL. THEN MAX  $T$  OCCURS

AT FRONT AND REAR - COMBINED BENDING AND TORSION. USE

EQUIVALENT TORQUE METHOD - CH. 4, EQ. 4-16 AND 4-17

$T = (200 \text{ LB})(18 \text{ IN}) = 3600 \text{ LB-IN}$   
 $M = (400 \text{ LB})(18 \text{ IN}) = 7200 \text{ LB-IN}$

NOTE: BENDING DUE TO 200 LB LOAD IS ONLY 4000 LB-IN AND ACTS AT A DIFFERENT POINT.

$T_e = \sqrt{M^2 + T^2} = \sqrt{7200^2 + 3600^2} = 8050 \text{ LB-IN}$

$T_{MAX} = T_e / Z_p = T_d ; \text{ THEN } Z_p = \frac{T_e}{T_d} = \frac{8050 \text{ LB-IN}}{5000 \text{ LB-IN}^2} = 1.61 \text{ IN}^3$

BUT  $S = Z_p / 2 = 0.805 \text{ IN}^3 \rightarrow$  FROM APP. A-15 USE  $2\frac{1}{2}$  IN SCH. 40

$S = 1.06 \text{ IN}^3 ; Z_p = 2.128 \text{ IN}^3 ; A = 1.70 \text{ IN}^2$

PIPE

CHECK  $T_{MAX} = \sqrt{\left(\frac{M}{Z}\right)^2 + T^2} = \sqrt{\left(\frac{7200}{2.128}\right)^2 + (1698)^2} = 3903 \text{ psi OK}$

$< T_d$

$\sigma = \sigma_b - \sigma_c = \frac{-M}{S} - \frac{400 \text{ LB}}{A} = \frac{-7200}{1.064} - \frac{400}{1.704} = -7002 \text{ psi}$

$T = \frac{T}{Z_p} = \frac{3600}{2.128} = 1692 \text{ psi}$

32.

FLUCTUATING SHEAR STRESS:

EQN 5-22:  $\frac{1}{N} = \frac{T_m}{S_{SM}} + \frac{K_t T_a}{S_{SM}}$

$T_m = \frac{T_m}{Z_p} ; T_a = \frac{T_a}{Z_p}$

$\frac{1}{N} = \frac{T_m}{Z_p S_{SM}} + \frac{K_t T_a}{Z_p S_{SM}} = \frac{1}{Z_p} \left[ \frac{T_m}{S_{SM}} + \frac{K_t T_a}{S_{SM}} \right]$

$Z_p = \frac{\pi D^3}{16} = N \left[ \frac{T_m}{S_{SM}} + \frac{K_t T_a}{S_{SM}} \right] = 2 \left[ \frac{47.5 \times 10^3 \text{ N}\cdot\text{mm}}{870 \text{ N/mm}^2} + \frac{2.5(17.5 \times 10^3) \text{ N}\cdot\text{mm}}{146 \text{ MPa}} \right]$

$S_{SM} = 0.75 S_u = 0.75(1160 \text{ MPa}) = 870 \text{ MPa}$

USE  $S_{SM} = 0.577 S_u$  SAE 4140 OQT 1000;  $S_u = 1160 \text{ MPa}, S_y = 1056 \text{ MPa}$   
 FOR  $S_u = 1160 \text{ MPa} ; S_m = 400 \text{ MPa}$  (FIG 5-8): LET  $C_s = 0.90, C_r = 0.81$   
 $S_{SM} = (0.577)(0.9)(0.81)(1160 \text{ MPa}) = 146 \text{ MPa}$

REQD  $Z_p = 708 \text{ mm}^3 = \pi D^3 / 16$

$D_{MIN} = \sqrt[3]{16 Z_p / \pi} = \sqrt[3]{16(708) \text{ mm}^3 / \pi} = 15.31 \text{ mm}$

SPECIFY  $D = 16.0 \text{ mm}$   
 ACTUAL  $C_s = 0.92$  OK  
 $Z_p = \pi(16)^3 / 16 = 804 \text{ mm}^3$

CHECK YIELDING: EQN 5-23

$T_a = \frac{T_a}{Z_p} = \frac{17500 \text{ N}\cdot\text{mm}}{804 \text{ mm}^3} = 21.77 \text{ MPa} ; T_m = \frac{T_m}{Z_p} = \frac{47500}{804} = 59.06 \text{ MPa}$

$N_y = \frac{S_y / 2}{K_t(T_a + T_m)} = \frac{1056 / 2}{2.5(21.77 + 59.06)} = 2.60 > 2.0$  OK

33.

FLUCTUATING NORMAL STRESS:

CASE 5:  $\frac{1}{N} = \frac{\sigma_m}{S_m} + \frac{K_t \sigma_a}{S_m'}$  FIG. 5-8

$S_y = 58 \text{ ksi}$ ;  $S_m = 75 \text{ ksi}$ ;  $N = 3$ ;  $S_m = 28 \text{ ksi}$   
 ASSUME MACHINED SURFACE AND  $C_s = 0.9$ ,  $C_a = 0.81$

$S_m' = (0.9)(0.81)(28 \text{ ksi}) = 20.4 \text{ ksi} = 20,400 \text{ psi}$

$M_{max} = \frac{FL}{4} = \frac{800(48)}{4} = 9600 \text{ LB}\cdot\text{IN.}$

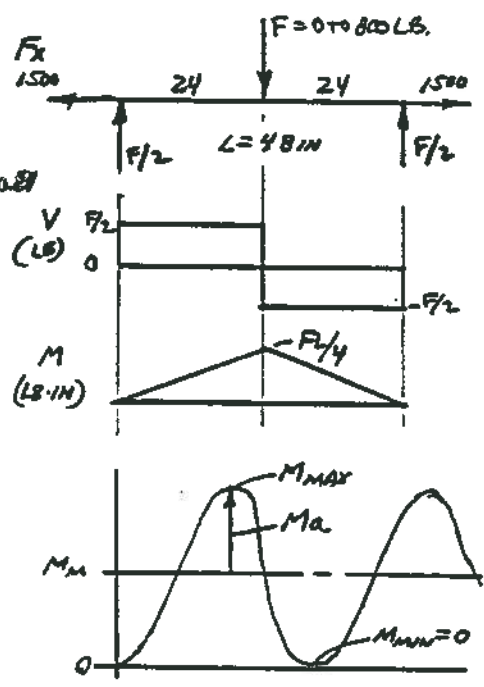
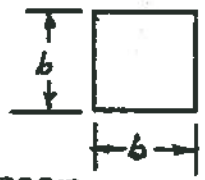
$M_m = M_n = 4800 \text{ LB}\cdot\text{IN}$

$S = b^3/6$ ;  $A = b^2$

$\sigma_m = \frac{F_x}{A} + \frac{M_m}{S}$

$\sigma_m = \frac{1500}{b^2} + \frac{4800}{b^3/6} = \frac{1500}{b^2} + \frac{28800}{b^3}$

$\sigma_a = \frac{M_a}{S} = \frac{4800}{b^3/6} = \frac{28800}{b^3}$



①  $\frac{1}{N} = \frac{\frac{1500}{b^2} + \frac{28800}{b^3}}{75000} + \frac{\frac{28800}{b^3}}{20,400} = \frac{0.020}{b^2} + \frac{0.384}{b^3} + \frac{1.412}{b^3} = \frac{0.020}{b^2} + \frac{1.796}{b^3}$

TERM INVOLVING  $b^2$  IS SMALL:  $b \approx \sqrt[3]{N(1.796)} = 1.75 \text{ IN}$  → USE  $b = 1.80 \text{ IN}$

RECHECK:  $C_s$  FOR  $b = 1.80 \text{ IN}$  SQUARE,  $P_c = 0.808 \sqrt{bh} = 0.808 \sqrt{b^2} = 0.808 b$   
 $P_c = 0.808(1.80) = 1.454 \text{ IN}$ . THEN  $C_c = \left(\frac{1.454}{0.3}\right)^{-0.11} = 0.84$  PREFERRED SIZE

$S_m' = (0.84)(0.81)(28 \text{ ksi}) = 19.05 \text{ ksi} = 19,050 \text{ psi}$

EQUATION ① BECOMES:

$\frac{1}{N} = \frac{\frac{1500}{b^2} + \frac{28800}{b^3}}{19,050} + \frac{\frac{28800}{b^3}}{19,050} = \frac{0.020}{b^2} + \frac{0.384}{b^3} + \frac{1.512}{b^3} = \frac{0.020}{b^2} + \frac{1.896}{b^3}$

$\frac{1}{N} = \frac{0.020}{(1.80)^2} + \frac{1.896}{(1.80)^3} = 0.331$ ;  $N = 3.02 \text{ OK}$  SPECIFY  $b = 1.80 \text{ IN}$



34 FLUCTUATING COMBINED STRESS:

CASE 5:  $\frac{1}{N} = \frac{(T_m)_{max}}{S_{SM}} + K_e \frac{(T_a)_{max}}{S_{SA}}$

T DUE TO TORQUE: (FIG. 3-10)

$T = \frac{I}{\phi} = \frac{1200}{0.2086^3} = \frac{1200}{.2086^3} = 5769$

FROM PROB. 33, MEAN STRESS

$\sigma_m = \frac{1500}{b^2} + \frac{28800}{b^3} \approx \frac{28800}{b^3}$   
(SMALL)

$R(T_m)_{max} = \sqrt{\left(\frac{5769}{b^3}\right)^2 + \left(\frac{14400}{b^3}\right)^2} = \frac{15573}{b^3}$

ALT. STRESS:  $T_a = \sigma_m/2$  FOR BENDING ONLY

$(T_a)_{max} = \frac{1}{2} \frac{28800}{b^3} = \frac{14400}{b^3}$

THEN  $\frac{1}{N} = \frac{15573}{b^3 \cdot 56250} + \frac{14400}{b^3 \cdot 10860} = \frac{1.602}{b^3}$

FOR  $N = 3$ ,  $b = 1.69$  IN  $\rightarrow$  USE  $b = 1.80$

CHECK:

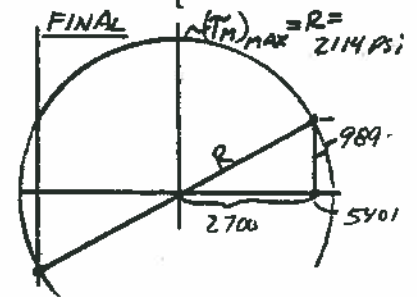
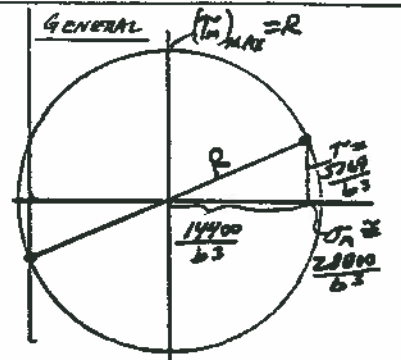
$\sigma_m = \frac{1500}{(1.80)^2} + \frac{28800}{(1.80)^3} = 5401$  PSI } FROM MONR CIRCLE!

$T_m = \frac{5769}{(1.80)^3} = 989$  PSI

$T_a = \frac{14400}{(1.80)^3} = 2469$  PSI

$\frac{1}{N} = \frac{2876}{56250} + \frac{2469}{10860} = 0.279 \rightarrow N = 3.59$  OK

MONR'S CIRCLE-MEAN STRESS



$S_{SM} = 0.75 S_m = 0.75(7250) = 5437.5$  PSI  
 $S_{SA} = 0.577 S_m' = (0.577)(0.83)(0.81)(2045) = 10860$  PSI

PROBLEMS 35, 36 AND 37 ALL DUCTILE MATERIALS - STEADY LOAD CASE 2.

35.  $\sigma = \frac{F}{A} = \frac{4500 N}{\pi(18^2 - 12^2)/4 \text{ mm}^2} = 31.8 \text{ MPa}; N = S_y/\sigma = 290/31.8 = 9.11$   
SAE 1040 HR;  $S_y = 290 \text{ MPa}$

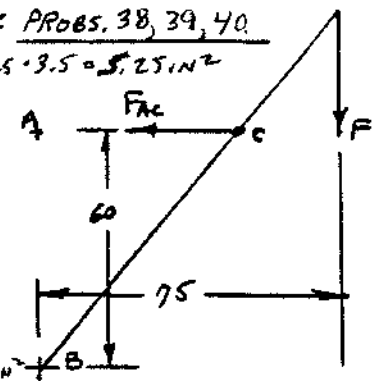
36.  $\sigma = F/A = 5000 N / (2)^2 \text{ mm}^2 = 34.7 \text{ MPa}; N = S_y/\sigma$  CASE 2.

- a) 1020 HR:  $N = 207/34.7 = 5.96$
  - b) 8650-OQT 1000;  $N = 1070/34.7 = 30.8$
  - c) DUCTILE IRON, 60-40-18:  $N = 276/34.7 = 7.95$
  - d) ALUM. 6061-T6:  $N = 276/34.7 = 7.95$
  - e) T1-6Al-4V:  $N = 827/34.7 = 23.8$
  - f) PVC:  $N$  BASED ON TENSILE STRENGTH;  $N = 41/34.7 = 1.18$
  - g) PHENOLIC:  $N = 45/34.7 = 1.30$
- ALL HIGH-REDUCE }  
LOW-INCREASE }

37.  $\sigma = \frac{F}{A} = \frac{12600 \text{ LB}}{[(2.25)^2 - (2.00)^2] \text{ in}^2} = 11859 \text{ PSI}; N = S_y/\sigma = 40000/11859 = 3.37$

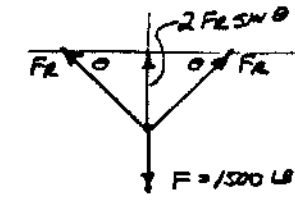
38 DUCTILE MATERIAL - STEADY LOAD - CASE 2 : PROBS. 38, 39, 40

SAE 1144 CD  $S_y = 90 \text{ ksi} = 90,000 \text{ psi}$   
 $A = 15 \cdot 3.5 = 52.5 \text{ in}^2$   
 $\Sigma M_B = 0 = 75F - 60C ; C = 75(2500)/60 = 3125 \text{ LB} = F_{AC}$   
 $\sigma = \frac{F_{AC}}{A} = \frac{3125 \text{ LB}}{(15)(3.5)} = 595 \text{ psi}$   
 $N = S_y / \sigma = 90,000 / 595 = 151 \text{ VERY HIGH}$   
 CONSIDER A SMALL ROD OF SAE 1020 HR STEEL  
 $\sigma_u = S_y / N = 3000 / 3 = 10,000 \text{ psi} ; A = \frac{F_{AC}}{\sigma} = 0.3125 \text{ in}^2$   
 A SQUARE BAR  $3/4 \text{ IN}$  ON A SIDE WOULD DO.  $A = 0.44 \text{ in}^2$   
 OR FRM APP. 15-15: RECT. TUBE -  $1.00 \times 7.00 \times 0.065 ; A = 0.373 \text{ in}^2$



39  $\Sigma F_v = 0 = 1500 \text{ LB} - 2 F_R \sin 45^\circ$  CASE 2

$F_R = 1500 / 2 (\sin 45^\circ) = 1061 \text{ LB}$   
 $\sigma_u = S_y / N = 42,000 / 3 = 14,000 \text{ psi} = F_R / A$   
 $A = \frac{F_R}{\sigma_u} = \frac{1061 \text{ LB}}{14,000 \text{ psi}} = 0.0758 \text{ in}^2 = 770^2 / 4 ; D = 0.311 \text{ in}$   
 USE  $5/16 \text{ IN.}$



40  $F_R = 1500 / 2 (\sin 15^\circ) = 2898 \text{ LB} ; A = \frac{F_R}{\sigma_u} = \frac{2898}{14,000} = 0.207 \text{ in}^2 ; D = 0.513 \text{ IN}$   
 USE  $9/16 \text{ IN.}$

41 REPEATED REVERSED AXIAL LOAD:  $C_s = 1.00 ; C_{ST} = 0.8$  AXIAL LOAD  $C_R = 0.81$

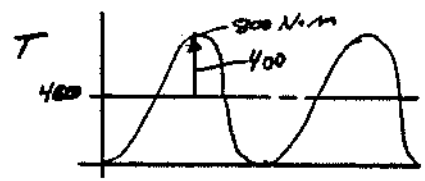
CASE 4:  $N = S_m' / \sigma_{MAX} ; \sigma_{MAX} = K_t F / A = \frac{1.83(7500)}{(6)(9) \text{ mm}^2} = 254 \text{ MPa}$   
 $r/b = 15/9 = 0.167 ; K_t = 1.83$   
 $r_m = 9/9 = 1.0 ; \text{USING FIG. 3-26(a)}$   
 $S_{uL} = 1160 \text{ MPa} \rightarrow S_m = 400 \text{ MPa (FIG. 5-9)} ; S_m' = (1.00)(0.8)(400)(0.81) = 259 \text{ MPa}$   
 $N = 259 / 254 = 1.02 \text{ LOW} \rightarrow \text{USE LARGER BAR AND/OR STRONGER MATERIAL.}$

42 REPEATED REVERSED SHEAR STRESS: CASE 4:  $C_s = 0.81 ; C_R = 0.81$

$T_{MAX} = \frac{T}{Z_p} = \frac{800 \times 10^3 \text{ N}\cdot\text{mm}}{\pi (30 \text{ mm})^3 / 16} = 32.6 \text{ MPa} ; SAE 1040 \text{ WQT 1000} ; S_{uL} = 780 \text{ MPa} ; S_m = 280 \text{ MPa}$   
 $S_{sm}' = (0.5)(0.81)(0.81)(280) = 91.9 \text{ MPa} ; S_{sm} = 0.5 S_m'$   
 $N = S_{sm}' / T_{MAX} = 91.9 / 32.6 = 2.82 \text{ OK}$

43 REPEATED - ONE DIRECTION SHEAR STRESS: FLUCTUATING SHEAR STRESS

CASE 5: EQ. 5-22:  $\frac{1}{N} = \frac{T_m}{S_{sm}'} + \frac{K_t T_a}{S_{sm}'}$   
 $Z_p = \pi D^3 / 16 = 24544 \text{ mm}^3$   
 $T_m = T_a = 400 \times 10^3 \text{ N}\cdot\text{mm} / 24544 \text{ mm}^3 = 16.30 \text{ MPa}$   
 $S_{sm} = 0.75 S_m = 0.75(780 \text{ MPa}) = 585 \text{ MPa}$   
 $\frac{1}{N} = \frac{16.30}{585} + \frac{(1.0)(16.30)}{91.9} = 0.205 ; N = 4.87$



( $S_{sm}'$  FROM PROB. 42)

44

DUCTILE MATERIAL - STATIC LOAD - CASE 2 :  $N = 0.5 S_y / T_{MAX}$

$$T_{MAX} = \frac{T}{Z_p} = \frac{88.0 \text{ LB-IN}}{\pi (0.40 \text{ IN})^3 / 6} = 7003 \text{ PSI}$$

LET  $N = 3.0$  DESIGN DECISION

$$\text{REQ'D } S_y = N T_{MAX} / 0.5 = 3(7003) / 0.5 = 42018 \text{ PSI}$$

ALUMINUM 2024-T4 HAS  $S_y = 47000 \text{ PSI}$

45

FLUCTUATING SHEAR STRESS: CASE 5

ASSUME  $C_s = 0.82$   
 $C_R = 0.81$   
 $N = 3$

$$T_{MAX} = \frac{63000 \text{ (HP)}}{m} = \frac{63000(1/10)}{560} = 12375 \text{ LB-IN.}$$

$$T_a = T_m = T_{MAX} / 2 = 6188 \text{ LB-IN} ; T_m = T_a = \frac{T}{2}$$

$$S_{SM} = 0.75 S_u = 0.75(208) = 156 \text{ KSI} ; S_u = 208 \text{ KSI} \rightarrow S_m = 64 \text{ KSI (FIG. 5-8)}$$

$$S_{SM}' = (0.50)(0.82)(0.81)(64) = 21.25 \text{ KSI} = 21250 \text{ PSI}$$

$$\frac{1}{N} = \frac{T_m}{Z_p S_{SM}} + \frac{T_a}{Z_p S_{SM}'} = \frac{1}{Z_p} \left[ \frac{T_m}{S_{SM}} + \frac{T_a}{S_{SM}'} \right] ; Z_p = N \left[ \frac{T_m}{S_{SM}} + \frac{T_a}{S_{SM}'} \right]$$

$$Z_p = 3 \left[ \frac{6188}{156000} + \frac{6188}{21250} \right] = 0.993 \text{ IN}^3 ; D = \sqrt[3]{6 Z_p / \pi} = 1.72 \text{ IN} \quad \text{USE } D = 1.75 \text{ IN.}$$

46

STEADY LOAD - CASE 3 :  $N = 0.5 S_y / T_{MAX}$

$$T = \frac{P}{m} = \frac{28 \times 10^3 \text{ N-mm/s}}{45 \text{ RAD/S}} = 622 \text{ N-mm} = 622 \times 10^3 \text{ N-mm}$$

$$Z_p = \frac{\pi (D^4 - d^4)}{16 D} = \frac{\pi (40^4 - 30^4)}{16(40)} \text{ mm}^3 = 8590 \text{ mm}^3$$

$$T_{MAX} = T / Z_p = 622 \times 10^3 \text{ N-mm} / 8590 \text{ mm}^3 = 72.4 \text{ MPa}$$

$$N = 0.5 S_y / T_{MAX} ; S_y = N T_{MAX} / 0.5 = 3(72.4) / 0.5 = 434 \text{ MPa}$$

AISI 1040 COLD DRAWN HAS  $S_y = 490 \text{ MPa}$ .

47

FLUCTUATING SHEAR STRESS - CASE 5: EQ. 5-22

(SEE PROBLEM 46.)

$$T_m = P_m / m = 21500 / 45 = 478 \text{ N-mm}$$

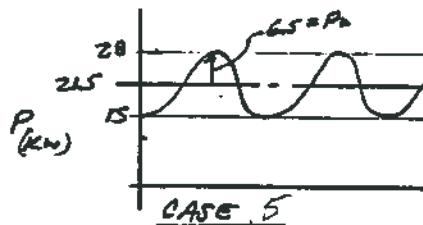
$$T_a = P_a / m = 6500 / 45 = 144 \text{ N-mm}$$

$$T_m = T_m / Z_p = 478000 / 8590 = 55.6 \text{ MPa}$$

$$T_a = T_a / Z_p = 144000 / 8590 = 16.8 \text{ MPa}$$

$$\frac{1}{N} = \frac{T_m}{S_{SM}} + \frac{K_t T_a}{S_{SM}'} = \frac{55.6}{S_{SM}} + \frac{(1.0)(16.8)}{S_{SM}'}$$

$C_s = 0.83$   
 $C_R = 0.81$



AFTER TRIALS:

AISI 1144 C.D.;  $S_y = 621 \text{ MPa}$ ;  $S_u = 690 \text{ MPa} \rightarrow S_m = 253 \text{ MPa (FIG. 5-8)}$

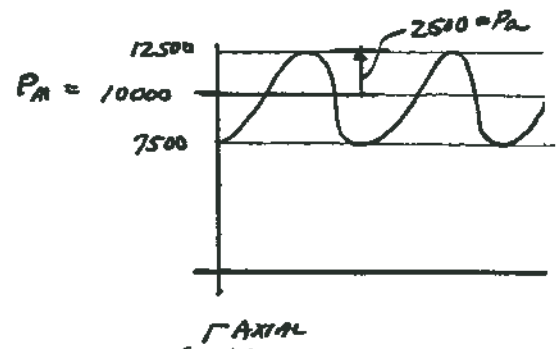
$$S_{SM} = 0.75 S_u = 0.75(690) = 518 \text{ MPa} ; S_{SM}' = 0.50 S_u' = (0.50)(0.83)(0.81)(690) = 85.0 \text{ MPa}$$

$$\frac{1}{N} = \frac{55.6}{518} + \frac{16.8}{85.0} = 0.305 ; N = 1 / 0.305 = 3.28 \quad \text{OK}$$

FLUCTUATING NORMAL STRESS: CASE 5: EQ. 5-20  
 PROBLEMS 48, 49, 50, 51.

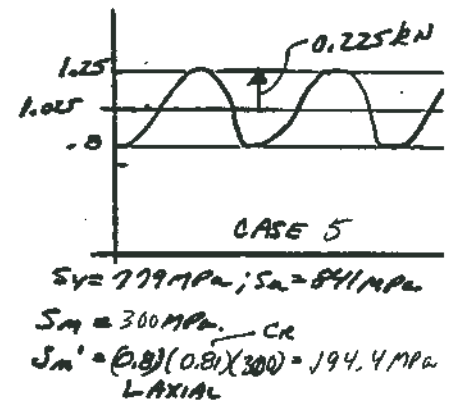
48

CASE 5:  $\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{k_t \sigma_a}{S_m'}$   
 $A = (w-d)t = (1.50 - 0.50)(0.50) = 0.50 \text{ in}^2$   
 $\sigma_m = \frac{10000}{.5} = 20000 \text{ psi}$   
 $\sigma_a = \frac{2500}{0.5} = 5000 \text{ psi}$   
 $d/w = .5/1.50 = 0.333 \rightarrow k_t = 2.31$   
 $S_y = 107 \text{ Kpsi}; S_u = 119 \text{ Kpsi} \rightarrow S_m = 43 \text{ Kpsi}; S_m' = (0.8)(0.81)(43) = 27.9 \text{ ksi}$   
 $\frac{1}{N} = \frac{20000}{118000} + \frac{2.31(5000)}{27900} = 0.583 \rightarrow N = 1.71 \text{ (Low)}$



49

$A = \pi(6)^2/4 = 28.27 \text{ mm}^2$   
 $\sigma_m = \frac{F_m}{A} = \frac{1025 \text{ N}}{28.27 \text{ mm}^2} = 36.25 \text{ MPa}$   
 $\sigma_a = \frac{F_a}{A} = \frac{225 \text{ N}}{28.27 \text{ mm}^2} = 7.96 \text{ MPa}$   
 $r/d = 0.5/6 = 0.083$   
 $D/d = 9/6 = 1.50$   
 $\frac{1}{N} = \frac{36.25}{841} + \frac{(2.03)(7.96)}{194.4} = 0.128; N = 7.92$   
 FOR YIELD:  $N = 8.68 \text{ OK}$   
 COULD USE WEATHER MATERIAL

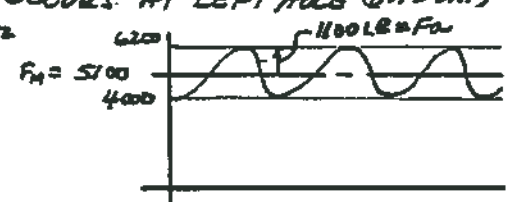


50

FROM PROB. 62, CHAPTER 3, MAX STRESS OCCURS AT BOTTOM  
 $k_t = 1.86; S_y = 86000 \text{ psi}; S_u = 121000 \text{ psi}; S_m = 43000 \text{ psi}$   
 $S_m' = (0.8)(81)(43000) = 27864 \text{ psi}; \sigma_m = \frac{F_m}{A} = \frac{600 \text{ LB}}{\pi(0.5)^2/4} = 3056 \text{ psi}$   
 $\sigma_a = \sigma_m$   
 CASE 5:  
 $\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{k_t \sigma_a}{S_m'} = \frac{3056}{121000} + \frac{(1.86)(3056)}{27864} = 0.224; N = 4.56 \text{ OK}$   
 FOR YIELD:  $N = 7.56 \text{ OK}$

51

FROM PROB. 63, CHAPTER 3, MAX STRESS OCCURS AT LEFT HOLE (0.72 DIA)  
 $k_t = 2.15; A = (1.40 - 0.72)(0.50) = 0.34 \text{ in}^2$   
 $\sigma_m = \frac{F_m}{A} = \frac{5100}{.34} = 15000 \text{ psi}$   
 $\sigma_a = \frac{F_a}{A} = \frac{1100}{.34} = 3235 \text{ psi}$   
 CASE 5  
 $\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{k_t \sigma_a}{S_m'} = \frac{15000}{145000} + \frac{2.15(3235)}{33050} = 0.314$   
 $N = 3.19$   
 FOR YIELD:  $N = \frac{S_y}{k_t(\sigma_a + \sigma_m)} = \frac{125000}{2.15(3235 + 15000)} = 3.19 \text{ O.K.}$



$S_y = 125 \text{ Kpsi}$   
 $S_u = 145 \text{ Kpsi}$   
 $S_m = 51 \text{ Kpsi}$   
 $S_m' = (0.8)(0.81)(51) = 33.05 \text{ Kpsi}$   
 CR CR

52 FROM PROB 3-64,  $\sigma_{max} = 16650 \text{ PSI}$  INCLUDING  $K_t$ ; CASE 1 :  $N = S_u / \sigma$   
 REQ'D  $S_{uc} = N \sigma = 3(18281) = 54843 \text{ PSI} \rightarrow$  USE GRADE 60 CAST IRON

53 CASE 5:  $\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t \sigma_a}{S_m'}$  : NOTE THAT A DIRECT SOLUTION IS NOT POSSIBLE BECAUSE BOTH  $S_u$  AND  $S_m'$  ARE UNKNOWN. ALSO DATA FOR ENDURANCE FOR TITANIUM ARE NOT DIRECTLY AVAILABLE HERE. AS AN ESTIMATE WE WILL USE FIG 5-8 AND THE DISCUSSION FOR STEEL TO OBTAIN  $S_m'$ . ALSO NOTE FROM PREVIOUS PROBLEMS,  $S_m' \approx S_u/4$ . THIS PERMITS SOLUTION FOR  $S_u$ . AFTER MATERIAL SELECTION, FINAL "N" CAN BE COMPUTED.

FROM PROBLEM 3-65,  $K_t = 2.30$

$$A = \pi(30)^2/4 = 707 \text{ mm}^2$$

$$\sigma_m = \frac{F_m}{A} = \frac{25.15 \times 10^3 \text{ N}}{707 \text{ mm}^2} = 35.57 \text{ MPa}$$

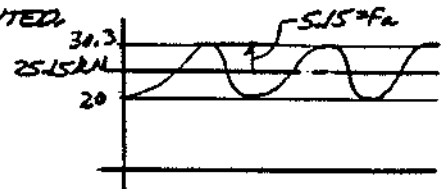
$$\sigma_a = \frac{F_a}{A} = \frac{5.15 \times 10^3 \text{ N}}{707 \text{ mm}^2} = 7.28 \text{ MPa}$$

$$\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t \sigma_a}{S_m'} = \frac{35.57}{S_u} + \frac{(2.30)(7.28)}{S_u/4} = \frac{102.5}{S_u}$$

THEN  $S_u = N(102.5) \approx 3(102.5) = 308 \text{ MPa}$ ; TRY Ti-50A;  $S_y = 276 \text{ MPa}$ ;  $S_u = 345 \text{ MPa}$   
 FROM FIG. 5-8:  $S_m \approx 120 \text{ MPa}$  (ESTIMATE)

$$S_m' = (0.8)(0.81)(120) = 84.2 \text{ MPa}$$

$$\frac{1}{N} = \frac{35.57}{345} + \frac{(2.30)(7.28)}{84.2} = 0.302 \rightarrow N = 3.31 \text{ OK Ti-50A}$$



54 FROM PROB. 3-66,  $K_t = 1.43$ ;  $T_m = 1100 \text{ LB}\cdot\text{IN} = T_a$   
 $Z_p = \pi(1.25)^3/16 = 0.383 \text{ in}^3$   
 $T_m = T_a/Z_p = 1100/0.383 = 2868 \text{ PSI} = T_a \rightarrow$  CASE 5  
 $\frac{1}{N} = \frac{T_m}{S_{sm}} + \frac{K_t T_a}{S_{sm}'}$  ASSUME  $S_{sm}' \approx S_{sm}/4$  [SEE PROB 53]

$$\frac{1}{N} = \frac{2868}{S_{sm}} + \frac{1.43(2868)}{S_{sm}/4} = \frac{19273}{S_{sm}} ; S_{sm} = 309273 \approx 57819 \text{ psi}$$

$$\text{BUT } S_{sm} = 0.75 S_u ; S_u = S_{sm}/0.75 = 57819/0.75 = 77092 \text{ PSI}$$

TRY AISI 1137 OQT 1300;  $S_y = 60 \text{ KSI}$ ;  $S_u = 87 \text{ KSI}$ ;  $S_m = 33 \text{ KSI}$

$$S_{sm}' = 0.50 S_m' ; S_{sm}' = (0.50)(0.81)(0.85)(33000) = 11360 \text{ psi}$$

$$S_{sm} = 0.75 S_u = 0.75(87000 \text{ PSI}) = 65250 \text{ PSI}$$

$$\frac{1}{N} = \frac{2868}{65250} + \frac{1.43(2868)}{11360} = 0.405 ; N = 2.47 \text{ LOW}$$

TRY SAE 1046 WQT 1000;  $S_u = 113 \text{ KSI}$ ,  $S_y = 88 \text{ KSI}$ ,  $S_m = 42 \text{ KSI}$

$$S_{sm} = 0.75(113) = 84.75 \text{ KSI}; S_{sm}' = (0.50)(0.81)(0.85)(42) = 14458 \text{ KSI}$$

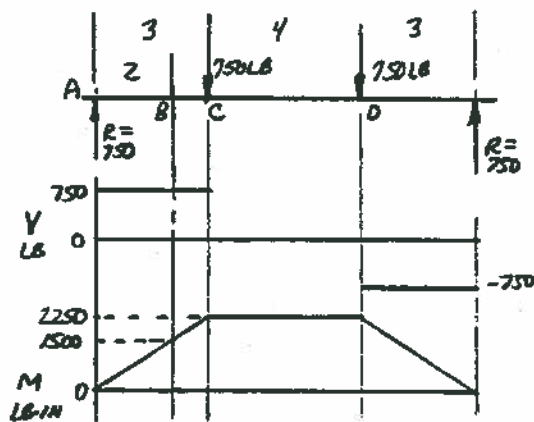
$$\frac{1}{N} = \frac{2868}{84750} + \frac{1.43(2868)}{14458} = 0.3175 ; N = 3.15 \text{ OK}$$

55. USE CASE 1 BECAUSE HIGHER STRENGTH DUCTILE IRONS ARE FAIRLY BRITTLE.  
 FROM PROB. 3-67,  $\sigma = 32,564$  PSI INCLUDING  $K_t$ .  $N = S_{ut}/\sigma$   
 REQ'D.  $S_{ut} = N\sigma = 3(32,564) = 97,692$  PSI  $\rightarrow$  USE GRADE 100-70-03.

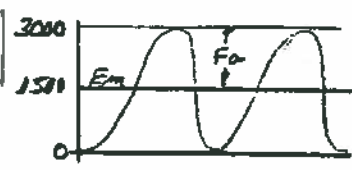
56. FROM PROB. 3-68,  $\sigma = 49,323$  PSI INCLUDING  $K_t$ : CASE 4:  $N = S_{ut}'/\sigma$   
 REQ'D  $S_{ut}' = N\sigma = 3(49,323) = 147,969$  PSI  
 REFERRING TO FIG. 5-8, THIS IS VERY HIGH. NO PRACTICAL MATERIAL  
RED-SIGN THE MEMBER.

57. LOAD IS REPEATED - ONE DIRECTION - TORSIONAL SHEAR STRESS;  
 FLUCTUATING SHEAR STRESS - CASE 5:  $T_m = T_a = 100 \text{ LB} \cdot \text{IN} / 2 = 50 \text{ LB} \cdot \text{IN}$ .  
 AT FILLET:  $r/d = 0.50/0.30 = 1.67$ ;  $\lambda/d = 0.025/0.30 = 0.083$ ;  $K_t = 1.43$   
 $Z_p = \pi d^3/16 = \pi(0.30)^3/16 = 0.00530 \text{ IN}^3$   
 $T_m = T_a = \frac{T}{Z_p} = \frac{50 \text{ LB} \cdot \text{IN}}{0.00530 \text{ IN}^3} = 9431 \text{ PSI}$   
 SAE 8740 Q&T 1000:  $S_y = 167 \text{ KSI}$ ;  $S_u = 175 \text{ KSI}$ ; THEN  $S_m = 60 \text{ KSI}$  (FIG. 5-8)  
 $S_{sm}' = (0.50)S_m = (0.50)(60) = 30 \text{ KSI}$  OR  $S_{sm}' = 0.75S_u = 0.75(175) = 131.3 \text{ KSI}$   
 $\frac{1}{N} = \frac{T_m}{S_{sm}} + \frac{K_t T_m}{S_{sm}'} = \frac{9431}{131300} + \frac{1.43(9431)}{24360} = 0.627$ ;  $N = 1.60$  LOW

58. STEADY LOAD - BRITTLE MAT'L.: CASE 1  
 AT MIDDLE - BETWEEN C AND D:  
 $Z = bh^2/6 = (0.75 \times 2.25)^2/6 = 0.633 \text{ IN}^3$   
 $\sigma = M/Z = 2250 \text{ LB} \cdot \text{IN} / 0.633 \text{ IN}^3 = 3556 \text{ PSI}$   
 AT STEP - POINT B:  
 $Z = bh^2/6 = (0.75 \times 1.25)^2/6 = 0.1953 \text{ IN}^3$   
 $M/h = 0.20/1.25 = 0.16$  }  $K_t = 1.63$   
 $M/h = 2.25/1.25 = 1.80$  }  
 $\sigma_{max} = \frac{K_t M}{Z} = \frac{1.63(1500)}{0.1953} = 12,519$   
 $N = S_{ut}/\sigma_{max} = 40,000/12,519 = 3.19$



59. REPEATED ONE DIRECTION: CASE 5: EQ. 5-20:  
 $D = 0.50 \text{ IN}$ :  $A = \pi D^2/4 = 0.196 \text{ IN}^2$ ; ASSUME  $K_t = 1.0$   
 $\sigma_m = \sigma_a = F/A = 1500 \text{ LB} / 0.196 \text{ IN}^2 = 7639 \text{ PSI}$



1ST TRIAL: SAE 1040 CD:  $S_y = 71 \text{ KSI}$ ;  $S_u = 80 \text{ KSI}$   
 $S_m = 30 \text{ KSI}$  (FIG. 5-8);  $S_m' = (C_R)(C_S)S_m = (0.81)(0.8)(30) = 19.4 \text{ KSI}$   
 $\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t \sigma_a}{S_m'} = \frac{7639}{80000} + \frac{1.0(7639)}{19400} = 0.489$ ;  $N = 2.04$  LOW  
 2ND: SAE 1040 W&T 1000:  $S_{ut} = 112 \text{ KSI}$ ;  $S_y = 87 \text{ KSI}$ ; 23% EL.;  $S_m = 42 \text{ KSI}$ ;  $S_m' = 27.2 \text{ KSI}$   
 $\frac{1}{N} = \frac{7639}{112000} + \frac{(1)(7639)}{27200} = 0.349$ ;  $N = 2.86$  OK  
 CHECK YIELD: EQ. 5-21  
 $N = \frac{S_y}{K_t(\sigma_m + \sigma_a)} = \frac{87000}{1(7639 + 7639)} = 5.69$  OK

60

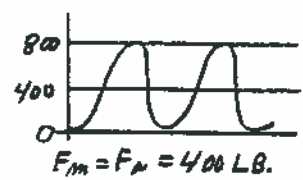
REPEATED - ONE DIRECTION; CASE 5; SPECIFY A STEEL

$F_{MAX} = 800 \text{ LB}$ ;  $F_{MIN} = 0$ ;  $\delta_{MAX} = 0.010 \text{ IN}$ ;  $L = 25.0 \text{ IN}$ .

CONSIDER DEFLECTION FIRST;

REQ'D.  $A = PL/ES = \frac{(800)(25.0)}{(30 \times 10^6)(0.010)} = 0.0667 \text{ IN}^2 = S^2$

$S = \sqrt{A} = \sqrt{0.0667 \text{ IN}^2} = 0.258 \text{ IN}$ ; TRY  $S = 0.300 \text{ IN}$ .



STRESS ANALYSIS & ASSUME  $K_t = 1.0$

$\sigma_m = \sigma_a = \frac{F}{A} = \frac{400 \text{ LB}}{(0.30 \text{ IN})^2} = 4444 \text{ PSI}$

TRY SAE 1040 CD:  $S_y = 71 \text{ KSI}$ ;  $S_u = 80 \text{ KSI}$ ; 12% ELONGATION

FROM FIG 5-8:  $S_m = 30 \text{ KSI}$ ; LET  $C_s = 1.0$ ;  $C_{ST} = 0.80$  (AXIAL);  $C_R = 0.81$

$S_m' = (1.0)(0.80)(0.81)(30) = 19,400 \text{ PSI}$

EQ. 5-20:  $\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t \sigma_a}{S_m'} = \frac{4444}{80000} + \frac{1.0(4444)}{19400} = 0.285$ ;  $N = 3.51 \text{ OK}$ .

61

REPEATED - ONE DIRECTION; CASE 5;  $F_{MAX} = 1200 \text{ LB}$ ;  $F_{MIN} = F_a = 600 \text{ LB}$ .

a)  $\frac{1}{N} = \frac{\sigma_m}{S_y} + \frac{K_t \sigma_a}{S_m'}$

FOR ILLUSTRATION USE SAME MATERIAL AS IN PROB. 60: SAE 1040 CD

$S_u = 80 \text{ KSI}$ ;  $S_m = 30 \text{ KSI}$

$S_m' = (0.90)(1.0)(0.81)(30) = 21.9 \text{ KSI}$

SIZE L BEAM

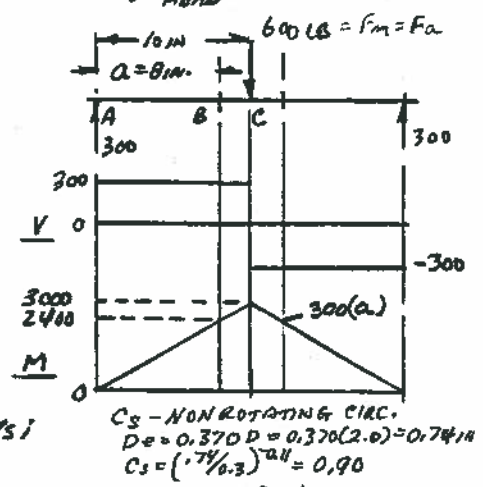
AT C:  $M = 3000 \text{ LB}\cdot\text{IN}$ ;  $K_t = 1.0$

$S = \pi D^3/32 = \pi(2.0)^3/32 = 0.785 \text{ IN}^3$

EQ. 5-20:  $\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t \sigma_a}{S_m'}$

$\sigma_m = \sigma_a = M/S = 3000 \text{ LB}\cdot\text{IN} / 0.785 \text{ IN}^3 = 3820 \text{ PSI}$

$\frac{1}{N} = \frac{3820}{80000} + \frac{1.0(3820)}{21900} = 0.222$ ;  $N = 4.50$  HIGHER THAN (b).



b) AT SECTION B:  $M = 2400 \text{ LB}\cdot\text{IN}$ ;  $M/D = 0.20/2.0 = 0.10$ ;  $D/d = 3.0/2.0 = 1.50$

$K_t = 1.74$ ;  $\sigma_m = \sigma_a = \frac{M}{S} = \frac{2400}{0.785} = 3056 \text{ PSI}$

$\frac{1}{N} = \frac{3056}{80000} + \frac{(1.74)(3056)}{21900} = 0.281$ ;  $N = 3.56$  - LOWER THAN (a).

62

REDESIGN BEAM IN PROB. 61. NOTE THAT  $N$  IS INVERSELY PROPORTIONAL TO MOMENT  $M$  AND DISTANCE  $a$ . THEN  $a$  MUST BE

REDUCED BY:  $a' = a \times 3.51/4.50 = 0.787(a) = 0.787(8.0) = 6.30 \text{ IN}$  (SAY 6.25 IN)

THEN  $M = 300(a) = 1875 \text{ LB}\cdot\text{IN}$ ;  $\sigma_m = \sigma_a = M/S = 1875/0.785 = 2389 \text{ PSI}$

$\frac{1}{N} = \frac{2389}{80000} + \frac{1.74(2389)}{21900} = 0.220$ ;  $N = 4.55 \text{ OK}$  HIGHER THAN (a).

SPECIFY  $a = 6.25 \text{ IN}$ .



63

REFER TO PROBS. 61, 62. ; NEW  $r = 0.40$ ;  $r/h = 0.40/2.0 = 0.20$

$d/d = 3.0/2.0 = 1.50$ ;  $K_t = 1.47$

IF  $d = 8.00$  IN AS GIVEN;  $M = 2400$  LB-IN AT B;  $\sigma_m = \sigma_r = 3056$  PSI

$\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t \sigma_r}{S_m'} = \frac{3056}{80000} + \frac{1.47(3056)}{21900} = 0.305$ ;  $N = 3.27$

DIMENSION  $d$  MUST BE REVISED TO GET  $N \geq 4.50$  AS AT C.

$d' = d \times 4.2/4.50 = 8.0(0.936) = 7.49$  IN; LET  $d = 7.25$  IN.

THEN  $M = (7.25)(300) = 2175$  LB-IN;  $\sigma = M/S = 2771$  PSI

$\frac{1}{N} = \frac{2771}{80000} + \frac{(1.47)(2771)}{21900} = 0.2206$ ;  $N = 4.53$  OK

64

REPEATED - ONE DIRECTION; FLUCTUATING STRESS: CASE S, EQ 5-20.

SAE 1040 HR:  $S_y = 42$  KSI;  $S_u = 72$  KSI;  $S_m = 23$  KSI FIG 5-8 HOT ROLLED.

$C_s = 1.0$  DIRECT TENSION;  $C_a = 0.81$ ;  $C_{sc} = 0.80$  AXIAL LOAD

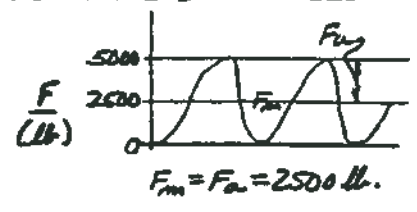
$S_m' = (0.8)(0.81)(23 \text{ KSI}) = 14.9$  KSI

a) AT PIN HOLE:  $d = 0.25$  IN. DIA.,  $w = 1.00$  IN.

$r/w = 0.25$ ;  $K_t = 4.40$

$\sigma_{nom} = \frac{F}{(w-d)t} = \frac{2500 \text{ LB}}{(1.0 - 0.25)(0.25) \text{ IN}^2} = 13333$  PSI

$\frac{1}{N} = \frac{\sigma_m}{S_u} + \frac{K_t \sigma_r}{S_m'} = \frac{13333}{72000} + \frac{(4.40)(13333)}{14900} = 4.12$ ;  $N = 0.243$



INDICATES FAILURE.

b) AT FILLETS:  $r/h = 0.02/1.00 = 0.02$ ;  $r/h = 2.0/1.0 = 2.0$

ON EPATIGUE: MIN.  $r/h = 0.04$  THEN  $K_t = 3.68$

$\sigma_{nom} = \frac{F}{A} = \frac{2500 \text{ LB}}{(1.00)(0.25) \text{ IN}^2} = 10000$  PSI

$\frac{1}{N} = \frac{10000}{72000} + \frac{(3.68)(10000)}{14900} = 2.61$ ;  $N = 0.383$  FAILURE.

65

IMPROVEMENTS: 1.) INCREASE THICKNESS, 2.) INCREASE FILLET RADIUS, 3.) USE STRONGER MATERIAL, 4.) INCREASE PIN HOLE SIZE - OR - CHANGE MANNER OF APPLYING FORCE TO THE PART TO ELIMINATE HOLE - OR - MAKE PART THICKER AT THE HOLES THAN IN MIDDLE OF THE PART. MATERIAL MAY BE REMOVED IN 2.00 IN. SECTION NEAR MIDDLE OF PART TO OFFSET ADDED MATERIAL ELSEWHERE. COULD TRY TITANIUM WITH LOWER DENSITY THAN STEEL. THIS PROBLEM MAY BE TOO RESTRICTIVE TO PERMIT A PRACTICAL SOLUTION WITH DATA IN THIS BOOK. MAY HAVE TO ACCEPT LOWER  $N < 3.0$  OR SOME INCREASE IN WEIGHT OF THE COMPONENT.



66

**FLUCTUATING NORMAL STRESS - CASE 5 - EQ. 5-20.**

SAE 1040 CD:  $S_y = 71 \text{ ksi}$ ;  $S_u = 80 \text{ ksi}$ ;  $S_m = 30 \text{ ksi}$  FIG. 5-8:  $C_R = 0.81$

$C_S = 1.0$ ,  $C_{ST} = 0.80$  AXIAL:  $S_m' = 0.8(0.81)(30) = 19.400 \text{ psi}$

$F_m = (24.8 + 3.0)/2 = 13.9 \text{ kN} \left( \frac{1.0 \text{ lb}}{4.448 \text{ N}} \right) = 3125 \text{ LB}$

$F_a = 24.8 - 13.9 = 10.9 \text{ kN} \left( \frac{1.0 \text{ lb}}{4.448 \text{ N}} \right) = 2450 \text{ LB}$

$t = 0.375 \text{ in.}$ ;  $w = 1.50 \text{ in.}$ ;  $d = 0.625 \text{ in.}$ ;  $d/w = 0.417$

AT PIN HOLES:  $K_t = 2.84$

$$\sigma_m = \frac{F_m}{(w-d)t} = \frac{3125}{(1.50-0.625)(0.375)} = 9524 \text{ psi}$$

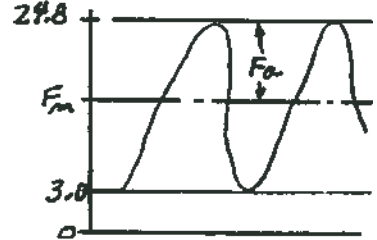
$$\sigma_a = \frac{F_a}{(w-d)t} = \frac{2450}{(1.50-0.625)(0.375)} = 7467 \text{ psi}$$

$$\frac{1}{N} = \frac{\sigma_m}{S_m} + \frac{K_t \sigma_a}{S_m'} = \frac{9524}{80000} + \frac{(2.84)(7467)}{19400} = 1.22; N = 0.825 \text{ FAILURE.}$$

AT MIDDLE HOLE:  $K_t = 2.22$ ; SAME NOMINAL STRESSES.

$$\frac{1}{N} = \frac{9524}{80000} + \frac{(2.22)(7467)}{19400} = 0.854; N = 1.17 \text{ LOW.}$$

PART MUST BE REDESIGNED.



67

**REPEATED REVERSED LOAD - BENDING - CASE 4:  $N = S_m' / \sigma_{MAX}$**

SAE 1340 OQT 1300:  $S_y = 517 \text{ MPa}$ ;  $S_u = 690 \text{ MPa}$ ;  $S_m = 260 \text{ MPa}$ ;  $C_R = 0.81$

$C_S = 0.98$ ,  $C_{ST} = 1.0$ ,  $C_m = 1.0$ :  $S_m' = (0.98)(0.81)(260) = 206 \text{ MPa}$

$R_F = 400 \text{ N} (250/400) = 250 \text{ N}$  |  $C_S$  AT B:  $D_e = 0.808(12) = 9.70 \text{ mm}$

$R_A = 400 \text{ N} (150/400) = 150 \text{ N}$  |  $C_S = 0.98$

AT B:  $S = S^3/6 = 12^3/6 = 288 \text{ mm}^3$

$M/h = 2.0/12.0 = 0.167$ ;  $M/h = 2.0/12 = 0.167$

$K_t = 1.60$

$$\sigma_{MAX} = \frac{K_t M}{S} = \frac{(1.60)(22500)}{288} = 125 \text{ MPa}$$

$$N = \frac{S_m'}{\sigma_{MAX}} = \frac{206}{125} = 1.648$$

AT C:  $S = bh^2/6 = 12(20)^2/6 = 800 \text{ mm}^3$

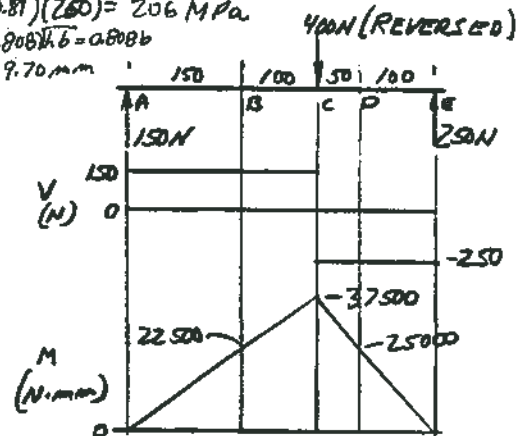
$$\sigma_{MAX} = \frac{37500(1.0)}{800} = 46.9 \text{ MPa} (< \text{AT B.})$$

AT D: FROM REFERENCE:  $d/w = 14/20 = 0.70$  -  $K_t = 1.40$

$$\sigma_{MIN} = \frac{6Mw}{(w^2-d^2)(t)} = \frac{6(25000)(20)}{(20^2-14^2)(12)} = 47.6 \text{ MPa}; \sigma_{MAX} = K_t \sigma_{MIN}$$

$$\sigma_{MAX} = K_t \sigma_{MIN} = (1.40)(47.6) = 66.6 \text{ MPa} (< \text{AT B.})$$

MINIMUM N = 1.648 AT B (Low)



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SEE PROB 67: FOR  $N = 2.5$ ;  $S_m' = N \sigma_{MAX} = 2.5(125) = 312.5 \text{ MPa} = (0.98)(0.81) S_m$

THEN SURREQ'D = 2.1150 MPa

(FIG 5-8)

$S_m = 394 \text{ MPa}$

FROM APPA 4-3: SAE 1340 OQT 900 HAS  $S_m = 1150 \text{ MPa}$

69. FLUCTUATING NORMAL STRESS - CASE 5:  $F_{min} = 300 \text{ LB}$ ;  $F_{max} = 700 \text{ LB}$

$$F_m = (300 + 700)/2 = 500 \text{ LB}; F_a = 700 - 500 = 200 \text{ LB}$$

$$\text{AT FILLET: } r/d = 0.06/1.25 = 0.048; D/d = 2.0/1.25 = 1.60$$

$$K_t = 2.35$$

$$S = \pi d^3/32 = \pi (1.25)^3/32 = 0.192 \text{ IN}^3$$

$$\sigma_m = \frac{M_m}{S} = \frac{4F_m}{S} = \frac{4(500) \text{ LB}\cdot\text{IN}}{0.192 \text{ IN}^3} = 10430 \text{ PSI}$$

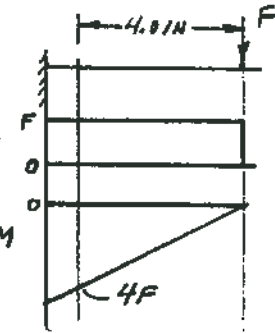
$$\sigma_a = \frac{M_a}{S} = \frac{4F_a}{S} = \frac{4(200) \text{ LB}\cdot\text{IN}}{0.192 \text{ IN}^3} = 4172 \text{ PSI}$$

$$\text{SAE 1050 HR: } S_y = 49 \text{ KSI}; S_u = 90 \text{ KSI}$$

$$S_m = 26 \text{ KSI (FIG 5-8 HR CURVE)}; S_m' = (0.95)(0.81)(26) = 20.0 \text{ KSI}$$

$$\frac{1}{N} = \frac{\sigma_m}{S} + \frac{K_t \sigma_a}{S_m'} = \frac{10430}{90000} + \frac{(2.35)(4172)}{20000} = 0.606; N = 1.65 \text{ LOW}$$

$C_s$  - NON-ROTATING CIRC. SHAFT  
 $D_e = .37D = 0.463 \text{ IN}$   
 $C_s = 0.95$



70. SEE PROB. 69: INCREASE  $N$  TO 3.0 OR HIGHER BY USING LARGER  $r$ . BEST POSSIBLE IMPROVEMENT WOULD BE  $K_t = 1.0$  WITH LARGE  $r$ .

$$\frac{1}{N} = \frac{\sigma_m}{S} + \frac{K_t \sigma_a}{S_m'} = \frac{10430}{90000} + \frac{(1.0)(4172)}{20000} = 0.324; N = 3.08 \text{ OK}$$

CONSIDER A RADIAL TAPER.

71. SEE PROB. 69: INCREASE MATERIAL STRENGTH TO GET  $N \geq 3.0$ .

TRY SAE 1340 OQT 900:  $S_y = 158 \text{ KSI}; S_u = 169 \text{ KSI}$  (APP 4-3) 17% ELONG.

USE MACHINED SURFACE:  $S_m = 58 \text{ KSI}; S_m' = (0.95)(0.81)(58) = 44.63 \text{ KSI}$

$$\frac{1}{N} = \frac{\sigma_m}{S} + \frac{K_t \sigma_a}{S_m'} = \frac{10430}{169000} + \frac{(2.35)(4172)}{44630} = 0.2814; N = 3.55 \text{ OK}$$

72. REPEATED REVERSED STRESS: CASE 4: DESIGN FOR  $N \geq 3.0$ .

SPECIFY MATERIAL:  $\sigma_B = S_m'/N$ .

$\sigma_{max, AT B}: M_B = 600 \text{ LB}\cdot\text{IN}$

$$S = \pi d^3/32 = \pi (1.00)^3/32 = 0.0982 \text{ IN}^3$$

$$\text{AT FILLET: } r/d = 0.06/1.0 = 0.06; D/d = 1.38/1.0 = 1.38$$

$$K_t = 1.99$$

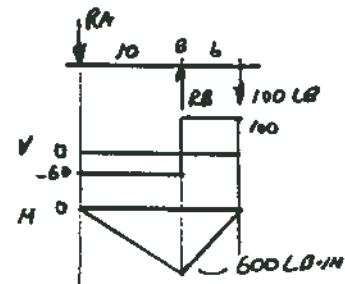
$$\text{LET } \sigma_B = \sigma_{max} = \frac{K_t M}{S} = \frac{1.99(600) \text{ LB}\cdot\text{IN}}{0.0982 \text{ IN}^3} = 12159 \text{ PSI}$$

$$\text{REQ'D } S_m' = N \sigma_B = (3.0)(12159) = 36477 \text{ PSI}$$

$$C_e = 0.8; C_s = 0.88; S_m = S_m' / (0.88)(0.81) = 51174 \text{ PSI}$$

FROM FIG. 5-8, REQ'D  $S_m = 145,000 \text{ PSI}$  (MACHINED SURFACE)

ONE POSSIBLE SOLUTION: SAE 3140 OQT 1000,  $S_m = 152,000 \text{ PSI}$ , 17% ELONG.



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## FLUCTUATING NORMAL STRESS - CASE 5:

SAE 1340 OQT 700;  $S_y = 197 \text{ ksi}$ ;  $S_m = 221 \text{ ksi}$ 

$$S_m' = (0.80)(0.81)(65 \text{ ksi}) = 42.1 \text{ ksi}$$

AXIAL    SUR    LFIG 5-8

$$F_m = (8500 + 16000)/2 = 12250 \text{ LB}$$

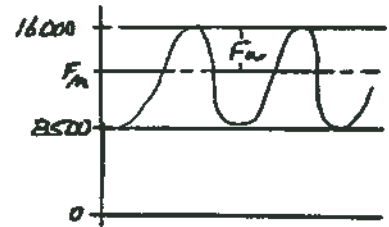
$$F_a = 16000 - 12250 = 3750 \text{ LB}$$

$$\text{AT FILLET: } r/d = 0.05/0.63 = 0.079; \quad d/D = 1.00/0.63 = 1.57; \quad K_t = 2.10$$

$$A = \pi d^2/4 = \pi (0.63)^2/4 = 0.312 \text{ IN}^2$$

$$\sigma_m = F_m/A = \frac{12250 \text{ LB}}{0.312 \text{ IN}^2} = 39300 \text{ psi}; \quad \sigma_a = F_a/A = \frac{3750 \text{ LB}}{0.312 \text{ IN}^2} = 12030 \text{ psi}$$

$$\frac{1}{N} = \frac{\sigma_m}{S_m} + \frac{K_t \sigma_a}{S_m'} = \frac{39300}{221000} + \frac{2.10(12030)}{42100} = 0.1778; \quad N = 1.29 \text{ LOW}$$



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SEE PROB. 73: TRY TO REDESIGN TO ACHIEVE  $N \geq 3.0$ .INCREASE FILLET RADIUS TO  $r = 0.185 \text{ IN}$ . FILLET WOULD THEN JUST BLEND WITH OUTSIDE OF 1.00 IN DIA.

$$r/d = \frac{0.185}{0.630} = 0.29; \quad d/D = \frac{1.00}{0.63} = 1.57; \quad K_t = 1.36$$

USE STRONGER MATERIAL: TRY SAE 8650 OQT 700

$$S_u = 240 \text{ ksi}; \quad S_y = 222 \text{ ksi}; \quad 12\% \text{ ELONGATION}$$

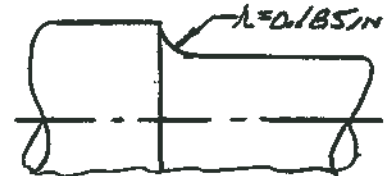
GRIND ALL CRITICAL SURFACES GENTLY.  $S_m = 88 \text{ ksi}$ ; USE  $C_r = 0.81$ 

$$\text{THEN } S_m' = (0.81)(0.8)(88 \text{ ksi}) = 57.0 \text{ ksi}$$

$$\frac{1}{N} = \frac{39300 + (1.36)(12030)}{57000} = 0.451; \quad N = 2.22 \text{ STILL TOO LOW}$$

EVEN IF  $K_t = 1.0$ ,  $N = 2.67$  - SLIGHTLY LOW

DIAMETERS MAY HAVE TO BE INCREASED.



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## REPEATED REVERSED LOAD - CASE 4:

BENDING MOMENT AT FILLETS =  $F(4.0 \text{ IN}) = (800)(4.0) = 3200 \text{ LB}\cdot\text{IN}$ .

$$S = \frac{M}{I} = \frac{(2.00)(1.25)^2}{6} = 0.5208 \text{ IN}^3; \quad \sigma_{nom} = \frac{M}{S} = \frac{3200 \text{ LB}\cdot\text{IN}}{0.5208 \text{ IN}^3} = 6144 \text{ psi}$$

$$N = \frac{S_m'}{\sigma_{max}} = \frac{S_m'}{K_t \sigma_{nom}}; \quad \text{THEN REQ'D } K_t = \frac{S_m'}{N \sigma_{nom}}$$

 $C_s$  FOR RECTANGLE  $1.25 \times 2.00 \text{ IN}$ .  $D_e = 0.808 \sqrt{hb} = 0.808 \sqrt{(1.25)(2.00)} = 1.28$ 

$$C_s = (1.28/0.3)^{-0.11} = 0.85$$

AISI 1144 OQT 1100;  $S_u = 112 \text{ ksi}$ ,  $S_m = 42 \text{ ksi}$ . USE  $C_r = 0.81$ 

$$S_m' = C_s C_r S_m = (0.85)(0.81)(42) = 28.9 \text{ ksi} = 28900 \text{ psi}$$

$$\text{THEN } K_{t,max} = \frac{S_m'}{N \sigma_{nom}} = \frac{28900 \text{ psi}}{(3)(6144) \text{ psi}} = 1.57$$

FROM FATIGUE:  $h/h_0 = 2.00/1.25 = 1.6$ ; FOR  $K_t = 1.57$ THEN  $r = 0.220 \text{ IN}$  GIVES  $K_t = 1.57$

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DESIGN SECTION AT 'B' FOR  $N \geq 3.0$ .

$$\sum M_{PIN} = 0 = 800(4.75) - R_1(2.25)$$

$$R_1 = 800(4.75)/2.25 = 1689 \text{ LB}$$

MULTIPLE SOLUTIONS POSSIBLE:

LET WIDTH = 2.00 IN; SAME AS OTHER SECTIONS

DESIGN FOR  $K_t \approx 1.50$ 

$$\text{CASE 4: } N = \frac{S_m' \sigma_{MAX}}{k_t \sigma_{NOM}} = \frac{S_m'}{k_t \sigma_{NOM}}$$

$$\text{BUT } \sigma_{MAX} = \frac{M}{S} = \frac{M}{bh^2/6} = \frac{6M}{bh^2}$$

$$\text{THEN } N = \frac{S_m' bh^2}{k_t (6M)}$$

$$t = \sqrt{\frac{N k_t (6M)}{S_m' b}} = \left[ \frac{3.0 \times 1.50 \times 6 \times 2533}{(28900)(2.00)} \right]^{1/2} = 0.444 \text{ IN. MIN.}$$

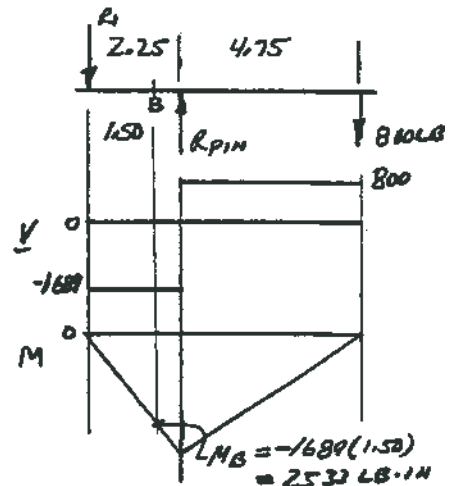
$$\text{LET } t = 0.85 \text{ IN ; } W = 2.00$$

$$\text{: FOR } K_t = 1.50, N/t = 0.3 \text{ EST.}$$

$$\text{THEN } r_L = 0.30 t = 0.30(0.85) = 0.255 \text{ IN 1ST. ESTIMATE.}$$

BY TRIAL

$$\text{FOR } r_L = 0.19, K_t = 1.50$$



PROBLEMS 77-83 ARE DESIGN PROBLEMS FOR WHICH THERE ARE MULTIPLE SOLUTIONS POSSIBLE.

## CHAPTER 6 COLUMNS

1.  $r = D/4 = 0.75/4 = 0.188 \text{ in} ; KL/r = 1.0(32)/0.188 = 171$   
 $S_y = 42,000 \text{ psi} ; C_c = \sqrt{\frac{2\pi^2 E}{S_y}} = \sqrt{\frac{2\pi^2(30 \times 10^6)}{42,000}} = 119 \rightarrow \text{LONG COLUMN}$   
 EULER  
 $A = \pi D^2/4 = 0.442 \text{ in}^2$   
 $P_{cr} = \frac{\pi^2 EA}{(KL/r)^2} = \frac{\pi^2(30 \times 10^6)(0.442)}{(171)^2} = 4473 \text{ LB}$
2.  $KL/r = 1.0(15)/0.188 = 79.8 < C_c \rightarrow \text{SHORT-JOHNSON FORMULA}$   
 $P_{cr} = AS_y \left[ 1 - \frac{S_y (KL/r)^2}{4\pi^2 E} \right] = (0.442)(42,000) \left[ 1 - \frac{42,000(79.8)^2}{4\pi^2(30 \times 10^6)} \right] = 14,373 \text{ LB}$
3.  $r = 0.188 \text{ in} ; KL/r = 171 ; S_y = 21,000 \text{ psi} ; E = 10 \times 10^6 \text{ psi}$   
 $C_c = \sqrt{\frac{2\pi^2(10 \times 10^6)}{21,000}} = 97 \rightarrow \text{LONG COLUMN}$   
 $P_{cr} = \frac{\pi^2 EA}{(KL/r)^2} = \frac{\pi^2(10 \times 10^6)(0.442)}{(171)^2} = 1492 \text{ LB}$
4.  $KL/r = (0.65)(32)/0.188 = 111 < C_c \rightarrow \text{SHORT-JOHNSON FORMULA}$   
 $P_{cr} = (0.442)(42,000) \left[ 1 - \frac{42,000(111)^2}{4\pi^2(30 \times 10^6)} \right] = 10,500 \text{ LB}$
5. SQUARE:  $r = S/\sqrt{12} = 0.65/\sqrt{12} = 0.188 \text{ in}$  SAME AS ROUND-PRB. 1.  
 $P_{cr} = 4473 \text{ LB.}$
6. ACRYLIC: LET  $S_y = \text{TENSILE STRENGTH} = 5400 \text{ psi} ; E = 220,000 \text{ psi}$   
 $C_c = \sqrt{\frac{2\pi^2 E}{S_y}} = \sqrt{\frac{2\pi^2(220,000)}{5400}} = 28.4 ; KL/r = 171 > C_c - \text{LONG}$   
 $P_{cr} = \frac{\pi^2 EA}{(KL/r)^2} = \frac{\pi^2(220,000)(0.442)}{(171)^2} = 32.8 \text{ LB}$

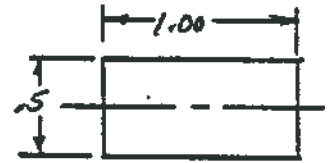
7.

$$r = 0.5 \text{ in} / \sqrt{12} = 0.144 \text{ in.}$$

$$KL/r = 1.0(8.5) / 0.144 = 58.9$$

$$S_y = 181,000 \text{ psi} \rightarrow C_c = 57 \text{ (FIG. 6-4) LONG COL.}$$

$$P_{CR} = \frac{\pi^2 EA}{(KL/r)^2} = \frac{\pi^2 (30 \times 10^6)(0.5)}{(58.9)^2} = \underline{42,675 \text{ LB}}$$



8.

$$r = \sqrt{(D^2 + d^2)/4} = \sqrt{(0.60^2 + (0.382)^2)/4} = 0.529 \text{ in.}; L = 6.25 \text{ ft} \left(\frac{12 \text{ in}}{\text{ft}}\right) = 75 \text{ in}$$

$$A = \pi(D^2 - d^2)/4 = 0.5106 \text{ in}^2; L/r = 75/0.529 = 142$$

$$S_y = 30,000 \text{ psi}; C_c = 140 \text{ (FIG. 6-5)}$$

a) PINNED ENDS:  $KL/r = 1.0(L/r) = 142 > C_c$  LONG-EULER

$$P_{CR} = \frac{\pi^2 (30 \times 10^6)(0.5106)}{(142)^2} = \underline{7498 \text{ LB}}$$

b) FIXED-FIXED:  $KL/r = 0.65(L/r) = 92.3 < C_c$  - SHORT-JOHNSON

$$P_{CR} = (0.5106)(30,000) \left[ 1 - \frac{30,000(92.3)^2}{4\pi^2(30 \times 10^6)} \right] = \underline{12,000 \text{ LB}}$$

c) FIXED-PINNED:  $KL/r = 0.8(L/r) = 114 < C_c$  - SHORT-JOHNSON

$$P_{CR} = (0.5106)(30,000) \left[ 1 - \frac{30,000(114)^2}{4\pi^2(30 \times 10^6)} \right] = \underline{10,300 \text{ LB}}$$

d) FIXED/FREE:  $KL/r = 2.10(L/r) = 298 > C_c$  - LONG

$$P_{CR} = \frac{\pi^2 (30 \times 10^6)(0.5106)}{(298)^2} = \underline{1,700 \text{ LB}}$$

9.

$$S_y = 152 \text{ ksi}; C_c \approx 60 \text{ (FIG. 6-5)}; \text{ ASSUME COLUMN IS LONG}; K=1.0$$

$$\text{(EQ. 6-9)} \quad D = \left[ \frac{64NP(KL)}{\pi^3 E} \right]^{1/4} = \left[ \frac{64(3)(8500)(50)}{\pi^3 (30 \times 10^6)} \right]^{1/4} = [4.39]^{1/4} = 1.45 \text{ in.}$$

$$r = D/4 = 1.50/4 = 0.375; KL/r = (1.0)(50)/0.375 = 133 > C_c \text{ LONG, O.K.}$$

10.

$$S_y = 30 \text{ ksi}; C_c = 140 \text{ (FIG. 6-5)}; \text{ ASSUME COLUMN IS LONG}$$

$$D = 1.45 \text{ in (SAME AS PROB. 9)} \text{ USE } D = 1.50; KL/r = 133 < C_c \text{ - JOHNSON}$$

$$\text{(EQ. 6-10)} \quad D = \left[ \frac{4(3)(8500)}{\pi(30,000)} + \frac{4(30,000)(50)^2}{\pi^2(30 \times 10^6)} \right]^{1/2} = 1.45 \text{ in USE } 1.50 \text{ in.}$$

NO ADVANTAGE TO 4140 STEEL

11.

$$\text{ALUM. 2014-T4}; S_y = 42,000 \text{ psi}; C_c \approx 69; \text{ ASSUME LONG COLUMN}$$

$$D = \left[ \frac{64(3)(8500)(50)}{\pi^3 (10 \times 10^6)} \right]^{1/4} = 1.90 \text{ in. - USED } 2.00 \text{ in.}, r = D/4 = 0.50 \text{ in.}$$

$$KL/r = (1.0)(50)/0.50 = 100 > C_c \text{ LONG. D O.K.}$$

12.

$$\text{SQUARE}; I = S^4/12; A = S^2; \text{ FROM EQ. 6-8 - EULER}$$

$$I = S^4/12 = NP(KL)^2 / \pi^2 E$$

$$S = \left[ \frac{12NP(KL)^2}{\pi^2 E} \right]^{1/4}$$

(CONTINUE NEXT PAGE)

12. (CONTINUED) JOHNSON:  $\lambda^2 = S^2/l^2$

$$\text{EQ. 6-7: } P_{CR} = NP = AS_y \left[ 1 - \frac{S_y (KL)^2}{4\pi^2 E I^2} \right] = S^2 S_y \left[ 1 - \frac{S_y (KL)^2}{4\pi^2 E (S^2/l^2)} \right]$$

$$NP = S^2 S_y - \frac{S^2 S_y^2 (KL)^2 l^2}{4\pi^2 E S^2} = S^2 S_y - \frac{3S_y^2 (KL)^2}{\pi^2 E} \quad \text{SOLVE FOR } S$$

$$S^2 S_y = NP + \frac{3S_y^2 (KL)^2}{\pi^2 E} \quad \text{DIVIDE BY } S_y \text{ AND TAKE } \sqrt{\quad}$$

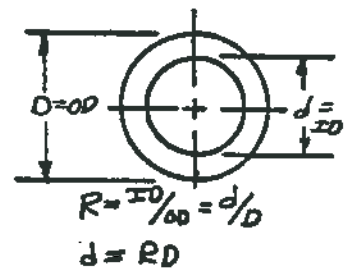
$$S = \left[ \frac{NP}{S_y} + \frac{3S_y (KL)^2}{\pi^2 E} \right]^{1/2}$$

13. EULER: EQ. (6-8)  $I = NP(KL)^2 / \pi^2 E$

$$I = \frac{\pi(D^4 - d^4)}{64} = \frac{\pi(D^4 - R^4 D^4)}{64} = \frac{\pi D^4 (1 - R^4)}{64}$$

$$\frac{\pi D^4 (1 - R^4)}{64} = \frac{NP(KL)^2}{\pi^2 E}$$

$$D = \left[ \frac{64 NP (KL)^2}{\pi^3 E (1 - R^4)} \right]^{1/4}$$



JOHNSON: EQ. (6-7)

$$P_{CR} = NP = AS_y \left[ 1 - \frac{S_y (KL/l)^2}{4\pi^2 E} \right]$$

$$\frac{NP}{S_y} = A - \frac{AS_y (KL)^2}{4\pi^2 E I^2}$$

$$= \frac{\pi D^4 (1 - R^4)}{4} - \frac{\pi D^4 (1 - R^4) S_y (KL)^2}{4\pi^2 E D^4 (1 + R^2) / 16}$$

$$\frac{NP}{S_y} = \frac{\pi D^4 (1 - R^4)}{4} - \frac{(1 - R^4) S_y (KL)^2}{\pi E (1 + R^2)}$$

$$\frac{\pi D^4 (1 - R^4)}{4} = \frac{NP}{S_y} + \frac{(1 - R^4) S_y (KL)^2}{\pi E (1 + R^2)}$$

$$D = \left[ \frac{4NP}{\pi S_y (1 - R^4)} + \frac{4(1 - R^4) S_y (KL)^2}{\pi^2 E (1 + R^2) (1 - R^4)} \right]^{1/2}$$

$$D = \left[ \frac{4NP}{\pi S_y (1 - R^4)} + \frac{4 S_y (KL)^2}{\pi^2 E (1 + R^2)} \right]^{1/2}$$

$$A = \frac{\pi(D^2 - d^2)}{4} = \frac{\pi(D^2 - R^2 D^2)}{4}$$

$$A = \pi D^2 (1 - R^2) / 4$$

$$I^2 = \frac{D^2 d^2}{16} = \frac{D^2 R^2 D^2}{16}$$

$$I^2 = \frac{D^2 (1 + R^2)}{16}$$

14. ASSUME COLUMN IS LONG: FROM PROB. 12:  $S = \left[ \frac{12NP(KL)^2}{\pi^2 E} \right]^{1/4}$   
 $KL = 0.65(64) = 41.6 \text{ IN (FIXED ENDS)}$   
 $S = \left[ \frac{12(3)(6500)(41.6)^2}{\pi^2 (11 \times 10^6)} \right]^{1/4} = 1.423 \text{ IN} \text{ --- USE } S = 1.500 \text{ IN}$   
 CHECK:  $\lambda = S/\sqrt{I} = 1.50/\sqrt{I} = 0.433$ ;  $KL/\lambda = 41.6/0.433 = 96.1$   
 FOR 6061-T6,  $S_y = 40 \text{ KSI}$ ; FROM FIG (6-6)  $C_c = 70$  LONG OK

15.  $R = ID/OD = 0.8$ ;  $(1-R^4) = 0.5904$   
 ASSUME LONG; FROM PROB. 13  
 $D = \left[ \frac{64NP(KL)^2}{4\pi^2 E (1-R^4)} \right]^{1/4} = \left[ \frac{64(3)(6500)(41.6)^2}{4\pi^2 (10 \times 10^6)(0.5904)} \right]^{1/4} = 1.31 \text{ IN}$   
 USE  $D = 1.50 \text{ IN}$ ;  $d = 0.8D = 1.20 \text{ IN}$ ;  $r = \sqrt{\frac{D^2 - d^2}{4}} = 0.48 \text{ IN}$ ;  $KL/r = 87$  LONG  
 WEIGHT COMPARISON: WT. PROPORTIONAL TO AREA  
 SQUARE:  $A = S^2 = (1.50)^2 = 2.25 \text{ IN}^2$   
 TUBE:  $A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(1.50^2 - 1.20^2) = 0.636 \text{ IN}^2$   
 $W_S/W_T = 2.25/0.636 = 3.54$  --- TUBE MUCH MORE EFFICIENT

16. ASSUME COLUMN IS LONG: EQ. (6-9)

$$D = \left[ \frac{64NP(KL)^2}{\pi^2 E} \right]^{1/4}$$

$$D = \left[ \frac{64(3.5)(5176)(60)^2}{\pi^2 (30 \times 10^6)} \right]^{1/4} = 1.46 \text{ IN}$$

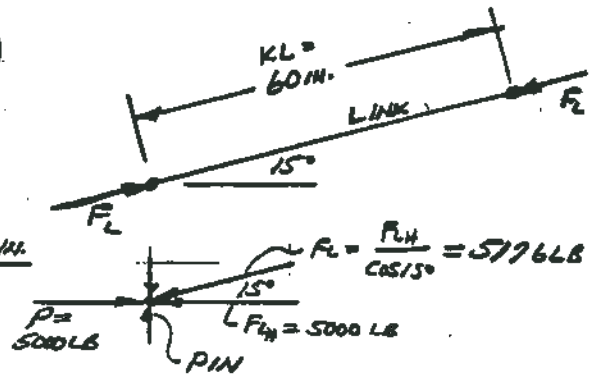
USE  $D = 1.50 \text{ IN}$

CHECK  $\lambda = D/4 = 0.375$

$KL/\lambda = 60/0.375 = 160$  LONG OK

$C_c \approx 60$  FIG. (6-5);  $S_y = 157 \text{ KSI}$

NOTE: CARE WOULD HAVE TO BE USED AT CONNECTIONS TO ENSURE AXIAL LOAD.



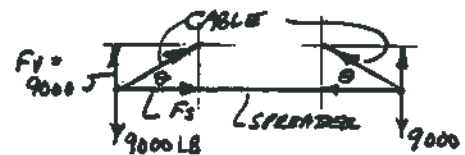
17. MULTIPLE DESIGNS POSSIBLE. CONSIDER HOLLOW TUBE - ROUND OR SQUARE. CHEAPER MATERIAL MAY ALSO BE USED.

18. MULTIPLE DESIGNS POSSIBLE

$$F_s = F_v / \tan \theta = \frac{9000}{\tan 30^\circ} = 15588 \text{ LB}$$

19. MULTIPLE DESIGNS POSSIBLE

$$F_s = \frac{9000}{\tan 15^\circ} = 33588 \text{ LB}$$





20.  $L = 10.75 \text{ FT} \times \frac{12 \text{ IN}}{\text{FT}} = 129 \text{ IN.} \therefore KL = 2.10(129) = 271 \text{ IN.} \therefore S_y = 68 \text{ KSI}; C_c = 93$   
 ASSUME COLUMN IS LONG-EULER-EQ. (6-9)  
 $D = \left[ \frac{64 N P (KL)^2}{\pi^3 E} \right]^{1/4} = \left[ \frac{64(25)(25000)(271)^2}{\pi^3 (30 \times 10^6)} \right]^{1/4} = 4.21 \text{ IN.}$   
 USE  $D = 4.25 \text{ IN.}$   
 CHECK  $\lambda = D/4 = 1.063 \text{ IN.} \therefore KL/\lambda = 271 / 1.063 = 255$  — LONG. OK

21. 22 MULTIPLE SOLUTIONS POSSIBLE

23. CROOKED COLUMN:  $a = 0.08 \text{ IN.}$ ,  $D = 0.75 \text{ IN.}$ ,  $c = D/2 = 0.375 \text{ IN.}$   
 $\lambda = D/4 = 0.188 \text{ IN.}$ ,  $\lambda^2 = 0.0352 \text{ IN}^2$ ;  $A = 0.442 \text{ IN}^2$ ;  $P_{cr} = 4473 \text{ LB}$   
 IN EQ. 6-11:  $C_1 = -\frac{1}{N} \left[ S_y A + \left( 1 + \frac{ac}{\lambda^2} \right) P_{cr} \right]$  [PROB. 6-1]  
 $C_1 = -\frac{1}{3} \left[ (42000)(0.442) + \left( 1 + \frac{(0.08)(0.375)}{0.0352} \right) (4473) \right] = -8951$

$$C_2 = \frac{S_y A P_{cr}}{N^2} = \frac{42000(0.442)(4473)}{(3)^2} = 9.226 \times 10^6$$

$$P = 0.5 \left[ -(-8951) - \sqrt{(-8951)^2 - 4(9.226 \times 10^6)} \right] = \underline{1189 \text{ LB}}$$

24. CROOKED COLUMN:  $a = 0.04 \text{ IN.}$ ;  $c = 0.5/2 = 0.25 \text{ IN.}$ ;  $P_{cr} = 42675 \text{ LB}$  (PROB. 6-9)  
 $\lambda = 0.144 \text{ IN.}$ ;  $\lambda^2 = 0.0208 \text{ IN}^2$ ;  $A = 0.50 \text{ IN}^2$ ;  $S_y = 181000 \text{ PSI}$

$$C_1 = -\frac{1}{3} \left[ (181000)(0.50) + \left( 1 + \frac{(0.04)(0.25)}{0.0208} \right) (42675) \right] = -51220$$

$$C_2 = \frac{(181000)(0.50)(42675)}{(3)^2} = 4.291 \times 10^8$$


$$P = 0.5 \left[ -(-51220) - \sqrt{(-51220)^2 - 4(4.291 \times 10^8)} \right] = \underline{10552 \text{ LB}}$$

25. CROOKED COLUMN:  $a = 0.15 \text{ IN.}$ ;  $c = 0.80/2 = 0.40 \text{ IN.}$ ;  $P_{cr} = 7498 \text{ LB}$  (PROB. 6-9)  
 $\lambda = 0.529 \text{ IN.}$ ;  $\lambda^2 = 0.280 \text{ IN}^2$ ;  $A = 0.5106 \text{ IN}^2$ ;  $S_y = 30000 \text{ PSI}$


$$C_1 = -\frac{1}{3} \left[ (30000)(0.5106) + \left( 1 + \frac{(0.15)(0.40)}{0.280} \right) (7498) \right] = -8677$$


$$C_2 = \frac{(30000)(0.5106)(7498)}{(3)^2} = 1.276 \times 10^7$$

$$P = 0.5 \left[ -(-8677) - \sqrt{(-8677)^2 - 4(1.276 \times 10^7)} \right] = \underline{1877 \text{ LB}}$$

26 ECCENTRIC COLUMN:  $L = 42 \text{ in.}$ ;  $S = 1.25 \text{ in.}$ ;  $C = \frac{S}{2} = 0.625 \text{ in.}$   
 $r = \frac{S}{\sqrt{12}} = 0.361 \text{ in.}$ ;  $r^2 = 0.130 \text{ in}^2$ ;  $A = S^2 = 1.563 \text{ in}^2$ ;  $e = 0.60 \text{ in.}$    
 $I = \frac{S^4}{12} = 0.2035 \text{ in}^4$ ;  $S_y = 13000 \text{ psi}$  FOR AL. 6063-T4.  
 FROM EQ.(6-12)  $\sigma_{\text{MAX}} = \sigma_{1/2} = \frac{1250}{1.563} \left[ 1 + \frac{(0.60)(0.625)}{0.130} \text{SEC} \left( \frac{42}{2(0.361)} \sqrt{\frac{1250}{(1.563)(0.2035)}} \right) \right]$   
 $\sigma_{\text{MAX}} = 3458 \text{ psi}$   
 EQ.(6-14):  $\gamma_{\text{MAX}} = 0.60 \left[ \text{SEC} \left( \frac{42}{2(0.361)} \sqrt{\frac{1250}{(1.563)(0.2035)}} \right) - 1 \right] = 0.0915 \text{ in.}$

27 ECCENTRIC COLUMN:  $L = 3.2 \text{ m} = 3200 \text{ mm}$ ;  $P = 30.5 \text{ kN} = 30500 \text{ N}$   
 3-IN SCH. 40;  $D_o = 3.50 \text{ in.}$ ,  $C = \frac{D_o}{2} = 1.75 \text{ in.}$  ( $25.4 \text{ mm}$ )  $= 44.5 \text{ mm}$   
 $r = 1/16 \text{ in.}$  ( $25.4$ )  $= 29.46 \text{ mm}$ ;  $r^2 = 868 \text{ mm}^2$ ;  $A = 2.23 \text{ in}^2$  ( $25.4$ )  $= 1439 \text{ mm}^2$   
 $I = 3.02 \text{ in}^4$  ( $25.4$ )  $= 1.251 \times 10^6 \text{ mm}^4$ ;  $e = 150 \text{ mm}$   
 AISI 1020 HR;  $E = 207 \text{ GPa} = 207 \times 10^9 \text{ Pa} = 207000 \text{ MPa} = 207000 \text{ N/mm}^2$   
 EQ.(6-12):  $\sigma_{\text{MAX}} = \sigma_{1/2} = \frac{30500}{1439} \left[ 1 + \frac{(150)(44.5)}{868} \text{SEC} \left( \frac{3200}{2(29.46)} \sqrt{\frac{30500}{(1439)(207000)}} \right) \right]$   
 $\sigma_{\text{MAX}} = 212 \text{ MPa}$ ; BUT  $S_y = 207 \text{ MPa}$ , THEN STRESS IS TOO HIGH.  
 $\gamma_{\text{MAX}} = (150) \left[ \text{SEC}(\quad) - 1 \right] = 25.9 \text{ mm}$  IF MATL. DOES NOT YIELD.

28 ECCENTRIC COLUMN:  $L = 14.75 \text{ in.}$ ;  $C = 0.30 \text{ in.}$ ;  $S = 0.250 \text{ in.}$   
 $A = S^2 = 0.0625 \text{ in}^2$ ;  $r = \frac{S}{\sqrt{12}} = \frac{0.25}{\sqrt{12}} = 0.0722 \text{ in.}$ ;  $r^2 = 0.00521 \text{ in}^2$    
 $P = 45 \text{ LB}$ ;  $E = 28 \times 10^6 \text{ psi}$ ;  $e = \frac{S}{2} = 0.125 \text{ in.}$   
 EQ.(6-12):  $\sigma_{\text{MAX}} = \frac{45}{0.0625} \left[ 1 + \frac{(0.30)(0.125)}{0.00521} \text{SEC} \left( \frac{14.75}{2(0.0722)} \sqrt{\frac{45}{(0.0625)(28 \times 10^6)}} \right) \right] = 6685 \text{ psi}$   
 EQ.(6-14):  $\gamma_{\text{MAX}} = (0.30) \left[ \text{SEC}(\quad) - 1 \right] = 0.045 \text{ in.}$

29 ECCENTRIC COLUMN:  $L = 40 \text{ in.}$ ;  $C = 0.50 \text{ in.}$ ;  $P = 75000 \text{ LB}$    
 FROM APP. 15-41:  $A = 3.37 \text{ in}^2$ ;  $r = 1.50 \text{ in.}$ ;  $r^2 = 2.31 \text{ in}^2$ ;  $C = \frac{4}{2} = 2.00 \text{ in.}$   
 ASTM A242:  $S_y = 50000 \text{ psi}$ ;  $E = 30 \times 10^6 \text{ psi}$   
 LET  $N = 3$ , THEN  $\sigma_3 = \frac{S_y}{3} = 16667 \text{ psi}$   
 LET  $\sigma_3'$  = RIGHT SIDE OF EQ.(6-13)  $= \frac{75000}{3.37} \left[ 1 + \frac{(0.50)(2.00)}{2.31} \text{SEC} \left( \frac{40}{2(1.50)} \sqrt{\frac{75000(3)}{(3.37)(30 \times 10^6)}} \right) \right]$   
 $\sigma_3' = 34099 \text{ psi} > \sigma_3$  UNSAFE / NOTE:  $6 \times 6 \times \frac{1}{2}$  REQ'D:  $\sigma_3' = 9493 \text{ psi}$

30 CENTRAL LOAD:  $L = 16.0 \text{ FT}$  ( $\frac{12 \text{ in.}}{\text{ft}}$ )  $= 192 \text{ in.}$ ; ASTM A36,  $S_y = 36 \text{ ksi}$   
 FROM APP. 15-91:  $A = 5.54 \text{ in}^2$ ;  $I_y = 9.13 \text{ in}^4$ ;  $r = \sqrt{I/A} = 1.28$ ;  $KL = \frac{(0.8)(192)}{1.28} = 119.7$   
 FROM FIG. 6-5,  $C_c = 125$ ; SHORT COLUMN - JOHNSON FORMULA. LET  $N = 3$   
 $P_a = \frac{P_{ce}}{N} = \frac{(5.54)(36000)}{3} \left[ 1 - \frac{(36000)(119.7)^2}{4\pi^2(30 \times 10^6)} \right] = 37500 \text{ LB}$

31

CENTRAL LOAD: FIXED-END,  $K=0.65$ ,  $L_e=0.65(66)=42.9$  IN.

S4x7.7:  $A=2.26$  IN<sup>2</sup>;  $r_{MN}=r_x=0.581$  IN;  $L_e/r_x=73.8$ ;  $N=3$ ,  $P_a=P_{CR}/N$

ASTM A36:  $S_y=36000$  PSI;  $E=30 \times 10^6$  PSI;  $C_c \approx 130$  - SHORT COLUMN, JOHNSON EQ (6-7)

$$P_a = [(2.26)(36000)/3] \left[ 1 - \frac{(36000)(73.8)^2}{4\pi^2(30 \times 10^6)} \right] = \underline{22600 \text{ LB}}$$

32

ECCENTRIC LOAD:  $P=1000$  LB;  $e=0.50+0.30/2=0.90$  IN;  $L=72$  IN.

$A=(1.60)(0.80)=1.28$  IN<sup>2</sup>;  $C=0.80/2=0.40$  IN;  $r_x=0.80/\sqrt{12}=0.2309$  IN.

USE STEEL -  $E=30 \times 10^6$  PSI

$$\sigma_{MAX} = \sigma_{1/2} = \frac{1000}{1.28} \left[ 1 + \frac{(0.90)(0.40)}{(0.2309)^2} \text{SEC} \left( \frac{72}{2(0.2309)} \sqrt{\frac{1000}{(1.28)(30 \times 10^6)}} \right) \right] = \underline{83150 \text{ PSI}}$$

$$M_{MAX} = 0.90 [\text{SEC}(1.7953) - 1] = \underline{0.386 \text{ IN}}$$

SPECIFY A MATERIAL TO PROVIDE  $N \geq 3$ .

$$\text{USING EQ (6-13)}: \sigma_a = \frac{1000}{1.28} \left[ 1 + \frac{(0.90)(0.40)}{(0.2309)^2} \text{SEC} \left( \frac{72}{2(0.2309)} \sqrt{\frac{3000}{AE}} \right) \right]$$

$$\sigma_a = 28280 \text{ PSI} = S_y/N: \text{ THEN } S_y = N \sigma_a = 84900 \text{ PSI}$$

SPECIFY AISI 1040 WQT 1000,  $S_y=86000$  PSI (OTHER SOLUTIONS POSSIBLE)

33

CENTRAL LOAD: SPECIFY A STEEL TUBE:  $S_y=36000$  PSI; LET  $N=3$

ASSUME COLUMN SUPPORTS  $1/2$  TOTAL LOAD:  $P_a=55000 \text{ LB}/2=27500 \text{ LB}$

ASSUME COLUMN IS LONG: EQ. 6-8:  $I = \frac{N P_a (KL)^2}{\pi^2 E}$

ASSUME FIXED-PINNED COLUMN,  $K=0.8$ :  $KL=(0.8)(18 \text{ SFT}) \frac{12 \text{ IN}}{\text{FT}}=177.6$  IN.  $C_c \approx 130$  (LONG)

$$I = \frac{3(27500)(177.6)^2}{\pi^2(30 \times 10^6)} = 8.79 \text{ IN}^4; \text{ USE SQ. TUBE } 4 \times 4 \times 1/2, I_x=11.9 \text{ IN}^4$$

OR RECT. TUBE  $6 \times 4 \times 1/4, I_y=11.1 \text{ IN}^4$  - LIGHTER

34

CENTRAL LOAD: C5x9 STEEL CHANNEL:  $A=2.64$  IN<sup>2</sup>;  $r_x=0.489$  IN;  $N=3$

$KL/r_x=1.0(12)/0.489=229$ : ASTM A36 -  $S_y=36000$  PSI,  $C_c \approx 130$  - LONG (L).

$$P_a = P_{CR}/N = \frac{\pi^2 EA}{N(KL/r_x)^2} = \frac{\pi^2(30 \times 10^6)(2.64)}{3(229)^2} = \underline{4967 \text{ LB}}$$

35

CENTRAL LOAD: SAME AS 34 EXCEPT FIXED ENDS,  $K=0.65$

$KL/r_x=(0.65)(12)/0.489=198.9$  - LONG COLUMN.

$$P_a = \frac{\pi^2(30 \times 10^6)(2.64)}{3(198.9)^2} = \underline{11750 \text{ LB}}$$

36

ECCENTRIC LOAD:  $e=x$  FROM APP. 15<sup>4</sup> =  $0.478$  IN. AND  $c=e$

USE EQ (6-13):  $\sigma_a = S_y/N = 36000/3 = 12000$  PSI

$$\sigma_a = \frac{P_a}{2.64} \left[ 1 + \frac{(0.478)(0.478)}{(0.489)^2} \text{SEC} \left( \frac{112}{2(0.489)} \sqrt{\frac{3(P_a)}{(2.64)(30 \times 10^6)}} \right) \right]$$

BY ITERATION: FOR  $P_a=4100$  LB,  $\sigma_a=11920$  PSI

<b>ECCENTRIC COLUMN ANALYSIS</b>		<i>Data from: Problem 6-37</i>	
Solves Equation 6-13 for design stress and Equation 6-14 for maximum deflection			
Enter data for variables in <i>italics in shaded boxes</i>		Use consistent U.S. Customary units.	
<b>Data To Be Entered:</b>		<b>Computed Values:</b>	
<b>Length and End Fixity:</b>			
Column length, <i>L</i> =	126 in	Eq. Length, $L_e = KL =$	126.0 in
End fixity, <i>K</i> =	1		
<b>Material Properties:</b>			
Yield strength, <i>s<sub>y</sub></i> =	46000 psi	Column constant, <i>C<sub>0</sub></i> =	111.6
Modulus of Elasticity, <i>E</i> =	2.90E+07 psi	Argument for secant =	0.777 for strength
		Value of secant =	1.4025
		Argument for secant =	0.449 for deflection
		Value of secant =	1.1098
<b>Cross Section Properties:</b>			
[Note: Enter <i>r</i> or compute $r = \sqrt{I/A}$ ]			
[Always enter Area]			
[Enter zero for <i>I</i> or <i>r</i> if not used]			
Area, <i>A</i> =	6.020 in <sup>2</sup>	Slenderness ratio, $KL/r =$	89.4
Moment of Inertia, <i>I</i> =	0 in <sup>4</sup>		
Radius of Gyration, <i>r</i> =	1.410 in		
<b>Values for Eqns. 6-13 and 6-14:</b>		Column is: <i>short</i>	
Eccentricity = <i>e</i> =	3 in	Req'd yield strength =	45,896 psi
Neutral axis to outside = <i>c</i> =	2 in	Must be less than actual yield strength:	
Allowable load = <i>P<sub>a</sub></i> =	17600 lb	<i>s<sub>y</sub></i> =	46,000 psi
<b>Design Factor</b>		Max. deflection, <i>y<sub>max</sub></i> =	0.329 in
Design factor on load, <i>N</i> =	3		

Note: *A* and *r* from Appendix 15-14

# ECCENTRIC COLUMN ANALYSIS

Data from: Problem 38A US

Solves Equation 6-13 for design stress and Equation 6-14 for maximum deflection

Enter data for variables in *kais* in shaded boxes

Use consistent U.S. Customary units

## Data To Be Entered:

### Length and End Fixity:

Column length,  $L = 40$  in

End fixity,  $K = 1$

### Material Properties:

Yield strength,  $s_y = 40000$  psi

Modulus of Elasticity,  $E = 1.00E+07$  psi

### Gross Section Properties:

[Note: Enter  $r$  or compute  $r = \sqrt{I/A}$ ]

[Always enter Area]

[Enter zero for  $I$  or  $r$  if not used]

Area,  $A = 0.600$  in<sup>2</sup>

Moment of Inertia,  $I = 0$  in<sup>4</sup>

OR

Radius of Gyration,  $r = 0.433$  in

### Values for Eqns. 6-13 and 6-14:

Eccentricity,  $e = 1.75$  in

Neutral axis to outside,  $c = 0.75$  in

Allowable load,  $P_n = 685$  lb

### Design Factor:

Design factor on load,  $N = 3$

## Computed Values:

NOTE: This solution considers the eccentric load with bending about the horizontal axis.

Eq. Length,  $L_e = KL = 40.0$  in

Column const.,  $C_c = 70.2$

Argument of sec = 0.855 for strength

Value of secant = 1.5236

Argument of sec = 0.494 for deflection

Value of secant = 1.1355

Slender. ratio,  $KL/r = 92.4$

Column is: *long*

## FINAL RESULTS

Req'd yield strength = 39,954 psi

Must be less than actual yield strength:

$s_y = 40,000$  psi

Max Deflection,  $y_{max} = 0.237$  in

See also Solution 38B for buckling about the thinner vertical axis.

NOTE!  $A = (1.50 \text{ in})(0.40 \text{ in}) = 0.600 \text{ in}^2$

$$r = \frac{h}{\sqrt{12}} = \frac{1.50 \text{ in}}{\sqrt{12}} = 0.433 \text{ in} ; P_n = 685 \text{ LB.}$$

ECCENTRIC LOAD TENDS TO BUCKLE THE BAR ABOUT ITS STRONG AXIS. BUT SEE SOLUTION 38B. LIMITING LOAD IS 163 LB FOR BUCKLING ABOUT THIN AXIS.



# COLUMN ANALYSIS PROGRAM

Data from: Problem 6-38B US

Refer to Figure 6-4 for analysis logic

Enter data for variables in *italics in shaded boxes*

Use consistent U.S. Customary units.

## Data To Be Entered:

## Computed Values:

### Length and End Fixity:

Column length,  $L = 40$  in

End fixity,  $K = 1$

Eq. Length,  $L_e = KL = 40.0$  in

### Material Properties:

Yield strength,  $s_y = 40000$  psi

Modulus of Elasticity,  $E = 1.00E+07$  psi

Column const.,  $C_c = 70.2$

### Cross-Section Properties:

[Note: Enter  $r$  or compute  $r = \sqrt{I/A}$ ]

[Always enter Area]

[Enter zero for  $I$  or  $r$  if not used]

Area,  $A = 0.6$  in<sup>2</sup>

Moment of Inertia,  $I = 0$  in<sup>4</sup>

$C_r$

Radius of Gyration,  $r = 0.115$  in

NOTE: Cross section properties taken with respect to the vertical axis because the load is central to that axis. But buckling is expected about the axis through the thin (0.40 in) section.

Slender. ratio,  $KL/r = 347.8$

Column is: *long*

Critical Buckling Load = 489 lb

Allowable Load = 163 lb

### Design Factor

Design factor on load,  $N = 3$

**This value governs the design, not solution 38A**

ANALYSIS AS A STRAIGHT CENTRALLY LOADED COLUMN  
THAT TENDS TO BUCKLE ABOUT THIN AXIS,  $t = 0.40$  IN

$$r = \frac{t}{\sqrt{12}} = \frac{0.40 \text{ IN}}{\sqrt{12}} = 0.115 \text{ IN}$$

FROM EULER FORMULA WITH  $N=3$   $P_a = 163 \text{ LB.}$

# ECCENTRIC COLUMN ANALYSIS

Data from: Problem 49-SI

Solves Equation 6-13 for design stress and Equation 6-14 for maximum deflection

Enter data for variables in *italics* in shaded boxes

Use consistent SI Metric units

## Data To Be Entered:

### Length and End Fixity:

Column length,  $L = 750$  mm  
End fixity,  $K = 1$

### Material Properties:

Yield strength,  $s_y = 966$  MPa  
Modulus of Elasticity,  $E = 200$  GPa

### Cross Section Properties:

[Note: Enter  $r$  or compute  $r = \sqrt{I/A}$ ]

[Always enter Area]

[Enter zero for  $I$  or  $r$  if not used]

Area,  $A = 314$  mm<sup>2</sup>  
Moment of Inertia,  $I = 0$  mm<sup>4</sup>  
OR  
Radius of Gyration,  $r = 7.289$  mm

### Values for Eqs. 6-13 and 6-14:

Eccentricity,  $e = 20$  mm  
Neutral axis to outside,  $c = 12.5$  mm  
Allowable load,  $P_a = 5200$  N

### Design Factor

Design factor on load,  $N = 3$

## Computed Values:

Eq. Length,  $L_e = KL = 750.0$  mm

Column const.,  $C_c = 63.9$

Argument of sec = 0.811 for strength  
Value of secant = 1.4512

Argument of sec = 0.468 for deflection  
Value of secant = 1.1205

Slender. ratio,  $KL/r = 102.9$

Column is: *long*

## FINAL RESULTS

Req'd yield strength = 389 MPa

Must be less than actual yield strength:

$s_y = 966$  MPa

Max Deflection,  $y_{max} = 2.41$  mm

Piston rod is safe for  $P_a = 5200$  N.



# COLUMN ANALYSIS PROGRAM

Data from: Problem 40A-Straight

Refer to Figure 6-4 for analysis logic

Enter data for variables in italics in shaded boxes

Use consistent U.S. Customary units.

## Data To Be Entered:

### Length and End Fixity:

Column length,  $L = 156$  in

End fixity,  $K = 1$

Eq. Length,  $L_e = KL = 156.0$  in

### Material Properties:

Yield strength,  $s_y = 36000$  psi

Modulus of Elasticity,  $E = 3.00E+07$  psi

Column const.,  $C_c = 128.3$

### Cross Section Properties:

[Note: Enter  $r$  or compute  $r = \sqrt{I/A}$ ]

[Always enter Area]

[Enter zero for  $I$  or  $r$  if not used]

Area,  $A = 1.075$  in<sup>2</sup>

Moment of Inertia,  $I = 0$  in<sup>4</sup>

Or

Radius of Gyration,  $r = 0.787$  in

Slender. ratio,  $KL/r = 198.2$

Column is: *long*

### Design Factor

Design factor on load,  $N = 3$

Critical Buckling Load = **8,101 lb**

Straight Pipe

Allowable Load = **2,700 lb**

See also Solution 40B for crooked pipe.

# CROOKED COLUMN ANALYSIS

Solves Equation 6-11 for Allowable Load

Enter data for variables in *italics* in shaded boxes

## Data To Be Entered:

### Length and End Fixity:

Column length,  $L = 156$  in  
End fixity,  $K = 1$

### Material Properties:

Yield strength,  $s_y = 36000$  psi  
Modulus of Elasticity,  $E = 3.00E+07$  psi

### Cross Section Properties:

{ Note: Enter  $r$  or compute  $r = \sqrt{I/A}$  }

{ Always enter Area }

{ Enter zero for  $I$  or  $r$  if not used }

Area,  $A = 1.075$  in<sup>2</sup>  
Moment of Inertia,  $I = 0$  in<sup>4</sup>  
Radius of Gyration,  $r = 0.787$  in

### Values for Eqn. 6-11:

Initial crookedness =  $a = 1.25$  in  
Neutral axis to outside =  $c = 1.188$  in

### Design Factor

Design factor on load,  $N = 3$

Data from: Problem 40B-Crooked

Use consistent U.S. Customary units.

## Computed Values:

Eq. Length,  $L_e = KL = 156.0$  in

Column const.,  $C_c = 128.3$

Euler buckling load = 8101 lb

$C_1$  in Eqn. 6-11 = -22074

$C_2$  in Eqn. 6-11 = 3.483E+07

Slender. ratio,  $KL/r = 198.2$

Column is: *long*

*Straight Column*

Critical Buckling Load = 8,101 lb

*Crooked Column*

Allowable Load = 1,711 lb

This value governs the use of the pipe.

See solution for straight pipe; Problem 40A.

## CHAPTER 7 BELT DRIVES AND CHAIN DRIVES

### V-BELTS

1.  $C \leq 24.0 \text{ IN}$ ;  $D_2 = 13.95 \text{ IN}$ ;  $D_1 = 5.25 \text{ IN}$ ; 3V BELT  
EQ. 7-31  
$$L = 2(24) + 1.57(13.95 + 5.25) + \frac{(13.95 - 5.25)^2}{4(24)} = 78.73 \text{ IN}$$
  
USE  $L = 75 \text{ IN}$  - STANDARD LENGTH.
2. ACTUAL  $C$  FROM EQ. 7-4:  $L = 75$   
 $B = 4(75) - 6.28(13.95 + 5.25) = 179.4$   
$$C = \frac{179.4 + \sqrt{(179.4)^2 - 32(13.95 - 5.25)^2}}{16} = 22.00 \text{ IN}$$
3.  $\theta_1 = 180^\circ - 2 \sin^{-1} \left[ \frac{13.95 - 5.25}{2(22.0)} \right] = 157.2^\circ$  (EQ. 7-5)  
 $\theta_2 = 180^\circ + 2 \sin^{-1} \left[ \frac{13.95 - 5.25}{2(22.0)} \right] = 202.8^\circ$  (EQ. 7-6)
4.  $C \leq 60.0 \text{ IN}$ ;  $D_2 = 27.7 \text{ IN}$ ;  $D_1 = 8.4 \text{ IN}$ ; 5V BELT (EQ. 7-3)  
COMPUTED  $L = 178.2 \text{ IN}$ . USE  $L = 170 \text{ IN}$
5. ACTUAL  $C = 55.83 \text{ IN}$ . (EQ. 7-4)
6.  $\theta_1 = 160.1^\circ$ ;  $\theta_2 = 199.9^\circ$  (EQ. 7-5), (EQ. 7-6)
7.  $C \leq 144 \text{ IN}$ ;  $D_2 = 94.8 \text{ IN}$ ;  $D_1 = 13.8 \text{ IN}$ ; 8V BELT (EQ. 7-3)  
COMPUTED  $L = 469.9 \text{ IN}$ . USE  $L = 450 \text{ IN}$ .
8. ACTUAL  $C = 133.6 \text{ IN}$  (EQ. 7-4)
9.  $\theta_1 = 144.7^\circ$ ;  $\theta_2 = 215.3^\circ$  (EQ. 7-5, 7-6)
10.  $N_b = R_1 \omega_1 = \frac{5.25 \text{ IN}}{2} \times \frac{1750 \text{ REV}}{\text{MIN}} \times \frac{2\pi \text{ RAD}}{\text{REV}} \times \frac{1 \text{ FT}}{12 \text{ IN}} = 2405 \text{ FT/MIN}$
11.  $N_b = R_1 \omega_1 = \frac{8.4 \text{ IN}}{2} \times \frac{1160 \text{ REV}}{\text{MIN}} \times \frac{2\pi \text{ RAD}}{\text{REV}} \times \frac{1 \text{ FT}}{12 \text{ IN}} = 2551 \text{ FT/MIN}$
12.  $N_b = R_1 \omega_1 = \frac{13.8 \text{ IN}}{2} \times \frac{870 \text{ REV}}{\text{MIN}} \times \frac{2\pi \text{ RAD}}{\text{REV}} \times \frac{1 \text{ FT}}{12 \text{ IN}} = 3143 \text{ FT/MIN}$
13. RATIO =  $13.95/5.25 = 2.66$ ;  $P \cong 6.25 \text{ HP}$ ;  $C_\theta = 0.94$ ;  $C_L = 1.03$   
CORRECTED POWER =  $C_\theta C_L P = (0.94)(1.03)(6.25) = 6.05 \text{ HP}$

## V-BELTS

14. RATIO =  $27.7/8.4 = 3.30$ ;  $P = 15.5 + 1.26 = 16.76 \text{ hp}$ ;  $C_0 = .95$ ;  $C_L = 1.05$   
CORRECTED POWER =  $C_0 C_L P = (.95)(1.05)(16.76) = 16.72 \text{ hp}$
15. RATIO =  $94.8/13.8 = 6.87$ ;  $P = 48 \text{ hp}$ ;  $C_0 = .904$ ;  $C_L = 1.09$   
CORRECTED POWER =  $C_0 C_L P = (.904)(1.09)(48) = 47.3 \text{ hp}$
16. A 15N BELT IS A METRIC SIZE HAVING A TOP WIDTH OF 15MM SIMILAR TO A 5V BELT.
17. A 17A BELT IS A METRIC AUTOMOTIVE BELT HAVING A TOP WIDTH OF 17MM. SIMILAR TO A "3/4 IN." AUTOMOTIVE BELT
18. DESIGN: SERVICE FACTOR = 1.5; DESIGN POWER =  $1.5(25) = 37.5 \text{ hp}$   
FROM FIG. 7-9 — USE 5V BELT  
RATIO =  $870/310 = 2.81$   
FOR  $N_2 \approx 4000 \text{ FT/IN}$ ;  $D_1 = \frac{12(4000)}{\pi(870)} = 17.56 \text{ IN}$   
FOR  $D_1 = 13.1 \text{ IN}$ ;  $D_2 = 37.4 \text{ IN}$ ;  $M_2 = 870 \times \frac{13.1}{324} = 305 \text{ RPM OK}$   
RATED POWER =  $22.5 + .94 = 23.44 \text{ hp}$   
CENTER DISTANCE:  
 $D_2 < C < 3(D_2 + D_1)$   
 $37.4 < C < 3(37.4 + 13.1) = 151.5$   
TRY  $C = 48 \text{ IN}$ . — NOMINAL  $L = 178 \text{ IN}$  (EQ. 7-3)  
USE  $L = 180 \text{ IN}$ . — ACTUAL  $C = 48.85 \text{ IN}$  (EQ. 7-4)  
 $\theta_1 = 157.2^\circ$ ;  $\theta_2 = 208.8^\circ$  (EQS. 7-5, 7-6)  
 $C_0 = 0.92$ ;  $C_L = 1.06$ ; CORR. POWER =  $(.92)(1.06)(23.44) = 22.86 \text{ hp/BELT}$   
NO. OF BELTS =  $37.5 \text{ hp} / 22.86 \text{ hp/BELT} = 1.64 \text{ BELTS} \rightarrow 2 \text{ BELTS}$
19. DESIGN: S.F. = 1.2; DES. POWER =  $1.2(5) = 6.0 \text{ hp} \rightarrow 3V \text{ BELT}$   
RATIO =  $1750/725 = 2.41$ ;  $D_1 \approx 12(4000)/\pi(1750) = 8.7 \text{ IN}$   
FOR  $D_1 = 7.95$ ;  $D_2 = 18.95$ ;  $M_2 = 1750 \frac{7.95}{18.95} = 734 \text{ RPM OK}$   
RATED POWER  $\approx 10.3 \text{ hp}$ ;  $18.95 < C < 80.7$ ; TRY  $C = 30 \text{ IN}$ .  
 $L = 103 \text{ IN} \rightarrow$  USE  $L = 100 \text{ IN}$ ; ACTUAL  $C = 28.35 \text{ IN}$   
 $\theta_1 = 157.6^\circ$ ;  $\theta_2 = 202.4^\circ$ ;  $C_0 = .94$ ;  $C_L = 1.09$   
CORR. POWER =  $(.94)(1.09)(10.3) = 10.55 \text{ hp/BELT} - \text{ONE BELT REQ'D}$



## V-BELTS

20. DESIGN:  $S.F. = 1.4$ ;  $DES. POWER = 1.4(40) = 56 \text{ hp} \rightarrow \text{5V BELT}$   
 $RATIO = 1600/550 = 2.73$ ;  $D_1 = 12(4000)/\pi(1500) = 10.2 \text{ in.}$   
 FOR  $D_1 = 10.2 \text{ in.}$ ;  $D_2 = 27.7 \text{ in.}$ ;  $M_2 = 1500 \times 10.2/27.7 = 552 \text{ RPM OK}$   
 $RATED POWER \approx 25.7 \text{ hp}$  BY INTERPOLATION ON FIG 13-10 AT 1500 RPM  
 $27.7 < C < 114$ ; USE  $C \approx 36 \text{ in.}$ ;  $L \approx 133.6 \text{ in.} \rightarrow \text{USE } L = 132 \text{ in.}$   
 $ACTUAL C = 35.16 \text{ in.}$ ;  $\theta_1 = 157.2^\circ$ ;  $\theta_2 = 208.8^\circ$ ;  $C_D = .92$ ;  $CL = 1.01$   
 $CORR. POWER = (.92)(1.01)(25.7) = 23.9 \text{ hp/BELT}$   
 $NO. OF BELTS = 56/23.9 = 2.35 \rightarrow \text{USE 3 BELTS}$

21. DESIGN:  $S.F. = 1.4$ ;  $DES. POWER = 1.4(20) = 28 \text{ hp} \rightarrow \text{3V BELT}$   
 $RATIO = 1250/695 = 1.80$ ;  $D_1 = 12(4000)/\pi(1250) = 12.2 \text{ in.}$   
 FOR  $D_1 = 10.55 \text{ in.}$ ;  $D_2 = 18.95 \text{ in.}$ ;  $M_2 = 1250 \times 10.55/18.95 = 695.9 \text{ RPM OK}$   
 $RATED POWER \approx 10.4 \text{ hp}$  BY INTERPOLATION ON FIG. 13-9 AT 1250 RPM  
 $18.95 < C < 88.5$ ; USE  $C \approx 20 \text{ in.}$ ;  $L \approx 87.2 \text{ in.} \rightarrow \text{USE } L = 90 \text{ in.}$   
 $ACTUAL C = 21.43 \text{ in.}$ ;  $\theta_1 = 157.4^\circ$ ;  $\theta_2 = 202.6^\circ$ ;  $C_D = .94$ ;  $CL = 1.07$   
 $CORR. POWER = (.94)(1.07)(10.4) = 10.46 \text{ hp/BELT}$   
 $NO. OF BELTS = 28/10.46 = 2.68 \text{ BELTS} \rightarrow \text{USE 3 BELTS}$

22. DESIGN:  $S.F. = 2.0$  (CHOKING);  $DES. P = 2.0(100) = 200 \text{ hp} \rightarrow \text{5V BELT}$   
 $RATIO = 870/625 = 1.39$ ;  $D_1 = 12(4000)/\pi(870) = 17.6 \text{ in.}$   
 FOR  $D_1 = 10.8 \text{ in.}$ ;  $D_2 = 14.9 \text{ in.}$ ;  $M_2 = 870 \times 10.8/14.9 = 631 \text{ RPM OK}$   
 $RATED POWER = 17.6 + .77 = 18.37 \text{ hp}$ ;  $14.9 < C < 77.1$ ; USE  $C \approx 48 \text{ in.}$   
 $L \approx 136 \text{ in.} \rightarrow \text{USE } L = 132 \text{ in.}$ ;  $ACTUAL C = 45.78 \text{ in.}$   
 $\theta_1 = 174.9^\circ$ ;  $\theta_2 = 185.1^\circ$ ;  $C_D = .988$ ;  $CL = 1.01$   
 $CORR. POWER = (.988)(1.01)(18.37) = 18.33 \text{ hp/BELT}$   
 $NO. OF BELTS = 200/18.33 = 10.9 \rightarrow \text{USE 11 BELTS, NOT ACCEPTABLE}$

TRY 8V BELT: FOR  $D_1 = 17.8 \text{ in.}$ ;  $D_2 = 24.8 \text{ in.}$ ;  $M_2 = 624.4 \text{ RPM OK}$   
 $RATED POWER = 66 \text{ hp}$ ;  $24.8 < C < 127.8$ ; USE  $C \approx 48 \text{ in.}$   
 $L \approx 163 \text{ in.} \rightarrow \text{USE } L = 160 \text{ in.}$ ;  $ACTUAL C = 46.43 \text{ in.}$   
 $\theta_1 = 171.4^\circ$ ;  $\theta_2 = 188.6^\circ$ ;  $C_D = .98$ ;  $CL = .94$   
 $CORR. POWER = (.98)(.94)(66) = 60.8 \text{ hp/BELT}$   
 $NO. OF BELTS = 200/60.8 = 3.3 \rightarrow \text{USE 4 BELTS}$

## ROLLER CHAIN

23. CHAIN NO. 140: PITCH =  $1\frac{1}{8}$  IN.
24. CHAIN NO. 60: PITCH =  $\frac{3}{4}$  IN
25. STATIC LOAD = 1250 LB; AVG. TENSILE STRENGTH =  $10W = 12500$  LB.  
USE A NO. 80 CHAIN (1.00 IN. PITCH); T.S. = 14500, TABLE 7-5
26. LOAD ON EACH CHAIN = 2500 LB; T.S. =  $10W = 25000$  LB  
USE NO. 120 CHAIN  $1\frac{1}{2}$  IN. PITCH; T.S. = 34000 LB
27. FATIGUE OF LINK PLATES; IMPACT OF ROLLERS ON SPROCKET TEETH; GALLING BETWEEN PINS AND BUSHINGS
28. TABLE 7-8: GIVEN NO. 60 CHAIN, 20 TEETH, 750 RPM  
RATED POWER = 21.69 HP (INTERPOLATION), TYPE B LUBE (BATH)  
SERVICE FACTOR = 1.2 FOR HYDRAULIC DRIVE  
DESIGN POWER RATING =  $21.69 / 1.2 = 18.08$  HP.
29. 3 STRANDS: FACTOR = 2.5  
POWER RATING =  $2.5(18.08) = 45.2$  HP
30. TABLE 7-7: GIVEN NO. 40 CHAIN, 12 TEETH, 860 RPM  
RATED POWER = 4.44 HP (INTERPOLATION), TYPE B LUBE (BATH)  
S.F. = 1.2; DESIGN POWER RATING =  $4.44 / 1.2 = 3.70$  HP.
31. 4 STRANDS: FACTOR = 3.3; POWER RATING =  $3.3(3.70) = 12.21$  HP
32. TABLE 7-9: GIVEN NO. 80 CHAIN; 32 TEETH; 1160 RPM  
RATED POWER = 78.69 HP (INTERPOLATION), TYPE C LUBE (OIL STREAM)  
SF = 1.2; DESIGN POWER =  $78.69 / 1.2 = 65.57$  HP
33. 2 STRANDS: FACTOR = 1.7; POWER RATING =  $(1.7)65.57$  HP = 111.5 HP
34. NO. 60 CHAIN;  $N_1 = 15$ ;  $N_2 = 50$ ;  $C \leq 36W$  - USE EQ. 7-9  
 $L_p = \frac{3}{4}$  IN = 0.75 IN.  
 $C = 36W / 0.75W / \text{PITCH} = 48$  PITCHES  
 $L = 2(48) + \frac{50+15}{2} + \frac{(50-15)^2}{4(48)} = 129.1$  PITCHES  
USE 128 PITCHES (EVEN NUMBER);  $L = 128(0.75) = 96$  IN.

35. FOR  $L=128$  PITCHES;  $C$  FROM EQ. 7-10

$$C = \frac{1}{4} \left[ 128 - \frac{50+15}{2} + \sqrt{\left[ 128 - \frac{50+15}{2} \right]^2 - \frac{8(50-15)^2}{4\pi^2}} \right] = 47.42 \text{ PITCHES}$$

$$C = 47.42 \text{ PITCHES} \times 0.75 \text{ IN/PITCH} = \underline{35.57 \text{ IN.}}$$

36. NO. 40 CHAIN;  $N_1=11$ ;  $N_2=45$ ;  $C \leq 24 \text{ IN.}$   
 $L-P = 1/2 \text{ IN.} = 0.50 \text{ IN.}; C = 24/0.5 = 48 \text{ PITCHES}$

$$L = 2(48) + \frac{45+11}{2} + \frac{(45-11)^2}{4\pi^2(48)} = 124.6 \text{ PITCHES} \rightarrow \text{USE } 124 \text{ PITCHES}$$

$$\frac{124(0.5)}{124(0.5)} = 62 \text{ IN.}$$

37.

$$C = \frac{1}{4} \left[ 124 - \frac{45+11}{2} + \sqrt{\left[ 124 - \frac{45+11}{2} \right]^2 - \frac{8(45-11)^2}{4\pi^2}} \right] = 47.69 \text{ PITCHES}$$

$$C = 47.69 \text{ PITCHES} \times 0.5 \text{ IN/PITCH} = \underline{23.85 \text{ IN.}}$$

38. DESIGN: 25 HP;  $M_1=310 \text{ RPM}$ ;  $M_2=160 \text{ RPM}$ ; NOM. RATIO =  $310/160 = 1.94$   
 $SF = 1.5$ ; DESIGN POWER =  $1.5(25) = 37.5 \text{ HP}$   
 USE 3 STRANDS & RATING =  $37.5/2.5 = 15.0 \text{ HP PER STRAND}$   
 NO. 80 CHAIN; 15 TEETH RATED  $> 15.8 \text{ HP AT } 310 \text{ RPM}$ ; TYPE B LUBE  
 $N_2 = N_1 \times \text{RATIO} = 15(1.94) = 29.1 \rightarrow 29 \text{ TEETH}$   
 $M_2 = M_1 \times N_1/N_2 = 310 \times 15/29 = 160.3 \text{ RPM OK}$   
 $D_1 = 1.00/\text{diam} (15/15) = 4.810 \text{ IN.}; D_2 = 1.00/\text{diam} (15/29) = 9.249 \text{ IN.}$   
 USE  $C \approx 40$  PITCHES WITH  $P=1.00 \text{ IN}$  FOR NO. 80 CHAIN

$$L = 2(40) + \frac{29+15}{2} + \frac{(29-15)^2}{4\pi^2(40)} = 102.1 \text{ PITCHES} \rightarrow \text{USE } 102 \text{ PITCHES} = L$$

$$C = \frac{1}{4} \left[ 102 - \frac{29+15}{2} + \sqrt{\left[ 102 - \frac{29+15}{2} \right]^2 - \frac{8(29-15)^2}{4\pi^2}} \right] = 39.94 \text{ PITCHES} = C$$

$$C = 39.94 \text{ IN.}$$

Problems 38-42 are design problems for chain drives for which there are no unique solutions. The general procedure is illustrated above for one possible solution for Problem 38. This and the other design problems are shown on the following pages using the spreadsheet that is available from the publisher's website for this book. Data for design power per strand from Tables 7-7, 7-8, or 7-9 must be used to ensure that the selected chain design has sufficient capacity.



## CHAIN DRIVE DESIGN

### Initial Input Data:

Problem 38 Multiple strands

Application: Hammer Mill  
 Drive type: Electric motor  
 Driven machine: Hammer Mill  
 Power input: 25 hp  
 Service factor: 1.5  
 Input speed: 310 rpm  
 Desired output speed: 160 rpm

Table 7-1

### Computed Data:

Design power: 37.5 hp  
 Speed ratio: 1.94

### Design Decisions-Chain Type and Teeth Numbers:

Number of strands:	3	1	2	3	4
Strand factor:	2.5	1.0	1.7	2.5	3.3
Required power per strand:	15.00 hp				

Chain number: 80 Tables 7-7, 7-8 or 7-9

Chain pitch: 1 in  
 Number of teeth-Driver sprocket: 15

Computed no. of teeth-Driven sprocket: 29.06

Enter Chosen number of teeth: 29

### Computed Data:

Actual output speed: 160.3 rpm  
 Pitch diameter-Driver sprocket: 4.810 in  
 Pitch diameter-Driven sprocket: 9.249 in

### Center Distance, Chain Length and Angle of Wrap:

Enter Nominal center distance: 40 pitches 30 to 50 pitches recommended

Computed nominal chain length: 102.1 pitches

Enter Specified no. of pitches: 102 pitches Even number recommended

Actual chain length: 102.00 in  
 Computed actual center distance: 39.938 pitches  
 Actual center distance: 39.938 in

Angle of wrap-Driver sprocket: 173.6 degrees Should be greater than 120 degrees  
 Angle of wrap-Driven sprocket: 186.4 degrees

## CHAIN DRIVE DESIGN

**Initial Input Data:** *Problem 39 - Single strand*

Application: *Agitator*  
 Drive type: *Electric motor*  
 Driven machine: *Agitator*  
 Power input: 5 hp  
 Service factor: 1  
 Input speed: 750 rpm  
 Desired output speed: 325 rpm

Table 7-1

**Computed Data:**

Design power: 5 hp  
 Speed ratio: 2.31

**Design Decisions-Chain Type and Teeth Numbers:**

Number of strands:	1	1	2	3	4
Strand factor:	1.0	1.0	1.7	2.5	3.3

Required power per strand: 5.00 hp

Chain number: 40 *Tables 7-7, 7-8, or 7-9*

Chain pitch: 0.5 in

Number of teeth-Driver sprocket: 18

Computed no. of teeth-Driven sprocket: 36.92

Enter: Chosen number of teeth: 37

**Computed Data:**

Actual output speed: 324.3 rpm

Pitch diameter-Driver sprocket: 2.563 in

Pitch diameter-Driven sprocket: 5.896 in

**Center Distance, Chain Length and Angle of Wrap:**

Enter: Nominal center distance: 32 pitches *30 to 50 pitches recommended*

Computed nominal chain length: 90.8 pitches

Enter: Specified no. of pitches: 90 pitches *Even number recommended*

Actual chain length: 45.00 in

Computed actual center distance: 31.573 pitches

Actual center distance: 15.787 in

Angle of wrap-Driver sprocket: 167.9 degrees *Should be greater than 120 degrees*

Angle of wrap-Driven sprocket: 192.1 degrees



## CHAIN DRIVE DESIGN

### Initial Input Data:

Problem 40 - Multiple strands

Application: Conveyor  
 Drive type: Engine  
 Driven machine: Heavy conveyor  
 Power input: 40 hp  
 Service factor: 1.4  
 Input speed: 500 rpm  
 Desired output speed: 250 rpm

Table 7-1

### Computed Data:

Design power: 56 hp  
 Speed ratio: 2.00

### Design Decisions-Chain Type and Teeth Numbers:

Number of strands	3	1	2	3	4
Strand factor	2.5	1.0	1.7	2.5	3.3

Required power per strand: 22.40 hp

Chain number: 50  
 Chain pitch: 1.60 in

Tables 7-7, 7-8 or 7-9

Number of teeth-Driver sprocket: 14

Computed no. of teeth-Driven sprocket: 28.00

Enter Chosen number of teeth: 28

### Computed Data:

Actual output speed: 250.0 rpm  
 Pitch diameter-Driver sprocket: 4.494 in  
 Pitch diameter-Driven sprocket: 8.931 in

### Center Distance, Chain Length and Angle of Wrap:

Enter Nominal center distance: 38 pitches 30 to 50 pitches recommended

Computed nominal chain length: 93.1 pitches

Enter Specified no. of pitches: 94 pitches even number recommended

Actual chain length: 94.00 in

Computed actual center distance: 36.432 pitches

Actual center distance: 36.432 in

Angle of wrap-Driver sprocket: 173.0 degrees Should be greater than 120 degrees

Angle of wrap-Driven sprocket: 187.0 degrees

## CHAIN DRIVE DESIGN

### Initial Input Data:

Problem 47 - Multiple strands

Application:	Pump drive
Drive type:	Steam turbine
Driven machine:	Centrifugal pump
Power input:	20 hp
Service factor:	1
Input speed:	2200 rpm
Desired output speed:	775 rpm

Table 7-1

### Computed Data:

Design power:	20 hp
Speed ratio:	2.84

### Design Decisions-Chain Type and Teeth Numbers:

Number of strands:	2	1	2	3	4
Strand factor:	1.7	1.0	1.7	2.3	3.3
Required power per strand:	11.76 hp				

Chain number: 40 Tables 7-7, 7-8, or 7-9

Chain pitch: 0.50 in

Number of teeth-Driver sprocket: 25 11.95 hp rating at 2200 rpm

Computed no. of teeth-Driver sprocket: 70.97

Enter Chosen number of teeth: 71 Check availability from vendor

### Computed Data:

Actual output speed:	774.6 rpm
Pitch diameter-Driver sprocket:	3.989 in
Pitch diameter-Driven sprocket:	11.304 in

### Center Distance, Chain Length and Angle of Wrap:

Enter Nominal center distance: 30 pitches 30 to 50 pitches recommended

Computed nominal chain length: 109.8 pitches

Enter Specified no. of pitches: 110 pitches Even number recommended

Actual chain length: 55.00 in

Computed actual center distance: 30.110 pitches

Actual center distance: 15.055 in

Angle of wrap-Driver sprocket: 151.9 degrees Should be greater than 120 degrees

Angle of wrap-Driven sprocket: 208.1 degrees



## CHAIN DRIVE DESIGN

<b>Initial Input Data:</b>		<i>Problem 42 - Multiple strands</i>			
Application:	Rock Crusher				
Drive type:	Hydraulic drive				
Driven machine:	Rock Crusher				
Power input:	100 hp				
Service factor:	1.4	Table 7-1			
Input speed:	825 rpm				
Desired output speed:	225 rpm				
<b>Computed Data:</b>					
Design power:	140 hp				
Speed ratio:	2.78				
<b>Design Decisions-Chain Type and Teeth Numbers:</b>					
Number of strands:	4	1	2	3	4
Strand factor:	3.3	1.0	1.7	2.5	3.3
Required power per strand:	42.42 hp				
Chain number:	80	Tables 7-7, 7-8 or 7-9			
Chain pitch:	1.00 in				
Number of teeth-Driver sprocket:	21	>42.94 hp per strand			
Computed no. of teeth-Driver sprocket:	58.33				
Enter: Chosen number of teeth:	58	Check availability from vendor			
<b>Computed Data:</b>					
Actual output speed:	226.3 rpm				
Pitch diameter-Driver sprocket:	6.710 in				
Pitch diameter-Driven sprocket:	18.471 in				
<b>Center Distance, Chain Length and Angle of Wrap:</b>					
Enter: Nominal center distance:	40 pitches	30 to 50 pitches recommended			
Computed nominal chain length:	120.4 pitches				
Enter: Specified no. of pitches:	120 pitches	Even number recommended			
Actual chain length:	120.00 in				
Computed actual center distance:	39.815 pitches				
Actual center distance:	39.815 in				
Angle of wrap-Driver sprocket:	163.0 degrees	Should be greater than 120 degrees			
Angle of wrap-Driven sprocket:	197.0 degrees				

## CHAPTER 8 KINEMATICS OF GEARS

### Gear Geometry

1.  $N = 44; P_d = 12$

<p>a. <math>D = N/P_d = 44/12 = 3.667 \text{ in.}</math></p> <p>b. <math>P_c = \pi/P_d = \pi/12 = 0.2618 \text{ in.}</math></p> <p>c. <math>m = 25.4/P_d = 25.4/12 = 2.117 \text{ mm}</math></p> <p>d. <math>m = 2.00 \text{ mm}</math></p> <p>e. <math>a = 1/P_d = 1/12 = 0.0833 \text{ in.}</math></p>	<p>f. <math>b = 1.25/P_d = 1.25/12 = 0.1042 \text{ in.}</math></p> <p>g. <math>c = 0.25/P_d = 0.25/12 = 0.0208 \text{ in.}</math></p> <p>h. <math>h_o = a + b = 2.25/P_d = 0.1875 \text{ in.}</math></p> <p>i. <math>h_k = 2a = 2/P_d = 2/12 = 0.1667 \text{ in.}</math></p> <p>j. <math>t = \pi/2P_d = \pi/2(12) = 0.131 \text{ in.}</math></p> <p>k. <math>D_o = (N+2)/P_d = 46/12 = 3.833 \text{ in.}</math></p>
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2.  $N = 34; P_d = 24$

<p>a. <math>D = 34/24 = 1.417 \text{ in.}</math></p> <p>b. <math>P_c = \pi/24 = 0.131 \text{ in.}</math></p> <p>c. <math>m = 25.4/24 = 1.058 \text{ mm}</math></p> <p>d. <math>m = 1.00 \text{ mm}</math></p> <p>e. <math>a = 1/24 = 0.0417 \text{ in.}</math></p>	<p>f. <math>b = \frac{1.200}{P_d} + 0.002 = 0.0520 \text{ in.}</math></p> <p>g. <math>c = 0.200/P_d + 0.002 = 0.0103 \text{ in.}</math></p> <p>h. <math>h_o = a + b = 0.0417 + 0.0520 = 0.0937 \text{ in.}</math></p> <p>i. <math>h_k = 2a = 2/24 = 0.0833 \text{ in.}</math></p> <p>j. <math>t = \pi/2(24) = 0.0654 \text{ in.}</math></p> <p>k. <math>D_o = (N+2)/P_d = 36/24 = 1.500 \text{ in.}</math></p>
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3.  $N = 45; P_d = 2$

<p>a. <math>D = 45/2 = 22.500 \text{ in.}</math></p> <p>b. <math>P_c = \pi/2 = 1.571 \text{ in.}</math></p> <p>c. <math>m = 25.4/2 = 12.7 \text{ mm}</math></p> <p>d. <math>m = 12 \text{ mm}</math></p> <p>e. <math>a = 1/2 = 0.500 \text{ in.}</math></p>	<p>f. <math>b = 1.25/2 = 0.625 \text{ in.}</math></p> <p>g. <math>c = 0.25/2 = 0.125 \text{ in.}</math></p> <p>h. <math>h_o = 2.25/2 = 1.125 \text{ in.}</math></p> <p>i. <math>h_k = 2/2 = 1.000 \text{ in.}</math></p> <p>j. <math>t = \pi/2(2) = 0.7854 \text{ in.}</math></p> <p>k. <math>D_o = 47/2 = 23.50 \text{ in.}</math></p>
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4.  $N = 18; P_d = 8$

<p>a. <math>D = 18/8 = 2.250 \text{ in.}</math></p> <p>b. <math>P_c = \pi/8 = 0.3927 \text{ in.}</math></p> <p>c. <math>m = 25.4/8 = 3.175 \text{ mm}</math></p> <p>d. <math>m = 3.0 \text{ mm}</math></p> <p>e. <math>a = 1/8 = 0.125 \text{ in.}</math></p>	<p>f. <math>b = 1.25/8 = 0.1563 \text{ in.}</math></p> <p>g. <math>c = 0.25/8 = 0.0313 \text{ in.}</math></p> <p>h. <math>h_o = 2.25/8 = 0.2813 \text{ in.}</math></p> <p>i. <math>h_k = 2/8 = 0.250 \text{ in.}</math></p> <p>j. <math>t = \pi/2(8) = 0.1963 \text{ in.}</math></p> <p>k. <math>D_o = 20/8 = 2.500 \text{ in.}</math></p>
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5.  $N = 22; P_d = 1.75$

<p>a. <math>D = 22/1.75 = 12.571 \text{ in.}</math></p> <p>b. <math>P_c = \pi/1.75 = 1.795 \text{ in.}</math></p> <p>c. <math>m = 25.4/1.75 = 14.514 \text{ mm}</math></p> <p>d. <math>m = 16 \text{ mm}</math></p> <p>e. <math>a = 1/1.75 = 0.5714 \text{ in.}</math></p>	<p>f. <math>b = 1.25/1.75 = 0.7143 \text{ in.}</math></p> <p>g. <math>c = 0.25/1.75 = 0.1429 \text{ in.}</math></p> <p>h. <math>h_o = 2.25/1.75 = 1.2857 \text{ in.}</math></p> <p>i. <math>h_k = 2/1.75 = 1.1429 \text{ in.}</math></p> <p>j. <math>t = \pi/2(1.75) = 0.8976 \text{ in.}</math></p> <p>k. <math>D_o = 24/1.75 = 13.714 \text{ in.}</math></p>
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6.  $N=20; P_d=64$

a.  $D = 20/64 = 0.3125 \text{ in.}$   
 b.  $P_c = \pi/64 = 0.0491 \text{ in.}$   
 c.  $m = 25.4/64 = 0.397 \text{ mm}$   
 d.  $m = 0.40 \text{ mm}$   
 e.  $a = 1/64 = 0.0156 \text{ in.}$

f.  $b = 1.200/64 + 0.002 = 0.0208 \text{ in.}$   
 g.  $c = 0.200/64 + 0.002 = 0.0051 \text{ in.}$   
 h.  $h_t = a + b = 0.0364 \text{ in.}$   
 i.  $h_k = 2/64 = 0.0313 \text{ in.}$   
 j.  $t = \pi/2(64) = 0.0245 \text{ in.}$   
 k.  $D_o = 22/64 = 0.3438 \text{ in.}$

7.  $N=180; P_d=80$

a.  $D = 180/80 = 2.250 \text{ in.}$   
 b.  $P_c = \pi/80 = 0.0393 \text{ in.}$   
 c.  $m = 25.4/80 = 0.318 \text{ mm}$   
 d.  $m = 0.30 \text{ mm}$   
 e.  $a = 1/80 = 0.0125 \text{ in.}$

f.  $b = 1.200/80 + 0.002 = 0.0170 \text{ in.}$   
 g.  $c = 0.200/80 + 0.002 = 0.0045 \text{ in.}$   
 h.  $h_t = a + b = 0.0295 \text{ in.}$   
 i.  $h_k = 2/80 = 0.025$   
 j.  $t = \pi/2(80) = 0.0196 \text{ in.}$   
 k.  $D_o = 182/80 = 2.275 \text{ in.}$

8.  $N=28; P_d=18$

a.  $D = 28/18 = 1.556 \text{ in.}$   
 b.  $P_c = \pi/18 = 0.1745 \text{ in.}$   
 c.  $m = 25.4/18 = 1.411 \text{ mm}$   
 d.  $m = 1.5 \text{ mm}$   
 e.  $a = 1/18 = 0.0556 \text{ in.}$

f.  $b = 1.25/18 = 0.0694 \text{ in.}$   
 g.  $c = 0.25/18 = 0.0139 \text{ in.}$   
 h.  $h_t = 2.25/18 = 0.125 \text{ in.}$   
 i.  $h_k = 2/18 = 0.111 \text{ in.}$   
 j.  $t = \pi/2(18) = 0.0873 \text{ in.}$   
 k.  $D_o = 30/18 = 1.667 \text{ in.}$

9.  $N=28; P_d=20$

a.  $D = 28/20 = 1.400 \text{ in.}$   
 b.  $P_c = \pi/20 = 0.1571 \text{ in.}$   
 c.  $m = 25.4/20 = 1.27 \text{ mm}$   
 d.  $m = 1.25 \text{ mm}$   
 e.  $a = 1/20 = 0.050 \text{ in.}$

f.  $b = 1.200/20 + 0.002 = 0.0620 \text{ in.}$   
 g.  $c = 0.200/20 + 0.002 = 0.012 \text{ in.}$   
 h.  $h_t = a + b = 0.1120 \text{ in.}$   
 i.  $h_k = 2/20 = 0.1000 \text{ in.}$   
 j.  $t = \pi/2(20) = 0.0785 \text{ in.}$   
 k.  $D_o = 30/20 = 1.500 \text{ in.}$

10.  $N=34; m=3=D/N$

a.  $D = mN = 3(34) = 102 \text{ mm}$   
 b.  $P_c = \pi D/N = \pi m = \pi(3) = 9.425 \text{ mm}$   
 c.  $P_d = 25.4/m = 25.4/3 = 8.47$   
 d.  $P_d = 8$   
 e.  $a = m = 3.00 \text{ mm}$

f.  $b = 1.25m = 1.25(3) = 3.750 \text{ mm}$   
 g.  $c = 0.25m = 0.25(3) = 0.750 \text{ mm}$   
 h.  $h_t = a + b = 2.25m = 6.750 \text{ mm}$   
 i.  $h_k = 2a = 2m = 6.00 \text{ mm}$   
 j.  $t = P_c/2 = \pi m/2 = 4.712 \text{ mm}$   
 k.  $D_o = m(N+2) = 3(36) = 108 \text{ mm}$



11.  $N = 45; m = 1.25$

a.  $D = mN = 1.25(45) = 56.25 \text{ mm}$   
 b.  $P_c = \pi m = \pi(1.25) = 3.927 \text{ mm}$   
 c.  $P_d = 25.4/m = 25.4/1.25 = 20.3$   
 d.  $P_d = 20$   
 e.  $a = m = 1.25 \text{ mm}$

f.  $b = 1.25(m) = 1.25(1.25) = 1.563 \text{ mm}$   
 g.  $C = 0.25m = 0.25(1.25) = 0.313 \text{ mm}$   
 h.  $h_t = 2.25m = 2.25(1.25) = 2.813 \text{ mm}$   
 i.  $h_k = 2m = 2(1.25) = 2.500 \text{ mm}$   
 j.  $t = P_c/2 = \pi m/2 = 1.963 \text{ mm}$   
 k.  $D_o = m(N+2) = 1.25(47) = 58.75 \text{ mm}$

12.  $N = 18; m = 12$

a.  $D = mN = 12(18) = 216 \text{ mm}$   
 b.  $P_c = \pi(12) = 37.70 \text{ mm}$   
 c.  $P_d = 25.4/12 = 2.117$   
 d.  $P_d = 2$   
 e.  $a = m = 12 \text{ mm}$

f.  $b = 1.25(12) = 15.00 \text{ mm}$   
 g.  $C = 0.25(12) = 3.00 \text{ mm}$   
 h.  $h_t = 2.25(12) = 27.00 \text{ mm}$   
 i.  $h_k = 2m = 24.00 \text{ mm}$   
 j.  $t = \pi(12)/2 = 18.85 \text{ mm}$   
 k.  $D_o = m(N+2) = 12(20) = 240 \text{ mm}$

13.  $N = 22; m = 20$

a.  $D = mN = 20(22) = 440 \text{ mm}$   
 b.  $P_c = \pi(20) = 62.83 \text{ mm}$   
 c.  $P_d = 25.4/20 = 1.27$   
 d.  $P_d = 1.25$   
 e.  $a = m = 20 \text{ mm}$

f.  $b = 1.25(20) = 25.00 \text{ mm}$   
 g.  $C = 0.25(20) = 5.00 \text{ mm}$   
 h.  $h_t = 2.25(20) = 45.00 \text{ mm}$   
 i.  $h_k = 2m = 2(20) = 40.00 \text{ mm}$   
 j.  $t = \pi(20)/2 = 31.42 \text{ mm}$   
 k.  $D_o = 20(24) = 480 \text{ mm}$

14.  $N = 20; m = 1$

a.  $D = mN = 20.00 \text{ mm}$   
 b.  $P_c = \pi m = 3.14 \text{ mm}$   
 c.  $P_d = 25.4/1 = 25.4$   
 d.  $P_d = 24$   
 e.  $a = m = 1.00 \text{ mm}$

f.  $b = 1.25(1) = 1.25 \text{ mm}$   
 g.  $C = 0.25(1) = 0.25 \text{ mm}$   
 h.  $h_t = 2.25(1) = 2.25 \text{ mm}$   
 i.  $h_k = 2(1) = 2.00 \text{ mm}$   
 j.  $t = \pi(1)/2 = 1.571 \text{ mm}$   
 k.  $D_o = (1)(22) = 22.00 \text{ mm}$

15.  $N = 180; m = 0.4$

a.  $D = 0.4(180) = 72.00 \text{ mm}$   
 b.  $P_c = \pi(0.4) = 1.26 \text{ mm}$   
 c.  $P_d = 25.4/0.4 = 63.5$   
 d.  $P_d = 64$   
 e.  $a = m = 0.40 \text{ mm}$

f.  $b = 1.25(0.4) = 0.50 \text{ mm}$   
 g.  $C = 0.25(0.4) = 0.10 \text{ mm}$   
 h.  $h_t = 2.25(0.4) = 0.900 \text{ mm}$   
 i.  $h_k = 2(0.4) = 0.80 \text{ mm}$   
 j.  $t = \pi(0.4)/2 = 0.628 \text{ mm}$   
 k.  $D_o = 0.4(182) = 72.80 \text{ mm}$

16.  $N = 28 ; m = 1.5$

- a.  $D = mN = 1.5(28) = 42.00 \text{ mm}$
- b.  $P_c = \pi m = \pi(1.5) = 4.71 \text{ mm}$
- c.  $P_d = 25.4/1.5 = 16.93$
- d.  $P_d = 16$
- e.  $a = m = 1.50 \text{ mm}$

- f.  $b = 1.25(1.5) = 1.875 \text{ mm}$
- g.  $c = 0.25(1.5) = 0.375 \text{ mm}$
- h.  $h_t = 2.25(1.5) = 3.375 \text{ mm}$
- i.  $h_k = 2(1.5) = 3.00 \text{ mm}$
- j.  $t = \pi(1.5)/2 = 2.36 \text{ mm}$
- k.  $D_o = 1.5(30) = 45.00 \text{ mm}$

17.  $N = 28 ; m = 0.8$

- a.  $D = 0.8(28) = 22.40 \text{ mm}$
- b.  $P_c = \pi(0.8) = 2.51 \text{ mm}$
- c.  $P_d = 25.4/0.8 = 31.75$
- d.  $P_d = 32$
- e.  $a = m = 0.80 \text{ mm}$

- f.  $b = 1.25(0.8) = 1.00 \text{ mm}$
- g.  $c = 0.25(0.8) = 0.20 \text{ mm}$
- h.  $h_t = 2.25(0.8) = 1.80 \text{ mm}$
- i.  $h_k = 2(0.8) = 1.60 \text{ mm}$
- j.  $t = \pi(0.8)/2 = 1.257 \text{ mm}$
- k.  $D_o = 0.8(30) = 24.00 \text{ mm}$

18. BACKLASH - SEE P. 389.

19. GEAR FROM PROB. 1:  $P_d = 12$ : BACKLASH 0.006 TO 0.009 IN  
 GEAR FROM PROB. 12:  $m = 12$ : BACKLASH 0.52 TO 0.82 MM  
 DEPENDING ON CENTER DISTANCE.

Velocity Ratio

20. a.  $C = \frac{N_p + N_g}{2P_d} = \frac{18 + 64}{2(12)} = 5.125 \text{ IN.}$

b.  $VR = N_g/N_p = 64/18 = 3.556$

c.  $M_g = M_p(N_p/N_g) = 2450(18/64) = 689 \text{ RPM}$

d.  $V_g = \frac{\pi d m_p}{12} = \frac{\pi N_p m_p}{12 P_d} = \frac{\pi(18)(2450)}{12(12)} = 1443 \text{ ft/min}$

21. a.  $C = (20 + 92)/2(4) = 14.000 \text{ IN.}$

b.  $VR = 92/20 = 4.60$

c.  $M_g = 225(20/92) = 48.9 \text{ RPM}$

d.  $V_g = \frac{\pi(20)(225)}{12(4)} = 294.5 \text{ ft/min}$

22. a.  $C = (30 + 68)/2(20) = 2.450 \text{ IN.}$

b.  $VR = 68/30 = 2.267$

c.  $M_g = 850(30/68) = 375 \text{ RPM}$

d.  $V_g = \frac{\pi(30)(850)}{12(20)} = 334 \text{ ft/min}$

23. a.  $C = (40 + 250)/2(64) = 2.266 \text{ IN.}$

b.  $VR = 250/40 = 6.25$

c.  $M_g = 3450(40/250) = 552 \text{ RPM}$

d.  $V_g = \frac{\pi(40)(3450)}{12(64)} = 565 \text{ ft/min}$

24. a.  $C = (24 + 88) / 2(12) = 4.667 \text{ IN.}$  | c.  $M_G = 1750 (24/88) = 477 \text{ RPM}$   
 b.  $VR = 88/24 = 3.667$  | d.  $N_G = \frac{\pi(24)(1750)}{12(12)} = 916 \text{ ft/min}$

25. a.  $C = (N_G + N_P) m / 2 = (68 + 22)(2) / 2 = 90.00 \text{ mm}$   
 b.  $VR = N_G / N_P = 68 / 22 = 3.091$   
 c.  $M_G = m_P \cdot N_P / N_G = 1750 (22/68) = 566 \text{ RPM}$   
 d.  $N_G = R\omega = \frac{d\omega}{2} = \left[ \frac{m N_P}{2} \frac{m_P}{1} \right] \frac{(m \text{ mm})(\text{rad})}{\text{min}} \times \frac{2\pi \text{ RAD}}{180} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{m}{10^3 \text{ mm}}$   
 $N_G = \frac{m N_P m_P}{19099} \text{ m/s} = \frac{(2)(22)(1750)}{19099} = 4.03 \text{ m/s}$

26. a.  $C = (48 + 18)(0.8) / 2 = 26.40 \text{ mm}$  | d.  $N_G = \frac{(0.8)(18)(1150)}{19099}$   
 b.  $VR = 48/18 = 2.667$  |  $N_G = 0.867 \text{ m/s}$   
 c.  $M_G = 1150 (8/48) = 431 \text{ RPM}$

27. a.  $C = (45 + 36)(4) / 2 = 162 \text{ mm}$  | d.  $N_G = \frac{(4)(36)(150)}{19099} = 1.13 \text{ m/s}$   
 b.  $VR = 45/36 = 1.25$   
 c.  $M_G = 150 (36/45) = 120 \text{ RPM}$

28. a.  $C = (36 + 15)(12) / 2 = 306 \text{ mm}$  | d.  $N_G = \frac{(12)(15)(480)}{19099} = 4.52 \text{ m/s}$   
 b.  $VR = 36/15 = 2.40$   
 c.  $M_G = 480 (15/36) = 200 \text{ RPM}$

Errors in statements for Problems 29 - 32

29. PINION AND GEAR CANT HAVE DIFFERENT PITCHES

30.  $C = \frac{N_P + N_G}{2P_d} = \frac{18 + 82}{2(6)} = 8.333 \text{ IN.}$  GIVEN C IS INACCURATE BY 0.033 IN

31. TOO FEW TEETH IN THE PINION, ASSUMING 20° P.D. TEETH INTERFERENCE WOULD OCCUR.

32.  $C = \frac{N_P + N_G}{2P_d} = \frac{24 + 45}{2(16)} = 2.156 \text{ IN.}$  D<sub>o</sub> CANNOT BE USED TO FIND C.

Housing Dimensions

33. HOUSING MUST CLEAR ADDENDUM CIRCLE OF ALL GEARS BY 0.10 IN/SIDE  
 $a = 1/P_d = 1/8 = 0.125 \text{ IN.}$ ;  $D_{oG} = (N_G + 2) / P_d = 66/8 = 8.25 \text{ IN}$   
 $Y = 8.25 \text{ IN} + 2(0.10) = 8.45 \text{ IN.}$   
 $X = d + 0 + 2a + 2(0.10) = \frac{N_P}{P_d} + \frac{N_G}{P_d} + \frac{2}{P_d} + 2(0.1) = 2.250 + 8.000 + 25 + .20$   
 $X = 10.700 \text{ IN.}$

$$34. \quad D_{og} = (N_g + 2)/P_d = 252/64 = 3.9375 \text{ in}; \quad Y = 3.9375 + 0.2 = 4.1375 \text{ in} = Y$$

$$X = d + D + 2a + 2(0.1) = \frac{40}{64} + \frac{256}{64} + \frac{2}{64} + 0.20 = 4.763 \text{ in} = X$$

$$35. \quad D_{og} = (N_g + 2)m = 50(0.8) = 40.0 \text{ mm}; \quad Y = 40.0 + 2(2 \text{ mm}) = 44.00 \text{ mm}$$

$$X = d + D + 2a + 2(2) = m N_f + m N_g + 2m + 2(2)$$

$$X = 0.8(18) + 0.8(48) + 2(0.8) + 4.0 = 58.40 \text{ mm} = X$$

$$36. \quad D_{og} = 47(4) = 188 \text{ mm}; \quad Y = 188 + 2(2) = 192 \text{ mm}$$

$$X = d + D + 2a + 2(2) = m N_f + m N_g + 2m + 4 =$$

$$X = 144 + 180 + 8 + 4 = 336 \text{ mm} = X$$

### Gear Trains - Analysis

$$37. \quad TV = - \frac{N_B}{N_A} \cdot \frac{N_D}{N_C} \cdot \frac{N_F}{N_E} = - \frac{42}{18} \cdot \frac{54}{18} \cdot \frac{54}{24} = -15.75 = M_{in}/M_{out}$$

$$M_{out} = M_{in}/TV = 1750 \text{ RPM}/(-15.75) = \underline{-111 \text{ RPM CCW}}$$

$$38. \quad TV = - \frac{N_B}{N_A} \cdot \frac{N_C}{N_D} \cdot \frac{N_E}{N_F} = - \frac{68}{22} \cdot \frac{68}{25} = -8.407 = M_{in}/M_{out}$$

$$M_{out} = M_{in}/TV = 1750 \text{ RPM}/-8.407 = \underline{-208 \text{ RPM CCW}}$$

$$39. \quad TV = + \frac{D_B}{D_A} \cdot \frac{D_D}{D_C} \cdot \frac{D_F}{D_E} \cdot \frac{N_H}{N_G} = \frac{2.875}{1.250} \cdot \frac{2.375}{1.125} \cdot \frac{2.250}{1.500} \cdot \frac{30}{18} = 12.139$$

$$D_A = N_A/P_d = 20/16 = 1.250 \text{ in}$$

$$D_D = N_D/P_d = 38/16 = 2.375$$

$$D_E = N_E/P_d = 18/12 = 1.500$$

$$M_{out} = \frac{M_{in}}{TV} = \frac{1750}{12.139} = \underline{144 \text{ RPM CW}}$$

$$40. \quad TV = + \frac{N_B}{N_A} \cdot \frac{N_D}{N_C} = \frac{24}{80} \cdot \frac{18}{60} = +0.09$$

$$M_{out} = M_{in}/TV = 1750/0.09 = \underline{19444 \text{ RPM CW}}$$

Helical Gears

41 HELICAL GEAR  $P_d = 8$ ,  $\phi_t = 14\frac{1}{2}^\circ$ ,  $N = 45$  TEETH,  $F = 2.00$  IN  
 HELIX ANGLE =  $\psi = 30^\circ$ .  
 CIRCULAR PITCH =  $\underline{p} = \pi/P_d = \pi/8 = 0.3927$  IN.  
 NORMAL CIRCULAR PITCH =  $\underline{p}_n = p \cos \psi = (0.3927) \cos(30^\circ) = 0.340$  IN.  
 NORMAL DIAMETRAL PITCH =  $\underline{P}_{nd} = P_d / \cos \psi = 8 / \cos(30^\circ) = 9.238$   
 AXIAL PITCH =  $\underline{P}_x = \frac{\pi}{P_d \tan \psi} = \frac{\pi}{8 \tan(30^\circ)} = 0.680$  IN  
 PITCH DIAMETER =  $\underline{D}_G = N/P_d = 45/8 = 5.625$  IN.  
 NORMAL PRESSURE ANGLE =  $\underline{\phi}_m = \tan^{-1} [\tan \phi_t \cos \psi]$   
 $\underline{\phi}_m = \tan^{-1} [\tan(14.5^\circ) \cos(30^\circ)] = 12.62^\circ$   
 $F/P_x = 2.00 \text{ IN} / 0.680 \text{ IN} = 2.94$  AXIAL PITCHES IN FACEWIDTH

42 HELICAL GEAR  $N = 48$ ,  $P_{nd} = 12$ ,  $\phi_n = 20^\circ$ ,  $F = 1.50$  IN,  $\psi = 45^\circ$ .  
 $p = \pi/P_d$  - BUT  $P_d = P_{nd} \cos \psi = 12 \cdot \cos(45^\circ) = 8.485$   
 $\underline{p} = \pi/8.485 = 0.370$  IN ;  $\underline{p}_n = p \cos \psi = \pi/P_{nd} = \pi/12 = 0.2618$  IN.  
 $\underline{P}_x = p / \tan \psi = \frac{0.370 \text{ IN}}{\tan 45^\circ} = 0.370$  IN ;  $\underline{D}_G = N/P_d = 48/8.485 = 5.657$  IN  
 $\underline{\phi}_t = \tan^{-1} \left[ \frac{\tan \phi_m}{\cos \psi} \right] = \tan^{-1} \left[ \frac{\tan 20^\circ}{\cos 45^\circ} \right] = 27.2^\circ$   
 $F/P_x = 1.500 \text{ IN} / 0.370 \text{ IN} = 4.05$  AXIAL PITCHES IN FACE WIDTH

43 HELICAL GEAR  $N = 36$ ,  $P_d = 6$ ,  $\phi_t = 14\frac{1}{2}^\circ$ ,  $\psi = 45^\circ$ ,  $F = 1.00$  IN  
 $\underline{p} = \pi/P_d = \pi/6 = 0.5236$  IN. ;  $\underline{p}_n = p \cos \psi = \frac{\pi}{6} \cdot \cos(45^\circ) = 0.370$  IN.  
 $\underline{P}_{nd} = P_d / \cos \psi = 6 / \cos 45^\circ = 8.485$  ;  $\underline{P}_x = \frac{\pi}{P_d \tan \psi} = \frac{\pi}{6 \cdot \tan 45^\circ} = 0.5236$  IN.  
 $\underline{D} = N/P_d = 36/6 = 6.000$  IN. ;  $\underline{\phi}_m = \tan^{-1} [\tan \phi_t \cos \psi] = 10.36^\circ$   
 $F/P_x = 1.00 \text{ IN} / 0.5236 \text{ IN} = 1.91$  AXIAL PITCHES IN FACE WIDTH (LOW)

44 HELICAL GEAR  $N = 72$  ;  $P_{nd} = 24$  ;  $\phi_n = 14\frac{1}{2}^\circ$  ;  $F = 0.25$  IN,  $\psi = 45^\circ$ .  
 $p = \pi/P_d$ , BUT  $P_d = P_{nd} \cos \psi = 24 \cos 45^\circ = 16.97$   
 $\underline{p} = \pi/16.97 = 0.1851$  IN. ;  $\underline{p}_n = p \cos \psi = 0.1851 \text{ IN} \cdot \cos 45^\circ = 0.1309$  IN.  
 $\underline{P}_x = p / \tan \psi = 0.1851 / \tan 45^\circ = 0.1851$  IN. ;  $\underline{D}_G = N/P_d = 72/16.97 = 4.243$  IN.  
 $\underline{\phi}_t = \tan^{-1} \left[ \frac{\tan \phi_m}{\cos \psi} \right] = \tan^{-1} \left[ \frac{\tan(14.5^\circ)}{\cos 45^\circ} \right] = 20.0^\circ$  ;  $F/P_x = 0.25 / 0.1851 = 1.35$   
 LOW

SEE PROBLEM 49 ON NEXT PAGE FOR FORMULAS AND SYMBOLS

BEVEL GEAR GEOMETRY	
<b>PROBLEM 46</b>	
<b>GIVEN DATA</b>	
No of teeth in pinion	15
No of teeth in gear	45
Diametral pitch	5
Pressure angle	20 degrees
<b>COMPUTED VALUES</b>	
Gear ratio	3.000
Pitch diameter: Pinion	2.500 in
Pitch diameter: Gear	7.500 in
Pitch cone angle: Pinion	18.435 degrees
Pitch cone angle: Gear	71.565 degrees
Outer cone distance	3.953 in
Nominal face width	1.186 in
Maximum face width (a)	1.318 in
Maximum face width (b)	1.667 in
INPUT Face width	1.250 in
Mean cone distance	3.328 in
Ratio $A_m/A_o$	0.842
Mean circular pitch	0.441 in
mean working depth	0.281 in
Clearance	0.035 in
Mean whole depth	0.316 in
mean addendum factor	0.242
Gear mean addendum	0.068 in
Pinion mean addendum	0.213 in
Gear mean dedendum	0.248 in
Pinion mean dedendum	0.103 in
Gear dedendum angle	4.257 degrees
Pinion dedendum angle	1.774 degrees
Gear outer addendum	0.087 in
Pinion outer addendum	0.259 in
Gear outside diameter	7.555 in
Pinion outside diameter	2.992 in
<b>EXAMPLE PROBLEM 47</b>	
<b>GIVEN DATA</b>	
No of teeth in pinion	25
No of teeth in gear	50
Diametral pitch	10
Pressure angle	20 degrees
<b>COMPUTED VALUES</b>	
Gear ratio	2.000
Pitch diameter: Pinion	2.500 in
Pitch diameter: Gear	5.000 in
Pitch cone angle: Pinion	26.565 degrees
Pitch cone angle: Gear	63.435 degrees
Outer cone distance	2.795 in
Nominal face width	0.839 in
Maximum face width (a)	0.932 in
Maximum face width (b)	1.000 in
INPUT Face width	0.900 in
Mean cone distance	2.345 in
Ratio $A_m/A_o$	0.839
Mean circular pitch	0.264 in
mean working depth	0.168 in
Clearance	0.021 in
Mean whole depth	0.189 in
mean addendum factor	0.283
Gear mean addendum	0.047 in
Pinion mean addendum	0.120 in
Gear mean dedendum	0.141 in
Pinion mean dedendum	0.068 in
Gear dedendum angle	3.450 degrees
Pinion dedendum angle	1.670 degrees
Gear outer addendum	0.061 in
Pinion outer addendum	0.148 in
Gear outside diameter	5.054 in
Pinion outside diameter	2.764 in

**NOTE:** Maximum face width is the smallest of (a) or (b)



**Given:**  $N_P = 18$ ;  $N_G = 72$ ;  $P_d = 12$ ;  $20^\circ$  pressure angle.

**Computed values:**

Gear ratio	$m_G = N_G/N_P = 72/18 = 4.000$
Pitch diameter: Pinion	$d = N_P/P_d = 18/12 = 1.500$ in
Pitch diameter: Gear	$D = N_G/P_d = 72/12 = 6.000$ in
Pitch cone angle: Pinion	$\gamma = \tan^{-1}(N_P/N_G) = \tan^{-1}(18/72) = 14.03^\circ$
Pitch cone angle: Gear	$\Gamma = \tan^{-1}(N_G/N_P) = \tan^{-1}(72/18) = 75.96^\circ$
Outer cone distance	$A_o = 0.5D/\sin(\Gamma) = 0.5(6.00 \text{ in})/\sin(75.96^\circ) = 3.092$ in

Face width must be specified:  $F = 0.800$  in Based on the following guidelines:

Nominal face width:  $F_{nom} = 0.30 A_o = 0.30(3.092 \text{ in}) = 0.928$  in

Maximum face width:  $F_{max} = A_o/3 = (3.092 \text{ in})/3 = 1.031$  in

or  $F_{max} = 10/P_d = 10/12 = 0.833$  in

Mean cone distance  $A_m = A_{mG} = A_o - 0.5F = 3.092 \text{ in} - 0.5(0.80 \text{ in}) = 2.692$  in

Ratio  $(A_m/A_o) = (2.692/3.092) = 0.871$  [This ratio occurs in several following calculations]

Mean circular pitch  $p_m = (\pi/P_d)(A_m/A_o) = (\pi/12)(0.871) = 0.228$  in

Mean working depth  $h = (2.00/P_d)(A_m/A_o) = (2.00/12)(0.871) = 0.145$  in

Clearance  $c = 0.125h = 0.125(0.145 \text{ in}) = 0.018$  in

Mean whole depth  $h_m = h + c = 0.145 \text{ in} + 0.018 \text{ in} = 0.163$  in

Mean addendum factor  $c_1 = 0.210 + 0.290/(m_G)^2 = 0.210 + 0.290/(4.00)^2 = 0.228$

Gear mean addendum  $a_G = c_1 h = (0.228)(0.145 \text{ in}) = 0.033$  in

Pinion mean addendum  $a_P = h - a_G = 0.145 \text{ in} - 0.033 \text{ in} = 0.112$  in

Gear mean dedendum  $b_G = h_m - a_G = 0.163 \text{ in} - 0.033 \text{ in} = 0.130$  in

Pinion mean dedendum  $b_P = h_m - a_P = 0.163 \text{ in} - 0.112 \text{ in} = 0.051$  in

Gear dedendum angle  $\delta_G = \tan^{-1}(b_G/A_{mG}) = \tan^{-1}(0.130/2.692) = 2.76^\circ$

Pinion dedendum angle  $\delta_P = \tan^{-1}(b_P/A_{mG}) = \tan^{-1}(0.051/2.692) = 1.09^\circ$

Gear outer addendum  $a_{oG} = a_G + 0.5F \tan \delta_P$

$a_{oG} = (0.033 \text{ in}) + (0.5)(0.80 \text{ in})\tan(1.09^\circ) = 0.0406$  in

Pinion outer addendum  $a_{oP} = a_P + 0.5F \tan \delta_G$

$a_{oP} = (0.112 \text{ in}) + (0.5)(0.80 \text{ in})\tan(2.76^\circ) = 0.1313$  in

Gear outside diameter  $D_o = D + 2a_{oG} \cos \Gamma$

$D_o = 6.000 \text{ in} + 2(0.0406 \text{ in})\cos(75.96^\circ) = 6.020$  in

Pinion outside diameter  $d_o = d + 2a_{oP} \cos \gamma$

$d_o = 1.500 \text{ in} + 2(0.1313 \text{ in})\cos(14.04^\circ) = 1.755$  in



## BEVEL GEAR GEOMETRY

PROBLEM: 49	
<b>GIVEN DATA</b>	
No. of teeth in pinion	18
No. of teeth in gear	72
Diametral pitch	12
Pressure angle	20 degrees
<b>COMPUTED VALUES</b>	
Gear ratio	4.000
Pitch diameter: Pinion	1.500 in
Pitch diameter: Gear	6.000 in
Pitch cone angle: Pinion	14.036 degrees
Pitch cone angle: Gear	75.964 degrees
Outer cone distance	3.092 in
Nominal face width	0.928 in
Maximum face width (a)	1.031 in
Maximum face width (b)	0.833 in
INPUT Face width	0.800 in
Mean cone distance	2.692 in
Ratio $A_m/A_o$	0.871
Mean circular pitch	0.228 in
mean working depth	0.145 in
Clearance	0.018 in
Mean whole depth	0.163 in
mean addendum factor	0.228
Gear mean addendum	0.033 in
Pinion mean addendum	0.112 in
Gear mean dedendum	0.130 in
Pinion mean dedendum	0.051 in
Gear dedendum angle	2.767 degrees
Pinion dedendum angle	1.090 degrees
Gear outer addendum	0.041 in
Pinion outer addendum	0.131 in
Gear outside diameter	6.020 in
Pinion outside diameter	1.755 in

PROBLEM: 50	
<b>GIVEN DATA</b>	
No. of teeth in pinion	16
No. of teeth in gear	64
Diametral pitch	32
Pressure angle	20 degrees
<b>COMPUTED VALUES</b>	
Gear ratio	4.000
Pitch diameter: Pinion	0.500 in
Pitch diameter: Gear	2.000 in
Pitch cone angle: Pinion	14.036 degrees
Pitch cone angle: Gear	75.964 degrees
Outer cone distance	1.031 in
Nominal face width	0.309 in
Maximum face width (a)	0.344 in
Maximum face width (b)	0.313 in
INPUT Face width	0.300 in
Mean cone distance	0.881 in
Ratio $A_m/A_o$	0.854
Mean circular pitch	0.084 in
mean working depth	0.053 in
Clearance	0.007 in
Mean whole depth	0.060 in
mean addendum factor	0.228
Gear mean addendum	0.012 in
Pinion mean addendum	0.041 in
Gear mean dedendum	0.048 in
Pinion mean dedendum	0.019 in
Gear dedendum angle	3.113 degrees
Pinion dedendum angle	1.227 degrees
Gear outer addendum	0.015 in
Pinion outer addendum	0.049 in
Gear outside diameter	2.007 in
Pinion outside diameter	0.596 in

**NOTE:** Maximum face width is the smallest of (a) or (b)

## BEVEL GEAR GEOMETRY

### PROBLEM 8-1

#### GIVEN DATA

No. of teeth in pinion	12
No. of teeth in gear	36
Diametral pitch	48
Pressure angle	20 degrees

#### COMPUTED VALUES

Gear ratio	3.000
Pitch diameter: Pinion	0.250 in
Pitch diameter: Gear	0.750 in
Pitch cone angle: Pinion	18.435 degrees
Pitch cone angle: Gear	71.565 degrees
Outer cone distance	0.395 in

Nominal face width	0.119 in
Maximum face width (a)	0.132 in
Maximum face width (b)	0.208 in

INPUT Face width = 0.125 in

Mean cone distance	0.333 in
Ratio $A_m/A_o$	0.842
Mean circular pitch	0.055 in
mean working depth	0.035 in
Clearance	0.004 in
Mean whole depth	0.039 in
mean addendum factor	0.242
Gear mean addendum	0.008 in
Pinion mean addendum	0.027 in
Gear mean dedendum	0.031 in
Pinion mean dedendum	0.013 in
Gear dedendum angle	5.316 degrees
Pinion dedendum angle	2.217 degrees
Gear outer addendum	0.011 in
Pinion outer addendum	0.032 in
Gear outside diameter	0.757 in
Pinion outside diameter	0.311 in

### EXAMPLE PROBLEM 8-1

#### GIVEN DATA

No. of teeth in pinion	16
No. of teeth in gear	48
Diametral pitch	8
Pressure angle	20 degrees

#### COMPUTED VALUES

Gear ratio	3.000
Pitch diameter: Pinion	2.000 in
Pitch diameter: Gear	6.000 in
Pitch cone angle: Pinion	18.435 degrees
Pitch cone angle: Gear	71.565 degrees
Outer cone distance	3.162 in

Nominal face width	0.949 in
Maximum face width (a)	1.054 in
Maximum face width (b)	1.250 in

INPUT Face width = 1.000 in

Mean cone distance	2.662 in
Ratio $A_m/A_o$	0.842
Mean circular pitch	0.331 in
mean working depth	0.210 in
Clearance	0.026 in
Mean whole depth	0.237 in
mean addendum factor	0.242
Gear mean addendum	0.051 in
Pinion mean addendum	0.159 in
Gear mean dedendum	0.186 in
Pinion mean dedendum	0.077 in
Gear dedendum angle	3.992 degrees
Pinion dedendum angle	1.663 degrees
Gear outer addendum	0.065 in
Pinion outer addendum	0.194 in
Gear outside diameter	6.041 in
Pinion outside diameter	2.369 in

NOTE: Maximum face width is the smallest of (a) or (b)

Wormgearing

52

WORM GEARING :  $D_w = 1.250 \text{ IN.}$ ,  $N_w = 1$ ,  $P_d = 10$ ;  $\phi_n = 14.5^\circ$

$N_g = 40$ ;  $F = 0.625 \text{ IN.}$

SINGLE THREAD

LEAD = AXIAL PITCH = CIRCULAR PITCH =  $\frac{\pi}{P_d} = \frac{\pi}{10} = 0.3142 \text{ IN.}$

LEAD ANGLE =  $\lambda = \tan^{-1} \left( \frac{L}{\pi D_w} \right) = \tan^{-1} \left( \frac{0.3142}{\pi(1.250)} \right) = 4.57^\circ$

ADDENDUM =  $a = \frac{1}{P_d} = \frac{1}{10} = 0.100 \text{ IN.}$ ; DEDENDUM =  $\frac{1.157}{P_d} = 0.1157 \text{ IN.}$

WORM OUTSIDE DIA. =  $D_{oW} = D_w + 2a = 1.250 + 2(0.100) = 1.450 \text{ IN.}$

WORM ROOT DIA. =  $D_{rW} = D_w - 2b = 1.250 - 2(0.1157) = 1.0186 \text{ IN.}$

GEAR PITCH DIA. =  $D_g = N_g / P_d = 40 / 10 = 4.000 \text{ IN.}$

CENTER DISTANCE =  $C = (D_g + D_w) / 2 = (4.000 + 1.250) / 2 = 2.625 \text{ IN.}$

VELOCITY RATIO =  $VR = N_g / N_w = 40 / 1 = 40$

NOTE: On the following two pages are the results of Problems 52-57 giving pertinent geometric properties of worms and wormgears and their velocity ratios. The detailed calculations follow the pattern illustrated above for Problem 52. The equations come from Section 8-9, Equations 8-20 to 8-25.

Compare the results to discern how variations in geometry such as diametral pitch and the number of threads in the worm affect the overall results. This is especially pertinent to Problem 53 in which three different designs for worm/wormgear sets provide the same velocity ratio. The single threaded worm produces the smallest center distance and overall size of the reducer. But note, also, that it has the smallest lead angle. The lead angle increases as the number of threads is increased. On the positive side, the small lead angle makes the reducer self-locking. On the negative side, the small lead angle results in lower mechanical efficiency as will be shown in Chapter 10, Section 10-11. The designer must balance these advantages and disadvantages for each application.

**WORMGEARING**                      **PROBLEM: 52****INPUT DATA**

Worm pitch diameter = 1.250 in  
 Diametral pitch = 10  
 No. of worm threads = 1  
 No. of gear teeth = 40  
 Face width of gear = 0.625 in

**COMPUTED RESULTS**

Circular pitch of gear = 0.3142 in  
 Axial pitch of worm = 0.3142 in  
 Lead of the worm = 0.3142 in  
 Lead angle = 4.574 deg  
 Addendum = 0.100 in  
 Dedendum = 0.116 in  
 Worm outside diameter = 1.450 in  
 Worm root diameter = 1.019 in  
 Gear pitch diameter = 4.000 in  
 Center distance = 2.625 in  
 Velocity ratio = 40.00

**WORMGEARING**                      **PROBLEM: 53A****INPUT DATA**

Worm pitch diameter = 1.000 in  
 Diametral pitch = 12  
 No. of worm threads = 1  
 No. of gear teeth = 20  
 Face width of gear = 0.500 in

**COMPUTED RESULTS**

Circular pitch of gear = 0.2618 in  
 Axial pitch of worm = 0.2618 in  
 Lead of the worm = 0.2618 in  
 Lead angle = 4.764 deg  
 Addendum = 0.083 in  
 Dedendum = 0.096 in  
 Worm outside diameter = 1.167 in  
 Worm root diameter = 0.807 in  
 Gear pitch diameter = 1.667 in  
 Center distance = 1.333 in  
 Velocity ratio = 20.00

**WORMGEARING**                      **PROBLEM: 53B****INPUT DATA**

Worm pitch diameter = 1.000 in  
 Diametral pitch = 12  
 No. of worm threads = 2  
 No. of gear teeth = 40  
 Face width of gear = 0.500 in

**COMPUTED RESULTS**

Circular pitch of gear = 0.2618 in  
 Axial pitch of worm = 0.2618 in  
 Lead of the worm = 0.5236 in  
 Lead angle = 9.462 deg  
 Addendum = 0.083 in  
 Dedendum = 0.096 in  
 Worm outside diameter = 1.167 in  
 Worm root diameter = 0.807 in  
 Gear pitch diameter = 3.333 in  
 Center distance = 2.167 in  
 Velocity ratio = 20.00

**WORMGEARING**                      **PROBLEM: 53C****INPUT DATA**

Worm pitch diameter = 1.000 in  
 Diametral pitch = 12  
 No. of worm threads = 4  
 No. of gear teeth = 80  
 Face width of gear = 0.500 in

**COMPUTED RESULTS**

Circular pitch of gear = 0.2618 in  
 Axial pitch of worm = 0.2618 in  
 Lead of the worm = 1.0472 in  
 Lead angle = 18.435 deg  
 Addendum = 0.083 in  
 Dedendum = 0.096 in  
 Worm outside diameter = 1.167 in  
 Worm root diameter = 0.807 in  
 Gear pitch diameter = 6.667 in  
 Center distance = 3.833 in  
 Velocity ratio = 20.00

**WORMGEARING**                      **PROBLEM: 54****INPUT DATA**

Worm pitch diameter = 0.625 in  
 Diametral pitch = 16  
 No. of worm threads = 2  
 No. of gear teeth = 100  
 Face width of gear = 0.313 in

**COMPUTED RESULTS**

Circular pitch of gear = 0.1963 in  
 Axial pitch of worm = 0.1963 in  
 Lead of the worm = 0.3927 in  
 Lead angle = 11.310 deg  
 Addendum = 0.063 in  
 Dedendum = 0.072 in  
 Worm outside diameter = 0.750 in  
 Worm root diameter = 0.480 in  
 Gear pitch diameter = 6.250 in  
 Center distance = 3.438 in  
 Velocity ratio = 50.00

**WORMGEARING**                      **PROBLEM: 55****INPUT DATA**

Worm pitch diameter = 2.000 in  
 Diametral pitch = 6  
 No. of worm threads = 4  
 No. of gear teeth = 72  
 Face width of gear = 1.000 in

**COMPUTED RESULTS**

Circular pitch of gear = 0.5236 in  
 Axial pitch of worm = 0.5236 in  
 Lead of the worm = 2.0944 in  
 Lead angle = 18.435 deg  
 Addendum = 0.167 in  
 Dedendum = 0.193 in  
 Worm outside diameter = 2.333 in  
 Worm root diameter = 1.614 in  
 Gear pitch diameter = 12.000 in  
 Center distance = 7.000 in  
 Velocity ratio = 18.00

**WORMGEARING**                      **PROBLEM: 56****INPUT DATA**

Worm pitch diameter = 4.000 in  
 Diametral pitch = 3  
 No. of worm threads = 1  
 No. of gear teeth = 54  
 Face width of gear = 2.000 in

**COMPUTED RESULTS**

Circular pitch of gear = 1.0472 in  
 Axial pitch of worm = 1.0472 in  
 Lead of the worm = 1.0472 in  
 Lead angle = 4.764 deg  
 Addendum = 0.333 in  
 Dedendum = 0.386 in  
 Worm outside diameter = 4.667 in  
 Worm root diameter = 3.229 in  
 Gear pitch diameter = 18.000 in  
 Center distance = 11.000 in  
 Velocity ratio = 54.00

**WORMGEARING**                      **PROBLEM: 57****INPUT DATA**

Worm pitch diameter = 0.333 in  
 Diametral pitch = 48  
 No. of worm threads = 4  
 No. of gear teeth = 80  
 Face width of gear = 0.156 in

**COMPUTED RESULTS**

Circular pitch of gear = 0.0654 in  
 Axial pitch of worm = 0.0654 in  
 Lead of the worm = 0.2618 in  
 Lead angle = 14.050 deg  
 Addendum = 0.021 in  
 Dedendum = 0.024 in  
 Worm outside diameter = 0.375 in  
 Worm root diameter = 0.285 in  
 Gear pitch diameter = 1.667 in  
 Center distance = 1.000 in  
 Velocity ratio = 20.00

Gear Trains - Analysis

FOR PROBLEM 58 - ASSUME THAT THE INPUT SHAFT ROTATES CLOCKWISE.

58

TRAIN VALUE = TV =  $m_1/m_6$  ;  $m_1 = 3450 \text{ RPM}$

$$TV = \frac{-N_B}{N_A} \cdot \frac{N_D}{N_C} \cdot \frac{N_F}{N_E} \cdot \frac{N_H}{N_G} \cdot \frac{N_I}{N_H} = \frac{-82}{18} \cdot \frac{64}{17} \cdot \frac{110}{20} \cdot \frac{18}{18} \cdot \frac{38}{18} = -119.1$$

$$m_6 = \frac{m_1}{TV} = \frac{3450 \text{ RPM}}{-119.1} = \underline{-17.32 \text{ RPM}} \text{ COUNTERCLOCKWISE}$$

GEAR H IS AN IDLER. IT DOES NOT AFFECT THE TV BUT CHANGES THE DIRECTION OF THE OUTPUT SHAFT.

59

$m_1 = 12200 \text{ RPM}$  ; FIND  $m_5$  :  $TV = m_1/m_5$

$$TV = \frac{N_B}{N_A} \cdot \frac{N_D}{N_C} \cdot \frac{N_F}{N_E} \cdot \frac{N_H}{N_G} = \frac{50}{12} \cdot \frac{40}{12} \cdot \frac{60}{1} \cdot \frac{72}{2} = 30000$$

$$m_5 = \frac{m_1}{TV} = \frac{12200 \text{ RPM}}{30000} = \underline{0.4067 \text{ RPM}}$$

60

$m_1 = 6840 \text{ RPM}$  ; FIND  $m_4$  ;  $TV = m_1/m_4$

$$TV = \frac{N_B}{N_A} \cdot \frac{N_D}{N_C} \cdot \frac{N_F}{N_E} = \frac{48}{16} \cdot \frac{48}{18} \cdot \frac{60}{12} = 40 \text{ EXACTLY}$$

$$m_4 = \frac{m_1}{TV} = \frac{6840}{40} = \underline{171 \text{ RPM EXACTLY}}$$

61

$m_1 = 2875 \text{ RPM}$  ; FIND  $m_4$  :  $TV = m_1/m_4$

$$TV = \frac{N_B}{N_A} \cdot \frac{N_D}{N_C} \cdot \frac{N_F}{N_E} = \frac{100}{3} \cdot \frac{80}{2} \cdot \frac{85}{20} = 5666.7$$

$$m_4 = \frac{m_1}{TV} = \frac{2875 \text{ RPM}}{5666.7} = \underline{0.5074 \text{ RPM}}$$



**Gear Trains - Kinematic Design**

VELOCITY RATIO FOR GEARS <i>PROBLEM 62</i>						
DESIRED VR = 3.1416 = $\pi$						
NP	NG	NG Act	VR-Act	DIFF =		
				Des VR - VR Act		
16	50.27	50	3.1250	0.01659		
17	53.41	53	3.1176	0.02395		
18	56.55	57	3.1667	0.02507		
19	59.69	60	3.1579	0.01630		
20	62.83	63	3.1500	0.00841		
XX	21	65.97	66	3.1429	0.00126	XX
22	69.12	69	3.1364	0.00523		
23	72.26	72	3.1304	0.01116		
24	75.40	75	3.1250	0.01659		
				Min diff =	0.00126	

VELOCITY RATIO FOR GEARS <i>PROBLEM 63</i>						
DESIRED VR = 1.7321 = $\sqrt{3}$						
NP	NG	NG	VR	DIFF =		
		Actual	Actual	Des VR - VR Act		
16	27.71	28	1.7500	0.01795		
17	29.44	29	1.7059	0.02617		
18	31.18	31	1.7222	0.00983		
19	32.91	33	1.7368	0.00479		
20	34.64	35	1.7500	0.01795		
21	36.37	36	1.7143	0.01777		
XX	22	38.11	38	1.7273	0.00478	XX
23	39.84	40	1.7391	0.00708		
24	41.57	42	1.7500	0.01795		
				Min diff =	0.00478	

VELOCITY RATIO FOR GEARS <i>PROBLEM 64</i>						
DESIRED VR = 6.1644 = $\sqrt{38}$						
NP	NG	NG Act	VR-Act	DIFF =		
				Des VR - VR Act		
16	98.63	99	6.1875	0.02309		
17	104.80	105	6.1765	0.01206		
XX	18	110.96	111	6.1667	0.00225	XX
19	117.12	117	6.1579	0.00652		
20	123.29	123	6.1500	0.01441		
21	129.45	129	6.1429	0.02156		
22	135.62	136	6.1818	0.01740		
23	141.78	142	6.1739	0.00950		
XX	24	147.95	148	6.1667	0.00225	XX
<i>Two EQUAL SOLUTIONS</i>				Min diff =	0.00225	

VELOCITY RATIO FOR GEARS <i>PROBLEM 65</i>						
DESIRED VR = 7.42						
NP	NG	NG	VR	DIFF =		
		Actual	Actual	Des VR - VR Act		
16	118.72	119	7.4375	0.01750		
17	126.14	126	7.4118	0.00824		
18	133.56	134	7.4444	0.02444		
XX	19	140.98	141	7.4211	0.00105	XX
20	148.40	148	7.4000	0.02000		
21	155.82	156	7.4286	0.00857		
22	163.24	163	7.4091	0.01091		
23	170.66	171	7.4348	0.01478		
24	178.08	178	7.4167	0.00333		
				Min diff =	0.00105	

66

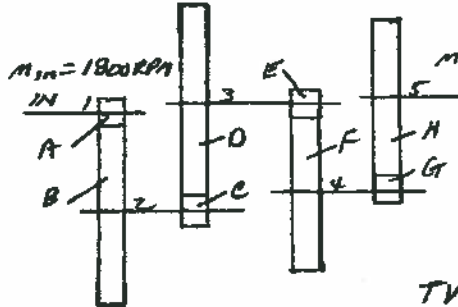
DESIGN:  $M_{in} = 1800 \text{ RPM}$   $M_{out} = 2 \text{ RPM}$  EXACT RATIO RED'D.

$TV = 1800/2 = 900$  EXACT; USE FACTORING:  $N_{max} = 150$

2 | 900  
 2 | 450  
 5 | 225  
 5 | 45  
 3 | 9  
 3

FACTORS ARE: 2-2-5-5-3-3  
 SEE TABLE B-6 FOR INTERFERENCE DATA  
 FOR 20° F.D. TEETH USE  $N_{min} = 16$  OR 17  
 NOMINAL  $VR_{max}$  PER PAIR =  $150/17 = 8.82$  TOO SMALL  
 TWO PAIRS:  $(8.82)^2 = 77.8$  SMALL  
 THREE PAIRS:  $(8.82)^3 = 687$  SMALL  
 FOUR PAIRS:  $(8.82)^4 = 6061$  RED'D.

RECOMBINE FACTORS:  $VR1 = 6, VR2 = 6, 1:3 = 5, VR4 = 5$



$N_A = 16$		$N_E = 16$		
$N_B = 96$		$N_F = 80$		5
$N_C = 16$		$N_G = 16$		5
$N_D = 96$		$N_H = 80$		

TABLE B-6 SAYS NO INTERFERENCE FOR  $N_p = 16$  IF  $N_g < 101$ .

$TV = \frac{96}{16} \frac{96}{16} \frac{80}{16} \frac{80}{16} = 900$  EXACT

67

DESIGN:  $M_{in} = 1800 \text{ RPM}$  EXACT;  $21 < M_{out} < 22$ ;  $N_{max} = 150$  & 20° F.D.

$TV_{Nom} = 1800/21.5 = 83.7$   $TV_{min} = 1800/22 = 81.8$   $TV_{max} = 1800/21 = 85.7$

FROM TABLE B-6, NO INTERFERENCE WITH  $N_p \geq 17$  FOR 20° F.D. TEETH

$VR_{max}$  PER PAIR =  $150/17 = 8.82$  SMALL; TWO PAIRS  $VR_{max} = (8.82)^2 = 77.9$  LOW

LAYOUT AS IN FIG. B-31 IN TEXT - TRIPLE REDUCTION.

TRY EQUAL REDUCTION RATIO:  $VR1 = VR2 = VR3 = \sqrt[3]{83.7} = 4.37$

LET  $N_A = N_C = N_E = 17$ ; LET  $VR1 = 5, VR2 = 4$ ; THEN  $VR3 = 83.7/20 = 4.19$

$N_F = (17)(4.19) = 71.2 \Rightarrow$  SPECIFY  $N_F = 71$

FINAL  $TV = \frac{85}{17} \frac{68}{17} \frac{71}{17} = 83.53$

$M_{out} = \frac{M_{in}}{TV} = \frac{1800}{83.53} = 21.55 \text{ RPM}$   
 OK

68

DESIGN:  $M_{in} = 3360 \text{ RPM}$  EXACT;  $M_{out} = 12 \text{ RPM}$  EXACT;  $N_{max} = 150$

20° F.D. TEETH. FROM TABLE B-6 LET  $N_{min} = 17$  FOR NO INTERFERENCE

$VR_{max}$  PER PAIR =  $150/17 = 8.82$ ; 2 PAIRS  $VR_{max} = (8.82)^2 = 77.8$ ; 3 PAIRS = 686

$TV = 3360/12 = 280$  EXACT; USE 3 PAIRS SIMILAR TO FIG B-31 IN TEXT

FACTORING: 2 | 280 2-2-5-2-7 = 280. RECOMBINE 8-7-5 = 280

2 | 280  
 2 | 140  
 5 | 70  
 2 | 14  
 7

$VR1 = 8$ ; $N_A = 17, N_B = 136$
$VR2 = 7$ ; $N_C = 17, N_D = 119$
$VR3 = 5$ ; $N_E = 17, N_F = 85$

$TV = \frac{136}{17} \frac{119}{17} \frac{85}{17} = 280$  EXACTLY

(OTHER DESIGNS POSSIBLE)

69

DESIGN:  $M_{in} = 4200 \overset{CW}{RPM}$  EXACTLY:  $13.0 < M_{out} < 13.5 \overset{CW}{RPM}$ : POSITIVE TV

$$TV_{min} = \frac{4200}{13.5} = 311.11; TV_{nom} = \frac{4200}{13.25} = 316.98; TV_{max} = \frac{4200}{13.0} = 323.08$$

FROM PROB 68, 3 PAIRS REQ'D. LAYOUT IN FIG 8-31 PRODUCES A NEGATIVE TV. USE IDLER IN ANY PAIR.

TRY RESIDUAL RATIO METHOD. NOMINAL VR =  $\sqrt[3]{317} = 6.82$  PER PAIR

TRY  $VR_1 = 7; VR_2 = 6$ ; THEN  $VR_3 \approx 317/42 \approx 7.55$ : USE  $VR_3 = 7.50$

$$FINAL TV = VR_1 \cdot VR_2 \cdot VR_3 = (7)(6)(7.50) = 315 \text{ OK}$$

IT IS PREFERRED TO PLACE HIGHER RATIOS EARLY IN THE TRAIN. LET  $VR_1 = 7.5, VR_2 = 7, VR_3 = 6$ .

LET  $N_A = 18; N_B = 7.5(18) = 135$ ; LET  $N_C = 17, N_D = 7(17) = 119$

LET  $N_E = 17, N_F = 17(6) = 102$ .

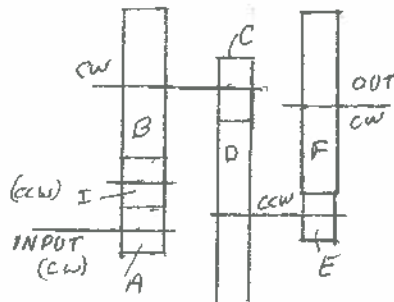
IDLER NEEDED FOR POSITIVE TRAIN

LET  $N_I = 17$ . PLACE IN FIRST PAIR, FINAL TRAIN VALUE:

$$TV = \frac{135}{18} \times \frac{119}{17} \times \frac{102}{17} = 315$$

$$M_{out} = \frac{M_{in}}{TV} = \frac{4200}{315} = 13.33 \text{ RPM CW}$$

OK



NOTE: CHAPTER 9 GIVES INFORMATION ON SELECTION OF  $P_d$ -DIAMETRAL PITCH. BECAUSE OF SPEED/TORQUE CHANGES,

$P_{d1} > P_{d2} > P_{d3}$ , LARGER  $P_d$  GIVES SMALLER GEARS.

THIS IS THE REASON THAT LARGER RATIOS SHOULD BE PLACED EARLIER IN THE TRAIN.

70

DESIGN:  $M_{in} = 5500 \text{ RPM}$  EXACTLY:

$$221 < M_{out} < 225$$

DESIGN: TWO DOUBLE REDUCTION WITH ALL EXTERNAL GEARS

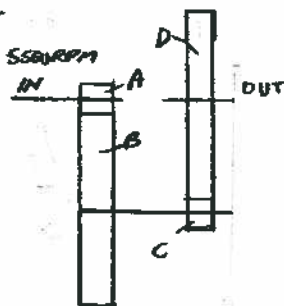
$$TV_{max} = \frac{5500}{221} = 24.88; TV_{min} = \frac{5500}{225} = 24.44; TV_{nom} = \frac{5500}{223} = 24.66$$

NOMINAL RATIO FOR EACH PAIR =  $\sqrt{24.66} = 4.97$ . TRY  $VR_1 = 5$

THEN  $VR_2 \approx 24.66/5 = 4.93$ : FOR  $N_F = 16, N_G = 78.9$  - USE  $N_G = 79$

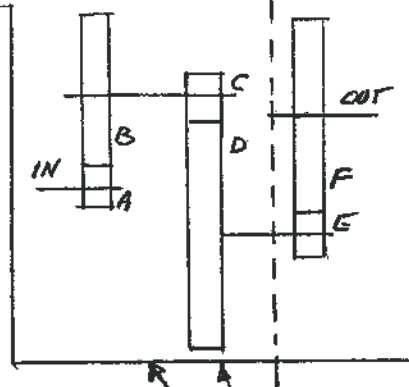
$$FINAL TV = \frac{N_B \cdot N_D}{N_A \cdot N_C} = \frac{80 \cdot 79}{16 \cdot 16} = 24.6875$$

$$M_{out} = \frac{5500}{TV} = 222.78 \text{ RPM OK}$$



71 DESIGN:  $M_{IN} = 5500 \text{ RPM}$      $13.0 < M_{OUT} < 14.0 \text{ RPM}$   
 $TV_{NOM} = 5500/13.5 = 407.4$     SKETCH AS IN 70.  
 MAX RATIO FOR ONE PAIR =  $150/17 = 8.82$   
 TWO PAIRS - MAX = 77.85; THREE PAIRS 687 - OK  
 NOMINAL RATIO PER PAIR:  $\sqrt[3]{407.4} = 7.41$   
 TRY  $VR_1 = 8, VR_2 = 8$  - BUT USE HUNTING TOOTH APPROACH.  
 $VR = 8: N_A = 17, N_B = 17(8) = 136$ ; USE  $N_B = 135$   
 SAME FOR  $N_C, N_D$ .  
 $(VR_1)(VR_2) = \left(\frac{135}{17}\right)^2 = 7.94^2 = 63.06$   
 RESIDUAL RATIO:  $407.4/63.06 = 6.46 = N_F/N_E$   
 LET  $N_E = 17$ ;  $N_F = 6.46(17) = 109.82 \Rightarrow$  USE 110 TEETH  
 $FINAL TV = \frac{135}{17} \times \frac{135}{17} \times \frac{110}{17} = 408.05$   
 FINAL OUTPUT SPEED =  $5500/408.05 = 13.48 \text{ RPM} - OK$

72 DESIGN:  $M_{IN} = 1750$ ;     $146 < M_{OUT} < 150$   
 $TV_{NOM} = 1750/148 = 11.82$   
 LET  $VR_1 = N_B/N_A = 75/18 = 4.167$   
 RESIDUAL RATIO =  $11.82/4.167 = 2.837$   
 LET  $N_C = 18$ ;  $N_D = 18(2.837) = 51.06 \Rightarrow 51$   
 $N_A = 18, N_B = 75, N_C = 18, N_D = 51$   
 $M_{OUT} = 1750 \times \frac{18}{75} \times \frac{18}{51} = 148.2 \text{ RPM} OK$



SKETCH SAME AS 71 WITH ONLY TWO PAIRS  
 [THESE RESULTS USED IN PROBLEM 9-74.]

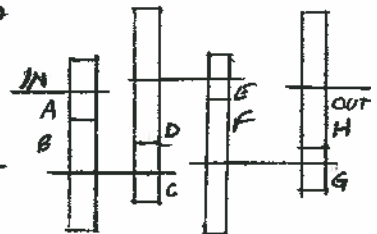
73 DESIGN:  $M_{IN} = 850 \text{ RPM}$ ;     $40 < M_{OUT} < 44$ ; USE 2 PAIRS  
 $TV_{NOM} = 850/42 = 20.24$ , LET  $VR_1 = N_B/N_A = 81/18 = 4.50$   
 RESIDUAL RATIO =  $VR_2 = 20.24/4.50 = 4.50$ ;  $N_C = 18, N_D = 81$   
 $M_{OUT} = 850 \times \frac{18}{81} \times \frac{18}{81} = 41.98 \text{ RPM} OK$   
 [THESE RESULTS USED IN PROBLEM 9-75.]

74 DESIGN: USE TWO PAIRS:  $M_{IN} = 3000 \text{ RPM}$ ;     $548 < M_{OUT} < 552$   
 $TV_{NOM} = 3000/550 = 5.4545$ ; LET  $VR_1 = VR_2 = \sqrt{5.4545} = 2.335$   
 LET  $N_A = 15$ ;  $N_B = 15(2.335) = 35.03 \Rightarrow 35$ . LET  $N_C = 15, N_D = 35$   
 $M_{OUT} = 3000 \times \frac{15}{35} \times \frac{15}{35} = 551 \text{ RPM} OK$   
 [THESE RESULTS USED IN PROBLEM 9-76.]

75 DESIGN:  $M_{IN} = 3600 \text{ RPM}$   $3.0 < M_{OUT} < 5.0$

$TV_{NOM} = 3600/4.0 = 900$  USE 4 PAIRS

FACTORING:  $2 \overline{) 900}$  USE  $VR_1 = 6 = 96/16 = N_B/N_A$   
 $2 \overline{) 450}$   $VR_2 = 6 = 96/16 = N_D/N_C$   
 $5 \overline{) 225}$   $VR_3 = 5 = 80/16 = N_E/N_F$   
 $5 \overline{) 45}$   $VR_4 = 5 = 80/16 = N_H/N_G$   
 $3 \overline{) 9}$



ALTERNATE SOLUTION USING HUNTING TOOTH:

LET  $N_A = N_C = N_E = N_G = 16$ . LET  $N_B = N_D = 96$ . LET  $N_F = 81$

$VR_1 = VR_2 = 96/16 = 6$ ;  $VR_3 = 81/16 = 5.0625$

$VR_1 \times VR_2 \times VR_3 = 178.47$ , RESIDUAL RATIO =  $900/178.47 = 5.043$

LET  $N_H = 81$ ,  $VR_4 = VR_3 = 81/16 = 5.0625$

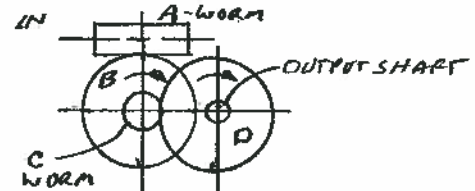
TOTAL  $TV = (178.47)(5.0625) = 903.5$

FINAL  $M_{OUT} = 3600/903.5 = 3.984 \text{ RPM OK}$

76 DESIGN:  $M_{IN} = 3600$   $3.0 < M_{OUT} < 5.0 \text{ RPM}$

$TV_{NOM} = 3600/4.0 = 900$  USE TWO PAIRS OF WORM/WORM GEARS

$N_A = N_C = 1$ ;  $N_B = N_D = 30$



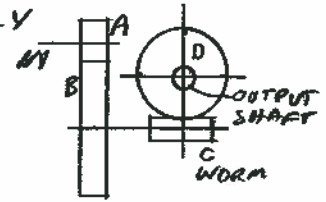
77 DESIGN:  $M_{IN} = 1800 \text{ RPM}$   $M_{OUT} = 8.0 \text{ EXACTLY}$

LET  $VR_2 = 50$  - WORM GEAR DRIVE.

$TV = 1800/8 = 225$ ;  $VR_1 = 225/50 = 4.50$   
 HELICAL GEARS

LET  $N_A = 16$ ,  $N_B = 72$ ,  $N_C = 1$ ,  $N_D = 50$

$M_{OUT} = 1800 \times \frac{16}{72} \times \frac{1}{50} = 8.0 \text{ RPM}$



78 DESIGN:  $M_{IN} = 3360 \text{ RPM}$ .  $M_{OUT} = 12.0 \text{ EXACTLY}$ ,  
 USE TWO PAIRS OF WORM GEAR DRIVES AS IN PROBLEM 76.  
 LET  $VR_1 = 20$ ,  $VR_2 = 14$ :  $N_A = 2$ ,  $N_B = 40$ ,  $N_C = 2$ ,  $N_D = 28$ .

79 DESIGN:  $m_{IN} = 4200 \text{ RPM}$ ,  $13.0 < m_{OUT} < 13.5$

USE COMBINED HELICAL WITH WORM GEAR AS IN PROBLEM 77.

LET  $VR_2 = 50$ . WORM GEAR DRIVE,  $N_C = 1, N_D = 50$

$$TV_{NOM} = 4200/13.25 = 316.98. VR_1 = 316.98/50 = 6.34$$

LET  $N_A = 18$ .  $N_B = 18(6.34) = 114.1 \Rightarrow \text{USE } 114 = N_B$

$$\text{FINAL OUTPUT SPEED} = 4200 \times \frac{18}{114} \times \frac{1}{50} = \underline{13.26 \text{ RPM OK}}$$

80. DESIGN:  $m_{IN} = 5500 \text{ RPM}$   $13.0 < m_{OUT} < 14.0 \text{ RPM}$

USE TWO WORM GEAR DRIVES AS IN PROBLEM 76.

$$TV_{NOM} = 5500/13.5 = 407.4.$$

$$\text{LET } VR_1 = 20. \text{ THEN } VR_2 = 407.4/20 = 20.37$$

$$\text{TRY } N_C = 3. N_D = 3(20.37) = 61.11 \Rightarrow \text{USE } 61$$

$$N_A = 3, N_B = 60, N_C = 3, N_D = 61$$

$$\text{FINAL OUTPUT SPEED} = 5500 \times \frac{3}{60} \times \frac{3}{61} = \underline{13.52 \text{ RPM OK}}$$

CAN ALSO USE  $N_D = 62$ . THEN  $m_{OUT} = \underline{13.30 \text{ RPM}}$

$N_D = 60$ . THEN  $m_{OUT} = \underline{13.75 \text{ RPM}}$



## CHAPTER 9 SPUR GEAR DESIGN

### Forces on Spur Gear Teeth

- 1 GIVEN:  $\phi = 20^\circ$ ,  $P = 7.5 \text{ kP}$ ,  $M_P = 1750 \text{ RPM}$ ,  $N_P = 20$ ,  $N_G = 72$ ,  $P_d = 12$
- a)  $m_G = M_P \cdot \frac{N_P}{N_G} = 1750 \text{ RPM} \cdot \frac{20}{72} = \underline{486.1 \text{ RPM}}$
- b)  $V_R = m_G = N_G / N_P = 72 / 20 = \underline{3.600}$
- c)  $D_P = N_P / P_d = 20 / 12 = \underline{1.667 \text{ IN}}$ ;  $D_G = N_G / P_d = 72 / 12 = \underline{6.000 \text{ IN}}$
- d)  $C = \frac{N_P + N_G}{2 P_d} = \frac{20 + 72}{2(12)} = \underline{3.833 \text{ IN}}$
- e)  $N_t = \pi D_P m_P / 12 = \pi (1.667)(1750) / 12 = \underline{764 \text{ FT/MIN}}$
- f)  $T_P = \frac{63000(P)}{M_P} = \frac{63000(7.5)}{1750} = \underline{270 \text{ LB}\cdot\text{IN}}$   
 $T_G = \frac{63000(P)}{M_G} = \frac{63000(7.5)}{486.1} = \underline{972 \text{ LB}\cdot\text{IN}}$
- g)  $W_t = \frac{T_P}{D_P/2} = \frac{270 \text{ LB}\cdot\text{IN}}{1.667 \text{ IN}/2} = \underline{324 \text{ LB}}$   
 OR  $W_t = \frac{33000(P)}{N_t} = \frac{33000(7.5)}{764} = \underline{324 \text{ LB}}$
- h)  $W_R = W_t \tan \phi = (324 \text{ LB}) \tan 20^\circ = \underline{118 \text{ LB}}$
- i)  $W_N = W_t / \cos \phi = 324 \text{ LB} / \cos 20^\circ = \underline{345 \text{ LB}}$

A SIMILAR METHOD IS USED FOR PROBLEMS 2-6.  
 SPREADSHEET SOLUTIONS ARE SHOWN ON THE FOLLOWING  
 PAGES. THE SOLUTION FOR PROBLEM 1 IS ALSO SHOWN  
 FOR COMPARISON TO THE SOLUTION SHOWN ABOVE.

### Forces on Spur Gear Teeth

<b>Problem 1</b>	<i>Pressure angle =</i>	<i>20</i>	<i>degrees</i>
<b>Chapter 9</b>	<i>Power =</i>	<i>7.5</i>	<i>hp</i>
	<i>pinion speed =</i>	<i>1750</i>	<i>rpm</i>
	<i>teeth in pinion =</i>	<i>20</i>	
	<i>teeth in gear =</i>	<i>72</i>	
	<i>diametral pitch =</i>	<i>12</i>	
<b>RESULTS:</b>			
a	Gear speed =	486.1	rpm
b	VR = $m_G$ =	3.600	
c	pinion PD =	1.667	in
	gear PD =	6.000	in
d	center distance = C =	3.833	in
e	pitch line speed =	764	ft/min
f	torque on pinion shaft =	270	lb in
	torque on gear shaft =	972	lb in
g	tangential force =	324	lb
h	radial force =	118	lb
i	normal force =	345	lb

<b>Problem 2</b>	<i>Pressure angle =</i>	<i>20</i>	<i>degrees</i>
<b>Chapter 9</b>	<i>Power =</i>	<i>50</i>	<i>hp</i>
	<i>pinion speed =</i>	<i>1150</i>	<i>rpm</i>
	<i>teeth in pinion =</i>	<i>18</i>	
	<i>teeth in gear =</i>	<i>68</i>	
	<i>diametral pitch =</i>	<i>5</i>	
<b>RESULTS:</b>			
a	Gear speed =	304.4	rpm
b	VR = $m_G$ =	3.778	
c	pinion PD =	3.600	in
	gear PD =	13.600	in
d	center distance = C =	8.600	in
e	pitch line speed =	1084	ft/min
f	torque on pinion shaft =	2739	lb in
	torque on gear shaft =	10348	lb in
g	tangential force =	1522	lb
h	radial force =	554	lb
i	normal force =	1620	lb

**Forces on Spur Gear Teeth**

<b>Problem 3</b>	Pressure angle = 20 degrees
<b>Chapter 9</b>	Power = 0.75 hp
	pinion speed = 3450 rpm
	teeth in pinion = 24
	teeth in gear = 110
	diametral pitch = 24
<b>RESULTS:</b>	
a	Gear speed = 752.7 rpm
b	$VR = m_G = 4.583$
c	pinion PD = 1.000 in
	gear PD = 4.583 in
d	center distance = C = 2.792 in
e	pitch line speed = 903 ft/min
f	torque on pinion shaft = 13.70 lb in
	torque on gear shaft = 62.77 lb in
g	tangential force = 27.40 lb
h	radial force = 9.97 lb
i	normal force = 29.16 lb

<b>Problem 4</b>	Pressure angle = 26 degrees
<b>Chapter 9</b>	Power = 7.5 hp
	pinion speed = 1750 rpm
	teeth in pinion = 20
	teeth in gear = 72
	diametral pitch = 12
<b>RESULTS:</b>	
a	Gear speed = 486.1 rpm
b	$VR = m_G = 3.600$
c	pinion PD = 1.667 in
	gear PD = 6.000 in
d	center distance = C = 3.833 in
e	pitch line speed = 764 ft/min
f	torque on pinion shaft = 270 lb in
	torque on gear shaft = 972 lb in
g	tangential force = 324 lb
h	radial force = 151 lb
i	normal force = 358 lb

### Forces on Spur Gear Teeth

<b>Problem 5</b>	<i>Pressure angle =</i>	<i>25</i>	<i>degrees</i>
<b>Chapter 9</b>	<i>Power =</i>	<i>50</i>	<i>hp</i>
	<i>pinion speed =</i>	<i>1150</i>	<i>rpm</i>
	<i>teeth in pinion =</i>	<i>18</i>	
	<i>teeth in gear =</i>	<i>68</i>	
	<i>diametral pitch =</i>	<i>5</i>	
<b>RESULTS:</b>			
a	Gear speed =	304.4	rpm
b	$VR = m_G =$	3.778	
c	pinion PD =	3.600	in
	gear PD =	13.600	in
d	center distance = C =	8.600	in
e	pitch line speed =	1084	ft/min
f	torque on pinion shaft =	2739	lb in
	torque on gear shaft =	10348	lb in
g	tangential force =	1522	lb
h	radial force =	710	lb
i	normal force =	1680	lb

<b>Problem 6</b>	<i>Pressure angle =</i>	<i>25</i>	<i>degrees</i>
<b>Chapter 9</b>	<i>Power =</i>	<i>0.75</i>	<i>hp</i>
	<i>pinion speed =</i>	<i>3450</i>	<i>rpm</i>
	<i>teeth in pinion =</i>	<i>24</i>	
	<i>teeth in gear =</i>	<i>110</i>	
	<i>diametral pitch =</i>	<i>24</i>	
<b>RESULTS:</b>			
a	Gear speed =	752.7	rpm
b	$VR = m_G =$	4.583	
c	pinion PD =	1.000	in
	gear PD =	4.583	in
d	center distance = C =	2.792	in
e	pitch line speed =	903	ft/min
f	torque on pinion shaft =	13.70	lb in
	torque on gear shaft =	62.77	lb in
g	tangential force =	27.40	lb
h	radial force =	12.78	lb
i	normal force =	30.24	lb

## Gear Manufacture and Quality

7. See Section 9-4. Form milling, shaping, hobbing, grinding.

For Problems 8-16, refer to Section 9-5 and Table 9-3 for recommended quality numbers in the  $A_v$  system according to AGMA Standard 2015. Grain harvester:  $A_v = 10$ .

8. Grain harvester:  $A_v = 10$ .
9. Printing press:  $A_v = 7$ .
10. Auto transmission:  $A_v = 6$ .
11. Gyroscope:  $A_v = 2$ .
12. Analytical quality measurements include *index variation, tooth alignment, tooth profile, root radius, and runout*.
13. AGMA Standard 2015 is currently used. See Table 9-2 for the range of quality numbers in this system and the comparisons with prior systems.

For Problems 14-16, for precision machinery, use the recommendations for machine tool drives in the lower part of Table 9-3. The choice of quality number is based on the pitch line speed of the gears.

14. (From Problem 1). Pitch line speed = 764 ft/min Use  $A_v = 10$ .
15. (From Problem 2). Pitch line speed = 1084 ft/min Use  $A_v = 8$ .
16. (From Problem 3). Pitch line speed = 903 ft/min Use  $A_v = 8$ .

## Gear Materials

Answers for Problems 17 – 25 are found in Sections 9-6 and 9-7. Only brief statements are given here.

17. Bending stresses are created by the tangential force on the gear teeth acting in a manner similar to that on a cantilever. The maximum bending stress occurs in the root of the tooth where it blends with the involute tooth form. High levels of contact stress, called Hertz stress, occur in the face of the teeth near the pitch line as forces are exerted between the pinion and the gear teeth. The probable mode of failure is pitting of the tooth surface.
18. AGMA standards give allowable bending stress numbers and allowable contact stress numbers related to the hardness of the material of the teeth. See Figures 9-11 and 9-12.
19. Gear steels are typically medium carbon plain or alloy steels that are heat treated by through-hardening using a quenching and tempering process. For examples, see Table 9-4, Section 9-7.
20. The AGMA recommends hardness values from HB 180 to HB 400. See Figures 9-11 and 9-12.
21. Grade 1 steel is typical commercial quality and is recommended for use in this book. Grades 2 and 3 require progressively more stringent quality controls on the alloy content and cleanliness of the materials. Cost increases dramatically for the higher grades. See AGMA Standard 2004-C08 or the latest revision.
22. Grades 2 and 3 may be specified for high-speed aerospace applications, turbine engine driven systems, ship propulsion drives, and high-capacity industrial drives such as those in steel rolling mills.

23. Case hardening by flame hardening, induction hardening, and carburizing are three processes that produce harder surfaces than typical through-hardening.
24. See AGMA Standard 2001-D04 or the latest revision.
25. AGMA Standard 2001-D04 provides data for gray cast iron, ductile iron, and bronze. Table 9-6.
26. From Figures 9-11 and 9-12:
  - a. Grade 1; 200 HB:  $s_{at} = 28.26$  ksi;  $s_{ac} = 93.50$  ksi – U.S.:  $s_{at} = 194.9$  MPa;  $s_{ac} = 644.6$  MPa – SI
  - b. Grade 1; 300 HB:  $s_{at} = 36.0$  ksi;  $s_{ac} = 125.7$  ksi – U.S.:  $s_{at} = 248.1$  MPa;  $s_{ac} = 866.6$  MPa – SI
  - c. Grade 1; 400 HB:  $s_{at} = 43.72$  ksi;  $s_{ac} = 157.9$  ksi – U.S.:  $s_{at} = 301.5$  MPa;  $s_{ac} = 1088.6$  MPa – SI
  - d. Using HB > 400 is not recommended.
  - e. Grade 2; 200 HB:  $s_{at} = 36.80$  ksi;  $s_{ac} = 104.1$  ksi – U.S.:  $s_{at} = 253.7$  MPa;  $s_{ac} = 718.5$  MPa – SI
  - f. Grade 2; 300 HB:  $s_{at} = 47.0$  ksi;  $s_{ac} = 139.0$  ksi – U.S.:  $s_{at} = 324.0$  MPa;  $s_{ac} = 959.5$  MPa – SI
  - g. Grade 2; 400 HB:  $s_{at} = 57.20$  ksi;  $s_{ac} = 173.9$  ksi – U.S.:  $s_{at} = 394.3$  MPa;  $s_{ac} = 1200.5$  MPa – SI
27. From Figure 9-11: Grade 1: 300 HB. Grade 2: 192 HB
28. From Table 9-5: Case hardening by carburizing produces 55-64 HRC
29. From Appendix 5: SAE 1020, 4118, 8620, and others
30. From Table 9-5: Flame or induction hardening produces 50-54 HRC with materials having high hardenability
31. SAE 4140, 4340, 6150. All have good hardenability
32. ASTM A536, Grade 80-55-06 has a minimum hardness of 179 HB.
33.
  - a.  $s_{at} = 45.0$  ksi;  $s_{ac} = 170.0$  ksi – U.S.:  $s_{at} = 310$  MPa;  $s_{ac} = 1172$  MPa – SI [Table 9-5]
  - b.  $s_{at} = 45.0$  ksi;  $s_{ac} = 175.0$  ksi – U.S.:  $s_{at} = 310$  MPa;  $s_{ac} = 1207$  MPa – SI [Table 9-5]
  - c.  $s_{at} = 55.0$  ksi;  $s_{ac} = 180.0$  ksi – U.S.:  $s_{at} = 379$  MPa;  $s_{ac} = 1241$  MPa – SI [Table 9-5]
  - d. Not listed
  - e.  $s_{at} = 55.0$  ksi;  $s_{ac} = 180.0$  ksi – U.S.:  $s_{at} = 379$  MPa;  $s_{ac} = 1241$  MPa – SI [Table 9-5]
  - f.  $s_{at} = 5.00$  ksi;  $s_{ac} = 50.0$  ksi – U.S.:  $s_{at} = 35.0$  MPa;  $s_{ac} = 345$  MPa – SI [Table 9-6]
  - g.  $s_{at} = 13.0$  ksi;  $s_{ac} = 75.0$  ksi – U.S.:  $s_{at} = 90.0$  MPa;  $s_{ac} = 517$  MPa – SI [Table 9-6]
  - h.  $s_{at} = 27.0$  ksi;  $s_{ac} = 92.0$  ksi – U.S.:  $s_{at} = 186$  MPa;  $s_{ac} = 634$  MPa – SI [Table 9-6]
  - i.  $s_{at} = 5.70$  ksi;  $s_{ac} = 30.0$  ksi – U.S.:  $s_{at} = 39.0$  MPa;  $s_{ac} = 207$  MPa – SI [Table 9-6]
  - j.  $s_{at} = 23.6$  ksi;  $s_{ac} = 65.0$  ksi – U.S.:  $s_{at} = 163$  MPa;  $s_{ac} = 448$  MPa – SI [Table 9-6]
  - k.  $s_{at} = 12.0$  ksi;  $s_{ac}$  not listed:  $s_{at} = 83.0$  MPa;  $s_{ac}$  not listed [Table 9-14]
  - l.  $s_{at} = 9.0$  ksi;  $s_{ac}$  not listed:  $s_{at} = 62.0$  MPa;  $s_{ac}$  not listed [Table 9-14]
34. Depth = 0.027 in [Figure 9-13.]
35. Depth = 0.90 mm. [Figure 9-13.]



**DESIGN OF SPUR GEARS**

<b>APPLICATION: Problems 36, 42, 48, 54</b>	
<i>Industrial conveyor driven by an electric motor</i>	
<b>Initial Input Data:</b>	
Overload Factor: $K_o = 1.50$	Table 9-7
Transmitted Power: $P = 10$ hp	
Design Power $P_{des} = 15$ hp	
Diametral Pitch: $P_d = 12$	Fig. 9-24
Input Speed: $n_P = 1750$ rpm	
Number of Pinion Teeth: $N_P = 18$	
Desired Output Speed: $n_G = 370$ rpm	
Computed number of gear teeth: 85.1	
Enter: Chosen No. of Gear Teeth: $N_G = 85$	
<b>Computed data:</b>	
Actual Output Speed: $n_G = 370.6$ rpm	
Gear Ratio: $m_G = 4.72$	
Pitch Diameter - Pinion: $D_P = 1.500$ in	
Pitch Diameter - Gear: $D_G = 7.083$ in	
Center Distance: $C = 4.292$ in	
Pitch Line Speed: $v_t = 687$ ft/min	
Transmitted Load: $W_t = 480$ lb	
<b>Secondary Input Data:</b>	
Face Width Guidelines (in): 0.667 1.000 1.333	Min Norm Max
Enter: Face Width: $F = 1.250$ in	
Ratio: Face width/pinion diameter: $F/D_P = 0.83$	
Recommended range of ratio: $0.50 < F/D_P < 2.00$	
Enter: Elastic Coefficient: $C_p = 2300$	Table 9-10
Enter: Quality Number: $A_v = 11$	Table 9-3
Dynamic Factor: $K_v = 1.35$	Table 9-9
[Factors for computing $K_v$ ]: $B = 0.826$ $C = 59.75$	
Reference: $N_P = 18$ $N_G = 85$	
Bending Geometry Factor-Pinion: $J_P = 0.320$	Fig. 9-15
Bending Geometry Factor-Gear: $J_G = 0.410$	Fig. 9-15
Reference: $m_G = 4.72$	
Enter: Pitting Geometry Factor: $I = 0.108$	Fig. 9-21
<b>Factors in Design Analysis:</b>	
Alignment Factor, $K_m = 1.0 \cdot C_{pr} \cdot C_{ma}$	IF $\leq 1.0$ IF $> 1.0$
Pinion Proportion Factor, $C_{pr} = 0.058$	0.061 [0.50 < $F/D_P$ < 2.00]
Enter: $C_{pr} = 0.061$	Figure 9-16
Type of gearing: Open	Commer. Precision Ex. Prec.
Mesh Alignment Factor, $C_{ma} = 0.268$	0.147 0.083 0.051
Enter: $C_{ma} = 0.147$	Figure 9-17
Alignment Factor: $K_m = 1.21$	[Computed]
Size Factor: $K_s = 1.00$	Table 9-8: Use 1.00 if $P_d \geq 5$
Pinion Rim Thickness Factor: $K_{RP} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Gear Rim Thickness Factor: $K_{RG} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Service Factor: $SF = 1.00$	Use 1.00 if no unusual conditions
Reliability Factor: $K_R = 1.00$	Table 9-11 Use 1.00 for $R = .99$
Enter: Design Life: 20000 hours	See Table 9-12
Pinion - Number of load cycles: $N_P = 2.1E+09$	Guidelines: $Y_N, Z_N$
Gear - Number of load cycles: $N_G = 4.4E+08$	10 <sup>6</sup> cycles > 10 <sup>7</sup> < 10 <sup>8</sup>
Bending Stress Cycle Factor: $Y_{NP} = 0.93$	1.00 0.93 Fig. 9-22
Bending Stress Cycle Factor: $Y_{NG} = 0.95$	1.00 0.95 Fig. 9-22
Pitting Stress Cycle Factor: $Z_{NP} = 0.88$	1.00 0.88 Fig. 9-23
Pitting Stress Cycle Factor: $Z_{NG} = 0.92$	1.00 0.92 Fig. 9-23
<b>Stress Analysis: Bending</b>	
Pinion: Required $s_{bt} = 37,906$ psi	See Fig. 9-11 or Table 9-5
Gear: Required $s_{gt} = 28,963$ psi	
<b>Stress Analysis: Pitting</b>	
Pinion: Required $s_{bc} = 199,099$ psi	See Fig. 9-12 or Table 9-5
Gear: Required $s_{gc} = 190,443$ psi	
<b>Required hardness of pinion HB: 528</b>	
<b>Required hardness of gear HB: 501</b>	
<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>	
<b>One possible material specification:</b>	
Requires Grade 2 carburized. Suggest redesign to lower stress levels.	

Computed bending stress number, $s_t = 35253$ psi	Pinion
Computed bending stress number, $s_t = 27514$ psi	Gear
Computed contact stress number, $s_c = 175,207$ psi	Pinion
Computed contact stress number, $s_c = 175,207$ psi	Gear

**DESIGN OF SPUR GEARS**

<b>APPLICATION:</b> Problems 37, 43, 49, 55 Cement kiln driven by an electric motor	
<b>Initial Input Data:</b> Overload Factor: $K_o = 1.75$ Table 9-7 Transmitted Power: $P = 40$ hp Design Power $P_{des} = 70$ hp Diametral Pitch: $P_d = 6$ Fig. 9-24 Input Speed: $n_P = 1150$ rpm Number of Pinion Teeth: $N_P = 20$ Desired Output Speed: $n_G = 479$ rpm Computed number of gear teeth: 48.0 Enter: Chosen No. of Gear Teeth: $N_G = 48$	
<b>Computed Data:</b> Actual Output Speed: $n_G = 479.2$ rpm Gear Ratio: $m_G = 2.40$ Pitch Diameter - Pinion: $D_P = 3.333$ in Pitch Diameter - Gear: $D_G = 8.000$ in Center Distance: $C = 5.667$ in Pitch Line Speed: $v_l = 1004$ ft/min Transmitted Load: $W_t = 1315$ lb	
<b>Secondary Input Data:</b> Face Width Guidelines (in): 1.333 2.000 2.667 Enter: Face Width: $F = 2.250$ in Ratio: Face width/pinion diameter: $F/D_P = 0.68$ Recommended range of ratio: $0.50 < F/D_P < 2.00$ Enter: Elastic Coefficient: $C_p = 2300$ Table 9-10 Enter: Quality Number: $A_v = 11$ Table 9-3 Dynamic Factor: $K_v = 1.42$ Table 9-9 [Factors for computing $K_v$ ]: $B = 0.828$ $C = 59.75$ Reference: $N_P = 20$ $N_G = 48$ Bending Geometry Factor-Pinion: $J_P = 0.325$ Fig. 9-15 Bending Geometry Factor-Gear: $J_G = 0.395$ Fig. 9-15 Reference: $m_G = 2.40$ Enter: Pitting Geometry Factor: $I = 0.095$ Fig. 9-21	
<b>Factors in Design Analysis:</b> Alignment Factor, $K_a = 1.0 + C_{pr} + C_{ma}$ If $F < 1.0$ If $F > 1.0$ Pinion Proportion Factor, $C_{pr} = 0.043$ $0.058$ [ $0.50 < F/D_P < 2.00$ ] Enter: $C_{pr} = 0.058$ Figure 9-16 Type of gearing: Open Commer. Precision Ex. Prec. Mesh Alignment Factor, $C_{ma} = 0.284$ $0.162$ $0.096$ $0.061$ Enter: $C_{ma} = 0.162$ Figure 9-17 Alignment Factor: $K_a = 1.22$ [Computed]	
Size Factor: $K_s = 1.00$ Table 9-8: Use 1.00 if $P_d >= 5$ Pinion Rim Thickness Factor: $K_{rp} = 1.00$ Fig. 9-18: Use 1.00 if solid blank Gear Rim Thickness Factor: $K_{rg} = 1.00$ Fig. 9-18: Use 1.00 if solid blank Service Factor: $S_F = 1.00$ Use 1.00 if no unusual conditions Reliability Factor: $K_R = 1.00$ Table 9-11 Use 1.00 for $R = .99$	
Enter: Design Life: 8000 hours See Table 9-12 Pinion - Number of load cycles: $N_p = 5.5E+08$ Guidelines: $Y_N, Z_N$ Gear - Number of load cycles: $N_G = 2.3E+08$ $10^7$ cycles $> 10^8$ $< 10^7$	
Bending Stress Cycle Factor: $Y_{NP} = 0.95$ Fig. 9-22 Bending Stress Cycle Factor: $Y_{NG} = 0.96$ Fig. 9-22 Pitting Stress Cycle Factor: $Z_{NP} = 0.91$ Fig. 9-23 Pitting Stress Cycle Factor: $Z_{NG} = 0.93$ Fig. 9-23	
<b>Stress Analysis: Bending</b> Pinion: Required $s_{at} = 34,466$ psi See Fig. 9-11 or Gear: Required $s_{at} = 28,063$ psi Table 9-5	
<b>Stress Analysis: Pitting</b> Pinion: Required $s_{ec} = 189,152$ psi See Fig. 9-12 or Gear: Required $s_{ec} = 185,084$ psi Table 9-5	
Required hardness of pinion HB: 497 Equations in Fig. 9-12-Grade 1 Required hardness of gear HB: 484 Equations in Fig. 9-12-Grade 1 Specify materials, alloy and heat treatment, for most severe requirement.	
One possible material specification: Requires Grade 2 flame or induction harden. Suggest redesign to lower stress levels.	

Computed bending stress number, $s_t =$	32743 psi	Pinion
Computed bending stress number, $s_t =$	26940 psi	Gear
Computed contact stress number, $s_c =$	172,128 psi	Pinion
Computed contact stress number, $s_c =$	172,128 psi	Gear

**DESIGN OF SPUR GEARS**

<b>APPLICATION: Problems 38, 44, 50, 56</b>		<b>Factors in Design Analysis:</b>	
Small machine tool driven by an electric motor		Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$ If $F < 1.0$ 0.042 0.035 [0.50 < F/Dp < 2.00] If $F > 1.0$	
<b>Initial Input Data:</b>		Pinion Proportion Factor, $C_{pf} =$ 0.042 0.042 Figure 9-16	
Overload Factor: $K_o =$ 1.50 Table 9-7	Transmitted Power: $P =$ 0.5 hp	Type of gearing: Open Commer. Precision Ex. Prec.	
Design Power $P_{des} =$ 0.75 hp	Diametral Pitch: $P_d =$ 32 Fig. 9-24	Mesh Alignment Factor, $C_{ma} =$ 0.255 0.135 0.074 0.043	
Input Speed: $n_P =$ 9450 rpm	Number of Pinion Teeth: $N_P =$ 24	Enter: $C_{mp} =$ 0.074 Figure 9-17	
Desired Output Speed: $n_G =$ 690 rpm	Computed number of gear teeth: 120.0	Alignment Factor: $K_m =$ 1.12 [Computed]	
Enter: Chosen No. of Gear Teeth: $N_G =$ 120		Size Factor: $K_s =$ 1.00 Table 9-6: Use 1.00 if $P_d >= 5$	
<b>Computed data:</b>		Pinion Rim Thickness Factor: $K_{rp} =$ 1.00 Fig. 9-18: Use 1.00 if solid blank	
Actual Output Speed: $n_G =$ 690.0 rpm	Gear Ratio: $m_G =$ 5.00	Gear Rim Thickness Factor: $K_{ge} =$ 1.00 Fig. 9-18: Use 1.00 if solid blank	
Pitch Diameter - Pinion: $D_P =$ 0.750 in	Pitch Diameter - Gear: $D_G =$ 3.750 in	Service Factor: $SF =$ 1.25 Use 1.00 if no unusual conditions	
Center Distance: $C =$ 2.250 in	Pitch Line Speed: $v_t =$ 577 ft/min	Reliability Factor: $K_R =$ 1.50 Table 9-11 Use 1.00 for $R = .99$	
Transmitted Load: $W_t =$ 24 lb		Enter: Design Life: 12000 hours See Table 9-12	
<b>Secondary Input Data:</b>		Pinion - Number of load cycles: $N_P =$ 2.5E+08	Guidelines: $Y_N, Z_N$
Face Width Guidelines (in): 0.250 0.375 0.500	Enter: Face Width: $F =$ 0.500 in	Gear - Number of load cycles: $N_G =$ 5.0E+08	10 <sup>7</sup> cycles >10 <sup>8</sup> <10 <sup>9</sup>
Ratio: Face width/pinion diameter: $F/D_P =$ 0.67	Recommended range of ratio: 0.50 < F/Dp < 2.00	Bending Stress Cycle Factor: $Y_{NP} =$ 0.92	1.00 0.92 Fig. 9-22
Enter: Elastic Coefficient: $C_p =$ 2300 Table 9-10	Enter: Quality Number: $A_v =$ 7 Table 9-3	Bending Stress Cycle Factor: $Y_{NG} =$ 0.95	1.00 0.95 Fig. 9-22
Enter: Dynamic Factor: $K_v =$ 1.11 Table 9-9	Dynamic Factor: $K_v =$ 1.11 Table 9-9	Pitting Stress Cycle Factor: $Z_{NP} =$ 0.88	1.00 0.88 Fig. 9-23
[Factors for computing $K_v$ ]: $B =$ 0.397 $C =$ 83.77	Reference: $N_P =$ 24 $N_G =$ 120	Pitting Stress Cycle Factor: $Z_{NG} =$ 0.91	1.00 0.91 Fig. 9-23
Bending Geometry Factor-Pinion: $J_P =$ 0.360 Fig. 9-15	Bending Geometry Factor-Gear: $J_G =$ 0.440 Fig. 9-15	<b>Stress Analysis: Bending</b>	
Bending Geometry Factor-Gear: $J_G =$ 0.440 Fig. 9-15	Reference: $m_G =$ 5.00	Pinion: Required $s_{bt} =$ 16,448 psi See Fig. 9-11 or	
Enter: Pitting Geometry Factor: $I =$ 0.118 Fig. 9-21		Gear: Required $s_{gt} =$ 13,033 psi Table 9-5	
		<b>Stress Analysis: Pitting</b>	
		Pinion: Required $s_{ps} =$ 156,966 psi See Fig. 9-12 or	
		Gear: Required $s_{gs} =$ 151,791 psi Table 9-5	
		Required hardness of pinion HB: 397 Equations in Fig. 9-12-Grade 1	
		Required hardness of gear HB: 381 Equations in Fig. 9-12-Grade 1	
		Specify materials, alloy and heat treatment, for most severe requirement.	
		One possible material specification:	
		SAE 4140 OQT 800, 429 HB, 14% elongation, good ductility	
		SAE 4140 OQT 900, 429 HB, 14% elongation, good ductility	

Computed bending stress number, $s_t =$	8071 psi	Pinion
Computed bending stress number, $s_t =$	6603 psi	Gear
Computed contact stress number, $s_c =$	73,669 psi	Pinion
Computed contact stress number, $s_c =$	73,669 psi	Gear



**DESIGN OF SPUR GEARS**

<p><b>APPLICATION:</b> Problems 38, 45, 51, 57 Aircraft actuator driven by a universal electric motor</p>		<p><b>Factors in Design Analysis:</b> Alignment Factor, <math>K_{an} = 1.0 + C_{pr} + C_{ma}</math> If <math>F &lt; 1.0</math> 0.025 0.031 If <math>F &gt; 1.0</math> [0.50 &lt; <math>F/D_p</math> &lt; 2.00]</p>	
<p><b>Initial Input Data:</b> Overload Factor: <math>K_o = 1.50</math> Table 9-7 Transmitted Power: <math>P = 15</math> hp Design Power <math>P_{des} = 22.5</math> hp Diametral Pitch: <math>P_d = 10</math> Fig. 9-24 Input Speed: <math>n_P = 6500</math> rpm Number of Pinion Teeth: <math>N_P = 30</math> Desired Output Speed: <math>n_G = 2216</math> rpm Computed number of gear teeth: 88.0 Enter: Chosen No. of Gear Teeth: <math>N_G = 88</math></p>		<p>Pinion Proportion Factor, <math>C_{pr} = 0.031</math> Figure 9-16 Type of gearing: Open Commer. Precision Ex. Prec. Mesh Alignment Factor, <math>C_{ma} = 0.272</math> 0.150 0.086 0.053 Enter: <math>C_{pr} = 0.031</math> Figure 9-16 Enter: <math>C_{ma} = 0.272</math> 0.150 0.086 0.053 Alignment Factor: <math>K_{an} = 1.08</math> [Computed]</p>	
<p><b>Computed data:</b> Actual Output Speed: <math>n_G = 2215.9</math> rpm Gear Ratio: <math>m_G = 2.93</math> Pitch Diameter - Pinion: <math>D_P = 3.000</math> in Pitch Diameter - Gear: <math>D_G = 8.800</math> in Center Distance: <math>C = 5.900</math> in Pitch Line Speed: <math>v_t = 5105</math> ft/min Transmitted Load: <math>W_t = 97</math> lb</p>		<p>Size Factor: <math>K_s = 1.00</math> Table 9-8; Use 1.00 if <math>P_d &gt;= 5</math> Pinion Rim Thickness Factor: <math>K_{rp} = 1.00</math> Fig. 9-18; Use 1.00 if solid blank Gear Rim Thickness Factor: <math>K_{rg} = 1.00</math> Fig. 9-18; Use 1.00 if solid blank Service Factor: <math>S_F = 1.00</math> Use 1.00 if no unusual conditions Reliability Factor: <math>K_R = 1.50</math> Table 9-11 Use 1.00 for <math>R = .99</math> Enter: Design Life: 4000 hours See Table 9-12 Pinion - Number of load cycles: <math>N_P = 1.6E+09</math> Guidelines: <math>Y_N, Z_N</math> Gear - Number of load cycles: <math>N_G = 5.3E+08</math> 10<sup>7</sup> cycles &gt;10<sup>7</sup> &lt;10<sup>7</sup></p>	
<p><b>Secondary Input Data:</b> Face Width Guidelines (in): Min 0.800 1.200 Max 1.600 Enter: Face Width: <math>F = 1.500</math> in Ratio: Face width/pinion diameter: <math>F/D_P = 0.50</math> Recommended range of ratio: <math>0.50 &lt; F/D_P &lt; 2.00</math> Enter: Elastic Coefficient: <math>C_p = 2300</math> Table 9-10 Enter: Quality Number: <math>A_v = 5</math> Table 9-3 Dynamic Factor: <math>K_v = 1.00</math> Table 9-9 [Factors for computing <math>K_v</math>]: <math>B = 0.000</math> <math>C = 106.00</math> Reference: <math>N_P = 30</math> <math>N_G = 88</math> Bending Geometry Factor-Pinion: <math>J_P = 0.460</math> Fig. 9-15 Bending Geometry Factor-Gear: <math>J_G = 0.530</math> Fig. 9-15 Reference: <math>m_G = 2.93</math> Enter: Pitting Geometry Factor: <math>I = 0.130</math> Fig. 9-21</p>		<p>Bending Stress Cycle Factor: <math>Y_{HP} = 0.93</math> Fig. 9-22 Bending Stress Cycle Factor: <math>Y_{HG} = 0.95</math> Fig. 9-22 Pitting Stress Cycle Factor: <math>Z_{HP} = 0.89</math> Fig. 9-23 Pitting Stress Cycle Factor: <math>Z_{HG} = 0.91</math> Fig. 9-23 <b>Stress Analysis: Bending</b> Pinion: Required <math>s_{bl} = 3,685</math> psi See Fig. 9-11 or Gear: Required <math>s_{bl} = 3,131</math> psi Table 9-5 <b>Stress Analysis: Pitting</b> Pinion: Required <math>s_{pc} = 63,637</math> psi See Fig. 9-12 or Gear: Required <math>s_{pc} = 62,239</math> psi Table 9-5 Required hardness of pinion HB: 107 Equations in Fig. 9-12-Grade 1 Required hardness of gear HB: 103 Equations in Fig. 9-12-Grade 1 <b>Specify materials, alloy and heat treatment, for most severe requirement.</b> <b>One possible material specification:</b> Stresses are quite low for steel gears. Suggest redesign.</p>	
<p>Ans. Problem: 39 Ans. Problem: 39 Ans. Problem: 51 Ans. Problem: 51</p>		<p>Computed bending stress number, <math>s_t = 2285</math> psi Pinion Computed bending stress number, <math>s_t = 1983</math> psi Gear Computed contact stress number, <math>s_c = 37,758</math> psi Pinion Computed contact stress number, <math>s_c = 37,758</math> psi Gear</p>	

**DESIGN OF SPUR GEARS**

<p><b>APPLICATION:</b> Problems 40, 46, 52, 58 Portable industrial water pump driven by a gasoline engine</p> <p><b>Initial Input Data:</b></p> <p>Overload Factor: <math>K_o = 1.70</math> Table 9-7                  Transmitted Power: <math>P = 125</math> hp                  Design Power <math>P_{des} = 212.5</math> hp                  Diametral Pitch: <math>P_d = 4</math> Fig. 9-24                  Input Speed: <math>n_P = 2500</math> rpm                  Number of Pinion Teeth: <math>N_P = 32</math>                  Desired Output Speed: <math>n_G = 1050</math> rpm                  Computed number of gear teeth: 76.2                  Enter: Chosen No. of Gear Teeth: <math>N_G = 76</math></p> <p><b>Computed data:</b></p> <p>Actual Output Speed: <math>n_G = 1052.6</math> rpm                  Gear Ratio: <math>m_G = 2.38</math>                  Pitch Diameter - Pinion: <math>D_P = 6.000</math> in                  Pitch Diameter - Gear: <math>D_G = 19.000</math> in                  Center Distance: <math>C = 13.500</math> in                  Pitch Line Speed: <math>v_t = 5236</math> ft/min                  Transmitted Load: <math>W_t = 788</math> lb</p> <p><b>Secondary Input Data:</b></p> <p>Face Width Guidelines (in): Min 3.000, Nom 4.000, Max 4.000                  Enter: Face Width: <math>F = 1.500</math> in                  Ratio: Face width/pinion diameter: <math>F/D_P = 0.50</math> Entered                  Recommended range of ratio: <math>0.50 &lt; F/D_P &lt; 2.00</math></p> <p>Enter: Elastic Coefficient: <math>C_p = 2300</math> Table 9-10                  Enter: Quality Number: <math>A_v = 9</math> Table 9-3                  Dynamic Factor: <math>K_v = 1.56</math> Table 9-9                  [Factors for computing <math>K_v</math>] <math>B = 0.630</math> <math>C = 70.71</math></p> <p>Reference: <math>N_P = 32</math> <math>N_G = 76</math>                  Bending Geometry Factor-Pinion: <math>J_P = 0.465</math> Fig. 9-15                  Bending Geometry Factor-Gear: <math>J_G = 0.520</math> Fig. 9-15</p> <p>Reference: <math>m_G = 2.38</math>                  Enter: Pitting Geometry Factor: <math>I = 0.124</math> Fig. 9-21</p>	<p><b>Factors in Design Analysis:</b></p> <p>Alignment Factor, <math>K_m = 1.0 + C_{pr} + C_{ma}</math> If <math>F &lt; 1.0</math> 0.025 If <math>F &gt; 1.0</math> 0.031 <math>[0.50 &lt; F/D_P &lt; 2.00]</math></p> <p>Pinion Proportion Factor, <math>C_{pr} = 0.031</math> Figure 9-16                  Enter: <math>C_{pr} = 0.031</math> Figure 9-16                  Type of gearing: Open Commer. Precision Ex. Prec.                  Mesh Alignment Factor, <math>C_{ma} = 0.272</math> 0.150 0.086 0.053                  Enter: <math>C_{ma} = 0.15</math> Figure 9-17                  Alignment Factor: <math>K_m = 1.18</math> [Computed]</p> <p>Size Factor: <math>K_s = 1.05</math> Table 9-8; Use 1.00 if <math>P_d \geq 5</math>                  Pinion Rim Thickness Factor: <math>K_{ar} = 1.00</math> Fig. 9-18; Use 1.00 if solid blank                  Gear Rim Thickness Factor: <math>K_{gr} = 1.00</math> Fig. 9-18; Use 1.00 if solid blank                  Service Factor: <math>SF = 1.00</math> Use 1.00 if no unusual conditions                  Reliability Factor: <math>K_R = 1.00</math> Table 9-11 Use 1.00 for <math>R = .99</math></p> <p>Enter: Design Life: 8000 hours See Table 9-12                  Pinion - Number of load cycles: <math>N_P = 1.2E+09</math> Guidelines: <math>Y_N, Z_N</math>                  Gear - Number of load cycles: <math>N_G = 5.1E+08</math> <math>10^7</math> cycles <math>&gt; 10^7</math> <math>&lt; 10^7</math></p> <p>Bending Stress Cycle Factor: <math>Y_{BP} = 0.93</math> 1.00 0.93 Fig. 9-22                  Bending Stress Cycle Factor: <math>Y_{BG} = 0.95</math> 1.00 0.95 Fig. 9-22                  Pitting Stress Cycle Factor: <math>Z_{RP} = 0.90</math> 1.00 0.90 Fig. 9-23                  Pitting Stress Cycle Factor: <math>Z_{RG} = 0.91</math> 1.00 0.91 Fig. 9-23</p> <p><b>Stress Analysis: Bending</b></p> <p>Pinion: Required <math>s_{bl} = 15,968</math> psi See Fig. 9-11 or Table 9-5                  Gear: Required <math>s_{bl} = 13,979</math> psi Table 9-5</p> <p><b>Stress Analysis: Pitting</b></p> <p>Pinion: Required <math>s_{po} = 109,809</math> psi See Fig. 9-12 or Table 9-5                  Gear: Required <math>s_{po} = 105,438</math> psi Table 9-5</p> <p>Required hardness of pinion HB: 241 Equations in Fig. 9-12-Grade 1                  Required hardness of gear HB: 237 Equations in Fig. 9-12-Grade 1                  Specify materials, alloy and heat treatment, for most severe requirement.</p> <p><b>One possible material specification:</b>                  SAE 1040 WQT 1000, 269 HB, 22% elongation                  SAE 1040 WQT 1000, 269 HB, 22% elongation</p>
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Ans. Problem:	40	Computed bending stress number, $s_t =$	14850 psi	Pinion
Ans. Problem:	40	Computed bending stress number, $s_t =$	13280 psi	Gear
Ans. Problem:	52	Computed contact stress number, $s_c =$	95,948 psi	Pinion
Ans. Problem:	52	Computed contact stress number, $s_c =$	95,948 psi	Gear

**DESIGN OF SPUR GEARS**

<p><b>APPLICATION:</b> Problems 41, 47, 53, 59 Lawn and garden tractor with fluid motor drive</p>		<p><b>Factors in Design Analysis:</b> Alignment Factor, <math>K_a = 1.0 \cdot C_{pf} \cdot C_{ma}</math> <math>1F &lt; 1.0</math> <math>1F &gt; 1.0</math></p>	
<p><b>Initial Input Data:</b></p>		<p>Pinion Proportion Factor, <math>C_{pf} = 0.027</math> <math>0.030</math> <math>[0.50 &lt; F/D_p &lt; 2.00]</math></p>	
<p>Overload Factor: <math>K_o = 1.75</math> Table 9-7 Transmitted Power: <math>P = 2.5</math> hp Design Power <math>P_{des} = 4.375</math> hp Diametral Pitch: <math>P_d = 10</math> Fig. 9-24 input Speed: <math>n_P = 680</math> rpm</p>		<p>Enter: <math>C_{pf} = 0.030</math> Figure 9-16 Type of gearing: Open Commer. Precision Ex. Prec. Mesh Alignment Factor, <math>C_{ma} = 0.268</math> <math>0.147</math> <math>0.083</math> <math>0.051</math> Enter: <math>C_{ma} = 0.147</math> Figure 9-17 Alignment Factor: <math>K_a = 1.18</math> [Computed]</p>	
<p>Number of Pinion Teeth: <math>N_P = 24</math> Desired Output Speed: <math>n_G = 263</math> rpm 62.1</p>		<p>Size Factor: <math>K_s = 1.00</math> Table 9-8: Use 1.00 if <math>P_d \geq 5</math> Pinion Rim Thickness Factor: <math>K_{RP} = 1.00</math> Fig. 9-18: Use 1.00 if solid blank Gear Rim Thickness Factor: <math>K_{RG} = 1.00</math> Fig. 9-18: Use 1.00 if solid blank Service Factor: <math>S_F = 1.25</math> Use 1.00 if no unusual conditions Reliability Factor: <math>K_R = 0.95</math> Table 9-11 Use 1.00 for <math>R = .99</math></p>	
<p>Computed number of gear teeth: Enter: Chosen No. of Gear Teeth: <math>N_G = 62</math></p>		<p>Enter: Design Life: 2000 hours See Table 9-12 Pinion - Number of load cycles: <math>N_P = 8.2E+07</math> Guidelines: <math>Y_N, Z_N</math> Gear - Number of load cycles: <math>N_G = 3.2E+07</math> <math>&gt;10^7</math> cycles <math>&lt;10^7</math></p>	
<p><b>Computed data:</b></p>		<p>Bending Stress Cycle Factor: <math>Y_{RP} = 0.98</math> Fig. 9-22 Bending Stress Cycle Factor: <math>Y_{NG} = 1.00</math> Fig. 9-22 Pitting Stress Cycle Factor: <math>Z_{RP} = 0.95</math> Fig. 9-23 Pitting Stress Cycle Factor: <math>Z_{NG} = 0.97</math> Fig. 9-23</p>	
<p>Actual Output Speed: <math>n_G = 263.2</math> rpm Gear Ratio: <math>m_G = 2.58</math> Pitch Diameter - Pinion: <math>D_P = 2.400</math> in Pitch Diameter - Gear: <math>D_G = 6.200</math> in Center Distance: <math>C = 4.300</math> in Pitch Line Speed: <math>v_t = 427</math> ft/min Transmitted Load: <math>W_t = 183</math> lb</p>		<p><b>Stress Analysis: Bending</b> Pinion: Required <math>s_{bt} = 10,254</math> psi See Fig. 9-11 or Gear: Required <math>s_{gt} = 8,642</math> psi Table 9-5</p>	
<p><b>Secondary Input Data:</b></p>		<p><b>Stress Analysis: Pitting</b> Pinion: Required <math>s_{pc} = 87,531</math> psi See Fig. 9-12 or Gear: Required <math>s_{gc} = 85,727</math> psi Table 9-5</p>	
<p>Face Width Guidelines (in): 0.800 1.200 1.600 Enter: Face Width: <math>F = 1.250</math> in Ratio: Face width/pinion diameter: <math>F/D_P = 0.52</math> Recommended range of ratio: <math>0.50 &lt; F/D_P &lt; 2.00</math></p>		<p>Required hardness of pinion HB: 161 Equations in Fig. 9-12-Grade 1 Required hardness of gear HB: 176 Equations in Fig. 9-12-Grade 1 Specify materials, alloy and heat treatment, for most severe requirement.</p>	
<p>Enter: Elastic Coef. (Ductile iron) <math>C_p = 2160</math> Table 9-10 Enter: Quality Number: <math>A_v = 11</math> Table 9-3 Dynamic Factor: <math>K_v = 1.28</math> Table 9-9 [Factors for computing <math>K_v</math>] <math>B = 0.826</math> <math>C = 58.75</math></p>		<p>One possible material specification: Pinion: Ductile iron 100-70-03 Q&amp;T; <math>s_{at} = 27,000</math> psi; <math>s_{ac} = 92,000</math> psi Gear: Ductile iron 100-70-03 Q&amp;T; <math>s_{at} = 27,000</math> psi; <math>s_{ac} = 92,000</math> psi</p>	
<p>Reference: <math>N_P = 24</math> <math>N_G = 62</math> Bending Geometry Factor-Pinion: <math>J_P = 0.430</math> Fig. 9-15 Bending Geometry Factor-Gear: <math>J_G = 0.500</math> Fig. 9-15 Reference: <math>m_G = 2.58</math> Enter: Pitting Geometry Factor: <math>I = 0.122</math> Fig. 9-21</p>		<p>Computed bending stress number, <math>s_t = 9458</math> psi Pinion Computed bending stress number, <math>s_t = 8134</math> psi Gear Computed contact stress number, <math>s_c = 78,263</math> psi Pinion Computed contact stress number, <math>s_c = 78,263</math> psi Gear</p>	



**DESIGN OF SPUR GEARS**

<b>APPLICATION: Problem 60</b>		<b>Factors in Design Analysis:</b>	
<b>Reciprocating compressor driven by an electric motor</b>		Alignment Factor, $K_m = 1.0 + C_{pt} + C_{ma}$ If $F < 1.0$ If $F > 1.0$	
<b>Initial Input Data:</b>		Pinion Proportion Factor, $C_{pf} = 0.044$ $0.048$ $[0.50 < F/D_p < 2.00]$	
Overload Factor: $K_o = 1.50$	Table 9-7	Enter: $C_{pf} = 0.048$ Figure 9-16	
Transmitted Power: $P = 5$ hp		Type of gearing: Open    Commer.    Precision    Ex. Prec.	
Design Power $P_{des} = 7.5$ hp		Mesh Alignment Factor, $C_{ma} = 0.268$ $0.147$ $0.083$ $0.051$	
Diametral Pitch: $P_d = 10$	Fig. 9-24	Enter: $C_{me} = 0.147$ Figure 9-17	
Input Speed: $n_P = 1200$ rpm		Alignment Factor: $K_m = 1.20$ [Computed]	
Number of Pinion Teeth: $N_P = 18$		Size Factor: $K_s = 1.00$ Table 9-8; Use 1.00 if $P_d > 5$	
Desired Output Speed: $n_G = 387.5$ rpm		Pinion Rim Thickness Factor: $K_{sp} = 1.00$ Fig. 9-18; Use 1.00 if solid blank	
Computed number of gear teeth: 55.7		Gear Rim Thickness Factor: $K_{ag} = 1.00$ Fig. 9-18; Use 1.00 if solid blank	
Enter: Chosen No. of Gear Teeth: $N_G = 56$		Service Factor: $SF = 1.00$ Use 1.00 if no unusual conditions	
<b>Computed data:</b>		Reliability Factor: $K_R = 1.00$ Table 9-11 Use 1.00 for $R = .99$	
Actual Output Speed: $n_G = 385.7$ rpm		Enter: Design Life: 20000 hours    See Table 9-12	
Gear Ratio: $m_G = 3.11$		Pinion - Number of load cycles: $N_p = 1.4E+09$ Guidelines: $Y_N, Z_N$	
Pitch Diameter - Pinion: $D_P = 1.800$ in		Gear - Number of load cycles: $N_G = 4.6E+08$ $> 10^7$ $< 10^7$	
Pitch Diameter - Gear: $D_G = 5.600$ in		Bending Stress Cycle Factor: $Y_{NP} = 0.93$ $1.00$ $0.93$ Fig. 9-22	
Center Distance: $C = 3.700$ in		Bending Stress Cycle Factor: $Y_{NG} = 0.95$ $1.00$ $0.95$ Fig. 9-22	
Pitch Line Speed: $v_t = 565$ ft/min		Pitting Stress Cycle Factor: $Z_{NP} = 0.89$ $1.00$ $0.89$ Fig. 9-23	
Transmitted Load: $W_t = 292$ lb		Pitting Stress Cycle Factor: $Z_{NG} = 0.92$ $1.00$ $0.92$ Fig. 9-23	
<b>Secondary Input Data:</b>		<b>Stress Analysis: Bending</b>	
Face Width Guidelines (in): 0.800    1.200    1.600	Min    Nom    Max	Pinion: Required $s_{at} = 18,543$ psi    See Fig. 9-11 or	
Enter: Face Width: $F = 1.250$ in		Gear: Required $s_{at} = 14,522$ psi    Table 9-5	
Ratio: Face width/pinion diameter: $F/D_p = 0.69$		<b>Stress Analysis: Pitting</b>	
Recommended range of ratio: $0.50 < F/D_p < 2.00$		Pinion: Required $s_{ac} = 143,088$ psi    See Fig. 9-12 or	
Enter: Elastic Coefficient: $C_p = 2300$ Table 9-10		Gear: Required $s_{ac} = 138,422$ psi    Table 9-5	
Enter: Quality Number: $A_v = 11$ Table 9-3		Required hardness of pinion HB: 354    Equations in Fig. 9-12-Grade 1	
Dynamic Factor: $K_v = 1.32$ Table 9-9		Required hardness of gear HB: 340    Equations in Fig. 9-12-Grade 1	
[Factors for computing $K_v$ :] $B = 0.828$ $C = 59.75$		<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>	
Reference: $N_P = 18$ $N_G = 56$		<b>One possible material specification:</b>	
Bending Geometry Factor-Pinion: $J_P = 0.320$ Fig. 9-15		Pinion requires HB 354; SAE 4140 OQT 900; HB 388, 16% Elongation	
Bending Geometry Factor-Gear: $J_G = 0.400$ Fig. 9-15		Gear requires HB 340; SAE 4140 OQT 1000; HB 340, 18% Elongation	
Reference: $m_G = 3.11$		<b>Comments:</b>	
Enter: Pitting Geometry Factor: $I = 0.100$ Fig. 9-21		It would be reasonable to specify the same heat treatment for both the pinion and the gear because their contact stresses are very similar.	

**DESIGN OF SPUR GEARS**

APPLICATION: Problem 61	
Milling machine driven by an electric motor	
<b>Initial Input Data:</b>	
Overload Factor: $K_o = 1.50$	Table 9-7
Transmitted Power: $P = 20$ hp	
Design Power $P_{des} = 30$ hp	
Diametral Pitch: $P_d = 6$	Fig. 9-24
Input Speed: $n_P = 550$ rpm	
Number of Pinion Teeth: $N_P = 24$	
Desired Output Speed: $n_G = 185$ rpm	
Computed number of gear teeth: 71.4	
Enter: Chosen No. of Gear Teeth: $N_G = 71$	
<b>Computed data:</b>	
Actual Output Speed: $n_G = 185.9$ rpm	
Gear Ratio: $m_G = 2.96$	
Pitch Diameter - Pinion: $D_P = 4.000$ in	
Pitch Diameter - Gear: $D_G = 11.833$ in	
Center Distance: $C = 7.917$ in	
Pitch Line Speed: $v_t = 576$ ft/min	
Transmitted Load: $W_t = 1146$ lb	
<b>Secondary Input Data:</b>	
Face Width Guidelines (in): Min 1.333, Norm 2.000, Max 2.667	
Enter: Face Width: $F = 2.000$ in	
Ratio: Face width/pinion diameter: $F/D_P = 0.50$	
Recommended range of ratio: $0.50 < F/D_P < 2.00$	
Enter: Elastic Coefficient: $C_p = 2300$	Table 9-10
Enter: Quality Number: $A_v = 9$	Table 9-3
Dynamic Factor: $K_v = 1.20$	Table 9-9
[Factors for computing $K_v$ ]: $B = 0.630$ , $C = 70.71$	
References: $N_P = 24$ , $N_G = 71$	
Bending Geometry Factor-Pinion: $J_P = 0.350$	Fig. 9-15
Bending Geometry Factor-Gear: $J_G = 0.420$	Fig. 9-15
Reference: $m_G = 2.96$	
Enter: Pitting Geometry Factor: $I = 0.108$	Fig. 9-21
<b>Factors in Design Analysis:</b>	
Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$	If $f < 1.0$ 0.025, If $f > 1.0$ 0.038
Pinion Proportion Factor, $C_{pf}$	0.025
Enter: $C_{pf} = 0.038$	Figure 9-16
Type of gearing: Open	Commer.
Mesh Alignment Factor, $C_{ma} = 0.280$	0.158
Enter: $C_{ma} = 0.093$	Figure 9-17
Alignment Factor: $K_m = 1.13$	[Computed]
Size Factor: $K_s = 1.00$	Table 9-8: Use 1.00 if $P_d >= 5$
Pinion Rim Thickness Factor: $K_{sp} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Gear Rim Thickness Factor: $K_{sg} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Service Factor: $SF = 1.00$	Use 1.00 if no unusual conditions
Reliability Factor: $K_R = 1.25$	Table 9-11 Use 1.00 for $R = .99$
Enter: Design Life: 20000 hours	See Table 9-12
Pinion - Number of load cycles: $N_P = 6.6E+08$	Guidelines: $Y_N, Z_N$
Gear - Number of load cycles: $N_G = 2.2E+08$	$> 10^7$ cycles $< 10^7$
Bending Stress Cycle Factor: $Y_{NP} = 0.94$	1.00
Bending Stress Cycle Factor: $Y_{NG} = 0.96$	1.00
Pitting Stress Cycle Factor: $Z_{NP} = 0.91$	1.00
Pitting Stress Cycle Factor: $Z_{NG} = 0.93$	1.00
<b>Stress Analysis: Bending</b>	
Pinion: Required $s_{bt} = 26,640$ psi	See Fig. 9-11 or
Gear: Required $s_{bt} = 21,737$ psi	Table 9-5
<b>Stress Analysis: Pitting</b>	
Pinion: Required $s_{ac} = 164,319$ psi	See Fig. 9-12 or
Gear: Required $s_{ac} = 160,785$ psi	Table 9-5
Required hardness of pinion HB: 420	Equations in Fig. 9-12-Grade 1
Required hardness of gear HB: 409	Equations in Fig. 9-12-Grade 1
<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>	
<b>One possible material specification:</b>	
Pinion and gear require flame or induction hardening	
SAE 4140 OQT 800; HB 352, 21% Elongation-Core: Case harden to HRC 50 min.	
<b>Comments:</b>	
It would be reasonable to specify the same heat treatment for both the pinion and the gear because their contact stresses are very similar.	

**DESIGN OF SPUR GEARS**

<b>APPLICATION: Problem 62</b>		<b>Factors in Design Analysis:</b>	
<b>Punch press driven by an electric motor</b>		Alignment Factor, $K_m = 1.0 + C_{pt} + C_{ma}$ If $F < 1.0$ If $F > 1.0$	
<b>Initial Input Data:</b>		Pinion Proportion Factor, $C_{pt} = 0.025$ $0.050$ $[0.50 < F/D_p < 2.00]$	
Overload Factor: $K_o = 1.75$ Table 9-7	Transmitted Power: $P = 50$ hp	Enter: $C_{pf} = 0.05$ Figure 9-16	Commer. Precision    Ex. Prec.
Design Power $P_{des} = 87.5$ hp	Diametral Pitch: $P_d = 4$ Fig. 9-24	Type of gearing: Open	0.173    0.105    0.068
Input Speed: $n_P = 900$ rpm	Number of Pinion Teeth: $N_P = 24$	Mesh Alignment Factor, $C_{ma} = 0.296$	
Desired Output Speed: $n_G = 227.5$ rpm	Computed number of gear teeth: 94.9	Enter: $C_{me} = 0.173$ Figure 9-17	
Enter: Chosen No. of Gear Teeth: $N_G = 95$	Enter: Chosen No. of Gear Teeth: $N_G = 95$	Alignment Factor: $K_m = 1.22$ [Computed]	
<b>Computed data:</b>		Size Factor: $K_s = 1.05$ Table 9-8: Use 1.00 if $P_d \geq 5$	
Actual Output Speed: $n_G = 227.4$ rpm	Gear Ratio: $m_G = 3.96$	Pinion Rim Thickness Factor: $K_{BP} = 1.00$ Fig. 9-18: Use 1.00 if solid blank	
Pitch Diameter - Pinion: $D_P = 6.000$ in	Pitch Diameter - Gear: $D_G = 23.750$ in	Gear Rim Thickness Factor: $K_{BG} = 1.00$ Fig. 9-18: Use 1.00 if solid blank	
Center Distance: $C = 14.875$ in	Pitch Line Speed: $v_t = 1414$ ft/min	Service Factor: $SF = 1.00$ Use 1.00 if no unusual conditions	
Transmitted Load: $W_t = 1167$ lb	<b>Secondary Input Data:</b>		
Face Width Guidelines (in): 2.000    3.000    4.000	Min	Norm	Max
Enter: Face Width: $F = 3.000$ in			
Ratio: Face width/pinion diameter: $F/D_P = 0.50$			
Recommended range of ratio: $0.50 < F/D_P < 2.00$			
Enter: Elastic Coefficient: $C_p = 2900$ Table 9-10	Enter: Quality Number: $A_v = 11$ Table 9-3	Dynamic Factor: $K_v = 1.50$ Table 9-9	
Reference: $N_P = 24$ $N_G = 95$	Reference: $N_P = 24$ $N_G = 95$	Reference: $N_P = 24$ $N_G = 95$	
Bending Geometry Factor-Pinion: $J_P = 0.360$ Fig. 9-15	Bending Geometry Factor-Gear: $J_G = 0.420$ Fig. 9-15	Reference: $m_G = 3.96$	
Enter: Pitting Geometry Factor: $I = 0.114$ Fig. 9-21			
<b>Stress Analysis: Bending</b>		Pinion: Required $s_{at} = 19,333$ psi    See Fig. 9-11 or Table 9-5	
<b>Stress Analysis: Pitting</b>		Gear: Required $s_{at} = 16,226$ psi	
<b>Stress Analysis: Bending</b>		Pinion: Required $s_{bc} = 139,718$ psi    See Fig. 9-12 or Table 9-5	
<b>Stress Analysis: Pitting</b>		Gear: Required $s_{bc} = 135,211$ psi	
<b>Stress Analysis: Bending</b>		Required hardness of pinion HB: 344    Equations in Fig. 9-12-Grade 1	
<b>Stress Analysis: Pitting</b>		Required hardness of gear HB: 330    Equations in Fig. 9-12-Grade 1	
<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>			
<b>One possible material specification:</b>			
Pinion requires HB 344; SAE 1040 WQT 800; HB 352; 21% elongation			
Gear requires HB 330; SAE 1040 WQT 800; HB 352; 21% elongation			
<b>Comments:</b>			
It would be reasonable to specify the same heat treatment for both the pinion and the gear because their contact stresses are very similar.			



**DESIGN OF SPUR GEARS**

<b>APPLICATION: Problem 63</b>		<b>Factors in Design Analysis:</b>	
<b>Cement mixer driven by a gasoline engine</b>		Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$	If $F < 1.0$ If $F > 1.0$
<b>Initial Input Data:</b>		Pinion Proportion Factor, $C_{pf} = 0.075$	0.091    [0.50 < F/D <sub>p</sub> < 2.00]
Overload Factor: $K_o = 2.00$	Table 9-7	Enter: $C_{ma} = 0.091$	Figure 9-16
Transmitted Power: $P = 2.5$ hp		Type of gearing: Open	Commer.    Precision    Ex. Prec.
Design Power $P_{des} = 5$ hp		Mesh Alignment Factor, $C_{ma} = 0.284$	0.162    0.096    0.061
Diametral Pitch: $P_d = 8$	Fig. 9-24	Enter: $C_{ma} = 0.284$	Figure 9-17
Input Speed: $n_p = 900$ rpm		Alignment Factor: $K_m = 1.38$	[Computed]
Number of Pinion Teeth: $N_p = 18$		Size Factor: $K_s = 1.00$	Table 9-8: Use 1.00 if $P_d > 5$
Desired Output Speed: $n_g = 75$ rpm		Pinion Rim Thickness Factor: $K_{gp} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Computed number of gear teeth: 216.0		Gear Rim Thickness Factor: $K_{gg} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Enter: Chosen No. of Gear Teeth: $N_g = 216$		Service Factor: $SF = 1.00$	Use 1.00 if no unusual conditions
<b>Computed data:</b>		Reliability Factor: $K_R = 1.00$	Table 9-11 Use 1.00 for $R = .99$
Actual Output Speed: $n_g = 75.0$ rpm		Enter: Design Life: 8000 hours	See Table 9-12
Gear Ratio: $m_g = 12.00$		Pinion - Number of load cycles: $N_p = 4.3E+08$	Guidelines: $Y_N, Z_N$
Pitch Diameter - Pinion: $D_p = 2.250$ in		Gear - Number of load cycles: $N_g = 3.6E+07$	$10^7$ cycles $> 10^7$ $< 10^7$
Pitch Diameter - Gear: $D_g = 27.000$ in		Bending Stress Cycle Factor: $Y_{NP} = 0.95$	1.00    0.95    Fig. 9-22
Center Distance: $C = 14.625$ in		Bending Stress Cycle Factor: $Y_{NG} = 0.99$	1.00    0.99    Fig. 9-22
Pitch Line Speed: $v_t = 530$ ft/min		Pitting Stress Cycle Factor: $Z_{NP} = 0.92$	1.00    0.92    Fig. 9-23
Transmitted Load: $W_t = 156$ lb		Pitting Stress Cycle Factor: $Z_{NG} = 0.97$	1.00    0.97    Fig. 9-23
<b>Secondary Input Data:</b>		<b>Stress Analysis: Bending</b>	
Face Width Guidelines (in): Min 1.00    Nom 1.500    Max 2.000		Pinion: Required $s_{bt} = 6,796$ psi	See Fig. 9-11 or Table 9-5
Enter: Face Width: $F = 2.250$ in		Gear: Required $s_{gt} = 4,929$ psi	Table 9-5
Ratio: Face width/pinion diameter: $F/D_p = 1.00$		<b>Stress Analysis: Pitting</b>	
Recommended range of ratio: $0.50 < F/D_p < 2.00$		Pinion: Required $s_{ac} = 72,362$ psi	See Fig. 9-12 or Table 9-5
Enter: Elastic Coefficient: $C_p = 2100$	Table 9-10	Gear: Required $s_{gc} = 68,632$ psi	Table 9-5
Enter: Quality Number: $A_v = 12$	Table 9-3	Required hardness of pinion HB: 134    Equations in Fig. 9-12-Grade 1	
Dynamic Factor: $K_v = 1.38$	Table 9-9	Required hardness of gear HB: 123    Equations in Fig. 9-12-Grade 1	
[Factors for computing $K_v$ ]: $B = 0.915$ $C = 54.74$		<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>	
Reference: $N_p = 18$ $N_g = 216$		<b>One possible material specification:</b>	
Bending Geometry Factor-Pinion: $J_p = 0.325$	Fig. 9-15	Pinion requires less than HB 190; SAE 1040 CD; HB 160; 12% elongation	
Bending Geometry Factor-Gear: $J_g = 0.430$	Fig. 9-15	Gear requires grey cast iron, ASTM A48, Class 40 [Table 9-6]	
Reference: $m_g = 12.00$		<b>Comments:</b>	
Enter: Pitting Geometry Factor: $I = 0.116$	Fig. 9-21	Large gear can be conveniently cast and affixed to the drum of the cement mixer.	
		Steel gear can be mounted on engine shaft.	

**DESIGN OF SPUR GEARS**

<b>APPLICATION: Problem 64</b>	
<b>Wood chipper driven by a gasoline engine: Speed Increase</b>	
<b>Initial Input Data:</b>	
Overload Factor: $K_o = 2.75$	Table 9-7
Transmitted Power: $P = 75$ hp	
Design Power $P_{des} = 206.25$ hp	
Diametral Pitch: $P_d = 6$	Fig. 9-24
Input Speed: $n_P = 2200$ rpm	
Number of Pinion Teeth: $N_P = 41$	
Desired Output Speed: $n_G = 4550$ rpm	
Computed number of gear teeth: 19.8	
Enter: Chosen No. of Gear Teeth: $N_G = 20$	
<b>Computed data: Note - Gear drives pinion</b>	
Actual Output Speed: $n_G = 4510.0$ rpm	
Gear Ratio: $m_G = 2.05$	
Pitch Diameter - Pinion: $D_P = 6.833$ in	
Pitch Diameter - Gear: $D_G = 3.333$ in	
Center Distance: $C = 5.083$ in	
Pitch Line Speed: $v_t = 3936$ ft/min	
Transmitted Load: $W_t = 629$ lb	
<b>Secondary Input Data:</b>	
Face Width Guidelines (in): 1.333	Min Norm Max
Enter: Face Width: $F = 3.000$ in	2.000 2.687
Ratio: Face width/pinion diameter: $F/D_P = 0.90$	
Recommended range of ratio: $0.50 < F/D_P < 2.00$	
Enter: Elastic Coefficient: $C_p = 2300$	Table 9-10
Enter: Quality Number: $A_v = 11$	Table 9-3
Dynamic Factor: $K_v = 1.81$	Table 9-9
[Factors for computing $K_v$ ]: $B = 0.826$	$C = 59.75$
Reference: $N_P = 41$	$N_G = 20$
Bending Geometry Factor-Pinion: $J_P = 0.380$	Fig. 9-15
Bending Geometry Factor-Gear: $J_G = 0.330$	Fig. 9-15
Reference: $m_G = 2.05$	
Enter: Pitting Geometry Factor: $I = 0.096$	Fig. 9-21
<b>Factors in Design Analysis:</b>	
Alignment Factor, $K_m = 1.0 + C_{pr} + C_{ma}$	If $F < 1.0$ If $F > 1.0$
Pinion Proportion Factor, $C_{pf} = 0.065$	0.090 [0.50 < F/D <sub>P</sub> < 2.00]
Enter: $C_{pr} = 0.09$	Figure 9-16
Type of gearing: Open	Commer. Precision Ex. Prec.
Mesh Alignment Factor, $C_{ma} = 0.296$	0.173 0.105 0.068
Enter: $C_{me} = 0.296$	Figure 9-17
Alignment Factor: $K_m = 1.39$	[Computed]
Size Factor: $K_s = 1.00$	Table 9-8: Use 1.00 if $P_d \geq 5$
Pinion Rim Thickness Factor: $K_{sp} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Gear Rim Thickness Factor: $K_{sg} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Service Factor: $SF = 1.00$	Use 1.00 if no unusual conditions
Reliability Factor: $K_R = 1.00$	Table 9-11 Use 1.00 for $R = .99$
Enter: Design Life: 8000 hours	See Table 9-12
Pinion - Number of load cycles: $N_P = 1.1E+09$	Guidelines: $Y_N, Z_N$
Gear - Number of load cycles: $N_G = 5.2E+08$	$10^7$ cycles $>10^7$ $<10^7$
Bending Stress Cycle Factor: $Y_{MP} = 0.94$	1.00 0.94 Fig. 9-22
Bending Stress Cycle Factor: $Y_{MG} = 0.95$	1.00 0.95 Fig. 9-22
Pitting Stress Cycle Factor: $Z_{MP} = 0.90$	1.00 0.90 Fig. 9-23
Pitting Stress Cycle Factor: $Z_{MG} = 0.91$	1.00 0.91 Fig. 9-23
<b>Stress Analysis: Bending - Note - Gear drives pinion</b>	
Gear: Required $s_{at} = 24,281$ psi	See Fig. 9-11 or
Pinion: Required $s_{at} = 27,666$ psi	Table 9-5
<b>Stress Analysis: Pitting - Adjusted equation to use <math>D_g</math> in place of <math>D_p</math></b>	
Pinion: Required $s_{pc} = 171,762$ psi	See Fig. 9-12 or
Gear: Required $s_{pc} = 169,875$ psi	Table 9-5
Required hardness of pinion HB: 443	Equations in Fig. 9-12-Grade 1
Required hardness of gear HB: 437	Equations in Fig. 9-12-Grade 1
<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>	
<b>One possible material specification:</b>	
Pinion and gear require flame or induction hardening	
SAE 4140 OQT 800; HB 352, 21% Elongation-Core; Case harden to HRC 54 min.	
<b>Comments:</b>	
It would be reasonable to specify the same heat treatment for both the pinion and the gear because their contact stresses are very similar.	



DESIGN OF SPUR GEARS		APPLICATION:	
NOTE: SI Metric data		Small tractor driven by a gasoline engine Problem 65	
<b>Initial Input Data:</b>		<b>Factors in Design Analysis:</b>	
Input Power: $P = 3.0$ kW		Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$	Pinion Proportion Factor, $C_{pf} = 0.042$ if $F < 25$ [0.50 < $F/D_p < 2.00$ ]
Input Speed: $n_p = 600$ rpm		Enter: $C_{pf} = 0.049$	Figure 9-16
[See Table 8-3] Module: $m = 3.00$ mm		Type of gearing: Open	Commer. Precision Ex. Prec.
Number of Pinion Teeth: $N_p = 20$		Mesh Alignment Factor, $C_{ma} = 0.274$	0.152 0.088 0.054
Desired Output Speed: $n_g = 175$ rpm		Enter: $C_{ma} = 0.274$	Figure 9-17
Computed number of gear teeth: 68.6		Alignment Factor: $K_m = 1.32$	[Computed]
Enter: Chosen No. of Gear Teeth: $N_g = 68$		Overload Factor: $K_o = 2.00$	Table 9-7
<b>Computed data:</b>		Size Factor: $K_s = 1.00$	Table 9-8: Use 1.00 if $P_d \geq 5$
Actual Output Speed: $n_g = 176.5$ rpm		Pinion Rim Thickness Factor: $K_{RP} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Gear Ratio: $m_G = 3.40$		Gear Rim Thickness Factor: $K_{RG} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Pitch Diameter - Pinion: $D_p = 60.00$ mm		Dynamic Factor: $K_v = 1.32$	[Computed: See Fig. 9-19]
Pitch Diameter - Gear: $D_g = 204.00$ mm		Service Factor: $SF = 1.00$	Use 1.00 if no unusual conditions
Center Distance: $C = 132.00$ mm		Reliability Factor: $K_R = 1.00$	Table 9-11 Use 1.00 for $R = .99$
Pitch Line Speed: $v_t = 1.88$ m/s		Enter: Design Life: 5000 hours	See Table 9-12
Transmitted Load: $W_t = 1592$ N		Pinion - Number of load cycles: $N_p = 1.8E+08$	Guidelines: $Y_N, Z_N < 10^7$
		Gear - Number of load cycles: $N_g = 5.3E+07$	$10^7$ cycles $> 10^7$
<b>Secondary Input Data:</b>		Bending Stress Cycle Factor: $Y_{NP} = 0.97$	1.00 0.97 Fig. 9-22
Min 24 mm	Nom 36 mm	Bending Stress Cycle Factor: $Y_{NG} = 0.99$	1.00 0.99 Fig. 9-22
Max 46 mm		Pitting Stress Cycle Factor: $Z_{NP} = 0.94$	1.00 0.94 Fig. 9-23
Enter: Face Width: $F = 40.0$ mm		Pitting Stress Cycle Factor: $Z_{NG} = 0.96$	1.00 0.96 Fig. 9-23
Ratio: Face width/pinion diameter: $F/D_p = 0.67$		<b>Stress Analysis: Bending</b>	
Recommended range of ratio: $0.50 < F/D_p < 2.00$		Pinion: Required $s_{at} = 144$ MPa	See Fig. 9-11 or Table 9-5
Enter: Elastic Coefficient: $C_p = 197$		Gear: Required $s_{at} = 113$ MPa	
Enter: Quality Number: $A_v = 12$		<b>Stress Analysis: Pitting</b>	
REF: $N_p, N_g = 20$		Pinion: Required $s_{ac} = 958$ MPa	See Fig. 9-12 or Table 9-5
Enter: Bending Geometry Factors: Press. angle = 20 deg		Gear: Required $s_{ac} = 938$ MPa	Table 9-5
Pinion: $J_p = 0.330$	Fig. 9-15	<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>	
Gear: $J_g = 0.415$	Fig. 9-15	<b>One possible material specification:</b> Steel pinion, Steel gear	
Enter: Pitting Geometry Factor: $I = 0.104$	Fig. 9-21	Pinion requires HB 341; SAE 4340 OQT 1000; HB 363	
REF: $m_G = 3.40$		Gear requires HB 332; SAE 4340 OQT 1000; HB 363 (Same as pinion)	

For $K_v$ :	
B	0.915
C	3.90

Through-Hardened	
Grade 1 Steel	
HB 105	Fig. 9-11
HB 46	Fig. 9-11
HB 341	Fig. 9-12
HB 332	Fig. 9-12



**DESIGN OF SPUR GEARS**

NOTE: SI Metric data

APPLICATION:

Electric power generator driven by a water turbine  
Problem 66

**Initial Input Data:**

Input Power:  $P = 75.0$  kW  
 Input Speed:  $n_P = 4500$  rpm  
 [See Table 8-3] Module:  $m = 4.00$  mm  
 Number of Pinion Teeth:  $N_P = 24$   
 Desired Output Speed:  $n_G = 3600$  rpm  
 Computed number of gear teeth: 30.0  
 Enter: Chosen No. of Gear Teeth:  $N_G = 30$

**Computed data:**

Actual Output Speed:  $n_G = 3600.0$  rpm  
 Gear Ratio:  $m_G = 1.25$   
 Pitch Diameter - Pinion:  $D_P = 96.00$  mm  
 Pitch Diameter - Gear:  $D_G = 120.00$  mm  
 Center Distance:  $C = 108.00$  mm  
 Pitch Line Speed:  $v_t = 22.62$  m/s  
 Transmitted Load:  $W_t = 3316$  N

**Secondary Input Data:**

	Min	Nom	Max
mm	32	48	64
Enter: Face Width: $F = 50.0$ mm			
Ratio: Face width/pinion diameter: $F/D_P = 0.52$			
Recommended range of ratio: $0.50 < F/D_P < 2.00$			
Enter: Elastic Coefficient: $C_p = 197$			
Enter: Quality Number: $A_v = 7$			
REF: $N_P, N_G = 24, 30$			

Enter: Bending Geometry Factors: Press, angle = 20 deg  
 Pinion:  $J_P = 0.347$  Fig. 9-15  
 Gear:  $J_G = 0.365$  Fig. 9-15  
 Enter: Pitting Geometry Factor:  $I = 0.084$  Fig. 9-21  
 REF:  $m_G = 1.25$

**Factors in Design Analysis:**

Alignment Factor,  $K_m = 1.0 + C_{pf} + C_{ma}$  If  $F < 25$  if  $F > 25$  mm  
 Pinion Proportion Factor,  $C_{pf} = 0.027$  0.040 [0.50 <  $F/D_P < 2.00$ ]  
 Enter:  $C_{pf} = 0.040$  Figure 9-16  
 Type of gearing: Open Commer. Precision Ex. Prec.  
 Mesh Alignment Factor,  $C_{ma} = 0.280$  0.158 0.093 0.058  
 Enter:  $C_{ma} = 0.158$  Figure 9-17  
 Alignment Factor:  $K_m = 1.20$  [Computed]

**Overload Factor:  $K_o = 1.20$  Table 9-7**

Size Factor:  $K_s = 1.00$  Table 9-8; Use 1.00 if  $P_d > = 5$   
 Pinion Rim Thickness Factor:  $K_{rp} = 1.00$  Fig. 9-18; Use 1.00 if solid blank  
 Gear Rim Thickness Factor:  $K_{rg} = 1.00$  Fig. 9-18; Use 1.00 if solid blank

Dynamic Factor:  $K_v = 1.26$  [Computed; See Fig. 9-19]  
 Service Factor:  $SF = 1.00$  Use 1.00 if no unusual conditions  
 Reliability Factor:  $K_R = 1.00$  Table 9-11 Use 1.00 for  $R = .99$   
 Enter: Design Life: 100000 hours See Table 9-12

Pinion - Number of load cycles:  $N_P = 2.7E+10$  Guidelines:  $Y_N, Z_N$   
 Gear - Number of load cycles:  $N_G = 2.2E+10$   $10^7$  cycles  $> 10^8$   $< 10^7$

Bending Stress Cycle Factor:  $Y_{MP} = 0.88$  Fig. 9-22  
 Bending Stress Cycle Factor:  $Y_{NG} = 0.89$  Fig. 9-22  
 Pitting Stress Cycle Factor:  $Z_{MP} = 0.83$  Fig. 9-23  
 Pitting Stress Cycle Factor:  $Z_{NG} = 0.84$  Fig. 9-23

**Stress Analysis: Bending**

Pinion: Required  $s_{bt} = 98$  MPa See Fig. 9-11 or  
 Gear: Required  $s_{gt} = 93$  MPa Table 9-5

**Stress Analysis: Pitting**

Pinion: Required  $s_{ap} = 889$  MPa See Fig. 9-12 or  
 Gear: Required  $s_{ag} = 878$  MPa Table 9-5

Specify materials, alloy and heat treatment, for most severe requirement.  
 One possible material specification: Steel pinion, Steel gear  
 Pinion requires HB 310; SAE 4340 OQT 1100; HB 321  
 Gear requires HB 305; SAE 4340 OQT 1100; HB 321 (Same as pinion)

For  $K_v$ :

B	0.397
C	5.97

Through-Hardened

Grade 1 Steel	
HB 19	Fig. 9-11
HB 8	Fig. 9-11
HB 310	Fig. 9-12
HB 305	Fig. 9-12

**DESIGN OF SPUR GEARS**

<b>APPLICATION: Problem 67</b>	
<b>Commercial band saw driven by an electric motor</b>	
<b>Initial Input Data:</b>	
Overload Factor: $K_o = 1.50$	Table 9-7
Transmitted Power: $P = 12$ hp	
Design Power $P_{des} = 18$ hp	
Diametral Pitch: $P_d = 10$	Fig. 9-24
Input Speed: $n_P = 3450$ rpm	
Number of Pinion Teeth: $N_P = 18$	
Desired Output Speed: $n_G = 730$ rpm	
Computed number of gear teeth: 85.1	
Enter: Chosen No. of Gear Teeth: $N_G = 85$	
<b>Computed data:</b>	
Actual Output Speed: $n_G = 730.6$ rpm	
Gear Ratio: $m_G = 4.72$	
Pitch Diameter - Pinion: $D_P = 1.800$ in	
Pitch Diameter - Gear: $D_G = 8.500$ in	
Center Distance: $C = 5.150$ in	
Pitch Line Speed: $v_t = 1626$ ft/min	
Transmitted Load: $W_t = 244$ lb	
<b>Secondary Input Data:</b>	
Face Width Guidelines (in): Min 0.800, Nom 1.200, Max 1.600	
Enter: Face Width: $F = 1.250$ in	
Ratio: Face width/pinion diameter: $F/D_P = 0.69$	
Recommended range of ratio: $0.50 < F/D_P < 2.00$	
Enter: Elastic Coefficient: $C_p = 2300$	Table 9-10
Enter: Quality Number: $A_v = 9$	Table 9-3
Dynamic Factor: $K_v = 1.33$	Table 9-9
[Factors for computing $K_v$ ]: $B = 0.630$ , $C = 70.71$	
Reference: $N_P = 18$ , $N_G = 85$	
Bending Geometry Factor-Pinion: $J_P = 0.310$	Fig. 9-15
Bending Geometry Factor-Gear: $J_G = 0.400$	Fig. 9-15
Reference: $m_G = 4.72$	
Enter: Pitting Geometry Factor: $I = 0.106$	Fig. 9-21
<b>Factors in Design Analysis:</b>	
Alignment Factor, $K_m = 1.0 + C_{pf} + C_{me}$	If $F < 1.0$ , If $F > 1.0$
Pinion Proportion Factor, $C_{pf} = 0.044$	0.048 [0.50 < $F/D_P < 2.00$ ]
Enter: $C_{me} = 0.048$	Figure 9-16
Type of gearing: Open	Commer.
Mesh Alignment Factor, $C_{ma} = 0.268$	0.147
Enter: $C_{me} = 0.147$	Figure 9-17
Alignment Factor: $K_m = 1.20$	[Computed]
Size Factor: $K_s = 1.00$	Table 9-8: Use 1.00 if $P_d >= 5$
Pinion Rim Thickness Factor: $K_{sp} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Gear Rim Thickness Factor: $K_{sg} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Service Factor: $SF = 1.00$	Use 1.00 if no unusual conditions
Reliability Factor: $K_R = 1.00$	Table 9-11 Use 1.00 for $R = .99$
Enter: Design Life: 8000 hours	See Table 9-12
Pinion - Number of load cycles: $N_P = 1.7E+09$	Guidelines: $Y_N, Z_N$
Gear - Number of load cycles: $N_G = 3.5E+08$	$10^7$ cycles $> 10^7$ $< 10^7$
Bending Stress Cycle Factor: $Y_{NP} = 0.93$	1.00 0.93 Fig. 9-22
Bending Stress Cycle Factor: $Y_{NG} = 0.96$	1.00 0.96 Fig. 9-22
Pitting Stress Cycle Factor: $Z_{NP} = 0.89$	1.00 0.89 Fig. 9-23
Pitting Stress Cycle Factor: $Z_{NG} = 0.92$	1.00 0.92 Fig. 9-23
<b>Stress Analysis: Bending</b>	
Pinion: Required $s_{at} = 16,101$ psi	See Fig. 9-11 or
Gear: Required $s_{at} = 12,088$ psi	Table 9-5
<b>Stress Analysis: Pitting</b>	
Pinion: Required $s_{ac} = 127,467$ psi	See Fig. 9-12 or
Gear: Required $s_{ac} = 123,310$ psi	Table 9-5
Required hardness of pinion HB: 305	Equations in Fig. 9-12-Grade 1
Required hardness of gear HB: 293	Equations in Fig. 9-12-Grade 1
<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>	
<b>One possible material specification:</b>	
Pinion requires HB 305; SAE 4140 OQT 1100; HB 321, 19% Elongation	
Gear requires HB 293; SAE 4140 OQT 1200; HB 293, 20% Elongation	
<b>Comments:</b>	
It would be reasonable to specify the same heat treatment for both the pinion and the gear because their contact stresses are very similar.	

**DESIGN OF SPUR GEARS**

<b>APPLICATION: Problem 68</b>	
<b>Commercial bend saw driven by an electric motor</b>	
<i>Initial Input Data:</i>	
Overload Factor: $K_o = 1.50$	Table 9-7
Transmitted Power: $P = 12$ hp	
Design Power $P_{des} = 18$ hp	
Diametral Pitch: $P_d = 14$	Fig. 9-24
Input Speed: $n_P = 3450$ rpm	
Number of Pinion Teeth: $N_P = 18$	
Desired Output Speed: $n_G = 730$ rpm	
Computed number of gear teeth: 85.1	
Enter: Chosen No. of Gear Teeth: $N_G = 85$	
<i>Computed data:</i>	
Actual Output Speed: $n_G = 730.6$ rpm	
Gear Ratio: $m_G = 4.72$	
Pitch Diameter - Pinion: $D_P = 1.286$ in	
Pitch Diameter - Gear: $D_G = 6.071$ in	
Center Distance: $C = 3.678$ in	
Pitch Line Speed: $v_t = 1161$ ft/min	
Transmitted Load: $W_t = 341$ lb	
<i>Secondary Input Data:</i>	
Face Width Guidelines (in):	Min      Nom      Max
Enter: Face Width: $F = 1.125$ in	0.571    0.857    1.143
Ratio: Face width/pinion diameter: $F/D_P = 0.88$	
Recommended range of ratio: $0.50 < F/D_P < 2.00$	
Enter: Elastic Coefficient: $C_p = 2300$	Table 9-10
Enter: Quality Number: $A_v = 7$	Table 9-3
Dynamic Factor: $K_v = 1.15$	Table 9-9
[Factors for computing $K_v$ ]: $B = 0.397$ $C = 83.77$	
Reference: $N_P = 18$ $N_G = 85$	
Bending Geometry Factor-Pinion: $J_P = 0.310$	Fig. 9-15
Bending Geometry Factor-Gear: $J_G = 0.400$	Fig. 9-15
Reference: $m_G = 4.72$	
Enter: Pitting Geometry Factor: $I = 0.106$	Fig. 9-21
<i>Factors in Design Analysis:</i>	
Alignment Factor, $K_{a1} = 1.0 + C_{pr} + C_{ma}$	If $F < 1.0$ 0.063    0.064    [0.50 < $F/D_P < 2.00$ ]
Pinion Proportion Factor, $C_{pr}$	Enter: $C_{pr} = 0.064$ Figure 9-16
Type of gearing:	Open    Commer.    Precision    Ex. Prec.
Mesh Alignment Factor, $C_{ma}$	$= 0.266$ 0.145    0.082    0.049
Enter: $C_{ma} = 0.145$	Figure 9-17
Alignment Factor: $K_m = 1.21$	[Computed]
Size Factor: $K_s = 1.00$	Table 9-8: Use 1.00 if $P_d \geq 5$
Pinion Rim Thickness Factor: $K_{BP} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Gear Rim Thickness Factor: $K_{BG} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Service Factor: $SF = 1.00$	Use 1.00 if no unusual conditions
Reliability Factor: $K_R = 1.00$	Table 9-11 Use 1.00 for $R = .99$
Enter: Design Life: 8000 hours	See Table 9-12
Pinion - Number of load cycles: $N_P = 1.7E+09$	Guidelines: $Y_N, Z_N$
Gear - Number of load cycles: $N_G = 3.5E+08$	$> 10^7$ cycles $< 10^7$
Bending Stress Cycle Factor: $Y_{NP} = 0.93$	1.00    0.93    Fig. 9-22
Bending Stress Cycle Factor: $Y_{NG} = 0.96$	1.00    0.96    Fig. 9-22
Pitting Stress Cycle Factor: $Z_{NP} = 0.89$	1.00    0.89    Fig. 9-23
Pitting Stress Cycle Factor: $Z_{NG} = 0.92$	1.00    0.92    Fig. 9-23
<i>Stress Analysis: Bending</i>	
Pinion: Required $s_{bt} = 30,567$ psi	See Fig. 9-11 or
Gear: Required $s_{bt} = 22,949$ psi	Table 9-5
<i>Stress Analysis: Pitting</i>	
Pinion: Required $s_{ac} = 175,629$ psi	See Fig. 9-12 or
Gear: Required $s_{ac} = 169,902$ psi	Table 9-5
Required hardness of pinion HB: 455	Equations in Fig. 9-12-Grade 1
Required hardness of gear HB: 437	Equations in Fig. 9-12-Grade 1
<i>Specify materials, alloy and heat treatment, for most severe requirement.</i>	
<i>One possible material specification:</i>	
Pinion requires case hardening by carburizing	
Gear requires case hardening by carburizing	
<i>Specifications: Example selection</i>	
Specify SAE 4620 DOQT 300; Case hardness HRC 62; ductile core; HB 248	
For both pinion and gear	



DESIGN OF SPUR GEARS

<b>APPLICATION: Problem 69</b>		
Machine tool driven by an electric motor		
<b>Initial Input Data:</b>		
Overload Factor: $K_o = 1.50$	Table 9-7	
Transmitted Power: $P = 20$ hp		
Design Power $P_{des} = 30$ hp		
Diametral Pitch: $P_d = 8$	Fig. 9-24	
Input Speed: $n_P = 650$ rpm		
Number of Pinion Teeth: $N_P = 22$		
Desired Output Speed: $n_G = 112.5$ rpm		
Computed number of gear teeth: 127.1		
Enter: Chosen No. of Gear Teeth: $N_G = 128$		
<b>Computed data:</b>		
Actual Output Speed: $n_G = 111.7$ rpm		
Gear Ratio: $m_G = 5.82$		
Pitch Diameter - Pinion: $D_P = 2.750$ in		
Pitch Diameter - Gear: $D_G = 16.000$ in		
Center Distance: $C = 9.375$ in		
Pitch Line Speed: $v_t = 468$ ft/min		
Transmitted Load: $W_t = 1410$ lb		
<b>Secondary Input Data:</b>		
Min	Nom	Max
Face Width Guidelines (in): 1.000	1.500	2.000
Enter: Face Width: $F = 2.000$ in		
Ratio: Face width/pinion diameter: $F/D_P = 0.73$		
Recommended range of ratio: $0.50 < F/D_P < 2.00$		
Enter: Elastic Coefficient: $C_p = 2300$	Table 9-10	
Enter: Quality Number: $A_v = 7$	Table 9-3	
Dynamic Factor: $K_v = 1.10$	Table 9-9	
[Factors for computing $K_v$ ]: $B = 0.397$	$C = 83.77$	
Reference: $N_P = 22$	$N_G = 128$	
Bending Geometry Factor-Pinion: $J_P = 0.345$	Fig. 9-15	
Bending Geometry Factor-Gear: $J_G = 0.440$	Fig. 9-15	
Reference: $m_G = 5.82$		
Enter: Pitting Geometry Factor: $I = 0.106$	Fig. 9-21	

<b>Factors in Design Analysis:</b>	
Alignment Factor, $K_{m1} = 1.0 + C_{pt} + C_{ma}$	If $F < 1.0$ 0.048 0.060 [0.50 < $F/D_P < 2.00$ ]
Pinion Proportion Factor, $C_{pt}$	Enter: $C_{pt} = 0.060$ Figure 9-16
Type of gearing: Open	Commer. Precision Ex. Prec.
Mesh Alignment Factor, $C_{ma}$	$0.280$ 0.158 0.093 0.058
Enter: $C_{ma} = 0.093$	Figure 9-17
Alignment Factor: $K_m = 1.15$	[Computed]
Size Factor: $K_s = 1.00$	Table 9-8: Use 1.00 if $P_d >= 5$
Pinion Rim Thickness Factor: $K_{sp} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Gear Rim Thickness Factor: $K_{sg} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Service Factor: $SF = 1.00$	Use 1.00 if no unusual conditions
Reliability Factor: $K_R = 1.00$	Table 9-11 Use 1.00 for $R = .99$
Enter: Design Life: 25000 hours	See Table 9-12
Pinion - Number of load cycles: $N_P = 9.8E+08$	Guidelines: $Y_N, Z_N$
Gear - Number of load cycles: $N_G = 1.7E+08$	$10^7$ cycles $> 10^7$ $< 10^7$
Bending Stress Cycle Factor: $Y_{NP} = 0.94$	1.00 0.94 Fig. 9-22
Bending Stress Cycle Factor: $Y_{NG} = 0.97$	1.00 0.97 Fig. 9-22
Pitting Stress Cycle Factor: $Z_{NP} = 0.90$	1.00 0.90 Fig. 9-23
Pitting Stress Cycle Factor: $Z_{NG} = 0.94$	1.00 0.94 Fig. 9-23
<b>Stress Analysis: Bending</b>	
Pinion: Required $s_{at} = 32,958$ psi	See Fig. 9-11 or
Gear: Required $s_{at} = 26,043$ psi	Table 9-5
<b>Stress Analysis: Pitting</b>	
Pinion: Required $s_{ac} = 173,012$ psi	See Fig. 9-12 or
Gear: Required $s_{ac} = 165,650$ psi	Table 9-5
Required hardness of pinion HB: 447	Equations in Fig. 9-12-Grade 1
Required hardness of gear HB: 424	Equations in Fig. 9-12-Grade 1
<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>	
<b>One possible material specification:</b>	
Pinion requires case hardening by carburizing	
Gear requires case hardening by carburizing	
<b>Specifications: Example selection</b>	
Specify SAE 4620 DOQT 300; Case hardness HRC 62; ductile core; HB 248	
For both pinion and gear	

**DESIGN OF SPUR GEARS**

**APPLICATION: Problem 70**

Crane cable drum driven by an electric motor

**Initial Input Data:**

Overload Factor:  $K_o = 1.50$  Table 9-7  
 Transmitted Power:  $P = 25$  hp  
 Design Power  $P_{des} = 37.5$  hp  
 Diametral Pitch:  $P_d = 6$  Fig. 9-24  
 Input Speed:  $n_P = 925$  rpm  
 Number of Pinion Teeth:  $N_P = 17$   
 Desired Output Speed:  $n_G = 163$  rpm

Computed number of gear teeth: 96.5

Enter: Chosen No. of Gear Teeth:  $N_G = 96$

**Computed data:**

Actual Output Speed:  $n_G = 163.8$  rpm  
 Gear Ratio:  $m_G = 5.65$   
 Pitch Diameter - Pinion:  $D_P = 2.833$  in  
 Pitch Diameter - Gear:  $D_G = 16.000$  in  
 Center Distance:  $C = 9.417$  in  
 Pitch Line Speed:  $v_1 = 696$  ft/min  
 Transmitted Load:  $W_1 = 1202$  lb

**Secondary Input Data:**

Face Width Guidelines (in): Min 1.333 Nom 2.000 Max 2.667  
 Enter: Face Width:  $F = 2.600$  in  
 Ratio: Face width/pinion diameter:  $F/D_P = 0.92$   
 Recommended range of ratio:  $0.50 < F/D_P < 2.00$

Enter: Elastic Coefficient:  $C_p = 2300$  Table 9-10  
 Enter: Quality Number:  $A_v = 7$  Table 9-3  
 Dynamic Factor:  $K_v = 1.11$  Table 9-9  
 [Factors for computing  $K_v$ ]:  $B = 0.397$   $C = 83.77$

Reference:  $N_P = 17$   $N_G = 96$   
 Bending Geometry Factor-Pinion:  $J_P = 0.295$  Fig. 9-15  
 Bending Geometry Factor-Gear:  $J_G = 0.420$  Fig. 9-15

Reference:  $m_G = 5.65$   
 Enter: Pitting Geometry Factor:  $I = 0.109$  Fig. 9-21

**Factors in Design Analysis:**

Alignment Factor,  $K_{m1} = 1.0 + C_{pf} + C_{ma}$  IF  $F < 1.0$  IF  $F > 1.0$   
 Pinion Proportion Factor,  $C_{pf} = 0.067$  0.087 Figure 9-16  
 Enter:  $C_{pf} = 0.087$   
 Type of gearing: Open Commer. Precision Ex. Prec.  
 Mesh Alignment Factor,  $C_{ma} = 0.290$  0.167 0.100 0.064  
 Enter:  $C_{ma} = 0.167$  Figure 9-17  
 Alignment Factor:  $K_m = 1.25$  [Computed]

Size Factor:  $K_s = 1.00$  Table 9-8: Use 1.00 if  $P_d \geq 5$   
 Pinion Rim Thickness Factor:  $K_{BP} = 1.00$  Fig. 9-18: Use 1.00 if solid blank  
 Gear Rim Thickness Factor:  $K_{BG} = 1.00$  Fig. 9-18: Use 1.00 if solid blank  
 Service Factor:  $SF = 1.00$  Use 1.00 if no unusual conditions  
 Reliability Factor:  $K_R = 1.00$  Table 9-11 Use 1.00 for  $R = .99$

Enter: Design Life: 31200 hours See Table 9-12  
 Pinion - Number of load cycles:  $N_P = 1.7E+09$  Guidelines:  $Y_N, Z_N$   
 Gear - Number of load cycles:  $N_G = 3.1E+08$   $> 10^7$   $< 10^8$   
 Bending Stress Cycle Factor:  $Y_{NP} = 0.93$  1.00 0.93 Fig. 9-22  
 Bending Stress Cycle Factor:  $Y_{NG} = 0.96$  1.00 0.96 Fig. 9-22  
 Pitting Stress Cycle Factor:  $Z_{NP} = 0.89$  1.00 0.89 Fig. 9-23  
 Pitting Stress Cycle Factor:  $Z_{NG} = 0.92$  1.00 0.92 Fig. 9-23

**Stress Analysis: Bending**

Pinion: Required  $s_{bt} = 21,194$  psi See Fig. 9-11 or  
 Gear: Required  $s_{gt} = 14,421$  psi Table 9-5  
**Stress Analysis: Pitting**  
 Pinion: Required  $s_{ac} = 144,763$  psi See Fig. 9-12 or  
 Gear: Required  $s_{ac} = 140,043$  psi Table 9-5

Required hardness of pinion HB: 359 Equations in Fig. 9-12-Grade 1  
 Required hardness of gear HB: 345 Equations in Fig. 9-12-Grade 1  
**Specify materials, alloy and heat treatment, for most severe requirement.**

**One possible material specification:**

Pinion requires HB 359; SAE 8650 OQT 1000; HB 363; 14% elongation  
 Pinion requires HB 345; SAE 8650 OQT 1000; HB 363; 14% elongation

**Comments:**

SPUR GEARS		APPLICATION: Centrifugal pump driven by an electric motor Chapter 9-Problem 71	
<b>POWER TRANSMITTING CAPACITY</b>			
<b>Initial Input Data:</b>			
Enter: Face Width:	F = 1.250 in		
Input Speed:	$n_p = 1725$ rpm		
Diametral Pitch:	$P_d = 10$		
Number of Pinion Teeth:	$N_p = 25$		
Number of Gear Teeth:	$N_g = 60$		
<b>Computed data:</b>			
Actual Output Speed:	$n_g = 718.8$ rpm		
Gear Ratio:	$m_G = 2.40$		
Pitch Diameter - Pinion:	$D_p = 2.500$ in		
Pitch Diameter - Gear:	$D_g = 6.000$ in		
Center Distance:	C = 4.250 in		
Pitch Line Speed:	$v_t = 1129$ ft/min		
Transmitted Load at $P_{min}$ Capacity:	$W_t = 377$ lb		
<b>Power Transmitting Capacity: (Using Eq. 9-32, 9-34)</b>			
Pinion: Based on Bending Stress:	25.08 hp		
Gear: Based on Bending Stress:	27.27 hp		
Pinion: Based on Contact Stress:	14.21 hp		
Gear: Based on Contact Stress:	12.91 hp		
<b>Power Transmitting Capacity:</b>	<b>12.91 hp</b>		
Enter: Elastic Coefficient:	$C_p = 2300$	Table 9-10	
Enter: Quality Number:	$A_v = 9$	Table 9-3	
REF: $N_p, N_g =$	25 60		
<b>Enter: Bending Geometry Factors: Press. angle = 20 deg</b>			
Pinion:	$J_p = 0.363$	Fig. 9-15	
Gear:	$J_g = 0.415$	Fig. 9-15	
<b>Enter: Pitting Geometry Factor: <math>I = 0.104</math> Fig. 9-21</b>			
REF: $m_G =$	2.40		
<b>ALLOWABLE BENDING STRESS NUMBERS: (Input)</b>			
Pinion: $S_{at} =$	39,100 psi	See Fig. 9-11 or Table 9-5	
Gear: $S_{at} =$	36,900 psi	Table 9-5	
<b>ALLOWABLE CONTACT STRESS NUMBERS: (Input)</b>			
Pinion: $s_{ac} =$	138,600 psi	See Fig. 9-12 or Table 9-5	
Gear: $s_{ac} =$	129,200 psi	Table 9-5	
<b>Material specification: Steel pinion; Steel gear: through hardened</b>			
Pinion material: SAE 4140 OQT 1000	340 HB		
Gear material: SAE 4140 OQT 1100	311 HB		
<b>Factors in Design Analysis:</b>			
Alignment Factor, $K_m = 1.0 + C_{pt} + C_{ma}$	IF < 1.0	IF > 1.0	F/D <sub>p</sub> = 0.50
Pinion Proportion Factor, $C_{pt} =$	0.025	0.028	[0.50 < F/D <sub>p</sub> < 2.00]
Enter: $C_{pt} =$	0.028	Figure 9-16	
Type of gearing:	Open	Commer.	Precision
Mesh Alignment Factor, $C_{ma} =$	0.268	0.147	0.083
Enter: $C_{ma} =$	0.147	Figure 9-17	
Alignment Factor: $K_m =$	1.18	[Computed]	
Overload Factor: $K_o =$	1.50	Table 9-7	
Size Factor: $K_s =$	1.00	Table 9-8: Use 1.00 if $P_d \geq 5$	
Pinion Rim Thickness Factor: $K_{sp} =$	1.00	Fig. 9-18: Use 1.00 if solid blank	
Gear Rim Thickness Factor: $K_{sg} =$	1.00	Fig. 9-18: Use 1.00 if solid blank	
Dynamic Factor: $K_y =$	1.28	[Computed: See Fig. 9-21]	
Service Factor: SF =	1.00	Use 1.00 if no unusual conditions	
Reliability Factor: $K_R =$	1.00	Table 9-8 Use 1.00 for R = .99	
<b>Enter: Design Life: 16000 hours See Table 9-7</b>			
Pinion - Number of load cycles: $N_p =$	1.8E+09	Guidelines: $Y_N, Z_N$	
Gear - Number of load cycles: $N_g =$	6.5E+08	10 <sup>7</sup> cycles > 10 <sup>8</sup> < 10 <sup>9</sup>	
Bending Stress Cycle Factor: $Y_{Np} =$	0.93	1.00	0.93 Fig. 9-22
Bending Stress Cycle Factor: $Y_{Ng} =$	0.94	1.00	0.94 Fig. 9-22
Pitting Stress Cycle Factor: $Z_{Np} =$	0.89	1.00	0.89 Fig. 9-24
Pitting Stress Cycle Factor: $Z_{Ng} =$	0.91	1.00	0.91 Fig. 9-24
<b>Through-Hardened</b>			
<b>Grade 1 Steel</b>			
		39.1 ksi	Fig. 9-11
		36.8 ksi	Fig. 9-11
		138.6 ksi	Fig. 9-12
		129.2 ksi	Fig. 9-12



SPUR GEARS		APPLICATION: Heavy duty conveyor driven by a gasoline engine Chapter 9-Problem 72	
<b>POWER TRANSMITTING CAPACITY</b>		<b>Initial Input Data:</b>	
Enter: Face Width:	$F = 2.000$ in	Enter: $F < 1.0$ If $F > 1.0$ $F/D_p = 0.50$	
Input Speed:	$n_p = 1500$ rpm	Pinion Proportion Factor, $C_{pt} = 0.025$ 0.038 [0.50 < $F/D_p$ < 2.00]	
Diametral Pitch:	$P_d = 6$	Enter: $C_{pr} = 0.038$ Figure 9-16	
Number of Pinion Teeth:	$N_p = 35$	Type of gearing: Open Commer. Precision Ex. Prec.	
Number of Gear Teeth:	$N_g = 100$	Mesh Alignment Factor, $C_{ma} = 0.280$ 0.158 0.093 0.058	
<b>Computed data:</b>		Enter: $C_{ma} = 0.158$ Figure 9-17	
Actual Output Speed:	$n_g = 525.0$ rpm	Alignment Factor: $K_m = 1.20$ [Computed]	
Gear Ratio:	$m_G = 2.86$	Overload Factor: $K_o = 2.00$ Table 9-7	
Pitch Diameter - Pinion:	$D_p = 5.833$ in	Size Factor: $K_s = 1.00$ Table 9-6: Use 1.00 if $P_d \geq 5$	
Pitch Diameter - Gear:	$D_g = 16.667$ in	Pinion Rim Thickness Factor: $K_{gr} = 1.00$ Fig. 9-18: Use 1.00 if solid blank	
Center Distance:	$C = 11.250$ in	Gear Rim Thickness Factor: $K_{gs} = 1.00$ Fig. 9-18: Use 1.00 if solid blank	
Pitch Line Speed:	$v_t = 2291$ ft/min	Dynamic Factor: $K_v = 1.63$ [Computed; See Fig. 9-21]	
Transmitted Load at $P_{min}$ Capacity:	$W_t = 277$ lb	Service Factor: $SF = 1.00$ Use 1.00 if no unusual conditions	
<b>Power Transmitting Capacity: (Using Eq. 9-32, 9-34)</b>		Reliability Factor: $K_R = 1.00$ Table 9-8 Use 1.00 for $R = .99$	
Pinion: Based on Bending Stress:	90.79 hp	Enter: Design Life: 15000 hours See Table 9-7	
Gear: Based on Bending Stress:	21.63 hp	Pinion - Number of load cycles: $N_p = 1.4E+09$ Guidelines: $Y_N, Z_N$	
Pinion: Based on Contact Stress:	86.50 hp	Gear - Number of load cycles: $N_g = 4.7E+08$ $10^7$ cycles $> 10^7$ $< 10^7$	
Gear: Based on Contact Stress:	19.26 hp	Bending Stress Cycle Factor: $Y_{Np} = 0.93$ 1.00 0.93 Fig. 9-22	
<b>Power Transmitting Capacity: 19.26 hp</b>		Bending Stress Cycle Factor: $Y_{Ng} = 0.95$ 1.00 0.95 Fig. 9-22	
Enter: Elastic Coefficient: $C_p = 2100$ Table 9-10		Pitting Stress Cycle Factor: $Z_{Np} = 0.89$ 1.00 0.89 Fig. 9-24	
Enter: Quality Number: $A_v = 11$ Table 9-3		Pitting Stress Cycle Factor: $Z_{Ng} = 0.92$ 1.00 0.92 Fig. 9-24	
REF: $N_p, N_g = 35, 100$		<b>Allowable Bending Stress Numbers: (Input)</b>	
Enter: Bending Geometry Factors: Press. angle = 20 deg		Pinion: $s_{at} = 40,000$ psi See Fig. 9-11	
Pinion: $J_p = 0.410$ Fig. 9-15		Gear: $s_{at} = 8,500$ psi Table 9-6	
Gear: $J_g = 0.450$ Fig. 9-15		<b>Allowable Contact Stress Numbers: (Input)</b>	
Enter: Pitting Geometry Factor: $I = 0.114$ Fig. 9-21		Pinion: $s_{ac} = 142,400$ psi See Fig. 9-12	
REF: $m_G = 2.86$		Gear: $s_{ac} = 65,000$ psi Table 9-6	
<b>Material specification: Steel pinion; Steel gear: through hardened</b>		Pinion material: SAE 1040 WQT 800 352 HB	
		Gear: Gray cast iron, ASTM A48, Class 30	

For $K_v$ :	
B	0.826
A	59.75

Through-Hardened	
Grade 1 Steel	
40.0 ksi	Fig. 9-11
-43.8 ksi	Fig. 9-11
142.4 ksi	Fig. 9-12
-29.1 ksi	Fig. 9-12

SPUR GEARS		APPLICATION: Heavy duty conveyor driven by a gasoline engine	
POWER TRANSMITTING CAPACITY		Chapter 9-Problem 73 - Redesign of system in Problem 72 to get capacity > 25 hp	
<b>Initial Input Data:</b>			
Enter: Face Width:	$F = 2.420$ in	Enter: $F < 1.0$	IF $F > 1.0$
Input Speed:	$n_p = 1500$ rpm	Pinion Proportion Factor, $C_{pf} = 0.025$	$0.043$ [0.50 < $F/D_p < 2.00$ ]
Diametral Pitch:	$P_d = 6$	Enter: $C_{pf} = 0.043$	Figure 9-16
Number of Pinion Teeth:	$N_p = 35$	Type of gearing:	Open Commer. Precision Ex. Prec.
Number of Gear Teeth:	$N_g = 100$	Mesh Alignment Factor, $C_{ma} = 0.287$	0.165 0.098 0.062
Enter: $C_{ma} = 0.165$ Figure 9-17			
Alignment Factor: $K_m = 1.21$ [Computed]			
<b>Computed data:</b>			
Actual Output Speed:	$n_g = 525.0$ rpm	Overload Factor: $K_o = 2.00$	Table 9-7
Gear Ratio:	$m_g = 2.86$	Size Factor: $K_s = 1.00$	Table 9-8: Use 1.00 if $P_d \geq 5$
Pitch Diameter - Pinion:	$D_p = 5.833$ in	Pinion Rim Thickness Factor: $K_{rp} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Pitch Diameter - Gear:	$D_g = 16.667$ in	Gear Rim Thickness Factor: $K_{rg} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Center Distance:	$C = 11.250$ in	Dynamic Factor: $K_v = 1.50$	[Computed: See Fig. 9-21]
Pitch Line Speed:	$v_t = 2291$ ft/min	Service Factor: $SF = 1.00$	Use 1.00 if no unusual conditions
Transmitted Load at $P_{min}$ Capacity:	$W_t = 361$ lb	Reliability Factor: $K_R = 1.00$	Table 9-8 Use 1.00 for $R = .99$
Enter: Design Life: 15000 hours See Table 9-7			
<b>Power Transmitting Capacity: (Using Eq. 9-32, 9-34)</b>			
Pinion: Based on Bending Stress:	118.13 hp	Pinion - Number of load cycles: $N_p = 1.4E+09$	Guidelines: $Y_{N1}, Z_N$
Gear: Based on Bending Stress:	28.14 hp	Gear - Number of load cycles: $N_g = 4.7E+08$	10 <sup>7</sup> cycles > 10 <sup>7</sup>
Pinion: Based on Contact Stress:	112.55 hp	Bending Stress Cycle Factor: $Y_{NP} = 0.93$	1.00 0.93 Fig. 9-22
Gear: Based on Contact Stress:	25.06 hp	Bending Stress Cycle Factor: $Y_{NG} = 0.95$	1.00 0.95 Fig. 9-22
Power Transmitting Capacity:	25.06 hp	Pitting Stress Cycle Factor: $Z_{NP} = 0.89$	1.00 0.89 Fig. 9-24
		Pitting Stress Cycle Factor: $Z_{NG} = 0.92$	1.00 0.92 Fig. 9-24
<b>Allowable Bending Stress Numbers: (Input)</b>			
Enter: Elastic Coefficient:	$C_p = 2100$ Table 9-10	Pinion: $s_{at} = 40,000$ psi	See Fig. 9-11
Enter: Quality Number:	$A_v = 10$ Table 9-3	Gear: $s_{at} = 8,500$ psi	Table 9-6
REF: $N_b, N_g = 35$ 100			
<b>Allowable Contact Stress Numbers: (Input)</b>			
Enter: Bending Geometry Factors: Press. angle = 20 deg		Pinion: $s_{ec} = 142,400$ psi	See Fig. 9-12
Pinion: $J_p = 0.410$ Fig. 9-15		Gear: $s_{ec} = 66,000$ psi	Table 9-6
Gear: $J_g = 0.450$ Fig. 9-15			
Enter: Pitting Geometry Factor: $I = 0.114$ Fig. 9-21			
REF: $m_g = 2.86$			
<b>Note: Increased face width from 2.00 to 2.42 in.</b>			
<b>Changed quality number from <math>A_v = 11</math> to 10 (More precise)</b>			
<b>Material specification: Steel pinion; Steel gear: through hardened</b>			
Pinion material: SAE 1040 WQT 800 352 HB			
Gear: Gray cast iron, ASTM A48, Class 30			

Set = 0.5

For $K_v$ :	
B	0.731
A	65.04

Through-Hardened	
Grade 1 Steel	
40.0 ksi	Fig. 9-11
42.8 ksi	Fig. 9-11
142.4 ksi	Fig. 9-12
29.1 ksi	Fig. 9-12

**DESIGN OF SPUR GEARS**

APPLICATION: Problem 74 - First pair		Factors in Design Analysis:	
Assembly conveyor driven by an electric motor		Alignment Factor, $K_m = 1.0 + C_{pt} + C_{ma}$	If $F < 1.0$ If $F > 1.0$
Double reduction - First pair - Input Data:		Pinion Proportion Factor, $C_{pt} = 0.048$	0.048 [0.50 < $F/D_p < 2.00$ ]
Overload Factor: $K_o = 1.50$	Table 9-7	Enter: $C_{pt} = 0.048$	Figure 9-16
Transmitted Power: $P = 10$ hp		Type of gearing: Open	Commer. Precision Ex. Prec.
Design Power $P_{des} = 15$ hp		Mesh Alignment Factor, $C_{ma} = 0.272$	0.150 0.086 0.053
Diametral Pitch: $P_d = 8$	Fig. 9-24	Enter: $C_{ma} = 0.150$	Figure 9-17
Input Speed: $n_P = 1750$ rpm		Alignment Factor: $K_m = 1.20$	[Computed]
Number of Pinion Teeth: $N_P = 18$		Size Factor: $K_s = 1.00$	Table 9-8: Use 1.00 if $P_d \geq 5$
Desired Output Speed: $n_G = 425$ rpm		Pin Rim Thickness Factor: $K_{BP} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Computed number of gear teeth: 74.1		Gear Rim Thickness Factor: $K_{BG} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Enter: Chosen No. of Gear Teeth: $N_G = 75$		Service Factor: $SF = 1.00$	Use 1.00 if no unusual conditions
Computed data:		Reliability Factor: $K_R = 1.00$	Table 9-11 Use 1.00 for $R = .99$
Actual Output Speed: $n_G = 420.0$ rpm		Enter: Design Life: 15000 hours	See Table 9-12
Gear Ratio: $m_G = 4.17$		Pinion - Number of load cycles: $N_p = 1.6E+09$	Guidelines: $Y_N, Z_N$
Pitch Diameter - Pinion: $D_P = 2.250$ in		Gear - Number of load cycles: $N_G = 3.8E+08$	$10^7$ cycles $> 10^7$ $< 10^7$
Pitch Diameter - Gear: $D_G = 9.375$ in		Bending Stress Cycle Factor: $Y_{NP} = 0.93$	1.00 0.93 Fig. 9-22
Center Distance: $C = 5.813$ in		Bending Stress Cycle Factor: $Y_{NG} = 0.95$	1.00 0.95 Fig. 9-22
Pitch Line Speed: $v_t = 1031$ ft/min		Pitting Stress Cycle Factor: $Z_{NP} = 0.89$	1.00 0.89 Fig. 9-23
Transmitted Load: $W_t = 320$ lb		Pitting Stress Cycle Factor: $Z_{NG} = 0.92$	1.00 0.92 Fig. 9-23
Secondary Input Data:		<b>Stress Analysis: Bending</b>	
Face Width Guidelines (in): 1.000	Min Nom Max	Pinion: Required $s_{bt} = 14,940$ psi	See Fig. 9-11 or
Enter: Face Width: $F = 1.500$ in		Gear: Required $s_{gt} = 11,237$ psi	Table 9-5
Ratio: Face width/pinion diameter: $F/D_p = 0.67$		<b>Stress Analysis: Pitting</b>	
Recommended range of ratio: $0.50 < F/D_p < 2.00$		Pinion: Required $s_{ac} = 123,772$ psi	See Fig. 9-12 or
Enter: Elastic Coefficient: $C_p = 2300$	Table 9-10	Gear: Required $s_{gc} = 119,736$ psi	Table 9-5
Enter: Quality Number: $A_v = 11$	Table 9-3	Required hardness of pinion HB: 294 Equations in Fig. 9-12-Grade 1	
Dynamic Factor: $K_v = 1.43$	Table 9-9	Required hardness of gear HB: 281 Equations in Fig. 9-12-Grade 1	
{Factors for computing $K_v$ :} $B = 0.826$ $C = 59.75$		<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>	
Reference: $N_P = 18$ $N_G = 75$		<b>One possible material specification:</b>	
Bending Geometry Factor-Pinion: $J_P = 0.315$	Fig. 9-15	Pinion requires HB 294; SAE 4340 OQT 1100; HB 321; 19% elongation	
Bending Geometry Factor-Gear: $J_G = 0.410$	Fig. 9-15	Gear requires HB 281; SAE 4340 OQT 1200; HB 293; 20% elongation	
Enter: Pitting Geometry Factor: $I = 0.106$	Fig. 9-21	<b>Comments:</b>	
Reference: $m_G = 4.17$			

Note: Larger part of the total reduction (4.17) in this pair Higher diametral pitch - 8 compared to 6 in pair 2

**DESIGN OF SPUR GEARS**

<b>APPLICATION: Problem 74 - Second pair</b>	
<i>Assembly conveyor driven by an electric motor</i>	
<b>Double reduction - Second pair - Input Data:</b>	
Overload Factor: $K_o = 1.50$	Table 9-7
Transmitted Power: $P = 10$ hp	
Design Power $P_{des} = 15$ hp	
Diametral Pitch: $P_d = 6$	Fig. 9-24
Input Speed: $n_p = 420$ rpm	
Number of Pinion Teeth: $N_p = 18$	
Desired Output Speed: $n_g = 148$ rpm	
Computed number of gear teeth: $51.1$	
Enter: Chosen No. of Gear Teeth: $N_g = 51$	
<b>Computed data:</b>	
Actual Output Speed: $n_g = 148.2$ rpm	
Gear Ratio: $m_g = 2.83$	
Pitch Diameter - Pinion: $D_p = 3.000$ in	
Pitch Diameter - Gear: $D_g = 8.500$ in	
Center Distance: $C = 5.750$ in	
Pitch Line Speed: $v_t = 330$ ft/min	
Transmitted Load: $W_t = 1000$ lb	
<b>Secondary Input Data:</b>	
Face Width Guidelines (in): 1.333 2.000 2.667	Min Nom Max
Enter: Face Width: $F = 2.000$ in	
Ratio: Face width/pinion diameter: $F/D_p = 0.67$	
Recommended range of ratio: $0.50 < F/D_p < 2.00$	
Enter: Elastic Coefficient: $C_p = 2300$	Table 9-10
Enter: Quality Number: $A_v = 11$	Table 9-3
Dynamic Factor: $K_v = 1.25$	Table 9-9
[Factors for computing $K_v$ ]: $B = 0.826$ $C = 59.75$	
Reference: $N_p = 18$ $N_g = 51$	
Bending Geometry Factor-Pinion: $J_p = 0.315$	Fig. 9-15
Bending Geometry Factor-Gear: $J_g = 0.395$	Fig. 9-15
Reference: $m_g = 2.83$	
Enter: Pitting Geometry Factor: $I = 0.108$	Fig. 9-21
<b>Note: Smaller part of the total reduction (42.83) in this pair</b>	
<b>Lower diametral pitch - 6 compared to 8 in pair 1</b>	
<b>Factors in Design Analysis:</b>	
Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$	If $F < 1.0$ If $F > 1.0$
Pinion Proportion Factor, $C_{pf} = 0.042$	0.054 [0.50 < $F/D_p < 2.00$ ]
Enter: $C_{pf} = 0.054$	Figure 9-16
Type of gearing: Open	Commer. Precision Ex. Prec.
Mesh Alignment Factor, $C_{ma} = 0.280$	0.158 0.093 0.058
Enter: $C_{ma} = 0.158$	Figure 9-17
Alignment Factor: $K_m = 1.21$	[Computed]
Size Factor: $K_s = 1.00$	Table 9-8: Use 1.00 if $P_d \geq 5$
Pinion Rim Thickness Factor: $K_{BP} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Gear Rim Thickness Factor: $K_{BG} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Service Factor: $SF = 1.00$	Use 1.00 if no unusual conditions
Reliability Factor: $K_R = 1.00$	Table 9-11 Use 1.00 for $R = .99$
Enter: Design Life: 15000 hours	See Table 9-12
Pinion - Number of load cycles: $N_p = 3.6E+08$	Guidelines: $Y_N, Z_N$
Gear - Number of load cycles: $N_g = 1.3E+08$	$> 10^7$ $< 10^8$
Bending Stress Cycle Factor: $Y_{NP} = 0.95$	1.00 0.95 Fig. 9-22
Bending Stress Cycle Factor: $Y_{NG} = 0.97$	1.00 0.97 Fig. 9-22
Pitting Stress Cycle Factor: $Z_{NP} = 0.92$	1.00 0.92 Fig. 9-23
Pitting Stress Cycle Factor: $Z_{NG} = 0.94$	1.00 0.94 Fig. 9-23
<b>Stress Analysis: Bending</b>	
Pinion: Required $s_{at} = 22,702$ psi	See Fig. 9-11 or
Gear: Required $s_{at} = 17,731$ psi	Table 9-5
<b>Stress Analysis: Pitting</b>	
Pinion: Required $s_{ac} = 147,790$ psi	See Fig. 9-12 or
Gear: Required $s_{ac} = 144,845$ psi	Table 9-5
Required hardness of pinion HB: 369	Equations in Fig. 9-12-Grade 1
Required hardness of gear HB: 359	Equations in Fig. 9-12-Grade 1
<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>	
<b>One possible material specification:</b>	
Pinion requires HB 369; SAE 4340 OQT 900; HB 388; 15% elongation	
Gear requires HB 359; SAE 4340 OQT 1000; HB 363; 17% elongation	
<b>Comments:</b>	
Center Distances and sizes of gears are well balanced	

**DESIGN OF SPUR GEARS**

<b>APPLICATION:</b> [Problem 75 - First pair Food waste grinder driven by an electric motor]	
<b>Double reduction - First pair - Input Data:</b>	
Overload Factor: $K_o = 1.50$	Table 9-7
Transmitted Power: $P = 0.5$ hp	
Design Power $P_{des} = 0.75$ hp	
Diametral Pitch: $P_d = 16$	Fig. 9-24
Input Speed: $n_P = 850$ rpm	
Number of Pinion Teeth: $N_P = 18$	
Desired Output Speed: $n_G = 190$ rpm	
Computed number of gear teeth: 80.5	
Enter: Chosen No. of Gear Teeth: $N_G = 81$	
<b>Computed data:</b>	
Actual Output Speed: $n_g = 188.9$ rpm	
Gear Ratio: $m_g = 4.50$	
Pitch Diameter - Pinion: $D_P = 1.125$ in	
Pitch Diameter - Gear: $D_G = 5.063$ in	
Center Distance: $C = 3.094$ in	
Pitch Line Speed: $v_l = 290$ ft/min	
Transmitted Load: $W_t = 66$ lb	
<b>Secondary Input Data:</b>	
Face Width Guidelines (in): 0.500 0.750 1.000	Min Nom Max
Enter: Face Width: $F = 0.500$ in	
Ratio: Face width/pinion diameter: $F/D_P = 0.44$	
Recommended range of ratio: $0.50 < F/D_P < 2.00$	
Enter: Elastic Coefficient: $C_p = 2300$	Table 9-10
Enter: Quality Number: $A_v = 9$	Table 9-3
Dynamic Factor: $K_v = 1.14$	Table 9-9
[Factors for computing $K_v$ ]: $B = 0.630$ $C = 70.71$	
Reference: $N_P = 18$ $N_G = 81$	
Bending Geometry Factor-Pinion: $J_P = 0.320$	Fig. 9-15
Bending Geometry Factor-Gear: $J_G = 0.415$	Fig. 9-15
Reference: $m_g = 4.50$	
Enter: Pitting Geometry Factor: $I = 0.106$	Fig. 9-21

Note: Equal reduction ratios used for pairs 1 and 2  
Equal diametral pitches (16) used for both pairs

<b>Factors in Design Analysis:</b>	
Alignment Factor, $K_m = 1.0 + C_{pt} + C_{ma}$	If $F < 1.0$ 0.019 If $F > 1.0$ 0.013 [0.50 < $F/D_P < 2.00$ ]
Pinion Proportion Factor, $C_{pf}$	Enter: $C_{pf} = 0.019$ Figure 9-16
Type of gearing:	Open Commer. Precision Ex. Prec.
Mesh Alignment Factor, $C_{ma}$	0.255 0.135 0.074 0.043
Enter: $C_{ma} = 0.135$	Figure 9-17
Alignment Factor: $K_m = 1.15$	[Computed]
Size Factor: $K_s = 1.00$	Table 9-8: Use 1.00 if $P_d \geq 5$
Pinion Rim Thickness Factor: $K_{sp} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Gear Rim Thickness Factor: $K_{sg} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Service Factor: $SF = 1.00$	Use 1.00 if no unusual conditions
Reliability Factor: $K_R = 1.00$	Table 9-11 Use 1.00 for $R = .99$
Enter: Design Life: 8000 hours	See Table 9-12
Pinion - Number of load cycles: $N_p = 4.1E+08$	Guidelines: $Y_N, Z_N$
Gear - Number of load cycles: $N_g = 9.1E+07$	$10^7$ cycles $> 10^7$ $< 10^7$
Bending Stress Cycle Factor: $Y_{NP} = 0.95$	1.00 0.95 Fig. 9-22
Bending Stress Cycle Factor: $Y_{NG} = 0.98$	1.00 0.98 Fig. 9-22
Pitting Stress Cycle Factor: $Z_{NP} = 0.92$	1.00 0.92 Fig. 9-23
Pitting Stress Cycle Factor: $Z_{NG} = 0.95$	1.00 0.95 Fig. 9-23
<b>Stress Analysis: Bending</b>	
Pinion: Required $s_{at} = 13,639$ psi	See Fig. 9-11 or
Gear: Required $s_{at} = 10,195$ psi	Table 9-5
<b>Stress Analysis: Pitting</b>	
Pinion: Required $s_{ac} = 116,542$ psi	See Fig. 9-12 or
Gear: Required $s_{ac} = 112,862$ psi	Table 9-5
Required hardness of pinion HB: 272	Equations in Fig. 9-12-Grade 1
Required hardness of gear HB: 260	Equations in Fig. 9-12-Grade 1
<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>	
<b>One possible material specification:</b>	
Pinion requires HB 272; SAE 4340 OQT 1200; HB 293; 21% elongation	
Gear requires HB 260; SAE 4340 OQT 1200; HB 293; 20% elongation	
<b>Comments:</b>	

Larger face width required for pair 2 (1.15 in) vs. 0.50 in for pair 1  
Stresses higher for pair 1, requiring higher hardness



**DESIGN OF SPUR GEARS**

<b>APPLICATION:</b> [Problem 75 - Second pair Food waste grinder driven by an electric motor]		
<b>Double reduction - Second pair - Input Data:</b>		
Overload Factor: $K_o = 1.50$	Table 9-7	
Transmitted Power: $P = 0.5$ hp		
Design Power $P_{des} = 0.75$ hp		
Diametral Pitch: $P_d = 16$	Fig. 9-24	
Input Speed: $n_P = 188.9$	rpm	
Number of Pinion Teeth: $N_P = 18$		
Desired Output Speed: $n_G = 42$	rpm	
Computed number of gear teeth: 81.0		
Enter: Chosen No. of Gear Teeth: $N_G = 81$		
<b>Computed data:</b>		
Actual Output Speed: $n_G = 42.0$	rpm	
Gear Ratio: $m_G = 4.50$		
Pitch Diameter - Pinion: $D_P = 1.125$	in	
Pitch Diameter - Gear: $D_G = 5.063$	in	
Center Distance: $C = 3.094$	in	
Pitch Line Speed: $v_t = 56$	ft/min	
Transmitted Load: $W_t = 297$	lb	
<b>Secondary Input Data:</b>		
Min	Nom	Max
Face Width Guidelines (in): 0.500	0.750	1.000
Enter: Face Width: $F = 1.150$	in	
Ratio: Face width/pinion diameter: $F/D_P = 1.02$		
Recommended range of ratio: $0.50 < F/D_P < 2.00$		
Enter: Elastic Coefficient: $C_p = 2300$		Table 9-10
Enter: Quality Number: $A_v = 9$		Table 9-3
Dynamic Factor: $K_v = 1.07$		Table 9-9
[Factors for computing $K_v$ ]: $B = 0.630$	$C = 70.71$	
Reference: $N_P = 18$	$N_G = 81$	
Bending Geometry Factor-Pinion: $J_P = 0.320$	Fig. 9-15	
Bending Geometry Factor-Gear: $J_G = 0.415$	Fig. 9-15	
Reference: $m_G = 4.50$		
Enter: Pitting Geometry Factor: $I = 0.106$	Fig. 9-21	

Note: Equal reduction ratios used for pairs 1 and 2  
Equal diametral pitches (16) used for both pairs

<b>Factors in Design Analysis:</b>		
Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$	IF $F < 1.0$	IF $F > 1.0$
Pinion Proportion Factor, $C_{pf} = 0.077$	0.079	$[0.50 < F/D_P < 2.00]$
Enter: $C_{pf} = 0.079$	Figure 9-16	
Type of gearing: Open	Commer.	Precision
Mesh Alignment Factor, $C_{ma} = 0.286$	0.145	0.082
Enter: $C_{ma} = 0.145$	Figure 9-17	
Alignment Factor: $K_m = 1.22$	[Computed]	
Size Factor: $K_s = 1.00$	Table 9-8: Use 1.00 if $P_d \geq 5$	
Pinion Rim Thickness Factor: $K_{BP} = 1.00$	Fig. 9-18: Use 1.00 if solid blank	
Gear Rim Thickness Factor: $K_{BG} = 1.00$	Fig. 9-18: Use 1.00 if solid blank	
Service Factor: $SF = 1.00$	Use 1.00 if no unusual conditions	
Reliability Factor: $K_R = 1.00$	Table 9-11 Use 1.00 for $R = .99$	
Enter: Design Life: 8000	hours	See Table 9-12
Pinion - Number of load cycles: $N_P = 9.1E+07$		Guidelines: $Y_N, Z_N$
Gear - Number of load cycles: $N_G = 2.0E+07$	$> 10^7$ cycles	$< 10^7$
Bending Stress Cycle Factor: $Y_{NP} = 0.98$	1.00	0.98
Bending Stress Cycle Factor: $Y_{NG} = 1.01$	1.00	1.01
Pitting Stress Cycle Factor: $Z_{NP} = 0.95$	1.00	0.95
Pitting Stress Cycle Factor: $Z_{NG} = 0.98$	1.00	0.98
<b>Stress Analysis: Bending</b>		
Pinion: Required $s_{at} = 25,734$	psi	See Fig. 9-11 or
Gear: Required $s_{at} = 19,253$	psi	Table 9-5
<b>Stress Analysis: Pitting</b>		
Pinion: Required $s_{ac} = 157,454$	psi	See Fig. 9-12 or
Gear: Required $s_{ac} = 152,634$	psi	Table 9-5
Required hardness of pinion HB: 399	Equations in Fig. 9-12-Grade 1	
Required hardness of gear HB: 384	Equations in Fig. 9-12-Grade 1	
<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>		
<b>One possible material specification:</b>		
Pinion requires HB 399; SAE 4340 OQT 800; HB 415; 12% elongation		
Gear requires HB 384; SAE 4340 OQT 800; HB 415; 12% elongation		
<b>Comments:</b>		
Pinion and gear made from same material and heat treatment for processing convenience.		
Larger face width required for pair 2 (1.15 in) vs. 0.50 in for pair 1		
Stresses higher for pair 2 than for pair 1, requiring higher hardness		



**DESIGN OF SPUR GEARS**

APPLICATION: Problem 76 - First pair	
Small hand drill driven by an electric motor	
<b>Double reduction - First pair - Input Data:</b>	
Overload Factor: $K_o = 1.50$	Table 9-7
Transmitted Power: $P = 0.25$ hp	
Design Power $P_{des} = 0.375$ hp	
Diametral Pitch: $P_d = 24$	Fig. 9-24
Input Speed: $n_P = 3000$ rpm	
Number of Pinion Teeth: $N_P = 15$	
Desired Output Speed: $n_G = 1300$ rpm	
Computed number of gear teeth: 34.6	
Enter: Chosen No. of Gear Teeth: $N_G = 35$	
<b>Computed data:</b>	
Actual Output Speed: $n_g = 1285.7$ rpm	
Gear Ratio: $m_g = 2.33$	
Pitch Diameter - Pinion: $D_P = 0.625$ in	
Pitch Diameter - Gear: $D_G = 1.458$ in	
Center Distance: $C = 1.042$ in	
Pitch Line Speed: $v_l = 491$ ft/min	
Transmitted Load: $W_t = 17$ lb	
<b>Secondary Input Data:</b>	
Face Width Guidelines (in): 0.333	Min
Enter: Face Width: $F = 0.250$ in	Nom
Ratio: Face width/pinion diameter: $F/D_P = 0.40$	Max
Recommended range of ratio: $0.50 < F/D_P < 2.00$	
Enter: Elastic Coefficient: $C_p = 2300$	Table 9-10
Enter: Quality Number: $A_v = 8$	Table 9-3
Dynamic Factor: $K_v = 1.19$	Table 9-9
[Factors for computing $K_v$ ]: $B = 0.630$	$C = 70.71$
Reference: $N_P = 15$	$N_G = 35$
Bending Geometry Factor-Pinion: $J_P = 0.250$	Fig. 9-15
Bending Geometry Factor-Gear: $J_G = 0.355$	Fig. 9-15
Reference: $m_g = 2.33$	
Enter: Pitting Geometry Factor: $I = 0.088$	Fig. 9-21
<b>Factors in Design Analysis:</b>	
Alignment Factor, $K_m = 1.0 + C_{pt} + C_{ma}$	If $F < 1.0$
Pinion Proportion Factor, $C_{pt} = 0.015$	0.006
Enter: $C_{pt} = 0.015$	Figure 9-16
Type of gearing: Open	Commer.
Mesh Alignment Factor, $C_{ma} = 0.251$	0.131
Enter: $C_{ma} = 0.131$	Figure 9-17
Alignment Factor: $K_m = 1.15$	[Computed]
Size Factor: $K_s = 1.00$	Table 9-8: Use 1.00 if $P_d \geq 5$
Pinion Rim Thickness Factor: $K_{BP} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Gear Rim Thickness Factor: $K_{BG} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Service Factor: $SF = 1.00$	Use 1.00 if no unusual conditions
Reliability Factor: $K_R = 1.00$	Table 9-11 Use 1.00 for $R = .99$
Enter: Design Life: 5000 hours	See Table 9-12
Pinion - Number of load cycles: $N_P = 9.0E+08$	Guidelines: $Y_N, Z_N$
Gear - Number of load cycles: $N_G = 3.9E+08$	$> 10^7$ cycles
Bending Stress Cycle Factor: $Y_{NP} = 0.94$	1.00
Bending Stress Cycle Factor: $Y_{NG} = 0.95$	0.94
Pitting Stress Cycle Factor: $Z_{NP} = 0.90$	1.00
Pitting Stress Cycle Factor: $Z_{NG} = 0.92$	0.95
<b>Stress Analysis: Bending</b>	
Pinion: Required $s_{at} = 14,014$ psi	See Fig. 9-11 or
Gear: Required $s_{at} = 9,765$ psi	Table 9-5
<b>Stress Analysis: Pitting</b>	
Pinion: Required $s_{ac} = 127,649$ psi	See Fig. 9-12 or
Gear: Required $s_{ac} = 124,874$ psi	Table 9-5
Required hardness of pinion HB: 306	Equations in Fig. 9-12-Grade 1
Required hardness of gear HB: 297	Equations in Fig. 9-12-Grade 1
<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>	
<b>One possible material specification:</b>	
Pinion requires HB 306; SAE 4340 OQT 1100; HB 321; 19% elongation	
Gear requires HB 297; SAE 4340 OQT 1100; HB 321; 19% elongation	
<b>Comments:</b>	
Larger face width required for pair 2 (0.66 in) vs. 0.25 in for pair 1	
Stresses nearly equal for pair 2 and for pair 1, requiring equal hardnesses	

**DESIGN OF SPUR GEARS**

**APPLICATION: Problem 76 - Second pair**  
 Small hand drill driven by an electric motor

**Double reduction - Second pair - Input Data:**

Overload Factor:  $K_o = 1.50$  Table 9-7  
 Transmitted Power:  $P = 0.25$  hp  
 Design Power  $P_{des} = 0.375$  hp  
 Diametral Pitch:  $P_d = 24$  Fig. 9-24  
 Input Speed:  $n_p = 1285.7$  rpm  
 Number of Pinion Teeth:  $N_p = 15$   
 Desired Output Speed:  $n_g = 550$  rpm  
 Computed number of gear teeth: 35.1  
 Enter: Chosen No. of Gear Teeth:  $N_g = 35$

**Computed data:**

Actual Output Speed:  $n_g = 551.0$  rpm  
 Gear Ratio:  $m_g = 2.33$   
 Pitch Diameter - Pinion:  $D_p = 0.625$  in  
 Pitch Diameter - Gear:  $D_g = 1.458$  in  
 Center Distance:  $C = 1.042$  in  
 Pitch Line Speed:  $v_t = 210$  ft/min  
 Transmitted Load:  $W_t = 39$  lb

**Secondary Input Data:**

Face Width Guidelines (in): Min 0.333 Nom 0.500 Max 0.667  
 Enter: Face Width:  $F = 0.560$  in  
 Ratio: Face width/pinion diameter:  $F/D_p = 0.90$   
 Recommended range of ratio:  $0.50 < F/D_p < 2.00$   
 Enter: Elastic Coefficient:  $C_p = 2300$  Table 9-10  
 Enter: Quality Number:  $A_v = 9$  Table 9-3  
 Dynamic Factor:  $K_v = 1.12$  Table 9-9  
 [F-factors for computing  $K_v$ ]  $B = 0.630$   $C = 70.71$   
 Reference:  $N_p = 15$   $N_g = 35$   
 Bending Geometry Factor-Pinion:  $J_p = 0.250$  Fig. 9-15  
 Bending Geometry Factor-Gear:  $J_g = 0.355$  Fig. 9-15  
 Reference:  $m_g = 2.33$   
 Enter: Pitting Geometry Factor:  $I = 0.088$  Fig. 9-21

Note: Equal reduction ratios used for pairs 1 and 2  
 Equal diametral pitches (24) used for both pairs

**Factors in Design Analysis:**

Alignment Factor,  $K_m = 1.0 + C_{pf} + C_{ma}$  If  $F < 1.0$  0.065 If  $F > 1.0$  0.059 [0.50 <  $F/D_p < 2.00$ ]  
 Pinion Proportion Factor,  $C_{pf} = 0.065$  Figure 9-16  
 Enter:  $C_{pf} = 0.065$   
 Type of gearing: Open Commer. Precision Ex. Prec.  
 Mesh Alignment Factor,  $C_{ma} = 0.256$  0.136 0.075 0.044  
 Enter:  $C_{ma} = 0.136$  Figure 9-17  
 Alignment Factor:  $K_m = 1.20$  [Computed]

Size Factor:  $K_s = 1.00$  Table 9-8: Use 1.00 if  $P_d \geq 5$   
 Pinion Rim Thickness Factor:  $K_{ap} = 1.00$  Fig. 9-18: Use 1.00 if solid blank  
 Gear Rim Thickness Factor:  $K_{ag} = 1.00$  Fig. 9-18: Use 1.00 if solid blank  
 Service Factor:  $SF = 1.00$  Use 1.00 if no unusual conditions  
 Reliability Factor:  $K_R = 1.00$  Table 9-11 Use 1.00 for  $R = .99$   
 Enter: Design Life: 5000 hours See Table 9-12

Pinion - Number of load cycles:  $N_p = 3.9E+08$  Guidelines:  $Y_N, Z_N$   
 Gear - Number of load cycles:  $N_g = 1.7E+08$   $> 10^7$  cycles  $< 10^8$

Bending Stress Cycle Factor:  $Y_{NP} = 0.95$  Fig. 9-22  
 Bending Stress Cycle Factor:  $Y_{NG} = 0.97$  Fig. 9-22  
 Pitting Stress Cycle Factor:  $Z_{NP} = 0.92$  Fig. 9-23  
 Pitting Stress Cycle Factor:  $Z_{NG} = 0.94$  Fig. 9-23

**Stress Analysis: Bending**

Pinion: Required  $s_{bt} = 14,339$  psi See Fig. 9-11 or  
 Gear: Required  $s_{gt} = 9,890$  psi Table 9-5

**Stress Analysis: Pitting**

Pinion: Required  $s_{sc} = 126,985$  psi See Fig. 9-12 or  
 Gear: Required  $s_{sc} = 124,283$  psi Table 9-5

Required hardness of pinion HB: 304 Equations in Fig. 9-12-Grade 1  
 Required hardness of gear HB: 296 Equations in Fig. 9-12-Grade 1  
 Specify materials, alloy and heat treatment, for most severe requirement.

**One possible material specification:**  
 Pinion requires HB 304; SAE 4340 OQT 1100; HB 321; 19% elongation  
 Gear requires HB 296; SAE 4340 OQT 1100; HB 321; 19% elongation

**Comments:**

Larger face width required for pair 2 (0.56 in) vs. 0.25 in for pair 1  
 Stresses nearly equal for pair 2 and for pair 1, requiring equal hardnesses

## DESIGN OF PLASTIC SPUR GEARS

### Application:

Band saw driven by electric motor  
Problem 77

### Initial Input Data:

Input Power:  $P = 0.25$  hp  
 Input Speed:  $n_P = 351$  rpm  
 Diametral Pitch:  $P_d = 12$   
 Number of Pinion Teeth:  $N_P = 18$   
 Desired Output Speed:  $n_G = 159.15$  rpm

Computed number of gear teeth: 62.319

Enter Chosen No. of Gear Teeth  $N_G = 62$

### Computed data:

Actual Output Speed:  $n_G = 160.0$  rpm  
 Gear Ratio:  $m_G = 3.444$   
 Pitch Diameter - Pinion:  $D_P = 1.500$  in  
 Pitch Diameter - Gear:  $D_G = 5.167$  in  
 Center Distance:  $C = 6.667$  in  
 Pitch Line Speed:  $v_t = 216.4$  ft/min  
 Transmitted Load:  $W_t = 38.11$  lb

### Secondary Input Data - Pinion:

Tooth Form: 20 degree full depth  
 Lewis Form Factor:  $Y = 0.521$  Table 9-15  
 Safety Factor:  $SF = 1.50$  Ref. Table 9-2  
 Material: Unfilled Nylon  
 Allowable Bending Stress:  $s_{at} = 6000$  psi Table 9-14 or Fig. 9-28

Required Face Width:  $F = 0.219$  in

Enter Specified Face Width:  $F = 0.220$  in

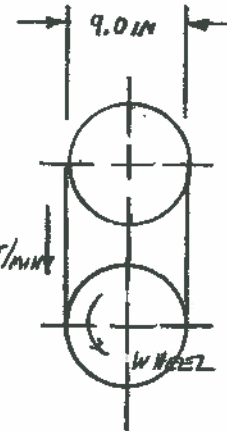
Actual Bending Stress in Pinion:  $s_t = 5985$  psi

### Secondary Input Data - Gear:

Tooth Form: 20 degree full depth Same as for pinion  
 Lewis Form Factor:  $Y = 0.716$  Table 9-15  
 Safety Factor:  $SF = 1.50$  Same as for pinion  
 Material: Unfilled Acetal  
 Allowable Bending Stress:  $s_{at} = 5000$  psi Table 9-14 or Fig. 9-28

Face Width - Gear:  $F = 0.220$  in Same as for pinion

Actual Bending Stress in Gear:  $s_t = 4355$  psi Must be  $< s_{at}$



From Problem 76

$$N_{\text{WHEEL}} = N_{\text{BLADE}} = 375 \text{ FT/MIN}$$

To produce blade speed of 375 ft/min

$$N_{\text{WHEEL}} = \pi D_W n_W / 12$$

$$n_W = \frac{12 N_{\text{WHEEL}}}{\pi D_W} = \frac{12(375)}{\pi(9.0)}$$

$$n_W = 159.2 \text{ RPM}$$

CONNECT WHEEL ON OUTPUT  
 SHAFT OF GEAR REDUCER.  
 ONE PAIR OF GEARS.

7B.

RACK DRIVES FURNACE DOOR.  $N_{RACK} \geq 2.0 \text{ FT/S} = 120 \text{ FT/MIN}$

$$N_R = N_G = \pi D_D m_D / 12$$

$$m_D \geq \frac{12 N_{RD}}{\pi D_D} = \frac{(12)(120)}{\pi D_D} = \frac{458.4}{D_D}$$

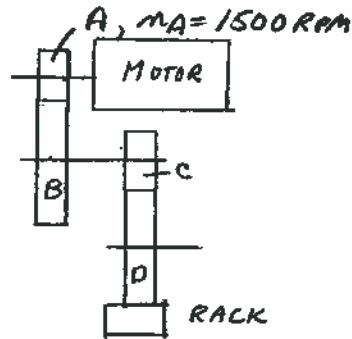
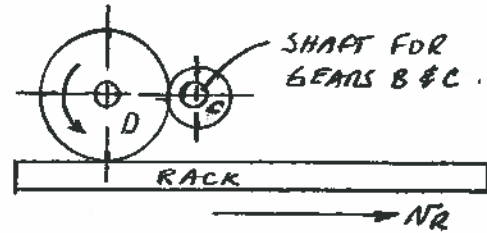
POSSIBLE VALUES FOR  $D_G$ :

$D_D$	$m_D$	$TV = m_A/m_D$
10.0 IN	45.84 RPM	22.72
9.0 IN	50.93	29.45
12.167	37.76	39.72 <u>USED THIS</u>

PAIR 1:  $m_A = 1500 \text{ RPM}$ ,  $m_B = 172.3 \text{ RPM}$   
 $N_A = 17$ ,  $N_B = 148$ ,  $VR_1 = 8.71$

PAIR 2:  $m_C = m_B = 172.3 \text{ RPM}$ ;  $m_D = 37.76$   
 $N_C = 16$ ,  $N_D = 73$ ,  $VR_2 = 4.56$

SEE TWO SPREADSHEET SOLUTIONS



DESIGNED FOR OPEN GEARING

DESIGN LIFE: IN EACH CYCLE, RACK MOVES 6 FT EACH WAY

TOTAL OF 12 FT FOR FULL CYCLE. AT 2.0 FT/S, CYCLE TIME IS:

$$\frac{1 \text{ SEC}}{2.0 \text{ FT}} \times 12 \text{ FT} = 6.0 \text{ SEC/CYCLE}$$

$$\frac{6.0 \text{ SEC}}{\text{CYCLE}} \times \frac{6 \text{ CYCLES}}{\text{HR}} \times \frac{24 \text{ HR}}{\text{DAY}} \times \frac{365 \text{ DAYS}}{\text{YR}} \times \frac{15 \text{ YRS}}{1} \times \frac{\text{HR}}{3600 \text{ S}} = 1314 \text{ HR}$$

USE 1500 HR DESIGN LIFE.

SUMMARY: POWER = 5.0 HP,  $K_o = 1.50$ ,  $P_{DES} = 7.5 \text{ HP}$ , QUALITY =  $AV = 11$

	$P_d$	$N_p$	$N_g$	$D_p$	$D_g$	$F$	$L$	WELL BALANCED IN SIZE.
PAIR 1:	10	17	172	1.700	14.800	12.00	8.250	
PAIR 2:	6	16	73	2.667	12.167	2.500	7.417	

ALL GEARS MADE FROM SAE 4340: PAIR 1 - OQT 1100; HB 321

PAIR 2 - OQT 900; HB 388



**DESIGN OF SPUR GEARS**

<b>APPLICATION: Problem 78 - First pair</b>		<b>Factors in Design Analysis:</b>	
<b>Rack and pinion drive driven by a fluid power motor</b>		Alignment Factor, $K_m = 1.0 + C_{pt} + C_{ma}$ If $F < 1.0$ If $F > 1.0$	
<b>Double reduction - First pair - Input Data:</b>		Pinion Proportion Factor, $C_{pt} = 0.048$ $[0.50 < F/D_p < 2.00]$	
Overload Factor: $K_o = 1.50$	Table 9-7	Enter: $C_{pt} = 0.048$	Figure 9-16
Transmitted Power: $P = 5$ hp		Type of gearing: Open	Commer. Precision Ex. Prec.
Design Power $P_{des} = 7.5$ hp		Mesh Alignment Factor, $C_{ma} = 0.267$	0.146    0.083    0.050
Diametral Pitch: $P_d = 10$	Fig. 9-24	Enter: $C_{ma} = 0.146$	Figure 9-17
Input Speed: $n_p = 1500$ rpm		Alignment Factor: $K_m = 1.19$	[Computed]
Number of Pinion Teeth: $N_p = 17$		Size Factor: $K_s = 1.00$	Table 9-8: Use 1.00 if $P_d \geq 5$
Desired Output Speed: $n_g = 172$ rpm		Pinion Rim Thickness Factor: $K_{BP} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Computed number of gear teeth: $N_g = 148.3$		Gear Rim Thickness Factor: $K_{BG} = 1.00$	Fig. 9-18: Use 1.00 if solid blank
Enter: Chosen No. of Gear Teeth: $N_g = 148$		Service Factor: $SF = 1.00$	Use 1.00 if no unusual conditions
<b>Computed data:</b>		Reliability Factor: $K_R = 1.00$	Table 9-11 Use 1.00 for $R = .99$
Actual Output Speed: $n_g = 172.30$ rpm		Enter: Design Life: 1500 hours	See Table 9-12
Gear Ratio: $m_g = 8.71$		Pinion - Number of load cycles: $N_p = 1.4E+08$	Guidelines: $Y_N, Z_N$
Pitch Diameter - Pinion: $D_p = 1.700$ in		Gear - Number of load cycles: $N_g = 1.6E+07$	$10^7$ cycles $> 10^7$ $< 10^7$
Pitch Diameter - Gear: $D_g = 14.800$ in		Bending Stress Cycle Factor: $Y_{NP} = 0.97$	1.00    0.97    Fig. 9-22
Center Distance: $C = 8.250$ in		Bending Stress Cycle Factor: $Y_{NG} = 1.01$	1.00    1.01    Fig. 9-22
Pitch Line Speed: $v_t = 668$ ft/min		Pitting Stress Cycle Factor: $Z_{NP} = 0.94$	1.00    0.94    Fig. 9-23
Transmitted Load: $W_t = 247$ lb		Pitting Stress Cycle Factor: $Z_{NG} = 0.99$	1.00    0.99    Fig. 9-23
<b>Secondary Input Data:</b>		<b>Stress Analysis: Bending</b>	
Face Width Guidelines (in): 0.800    1.200    1.600	Min    Nom    Max	Pinion: Required $s_{bt} = 17,347$ psi	See Fig. 9-11 or Table 9-5
Enter: Face Width: $F = 1.200$ in		Gear: Required $s_{gt} = 11,564$ psi	Table 9-5
Ratio: Face width/pinion diameter: $F/D_p = 0.71$		<b>Stress Analysis: Pitting</b>	
Recommended range of ratio: $0.50 < F/D_p < 2.00$		Pinion: Required $s_{ac} = 126,062$ psi	See Fig. 9-12 or Table 9-5
Enter: Elastic Coefficient: $C_p = 2300$	Table 9-10	Gear: Required $s_{ac} = 119,695$ psi	Table 9-5
Enter: Quality Number: $A_v = 11$	Table 9-3	Required hardness of pinion HB: 301    Equations in Fig. 9-12-Grade 1	
Dynamic Factor: $K_v = 1.35$	Table 9-9	Required hardness of gear HB: 281    Equations in Fig. 9-12-Grade 1	
[Factors for computing $K_v$ ]: $B = 0.826$ $C = 59.75$		<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>	
Reference: $N_p = 17$ $N_g = 148$		<b>One possible material specification:</b>	
Bending Geometry Factor-Pinion: $J_p = 0.295$	Fig. 9-15	Pinion requires HB 301; SAE 4340 OQT 1100; HB 321; 19% elongation	
Bending Geometry Factor-Gear: $J_g = 0.425$	Fig. 9-15	Gear requires HB 281; SAE 4340 OQT 1100; HB 321; 19% elongation	
Reference: $m_g = 8.71$		<b>Comments:</b>	
Enter: Pitting Geometry Factor: $I = 0.110$	Fig. 9-21	Same material used for pinion and gear because contact stresses are close.	

**DESIGN OF SPUR GEARS**

APPLICATION: [Problem 78 - Second pair Rack and pinion drive driven by a fluid power motor]		
<b>Double reduction - Second pair - Input Data:</b>		
Overload Factor: $K_o = 1.50$	Table 9-7	
Transmitted Power: $P = 5$ hp		
Design Power $P_{des} = 7.5$ hp		
Diametral Pitch: $P_d = 6$	Fig. 9-24	
Input Speed: $n_p = 172.3$ rpm		
Number of Pinion Teeth: $N_p = 16$		
Desired Output Speed: $n_g = 38.2$ rpm		
Computed number of gear teeth: 72.2		
Enter: Chosen No. of Gear Teeth: $N_g = 73$		
<b>Computed data:</b>		
Actual Output Speed: $n_g = 37.76$ rpm		
Gear Ratio: $m_g = 4.56$		
Pitch Diameter - Pinion: $D_p = 2.667$ in		
Pitch Diameter - Gear: $D_g = 12.167$ in		
Center Distance: $C = 7.417$ in		
Pitch Line Speed: $v_t = 120$ ft/min		
Transmitted Load: $W_t = 1372$ lb		
<b>Secondary Input Data:</b>		
Face Width Guidelines (in): 1.333	2.000	Max
Enter: Face Width: $F = 2.500$ in		
Ratio: Face width/pinion diameter: $F/D_p = 0.94$		
Recommended range of ratio: $0.50 < F/D_p < 2.00$		
Enter: Elastic Coefficient: $C_p = 2300$	Table 9-10	
Enter: Quality Number: $A_v = 11$	Table 9-3	
Dynamic Factor: $K_v = 1.15$	Table 9-9	
[Factors for computing $K_v$ ]: $B = 0.826$	$C = 59.75$	
Reference: $N_p = 16$	$N_g = 73$	
Bending Geometry Factor-Pinion: $J_p = 0.265$	Fig. 9-15	
Bending Geometry Factor-Gear: $J_g = 0.400$	Fig. 9-15	
Reference: $m_g = 4.56$		
Enter: Pitting Geometry Factor: $I = 0.102$	Fig. 9-21	
<b>Factors in Design Analysis:</b>		
Alignment Factor, $K_m = 1.0 + C_{pt} + C_{ma}$	If $F < 1.0$	If $F > 1.0$
Pinion Proportion Factor, $C_{pt} = 0.069$	0.088	$[0.50 < F/D_p < 2.00]$
Enter: $C_{pt} = 0.088$	Figure 9-16	
Type of gearing: Open	Commer.	Precision
Mesh Alignment Factor, $C_{ma} = 0.288$	0.166	0.099
Enter: $C_{ma} = 0.166$	Figure 9-17	
Alignment Factor: $K_m = 1.25$	[Computed]	
Size Factor: $K_s = 1.00$	Table 9-8; Use 1.00 if $P_d \geq 5$	
Pinion Rim Thickness Factor: $K_{BP} = 1.00$	Fig. 9-18; Use 1.00 if solid blank	
Gear Rim Thickness Factor: $K_{BG} = 1.00$	Fig. 9-18; Use 1.00 if solid blank	
Service Factor: $SF = 1.00$	Use 1.00 if no unusual conditions	
Reliability Factor: $K_R = 1.00$	Table 9-11 Use 1.00 for $R = .99$	
Enter: Design Life: 1500 hours	See Table 9-12	
Pinion - Number of load cycles: $N_p = 1.6E+07$	Guidelines: $Y_N, Z_N$	
Gear - Number of load cycles: $N_g = 3.4E+06$	$10^7$ cycles	$> 10^7$
Bending Stress Cycle Factor: $Y_{NP} = 1.01$	1.00	1.01
Bending Stress Cycle Factor: $Y_{NG} = 1.04$	1.00	1.04
Pitting Stress Cycle Factor: $Z_{NP} = 0.99$	1.00	0.99
Pitting Stress Cycle Factor: $Z_{NG} = 1.03$	1.00	1.03
<b>Stress Analysis: Bending</b>		
Pinion: Required $s_{at} = 26,592$ psi	See Fig. 9-11 or	
Gear: Required $s_{at} = 17,109$ psi	Table 9-5	
<b>Stress Analysis: Pitting</b>		
Pinion: Required $s_{ac} = 153,423$ psi	See Fig. 9-12 or	
Gear: Required $s_{ac} = 147,465$ psi	Table 9-5	
Required hardness of pinion HB: 386	Equations in Fig. 9-12-Grade 1	
Required hardness of gear HB: 368	Equations in Fig. 9-12-Grade 1	
<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>		
<b>One possible material specification:</b>		
Pinion requires HB 386; SAE 4340 OQT 900; HB 388; 14% elongation		
Gear requires HB 368; SAE 4340 OQT 900; HB 368; 14% elongation		
<b>Comments:</b>		
Same material and heat treatment used for pinion and gear.		
Same material used for both Pair 1 and Pair2; Different tempering temperatures.		



79.

GEAR DRIVE FOR A LIFT TRUCK  
 ROLLING WHEEL IS THE INVERSE OF  
 A PINION DRIVING A RACK.

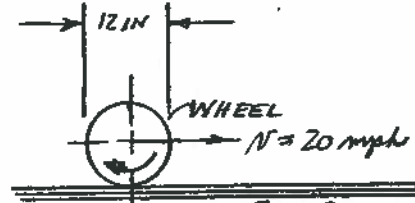
$N$  AT CENTER OF WHEEL EQUALS  
 SPEED OF LIFT TRUCK.

$$N = \pi D_w M_w / 12$$

$$M_w = \frac{12 N}{\pi D_w} = \frac{12 (1760 \text{ FT/MIN})}{\pi (12 \text{ IN})} = 560.23 \text{ RPM}$$

$$\text{TRAIN VALUE} = 3000 / 560.23 = 5.355$$

THIS TV COULD BE PROVIDED BY ONE PAIR OF GEARS.  
 HOWEVER, FOR POWER REQUIRED, GEAR WOULD BE TOO  
 LARGE TO ATTACH TO AXLE WITH 12-IN WHEEL.  
 USE DOUBLE REDUCTION. SEE TWO FOLLOWING PAGES.



$$N = \frac{20 \text{ MI}}{\text{hr}} \times \frac{5280 \text{ FT}}{\text{MI}} \times \frac{1 \text{ hr}}{60 \text{ MIN}} = 1760 \text{ FT/MIN.}$$

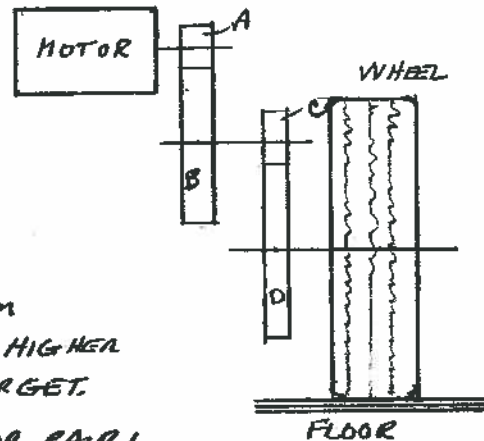
$$VR_1 = \frac{N_B}{N_A} = \frac{50}{17} = 2.94$$

$$VR_2 = \frac{N_D}{N_C} = \frac{38}{21} = 1.81$$

$$M_w = M_D = M_A \cdot \frac{N_A N_C}{N_B N_D}$$

$$M_w = 3000 \cdot \frac{17}{50} \cdot \frac{21}{38} = 563.7 \text{ RPM}$$

SLIGHTLY HIGHER  
 THAN TARGET.



NOTES GEARS HAVE  $P_d = 6$  FOR PAIR 1

AND  $P_d = 5$  FOR PAIR 2. FACE WIDTHS ARE  
 RELATIVELY LARGE.  $F_1 = 2.50 \text{ IN}$ ,  $F_2 = 3.00 \text{ IN}$ .

REDESIGN WITH HELICAL GEARS SHOULD ALLOW  
 A SMALLER SYSTEM.

ALSO ALL GEARS USE SAME MATERIAL AND  
 HEAT TREATMENT.

LIFE CALCULATION:

$$\frac{16 \text{ h}}{\text{DAY}} \times \frac{6 \text{ DAYS}}{\text{WK}} \times \frac{52 \text{ WKS}}{\text{YR}} \times 20 \text{ YR} = 99840 \text{ h} \quad \text{USE } L = 100000 \text{ h}$$

DESIGN DECISIONS:  $SF = 1.00$ ,  $KR = 1.50$  [FAILURE IN 10000;  $R = 0.9999$ ]

CONTINUED ON NEXT TWO PAGES

**DESIGN OF SPUR GEARS**

<p><b>APPLICATION:</b> [Problem 79 - First pair Industrial lift truck drive to wheels: driven by DC motor]</p>	
<p><b>Double reduction - First pair - Input Data:</b></p> <p>Overload Factor: <math>K_o = 1.50</math> Table 9-7                  Transmitted Power: <math>P = 20</math> hp                  Design Power <math>P_{des} = 30</math> hp                  Diametral Pitch: <math>P_d = 6</math> Fig. 9-24                  Input Speed: <math>n_p = 3000</math> rpm                  Number of Pinion Teeth: <math>N_p = 17</math>                  Desired Output Speed: <math>n_g = 1000</math> rpm                  Computed number of gear teeth: 51.0                  Enter: Chosen No. of Gear Teeth: <math>N_g = 50</math></p>	
<p><b>Computed data:</b></p> <p>Actual Output Speed: <math>n_g = 1020.00</math> rpm                  Gear Ratio: <math>m_g = 2.94</math>                  Pitch Diameter - Pinion: <math>D_p = 2.833</math> in                  Pitch Diameter - Gear: <math>D_g = 8.333</math> in                  Center Distance: <math>C = 5.583</math> in                  Pitch Line Speed: <math>v_t = 2225</math> ft/min                  Transmitted Load: <math>W_t = 297</math> lb</p>	
<p><b>Secondary Input Data:</b></p> <p>Min Nom Max                  Face Width Guidelines (in): 1.333 2.000 2.667                  Enter: Face Width: <math>F = 2.500</math> in                  Ratio: Face width/pinion diameter: <math>F/D_p = 0.88</math>                  Recommended range of ratio: <math>0.50 &lt; F/D_p &lt; 2.00</math></p>	
<p>Enter: Elastic Coefficient: <math>C_p = 2300</math> Table 9-10                  Enter: Quality Number: <math>A_v = 7</math> Table 9-3                  Dynamic Factor: <math>K_v = 1.19</math> Table 9-9                  [Factors for computing <math>K_v</math>]: <math>B = 0.397</math> <math>C = 83.77</math>                  Reference: <math>N_p = 17</math> <math>N_g = 50</math>                  Bending Geometry Factor-Pinion: <math>J_p = 0.293</math> Fig. 9-15                  Bending Geometry Factor-Gear: <math>J_g = 0.380</math> Fig. 9-15                  Reference: <math>m_g = 2.94</math>                  Enter: Pitting Geometry Factor: <math>I = 0.097</math> Fig. 9-21</p>	
<p><b>Factors in Design Analysis:</b></p> <p>Alignment Factor, <math>K_{at} = 1.0 + C_{pt} C_{ma}</math> If <math>F &lt; 1.0</math> If <math>F &gt; 1.0</math>                  Pinion Proportion Factor, <math>C_{pt} = 0.063</math> <math>0.082</math> <math>[0.50 &lt; F/D_p &lt; 2.00]</math>                  Enter: <math>C_{pt} = 0.082</math> Figure 9-16                  Type of gearing: Open Commer. Precision Ex. Prec.                  Mesh Alignment Factor, <math>C_{ma} = 0.288</math> <math>0.166</math> <math>0.099</math> <math>0.063</math>                  Enter: <math>C_{ma} = 0.166</math> Figure 9-17                  Alignment Factor: <math>K_m = 1.25</math> [Computed]</p>	
<p>Size Factor: <math>K_s = 1.00</math> Table 9-8: Use 1.00 if <math>P_d \geq 5</math>                  Pinion Rim Thickness Factor: <math>K_{RP} = 1.00</math> Fig. 9-18: Use 1.00 if solid blank                  Gear Rim Thickness Factor: <math>K_{RG} = 1.00</math> Fig. 9-18: Use 1.00 if solid blank                  Service Factor: <math>SF = 1.00</math> Use 1.00 if no unusual conditions                  Reliability Factor: <math>K_R = 1.50</math> Table 9-11 Use 1.00 for <math>R = .99</math></p>	
<p>Enter: Design Life: 100000 hours See Table 9-12</p>	
<p>Pinion - Number of load cycles: <math>N_p = 1.8E+10</math> Guidelines: <math>Y_N, Z_N</math>                  Gear - Number of load cycles: <math>N_g = 6.1E+09</math> <math>10^7</math> cycles <math>&gt; 10^8</math> <math>&lt; 10^9</math></p>	
<p>Bending Stress Cycle Factor: <math>Y_{NP} = 0.89</math> Fig. 9-22                  Bending Stress Cycle Factor: <math>Y_{NG} = 0.91</math> Fig. 9-22                  Pitting Stress Cycle Factor: <math>Z_{NP} = 0.84</math> Fig. 9-23                  Pitting Stress Cycle Factor: <math>Z_{NG} = 0.86</math> Fig. 9-23</p>	
<p><b>Stress Analysis: Bending</b></p> <p>Pinion: Required <math>s_{at} = 9,152</math> psi See Fig. 9-11 or                  Gear: Required <math>s_{at} = 6,901</math> psi Table 9-5</p>	
<p><b>Stress Analysis: Pitting</b></p> <p>Pinion: Required <math>s_{ac} = 127,576</math> psi See Fig. 9-12 or                  Gear: Required <math>s_{ac} = 124,609</math> psi Table 9-5</p>	
<p>Required hardness of pinion HB: 306 Equations in Fig. 9-12: Grade 1                  Required hardness of gear HB: 297 Equations in Fig. 9-12: Grade 1  <b>Specify materials, alloy and heat treatment, for most severe requirement.</b>                  One possible material specification:                  Pinion requires HB 306; SAE 4340 OQT 1100; HB 321; 19% elongation                  Gear requires HB 297; SAE 4340 OQT 1100; HB 321; 19% elongation</p>	
<p><b>Comments:</b>                  Same material used for pinion and gear because contact stresses are close.</p>	

**DESIGN OF SPUR GEARS**

APPLICATION: [Problem 79 - Second pair Industrial lift truck drive to wheels: driven by DC motor]		Factors in Design Analysis:	
<b>Double reduction - Second pair - Input Data:</b> Overload Factor: $K_o = 1.50$ Table 9-7 Transmitted Power: $P = 20$ hp Design Power $P_{des} = 30$ hp Diametral Pitch: $P_d = 5$ Fig. 9-24 Input Speed: $n_p = 1020$ rpm Number of Pinion Teeth: $N_p = 21$ Desired Output Speed: $n_g = 560.23$ rpm Computed number of gear teeth: 38.2 Enter: Chosen No. of Gear Teeth: $N_g = 38$		Alignment Factor, $K_m = 1.0 + C_{pr}C_{ma}$ If $F < 1.0$ If $F > 1.0$ Pinion Proportion Factor, $C_{pf} = 0.046$ 0.071 [0.50 < $F/D_p < 2.00$ ] Enter: $C_{pf} = 0.071$ Figure 9-16 Type of gearing: Open Commer. Precision Ex. Prec. Mesh Alignment Factor, $C_{ma} = 0.296$ 0.173 0.105 0.068 Enter: $C_{ma} = 0.173$ Figure 9-17 Alignment Factor: $K_m = 1.24$ [Computed]	
<b>Computed data:</b> Actual Output Speed: $n_g = 563.68$ rpm Gear Ratio: $m_g = 1.81$ Pitch Diameter - Pinion: $D_p = 4.200$ in Pitch Diameter - Gear: $D_g = 7.600$ in Center Distance: $C = 5.900$ in Pitch Line Speed: $v_t = 1122$ ft/min Transmitted Load: $W_t = 598$ lb		Size Factor: $K_s = 1.00$ Table 9-8; Use 1.00 if $P_d \geq 5$ Pinion Rim Thickness Factor: $K_{BP} = 1.00$ Fig. 9-18; Use 1.00 if solid blank Gear Rim Thickness Factor: $K_{BS} = 1.00$ Fig. 9-18; Use 1.00 if solid blank Service Factor: $SF = 1.00$ Use 1.00 if no unusual conditions Reliability Factor: $K_R = 1.50$ Table 9-11 Use 1.00 for $R = .99$ Enter: Design Life: 100000 hours See Table 9-12	
<b>Secondary Input Data:</b> Min Nom Max Face Width Guidelines (in): 1.600 2.400 3.200 Enter: Face Width: $F = 3.000$ in Ratio: Face width/pinion diameter: $F/D_p = 0.71$ Recommended range of ratio: $0.50 < F/D_p < 2.00$ Enter: Elastic Coefficient: $C_p = 2300$ Table 9-10 Enter: Quality Number: $A_v = 7$ Table 9-3 Dynamic Factor: $K_v = 1.14$ Table 9-9 [Factors for computing $K_v$ ]: $B = 0.397$ $C = 63.77$ Reference: $N_p = 21$ $N_g = 38$ Bending Geometry Factor-Pinion: $J_p = 0.330$ Fig. 9-15 Bending Geometry Factor-Gear: $J_g = 0.380$ Fig. 9-15 Reference: $m_g = 1.81$ Enter: Pitting Geometry Factor: $I = 0.092$ Fig. 9-21		Pinion - Number of load cycles: $N_p = 6.1E+09$ Guidelines: $Y_N, Z_N$ Gear - Number of load cycles: $N_g = 3.4E+09$ $10^7$ cycles $> 10^7$ $< 10^7$ Bending Stress Cycle Factor: $Y_{NP} = 0.91$ 1.00 0.91 Fig. 9-22 Bending Stress Cycle Factor: $Y_{NG} = 0.92$ 1.00 0.92 Fig. 9-22 Pitting Stress Cycle Factor: $Z_{NP} = 0.86$ 1.00 0.86 Fig. 9-23 Pitting Stress Cycle Factor: $Z_{NG} = 0.87$ 1.00 0.87 Fig. 9-23 <b>Stress Analysis: Bending</b> Pinion: Required $s_{bt} = 10,447$ psi See Fig. 9-11 or Gear: Required $s_{gt} = 8,974$ psi Table 9-5 <b>Stress Analysis: Pitting</b> Pinion: Required $s_{ac} = 131,992$ psi See Fig. 9-12 or Gear: Required $s_{gc} = 130,475$ psi Table 9-5 Required hardness of pinion HB: 320 Equations in Fig. 9-12-Grade 1 Required hardness of gear HB: 315 Equations in Fig. 9-12-Grade 1 <b>Specify materials, alloy and heat treatment, for most severe requirement.</b> One possible material specification: Pinion requires HB 320; SAE 4340 OQT 1100; HB 321; 19% elongation Gear requires HB 315; SAE 4340 OQT 1100; HB 388; 19% elongation <b>Comments:</b> Same material and heat treatment used for pinion and gear. Same material used for both Pair 1 and Pair 2; Same tempering temperature.	



## DESIGN OF PLASTIC SPUR GEARS

### Application:

Small band saw driven by an electric motor  
 Problem 80 - One possible design

### Initial Input Data:

Input Power:  $P = 0.5$  hp  
 Input Speed:  $n_P = 860$  rpm  
 Diametral Pitch:  $P_d = 12$   
 Number of Pinion Teeth:  $N_P = 18$   
 Desired Output Speed:  $n_G = 266$  rpm

Computed number of gear teeth:  $58.195$

Enter: Chosen No. of Gear Teeth:  $N_G = 58$

### Computed data:

Actual Output Speed:  $n_G = 266.9$  rpm  
 Gear Ratio:  $m_G = 3.222$   
 Pitch Diameter - Pinion:  $D_P = 1.500$  in  
 Pitch Diameter - Gear:  $D_G = 4.833$  in  
 Center Distance:  $C = 6.333$  in  
 Pitch Line Speed:  $v_t = 337.7$  ft/min  
 Transmitted Load:  $W_t = 48.84$  lb

### Secondary Input Data - Pinion:

Tooth Form: *20 degree full depth*  
 Lewis Form Factor:  $Y = 0.521$  Table 9-15  
 Safety Factor:  $SF = 1.50$  Ref. Table 9-7  
 Material: Nylon  
 Allowable Bending Stress:  $s_{at} = 6000$  psi Table 9-14 or Fig. 9-28

Required Face Width:  $F = 0.281$  in

Enter: Specified Face Width:  $F = 0.300$  in

Actual Bending Stress in Pinion:  $s_t = 5624$  psi

### Secondary Input Data - Gear:

Tooth Form: *20 degree full depth* Same as for pinion  
 Lewis Form Factor:  $Y = 0.700$  Table 9-15  
 Safety Factor:  $SF = 1.50$  Same as for pinion  
 Material: Acetal  
 Allowable Bending Stress:  $s_{at} = 5000$  psi Table 9-14 or Fig. 9-28

Face Width - Gear:  $F = 0.300$  in Same as for pinion

Actual Bending Stress in Gear:  $s_t = 4133$  psi Must be  $< s_{at}$

## DESIGN OF PLASTIC SPUR GEARS

### Application:

Paper feed roll driven by an electric motor  
 Problem 81 - One possible design

### Initial Input Data:

Input Power:  $P = 0.06$  hp  
 Input Speed:  $n_P = 88$  rpm  
 Diametral Pitch:  $P_D = 20$   
 Number of Pinion Teeth:  $N_P = 18$   
 Desired Output Speed:  $n_G = 21$  rpm

Computed number of gear teeth: 67.048

Enter: Chosen No. of Gear Teeth:  $N_G = 67$

### Computed data:

Actual Output Speed:  $n_G = 21.0$  rpm  
 Gear Ratio:  $m_G = 4.188$   
 Pitch Diameter - Pinion:  $D_P = 0.800$  in  
 Pitch Diameter - Gear:  $D_G = 3.350$  in  
 Center Distance:  $C = 4.150$  in  
 Pitch Line Speed:  $v_t = 18.4$  ft/min  
 Transmitted Load:  $W_t = 107.4$  lb

### Secondary Input Data - Pinion:

Tooth Form: 20 degree stub  
 Lewis Form Factor:  $Y = 0.578$  Table 9-15  
 Safety Factor:  $SF = 1.25$  Ref. Table 9-7  
 Material: Nylon-glass filled  
 Allowable Bending Stress:  $s_{at} = 12000$  psi Table 9-14 or Fig. 9-28

Required Face Width:  $F = 0.387$  in

Enter: Specified Face Width:  $F = 0.400$  in

Actual Bending Stress in Pinion:  $s_t = 11612$  psi

### Secondary Input Data - Gear:

Tooth Form: 20 degree stub Same as for pinion  
 Lewis Form Factor:  $Y = 0.782$  Table 9-15  
 Safety Factor:  $SF = 1.25$  Same as for pinion  
 Material: Nylon-glass filled  
 Allowable Bending Stress:  $s_{at} = 12000$  psi Table 9-14 or Fig. 9-28

Face Width - Gear:  $F = 0.400$  in Same as for pinion

Actual Bending Stress in Gear:  $s_t = 8583$  psi Must be  $< s_{at}$



## DESIGN OF PLASTIC SPUR GEARS

### Application:

Wheels of remote control car - Electric motor drive  
 Problem 82 - One possible design

### Initial Input Data:

Input Power:  $P = 0.025$  hp  
 Input Speed:  $n_P = 430$  rpm  
 Diametral Pitch:  $P_d = 48$   
 Number of Pinion Teeth:  $N_P = 14$   
 Desired Output Speed:  $n_G = 121$  rpm

Computed number of gear teeth:  $49.752$

Enter: Chosen No. of Gear Teeth:  $N_G = 50$

### Computed data:

Actual Output Speed:  $n_G = 120.4$  rpm  
 Gear Ratio:  $m_G = 3.571$   
 Pitch Diameter - Pinion:  $D_P = 0.292$  in  
 Pitch Diameter - Gear:  $D_G = 1.042$  in  
 Center Distance:  $C = 1.333$  in  
 Pitch Line Speed:  $v_t = 32.8$  ft/min  
 Transmitted Load:  $W_t = 25.1$  lb

### Secondary Input Data - Pinion:

Tooth Form: *20 degree stub*  
 Lewis Form Factor:  $Y = 0.54$  Table 9-15  
 Safety Factor:  $SF = 1.25$  Ref: Table 9-7  
 Material: Nylon-unfilled  
 Allowable Bending Stress:  $s_{at} = 6000$  psi Table 9-14 or Fig. 9-28

Required Face Width:  $F = 0.465$  in

Enter: Specified Face Width:  $F = 0.470$  in

Actual Bending Stress in Pinion:  $s_t = 5938$  psi

### Secondary Input Data - Gear:

Tooth Form: *20 degree stub* Same as for pinion  
 Lewis Form Factor:  $Y = 0.756$  Table 9-15  
 Safety Factor:  $SF = 1.25$  Same as for pinion  
 Material: Nylon-unfilled  
 Allowable Bending Stress:  $s_{at} = 6000$  psi Table 9-14 or Fig. 9-28

Face Width - Gear:  $F = 0.470$  in Same as for pinion

Actual Bending Stress in Gear:  $s_t = 4230$  psi Must be  $< s_{at}$



**DESIGN OF PLASTIC SPUR GEARS**

**Application:**

Food-chopping machine driven by electric motor  
 Problem 83 - One possible design

**Initial Input Data:**

Input Power:  $P = 0.65$  hp  
 Input Speed:  $n_P = 1560$  rpm  
 Diametral Pitch:  $P_d = 16$   
 Number of Pinion Teeth:  $N_P = 18$   
 Desired Output Speed:  $n_G = 468$  rpm

Computed number of gear teeth: 59.872

Enter: Chosen No. of Gear Teeth:  $N_G = 60$

**Computed data:**

Actual Output Speed:  $n_G = 468.0$  rpm  
 Gear Ratio:  $m_G = 3.333$   
 Pitch Diameter - Pinion:  $D_P = 1.125$  in  
 Pitch Diameter - Gear:  $D_G = 3.750$  in  
 Center Distance:  $C = 4.875$  in  
 Pitch Line Speed:  $v_t = 459.5$  ft/min  
 Transmitted Load:  $W_t = 46.7$  lb

**Secondary Input Data - Pinion:**

Tooth Form: 20 degree full depth  
 Lewis Form Factor:  $Y = 0.521$  Table 9-15  
 Safety Factor:  $SF = 1.75$  Ref: Table 9-7  
 Material: Nylon-unfilled  
 Allowable Bending Stress:  $s_{at} = 6000$  psi Table 9-14 or Fig. 9-28

Required Face Width:  $F = 0.418$  in

Enter: Specified Face Width:  $F = 0.420$  in

Actual Bending Stress in Pinion:  $s_t = 5971$  psi

**Secondary Input Data - Gear:**

Tooth Form: 20 degree full depth Same as for pinion  
 Lewis Form Factor:  $Y = 0.713$  Table 9-15  
 Safety Factor:  $SF = 1.75$  Same as for pinion  
 Material: Acetal-unfilled  
 Allowable Bending Stress:  $s_{at} = 5000$  psi Table 9-14 or Fig. 9-28

Face Width - Gear:  $F = 0.420$  in Same as for pinion

Actual Bending Stress in Gear:  $s_t = 4363$  psi Must be  $< s_{at}$

## CHAPTER 10

### HELICAL GEARS, BEVEL GEARS, AND WORMGEARING

1. HELICAL GEARS:  $P_d = 8$ ,  $\phi_c = 14\frac{1}{2}^\circ$ ,  $N_g = 45$ ,  $F = 2.011N$ ,  $\psi = 30^\circ$

a.  $P = 5.0 \text{ HP}$ ,  $M_G = 1250 \text{ RPM}$ ; TORQUE =  $T = \frac{63000(5.0)}{1250} = 252 \text{ LB}\cdot\text{IN}$

$$W_t = \frac{T}{D_g/2} : D_g = \frac{N_g}{P_d} = \frac{45}{8} = 5.625 \text{ IN}$$

$$W_t = \frac{252 \text{ LB}\cdot\text{IN}}{5.625 \text{ IN}/2} = \underline{89.6 \text{ LB}}$$

$$W_x = W_t \tan \psi = 89.6 \tan 30^\circ = \underline{51.7 \text{ LB}}$$

$$W_r = W_t \tan \phi_c = 89.6 \tan 14\frac{1}{2}^\circ = \underline{23.2 \text{ LB}}$$

b.  $N_p = 15$ ; DRIVE TO A RECIRCULATING PUMP,  $K_b = 1.50$

$$S_{tP} = \frac{W_t P_d}{F J_p} \cdot \frac{K_o K_s K_m K_B K_v}{K_t} ; \text{ASSUME } K_s = K_B = 1.0$$

APPROXIMATE  $J_p$  FROM FIG 10-5. DATA ARE FOR  $\phi_m = 15^\circ$

$$\text{ACTUAL } \phi_m = \tan^{-1}[\tan \phi_c \cos \psi] = \tan^{-1}[\tan 14\frac{1}{2}^\circ \cos 30^\circ] = 12.6^\circ$$

$$J_p \approx (0.38)(0.97) = 0.369$$

$$D_p = N_p/P_d = 15/8 = 1.875 \text{ IN}; F/D_p = 1.067; C_{pf} = 0.075; C_{ma} = 1.55$$

$$K_m = 1.0 + C_{pf} + C_{ma} = 1.00 + 0.075 + 0.155 = 1.23$$

$$v_x = \pi D_g M_G / 12 = \pi(5.625)(1250)/12 = 1841 \text{ FT/MIN}$$

SPECIFY  $A_v = 11$ ; THE  $K_v = 1.55$  (FIG 9-20)

$$S_{tP} = \frac{(89.6)(8)}{(2.00)(0.369)} (1.50)(1.0)(1.23)(1.0)(1.55) = \underline{2778 \text{ PSI}}$$

$$S_c = C_p \sqrt{\frac{W_t K_o K_B K_m K_v}{F D_p I}} = 1960 \sqrt{\frac{(89.6)(1.50)(1.0)(1.23)(1.55)}{(2.00)(1.875)(0.20)}}$$

$$S_c = \underline{3622 \text{ PSI}} ; \text{I EST. FROM TABLE 10-1.}$$

c. SPECIFIED CAST IRON BECAUSE OF LOW STRESSES

$$\text{GRAY CAST IRON: } S_{at} = 5000 \text{ PSI}, S_{ac} = 50000 \text{ PSI}$$

CLASS 20

2 HELICAL GEARS:  $P = 2.50 \text{ HP}$ ,  $N_P = 16$ ,  $N_G = 48$ ,  $P_d = 12$ ,  $\phi_n = 20^\circ$ ,  $\psi = 45^\circ$

$F = 1.50 \text{ IN}$ : TORQUE =  $T = \frac{63000(2.50)}{1750} = 90.0 \text{ LB}\cdot\text{IN}$

$M_G = 1750 \text{ RPM}$ :  $W_t = \frac{T}{D_G/2} = \frac{90.0}{(5.657/2)} = 31.8 \text{ LB}$

FROM PROBLEM 8-42:  $P_d = 8.485$ ;  $D_G = 5.657 \text{ IN}$ ,  $\phi_t = 27.2^\circ$

$D_P = \frac{N_P}{P_d} = \frac{16}{8.485} = 1.887 \text{ IN}$

$W_x = W_t \tan \psi = 31.8 \text{ LB} \cdot \tan 45^\circ = 31.8 \text{ LB}$

$W_n = W_t \tan \phi_t = 31.8 \cdot \tan 27.2^\circ = 16.4 \text{ LB}$

b.  $K_0 = 1.25$  (L.F. SHOCK);  $K_s = K_B = 1.00$ ;  $J_P = 0.30$

$N_G = \pi D_G M_G / 12 = \pi(5.657)(1750) / 12 = 2592 \text{ FT}/\text{MIN}$

CENTRIFUGAL BLOWER LET  $A_r = 9$ ;  $K_r = 1.40$

$F/D_P = 1.50/1.887 = 0.795$   $C_{pt} = 0.05$ ,  $C_{md} = 0.15$ ;  $K_m = 1.20$

$S_t = \frac{W_t P_d K_0 K_s K_m K_B K_r}{F J_P} = \frac{(31.8)(8.485)(1.25)(1.0)(1.2)(1.40)}{(1.50)(0.30)} = 1259 \text{ PSI}$

PITTING:  $I = 0.21$ ,  $C_P = 1960$  CAST IRON

$S_c = C_P \sqrt{\frac{W_t K_0 K_s K_m K_r}{F D_P I}} = 1960 \sqrt{\frac{(31.8)(1.25)(1.0)(1.2)(1.40)}{(1.5)(1.887)(0.21)}} = 20,775 \text{ PSI}$

c.  $S_c = 20,775 \text{ PSI}$ : SPECIFY CLASS 20 CAST IRON

$S_{at} = 5000 \text{ PSI}$ ,  $S_{ac} = 5000 \text{ PSI}$

3 HELICAL GEARS:  $P = 15 \text{ HP}$ ,  $N_P = 12$ ,  $N_G = 36$ ,  $P_d = 6$ ,  $\phi_n = 14\frac{1}{2}^\circ$ ,  $\psi = 45^\circ$ ,  $F = 1.00 \text{ IN}$

$M_G = 2200 \text{ RPM}$ ;  $T = \frac{63000 P}{m} = \frac{63000(15)}{2200} = 430 \text{ LB}\cdot\text{IN}$

FROM PROB. 8-43:  $D_G = 6.00 \text{ IN}$ ;  $D_P = \frac{N_P}{P_d} = \frac{12}{6} = 2.00 \text{ IN}$

a.  $W_t = T / (D_G/2) = 430 / (6.00/2) = 143 \text{ LB}$

$W_x = W_t \tan \psi = 143 \text{ LB} \cdot \tan 45^\circ = 143 \text{ LB}$

$W_n = W_t \tan \phi_t = 143 \text{ LB} \cdot \tan 14\frac{1}{2}^\circ = 37.0 \text{ LB}$

$K_0 = 2.00$  CONC. MIXER

$K_s = 1.00 = K_B$

$\frac{F}{D_P} = \frac{1.00}{2.00} = 0.50$ ;  $C_{pt} = 0.025$

$C_{md} = 0.15$ ;  $K_m = 1.10$

b.  $N_G = \pi D_G M_G / 12 = \pi(6.00)(2200) / 12 = 3456 \text{ FT}/\text{MIN}$

$A_r = 9$ ;  $K_r = 1.44$ ;  $J_P = 0.30$  (EST.),  $I = 0.190$  (EST)

$S_t = \frac{W_t P_d K_0 K_s K_m K_B K_r}{F J_P} = \frac{(143)(6)(1.0)(1.10)(1.0)(1.44)(2.0)}{(1.00)(0.30)} = 9720 \text{ PSI}$

USE MODULAR (DUCTILE) IRON  $C_P = 2050$

$S_c = C_P \sqrt{\frac{W_t K_0 K_s K_m K_r}{F D_P I}} = 2050 \sqrt{\frac{(143)(2.0)(1.0)(1.10)(1.44)}{(1.0)(2.00)(0.19)}} = 73300 \text{ PSI}$

c. SPECIFY: DUCTILE IRON ASTM A536 60-40-18 OR OF CLASS 40  
 $S_{at} = 22000 \text{ PSI}$ ,  $S_{ac} = 77000 \text{ PSI}$   $S_{at} = 13 \text{ KSI}$ ,  $S_{ac} = 75 \text{ KSI}$

NOTE: PROB. 8-43 GIVES  $P_x = \text{AXIAL PITCH} = 0.5236 \text{ IN}$

THEN  $\psi/P_x = 1.00/0.5236 = 1.91$  - LOW - SHOULD BE 72.0 FOR FULL HELIX ACTION.

4

HELICAL GEARS:  $P = 0.50 \text{ HP}$ ,  $m_g = 3450 \text{ RPM}$ ,  $P_{nd} = 24$ ,  $\phi_m = 14\frac{1}{2}^\circ$   
 $\psi = 45^\circ$ ,  $N_g = 72$ ,  $N_p = 16$ ,  $F = 0.25 \text{ IN}$ , WINCH-MOD. SHOCK  $K_0 = 1.50$   
 FROM PROB B-44:  $P_d = 16.97$ ,  $D_g = 4.243 \text{ IN}$   $D_p = \frac{N_p}{P_d} = \frac{16}{16.97} = 0.943 \text{ IN}$

$$T = \frac{63000(P)}{m_g} = \frac{63000(0.50)}{3450} = 9.13 \text{ LB}\cdot\text{IN}$$

$$a. \quad W_t = T / (D_g/2) = (9.13) / (4.243/2) = 4.30 \text{ LB}$$

$$W_x = W_t \tan \psi = 4.30 \text{ LB} \cdot \tan 45^\circ = 4.30 \text{ LB}$$

$$W_n = W_t \tan \phi_c = 4.30 \tan 20^\circ = 1.57 \text{ LB}; \quad \phi_c = 20.0^\circ \text{ FROM PROB B-44.}$$

$$b. \quad N_e = \pi D_g m_g / 12 = \pi (4.243)(3450) / 12 = 3832 \text{ FT/MIN}$$

$$\text{LET } A_N = 9, \quad K_N = 1.48, \quad K_S = 1.00 = K_B, \quad J_P = 0.32 \text{ (EST)}, \quad I = 0.72 \text{ (EST)}$$

$$F/D_p = 0.25 / 0.943 = 0.265; \quad C_P = 0; \quad C_m = 0.14; \quad K_m = 1.14$$

$$S_e = \frac{W_t K_0 K_S K_m K_B K_N}{F J} = \frac{(4.30)(16.97)(1.50)(1.00)(1.14)(1.48)}{(0.25)(0.72)}$$

$$S_e = 2308 \text{ PSI}$$

TRY CAST IRON

$$S_c = C_P \sqrt{\frac{W_t K_0 K_S K_m K_B K_N}{F D_p I}} = 1960 \sqrt{\frac{(4.30)(1.50)(1.00)(1.14)(1.48)}{(0.25)(0.943)(0.72)}}$$

$$S_c = 26633 \text{ PSI}$$

SPECIFY CLASS 20 CAST IRON  $S_{AT} = 5000 \text{ PSI}$   $S_{CC} = 50000 \text{ PSI}$

NOTE: PROBLEM B-44 GIVES  $P_x = \text{AXIAL PITCH} = 0.1815 \text{ IN}$   
 THEN  $F/P_x = 0.25 \text{ IN} / 0.1815 \text{ IN} = 1.35$  - LOW - SHOULD BE 72.0  
 FOR FULL HELICAL ACTION.

THE FOLLOWING PAGES GIVE SAMPLE DESIGNS FOR PROBLEMS 5-11. THE PROCEDURE IS SIMILAR TO THAT USED IN EXAMPLE PROBLEM 10-2. OTHER DESIGNS ARE POSSIBLE. READER IS ENCOURAGED TO WORK TOWARD A PARTICULAR GOAL OF MATERIAL TYPE, CENTER DISTANCE, OVERALL SIZE OR OTHER APPLICATION-SPECIFIC GOAL.

NOTE THAT TRANSVERSE DIAMETRAL PITCH MUST BE INPUT. IF NORMAL DIAMETRAL PITCH IS ORIGINALLY SPECIFIED, COMPUTE  $P_d = P_{nd} \cos \psi$ .



DESIGN OF HELICAL GEARS-U.S.		APPLICATION:		Reciprocating compressor driven by an electric motor Problem 10-5	
<b>Initial Input Data:</b>				<b>Factors in Design Analysis:</b>	
Input Power:	$P = 5$ hp	Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$	If $F < 1.0$	If $F > 1.0$	$F/D_p = 1.20$
Input Speed:	$n_P = 1200$ rpm	Pinion Proportion Factor, $C_{pf}$	$0.095$	$0.098$	$[0.50 < F/D_p < 2.00]$
Transverse Diametral Pitch, $P_d$ :	$18$	Enter: $C_{pf} = 0.098$	Figure 9-16		
Number of Pinion Teeth:	$N_P = 18$	Type of gearing:	Open	Commer.	Precision
Desired Output Speed:	$n_G = 387.5$ rpm	Mesh Alignment Factor, $C_{ma}$	$0.267$	$0.146$	$0.083$
Computed number of gear teeth:	$55.7$	Enter: $C_{ma} = 0.146$	Figure 9-17		
Enter: Chosen No. of Gear Teeth:	$N_G = 56$	Alignment Factor: $K_m = 1.24$	[Computed]		
<b>Computed data:</b>				Overload Factor: $K_o = 1.50$ Table 9-7	
Actual Output Speed:	$n_G = 385.7$ rpm	Size Factor: $K_s = 1.00$	Table 9-8: Use 1.00 if $P_d \geq 5$		
Gear Ratio:	$m_G = 3.11$	Pinion Rim Thickness Factor: $K_{ap} = 1.00$	Fig. 9-18: Use 1.00 if solid blank		
Pitch Diameter - Pinion:	$D_P = 1.000$ in	Gear Rim Thickness Factor: $K_{ag} = 1.00$	Fig. 9-18: Use 1.00 if solid blank		
Pitch Diameter - Gear:	$D_G = 3.111$ in	Dynamic Factor: $K_v = 1.24$	[Computed: See Fig. 9-20]		
Center Distance:	$C = 2.056$ in	Service Factor: $SF = 1.00$	Use 1.00 if no unusual conditions		
Pitch Line Speed:	$V_l = 314$ ft/min	Reliability Factor: $K_R = 1.00$	Table 9-11 Use 1.00 for $R = .99$		
Transmitted Load:	$W_t = 525$ lb	Enter: Design Life: $15000$ hours	See Table 9-12		
<b>Secondary Input Data:</b>				Pinion - Number of load cycles: $N_P = 1.1E+09$	
Transverse pressure angle:	$\phi_t = 20.0$ deg	Gear - Number of load cycles: $N_G = 3.5E+08$	Guidelines: $Y_N, Z_N$		
Helix angle:	$\psi = 25.0$ deg	Bending Stress Cycle Factor: $Y_{NP} = 0.94$	$1.00$	$0.94$	Fig. 9-22
Axial Pitch:	$P_x = 0.3743$ in	Bending Stress Cycle Factor: $Y_{NG} = 0.96$	$1.00$	$0.96$	Fig. 9-22
Min. Face Width (2 x Axial Pitch):	$F_{min} = 0.749$ in	Pitting Stress Cycle Factor: $Z_{NP} = 0.90$	$1.00$	$0.90$	Fig. 9-24
Enter: Face Width:	$F = 1.200$ in	Pitting Stress Cycle Factor: $Z_{NG} = 0.92$	$1.00$	$0.92$	Fig. 9-24
Enter: Elastic Coefficient:	$C_p = 2300$ Table 9-10	<b>Stress Analysis: Bending</b>			
Enter: Quality Number:	$A_v = 11$ Table 9-3	Pinion: Required $s_{at} = 42,786$ psi	See Fig. 9-11 or Table 9-5		
REF: $n_P, n_G = 18, 56$		Gear: Required $s_{at} = 39,050$ psi			
<b>Enter: Bending Geometry Factors:</b>				<b>Stress Analysis: Pitting</b>	
Pinion:	$J_P = 0.453$ Fig 10-5,6,7	Pinion: Required $s_{ec} = 179,572$ psi	See Fig. 9-12 or Table 9-5		
Gear:	$J_G = 0.486$ Fig 10-5,6,7	Gear: Required $s_{ec} = 175,668$ psi			
Enter: Pitting Geometry Factor:	$I = 0.205$ Tab. 10-1,2	<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>			
REF: $m_G = 3.11$		<b>One possible material specification:</b> Steel pinion and gear: Carburized, Grade 1			
Axial Force:	$W_x = 245$ lb	Pinion requires HRC 58 min.; SAE 4320 SOQT 450; HRC 59; Carburized			
Radial Force:	$W_r = 191$ lb	Gear requires HRC 58 min.; SAE 4320 SOQT 450; HRC 59; Carburized			



DESIGN OF HELICAL GEARS-U.S.		APPLICATION:	
<b>Initial Input Data:</b> Input Power: $P = 20$ hp Input Speed: $n_P = 550$ rpm Transverse Diametral Pitch, $P_d = 10$ Number of Pinion Teeth: $N_P = 24$ Desired Output Speed: $n_G = 185$ rpm Computed number of gear teeth: 71.4 Enter: Chosen No. of Gear Teeth: $N_G = 72$		Milling machine driven by an electric motor Problem 10-6	
<b>Computed data:</b> Actual Output Speed: $n_G = 183.3$ rpm Gear Ratio: $m_G = 3.00$ Pitch Diameter - Pinion: $D_P = 2.400$ in Pitch Diameter - Gear: $D_G = 7.200$ in Center Distance: $C = 4.800$ in Pitch Line Speed: $v_t = 346$ ft/min Transmitted Load: $W_t = 1910$ lb		<b>Factors in Design Analysis:</b> Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ms}$ If $F < 1.0$ 0.079    0.098    If $F > 1.0$ $F/D_p = 1.04$ Pinion Proportion Factor, $C_{pf} =$ 0.079    0.098    [0.50 < $F/D_p$ < 2.00] Enter: $C_{pf} = 0.098$ Figure 9-16 Type of gearing: Open    Commer.    Precision    Ex. Prec. Mesh Alignment Factor, $C_{ma} = 0.288$ 0.166    0.099    0.063 Enter: $C_{ma} = 0.099$ Figure 9-17 Alignment Factor: $K_m = 1.20$ [Computed]	
Overload Factor: $K_o = 1.50$ Table 9-7 Size Factor: $K_s = 1.00$ Table 9-8: Use 1.00 if $P_d \geq 5$ Pinion Rim Thickness Factor: $K_{BP} = 1.00$ Fig. 9-18: Use 1.00 if solid blank Gear Rim Thickness Factor: $K_{BG} = 1.00$ Fig. 9-18: Use 1.00 if solid blank Dynamic Factor: $K_v = 1.16$ [Computed: See Fig. 9-20]		Service Factor: $SF = 1.00$ Use 1.00 if no unusual conditions Reliability Factor: $K_R = 1.25$ Table 9-11    Use 1.00 for $R = .99$ Enter: Design Life: 15000 hours    See Table 9-12	
<b>Secondary Input Data:</b> Transverse pressure angle: $\phi_t = 20.0$ deg Helix angle: $\psi = 15.0$ deg Axial Pitch: $p_x = 1.1725$ in Min. Face Width (2 x Axial Pitch): $F_{min} = 2.345$ in Enter: Face Width: $F = 2.500$ in Enter: Elastic Coefficient: $C_p = 2300$ Table 9-10 Enter: Quality Number: $A_v = 9$ Table 9-3 REF: $N_P, N_G = 24, 72$		Pinion - Number of load cycles: $N_P = 5.0E+08$ Gear - Number of load cycles: $N_G = 1.7E+08$ Bending Stress Cycle Factor: $Y_{NP} = 0.95$ 1.00    0.95    Fig. 9-22 Bending Stress Cycle Factor: $Y_{NG} = 0.97$ 1.00    0.97    Fig. 9-22 Pitting Stress Cycle Factor: $Z_{NP} = 0.91$ 1.00    0.91    Fig. 9-24 Pitting Stress Cycle Factor: $Z_{NG} = 0.94$ 1.00    0.94    Fig. 9-24 Guidelines: $Y_N, Z_N$ 10 <sup>7</sup> cycles    >10 <sup>7</sup> <10 <sup>7</sup>	
Enter: Bending Geometry Factors: Pinion: $J_P = 0.480$ Fig 10-5,6,7 Gear: $J_G = 0.526$ Fig 10-5,6,7 Enter: Pitting Geometry Factor: $I = 0.220$ Tab. 10-1,2 REF: $m_G = 3.00$ Axial Force: $W_x = 512$ lb Radial Force: $W_r = 695$ lb		<b>Stress Analysis: Bending</b> Pinion: Required $s_{at} = 43,560$ psi    See Fig. 9-11 or Gear: Required $s_{at} = 38,931$ psi    Table 9-5 <b>Stress Analysis: Pitting</b> Pinion: Required $s_{ec} = 173,320$ psi    See Fig. 9-12 or Gear: Required $s_{ec} = 167,788$ psi    Table 9-5 <b>Specify materials, alloy and heat treatment, for most severe requirement.</b> <b>One possible material specification:</b> Steel pinion and gear: Carburized, Grade 1 Pinion requires HRC 58 min.: SAE 4320 SOQT 450; HRC 59; Carburized Gear requires HRC 58 min.: SAE 4320 SOQT 450; HRC 59; Carburized	

**DESIGN OF HELICAL GEARS-U.S.** APPLICATION: **Punch press driven by an electric motor** Problem 10-7

**Initial Input Data:**

Input Power:	$P = 50$ hp
Input Speed:	$n_p = 900$ rpm
Transverse Diametral Pitch, $P_d$ :	$P_d = 6$
Number of Pinion Teeth:	$N_p = 24$
Desired Output Speed:	$n_g = 227.5$ rpm
Computed number of gear teeth:	94.9
Enter: Chosen No. of Gear Teeth:	$N_g = 95$

**Computed data:**

Actual Output Speed:	$n_g = 227.4$ rpm
Gear Ratio:	$m_g = 3.96$
Pitch Diameter - Pinion:	$D_p = 4.000$ in
Pitch Diameter - Gear:	$D_g = 15.833$ in
Center Distance:	$C = 9.917$ in
Pitch Line Speed:	$V_l = 942$ ft/min
Transmitted Load:	$W_t = 1751$ lb

**Secondary Input Data:**

Transverse pressure angle:	$\phi_t = 20.0$ deg
Helix angle:	$\psi = 25.0$ deg
Axial Pitch:	$p_x = 1.1229$ in
Min. Face Width (2 x Axial Pitch):	$F_{min} = 2.246$ in
Enter: Face Width:	$F = 2.500$ in
Enter: Elastic Coefficient:	$C_p = 2300$ Table 9-10
Enter: Quality Number:	$A_v = 9$ Table 9-3
REF: $N_p, N_g =$	24 95

**Enter: Bending Geometry Factors:**

Pinion:	$J_p = 0.465$ Fig 10-5,6,7
Gear:	$J_g = 0.496$ Fig 10-5,6,7
Enter: Pitting Geometry Factor:	$I = 0.220$ Tab. 10-1,2
REF: $m_g =$	3.96
Axial Force:	$W_x = 816$ lb
Radial Force:	$W_r = 637$ lb

**Factors in Design Analysis:**

Alignment Factor, $K_m = 1.0 + C_{pt} + C_{ma}$	IF $F < 1.0$	IF $F > 1.0$	$F/D_p = 0.63$
Pinion Proportion Factor, $C_{pt}$	0.038	0.056	[0.50 < $F/D_p$ < 2.00]
Enter: $C_{pt} =$	0.056	Figure 9-16	
Type of gearing:	Open	Commer.	Precision
Mesh Alignment Factor, $C_{ma}$	0.288	0.166	0.099
Enter: $C_{ma} =$	0.166	Figure 9-17	
Alignment Factor: $K_m =$	1.22	[Computed]	
Overload Factor: $K_o =$	1.75	Table 9-7	
Size Factor: $K_s =$	1.00	Table 9-8: Use 1.00 if $P_d \geq 5$	
Pinion Rim Thickness Factor: $K_{ap} =$	1.00	Fig. 9-18: Use 1.00 if solid blank	
Gear Rim Thickness Factor: $K_{ag} =$	1.00	Fig. 9-18: Use 1.00 if solid blank	
Dynamic Factor: $K_v =$	1.28	[Computed: See Fig. 9-20]	
Service Factor: $SF =$	1.00	Use 1.00 if no unusual conditions	
Reliability Factor: $K_R =$	1.25	Table 9-11 Use 1.00 for $R = .99$	
Enter: Design Life:	15000	hours	See Table 9-12
Pinion - Number of load cycles: $N_p =$	8.1E+08		Guidelines: $Y_N, Z_N$
Gear - Number of load cycles: $N_g =$	2.0E+08		$10^7$ cycles > $10^8$ < $10^9$
Bending Stress Cycle Factor: $Y_{NP} =$	0.94	1.00	0.94 Fig. 9-22
Bending Stress Cycle Factor: $Y_{NG} =$	0.96	1.00	0.96 Fig. 9-22
Pitting Stress Cycle Factor: $Z_{NP} =$	0.90	1.00	0.90 Fig. 9-24
Pitting Stress Cycle Factor: $Z_{NG} =$	0.93	1.00	0.93 Fig. 9-24

**Stress Analysis: Bending**  
 Pinion: Required  $s_{at} = 32,253$  psi See Fig. 9-11 or  
 Gear: Required  $s_{at} = 29,607$  psi Table 9-5

**Stress Analysis: Pitting**  
 Pinion: Required  $s_{ec} = 147,637$  psi See Fig. 9-12 or  
 Gear: Required  $s_{ec} = 142,875$  psi Table 9-5

**Specify materials, alloy and heat treatment, for most severe requirement.**  
**One possible material specification:** Steel pinion and gear. Through hardened  
 Pinion requires HB 368; SAE 4140 OQT 900; HB 388  
 Gear requires HB 353; SAE 4140 OQT 900; HB 388

DESIGN OF HELICAL GEARS-U.S.		APPLICATION:	
<b>Initial Input Data:</b> Input Power: $P = 2.5$ hp Input Speed: $n_p = 900$ rpm Transverse Diametral Pitch, $P_d = 12$ Number of Pinion Teeth: $N_p = 20$ Desired Output Speed: $n_g = 75$ rpm Computed number of gear teeth: 240.0 Enter: Chosen No. of Gear Teeth: $N_g = 240$		Small cement mixer driven by a gasoline engine Problem 10-8 Use steel pinion with cast iron gear	
<b>Computed data:</b> Actual Output Speed: $n_g = 75.0$ rpm Gear Ratio: $m_g = 12.00$ Pitch Diameter - Pinion: $D_p = 1.667$ in Pitch Diameter - Gear: $D_g = 20.000$ in Center Distance: $C = 10.833$ in Pitch Line Speed: $v_l = 393$ ft/min Transmitted Load: $W_t = 210$ lb		<b>Factors in Design Analysis:</b> Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$ IF $F < 1.0$ 0.080 IF $F > 1.0$ 0.089 $F/D_p = 1.05$ Pinion Proportion Factor, $C_{pf} = 0.089$ [0.50 < $F/D_p$ < 2.00] Enter: $C_{pf} = 0.089$ Figure 9-16 Type of gearing: Open Commer. Precision Ex. Prec. Mesh Alignment Factor, $C_{ma} = 0.276$ 0.154 0.090 0.056 Enter: $C_{ma} = 0.276$ Figure 9-17 Alignment Factor: $K_m = 1.37$ [Computed]	
Overload Factor: $K_o = 2.00$ Table 9-7 Size Factor: $K_s = 1.00$ Table 9-8: Use 1.00 if $P_d \geq 5$ Pinion Rim Thickness Factor: $K_{BP} = 1.00$ Fig. 9-18: Use 1.00 if solid blank Gear Rim Thickness Factor: $K_{BG} = 1.00$ Fig. 9-18: Use 1.00 if solid blank		Dynamic Factor: $K_v = 1.33$ [Computed: See Fig. 9-20] Service Factor: $SF = 1.00$ Use 1.00 if no unusual conditions Reliability Factor: $K_R = 1.00$ Table 9-11 Use 1.00 for $R = .99$	
<b>Secondary Input Data:</b> Transverse pressure angle: $\phi_t = 20.0$ deg Helix angle: $\psi = 25.0$ deg Axial Pitch: $P_x = 0.5614$ in Min. Face Width (2 x Axial Pitch): $F_{min} = 1.123$ in Enter: Face Width: $F = 1.750$ in Enter: Elastic Coefficient: $C_p = 2100$ Table 9-10 Enter: Quality Number: $A_v = 12$ Table 9-3 REF: $N_p, N_g = 20, 240$		Enter: Design Life: 8000 hours See Table 9-12 Pinion - Number of load cycles: $N_p = 4.3E+08$ Gear - Number of load cycles: $N_g = 3.6E+07$ Bending Stress Cycle Factor: $Y_{NP} = 0.95$ Fig. 9-22 Bending Stress Cycle Factor: $Y_{NG} = 0.99$ Fig. 9-22 Pitting Stress Cycle Factor: $Z_{NP} = 0.92$ Fig. 9-24 Pitting Stress Cycle Factor: $Z_{NG} = 0.97$ Fig. 9-24 Stress Analysis: Bending Pinion: Required $s_{af} = 12,180$ psi See Fig. 9-11 or Gear: Required $s_{af} = 10,295$ psi Table 9-5	
Enter: Bending Geometry Factors: Pinion: $J_p = 0.451$ Fig 10-5,6,7 Gear: $J_g = 0.512$ Fig 10-5,6,7 Enter: Pitting Geometry Factor: $I = 0.260$ Tab. 10-1,2 REF: $m_g = 12.00$ Axial Force: $W_x = 96$ lb Radial Force: $W_r = 76$ lb		Stress Analysis: Pitting Pinion: Required $s_{ac} = 72,310$ psi See Fig. 9-12 or Gear: Required $s_{ac} = 68,583$ psi Table 9-5 Specify materials, alloy and heat treatment, for most severe requirement. One possible material specification: Steel pinion and cast iron gear Pinion requires HB 134; SAE 1020 CD; HB 160 - Or almost any steel Gear: Grade 40 gray cast iron; HB 201; $s_{af} = 13$ ksi; $s_{ac} = 75$ ksi (Table 9-6)	



DESIGN OF HELICAL GEARS-U.S.	APPLICATION:	Wood chipper driven by a gasoline engine Problem 10-9	Speed increaser - cells changed
<b>Initial Input Data:</b>		Input Power: $P = 75$ hp Input Speed: $n_g = 2200$ rpm Transverse Diametral Pitch, $P_d = 10$ Number of Pinion Teeth: $N_p = 26$ Desired Output Speed: $n_p = 4550$ rpm Computed number of gear teeth: 53.8 Enter: Chosen No. of Gear Teeth: $N_g = 54$	
<b>Computed data:</b>		Actual Output Speed: $n_g = 4569.2$ rpm Gear Ratio: $m_g = 2.08$ Pitch Diameter - Pinion: $D_p = 2.600$ in Pitch Diameter - Gear: $D_g = 5.400$ in Center Distance: $C = 4.000$ in Pitch Line Speed: $v_t = 3097$ ft/min Transmitted Load: $W_t = 799$ lb	
<b>Secondary Input Data:</b>		Transverse pressure angle: $\phi_t = 20.0$ deg Helix angle: $\psi = 25.0$ deg Axial Pitch: $p_x = 0.67372$ in Min. Face Width (2 x Axial Pitch): $F_{min} = 1.347$ in Enter: Face Width: $F = 1.800$ in Enter: Elastic Coefficient: $C_p = 2300$ Enter: Quality Number: $A_v = 9$ REF: $N_p, N_g = 26$	
<b>Enter: Bending Geometry Factors:</b>		Pinion: $J_p = 0.453$ Fig 10-5.6,7 Gear: $J_g = 0.463$ Fig 10-5.6,7 Enter: Pitting Geometry Factor: $I = 0.188$ Tab. 10-1.2 REF: $m_g = 2.08$	
Axial Force: $W_x = 373$ lb			
Radial Force: $W_r = 291$ lb			
<b>Factors in Design Analysis:</b>			
Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$		if $F < 1.0$ 0.044 if $F > 1.0$ 0.054 Enter: $C_{pf} = 0.054$ Figure 9-16	$F/D_p = 0.69$ $[0.50 < F/D_p < 2.00]$
Mesh Alignment Factor, $C_{ma}$		0.277	0.155
Type of gearing:		Open	Commer.
Precision		Ex. Prec.	
Enter: $C_{ma} = 0.277$ Figure 9-17			
Alignment Factor: $K_m = 1.33$ [Computed]			
Overload Factor: $K_o = 2.75$ Table 9-7			
Size Factor: $K_s = 1.00$ Table 9-8: Use 1.00 if $P_d \geq 5$			
Pinion Rim Thickness Factor: $K_{ap} = 1.00$ Fig. 9-18: Use 1.00 if solid blank			
Gear Rim Thickness Factor: $K_{ag} = 1.00$ Fig. 9-18: Use 1.00 if solid blank			
Dynamic Factor: $K_v = 1.44$ [Computed: See Fig. 9-20]			
Service Factor: $SF = 1.00$ Use 1.00 if no unusual conditions			
Reliability Factor: $K_R = 1.00$ Table 9-11 Use 1.00 for $R = .99$			
Enter: Design Life: 8000 hours		See Table 9-12	
Pinion - Number of load cycles: $N_p = 1.1E+09$		Guidelines: $Y_N, Z_N$	
Gear - Number of load cycles: $N_g = 5.1E+08$		$10^7$ cycles $> 10^8$ $< 10^9$	
Bending Stress Cycle Factor: $Y_{NP} = 0.94$		1.00	0.94 Fig. 9-22
Bending Stress Cycle Factor: $Y_{NG} = 0.95$		1.00	0.95 Fig. 9-22
Pitting Stress Cycle Factor: $Z_{NP} = 0.90$		1.00	0.90 Fig. 9-24
Pitting Stress Cycle Factor: $Z_{NG} = 0.91$		1.00	0.91 Fig. 9-24
<b>Stress Analysis: Bending</b>			
Pinion: Required $s_{at} = 55,024$ psi			See Fig. 9-11 or Table 9-5
Gear: Required $s_{at} = 53,268$ psi			
<b>Stress Analysis: Pitting</b>			
Pinion: Required $s_{ec} = 176,932$ psi			See Fig. 9-12 or Table 9-5
Gear: Required $s_{ec} = 174,988$ psi			
<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>			
<b>One possible material specification: Steel pinion+gear: Carburized-Case hard.</b>			
Pinion requires HRC 55: SAE 4118 DOQT 300; HRC 62			
Gear requires HRC 55: SAE 4118 DOQT 300; HRC 62			

DESIGN OF HELICAL GEARS-U.S.		APPLICATION:	
<b>Initial Input Data:</b> Input Power: $P = 20$ hp Input Speed: $n_P = 450$ rpm Transverse Diametral Pitch, $P_d = 6$ Number of Pinion Teeth: $N_P = 21$ Desired Output Speed: $n_G = 77.5$ rpm Computed number of gear teeth: 121.9 Enter: Chosen No. of Gear Teeth: $N_G = 122$		Small tractor driven by a gasoline engine Problem 10-10 Use steel pinion with steel gear	
<b>Computed data:</b> Actual Output Speed: $n_G = 77.5$ rpm Gear Ratio: $m_G = 5.81$ Pitch Diameter - Pinion: $D_P = 3.500$ in Pitch Diameter - Gear: $D_G = 20.333$ in Center Distance: $C = 11.917$ in Pitch Line Speed: $v_t = 412$ ft/min Transmitted Load: $W_t = 1601$ lb		<b>Factors in Design Analysis:</b> Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$ If $F > 1.0$ $F/D_p = 0.64$ Pinion Proportion Factor, $C_{pf} = 0.039$ If $F < 1.0$ $0.055$ $[0.50 < F/D_p < 2.00]$ Enter: $C_{pf} = 0.055$ Figure 9-16 Type of gearing: Open Commer. Precision Ex. Prec. Mesh Alignment Factor, $C_{ma} = 0.284$ $0.162$ $0.098$ $0.061$ Enter: $C_{ma} = 0.162$ Figure 9-17 Alignment Factor: $K_m = 1.22$ [Computed]	
<b>Secondary Input Data:</b> Transverse pressure angle: $\phi_t = 20.0$ deg Helix angle: $\psi = 25.0$ deg Axial Pitch: $P_x = 1.1229$ in Min. Face Width (2 x Axial Pitch): $F_{min} = 2.246$ in Enter: Face Width: $F = 2.250$ in Enter: Elastic Coefficient: $C_p = 2300$ Table 9-10 Enter: Quality Number: $A_v = 11$ Table 9-3 REF: $N_P, N_G = 21, 122$		Overload Factor: $K_o = 2.75$ Table 9-7 Size Factor: $K_s = 1.00$ Table 9-8: Use 1.00 if $P_d \geq 5$ Pinion Rim Thickness Factor: $K_{BP} = 1.00$ Fig. 9-18: Use 1.00 if solid blank Gear Rim Thickness Factor: $K_{BG} = 1.00$ Fig. 9-18: Use 1.00 if solid blank Dynamic Factor: $K_v = 1.27$ [Computed: See Fig. 9-20] Service Factor: $SF = 1.00$ Use 1.00 if no unusual conditions Reliability Factor: $K_R = 1.00$ Table 9-11 Use 1.00 for $R = .99$ Enter: Design Life: 8000 hours See Table 9-12 Pinion - Number of load cycles: $N_P = 2.2E+08$ Guidelines: $Y_N, Z_N$ Gear - Number of load cycles: $N_G = 3.7E+07$ $10^7$ cycles $> 10^8$ $< 10^7$	
<b>Enter: Bending Geometry Factors:</b> Pinion: $J_P = 0.444$ Fig 10-5,6,7 Gear: $J_G = 0.494$ Fig 10-5,6,7 Enter: Pitting Geometry Factor: $I = 0.240$ Tab. 10-1,2 REF: $m_G = 5.81$ Axial Force: $W_x = 746$ lb Radial Force: $W_r = 563$ lb		Bending Stress Cycle Factor: $Y_{NP} = 0.96$ Fig. 9-22 Bending Stress Cycle Factor: $Y_{NG} = 0.99$ Fig. 9-22 Pitting Stress Cycle Factor: $Z_{NP} = 0.93$ Fig. 9-24 Pitting Stress Cycle Factor: $Z_{NG} = 0.97$ Fig. 9-24 <b>Stress Analysis: Bending</b> Pinion: Required $s_{al} = 42,676$ psi See Fig. 9-11 or Gear: Required $s_{al} = 37,194$ psi Table 9-5 <b>Stress Analysis: Pitting</b> Pinion: Required $s_{ec} = 148,576$ psi See Fig. 9-12 or Gear: Required $s_{ec} = 142,449$ psi Table 9-5 <b>Specify materials, alloy and heat treatment, for most severe requirement.</b> <b>One possible material specification:</b> Steel pinion and gear. Through hardened Pinion requires HB 371: SAE 4340 OQT 900; HB 388 Gear requires HB 352: SAE 4340 OQT 1000; HB 363	



DESIGN OF HELICAL GEARS-U.S.		APPLICATION:	
<b>Initial Input Data:</b>		Electric power generator driven by a water turbine Problem 10-11 Use steel pinion with steel gear	
Input Power:	$P = 15$ hp		
Input Speed:	$n_P = 4500$ rpm		
Transverse Diametral Pitch, $P_d$ :	$P_d = 12$		
Number of Pinion Teeth:	$N_P = 20$		
Desired Output Speed:	$n_G = 3600$ rpm		
Computed number of gear teeth:	$25.0$		
Enter: Chosen No. of Gear Teeth:	$N_G = 25$		
<b>Computed data:</b>			
Actual Output Speed:	$n_G = 3600.0$ rpm		
Gear Ratio:	$m_G = 1.25$		
Pitch Diameter - Pinion:	$D_P = 1.667$ in		
Pitch Diameter - Gear:	$D_G = 2.083$ in		
Center Distance:	$C = 1.875$ in		
Pitch Line Speed:	$V_l = 1963$ ft/min		
Transmitted Load:	$W_l = 252$ lb		
<b>Secondary Input Data:</b>			
Transverse pressure angle:	$\phi_t = 20.0$ deg		
Helix angle:	$\psi = 25.0$ deg		
Axial Pitch:	$p_x = 0.5614$ in		
Min. Face Width (2 x Axial Pitch):	$F_{min} = 1.123$ in		
Enter: Face Width:	$F = 1.250$ in		
Enter: Elastic Coefficient:	$C_p = 2300$	Table 9-10	
Enter: Quality Number:	$A_v = 9$	Table 9-3	
REF: $N_P, N_G = 20, 25$			
<b>Enter: Bending Geometry Factors:</b>			
Pinion:	$J_P = 0.418$	Fig 10-5,6,7	
Gear:	$J_G = 0.432$	Fig 10-5,6,7	
Enter: Pitting Geometry Factor:	$I = 0.150$	Tab. 10-1,2	
REF: $m_G = 1.25$			
Axial Force:	$W_x = 118$ lb		
Radial Force:	$W_r = 92$ lb		
<b>Factors in Design Analysis:</b>			
Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$	$0.050$	IF $F < 1.0$	$0.053$
Pinion Proportion Factor, $C_{pf}$	$0.053$	IF $F > 1.0$	$0.053$
Enter: $C_{pf} = 0.053$		Figure 9-16	
Type of gearing:	Open	Commer.	Precision
Mesh Alignment Factor, $C_{ma}$	$0.268$	$0.147$	$0.083$
Enter: $C_{ma} = 0.147$		Figure 9-17	
Alignment Factor: $K_m = 1.20$	[Computed]		
Overload Factor: $K_o = 1.20$	Table 9-7		
Size Factor: $K_s = 1.00$	Table 9-8: Use 1.00 if $P_d \geq 5$		
Pinion Rim Thickness Factor: $K_{ap} = 1.00$	Fig. 9-18: Use 1.00 if solid blank		
Gear Rim Thickness Factor: $K_{ag} = 1.00$	Fig. 9-18: Use 1.00 if solid blank		
Dynamic Factor: $K_v = 1.96$	[Computed: See Fig. 9-20]		
Service Factor: $SF = 1.00$	Use 1.00 if no unusual conditions		
Reliability Factor: $K_R = 1.25$	Table 9-11 Use 1.00 for $R = .99$		
Enter: Design Life: $100000$ hours	See Table 9-12		
Pinion - Number of load cycles: $N_P = 2.7E+10$	Guidelines: $Y_N, Z_N$		
Gear - Number of load cycles: $N_G = 2.2E+10$	$10^7$ cycles $> 10^8$ $< 10^7$		
Bending Stress Cycle Factor: $Y_{NP} = 0.88$	1.00	0.88	Fig. 9-22
Bending Stress Cycle Factor: $Y_{NG} = 0.89$	1.00	0.89	Fig. 9-22
Pitting Stress Cycle Factor: $Z_{NP} = 0.83$	1.00	0.83	Fig. 9-24
Pitting Stress Cycle Factor: $Z_{NG} = 0.84$	1.00	0.84	Fig. 9-24
<b>Stress Analysis: Bending</b>			
Pinion: Required $s_{at} = 16,093$ psi	See Fig. 9-11 or		
Gear: Required $s_{at} = 15,397$ psi	Table 9-5		
<b>Stress Analysis: Pitting</b>			
Pinion: Required $s_{ec} = 137,623$ psi	See Fig. 9-12 or		
Gear: Required $s_{ec} = 135,985$ psi	Table 9-5		
<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>			
<b>One possible material specification:</b> Steel pinion and gear: Through hardened			
Pinion requires HB 337: SAE 4340 OQT 1000; HB 363			
Gear requires HB 332: SAE 4340 OQT 1000; HB 363			



**HELICAL GEARS** APPLICATION: Centrifugal pump driven by an electric motor

**POWER TRANSMISSION CAPACITY** Chapter 10-Problem 13 Used:  $A_v = 12$ ;  $L = 15000$  h

**Initial Input Data:**  
 Enter: Face Width:  $F = 2.500$  in  
 Input Speed:  $n_p = 1725$  rpm  
 Diametral Pitch:  $P_d = 9.659$   
 Number of Pinion Teeth:  $N_p = 20$   
 Number of Gear Teeth:  $N_g = 75$

**Computed data:**  
 Actual Output Speed:  $n_g = 460.0$  rpm  
 Gear Ratio:  $m_g = 3.75$   
 Pitch Diameter - Pinion:  $D_p = 2.071$  in  
 Pitch Diameter - Gear:  $D_g = 7.765$  in  
 Center Distance:  $C = 4.918$  in  
 Pitch Line Speed:  $v_l = 935$  ft/min  
 Transmitted Load at  $P_{min}$  Capacity:  $W_l = 1338$  lb

**Power Transmission Capacity: (Using Eq. 9-32, 9-34)**  
 Pinion: Based on Bending Stress:  $58.12$  hp  
 Gear: Based on Bending Stress:  $66.52$  hp  
 Pinion: Based on Contact Stress:  $37.94$  hp  
 Gear: Based on Contact Stress:  $40.54$  hp  
**Power Transmission Capacity:  $37.94$  hp**

Enter: Elastic Coefficient:  $C_p = 2300$  Table 9-10  
 Enter: Quality Number:  $A_v = 12$  Table 9-3  
 REF:  $N_p, N_g = 20, 75$

Enter: Bending Geometry Factors: Press. angle =  $20$  deg  
 Pinion:  $J_p = 0.465$  Fig. 10-5, 6, 7  
 Gear:  $J_g = 0.521$  Fig. 10-5, 6, 7  
 Enter: Pitting Geometry Factor:  $I = 0.200$  Tables 10-1, 2  
 REF:  $m_g = 3.75$

Enter:  $s_{at} = 55,000$  psi See Fig. 9-11 or Table 9-5  
 Gear:  $s_{at} = 55,000$  psi Table 9-5  
 Pinion:  $s_{ec} = 180,000$  psi See Fig. 9-12 or Table 9-5  
 Gear:  $s_{ec} = 180,000$  psi Table 9-5

**Material specification: Steel pinion+gear, Carburized case hard.**  
 Pinion material: SAE 4620 DOQT 300 62 HRC  
 Gear material: SAE 4620 DOQT 300 62 HRC

**Factors in Design Analysis:**  
 Alignment Factor,  $K_m = 1.0 + C_{pf} + C_{ma}$  If  $F < 1.0$  If  $F > 1.0$   $F/D_p = 1.21$   
 Pinion Proportion Factor,  $C_{pf} = 0.096$  0.114 [0.50 <  $F/D_p$  < 2.00]  
 Enter:  $C_{ma} = 0.114$  Figure 9-16  
 Type of gearing: Open Commer. Precision Ex. Prec.  
 Mesh Alignment Factor,  $C_{ma} = 0.288$  0.166 0.099 0.063  
 Enter:  $C_{ma} = 0.166$  Figure 9-17  
 Alignment Factor:  $K_m = 1.28$  [Computed]

Overload Factor:  $K_o = 1.25$  Table 9-7  
 Size Factor:  $K_s = 1.00$  Table 9-8: Use 1.00 if  $P_d \geq 5$   
 Pinion Rim Thickness Factor:  $K_{Rp} = 1.00$  Fig. 9-18: Use 1.00 if solid blank  
 Gear Rim Thickness Factor:  $K_{Rg} = 1.00$  Fig. 9-18: Use 1.00 if solid blank  
 Dynamic Factor:  $K_v = 1.50$  [Computed: See Fig. 9-20]  
 Service Factor:  $SF = 1.00$  Use 1.00 if no unusual conditions

Reliability Factor:  $K_R = 1.25$  Table 9-9 Use 1.00 for  $R = .99$   
 Enter: Design Life: 15000 hours See Table 9-7  
 Pinion - Number of load cycles:  $N_p = 1.6E+08$  Guidelines:  $Y_N, Z_N$   
 Gear - Number of load cycles:  $N_g = 4.1E+08$   $10^7$  cycles  $> 10^7$   $< 10^7$

Bending Stress Cycle Factor:  $Y_{NP} = 0.93$  1.00 0.93 Fig. 9-20  
 Bending Stress Cycle Factor:  $Y_{NG} = 0.95$  1.00 0.95 Fig. 9-20  
 Pitting Stress Cycle Factor:  $Z_{NP} = 0.89$  1.00 0.89 Fig. 9-22  
 Pitting Stress Cycle Factor:  $Z_{NG} = 0.92$  1.00 0.92 Fig. 9-22

**Allowable Bending Stress Numbers: (Input)**  
 Pinion:  $s_{at} = 55,000$  psi See Fig. 9-11 or Table 9-5  
 Gear:  $s_{at} = 55,000$  psi Table 9-5  
**Allowable Contact Stress Numbers: (Input)**  
 Pinion:  $s_{ec} = 180,000$  psi See Fig. 9-12 or Table 9-5  
 Gear:  $s_{ec} = 180,000$  psi Table 9-5

For  $K_v$ :  
 B 0.915  
 C 54.74

Through-Hardened  
 Grade 1 Steel  
 17.6 ksi Fig. 9-11  
 17.6 ksi Fig. 9-11  
 49.1 ksi Fig. 9-12  
 49.1 ksi Fig. 9-12

**BEVEL GEARS**

**Forces and torque for shaft and bearing load analysis:**

**On Pinion Shaft - Torque:**  $T_P = 630$  lb-in

Mean radius of pinion:  $r_m = 1.052$  in

*Enter:* pressure angle:  $\phi = 20$  degrees

On Pinion - Tangential load:  $W_{tP} = 598.7$  lb

On Pinion - Radial load:  $W_{rP} = 206.7$  lb

On Pinion - Axial load:  $W_{xP} = 68.9$  lb

**On Gear Shaft - Torque:**  $T_G = 1890.0$  lb-in

On Gear - Tangential load:  $W_{tG} = 598.7$  lb

On Gear - Radial load:  $W_{rG} = 68.9$  lb

On Gear - Axial load:  $W_{xG} = 206.7$  lb

**See following page for stress analysis and design details.**

**DESIGN OF BEVEL GEARS**

**APPLICATION:** Concrete mixer with moderate shock driven by a gasoline engine  
**Problem 10-14** Neither gear straddle mounted

**Initial Input Data:**  
 Input Power:  $P = 3$  hp  
 Input Speed:  $n_P = 300$  rpm  
 Diametral Pitch:  $P_d = 6$   
 Number of Pinion Teeth:  $N_P = 15$   
 Desired Output Speed:  $n_G = 100$  rpm  
 Computed number of gear teeth: 45.0  
 Enter: Chosen No. of Gear Teeth:  $N_G = 45$

**Computed data:**  
 Actual Output Speed:  $n_G = 100.0$  rpm  
 Gear Ratio:  $m_G = 3.00$   
 Pitch Diameter - Pinion:  $D_P = 2.500$  in  
 Pitch Diameter - Gear:  $D_G = 7.500$  in  
 Pitch cone angle - Pinion:  $\gamma = 18.43$  degrees  
 Pitch cone angle - Gear:  $\Gamma = 71.57$  degrees  
 Outer cone distance:  $A_o = 3.9528$  in  
 Pitch Line Speed:  $V_t = 196$  ft/min  
 Transmitted Load:  $W_t = 504$  lb

**Secondary Input Data:**

	Norm	Max	Max
Face Width Guidelines (in):	1.186	1.318	1.667
Enter: Face Width: $F =$	1.250 in		
Enter: Elastic Coefficient: $C_p =$	2300 Table 9-10		
Enter: Quality Number: $A_v =$	9 Table 9-3		

**Enter: Bending Geometry Factors:**  
 Pinion:  $J_P = 0.228$  Fig. 10-15  
 Gear:  $J_G = 0.190$  Fig. 10-15  
 Enter: Pitting Geometry Factor:  $I = 0.078$  Fig. 10-19

**Factors in Design Analysis:**  
 Load distribution factor,  $K_m$ : From Equation 10-18  
 Select from:  
 Both gears straddle mounted: 1.00  
 One gear straddle mounted: 1.10  
 Neither gear straddle mounted: 1.25  
 Enter Factor,  $K_{mb} = 1.25$   
 $K_m = 1.26$

Overload Factor:  $K_o = 2.00$  Table 9-7  
 Bending Size Factor:  $K_s = 0.52$  Figure 10-13 (for  $P_d < 16$ )  
 Dynamic Factor:  $K_v = 1.121$  Computed: Table 9-9  
 Pitting Size Factor:  $C_s = 0.59$  For  $0.60 < F < 3.14$   
 For  $F > 3.14$ ,  $C_s = 0.83$   
 Enter  $C_s = 0.59$

**Service Factor:  $S_F = 1.00$**  Use 1.00 if no unusual conditions  
 Bending Reliability Factor:  $K_R = 1.00$  Pitting:  $C_R = 1.00$  hours See Table 9-7

Pinion - Number of load cycles:  $N_P = 1.80E+07$   $Y_{N, Z_N}$  Pinion-Fig. 10-18; Gear-Fig. 10-20  
 Gear - Number of load cycles:  $N_G = 6.00E+06$   $10^{-7} < N < 10^7$   $N > 10^7$   
 Bending Stress Cycle Factor:  $K_L = 1.01$  2.70 0.84 1.01  
 Bending Stress Cycle Factor:  $K_L = 0.96$  2.70 0.96 1.03  
 Pitting Stress Cycle Factor:  $C_L = 1.27$   $N < 10^6$   $N > 10^6$   
 Pitting Stress Cycle Factor:  $C_L = 1.36$  2.00 1.27  
 2.00 1.36

**Stress Analysis - Bending:**  
 Pinion: Required  $s_{at} = 15,447$  psi See Fig. 10-17 or  
 Gear: Required  $s_{at} = 19,502$  psi Table 10-4  
**Stress Analysis - Pitting:** Assumes  $C_{xc} = 1.5$  for properly crowned teeth  
 Pinion: Required  $s_{wc} = 129,992$  psi See Fig. 10-21 or  
 Gear: Required  $s_{wc} = 121,389$  psi Table 10-4

**Specify materials, alloy and heat treatment, for most severe requirement.**  
 One possible material specification:  
 Pinion: HB 312 required: SAE 6150 OQT 1100; HB = 341  
 Gear: HB 395 required: SAE 6150 OQT 900; HB = 401

For  $K_v$   
 $B = 0.630$   
 $C = 70.7$

For $R =$	$K_R$	$C_R$
0.9	0.85	0.62
0.99	1.00	1.00
0.999	1.25	1.12
0.9999	1.50	1.22

Through Hardened  
 Grade 1 Steel  
 HB 303 Fig. 10-17  
 HB 395 Fig. 10-17  
 HB 312 Fig. 10-21  
 HB 287 Fig. 10-21



**BEVEL GEARS**

**Forces and torque for shaft and bearing load analysis:**

**On Pinion Shaft - Torque:**  $T_P = 176.4$  lb-in

Mean radius of pinion:  $r_m = 1.093$  in

*Enter:* pressure angle:  $\phi = 20$  degrees

On Pinion - Tangential load:  $W_{tP} = 161.3$  lb

On Pinion - Radial load:  $W_{rP} = 52.5$  lb

On Pinion - Axial load:  $W_{xP} = 26.3$  lb

**On Gear Shaft - Torque:**  $T_G = 352.8$  lb-in

On Gear - Tangential load:  $W_{tG} = 161.3$  lb

On Gear - Radial load:  $W_{rG} = 26.3$  lb

On Gear - Axial load:  $W_{xG} = 52.5$  lb

**See following page for stress analysis and design details.**

DESIGN OF BEVEL GEARS		APPLICATION:	
<b>Initial Input Data:</b>		<b>Factors in Design Analysis:</b>	
Input Power: $P = 3.5$ hp		Neither gear straddle mounted	
Input Speed: $n_p = 1250$ rpm		From Equation 10-16	
Diametral Pitch: $P_d = 10$		Factor, $K_{mb}$	
Number of Pinion Teeth: $N_p = 25$		1.00	
Desired Output Speed: $n_g = 625$ rpm		1.10	
		1.25	
Computed number of gear teeth: 50.0		1.25	
Enter: Chosen No. of Gear Teeth: $N_g = 50$		$K_m = 1.25$	
<b>Computed data:</b>		Table 9-7	
Actual Output Speed: $n_g = 625.0$ rpm		Figure 10-13 (for $P_d < 16$ )	
Gear Ratio: $m_g = 2.00$		Computed: Table 9-9	
Pitch Diameter - Pinion: $D_p = 2.500$ in		For $0.50 < F < 3.14$	
Pitch Diameter - Gear: $D_g = 5.000$ in		For $F > 3.14$ , $C_s = 0.83$	
Pitch cone angle - Pinion: $\gamma = 26.57$ degrees			
Pitch cone angle - Gear: $\Gamma = 63.43$ degrees			
Outer cone distance: $A_o = 2.7951$ in			
Pitch Line Speed: $v_t = 818$ ft/min			
Transmitted Load: $W_t = 141$ lb			
<b>Secondary Input Data:</b>		See Table 9-7	
Face Width Guidelines (in):	Norm Max		
Enter: Face Width: $F = 0.839$	0.832		
Enter: Elastic Coefficient: $C_p = 2300$	Table 9-10		
Enter: Quality Number: $A_v = 10$	Table 9-3		
<b>Enter: Bending Geometry Factors:</b>			
Pinion: $J_p = 0.258$	Fig. 10-15		
Gear: $J_g = 0.220$	Fig. 10-15		
<b>Enter: Pitting Geometry Factor:</b>			
$I = 0.083$	Fig. 10-19		
<p><b>Problem 10-15</b> Conveyor with moderate shock driven by a gasoline engine</p> <p>Neither gear straddle mounted</p>			
Load distribution factor, $K_m$ :		From Equation 10-16	
Select from:		Factor, $K_{mb}$	
Both gears straddle mounted:		1.00	
One gear straddle mounted:		1.10	
Neither gear straddle mounted:		1.25	
Enter Factor, $K_{mb}$		1.25	
$K_m = 1.25$			
Overload Factor: $K_o = 2.00$		Table 9-7	
Bending Size Factor: $K_s = 0.51$		Figure 10-13 (for $P_d < 16$ )	
Dynamic Factor: $K_v = 1.305$		Computed: Table 9-9	
Pitting Size Factor: $C_s = 0.53$		For $0.50 < F < 3.14$	
For $F > 3.14$ , $C_s = 0.83$			
Service Factor: $S_F = 1.00$		Use 1.00 if no unusual conditions	
Reliability Factor: $K_R = 1.00$		Pitting: $C_R = 1.00$	
Bending Reliability Factor: $K_R = 1.00$		hours	
Enter: Design Life: 15000		See Table 9-7	
Pinion - Number of load cycles: $N_p = 1.13E+09$		$Y_N, Z_N, Pinion-Fig. 10-18, Gear-Fig. 10-20$	
Gear - Number of load cycles: $N_g = 5.63E+08$		$N < 10^3$ $10^3 < N \leq 10^7$ $N > 10^7$	
Bending Stress Cycle Factor: $K_L = 0.94$		2.70 0.51 0.94	
Bending Stress Cycle Factor: $K_L = 0.95$		2.70 0.56 0.95	
Pitting Stress Cycle Factor: $C_L = 1.27$		$N < 10^3$ $N > 10^3$	
Pitting Stress Cycle Factor: $C_L = 1.36$		2.00 0.99	
		2.00 1.04	
<b>Stress Analysis - Bending:</b>		Through Hardened	
Pinion: Required $s_w = 13,808$ psi		Grade 1 Steel	
Gear: Required $s_w = 16,023$ psi		HB 266 Fig. 10-17	
<b>Stress Analysis - Pitting:</b> Assumes $C_{xc} = 1.5$ for properly crowned teeth		HB 316 Fig. 10-17	
Pinion: Required $s_{wc} = 91,011$ psi		HB 198 Fig. 10-21	
Gear: Required $s_{wc} = 84,988$ psi		HB 180 Fig. 10-21	
Specify materials, alloy and heat treatment, for most severe requirement.			
One possible material specification:			
Pinion: HB 266 required; SAE 6150 OQT 1200; HB = 293			
Gear: HB 316 required; SAE 6150 OQT 1100; HB = 341			

Problem 10-16

**BEVEL GEARS**

**Forces and torque for shaft and bearing load analysis:**

**On Pinion Shaft - Torque:**  $T_P = 370.59$  lb-in

Mean radius of pinion:  $r_m = 0.938$  in

*Enter:* pressure angle:  $\phi = 20$  degrees

On Pinion - Tangential load:  $W_{tP} = 395.2$  lb

On Pinion - Radial load:  $W_{rP} = 135.6$  lb

On Pinion - Axial load:  $W_{xP} = 47.9$  lb

**On Gear Shaft - Torque:**  $T_G = 1050.0$  lb-in

On Gear - Tangential load:  $W_{tG} = 395.2$  lb

On Gear - Radial load:  $W_{rG} = 47.9$  lb

On Gear - Axial load:  $W_{xG} = 135.6$  lb

**See following page for stress analysis and design details.**

DESIGN OF BEVEL GEARS		APPLICATION:	
<b>Initial Input Data:</b>		Input Power: $P = 5$ hp	
		Input Speed: $n_p = 850$ rpm	
		Diametral Pitch: $P_d = 8$	
		Number of Pinion Teeth: $N_p = 18$	
		Desired Output Speed: $n_g = 300$ rpm	
		Computed number of gear teeth: 51.0	
		Enter: Chosen No. of Gear Teeth: $N_g = 51$	
<b>Computed data:</b>			
		Actual Output Speed: $n_g = 300.0$ rpm	
		Gear Ratio: $m_g = 2.83$	
		Pitch Diameter - Pinion: $D_p = 2.250$ in	
		Pitch Diameter - Gear: $D_g = 6.375$ in	
		Pitch cone angle - Pinion: $\gamma = 19.44$ degrees	
		Pitch cone angle - Gear: $\Gamma = 70.56$ degrees	
		Outer cone distance: $A_o = 3.3802$ in	
		Pitch Line Speed: $v_t = 501$ ft/min	
		Transmitted Load: $W_t = 330$ lb	
<b>Secondary Input Data:</b>			
		Face Width Guidelines (in):	Norm Max Max
		Enter: Face Width: $F = 1.014$	1.127 1.250
		Enter: Elastic Coefficient: $C_p = 2300$	Table 9-10
		Enter: Quality Number: $A_v = 10$	Table 9-3
<b>Enter: Bending Geometry Factors:</b>			
		Pinion: $J_p = 0.240$	Fig. 10-15
		Gear: $J_g = 0.200$	Fig. 10-15
<b>Enter: Pitting Geometry Factor:</b>		$I = 0.081$	Fig. 10-19
<b>Problem 10-16</b>		<b>Both gears straddle mounted</b>	
<b>Conveyor with heavy shock driven by a gasoline engine</b>			
<b>Factors in Design Analysis:</b>		From Equation 10-16	
Load distribution factor, $K_m$ :		Factor, $K_{mb}$	
Select from:		Both gears straddle mounted: 1.00	
		One gear straddle mounted: 1.10	
		Neither gear straddle mounted: 1.25	
Enter Factor, $K_{mb}$		1.00	
$K_m = 1.00$			
Overload Factor: $K_o = 2.00$		Table 9-7	
Bending Size Factor: $K_s = 0.51$		Figure 10-13 (for $P_d < 16$ )	
Dynamic Factor: $K_v = 1.241$		Computed: Table 9-9	
Pitting Size Factor: $C_s = 0.58$		For $0.50 < F < 3.14$	
For $F < 0.50$ , $C_s = 0.5$		For $F > 3.14$ , $C_s = 0.83$	
Enter $C_s = 0.58$			
Service Factor: $S_F = 1.00$		Use 1.00 if no unusual conditions	
Bending Reliability Factor: $K_R = 1.00$		Pitting: $C_R = 1.00$	
Enter: Design Life: 15000 hours		See Table B-7	
Pinion - Number of load cycles: $N_p = 7.65E+08$		$Y_N, Z_N$ : Pinion-Fig. 10-16; Gear-Fig. 10-20	
Gear - Number of load cycles: $N_g = 2.70E+08$		$N < 10^3$ $10^3 < N < 10^7$ $N > 10^7$	
Bending Stress Cycle Factor: $K_L = 0.94$		2.70 0.54 0.94	
Bending Stress Cycle Factor: $K_L = 0.96$		2.70 0.61 0.96	
Enter: $N < 10^3$		$N > 10^3$	
Pitting Stress Cycle Factor: $C_L = 1.02$		2.00 1.02	
Pitting Stress Cycle Factor: $C_L = 1.08$		2.00 1.08	
<b>Stress Analysis - Bending:</b>			
Pinion: Required $s_{at} = 13,300$ psi		See Fig. 10-17 or	
Gear: Required $s_{at} = 15,628$ psi		Table 10-4	
<b>Stress Analysis - Pitting:</b> Assumes $C_{xc} = 1.5$ for properly crowned teeth			
Pinion: Required $s_{pc} = 133,166$ psi		See Fig. 10-21 or	
Gear: Required $s_{pc} = 125,768$ psi		Table 10-4	
<b>Specify materials, alloy and heat treatment, for most severe requirement.</b>			
<b>One possible material specification:</b>			
Pinion: HB 321 required: SAE 6150 OQT 1100; HB = 341		Fig. 10-21	
Gear: HB 307 required: SAE 6150 OQT 1100; HB = 341		Fig. 10-21	

**BEVEL GEARS**

**Forces and torque for shaft and bearing load analysis:**

<b>On Pinion Shaft - Torque:</b>	$T_P =$	26.25	lb-in
Mean radius of pinion:	$r_m =$	0.386	in
<i>Enter:</i> pressure angle:	$\phi =$	20	degrees
On Pinion - Tangential load:	$W_{tP} =$	68.0	lb
On Pinion - Radial load:	$W_{rP} =$	23.9	lb
On Pinion - Axial load:	$W_{xP} =$	6.3	lb
<b>On Gear Shaft - Torque:</b>	$T_G =$	99.2	lb-in
On Gear - Tangential load:	$W_{tG} =$	68.0	lb
On Gear - Radial load:	$W_{rG} =$	6.3	lb
On Gear - Axial load:	$W_{xG} =$	23.9	lb
<b>See following page for stress analysis and design details.</b>			



**DESIGN OF BEVEL GEARS** **APPLICATION:**

**Initial Input Data:**  
 Input Power:  $P = 0.75$  hp  
 Input Speed:  $n_P = 1800$  rpm  
 Diametral Pitch:  $P_d = 20$   
 Number of Pinion Teeth:  $N_P = 18$   
 Desired Output Speed:  $n_G = 475$  rpm  
 Computed number of gear teeth: 68.2  
 Enter: Chosen No. of Gear Teeth:  $N_G = 68$

**Computed data:**  
 Actual Output Speed:  $n_G = 476.5$  rpm  
 Gear Ratio:  $m_G = 3.78$   
 Pitch Diameter - Pinion:  $D_P = 0.900$  in  
 Pitch Diameter - Gear:  $D_G = 3.400$  in  
 Pitch cone angle - Pinion:  $\gamma = 14.83$  degrees  
 Pitch cone angle - Gear:  $\Gamma = 75.17$  degrees  
 Outer cone distance:  $A_o = 1.7586$  in

**Secondary Input Data:**

	Norm	Max	Max
Face Width Guidelines (in):	0.528	0.586	0.500
Enter: Face Width: $F =$	0.500 in		
Enter: Elastic Coefficient: $C_p =$	2300 Table 9-10		
Enter: Quality Number: $A_v =$	11 Table 9-3		

**Enter: Bending Geometry Factors:**  
 Pinion:  $J_P = 0.250$  Fig. 10-15  
 Gear:  $J_G = 0.210$  Fig. 10-15  
 Enter: Pitting Geometry Factor:  $I = 0.085$  Fig. 10-19

**Reciprocating saw driven by an electric motor**  
**Problem 10-17** **Both gears straddle mounted**

**Factors in Design Analysis:**  
 Load distribution factor,  $K_m$ :  
 Select from:  
 Both gears straddle mounted: 1.00  
 One gear straddle mounted: 1.10  
 Neither gear straddle mounted: 1.25  
 Enter Factor,  $K_{mb} = 1.00$   
 From Equation 10-16  
 Factor,  $K_{mb} = 1.00$

Overload Factor:  $K_o = 1.75$  Table 9-7  
 Bending Size Factor:  $K_s = 0.50$  Figure 10-13 (for  $P_d < 16$ )  
 Dynamic Factor:  $K_v = 1.277$  Computed: Table 9-9  
 Pitting Size Factor:  $C_p = 0.50$  For  $0.50 < F < 3.14$   
 For  $F > 3.14$ ,  $C_p = 0.83$

Service Factor:  $S_F = 1.00$  Use 1.00 if no unusual conditions  
 Reliability Factor:  $K_R = 1.00$  Pitting:  $C_R = 1.00$   
 Enter: Design Life: 15000 hours See Table 9-7

Pinion - Number of load cycles:  $N_P = 1.62E+09$   
 Gear - Number of load cycles:  $N_G = 4.29E+08$   
 Bending Stress Cycle Factor:  $K_L = 0.93$   
 Bending Stress Cycle Factor:  $K_L = 0.95$   
 Pitting Stress Cycle Factor:  $C_L = 0.97$   
 Pitting Stress Cycle Factor:  $C_L = 1.05$

**Stress Analysis - Bending:**  
 Pinion: Required  $s_{at} = 11,171$  psi See Fig. 10-17 or  
 Gear: Required  $s_{at} = 13,018$  psi Table 10-4

**Stress Analysis - Pitting:** Assumes  $C_{xc} = 1.5$  for properly crowned teeth  
 Pinion: Required  $s_{ac} = 119,964$  psi See Fig. 10-21 or  
 Gear: Required  $s_{ac} = 110,824$  psi Table 10-4

**Specify materials, alloy and heat treatment, for most severe requirement.**  
 One possible material specification:  
 Pinion: HB 283 required; SAE 6150 OQT 1200; HB = 293  
 Gear: HB 256 required; SAE 6150 OQT 1200; HB = 293

For $R =$	$K_R$	$C_R$
0.9	0.85	0.62
0.99	1.00	1.00
0.999	1.25	1.12
0.9999	1.50	1.22

Through Hardened	Grade 1 Steel
HB 208	Fig. 10-17
HB 248	Fig. 10-17
HB 283	Fig. 10-21
HB 256	Fig. 10-21

WORMGEARING DATA FROM PROB. 8-52:  $T_o = 924 \text{ LB}\cdot\text{IN.}$ ,  $n_g = 30 \text{ RPM}$

FORCES:  $W_{tG} = W_{xW} = T_o / (D_g/2) = 924 \text{ LB}\cdot\text{IN.} / 2.00 \text{ IN.} = 462 \text{ LB}$

PITCH LINE SPEED OF GEAR  $= N_{tG} = \pi D_g n_g / 12 = \pi (4.00 \times 30) / 12 = 31.4 \text{ FT/MIN.}$

SLIDING VELOCITY  $= N_s = N_{tG} / \sin \lambda = 31.4 / \sin(4.5^\circ) = 394 \text{ FT/MIN.}$

FROM FIG. 10-25;  $\mu = 0.0323$  [COMPUTED FROM EQ. 10-27]

$W_{xG} = W_{tW} = 462 \times \frac{\cos(4.5^\circ) \sin(4.57^\circ) + 0.0323 \cos(4.57^\circ)}{\cos(4.5^\circ) \cos(4.57^\circ) - 0.0323 \sin(4.57^\circ)} = 53 \text{ LB}$   
(EQ. 10-30)

$W_{rG} = W_{rW} = \frac{462 \cdot \sin(4.5^\circ)}{\cos(4.5^\circ) \cos(4.57^\circ) - 0.0323 \sin(4.57^\circ)} = 120 \text{ LB}$   
(EQ. 10-31)

FRICION FORCE  $= W_f = \frac{(0.0323)(462)}{\cos(4.57^\circ) \cos(4.5^\circ) - 0.0323 \sin(4.57^\circ)} = 15.5 \text{ LB}$   
(EQ. 10-32)

FRICION POWER LOSS  $= P_L = \frac{N_s W_f}{33000} = \frac{(394)(15.5)}{33000} = 0.185 \text{ HP}$

INPUT POWER  $= P_{iH} = P_o + P_L = \frac{T_o n_g}{63000} + 0.185 = 0.44 + 0.185 = 0.625 \text{ HP}$

EFFICIENCY  $= P_o / P_{iH} \times 100\% = 70.4\%$ ; INPUT SPEED  $= n_g \cdot V_R = (30)(40) = 1200 \text{ RPM}$

STRESS  $\sigma_G = \frac{W_d}{K_v F_p m} = \frac{W_{tG} Q}{K_v n_g F \pi \cos \lambda} = \frac{(462)(10)}{(0.974)(0.100)(0.625)(\pi)(\cos 4.5^\circ)} = 24235 \text{ PSI}$

$K_v = 1200 / (1200 + N_{tG}) = 1200 / (1200 + 31.4) = 0.974$

PITTING  $W_{tR} = C_s D_g^{0.8} F_e C_m C_v = (1000)(4.00)^{0.8} (0.625)(0.814)(0.427) = 659 \text{ LB}$   
(FK 12-21)

BECAUSE  $W_{tR} > W_{tG}$  - OK FOR PITTING.

$\sigma_G = 24235 \text{ PSI}$  SLIGHTLY HIGHER THAN  $S_{ax} = 24000 \text{ PSI}$  FOR PHOSPHOR BRONZE

(SEE SPREADSHEET SOLUTION ON FOLLOWING PAGE.)

Wormgearing - Design		Problem: 10-18	
<b>Input Data:</b>			
Desired output torque:	$T_o =$	924 lb-in	
Output speed:	$n_g =$	30 rpm	
Velocity Ratio:	$VR =$	40	
<b>Design Decisions:</b>			
Diametral pitch:	$P_d =$	10	
No. of worm threads:	$N_w =$	1	
Required No. of gear teeth:	$N_g =$	40	
Specify No. of gear teeth:	$N_g =$	40	
Normal pressure angle:	$\phi_n =$	14.5 degrees	
<b>Computed Results and Additional Inputs:</b>			
Actual input speed:	$n_w =$	1200 rpm	
Actual velocity ratio:	$VR =$	40	
Gear pitch diameter:	$D_g =$	4 in	
Specify worm diameter:	$D_w =$	1.25 in	
Actual center distance:	$C =$	2.625 in	
	$C^{0.875}/D_w =$	1.86	
	Should be $>1.6$ and $\leq 3.0$		
Circular pitch of gear:	$P_g =$	0.314 in	
Axial pitch of worm:	$p_{xw} =$	0.314 in	
Lead of the worm:	$L =$	0.314 in	
Lead angle:	$\lambda =$	4.574 deg	
Addendum:	$a =$	0.100 in	
Dedendum:	$b =$	0.116 in	
Worm outside diameter:	$D_{ow} =$	1.450 in	
Worm root diameter:	$D_{rw} =$	1.019 in	
Nominal worm face length:	$F_{fnom} =$	1.789 in	
Gear throat diameter:	$D_{tg} =$	4.200 in	
Nominal gear face width:	$F_{eg} =$	0.735 in	
Max effective gear face width:	$0.67 \cdot D_w =$	0.8375 in	
Effective gear face width:	$F_o =$	0.625 in	
	<i>(Used given face width)</i>		
<b>Additional Computed Results:</b>			
Pitch line speed - Gear:	31.42 ft/min		
Sliding velocity $v_s =$	394 ft/min		
Coefficient of friction:	0.032 if $v_s > 10$ ft/min		
<b>Forces: (lb)</b>	Gear	Worm	
Tangential:	462	53	
Radial:	120	120	
Axial:	53	462	
Friction force, $W_f =$	15.6 lb		
<b>Power:</b>			
Power output from gear:	0.440 hp		
Power loss - friction:	0.186 hp		
Power Input:	0.626 hp		
<b>Efficiency:</b>	70.3 %		
<b>Stresses:</b>			
Normal pressure angle, $\phi_n$	14.5	20	25
30			
<b>Bending Stress on Gear:</b>			
Enter: Lewis form factor: $y =$	0.100	----->	
Normal circular pitch:	0.313 in		
Dynamic factor: $K_v =$	0.974		
<b>Bending stress on gear:</b>	24223 psi [Using effective gear face width]		
Allowable stresses-Bronze: Manganese = 17000 psi; Phosphor = 24000 psi			
<b>Surface Durability: [Hardened steel worm; bronze gear]</b>			
Type of bronze/ $D_g \rightarrow$	$<2.5$ in	$>2.5$ in	$>8$ in
	$<8$ in	$>8$ in	$<25$ in
Sand cast: $C_s =$	903	1000	
Chill cast or forged: $C_s =$			1137
Centrifugally cast: $C_s =$			1000
Enter: Materials factor: $C_s =$	1000		
Gear Ratio: $m_g =$	6 to 20	20 to 76	$>76$ Actual $m_g = 40$
Ratio correction factor: $C_m =$	#NUM!	0.814	0.685
Enter: $C_m =$	0.814		
Sliding velocity:	$<700$	$700-3000$	$>3000$ Actual $v_s = 394$
Velocity factor: $C_v =$	0.427	0.439	0.642
Enter: $C_v =$	0.427		
<b>Rated tangential load: <math>W_{TR} =</math></b>	659 lb		
<b>Must be <math>&gt; W_f =</math></b>	462 lb		
3. Equation for $C_m$ produces invalid result for large gear ratio as $VR = 40$			

Notes: 1. Bending stress on gear slightly high  
2. Suggest using larger face width; Say  $F = F_o = 0.75$  in.

Wormgearing - Design		Problem: 10-18A Adjusted $F_o = 0.750$ in	
<b>Input Data:</b>			
Desired output torque:	$T_o =$	924 lb-in	
Output speed:	$n_o =$	30 rpm	
Velocity Ratio:	$VR =$	40	
<b>Design Decisions:</b>			
Diametral pitch:	$P_d =$	10	
No. of worm threads:	$N_w =$	1	
Required No. of gear teeth:	$N_g =$	40	
Specify No. of gear teeth:	$N_g =$	40	
Normal pressure angle:	$\phi_n =$	14.5 degrees	
<b>Computed Results and Additional Inputs:</b>			
Actual input speed:	$n_w =$	1200 rpm	
Actual velocity ratio:	$VR =$	40	
Gear pitch diameter:	$D_g =$	4 in	
Specify worm diameter:	$D_w =$	1.25 in	
Actual center distance:	$C =$	2.625 in	
	$C^{0.875}/D_w =$	1.86	
		Should be $>1.6$ and $<3.0$	
Circular pitch of gear:	$P_g =$	0.314 in	
Axial pitch of worm:	$P_{xw} =$	0.314 in	
Lead of the worm:	$L =$	0.314 in	
Lead angle:	$\lambda =$	4.574 deg	
Addendum:	$a =$	0.100 in	
Dedendum:	$b =$	0.116 in	
Worm outside diameter:	$D_{oW} =$	1.450 in	
Worm root diameter:	$D_{rW} =$	1.019 in	
Nominal worm face length:	$F_{wnom} =$	1.789 in	
Gear throat diameter:	$D_{tG} =$	4.200 in	
Nominal gear face width:	$F_{oG} =$	0.735 in	
Max effective gear face width:	$0.67 \cdot D_w =$	0.8375 in	
Effective gear face width:	$F_o =$	0.750 in	
		(Used given face width)	
<b>Additional Computed Results:</b>			
Pitch line speed - Gear:		31.42 ft/min	
Sliding velocity $v_s =$		394 ft/min	
Coefficient of friction:		0.032 if $v_s > 10$ ft/min	
<b>Forces: (lb)</b>			
	Gear	Worm	
Tangential:	462	53	
Radial:	120	120	
Axial:	53	462	
Friction force, $W_f =$	15.6 lb		
<b>Power:</b>			
Power output from gear:	0.440 hp		
Power loss - friction:	0.186 hp		
Power input:	0.626 hp		
Efficiency:	70.3 %		
<b>Stresses:</b>			
<b>Bending Stress on Gear:</b>			
Enter: Lewis form factor: $y =$	0.100	----->	Normal pressure angle, $\phi_n$
			14.5 20 25 30
Normal circular pitch:	0.313 in		Lewis form factor, $y$
Dynamic factor: $K_v =$	0.974		0.100 0.125 0.150 0.175
Bending stress on gear:	20186 psi [Using effective gear face width]		
Allowable stresses-Bronze: Manganese = 17000 psi; Phosphor = 24000 psi			
<b>Surface Durability: [Hardened steel worm; bronze gear]</b>			
Type of bronze/ $D_g \rightarrow$	$>2.5$ in	$<2.5$ in	$>8$ in
	$<8$ in	$>25$ in	$<25$ in
Sand cast: $C_s =$	903	1000	
Chill cast or forged: $C_s =$			1137 1000
Centrifugally cast: $C_s =$			1143 1000
Enter: Materials factor: $C_s =$	1000		
Gear Ratio: $m_g =$	6 to 20	20 to 76	$>76$ Actual $m_g = 40$
Ratio correction factor: $C_m =$	#NUM!	0.814	0.885
Enter: $C_m =$	0.814		
Sliding velocity:	$<700$	700-3000	$>3000$ Actual $v_s = 394$
Velocity factor: $C_v =$	0.427	0.439	0.642
Enter: $C_v =$	0.427		
Rated tangential load: $W_{tR} =$	790 lb		
Must be $> W_t =$	462 lb		
3. Equation for $C_m$ produces invalid result for large gear ratio as $VR = 40$			

Notes: 1. Bending stress on gear OK for Phosphor Bronze  
2. Using  $F = F_o = 0.75$  in.



Problems 10-19 and 10-20

**COMPARISON OF THREE PROPOSED DESIGNS**

See details on following three spreadsheets

**Given data:**

Diametral pitch, $P_d =$	12	
Velocity ratio, $VR =$	20	
Output speed (Gear) =	90 rpm	
Worm pitch diameter, $D_w =$	1.000 in	See comment
Gear face width, $F =$	0.500 in	See comment
Normal pressure angle, $\phi_n =$	14.5 degrees	

Assumed gear is made from chilled cast phosphor bronze

Allowable bending stress = 24,000 psi

**Results:**

**DESIGN**

	A	B	C	
<b>Number of threads in worm:</b>	<b>1</b>	<b>2</b>	<b>4</b>	
Output torque (lb-in), $T_o =$	202	484	878	
Output power (hp), $P_o =$	0.289	0.691	1.254	
Gear bending stress (psi)	19190	<b>23963</b>	<b>23987</b>	Limits in <b>Bold</b>
Allowable bending stress (psi)	24000	24000	24000	
Rated load for surface durability (lb)	<b>242</b>	418	714	Limits in <b>Bold</b>
Gear transmitted load (lb)	242	290	263	
Efficiency (%) [Problem 20]	72.9	84.1	90.8	
Power input (hp)	0.396	0.822	1.381	
Lead angle (degrees)	4.76	9.46	18.4	
Gear pitch diameter (in)	1.667	3.333	6.667	
Center distance (in)	1.333	2.167	3.833	

**Comments on results:**

The given face width is small. Could use  $F > 0.601$  in to maximize effective face width.

Worm diameter is too large for Design A. See Equations 10-46 and 10-47

Worm diameter is too small for Design C. See Equations 10-46 and 10-47

Design A is limited by surface durability

Designs B and C are limited by bending stress in gear teeth.

**As number of threads in worm increases:**

Lead angle increases

Efficiency increases

Torque and power capacity increase

**BUT:** Gear size and center distance increase



**Wormgearing - Design**

Problem: 16-156 and 20a

**Input Data:**

Desired output torque:  $T_o = 202$  lb-in  
 Output speed:  $n_o = 90$  rpm  
 Velocity Ratio:  $VR = 20$

**Design Decisions:**

Diametral pitch:  $P_d = 12$   
 No. of worm threads:  $N_w = 1$

Required No. of gear teeth:  $N_g = 20$   
 Specify No. of gear teeth:  $N_g = 20$   
 Normal pressure angle:  $\phi_n = 14.5$  degrees

**Computed Results and Additional Inputs:**

Actual input speed:  $n_w = 1800$  rpm  
 Actual velocity ratio:  $VR = 20$   
 Gear pitch diameter:  $D_g = 1.66667$  in  
 Specify worm diameter:  $D_w = 1.000$  in  
 Actual center distance:  $C = 1.333$  in  
 $C^{0.875}/D_w = 1.28$  **LOW**  
 Use smaller worm diameter Should be  $>1.6$  and  $<3.0$

Circular pitch of gear:  $p_g = 0.262$  in  
 Axial pitch of worm:  $p_{aw} = 0.262$  in  
 Lead of the worm:  $L = 0.262$  in  
 Lead angle:  $\lambda = 4.764$  deg  
 Addendum:  $a = 0.083$  in  
 Dedendum:  $b = 0.096$  in  
 Worm outside diameter:  $D_{ow} = 1.167$  in  
 Worm root diameter:  $D_{rw} = 0.807$  in  
 Nominal worm face length:  $F_{whom} = 1.054$  in  
 Gear throat diameter:  $D_{tg} = 1.833$  in  
 Nominal gear face width:  $F_{ag} = 0.601$  in  
 Max effective gear face width:  $0.67 \cdot D_w = 0.670$  in  
 Effective gear face width:  $F_e = 0.500$  in  
 [Lead given face width]  
 Given face width is small; Could use  $F > 0.601$

**Additional Computed Results:**

Pitch line speed - Gear:  $39.27$  ft/min  
 Sliding velocity  $v_s = 473$  ft/min  
 Coefficient of friction:  $0.030$  if  $v_s > 10$  ft/min

**Forces: (lb)**

Tangential: 242  
 Radial: 63  
 Axial: 28

Friction force,  $W_f = 7.5$  lb

**Power:**

Power output from gear:  $0.289$  hp  
 Power loss - friction:  $0.107$  hp  
 Power input:  $0.396$  hp

**Efficiency:**

$72.9\%$

**Stresses:**

Normal pressure angle,  $\phi_n$   
 14.5 20 25 30

**Bending Stress on Gear:**

Enter Lewis form factor  $y = 0.100$  →  
 Lewis form factor,  $y$   
 0.100 0.125 0.150 0.175

Normal circular pitch:  $0.261$  in  
 Dynamic factor:  $K_v = 0.968$

Bending stress on gear:  $19190$  psi [Using effective gear face width]  
 Allowable stresses-Bronze: Manganese =  $17000$  psi; Phosphor =  $24000$  psi

**Surface Durability: [Hardened steel worm; bronze gear]**

Type of bronze/ $D_g$  →  $>2.5$  in  $<2.5$  in  $<8$  in  $>25$  in  $<25$  in

Sand cast:  $C_s = 1084$  1000  
 Chill cast or forged:  $C_s = 1311$  1000  
 Centrifugally cast:  $C_s = 1211$  1000

Enter Material factor  $C_m = 1000$

Gear Ratio:  $m_g = 6$  to  $20$   $20$  to  $76$   $>76$  Actual  $m_g = 20$

Ratio correction factor:  $C_m = 0.820$   $0.819$   $1.017$

Enter  $C_w = 0.819$

Sliding velocity:  $<700$   $700$ - $3000$   $>3000$  Actual  $v_s = 473$

Velocity factor:  $C_v = 0.392$   $0.395$   $0.557$

Enter  $C_p = 0.392$

Rated tangential load:  $W_{st} = 242$  lb  
 Must be  $> W_f = 242$  lb

Adjusted output torque until limits reached on either bending or surface durability  
 Surface durability controls this design

**Wormgearing - Design**

Problem: 10-19b and 20b

**Input Data:**

Measured output torque:  $T_g = 494$  lb-in  
 Output speed:  $n_g = 90$  rpm  
 Velocity Ratio:  $VR = 20$

**Design Decisions:**

Diametral pitch:  $P_d = 12$   
 No. of worm threads:  $N_w = 2$   
 Required No. of gear teeth:  $N_g = 40$   
 Specify No. of gear teeth:  $N_g = 40$   
 Normal pressure angle:  $\phi_n = 14.5$  degrees

**Computed Results and Additional Inputs:**

Actual input speed:  $n_w = 1800$  rpm  
 Actual velocity ratio:  $VR = 20$   
 Gear pitch diameter:  $D_g = 3.33333$  in  
 Specify worm diameter:  $D_w = 1.000$  in  
 Actual center distance:  $C = 2.167$  in  
 $C_{opt}/D_w = 1.97$   
 Should be  $>1.6$  and  $<3.0$

Circular pitch of gear:  $P_g = 0.262$  in  
 Axial pitch of worm:  $P_{xw} = 0.262$  in  
 Lead of the worm:  $L = 0.524$  in  
 Lead angle:  $\lambda = 9.462$  deg  
 Addendum:  $a = 0.083$  in  
 Dedendum:  $b = 0.096$  in  
 Worm outside diameter:  $D_{ow} = 1.167$  in  
 Worm root diameter:  $D_{rw} = 0.807$  in  
 Nominal worm face length:  $F_{worm} = 1.491$  in  
 Gear throat diameter:  $D_{tg} = 3.500$  in  
 Nominal gear face width:  $F_{og} = 0.601$  in  
 Max effective gear face width:  $0.67 \cdot D_w = 0.670$  in  
 Effective gear face width:  $F_g = 0.500$  in.  
 [Used given face width]  
 Given face width is small; Could use  $F > 0.601$  in

**Additional Computed Results:**

Pitch line speed - Gear:  $78.54$  ft/min  
 Sliding velocity  $v_s = 478$  ft/min  
 Coefficient of friction:  $0.030$  if  $v_s > 10$  ft/min

**Forces: (lb)**

Tangential: 290 58  
 Radial: 77 77  
 Axial: 58 290

**Friction force,  $W_f = 9.1$  lb**

**Power:**

Power output from gear:  $0.691$  hp  
 Power loss - friction:  $0.131$  hp  
 Power input:  $0.822$  hp  
 Efficiency:  $84.1\%$

**Stresses:**

Normal pressure angle,  $\phi_n$   
 14.5 20 25 30

**Bending Stress on Gear:**

Enter Lewis form factor  $y = 0.100$   
 Normal circular pitch:  $0.258$  in  
 Dynamic factor:  $K_v = 0.939$

**Bending stress on gear:  $23963$  psi [Using effective gear face width]**

Allowable stresses-Bronze: Manganese =  $17000$  psi; Phosphor =  $24000$  psi

**Surface Durability: [Hardened steel worm; bronze gear]**

Type of bronze/ $D_g \rightarrow$   $>2.5$  in  $<2.5$  in  $<2.5$  in  $<25$  in  $>8$  in  $>25$  in  $<25$  in  
 Sand cast:  $C_s = 940$  1000  
 Chill cast or forged:  $C_s = 1173$  1000  
 Centrifugally cast:  $C_s = 1157$  1000  
 Enter Materials factor  $C_m = 1000$  Chilled Cast - Phosphor bronze  
 Gear Ratio:  $m_g = 6$  to  $20$   $20$  to  $76$   $>76$  Actual  $m_g = 20$   
 Ratio correction factor:  $C_m = 0.820$   $0.819$   $1.017$   
 Enter  $C_m = 0.819$

**Sliding velocity:**

$<700$   $700-3000$   $>3000$  Actual  $v_s = 478$

**Velocity factor:  $C_v = 0.390$   $0.393$   $0.553$**

**Enter  $C_v = 0.390$**

**Rated tangential load:  $W_{tR} = 418$  lb**

Must be  $> W_t = 290$  lb

Adjusted output torque until limits reached on either bending or surface durability  
 Bending stress controls this design



Wormgearing - Design		Problem: (P-19e and 30)	
<b>Input Data:</b>			
Desired output torque:	$T_o =$	878	lb-in
Output speed:	$n_o =$	90	rpm
Velocity Ratio:	$VR =$	20	
<b>Design Decisions:</b>			
Diametral pitch:	$P_d =$	1/2	
No. of worm threads:	$N_w =$	4	
Required No. of gear teeth:	$N_g =$	80	
Speedy No. of gear teeth:	$N_g =$	90	
Normal pressure angle:	$\phi_n =$	14.5	degrees
<b>Computed Results and Additional Inputs:</b>			
Actual input speed:	$n_w =$	1800	rpm
Actual velocity ratio:	$VR =$	20	
Gear pitch diameter:	$D_g =$	6.6667	in
Specific worm diameter:	$D_w =$	1.000	in
Actual center distance:	$C =$	3.833	in
	$C^{0.875}/D_w =$	3.24	HIGH
<b>Use larger worm diameter</b> Should be >1.6 and <3.0			
Circular pitch of gear:	$P_g =$	0.262	in
Axial pitch of worm:	$P_{xw} =$	0.262	in
Lead of the worm:	$L =$	1.047	in
Lead angle:	$\lambda =$	18.435	deg
Addendum:	$a =$	0.083	in
Dedendum:	$b =$	0.096	in
Worm outside diameter:	$D_{ow} =$	1.167	in
Worm root diameter:	$D_{rw} =$	0.807	in
Nominal worm face length:	$F_{worm} =$	2.108	in
Gear throat diameter:	$D_{tg} =$	6.833	in
Nominal gear face width:	$F_{ag} =$	0.601	in
Max effective gear face width:	$0.67 \cdot D_w =$	0.670	in
Effective gear face width:	$F_g =$	0.500	in
			[Used given face width]
Given face width is small; Could use $F > 0.601$ in			
<b>Additional Computed Results:</b>			
Pitch line speed - Gear: 157.08 ft/min			
Sliding velocity $v_s =$ 497 ft/min			
Coefficient of friction: 0.029 if $v_s > 10$ ft/min			
<b>Forces: (lb)</b>			
	Gear	Worm	
Tangential:	263	97	
Radial:	73	73	
Axial:	97	263	
Friction force, $W_f =$ 8.4 lb			
<b>Power:</b>			
Power output from gear: 1.254 hp			
Power loss - friction: 0.127 hp			
Power input: 1.381 hp			
<b>Efficiency:</b> 90.8 %			
<b>Stresses:</b>			
<b>Bending Stress on Gear:</b>			
Enter Lewis form factor $y =$ 0.100 $\rightarrow$			
Normal circular pitch: 0.248 in			
Dynamic factor: $K_v =$ 0.894			
Bending stress on gear: 23987 psi [Using effective gear face width]			
Allowable stresses-Bronze: Manganese = 17000 psi; Phosphor = 24000 psi			
<b>Surface Durability: [Hardened steel worm; bronze gear]</b>			
Type of bronze/ $D_o \rightarrow$ >2.5 in <2.5 in >8 in <8 in >25 in <25 in			
Sand cast: $C_s =$ 797 1000			
Chill cast or forged: $C_s =$ 1036 1000			
Centrifugally cast: $C_s =$ 1103 1000			
Enter Materials factor: $C_m =$ 1000 Chilled Cast - Phosphor bronze			
Ratio correction factor: $C_m =$ 6 to 20 20 to 76 >76 Actual $m_g =$ 20			
Ratio correction factor: $C_m =$ 0.820 0.819 1.017			
Enter $C_m =$ 0.819			
Sliding velocity: <700 700-3000 >3000 Actual $v_s =$ 497			
Velocity factor: $C_v =$ 0.382 0.384 0.537			
Enter $C_v =$ 0.382			
Rated tangential load: $W_{Rt} =$ 714 lb			
Must be $> W_f =$ 263 lb			
Adjusted output torque until limits reached on either bending or surface durability			
Bending stress controls this design			

Wormgearing - Design		Problem: 10-27
<b>Input Data:</b>		
Desired output torque:	904 lb-in	
Output speed:	80 rpm	
Velocity Ratio:	VR = 7.5	
<b>Design Decisions:</b>		
Diametral pitch:	$P_d = 8$	
No. of worm threads:	$N_w = 4$	
Required No. of gear teeth:	$N_g = 30$	
Specify No. of gear teeth:	$N_g = 30$	
Normal pressure angle:	$\phi_n = 14.5$ degrees	
<b>Computed Results and Additional Inputs:</b>		
Actual input speed:	$n_w = 600$ rpm	
Actual velocity ratio:	VR = 7.5	
Gear pitch diameter:	$D_g = 3.75$ in	
Specify worm diameter:	$D_w = 1.375$ in	
Actual center distance:	$C = 2.563$ in	
	$C^{0.875}/D_w = 1.66$	
	Should be >1.6 and <3.0	
Circular pitch of gear:	$P_g = 0.393$ in	
Axial pitch of worm:	$P_{xw} = 0.393$ in	
Lead of the worm:	$L = 1.571$ in	
Lead angle:	$\lambda = 19.983$ deg	
Addendum:	$a = 0.125$ in	
Dedendum:	$b = 0.145$ in	
Worm outside diameter:	$D_{ow} = 1.625$ in	
Worm root diameter:	$D_{rw} = 1.086$ in	
Nominal worm face length:	$F_{wnom} = 1.936$ in	
Gear throat diameter:	$D_{tg} = 4.000$ in	
Nominal gear face width:	$F_{eg} = 0.866$ in	
Max effective gear face width:	$0.67 \cdot D_w = 0.92125$ in	
Effective gear face width:	$F_e = 0.866$ in	
<b>Additional Computed Results:</b>		
Pitch line speed - Gear:	78.54 ft/min	
Sliding velocity $v_s$ :	230 ft/min	
Coefficient of friction:	0.041 if $v_s > 10$ ft/min	
<b>Forces: (lb)</b>		
	Gear	Worm
Tangential:	525	216
Radial:	147	147
Axial:	216	525
Friction force, $W_f$ :	24.0 lb	
<b>Power:</b>		
Power output from gear:	1.250 hp	
Power loss - friction:	0.167 hp	
Power input:	1.416 hp	
<b>Efficiency:</b> 88.2 %		
<b>Stresses:</b>		
Normal pressure angle, $\phi_n$	14.5	20 25 30
Lewis form factor, $y$	0.100	0.125 0.150 0.175
<b>Bending Stress on Gear:</b>		
Enter Lewis form factor $y$	0.100	→
Normal circular pitch:	0.369 in	
Dynamic factor: $K_v$ :	0.939	
Bending stress on gear:	17495 psi [Using effective gear face width]	
Allowable stresses-Bronze: Manganese = 17000 psi; Phosphor = 24000 psi		
<b>Surface Durability: [Hardened steel worm; bronze gear]</b>		
Type of bronze/ $D_g$ →	>2.5 in <2.5 in	>8 in <8 in >25 in <25 in
Sand cast: $C_s$ :	916	1000
Chill cast or forged: $C_f$ :		1150 1000
Centrifugally cast: $C_a$ :		1148 1000
Enter Material factor $C_p$ (0.01)		
Gear Ratio: $m_G$ :	6 to 20	20 to 76 >76 Actual $m_G = 7.5$
Ratio correction factor: $C_m$ :	0.719	0.794 1.099
Enter $C_m$ (0.719)		
Sliding velocity:	<700	700-3000 >3000 Actual $v_s = 230$
Velocity factor: $C_v$ :	0.512	0.597 0.974
Enter $C_v$ (0.512)		
Rated tangential load: $W_{st}$ :	918 lb	
Must be > $W_f$ :	525 lb	



**Wormgearing - Design**

Problem: 16-22

**Input Data:**

Desired output torque:  $T_o = 52.6 \text{ lb-in}$   
 Output speed:  $n_g = 600 \text{ rpm}$   
 Velocity Ratio:  $VR = 3$

**Design Decisions:**

Diametral pitch:  $P_d = 12$   
 No. of worm threads:  $N_w = 6$   
 Required No. of gear teeth:  $N_g = 18$   
 Speedy No. of gear teeth:  $N_g = 76$   
 Normal pressure angle:  $\phi_n = 25 \text{ degrees}$

**Computed Results and Additional Inputs:**

Actual input speed:  $n_w = 1800 \text{ rpm}$   
 Actual velocity ratio:  $VR = 3$   
 Gear pitch diameter:  $D_g = 1.5 \text{ in}$   
 Speedy worm diameter:  $D_w = 0.5 \text{ in}$   
 Actual center distance:  $C = 1.000 \text{ in}$   
 $C^{0.875}/D_w = 2.00$   
 Should be  $>1.6$  and  $<3.0$   
 Circular pitch of gear:  $P_g = 0.262 \text{ in}$   
 Axial pitch of worm:  $P_{xw} = 0.262 \text{ in}$   
 Lead of the worm:  $L = 1.571 \text{ in}$   
 Lead angle:  $\lambda = 45.000 \text{ deg}$   
 Addendum:  $a = 0.083 \text{ in}$   
 Dedendum:  $b = 0.096 \text{ in}$   
 Worm outside diameter:  $D_{ow} = 0.667 \text{ in}$   
 Worm root diameter:  $D_{rw} = 0.307 \text{ in}$   
 Nominal worm face length:  $F_{worm} = 1.000 \text{ in}$   
 Gear throat diameter:  $D_{ts} = 1.667 \text{ in}$   
 Nominal gear face width:  $F_{es} = 0.441 \text{ in}$   
 Max effective gear face width:  $0.67 \cdot D_w = 0.335 \text{ in}$   
 Effective gear face width:  $F_e = 0.335 \text{ in}$

**Additional Computed Results:**

Pitch line speed - Gear:  $235.62 \text{ ft/min}$   
 Sliding velocity  $v_s = 333 \text{ ft/min}$   
 Coefficient of friction:  $0.035$  if  $v_s > 10 \text{ ft/min}$

**Forces: (lb)**

Tangential: 70 Gear Worm  
 Radial: 48 48  
 Axial: 76 70  
 Friction force,  $W_f = 4.0 \text{ lb}$

**Power:**

Power output from gear: 0.500 hp  
 Power loss - friction: 0.040 hp  
 Power input: 0.540 hp

**Efficiency:**

92.6 %

**Stresses:**

Normal pressure angle,  $\phi_n$   
 14.5 20 25 30

**Bending Stress on Gear:**

Enter Lewis form factor  $y$  →  
 0.100 0.125 0.150 0.175

Normal circular pitch: 0.185 in  
 Dynamic factor:  $K_v = 0.836$

Bending stress on gear: 9003 psi [Using effective gear face width]

Allowable stresses-Bronze: Manganese = 17000 psi; Phosphor = 24000 psi

**Surface Durability: [Hardened steel worm; bronze gear]**

Type of bronze/ $D_g$  →  $>2.5 \text{ in}$   $<2.5 \text{ in}$   $>8 \text{ in}$   $<8 \text{ in}$   $>25 \text{ in}$   $<25 \text{ in}$

Sand cast:  $C_s = 1106$  1000

Chill cast or forged:  $C_s = 1331$  1000

Centrifugally cast:  $C_s = 1220$  1000

Enter Materials factor  $C_m = 1000$

Gear Ratio:  $m_g = 6 \text{ to } 20$  20 to 76  $>76$  Actual  $m_g = 3$

Ratio correction factor:  $C_m = 0.578$  0.779 1.129

Enter  $C_p = 0.578$

Sliding velocity:  $<700$  700-3000  $>3000$  Actual  $v_s = 333$

Velocity factor:  $C_v = 0.457$  0.483 0.731

Enter  $C_v = 0.457$

Rated tangential load:  $W_m = 122 \text{ lb}$

Must be  $> W_t = 70 \text{ lb}$



Wormgearing - Design		Problem: 10-23
<b>Input Data:</b>		
Desired output torque:	$T_o =$	4200 ft-lb
Output speed:	$n_o =$	45 rpm
Velocity Ratio:	$VR =$	10
<b>Design Decisions:</b>		
Diametral pitch:	$P_d =$	8
No. of worm threads:	$N_w =$	2
Required No. of gear teeth:	$N_g =$	80
Specify No. of gear teeth:	$N_g =$	80
Normal pressure angle:	$\phi_n =$	14.5 degrees
<b>Computed Results and Additional Inputs:</b>		
Actual input speed:	$n_w =$	1800 rpm
Actual velocity ratio:	$VR =$	40
Gear pitch diameter:	$D_g =$	10 in
Specify worm diameter:	$D_w =$	2.25 in
Actual center distance:	$C =$	6.125 in
	$C^{0.875}/D_w =$	2.17
	Should be	>1.6 and <3.0
Circular pitch of gear:	$p_g =$	0.393 in
Axial pitch of worm:	$p_{xw} =$	0.393 in
Lead of the worm:	$L =$	0.785 in
Lead angle:	$\lambda =$	6.340 deg
Addendum:	$a =$	0.125 in
Dedendum:	$b =$	0.145 in
Worm outside diameter:	$D_{oW} =$	2.500 in
Worm root diameter:	$D_{rW} =$	1.961 in
Nominal worm face length:	$F_{wnom} =$	3.162 in
Gear throat diameter:	$D_{tG} =$	10.250 in
Nominal gear face width:	$F_{ac} =$	1.090 in
Max effective gear face width:	$0.67 \cdot D_w =$	1.5075 in
Effective gear face width:	$F_e =$	1.090 in
<b>Additional Computed Results:</b>		
Pitch line speed - Gear:	$117.81$	ft/min
Sliding velocity $v_s =$	$1067$	ft/min
Coefficient of friction:	$0.020$	if $v_s > 10$ ft/min
<b>Forces: (lb)</b>		
	Gear	Worm
Tangential:	840	111
Radial:	219	219
Axial:	111	840
Friction force, $W_f =$	17.6 lb	
<b>Power:</b>		
Power output from gear:	3.000 hp	
Power loss - friction:	0.570 hp	
Power input:	3.570 hp	
Efficiency:	84.0 %	
Stresses:		Normal pressure angle, $\phi_n$
		14.5 20 25 30
Bending Stress on Gear:		Lewis form factor, $y$
Enter Lewis form factor, $Y$	0.100	0.125 0.150 0.175
Normal circular pitch:	0.390 in	
Dynamic factor: $K_v =$	0.911	
Bending stress on gear:	21689 psi [Using effective gear face width]	
Allowable stresses-Bronze:	Manganese = 17000 psi; Phosphor = 24000 psi	
<b>Surface Durability: [Hardened steel worm; bronze gear]</b>		
Type of bronze/ $D_g \rightarrow$	>2.5 in	<2.5 in
	>8 in	<8 in
	>25 in	<25 in
Sand cast: $C_s =$	713	1000
Chill cast or forged: $C_s =$		956 1000
Centrifugally cast: $C_s =$		1072 1000
Enter Materials factor: $C_m =$	Sand Cast	
Gear Ratio: $m_G =$	6 to 20	20 to 76 >76
Actual $m_G =$	40	
Ratio correction factor: $C_m =$	#NUM!	
Enter $C_m =$	0.814 0.885	
Sliding velocity:	<700	700-3000 >3000
Velocity factor: $C_v =$	0.204	0.248 0.297
Enter $C_v =$	0.248	
Rated tangential load: $W_{tr} =$	990 lb	
Must be > $W_f =$	840 lb	

**PROBLEM 10-24**

**COMPARISON OF DESIGNS A and B.**

*Note: Also includes a revised Design A with a smaller worm diameter and larger gear face width*

<b>Given data:</b>	<b>Design</b>			
	<b>A</b>	<b>B</b>	<b>A Revised</b>	
Diametral pitch:	6	10	6	
Threads in worm:	1	2	1	
Teeth in gear:	30	60	30	
Worm diameter (in):	2.000	1.250	1.750	Design A Rev. - Smaller diameter
Face width - Gear (in):	1.000	0.625	1.130	Design A Rev. - Larger face width
Pressure angle (deg):	14.5	14.5	14.5	
<b>Results:</b>				
<b>Forces - Gear (lb):</b>				
Tangential:	480	400	480	
Radial:	125	106	125	
Axial:	58	82	65	
<b>Forces - Worm (lb):</b>				
Tangential:	58	82	65	
Radial:	125	106	125	
Axial:	480	400	480	
Lead angle (degrees):	4.76	9.09	5.44	Design B - Higher lead angle
Efficiency (%):	69.1	77.6	70.6	Design B - Higher efficiency
Power output (hp):	0.381	0.381	0.381	
Power input (hp):	0.552	0.491	0.54	Design B OK for 0.50 hp motor
Gear pitch diameter (in):	5.000	6.000	5.000	Design A smaller
Center distance (in):	3.500	3.625	3.375	Design A smaller
Stress - Wormgear (psi):	9400	21171	8324	Design A - Lower bending stress
Rated load - Durability (lb):	1193	937	1406	
				Design A - OK for sand cast Manganese Bronze
				Design B - OK for sand cast Phosphor bronze

Wormgearing - Design		Problems 10-244	
<b>Input Data:</b>			
Desired output torque:	$T_o =$	1200 lb-in	
Output speed:	$n_o =$	20 rpm	
Velocity Ratio:	$VR =$	30	
<b>Design Decisions:</b>			
Diametral pitch:	$P_d =$	6	
No. of worm threads:	$N_w =$	1	
Required No. of gear teeth:	$N_g =$	30	
Specify No. of gear teeth:	$N_g =$	30	
Normal pressure angle:	$\phi_n =$	14.5 degrees	
<b>Computed Results and Additional Inputs:</b>			
Actual input speed:	$n_w =$	600 rpm	
Actual velocity ratio:	$VR =$	30	
Gear pitch diameter:	$D_g =$	5 in	
Specify worm diameter:	$D_w =$	2.000 in	
Actual center distance:	$C =$	3.500 in	
$C^{0.917}/D_w =$	1.50	LOW	
<b>Use smaller worm diameter</b> Should be $>1.6$ and $<3.0$			
Circular pitch of gear:	$P_c =$	0.524 in	
Axial pitch of worm:	$P_{xw} =$	0.524 in	
Lead of the worm:	$L =$	0.524 in	
Lead angle:	$\lambda =$	4.764 deg	
Addendum:	$a =$	0.167 in	
Dedendum:	$b =$	0.193 in	
Worm outside diameter:	$D_{ow} =$	2.333 in	
Worm root diameter:	$D_{rw} =$	1.614 in	
Nominal worm face length:	$F_{wfm} =$	2.582 in	
Gear throat diameter:	$D_{ts} =$	5.333 in	
Nominal gear face width:	$F_{og} =$	1.202 in	
Max effective gear face width:	$0.97 \cdot D_w =$	1.340 in	
Effective gear face width:	$F_e =$	1.000 in	
[Used given face width]			
Given face width is small; Could use $F > 1.202$			
<b>Additional Computed Results:</b>			
Pitch line speed - Gear:	26.18 ft/min		
Sliding velocity $v_s =$	315 ft/min		
Coefficient of friction:	0.036 if $v_s > 10$ ft/min		
<b>Forces: (lb)</b>			
Tangential:	480	Worm	
Radial:	125		
Axial:	58		
Friction force, $W_f =$	17.9 lb		
<b>Power:</b>			
Power output from gear:	0.381 hp		
Power loss - friction:	0.171 hp		
Power input:	0.552 hp		
Efficiency:	69.1 %		
<b>Stresses:</b>			
<b>Bending Stress on Gear:</b>			
Enter Lewis form factor, $y =$	0.100		
Normal circular pitch:	0.522 in		
Dynamic factor, $K_v =$	0.979		
Bending stress on gear:	9400 psi [Using effective gear face width]		
Allowable stresses-Bronze: Manganese = 17000 psi; Phosphor = 24000 psi			
<b>Surface Durability: [Hardened steel worm; bronze gear]</b>			
Type of bronze/ $D_g \rightarrow$	$>2.5$ in	$<2.5$ in	$>8$ in
Sand cast: $C_s =$	857	1000	
Chill cast or forged: $C_s =$			1093
Centrifugally cast: $C_s =$			1000
Enter Materials factor, $C_m =$	557	Sand cast	1126
Ratio correction factor: $C_m =$	0.759	0.824	0.951
Enter $C_m =$	0.824		
Gear Ratio: $m_g =$	6 to 20	20 to 76	$>76$ Actual $m_g = 30$
Sliding velocity:	$<700$	700-3000	$>3000$ Actual $v_s = 315$
Velocity factor: $C_v =$	0.468	0.498	0.763
Enter $C_v =$	0.468		
Rated tangential load: $W_{tr} =$	1193 lb		OK For sand cast bronze
Must be $> W_f =$	480 lb		
Can use Manganese bronze based on bending stress in gear. Worm diameter is too large.			



Wormgearing - Design		Problem 10-24D	
<b>Input Data:</b>			
Desired output torque	$T_o = 1200$ lb-in		
Output speed	$n_g = 20$ rpm		
Velocity ratio	$VR = 30$		
<b>Design Decisions:</b>			
Pressure angle	$\phi_n = 10$		
No. of worm threads	$N_w = 2$		
Required No. of gear teeth:	$N_g = 60$		
Specify No. of gear teeth	$N_g = 60$		
Normal pressure angle	$\phi = 17.5$ degrees		
<b>Computed Results and Additional Inputs:</b>			
Actual input speed:	$n_w = 600$ rpm		
Actual velocity ratio:	$VR = 30$		
Gear pitch diameter:	$D_g = 6$ in		
Specify worm diameter	$D_w = 1.250$ in		
Actual center distance:	$C = 3.625$ in		
	$C_{std}/D_w = 2.47$ OK		
	Should be $>1.6$ and $<3.0$		
Circular pitch of gear:	$p_g = 0.314$ in		
Axial pitch of worm:	$p_w = 0.314$ in		
Lead of the worm:	$L = 0.628$ in		
Lead angle:	$\lambda = 9.090$ deg		
Addendum:	$a = 0.100$ in		
Dedendum:	$b = 0.116$ in		
Worm outside diameter:	$D_{ow} = 1.450$ in		
Worm root diameter:	$D_{rw} = 1.019$ in		
Nominal worm face length:	$F_{worm} = 2.191$ in		
Gear throat diameter:	$D_{tg} = 6.200$ in		
Nominal gear face width:	$F_{eg} = 0.735$ in		
Max effective gear face width:	$0.67 \cdot D_w = 0.838$ in		
Effective gear face width:	$F_e = 0.625$ in		
	[Used given face width]		
	Given face width is small; Could use $F > 0.735$		
<b>Additional Computed Results:</b>			
Pitch line speed - Gear:	$31.42$ ft/min		
Sliding velocity $v_s =$	$199$ ft/min		
Coefficient of friction:	$0.043$ If $v_s > 10$ ft/min		
<b>Forces: (lb)</b>			
	Gear	Worm	
Tangential:	400	82	
Radial:	106	106	
Axial:	82	400	
Friction force, $W_f =$	$18.3$ lb		
<b>Power:</b>			
Power output from gear:	$0.381$ hp		
Power loss - friction:	$0.110$ hp		
Power input:	$0.491$ hp		
Efficiency:	$77.6\%$		
<b>Stresses:</b>			
	14.5	20	25
	30		
<b>Bending Stress on Gear:</b>			
Enter Lewis form factor, $y$	$0.100$		
Normal circular pitch:	$0.310$ in		
Dynamic factor: $K_v =$	$0.974$		
Bending stress on gear:	$21171$ psi [Using effective gear face width]		
Allowable stresses - Bronze: Manganese = 17000 psi; Phosphor = 24000 psi			
<b>Surface Durability: [Hardened steel worm; bronze gear]</b>			
Type of bronze: $D_g \rightarrow$	$>2.5$ in	$<2.5$ in	$>8$ in
	$>8$ in	$<8$ in	$>25$ in
	$<25$ in		
Sand cast: $C_s =$	819	1000	
Chill cast or forged: $C_s =$			1057
Centrifugally cast: $C_s =$			1000
Enter Materials factor $C_m =$	819	Sand cast	1111
			1000
Gear Ratio: $m_g =$	6 to 20	20 to 76	$>76$ Actual $m_g = 30$
Ratio correction factor: $C_m =$	0.759	0.824	0.951
Enter $C_v =$	0.625		
Sliding velocity:	$<700$	700-3000	$>3000$ Actual $v_s = 199$
Velocity factor: $C_v =$	0.530	0.648	1.090
Enter $C_p =$	0.530		
Rated tangential load: $W_{gr} =$	937 lb		OK For sand cast bronze
Must be $> W_t =$	400 lb		
Can use Phosphor bronze based on bending stress in gear.			

**Wormgearing - Design**

Problem: 18.27a, Revised

**Input Data:**

Desired output torque:  $T_o = 1200 \text{ lb-in}$   
 Input speed:  $n_p = 20 \text{ rpm}$   
 Velocity Ratio:  $VR = 30$

**Design Decisions:**

Diametral pitch:  $P_d = 6$   
 No. of worm threads:  $N_w = 1$   
 Required No. of gear teeth:  $N_g = 30$   
 Specify No. of gear teeth:  $N_g = 30$   
 Normal pressure angle:  $\phi_n = 14.5 \text{ degrees}$

**Computed Results and Additional Inputs:**

Actual input speed:  $n_w = 600 \text{ rpm}$   
 Actual velocity ratio:  $VR = 30$   
 Gear pitch diameter:  $D_g = 5 \text{ in}$   
 Specify worm diameter:  $D_w = 1.750 \text{ in}$   
 Actual center distance:  $C = 3.375 \text{ in}$   
 $C^{0.875}/D_w = 1.66 \text{ OK}$   
 Should be  $>1.6$  and  $<3.0$

Circular pitch of gear:  $P_g = 0.524 \text{ in}$   
 Axial pitch of worm:  $P_{xw} = 0.524 \text{ in}$   
 Lead of the worm:  $L = 0.524 \text{ in}$   
 Lead angle:  $\lambda = 5.440 \text{ deg}$   
 Addendum:  $a = 0.167 \text{ in}$   
 Dedendum:  $b = 0.193 \text{ in}$   
 Worm outside diameter:  $D_{ow} = 2.083 \text{ in}$   
 Worm root diameter:  $D_{rw} = 1.364 \text{ in}$   
 Nominal worm face length:  $F_{wnom} = 2.582 \text{ in}$   
 Gear throat diameter:  $D_{gs} = 5.333 \text{ in}$   
 Nominal gear face width:  $F_{ge} = 1.130 \text{ in}$   
 Max effective gear face width:  $0.67 \cdot D_w = 1.173 \text{ in}$   
 Effective gear face width:  $F_g = 1.130 \text{ in}$

**[Use nominal face width]**

**Additional Computed Results:**

Pitch line speed - Gear:  $26.18 \text{ ft/min}$   
 Sliding velocity  $v_s = 276 \text{ ft/min}$   
 Coefficient of friction:  $0.038 \text{ if } v_s > 10 \text{ ft/min}$

**Forces: (lb)**

Tangential: 480  
 Radial: 125  
 Axial: 65  
 Friction force,  $W_f = 19.0 \text{ lb}$

**Power:**

Power output from gear:  $0.381 \text{ hp}$   
 Power loss - friction:  $0.159 \text{ hp}$   
 Power Input:  $0.540 \text{ hp}$   
 Efficiency:  $70.6 \%$

**Stresses:**

Normal pressure angle,  $\phi_n$   
 14.5 20 25 30  
 Lewis form factor,  $y$   
 0.100 0.125 0.150 0.175

**Bending Stress on Gear:**

Enter Lewis form factor,  $y = 0.100$   
 Normal circular pitch:  $0.521 \text{ in}$   
 Dynamic factor:  $K_v = 0.979$

**Bending stress on gear:**

$8324 \text{ psi}$  [Using effective gear face width]

Allowable stresses-Bronze: Manganese =  $17000 \text{ psi}$ ; Phosphor =  $24000 \text{ psi}$

**Surface Durability: [Hardened steel worm; bronze gear]**

Type of bronze/ $D_g \rightarrow$   $>2.5 \text{ in}$   $<2.5 \text{ in}$   $>8 \text{ in}$   $<8 \text{ in}$   $>25 \text{ in}$   $<25 \text{ in}$   
 Sand cast:  $C_s = 857$  1000  
 Chill cast or forged:  $C_s = 1093$  1000  
 Centrifugally cast:  $C_s = 1126$  1000

**Enter Materials factor,  $C_m = 0.857$  Sand cast**

Gear Ratio:  $m_g = 6 \text{ to } 20$   $20 \text{ to } 76$   $>76$  Actual  $m_g = 30$   
 Ratio correction factor:  $C_m = 0.759$   $0.824$   $0.951$

**Enter  $C_p = 0.80$**

Sliding velocity:  $<700$   $700\text{-}3000$   $>3000$  Actual  $v_s = 276$   
 Velocity factor:  $C_v = 0.486$   $0.537$   $0.845$

**Enter  $C_e = 0.486$**

Rated tangential load:  $W_{tr} = 1406 \text{ lb}$  OK For sand cast bronze  
 Must be  $> W_f = 480 \text{ lb}$

Can use Manganese bronze based on bending stress in gear.  
 Changed worm diameter to  $1.75 \text{ in}$ ; Changed face width to  $1.13 \text{ in}$



## CHAPTER 11 KEYS, COUPLINGS, AND SEALS

GENERAL NOTES FOR KEY DESIGN PROBLEMS 1-15:

FOR SAE 1018 CD STEEL:  $S_y = 54,000$  psi. IF KEY MATERIAL IS WEAKEST -

$$\tau_d = \frac{0.5 S_y}{N} = \frac{0.5(54,000)}{3} = 9,000 \text{ psi}$$

$$\sigma_d = \frac{S_y}{N} = \frac{54,000}{3} = 18,000 \text{ psi}$$

1.  $D_{\text{SHAFT}} = 2.00$  IN; USE  $\frac{1}{2}$  IN SQ. KEY; SAE 1018 CD  
 $L = \frac{2T}{\tau_d D W} = \frac{2(21,000 \text{ LB}\cdot\text{IN})}{(9,000 \text{ LB}/\text{IN}^2)(2.00 \text{ IN})(0.5 \text{ IN})} = 4.667$  IN  
 BUT HUB LENGTH = 4.00 IN. USE SAE 1045 CD;  $S_y = 77,000$  psi  
 $\tau_d = 0.5 S_y / N = 0.5(77,000) / 3 = 12,833$  psi  
 $L = \frac{2(21,000)}{(12,830)(2.00)(0.5)} = 3.27$  IN; USE  $L = 3\frac{3}{4} = 3.75$  IN.
2.  $D_s = 3.60$  IN; USE  $\frac{7}{8}$  IN SQUARE KEY; SAE 1018 CD  
 $L = \frac{2(21,000)}{(9,000)(3.60)(0.875)} = 1.48$  IN USE  $L = 3.50$  IN TO MORE NEARLY MATCH HUB LENGTH
3.  $D_s = 1.75$  IN; USE  $\frac{3}{8}$  IN. SQ. KEY; SAE 1018 CD;  $\tau_d = 9,000$  psi  
 FOR HUB; CLASS 20 CI;  $S_u = 20,000$  psi  
 FOR BEARING  $\sigma_d = \frac{20,000}{3} = 6,667$  psi (CONSERVATIVE)  
 BECAUSE COMPRESSIVE STRENGTH OF CI IS MUCH GREATER THAN TENSILE STRENGTH.  
 SHEAR OF KEY:  $L = \frac{2T}{\tau_d D W} = \frac{2(1112)}{(9,000)(1.75)(.375)} = 0.377$  IN  
 BEARING ON HUB:  $L = \frac{4T}{\sigma_d D H} = \frac{4(1112)}{(6,667)(1.75)(.375)} = 1.02$  IN  
 USE  $L = 1.50$  IN TO MATCH HUB LENGTH.
4.  $T = 63,000(110) / 1700 = 4,076$  LB $\cdot$ IN;  $D_s = 2.50$  IN; USE  $\frac{5}{8}$  SQ. KEY  
 $L = \frac{2T}{\tau_d D W} = \frac{2(4,076)}{(9,000)(2.50)(0.625)} = 0.580$  IN  
 USE  $L = 2.50$  IN TO MORE NEARLY MATCH HUB LENGTH.

5.

EXPRESS DATA FROM TABLE 11-6 AS  $T = K D^2 L$   
 REQ'D  $K = T / D^2 L$  ( $L = \text{HUB LENGTH}$ ) USE B-FIT

a) PROB. 1 DATA;  $T = 21000$ ;  $D = 2.00 \text{ IN}$ ,  $L = 4.00 \text{ IN}$ .

$$K = \frac{21000}{(2.00)^2(4.0)} = 1313 \quad \text{TOO HIGH FOR ANY SPLINE IN TABLE 11-6}$$

b) PROB. 2 DATA;  $T = 21000 \text{ LB-IN}$ ;  $D = 3.60 \text{ IN}$ ;  $L = 4.00 \text{ IN}$ .

$$K = \frac{21000}{(3.60)^2(4.0)} = 405; \text{ USE 16 SPLINES; } K = 521$$

c) PROB. 3 DATA:  $T = 1112 \text{ LB-IN}$ ;  $D = 1.75 \text{ IN}$ ;  $L = 1.75 \text{ IN}$

$$K = \frac{1112}{(1.75)^2(1.75)} = 208 \quad \text{USE 6 SPLINES}$$

d) PROB. 4 DATA:  $T = 4076 \text{ LB-IN}$ ;  $D = 2.50 \text{ IN}$ ;  $L = 3.25 \text{ IN}$

$$K = \frac{4076}{(2.50)^2(3.25)} = 201 \quad \text{USE 6 SPLINES}$$

6.

$$\text{AT } 220 \text{ HP; } T_2 = 63000(220)/1700 = 8153 \text{ LB-IN}$$

$$\text{AT } 110 \text{ HP; } T_1 = 63000(110)/1700 = 4076 \text{ LB-IN}$$

PIN SHOULD SHEAR AT  $T_2$ ;  $T_2 = S_{MS} = S_u(0.75)$  (SECT. 2-2)

PIN SHOULD NOT YIELD AT  $T_1$ ;  $T_1 \leq S_{YS} = S_y(0.5)$

FOR A GIVEN PIN  $d$  AND SHAFT  $D$  IN EQ. 11-18

$$S_{YS} = T_1 = \frac{4T_1}{D\pi d^2} \quad ; \quad T_2 = \frac{4T_2}{D\pi d^2} = 0.75 S_u$$

$$\text{RATIO} \quad \frac{0.5 S_y}{0.75 S_u} = \frac{\frac{4T_1}{D\pi d^2}}{\frac{4T_2}{D\pi d^2}} = \frac{T_1}{T_2} \quad ; \quad S_y = \frac{0.75 T_2 S_u}{0.5 T_1}$$

$$\text{MIN. } S_y = \frac{0.75(4076) S_u}{0.5(8153)} = 0.75 S_u$$

MOST COLD DRAWN STEELS

HAVE  $S_y \geq 0.75 S_u$

$$\text{FOR SAE 1018 CD. } d = \sqrt{\frac{4T_2}{D\pi T_2}} = \sqrt{\frac{4(8153)}{(2.50)(\pi)(115)(63000)}} = 0.294 \text{ in.}$$

$S_y = 54 \text{ KSI}$ ;  $S_u = 64 \text{ KSI}$

$$\text{AT } T_1 = 4076 \text{ LB-IN; } T = \frac{4(4076)}{(2.50)(\pi)(0.294)^2} = 24009 \text{ psi} < \frac{S_y = 54000 \text{ PSI}}{2} = 27000 \text{ PSI}$$

7. FROM EX. PROB. 12-3:  $T = 4168 \text{ LB}\cdot\text{IN}$ ;  $D_L = 2\frac{1}{4} \text{ IN}$ ;  $D_S = 1\frac{1}{2} \text{ IN}$ .  
 SPROCKET:  $\frac{1}{2} \text{ IN}$  SQUARE KEY; SAE 1018 CD  
 $L = \frac{2T}{T_s DW} = \frac{2(4168)}{(9000)(2.25)(0.5)} = 0.823 \text{ IN}$  USE  $L = 1.00 \text{ IN}$   
 WORM GEAR:  $\frac{3}{8} \text{ SQ. KEY}$   
 $L = \frac{2(4168)}{(9000)(1.50)(0.375)} = 1.649 \text{ IN}$  USE  $L = 1.75 \text{ IN}$

8. WOODRUFF KEY 204: NOMINAL  $W = \frac{2}{32} = \frac{1}{16} \text{ IN}$ ; NOMINAL  $B = \frac{4}{8} = \frac{1}{2} \text{ IN}$   
 ACTUAL DIMS. IN TABLE 11-3

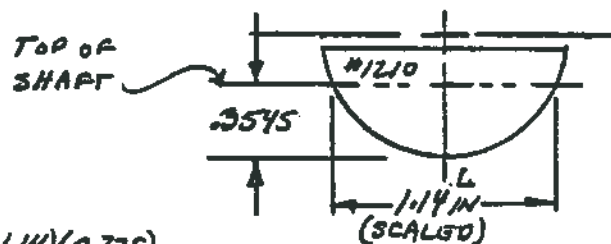
9. WOODRUFF KEY 162B: NOM.  $W = \frac{16}{32} = \frac{1}{2} \text{ IN}$ ; NOM.  $B = \frac{28}{8} = 3\frac{1}{2} \text{ IN}$

10, 11, 12 ARE DRAWINGS

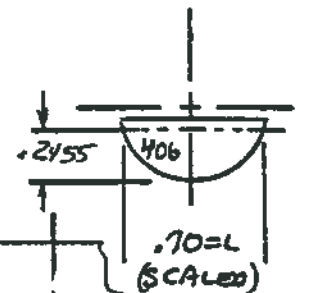
13.  $T = F D / 2 = F = 2T / D$   
 KEY  $W = \frac{3}{8}$

$A_s = LW$ ;  $T = \frac{F}{A_s} = \frac{2T}{DLW}$

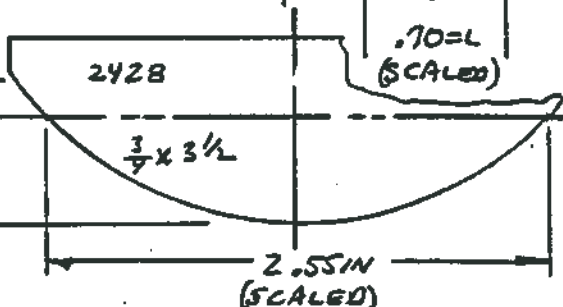
$T = \frac{T_s DLW}{2} = \frac{(9000)(1.500)(1.14)(0.375)}{2} = 2886 \text{ LB}\cdot\text{IN}$



14.  $T = \frac{T_s DLW}{2} = \frac{(9000)(.500)(.70)(.125)}{2} = 197 \text{ LB}\cdot\text{IN}$   
 KEY  $W = \frac{1}{8} \text{ IN}$



15.  $T = \frac{(9000)(3.25)(2.55)(0.75)}{2} = 27,970 \text{ LB}\cdot\text{IN}$



19. a)  $T = 139D^2 = 139(1.50)^2$  (PROB 16)  
 $= 313 \text{ LB}\cdot\text{IN / IN OF HUB L.}$

b)  $T = 326D^2 = 326(3.50)^2$   
 $= 3994 \text{ LB}\cdot\text{IN / IN OF HUB L.}$  (PROB 17)

c)  $T = 688D^2 = 688(2.500)^2 = 4300 \text{ LB}\cdot\text{IN / IN OF HUB L.}$  (PROB 18)

**NOTE: Problems 20-46 call for narrative answers for which the proper information can be found in the text. Guidance is provided below for sections in which additional information can be found.**

20. Section 11-6 includes discussion of applying set screws to transmit torque. A table of approximate holding force capacity vs. set screw size is provided.
21. Press fit is described briefly in Section 11-6. More discussion follows in Chapter 13.
22. Section 11-7 describes both rigid and flexible couplings and compares their performance. Examples of commercially available couplings are shown and described.
23. Section 11-8 contains general information about universal joints.
24. Section 11-9 contains general information about retaining rings and other means of locating machine elements axially on shafts and in other devices. Included are collars, shoulders, spacers, and locknuts.
25. to 38. Section 11-10 contains general information about seals.
39. to 46. Section 11-11 contains general information about seal materials, including elastomers.
  40. to 45. A list of 14 elastomers is included in Section 11-11. Following the list of 14, elastomers, their general performance capabilities are described.
  46. The required conditions for shafts on which elastomeric seals operate are discussed in the last part of Section 11-11. Examples are:
    - Steels, hardened to HRC 30 with tolerances of less than  $\pm 0.005$  in (0.13 mm) are typically used for shafts on which seals operate. The surface must be free of burrs with a surface finish of 10 to 20 microinches is recommended. Lubrication is recommended.

## CHAPTER 12

# SHAFT DESIGN

### GENERAL NOTES CONCERNING SOLUTIONS TO SHAFT DESIGN PROBLEMS

- Design values for stress concentrations as given in Section 12-4 are used for the initial calculations. These values must be checked once final design details are specified for diameters, fillet radii, and other features.
- Estimates are originally used for the size factors used in calculations because they depend on the shaft sizes that are unknown at the start of a design problem. These values must be checked once final design decisions have been specified.
- The choice of the reliability factor,  $C_R$ , is a design decision. Other values may be preferred.
- In most cases, the proposed final values for diameters are expected to be safe because trial values are typically conservative and because final specified diameters are typically made to the next larger preferred size according to Appendix Table A-2.
- Final specifications for diameters where bearings are to be mounted must await the selections for suitable bearings that can accommodate the radial and thrust loads applied to them. This process is described in Chapter 14 and the MDESIGN – MOTT software is an excellent tool for making those decisions. The computed 'minimum required diameter' from the shaft design process should be used as to limit the bearing selection to only feasible sizes.

#### Shafts with Only Radial Loads Applied to Them:

Problems P1 through P30 relate to one of the Figures P12-1 through P12-17 showing shafts carrying a variety of combinations of gears, belt sheaves, chain sprockets, and a few other items such as a flywheel and a propeller-type fan. All of these elements apply only radial loads to the shafts on which they are mounted.

- **Problems 1-11 include only forces and torques exerted by gears on shafts. No separate solutions for these problems are included here.**
- **Problems 12-21 include only forces and torques exerted by belt drives and chain drives on shafts. No separate solutions are included here.**
- **Problems 22-30 are comprehensive design problems that use the same shaft assemblies that are used for Problems 1-21. The solutions to these problems include the analyses of forces and torques and should be used as the solutions for problems 1-21.**
- **The parts of the solutions for torques and forces give discrete, single-answers.**
- **The remaining parts of the comprehensive shaft designs include many design decisions and multiple solutions are possible. The given solutions should be considered examples only.**

There are multiple ways in which the problems P1 through P30 may be assigned. The following table may help instructors decide how to assign the problems for student solution and may help students



comprehend how the sets of problems lead to the more general shaft design. Any combination of problems may be chosen.

<u>Torques and Forces Acting Radial to Shaft</u>	<u>Comprehensive</u>
Figure P12-1: P1 – Gear <i>B</i> ; P14 – Sheave <i>D</i>	P22
Figure P12-2: P2 – Gear <i>C</i> ; P12 – Sprocket <i>D</i> ; P13 – Pulley <i>A</i>	P23
Figure P12-3: P3 – Gear <i>B</i> ; P15 – Sprocket <i>C</i> ; P16 – Sheaves <i>D, E</i>	P24
Figure P12-4: P4 – Gear <i>A</i> ; P19 – Sprockets <i>C, D</i>	P25
Figure P12-5: P5 – Gear <i>D</i> ; P20 – Sheave <i>A</i> ; P21 – Sprocket <i>E</i>	P26
Figure P12-6: P6 – Gear <i>E</i> ; (No separate analysis of Sheave <i>A</i> )	P27 (Includes Sheave <i>A</i> )
Figure P12-7: P7 – Gear <i>C</i> ; P8 – Gear <i>A</i>	P28 (Includes Sheaves <i>D, E</i> )
Figure P12-9: P9 – Gear <i>C</i> ; P10 – Gear <i>D</i> ; P11 – Gear <i>F</i>	P29 (Includes Sheave <i>B</i> )
Figure P12-17: P17 – Sheave <i>C</i> ; P18 – Pulley <i>D</i>	P30 (Includes Fan <i>A</i> )

#### **Shafts with both Radial and Axial Loads Applied to Them:**

Problems P31 to P34 deal with shafts carrying helical gears and wormgears that produce forces directed axially in addition to radial forces. Solutions are only shown for the comprehensive problems (12-32 and 12-34) in which the details of the analyses of torques and forces are included.

<u>Torques and Forces Acting Radial and Axial to Shaft</u>	<u>Comprehensive</u>
Figure P12-31: P31 – Helical Gear <i>B</i>	P32
Figure P12-33: P33 – Wormgear <i>C</i>	P34 (Includes Sheave <i>A</i> )

#### **Other Comprehensive Design Problems**

Problems 35 to 41 contain a variety of loading situations for which the general solution procedure must be adapted. Some of the problems involve more than one shaft, considering shafts for mating gears and multiple reductions.

Figure P12-35: P35 – Double reduction helical drive

Figure P10-8 in Chapter 10: P36 – Bevel gear drive

Figure P12-37: P37 – Bevel gear drive with two chain sprockets

Figure P12-38: P38 – Double reduction spur gear drive; design three shafts.

Figure P12-39: P39 – Drive system consisting of an electric motor, a V-belt drive, a double reduction spur gear type reducer, and a chain drive.

Figure P12-40: P40 – Shaft with three spur gears

Figure P12-41: P41 – Shaft for windshield wiper mechanism with two levers

FIGURE P12-1

22 TORQUE ON GEAR B:  $T_B = 63000(30)/550 = 3436 \text{ LB-IN}$

SAME TORQUE ON SHAFT D:  $T_D = 3436 \text{ LB-IN}$

TORQUE IN SHAFT:  $T_{A-B} = 0$ ;  $T_{B-D} = 3436 \text{ LB-IN}$

FORCES ON GEAR B:

$W_{tB} = \frac{T_B}{R_B} = \frac{3436 \text{ LB-IN}}{8 \text{ IN}} = 430 \text{ LB} \leftarrow = F_{Bx}$

$W_{rB} = t_B \tan 20^\circ = 156 \text{ LB} \downarrow = F_{By}$

FORCES ON SHAFT D

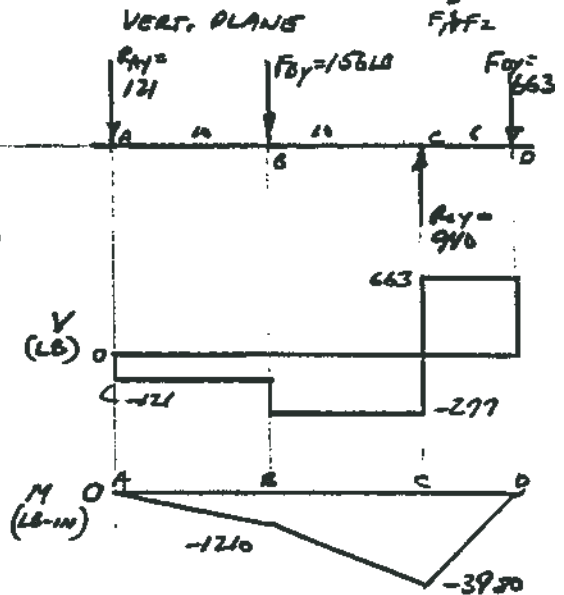
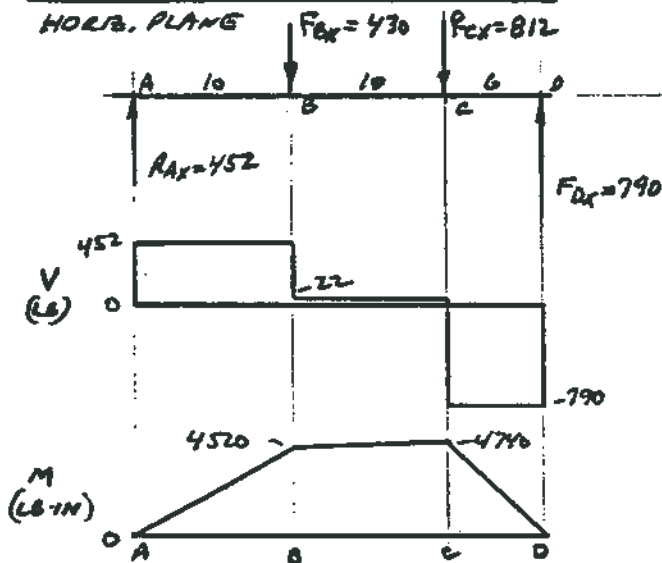
$F_1 - F_2 = \frac{T_D}{R_D} = \frac{3436 \text{ LB-IN}}{5.0 \text{ IN}} = 687 \text{ LB}$

$F_1 + F_2 = 1.5(F_1 - F_2) = 1.5(687) = 1031 \text{ LB} = F_D$

$F_{Dx} = F_D \cos 40^\circ = 790 \text{ LB} \rightarrow$

$F_{Dy} = F_D \sin 40^\circ = 663 \text{ LB} \downarrow$

BENDING MOMENT DIAGRAMS



$M_B = \sqrt{4520^2 + 1210^2} = 4679 \text{ LB-IN}$

$M_C = \sqrt{4740^2 + 3950^2} = 6189 \text{ LB-IN}$

$T_C = 3436 \text{ LB-IN}$

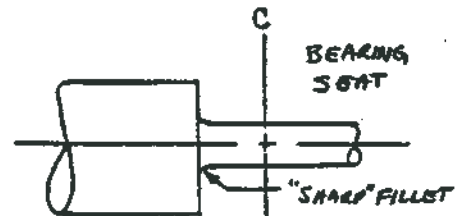
ASSUME  $K_t = 2.5$  AT LEFT OF C

SAE 1040 CD;  $S_u = 80 \text{ KSI}$ ;  $S_y = 71 \text{ KSI}$

$S_m = 30 \text{ KSI}$  (FIG. 5-8 MACHINED)  $C_A = 0.81$ ,  $C_s = 0.80$

$S_m' = C_A C_s S_m = (0.80)(0.81)(30) = 19440 \text{ PSI}$

$D_c = \frac{22(3)}{\pi} \sqrt{\frac{[2.5(6189)]^2}{19440} + \frac{3}{4} \left[ \frac{3436}{71000} \right]^2} = 2.90 \text{ IN.}$



DESIGN GEOMETRY AT C

USE  $N = 3$

SPECIFY  $D = 3.00 \text{ IN.}$   
 $C_s = 0.78$  OK

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FIGURE P12-2

TORQUE ON PULLEY A:  $T_A = 63000(10)/200 = 3150 \text{ LB-IN}$

" ON GEAR C:  $T_C = 63000(6)/200 = 1890 \text{ LB-IN}$

" ON SPROCKET D:  $T_D = 63000(4)/200 = 1260 \text{ LB-IN}$

TORQUE DISTRIBUTION IN SHAFT:  $T_{A-C} = 3150 \text{ LB-IN}$ ;  $T_{C-D} = 1260 \text{ LB-IN}$ ;  $T_{D-E} = 0$

FORCES ON PULLEY A:

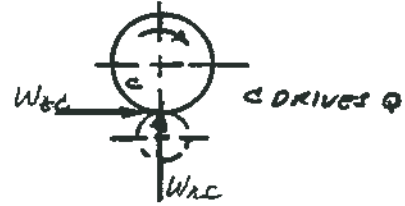
$F_1 - F_2 = T_A / R_A = 3150 / 10 = 315 \text{ LB}$

$F_1 + F_2 = 2.0(F_1 - F_2) = 2.0(315) = 630 \text{ LB} \uparrow = F_{AY}$ ;  $F_{AX} = 0$

FORCES ON GEAR C:

$W_{tc} = T_c / R_c = \frac{1890 \text{ LB-IN}}{5.0 \text{ IN}} = 378 \text{ LB} \rightarrow = F_{cx}$

$W_{nc} = W_{tc} \tan 20^\circ = 138 \text{ LB} \uparrow = F_{cy}$

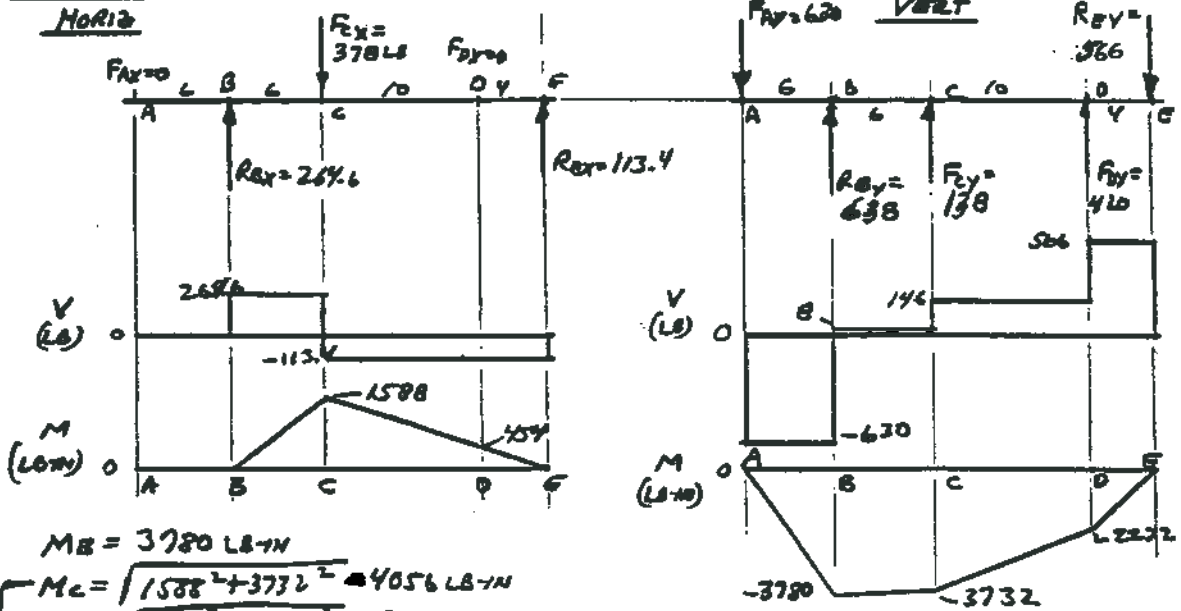


FORCES ON SPROCKET D:

$F_1 = F_D = F_{Dy} = \frac{T_D}{R_D} = \frac{1260}{3} = 420 \text{ LB} \uparrow$

$F_{Dx} = 0$

BENDING MOMENT DIAGRAMS



$M_B = 3780 \text{ LB-IN}$

$M_C = \sqrt{1588^2 + 3732^2} = 4056 \text{ LB-IN}$

$M_D = \sqrt{454^2 + 2272^2} = 2317 \text{ LB-IN}$

$T_c = 3150 \text{ LB-IN TO LEFT}$ ;  $M_c = 3.0 \text{ RING G.}$

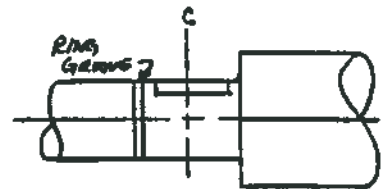
$T_d = 1260 \text{ LB-IN TO RIGHT}$ ;  $K_t = 2.0 \text{ KEY}$

SAE 1117 CD;  $S_u = 69 \text{ KSI}$ ;  $S_y = 57 \text{ KSI}$

$S_m = 28 \text{ KSI}$ ;  $S_m' = C_s C_r S_u = (0.85)(0.81)(28) = 17.9 \text{ KSI}$

$D_c = \left[ \frac{32(3)}{\pi} \sqrt{\left( \frac{3.0(4056)}{17900} \right)^2 + \frac{3}{4} \left( \frac{3150}{57000} \right)^2} \right]^{1/3} = 2.75 \text{ IN.}$

INCREASE BY 6% AT GROOVE  $D \approx 1.06(2.75) = 2.92 \text{ IN.}$



DESIGN AT C

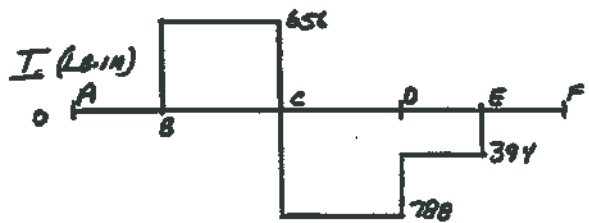
SPECIFY  $D_c = 3.00 \text{ IN}$

$C_s = 0.78 \text{ OK}$

FIGURE P12-3  
 24 TORQUE ON GEAR B:  $T_B = 63000(5)/480 = 656 \text{ LB}\cdot\text{IN}$   
 ON SHEAVES D AND E:  $T_D = T_E = 63000(3)/480 = 394 \text{ LB}\cdot\text{IN}$   
 ON SPROCKET C:  $T_C = 63000(11)/480 = 1444 \text{ LB}\cdot\text{IN}$

TORQUE DISTRIBUTION IN SHAFT:

$T_{AB} = 0$ ;  $T_{BC} = 656 \text{ LB}\cdot\text{IN}$   
 $T_{CD} = 788 \text{ LB}\cdot\text{IN}$   
 $T_{DE} = 394 \text{ LB}\cdot\text{IN}$ ;  $T_{EF} = 0$



FORCES ON GEAR B:

$W_{tB} = T_B / r_B = 656 / 1.5 = 437 \text{ LB}$   $\rightarrow = F_{Bx}$   
 $W_{nB} = W_{tB} \tan 20^\circ = 159 \text{ LB}$   $\uparrow = F_{By}$

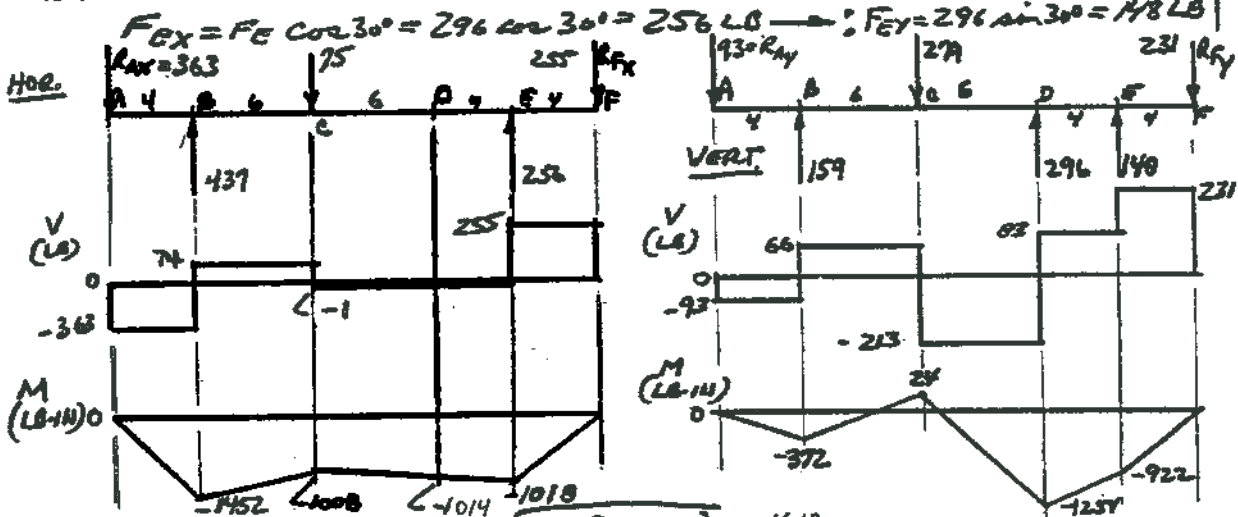
FORCES ON SPROCKET C:  $F_C = T_C / r_C = 1444 / 5.0 = 289 \text{ LB}$   $\nearrow 15^\circ$

$F_{Cx} = F_C \sin 15^\circ = 289 \sin 15^\circ = 75 \text{ LB}$   $\rightarrow$   
 $F_{Cy} = F_C \cos 15^\circ = 289 \cos 15^\circ = 279 \text{ LB}$   $\downarrow$

FORCES ON SHEAVE D:  $F_1 = F_2 = T_D / r_D = 394 / 2.0 = 197 \text{ LB}$

$F_D = F_1 + F_2 = 1.5(F_1 - F_2) = 1.5(197) = 296 \text{ LB}$   $\uparrow = F_{Dy}$ ;  $F_{Dx} = 0$

FORCES ON SHEAVE E:  $F_E = F_D = 296 \text{ LB}$   $\nearrow 30^\circ$



WORST CASE AT D:  $M_D = \sqrt{1014^2 + 1257^2} = 1613 \text{ LB}\cdot\text{IN}$

$T_D = 788 \text{ LB}\cdot\text{IN}$  (ATO & TO LEFT);  $K_t = 3.0$  RST. RING  
 SAE 1137 OQT 1300;  $S_u = 87 \text{ KSI}$ ;  $S_y = 60 \text{ KSI}$ ;  $S_m = 33 \text{ KSI}$  (FIG 5-8)

$S_m = 0.9 \times 0.81 \times 33 = 24.1 \text{ KSI}$  ( $C_s = 0.80$  EST.)

$D_D = \left[ \frac{32(3)}{\pi} \sqrt{\left( \frac{3.0(1613)}{24100} \right)^2 + 3 \left( \frac{788}{60000} \right)^2} \right]^{1/3} = 1.83 \text{ IN.}$

INCREASE BY 6%;  $D_D = 1.06(1.83) = 1.94 \text{ IN.}$ ; SPECIFY  $D_D = 2.00 \text{ IN}$

CHECK:  $C_s = 0.81$ ;  $S_m = 21700 \text{ PSI}$ ;  $D_{D \text{ req}} = 2.01 \text{ IN}$ ; ACCEPTABLE TO USE  $D_D = 2.00 \text{ IN}$

25

FIGURE P12-4

TORQUE ON GEAR A:  $T_A = 63000(40)/120 = 21000 \text{ LB}\cdot\text{IN}$

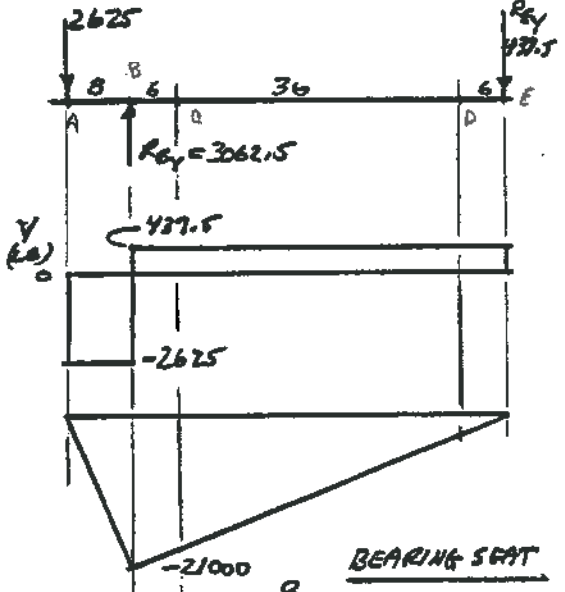
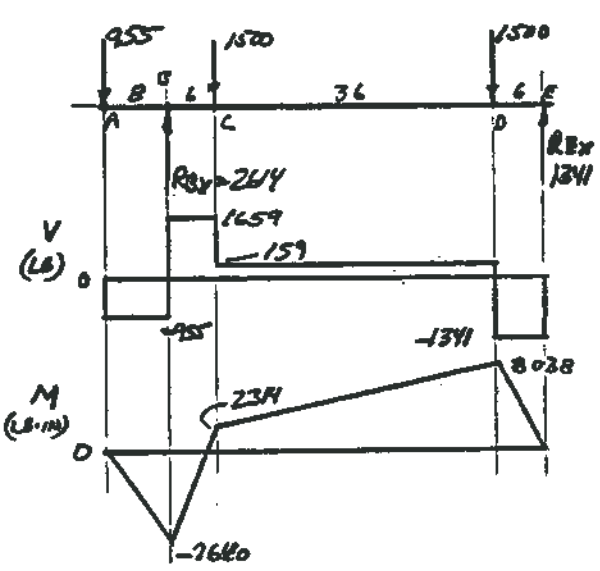
" " ON SPROCKETS C & D:  $T_C = T_D = 63000(20)/120 = 10500 \text{ LB}\cdot\text{IN}$

TORQUE IN SHAFT:  $T_{A-C} = 21000 \text{ LB}\cdot\text{IN}$ ;  $T_{C-D} = 10500 \text{ LB}\cdot\text{IN}$ ;  $T_{D-E} = 0$

FORCES ON GEAR A:  $W_{GA} = T_A/R_A = 21000/8 = 2625 \text{ LB} = F_{AY}$

$W_{HA} = W_{GA} \tan 20^\circ = 955 \text{ LB} = F_{AX}$

FORCES ON SPROCKETS C & D:  $F_{CX} = F_{DX} = T/R = 10500/7 = 1500 \text{ LB}$



AT B:  $T = 21000 \text{ LB}\cdot\text{IN}$   
 $M = \sqrt{7640^2 + 21000^2} = 22347 \text{ LB}\cdot\text{IN}$

TO RIGHT OF B:  $K_t = 2.5$  - SMALL FILLET RAD.

$$D_{BR} = \left[ \frac{32(3)}{\pi} \sqrt{\left( \frac{2.5(22347)}{13400} \right)^2 + \frac{3}{4} \left( \frac{21000}{51000} \right)^2} \right]^{1/3}$$

$D_{BR} = 5.05 \text{ IN.}$  SPECIFY 5.20 IN  $C_s = 0.73$  OK

TO LEFT OF B:  $K_t = 1.5$  - WELL RND. FILLET

$$D_{BL} = \left[ \frac{32(3)}{\pi} \sqrt{\left( \frac{1.5(22347)}{13400} \right)^2 + \frac{3}{4} \left( \frac{21000}{51000} \right)^2} \right]^{1/3}$$

$D_{BL} = 4.27 \text{ IN.}$  SPECIFY 4.50 IN  $C_s = 0.74$  OK

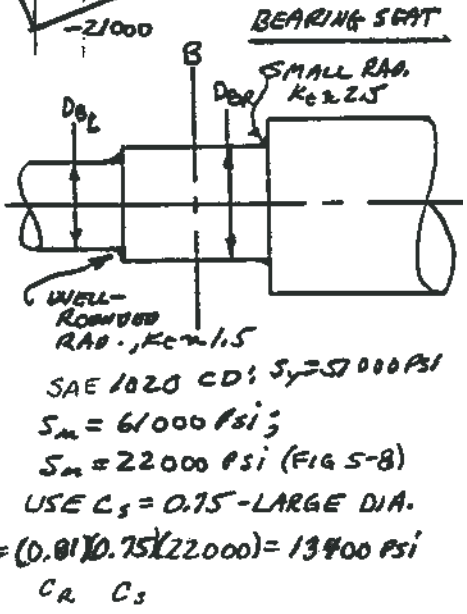




FIGURE P12-5

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TORQUE ON SHEAVE A:  $T_A = 63000(10)/240 = 2625 \text{ LB}\cdot\text{IN} = T_{AD} \text{ IN SHAFT}$

TORQUE ON GEAR D:  $T_D = 63000(15)/240 = 3938 \text{ LB}\cdot\text{IN}$

TORQUE ON SPROCKET E:  $T_E = 63000(5)/240 = 1313 \text{ LB}\cdot\text{IN} = T_{DE} \text{ IN SHAFT}$

FORCES ON SHEAVE A:  $F_1 - F_2 = T_A / r_A = 2625 / 6 = 438 \text{ LB}$ ;  $F_{AX} = 0$

$F_A = F_{AY} = F_1 + F_2 = 1.5(F_1 - F_2) = 1.5(438) = 657 \text{ LB}$

FORCES ON GEAR D:  $W_{tD} = T_D / r_D = 3938 / 4 = 985 \text{ LB}$

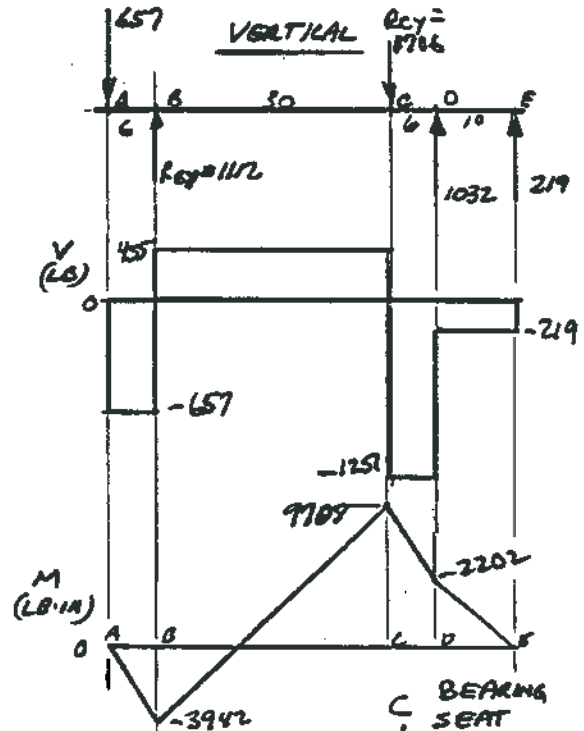
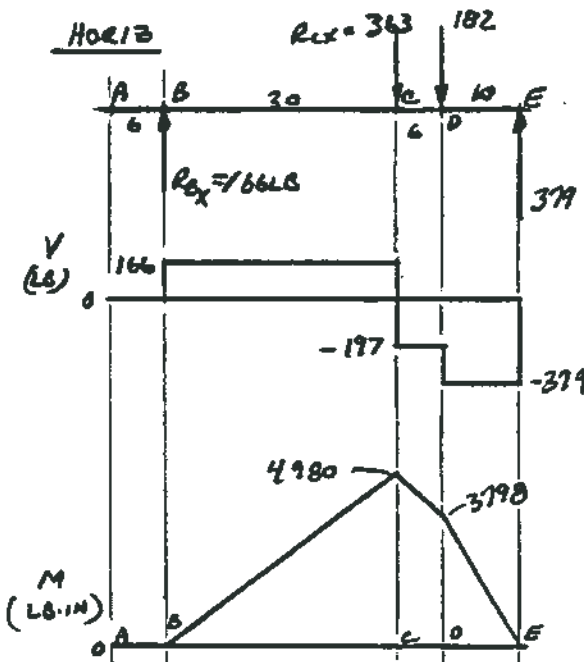
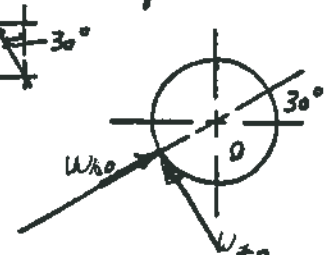
$W_{AD} = W_{tD} \tan 20^\circ = 358 \text{ LB}$

$F_{DX} = 985 \sin 30^\circ - 358 \cos 30^\circ = 182 \text{ LB}$

$F_{DY} = 985 \cos 30^\circ + 358 \sin 30^\circ = 1032 \text{ LB}$

FORCES ON SPROCKET E:  $F_E = T_E / r_E = 1313 / 3 = 438 \text{ LB}$

$F_{EX} = F_E \cos 30^\circ = 379 \text{ LB}$ ;  $F_{EY} = F_E \sin 30^\circ = 219 \text{ LB}$



AT C:  $T = 2625 \text{ LB}\cdot\text{IN}$ ; USE  $N = 4$  SHAFT

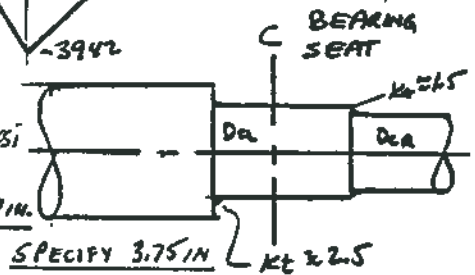
$M = \sqrt{4980^2 + 9708^2} = 10911 \text{ LB}\cdot\text{IN}$

CHOOSE 1137 CD -  $S_y = 82000 \text{ PSI}$

$S_m = 98 \text{ KSI}$ ;  $S_n = 37 \text{ KSI}$ ;  $S_m' = 0.8(0.8)(98) = 24.0 \text{ KSI}$

$D_{CL} = \left[ \frac{32(Y)}{\pi} \sqrt{\left( \frac{2.5(10911)}{24000} \right)^2 + \frac{3}{4} \left( \frac{2625}{82000} \right)^2} \right]^{1/3} = 3.59 \text{ IN}$


WITH  $K_t = 1.5$ ,  $D_{CR} = 3.03 \text{ IN}$ . SPECIFY 3.20 IN  
 $C_s = 0.77$  OK

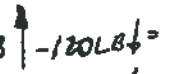



SPECIFY 3.75 IN  
 $C_s = 0.76$  OK

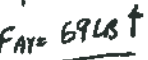
27 FIGURE P12-6  
TORQUE ON SHEAVE A AND GEAR E:  $T = 63000(20)/30 = 4065 \text{ LB}\cdot\text{IN}$

FORCES ON SHEAVE A:  $F_1 - F_2 = T/R_A = 4065/11 = 370 \text{ LB}$

$F_A = F_1 + F_2 = 1.5(F_1 - F_2) = 554 \text{ LB.}$  

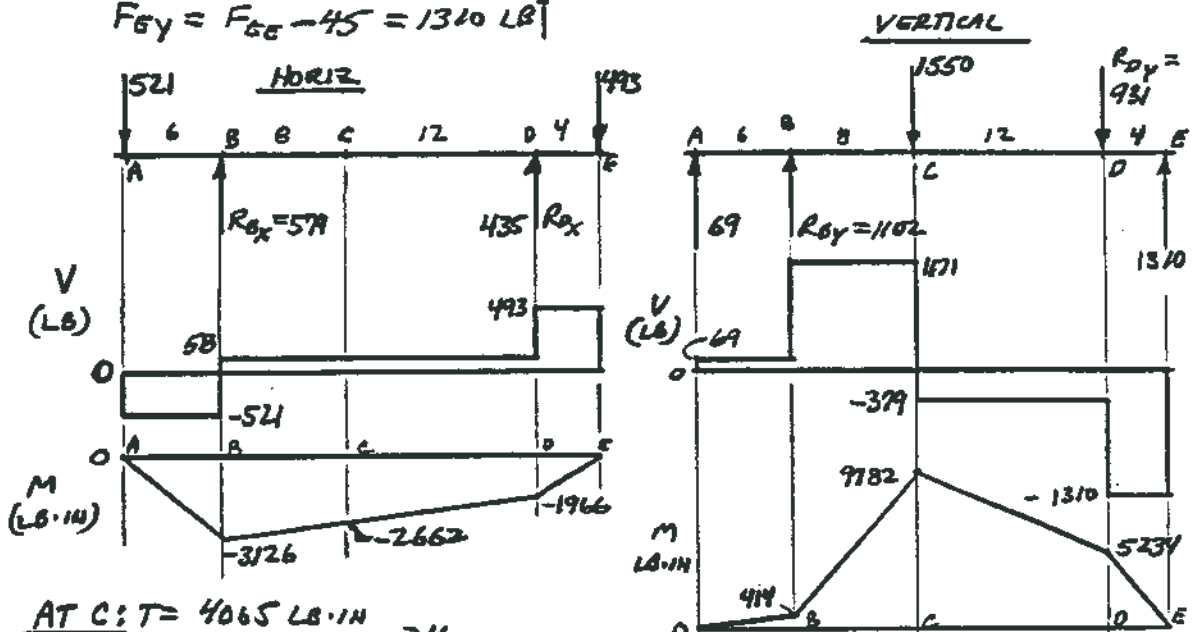
$F_{Ax} = F_A \cos 20^\circ = 521 \text{ LB}$  ;  $F_{Ay} = F_A \sin 20^\circ - 120 \text{ LB} = 189 \text{ LB}$  

FORCES ON GEAR E:  $W_{GE} = T/R_E = 4065/3 = 1355 \text{ LB}$  

$F_{Ay} = 69 \text{ LB}$  

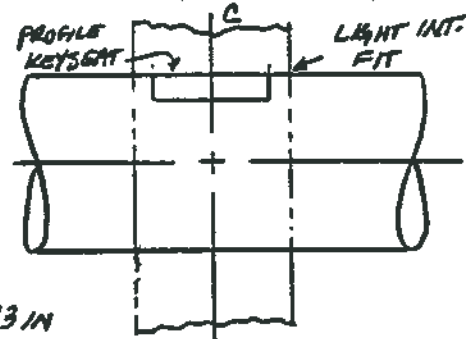
$W_{AE} = W_{GE} \tan 20^\circ = 493 \text{ LB}$   $\rightarrow = F_{Ex}$

$F_{Ey} = F_{GE} - 45 = 1310 \text{ LB}$  



AT C:  $T = 4065 \text{ LB}\cdot\text{IN}$   
 $M = [2662^2 + 9782^2]^{1/2} = 10138 \text{ LB}\cdot\text{IN}$   
 USE  $K_t = 2.0$  ;  $N = 3$   
 SAE 1050 CD ;  $S_y = 84 \text{ KSI}$   
 $S_u = 100 \text{ KSI}$  ;  $S_m = 38 \text{ KSI}$   
 $S_m' = 0.8(0.8)(70) = 24.6 \text{ KSI}$

$D_c = \left[ \frac{32(3)}{\pi} \sqrt{\left( \frac{2.0(10138)}{24600} \right)^2 + \frac{3(4065)}{4(84000)} \right]^{1/3} = 2.93 \text{ IN}$   
 SPECIFY  $D_c = 3.00 \text{ IN}$  ;  $C_s = 0.78 \text{ OK}$



28 TORQUE ON GEAR A:  $T_A = 63000(30)/480 = 3938 \text{ LB}\cdot\text{IN}$

TORQUE ON GEAR C:  $T_C = 63000(50)/480 = 6563 \text{ LB}\cdot\text{IN}$

TORQUE ON SHEAVES D AND E:  $T_D = T_E = 63000(10)/480 = 1313 \text{ LB}\cdot\text{IN}$

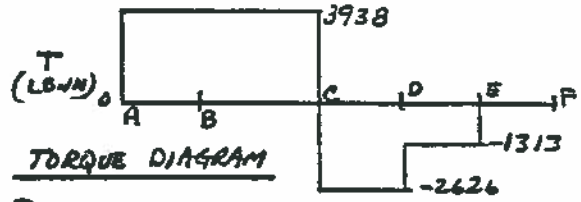
TORQUE IN SHAFT:  $T_{AC} = 3938 \text{ LB}\cdot\text{IN}$  ;  $T_{C-D} = 2626 \text{ LB}\cdot\text{IN}$  ;  $T_{D-E} = 1313 \text{ LB}\cdot\text{IN}$   
 $T_{E-F} = 0$

(CONTINUED ON NEXT PAGE)

28

(FIGURE P12-7)

(CONTINUED)



FORCES ON GEAR A:

$$W_{bA} = T_A / r_A = 3938 / 2.5 = 1575 \text{ LB} \uparrow = F_{Ay}$$

$$W_{tA} = W_{bA} \tan 20^\circ = 1575 \tan 20^\circ = 573 \text{ LB} \rightarrow = F_{Ax}$$

FORCES ON GEAR C:  $W_{bC} = T_C / r_C = 6563 / 5 = 1313 \text{ LB} \rightarrow = F_{Cx}$

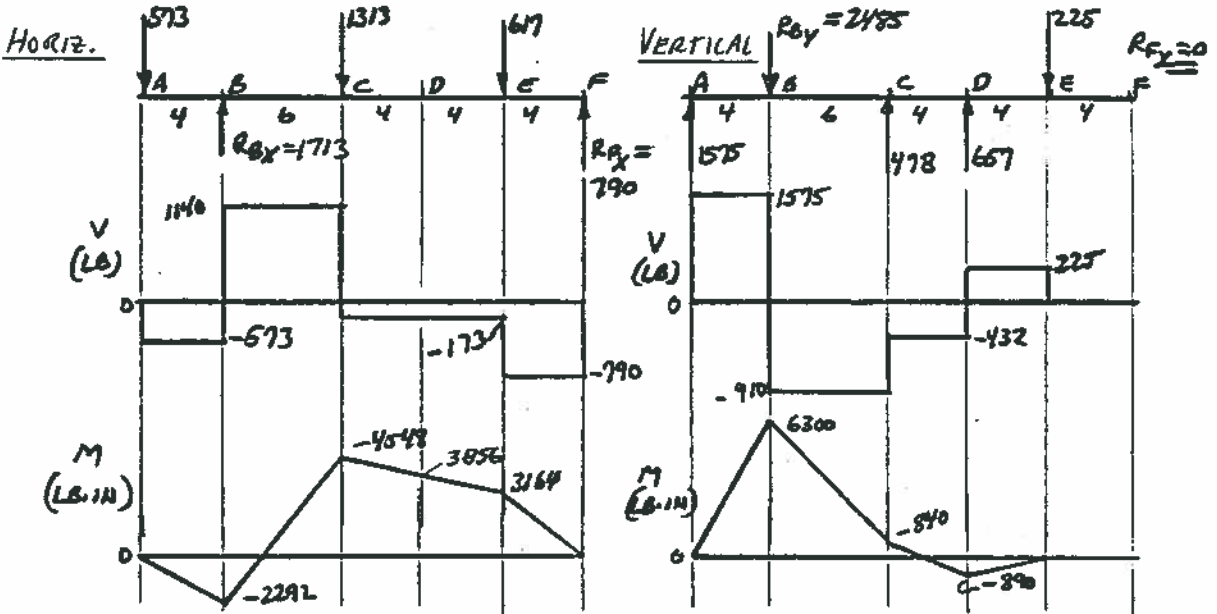
$$W_{tC} = W_{bC} \tan 20^\circ = 1313 \tan 20^\circ = 478 \text{ LB} \uparrow = F_{Cy}$$

FORCES ON SHEAVE D:  $F_1 - F_2 = T_D / r_D = 1313 / 3 = 438 \text{ LB}$

$$F_D = F_{Dy} = F_1 + F_2 = 1.5(F_1 - F_2) = 1.5(438) = 657 \text{ LB} \uparrow : F_{Dx} = 0$$

FORCES ON SHEAVE E:  $F_E = F_D = 657 \text{ LB}$

$$F_{Ex} = F_E \cos 20^\circ = 617 \text{ LB} \rightarrow : F_{Ey} = F_E \sin 20^\circ = 225 \text{ LB} \downarrow$$



At B:  $T = 3938 \text{ LB}\cdot\text{IN}$ ;  $N = 3$

$$M_B = \sqrt{(2292)^2 + (6300)^2} = 6704 \text{ LB}\cdot\text{IN}$$

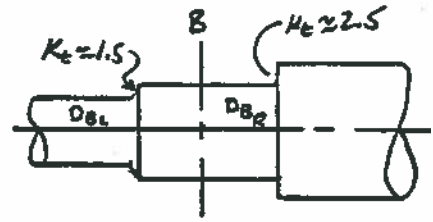
$$D_{BR} = \left[ \frac{32(3)}{\pi} \sqrt{\frac{2.5(6704)^2}{35,800} + \frac{3(3938)^2}{4(133,000)}} \right]^{1/3} = 2.43 \text{ IN}$$

$$D_{BL} = 2.05 \text{ IN WITH } K_t = 1.5$$

SPECIFY  $D = 2.20 \text{ IN}$

$$C_s = 0.80 \text{ OK}$$

SPECIFY  $D = 2.50 \text{ IN}$   
 $C_s = 0.78 \text{ OK}$



SAE 3140 OQT 1000

$$S_y = 133 \text{ KSI}; S_m = 152 \text{ KSI}$$


$$S_m = 52 \text{ KSI}; S_m' = C_s \cdot C_r \cdot S_m$$

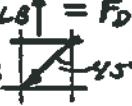

$$S_m' = (0.85)(0.81)(52) = 35.8 \text{ KSI}$$

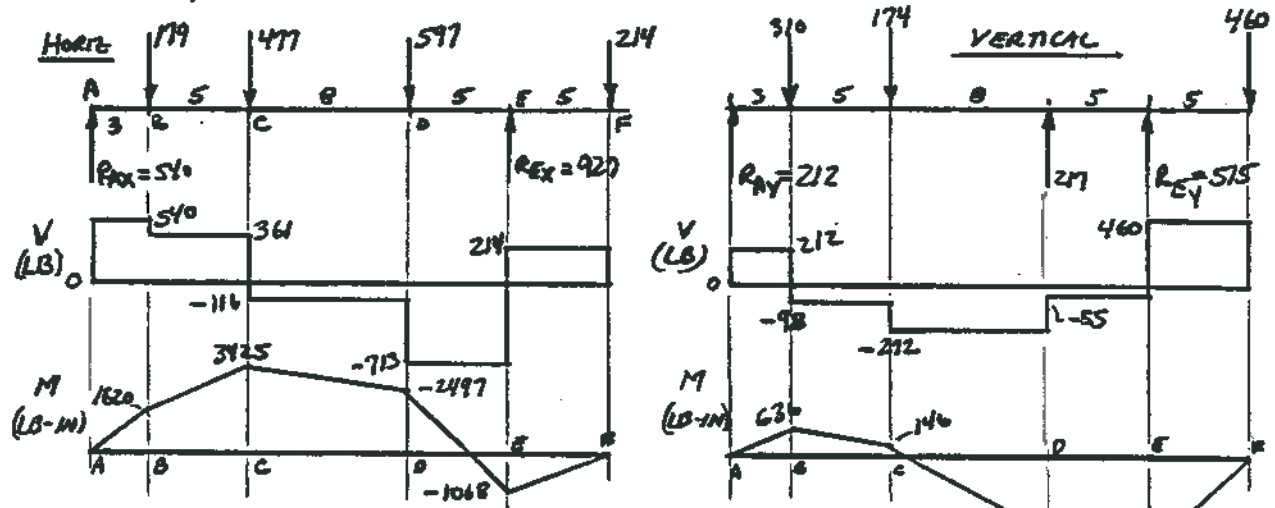
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FIGURE P12-9

TORQUE ON SHEAVE B:  $T_B = 63000(2.5)/220 = 716 \text{ LB}\cdot\text{IN}$   
 " ON GEARS C AND F:  $T_C = T_F = 63000(5)/220 = 1432 \text{ LB}\cdot\text{IN}$   
 " ON GEAR D:  $T_D = 63000(12.5)/220 = 3580 \text{ LB}\cdot\text{IN}$   
TORQUE IN SHAFT:  $T_{A-B} = 0$ ;  $T_{B-C} = 716 \text{ LB}\cdot\text{IN}$ ;  $T_{C-D} = 2148 \text{ LB}\cdot\text{IN}$   
 $T_{D-E} = 1432 \text{ LB}\cdot\text{IN}$

FORCES ON SHEAVE B:  $F_1 = F_2 = T_B/r_B = 716/3 = 239 \text{ LB}$   
 $F_B = F_1 + F_2 = 1.5(F_1 - F_2) = 1.5(239) = 358 \text{ LB}$    
 $F_{Bx} = F_B \sin 30^\circ = 179 \text{ LB} \leftarrow$ ;  $F_{By} = F_B \cos 30^\circ = 310 \text{ LB} \downarrow$   
FORCES ON GEAR C:  $W_{tc} = T_C/r_C = 1432/3 = 477 \text{ LB} \leftarrow = F_{cx}$   
 $W_{tc} = W_{tc} \tan 20^\circ = (477) \tan 20^\circ = 174 \text{ LB} \downarrow = F_{cy}$

FORCES ON GEAR D:  $W_{d0} = T_D/r_D = 3580/6 = 597 \text{ LB} \leftarrow = F_{dx}$   
 $W_{d0} = W_{d0} \tan 20^\circ = (597) \tan 20^\circ = 217 \text{ LB} \uparrow = F_{dy}$   
FORCES ON GEAR F:  $W_{cf} = W_{cc} = 477 \text{ LB}$    $W_{cf} = W_{cc} = 174 \text{ LB}$    
 $F_{Fx} = 477 \sin 45^\circ - 174 \cos 45^\circ = 214 \text{ LB} \leftarrow$   
 $F_{Fy} = 477 \cos 45^\circ + 174 \sin 45^\circ = 460 \text{ LB} \downarrow$

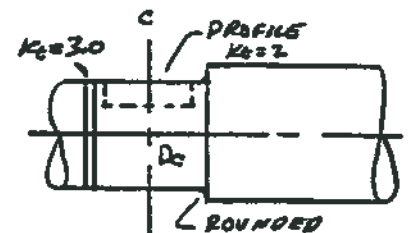


AT C:  $T = 2148 \text{ LB}\cdot\text{IN}$  TO RIGHT,  $k_c = 2.0$ ;  $M_c = 3428 \text{ LB}\cdot\text{IN}$   
 $T = 716 \text{ LB}\cdot\text{IN}$  TO LEFT,  $k_c = 3.0$  " " "

$$D_{ca} = \left[ \frac{32(3)}{\pi} \sqrt{\left( \frac{2.0(3428)}{15100} \right)^2 + \frac{3(2148)}{4(57000)}} \right]^{1/3} = 2.40 \text{ IN}$$

$$D_{cl} = \left[ \frac{32(3)}{\pi} \sqrt{\left( \frac{3.0(3428)}{15100} \right)^2 + \frac{3(716)}{4(57000)}} \right]^{1/3} = 2.75 \text{ IN}$$

CRITICAL



INCREASE BY 6%:  $D_c = 1.06(2.75)$   
 $D_c = 2.91 \text{ IN}$

SAC1020 CD:  $S_y = 57 \text{ ksi}$ ;  $S_u = 61 \text{ ksi}$   
 $S_m = 22 \text{ ksi}$   
 $S_m' = (0.85)(.81)(22) = 15.1 \text{ ksi}$   
 $C_s \quad C_R$

SPECIFY  $D_c = 3.00 \text{ IN}$   $C_s = 0.78$  OK

30

## FIGURE P12-17

TORQUE AT FAN:  $T_A = 63000(12)/475 = 1592 \text{ LB}\cdot\text{IN}$

" ON PULLEY D:  $T_D = 63000(3.5)/475 = 464 \text{ LB}\cdot\text{IN}$

" ON SHEAVE C:  $T_C = 63000(15.5)/475 = 2056 \text{ LB}\cdot\text{IN}$

TORQUE IN SHAFT:  $T_{AC} = 1592 \text{ LB}\cdot\text{IN}$ ;  $T_{CD} = 464 \text{ LB}\cdot\text{IN}$ ;  $T_{DE} = 0$

FORCES ON A:  $F_{Ax} = 0$ ;  $F_{Ay} = 37 \text{ LB}$  THE FAN WOULD ALSO PRODUCE AN AXIAL THRUST FORCE BUT ITS EFFECT ON SHAFT DIA. IS SMALL

FORCES ON C (V-BELT)  
 $F_1 - F_2 = T_C / R_C = 2056 / 5.0 = 411 \text{ LB}$

$F_C = F_{Cx} = F_1 + F_2 = 1.5(F_1 - F_2) = 1.5(411) = 617 \text{ LB}$   $\rightarrow$ ;  $F_{Cy} = 0$

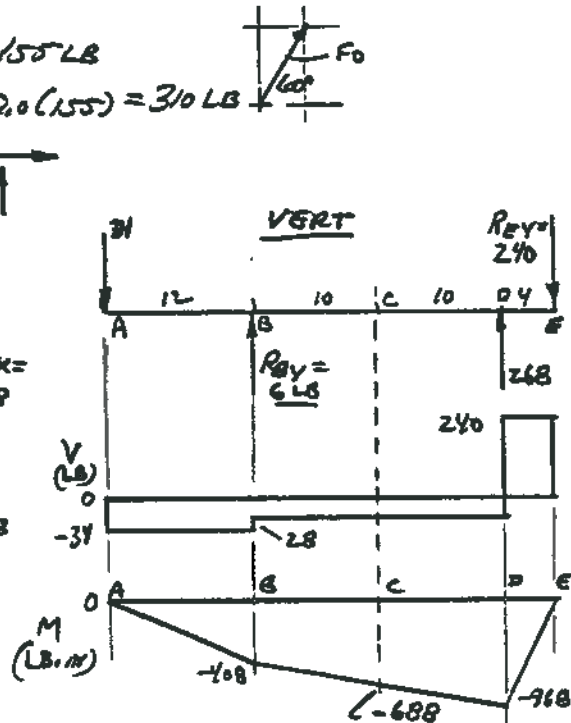
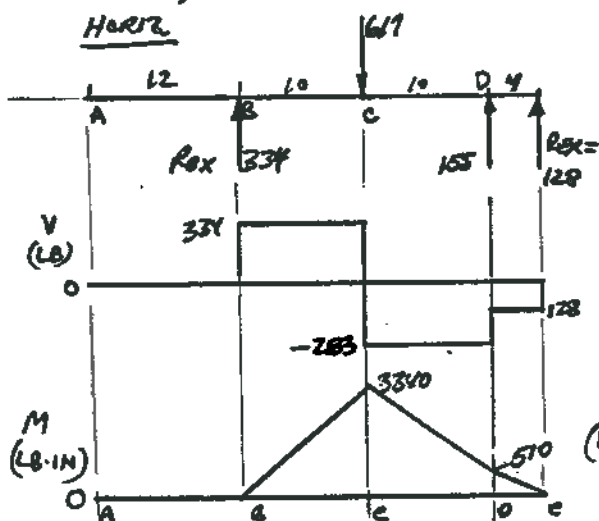
FORCES ON D (FLAT BELT)

$F_1 - F_2 = T_D / R_D = 464 / 3 = 155 \text{ LB}$

$F_D = F_1 + F_2 = 2.0(F_1 - F_2) = 2.0(155) = 310 \text{ LB}$

$F_{Dx} = F_D \cos 60^\circ = 155 \text{ LB}$   $\rightarrow$

$F_{Dy} = F_D \sin 60^\circ = 268 \text{ LB}$   $\uparrow$



ATC:  $T_{C \text{ LEFT}} = 1592 \text{ LB}\cdot\text{IN}$ ;  $T_{C \text{ RIGHT}} = 464 \text{ LB}\cdot\text{IN}$

$M_C = \sqrt{3340^2 + 688^2} = 3410 \text{ LB}\cdot\text{IN}$

ATC & TO LEFT:  $K_C = 2.0$  KEYS

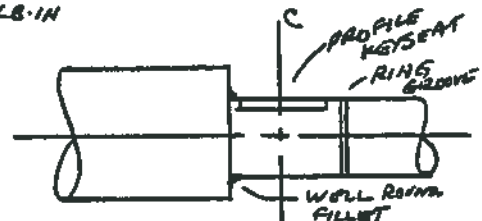
$D = \left[ \frac{32(3)}{\pi} \sqrt{\frac{2.0(3410)^2}{.00} + \frac{3}{4} \left( \frac{1592}{90000} \right)^2} \right]^{1/3} = 2.10 \text{ IN}$

AT RIGHT DFC:  $K_C = 3.0$  - RING GROOVE

$D = \left[ \frac{32(3)}{\pi} \sqrt{\frac{3.0(3410)^2}{.00} + \frac{3}{4} \left( \frac{464}{90000} \right)^2} \right]^{1/3} = 2.29 \text{ IN}$  CRITICAL

INCREASE BY 6%:  $D = 1.06(2.29) = 2.42 \text{ IN. MIN}$

SPECIFY  $D = 2.50 \text{ IN}$   $C_s = 0.79 \text{ OK}$



SAE 1144 CD:  $S_{ey} = 90 \text{ KSI}$

$S_{uw} = 100 \text{ KSI}$ ;  $S_m = 38 \text{ KSI}$

$S'_m = (0.85)(0.81)(38) = 26.2 \text{ KSI}$

$C_s \quad C_e$



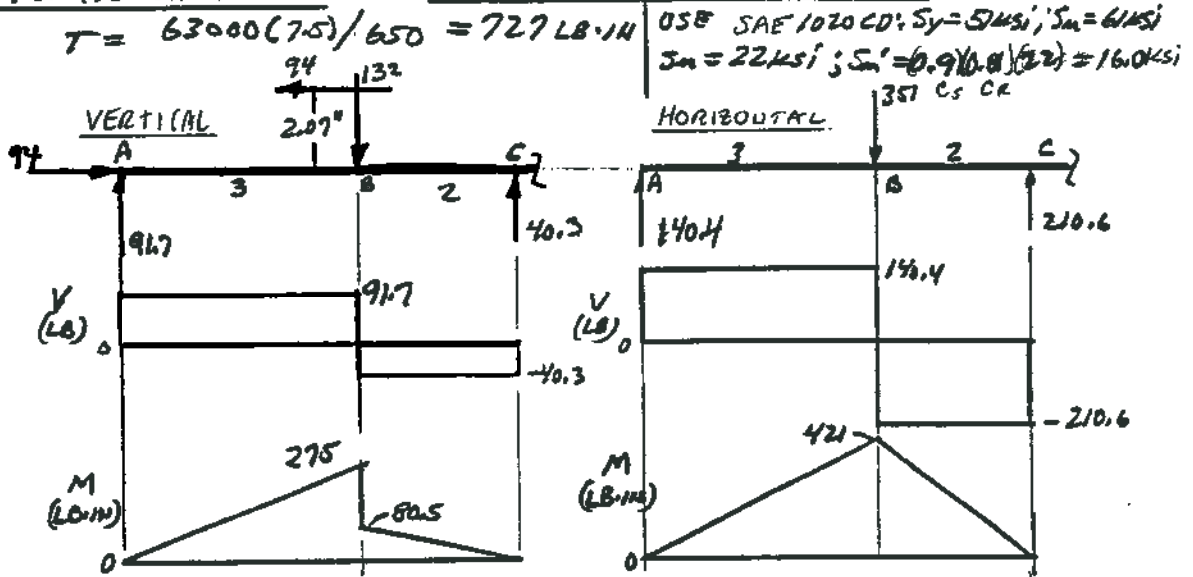
FIGURE P12-31

31

TORQUE ON GEAR B AND IN SHAFT FROM B TO COUPLING;

AND

32



AT B:  $M = \sqrt{275^2 + 421^2} = 503 \text{ LB}\cdot\text{IN}$ ; USE  $N = 3$ ;  $K_t = 2.0$  (KEYSEAT)

$D_B = \left[ \frac{32(3)}{\pi} \sqrt{\left( \frac{2.0(503)}{16000} \right)^2 + \frac{3(727)}{4(51000)}^2} \right]^{1/3} = 1.25 \text{ IN}$  C.S. 70.9 OK  
 SPECIFY

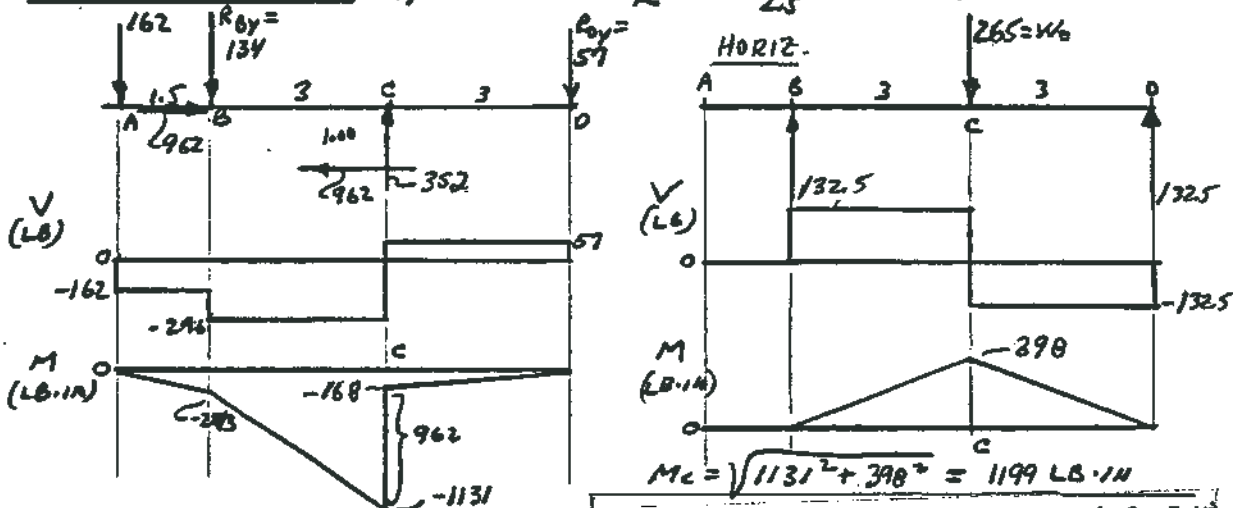
33

TORQUE =  $63000(7.5)/1750 = 270 \text{ LB}\cdot\text{IN}$

AND

34

FORCES ON SHEAVE:  $F_{Ay} = 1.5(F_1 + F_2) = \frac{1.5T}{R} = \frac{1.5(270)}{2.5} = 162 \text{ LB}$



ASSUME TORQUE IS NEGLIGIBLE; USE  $N = 4$

$\sigma = \frac{K_t M}{S} = \frac{1.5(1199)}{\pi(1.614)^3/32} = 4357 \text{ PSI} = \frac{S_m'}{N}$

$S_m' = C_s C_R S_m = 0.83(0.81) S_m = 0.67 S_m$   
 REQD  $S_m' = N \sigma = 4(4357) = 17428 \text{ PSI} = 0.67 S_m$   
 REQD  $S_m = S_m' / 0.67 = 26000 \text{ PSI}$   
 FROM FIG. 5-8;  $S_m \approx 68 \text{ KSI}$

SPECIFY SAE 1040 CD;  $S_u = 80 \text{ KSI}$ ;  $S_y = 70 \text{ KSI}$

$S_m = 30 \text{ KSI}$ ;  $S_m' = 0.67(30) = 20.1 \text{ KSI}$

CHECK WITH EQ. 12-24 WITH  $M$  AND  $T$ .

$D_c = \frac{32(4)}{\pi} \sqrt{\left( \frac{1.5(1199)}{20100} \right)^2 + \frac{3(270)}{4(70000)}^2} = 1.54 \text{ IN}$  OK

THIS IS LESS THAN ACTUAL  $D_c = 1.614 \text{ IN}$

FIGURE P12-35

35

TORQUES ON GEARS AND IN SHAFTS:

GEAR P, SHAFT 1:  $T_1 = 63000(5)/1800 = 175 \text{ LB}\cdot\text{IN}$

GEARS B & C, SHAFT 2:  $T_2 = 63000(5)/900 = 350 \text{ LB}\cdot\text{IN}$

GEAR Q, SHAFT 3:  $T_3 = 63000(5)/300 = 1050 \text{ LB}\cdot\text{IN}$

SHAFT 2: (SEE CH.10)  $\phi_m = 14\frac{1}{2}^\circ$ ;  $\psi = 45^\circ$

GEAR B

$$W_{tB} = T_2/r_b = 350/1.5 = 233 \text{ LB} \rightarrow \quad (\text{EQ 10-2})$$

$$W_{xB} = W_{tB} \tan \psi = 233 \tan 45^\circ = 233 \text{ LB} \quad (\text{EQ 10-8})$$

$$\phi_c = \tan^{-1} \left( \frac{\tan \phi_m}{\cos \psi} \right) = 20.1^\circ \text{ OR } \tan \phi_c = \frac{\tan \phi_m}{\cos \psi} = 0.366 \quad (\text{EQ 10-1})$$

$$W_{rB} = W_t \tan \phi_c = 233(0.366) = 85 \text{ LB} \quad (\text{EQ 10-7})$$

SIMILARLY:

GEAR C

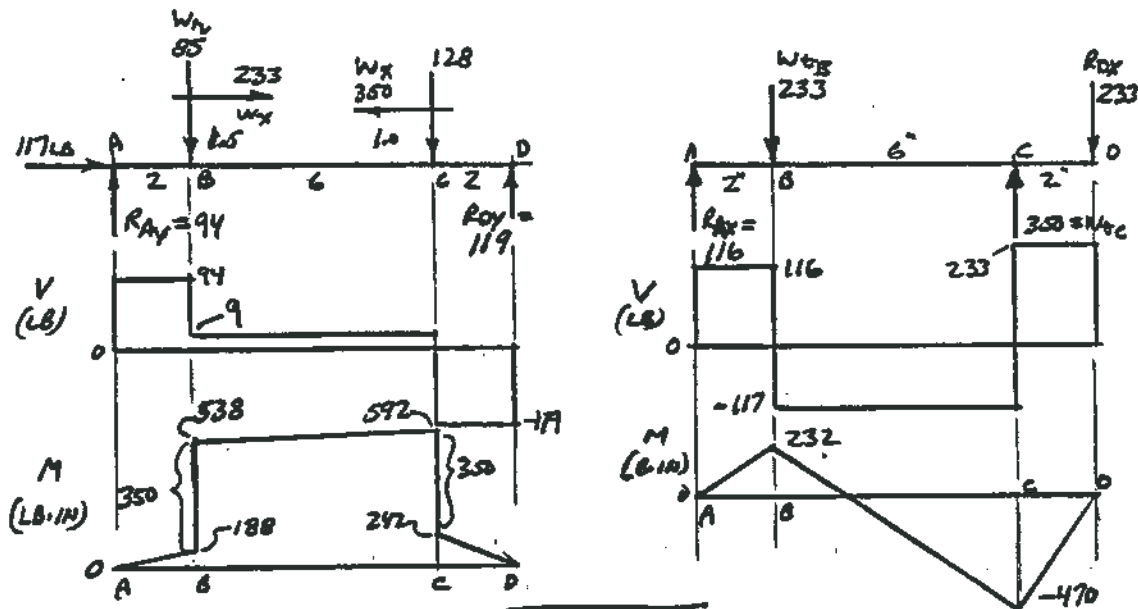
$$W_{tC} = T_2/r_c = 350/1 = 350 \text{ LB} \leftarrow$$

$$W_{xC} = W_{tC} \tan \psi = 350 \tan 45^\circ = 350 \text{ LB}$$

$$W_{rC} = W_t \tan \phi_c = 350(0.366) = 128 \text{ LB}$$

FOR SAE 4140 Q&T 1200;  $S_y = 114 \text{ KSI}$ ;  $S_u = 130 \text{ KSI}$ ;  $S_m = 46 \text{ KSI}$  (FIG. 5-8)

$$S_m' = C_s C_R S_m = (0.9)(0.81)(46 \text{ KSI}) = 33.5 \text{ KSI}$$



AT C:  $T = 350 \text{ LB}\cdot\text{IN}$ ;  $M_C = \sqrt{592^2 + 470^2} = 756 \text{ LB}\cdot\text{IN}$ ; USE  $K_t = 2.0$  (KEY)

$$D_c = \left[ \frac{32(3)}{\pi} \sqrt{\left( \frac{2.0(756)}{33500} \right)^2 + \frac{3(350)}{4(114000)}^2} \right]^{1/3} = 1.11 \text{ IN}$$

SPECIFY  $D_c = 1.25 \text{ IN}$ ;  $C_s = 0.85$  OK

SHAFTS 1 AND 3 CAN BE DESIGNED SIMILAR TO PROBLEM 33.

DATA FROM FIGURE 10-8 TO 10-12.

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PINION:  $T = 263 \text{ LB}\cdot\text{IN}$ ;  $M_E = \sqrt{220^2 + 95.8^2} = 240 \text{ LB}\cdot\text{IN}$ ;  $N = 3$ ;  $K_t = 2.0 (\text{KEY})$

SAE 1040 OQT 1200;  $S_y = 63 \text{ KSI}$ ;  $S_{ut} = 93 \text{ KSI}$ ;  $S_m = 36 \text{ KSI}$ ;  $S'_m = (0.8)(0.8)(36) =$

$$D_E = \left[ \frac{32(3)}{\pi} \sqrt{\left( \frac{2.0(240)}{25100} \right)^2 + \frac{3(263)^2}{4(63000)^2}} \right]^{1/3} = 0.84 \text{ IN} \quad S'_m = 25.1 \text{ KSI}$$

GEAR:  $T = 788 \text{ LB}\cdot\text{IN}$ ;  $M_F = 390 \text{ LB}\cdot\text{IN}$ ;  $N = 3$ ;  $K_t = 2.0 (\text{KEY})$

$$D_F = \left[ \frac{32(3)}{\pi} \sqrt{\left( \frac{2.0(390)}{25100} \right)^2 + \frac{3(788)^2}{4(63000)^2}} \right]^{1/3} = 1.00 \text{ IN} \quad C_s = 0.88 \text{ OK}$$

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$T_{\text{GEAR}} = 63000(4.0)/600 = 420 \text{ LB}\cdot\text{IN}$ ;  $T_{SP} = 420/2 = 210 \text{ LB}\cdot\text{IN}$

$F_{t1} = F_{t2} = T_{SP}/5 = 210/5 = 42 \text{ LB}$

$\Gamma = \tan^{-1}(15/15) = 71.6^\circ$

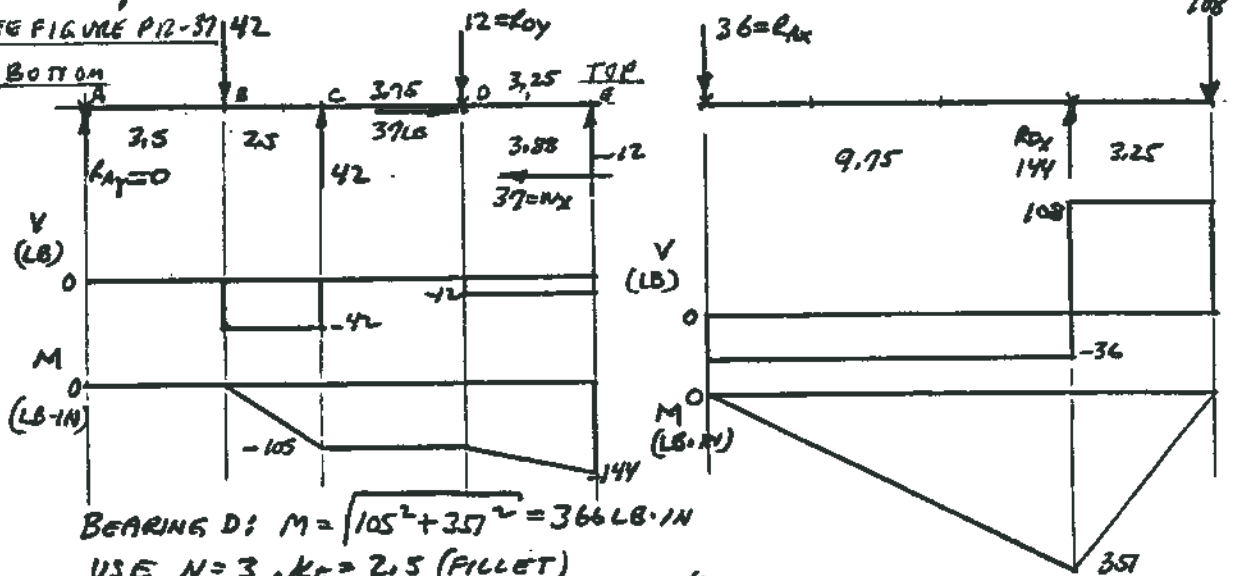
$W_{t2} = T_{SP}/R_m = 420/3.88 = 108 \text{ LB}$

$\gamma = \tan^{-1}(15/45) = 18.4^\circ$

$W_{t1} = W_t \tan \phi \cos \Gamma = (108) \tan 20^\circ \cos 71.6^\circ = 12 \text{ LB}$

$W_{x1} = W_t \tan \theta \sin \Gamma = (108) \tan 21^\circ \sin 71.6^\circ = 37 \text{ LB}$

SEE FIGURE P12-37 42



BEARING D:  $M = \sqrt{105^2 + 35^2} = 366 \text{ LB}\cdot\text{IN}$

USE  $N = 3$ ,  $K_t = 2.5 (\text{FILLET})$

$$D_0 = \left[ \frac{32(3)}{\pi} \sqrt{\left( \frac{2.5(366)}{44600} \right)^2 + \frac{3(420)}{4(152000)^2}} \right]^{1/3} = 0.86 \text{ IN}$$

SPECIFY  $D = 1.00 \text{ IN}$   $C_s = 0.88$  OK

SAE 4140 OQT 1000

$S_y = 152 \text{ KSI}$ ;  $S_{ut} = 168 \text{ KSI}$

$S_m = 58 \text{ KSI}$ ;  $S'_m = (0.95)(0.8)(58) =$

$S'_m = 44.6 \text{ KSI}$

38

$P = 15.0 \text{ HP}; m_1 = 1725 \text{ RPM}; m_2 = 575 \text{ RPM}; m_3 = 287.5 \text{ RPM}$

TORQUE ON SHAFT 1 =  $T_1 = T_A = 63000(15)/1725 = 548 \text{ LB}\cdot\text{IN.}$

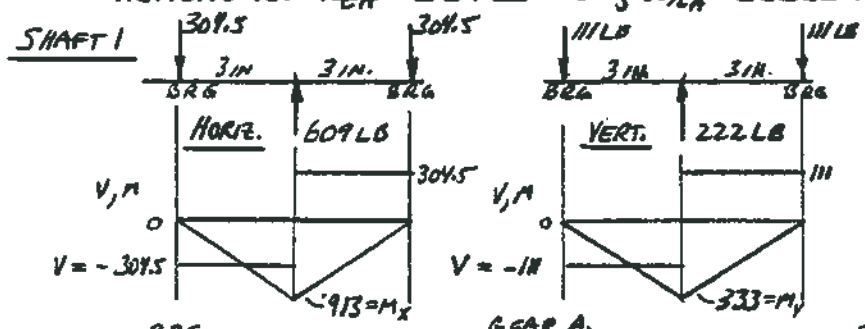
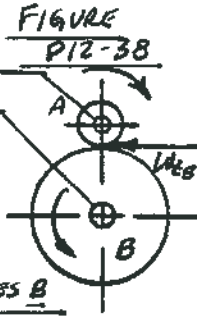
TORQUE ON SHAFT 2 =  $T_2 = T_B = T_C = 63000(15)/575 = 1643 \text{ LB}\cdot\text{IN.}$

TORQUE ON SHAFT 3 =  $T_3 = T_D = 63000(15)/287.5 = 3287 \text{ LB}\cdot\text{IN.}$

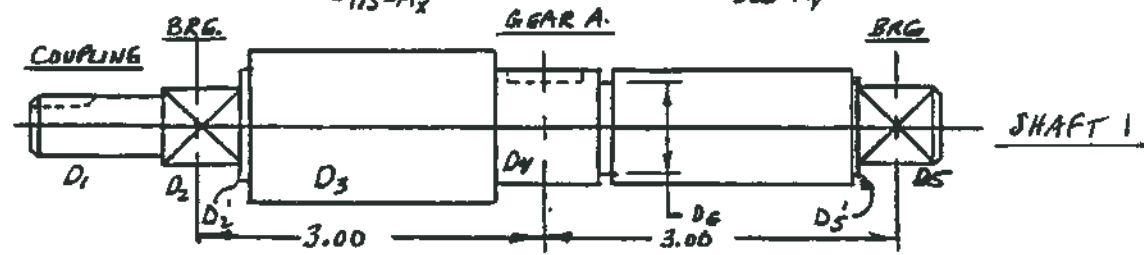
$W_{tB} = T_A/\lambda_A = \frac{548 \text{ LB}\cdot\text{IN.}}{0.90 \text{ IN.}} = 609 \text{ LB}$

$W_{rB} = W_{tB} \tan \phi = 609 \text{ LB} \tan 20^\circ = 222 \text{ LB} \downarrow$

REACTIONS:  $W_{tA} = 609 \text{ LB} \rightarrow$ ;  $W_{rA} = 222 \text{ LB} \uparrow$



AT MIDDLE OFS SHAFT:  
 $M = \sqrt{9713^2 + 333^2} = 9726 \text{ IN}$   
 $T = 548 \text{ LB}\cdot\text{IN}$  FROM COUPLING TO GEAR A.



EQUATION (2-24) USED TO COMPUTE ALL DIAMETERS.

DIAMETER D1:  $T = 548 \text{ LB}\cdot\text{IN}; M = 0$ ; DESIGN FOR 0.999 RELIABILITY -  $C_R = 0.75$

USE SAE 1040 CD STEEL:  $S_y = 71,000 \text{ PSI}; S_u = 80,000 \text{ PSI}; 12\% \text{ ELONGATION}$   
 $S_m = 30,000 \text{ PSI}; S_m' = S_m C_s C_R = (30,000)(0.9)(0.75) = 20,250 \text{ PSI}; \text{LET } N = 3.$

THEN  $D_{1 \text{ MIN}} = 0.589 \text{ IN.}$

DIAMETER D2: SAME CONDITIONS AS D1;  $D_{2 \text{ MIN}} = 0.589 \text{ IN.}$

DIAMETER D3: DEPENDS ON D4.

DIAMETER D4:  $M = 972 \text{ LB}\cdot\text{IN}; T = 548 \text{ LB}\cdot\text{IN}$  AT SHOULDER AND KEYSEAT.  
 $(K_t = 2.5) \quad (K_r = 2.0)$

AT SHOULDER:  $D_{4 \text{ MIN}} = 1.54 \text{ IN.}$

AT RING GROOVE:  $T = 0, K_t \approx 3.0; D_{G \text{ MIN}} = 1.64 \text{ IN.}$

INCREASE BY 6% FOR D4:  $D_4 \approx 1.06(1.64) = 1.74 \text{ IN}$  GOVERNING VALUE

DIAMETER D5:  $M = 0; T = 0$ : VERY SMALL DIA. REQD TO RESIST SHEAR.

BEARING SEATS D2, D5: ASSUME BEARINGS WITH BORE = 0.7874 IN (20mm)  
 CAN BE FOUND TO CARRY RADIAL LOADS. CHAPTER 14, TABLE 14-3

SPECIFICATIONS:

$D_1 = 0.750 \text{ IN.}$   $D_6$  SPECIFIED BY RETAINING RING MFR.

$D_2 = D_5 = 0.7874 \text{ IN.}$

$D_3 = 2.00 \text{ IN.}$   
 $D_4 = 1.80 \text{ IN.}$  } RELIEF PROVIDED ON LEFT SIDE OF D3 AND RIGHT END OF D4  
 TO ENSURE THAT OUTER RACE OF BEARING 6 DOES NOT CONTACT  
 ROTATING SHAFT.  $D_2' = D_4' = 0.969 \text{ IN}$  (SHAFT SHOULDER)

Cs CHECKED FOR ALL DIAMETERS - OK

(CONTINUED)

38

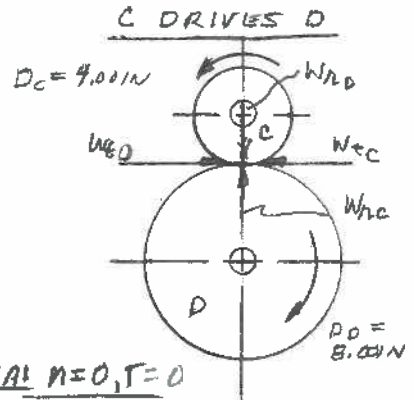
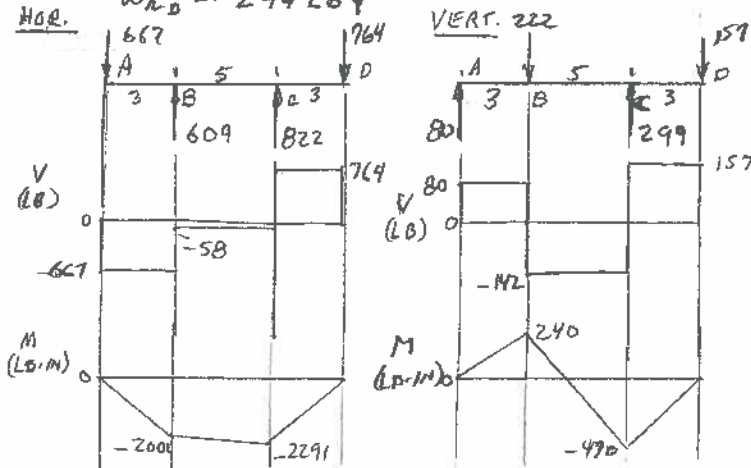
(CONTINUED) SHAFT Z  $T_2 = 1643 \text{ LB}\cdot\text{IN}$

$$W_{tc} = \frac{T_2}{R_c} = \frac{1643 \text{ LB}\cdot\text{IN}}{2.00 \text{ IN}} = 822 \text{ LB}$$

$$W_{td} = 822 \text{ LB} \rightarrow$$

$$W_{rc} = W_{tc} \tan \phi = 822 \tan 20^\circ = 299 \text{ LB} \uparrow$$

$$W_{rd} = 299 \text{ LB} \downarrow$$



AT A:  $M=0, T=0$

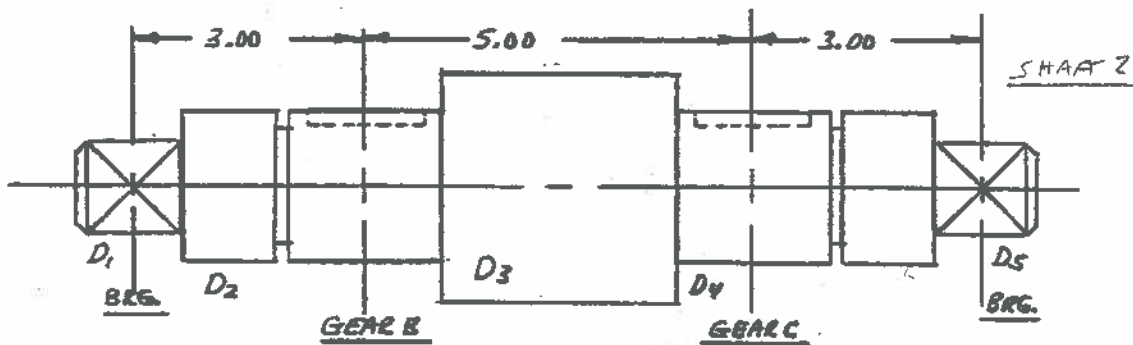
$$V_A = \sqrt{667^2 + 80^2} = 671 \text{ LB}$$

$$B: M_B = \sqrt{2000^2 + 240^2} = 2015 \text{ LB}\cdot\text{IN}$$

$$C: M_C = \sqrt{2291^2 + 470^2} = 2339 \text{ LB}\cdot\text{IN}$$

D:  $M=0, T=0$

$$V_D = \sqrt{764^2 + 157^2} = 780 \text{ LB}$$



SAE 1040 CD STEEL;  $S_y = 71000 \text{ PSI}$ ;  $S_u = 80000 \text{ PSI}$ ; 12% EL.

$S_m = 30080 \text{ PSI}$ ;  $S_m' = 20250 \text{ PSI}$  [SAME AS SHAFT 1]

D2:  $M_B = 2015 \text{ LB}\cdot\text{IN}$ ;  $T = 1643 \text{ LB}\cdot\text{IN}$  AT KEYSEAT + SHOULDER ( $K_t = 2.5$ )  
( $K_r = 2.0$ )

$$D_{2 \text{ MIN}} = 1.968 \text{ IN. AT SHOULDER}$$

AT RING GROOVE:  $D_{2 \text{ MIN}} = (1.06)(2.089) = 2.215 \text{ IN. GOVERNS}$

D4:  $M_C = 2339 \text{ LB}\cdot\text{IN}$ ;  $T = 1643 \text{ LB}\cdot\text{IN}$  AT KEYSEAT AND SHOULDER ( $K_t = 2.5$ )

$$D_{4 \text{ MIN}} = 2.068 \text{ IN}$$

AT RING GROOVE:  $T=0, K_r = 3.0$ ;  $M = 2339 \text{ LB}\cdot\text{IN}$

$$D_{4 \text{ MIN}} = (1.06)(2.1959) = 2.328 \text{ IN. GOVERNS}$$

D5: RIGHT BRG.:  $M=0, T=0, V_D = 780 \text{ LB.}$ ;  $K_t = 2.5$

$$D_{5 \text{ MIN}} = \sqrt{2.94 K_t V_N / S_m'} = \sqrt{2.94(2.5)(780)(3) / 20250} = 0.922 \text{ IN.}$$

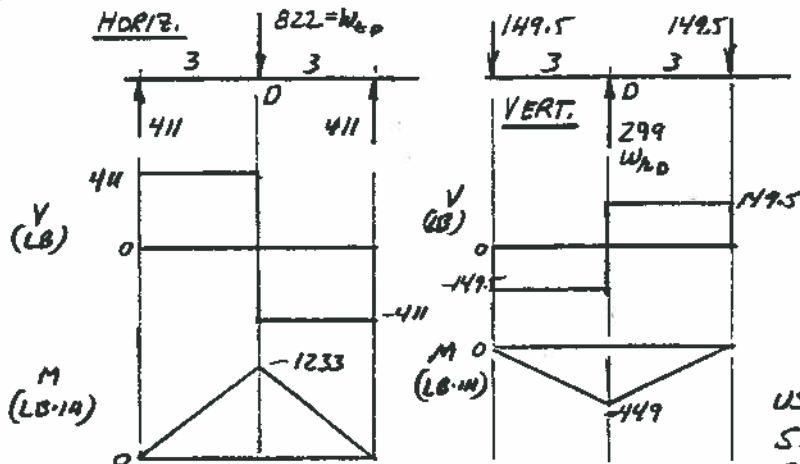
D1:  $V_A = 671 \text{ LB} \rightarrow D_{1 \text{ MIN}} = 0.855 \text{ IN.}$

LET:  $D_2 = 2.250 \text{ IN}$ ;  $D_4 = 2.400 \text{ IN}$ ;  $D_3 = 2.600 \text{ IN}$

D1 AND D5 DEPEND ON BEARING SELECTION.



38 (CONTINUED) SHAFT 3

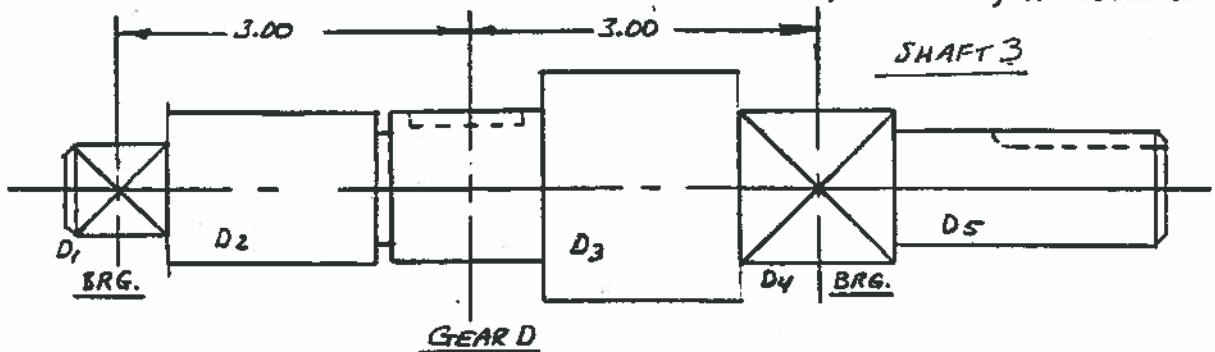


$$M_D = \sqrt{1233^2 + 449^2}$$

$$M_D = 1372 \text{ LB}\cdot\text{IN.}$$

$$T = 3287 \text{ LB}\cdot\text{IN FROM KEYSLOT TO RIGHT END OF SHAFT.}$$

USE SAME MATERIAL AS FOR SHAFTS 1 AND 2.  $C_s = 0.80$   
 $S_y = 71000 \text{ PSI}; S_m = 18000 \text{ PSI}$



DIAMETER  $D_2$ ;  $M = 1372 \text{ LB}\cdot\text{IN}$ ;  $T = 3287 \text{ LB}\cdot\text{IN}$ ,  $K_t = 2.5$  AT SHOULDER.

$D_2 = 1.79 \text{ IN. AT SHOULDER.}$

AT GROOVE:  $D_6 = 1.88 \text{ IN}$  FOR  $K_c = 3.0$ ;  $T = 0$ .

INCREASE BY 6%:  $D_2 \approx 1.06(1.88) = 2.00 \text{ IN. GOVERNS}$

DIAMETER  $D_{4 \text{ MIN}}$ ;  $D_{5 \text{ MIN}}$ :  $M = 0$ ;  $T = 3287 \text{ LB}\cdot\text{IN}$ ;  $D_{4 \text{ MIN}} = 1.07 \text{ IN.}$

SPECIFY!

$D_1 = 1.3780 \text{ IN (35 mm), BRG 6207}$

$D_2 = 2.000 \text{ IN}$

$D_3 = 2.250 \text{ IN}$

$D_4 = 1.3780 \text{ IN (35 mm) BRG. 6207}$

$D_5 = 1.25 \text{ IN}$

[ CHECKED  $C_s$  OK ]

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$P = 12.0 \text{ HP}$ ,  $n_{\text{motor}} = 1150 \text{ RPM}$ , FIGURE P12-39

MOTOR SHAFT:

NET DRIVING FORCE =  $F_N = T/D/2$

$T = 63000(P)/n = 63000(12)/1150 = 657 \text{ LB}\cdot\text{IN.}$

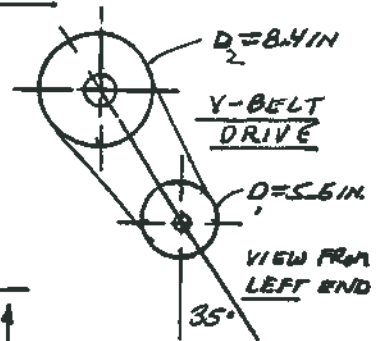
$F_N = 657 \text{ LB}\cdot\text{IN.} / (5.6 \text{ IN.} / 2) = 235 \text{ LB.}$

BENDING FORCE =  $F_B = 1.5 F_N = 352 \text{ LB}$

$F_{Bx} = F_B \sin 35^\circ = (352 \text{ LB}) (\sin 35^\circ) = 202 \text{ LB}$

$F_{By} = F_B \cos 35^\circ = (352 \text{ LB}) (\cos 35^\circ) = 288 \text{ LB}$

FORCES ACT ON MOTOR SHAFT. DIRECTIONS AS VIEWED FROM BEHIND THE LEFT END OF THE MOTOR.



REDUCER INPUT SHAFT: FORCES SHOWN AS VIEWED FROM

RIGHT END FOR CONSISTENTLY WITH VIEWS OF GEAR SYSTEM IN FIGURE P12-38.

SHAFT SPEED:  $n_1 = n_{\text{motor}} \left( \frac{D_1}{D_2} \right) = 1150 \text{ RPM} \left( \frac{5.6}{8.4} \right) = 767 \text{ RPM}$

$T_1 = 63000(12)/767 = 986 \text{ LB}\cdot\text{IN.}$

FORCES AT SHEAVES:  $F_N = T_1 / (D_1 / 2) = (986 \text{ LB}\cdot\text{IN.}) / (8.4 \text{ IN.} / 2) = 235 \text{ LB.}$

$F_B = 1.5 F_N = 352 \text{ LB}$ ;  $F_{Bx} = 202 \text{ LB}$ ;  $F_{By} = 288 \text{ LB}$   
(SEE MOTOR SHAFT ANALYSIS)

FROM PROBLEM 38:

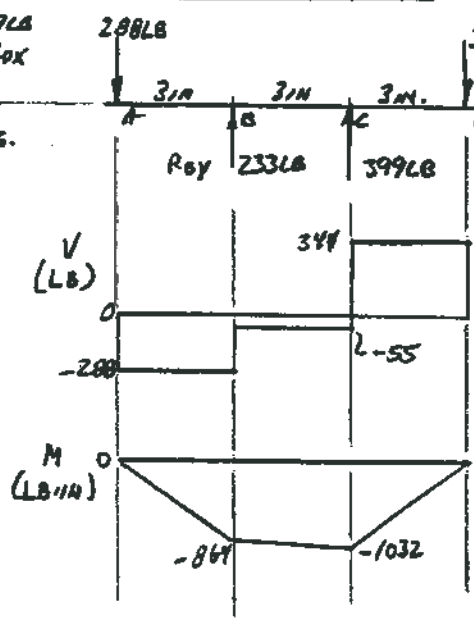
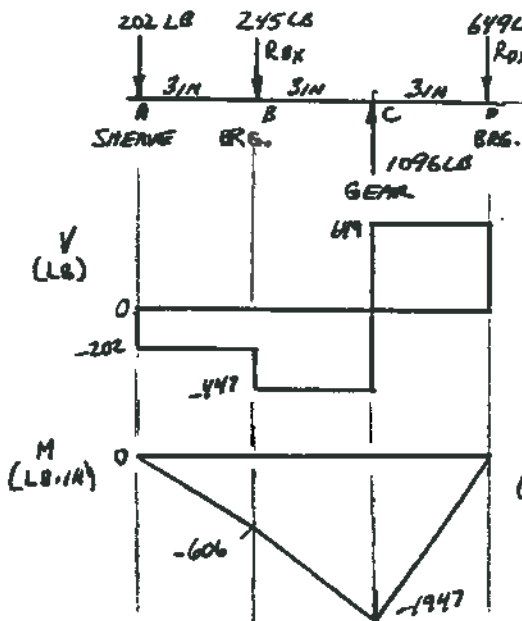
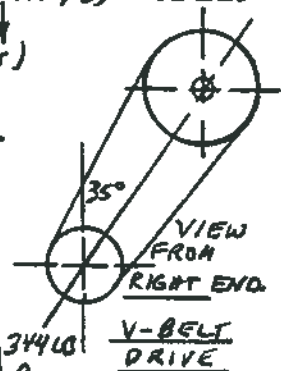
GEAR A:  $W_{GA} = T_1 / n_A = 986 \text{ LB}\cdot\text{IN.} / 0.90 \text{ IN.} = 1096 \text{ LB}$

$W_{Ax} = W_{GA} \tan \phi = (1096 \text{ LB}) \tan 20^\circ = 399 \text{ LB}$

POSITION OF SHEAVE ON SHAFT 1 ASSUMED

HORIZONTAL PLANE

VERTICAL PLANE



RESULTANT MOMENTS:

$M_B = \sqrt{606^2 + 867^2}$

$M_B = 1055 \text{ LB}\cdot\text{IN.}$

$M_C = \sqrt{1947^2 + 1032^2}$

$M_C = 2204 \text{ LB}\cdot\text{IN.}$

BENDING FORCES:

$R_B = \sqrt{245^2 + 288^2}$

$R_B = 338 \text{ LB.}$

$R_D = \sqrt{649^2 + 344^2}$

$R_D = 735 \text{ LB.}$

39.

CONTINUED

DESIGN OF SHAFT 1 WOULD BE COMPLETED IN A MANNER SIMILAR TO THAT SHOWN IN PROBLEM 38.

SHAFT 2: SEE PROBLEM 38 FOR ANALYSIS AND DESIGN PROC.

FORCES:  $W_{t_2} = W_{o_1} = 1096 \text{ LB.} \leftarrow$ ;  $W_{r_2} = W_{r_1} = 399 \text{ LB.} \downarrow$  GEAR B

SHAFT 2 SPEED =  $M_2 = M_1 \frac{N_1}{N_2} = 767 \times \frac{18}{54} = 256 \text{ RPM}$

$T_2 = 63000 (12) / 256 = 2958 \text{ LB}\cdot\text{IN}$

$W_{t_c} = T_2 / r_c = 2958 \text{ LB}\cdot\text{IN} / 2.00 \text{ IN.} = 1479 \text{ LB} \rightarrow$  GEAR C

$W_{r_c} = W_{t_c} \tan 20^\circ = 538 \text{ LB} \downarrow$

SHAFT 3: SEE PROBLEM 38 FOR ANALYSIS AND DESIGN PROCEDURE.

FORCES:  $W_{t_3} = W_{t_2} = 1479 \text{ LB} \rightarrow$ ;  $W_{r_3} = W_{r_2} = 538 \text{ LB} \downarrow$  GEAR D

CHAIN SPROCKET AT END OF SHAFT 3:

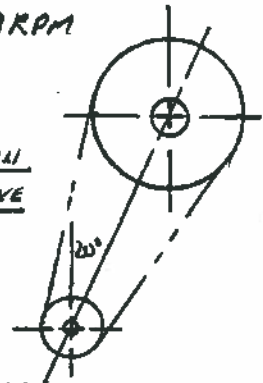
SPEED:  $M_3 = M_2 \cdot \frac{N_2}{N_3} = 256 \text{ RPM} \cdot \frac{24}{48} = 128 \text{ RPM}$

$T_3 = 63000 (12) / 128 = 5917 \text{ LB}\cdot\text{IN}$

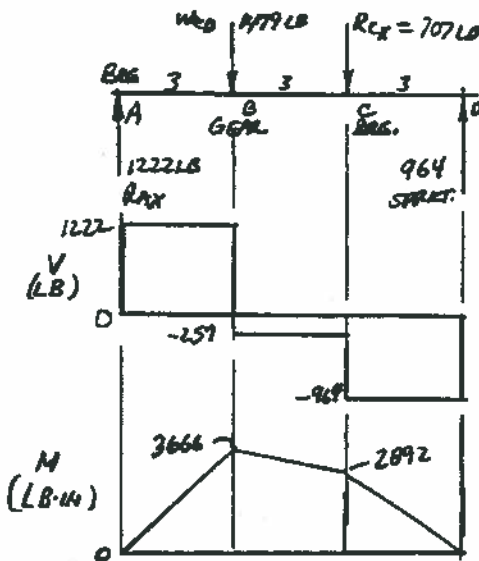
$F_t = F_b = \frac{T_3}{R} = \frac{5917 \text{ LB}\cdot\text{IN}}{(4.2 \text{ IN} / 2)} = 2817 \text{ LB} \uparrow$

$F_{B_x} = F_b \sin 20^\circ = (2817 \text{ LB}) \sin 20^\circ = 964 \text{ LB} \rightarrow$  CHAIN DRIVE

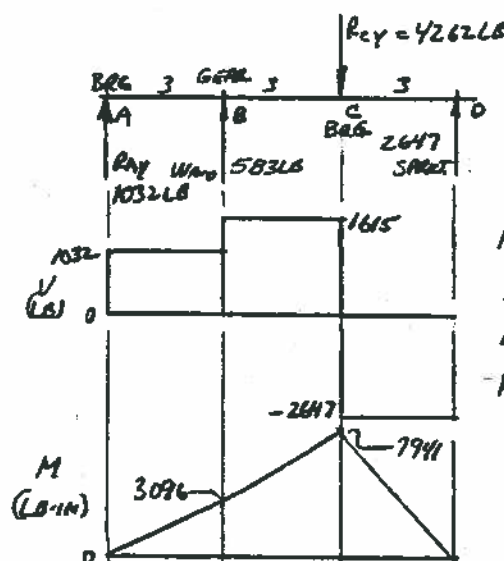
$F_{B_y} = F_b \cos 20^\circ = (2817 \text{ LB}) \cos 20^\circ = 2647 \text{ LB} \uparrow$



HORIZONTAL PLANE



VERTICAL PLANE



RESULTANTS:

$M_B = \sqrt{3666^2 + 3096^2}$

$M_B = 4798 \text{ LB}\cdot\text{IN.}$

$M_C = \sqrt{2892^2 + 7941^2}$

$M_C = 8451 \text{ LB}\cdot\text{IN.}$

BEARING FORCES:

$R_A = \sqrt{1222^2 + 1032^2}$

$R_A = 1600 \text{ LB}$

$R_C = \sqrt{707^2 + 4262^2}$

$R_C = 4320 \text{ LB}$

CONVEYOR SHAFT FORCES:  $F_{B_x} = 964 \text{ LB} \leftarrow$ ;  $F_{B_y} = 2647 \text{ LB} \uparrow$

SPEED:  $M_c = M_3 \cdot \frac{42}{10.6} = 128 \cdot \frac{42}{10.6} = 50.6 \text{ RPM}$ ;  $T_c = 63000 (12) / 50.6 = 14,932 \text{ LB}\cdot\text{IN}$

40

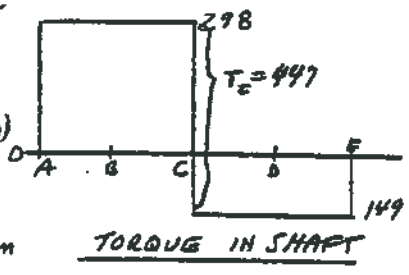
FIGURE P12-40: SHAFT 2:  $M = 480 \text{ RPM}$ ; POWER IN AT C =  $22.5 \text{ kW}$ .  
 POWER OUT AT A =  $15 \text{ kW}$ ; POWER OUT AT E =  $7.5 \text{ kW}$ .  
 COAL CRUSHER - USE  $N = 4$  BECAUSE OF IMPACT AND SHOCK.

$$m = 480 \frac{\text{REV}}{\text{MIN}} \times \frac{2\pi \text{ RAD}}{\text{REV}} \times \frac{1 \text{ MIN}}{60 \text{ SEC}} = 50.27 \text{ RAD/S}$$

$$T_{AC} = \frac{P}{m} = \frac{15 \times 10^3 \text{ N}\cdot\text{m/s}}{50.27 \text{ RAD/S}} = 298 \text{ N}\cdot\text{m}$$

$$T_{CE} = \frac{P}{m} = \frac{7.5 \times 10^3 \text{ N}\cdot\text{m/s}}{50.27 \text{ RAD/S}} = 149 \text{ N}\cdot\text{m}$$

$$\text{TORQUE ON GEAR C} = \frac{22.5 \times 10^3 \text{ N}\cdot\text{m/s}}{50.27 \text{ RAD/S}} = 447 \text{ N}\cdot\text{m}$$



FORCES:

$$\text{GEAR A: } W_{tA} = \frac{T_A}{r_A} = \frac{298 \text{ N}\cdot\text{m}}{50 \text{ mm}} \times \frac{10^3 \text{ mm}}{\text{m}} = 5960 \text{ N}$$

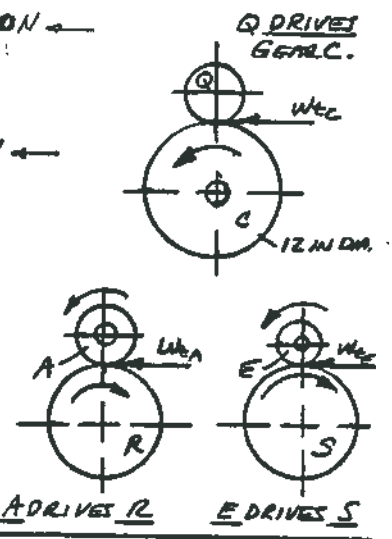
$$W_{nA} = W_{tA} \tan 20^\circ = (5960 \text{ N}) \tan 20^\circ = 2169 \text{ N}$$

$$\text{GEAR C: } W_{tC} = \frac{T_C}{r_C} = \frac{447 \text{ N}\cdot\text{m}}{150 \text{ mm}} \times \frac{10^3}{\text{m}} = 2980 \text{ N}$$

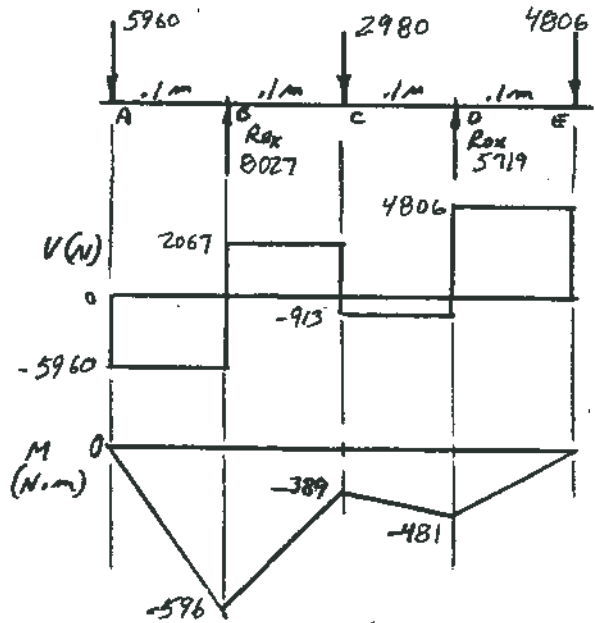
$$W_{nC} = W_{tC} \tan 20^\circ = (2980 \text{ N}) \tan 20^\circ = 1065 \text{ N}$$

$$\text{GEAR E: } W_{tE} = \frac{T_E}{r_E} = \frac{149 \text{ N}\cdot\text{m}}{31 \text{ mm}} \times \frac{10^3}{\text{m}} = 4806 \text{ N}$$

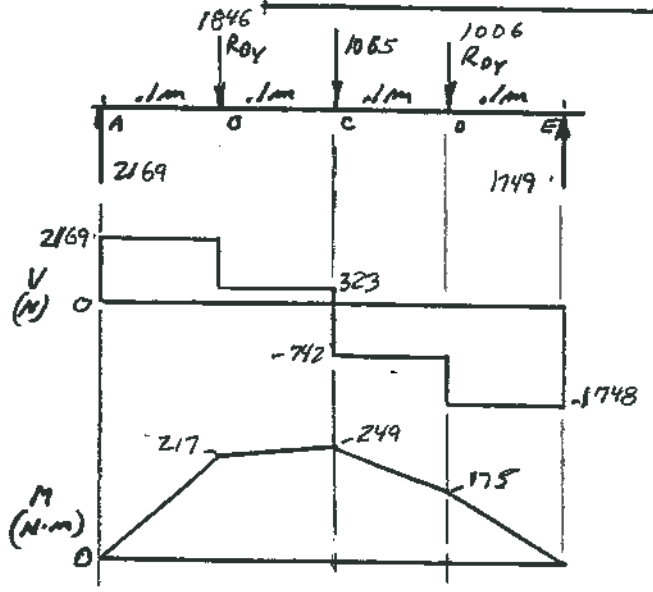
$$W_{nE} = W_{tE} \tan 20^\circ = (4806 \text{ N}) \tan 20^\circ = 1749 \text{ N}$$



HORIZONTAL PLANE



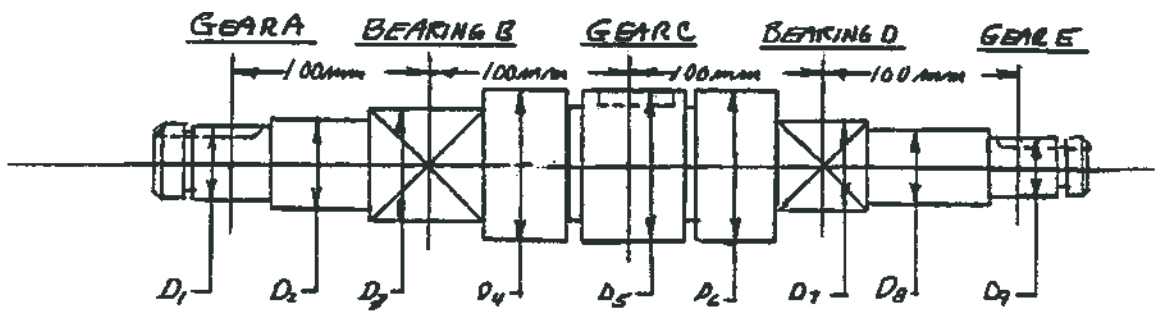
VERTICAL



$$M_B = [596^2 + 217^2]^{1/2} = 634 \text{ N}\cdot\text{m}; \quad M_C = [389^2 + 249^2]^{1/2} = 462 \text{ N}\cdot\text{m}; \quad M_D = [481^2 + 175^2]^{1/2} = 512 \text{ N}\cdot\text{m}$$

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BEARING FORCES:  $R_B = \sqrt{800^2 + 1846^2} = 2037 N$ ;  $R_D = \sqrt{5719^2 + 1006^2} = 5809 N$   
PROPOSED SHAFT DESIGN:



ASSUME ALL FILLETS ARE SMALL RADII WITH  $K_t \approx 2.5$ , EXCEPT  $r_2, r_6$   
 USE  $K_t = 3.0$  AT RING GROOVES WITH SHAFT DIA.  $= 1.06 \times$  GROOVE DIA.  
 USE  $K_t = 1.5$  AT  $r_2$  AND  $r_6$  (WELL ROUNDED)

MATERIAL SELECTION: SAE 4140 OBT 1000,  $S_u = 1160 MPa$ ,  $S_y = 1050 MPa$

17% ELONGATION - GOOD STRENGTH AND DUCTILITY.

FROM FIG. 5-9:  $S_m = 400 MPa$ .

SELECT  $C_s = 0.80$  (FRO  $\approx 65$  mm OR LESS);  $C_R = 0.81$  (0.99 RELIABILITY)

$S_m' = C_s C_R S_m = (0.80)(0.81)(400 MPa) = 259 MPa = 259 N/mm^2$

SOLUTION FOR DIAMETERS USING EQ. 9-22 - SUMMARY: ( $N=4$ )

LOCATION	$K_t$	$M(N \cdot mm)$	$T(N \cdot mm)$	$D_{MIN}$	SPECIFIED D
D1	2.5	0	298,000	21.56	50.0 mm
D2	1.5	634,000	298,000	53.13	60.0 mm
D3	2.5	634,000	298,000	62.96	65.0 mm
D4	3.0	525,000*	298,000	62.82 x 1.06 = 66.6	80.0 mm
D5	2.5	462,000	298,000	58.67	
D6	3.0	462,000	149,000	60.19 x 1.06 = 63.8	
D7	2.5	512,000	149,000	58.62	60.0 mm
D8	1.5	512,000	149,000	49.45	55.0 mm
D9	2.5	0	149,000	17.11	45.0 mm

NOTES: D2 AND D7 ARE STANDARD BEARING BORES FROM TABLE 14-3.

\* MOMENT AT D4 ESTIMATED BETWEEN POINTS B AND C.

80.0 mm USED FOR D4, D5, AND D6 TO PROVIDE SHOULDER FOR BEARINGS AT B AND D.

D2 AND D8 PROVIDE EASE OF INSTALLATION FOR BEARINGS.

D1 AND D9 MADE LARGER THAN REQUIRED FOR COMPATIBILITY WITH ADJACENT DIAMETERS AND TO WITHSTAND MOMENTS AT SHOULDERS.

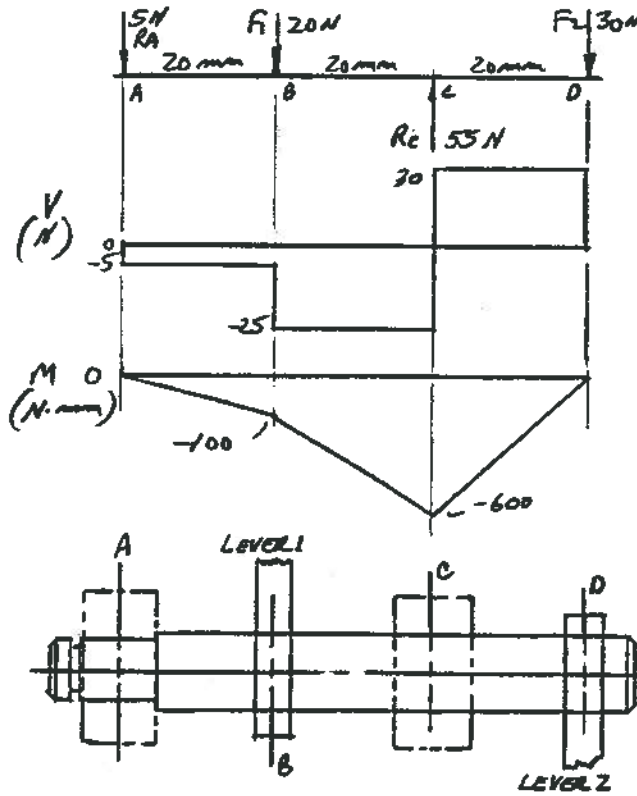
FINAL STRESSES MUST BE CHECKED AFTER SPECIFYING FILLET RADII, GROOVE GEOMETRY, AND BEARING SELECTION.



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FIGURE P12-41 DESIGN SHAFT AND LEVERS.  $F_1 = 20\text{ N}$

$F_2 = F_1(60\text{ mm})/40\text{ mm} = 30\text{ N}$ ; TORQUE =  $F_1 \cdot 60 = 1200\text{ N}\cdot\text{mm}$  FROM B TO D.



USE SAE 1137 CD STEEL

$S_u = 676\text{ MPa}$ ;  $S_y = 565\text{ MPa}$

$S_m = 260\text{ MPa}$  (FIG. 5-8)

LET  $C_s = 0.9$ ,  $C_r = 0.75$

$S_m' = (0.9)(0.75)(260) = 175\text{ MPa}$

WIPER MECHANISM HAS AN OSCILLATING MOTION. BOTH BENDING AND TORSION WILL BE VARYING.

SHAFT RESTRAINED IN BEARING AT A BUT CAN FLOAT IN BEARING C.

ASSUME  $K_c = 2.5$  AT FILLET TO RIGHT OF A. ASSUME  $K_t = 1.0$  AT C.

ASSUME LEVERS ARE INSTALLED WITH A LIGHT PRESS FIT AND TALK WELDED IN POSITION. USE  $K_t = 3.0$  FOR WELD AREA.

DESIGN EQUATION 12-24 MUST BE MODIFIED FOR VARYING TORQUE.

ADD  $K_t$  FOR TORSION. SUBSTITUTE  $S_m'$  FOR  $S_y$ . THEN:

$$D = \left[ \frac{32N}{\pi} \sqrt{\left( \frac{K_{tB} M}{S_m'} \right)^2 + \frac{3}{4} \left( \frac{K_{tT} T}{S_m'} \right)^2} \right]^{1/3}$$

AT C:

$$D = \left[ \frac{32(3)}{\pi} \sqrt{\left( \frac{10(600)}{175} \right)^2 + \frac{3}{4} \left( \frac{1.0(1200)}{175} \right)^2} \right]^{1/3} = 5.94\text{ mm}$$

AT B:

$$D = \left[ \frac{32(3)}{\pi} \sqrt{\left( \frac{3.0(100)}{175} \right)^2 + \frac{3}{4} \left( \frac{3.0(1200)}{175} \right)^2} \right]^{1/3} = 8.18\text{ mm}$$

USE  $D \geq 10.0\text{ mm}$   
AT B, C, D.  
USE  $D \geq 8.0\text{ mm}$   
AT A.

LEVER DESIGN: USE FLAT STOCK, 4.0 mm THICK =  $b$

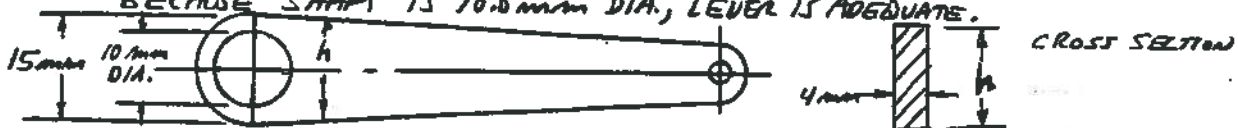
SAME MATERIAL AS SHAFT:  $S_m' = 175\text{ MPa}$ ;  $\sigma_b = \frac{S_m'}{N} = \frac{175}{3} = 58.3\text{ MPa}$

$M_{max} = (20\text{ N})(60\text{ mm}) = 1200\text{ N}\cdot\text{mm}$ ;  $\sigma = M/S$

REQ'D  $S = \frac{M}{\sigma} = \frac{1200\text{ N}\cdot\text{mm}}{58.3\text{ N/mm}} = 20.57\text{ mm}^3 = \frac{bh^2}{6}$

REQ'D  $h = \sqrt{6S/b} = \sqrt{6(20.57\text{ mm}^3)/4.0\text{ mm}} = 5.55\text{ mm}$  AT SHAFT.

BECAUSE SHAFT IS 10.0 mm DIA., LEVER IS ADEQUATE.



## CHAPTER 13 TOLERANCES AND FITS

1. LOOSE - RCB: HOLE  $\begin{matrix} +5 \\ -0 \end{matrix}$ ; SHAFT  $\begin{matrix} -7 \\ -10.5 \end{matrix}$ ; CLEARANCE  $\begin{matrix} +7 \\ +15.5 \end{matrix}$   
 LIMITS: HOLE  $\frac{3.5050}{3.5000}$       SHAFT  $\frac{3.4930}{3.4895}$       CL 0.0070 to 0.0155

2. PRECISION - RC2: HOLE  $\begin{matrix} +0.4 \\ -0 \end{matrix}$ ; SHAFT  $\begin{matrix} -0.25 \\ -0.55 \end{matrix}$ ; CL  $\begin{matrix} +0.25 \\ +0.95 \end{matrix}$   
 LIMITS: HOLE  $\frac{0.50040}{0.50000}$       SHAFT  $\frac{0.49975}{0.49945}$       CL 0.00025 to 0.00095

3. LOOSE - RCB: HOLE  $\begin{matrix} +2.8 \\ -0 \end{matrix}$ ; SHAFT  $\begin{matrix} -3.5 \\ -5.1 \end{matrix}$ ; CL  $\begin{matrix} +3.5 \\ +7.9 \end{matrix}$   
 ADJUST TOLERANCES FOR BASIC SHAFT SYSTEM - ADD 3.5.

HOLE  $\begin{matrix} +6.3 \\ +3.5 \end{matrix}$ ; SHAFT  $\begin{matrix} 0 \\ -1.6 \end{matrix}$ ; CL  $\begin{matrix} +3.5 \\ +7.9 \end{matrix}$   
 LIMITS: HOLE  $\frac{0.6313}{0.6285}$       SHAFT  $\frac{0.6250}{0.6234}$       CL 0.0035 to 0.0079

4. CLOSE FIT - RELIABLE MOTION - RC5  
 HOLE  $\begin{matrix} +1.2 \\ -0 \end{matrix}$ ; SHAFT  $\begin{matrix} -1.6 \\ -2.4 \end{matrix}$ ; CL  $\begin{matrix} +1.6 \\ +3.6 \end{matrix}$   
 LIMITS: HOLE  $\frac{0.8012}{0.8000}$       SHAFT  $\frac{0.7984}{0.7976}$       CL 0.0016 to 0.0036

5. LOOSE - RCB: HOLE  $\begin{matrix} +4.0 \\ -0 \end{matrix}$ ; PIN  $\begin{matrix} -5.0 \\ -7.5 \end{matrix}$ ; CL  $\begin{matrix} +5.0 \\ +11.5 \end{matrix}$   
 LIMITS: HOLE  $\frac{1.2540}{1.2500}$       PIN  $\frac{1.2450}{1.2425}$       CL 0.0050 to 0.0115

6. LOOSE - RCB: HOLE  $\begin{matrix} +5.0 \\ -0 \end{matrix}$ ; PIN  $\begin{matrix} -7.0 \\ -10.5 \end{matrix}$ ; CL  $\begin{matrix} +7.0 \\ +15.5 \end{matrix}$   
 ADJUST TOLERANCES FOR BASIC SHAFT SYSTEM - ADD 7.0

HOLE  $\begin{matrix} +12.0 \\ +7.0 \end{matrix}$ ; PIN  $\begin{matrix} 0 \\ -3.5 \end{matrix}$ ; CL  $\begin{matrix} +7.0 \\ +15.5 \end{matrix}$   
 LIMITS: HOLE  $\frac{4.0120}{4.0070}$       PIN  $\frac{4.0000}{3.9965}$       CL 0.0070 to 0.0155

7. PRECISION WITH WIDE TEMPERATURE VARIATIONS WOULD TYPICALLY CALL FOR RC3 OR RC4. RC2 PROBABLY TOO TIGHT FOR TEMP. CHANGE; RC5 PROBABLY TOO LOOSE FOR REQ'D PRECISION. RC3 OR RC4 NOT AVAILABLE IN TABLE B-6.

ILLUSTRATE LIMITS WITH RCS:

HOLE:  $\begin{matrix} +1.2 \\ -0 \end{matrix}$  ; PIN  $\begin{matrix} -1.6 \\ -2.4 \end{matrix}$  ; CL  $\begin{matrix} +1.6 \\ +3.6 \end{matrix}$

LIMITS: HOLE  $\frac{0.7512}{0.7500}$  PIN  $\frac{0.7487}{0.7476}$  CL 0.0016 TO 0.0036

8. LOOSE -RCB: HOLE  $\begin{matrix} +4.0 \\ -0 \end{matrix}$  ; SHAFT  $\begin{matrix} -5.0 \\ -7.5 \end{matrix}$  ; CL  $\begin{matrix} +5.0 \\ +11.5 \end{matrix}$

ADJUST TOLERANCES FOR BASIC SHAFT SYSTEM - ADD 5.0.

HOLE  $\begin{matrix} +9.0 \\ +5.0 \end{matrix}$  ; SHAFT  $\begin{matrix} 0 \\ -2.5 \end{matrix}$  ; CL  $\begin{matrix} +5.0 \\ +11.5 \end{matrix}$

LIMITS: HOLE  $\frac{1.5090}{1.5050}$  SHAFT  $\frac{1.5000}{1.4975}$  CL 0.0050 TO 0.0115

10.  $a = 0$ ;  $b = 3.25/2 = 1.625$  IN;  $c = 4.000/2 = 2.000$  IN : BOTH STEEL  
USE FNS - HEAVY FORCE FIT  $E = 30 \times 10^6$  PSI

HOLE:  $\begin{matrix} +2.2 \\ -0 \end{matrix}$  SHAFT  $\begin{matrix} +8.4 \\ +7.0 \end{matrix}$  INTERFERENCE:  $\begin{matrix} 4.8 \\ 8.4 \end{matrix}$

LIMITS: HOLE  $\frac{3.2522}{3.2500}$  SHAFT  $\frac{3.2587}{3.2570}$   $\delta_{MAX} = 0.0084$  IN.

(EQ. 13-2)  $p = \frac{E\delta}{2b} \left[ \frac{(c^2 - b^2)(b^2 - a^2)}{2b^2(c^2 - a^2)} \right] = \frac{(30 \times 10^6)(0.0084) \left[ \frac{2.00^2 - 1.625^2}{2(1.625)^2(2.00^2 - 0)} \right]}{2(1.625)}$

(EQ. 13-4)  $\sigma_o = p \left( \frac{c^2 + b^2}{c^2 - b^2} \right) = 13175 \left[ \frac{2.00^2 + 1.625^2}{2.00^2 - 1.625^2} \right] = 64363$  PSI NOT ACCEPTABLE FOR 1020 HR;  $S_u = 57251$

11.  $a = 3.50/2 = 1.75$  IN;  $b = 4.0/2 = 2.000$  IN;  $c = 4.50/2 = 2.250$  IN

FNS: HOLE  $\begin{matrix} +1.4 \\ -0 \end{matrix}$  SHAFT  $\begin{matrix} +4.9 \\ +4.0 \end{matrix}$  INT.  $\begin{matrix} 2.6 \\ 4.9 \end{matrix}$

LIMITS: STEEL SLEEVE ID  $\frac{4.0014}{4.0000}$  ; BRONZE BUSHING OD  $\frac{4.0049}{4.0040}$  ;  $\delta_{MAX} = 0.0049$  IN

$p = 1575$  PSI (EQ. 13-3) USING  $E_o = 30 \times 10^6$  PSI  $E_A = 15 \times 10^6$  PSI  
 $\nu_o = 0.27$   $\nu_A = 0.27$

$\sigma_o = 13438$  PSI (EQ. 13-4) INNER SURFACE OF STEEL SLEEVE

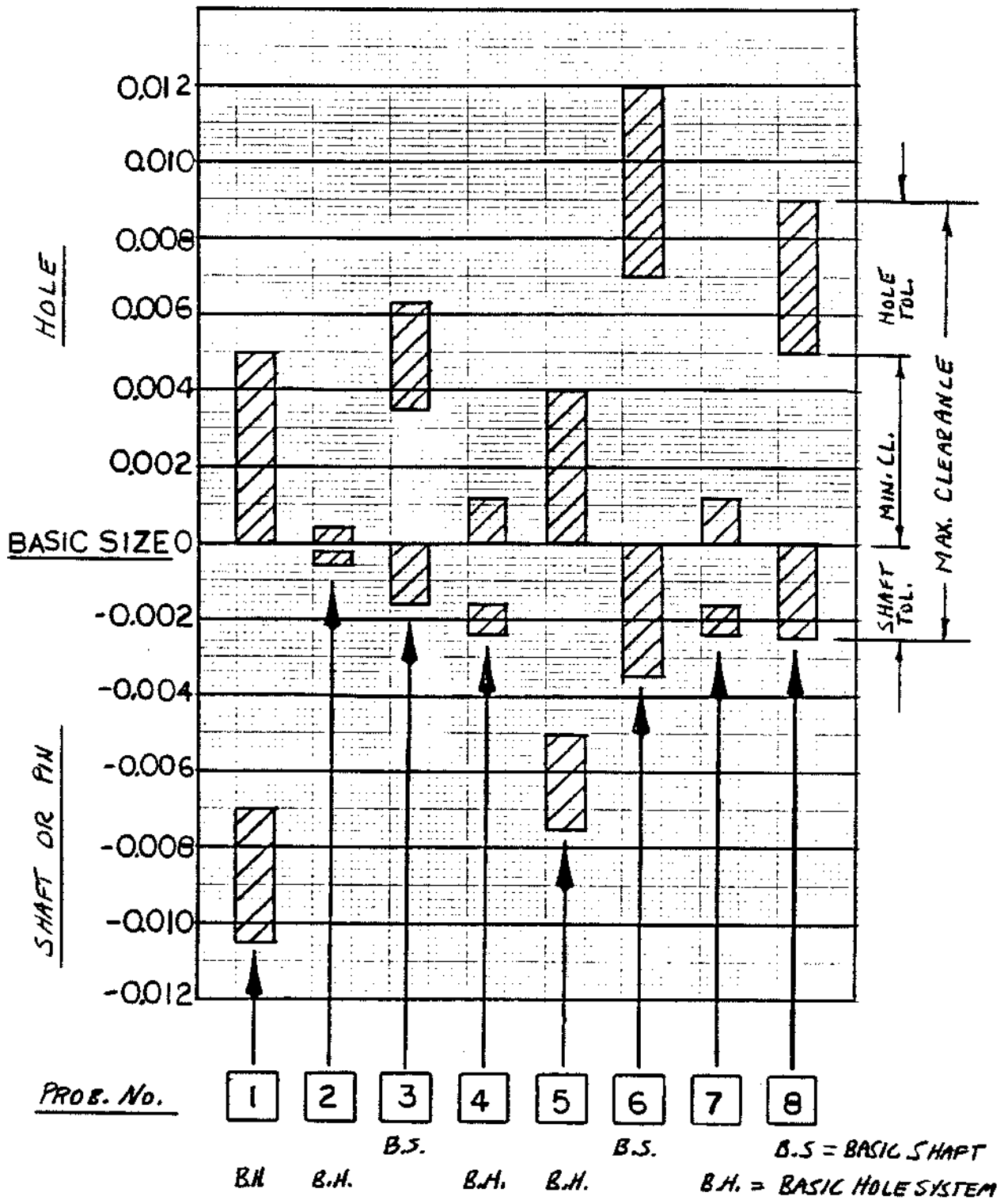
$\sigma_A = -11869$  PSI (EQ. 13-5) OUTER SURFACE OF BRONZE BUSHING

NOTE: APPENDIX A-7, BEARING BRONZE HAS A YIELD STRENGTH OF 13000 PSI.

$N = \frac{S_y}{\sigma_A} = \frac{13000}{11869} = 1.52$  LOW

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TOLERANCE DIAGRAMS FOR PROBLEMS 1-8.



STRESSES FOR FORCE FITS Refer to Figure 13-6 for geometry		Problem identification: Problem j2	
Input Data:		Numerical values in <i>italics</i> must be inserted for each problem	
Inside radius of inner member =	$a =$	0.0000 in	
Outside radius of inner member =	$b =$	1.5000 in	
Outside radius of outer member =	$c =$	2.5000 in	
Total interference =	$\delta =$	0.0072 in	
Modulus of outer member =	$E_o =$	1.00E+07 psi	ALUMINUM
Modulus of inner member =	$E_i =$	3.00E+07 psi	STEEL
Poisson's ratio for outer member =	$\nu_o =$	0.33	
Poisson's ratio for inner member =	$\nu_i =$	0.27	
Computed results:			
Pressure at Mating Surface:	$p =$	8894 psi	Using Equation (13-3)
Tensile Stress in the Outer Member:	$\sigma_o =$	18901 psi	Using Equation (13-4)
Compressive Stress in the Inner Member:	$\sigma_i =$	-8894 psi	Using Equation (13-5)
Increase in Diameter of Outer Member:	$\delta_o =$	0.0065 in	Using Equation (13-6)
Decrease in Diameter of Inner Member:	$\delta_i =$	0.0065 in	Using Equation (13-7)

EVALUATE FMS FIT FOR MAXIMUM INTERFERENCE.

HOLE TOLERANCE: +0.0018 SHAF: +0.0072 INTERFERENCE: 0.0092  
 -0 +0.0060 0.0072

MAY. INTERFERENCE = 0.0072 - 0 = 0.0072 SMALLEST HOLE  
 LARGEST SHAF

STRESS IN ALUMINUM = 18901 PSI TENSION

ASSUMING NO ADDITIONAL LOADS AND STATIC CONDITION.

LET  $N=2$ . REQ'D  $S_y = 2(18901 \text{ PSI}) = 37802 \text{ PSI}$

SPECIFY ANY ALUMINUM ALLOY WITH  $S_y > 37802 \text{ PSI}$

EXAMPLES:

2014-T4,  $S_y = 42 \text{ KSI}$  OR 6061-T6,  $S_y = 40 \text{ KSI}$

ANY STEEL FOR ROD WOULD BE SATISFACTORY, PROVIDED

$S_y > 2(8894 \text{ PSI}) = 17788$

ANY CARBON OR ALLOY STEEL WITH  $S_y > 18000 \text{ PSI}$

FROM APP. 3. EXAMPLES AISI 1020 HR,  $S_y = 30 \text{ KSI}$

AISI 1040 HR,  $S_y = 42 \text{ KSI}$



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STEEL SLEEVES ON ALUMINUM TUBE

a = [2.00 - 2(0.065)]/2 = 0.935 in; b = 2.00/2 = 1.000 in; c = 3.04/2 = 1.520 in

EQ. 13-5 sigma\_i = -p \* (b^2 + a^2) / (b^2 - a^2) = -p \* (1.00 + .8742) / (1.00 - .8742) = -p(14.898)

FOR sigma\_i = -8500 psi; p = -8500 psi / 14.898 = 570 psi MAX ALLOWABLE

FROM EQ 13-3 SOLVE FOR delta

delta\_max = p \* { 2b [ 1/E\_o \* (c^2 + b^2) / (c^2 - b^2) + 1/E\_i \* (b^2 + a^2) / (b^2 - a^2) - nu\_i ] }

E\_o = 30 x 10^6 psi; E\_i = 10 x 10^6 psi; nu\_o = 0.27; nu\_i = 0.33

delta\_max = 0.00177 in.

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NOMINAL DIA. = 3.250 in.; ASSUME MAX INTERFERENCE = 0.0084 in

FOR FINAL CLEARANCE = 0.002, CHANGE IN DIA. = 0.0084 + 0.002 = 0.0104

delta\_c = delta / alpha L = 0.0104 in / (6.5 x 10^-6) / F^2 (3.250 in) = 492 F; T\_f = 75 + 492 = 567 F

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BRONZE - SHRINK FROM +75 TO -20; delta T = 95 F

delta = alpha L (delta T) = (1.0 x 10^-6) (4.00) (-95) = -0.0038 in.

MAX INTERFERENCE = 0.0049 in.

-0.0038 / 0.0011 + 0.0040 DESIRED CLEARANCE = 0.0051 IN REQ'D EXPANSION OF STEEL.

delta T = delta / alpha L = 0.0051 / (6.1 x 10^-6) (4000) = 209 F; 75 F AMBIENT; 284 F REQ'D STEEL TEMP.

16

EQ. 13-7

delta\_i = -2b p / E\_i \* [ (b^2 + a^2) / (b^2 - a^2) - nu\_i ] = -2(2.00)(1575) / (17 x 10^6) \* [ (2.00^2 + 1.75^2) / (2.00^2 - 1.75^2) - 0.27 ]

delta\_i = -0.00269 in

FINAL ID = 3.500 - 0.00269 = 3.4973 in.

## CHAPTER 14 ROLLING CONTACT BEARINGS

1. EQ. 14-2:  $L_d = \left(\frac{C}{P_d}\right)^k (10^6) = \left(\frac{2350}{1675}\right)^{3.0} (10^6) = 2.76 \times 10^6 \text{ REV.}$

2.  $L_d = (20,000 \text{ HR})(880 \text{ RPM})(60 \text{ MIN/HR}) = 1.06 \times 10^9 \text{ REV.}$

EQ. 14-3:  $C = P_d (L_d / 10^6)^{1/k} = 1250 (1.06 \times 10^9 / 10^6)^{1/3} = 12,745 \text{ LB}$

3. EQ. 14-2 (a)  $L_d = \left(\frac{C}{P_d}\right)^k (10^6) = \left(\frac{3150}{2200}\right)^3 (10^6) = 2.94 \times 10^6 \text{ REV.}$

(b)  $L_d = \left(\frac{3150}{4500}\right)^3 (10^6) = 0.343 \times 10^6 = 3.43 \times 10^5 \text{ REV.}$

4. USE  $L_d = (5000 \text{ HR})(1150 \text{ RPM})(60 \text{ MIN/HR}) = 1.04 \times 10^9 \text{ REV}$

$C = P_d (L_d / 10^6)^{1/k} = 1450 (1.04 \times 10^9 / 10^6)^{1/3} = 14,667 \text{ LB}$

5. FROM FIGURE 12-12:

REACTION AT B:  $R_B = \sqrt{R_{Bx}^2 + R_{By}^2} = \sqrt{458^2 + 4620^2} = 4643 \text{ LB}$

REACTION AT D:  $R_D = \sqrt{R_{Dx}^2 + R_{Dy}^2} = \sqrt{1223^2 + 1680^2} = 2078 \text{ LB}$

FROM EX. PROB. 12-1; DIA. AT B (MIN.) =  $D_3 = 3.55 \text{ IN}$

DIA. AT D (MIN.) =  $D_6 = 1.09 \text{ IN}$

INDUSTRIAL BLOWER; USE  $L_d = (10,000 \text{ HR})(600 \text{ RPM})(60) = 3.60 \times 10^8 \text{ REV.}$

REQD.  $C$  VALUE AT B:  $C = R_B (3.6 \times 10^8 / 10^6)^{1/3} = 4643 (7.114) = 33,029 \text{ LB.}$

AT D:  $C = R_D (3.6 \times 10^8 / 10^6)^{1/3} = 2078 (7.114) = 14,782 \text{ LB.}$

FROM TABLE 14-3: BRG. 6319 HAS  $C = 34,397 \text{ LB}$ ; BORE = 3.7402 IN.

BRG. 6311 HAS  $C = 16,076 \text{ LB}$ ; BORE = 2.1654 IN.

6.

DATA OF EX. PROB. 12-2 : FROM FIG. 12-16

$$R_B = \sqrt{589^2 + 164^2} = 611 \text{ LB} ; R_D = \sqrt{393^2 + 188^2} = 436 \text{ LB}$$

$$D_{\text{MIN}} = 2.02 \text{ IN} ; D_{\text{MIN}} = 1.98 \text{ IN. FROM TABLE 12-2.}$$

TABLE 14-4 : AGRICULTURAL EQ. - LBT  $L_d = 5000 \text{ HR}$

$$L_d = (5000)(1700 \text{ RPM})(60) = 5.1 \times 10^8 \text{ REV}$$

$$\text{REQ'D } C \text{ VALUE AT B: } C = 611 \left( \frac{5.1 \times 10^8}{10^6} \right)^{1/3} = 4882 \text{ LB}$$

$$\text{AT D: } C = 436 \left( \frac{5.1 \times 10^8}{10^6} \right)^{1/3} = 3483 \text{ LB}$$

ATB: FROM TABLE 14-3 : BEARING 6011 HAS  $C = 6317 \text{ LB}$  AND A BORE OF 2.1654 IN.  $C$  IS HIGHER THAN REQ'D BUT SHAFT DIA. MUST BE  $> 2.02 \text{ IN.}$   
SPECIFY BRG. 6011 FOR BOTH B AND D.

7.

DATA OF EX. PROB. 12-3 AND FIGS. 12-17 AND 12-18

$$R_A = \sqrt{507^2 + 41^2} = 509 \text{ LB} ; R_C = \sqrt{1697^2 + 393^2} = 1742 \text{ LB RADIAL}$$

$R_A$  PURELY RADIAL ; BRG. C CARRIES 265 LB THRUST LOAD

$$D_{\text{MIN}} = 0.59 \text{ IN} ; D_{\text{MIN}} = 2.26 \text{ IN}$$

$$\text{USE } L_d = (20000 \text{ HR})(101 \text{ RPM})(60) = 1.2 \times 10^8 \text{ REV.}$$

$$\text{REQ'D } C \text{ VALUE AT A: } C_A = 509 \left( \frac{1.2 \times 10^8}{10^6} \right)^{1/3} = 2519 \text{ LB}$$

BRG. 6302 HAS  $C = 2563 \text{ LB}$  AND A BORE OF 0.5906 IN.

SHOULD BE COMPATIBLE WITH DIA.  $D_2$  (FIG. 12-16) TO PROVIDE A SHOULDER FOR THE BEARING. A LIGHTER BEARING WITH A LARGER BORE MAY BE PREFERRED.

BEARING C : COMBINED RADIAL & THRUST LOAD. (EQ. 14-5)

$$\text{ASSUME } Y = 1.5 ; P = (1.0)(0.50)(1742) + (1.5)(265) = 1373 \text{ LB.}$$

$$C_c = 1373 \left( \frac{1.2 \times 10^8}{10^6} \right)^{1/3} = 6772 \text{ LB.}$$

BRG. 6212 HAS  $C = 10679 \text{ LB}$ , BORE = 2.3622 IN,  $C_0 = 7307 \text{ LB}$

$$\text{CHECK: } T/C_0 = 265/7307 = 0.0363 \rightarrow C = 0.24$$

$$T/R = 265/1742 = 0.152 < e \text{ - USE EQ. 14-5; } P = 1.0 R_c = 1742 \text{ LB}$$

$$C_c = 1742 \left( \frac{1.2 \times 10^8}{10^6} \right)^{1/3} = 8592 \text{ LB} \rightarrow \text{BRG. 6212 OK,}$$

ROLLING CONTACT BEARING DESIGN CALCULATIONS - CHAPTER 14										Summary of Design Calculations					
USING DATA FROM TABLE 14-3					INNER RACE ROTATES IN ALL CASES					See manual solutions for details of calculations					
PROB NO. & BRG	RADIAL LOAD, R (LB)	THRUST LOAD, T (LB)	SPEED (RPM)	LIFE (HR)	LIFE (REV.)	EQUIV LOAD, P (LB)	DYNAMIC LOAD, P (LB)	BRG. NO.	RATED LOAD, C (LB)	BEARING BORE (mm)	BEARING BORE (in)	MIN. BORE (in)	X	Y	C <sub>0</sub>
5-BRG. B	4643	0	600	10000	3.60E+08	4643	33029	6319	34397	95	3.7402	3.55	1	0	48561
5-BRG. C	2078	0	600	10000	3.60E+08	2078	14782	6311	16076	55	2.1654	1.09	1	0	48561
6-BRG. B	611	0	1700	5000	5.1E+08	611	4882	6011	6317	55	2.1654	2.00	1	0	4766
6-BRG. D	436	0	1700	5000	5.1E+08	436	3483	6011	6317	55	2.1654	1.98	1	0	4766
7-BRG. A	509	0	101	20000	1.21E+08	509	2519	6302	2563	15	0.5906	0.59	1	0	1214
7-BRG. C	1742	265	101	20000	1.21E+08	1742	8621	6212	10679	60	2.3622	2.26	1	0	7307
9	455	0	1150	20000	1.38E+09	455	5066	6306	6317	30	1.1811	NS	1	0	10
10	857	0	450	30000	8.1E+08	857	7989	6308	9218	40	1.5748	NS	1	0	10
11	1265	645	210	5000	6.30E+07	1579	6284	6307	7464	35	1.3780	NS	0.56	1.35	4272
12	235	88	1750	20000	2.1E+09	301	3860	6305	5058	25	0.9843	NS	0.56	1.93	2608
13	2875	1350	600	15000	5.4E+08	3919	31909	6318	32149	90	3.5433	NS	0.56	1.71	24281
14(M->lb)	854	0	3450	15000	3.11E+09	854	12459	6310	13894	50	1.9685	NS	1	0	10
14(kN)	3.8	0	3450	15000	3.11E+09	3.80	55.44	6310	61.8	50	1.9685	NS	1	0	10
15(kN)	5.6	2.8	450	2000	5.40E+07	6.78	25.61	6306	28.1	30	1.1811	NS	0.56	1.3	16
16(kN)	10.5	0	1150	20000	1.38E+09	10.50	116.90	6316	124.0	80	3.1496	NS	1	0	10
16(M->lb)	2361	0	1150	20000	1.38E+09	2361	26286	6316	27878	80	3.1496	NS	1	0	10
17(kN)	1.2	0.85	860	20000	1.03E+09	2.04	20.62	6305	22.5	25	0.9843	NS	0.56	1.61	11.6
24-1	1750	350	101		0	1750	0	6211	9802	55	2.1654	NS	1	0	6520
24-2	600	250	101		0	809	0	6211	9802	55	2.1654	NS	0.56	1.89	6520
24-3	280	110	101		0	403	0	6211	9802	55	2.1654	NS	0.56	2.24	6520

See manual solution for combined loading and overall life for Problem 24.

19.

VARYING LOADS : (BRG. 6324)  $n = 600 \text{ rpm}$ ;  $C = 46783$

1	4500 LB	25 MIN	$F_m = \left( \frac{25(4500)^3 + 15(2500)^3}{40} \right)^{1/3} = 3975 \text{ LB}$
2	2500 LB	15 MIN	

$$L = \left( \frac{C}{F_m} \right)^3 = \left( \frac{46783}{3975} \right)^3 = 1630 \times 10^6 \text{ rev} \times \frac{\text{MIN}}{600 \text{ rev}} \times \frac{h}{60 \text{ MIN}}$$

$$L = 45285 \text{ h}$$

20.

BEARING 6314,  $n = 600 \text{ rpm}$ ;  $C = 23381 \text{ LB}$ .

1.	2500 LB	25 MIN	$F_m = \left( \frac{25(2500)^3 + 15(1500)^3}{40} \right)^{1/3} = 2226 \text{ LB}$
2.	1500 LB	15 MIN	

$$L = \left( \frac{C}{F_m} \right)^3 = \left( \frac{23381}{2226} \right)^3 = 1159 \times 10^6 \text{ rev} \times \frac{h}{(600 \text{ rev/min})(60 \text{ min/h})} = 32,189 \text{ h}$$

21.

BEARING 6209,  $n = 1700 \text{ rpm}$ ,  $C = 7464 \text{ LB}$

1.	600 LB	480 MIN
2.	200 LB	115 MIN
3.	100 LB	45 MIN
640 MIN		

$$F_m = \left( \frac{480(600)^3 + 115(200)^3 + 45(100)^3}{640} \right)^{1/3} = 547 \text{ LB}$$

$$L = \left( \frac{C}{F_m} \right)^3 = \left( \frac{7464 \text{ LB}}{547 \text{ LB}} \right)^3 = \frac{2580 \times 10^6 \text{ rev}}{(1700 \text{ rev/min})(60 \text{ min/h})} = 24,969 \text{ h}$$

22.

BEARING 6209,  $n = 1700 \text{ rpm}$ ,  $C = 7464 \text{ LB}$

1.	450 LB	480 MIN
2.	180 LB	115 MIN
3.	50 LB	45 MIN
640 MIN		

$$F_m = \left( \frac{480(450)^3 + 115(180)^3 + 45(50)^3}{640} \right)^{1/3} = 411 \text{ LB}$$

$$L = \left( \frac{C}{F_m} \right)^3 = \left( \frac{7464}{411} \right)^3 = \frac{5989 \times 10^6 \text{ rev}}{(1700 \text{ rev/min})(60 \text{ min/h})} = 58,720 \text{ h}$$



23. BEARING 6205,  $n = 101 \text{ rpm}$ ,  $C = 3147 \text{ LB}$

1. 500 LB 6.75 h

2. 800 LB 0.40 h

3. 100 LB 0.85 h

8.00 h

$$F_m = \left( \frac{6.75(500)^3 + 0.40(800)^3 + 0.85(100)^3}{8.0} \right)^{1/3} = 508 \text{ LB}$$

$$L = \left( \frac{C}{F_m} \right)^3 = \left( \frac{3147}{508} \right)^3 = \frac{237.7 \times 10^6 \text{ Rev}}{(101 \text{ Rev/min})(60 \text{ min/h})} = 39231 \text{ h}$$

24. BEARING 6211,  $n = 101 \text{ rpm}$ ,  $C = 9802 \text{ LB}$ ,  $C_0 = 6520 \text{ LB}$

COMBINED RADIAL AND THRUST LOADS:

COMPUTE EQUIVALENT LOAD P AS IN SECTION 14-10.

	$\frac{R}{1750 \text{ LB}}$	$\frac{T}{350 \text{ LB}}$	$\frac{T/R}{0.20}$	$\frac{T/C_0}{0.652}$	$\frac{e}{0.26}$	$\frac{Y}{1.89}$	$\frac{P}{1750 \text{ LB}} = R$
1. 6.75 h			0.20	0.652	0.26	1.89	809 LB
2. 0.40 h			0.417	0.0383	0.234	1.89	809 LB
3. 0.85 h			0.393	0.019	0.20	2.24	403 LB
8.00 h							

$$[P = 0.56R + YT] \rightarrow$$

USE EQUIVALENT LOADS TO COMPUTE  $F_m$ :

$$F_m = \left( \frac{6.75(1750)^3 + 0.40(809)^3 + 0.85(403)^3}{8.00} \right)^{1/3} = 1658 \text{ LB}$$

$$L = \left( \frac{C}{F_m} \right)^3 = \left( \frac{9802}{1658} \right)^3 = \frac{206.7 \times 10^6 \text{ Rev}}{(101 \text{ Rev/min})(60 \text{ min/h})} = 34,115 \text{ h}$$

25.

$$P_d = 1450 \text{ LB. } m = 1150 \text{ RPM. } L_d = 15000 \text{ h. } R = 0.95 \Rightarrow C_R = 0.62$$

$$\text{ACTUAL } L_d = (5000 \text{ h})(1150 \text{ RPM})(60 \text{ MIN/H}) = 1.04 \times 10^9 \text{ REV}$$

$$\text{ADJUSTED } L_{da} = L_d / C_R = 1.04 \times 10^9 / 0.62 = 1.68 \times 10^9 \text{ REV}$$

$$\text{EQ. 14-3: } C = P_d \left( \frac{L_{da}}{10^6} \right)^{1/3} = 1450 \left( \frac{1.68 \times 10^9}{10^6} \right)^{1/3} = \underline{17229 \text{ LB.}}$$

26.

$$P_d = 509 \text{ LB. } m = 101 \text{ RPM. } L_d = 26000 \text{ h. } R = 0.99 \Rightarrow C_R = 0.21$$

$$\text{ACTUAL } L_d = (20000)(101)(60) = 1.21 \times 10^8 \text{ REV.}$$

$$\text{ADJUSTED } L_{da} = L_d / C_R = 1.21 \times 10^8 / 0.21 = 5.77 \times 10^8 \text{ REV}$$

$$\text{EQ. 14-3: } C = 509 \left( \frac{5.77 \times 10^8}{10^6} \right)^{1/3} = \underline{4238 \text{ LB.}}$$

27.

$$P_d = 436 \text{ LB. } m = 1700 \text{ RPM. } L_d = 5000 \text{ h. } R = 0.97 \Rightarrow C_R = 0.44$$

$$\text{ACTUAL } L_d = (5000)(1700)(60) = 5.10 \times 10^8 \text{ REV.}$$

$$\text{ADJUSTED } L_{da} = L_d / C_R = 5.10 \times 10^8 / 0.44 = 1.16 \times 10^9 \text{ REV}$$

$$\text{EQ. 14-3: } C = 436 \left( \frac{1.16 \times 10^9}{10^6} \right)^{1/3} = \underline{4580 \text{ LB.}}$$

28.

$$P_d = 1250 \text{ LB. } m = 880 \text{ RPM. } L_d = 20000 \text{ h. } R = 0.95 \Rightarrow C_R = 0.62$$

$$\text{ACTUAL } L_d = (20000 \text{ h})(880 \text{ RPM})(60 \text{ MIN/H}) = 1.06 \times 10^9 \text{ REV}$$

$$\text{ADJUSTED } L_d = L_d / C_R = 1.06 \times 10^9 / 0.62 = 1.70 \times 10^9 \text{ REV}$$

$$\text{EQ. 14-3: } C = 1250 \left( \frac{1.70 \times 10^9}{10^6} \right)^{1/3} = \underline{14928 \text{ LB.}}$$

## CHAPTER 16 PLAIN SURFACE BEARINGS

All of the problems in this chapter are design problems with no unique solutions. A sample of each type of design problem is shown here.

1.  $F = 225 \text{ LB}; D = 3.00 \text{ IN}; M = 1750 \text{ RPM}$  BOUNDARY LUBRICATED  
BEARINGS  
 LET  $L = 1.5D = 1.5(3.00) = 4.50 \text{ IN.}$   
 $p = \frac{F}{LD} = \frac{225 \text{ LB}}{(4.50)(3.00) \text{ IN}^2} = 16.67 \text{ psi}$   
 $V = \pi D m / 12 = \pi(3.00)(1750) / 12 = 1374 \text{ FT/min}$   
 $pV = (16.67)(1374) = 22900 \text{ psi-fpm}$   
 REQ'D  $pV$ -RATING =  $2(pV) = 2(22900) = 45800 \text{ psi-fpm}$   
POROUS BRONZE BEARING MATERIAL/OIL IMPREGNATED  
 OR BU OR DU DRY LUBRICATED BEARING
4.  $F = 75 \text{ LB}; D = 0.50 \text{ in}; m = 600 \text{ rpm}$   
 LET  $L = 1.5D = 1.5(0.50) = 0.75 \text{ in}$   
 $p = F/LD = 75 / (0.75 \times 0.50) = 200.0 \text{ psi}$  }  $pV = 15708 \text{ psi-fpm}$   
 $V = \pi(0.50)(600) / 12 = 78.5 \text{ ft/min}$   
 REQ'D  $pV = 2(15708) = 31416 \text{ psi-fpm}$  POROUS BRONZE  
 OR BU BEARING
7.  $F = 800 \text{ LB}; D = 3.00 \text{ in}; m = 350 \text{ rpm}$   
 LET  $L = 1.5D = 1.5(3.00) = 4.50 \text{ in}$   
 $p = F/LD = 800 / (4.50)(3.00) = 59.3 \text{ psi}$  }  $pV = 16290 \text{ psi-fpm}$   
 $V = \pi(3.00)(350) / 12 = 275 \text{ fpm}$   
 REQ'D  $pV = 2(16290) = 32580 \text{ psi-fpm}$  POROUS BRONZE  
 OR BU BEARING
8.  $F = 60 \text{ LB}; D = 0.75 \text{ IN}; m = 750 \text{ RPM}; \text{TRY } L = 1.25D = 1.25(0.75) = 0.938 \text{ IN}$   
 LET  $L = 1.00 \text{ IN.}; L/D = 1.00 / 0.75 = 1.33 \text{ OK}$   
 $p = F/LD = 60 / (1.00 \times 0.75) = 80 \text{ psi}$  }  $pV = 11784 \text{ psi-fpm}$   
 $V = \pi D m / 12 = \pi(0.75)(750) / 12 = 147.3 \text{ ft/min}$   
 REQ'D  $pV$  RATING =  $2(11784) = 23568 \text{ psi-fpm}$  - USE BABBITT-HIGH TIN

## HYDRODYNAMIC LUBRICATION

9.  $F = 1250 \text{ LB}$ ;  $D_{\text{MIN}} = 2.60 \text{ IN.}$ ;  $n = 1750 \text{ RPM}$ ; ELECTRIC MOTOR

LET  $D = 2.75 \text{ IN}$ ,  $R = D/2 = 1.375 \text{ IN.}$

FOR  $\mu = 300 \text{ PSI}$ ;  $L = \frac{F}{\mu D} = \frac{1250 \text{ LB}}{(300 \text{ LB/IN}^2)(2.75 \text{ IN})} = 1.515 \text{ IN}$

$L/D = 1.515/2.75 = 0.55$ ; LET  $L = 0.5D = 1.375 \text{ IN}$ ;  $L/D = 0.50$

$\mu = \frac{F}{LD} = \frac{1250 \text{ LB}}{(1.375)(2.75) \text{ IN}^2} = 331 \text{ PSI OK}$

$C_d = 0.0036 \text{ IN}$ ;  $C_n = 0.0018 \text{ IN}$ ;  $R/C_n = 1.375/0.0018 = 754$   
(FIG 16-3)

SURFACE FINISH: 16-32  $\mu\text{N}$  AVG

$h_0 \approx 0.00025(2.75 \text{ IN}) = 0.00069 \text{ IN}$ ; USE  $h_0 = 0.0007 \text{ IN.}$

$h_0/C_n = 0.0007/0.0018 = 0.389 \rightarrow S = 0.29$   
(FIG. 16-7)

$n_s = n/60 = 1750/60 = 29.17 \text{ REV/S}$

REQ'D  $\mu = \frac{S \mu}{n_s (R/C_n)^2} = \frac{(0.29)(331)}{(29.17)(754)^2} = 5.79 \times 10^{-6} \text{ REYNS}$

SAE 50 OIL HAS  $\mu = 6.5 \times 10^{-6} \text{ REYNS @ } 160^\circ\text{F}$

$S$  IS PROPORTIONAL TO  $\mu$ :  $S = 0.29 (6.5/5.79) = 0.326$

FROM FIG 16-8:  $\xi(R/C_n) = 8.0$

$\xi = \frac{\xi(R/C_n)}{(R/C_n)} = \frac{8.0}{754} = 0.0106$

$T_f = \xi F R = (0.0106)(1250)(1.375) = 18.2 \text{ LB}\cdot\text{IN}$

$P_f = \frac{T_f n}{63000} = \frac{(18.2 \text{ LB}\cdot\text{IN})(1750 \text{ RPM})}{63000} = 0.506 \text{ HP}$

13.

$F = 500 \text{ LB}$ ;  $D_{\text{MIN}} = 1.15 \text{ IN}$ ;  $M = 2500 \text{ RPM}$ ; PRECISION SPINDLE

LET  $D = 1.200 \text{ IN}$ ;  $R = \frac{D}{2} = 0.600 \text{ IN}$ .

FOR  $\mu = 200 \text{ PSI}$ ;  $L = \frac{F}{\mu} = \frac{500 \text{ LB}}{(200 \text{ LB/IN}^2)(1.200 \text{ IN})} = 2.08$

$L/D = 2.08/1.200 = 1.73$ ; LET  $L/D = 1.50$ ;  $L = 1.50D = (1.5)(1.2) = 1.800 \text{ IN}$

$\mu = \frac{F}{L \cdot D} = \frac{500}{(1.800)(1.200)} = 232 \text{ PSI}$  OK

$C_d = 0.0014 \text{ IN}$ ;  $C_n = 0.0007 \text{ IN}$ ;  $R/C_n = 0.600/0.0007 = 857$

SURFACE FINISH: 8-16  $\mu\text{M}$  AVG.

$h_o \approx 0.00025(1.20) = 0.00030 \text{ IN}$ ;  $h_o/C_n = 0.0003/0.0007 = 0.429$

$S = 0.11$ ;  $M_s = 2500/60 = 41.67 \text{ REV/S}$

REQ'D  $\mu = \frac{S \cdot \mu}{M_s (R/C_n)^2} = \frac{(0.11)(232)}{(41.67)(857)^2} = 0.832 \times 10^{-6} \text{ REYNS}$

SAE 5W HAS  $\mu = 0.91 \times 10^{-6}$  @  $160^\circ\text{F}$

$S$  IS PROPORTIONAL TO  $\mu$ :  $S = 0.11 \left( \frac{0.91}{0.832} \right) = 0.120$

$f(R/C_n) = 2.80$  FROM FIG. 16-8:

$f = \frac{f(R/C_n)}{R/C_n} = \frac{2.80}{857} = 0.0033$

$T_f = f F R = (0.0033)(500 \text{ LB})(0.60 \text{ IN}) = 0.98 \text{ LB}\cdot\text{IN}$

$P_f = T_f M / 63000 = (0.98 \text{ LB}\cdot\text{IN})(2500 \text{ RPM}) / 63000 = 0.039 \text{ HP}$



16.

$$F = 18.7 \text{ kN}; D_{\text{MIN}} = 100 \text{ mm}; n = 500 \text{ RPM}; \text{ CONVEYOR}$$

$$D = 100 \text{ mm}; R = 50 \text{ mm}$$

$$\text{FOR } p = 2.0 \text{ MPa} = 2.0 \text{ N/mm}^2; L = \frac{F}{p \cdot D} = \frac{18.7 \times 10^3 \text{ N}}{(2.0 \text{ N/mm}^2) \cdot 100 \text{ mm}} = 93.5 \text{ mm}$$

$$\text{LET } L/D = 1.0; L = D = 100 \text{ mm}$$

$$p = \frac{F}{L \cdot D} = \frac{18.7 \times 10^3 \text{ N}}{(100)(100) \text{ mm}^2} = 1.87 \text{ N/mm}^2 = 1.87 \text{ MPa OK}$$

$$C_s = 0.15 \text{ mm}; C_r = 0.075 \text{ mm}; R/C_r = 50/0.075 = 667$$

(LARGE CLEARANCE DESIRED)

$$\text{SURFACE FINISH: NOTE: } 1.0 \mu\text{m} = 1.0 \times 10^{-6} \text{ m} \times \frac{0.0254 \text{ m}}{1 \text{ IN}} = 0.0254 \mu\text{m}$$

$$\text{THEN } 0.20 \mu\text{m}; 16 \mu\text{m} = 0.40 \mu\text{m}; 32 \mu\text{m} = 0.80 \mu\text{m}$$

$$63 \mu\text{m} = 1.60 \mu\text{m}$$

SPECIFY SURFACE FINISH = 0.8 TO 1.6 μm AVG.

$$h_0 = 0.00025(100) = 0.025 \text{ mm}; h_0/C_r = 0.025/0.075 = 0.333$$

$$S = 0.096; M_s = n/60 = 500/60 = 8.33 \text{ REV/S}$$

$$\text{REQ'D } \mu = \frac{S \cdot p}{M_s (R/C_r)^2} = \frac{(0.096)(1.87 \times 10^6 \text{ Pa})}{(8.33 \text{ REV/S})(667)^2} = 0.0485 \text{ Pa}\cdot\text{s}$$

AT 70°C, SAE 50 HAS  $\mu = 0.046 \text{ Pa}\cdot\text{s}$ ;  $h_0$  SLIGHTLY  $< 0.025 \text{ mm}$

$$S = (0.096) \frac{0.046}{0.0485} = 0.091 \rightarrow f(R/C_r) = 2.6$$

$$f = \frac{f(R/C_r)}{R/C_r} = \frac{2.6}{667} = 0.0039$$

$$T_f = f F R = (0.0039)(18.7 \times 10^3 \text{ N})(50 \times 10^{-3} \text{ m}) = 3.65 \text{ N}\cdot\text{m}$$

$$P_f = T_f n = 3.65 \text{ N}\cdot\text{m} \times 8.33 \frac{\text{REV}}{\text{S}} \times \frac{2\pi \text{ RAD}}{\text{REV}} = 191 \frac{\text{N}\cdot\text{m}}{\text{S}} = 191 \text{ WATTS}$$

17.

$$F = 225 \text{ kN}; D_{\text{min}} = 25 \text{ mm}; n = 2200 \text{ RPM}; \text{ MACHINE TOOL}$$

$$\text{LET } D = 25 \text{ mm}; R = D/2 = 12.5 \text{ mm} \quad \left| \begin{array}{l} C_d = 0.036 \text{ mm}; C_n = 0.018 \text{ mm} \\ R/C_n = \frac{12.5}{0.018} = 694 \end{array} \right.$$

$$\text{FOR } p = 2.0 \text{ MPa}; L = \frac{F}{p} = \frac{2.25 \times 10^3 \text{ N}}{(2.0 \text{ N/mm}^2)(25 \text{ mm})} = 45 \text{ mm}$$

$$\text{LET } L = 2D = 50 \text{ mm}; L/D = 2.0$$

$$p = \frac{F}{LD} = \frac{2.25 \times 10^3 \text{ N}}{(50)(25) \text{ mm}^2} = 1.80 \text{ N/mm}^2 = 1.80 \text{ MPa OK}$$

SURFACE FINISH: 16-32  $\mu\text{m}$  AVG. (0.4 TO 0.8  $\mu\text{m}$  AVG.)

$$l_0 = 0.0025(25) = 0.00625 \text{ mm} \approx 0.006 \text{ mm}$$

$$h_0/C_n = 0.006/0.018 = 0.333 \rightarrow S = 0.057$$

$$m_s = n/60 = 2200/60 = 36.7 \text{ REV/S}$$

$$\text{REQD. } \mu = \frac{S p}{m_s (R/C_n)^2} = \frac{(0.057)(1.80 \times 10^6 \text{ Pa})}{(36.7 \text{ REV/S})(694)^2} = 0.0058 \text{ Pa}\cdot\text{s}$$

SAE 5N HAS  $\mu = 0.0066 \text{ Pa}\cdot\text{s}$  @ 70 °C

$$S = 0.057 \left( \frac{0.0066}{0.0058} \right) = 0.065 \rightarrow f(R/C_n) = 1.6$$

$$f = \frac{f(R/C_n)}{R/C_n} = \frac{1.6}{694} = 0.0023$$

$$T_f = f F R = (0.0023)(2.25 \times 10^3 \text{ N})(12.5 \times 10^{-3} \text{ m}) = 0.065 \text{ N}\cdot\text{m}$$

$$P_f = T_f m = (0.065 \text{ N}\cdot\text{m}) \left( 36.7 \frac{\text{REV}}{\text{s}} \right) \left( \frac{2\pi \text{ RAD}}{\text{REV}} \right) = 15.0 \frac{\text{N}\cdot\text{m}}{\text{s}}$$

$$P_f = 15.0 \text{ WATTS}$$

19.

HYDROSTATIC LUBRICATION

$$F = 1250 \text{ LB}; p_s = 300 \text{ psi}; \text{LET } p_n = 250 \text{ psi}$$

$$\text{LET } R_n/R = 0.50; a_f = 0.55; q_f = 1.40$$

$$A_p = \frac{F}{a_f p_n} = \frac{1250 \text{ LB}}{(0.55)(250 \text{ LB/IN}^2)} = 9.09 \text{ IN}^2 = \pi D^2/4$$

$$D = \sqrt{4A_p/\pi} = \sqrt{4(9.09)/\pi} = 3.40 \text{ IN. OK}$$

$$R = D/2 = 1.70 \text{ IN}; R_n = 0.5R = 0.5(1.7) = 0.85 \text{ IN.}$$

$$\text{LET } h = 0.005 \text{ IN}; \text{SAE 30 OL @ } 120^\circ \text{F} - \mu = 8.3 \times 10^{-6} \text{ LB}\cdot\text{S/IN}^2$$

$$Q = q_f \frac{F R^3}{A_p \mu} = \frac{(1.40)(1250 \text{ LB})(0.005)^3 \text{ IN}^3}{(9.09 \text{ IN}^2)(8.3 \times 10^{-6} \text{ LB}\cdot\text{S/IN}^2)} = 2.90 \text{ IN}^3/\text{S}$$

$$P = p_n Q = (250 \text{ LB/IN}^2)(2.90 \text{ IN}^3/\text{S}) = 725 \text{ LB}\cdot\text{IN/S} \times \frac{1 \text{ LB}}{6600 \text{ LB}\cdot\text{IN/S}} = 0.11 \text{ HP}$$

21.

$$F = 3500 \text{ LB}; p_s = 500 \text{ psi}; \text{LET } p_n = 350 \text{ psi}$$

$$\text{LET } R_n/R = 0.60; a_f = 0.62; q_f = 1.60$$

$$A_p = \frac{F}{a_f p_n} = \frac{3500 \text{ LB}}{(0.62)(350 \text{ LB/IN}^2)} = 16.13 \text{ IN}^2$$

$$D = \sqrt{4A_p/\pi} = \sqrt{4(16.13)/\pi} = 4.53 \text{ IN} - \text{USE } D = 4.50 \text{ IN.}$$

$$A_p = \pi D^2/4 = \pi(4.50)^2/4 = 15.90 \text{ IN}^2$$

$$p_n = \frac{F}{a_f A_p} = \frac{3500 \text{ LB}}{(0.62)(15.90 \text{ IN}^2)} = 355 \text{ psi OK}$$

$$R = D/2 = 2.25 \text{ IN.}; R_n = 0.60R = 1.35 \text{ IN.}$$

$$\text{LET } h = 0.008 \text{ IN.}; \text{SAE 40 OL @ } 140^\circ \text{F}; \mu = 7.0 \times 10^{-6} \text{ LB}\cdot\text{S/IN}^2$$

$$Q = \frac{q_f F R^3}{A_p \mu} = \frac{(1.60)(3500 \text{ LB})(0.008)^3 \text{ IN}^3}{(15.90 \text{ IN}^2)(7.0 \times 10^{-6} \text{ LB}\cdot\text{S/IN}^2)} = 25.8 \text{ IN}^3/\text{S}$$

$$P = p_n Q = (355 \text{ LB/IN}^2)(25.8 \text{ IN}^3/\text{S}) = \frac{9145 \text{ LB}\cdot\text{IN/S}}{(6600 \text{ LB}\cdot\text{IN/S})/\text{HP}} = 1.39 \text{ HP}$$

25. HYDROSTATIC LUBRICATION - METRIC DATA

$$F = 22.5 \text{ kN}; p_s = 2.0 \text{ MPa}; \text{LET } p_n = 1.60 \text{ MPa} = 1.6 \times 10^6 \frac{\text{N}}{\text{m}^2} = 1.60 \text{ N/mm}^2$$

$$\underline{R_n/R = 0.60}; a_s = 0.62; q_f = 1.60$$

$$A_p = \frac{F}{a_s p_n} = \frac{22.5 \times 10^3 \text{ N}}{(62)(1.60 \text{ N/mm}^2)} = 2.27 \times 10^4 \text{ mm}^2$$

$$\underline{D = \sqrt{4A_p/\pi} = \sqrt{4(2.27 \times 10^4)/\pi} = 170 \text{ mm OK}}$$

$$R = D/2 = 85 \text{ mm}; \underline{R_n = 0.6R = 51.0 \text{ mm}}$$

$$\text{LET } h = 0.15 \text{ mm}; \text{ AT } 50^\circ\text{C, SAE 30 OIL HAS } \mu = 0.054 \text{ Pa}\cdot\text{s}$$

$$\underline{Q = \frac{q_f F h^3}{A_p \mu} = \frac{(1.60)(22500 \text{ N})(.15)^3 \text{ mm}^3}{(2.27 \times 10^4 \text{ mm}^2)(0.054 \text{ N}\cdot\text{s/m}^2)} \cdot \frac{1 \text{ m}}{10^3 \text{ mm}} = 9.91 \times 10^{-5} \text{ m}^3/\text{s}}$$

$$\underline{P = p_n Q = \left(1.60 \times 10^6 \frac{\text{N}}{\text{m}^2}\right) \left(9.91 \times 10^{-5} \frac{\text{m}^3}{\text{s}}\right) = 159 \frac{\text{N}\cdot\text{m}}{\text{s}} = 159 \text{ WATTS}}$$

26.

$$F = 1.20 \text{ kN}; p_s = 750 \text{ kPa} = 0.75 \text{ MPa}$$

$$\text{LET } p_n = 600 \text{ kPa} = 0.60 \text{ MPa} = 0.60 \times 10^6 \text{ N/m}^2 = 0.60 \text{ N/mm}^2$$

$$\underline{R_n/R = 0.60}; a_s = 0.62; q_f = 1.60$$

$$A_p = \frac{F}{a_s p_n} = \frac{1.20 \times 10^3 \text{ N}}{(62)(.60 \text{ N/mm}^2)} = 3226 \text{ mm}^2$$

$$\underline{D = \sqrt{4A_p/\pi} = \sqrt{4(3226)/\pi} = 64 \text{ mm OK}}$$

$$R = D/2 = 32 \text{ mm}; \underline{R_n = 0.6R = .6(32) = 19.2 \text{ mm}}$$

$$\text{LET } h = 0.10 \text{ mm}; \text{ AT } 60^\circ\text{C, SAE 10W OIL HAS } \mu = 0.014 \text{ Pa}\cdot\text{s}$$

$$\underline{Q = \frac{q_f F h^3}{A_p \mu} = \frac{(1.60)(1200 \text{ N})(.10)^3 \text{ mm}^3}{(3226 \text{ mm}^2)(0.014 \text{ N}\cdot\text{s/m}^2)} \cdot \frac{1 \text{ m}}{10^3 \text{ mm}} = 4.25 \times 10^{-5} \text{ m}^3/\text{s}}$$

$$\underline{P = p_n Q = 0.60 \times 10^6 \frac{\text{N}}{\text{m}^2} \cdot 4.25 \times 10^{-5} \frac{\text{m}^3}{\text{s}} = 25.5 \frac{\text{N}\cdot\text{m}}{\text{s}} = 25.5 \text{ WATTS}}$$

## CHAPTER 17 LINEAR MOTION ELEMENTS

5.

$$\text{REQ'D. } A_t = \frac{F}{\sigma_t} = \frac{30000 \text{ LB}}{10000 \text{ LB/IN}^2} = 3.0 \text{ IN}^2 \text{ USE } 2\frac{1}{2}\text{-3 ACME THREAD.}$$

$$A_t = 3.802 \text{ IN}^2$$

6.

$$\text{REQ'D } A_s = \frac{F}{T_u} = \frac{30000 \text{ LB}}{6000 \text{ LB/IN}^2} = 5.0 \text{ IN}^2$$

FOR AN ACME 2 1/2-3:  $A_s = 4.075 \text{ IN}^2/\text{IN}$  OF LENGTH  
 REQ'D.  $L = 5.0 / 4.075 = 1.23 \text{ IN}$ .

7.

$$\lambda = \tan^{-1} \left( \frac{L}{\pi D_p} \right) = \tan^{-1} \left( \frac{0.233}{\pi(2.2932)} \right) = \tan^{-1}(0.0463) = 2.65^\circ$$

$\uparrow \tan \lambda$

$$\cos \phi = \cos 14.5^\circ = 0.968$$

(EQ. 18-10)

$$T_u = \frac{F D_p}{2} \left[ \frac{\cos \phi \tan \lambda + f}{\cos \phi - f \tan \lambda} \right] = \frac{30000(2.2932)}{2} \left[ \frac{(0.968)(0.0463) + .15}{.968 - .15(0.0463)} \right]$$

$$T_u = 6974 \text{ lb}\cdot\text{in}$$

8.

$$T_d = \frac{F D_p}{2} \left[ \frac{f - \cos \phi \tan \lambda}{\cos \phi + f \tan \lambda} \right] = \frac{30000(2.2932)}{2} \left[ \frac{.15 - (0.968)(0.0463)}{.968 + .15(0.0463)} \right]$$

$$T_d = 3712 \text{ lb}\cdot\text{in}$$

9.

SQUARE THREAD: 3/4-6;  $F = 4000 \text{ LB}$ ,  $L = p = \frac{1}{6} = \frac{1}{6} = 0.1667 \text{ IN}$ .

$$T_u = \frac{F D_p}{2} \left[ \frac{L + \pi f D_p}{\pi D_p - f L} \right] = \frac{4000(.6424)}{2} \left[ \frac{.1667 + \pi(.15)(.6424)}{\pi(.6424) - .15(.1667)} \right]$$

$$T_u = 303 \text{ lb}\cdot\text{in}$$

10.

$$T_d = \frac{F D_p}{2} \left[ \frac{\pi f D_p - L}{\pi D_p + f L} \right] = \frac{4000(.6424)}{2} \left[ \frac{\pi(.15)(.6424) - .1667}{\pi(.6424) + .15(.1667)} \right] = 86 \text{ lb}\cdot\text{in}$$

11.

$$\lambda = \tan^{-1} (L / \pi D_p) = \tan^{-1} (1667 / \pi(6424)) = 4.72^\circ$$

$\tan \lambda = 0.083 < f \rightarrow$  SELF LOCKING



$$12. \quad e = \frac{FL}{2\pi T_m} = \frac{(4000)(0.1667)}{2\pi(303)} = 0.35 \text{ or } 35\%$$

$$13. \quad n = \frac{0.50 \text{ IN}}{5} \times \frac{1 \text{ REV.}}{0.1667 \text{ IN}} \times \frac{60 \text{ S}}{\text{MIN}} = 180 \text{ RPM}$$

$$P = T_m / 63000 = (303)(180) / 63000 = 0.866 \text{ hp}$$

$$14. \quad \text{TRAVEL} = \frac{2 \text{ IN}}{\text{CYCLE}} \times \frac{10 \text{ CYCLES}}{\text{HR}} \times \frac{2 \text{ HR}}{\text{DAY}} \times \frac{365 \text{ DAYS}}{\text{YR}} \times 10 \text{ YRS} = 2.10 \times 10^7 \text{ INCHES}$$

AT 600 LB; 3/4-2 SCREW REQ'D.; L=0.50 IN

$$15. \quad T = \frac{FL}{2\pi e} = \frac{(600)(.50)}{2\pi(1.9)} = 53.1 \text{ lb-in}$$

$$16. \quad n = \frac{1 \text{ REV}}{0.5 \text{ IN}} \times \frac{10.0 \text{ IN}}{\text{MIN.}} = 20.0 \text{ RPM}$$

$$P = \frac{T_m}{63000} = \frac{53.1(20.0)}{63000} = 0.017 \text{ hp}$$

17. FOR THE 3/4-2 SCREW AT 600 LB, TRAVEL LIFE =  $1.04 \times 10^8$  IN.

$$1.04 \times 10^8 \text{ IN.} \times \frac{\text{CYCLE}}{24 \text{ IN}} \times \frac{\text{HR}}{20 \text{ CYCLES}} \times \frac{\text{DAY}}{24 \text{ HR}} \times \frac{\text{YR}}{365 \text{ DAYS}} = 24.7 \text{ YRS}$$

METRIC TRAPEZOIDAL POWER SCREWS - TABLE 17-1 M

18. SPECIFY A SIZE: LOAD = 125 kN;  $\sigma_a = 75 \text{ MPa} = \frac{F}{A_T}$  PROBLEMS 18 TO 23 USE SAME DATA

$$\text{REQ'D. } A_T = \frac{F}{\sigma_a} = \frac{125000 \text{ N}}{75 \text{ N/mm}^2} = 1667 \text{ mm}^2 - \text{WE ASSY X 9 SCREW}$$

$$A_T = 1791 \text{ mm}^2$$

NOMINAL  $D_o = 55 \text{ mm}$ ; LEAD = PITCH = 9.0 mm FOR SINGLE THREAD  
 $D_p = 50.5 \text{ mm}$

19. FIND TORQUE TO RAISE 125 kN LOAD FOR  $f = 0.15$ .

$$T_u = \frac{FD_p}{2} \left[ \frac{L + \pi f D_p}{\pi D_p - fL} \right] = \frac{125000 \text{ N}(50.5 \text{ mm})}{2} \left[ \frac{9.0 + \pi(0.15)(50.5)}{\pi(50.5) - (0.15)(9.0)} \right] \text{ EQ 17-2}$$

$$T_u = 658559 \text{ N}\cdot\text{mm} = 658.6 \text{ N}\cdot\text{m}$$

20. FIND TORQUE TO LOWER LOAD.  $F = 125 \text{ kN}$ ,  $f = 0.15$

$$T_d = \frac{FD_p}{2} \left[ \frac{\pi f D_p - L}{\pi D_p + fL} \right] = \frac{125000 \text{ N}(50.5 \text{ mm})}{2} \left[ \frac{\pi(0.15)(50.5) - 9}{\pi(50.5) + 0.15(9)} \right] \text{ EQ 17-11}$$

$$T_d = 291905 \text{ N}\cdot\text{mm} = 291.9 \text{ N}\cdot\text{m}$$

21. FIND POWER TO RAISE 125 kN - 4250 mm IN 7.5 s. M55 X 9 SCREW

$$P = T_u \cdot \omega; T_u = 658.6 \text{ N}\cdot\text{m FROM PROB. 19.}$$

$$\omega = \frac{N}{L} = \frac{d/t}{L} = \frac{4250 \text{ mm}}{7.5 \text{ s}} \times \frac{1 \text{ REV}}{9 \text{ mm}} \times \frac{2\pi \text{ RAD}}{1 \text{ REV}} = 395.6 \text{ RAD/s}$$

$$P = T_u \omega = 658.6 \text{ N}\cdot\text{m} \times 395.6 \text{ RAD/s} = 260547 \text{ N}\cdot\text{m/s} = 260547 \text{ W} \text{ (VERY HIGH)}$$

$$P = 260.5 \text{ kW}$$

22. FIND LEAD ANGLE  $\lambda$ :  $\lambda = \tan^{-1}\left(\frac{L}{\pi D_p}\right) = \tan^{-1}\left(\frac{9}{\pi(50.5)}\right) = 3.25^\circ$  EQ. 17-3  
FOR  $\lambda < 5^\circ$  - SELF LOCKING.

23. FIND EFFICIENCY:  $e = \frac{FL}{2\pi T_u} = \frac{125000 \text{ N}(9 \text{ mm})}{2\pi(658559 \text{ N}\cdot\text{mm})} = 0.272; 27.2\%$

24. SPECIFY A POWER SCREW SIZE:  $F = 8500 \text{ N}; \sigma_a = 110 \text{ MPa}$  PROBLEM 24-29  
USE SAME DATA

$$\text{REQD } A_T = \frac{F}{\sigma_a} = \frac{8500 \text{ N}}{110 \text{ N/mm}^2} = 77.3 \text{ mm}^2 \text{ --- M14 X 3 SCREW}$$

$$D_p = 12.5 \text{ mm}, L = \phi = 3.0 \text{ mm} \quad A_T = 103.9 \text{ mm}^2$$

25. FIND TORQUE TO RAISE LOAD;  $f = 0.15$

$$T_u = \frac{F D_p}{2} \left[ \frac{L + \pi f D_p}{\pi D_p - f L} \right] = \frac{8500 \text{ N}(12.5 \text{ mm})}{2} \left[ \frac{3.0 + \pi(0.15)(12.5)}{\pi(12.5) - 0.15(3)} \right]$$

$$T_u = 12167 \text{ N}\cdot\text{mm} = 12.167 \text{ N}\cdot\text{m}$$

26. FIND TORQUE TO LOWER LOAD.  $F = 8500 \text{ N}; f = 0.15$

$$T_d = \frac{F D_p}{2} \left[ \frac{\pi f D_p - L}{\pi D_p + f L} \right] = \frac{8500 \text{ N}(12.5 \text{ mm})}{2} \left[ \frac{\pi(0.15)(12.5) - 3}{\pi(12.5) + 0.15(3)} \right]$$

$$T_d = 3956 \text{ N}\cdot\text{mm} = 3.956 \text{ N}\cdot\text{m}$$

27. FIND POWER TO RAISE LOAD: 240 mm IN 3.5 s.

$$P = T_u \cdot \omega; T_u = 12.167 \text{ N}\cdot\text{m} \text{ (PROB 25)}$$

$$\omega = \frac{N}{L} = \frac{d/t}{L} = \frac{240 \text{ mm}}{3.5 \text{ s}} \times \frac{1 \text{ REV}}{3.0 \text{ mm}} \times \frac{2\pi \text{ RAD}}{1 \text{ REV}} = 143.62 \text{ RAD/s}$$

$$P = T_u \cdot \omega = 12.167 \text{ N}\cdot\text{m} (143.62 \text{ RAD/s}) = 1747 \text{ N}\cdot\text{m/s} = 1747 \text{ W} = 1.747 \text{ kW}$$

28. LEAD ANGLE  $\lambda = \tan^{-1}\left(\frac{L}{\pi D_p}\right) = \tan^{-1}\left(\frac{3.0}{\pi(12.5)}\right) = 4.37^\circ$

29. EFFICIENCY  $e = \frac{FL}{2\pi T_u} = \frac{8500 \text{ N}(3.0 \text{ mm})}{2\pi(12167 \text{ N}\cdot\text{mm})} = 0.334 = 33.4\%$

30. BALL SCREW FROM PROBLEM 14,  $\frac{3}{4}-2$ , LENGTH = 28.0 IN  
 FIND ESTIMATE OF CRITICAL SPEED. EQ. 17-15

$$m_c (\text{RPM}) = \frac{476 \times 10^6 d K_c}{(SF) L^2}$$

$d$  = MINOR DIA. OF SCREW. THIS VALUE NOT AVAILABLE IN THIS BOOK, OR ON INTERNET SITE ID. AS AN ESTIMATE, USE MINOR DIA. FOR A  $\frac{3}{4}$  ACME SCREW -  $d \approx 0.55$  IN - FROM MACHINERY'S HANDBOOK, 28TH ED., P. 1830.

$L = 28.0$  IN - BETWEEN SINGLE BEARINGS AT ENDS

$$K_c = 1.00 \quad \checkmark$$

LET  $SF = 1.0$  TO ESTIMATE CRITICAL SPEED.

$$m_c = \frac{476 \times 10^6 (0.55 \text{ IN}) (1.0)}{(1) (28.0 \text{ IN})^2} = \underline{3339 \text{ RPM}}$$

SAFE OPERATING SPEED -  $SF = 3.0$

$$m_{\text{OPER}} = \frac{m_c}{SF} = \frac{3339 \text{ RPM}}{3} = 1113 \text{ RPM}$$

## CHAPTER 18 SPRINGS

$$1 \quad k = \Delta F / \Delta L = 12.0 / (2.75 - 1.85) = \underline{13.3 \text{ LB/IN.}}$$

$$2 \quad k = \frac{F_s - 0}{L_s - L_0} ; L_s - L_0 = \frac{F_s}{k} ; L_s = L_0 + \frac{F_s}{k} = 1.25 + \frac{4.65 \text{ LB}}{18.840/\text{IN.}} = \underline{1.497 \text{ IN.}}$$

$$3 \quad k = \frac{F_s - F_0}{L_s - L_0} ; F_s = k(L_0 - L_c) + F_0 = 76.7(0.830 - 0.626) + 32.2 = \underline{47.8 \text{ LB} = F_s}$$

$$L_s = L_0 + F_s/k = 0.830 + 32.2/76.7 = \underline{1.25 \text{ IN} = L_s}$$

$$4 \quad k = \Delta F / \Delta L = 99.2 \text{ N} / (63.5 - 37.1) \text{ mm} = \underline{3.76 \text{ N/mm}}$$

$$5 \quad L_s = L_0 + F_s/k = 39.47 \text{ mm} + 54.05 \text{ N} / 1.47 \text{ N/mm} = \underline{76.24 \text{ mm}}$$

$$6 \quad F_s = k(L_0 - L_c) + F_0 = 8.95(29.4 - 21.4) + 134 \text{ N} = \underline{205.6 \text{ N} = F_s}$$

$$L_s = L_0 + F_s/k = 29.4 + 134/8.95 = \underline{44.4 \text{ mm} = L_s}$$

$$7 \quad I D = O D - 2 D_w = 1.100 - 2(0.085) = \underline{0.93 \text{ IN} = I D}$$

$$D_m = O D - D_w = 1.100 - 0.085 = \underline{1.015 \text{ IN} = D_m}$$

$$C = D_m / D_w = 1.015 / 0.085 = \underline{11.94 = C}$$

$$L_s = N D_w ; N = L_s / D_w = 0.563 / 0.085 = \underline{6.6 \text{ COILS}}$$

$$8 \quad D_m = O D - D_w = 0.560 - 0.059 = 0.501 \text{ IN} ; C = D_m / D_w = 0.501 / 0.059 = \underline{8.49 = C}$$

$$L_s = p N_a + 2 D_w ; N_a = N - 2 = 19 - 2 = 17 \text{ ACTIVE COILS}$$

$$p = \frac{L_s - 2 D_w}{N_a} = \frac{4.22 - 2(0.059)}{17} = \underline{0.241 \text{ IN} = p}$$

$$\lambda = \tan^{-1} \left( \frac{p}{\pi D_m} \right) = \tan^{-1} \left( \frac{0.241}{\pi(0.501)} \right) = \underline{8.7 \text{ DEGREES} = \lambda}$$

$$L_s = N D_w = 19(0.059) = \underline{1.12 \text{ IN.} = L_s}$$

$$9 \quad \text{FROM EQ 19-6: } F_0 = \frac{F_0 G D_w}{8 C^3 N_a} = \frac{(4.22 - 3.00)(11.85 \times 10^6)(0.059)}{8(0.49)^3(17)} = 10.25 \text{ LB} = F_0$$

$$\text{EQ. 7-4: } T_s = \frac{8 k F_0 C}{\pi D_w^2} = \frac{8(1.17)(10.25)(8.49)}{\pi(0.059)^2} = 74500 \text{ PSI} = T_s$$

$$k = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(8.49)-1}{4(8.49)-4} + \frac{0.615}{8.49} = 1.17$$

FROM FIGURE 18-9: MUSIC WIRE:  $T_j = 132000 \text{ PSI}$  FOR AVG. SERVICE -OK.

$$10. \quad L_4/D_m = 4.22/0.501 = 8.42: \text{ FROM FIG 19-15, CURVE A, } \left(\frac{f_0}{L_4}\right)_{CR} = 0.18$$

$$(f_0)_{CR} = 0.18 L_4 = 0.18(4.22) = 0.76 \text{ IN}$$

ACTUAL  $f_0 = 4.22 - 3.00 = 1.22 \text{ IN} > 0.76 \text{ IN}$  -BUCKLING SHOULD OCCUR

$$11. \quad (\text{EQ. 19-3})$$

$$(OD)_S = \sqrt{D_m^2 + \frac{F - D_w^2}{\pi^2}} + D_w = \sqrt{(0.501)^2 + \frac{0.241 - (0.059)^2}{\pi^2}} + 0.059 = 0.583 \text{ IN}$$

$$12. \quad F_s = k(L_4 - L_3) = 8.40(4.22 - 1.112) = 26.05 \text{ LB}$$

$$\text{FROM PROB. 9: } k = \frac{F_0}{L_4 - L_0} = \frac{10.25}{4.22 - 3.00} = 8.40 \text{ LB/IN}$$

$$T_s = T_0 \times F_s/F_0 = 74500 \text{ PSI} \times 26.05/10.25 = 189300 \text{ PSI TOO HIGH}$$

FROM FIG 19-9, LIGHT SERVICE;  $T_{Sj} = 147000 \text{ PSI} = \text{APPROX. } S_y$

Notes concerning Problems 13 - 35: Most of these problems are design problems. No single unique solutions exist. Sample solutions are shown.

Problems 13 through 24 are compression springs. Problem 13 is done by both methods 1 and 2 as outlined in the text. Others are done by one or the other method. In some problems only summary results are shown.

Problems 25 through 31 are extension springs. Problem 25 is worked out in detail. Problems 26 through 30 were designed with the aid of a computer program using the same procedure. Only summary results are shown. Problem 31 is the stress analysis of the ends of the spring.

Problems 32 through 35 are torsion springs. In each case, the design of each end is required. It was assumed that ends would be straight with lengths  $L_1$  and  $L_2$  as shown. The effects of the ends on the spring rate were then included in the analysis.



13 METHOD 1  $F_0 = 220 \text{ LB}$ ;  $F_1 = 180 \text{ LB}$ ;  $\Delta F = 220 - 180 = 40 \text{ LB}$ ;  $\Delta L = 0.50 \text{ IN}$ .

STEP 1 - ASTM A229; SEVERE SERVICE;  $G = 11.2 \times 10^6 \text{ PSI}$

STEPS 2-5 - LET  $L_i = 3.00 \text{ IN}$ ;  $D_m = 3.00 \text{ IN}$ . (DESIGN DECISIONS)

$$R = \Delta F / \Delta L = 40 \text{ LB} / 0.50 \text{ IN} = 80 \text{ LB/IN}$$

$$L_f = L_i + F_1 / R = 3.00 + 180 / 80 = 5.25 \text{ IN}$$

$$L_o = L_i - \Delta L = 3.00 - 0.50 = 2.50 \text{ IN}$$

$$f_o = L_f - L_o = 5.25 - 2.50 = 2.75 \text{ IN}$$

STEP 6 - ASSUME  $T_0 = 80,000 \text{ PSI}$

STEP 7 - (EQ. 18-7)

$$D_w = \left[ \frac{(3.06)(F_0)(D_m)}{T_0} \right]^{1/3} = \left[ \frac{3.06(220)(3.00)}{80,000} \right]^{1/3} = 0.293 \text{ IN}$$

STEP 8 - SELECT  $D_w = 0.3065 \text{ IN}$  U.S. GAGE 0;  $T_0 = 78 \text{ KSI}$ ;  $T_{MAX} = 104 \text{ KSI}$

STEP 9 -  $C = D_m / D_w = 3.00 / 0.3065 = 9.79$ ;  $K = 1.15$  (FIG. 18-14)

STEP 10 -

$$T_0 = \frac{8 K F_0 D_m}{\pi D_w^3} = \frac{8(1.15)(220)(3.00)}{\pi(0.3065)^3} = 67,126 \text{ PSI} \quad \text{OK}$$

$$\text{STEP 11 - } N_a = \frac{G D_w}{8 K C^3} = \frac{(11.2 \times 10^6)(0.3065)}{8(80)(9.79)^3} = 5.72 \text{ COILS}$$

$$\text{STEP 12 - } L_s = D_w(N_a + 2) = 0.3065(5.72 + 2) = 2.365 \text{ IN}$$

$$F_s = k(L_f - L_s) = 80(5.25 - 2.365) = 231 \text{ LB}$$

$$T_s = T_0(F_s / F_0) = 67,126(231 / 220) = 70,420 \text{ PSI} \quad \text{OK}$$

$$\text{STEP 13 - } D_o = D_m + D_w = 3.00 + 0.3065 = 3.3065 \text{ IN}$$

$$D_i = D_m - D_w = 3.00 - 0.3065 = 2.694 \text{ IN}$$

SUMMARY:  $D_w = 0.3065 \text{ IN}$ ;  $L_f = 5.25 \text{ IN}$ ;  $L_o = 2.50 \text{ IN}$ ;  $D_m = 3.00 \text{ IN}$ .

13 METHOD 2 PROCESS STARTS SAME AS METHOD 1 WITHOUT  $D_m$ .

$$\text{STEP 2 - (EQ. 18-10) } D_w = \sqrt[3]{21.4 F_0 / T_0} = \sqrt[3]{21.4(220) / 80,000} = 0.243 \text{ IN}$$

STEP 3 - TRY  $D_w = 0.2625 \text{ IN}$  U.S. GAGE 2;  $T_0 = 80,000 \text{ PSI}$

STEP 4 - (EQ. 18-11)

$$N_{a \text{ MAX}} = (L_o - 2D_w) / D_w = [2.50 - 2(0.2625)] / 0.2625 = 7.5 \text{ COILS}$$

$$\text{STEP 6 - (EQ. 19-12) } C = \left[ \frac{G D_w}{8 K N_a} \right]^{1/3} = \left[ \frac{(11.2 \times 10^6)(0.2625)}{8(80)(6.0)} \right]^{1/3} = 9.15 \quad \text{USE } N_a = 6.0 \quad \text{STEPS 5}$$

$$K = 1.16 \quad \text{(FIG. 7-12)}$$

$$\text{STEP 8 - (EQ. 19-4) } T_0 = \left( \frac{8 K C F_0}{\pi D_w^2} \right) = \frac{2.546(1.16)(9.15)(220)}{(0.2625)^2} = 86,262 \text{ PSI} \quad \left\{ \begin{array}{l} \text{TOO} \\ \text{HIGH} \end{array} \right\}$$

REPEAT FROM STEP 3: TRY  $D_w = 0.2830 \text{ IN}$  U.S. GAGE 1;  $T_0 = 79,000 \text{ PSI}$

(CONTINUED - NEXT PAGE)

$T_{MAX} = 104,000 \text{ PSI}$

13. (CONTINUED) STEP 4 -  $N_{a, \max} = [2.50 - 2(0.2830)] / 0.2830 = 6.83$  USE  $N_a = 6$  COILS

STEP 6 -  $C = \left[ \frac{(14.2 \times 10^6)(0.2830)}{8(80)(6.0)} \right]^{1/3} = 9.38$ ;  $K = 1.155$  STEP 5

OPERATING STRESS -  $T_0 = \frac{(2.546)(1.155)(9.38)(220)}{(0.2830)^2} = 75770 \text{ PSI}$  OK

- SOLID LENGTH & STRESS AT SOLID LENGTH

$L_s = D_w(N_a + 2) = 0.2830(6.0 + 2) = 2.264 \text{ IN}$

$F_s = k(L_f - L_s) = (80 \text{ LB/IN})(5.25 - 2.264) \text{ IN} = 239 \text{ LB}$

STRESS AT SOLID HEIGHT  $T_s = T_0 (F_s / F_0) = 75770 (239 / 220) = 82272 \text{ PSI}$  OK

FINAL GEOMETRY

-  $D_m = C D_w = 9.38(0.2830) = 2.655 \text{ IN}$

$D_o = D_m + D_w = 2.655 + 0.2830 = 2.938 \text{ IN}$

$D_i = D_m - D_w = 2.655 - 0.2830 = 2.372 \text{ IN}$

$CC = (L_o - L_s) / N_a = (2.50 - 2.264) / 6.0 = 0.0393 \text{ IN} > \frac{D_w}{10}$  OK

SUMMARY:  $D_w = 0.2830 \text{ IN}$ ,  $L_f = 5.25 \text{ IN}$ ,  $L_o = 2.50 \text{ IN}$ ,  $D_m = 2.655 \text{ IN}$ .

14. METHOD 2  $L_o = 1.75 \text{ IN}$ ;  $F_o = 22.0 \text{ LB}$ ;  $L_i = 3.00 \text{ IN}$ ;  $F_i = 5.0 \text{ LB}$   
ASTM A401; SEVERE SERVICE;  $T_b \approx 125 \text{ KSI}$

$k = \frac{F_o - F_i}{L_i - L_o} = \frac{22.0 - 5.0}{3.00 - 1.75} = 13.6 \text{ LB/IN} = k$   $F_o = L_f - L_o = 3.368 - 1.75$   
 $F_o = 1.618 \text{ IN}$

$L_f = L_i + F_i/k = 3.00 + 5.0/13.6 = 3.368 \text{ IN}$

$D_w = \sqrt{21.4(F_o)/T_b} = \sqrt{21.4(22.0)/125000} = 0.061 \text{ IN}$

TRY  $16 \text{ GA.}$ ;  $D_w = 0.0625 \text{ IN} \rightarrow T_d = 130 \text{ KSI}$ ;  $T_{max} = 188 \text{ KSI}$

$N_{a, \max} = (L_o - 2D_w) / D_w = [1.75 - 2(0.0625)] / 0.0625 = 26$ ; TRY  $N_a = 20$

$C = \left[ \frac{G D_w}{8k N_a} \right]^{1/3} = \left[ \frac{(11.2 \times 10^6)(0.0625)}{8(13.6)(20)} \right]^{1/3} = 6.85 \rightarrow K = 1.225$

$T_0 = \frac{2.546 K C F_o}{D_w^2} = \frac{2.546(1.225)(6.85)(22.0)}{(0.0625)^2} = 120325 \text{ PSI}$  OK

$L_s = D_w(N_a + 2) = 0.0625(22) = 1.375 \text{ IN}$

$F_s = k(L_f - L_s) = 13.6(3.368 - 1.375) = 27.1 \text{ LB}$

$T_s = T_0 (F_s / F_o) = 120325 \text{ PSI} (27.1 / 22.0) = 148245 \text{ PSI}$  OK

$D_m = C D_w = 6.85(0.0625) = 0.428 \text{ IN} = D_m$

$D_o = D_m + D_w = 0.491$ ;  $D_i = D_m - D_w = 0.366 \text{ IN}$

$CC = (L_o - L_s) / N_a = (1.75 - 1.375) / 20 = 0.019 \text{ IN} > \frac{D_w}{10}$  OK

BUCKLING:  $L_f / D_m = 3.368 / 0.428 = 7.87$ ;  $(L_f / L_s)_{CR} = 0.21$  (FIG. 19-15)

$(f_o)_{CR} = 0.21 L_f = 0.21(3.368) = 0.71 \text{ IN}$ ; ACTUAL  $f_o > (f_o)_{CR}$  - WILL BUCKLE

14. (CONTINUED) AFTER SEVERAL ITERATIONS THIS DESIGN WAS PRODUCED WHICH WILL NOT BUCKLE.

$$D_w = 0.072 \text{ IN}; T_d = 129 \text{ KSI}; T_{\text{MAX}} = 185 \text{ KSI}$$

$$N_{a \text{ MAX}} = 22.3; \text{ USED } N_a = 12; C = 8.52; D_m = 0.613 \text{ IN.}$$

$$T_o = 108200 \text{ PSI}; L_s = 1.008 \text{ IN.}; F_s = 321 \text{ LB}; T_s = 157.9 \text{ KSI}$$

$$CC = 0.062 \text{ IN (4000)}; \text{BUCKLING - } L_e/L_o = 5.49; \left(\frac{L_e}{L_o}\right)_{\text{CR}} = 0.53$$

$$(f_o)_{\text{CR}} = 0.53 L_e = 1.78 \text{ IN}; \text{ACTUAL } f_o = 1.62 \text{ IN OK}$$

15. METHOD 2  $L_o = 1.25 \text{ IN}; F_o = 14.0 \text{ LB}; L_i = 2.00 \text{ IN}; F_i = 1.50 \text{ LB}$

ASTM A313, TYPE 302, AVG. SERVICE;  $T_d \approx 100 \text{ KSI}$

$$k = \frac{14 - 1.50}{2.00 - 1.25} = 16.67 \text{ LB/IN}; L_s = L_i + \frac{F_i}{k} = 2.00 + \frac{1.50}{16.67} = 2.09 \text{ IN.}$$

$$D_w = \sqrt{21.4(F_o)/T_d} = \sqrt{21.4(14.0)/100000} = 0.055 \text{ IN.}$$

$$\text{TRY 16 GA.}; D_w = 0.0625 \text{ IN}; T_d = 115 \text{ KSI}; T_m = 128 \text{ KSI}$$

$$N_{a \text{ MAX}} = (L_o - 2D_w)/D_w = [1.25 - 2(0.0625)]/0.0625 = 18.0 \rightarrow \text{USE } N_a = 16$$

$$C = \left[ \frac{G D_w}{8(k) N_a} \right]^{1/3} = \left[ \frac{(16 \times 10^6)(0.0625)}{8(16.67)(16)} \right]^{1/3} = 6.64 \text{ THEN } K = 6.23$$

$$T_o = \frac{2.546(1.23)(6.64)(14.0)}{(0.0625)^2} = 74540 \text{ PSI OK}$$

$$L_s = D_w(N_a + 2) = 0.0625(18) = 1.125 \text{ IN.}$$

$$F_s = k(L_s - L_i) = 16.67(2.09 - 1.125) = 16.09 \text{ LB}$$

$$T_s = T_o \left(\frac{F_s}{F_o}\right) = 74540 \left(\frac{16.09}{14.0}\right) = 85650 \text{ PSI OK}$$

$$D_m = C D_w = 6.64(0.0625) = 0.415$$

$$L_e/D_m = 2.09/0.415 = 5.04 \text{ NO BUCKLING}$$

$$CC = (L_o - L_s)/N_a = (1.25 - 1.125)/16 = 0.008 \text{ IN} > D_w/10 \text{ OK}$$

$$D_o = D_m + D_w = 0.415 + 0.0625 = 0.478 \text{ IN}$$

$$D_i = D_m - D_w = 0.415 - 0.0625 = 0.353 \text{ IN.}$$

16

METHOD 2:  $L_o = 4.00$  IN,  $F_o = 250$  LB,  $L_i = 10.50$  IN,  $F_i = 60$  LB

ASTM A231:  $T_u = 90$  KSI FOR SEVERE SERVICE

$$R = \frac{F_o - F_i}{L_i - L_o} = \frac{250 - 60}{10.50 - 4.00} = 29.23 \text{ LB/IN}; L_c = 10.50 + \frac{60 \text{ LB}}{29.23 \text{ LB/IN}} = 12.55 \text{ IN}$$

$$D_w = \sqrt{21.4 F_o / T_u} = \sqrt{21.4 (250) / 90000} = 0.244 \text{ IN.}$$

TRIALS WITH GAGES 2 AND 1 FAILED  $T_o$  TOO HIGH

FOR  $D_w = 0.3065$  IN (0 GAGE),  $T_u = 90000$  PSI,  $T_{max} = 130$  KSI

$$N_{a, max} = (L_o - 2D_w) / D_w = (4.00 - 2(0.3065)) / 0.3065 = 11.05; \text{ TRY } N_a = 10$$

$$C = \left[ \frac{4 D_w}{8 K N_a} \right]^{1/3} = \left[ \frac{4 (0.3065)}{8 (29.23) (10)} \right]^{1/3} = 11.37; \text{ THEN } K = 1.125$$

$$T_o = \frac{(2.546) K C F_o}{D_w^2} = \frac{(2.546) (1.125) (11.37) (250)}{(0.3065)^2} = 86,680 \text{ PSI OK}$$

$$L_s = D_w (N_a + 2) = 0.3065 (12) = 3.678 \text{ IN}$$

$$F_s = R (L_c - L_s) = 29.23 (12.55 - 3.678) = 259 \text{ LB}$$

$$T_s = T_o (F_s / F_o) = (86,680) (259 / 250) = 89,900 \text{ PSI} < T_{max} \text{ OK}$$

$$D_m = C D_w = (11.37) (0.3065) = 3.485 \text{ IN}$$

$$L_s / D_m = 12.55 / 3.485 = 3.60 \text{ NO BUCKLING}$$

$$CC = (L_o - L_s) / N_a = (4.00 - 3.678) / 10 = 0.032 \text{ IN} > D_w / 10 \text{ OK}$$

17

METHOD 2  $F_o = 14.0$  LB;  $L_o = 0.68$  IN;  $F_i = 0$ ;  $L_i = L_c = 1.75$  IN;  $R = \frac{14}{1.75 - 0.68} = 13.08 \text{ LB/IN.}$

MUSIC WIRE; AV. SERV.;  $T_u = 120$  KSI

$$D_w = \sqrt{21.4 (14) / 120000} = 0.050 \text{ IN}; \text{ USE } D_w = 0.055 \text{ IN, 24 GAGE} \left\{ \begin{array}{l} T_u = 135 \text{ KSI} \\ T_n = 150 \text{ KSI} \end{array} \right.$$

$$N_{a, max} = (L_o - 2D_w) / D_w = (0.68 - 2(0.055)) / 0.055 = 8.9 \text{ CIRCLES}$$

USE  $N_a = 8.0$

$$C = \left[ \frac{(11.85 \times 10^6) (0.055)}{8 (13.08) (8.0)} \right]^{1/3} = 9.20 \rightarrow K = 1.16$$

$$T_o = \frac{2.546 (1.16) (9.20) (14)}{(0.055)^2} = 125,750 \text{ PSI OK}$$

$$L_s = D_w (N_a + 2) = 0.055 (10) = 0.550 \text{ IN}$$

$$F_s = R (L_c - L_s) = 13.08 (1.75 - 0.55) = 15.70 \text{ LB}$$

$$T_s = T_o (F_s / F_o) = 125,750 (15.7 / 14.0) = 141,010 \text{ PSI OK}$$

$$D_m = C D_w = 9.20 (0.055) = 0.506 \text{ IN}$$

$$L_s / D_m = 1.75 / 0.506 = 3.46 \text{ NO BUCKLING}$$

$$CC = (0.68 - 0.55) / 8 = 0.016 \text{ IN} > D_w / 10 \text{ OK}$$

$$D_o = D_m + D_w = 0.561 \text{ IN.}$$

$$D_i = D_m - D_w = 0.451 \text{ IN.}$$

$$k = \frac{8.00}{1.75} = 4.57 \text{ LB/IN}$$

18.

METHOD 2  $L_t = 2.75$ ;  $f_0 = 1.75 \text{ IN}$ ;  $L_0 = L_t - f_0 = 1.00 \text{ IN}$ ;  $F_0 = 8.00 \text{ IN}$ .

ASTM A313, TYPE 316, AVG. SERV.;  $T_d \approx 90 \text{ KSI}$

$$D_{nr} = \sqrt{21.4 (8.0) / 90,000} = 0.044 \text{ IN} \rightarrow \text{USE } D_{nr} = 0.0475 \text{ IN}; \text{ R GAGE}$$

$$T_d = 0.85 (122 \text{ KSI}) = 104 \text{ KSI}; T_n = 0.85 (135) = 115 \text{ KSI}$$

$$N_{a, \text{MAX}} = (L_0 - 2D_{nr}) / D_{nr} = [1.00 - 2(0.0475)] / 0.0475 = 19.05 \rightarrow \text{USE } N_a = 17$$

$$C = \left[ \frac{(10.0 \times 10^6) (0.0475)}{8 (4.57) (17)} \right]^{1/3} = 9.14 \rightarrow R = 1.16$$

$$T_0 = \frac{2.546 (1.16) (9.14) (8.0)}{(0.0475)^2} = 95,740 \text{ PSI OK}$$

$$L_s = 0.0475 (17 + 2) = 0.9125 \text{ IN}; F_s = 4.57 \text{ LB/IN} (2.75 - 0.9125) = 8.44 \text{ LB}$$

$$T_s = T_0 (F_s / F_0) = 95,740 (8.44 / 8.0) = 101,050 \text{ PSI OK}$$

$$D_m = C D_{nr} = 9.14 (0.0475) = 0.434 \text{ IN}; L_t / D_m = 2.75 / 0.434 = 6.33 \text{ (HIGH)}$$

$$(L_t / L_s)_{CR} = 0.32; (f_0)_{CR} = 0.32 (2.75) = 0.88 \text{ IN}; f_0_{\text{ACT}} = 1.75 > 0.88$$

(WILL BUCKLE)

19.

SAME AS 18 EXCEPT  $D_{rod} = 0.625 \text{ IN}$  - USE METHOD 1

LET  $D_m = 0.75 \text{ IN}$ ;  $T_d \approx 100 \text{ KSI}$

$$D_{nr} = \left[ \frac{3.06 (F_0) (D_m)}{T_d} \right]^{1/3} = \left[ \frac{3.06 (8.0) (0.75)}{100,000} \right]^{1/3} = 0.057 \rightarrow \text{USE } D_{nr} = 0.0625 \text{ IN}$$

$$T_d = 0.85 (115 \text{ KSI}) = 97.8 \text{ KSI}; T_n = 0.85 (128) = 109 \text{ KSI}$$

$$C = D_m / D_{nr} = 0.75 / 0.0625 = 12.0 \rightarrow R = 1.12$$

$$T_0 = \frac{2.546 (1.12) (12.0) (8.0)}{(0.0625)^2} = 70,100 \text{ PSI OK}$$

$$N_a = \frac{G D_{nr}}{8 k C^3} = \frac{(10 \times 10^6) (0.0625)}{8 (4.57) (12)^3} = 9.89 \text{ COILS}$$

$$L_s = D_{nr} (N_a + 2) = 0.0625 (11.89) = 0.743 \text{ IN}$$

$$F_s = k (L_t - L_s) = 4.57 (2.75 - 0.743) = 9.17 \text{ LB}$$

$$T_s = T_0 (F_s / F_0) = 70,100 (9.17 / 8.0) = 80,400 \text{ PSI OK}$$

$$L_t / D_m = 2.75 / 0.75 = 3.67 \text{ OK NO BUCKLING}$$

$$CC = (L_0 - L_s) / N_a = (1.00 - 0.743) / 9.89 = 0.026 \text{ IN GOOD}$$

$$D_i = D_m + D_{nr} = 0.75 + 0.0625 = 0.8125 \text{ IN}$$

$$D_i - D_m - D_{nr} = 0.75 - 0.0625 = 0.6875 \text{ IN} > D_{rod} \text{ OK}$$

20.

SAME AS 17 EXCEPT  $D_{\text{HOLE}} = 0.75 \text{ IN.}$  METHOD 1LET  $D_m = 0.625 \text{ IN.}; T_s = 120 \text{ KSI}$ 

$$D_w = \left[ \frac{3.06(14.0)(0.625)}{120000} \right]^{1/3} = 0.061 \text{ IN.} \rightarrow \text{USE } D_w = 0.063 \text{ IN.} \quad T_s = 130 \text{ KSI} \\ T_M = 145 \text{ KSI}$$

$$C = D_m / D_w = 0.625 / 0.063 = 9.92 \rightarrow K = 1.15$$

$$T_0 = \frac{2.546(1.15)(9.92)(14.0)}{(0.063)^2} = 102,500 \text{ PSI OK}$$

$$N_a = \frac{(1.85 \times 10^4)(0.063)}{8(13.08)(9.92)^3} = 7.31 \text{ COILS}; L_c = 0.063(7.31 + 2) = 0.586 \text{ IN}$$

$$F_s = k(L_f - L_s) = 13.08(1.75 - 0.586) = 15.22 \text{ LB}; T_s = T_0 F_s / F_0 = 111,500 \text{ PSI OK}$$

$$L_f / D_m = 1.75 / 0.625 = 2.80 \text{ OK NO BUCKLING}$$

$$CC = (0.68 - 0.586) / 7.31 = 0.013 \text{ IN} > D_w / 10 \text{ OK}$$

$$D_o = 0.625 + 0.063 = 0.688 \text{ IN}$$

$$\text{CLEARANCE WITH HOLE} = 0.75 - 0.688 = 0.062 \text{ IN OK} > D_w / 10$$

21.

METHOD 2  $L_f = 3.05 \text{ IN.}; F_0 = 45 \text{ LB.}; L_i = 3.50 \text{ IN.}; F_i = 22 \text{ LB}$ 

$$k = \frac{45 - 22}{3.50 - 3.05} = 57.1 \text{ LB/IN.}; L_f = L_i + F_i / k = 3.50 + 22 / 57.1 = 3.93 \text{ IN}$$

$$D_w = \sqrt{21.4(45) / 90000} = 0.103 \text{ IN.} \rightarrow \text{USE } D_w = 0.1055 \text{ IN.}$$

ASTM A231;  $T_s = 109 \text{ KSI}; T_M = 157 \text{ KSI};$  SEVERE SERVICE

$$N_{a, \text{MAX}} = (L_f - 2D_w) / D_w = [3.05 - 2(0.1055)] / 0.1055 = 26.9 \text{ COILS} \rightarrow \text{USE } N_a = 18$$

$$C = \left[ \frac{G D_w}{8 k N_a} \right]^{1/3} = \left[ \frac{(11.2 \times 10^6)(0.1055)}{8(57.1)(18)} \right]^{1/3} = 5.44 \rightarrow K = 1.285$$

$$T_0 = \frac{2.546(1.285)(5.44)(45)}{(0.1055)^2} = 71,950 \text{ PSI} < T_s \text{ OK}$$

$$L_c = D_w(N_a + 2) = 0.1055(20) = 2.11 \text{ IN.}; F_s = 57.1 \text{ LB/IN.}(3.93 - 2.11) = 93.0 \text{ LB}$$

$$T_s = T_0 (F_s / F_0) = 71,950 (93.0 / 45.0) = 148,700 \text{ PSI} < T_{\text{MAX}} \text{ OK}$$

$$D_m = C D_w = 5.44(0.1055) = 0.574 \text{ IN}$$

$$L_f / D_m = 3.93 / 0.574 = 6.85; (F_0 / L_f)_{\text{CR}} = 0.27$$

$$(F_0)_{\text{CR}} = 0.27(3.93) = 1.061 \text{ IN.}$$

$$(F_0)_{\text{ACT}} = L_f - L_0 = 3.93 - 3.05 = 0.88 \text{ IN} < (F_0)_{\text{CR}} \text{ OK NO BUCKLING}$$

$$CC = (3.05 - 2.11) / 18 = 0.052 \text{ IN} > D_w / 10 \text{ OK}$$

$$D_o = D_m + D_w = 0.680 \text{ IN.}$$

$$D_i = D_m - D_w = 0.469 \text{ IN.}$$



22 COMPRESSION SPRING: METHOD 1

ASTM A227 STEEL WIRE;  $G = 11.5 \times 10^6 \text{ PSI}$  (TABLE 18-4)  
STRESSES FROM FIG. 18-8.

$$L_i = 2.50 \text{ IN}, F_i = 20.0 \text{ LB}, L_o = 2.10 \text{ IN}, F_o = 35 \text{ LB.}$$

INSTALL SPRING AROUND 1.50 IN DIA. SHAFT.

$$\text{SPRING SCALE} = k = \frac{F_o - F_i}{L_i - L_o} = \frac{35 - 20}{2.50 - 2.10} = 37.5 \text{ LB/IN}$$

$$\text{FREE LENGTH} = L_f = L_i + F_i/k = 2.50 + 20.0/37.5 = 3.033 \text{ IN.}$$

$$\text{TRY } D_m = 1.75 \text{ IN.}; D_{w \text{ MAX}} = D_m - D_{sh} = 1.75 - 1.50 = 0.25 \text{ IN.}$$

SEVERE SERVICE:  $T_d \approx 85,000 \text{ PSI}$

$$D_w \approx \left[ \frac{(3.06)(F_o)(D_m)}{T_d} \right]^{1/3} = \left[ \frac{(3.06)(35)(1.75)}{85,000} \right]^{1/3} = 0.130 \text{ IN.}$$

TRY U.S. WIRE GA. 9:  $D_w = 0.1483 \text{ IN.}$

$$T_d = 87,000 \text{ PSI (SEVERE SERV.)}; T_{\text{MAX}} = 116,000 \text{ PSI (LT. SERV.)}$$

$$C = D_m/D_w = 1.75/0.1483 = 11.80; K = 1.125 \text{ (FIG. 18-14)}$$

$$T_o = \frac{8KF_o D_m}{\pi D_w^3} = \frac{8(1.125)(35)(1.75)}{\pi (0.1483)^3} = 53,800 \text{ PSI OK}$$

$$N_a = \frac{G D_w}{8k C^3} = \frac{(11.5 \times 10^6)(0.1483)}{8(37.5)(11.80)^3} = 3.45 \text{ COILS}$$

$$L_s = D_w(N_a + 2) = (0.1483)(3.45 + 2) = 0.809 \text{ IN.}$$

$$F_s = k(L_f - L_s) = (37.5)(3.033 - 0.809) = 83.4 \text{ LB}$$

$$T_s = T_o \cdot F_s/F_o = (53,800 \text{ PSI}) \left( \frac{83.4}{35} \right) = 128,200 \text{ PSI TOO HIGH}$$

TRY  $D_w = 0.162 \text{ IN. - 8 GAUGE}; T_d = 86,000 \text{ PSI}; T_{\text{MAX}} = 114,000 \text{ PSI}$

SUMMARY OF RESULTS:

$$C = 10.80; K = 1.13$$

$$T_o = 41,455 \text{ PSI OK}$$

$$N_a = 4.93 \text{ COILS}; L_s = 1.122 \text{ IN.}; F_s = 71.7 \text{ LB}$$

$$T_s = 84,900 \text{ PSI OK}$$

$$ID = D_m - D_w = 1.75 - 0.162 = 1.588 \text{ IN OK}$$

23

COMPRESSION SPRING: METHOD 1: METRICASTM A 227 STEEL WIRE:  $G = 79.3 \text{ GPa} = 79.3 \times 10^3 \text{ MPa}$  (TABLE 18-4) $L_i = 60 \text{ mm}$ ,  $F_i = 90 \text{ N}$ ,  $L_o = 50 \text{ mm}$ ,  $F_o = 155 \text{ N}$ 

$$k = \frac{F_o - F_i}{L_i - L_o} = \frac{155 - 90}{60 - 50} = 6.5 \text{ N/mm}$$

$$L_s = L_i + F_i/k = 60 \text{ mm} + 90 \text{ N}/6.5 \text{ N/mm} = 73.85 \text{ mm}$$

SPRING INSTALLED ON 38 mm DIA. SHAFT: TRY  $D_m = 45 \text{ mm}$ SEVERE SERVICE:  $T_s \approx 550 \text{ MPa}$  (APP. A194)

$$\text{TRIAL } D_w = \left[ \frac{(3.06)(F_o)(D_m)}{T_s} \right]^{1/3} = \left[ \frac{(3.06)(155)(45)}{550} \right]^{1/3} = 3.39 \text{ mm}$$

TRY  $D_w = 3.80 \text{ mm}$  (TABLE 18-2);  $T_s = 600 \text{ MPa}$ ,  $T_{\text{max}} = 800 \text{ MPa}$ 

$$C = D_m/D_w = 45/3.80 = 11.84; K = 1.125 \text{ (Eq. 18-4)}$$

$$T_o = \frac{8 K F_o D_m}{\pi D_w^3} = \frac{8(1.125)(155)(45)}{\pi(3.80)^3} = 364 \text{ MPa OK}$$

$$N_a = \frac{G D_w}{8 K C^3} = \frac{(79.3 \times 10^3)(3.80)}{8(6.5)(11.84)^3} = 3.49 \text{ COILS}$$

$$L_s = D_w(N_a + 2) = (3.80)(3.49 + 2) = 20.86 \text{ mm}$$

$$F_s = k(L_s - L_o) = (6.5 \text{ N/mm})(20.86 - 50) \text{ mm} = 344 \text{ N}$$

$$T_s = T_o \frac{F_s}{F_o} = 364 \text{ MPa} \left( \frac{344 \text{ N}}{155 \text{ N}} \right) = 809 \text{ MPa (JUST SLIGHTLY HIGH)}$$

$$ID = D_m - D_w = 45 - 3.80 = 41.2 \text{ mm OK}$$

24

COMPRESSION SPRING: ANALYSIS: ASTM A 229,  $G = 11.2 \times 10^6 \text{ PSI}$ 17 GA.:  $D_w = 0.054 \text{ IN.}$ ;  $T_s = 106 \text{ KSI (SEVERE)}$ ;  $T_o = 128 \text{ KSI (AVG)}$ ;  $T_s = 144 \text{ KSI (LT)}$ OD = 0.531 IN:  $D_m = OD - D_w = 0.531 - 0.054 = 0.477 \text{ IN}$ 

$$C = D_m/D_w = 0.477/0.054 = 8.83; K = 1.17; N = 7.0 \text{ COILS}; N_a = 7.0 - 2 = 5.0$$

FOR  $F_o = 10.0 \text{ LB}$ :

$$T_o = \frac{8 K F_o C}{\pi D_w^3} = \frac{8(1.17)(10)(8.83)}{\pi(0.054)^3} = 90250 \text{ PSI OK FOR SEVERE SERV.}$$

$$f_o = \frac{8(F_o)(C^3)(N)}{G D_w} = \frac{8(10)(8.83)^3(5.0)}{(11.2 \times 10^6)(0.054)} = 0.456 \text{ IN}$$

$$L_o = L_i - f_o = 1.25 \text{ IN} - 0.456 = 0.794 \text{ IN}; L_s = D_w(N) = 0.054(7) = 0.378 \text{ IN}$$

$$k = \frac{F_o}{L_i - L_o} = \frac{10.0 \text{ LB}}{(1.25 - 0.794) \text{ IN}} = 21.93 \text{ LB/IN}$$

$$F_s = k(L_i - L_s) = (21.93)(1.25 - 0.378) = 19.12 \text{ LB}$$

$$T_s = T_o \frac{F_s}{F_o} = (90250 \text{ PSI}) \frac{19.12 \text{ LB}}{10.0 \text{ LB}} = 172,600 \text{ PSI TOO HIGH}$$

BUT  $T_{\text{max}} \approx T_o$  FOR LIGHT SERVICE = 141,000 PSI

25

EXTENSION SPRING  $F_o = 7.75 \text{ LB}$ ;  $L_o = 2.75 \text{ IN}$ ;  $L_i = 2.25 \text{ IN}$ ;  $F_i = 5.25 \text{ LB}$   
 $D_o < 3.00 \text{ IN}$ ; MUSIC WIRE, SEVERE SERVICE. INSTALLED

LET  $D_m = 0.250 \text{ IN}$ ;  $T_s \approx 120,000 \text{ PSI}$

$$D_{wr} = \left[ \frac{8K F_o D_m}{\pi T_s} \right]^{2/3} = \left[ \frac{8(1.20)(7.75)(0.250)}{\pi(120,000)} \right]^{2/3} = 0.0367 \text{ IN}$$

USE 16 GA.,  $D_{wr} = 0.037 \text{ IN}$ ;  $T_s = 120,000 \text{ PSI}$

$$D_o = D_m + D_{wr} = 0.287 \text{ IN } \underline{OK}; D_i = D_m - D_{wr} = 0.213 \text{ IN.}$$

$$C = D_m / D_{wr} = 0.25 / 0.037 = 6.76 \rightarrow K = 1.225$$

$$T_o = \frac{8K F_o D_m}{\pi D_{wr}^3} = \frac{8K F_o C}{\pi D_{wr}^2} = \frac{8(1.225)(7.75)(6.76)}{\pi(0.037)^2} = 119,320 \text{ PSI } \underline{OK}$$

$$k = \frac{\Delta F}{\Delta L} = \frac{7.75 - 5.25}{2.75 - 2.25} = 5.0 \text{ LB/IN.}$$

$$N_a = \frac{G D_{wr}}{8 C^3 k} = \frac{(11.85 \times 10^6)(0.037)}{8(6.76)^3(5.0)} = 35.5 \text{ COILS}$$

$$\text{BODY LENGTH} \approx D_{wr}(N_a + 1) = 0.037(36.5) = 1.350 \text{ IN.} = \text{B.L.}$$

ASSUME FULL LOOP AT EACH END

$$L_f = \text{B.L.} + 2D_i = 1.350 + 2(0.213) = 1.777 \text{ IN.}$$

$$\text{DEFLECTION TO } L_o: f_o = L_o - L_f = 2.750 - 1.777 = 0.973 \text{ IN}$$

$$\text{INITIAL FORCE} = F_i = F_o - k f_o = 7.75 - 5.0(0.973) = 2.88 \text{ LB}$$

$$\text{INITIAL STRESS} = T_i = T_o \times \frac{F_i}{F_o} = 119,320 \times \frac{2.88}{7.75} = 44,340 \text{ PSI}$$

FROM FIG 19-21 - STRESS SHOULD BE  $\approx 14-21 \text{ KSI}$  H16.H

USE SMALLER END LOOPS - SAY 0.06 EACH

$$L_f = \text{B.L.} + 2(0.06) = 1.35 + 0.12 = 1.47 \text{ IN}$$

$$f_o = L_o - L_f = 2.750 - 1.47 = 1.28 \text{ IN.}$$

$$F_i = 7.75 - 5.0(1.28) = 1.35 \text{ LB}$$

$$T_i = T_o \times \frac{F_i}{F_o} = 119,320 \times \frac{1.35}{7.75} = 20,800 \text{ PSI } \underline{OK}$$

### SUMMARY:

$D_{wr} = 0.037 \text{ IN}$  - 16 GA. MUSIC WIRE

$D_m = 0.250 \text{ IN}$ ;  $D_o = 0.287 \text{ IN}$ ;  $D_i = 0.213 \text{ IN.}$

35.5 COILS

2 LOOPS @ 0.06 IN. EACH

$F_o = 7.75 \text{ LB}$  @ 2.75 IN - OPERATING

$F_i = 5.25 \text{ LB}$  @ 2.25 IN - INSTALLED

$F_i = 1.35 \text{ LB}$  @ 1.47 IN - INITIAL

26 EXTENSION: MUSIC WIRE, AVG. SERVICE,  $T_u \approx 130 \text{ ksi}$   
 $F_0 = 15.0 \text{ LB}$ ;  $F_L = 5.20 \text{ LB}$ ;  $L_0 = 5.00 \text{ in}$ ;  $L_A = 3.75 \text{ in}$ .  
 $R = 7.84 \text{ LB/in}$ ;  $D_m = 0.60 \text{ in}$ ;  $D_{m_{\text{TRIAL}}} = 0.0596 \text{ in}$ ;  $D_m = 0.063 (26 \text{ Ga.})$   
 $T_u = 132,000 \text{ PSI}$ ;  $C = 9.52$ ;  $K = 1.153$ ;  $T_0 = 105,600 \text{ PSI OK}$ ;  $N_A = 13,78$   
 BODY LENGTH = B.L. =  $0.93 \text{ in}$ ; FOR ENO LOOP SIZE =  $0.537 \text{ in} (= ID)$ ;  $F_I = -8.48 \text{ LB}$   
 CHANGE E.L.S. =  $1.20 \text{ in} \rightarrow F_I = 1.92 \text{ LB}$ ;  $T_I = 13500 \text{ PSI (OK FIG. 18-21)}$   
 $OD = 0.663 \text{ in} < 0.75 \text{ in. OK}$

27 EXTENSION: MUSIC WIRE, SEVERE SERVICE;  $T_u \approx 100 \text{ ksi}$ ;  $F_0 = 10.0 \text{ LB}$ ;  $L_0 = 3.00 \text{ in}$ ,  
 $R = 6.80 \text{ LB/in}$ ;  $D_m = 0.65 \text{ in}$ ;  $D_{m_{\text{TRIAL}}} = 0.059 \text{ in} \rightarrow \text{USED } D_m = 0.059 \text{ in} (25 \text{ Ga.})$   
 $T_u = 110 \text{ ksi}$ ;  $OD = 0.709 \text{ in}$ ;  $ID = 0.591$ ;  $C = 11.02$ ;  $K = 1.131$ ;  $T_0 = 91.1 \text{ ksi OK}$   
 $N_A = 9.61 \text{ COILS}$ ; B.L. =  $0.626 \text{ in}$ ; E.L.S. =  $0.591 \text{ in} = ID$ ;  $F_I = 1.89 \text{ LB}$ ;  $T_I = 17.3 \text{ ksi HIGH}$   
 CHANGE E.L.S. =  $0.54 \text{ in}$ ;  $F_I = 1.20 \text{ LB}$ ;  $T_I = 10.95 \text{ ksi (OK FIG. 18-21)}$

28 EXTENSION: MUSIC WIRE, SEVERE SERV.;  $F_0 = 10.0 \text{ LB}$ ;  $L_0 = 6.10 \text{ in}$ ;  $T_u = 100 \text{ ksi}$   
 (SAME AS #24 FOR  $D_m$ ,  $D_m$ ,  $OD$ ,  $ID$ ,  $C$ ,  $K$ ,  $T_0$ );  $N_A = 25.14 \text{ COILS}$ ; B.L. =  $1.572 \text{ in}$   
 E.L.S. =  $0.591 \text{ in} = ID$ ;  $F_I = 1.48 \text{ LB}$ ;  $T_I = 13500 \text{ PSI (OK FIG. 18-21)}$

29 EXTENSION: MUSIC WIRE; AVERAGE SERV.;  $F_0 = 10.0 \text{ LB}$ ;  $L_0 = 9.4 \text{ in}$ ;  $R = 1.50 \text{ LB/in}$ .  
 $T_u \approx 130 \text{ ksi}$ ;  $D_m = 0.6875 \text{ in. (11/16)}$ ;  $D_{m_{\text{TRIAL}}} = 0.0545 \text{ in}$ ;  $\text{USED } D_m = 0.055 \text{ in}$ ,  
 $T_u = 135 \text{ ksi}$ ; (24 Ga.);  $OD = 0.7425 \text{ in}$ ;  $ID = 0.6325 \text{ in}$ ;  $C = 12.5$ ;  $K = 1.114$   
 $T_0 = 117.3 \text{ ksi}$ ;  $N_A = 27.81 \text{ COILS}$ ; B.L. =  $1.58 \text{ in}$ ; E.L.S. =  $0.6325 \text{ in} = ID \rightarrow$   
 $F_I = -0.14 \text{ LB}$ ; CHANGE E.L.S. =  $1.0 \text{ in}$ ;  $F_I = 0.96 \text{ LB}$ ;  $T_I = 113 \text{ ksi (OK)}$

30 EXTENSION: ASTM A313; TYPE 302;  $F_0 = 162 \text{ LB}$ ;  $L_0 = 10.80 \text{ in}$ ;  $R = 38.0 \text{ LB/in}$   
 $T_u \approx 80 \text{ ksi (AVG. SERV.)}$ ;  $D_m = 1.50 \text{ in}$ ;  $D_{m_{\text{TRIAL}}} = 0.210 \text{ in} \rightarrow D_m = 0.2253 \text{ in} (4 \text{ Ga.})$   
 $T_u = 81 \text{ ksi}$ ;  $OD = 1.7253 \text{ in}$ ;  $ID = 1.2747 \text{ in}$ ;  $C = 6.66$ ;  $K = 1.225$   
 $T_0 = 66.31 \text{ ksi}$ ;  $N_A = 25.11 \text{ COILS}$ ; B.L. =  $5.88 \text{ in}$ ; E.L.S. =  $1.27 \text{ in} = ID$ ,  
 $F_I = 71.7 \text{ LB}$ ;  $T_I = 29300 \text{ PSI (T0 HIGH)}$ .  
 CHANGE E.L.S. =  $0.90 \text{ in}$ ;  $F_I = 43.6 \text{ LB}$ ;  $T_I = 17.8 \text{ ksi (OK FIG. 18-21)}$

31

EXTENSION SPRING ENDSM G. :  $D_m = 0.041$  IN ;  $D_m = 0.28$  IN ;  $R_1 = 0.25$  IN ;  $R_2 = 0.094$  INL<sub>a</sub>  $\sigma_d = 146$  KSI ;  $T_d = 130$  KSI FOR AVERAGE SERVICE

BENDING (EQ. 19-13) (FIG. 18-22)

$$C_1 = 2R_1/D_m = 2(0.25)/0.041 = 12.20$$

$$K_1 = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} = 1.065$$

$$\sigma_A = \frac{16(0.28)(5.0)(1.065)}{\pi(0.041)^3} + \frac{4(5.0)}{\pi(0.041)^2} = 114000 \text{ PSI } \underline{OK}$$

TORSION (EQ. 18-15)

$$C_2 = 2R_2/D_m = 2(0.094)/0.041 = 4.59$$

$$K_2 = \frac{4C_2 - 1}{4C_2 - 4} = 1.209$$

$$T_B = \frac{8D_m F_0 K_2}{\pi(D_m)^3} = \frac{8(0.28)(5.0)(1.209)}{\pi(0.041)^3} = 62600 \text{ PSI } \underline{OK}$$

32

TORSION SPRING : ASTM A313, TYPE 302, AVG. SERVICEASSUME  $D_m = 0.420$  IN ;  $\sigma_d = 140000$  PSI ;  $K_1 = 1.15$  ;  $\theta_c = 180^\circ = 0.50$  REV.

$$D_m = \left[ \frac{32 M_0 K_1}{\pi \sigma_d} \right]^{1/3} = \left[ \frac{32(1.0)(1.15)}{\pi(140000)} \right]^{1/3} = 0.0437 \text{ IN } \left\{ \begin{array}{l} \text{USE } 18 \text{ GA. } D_m = 0.0475 \text{ IN} \\ \sigma_d = 160000 \text{ PSI} \end{array} \right.$$

$$C = D_m/D_w = 0.420/0.0475 = 8.84 ; k = \frac{4C^2 - C - 1}{4C(C - 1)} = 1.092$$

$$\text{ACTUAL } \sigma = \frac{32 M_0 K_1}{\pi D_m^3} = \frac{32(1.0)(1.15)}{\pi(0.0475)^3} = 103900 \text{ PSI } \underline{OK}$$

$$k_0 = 1.0 \text{ LB-IN} / 0.50 \text{ REV} = 2.00 \text{ LB-IN/REV}$$

$$N_a = \frac{E D_w^4}{10.2 D_m (k_0)} = \frac{(28 \times 10^6)(0.0475)^4}{10.2(0.420)(2.00)} = 16.64 \text{ COILS TOTAL}$$

ENDS: SPECIFY  $L_1 = L_2 = 0.75$  IN

$$N_e = \frac{L_1 + L_2}{3\pi D_m} = \frac{0.75 + 0.75}{3\pi(0.420)} = 0.38 \text{ COIL (ENDS)}$$

$$N_b = N_a - N_e = 16.64 - 0.38 = 16.26 \text{ COILS IN BODY}$$

$$\text{OPERATING } D_m = \frac{D_m N_a}{N_a + \theta_c} = \frac{0.420(16.64)}{16.64 + 0.50} = 0.408 \text{ IN}$$

$$\text{MINIMUM ID} = 0.408 \text{ IN} - D_w = 0.408 - 0.0475 = 0.360 \text{ IN}$$

$$\text{MAKE ROD } 90\% \text{ OF ID}_{\text{MIN}} \approx 0.9(0.360) = 0.324 \text{ IN}$$

USE  $D_{RW} = 0.300$  IN (STD. SIZE)

$$L = D_w(N_a + 1 + \theta_c) = 0.0475(16.64 + 1 + 0.5) = 0.862 \text{ IN IF COILS TOUCH.}$$

$$D_o = D_m + D_w = 0.420 \text{ IN} + 0.0475 = 0.4675 \text{ IN. } \underline{OK}$$

33

TORSION SPRING: ASTM A313, TYPE 302, SEVERE SERVICE

ASSUME  $D_m = 1.125$ ;  $\sigma_d = 110 \text{ KSI}$ ;  $K_b = 1.15$ ;  $\theta_c = 270^\circ = 0.75 \text{ REV}$ .

$$D_{nr} = \left[ \frac{32 M_o K_b}{\pi \sigma_d} \right]^{1/3} = \left[ \frac{32(12.0)(1.15)}{\pi(110000)} \right]^{1/3} = 0.1085 \text{ IN} \quad \left\{ \begin{array}{l} \text{USE 11 GA.; } D_{nr} = 0.1205 \text{ IN} \\ \sigma_d = 118000 \text{ PSI} \end{array} \right.$$

$$C = D_m / D_{nr} = 1.125 / 0.1205 = 9.34; K_b = \frac{4C^2 - C - 1}{4C(C-1)} = 1.087$$

$$\sigma = \frac{32 M_o K_b}{\pi D_{nr}^3} = \frac{32(12.0)(1.087)}{\pi(0.1205)^3} = 75920 \text{ PSI OK}$$

$$R_o = M_o / \theta_c = 12.0 \text{ LB-IN} / 0.75 \text{ REV} = 16.0 \text{ LB-IN/REV}$$

$$N_a = \frac{E D^4}{10.2 D_m (R_o)} = \frac{(29 \times 10^6)(0.1205)^4}{10.2(1.125)(16.0)} = 32.15 \text{ COILS TOTAL}$$

$$\text{ENDS: LET } L_1 = L_2 = 1.50 \text{ IN}; N_e = \frac{L_1 + L_2}{3\pi(D_m)} = 0.28 \text{ COIL (ENDS)}$$

$$\text{BODY } N_b = N_a - N_e = 32.15 - 0.28 = 31.87 \text{ COILS}$$

$$\text{OPERATING } D_m = \frac{D_m N_a}{N_a + \theta_c} = \frac{(1.125)(32.15)}{(32.15 + 0.75)} = 1.100 \text{ IN}$$

$$I D_{\text{MIN}} = 1.100 - 0.1205 = 0.9795 \text{ IN}; D_{r00} \approx 0.9(0.9795) = 0.882 \text{ IN}$$

$$\text{USE } D_{r00} = 7/8 \text{ IN} = 0.875 \text{ IN}; L = D_m(N_a + 1 + \theta_c) = 0.1205(32.15 + 1.75)$$

$$L = 4.08 \text{ IN}; O.D. = 1.125 + 0.1205 = 1.246 \text{ IN}$$

34

TORSION SPRING: MUSIC WIRE; SEVERE SERVICE;  $\theta_c = 1.0 \text{ REV}$  $D_m = 0.625 \text{ IN}$ ;  $\sigma_d \approx 140 \text{ KSI}$ ;  $K_b \approx 1.15$ 

$$D_{nr} = \left[ \frac{32 M_o K_b}{\pi \sigma_d} \right]^{1/3} = \left[ \frac{32(2.50)(1.15)}{\pi(140000)} \right]^{1/3} = 0.10594 \text{ IN} \quad \left\{ \begin{array}{l} \text{USE 26 GA.; } D_{nr} = 0.063 \\ \sigma_d = 156000 \text{ PSI} \end{array} \right.$$

$$C = D_m / D_{nr} = 0.625 / 0.063 = 9.92; K_b = \frac{4C^2 - C - 1}{4C(C-1)} = 1.081$$

$$\sigma = \frac{32(2.50)(1.081)}{\pi(0.063)^3} = 110100 \text{ PSI OK}$$

$$R_o = 2.50 \text{ LB-IN/REV}; N_a = \frac{(29 \times 10^6)(0.063)^4}{10.2(0.625)(2.50)} = 28.66 \text{ COILS}$$

SPECIFY ENDS:  $L_1 = L_2 = 1.50 \text{ IN}$ .

$$N_e = \frac{2(1.5)}{3\pi(0.625)} = 0.51 \text{ COIL}; N_b = 28.66 - 0.51 = 28.15 \text{ COILS}$$

$$\text{MINIMUM } D_m = \frac{D_m N_a}{N_a + \theta_c} = \frac{0.625(28.66)}{28.66 + 1.0} = 0.604 \text{ IN}$$

$$\text{MIN I.D.} = 0.604 - 0.063 = 0.541 \text{ IN}$$

$$D_{r00} \approx 0.9(0.541) = 0.487 \text{ IN} \rightarrow \text{USE } D_r = 7/16 \text{ IN} = 0.4375 \text{ IN}$$

$$O.D. = 0.625 + 0.063 = 0.688 \text{ IN}$$

$$L = 0.063(28.66 + 1 + 1.0) = 1.93 \text{ IN}$$



35

TORSION SPRING:  $D_m = 0.038$ ;  $D_o = 0.368$  IN;  $N_b = 9.50$  COILS  
 $L_1 = 0.50$  IN;  $L_2 = 1.125$  IN; A401 STEEL;  $\theta = 180^\circ = 0.50$  REV

FROM EQ. 7-21, SOLVE FOR  $M$ :

$$M_o = \frac{5D_m^4 \theta}{11.2(D_m)(N_a)} = \frac{(29.5 \times 10^6)(0.038)^4(0.50)}{11.2(0.330)(10.02)} = 0.91 \text{ LB-IN}$$

WHERE:  $D_m = D_o - D_w = 0.368 - 0.038 = 0.330$  IN

ENDS:  $N_e = \frac{L_1 + L_2}{3\pi(D_m)} = \frac{0.50 + 1.125}{3\pi(0.330)} = 0.52$  COIL

$N_a = N_b + N_e = 9.50 + 0.52 = 10.02$  COILS

$$\sigma = \frac{32M_o K_b}{\pi(D_w)^3} = \frac{32(0.91)(1.094)}{\pi(0.038)^3} = 184,800 \text{ PSI}$$

WHERE  $C = D_m/D_w = 0.330/0.038 = 8.68$

$$K_b = \frac{4C^2 - C - 1}{4C(C-1)} = 1.094$$

FOR SEVERE SERVICE  $\sigma = 200,000$  PSI;  $\sigma = 184,800$  PSI OK

## CHAPTER 19 FASTENERS

- 4 LOAD PER BOLT =  $6000 \text{ LB} / 4 = 1500 \text{ LB}$   
 SELECT GRADE 2:  $\sigma_u = .75 S_{\text{PROOF}} = .75(55000) = 41250 \text{ PSI}$   
[SAE GRADE 2]  
TABLE 19-2  
 $A_t = \frac{1500}{41250} = 0.0364 \text{ IN}^2$
- USE  $1/4$ -28 UNF OR  $5/16$ -18 UNC  
 FOR  $5/16$ -18:  $T = KDP = 0.15(.3125)(1500) = \underline{70.3 \text{ LB-IN}}$
- 5  $F = A_t S_{\text{pr}} = (0.0140 \text{ IN}^2)(85000 \text{ LB/IN}^2) = \underline{1190 \text{ LB}}$   
TABLE 19-2
- 6 M4x.5:  $A_t = 9.79 \text{ mm}^2$ ; GRADE 8.6 (SECTION 19-2)  
 TENSILE STRENGTH  $\approx 800 \text{ MPa}$   
 YIELD STRENGTH  $\approx 0.6 \text{ (T.S.)} = .6(800) = 480 \text{ MPa}$   
 PROOF STRENGTH  $\approx 0.9 \text{ (Y.S.)} = .9(480) = 432 \text{ MPa}$   
 $F = A_t \cdot S_{\text{pr}} = (9.79 \text{ mm}^2)(432 \text{ N/mm}^2) = 4229 \text{ N} = \underline{4.23 \text{ kN}}$   
TABLE 19-5
- 7  $7/8$ -14 UNF;  $D = 0.875 \text{ IN} \times 25.4 \text{ mm/IN} = 22.2 \text{ mm}$   
 $p = 1/m = 1/14 = 0.0714 \text{ IN} \times 25.4 \text{ mm/IN} = 1.81 \text{ mm}$   
 NEAREST METRIC THREAD = M24x2 TABLE 19-5  
 DIFFERENCE IN D =  $24.0 - 22.2 = 1.8 \text{ mm} = 0.071 \text{ IN}$   
 METRIC THREAD IS APPROX. 8% LARGER IN THIS SIZE
- 8 PITCH =  $.630/20 = 0.0315 \text{ IN} \times 25.4 \text{ mm/IN} = 0.800 \text{ mm}$   
 $D = 0.196 \text{ IN} \times 25.4 \text{ mm/IN} = 4.98 \text{ mm}$   
 CLOSEST STANDARD THREAD IS M5x0.8  
 AMERICAN STANDARD #10-32 IS SIMILAR BUT NOT  
 AS CLOSE TO MEASURED DIMENSIONS.
- 9 M10x1.5:  $A_t = 58.0 \text{ mm}^2$   
 NYLON 6/6 !T.S. =  $146 \text{ MPa}$ ; MAX.  $\sigma_u = 0.75 \text{ (T.S.)} = 109.5 \text{ MPa}$   
 $F = A_t \sigma_u = (58.0 \text{ mm}^2)(109.5 \text{ N/mm}^2) = 6351 \text{ N} = \underline{6.351 \text{ kN}}$

10

$$\frac{1}{4} - 20 \text{ UNC}; A_t = 0.0318 \text{ in}^2; \sigma_a = 0.5(TS)$$

$$a) \text{ SAE GRADE 2: } \sigma_a = 0.5(74) = 37 \text{ ksi}; F = A_t \sigma_a = (0.0318)(37000)$$

$$F = 1177 \text{ LB}$$

$$b) \text{ SAE GRADE 5: } \sigma_a = 0.5(120) = 60 \text{ ksi}; F = 1908 \text{ LB}$$

$$c) \text{ SAE GRADE 8: } \sigma_a = 0.5(150) = 75 \text{ ksi}; F = 2385 \text{ LB}$$

$$d) \text{ ASTM A307: } \sigma_a = 0.5(60) = 30 \text{ ksi}; F = 954 \text{ LB}$$

$$e) \text{ ASTM A574: } \sigma_a = 0.5(180) = 90 \text{ ksi}; F = 2862 \text{ LB}$$

$$f) \text{ METRIC GRADE 8.8; } \sigma_a = 0.5(830 \text{ MPa}) = 415 \text{ MPa} \left( \frac{1.0 \text{ ksi}}{6.895 \text{ MPa}} \right) = 60.2 \text{ ksi}$$

$$F = A_t \sigma_a = (0.0318 \text{ in}^2)(60200 \text{ LB/in}^2) = 1914 \text{ LB} \text{ SIMILAR TO SAE GR. 5}$$

$$F = 1914 \text{ LB} \times 4.448 \text{ N/LB} = 8513 \text{ N} = 8.51 \text{ kN}$$

$$g) \text{ ALUM, 2024-T4: } \sigma_a = 0.5(68) = 34 \text{ ksi}; F = 1081 \text{ LB}$$

$$h) \text{ S43000, ANN.; } \sigma_a = 0.5(75) = 37.5 \text{ ksi}; F = 1193 \text{ LB}$$

$$i) \text{ T1-602-4V, ANN.; } \sigma_a = 0.5(130) = 65 \text{ ksi}; F = 2067 \text{ LB}$$

$$j) \text{ NYLON 660X: } \sigma_a = 0.5(21) = 10.5 \text{ ksi}; F = 334 \text{ LB}$$

$$k) \text{ POLYCARBONATE: } \sigma_a = 0.5(9) = 4.5 \text{ ksi}; F = 143 \text{ LB}$$

$$l) \text{ ABS (HIGH IMPACT); } \sigma_a = 0.5(5) = 2.5 \text{ ksi}; F = 80 \text{ LB}$$

ALTERNATE FOR h): SCREW MAY BE FULL HARD - APP. 6

S43000

$$S_u = 90 \text{ ksi}; \sigma_a = 0.5(90) = 45 \text{ ksi}; F = 1431 \text{ LB}$$

**CHAPTER 20**  
**MACHINE FRAMES, BOLTED CONNECTIONS,**  
**AND WELDED JOINTS**

1. DIRECT SHEAR, 5 BOLTS, A307

$$F = \frac{12000 \text{ LB}}{5} = 2400 \text{ LB/BOLT}$$

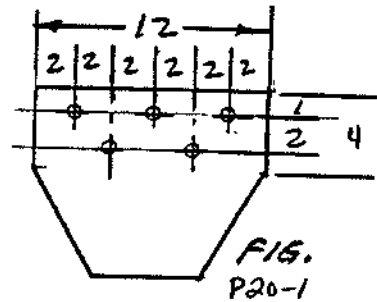
SINGLE SHEAR

$$\tau = F/A_s$$

$$\text{REQ'D } A_s = \frac{F}{\tau_s} = \frac{2400 \text{ LB}}{10000 \text{ LB/IN}^2} = 0.240 \text{ IN}^2$$

$$A_s = \pi D^2/4 ; D = \sqrt{4A_s/\pi} = \sqrt{4(0.240)/\pi} = 0.55 \text{ IN}$$

USE 9/16-12 UNC BOLTS, 1 3/4 IN LONG



2. DIRECT SHEAR, EACH BAR SUPPORTS 4 HANGERS

$$F = 4(750) = 3000 \text{ LB} = \text{TOTAL LOAD ON JOINT}$$

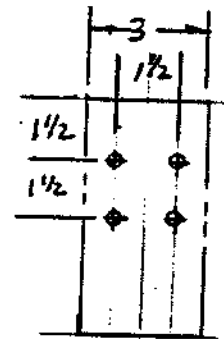
4 BOLTS IN DOUBLE SHEAR, A307

$$\text{TOTAL } A_s \text{ REQ'D} = \frac{F}{\tau_s} = \frac{3000 \text{ LB}}{10000 \text{ LB/IN}^2} = 0.30 \text{ IN}^2$$

$$A_s = (2)(4) \frac{\pi D^2}{4} = 2\pi D^2$$

$$D = \sqrt{A_s/(2\pi)} = \sqrt{0.30/(2\pi)} = 0.219 \text{ IN}$$

USE 1/4-20 UNC BOLTS, 2.00 IN LONG



3.

4 BOLTS

HORIZ. DIRECT SHEAR:  $F_1 = 5196/4 = 1299 \text{ LB/BOLT}$

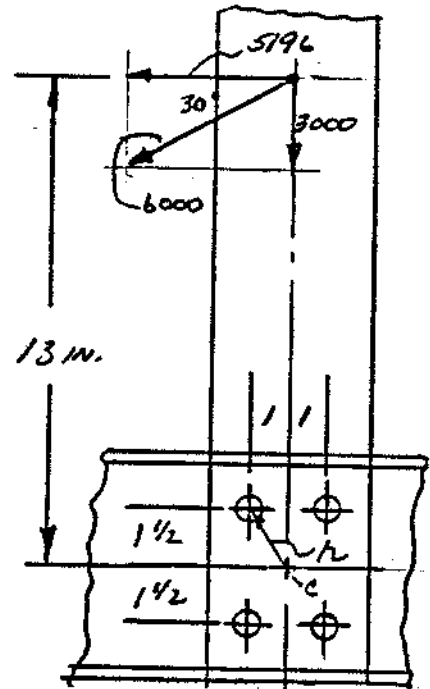
VERTICAL DIR. SHEAR:  $F_2 = 3000/4 = 750 \text{ LB} \downarrow$

$M = 5196(13) = 67548 \text{ LB}\cdot\text{IN.}$

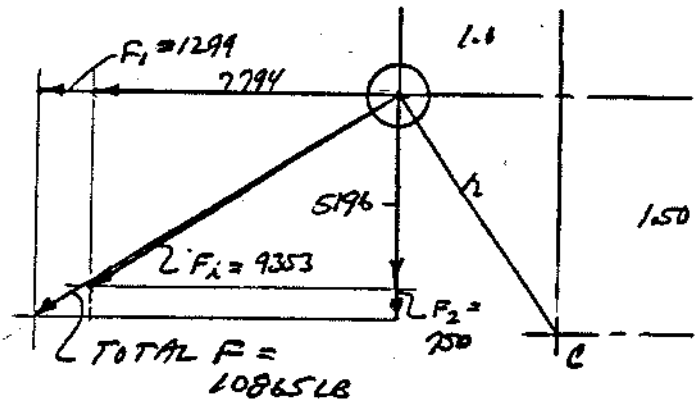
$r = \sqrt{1.50^2 + 1^2} = 1.80 \text{ IN - ALL BOLTS}$

$\Sigma r^2 = 4r^2 = 4(1.80)^2 = 13.0 \text{ IN}^2$

$F_1' = \frac{M r_i}{\Sigma r^2} = \frac{(67548)(1.80)}{13.0} = 9353 \text{ LB/BOLT}$



FORCES ON BOLT AT UPPER LEFT:



TOTAL  $F_x = 7794 + 1299 = 9093$

TOTAL  $F_y = 5196 + 750 = 5946$

TOTAL  $F = \sqrt{9093^2 + 5946^2}$

$F = 10865 \text{ LB}$

A 490 BOLTS, DOUBLE SHEAR

$A_s = \frac{F}{T_s} = \frac{10865 \text{ LB}}{22000 \text{ LB/IN}^2} = 0.494 \text{ IN}^2 = 2(\pi D^2/4) = \pi D^2/2$

REQ'D  $D = \sqrt{2A_s/\pi} = \sqrt{2(.494)/\pi} = 0.561 \text{ IN.}$

USE 9/16-12 UNC, 3 1/2 IN LONG

4. FIXED-END BEAM - CASE C APPENDIX 14-3.

$$M = \frac{Wl}{8} = \frac{(3000)(87.66)}{8} = 32873 \text{ LB}\cdot\text{IN}$$

EACH SIDE:  $M = 32873 \text{ LB}\cdot\text{IN}$ .

$$\text{DIRECT SHEAR} = \frac{1500 \text{ LB}}{8 \text{ BOLTS}} = 188 \text{ LB/BOLT}$$

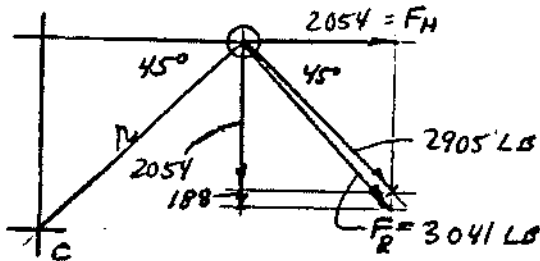
USE 4 BOLTS ON EACH FACE OF COLUMN THROUGH FLANGE AND PLATE.

FORCE ON EACH BOLT DUE TO MOMENT:

$$r = 1.414 \text{ IN}; \Sigma r^2 = 8(1.414)^2 = 16.00 \text{ IN}^2$$

$$F = \frac{M/r}{\Sigma r^2} = \frac{(32873)(1.414)}{16.00} = 2905 \text{ LB/BOLT}$$

FORCES ON BOLT AT UPPER RIGHT:



$$\text{TOTAL } F_v = 2054 + 188 = 2242 \text{ LB}$$

$$F_R = \sqrt{2242^2 + 2054^2} = 3041 \text{ LB}$$

SPECIFY ASTM A 325 HIGH STRENGTH BOLTS

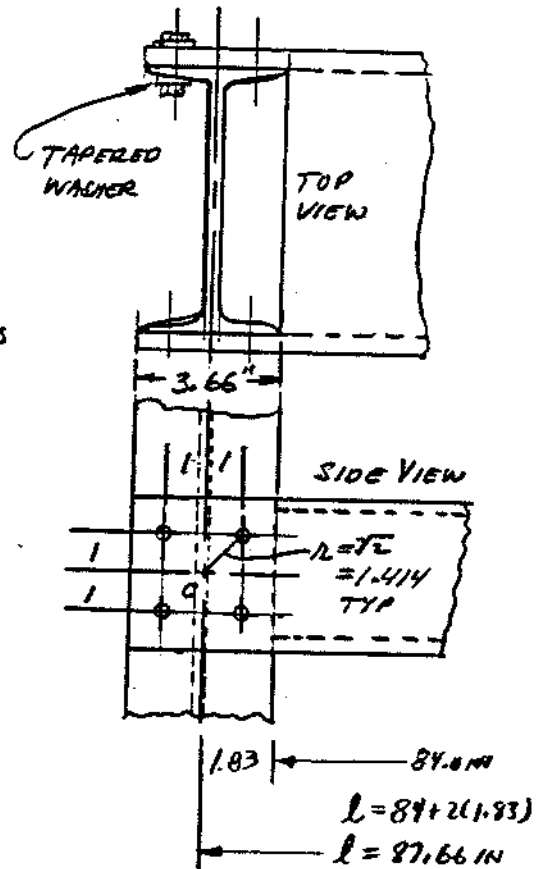
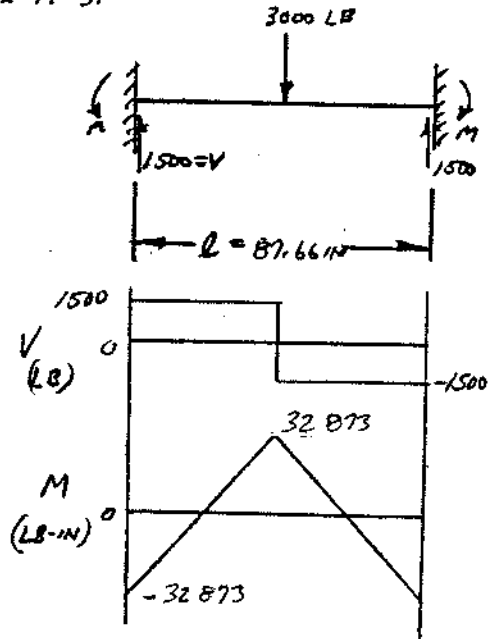
$$T_d = 17500 \text{ PSI} = F_R / A_s$$

$$\text{REQ'D. } A_s = F_R / T_d = \frac{3041 \text{ LB}}{17500 \text{ LB/IN}^2} = 0.174 \text{ IN}^2$$

$$A_s = \pi D^2 / 4; D = \sqrt{4A_s / \pi} = \sqrt{4(0.174) / \pi}$$

$$D = 0.470 \text{ IN}$$

USE 1/2-13 UNC BOLTS. NO THREADS IN THE SHEAR PLANE.





5.

6 BOLTS, A307, 1400 LB/SIDE

$$\text{SHEAR: } \frac{1400 \text{ LB}}{6} = 233 \text{ LB/BOLT } \downarrow$$

$$M = (1400 \text{ LB})(13 \text{ IN}) = 18200 \text{ LB}\cdot\text{IN}$$

$$r_1 = \sqrt{2^2 + 3^2} = 3.61 \text{ IN}$$

$$r_2 = 2.00 \text{ IN}$$

$$\Sigma A^2 = 4(3.61)^2 + 2(2.00)^2 = 60 \text{ IN}^2$$

$$\text{BOLT } \textcircled{D} \quad F = \frac{M r_1}{\Sigma A^2} = \frac{(18200)(3.61)}{60} = 1095 \text{ LB}$$

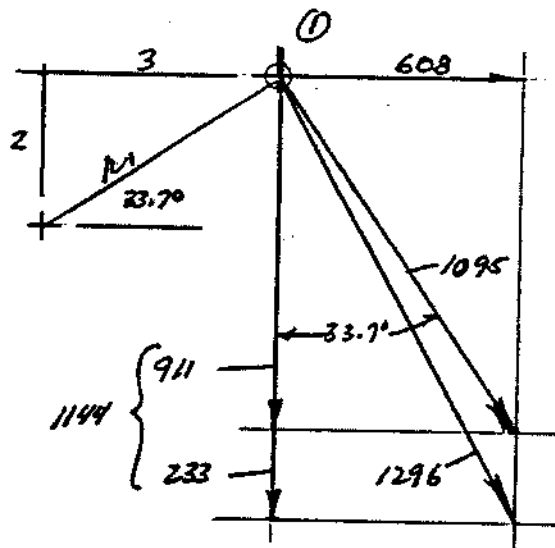
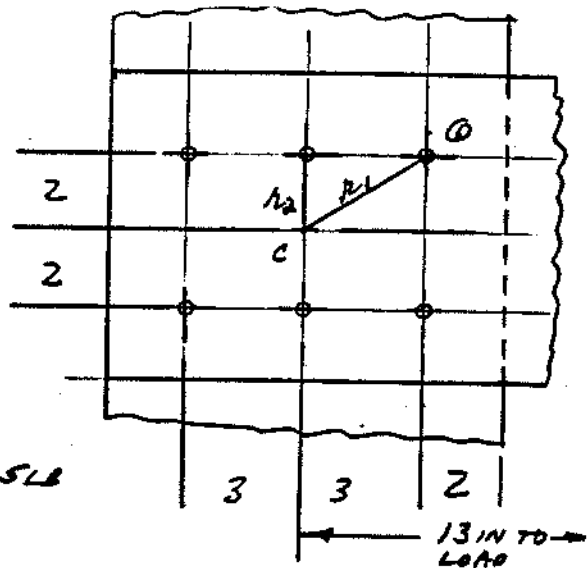
$$\text{TOTAL } F = 1296 \text{ LB}$$

$$A_s = \frac{F}{T_s} = \frac{1296 \text{ LB}}{10000 \text{ LB/IN}^2}$$

$$A_s = 0.130 \text{ IN}^2 = \pi D^2 / 4$$

$$D = \sqrt{\frac{4 A_s}{\pi}} = \sqrt{\frac{4(0.130)}{\pi}} = 0.406 \text{ IN.}$$

USE 7/16-14 BOLTS



6. FIG. P20-6

VERTICAL COMPONENT OF FORCE  
IN EACH CABLE MUST BE 2000 LB

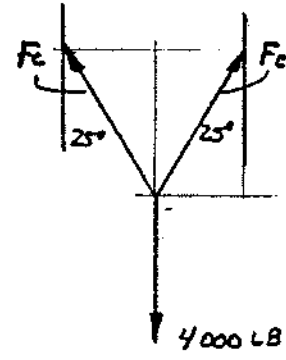
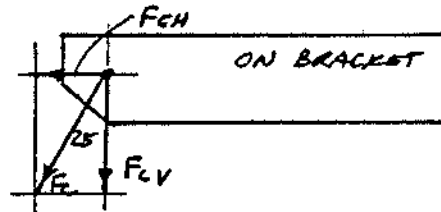
$$F_c \cos 25^\circ = 2000 \text{ LB} = F_{cv}$$

$$F_c = \frac{2000}{\cos 25^\circ} = 2207 \text{ LB}$$

$$F_{ch} = F_c \sin 25^\circ$$

$$= 2207 \sin 25^\circ$$

$$F_{ch} = 933 \text{ LB}$$



8 BOLTS,  $r = 3.00 \text{ IN}$

A325

$$\Sigma A^2 = 8(3.0)^2 = 72 \text{ IN}^2$$

$$F_s = \frac{M \cdot r}{\Sigma A^2} = \frac{60000(3)}{72}$$

$$F_s = 2500 \text{ LB}$$

SHEAR-HORIZ.  $F_1 = \frac{933}{8} = 117 \text{ LB} \leftarrow$

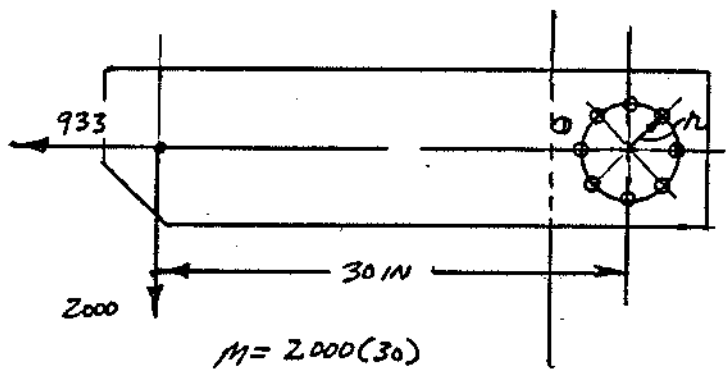
SHEAR-VERT.  $F_2 = \frac{2000}{8} = 250 \text{ LB} \downarrow$

$$F_T = \sqrt{2750^2 + 117^2} = 2753 \text{ LB}$$

$$A_s = \frac{F_T}{T_u} = \frac{2753}{17500} = 0.157 \text{ IN}^2 = \frac{\pi D^2}{4}$$

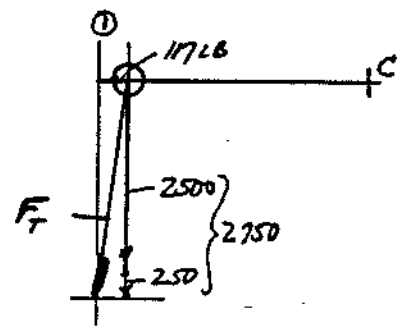
$$D = \sqrt{4A_s/\pi} = \sqrt{4(0.157)/\pi} = 0.448 \text{ IN}$$

USE 1/2-13 BOLTS



$$M = 2000(30)$$

$$= 60000 \text{ LB}\cdot\text{IN}$$



WELDED JOINTS

7. FIGURE P20-1 DIRECT SHEAR;  $f = V/A_w$

WELD BOTH VERTICAL SIDES, 4.0 IN. LONG.  $A_w = 2(4) = 8.0 \text{ IN}$

$$f = \frac{V}{A_w} = \frac{12000 \text{ LB}}{8.0 \text{ IN}} = 1500 \text{ LB/IN}$$

FOR A36 STEEL AND 560 ELECTRODE;  $f_w = 9600 \text{ LB/IN/IN}$

$$w = \frac{f}{f_w} = \frac{1500 \text{ LB/IN}}{9600 \text{ LB/IN/IN}} = 0.156 \text{ IN}; \text{ USE } w = \underline{\underline{3/16 \text{ IN LEG.}}}$$

8. FROM PROBLEM 4 - AT EACH END OF THE BEAM,  $V = 1500 \text{ LB}$   
 $T = 32873 \text{ LB}\cdot\text{IN}$  - SHARED EQUALLY ON FRONT AND BACK OF  
 PLATES THAT ARE WELDED TO  $57 \times 153$  COLUMN.

WELD ALONG TOP AND BOTTOM OF PLATE.

CASE 3 - FIG. 20-8:  $A_w = 2b = 2(3.66) = 7.32 \text{ IN}$ .

$$J_w = (b^3 + 3bd^2) / 6 = (3.66^2 + 3(3.66)(4.00)^2) / 6$$

$$J_w = 37.45 \text{ IN}^3 \text{ TORSION}$$

ON EACH WELD PATTERN:  $V = 750 \text{ LB}$ ,  $T = 16436 \text{ LB}\cdot\text{IN}$ .

AT (A):  $f_1 = \frac{V}{A_w} = \frac{750 \text{ LB}}{7.32 \text{ IN}} = 102 \text{ LB/IN} \downarrow$

$$f_2 = \frac{T c_y}{J_w} = \frac{(16436 \text{ LB}\cdot\text{IN})(2.0 \text{ IN})}{37.45 \text{ IN}^3} = 878 \text{ LB/IN} \rightarrow$$

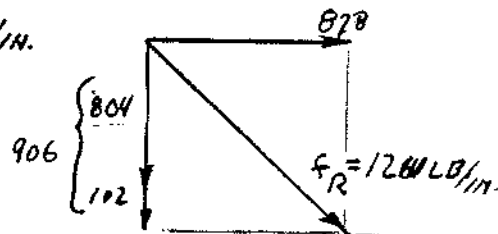
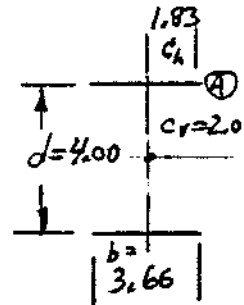
$$f_3 = \frac{T c_x}{J_w} = \frac{(16436)(1.83)}{37.45} = 804 \text{ LB/IN} \downarrow$$

VECTOR SUM OF FORCES;  $F_R = 1211 \text{ LB/IN}$ .

USE E60 ELECTRODE:

$$w = \frac{F_R}{f_w} = \frac{1211}{9600} = 0.121 \text{ IN}$$

USE  $w = \underline{\underline{3/16 \text{ IN}}} = 0.188 \text{ IN}$  - MINIMUM  
 FOR  $1/2 \text{ IN}$  PLATE.



9.

FIGURE P20-5: TRIM SIDES SO THAT 2.00 IN EXTEND ONTO RIGID STRUCTURE. WELD TOP AND BOTTOM ONLY.

EACH SIDE CARRIES 1400 LB

TORQUE ON WELD:  $T = 1400(8 + \frac{1}{2})$

$$T = 1400(8 + 1) = 12600 \text{ LB}\cdot\text{IN}$$

$$A_w = 2b = 2(2) = 4.00 \text{ IN}$$

$$J_w = \frac{b^3 + 3bd^2}{6} = \frac{(2)^3 + 3(2)(8)^2}{6} = 65.3 \text{ IN}^3$$

$$f_1 = \frac{V}{A_w} = \frac{1400 \text{ LB}}{4.00 \text{ IN}} = 350 \text{ LB/IN} \downarrow$$

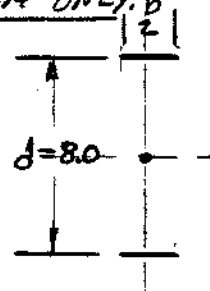
$$f_2 = \frac{T c_v}{J_w} = \frac{(12600 \text{ LB}\cdot\text{IN})(4.00 \text{ IN})}{65.3 \text{ IN}^3} = 771 \text{ LB/IN} \rightarrow$$

$$f_3 = \frac{T c_h}{J_w} = \frac{(12600)(1.00)}{65.3} = 193 \text{ LB/IN} \downarrow$$

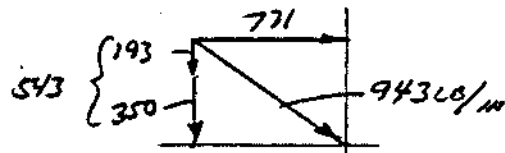
E60 ELECTRODES

$$w = \frac{943}{9600} = 0.098 \text{ IN}$$

USE  $w = \frac{3}{16} = 0.188 \text{ IN (MIN)}$



CASE 3 -  
FIGURE 20-8



10.

FIG. P20-6

$$A_w = 2(10) = 20 \text{ IN}$$

$$J_w = \frac{d(3b^2 + d^2)}{6}$$

$$J_w = \frac{10(3(10)^2 + 10^2)}{6} = 667 \text{ IN}^3$$

AT (A)

$$f_1 = \text{SHEAR} = \frac{933}{20} = 46.7 \text{ LB/IN} \leftarrow$$

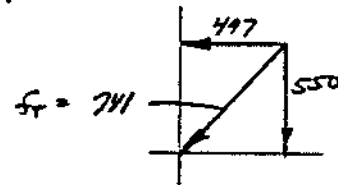
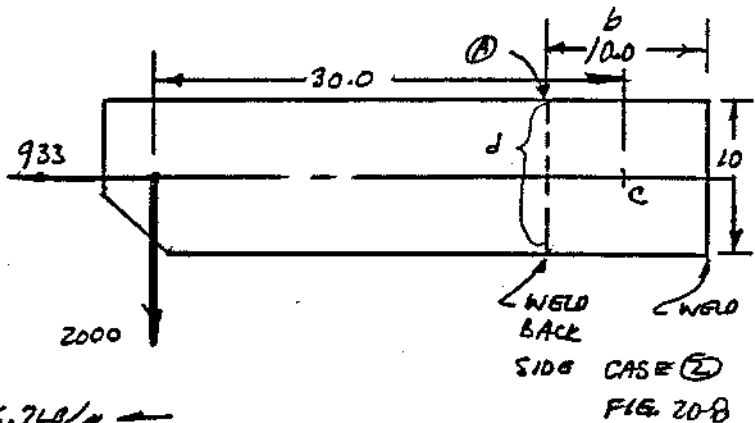
$$f_2 = \text{SHEAR} = \frac{2000}{20} = 100 \text{ LB/IN} \downarrow$$

$$f_3 = \text{TORSION} = \frac{T C_v}{J_w} = \frac{2000(30)(5)}{667} = 450 \text{ LB/IN} \leftarrow$$

$$f_4 = \text{TORSION} = \frac{T c_1}{J_w} = f_3 = 450 \text{ LB/IN} \downarrow$$

$$\left. \begin{aligned} f_1 + f_3 &= 497 \text{ LB/IN} \\ f_2 + f_4 &= 550 \text{ LB/IN} \end{aligned} \right\} f_r = 741 \text{ LB/IN}$$

$$n = \frac{f_r}{F_u} = \frac{741}{9600} = 0.0771 \text{ IN} \leftarrow \text{USE } n = \frac{3}{16} \text{ (MIN)}$$



11.

FIGURE P20-11

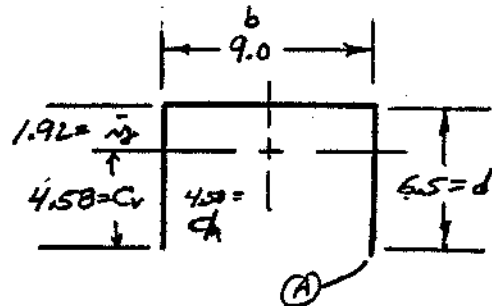
CASE (2), FIG. 20-8

$$A_w = b + 2d = 9 + 2(6.5) = 22.0 \text{ IN}$$

$$\bar{y} = \frac{d^2}{b + 2d} = \frac{(6.5)^2}{9 + 2(6.5)} = 1.92 \text{ IN}$$

$$S_w = \frac{d^2(2b + d)}{3(b + d)} = \frac{(6.5)^2(2(9) + 6.5)}{3(9 + 6.5)} = 22.3 \text{ IN}^2 \text{ (BOTTOM)}$$

$$J_w = \frac{(b + 2d)^3}{12} - \frac{d^2(b + d)^2}{b + 2d} = 887 - 461 = 426 \text{ IN}^2$$



11. (CONTINUED) AT (A):

$$f_1 = \text{SHEAR} = \frac{V}{A_w} = \frac{4200 \text{ LB}}{22.0 \text{ IN}} = 191 \text{ LB/IN} \downarrow$$

$$f_2 = \text{BENDING} = \frac{M}{S_w} = \frac{4200(21.5)}{22.3} = 4056 \text{ LB/IN (INTO WALL)}$$

$$f_3 = \text{TORSION (HORIZ.)} = \frac{T C_V}{J_w} = \frac{1200(165)(4.58)}{426} = 213 \text{ LB/IN} \leftarrow$$

$$f_4 = \text{TORSION (VERT)} = \frac{T C_A}{J_w} = \frac{1200(165)(4.50)}{426} = 209 \downarrow$$

$$f_1 + f_4 = 191 + 209 = 400 \text{ LB/IN} \downarrow$$

$$\text{RESULTANT} = f_T = \sqrt{f_2^2 + f_3^2 + (f_1 + f_4)^2} = \sqrt{4056^2 + 213^2 + 400^2}$$

$$f_T = 4081 \text{ LB/IN} \quad \text{USE E70 ELECTRODE}$$

$$\text{REQ'D } w = \frac{4081}{11200} = 0.364 \text{ IN} \quad \text{USE } \frac{3}{8} \text{ " } = w$$

12. FIG P20-11 WITH  $P_L = 0$ ; NO TORSION

$$f_1 = \text{SHEAR} = \frac{V}{A_w} = \frac{3000}{22.0} = 136 \text{ LB/IN} \downarrow$$

$$f_2 = \text{BENDING} = \frac{M}{S_w} = \frac{3000(21.5)}{22.7} = 2892 \text{ LB/IN (INTO WALL)}$$

$$f_T = \sqrt{f_1^2 + f_2^2} = \sqrt{136^2 + 2892^2} = 2896 \text{ LB/IN}$$

USE E60 ELECTRODE:

$$w = \frac{2896}{9600} = 0.302 \text{ IN} \quad \text{USE } \frac{5}{16} \text{ IN, } 0.312 \text{ IN.}$$



13.

FIGURE P20-1

P = 4000 LB ; 6061 ALUMINUM

DIRECT SHEAR - DATA FROM TABLE 19-2

$$T = \frac{P}{A_s} = \frac{P}{0.707 w L}$$

$A_s = \text{SHEAR AREA} = (0.707 w) \times \text{LENGTH OF WELD}$

FOR 4043 FILLER ALLOY,  $T_{\text{ALLOW}} = 5000 \text{ PSI}$

LET  $L = 2(4.00 \text{ IN}) = 8.00 \text{ IN}$  (VERTICAL SIDES OF BRACKET)

$$\text{REQ'D } w = \frac{P}{0.707 L T_{\text{ALLOW}}} = \frac{4000 \text{ LB}}{0.707(8.00 \text{ IN})(5000 \text{ LB/IN}^2)} = 0.141 \text{ IN.}$$

USE  $3/16 \text{ IN. WELD (MIN.)}$

14.

FIGURE P20-14

DIRECT SHEAR, 6061 ALUMINUM

4043 FILLER ALLOY

$$\text{LENGTH OF WELD} = L = \pi D = \pi(4.50) = 14.1 \text{ IN}$$

$$\text{AS IN 13: } w = \frac{P}{.707 L T_n} = \frac{1500}{(.707)(14.1)(5000)} = 0.030 \text{ IN.}$$

USE  $3/16 \text{ IN}$

COULD ALSO CONSIDER PARTIAL WELD.

15.

FIGURE P20-15

DIRECT SHEAR, 6063 ALUM.

4043 FILLER

$$\text{LET } w = 3/16 = 0.188 \text{ IN}$$

SOLVE FOR L

$$L = \frac{P}{0.707 w T_{\text{ALLOW}}} = \frac{800 \text{ LB}}{(0.707)(.188)(5000 \text{ LB/IN}^2)} = 1.20 \text{ IN.}$$

DISTRIBUTE PARTIAL WELDS TOTALING 1.20 IN  
EQUALLY ON BOTH SIDES OF TAB.

16. FIGURE P20-16 DIRECT SHEAR, 5003 ALUMINUM  
4043 FILLER ALLOY

AS IN PROBLEM 14:

$$\text{REQ'D. } w = \frac{P}{0.707 L T_w}$$

$$T_w = 5000 \text{ PSI}$$

$$L = 2(\pi D) = 2\pi(2.0 \text{ IN}) = 12.57 \text{ IN. } \underline{\text{WELD ALL AROUND}}$$

$$w = \frac{10000 \text{ LB}}{(0.707)(12.57) \text{ IN}(5000 \text{ LB/IN}^2)} = 0.225 \text{ IN}$$

$$\underline{\text{USE } w = 1/4 \text{ IN} = 0.250 \text{ IN}}$$

17. MATERIAL COMPARISON

$$F = 4800 \text{ LB}; N = 2; C_D = S_y/2$$

$$\text{REQ'D } A = F/\sigma_b; D = \sqrt{4A/\pi}$$

COMPUTE WEIGHT PER INCH OF LENGTH

$$V = A \times 1.0 \text{ IN}$$

$$W = \rho_w V$$

<u>MATERIAL</u>	<u>S<sub>y</sub>(KSI)</u>	<u>σ<sub>b</sub>(KSI)</u>	<u>A(IN<sup>2</sup>)</u>	<u>D(IN)</u>	<u>V(IN<sup>3</sup>)</u>	<u>ρ<sub>w</sub>(LB/IN<sup>3</sup>)</u>	<u>W(LB)</u>
a) 1020 HR	30	15	.32	.638	.32	.283	.0906
b) S160 OQT1300	100	50	.096	.350	.096	.283	.0272
c) ALUM. 2014-T6	60	30	.16	.451	.16	.100	.0160
d) ALUM 7075-T6	73	36.5	.132	.409	.132	.100	.0132
e) Ti-6AL-4V (ANNEALED)	120	60	.080	.319	.080	.160	.0128
f) Ti-3AL-7.5V-11CR	175	87.5	.055	.264	.055	.160	.0088

## CHAPTER 21

### ELECTRIC MOTORS AND CONTROLS

Questions 1 - 8: See Sections 21-2 and 21-3.

9. Standard frequency for AC power in the U.S. is 60 hertz.
10. Standard frequency for AC power in Europe is 50 hertz.
11. Single phase AC power at 115 and 230 volts.
12. Two conductors plus a ground wire.
13. 480V, three phase is preferred because the current would be lower and the size of the motor would be smaller.
14. Synchronous speed is the speed at which an AC motor tends to run at zero load.  $n_s = 120(f)/p$ , where  $f$  is the frequency of the power and  $p$  is the number of poles in the motor.
15. Full-load speed is the speed of the motor when it is delivering its rated torque.
16. In U.S.:  $n_s = 120(f)/p = 120(60)/4 = 1800$  rpm  
In France:  $n_s = 120(f)/p = 120(50)/4 = 1500$  rpm
17. 2-pole motor. Zero-load speed approximately 3600 rpm.
18.  $n_s = 120(f)/p = 120(400)/4 = 12000$  rpm
19. Two speed motor; 1725 rpm and 1140 rpm
20. Variable frequency control
- 21,22,24 - See Section 21-8.
23. National Electrical Manufacturers Association
25. TEFC - Totally enclosed - fan-cooled. See Section 21-8.
26. TENV - Totally enclosed - non-ventilated. Section 21-8.
27. NEMA Design 9 - Hazardous locations. Flour can explode.
28. TENV because motor may get bathed in water during cleaning and to protect food from contaminants from the motor.

30. Locked rotor torque is the torque that a motor can exert when the rotor is at rest. Also called starting torque.
31. A poorer speed regulation means that the motor would slow down more when subjected to an increase in torque.
32. Breakdown torque is the maximum torque a motor can develop during the increase in speed after start or the torque at which a motor would be stalled if the torque is increased after it is running.
33. Split-phase; capacitor-start; permanent-split capacitor; shaded pole.
34. a) Single phase, split-phase AC motor because of the moderate starting torque and the change of torque when the switch cuts out the starting winding.
- b) From Table 21-2, full-load speed = 1140 rpm  
 $T = 63000(P)/n = 63000(.75)/1140 = 41.4 \text{ lb-in}$
- c) Starting torque = 150%(F.L. torque)  
 $T_s = 1.5(41.4) = 62.2 \text{ lb-in (approximate)}$
- d) Breakdown = 350%(F.L. torque)  
 $T_b = 3.5(41.4) = 145 \text{ lb-in (approximate)}$
35. 2-pole; 1.50 kW rated power.
- b) From Table 21-2, full-load speed = 3450 rpm  
 $n = (3450 \text{ rev/min})(2\pi \text{ rad/rev})(1 \text{ min}/60 \text{ s}) = 361 \text{ rad/s}$   
 $T = P/n = (1.5 \times 10^3 \text{ N-m/s})/(361 \text{ rad/s}) = 4.15 \text{ N-m}$
- c) Starting torque = 1.5(4.15 N-m) = 6.23 N-m
- d) Breakdown torque = 3.5(4.15 N-m) = 14.5 N-m
36. Fan requires about 18 lb-in of torque at 1725 rpm; low starting torque; assume fan cools motor. Recommend 4-pole, single phase permanent split capacitor AC motor.  
 $\text{Power} = Tn/63000 = (18)(1725)/63000 = 0.49 \text{ hp (use } 1/2 \text{ hp)}$
37. Full load torque about 0.5 N-m at 3450 rpm; high starting torque (about 2.8xF.L.T.) due to starting compressor against high pressure in the system. Recommend capacitor start, single phase, 2-pole, AC motor.  
 $n = (3450 \text{ rev/min})(2\pi \text{ rad/rev})(1 \text{ min}/60 \text{ s}) = 361 \text{ rad/s}$   
 $P = Tn = (0.5 \text{ N-m})(361 \text{ rad/s}) = 181 \text{ N-m/s} = 181 \text{ watts}$
38. Speed is adjusted by varying the resistance in the rotor circuit through an external resistance control.
39. F.L. speed = synchronous speed = 720 rpm. (Table 21-2)

40. Pull-out torque is the torque that would disengage the motor from its synchronous speed and cause it to stop.
41. Universal motors are very small and light weight for a given power rating. Some vacuums, appliances, and hand tools utilize the high speed of rotation effectively.
42. A universal motor can operate on DC or almost any frequency of AC voltage when operating near its full-load point.
43. Batteries, generators, rectified AC, HYDROGEN FUEL CELLS.
44. See Table 21-7.
45. SCR - Silicon controlled rectifier. Used to produce DC power from AC.
46. SCR controls do not produce pure DC power; it has some variation, called ripple, due to the AC input. A low-ripple control would produce a nearly true DC power.
47. Could use a 90V DC motor powered from a NEMA Type K SCR power supply to convert 115 V AC to 90V DC power.
- 48-50. See Section 21-11.
51. The motor would speed up without limit and may fail catastrophically.
52. Speed is proportional to torque.  $T_2 = T_1 (n_1/n_2)$   
 $T_2 = (15.0 \text{ N-m}) (3000/2200) = 20.5 \text{ N-m}$
- 53, 56-61. See Sections 21-9, 21-11, and 21-12.
54. NEMA Size 2 motor starter for 10 hp, 220V AC, 3-phase.
55. NEMA Size 1 motor starter for 1.0 kW, 110V AC, single phase.

## CHAPTER 22

### MOTION CONTROL: CLUTCHES AND BRAKES

1. FROM EQ. 22-1:  $T = C PK / m = (63025)(5.0)(2.75) / 1750 = 495 \text{ LB}\cdot\text{IN}$   
 C AND K FROM SECTION 22-4.

SUMMARY OF RESULTS FOR PROBLEMS 2-7.

PROB.	C	P	K	m	TORQUE
2	5252	75HP	5.0	2500	788 LB·FT
3	63025	0.50HP	1.5	1150	41 LB·IN
4	63025	5.0HP	2.75	180	4814 LB·IN
5-1	63025	5.0HP	1.0	1750	180 LB·IN
5-2	5252	75HP	1.0	2500	158 LB·FT
5-3	63025	0.50HP	1.0	1150	27.4 LB·IN
5-4	63025	5.0HP	1.0	180	1751 LB·IN
6	9549	20 kW	2.75	3450	152 N·m
7(a)	9549	50 kW	4.0	900	2122 N·m CLUTCH
(b)	9549	50 kW	1.0	900	531 N·m BRAKE

8. DISK:  $D = 24.0 \text{ IN}$ ;  $R_1 = 12.0 \text{ IN}$ ;  $L = 2.50 \text{ IN}$ ;  $R_2 = 0$ ; STEEL

$$Wk^2 = \frac{L(R_1^4 - R_2^4)}{323.9} = \frac{2.50(12.0^4 - 0)}{323.9} \text{ LB}\cdot\text{FT}^2 = 160.0 \text{ LB}\cdot\text{FT}^2$$

$$T = \frac{Wk^2(\Delta\omega)}{308t} = \frac{(160)(550)}{308(2.0)} = 142.9 \text{ LB}\cdot\text{FT}$$

- 9.

	$R_1$	$R_2$	L	$Wk^2$
SHAFT	0.625	0	16.0	0.00754
COUPLING	1.50	.625	2.25	0.0341
BRG-1	1.00	.625	1.80	0.0047
HUB	2.00	.625	1.00	0.0489
GEAR	6.00	.625	3.00	12.0023
BRG-2	1.00	.625	1.80	0.0047
	TOTAL			12.102 LB·FT <sup>2</sup>
$T = \frac{Wk^2(\Delta\omega)}{308t} =$	$\frac{(12.102)(775)}{308(1.50)}$			$= 60.9 \text{ LB}\cdot\text{FT}$



10.

NEGLECT CLUTCH AND SHORT SHAFT BETWEEN CLUTCH AND GEAR A.

SPEED OF SHAFT 2:  $m_2 = 1750 (4.00/15.00) = 466.7 \text{ RPM}$

IN EQ. 22-4  $(m/m_c)^2 = (m_2/m_1)^2 = (466.7/1750)^2 = 0.07117$

	<u>R<sub>1</sub></u>	<u>R<sub>2</sub></u>	<u>L</u>	<u>WR<sup>2</sup></u>	<u>WR<sub>e</sub><sup>2</sup></u>
GEAR A	2.00	0	3.00	0.1482	0.1482
GEAR B	7.50	1.25	3.00	29.2833	2.0824
SHAFT 2	1.25	0	54.0	0.4070	0.0289
HUB 1	2.50	1.25	5.0	0.5653	0.0402
HUB 2	2.50	1.25	5.0	0.5653	0.0402
END PLATES	9.00	1.25	2.00	40.4974	2.8798
HOLLOW CYL	9.00	7.50	30.0	34.6284	22.3736
			TOTAL		27.5933 LB-FT <sup>2</sup>

$$T = \frac{WR_e^2(\omega_m)}{308t} = \frac{(27.5933)(1750)}{308(1.50)} = 104.5 \text{ LB-FT}$$

11.

LOAD SPEED = 50 FT/MIN = V

DRUM SPEED =  $\frac{V}{R} = \frac{50 \text{ FT}}{\text{MIN}} \times \frac{1}{4.00 \text{ IN}} \times \frac{12 \text{ IN}}{\text{FT}} = 150 \text{ RAD/MIN} = \omega$

RPM OF DRUM =  $M = \frac{150 \text{ RAD}}{\text{MIN}} \times \frac{60 \text{ SEC}}{2\pi \text{ RAD}} = 23.87 \text{ RPM}$

	<u>R<sub>1</sub></u>	<u>R<sub>2</sub></u>	<u>L</u>	<u>WR<sup>2</sup></u>
SHAFT	0.75	0	24.0	0.0234
END PLATES	4.00	.75	3.0	2.3682
HOLLOW CYL.	4.00	3.00	12.0	7.0238
			TOTAL	9.4154 LB-FT <sup>2</sup>

$$\text{LOAD: } WR_e^2 = W \left( \frac{V}{\omega} \right)^2 = 600 \text{ LB} \left( \frac{50 \text{ FT/MIN}}{150 \text{ RAD/MIN}} \right)^2 = 66.6667 \text{ LB-FT}^2$$

$$\text{TOTAL } WR^2 = 9.4154 + 66.6667 = 76.08 \text{ LB-FT}^2$$

$$\text{BRAKING TORQUE} = \frac{WR^2(\omega_m)}{308t} = \frac{(76.08)(23.87)}{308(0.25)} = 23.6 \text{ LB-FT}$$

$$\text{ADDITIONAL TORQUE TO HOLD LOAD} = 600 \text{ LB} (4 \text{ IN}) \left( \frac{12 \text{ IN}}{\text{FT}} \right) = 200 \text{ LB-FT}$$

$$\text{TOTAL BRAKE TORQUE} = 223.6 \text{ LB-FT}$$

12. (a) CLUTCH ON MOTOR SHAFT:

MOTOR SPEED = CLUTCH SPEED = 1150 RPM =  $m_c$

BARREL SPEED = 38 RPM =  $m$

$$(m/m_c)^2 = (38/1150)^2 = 0.001092$$

	$R_1$	$R_2$	$L$	$Wk^2$	$Wk^2$
WORM	1.75	0	8.00	0.2316	0.2316
WORM GEAR	8.00	0	2.50	31.6147	0.0344
2 HUBS	4.00	2.00	4.00	2.9639	0.0032
END PLATES	14.00	2.00	4.00	474.2204	0.5178
HOLLOW CYL.	14.00	12.00	18.00	982.5255	1.0728
SHAFT	2.00	0	36.0	1.7783	0.0019
					<u>1.8617 LB-FT<sup>2</sup></u>

$$T = \frac{Wk^2 (\Delta \omega)}{308 t} = \frac{(1.8617)(1150)}{308(2.0)} = \underline{\underline{3.48 \text{ LB-FT}}}$$

(b) CLUTCH ON WORM GEAR SHAFT. ONLY BARREL AND HUBS ARE ACCELERATED TO 38 RPM = CLUTCH SPEED.

	$Wk^2$	$T = \frac{Wk^2 (\Delta \omega)}{308 t}$
2 HUBS	2.9639	
END PLATES	474.2204	
HOLLOW CYL.	982.5255	
SHAFT	1.7783	
TOTAL	<u>1461.5 LB-FT<sup>2</sup></u>	$T = \frac{(1461.5)(38)}{308(2.0)} = \underline{\underline{90.2 \text{ LB-FT.}}}$

13.

$$T_f = F N R_m \quad ; \quad R_m = T_f / F N = (75 \text{ LB-IN}) / (1.25 \times 150) \text{ LB} = \underline{\underline{2.00 \text{ IN}}}$$

$$P_f = \frac{T_f m}{63000} \quad h_p = \frac{(75)(1150)}{63000} = \underline{\underline{1.37 \text{ hp}}}$$

$$\text{LET } W R = 0.10 \text{ hp/in}^2 = P_f / A$$

$$A = P_f / W R = 1.37 \text{ hp} / 0.10 \text{ hp/in}^2 = \underline{\underline{13.7 \text{ in}^2}}$$

$$A = \pi (R_o^2 - R_i^2), \quad R_m = (R_o + R_i) / 2 \quad \text{OR} \quad 2 R_m = R_o + R_i$$

$$\text{TRY } R_o = 1.5 R_i \quad ; \quad 2 R_m = 1.5 R_i + R_i = 2.5 R_i \quad ; \quad R_i = \frac{2 R_m}{2.5} = \frac{2.00}{1.25} = 1.60 \text{ in}$$

$$R_o = 1.5(1.60) = 2.40 \text{ in} \quad ; \quad A = \pi (2.40^2 - 1.60^2) = \underline{\underline{10.05 \text{ in}^2}}$$

SIMILARLY, FOR  $R_o = 1.75 R_i$

$$\underline{\underline{R_i = 1.45 \text{ in} \quad ; \quad R_o = 2.55 \text{ in} \quad ; \quad A = 13.75 \text{ in}^2 \text{ OK}}}$$

14. FROM PROB. 9:  $T_f = 64 \text{ LB}\cdot\text{IN}$ ; TRY  $R_m = 1.50 \text{ IN}$ ;  $f = 0.25$

$$N = \frac{T_f}{f R_m} = \frac{64.0}{(0.25)(1.50)} = \underline{170 \text{ LB}}$$

$$P_f = \frac{T_f M}{63000} = \frac{(64)(775)}{63000} = 0.79 \text{ HP}$$

FOR  $WR = 0.10 \text{ hp}/\text{in}^2$ ;  $A = P_f/WR = 7.9 \text{ in}^2$

TRY  $R_o = 1.50 R_i$ ;  $R_m = (R_o + R_i)/2$

$$2R_m = 1.5R_i + R_i = 2.5R_i$$

$$R_i = R_m/1.25 = 1.50 \text{ in}/1.25 = 1.20 \text{ IN}; R_o = 1.5(1.2) = 1.80 \text{ in}$$

$$A = \pi(R_o^2 - R_i^2) = 5.65 \text{ IN}^2 \text{ (LOW)}$$

TRY  $R_o = 2.0 R_i$ ;  $2R_m = 2.0R_i + R_i = 3.0R_i$ ;  $R_i = R_m/1.5 = 1.0 \text{ in}$

$$\underline{R_o = 2.0(1.0) = 2.0 \text{ in}}; A = \pi(2.0^2 - 1.0^2) = 9.42 \text{ in}^2 \text{ OK}$$

15. FROM EQ. 22-13:

$$F_a = \frac{T_f (\sin \alpha + f \cos \alpha)}{f R_m} = \frac{15 \text{ lb}\cdot\text{ft} (\sin 12^\circ + 0.25 \cos 12^\circ)}{(0.25)(3.0 \text{ in})} \frac{12 \text{ in}}{\text{ft}}$$

$$F_a = 109 \text{ lb}$$

16.  $T_f = 64 \text{ lb}\cdot\text{in}$ ; Let  $f = 0.25$ ;  $R_m = 2.0 \text{ in}$ ;  $\alpha = 12^\circ$

$$F_a = \frac{(64)(\sin 12^\circ + 0.25 \cos 12^\circ)}{(0.25)(1.0)} = 116 \text{ lb}$$

17.  $T_f = 150 \text{ lb}\cdot\text{ft} \times 12 \text{ in}/\text{ft} = 1800 \text{ lb}\cdot\text{in} = F_f D_o/2$

$$F_f = 2T_f/D_o = 2(1800)/12.0 = 300 \text{ lb}$$

$$W = \frac{F_f \left( \frac{a}{f} - b \right)}{L} = \frac{(300 \text{ lb}) \left( \frac{4.0}{0.25} - 5.0 \right) \text{ in}}{24.0 \text{ in}} = 135 \text{ lb}$$

18. FOR SELF ACTIVATION;  $W < 0$ .

FOR  $W = 0$ ,  $a/f - b = 0$  OR  $b = a/f = 4.0 \text{ in}/0.25 = 16.0 \text{ IN}$

$b > 16.0 \text{ IN}$  FOR SELF ACTIVATION

19.

$$T_f = 100 \text{ LB} \cdot \text{FT} \times 12 \text{ IN/FT} = 1200 \text{ LB} \cdot \text{IN}$$

TRY  $D_o = 10.0 \text{ IN}$ ;  $f = 0.25$

$$F_f = \frac{2T_f}{D_o} = \frac{2(1200)}{10} = 240 \text{ LB}$$

IN FIG 22-17 (C); LET  $a = 2.0 \text{ IN}$ ;  $b = 6.00 \text{ IN}$ ;  $f = 0.25$ ;  $L = 18 \text{ IN}$

$$W = \frac{F_f \left( \frac{a}{f} - b \right)}{L} = \frac{240 \left( \frac{2.0}{0.25} - 6.0 \right)}{18} = 26.7 \text{ LB}$$

20.

$$T_f = 100 \text{ LB} \cdot \text{FT} \times 12 \text{ IN/FT} = 1200 \text{ LB} \cdot \text{IN}$$

SELECT WOVEN ASBESTOS;  $f = 0.25$ ;  $p \approx 30 \text{ psi}$

IN FIG 22-19:  $r = 6.0 \text{ IN}$ ;  $C = 9.00 \text{ IN}$ ;  $L = 20.0 \text{ IN}$

$$\theta_1 = 45^\circ; \theta_2 = 135^\circ$$

FROM EQ. 22-18:

$$w = \frac{T_f}{r^2 f p (\cos \theta_1 - \cos \theta_2)} = \frac{1200 \text{ LB} \cdot \text{IN}}{(6.0)^2 (0.25) (30) (\cos 45^\circ - \cos 135^\circ)} = 3.25 \text{ IN}$$

$$\text{USE } w = 3.25 \text{ IN}; P = 30 \text{ psi} \left( \frac{3.14}{3.25} \right) = 29.0 \text{ psi}$$

$$\theta_2 - \theta_1 = 135^\circ - 45^\circ = 90^\circ \times \frac{\pi}{180} = \frac{\pi}{2} = 1.57 \text{ rad.}$$

(EQ. 22-20)

$$M_N = 0.25 (29.0) (3.25) (6.0) (9.0) [2(1.57) - \sin 270^\circ + \sin 90^\circ]$$

$$M_N = 6540 \text{ LL} \cdot \text{IN}$$

(EQ. 22-21)

$$M_F = -(0.25) (29.0) (3.25) (6.0) [6.0 (\cos 45^\circ - \cos 135^\circ) + (2.5) (9.0) (\cos 270^\circ - \cos 90^\circ)]$$

$$M_F = -1200 \text{ LL} \cdot \text{IN}$$

(EQ. 22-19)

$$W = (M_N - M_F) / L = (6540 - 1200) / 20 = 267 \text{ LB} = \text{ACTIVATION FORCE}$$

CHECK WEAR RATIO

$$P_f = T_f M / 63000 = 1200 (480) / 63000 = 9.14 \text{ hp}$$

(EQ. 22-23)

$$A = 2wr \sin \left( \frac{\theta_2 - \theta_1}{2} \right) = 2(3.25)(6.0) \sin 45^\circ = 27.6 \text{ in}^2$$

$$WR = \frac{P_f}{A} = \frac{9.14 \text{ hp}}{27.6 \text{ in}^2} = 0.33 \text{ hp/in}^2 \text{ SOMEWHAT HIGH - INTERMITTENT SERVICE ONLY.}$$

21. BAND BRAKE:  $T_f = 75 \text{ LB}\cdot\text{FT} (12 \text{ in}/\text{FT}) = 900 \text{ LB}\cdot\text{IN}$ ;  $n = 350 \text{ RPM}$   
 USE WOVEN ASBESTOS;  $p_{\text{MAX}} = 25.0 \text{ psi}$ ;  $f = 0.25$   
 TRY  $r = 6.0 \text{ IN}$ ;  $\theta = 210^\circ (3.67 \text{ RAD})$ ;  $w = 2.5 \text{ IN}$

$$P_1 = p_{\text{MAX}} r w = (25.0)(6.0)(2.5) = 375 \text{ LB}$$

$$P_2 = \frac{P_1}{e^{f\theta}} = \frac{375 \text{ LB}}{e^{(0.25)(3.67)}} = 150 \text{ LB}$$

$$T_f = (P_1 - P_2)r = (375 - 150)(6.0) = 1350 \text{ LB}\cdot\text{IN} \text{ (HIGH)}$$

TRY  $r = 5.50 \text{ IN}$ ;  $\theta = 210^\circ$ ;  $w = 2.0 \text{ IN}$

$$P_1 = (25.0)(5.50)(2.0) = 275 \text{ LB}$$

$$P_2 = \frac{275}{e^{(0.25)(3.67)}} = 110 \text{ LB}$$

$$T_f = (275 - 110)(5.5) = 908 \text{ LB}\cdot\text{IN} \text{ OK}$$

FOR SIMPLE BAND BRAKE: LET  $a = 5.50 \text{ IN}$ ;  $L = 12.0 \text{ IN}$

$$w = P_2 (a/L) = 110(5.5/12) = 50.4 \text{ LB}$$

WEAR RATIO:

$$A = 2\pi r w \frac{\theta}{360} = 2\pi(5.5)(2.0) \frac{210}{360} = 40.3 \text{ IN}^2$$

$$P_f = \frac{T_f n}{63000} = \frac{(908)(350)}{63000} = 5.04 \text{ HP}$$

$$\text{WR} = P_f/A = 5.04/40.3 = 0.125 \text{ HP}/\text{IN}^2 \text{ OK}$$