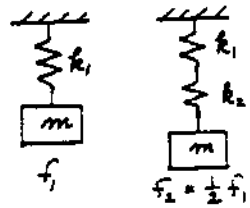
2-1

From Eq. 2.2-9

$$f = \frac{15.76}{\sqrt{\Delta_{mm}}} = \frac{15.76}{\sqrt{7.87}} = 5.62 \quad Hz$$

2-2

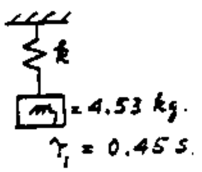


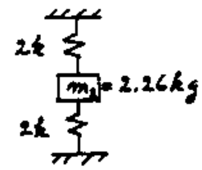
$$f_1 = \sqrt{\frac{k_1}{m}} \qquad f_2 = \sqrt{\frac{k_1 k_2}{m(k_1+k_2)}}$$

$$\frac{1}{2}\sqrt{\frac{k_1}{m}} = \sqrt{\frac{k_1 k_2}{m(k_1+k_2)}}$$

$$\frac{1}{4}\cancel{k_1} = \frac{\cancel{k_1} k_2}{k_1+k_2} \qquad \therefore \quad k_2 = \frac{1}{3} k_1$$

2-3





$$k = \left(\frac{2\pi}{\tau_1}\right)^2 m_1 = \left(\frac{2\pi}{.45}\right)^2 4.53$$

$$= 883.5 \quad N/m$$

$$\tau_2 = 2\pi\sqrt{\frac{m_2}{4k}} = 2\pi\sqrt{\frac{2.26}{4 \times 883.}}$$

$$= 0.159 \quad s$$

2 - 4

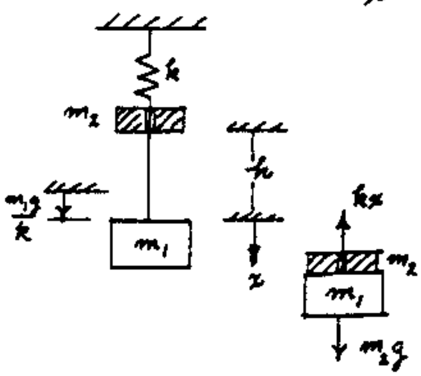
$$\frac{k}{m} = (2\pi f)^2 = \left(2\pi \frac{94}{60}\right)^2 \qquad \frac{k}{m+.453} = \left(2\pi \frac{76.7}{60}\right)^2$$

$$\frac{m+.453}{m} = \left(\frac{94}{76.7}\right)^2 \qquad \therefore \; m = 0.9028 \text{ kg}$$

$$k = 87.48 \text{ N/m}$$

---

2 - 5



x measured from static equilibrium position of m_1 k

Eq. of motion after impact

$$(m_1+m_2)\ddot{x} = -kx + m_2 g$$

Gen solution:

$$x(t) = \frac{m_2 g}{k} + A\sin\omega t + B\cos\omega t$$

Initial conditions

$$x(0) = 0 = \frac{m_2 g}{k} + B \quad \therefore B = -\frac{m_2 g}{k}$$

$$\dot{x}(0) = \frac{m_2\sqrt{2gh}}{m_1+m_2} = \omega A \qquad \therefore \; A = \frac{m_2\sqrt{2gh}}{(m_1+m_2)\omega}$$

$$\omega = \sqrt{\frac{k}{m_1+m_2}}$$

$$\therefore x(t) = \frac{m_2 g}{k} + \frac{m_2\sqrt{2gh}}{m_1+m_2}\sqrt{\frac{m_1+m_2}{k}}\sin\omega t - \frac{m_2 g}{k}\cos\omega t$$

$$= \frac{m_2 g}{k}(1-\cos\omega t) + \frac{m_2\sqrt{2gh}}{\sqrt{k(m_1+m_2)}}\sin\omega t$$

---

2 - 6

$$\omega_n^2 = \frac{k}{m} = 4.0 \qquad \omega_n = 2.0$$

$$x = x_0\cos\omega t + \frac{v_0}{\omega}\sin\omega t = 2\cos 2t - \frac{8}{2}\sin 2t$$

$$\dot{x} = -4\sin 2t - 8\cos 2t = 0 \qquad \therefore \tan 2t_p = -2$$

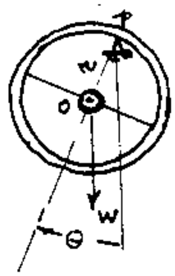
$$\therefore 2t_p = 116.57^\circ \qquad \sin 116.57^\circ = .8944 \qquad \cos 116.57 = -.4472$$

$$x_{max} = 2(-.4472) - 4(.8944) = -4.472 \text{ cm}$$

$$\ddot{x}_{max} = \omega^2 x_{max} = 4(\pm 4.472) = \pm 17.89 \text{ cm/s}^2$$

2-7

$$J_p \ddot{\theta} = -W r \theta \qquad \ddot{\theta} = -\omega^2 \theta$$



$$J_p = \frac{Wr}{\omega^2} = \frac{70 \times 6}{\left(\frac{2\pi}{1.22}\right)^2} = 15.83$$

$$J_o = J_p - \frac{W}{g} r^2 = 15.83 - \frac{70}{386} \times 6^2$$

$$= 9.30 \ \text{lb in sec}^2$$

2-8

$$\omega = 2\pi \ \frac{53}{60} = 5.55 \ r/s$$

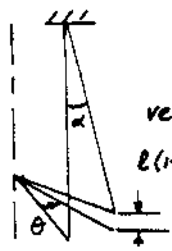
$$J_p = \frac{Wr}{\omega^2} = \frac{21.35 \times .254}{5.55^2} = 0.1761$$

$$J_{cg} = J_o - \frac{Wr^2}{g} = 0.1761 - \frac{21.35 \times .254^2}{9.81} = 0.0356 \ kg\, m^2$$

2-9

$$r\theta = l\alpha$$

Work done = change in KE



$$\alpha = \frac{r\theta}{l}$$

$$W \frac{l}{2} \frac{r^2}{l^2} \theta^2_{max} = \frac{1}{2} J \dot{\theta}^2_{max} = \frac{1}{2} J \omega^2 \theta^2_{max}$$

vertical displ =

$$l(1-\cos\alpha) = l\left[1-\left(1-\tfrac{1}{2}\alpha^2+\dots\right)\right]$$

$$= \frac{l\alpha^2}{2} = \frac{l}{2}\left(\frac{r\theta}{l}\right)^2$$

$$J = \frac{W}{g} k^2 \qquad k = \text{rad. of gyr.}$$

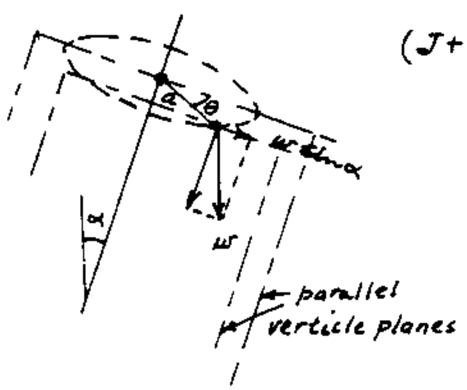
$$\frac{W r^2}{l} = \frac{W}{g} k^2 \omega^2$$

$$k^2 = \frac{r}{\omega}\sqrt{\frac{g}{l}} = \frac{.254 \times 2.17}{2\pi}\sqrt{\frac{9.81}{1.829}} = .2032$$

$$k = .4507 \ m$$

2-10

Moment about shaft $= (a \sin\theta)\, w \sin\alpha$



$$\left(J + \frac{w}{g} a^2\right)\ddot{\theta} = -(a \sin\theta)\, w \sin\alpha$$

$$\cong -(a w \sin\alpha)\,\theta$$

$$\therefore f_n = \frac{1}{2\pi}\sqrt{\frac{w a \sin\alpha}{J + \frac{w}{g} a^2}}$$

2-11

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J_0 \dot{\theta}^2 \qquad r\dot{\theta} = \dot{x}$$

$$= \frac{1}{2}\left(m + \frac{J_0}{r^2}\right)\dot{x}^2 \qquad \dot{x} = \omega x$$

$$U = \frac{1}{2} k x^2 \qquad \therefore\ \omega = \sqrt{\frac{k}{m + J_0/r^2}}$$

2-12

$$\tau = 2\pi\sqrt{\frac{L}{g}} \qquad L = g\left(\frac{\tau}{2\pi}\right)^2 = 9.81\left(\frac{2}{2\pi}\right)^2 = 0.994\ m$$

$$v_{max} = L(\omega\theta_0) = \frac{.003175}{.01}\ m/s \qquad \theta_0 = \frac{.3175}{.994\ \pi} = .1017\ r = 5.826^\circ$$

2-13 water weighs 9802. N/m^3 $\quad \therefore\ \rho = 1.2 \times 9802. = 11762.\ N/m^3$

$$\text{buoyant force} = \pi r^2 x \cdot \rho = m\ddot{x} = \omega^2 x$$

$$\frac{1}{\omega} = \frac{\tau}{2\pi} = \sqrt{\frac{m}{\pi r^2 \rho}} \qquad m = .0372\ kg \quad r = .0032\ m$$

$$\tau = 2\pi\sqrt{\frac{.0372}{\pi \times .0032^2 \times 11762}} = 1.97\ s$$

2-14 moment about geom. center

$$-W\,8\,\theta = J_0\ddot{\theta} = -\omega^2 J_0 \theta$$

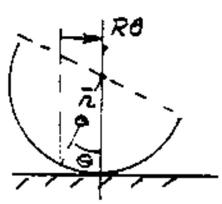
$$J_0 = \frac{8W}{\omega^2} = \frac{8W\,(1.3)^2}{(2\pi)^2} = 0.3428\ W$$

2-15

$$-W h \theta = J\ddot{\theta} = -\omega^2 J \theta$$

$$\frac{1}{\omega} = \frac{\tau}{2\pi} = \sqrt{\frac{J}{Wh}} \qquad \tau = 2\pi\sqrt{\frac{J}{Wh}}$$

2-16 Displ. of cg $= (R-\bar{r})\theta$ $\qquad T_{max} = U_{max}$



$$T_{max} = \frac{1}{2} m (R-\bar{r})^2 \dot{\theta}_{max}^2 + \frac{1}{2} J_{cg}\,\dot{\theta}_{max}^2$$

$$= \frac{1}{2} m\left[(R-\bar{r})^2 + (R^2 - \bar{r}^2)\right]\omega^2\theta_{max}^2$$

$$U_{max} = m g \bar{r}\,(1 - \cos\theta_{max}) \cong m g \bar{r}\,\frac{\theta_{max}^2}{2}$$

2-16 cont $\qquad T_{max} = U_{max}$

$$\omega^2 = \frac{\bar{r} g}{(R-\bar{r})^2 + (R^2 - \bar{r}^2)} = \frac{\bar{r} g}{2R(R-\bar{r})}$$

$$\tau = 2\pi \sqrt{\frac{2R(R-\bar{r})}{\bar{r} g}} \qquad \text{but } \bar{r} = \frac{2R}{\pi} \qquad \therefore \ \tau = 2\pi \sqrt{\frac{R(\pi - 2)}{g}}$$

2-17 $\qquad U = mgh(1-\cos\phi) \cong mgh \tfrac{1}{2}\phi^2 \qquad h\phi = \frac{a}{2}\theta$

$$= mg\frac{h}{2}\left(\frac{a\theta}{2h}\right)^2 = mg\frac{a^2}{8}\frac{\theta^2}{h}$$

$$T = \frac{1}{2}\left(m\frac{L^2}{12}\right)\dot{\theta}^2 = \frac{1}{2}\left(m\frac{L^2}{12}\right)\omega^2\theta^2$$

$$T_{max} = U_{max} \qquad \therefore \ \tau = 2\pi\frac{L}{a}\sqrt{\frac{h}{3g}}$$

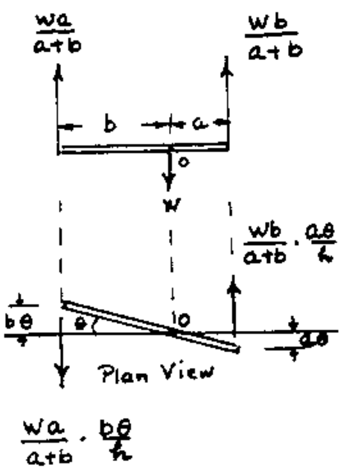
2-18 $\qquad \tau_1 = 2\pi\sqrt{\frac{h}{g}} \qquad$ for $\tau_2 \qquad T = \frac{1}{2} m k^2 \omega^2 \theta^2$

$$U = mg\frac{L^2}{8}\frac{\theta^2}{h}$$

$$\therefore \ \tau_2 = 2\pi\sqrt{\frac{4hk^2}{gL^2}} \qquad k = \frac{\tau_2 L}{2\pi}\sqrt{\frac{g}{4h}} = \frac{\tau_2}{\tau_1}\left(\frac{L}{2}\right)$$

2-19

$$\sum M_o = J_{cg.}\ddot{\theta} = -\frac{Wba}{a+b}\frac{\theta}{h}.a - \frac{Wab}{a+b}\frac{\theta}{h}.b$$



$$\frac{W}{g}k^2\ddot{\theta} + \frac{Wab}{h}\theta = 0$$

$$\ddot{\theta} + \left(\frac{g\,ab}{k^2 h}\right)\theta = 0$$

$$\therefore \ f = \frac{1}{2\pi}\sqrt{\frac{g\,ab}{k^2 h}}$$

$$\therefore \sum F_{horiz} = 0$$

2 - 20

J_o of wheel about torsion bar $= J_{cm} + m(24")^2$

$= m(k^2 + 24^2) = m(9^2 + 24^2) = 657\ m$

Stiffness of torsion bar $K = \frac{GI_p}{\ell}$

$I_p = \frac{\pi D^4}{32} = \frac{\pi}{32}(1.50)^4 = 0.497\ in^4$ = polar mom. inertia of torsion bar

$G = 11.2 \times 10^6\ lb/in^2$ = shear modulus of steel

$K = \frac{(11.2\times10^6)\times(0.497)}{50} = 0.1113 \times 10^6\ lb.in./rad.$

$J_o\ddot{\theta} + K\theta = 0 \qquad f = \frac{1}{2\pi}\sqrt{\frac{K}{J_o}} = \frac{1}{2\pi}\sqrt{\frac{0.1113\times10^6\times386}{38\times657}} = 6.60$ cps locked wheel.

With wheel free $J_o = m(24)^2 = 576\ m$

$f = \frac{1}{2\pi}\sqrt{\frac{0.1113\times10^6\times386}{38\times576}} = 7.05$ cps.

---

2 - 21

$$\frac{\ell\rho}{g}\ddot{x} = -2x\rho \qquad \ddot{x} + \frac{2g}{\ell}x = 0$$

$$\omega^2 = \frac{2g}{\ell} \qquad \tau = 2\pi\sqrt{\frac{\ell}{2g}}$$

---

2 - 22

$$k = 2\left(\frac{12EI}{\ell^3}\right) \qquad \tau = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m\ell^3}{24EI}}$$

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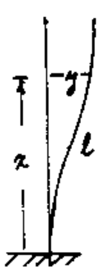
2 - 23



$y = \frac{1}{2}y_{max}(1-\cos\frac{\pi x}{\ell})\sin\omega t$

$\dot{y} = \frac{1}{2}\omega y_{max}(1-\cos\frac{\pi x}{\ell})\cos\omega t$

$T = \frac{1}{2}\int_0^\ell m(x)\dot{y}^2dx = \frac{1}{2}\frac{m}{4}y_{max}^2\int_0^\ell(1-\cos\frac{\pi x}{\ell})^2\omega^2 dx\cos^2\omega t$

$= \frac{1}{2}\cdot\frac{m}{4}y_{max}^2\omega^2\cos^2\omega t\int_0^\ell\left(1-2\cos\frac{\pi x}{\ell}+\cos^2\frac{\pi x}{\ell}\right)dx$

$= \quad " \quad \left[x - \frac{2\ell}{\pi}\sin\frac{\pi x}{\ell} + \frac{x}{2} + \frac{1}{2}\cdot\frac{\ell}{2\pi}\sin\frac{2\pi x}{\ell}\right]_0^\ell$

2-23 Cont.

$$T = \quad " \quad \left[\tfrac{3}{2}\ell - 0 + 0\right]$$

$$= \frac{1}{2}\left(\frac{m}{4}\cdot\frac{3\ell}{2}\right)\omega^2 y_{max}^2 \cos^2\omega t$$

$\therefore m_{eff} = \left(\frac{3}{8}\, m\ell\right)$ for each column, where $m\ell$ = total mass of each column

---

2-24

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J\left(\frac{\dot{x}}{b}\right)^2$$

where J = moment of inertia of Linkage about pivot.

$$x = \frac{b}{a} x_m$$

$$T = \frac{1}{2} m\left(\frac{b}{a}\right)^2 \dot{x}_m^2 + \frac{1}{2} J\left(\frac{b}{a}\right)^2 \frac{1}{b^2}\dot{x}_m^2$$

$$= \frac{1}{2}\left[m\left(\frac{b}{a}\right)^2 + \frac{J}{a^2}\right]\dot{x}_m^2 \qquad \therefore m_{eff} = \left[m\left(\frac{b}{a}\right)^2 + \frac{J}{a^2}\right]$$

2-25

$$T = \frac{1}{2}\left[J_0\dot{\theta}^2 + m_1(b\dot{\theta})^2\right] = \frac{1}{2}\left[J_0 + m_1 b^2\right]\dot{\theta}^2$$

$$\dot{\theta} = \dot{x}/b \qquad T = \frac{1}{2}\left[J_0/b^2 + m_1\right]\dot{x}^2 \qquad m_{eff} = J_0/b^2 + m_1$$

2-26 The kinetic energy is

$$T = \frac{1}{2}J\dot{\theta}^2 + \frac{1}{2}m_w(b\dot{\theta})^2 + \frac{1}{2}\left(\frac{m_s}{3}\right)(b\dot{\theta})^2 = \frac{1}{2}\left(J + m_w b^2 + \frac{1}{3}m_s b^2\right)\dot{\theta}^2$$

With the velocity at A equal to $\dot{x} = a\dot{\theta}$, T becomes

$$T = \frac{1}{2}\left(\frac{J + m_w b^2 + \frac{1}{3}m_s b^2}{a^2}\right)\dot{x}^2$$ and the effective mass at A is

$$m_A = \left(\frac{J + m_w b^2 + \frac{1}{3}m_s b^2}{a^2}\right)$$

2-27 $$y = \frac{1}{2} y_0\left[3\left(\frac{x}{\ell}\right)^2 - \left(\frac{x}{\ell}\right)^3\right]$$

$$T = \frac{1}{2} m\int_0^\ell \dot{y}^2 dx = \frac{1}{2}\frac{m}{4}\dot{y}_0^2\int_0^\ell\left[9\left(\frac{x}{\ell}\right)^4 - 6\left(\frac{x}{\ell}\right)^5 + \left(\frac{x}{\ell}\right)^6\right]dx$$

$$= \frac{1}{2} m\dot{y}_0^2\frac{\ell}{4}\left[\frac{9}{5} - 1 + \frac{1}{7}\right] = \frac{1}{2}\left(\frac{33}{140}\, m\ell\right)\dot{y}_0^2$$

<u>2-28</u> $y(x) = \frac{w\ell}{24}\frac{\ell^3}{EI}\left[\left(\frac{x}{\ell}\right)^4 - 4\left(\frac{x}{\ell}\right) + 3\right]$ x measured from free end.

$$y_0 = y_{max} = \frac{w\ell}{8}\frac{\ell^3}{EI} \quad \therefore \quad y(x) = \frac{1}{3}\, y_{max}\left[\left(\frac{x}{\ell}\right)^4 - 4\left(\frac{x}{\ell}\right) + 3\right]$$

$$T = \frac{1}{2}\, m \int_0^{\ell} \dot{y}^2(x)\, dx = \frac{1}{2}\, m\, \frac{1}{9}\, \dot{y}_{max}^2 \int_0^{\ell}\left[\left(\frac{x}{\ell}\right)^4 - 4\left(\frac{x}{\ell}\right) + 3\right]^2 dx$$

$$\int_0^{\ell}\left[\left(\frac{x}{\ell}\right)^4 - 4\left(\frac{x}{\ell}\right) + 3\right]^2 dx = \int_0^{\ell}\left[\left(\frac{x}{\ell}\right)^8 - 8\left(\frac{x}{\ell}\right)^5 + 6\left(\frac{x}{\ell}\right)^4 + 16\left(\frac{x}{\ell}\right)^2 - 24\left(\frac{x}{\ell}\right) + 9\right] dx$$

$$= \ell\left[\frac{1}{9}\left(\frac{x}{\ell}\right)^9 - \frac{8}{6}\left(\frac{x}{\ell}\right)^6 + \frac{6}{5}\left(\frac{x}{\ell}\right)^5 + \frac{16}{3}\left(\frac{x}{\ell}\right)^3 - \frac{24}{2}\left(\frac{x}{\ell}\right)^2 + 9\left(\frac{x}{\ell}\right)\right]_0^{\ell} = \frac{624}{270}\,\ell$$

$$\therefore \quad T = \frac{1}{2}\, m\, \frac{1}{9}\, \dot{y}_{max}^2\left(\frac{624}{270}\right)\ell = \frac{1}{2}\left(.2568\, m\ell\right)\dot{y}_{max}^2$$

Compare with Prob. 2-27

$$T = \frac{1}{2}\left(\frac{33}{140}\right) m\ell\, \dot{y}_0^2 = \frac{1}{2}\left(.2357\, m\ell\right)\dot{y}_{max}^2$$

Effective mass for the two problems are nearly equal

<u>2-29</u> Let θ_0 = rotation of J, T_0 = total torque

T_L = torque to left of J

T_R = " " right " "

$$\left(\frac{1}{K_1} + \frac{1}{K_2}\right) T_L = \theta_0 \qquad \left(\frac{1}{K_2}\right) T_R = \theta_0$$

2-29 Cont.

$$T_o = T_L + T_R = \left[\frac{1}{\left(\frac{1}{K_1}+\frac{1}{K_2}\right)} + \frac{1}{\left(\frac{1}{K_2}\right)}\right]\theta_o = K\,\theta_o$$

$$\therefore K = \left(\frac{K_1 K_2}{K_1+K_2} + K_2\right) \qquad \omega_n = \sqrt{\frac{K}{J}} = \frac{2\pi}{\tau}$$

2-30

$$T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} J_2 \left(\frac{\dot{x}}{R_2}\right)^2 + \frac{1}{2} J_3 \left(\frac{R_2}{R_1}\frac{\dot{x}}{R_2}\right)^2$$

$$= \frac{1}{2}\left[m_1 + J_2/R_2^2 + J_3/R_1^2\right]\dot{x}^2 = \frac{1}{2} m_{eff}\,\dot{x}^2$$

$$U = \frac{1}{2} k_1 x^2 + \frac{1}{2} K_3\left(\frac{x}{R_1}\right)^2 = \frac{1}{2}\left[k_1 + K_3/R_1^2\right]x^2 = \frac{1}{2} k_{eff}\, x^2$$

2-31

$$T = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} J_2 \left(\frac{n_1}{n_2}\dot{\theta}_1\right)^2$$

$$= \frac{1}{2}\left[J_1 + J_2\left(\frac{n_1}{n_2}\right)^2\right]\dot{\theta}_1^2 = \frac{1}{2} J_{eff}\,\dot{\theta}_1^2$$

2-32

$$T = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2}(m_0+m_2)\dot{x}^2 + \frac{1}{2} J_2 \dot{\theta}_2^2$$

$$\dot{\theta}_1 = \dot{x}/n_1 \qquad \dot{\theta}_2 = \dot{x}/n_2$$

$$T = \frac{1}{2}\left[J_1/n_1^2 + (m_0+m_2) + J_2/n_2^2\right]\dot{x}^2$$

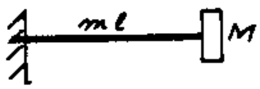
$$U = \frac{1}{2} k_1 (R\theta_1)^2 + \frac{1}{2} k_2 (n_2\theta_2)^2$$

$$= \frac{1}{2} k_1 \left(\frac{R}{n_1}x\right)^2 + \frac{1}{2} k_2 \left(n_2 \frac{x}{n_2}\right)^2$$

$$= \frac{1}{2}\left[k_1\left(\frac{R}{n_1}\right)^2 + k_2\right]x^2$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1\left(\frac{R}{n_1}\right)^2 + k_2}{J_1/n_1^2 + (m_0+m_2) + J_2/n_2^2}}$$

2 − 33 $\quad m_{eff} = M + \frac{33}{140} m\ell \quad$ (see Prob. 2-27)



beam vol $= (.1016 \times .635 \times 8.89) = .5735 \ cm^3$

wt. of steel $= 0.07655 \ N/cm^3$

wt. of beam $= .5735 \times .07655 = .04390 \ N$

mass of beam $= \dfrac{.04390}{9.81} = .00475 \ kg = m\ell$

$\frac{33}{140} m\ell = .001055 \qquad$ beam stiffness $= \dfrac{3EI}{\ell^3} = k$

$E = 200 \times 10^9 \ N/m^2 \qquad I = \dfrac{bh^3}{12} = \dfrac{.635 \times .1016^3}{12} = .0000555 \times 10^{-8} \ m^4$

$$k = \frac{3 \times 200 \times 10^9 \times 555 \times 10^{-15}}{(.0889)^3} = 473.96 \ N/m$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_{eff}}} \qquad m_{eff} = M + .001055 = \frac{3EI}{\ell^3 4\pi^2 f^2} = \frac{473.96}{4\pi^2 \times 400}$$

$\therefore M = 0.0289 \ kg$

---

2 − 34 $\quad c_c = 2\sqrt{mk} = 2\sqrt{.907 \times 7 \times 10^2} = 50.4 \ \frac{Ns}{m}$

---

2 − 35 $\quad F_d = cv \qquad c = \dfrac{F_d}{v} = \dfrac{0.50}{1.20} = 0.417 \ \frac{lb\ sec}{in}$

$$c = 0.417 \ \frac{lb.s}{in} \times 4.448 \frac{N}{lb} \times \frac{1}{2.54} \frac{in}{cm} = 0.7303 \ \frac{NS}{cm} = 73.03 \frac{NS}{m}$$

$$\zeta = \frac{c}{c_c} = \frac{73.03}{50.4} = 1.45$$

---

2 − 36

(a) $\zeta = 2. \quad$ Eq 2.6-20, 2.6-21 $\quad A = -B = \dfrac{v_0}{2\omega_n\sqrt{\zeta^2-1}}$

$$\frac{x\omega_n}{v_0} = \frac{1}{3.464}\left(e^{-0.268\omega_n t} - e^{-3.732\,\omega_n t}\right)$$

(b) $\zeta = 0.50 \qquad \dfrac{x\omega_n}{v_0} = \dfrac{e^{-0.50\omega_n t}}{0.865} \sin 0.865\omega_n t \quad$ Eq 2.6-17

(c) $\zeta = 1.0 \qquad \dfrac{x\omega_n}{v_0} = \omega_n t \ e^{-\omega_n t}$

Eq 2.6-23

2 – 37 Solve on computer using RUNGA. With ω_n not given, the equation

$$\frac{d^2x}{dt^2} + 2\zeta\frac{dx}{dt} + \omega_n^2 x = 0$$

can be rewritten for RUNGA as follows

Let $T = \omega_n t$ and $y = \frac{x\omega_n}{v_0}$

Then $x = \frac{v_0}{\omega_n} y$, $\frac{dx}{dt} = \frac{v_0}{\omega_n}\frac{dy}{dt} = v_0 \frac{dy}{dT}$

$$\frac{d^2x}{dt^2} = \frac{v_0}{\omega_n}\frac{d^2y}{dt^2} = \frac{v_0}{\omega_n}\frac{\omega_n^2}{\omega_n^2}\frac{d^2y}{dt^2} = v_0\omega_n\frac{d^2y}{dT^2}$$

The original equation then becomes

$$v_0\omega_n\left[\frac{d^2y}{dT^2} + 2\zeta\frac{dy}{dT} + y\right] = 0$$

and $v_0 = 1.0$ for all computations.

Computer solutions for (a), (b) and (c) are:

(a) $\zeta = 2.0$ $y_{max} = \frac{\omega_n x_{max}}{v_0} = 0.1953$ (critical damping)

(b) $\zeta = .50$ $y_{max} = 0.534$ (oscillatory)

(c) $\zeta = 1.0$ $y_{max} = 0.3666$ (aperiodic)

See Prob 2-36 for analytic solution.

» 237 Cont

Prob. 237(a)

```
 Enter the value of the mass (kg) : 1

m =

     1


 Enter the value of the damping coefficient (N.s/m) : 4

c =

     4


 Enter the value of the spring constant (N/m) : 1

k =

     1


 Enter the value of the initial position (m) : 0

x1 =

     0


 Enter the value of the initial velocity (m/s) : 1

y1 =

     1


ans =

Natural period


T =

    6.2832
```

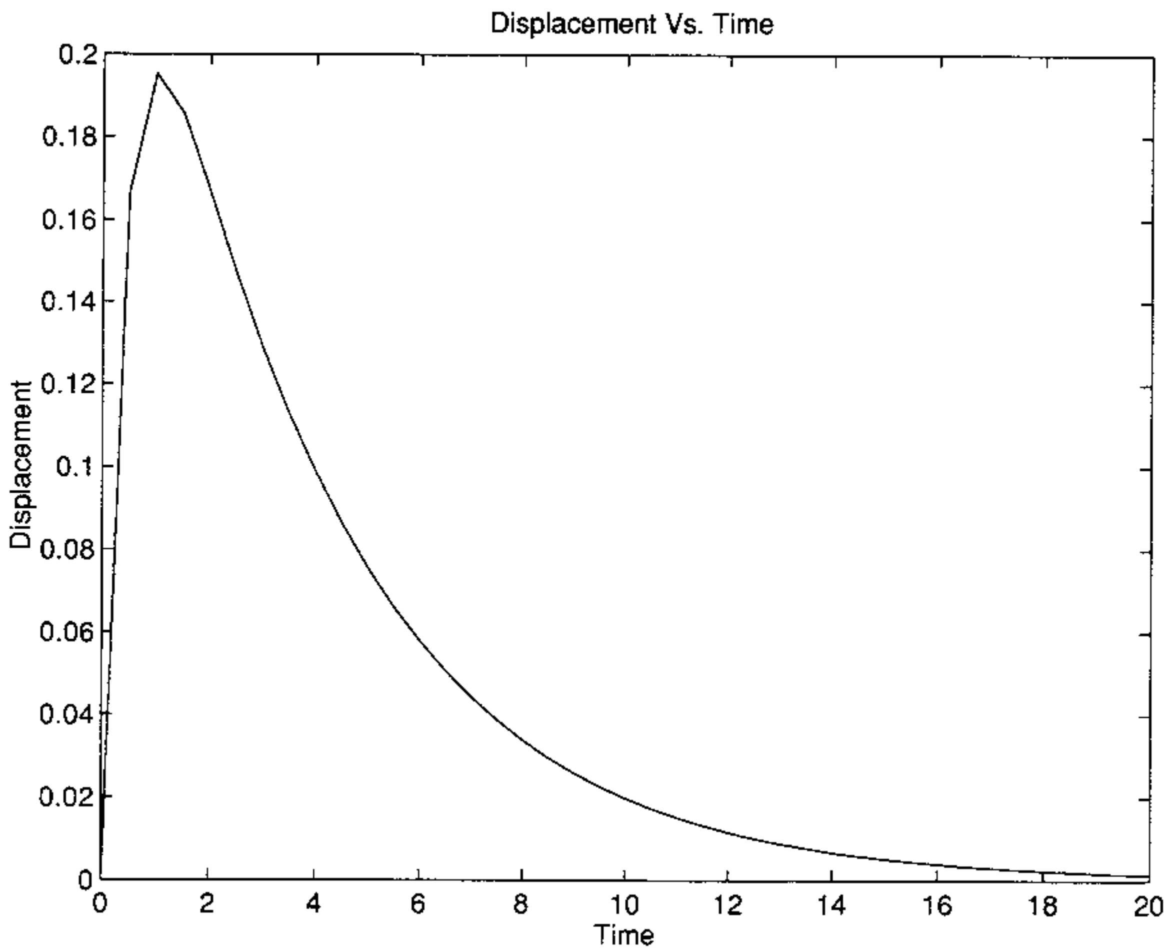
2-37 cont

```
 Enter the value of time increment in seconds ( < T as above ) : .5

dt =

    0.5000


 Enter the value of the initial time in seconds (0) : 0

t1 =

     0


 Enter the value of the final time in seconds : 10

tf =

    10


ans =

     Time           Displ.        Vel.


ans =

         0         0    1.0000
    0.5000    0.1667    0.2526
    1.0000    0.1953    0.0360
    1.5000    0.1856   -0.0235
    2.0000    0.1667   -0.0368
    2.5000    0.1471   -0.0371
    3.0000    0.1290   -0.0339
    3.5000    0.1130   -0.0301
    4.0000    0.0988   -0.0264
    4.5000    0.0864   -0.0231
    5.0000    0.0756   -0.0203
    5.5000    0.0661   -0.0177
    6.0000    0.0578   -0.0155
    6.5000    0.0506   -0.0136
    7.0000    0.0442   -0.0119
```

2-3 f cont

7.5000	0.0387	-0.0104
8.0000	0.0338	-0.0091
8.5000	0.0296	-0.0079
9.0000	0.0259	-0.0069
9.5000	0.0226	-0.0061
10.0000	0.0198	-0.0053

```
ans =

maximum amplitude


A =

    0.1953
```

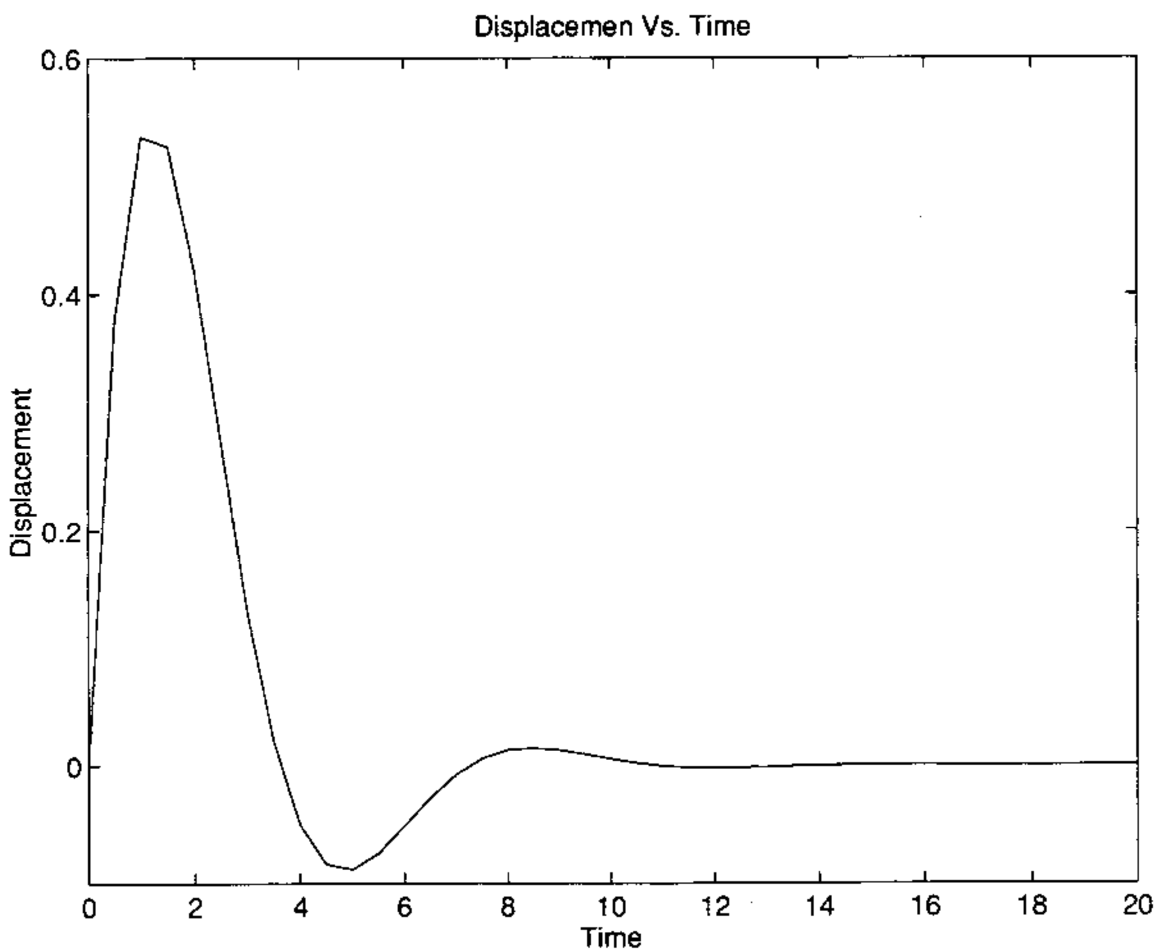


» 2 - 37 Cont   2.37(b)

```
Enter the value of the mass (kg) : 1

m =

     1


Enter the value of the damping coefficient (N.s/m) : 1

c =

     1


Enter the value of the spring constant (N/m) : 1

k =

     1


Enter the value of the initial position (m) : 0

x1 =

     0

Enter the value of the initial velocity (m/s) : 1

y1 =

     1


ans =

Natural period


T =

    6.2832
```

2-37 Cont

```
Enter the value of time increment in seconds ( < T as above ) : .5

dt =

    0.5000


Enter the value of the initial time in seconds (0) : 0

t1 =

     0


Enter the value of the final time in seconds : 20

tf =

    20
```



α -5+ Cont

Time	Displ.	Vel.
0	0	1.0000
0.5000	0.3776	0.5182
1.0000	0.5340	0.1260
1.5000	0.5259	-0.1363
2.0000	0.4196	-0.2692
2.5000	0.2743	-0.2980
3.0000	0.1332	-0.2580
3.5000	0.0219	-0.1840
4.0000	-0.0499	-0.1036
4.5000	-0.0838	-0.0349
5.0000	-0.0882	0.0136
5.5000	-0.0739	0.0404
6.0000	-0.0510	0.0488
6.5000	-0.0272	0.0446
7.0000	-0.0076	0.0334
7.5000	0.0058	0.0202
8.0000	0.0128	0.0082
8.5000	0.0146	-0.0006
9.0000	0.0129	-0.0058
9.5000	0.0093	-0.0079
10.0000	0.0054	-0.0076
10.5000	0.0020	-0.0060
11.0000	-0.0005	-0.0038
11.5000	-0.0019	-0.0018
12.0000	-0.0024	-0.0002
12.5000	-0.0022	0.0008
13.0000	-0.0017	0.0012
13.5000	-0.0010	0.0013
14.0000	-0.0004	0.0011
14.5000	-0.0000	0.0007
15.0000	0.0003	0.0004
15.5000	0.0004	0.0001
16.0000	0.0004	-0.0001
16.5000	0.0003	-0.0002
17.0000	0.0002	-0.0002
17.5000	0.0001	-0.0002
18.0000	0.0000	-0.0001
18.5000	-0.0000	-0.0001
19.0000	-0.0001	-0.0000
19.5000	-0.0001	0.0000
20.0000	-0.0001	0.0000

2-37 cont

```
maximum amplitude

A =

    0.5340

» prob 2.37(a)
 Enter the value of the mass (kg) : 1

m =

     1


 Enter the value of the damping coefficient (N.s/m) : 2

c =

     2


 Enter the value of the spring constant (N/m) : 1

k =

     1


 Enter the value of the initial position (m) : 0

x1 =

     0


 Enter the value of the initial velocity (m/s) : 1

y1 =

     1


ans =
```

2-37 cont

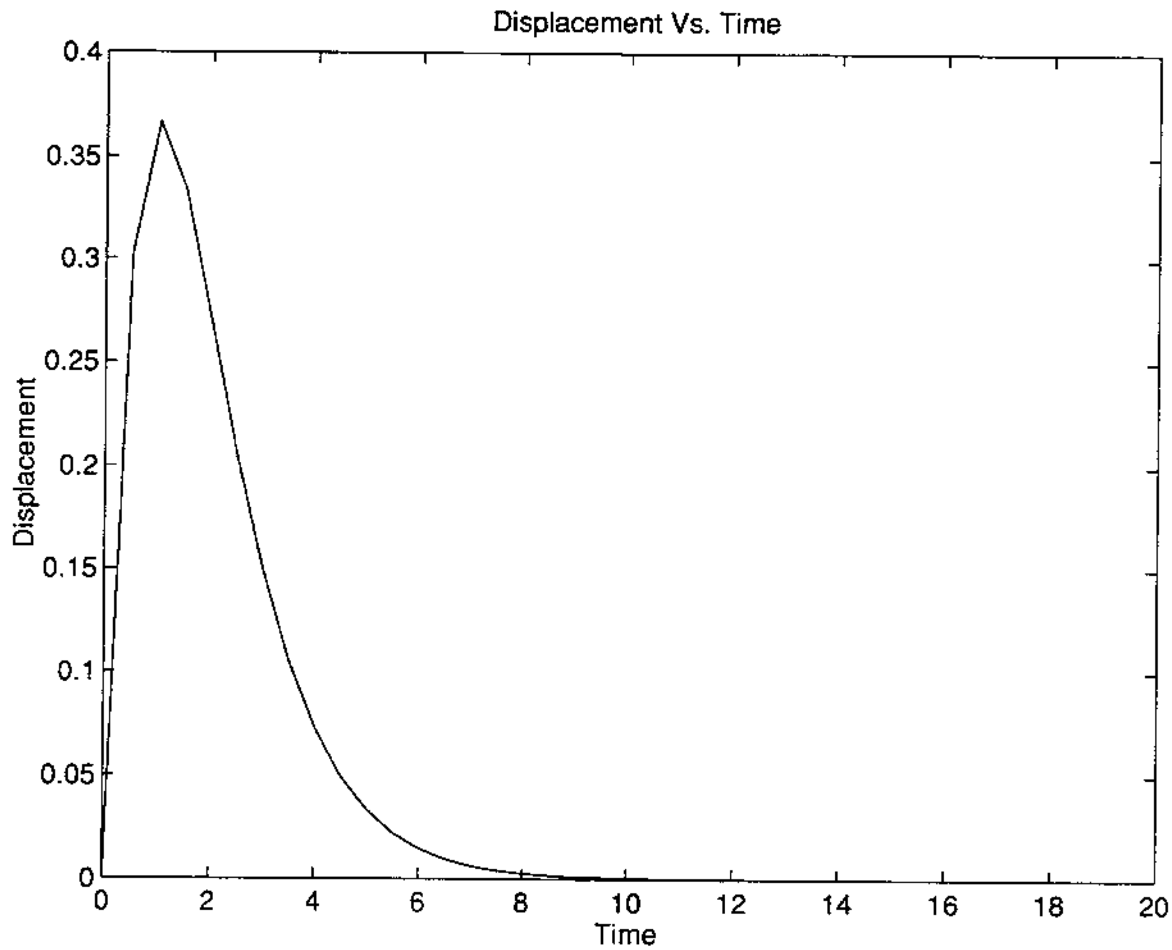
```
Natural period

T =

    6.2832


 Enter the value of time increment in seconds ( < T as above ) : .5

dt =

    0.5000


 Enter the value of the initial time in seconds (0) : 0

t1 =

     0


 Enter the value of the final time in seconds : 20

tf =

    20


ans =

     Time          Displ.        Vel.


ans =

         0         0    1.0000
    0.5000    0.3021    0.3047
    1.0000    0.3666    0.0016
    1.5000    0.3337   -0.1103
    2.0000    0.2699   -0.1344
    2.5000    0.2047   -0.1225
    3.0000    0.1491   -0.0992
    3.5000    0.1055   -0.0752
```

2-37 cont

4.0000	0.0732	-0.0548
4.5000	0.0500	-0.0388
5.0000	0.0337	-0.0269
5.5000	0.0225	-0.0184
6.0000	0.0149	-0.0124
6.5000	0.0098	-0.0083
7.0000	0.0064	-0.0055
7.5000	0.0042	-0.0036
8.0000	0.0027	-0.0024
8.5000	0.0017	-0.0015
9.0000	0.0011	-0.0010
9.5000	0.0007	-0.0006
10.0000	0.0005	-0.0004
10.5000	0.0003	-0.0003
11.0000	0.0002	-0.0002
11.5000	0.0001	-0.0001
12.0000	0.0001	-0.0001
12.5000	0.0000	-0.0000
13.0000	0.0000	-0.0000
13.5000	0.0000	-0.0000
14.0000	0.0000	-0.0000
14.5000	0.0000	-0.0000
15.0000	0.0000	-0.0000
15.5000	0.0000	-0.0000
16.0000	0.0000	-0.0000
16.5000	0.0000	-0.0000
17.0000	0.0000	-0.0000
17.5000	0.0000	-0.0000
18.0000	0.0000	-0.0000
18.5000	0.0000	-0.0000
19.0000	0.0000	-0.0000
19.5000	0.0000	-0.0000
20.0000	0.0000	-0.0000

```
ans =

maximum amplitude


A =

    0.3666

>>
```

2-57 cont


Displacement Vs. Time
Displacement
Time
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
2
4
6
8
10
12
14
16
18
20

2-38

$$\delta = \ln \frac{x_1}{x_2} = \ln \frac{1.00}{.980} = \ln 1.020408 = 0.0202$$

$$\zeta \cong \frac{\delta}{2\pi} = .003215$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1750}{2.267}} = 27.78 \cong \omega_d$$

$$c = 2 m \omega_n \zeta = 2 \times 2.267 \times 27.78 \times .003215 = .405 \ \frac{Ns}{m}$$

2-39

(a) $$\zeta = \frac{c}{2m}\sqrt{\frac{m}{k}} = \frac{12.43}{2 \times 4.534}\sqrt{\frac{4.534}{3500}} = 0.0493$$

(b) $$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = 0.3101$$

(c) $$\frac{x_n}{x_{n+1}} = e^{\delta} = (2.718)^{.3101} = 1.364$$

2-40

(a) $$\zeta = \frac{c}{2\sqrt{mk}} = \frac{70}{2\sqrt{17.5 \times 7000}} = 0.10$$

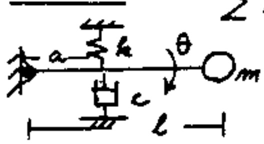
(b) $$f_d = \frac{1}{2\pi}\sqrt{1-\zeta^2}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{1-.01}\sqrt{\frac{7000}{17.5}} = 3.167 \text{ Hz}$$

(c) $$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = .6315$$

(d) $$x_n / x_{n+1} = e^{.6315} = 1.874$$

2-41

$$\sum M_o = -a\,c(a\dot\theta) - a\,k(a\theta) = m\ell^2\ddot\theta$$



$$\ddot\theta + \frac{c}{m}\left(\frac{a}{\ell}\right)^2\dot\theta + \frac{k}{m}\left(\frac{a}{\ell}\right)^2\theta = 0 \qquad \text{let } \theta = e^{st}$$

$$s_{1,2} = -\frac{c}{2m}\left(\frac{a}{\ell}\right)^2 \pm \sqrt{\left(\frac{ca^2}{2m\ell^2}\right)^2 - \frac{k}{m}\left(\frac{a}{\ell}\right)^2}$$

crit. damp. $\frac{c_c a^2}{2m\ell^2} = \frac{a}{\ell}\sqrt{\frac{k}{m}} \qquad c_c = 2\frac{\ell}{a}\sqrt{km}$

$$\omega_d = \frac{a}{\ell}\sqrt{\frac{k}{m} - \left(\frac{ca}{2m\ell}\right)^2} = \frac{a}{\ell}\sqrt{\frac{k}{m}}\sqrt{1 - \left(\frac{ca}{2\ell\sqrt{km}}\right)^2} = \omega_n\sqrt{1-\zeta^2}$$

$\therefore \ \omega_n = \frac{a}{\ell}\sqrt{\frac{k}{m}}, \quad \zeta = \frac{ca}{2\ell\sqrt{km}}$ identify from $\ddot\theta + 2\zeta\omega_n\dot\theta + \omega_n^2\theta = 0$

2-42

$$\Sigma M_o = ma^2\ddot{\theta} = -kb^2\theta - ca^2\dot{\theta}$$

$$\ddot{\theta} + \frac{c}{m}\dot{\theta} + \frac{k}{m}\left(\frac{b}{a}\right)^2\theta = 0$$

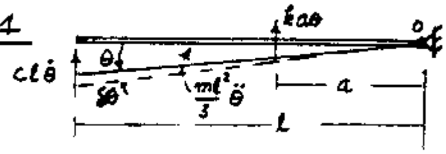
$$\therefore \quad \omega_n = \frac{b}{a}\sqrt{\frac{k}{m}} \qquad \omega_d = \sqrt{\frac{k}{m}\left(\frac{b}{a}\right)^2 - \left(\frac{c}{2m}\right)^2} \qquad c_c = \frac{2b}{a}\sqrt{km}$$

2-43

$$\delta = \ln\frac{1.0}{0.95} = \ln 1.0527 = .05129$$

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = .05129 \qquad \zeta = .00816$$

2-44



$$\delta W = -\frac{ml^2}{3}\ddot{\theta}\cancel{\delta\theta} - cl\dot{\theta}\,l\cancel{\delta\theta} - ka\theta\, a\cancel{\delta\theta} = 0$$

$$\ddot{\theta} + \frac{3c}{m}\dot{\theta} + \frac{3k}{m}\left(\frac{a}{l}\right)^2\theta = 0$$

$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = 0$$

$$\therefore \omega_n = \frac{a}{l}\sqrt{\frac{3k}{m}}, \qquad C_c = \frac{2a}{3l}\sqrt{3km}, \qquad \zeta = \frac{3}{2}\frac{c}{m\omega_n}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = \frac{a}{l}\sqrt{\frac{3k}{m}}\sqrt{1-\frac{9}{4}\left(\frac{c}{m\omega_n}\right)^2} = \frac{a}{l}\sqrt{\frac{3k}{m}}\sqrt{1-\frac{3}{4km}\left(\frac{cl}{a}\right)^2}$$

2-45.

$$\frac{W}{g}\ddot{x} + 2\mu A\dot{x} + kx = 0$$

$$\ddot{x} + \frac{2\mu A g}{W}\dot{x} + \frac{kg}{W}x = 0 \qquad \therefore \tau_1 = 2\pi\sqrt{\frac{W}{kg}}$$

$$f_2 = \frac{1}{\tau_2} = \frac{1}{2\pi}\sqrt{\frac{kg}{W} - \left(\frac{\mu A g}{W}\right)^2} = \frac{1}{2\pi}\sqrt{\left(\frac{2\pi}{\tau_1}\right)^2 - \left(\frac{\mu A g}{W}\right)^2}$$

square both sides

$$\left(\frac{2\pi}{\tau_2}\right)^2 - \left(\frac{2\pi}{\tau_1}\right)^2 = -\left(\frac{\mu A g}{W}\right)^2$$

$$\therefore \quad \mu = \frac{2\pi W}{Ag}\sqrt{\frac{\tau_2^2 - \tau_1^2}{\tau_1^2\,\tau_2^2}} = \frac{2\pi W}{Ag\,\tau_1\tau_2}\sqrt{\tau_2^2 - \tau_1^2}$$

2-46

$$\omega_n = \sqrt{\frac{20,000 \times 32.2}{1200}} = 23.17 \quad r/s$$

$$\frac{1}{2} m \dot{x}_{max}^2 = \frac{1}{2} k x_{max}^2 \qquad \dot{x}_{max} = 23.17 \times 4 = 92.66 \text{ ft/s}.$$

Eq.(2.3-19) $\quad x = e^{-\omega_n t}[0 + \omega_n x(0)]t + x(0) e^{-\omega_n t}$

$$= e^{-\omega_n t} x(0) [1 + \omega_n t]$$

$$\frac{2}{12}' = e^{-\omega_n t} 4 [1 - \omega_n t] \quad \text{or} \quad e^{-\omega_n t}[1 + \omega_n t] = .0417$$

solve by trial

$\omega_n t$	$e^{-\omega_n t}$	$e^{-\omega_n t}[1+\omega_n t]$	
4.90	.00745	.0439	
4.96	.007017	.04182	← close
4.97	.006947	.04147	

$$\therefore \omega_n t = 4.96 \qquad t = \frac{4.96}{23.17} = 0.214 \text{ s}$$

2-47

$$\omega_n = \sqrt{\frac{35000}{4.53}} = 87.89 \quad r/s$$

$$\tau = \frac{2\pi}{87.89} = .0715 \text{ s} \qquad c_c = 2\sqrt{km} = 797.04$$

$$\zeta = .2197$$

$$\tau_d = \sqrt{1-\zeta^2}\ \tau = .0697$$

Eq.(2.6-16)

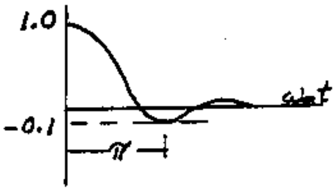
$$x = \frac{\dot{x}(0)}{\omega_n \sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin\sqrt{1-\zeta^2}\ \omega_n t$$

at x_{max}, $\sin\sqrt{1-\zeta^2}\ \omega_n t \cong 1.0$, also $\omega_n t = \pi/2$

$$x = \frac{15.24}{87.89 \times .9756} e^{-.2197(\pi/2)} = .1259 \quad m$$

$$t = \frac{1}{4} \tau_d = .0174 \text{ s}.$$

2 -48



Eq.(2.6-17) for $\dot{x}(0) = 0$

$$x = x(0)\, e^{-\zeta\omega_n t} \cos\sqrt{1-\zeta^2}\,\omega_n t$$

at $\sqrt{1-\zeta^2}\,\omega_n t = \pi \qquad \cos\sqrt{1-\zeta^2}\,\omega_n t = -1$

$$-.10 = 1\, e^{-\zeta\omega_n t}(-1) \qquad \text{solve by trial}$$

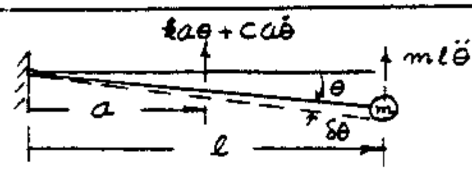
ζ	$\sqrt{1-\zeta^2}$	$\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}$	$e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$
.50	.866		.1630
.59	.8074	−2.2957	.1007 ←
.60	.800	−2.3562	.0948

$$\therefore\ \zeta_1 = 0.59$$

If $\zeta = \frac{1}{2}\zeta_1 = 0.295\ , \quad \sqrt{1-\zeta^2} = .9555$

$$x_{overshoot} = 1\, e^{-\frac{.295\,\pi}{.9555}} = 0.379 = 37.9\,\%$$

2-49(a) (Prob. 2-41 by V.W.)



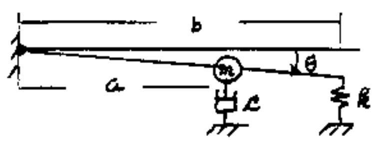
$$\delta W = -m\ell\ddot{\theta}\cdot\ell\delta\theta - (ka\theta + ca\dot{\theta})\,a\delta\theta = 0$$

$$\ddot{\theta} + \frac{ca^2}{m\ell^2}\dot{\theta} + \frac{ka^2}{m\ell^2}\theta = 0$$

$$\ddot{\theta} + 2\zeta\dot{\theta} + \omega_n^2\theta = 0 \qquad \therefore\ \omega_n = \frac{a}{\ell}\sqrt{\frac{k}{m}}, \quad \zeta = \frac{1}{2}\frac{c}{m}\left(\frac{a}{\ell}\right)^2\cdot\frac{\ell}{a}\sqrt{\frac{m}{k}} = \frac{ca}{2\ell\sqrt{km}}$$

$$\omega_d = \frac{a}{\ell}\sqrt{\frac{k}{m}}\sqrt{1 - \frac{1}{km}\left(\frac{ca}{2\ell}\right)^2}$$

2-49(b) (Prob. 2-42 by V.W.)



$$\delta W = -ma\ddot{\theta}\cdot a\delta\theta - ca\dot{\theta}\cdot a\delta\theta - kb\theta\cdot b\delta\theta = 0$$

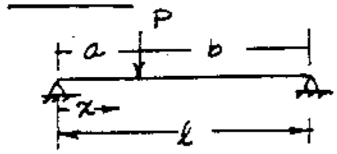
$$\therefore\ \ddot{\theta} + \frac{c}{m}\dot{\theta} + \frac{k}{m}\left(\frac{b}{a}\right)^2\theta = 0$$

2 - 50

$$k_{eff} = (k_1 + k_2) \text{ in series with } k_3$$

$$= \frac{(k_1+k_2)\,k_3}{k_1 + k_2 + k_3}$$

2-51



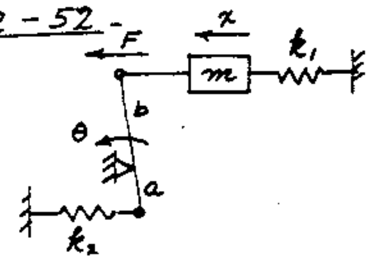
$$y(x) = \frac{Pbx}{6EIl}\left(l^2 - x^2 - b^2\right) \qquad 0 \le x \le a$$

Let $x = \frac{l}{3}$

$$y\left(\frac{l}{3}\right) = \frac{P\frac{2}{3}l \cdot \frac{1}{3}l}{6EIl}\left(l^2 - \frac{l^2}{9} - \frac{4}{9}l^2\right) = \frac{Pl^3}{EI}\cdot\frac{4}{243}$$

$$\text{Flexibility} = \frac{y}{P} = \frac{4}{243}\,\frac{l^3}{EI} \qquad \text{at } \frac{x}{l} = \frac{1}{3}$$

2-52



$$F = k_1 b\theta + \frac{a}{b}k_2 a\theta$$

$$x = b\theta$$

$$\therefore F = k_1 x + \left(\frac{a}{b}\right)^2 k_2 x$$

$$k_{eff} = \frac{F}{x} = k_1 + \left(\frac{a}{b}\right)^2 k_2$$

2-53 $\qquad k_{eff} = \dfrac{k_1 k_2}{k_1 + k_2}$

2-54 Eq. (2.6-17) $\qquad x(t) = e^{-\zeta\omega_n t}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\sqrt{1-\zeta^2}\,\omega_n t + \cos\sqrt{1-\zeta^2}\,\omega_n t\right)$

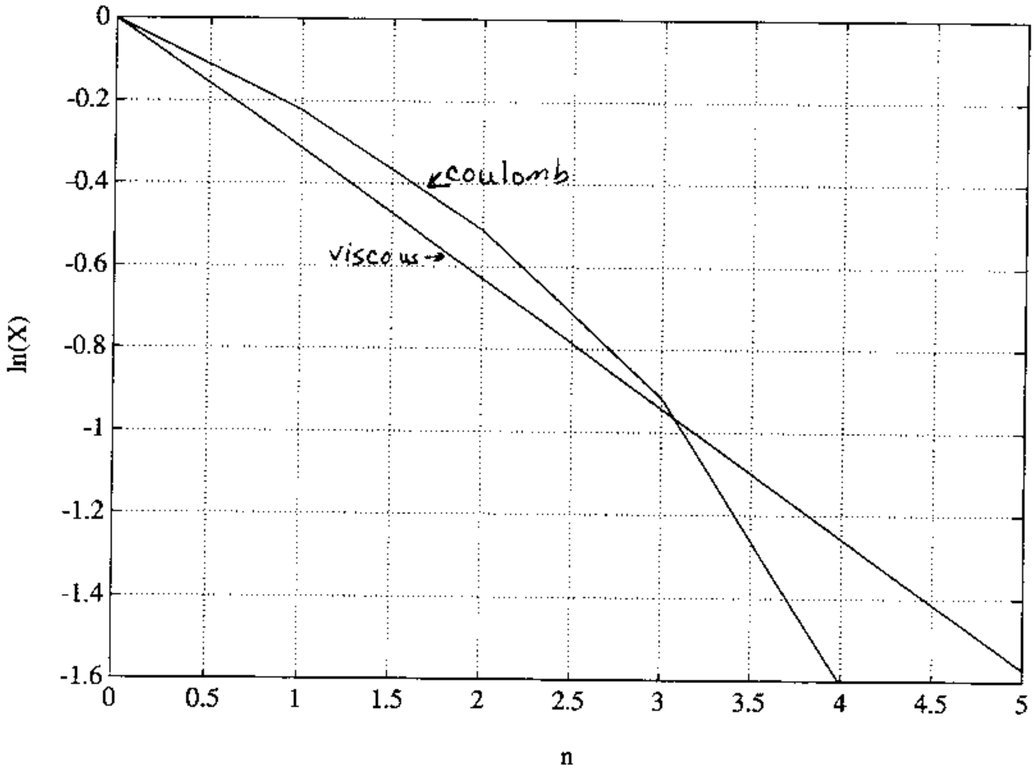
at $\omega_n t = 2\pi, 4\pi, 6\pi$, etc. $\qquad x(t) \cong e^{-\zeta\omega_n t}(0 + 1)$

n	$\omega_n t$	$e^{-.05\omega_n t}$	X_n
0	0	1.0	1.0
1	2π	.7304	.8
2	4π	.5335	.6
3	6π	.3896	.4
4	8π	.2845	.2
5	10π	.2078	0

For Coulomb friction

$$X_1 - X_2 = \frac{4F_d}{k} = \frac{4 \times .05k}{k} = .20$$

$$X_n = 1 - .2n$$



The two amplitudes are equal for $n \cong 3.1$

2 - 55

$$T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \dot{x}^2 + \frac{1}{2} I_o \left(\frac{\dot{x}}{r}\right)^2$$

$$U = \frac{1}{2} k_1 x^2 + \frac{1}{2} k_2 \left(a\frac{x}{r} + x\right)^2$$

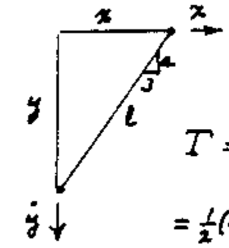
$$\frac{d}{dt}(T+U) = \left(m_1 \ddot{x} + m_2 \ddot{x} + \frac{I_o}{r^2}\ddot{x}\right)\dot{x}$$

$$+ \left[k_1 + k_2\left(1+\frac{a}{r}\right)^2\right] x \dot{x} = -c\,\dot{x}\,\dot{x} = \frac{dW}{dt}$$

$$\left(m_1 + m_2 + I_o/r^2\right)\ddot{x} + \left[k_1 + k_2\left(1+\frac{a}{r}\right)^2\right] x + c\dot{x} = 0$$

$$c_c = 2\sqrt{k_{eff}\, m_{eff}} = 2\sqrt{\left[k_1 + k_2\left(1+\frac{a}{r}\right)^2\right]\left(m_1 + m_2 + I_o/r^2\right)}$$

2 – 56



$$x + y = \ell$$

$$2x\,dx + 2y\,dy = 0 \qquad \therefore \frac{dy}{dx} = -\frac{x}{y}$$

$$\dot{y} = -\frac{3}{4}\dot{x}$$

$$T = \frac{1}{2}(m\ell)\frac{\ell^2}{3}\left(\frac{\dot{x}}{\ell}\right)^2 + \frac{1}{2}(m\ell)\left[\left(\frac{\dot{x}}{2}\right)^2 + \left(\frac{\dot{y}}{2}\right)^2\right] + \frac{1}{2}(m\ell)\frac{\ell^2}{12}\left[\frac{4\dot{x}}{5\ell} + \frac{3}{5}\frac{\dot{y}}{\ell}\right] + \frac{1}{2}M\dot{y}^2$$

$$= \frac{1}{2}(m\ell)\left\{\frac{\ell^2}{3}\left(\frac{\dot{x}}{\ell}\right)^2 + \left[\left(\frac{\dot{x}}{2}\right)^2 + \left(\frac{3\dot{x}}{8}\right)^2\right] + \frac{\ell^2}{12}\left[\frac{4}{5}\frac{\dot{x}}{\ell} + \frac{9}{20}\frac{\dot{x}}{\ell}\right]^2 + \frac{1}{2}M\left(\frac{3\dot{x}}{4}\right)^2\right\}$$

$$= \frac{1}{2}(m\ell)\left[\frac{1}{3} + \frac{1}{4} + \frac{9}{64} + \frac{1}{12}\left(\frac{16}{25} + \frac{81}{400} + \frac{72}{100}\right)\right]\dot{x}^2 + \frac{1}{2}M\frac{9}{16}\dot{x}^2$$

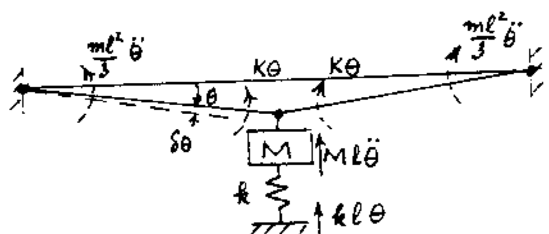
$$= \frac{1}{2}\left[(.854\,m\ell) + .5625\,M\right]\dot{x}^2$$

$$U = \frac{1}{2}ky^2 = \frac{1}{2}k\frac{9}{16}x^2$$

$$\frac{d}{dt}(T+U) = -c\,\frac{2}{3}\ell\,\frac{\dot{x}}{\ell} = -\frac{2}{3}c\dot{x}$$

$$(0.8541\,m\ell + .5625\,M)\ddot{x} + .5625\,kx + \frac{2}{3}c\dot{x} = 0$$

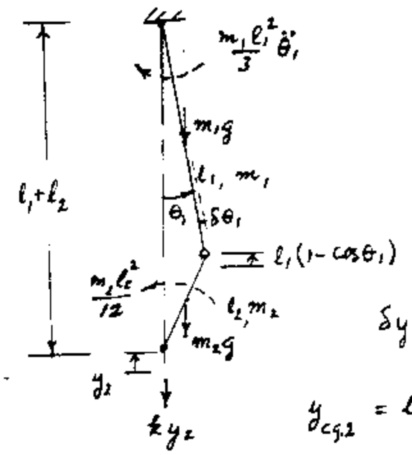
2 – 57



$$\delta W = \left[-M\ell\ddot{\theta}\,\ell\delta\theta - k\ell\theta\,\ell\delta\theta\right] - 2K\theta\cdot\delta\theta - 2\frac{m\ell^2}{3}\ddot{\theta}\,\delta\theta = 0$$

$$\left(M\ell^2 + \frac{2}{3}m\ell^2\right)\ddot{\theta} + (k\ell^2 + 2K)\theta = 0$$

2 - 58



Accel of c.g. in vertical direction $\cong 0$
but work done by gravity is not zero

$$\delta y_{cg.1} = \frac{l_1}{2}\sin\theta_1\delta\theta_1 \cong \frac{l_1}{2}\theta_1\delta\theta_1$$

$$y_2 = l_1(1-\cos\theta_1) + l_2(1-\cos\theta_2)$$

$$l_1\theta_1 \cong l_2\theta_2\,,\quad \theta_2 = \frac{l_1}{l_2}\theta_1\,,\quad \therefore\ \delta\theta_2 = \frac{l_1}{l_2}\delta\theta_1$$

$$\delta y_2 = l_1\left(\theta_1 + \frac{l_1}{l_2}\theta_1\right)\delta\theta_1 = l\left(1+\frac{l_1}{l_2}\right)\theta_1\delta\theta_1$$

$$y_{cg.2} = l_1(1-\cos\theta_1) + \frac{l_2}{2}(1-\cos\theta_2) \quad \therefore\ \delta y_{cg2} = l_1\theta_1\delta\theta_1 + \frac{l_2}{2}\theta_2\delta\theta_2$$

$$= l_1\theta_1\delta\theta_1 + \frac{l_2}{2}\frac{l_1}{l_2}\theta_1\frac{l_1}{l_2}\delta\theta_1$$

$$= l_1\left(1+\frac{l_1}{2l_2}\right)\theta_1\delta\theta_1$$

$$\delta W = -\left(\frac{m_1 l_1^2}{3}\right)\ddot\theta_1\delta\theta_1 - \left(\frac{m_2 l_2^2}{12}\ddot\theta_2\right)\delta\theta_2 - m_1 g\frac{l_1}{2}\theta_1\delta\theta_1 - m_2 g\, l_1\left(1+\frac{l_1}{2l_2}\right)\theta_1\delta\theta_1$$
$$- k\left[l_1(1-\cos\theta_1) + l_2(1-\cos\theta_2)\right]l\left(1+\frac{l_1}{l_2}\right)\theta_1\delta\theta_1 = 0$$

sub. $\ddot\theta_2 = \frac{l_1}{l_2}\ddot\theta_1\,,\quad \delta\theta_2 = \frac{l_1}{l_2}\delta\theta_1\,,\quad \theta_2 = \frac{l_1}{l_2}\theta_1$

$$\left[m_1\frac{l_1^2}{3} + \frac{m_2 l_2^2}{12}\left(\frac{l_1}{l_2}\right)^2\right]\ddot\theta_1 + \left[m_1 g\frac{l_1}{2} + m_2 g\, l_1\left(1+\frac{l_1}{2l_2}\right)\right]\theta_1 = 0$$

(spring force is 2nd order infinitesmal)

2-59 With the initial conditions $x(0) = x_o$ and $\dot{x}(0) = 0$, the solution must be considered for each half cycle. During the first half cycle, with m moving from right to left, $\dot{x}(t)$ is negative so that $sgn(\dot{x})$ is positive and the equations of motion are;

$$x(t) = A\sin\omega_n t + B\cos\omega_n t + \frac{\mu F}{k}$$
$$\dot{x}(t) = \omega_n A\cos\omega_n t - \omega_n B\sin\omega_n t$$

At $t=0$

$$x(0) = x_o = B + \frac{\mu F}{k} \qquad \therefore B = \left(x_o - \frac{\mu F}{k}\right)$$
$$\dot{x}(0) = 0 = \omega_n A \qquad \therefore A = 0$$

and the general solution becomes

$$x(t) = \left(x_o - \frac{\mu F}{k}\right)\cos\omega_n t + \frac{\mu F}{k}$$
$$\dot{x}(t) = -\omega_n\left(x_o - \frac{\mu F}{k}\right)\sin\omega_n t$$

For the second half cycle $\dot{x}$ is positive and the friction force is negative. The new equation must now be written with two other constants C and D

$$x(t) = C\sin\omega_n t + D\cos\omega_n t - \frac{\mu F}{k}$$
$$\dot{x}(t) = \omega_n C\cos\omega_n t - \omega_n D\sin\omega_n t$$

We again measure time $t=0$ at the beginning of the 2^{nd} half cycle, with new initial conditions found from the previous equations as $x(0) = \left(-x_o + 2\frac{\mu F}{k}\right)$ and $\dot{x}(0) = 0$

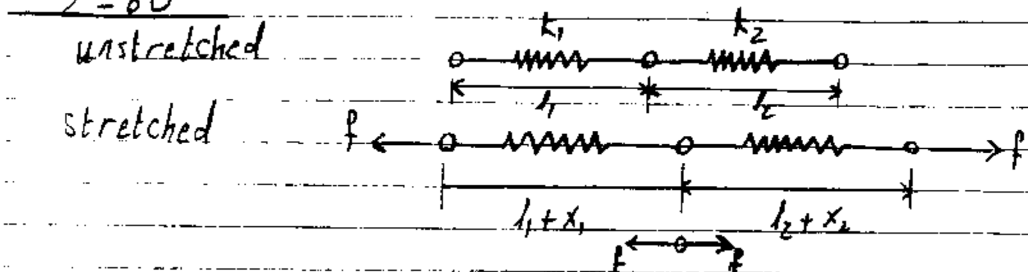
$$\therefore D = -x_o + 3\frac{\mu F}{k} \quad \text{and} \quad C = 0$$

$\therefore$ The new equations for the second half cycle

$$x(t) = \left(-x_o + \frac{3\mu F}{k}\right)\cos\omega_n t - \frac{\mu F}{k}$$

$$\dot{x}(t) = -\omega_n\left(-x_o + \frac{3\mu F}{k}\right)\sin\omega_n t$$

2-60



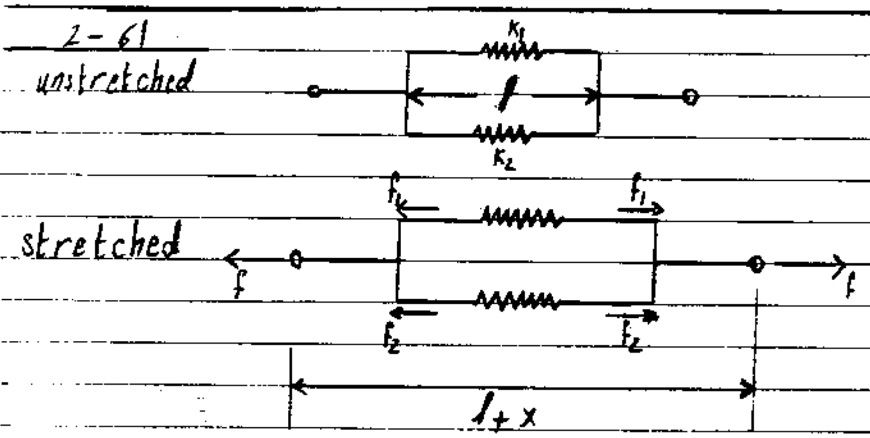
In the stretch position the total displacement of the two springs is $x_1 + x_2$. Since each spring exhibits the same force f, we have

$$X_{eq} = x_1 + x_2$$

$$\therefore \frac{f}{k_{eq}} = \frac{f}{k_1} + \frac{f}{k_2} \Rightarrow k_{eq} = \frac{1}{1/k_1 + 1/k_2},$$

where, x_{eq} and k_{eq} are the equivalent displacement and stiffness of the series combination.

2-61

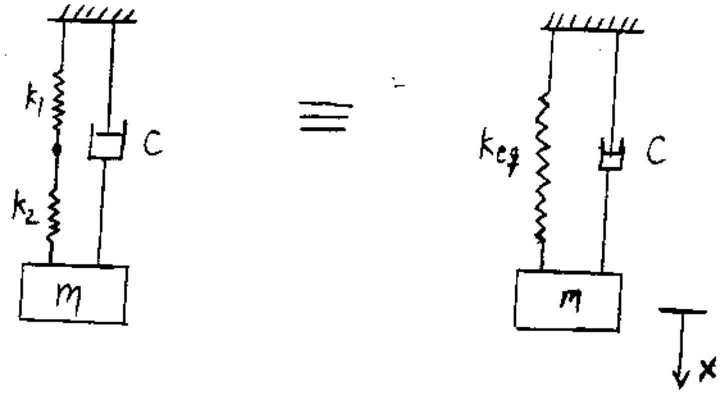


$$f = f_1 + f_2$$

$$\therefore k_{eq} x = k_1 x + k_2 x \Rightarrow k_{eq} = k_1 + k_2,$$

where, k_{eq} is the equivalent stiffness of the parallel combination.

2.62



Equation of motion

$$m\ddot{x} = -k_{eq}x - c\dot{x} \Longrightarrow m\ddot{x} + c\dot{x} + k_{eq}x = 0,$$

where, $k_{eq} = \dfrac{1}{1/k_1 + 1/k_2}$ (see problem 2.60) is the effective spring constant for the system.