Computer Exploration

Chapter 2 Section 7, Tangents and Derivatives

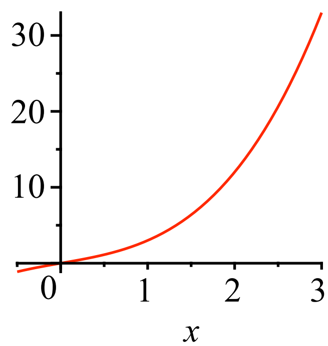
Problem 45.

Begin with the definition of the function f and the difference quotient function q .

> $f(x) := x^3 + 2x :\ q(h) := \frac{f(0+h) - f(0)}{h}$:

Here is a plot of f.

> $plot\left(f(x), x = -\frac{1}{2}..3\right)$



Let m_0 denote the limit of the difference quotient as $h \to 0$.

> $m_0 := \lim_{h \to 0} q(h)$

$$m_0 := 2 \quad \textbf{(1)}$$

The tangent line function is defined next.

> $T(x) := m_0 \cdot (x - 0) + f(0)$

$$T := x \to m_0 x + f(0) \quad \textbf{(2)}$$

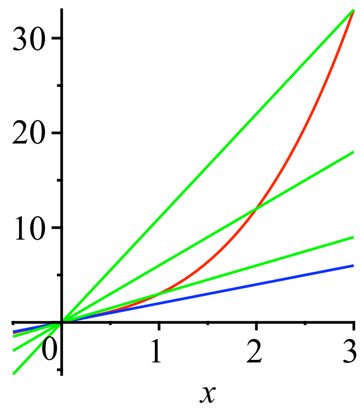
And the next entry defines the secant line at 0 as a function of x and h.

> $S(x, h) := q(h) \cdot (x - 0) + f(0)$

$$S := (x, h) \to q(h)\, x + f(0) \quad \textbf{(3)}$$

The plots follow. The tangent line is blue and the secant lines are green.

> $plot\left([f(x), T(x), S(x, 1), S(x, 2), S(x, 3)], x = -\frac{1}{2}..3, color = [red, blue, green\$3]\right)$


30
20
10
0
1
2
3
x

Computer Exploration

Chapter 2 Section 3, The Precise Definition of a Limit

Maple Preliminaries

To evaluate a limit type *limit* then press **[esc]-[enter]** to enter a limit template. Tab from position to position.

> $\lim_{x \to a} f(x)$

$$f(a) \tag{1}$$

If no information is given, then Maple assumes that the function f is continuous at a .

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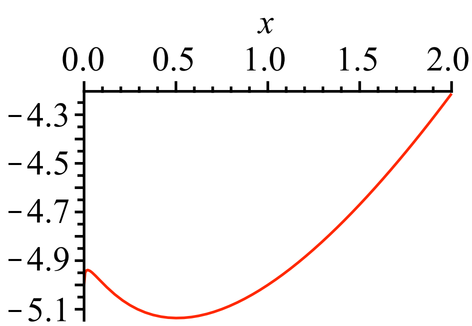
Using a plot of the function

> $f(x) := \dfrac{3x^2 - (7x+1)\sqrt{x} + 5}{x-1}$

$$f := x \rightarrow \frac{3x^2 - (7x+1)\sqrt{x} + 5}{x-1} \tag{2}$$

near to $x_0 = 1$ our guess is that the limiting value of f is $L = -5$.

> $plot(f(x), x = 0..2)$



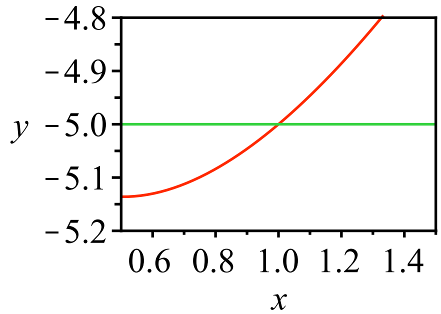
The following limit calculation confirms that this is the case.

> $\lim_{x \to 1} f(x)$

$$-5 \tag{3}$$

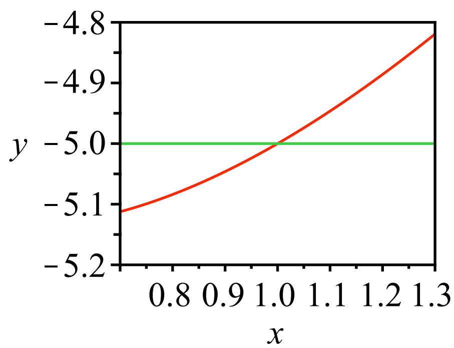
Based upon the picture above we can make the following preliminary plot. Note that a horizontal line at $y = 5$ has been added to the plot, and we have boxed the axes.

> $plot(\ [f(x), -5], x = 0.5..1.5, y = -5.2..-4.8, axes = boxed)$



It appears that $\delta = 0.3$ will do the job for $\epsilon = 0.2$. See the following picture.

> $plot(\ [f(x), -5], x = 1 - 0.3 .. 1 + 0.3, y = -5 - 0.2 .. -5 + 0.2, axes = boxed)$



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