

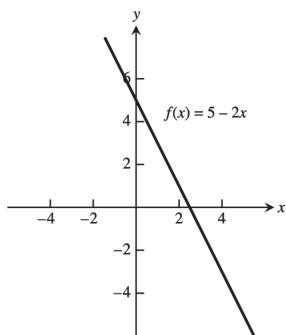
CHAPTER 1 FUNCTIONS

1.1 FUNCTIONS AND THEIR GRAPHS

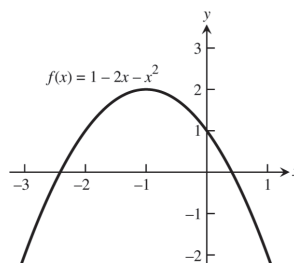
1. domain $= (-\infty, \infty)$; range $= [1, \infty)$
2. domain $= [0, \infty)$; range $= (-\infty, 1]$
3. domain $= [-2, \infty)$; y in range and $y = \sqrt{5x+10} \geq 0 \Rightarrow y$ can be any positive real number \Rightarrow range $= [0, \infty)$.
4. domain $= (-\infty, 0] \cup [3, \infty)$; y in range and $y = \sqrt{x^2 - 3x} \geq 0 \Rightarrow y$ can be any positive real number \Rightarrow range $= [0, \infty)$.
5. domain $= (-\infty, 3) \cup (3, \infty)$; y in range and $y = \frac{4}{3-t}$, now if $t < 3 \Rightarrow 3-t > 0 \Rightarrow \frac{4}{3-t} > 0$, or if $t > 3 \Rightarrow 3-t < 0 \Rightarrow \frac{4}{3-t} < 0 \Rightarrow y$ can be any nonzero real number \Rightarrow range $= (-\infty, 0) \cup (0, \infty)$.
6. domain $= (-\infty, -4) \cup (-4, 4) \cup (4, \infty)$; y in range and $y = \frac{2}{t^2-16}$, now if $t < -4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0$, or if $-4 < t < 4 \Rightarrow -16 \leq t^2 - 16 < 0 \Rightarrow -\frac{2}{16} \geq \frac{2}{t^2-16}$, or if $t > 4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0 \Rightarrow y$ can be any nonzero real number \Rightarrow range $= (-\infty, -\frac{1}{8}] \cup (0, \infty)$.
7. (a) Not the graph of a function of x since it fails the vertical line test.
(b) Is the graph of a function of x since any vertical line intersects the graph at most once.
8. (a) Not the graph of a function of x since it fails the vertical line test.
(b) Not the graph of a function of x since it fails the vertical line test.
9. base $= x$; $(\text{height})^2 + \left(\frac{x}{2}\right)^2 = x^2 \Rightarrow \text{height} = \frac{\sqrt{3}}{2}x$; area is $a(x) = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$;
perimeter is $p(x) = x + x + x = 3x$.
10. $s = \text{side length} \Rightarrow s^2 + s^2 = d^2 \Rightarrow s = \frac{d}{\sqrt{2}}$; and area is $a = s^2 \Rightarrow a = \frac{1}{2}d^2$
11. Let $D = \text{diagonal length of a face of the cube}$ and $\ell = \text{the length of an edge}$. Then $\ell^2 + D^2 = d^2$ and $D^2 = 2\ell^2 \Rightarrow 3\ell^2 = d^2 \Rightarrow \ell = \frac{d}{\sqrt{3}}$. The surface area is $6\ell^2 = \frac{6d^2}{3} = 2d^2$ and the volume is $\ell^3 = \left(\frac{d}{\sqrt{3}}\right)^3 = \frac{d^3}{3\sqrt{3}}$.
12. The coordinates of P are (x, \sqrt{x}) so the slope of the line joining P to the origin is $m = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}} (x > 0)$.
Thus, $(x, \sqrt{x}) = \left(\frac{1}{m^2}, \frac{1}{m}\right)$.
13. $2x + 4y = 5 \Rightarrow y = -\frac{1}{2}x + \frac{5}{4}$; $L = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + \left(-\frac{1}{2}x + \frac{5}{4}\right)^2} = \sqrt{x^2 + \frac{1}{4}x^2 - \frac{5}{4}x + \frac{25}{16}}$
 $= \sqrt{\frac{5}{4}x^2 - \frac{5}{4}x + \frac{25}{16}} = \sqrt{\frac{20x^2 - 20x + 25}{16}} = \frac{\sqrt{20x^2 - 20x + 25}}{4}$
14. $y = \sqrt{x-3} \Rightarrow y^2 + 3 = x$; $L = \sqrt{(x-4)^2 + (y-0)^2} = \sqrt{(y^2 + 3 - 4)^2 + y^2} = \sqrt{(y^2 - 1)^2 + y^2}$
 $= \sqrt{y^4 - 2y^2 + 1 + y^2} = \sqrt{y^4 - y^2 + 1}$

2 Chapter 1 Functions

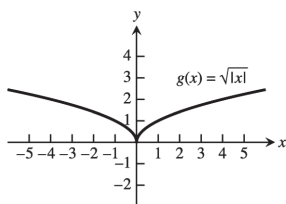
15. The domain is $(-\infty, \infty)$.



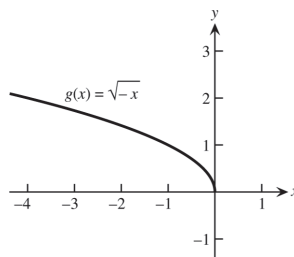
16. The domain is $(-\infty, \infty)$.



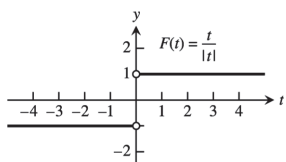
17. The domain is $(-\infty, \infty)$.



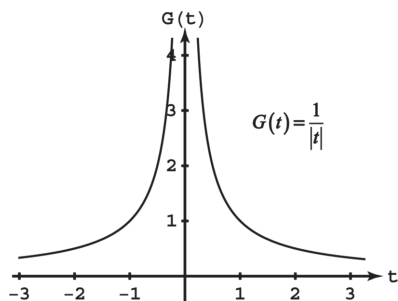
18. The domain is $(-\infty, 0]$.



19. The domain is $(-\infty, 0) \cup (0, \infty)$.



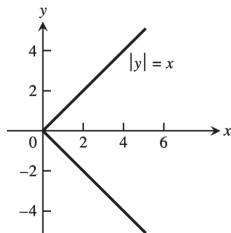
20. The domain is $(-\infty, 0) \cup (0, \infty)$.



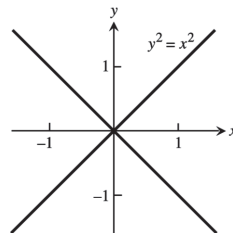
21. The domain is $(-\infty, -5) \cup (-5, -3] \cup [3, 5) \cup (5, \infty)$ 22. The range is $[2, 3)$.

23. Neither graph passes the vertical line test

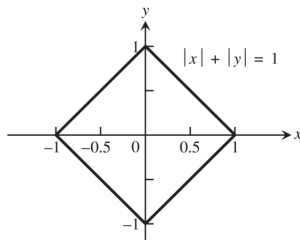
(a)



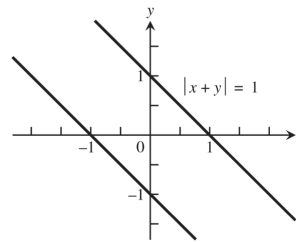
(b)



24. Neither graph passes the vertical line test
(a)



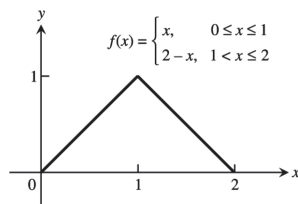
(b)



$$|x + y| = 1 \Leftrightarrow \begin{cases} x + y = 1 \\ \text{or} \\ x + y = -1 \end{cases} \Leftrightarrow \begin{cases} y = 1 - x \\ \text{or} \\ y = -1 - x \end{cases}$$

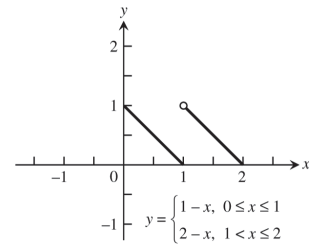
25.

x	0	1	2
y	0	1	0

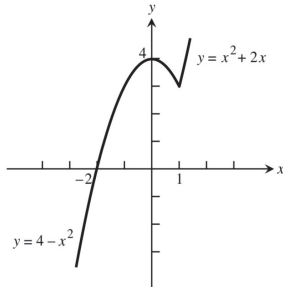


26.

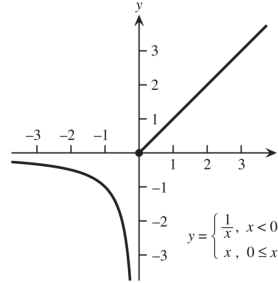
x	0	1	2
y	1	0	0



27. $F(x) = \begin{cases} 4 - x^2, & x \leq 1 \\ x^2 + 2x, & x > 1 \end{cases}$



28. $G(x) = \begin{cases} \frac{1}{x}, & x < 0 \\ x, & 0 \leq x \end{cases}$



29. (a) Line through (0, 0) and (1, 1): $y = x$; Line through (1, 1) and (2, 0): $y = -x + 2$

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ -x + 2, & 1 < x \leq 2 \end{cases}$$

(b) $f(x) = \begin{cases} 2, & 0 \leq x < 1 \\ 0, & 1 \leq x < 2 \\ 2, & 2 \leq x < 3 \\ 0, & 3 \leq x \leq 4 \end{cases}$

30. (a) Line through (0, 2) and (2, 0): $y = -x + 2$

Line through (2, 1) and (5, 0): $m = \frac{0-1}{5-2} = -\frac{1}{3}$, so $y = -\frac{1}{3}(x-2) + 1 = -\frac{1}{3}x + \frac{5}{3}$

$$f(x) = \begin{cases} -x + 2, & 0 < x \leq 2 \\ -\frac{1}{3}x + \frac{5}{3}, & 2 < x \leq 5 \end{cases}$$

(b) Line through $(-1, 0)$ and $(0, -3)$: $m = \frac{-3-0}{0-(-1)} = -3$, so $y = -3x - 3$

Line through $(0, 3)$ and $(2, -1)$: $m = \frac{-1-3}{2-0} = \frac{-4}{2} = -2$, so $y = -2x + 3$

$$f(x) = \begin{cases} -3x-3, & -1 < x \leq 0 \\ -2x+3, & 0 < x \leq 2 \end{cases}$$

31. (a) Line through $(-1, 1)$ and $(0, 0)$: $y = -x$

Line through $(0, 1)$ and $(1, 1)$: $y = 1$

Line through $(1, 1)$ and $(3, 0)$: $m = \frac{0-1}{3-1} = \frac{-1}{2} = -\frac{1}{2}$, so $y = -\frac{1}{2}(x-1) + 1 = -\frac{1}{2}x + \frac{3}{2}$

$$f(x) = \begin{cases} -x & -1 \leq x < 0 \\ 1 & 0 < x \leq 1 \\ -\frac{1}{2}x + \frac{3}{2} & 1 < x < 3 \end{cases}$$

(b) Line through $(-2, -1)$ and $(0, 0)$: $y = \frac{1}{2}x$

Line through $(0, 2)$ and $(1, 0)$: $y = -2x + 2$

Line through $(1, -1)$ and $(3, -1)$: $y = -1$

$$f(x) = \begin{cases} \frac{1}{2}x & -2 \leq x \leq 0 \\ -2x+2 & 0 < x \leq 1 \\ -1 & 1 < x \leq 3 \end{cases}$$

32. (a) Line through $(\frac{T}{2}, 0)$ and $(T, 1)$: $m = \frac{1-0}{T-(T/2)} = \frac{2}{T}$, so $y = \frac{2}{T}(x - \frac{T}{2}) + 0 = \frac{2}{T}x - 1$

$$f(x) = \begin{cases} 0, & 0 \leq x \leq \frac{T}{2} \\ \frac{2}{T}x - 1, & \frac{T}{2} < x \leq T \end{cases}$$

$$(b) f(x) = \begin{cases} A, & 0 \leq x < \frac{T}{2} \\ -A, & \frac{T}{2} \leq x < T \\ A, & T \leq x < \frac{3T}{2} \\ -A, & \frac{3T}{2} \leq x \leq 2T \end{cases}$$

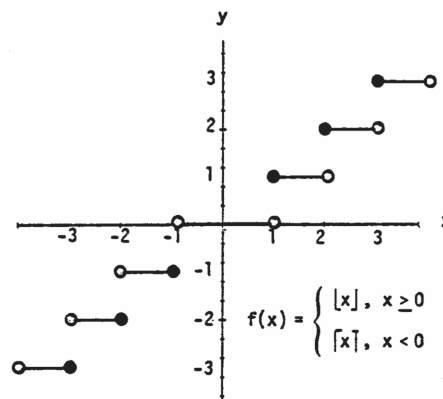
33. (a) $\lfloor x \rfloor = 0$ for $x \in [0, 1)$

(b) $\lceil x \rceil = 0$ for $x \in (-1, 0]$

34. $\lfloor x \rfloor = \lceil x \rceil$ only when x is an integer.

35. For any real number x , $n \leq x \leq n+1$, where n is an integer. Now: $n \leq x \leq n+1 \Rightarrow -(n+1) \leq -x \leq -n$. By definition: $\lceil -x \rceil = -n$ and $\lfloor x \rfloor = n \Rightarrow -\lfloor x \rfloor = -n$. So $\lceil -x \rceil = -\lfloor x \rfloor$ for all real x .

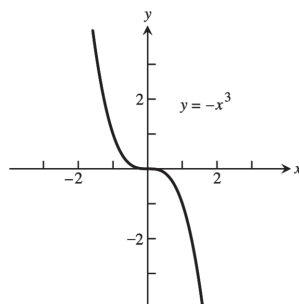
36. To find $f(x)$ you delete the decimal or fractional portion of x , leaving only the integer part.



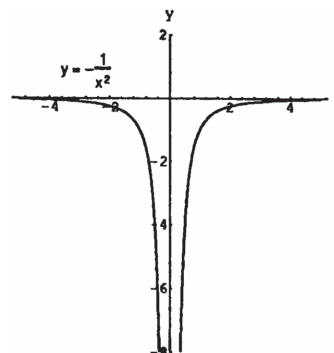
37. Symmetric about the origin

Dec: $-\infty < x < \infty$

Inc: nowhere

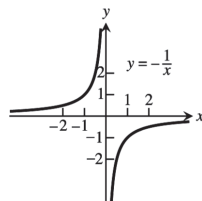


38. Symmetric about the y-axis

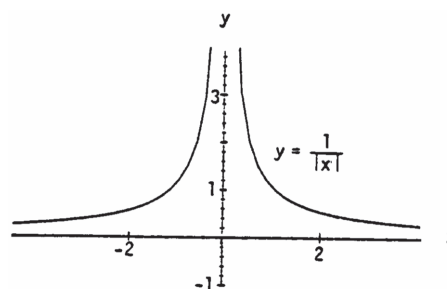
Dec: $-\infty < x < 0$ Inc: $0 < x < \infty$ 

39. Symmetric about the origin

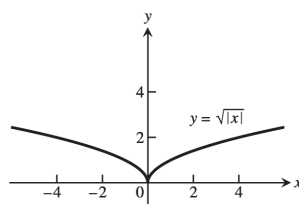
Dec: nowhere

Inc: $-\infty < x < 0$ $0 < x < \infty$ 

40. Symmetric about the y-axis

Dec: $0 < x < \infty$ Inc: $-\infty < x < 0$ 

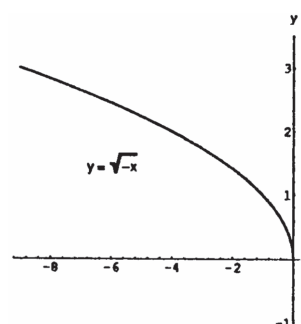
41. Symmetric about the y-axis

Dec: $-\infty < x \leq 0$ Inc: $0 \leq x < \infty$ 

42. No symmetry

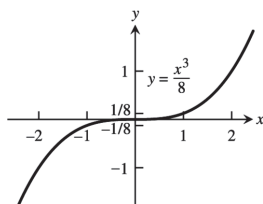
Dec: $-\infty < x \leq 0$

Inc: nowhere



43. Symmetric about the origin

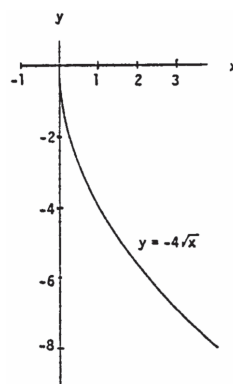
Dec: nowhere

Inc: $-\infty < x < \infty$ 

44. No symmetry

Dec: $0 \leq x < \infty$

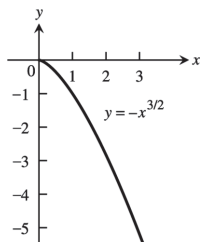
Inc: nowhere



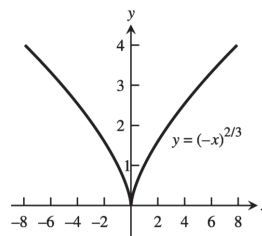
45. No symmetry

Dec: $0 \leq x < \infty$

Inc: nowhere



46. Symmetric about the y-axis

Dec: $-\infty < x \leq 0$ Inc: $0 \leq x < \infty$ 

47. Since a horizontal line not through the origin is symmetric with respect to the y-axis, but not with respect to the origin, the function is even.

- 48.
- $f(x) = x^{-5} = \frac{1}{x^5}$
- and
- $f(-x) = (-x)^{-5} = \frac{1}{(-x)^5} = -\left(\frac{1}{x^5}\right) = -f(x)$
- . Thus the function is odd.

49. Since
- $f(x) = x^2 + 1 = (-x)^2 + 1 = f(-x)$
- . The function is even.

50. Since
- $[f(x) = x^2 + x] \neq [f(-x) = (-x)^2 - x]$
- and
- $[f(x) = x^2 + x] \neq [-f(x) = -(x)^2 - x]$
- the function is neither even nor odd.

51. Since
- $g(x) = x^3 + x$
- ,
- $g(-x) = -x^3 - x = -(x^3 + x) = -g(x)$
- . So the function is odd.

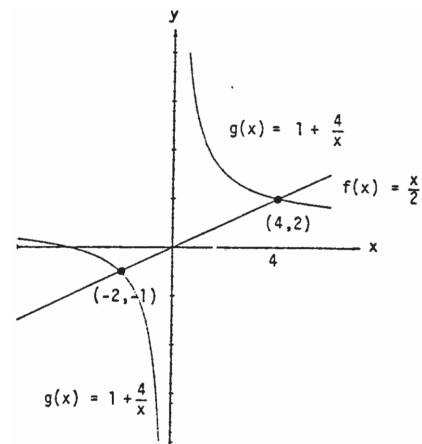
- 52.
- $g(x) = x^4 + 3x^2 - 1 = (-x)^4 + 3(-x)^2 - 1 = g(-x)$
- , thus the function is even.

- 53.
- $g(x) = \frac{1}{x^2 - 1} = \frac{1}{(-x)^2 - 1} = g(-x)$
- . Thus the function is even.

- 54.
- $g(x) = \frac{x}{x^2 - 1}$
- ;
- $g(-x) = -\frac{x}{x^2 - 1} = -g(x)$
- . So the function is odd.

- 55.
- $h(t) = \frac{1}{t-1}$
- ;
- $h(-t) = \frac{1}{-t-1}$
- ;
- $-h(t) = \frac{1}{1-t}$
- . Since
- $h(t) \neq -h(t)$
- and
- $h(t) \neq h(-t)$
- , the function is neither even nor odd.

56. Since $|t^3| = |(-t)^3|$, $h(t) = h(-t)$ and the function is even.
57. $h(t) = 2t + 1$, $h(-t) = -2t + 1$. So $h(t) \neq h(-t)$. $-h(t) = -2t - 1$, so $h(t) \neq -h(t)$. The function is neither even nor odd.
58. $h(t) = 2|t| + 1$ and $h(-t) = 2|-t| + 1 = 2|t| + 1$. So $h(t) = h(-t)$ and the function is even.
59. $s = kt \Rightarrow 25 = k(75) \Rightarrow k = \frac{1}{3} \Rightarrow s = \frac{1}{3}t$; $60 = \frac{1}{3}t \Rightarrow t = 180$
60. $K = c v^2 \Rightarrow 12960 = c(18)^2 \Rightarrow c = 40 \Rightarrow K = 40v^2$; $K = 40(10)^2 = 4000$ joules
61. $r = \frac{k}{s} \Rightarrow 6 = \frac{k}{4} \Rightarrow k = 24 \Rightarrow r = \frac{24}{s}$; $10 = \frac{24}{s} \Rightarrow s = \frac{12}{5}$
62. $P = \frac{k}{v} \Rightarrow 14.7 = \frac{k}{1000} \Rightarrow k = 14700 \Rightarrow P = \frac{14700}{v}$; $23.4 = \frac{14700}{v} \Rightarrow v = \frac{24500}{39} \approx 628.2$ in³
63. $v = f(x) = x(14 - 2x)(22 - 2x) = 4x^3 - 72x^2 + 308x$; $0 < x < 7$.
64. (a) Let h = height of the triangle. Since the triangle is isosceles, $(\overline{AB})^2 + (\overline{AB})^2 = 2^2 \Rightarrow \overline{AB} = \sqrt{2}$. So,
 $h^2 + 1^2 = (\sqrt{2})^2 \Rightarrow h = 1 \Rightarrow B$ is at $(0, 1) \Rightarrow$ slope of $AB = -1 \Rightarrow$ The equation of AB is
 $y = f(x) = -x + 1$; $x \in [0, 1]$.
 (b) $A(x) = 2xy = 2x(-x + 1) = -2x^2 + 2x$; $x \in [0, 1]$.
65. (a) Graph h because it is an even function and rises less rapidly than does Graph g .
 (b) Graph f because it is an odd function.
 (c) Graph g because it is an even function and rises more rapidly than does Graph h .
66. (a) Graph f because it is linear.
 (b) Graph g because it contains $(0, 1)$.
 (c) Graph h because it is a nonlinear odd function.
67. (a) From the graph, $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow x \in (-2, 0) \cup (4, \infty)$
 (b) $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow \frac{x}{2} - 1 - \frac{4}{x} > 0$
 $x > 0$: $\frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2 - 2x - 8}{2x} > 0 \Rightarrow \frac{(x-4)(x+2)}{2x} > 0$
 $\Rightarrow x > 4$ since x is positive;
 $x < 0$: $\frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2 - 2x - 8}{2x} < 0 \Rightarrow \frac{(x-4)(x+2)}{2x} < 0$
 $\Rightarrow x < -2$ since x is negative;
 sign of $(x-4)(x+2)$
- $\begin{array}{ccccccc} & + & & - & & + & \\ & & -2 & & 4 & & \\ \hline \end{array}$
- Solution interval: $(-2, 0) \cup (4, \infty)$



68. (a) From the graph, $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow x \in (-\infty, -5) \cup (-1, 1)$

(b) Case $x < -1$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} > 2$
 $\Rightarrow 3x+3 < 2x-2 \Rightarrow x < -5$.

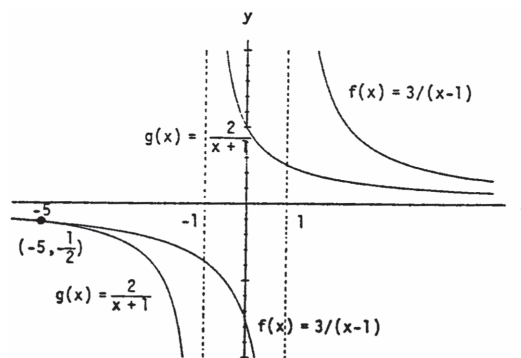
Thus, $x \in (-\infty, -5)$ solves the inequality.

Case $-1 < x < 1$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} < 2$

$\Rightarrow 3x+3 > 2x-2 \Rightarrow x > -5$ which is true if $x > -1$. Thus, $x \in (-1, 1)$ solves the inequality.

Case $1 < x$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow 3x+3 < 2x-2 \Rightarrow x < -5$ which is never true if $1 < x$, so no solution here.

In conclusion, $x \in (-\infty, -5) \cup (-1, 1)$.



69. A curve symmetric about the x -axis will not pass the vertical line test because the points (x, y) and $(x, -y)$ lie on the same vertical line. The graph of the function $y = f(x) = 0$ is the x -axis, a horizontal line for which there is a single y -value, 0, for any x .

70. price = $40 + 5x$, quantity = $300 - 25x \Rightarrow R(x) = (40 + 5x)(300 - 25x)$

71. $x^2 + x^2 = h^2 \Rightarrow x = \frac{h}{\sqrt{2}} = \frac{\sqrt{2}h}{2}$; cost = $5(2x) + 10h \Rightarrow C(h) = 10\left(\frac{\sqrt{2}h}{2}\right) + 10h = 5h(\sqrt{2} + 2)$

72. (a) Note that 2 mi = 10,560 ft, so there are $\sqrt{800^2 + x^2}$ feet of river cable at \$180 per foot and $(10,560 - x)$ feet of land cable at \$100 per foot. The cost is $C(x) = 180\sqrt{800^2 + x^2} + 100(10,560 - x)$.

(b) $C(0) = \$1,200,000$

$C(500) \approx \$1,175,812$

$C(1000) \approx \$1,186,512$

$C(1500) \approx \$1,212,000$

$C(2000) \approx \$1,243,732$

$C(2500) \approx \$1,278,479$

$C(3000) \approx \$1,314,870$

Values beyond this are all larger. It would appear that the least expensive location is less than 2000 feet from the point P .

1.2 COMBINING FUNCTIONS; SHIFTING AND SCALING GRAPHS

1. $D_f: -\infty < x < \infty, D_g: x \geq 1 \Rightarrow D_{f+g} = D_{fg}: x \geq 1. R_f: -\infty < y < \infty, R_g: y \geq 0, R_{f+g}: y \geq 1, R_{fg}: y \geq 0$

2. $D_f: x+1 \geq 0 \Rightarrow x \geq -1, D_g: x-1 \geq 0 \Rightarrow x \geq 1$. Therefore $D_{f+g} = D_{fg}: x \geq 1$.

$R_f = R_g: y \geq 0, R_{f+g}: y \geq \sqrt{2}, R_{fg}: y \geq 0$

3. $D_f: -\infty < x < \infty, D_g: -\infty < x < \infty, D_{f/g}: -\infty < x < \infty, D_{g/f}: -\infty < x < \infty, R_f: y = 2, R_g: y \geq 1, R_{f/g}: 0 < y \leq 2, R_{g/f}: \frac{1}{2} \leq y < \infty$

4. $D_f: -\infty < x < \infty, D_g: x \geq 0, D_{f/g}: x \geq 0, D_{g/f}: x \geq 0; R_f: y = 1, R_g: y \geq 1, R_{f/g}: 0 < y \leq 1, R_{g/f}: 1 \leq y < \infty$

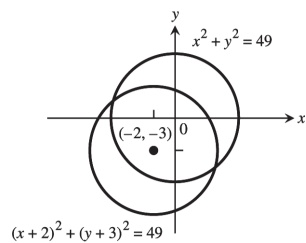
5. (a) 2 (b) 22 (c) $x^2 + 2$
 (d) $(x+5)^2 - 3 = x^2 + 10x + 22$ (e) 5 (f) -2
 (g) $x+10$ (h) $(x^2 - 3)^2 - 3 = x^4 - 6x^2 + 6$
6. (a) $-\frac{1}{3}$ (b) 2 (c) $\frac{1}{x+1} - 1 = \frac{-x}{x+1}$
 (d) $\frac{1}{x}$ (e) 0 (f) $\frac{3}{4}$
 (g) $x-2$ (h) $\frac{1}{\frac{1}{x+1} + 1} = \frac{1}{\frac{x+2}{x+1}} = \frac{x+1}{x+2}$
7. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(4-x)) = f(3(4-x)) = f(12-3x) = (12-3x) + 1 = 13-3x$
8. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x^2)) = f(2(x^2)-1) = f(2x^2-1) = 3(2x^2-1) + 4 = 6x^2 + 1$
9. $(f \circ g \circ h)(x) = f(g(h(x))) = f\left(g\left(\frac{1}{x}\right)\right) = f\left(\frac{1}{\frac{1}{x} + 4}\right) = f\left(\frac{x}{1+4x}\right) = \sqrt{\frac{x}{1+4x} + 1} = \sqrt{\frac{5x+1}{1+4x}}$
10. $(f \circ g \circ h)(x) = f(g(h(x))) = f\left(g\left(\sqrt{2-x}\right)\right) = f\left(\frac{(\sqrt{2-x})^2}{(\sqrt{2-x})^2 + 1}\right) = f\left(\frac{2-x}{3-x}\right) = \frac{\frac{2-x}{3-x} + 2}{3 - \frac{2-x}{3-x}} = \frac{8-3x}{7-2x}$
11. (a) $(f \circ g)(x)$ (b) $(j \circ g)(x)$ (c) $(g \circ g)(x)$
 (d) $(j \circ j)(x)$ (e) $(g \circ h \circ f)(x)$ (f) $(h \circ j \circ f)(x)$
12. (a) $(f \circ j)(x)$ (b) $(g \circ h)(x)$ (c) $(h \circ h)(x)$
 (d) $(f \circ f)(x)$ (e) $(j \circ g \circ f)(x)$ (f) $(g \circ f \circ h)(x)$
- 13.
- | $g(x)$ | $f(x)$ | $(f \circ g)(x)$ |
|---------------------|-------------------|---|
| (a) $x-7$ | \sqrt{x} | $\sqrt{x-7}$ |
| (b) $x+2$ | $3x$ | $3(x+2) = 3x+6$ |
| (c) x^2 | $\sqrt{x-5}$ | $\sqrt{x^2-5}$ |
| (d) $\frac{x}{x-1}$ | $\frac{x}{x-1}$ | $\frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1} = \frac{x}{x - (x-1)} = x$ |
| (e) $\frac{1}{x-1}$ | $1 + \frac{1}{x}$ | x |
| (f) $\frac{1}{x}$ | $\frac{1}{x}$ | x |
14. (a) $(f \circ g)(x) = |g(x)| = \frac{1}{|x-1|}$.
 (b) $(f \circ g)(x) = \frac{g(x)-1}{g(x)} = \frac{x}{x+1} \Rightarrow 1 - \frac{1}{g(x)} = \frac{x}{x+1} \Rightarrow 1 - \frac{x}{x+1} = \frac{1}{g(x)} \Rightarrow \frac{1}{x+1} = \frac{1}{g(x)}$, so $g(x) = x+1$.
 (c) Since $(f \circ g)(x) = \sqrt{g(x)} = |x|$, $g(x) = x^2$.
 (d) Since $(f \circ g)(x) = f(\sqrt{x}) = |x|$, $f(x) = x^2$. (Note that the domain of the composite is $[0, \infty)$.)

The completed table is shown. Note that the absolute value sign in part (d) is optional.

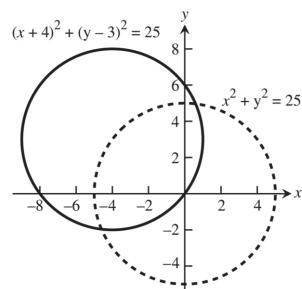
$g(x)$	$f(x)$	$(f \circ g)(x)$
$\frac{1}{x-1}$	$ x $	$\frac{1}{ x-1 }$
$x+1$	$\frac{x-1}{x}$	$\frac{x}{x+1}$
x^2	\sqrt{x}	$ x $
\sqrt{x}	x^2	$ x $

15. (a) $f(g(-1)) = f(1) = 1$ (b) $g(f(0)) = g(-2) = 2$ (c) $f(f(-1)) = f(0) = -2$
 (d) $g(g(2)) = g(0) = 0$ (e) $g(f(-2)) = g(1) = -1$ (f) $f(g(1)) = f(-1) = 0$
16. (a) $f(g(0)) = f(-1) = 2 - (-1) = 3$, where $g(0) = 0 - 1 = -1$
 (b) $g(f(3)) = g(-1) = -(-1) = 1$, where $f(3) = 2 - 3 = -1$
 (c) $g(g(-1)) = g(1) = 1 - 1 = 0$, where $g(-1) = -(-1) = 1$
 (d) $f(f(2)) = f(0) = 2 - 0 = 2$, where $f(2) = 2 - 2 = 0$
 (e) $g(f(0)) = g(2) = 2 - 1 = 1$, where $f(0) = 2 - 0 = 2$
 (f) $f\left(g\left(\frac{1}{2}\right)\right) = f\left(-\frac{1}{2}\right) = 2 - \left(-\frac{1}{2}\right) = \frac{5}{2}$, where $g\left(\frac{1}{2}\right) = \frac{1}{2} - 1 = -\frac{1}{2}$
17. (a) $(f \circ g)(x) = f(g(x)) = \sqrt{\frac{1}{x} + 1} = \sqrt{\frac{1+x}{x}}$
 $(g \circ f)(x) = g(f(x)) = \frac{1}{\sqrt{x+1}}$
 (b) Domain $(f \circ g)$: $(-\infty, -1] \cup (0, \infty)$, domain $(g \circ f)$: $(-1, \infty)$
 (c) Range $(f \circ g)$: $(1, \infty)$, range $(g \circ f)$: $(0, \infty)$
18. (a) $(f \circ g)(x) = f(g(x)) = 1 - 2\sqrt{x} + x$
 $(g \circ f)(x) = g(f(x)) = 1 - |x|$
 (b) Domain $(f \circ g)$: $[0, \infty)$, domain $(g \circ f)$: $(-\infty, \infty)$
 (c) Range $(f \circ g)$: $(0, \infty)$, range $(g \circ f)$: $(-\infty, 1]$
19. $(f \circ g)(x) = x \Rightarrow f(g(x)) = x \Rightarrow \frac{g(x)}{g(x)-2} = x \Rightarrow g(x) = (g(x)-2)x = x \cdot g(x) - 2x$
 $\Rightarrow g(x) - x \cdot g(x) = -2x \Rightarrow g(x) = -\frac{2x}{1-x} = \frac{2x}{x-1}$
20. $(f \circ g)(x) = x + 2 \Rightarrow f(g(x)) = x + 2 \Rightarrow 2(g(x))^3 - 4 = x + 2 \Rightarrow (g(x))^3 = \frac{x+6}{2} \Rightarrow g(x) = \sqrt[3]{\frac{x+6}{2}}$
21. (a) $y = -(x+7)^2$ (b) $y = -(x-4)^2$
22. (a) $y = x^2 + 3$ (b) $y = x^2 - 5$
23. (a) Position 4 (b) Position 1 (c) Position 2 (d) Position 3
24. (a) $y = -(x-1)^2 + 4$ (b) $y = -(x+2)^2 + 3$ (c) $y = -(x+4)^2 - 1$ (d) $y = -(x-2)^2$

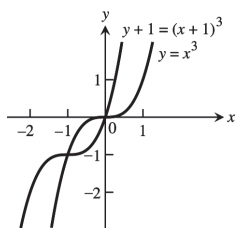
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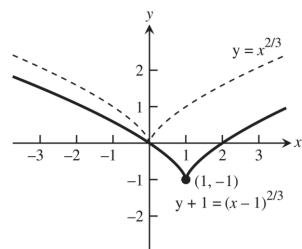
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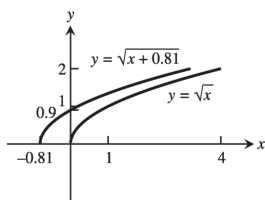
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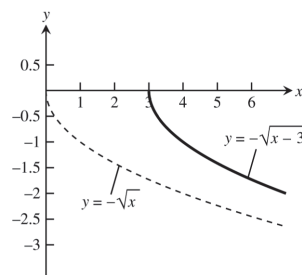
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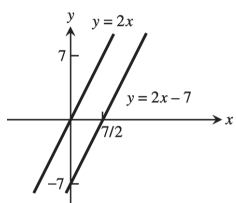
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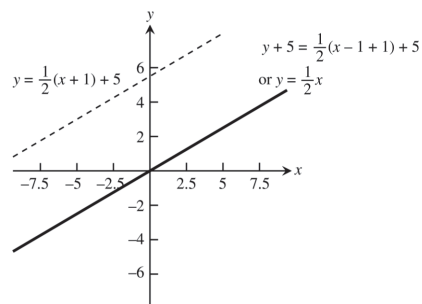
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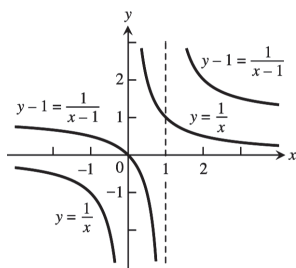
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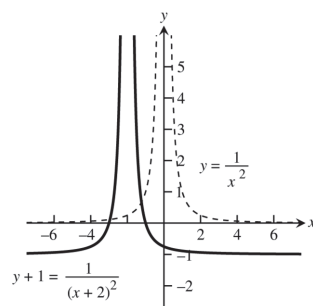
32.



33.

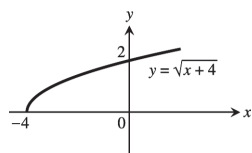


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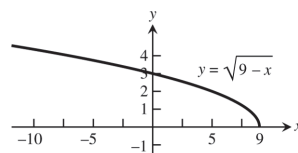


12 Chapter 1 Functions

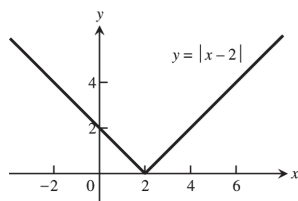
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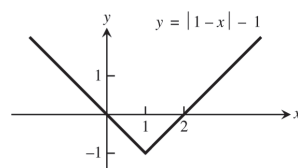
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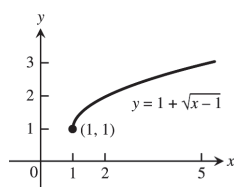
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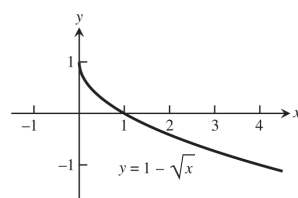
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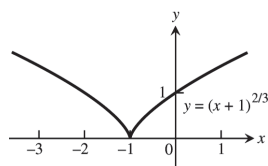
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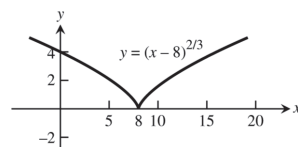
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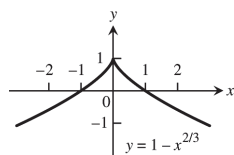
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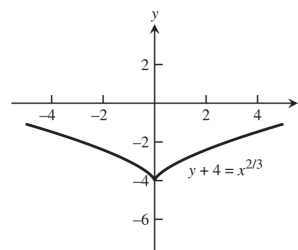
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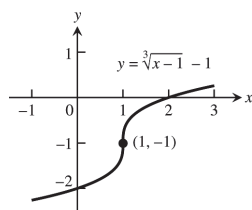
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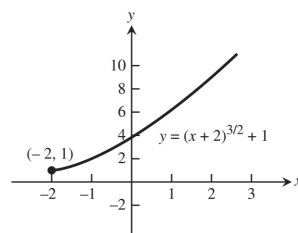
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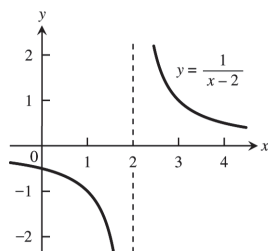
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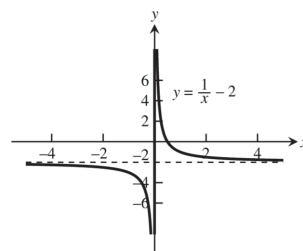
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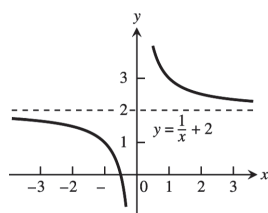
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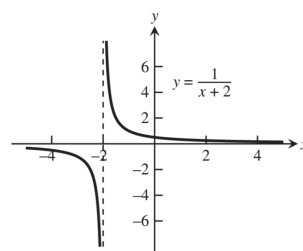
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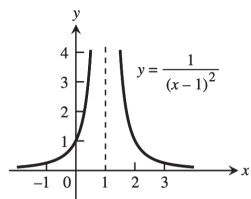
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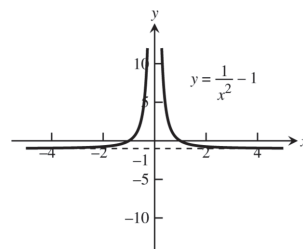
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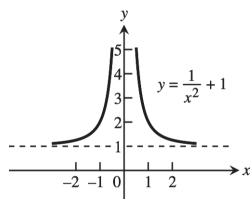
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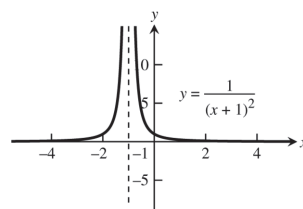
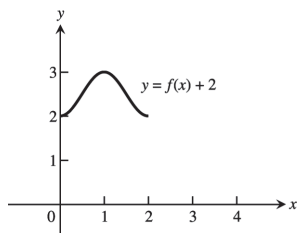
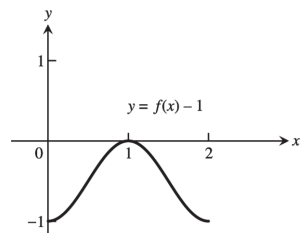
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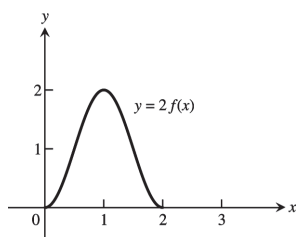
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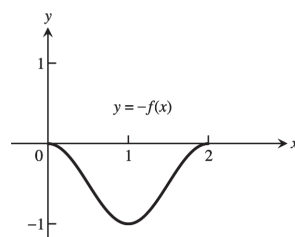
54.

55. (a) domain: $[0, 2]$; range: $[2, 3]$ (b) domain: $[0, 2]$; range: $[-1, 0]$ 

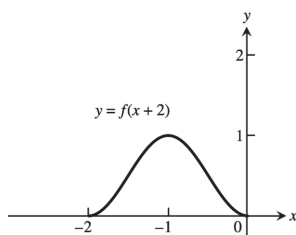
(c) domain: $[0, 2]$; range: $[0, 2]$



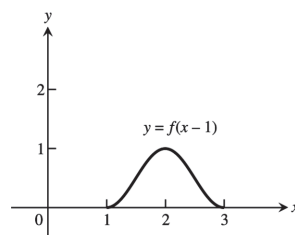
(d) domain: $[0, 2]$; range: $[-1, 0]$



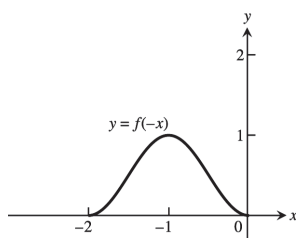
(e) domain: $[-2, 0]$; range: $[0, 1]$



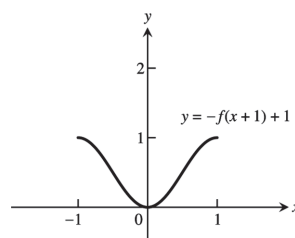
(f) domain: $[1, 3]$; range: $[0, 1]$



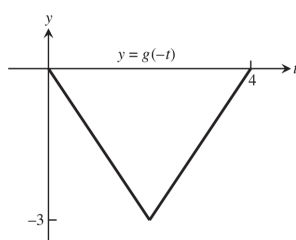
(g) domain: $[-2, 0]$; range: $[0, 1]$



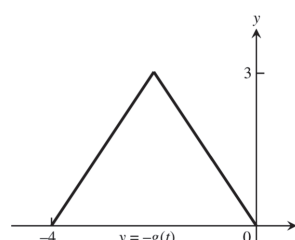
(h) domain: $[-1, 1]$; range: $[0, 1]$



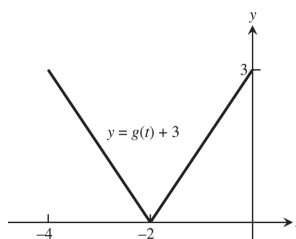
56. (a) domain: $[0, 4]$; range: $[-3, 0]$



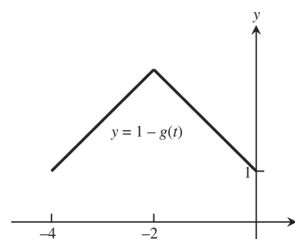
(b) domain: $[-4, 0]$; range: $[0, 3]$

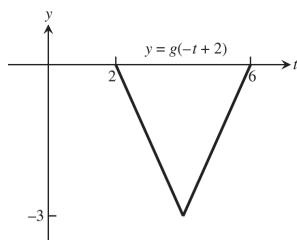
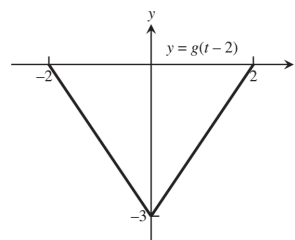
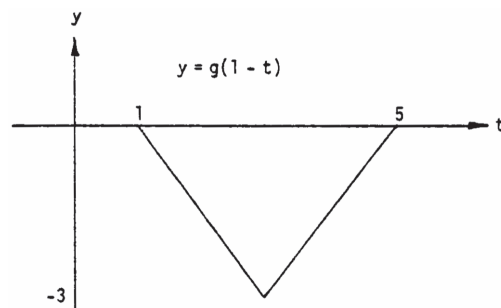
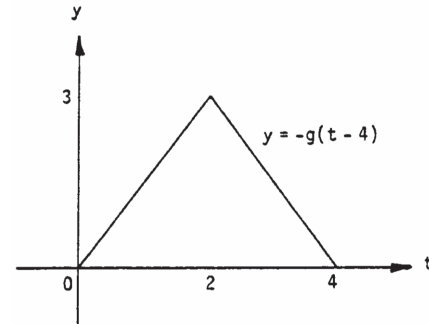


(c) domain: $[-4, 0]$; range: $[0, 3]$



(d) domain: $[-4, 0]$; range: $[1, 4]$



(e) domain: $[2, 4]$; range: $[-3, 0]$ (f) domain: $[-2, 2]$; range: $[-3, 0]$ (g) domain: $[1, 5]$; range: $[-3, 0]$ (h) domain: $[0, 4]$; range: $[0, 3]$ 

57. $y = 3x^2 - 3$

59. $y = \frac{1}{2} \left(1 + \frac{1}{x^2} \right) = \frac{1}{2} + \frac{1}{2x^2}$

61. $y = \sqrt{4x+1}$

63. $y = \sqrt{4 - \left(\frac{x}{2} \right)^2} = \frac{1}{2} \sqrt{16 - x^2}$

65. $y = 1 - (3x)^3 = 1 - 27x^3$

58. $y = (2x)^2 - 1 = 4x^2 - 1$

60. $y = 1 + \frac{1}{(x/3)^2} = 1 + \frac{9}{x^2}$

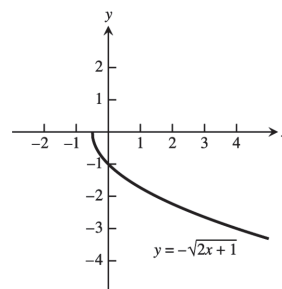
62. $y = 3\sqrt{x+1}$

64. $y = \frac{1}{3} \sqrt{4 - x^2}$

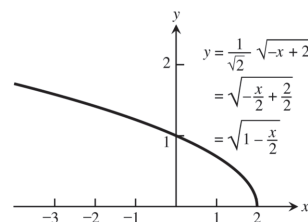
66. $y = 1 - \left(\frac{x}{2} \right)^3 = 1 - \frac{x^3}{8}$

67. Let $y = -\sqrt{2x+1} = f(x)$ and let $g(x) = x^{1/2}$,
 $h(x) = \left(x + \frac{1}{2}\right)^{1/2}$, $i(x) = \sqrt{2}\left(x + \frac{1}{2}\right)^{1/2}$, and
 $j(x) = -\left[\sqrt{2}\left(x + \frac{1}{2}\right)^{1/2}\right] = f(x)$. The graph of $h(x)$

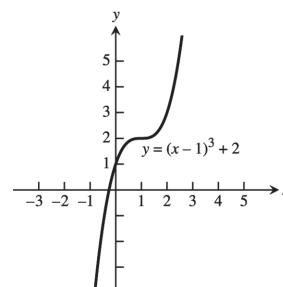
is the graph of $g(x)$ shifted left $\frac{1}{2}$ unit; the graph of $i(x)$ is the graph of $h(x)$ stretched vertically by a factor of $\sqrt{2}$; and the graph of $j(x) = f(x)$ is the graph of $i(x)$ reflected across the x -axis.



68. Let $y = \sqrt{1 - \frac{x}{2}} = f(x)$. Let $g(x) = (-x)^{1/2}$,
 $h(x) = (-x+2)^{1/2}$, and $i(x) = \frac{1}{\sqrt{2}}(-x+2)^{1/2} =$
 $\sqrt{1 - \frac{x}{2}} = f(x)$. The graph of $g(x)$ is the graph of $y = \sqrt{x}$ reflected across the x -axis. The graph of $h(x)$ is the graph of $g(x)$ shifted right two units. And the graph of $i(x)$ is the graph of $h(x)$ compressed vertically by a factor of $\sqrt{2}$.

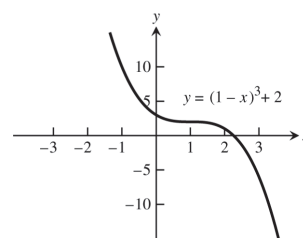


69. $y = f(x) = x^3$. Shift $f(x)$ one unit right followed by a shift two units up to get $g(x) = (x-1)^3 + 2$.

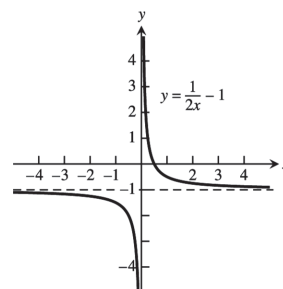


70. $y = (1-x)^3 + 2 = -[(x-1)^3 + (-2)] = f(x)$.

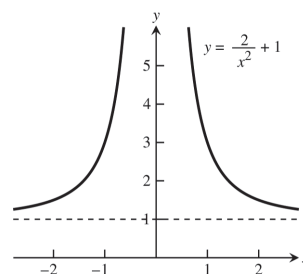
Let $g(x) = x^3$, $h(x) = (x-1)^3$, $i(x) = (x-1)^3 + (-2)$, and $j(x) = -[(x-1)^3 + (-2)]$. The graph of $h(x)$ is the graph of $g(x)$ shifted right one unit; the graph of $i(x)$ is the graph of $h(x)$ shifted down two units; and the graph of $f(x)$ is the graph of $i(x)$ reflected across the x -axis.



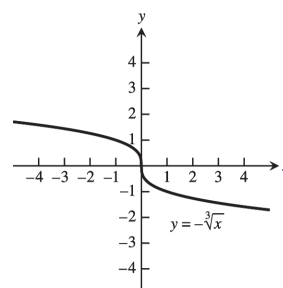
71. Compress the graph of $f(x) = \frac{1}{x}$ horizontally by a factor of 2 to get $g(x) = \frac{1}{2x}$. Then shift $g(x)$ vertically down 1 unit to get $h(x) = \frac{1}{2x} - 1$.



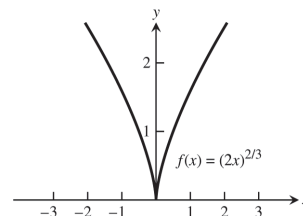
72. Let $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{2}{x^2} + 1 = \left(\frac{x^2}{2}\right) + 1$
 $= \frac{1}{(x/\sqrt{2})^2} + 1 = \frac{1}{[(1/\sqrt{2})x]^2} + 1$. Since $\sqrt{2} \approx 1.4$, we see
 that the graph of $f(x)$ stretched horizontally by a factor of 1.4 and shifted up 1 unit is the graph of $g(x)$.



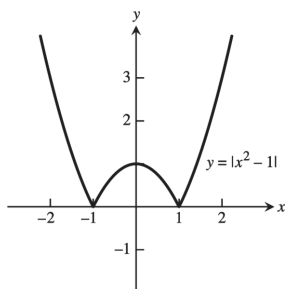
73. Reflect the graph of $y = f(x) = \sqrt[3]{x}$ across the x -axis to get $g(x) = -\sqrt[3]{x}$.



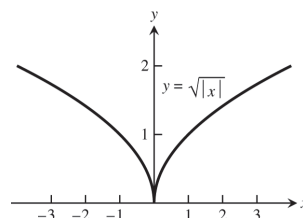
74. $y = f(x) = (-2x)^{2/3} = [(-1)(2x)]^{2/3} = (-1)^{2/3} (2x)^{2/3} = (2x)^{2/3}$. So the graph of $f(x)$ is the graph of $g(x) = x^{2/3}$ compressed horizontally by a factor of 2.



75.



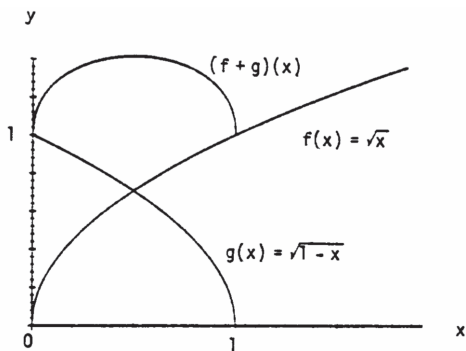
76.



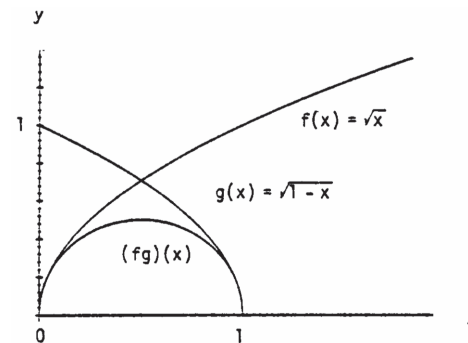
77. (a) $(fg)(-x) = f(-x)g(-x) = f(x)(-g(x)) = -(fg)(x)$, odd
 (b) $\left(\frac{f}{g}\right)(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\left(\frac{f}{g}\right)(x)$, odd
 (c) $\left(\frac{g}{f}\right)(-x) = \frac{g(-x)}{f(-x)} = \frac{-g(x)}{f(x)} = -\left(\frac{g}{f}\right)(x)$, odd
 (d) $f^2(-x) = f(-x)f(-x) = f(x)f(x) = f^2(x)$, even
 (e) $g^2(-x) = (g(-x))^2 = (-g(x))^2 = g^2(x)$, even
 (f) $(f \circ g)(-x) = f(g(-x)) = f(-g(x)) = f(g(x)) = (f \circ g)(x)$, even
 (g) $(g \circ f)(-x) = g(f(-x)) = g(f(x)) = (g \circ f)(x)$, even
 (h) $(f \circ f)(-x) = f(f(-x)) = f(f(x)) = (f \circ f)(x)$, even
 (i) $(g \circ g)(-x) = g(g(-x)) = g(-g(x)) = -g(g(x)) = -(g \circ g)(x)$, odd

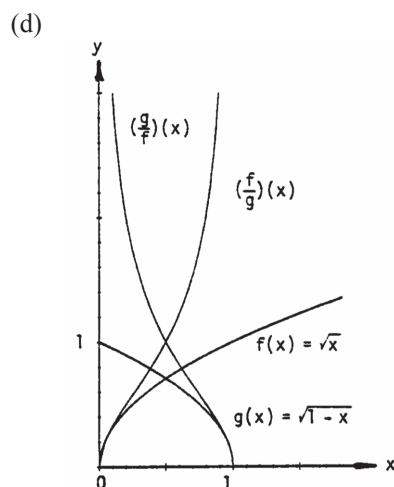
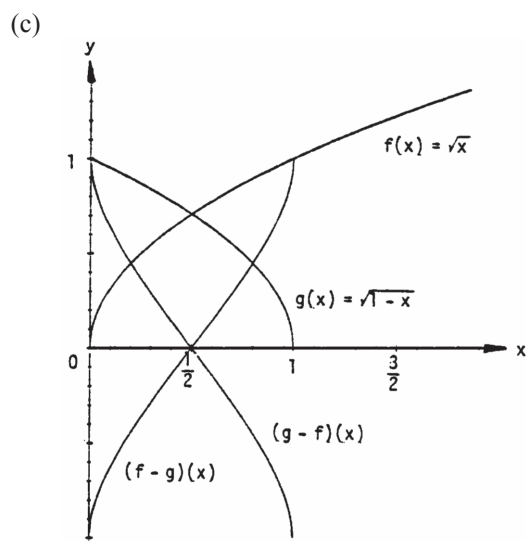
78. Yes, $f(x) = 0$ is both even and odd since $f(-x) = 0 = f(x)$ and $f(-x) = 0 = -f(x)$.

79. (a)

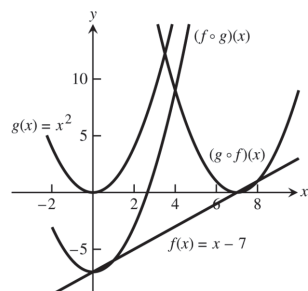


(b)





80.



1.3 TRIGONOMETRIC FUNCTIONS

- (a) $s = r\theta = (10)\left(\frac{4\pi}{5}\right) = 8\pi$ m (b) $s = r\theta = (10)(110^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{110\pi}{18} = \frac{55\pi}{9}$ m
- $\theta = \frac{s}{r} = \frac{10\pi}{8} = \frac{5\pi}{4}$ radians and $\frac{5\pi}{4}\left(\frac{180^\circ}{\pi}\right) = 225^\circ$
- $\theta = 80^\circ \Rightarrow \theta = 80^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{4\pi}{9} \Rightarrow s = (6)\left(\frac{4\pi}{9}\right) = 8.4$ in. (since the diameter = 12 in. \Rightarrow radius = 6 in.)
- $d = 1$ meter $\Rightarrow r = 50$ cm $\Rightarrow \theta = \frac{s}{r} = \frac{30}{50} = 0.6$ rad or $0.6\left(\frac{180^\circ}{\pi}\right) \approx 34^\circ$

5.

θ	$-\pi$	$-\frac{2\pi}{3}$	0	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$\sin \theta$	0	$-\frac{\sqrt{3}}{2}$	0	1	$\frac{1}{\sqrt{2}}$
$\cos \theta$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{\sqrt{2}}$
$\tan \theta$	0	$\sqrt{3}$	0	und.	-1
$\cot \theta$	und.	$\frac{1}{\sqrt{3}}$	und.	0	-1
$\sec \theta$	-1	-2	1	und.	$-\sqrt{2}$
$\csc \theta$	und.	$-\frac{2}{\sqrt{3}}$	und.	1	$\sqrt{2}$

6.

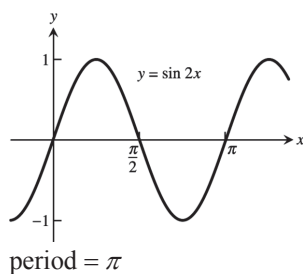
θ	$-\frac{3\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{5\pi}{6}$
$\sin \theta$	1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\cos \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$
$\tan \theta$	und.	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	1	$-\frac{1}{\sqrt{3}}$
$\cot \theta$	0	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	1	$-\sqrt{3}$
$\sec \theta$	und.	2	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	$-\frac{2}{\sqrt{3}}$
$\csc \theta$	1	$-\frac{2}{\sqrt{3}}$	-2	$\sqrt{2}$	2

7. $\cos x = -\frac{4}{5}, \tan x = -\frac{3}{4}$

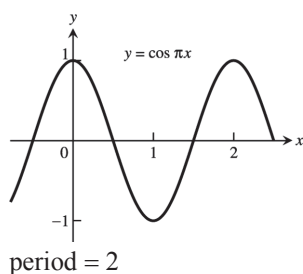
9. $\sin x = -\frac{\sqrt{8}}{3}, \tan x = -\sqrt{8}$

11. $\sin x = -\frac{1}{\sqrt{5}}, \cos x = -\frac{2}{\sqrt{5}}$

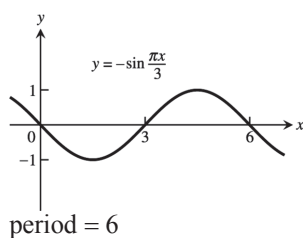
13.



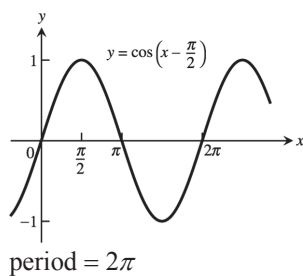
15.



17.



19.

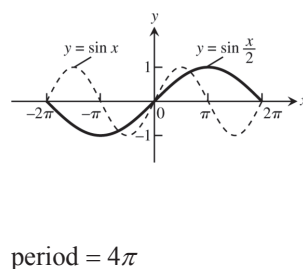


8. $\sin x = \frac{2}{\sqrt{5}}, \cos x = \frac{1}{\sqrt{5}}$

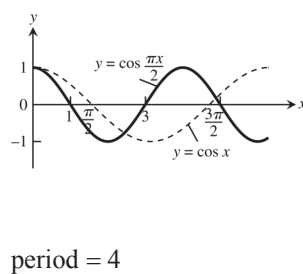
10. $\sin x = \frac{12}{13}, \tan x = -\frac{12}{5}$

12. $\cos x = -\frac{\sqrt{3}}{2}, \tan x = \frac{1}{\sqrt{3}}$

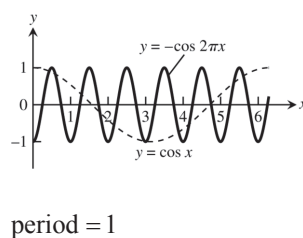
14.



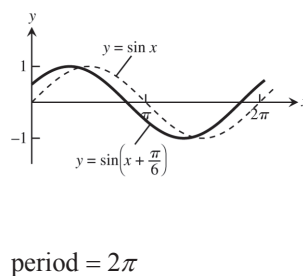
16.



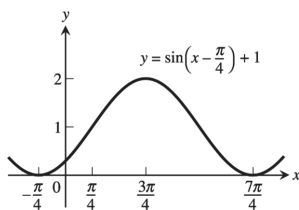
18.



20.

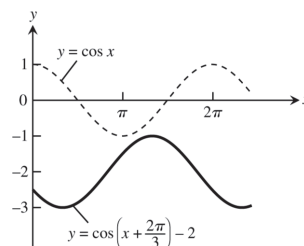


21.



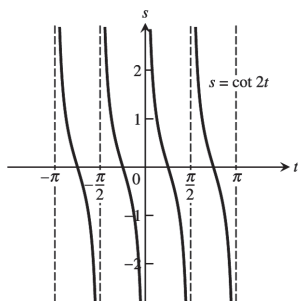
period = 2π

22.

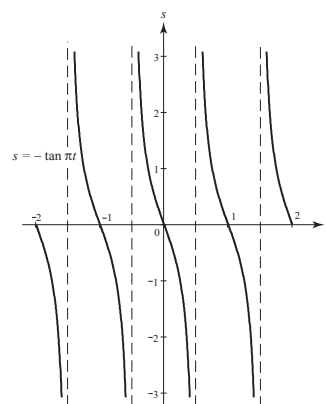


period = 2π

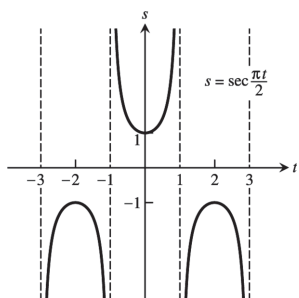
23. period = $\frac{\pi}{2}$, symmetric about the origin



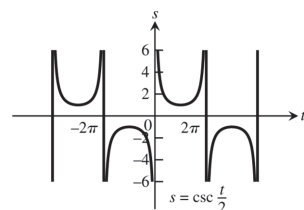
24. period = 1, symmetric about the origin



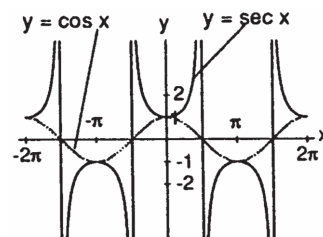
25. period = 4, symmetric about the s-axis



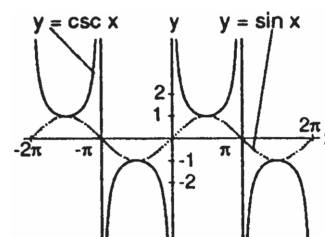
26. period = 4π , symmetric about the origin



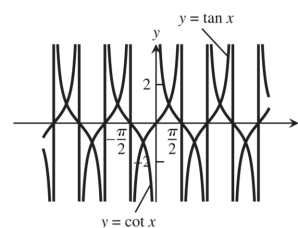
27. (a) $\cos x$ and $\sec x$ are positive for x in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$; and $\cos x$ and $\sec x$ are negative for x in the intervals $(-\frac{3\pi}{2}, -\frac{\pi}{2})$ and $(\frac{\pi}{2}, \frac{3\pi}{2})$. $\sec x$ is undefined when $\cos x$ is 0. The range of $\sec x$ is $(-\infty, -1] \cup [1, \infty)$; the range of $\cos x$ is $[-1, 1]$.



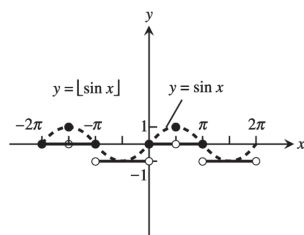
- (b) $\sin x$ and $\csc x$ are positive for x in the intervals $(-\frac{3\pi}{2}, -\pi)$ and $(0, \pi)$; and $\sin x$ and $\csc x$ are negative for x in the intervals $(-\pi, 0)$ and $(\pi, \frac{3\pi}{2})$. $\csc x$ is undefined when $\sin x$ is 0. The range of $\csc x$ is $(-\infty, -1] \cup [1, \infty)$; the range of $\sin x$ is $[-1, 1]$.



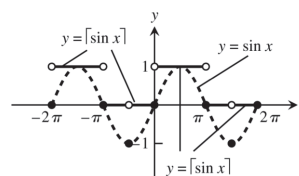
28. Since $\cot x = \frac{1}{\tan x}$, $\cot x$ is undefined when $\tan x = 0$ and is zero when $\tan x$ is undefined. As $\tan x$ approaches zero through positive values, $\cot x$ approaches infinity. Also, $\cot x$ approaches negative infinity as $\tan x$ approaches zero through negative values.



29. $D: -\infty < x < \infty$; $R: y = -1, 0, 1$



30. $D: -\infty < x < \infty$; $R: y = -1, 0, 1$



31. $\cos\left(x - \frac{\pi}{2}\right) = \cos x \cos\left(-\frac{\pi}{2}\right) - \sin x \sin\left(-\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(-1) = \sin x$
32. $\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos\left(\frac{\pi}{2}\right) - \sin x \sin\left(\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(1) = -\sin x$
33. $\sin\left(x + \frac{\pi}{2}\right) = \sin x \cos\left(\frac{\pi}{2}\right) + \cos x \sin\left(\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(1) = \cos x$
34. $\sin\left(x - \frac{\pi}{2}\right) = \sin x \cos\left(-\frac{\pi}{2}\right) + \cos x \sin\left(-\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(-1) = -\cos x$
35. $\cos(A - B) = \cos(A + (-B)) = \cos A \cos(-B) - \sin A \sin(-B) = \cos A \cos B - \sin A(-\sin B)$
 $= \cos A \cos B + \sin A \sin B$
36. $\sin(A - B) = \sin(A + (-B)) = \sin A \cos(-B) + \cos A \sin(-B) = \sin A \cos B + \cos A(-\sin B)$
 $= \sin A \cos B - \cos A \sin B$
37. If $B = A$, $A - B = 0 \Rightarrow \cos(A - B) = \cos 0 = 1$. Also $\cos(A - B) = \cos(A - A) = \cos A \cos A + \sin A \sin A$
 $= \cos^2 A + \sin^2 A$. Therefore, $\cos^2 A + \sin^2 A = 1$.
38. If $B = 2\pi$, then $\cos(A + 2\pi) = \cos A \cos 2\pi - \sin A \sin 2\pi = (\cos A)(1) - (\sin A)(0) = \cos A$ and
 $\sin(A + 2\pi) = \sin A \cos 2\pi + \cos A \sin 2\pi = (\sin A)(1) + (\cos A)(0) = \sin A$. The result agrees with the fact that the cosine and sine functions have period 2π .
39. $\cos(\pi + x) = \cos \pi \cos x - \sin \pi \sin x = (-1)(\cos x) - (0)(\sin x) = -\cos x$

$$40. \sin(2\pi - x) = \sin 2\pi \cos(-x) + \cos(2\pi) \sin(-x) = (0)(\cos(-x)) + (1)(\sin(-x)) = -\sin x$$

$$41. \sin\left(\frac{3\pi}{2} - x\right) = \sin\left(\frac{3\pi}{2}\right)\cos(-x) + \cos\left(\frac{3\pi}{2}\right)\sin(-x) = (-1)(\cos x) + (0)(\sin(-x)) = -\cos x$$

$$42. \cos\left(\frac{3\pi}{2} + x\right) = \cos\left(\frac{3\pi}{2}\right)\cos x - \sin\left(\frac{3\pi}{2}\right)\sin x = (0)(\cos x) - (-1)(\sin x) = \sin x$$

$$43. \sin \frac{7\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin \frac{\pi}{4} \cos \frac{\pi}{3} + \cos \frac{\pi}{4} \sin \frac{\pi}{3} = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$44. \cos \frac{11\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) = \cos \frac{\pi}{4} \cos \frac{2\pi}{3} - \sin \frac{\pi}{4} \sin \frac{2\pi}{3} = \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2} + \sqrt{6}}{4}$$

$$45. \cos \frac{\pi}{12} = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos \frac{\pi}{3} \cos\left(-\frac{\pi}{4}\right) - \sin \frac{\pi}{3} \sin\left(-\frac{\pi}{4}\right) = \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = \frac{1 + \sqrt{3}}{2\sqrt{2}}$$

$$46. \sin \frac{5\pi}{12} = \sin\left(\frac{2\pi}{3} - \frac{\pi}{4}\right) = \sin\left(\frac{2\pi}{3}\right)\cos\left(-\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{3}\right)\sin\left(-\frac{\pi}{4}\right) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = \frac{1 + \sqrt{3}}{2\sqrt{2}}$$

$$47. \cos^2 \frac{\pi}{8} = \frac{1 + \cos\left(\frac{2\pi}{8}\right)}{2} = \frac{1 + \frac{\sqrt{2}}{2}}{2} = \frac{2 + \sqrt{2}}{4}$$

$$48. \cos^2 \frac{5\pi}{12} = \frac{1 + \cos\left(\frac{10\pi}{12}\right)}{2} = \frac{1 + \left(-\frac{\sqrt{3}}{2}\right)}{2} = \frac{2 - \sqrt{3}}{4}$$

$$49. \sin^2 \frac{\pi}{12} = \frac{1 - \cos\left(\frac{2\pi}{12}\right)}{2} = \frac{1 - \frac{\sqrt{3}}{2}}{2} = \frac{2 - \sqrt{3}}{4}$$

$$50. \sin^2 \frac{3\pi}{8} = \frac{1 - \cos\left(\frac{6\pi}{8}\right)}{2} = \frac{1 - \left(-\frac{\sqrt{2}}{2}\right)}{2} = \frac{2 + \sqrt{2}}{4}$$

$$51. \sin^2 \theta = \frac{3}{4} \Rightarrow \sin \theta = \pm \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$52. \sin^2 \theta = \cos^2 \theta \Rightarrow \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{\cos^2 \theta}{\cos^2 \theta} \Rightarrow \tan^2 \theta = 1 \Rightarrow \tan \theta = \pm 1 \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$53. \sin 2\theta - \cos \theta = 0 \Rightarrow 2 \sin \theta \cos \theta - \cos \theta = 0 \Rightarrow \cos \theta (2 \sin \theta - 1) = 0 \Rightarrow \cos \theta = 0 \text{ or } 2 \sin \theta - 1 = 0 \\ \Rightarrow \cos \theta = 0 \text{ or } \sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \text{ or } \theta = \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

$$54. \cos 2\theta + \cos \theta = 0 \Rightarrow 2 \cos^2 \theta - 1 + \cos \theta = 0 \Rightarrow 2 \cos^2 \theta + \cos \theta - 1 = 0 \Rightarrow (\cos \theta + 1)(2 \cos \theta - 1) = 0 \\ \Rightarrow \cos \theta + 1 = 0 \text{ or } 2 \cos \theta - 1 = 0 \Rightarrow \cos \theta = -1 \text{ or } \cos \theta = \frac{1}{2} \Rightarrow \theta = \pi \text{ or } \theta = \frac{\pi}{3}, \frac{5\pi}{3} \Rightarrow \theta = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

$$55. \tan(A + B) = \frac{\sin(A + B)}{\cos(A + B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} = \frac{\frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} - \frac{\sin A \sin B}{\cos A \cos B}} = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$56. \tan(A - B) = \frac{\sin(A - B)}{\cos(A - B)} = \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B + \sin A \sin B} = \frac{\frac{\sin A \cos B}{\cos A \cos B} - \frac{\cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} + \frac{\sin A \sin B}{\cos A \cos B}} = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$57. \text{ According to the figure in the text, we have the following: By the law of cosines, } c^2 = a^2 + b^2 - 2ab \cos \theta \\ = 1^2 + 1^2 - 2 \cos(A - B) = 2 - 2 \cos(A - B). \text{ By distance formula, } c^2 = (\cos A - \cos B)^2 + (\sin A - \sin B)^2 \\ = \cos^2 A - 2 \cos A \cos B + \cos^2 B + \sin^2 A - 2 \sin A \sin B + \sin^2 B = 2 - 2(\cos A \cos B + \sin A \sin B). \text{ Thus } \\ c^2 = 2 - 2 \cos(A - B) = 2 - 2(\cos A \cos B + \sin A \sin B) \Rightarrow \cos(A - B) = \cos A \cos B + \sin A \sin B.$$

58. (a) $\cos(A - B) = \cos A \cos B + \sin A \sin B$
 $\sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)$ and $\cos \theta = \sin\left(\frac{\pi}{2} - \theta\right)$
 Let $\theta = A + B$
 $\sin(A + B) = \cos\left[\frac{\pi}{2} - (A + B)\right] = \cos\left[\left(\frac{\pi}{2} - A\right) - B\right] = \cos\left(\frac{\pi}{2} - A\right) \cos B + \sin\left(\frac{\pi}{2} - A\right) \sin B$
 $= \sin A \cos B + \cos A \sin B$
- (b) $\cos(A - B) = \cos A \cos B + \sin A \sin B$
 $\cos(A - (-B)) = \cos A \cos(-B) + \sin A \sin(-B)$
 $\Rightarrow \cos(A + B) = \cos A \cos(-B) + \sin A \sin(-B) = \cos A \cos B + \sin A(-\sin B) = \cos A \cos B - \sin A \sin B$
 Because the cosine function is even and the sine functions is odd.

59. $c^2 = a^2 + b^2 - 2ab \cos C = 2^2 + 3^2 - 2(2)(3) \cos(60^\circ) = 4 + 9 - 12 \cos(60^\circ) = 13 - 12\left(\frac{1}{2}\right) = 7$.
 Thus, $c = \sqrt{7} \approx 2.65$.

60. $c^2 = a^2 + b^2 - 2ab \cos C = 2^2 + 3^2 - 2(2)(3) \cos(40^\circ) = 13 - 12 \cos(40^\circ)$. Thus, $c = \sqrt{13 - 12 \cos 40^\circ} \approx 1.951$.

61. From the figures in the text, we see that $\sin B = \frac{h}{c}$. If C is an acute angle, then $\sin C = \frac{h}{b}$. On the other hand, if C is obtuse (as in the figure on the right in the text), then $\sin C = \sin(\pi - C) = \frac{h}{b}$. Thus, in either case,
 $h = b \sin C = c \sin B \Rightarrow ah = ab \sin C = ac \sin B$.

By the law of cosines, $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ and $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$. Moreover, since the sum of the interior angles of triangle is π , we have $\sin A = \sin(\pi - (B + C)) = \sin(B + C) = \sin B \cos C + \cos B \sin C$
 $= \left(\frac{h}{c}\right) \left[\frac{a^2 + b^2 - c^2}{2ab}\right] + \left[\frac{a^2 + c^2 - b^2}{2ac}\right] \left(\frac{h}{b}\right) = \left(\frac{h}{2abc}\right)(2a^2 + b^2 - c^2 + c^2 - b^2) = \frac{ah}{bc} \Rightarrow ah = bc \sin A$.

Combining our results we have $ah = ab \sin C$, $ah = ac \sin B$, and $ah = bc \sin A$. Dividing by abc gives
 $\frac{h}{bc} = \frac{\sin A}{a} = \frac{\sin C}{c} = \frac{\sin B}{b}$.
 law of sines

62. By the law of sines, $\frac{\sin A}{2} = \frac{\sin B}{3} = \frac{\sqrt{3}/2}{c}$. By Exercise 59 we know that $c = \sqrt{7}$. Thus $\sin B = \frac{3\sqrt{3}}{2\sqrt{7}} \approx 0.982$.

63. From the figure at the right and the law of cosines,
 $b^2 = a^2 + 2^2 - 2(2a) \cos B$

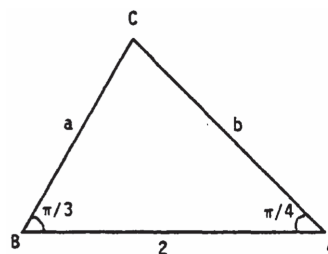
$$= a^2 + 4 - 4a\left(\frac{1}{2}\right) = a^2 - 2a + 4.$$

Applying the law of sines to the figure, $\frac{\sin A}{a} = \frac{\sin B}{b}$

$$\Rightarrow \frac{\sqrt{2}/2}{a} = \frac{\sqrt{3}/2}{b} \Rightarrow b = \frac{\sqrt{3}}{2}a. \text{ Thus, combining results,}$$

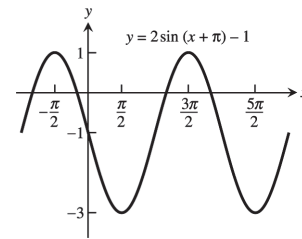
$$a^2 - 2a + 4 = b^2 = \frac{3}{2}a^2 \Rightarrow 0 = \frac{1}{2}a^2 + 2a - 4 \Rightarrow 0 = a^2 + 4a - 8. \text{ From the quadratic formula and the fact that}$$

$$a > 0, \text{ we have } a = \frac{-4 + \sqrt{4^2 - 4(1)(-8)}}{2} = \frac{4\sqrt{3} - 4}{2} \approx 1.464.$$

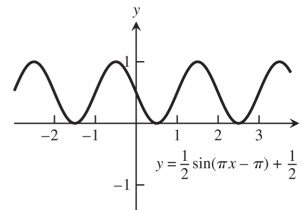


64. (a) The graphs of $y = \sin x$ and $y = x$ nearly coincide when x is near the origin (when the calculator is in radians mode).
 (b) In degree mode, when x is near zero degrees the sine of x is much closer to zero than x itself. The curves look like intersecting straight lines near the origin when the calculator is in degree mode.

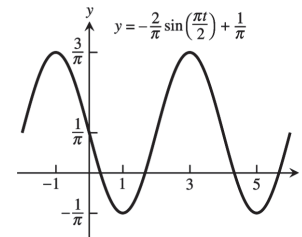
65. $A = 2, B = 2\pi, C = -\pi, D = -1$



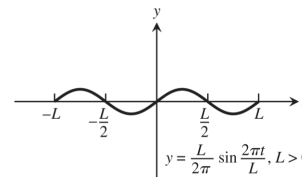
66. $A = \frac{1}{2}, B = 2, C = 1, D = \frac{1}{2}$



67. $A = -\frac{2}{\pi}, B = 4, C = 0, D = \frac{1}{\pi}$



68. $A = \frac{L}{2\pi}, B = L, C = 0, D = 0$



69–72. Example CAS commands:

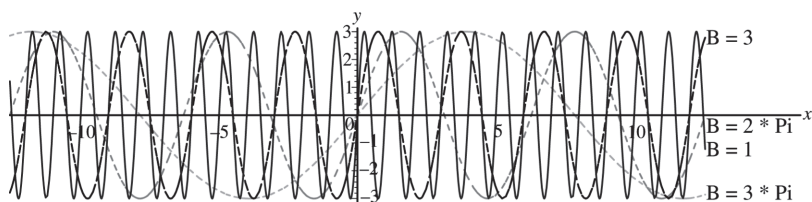
Maple:

```
f:=x->A*sin((2*Pi/B)*(x-C))+D1;
A:=3; C:=0; D1:=0;
f_list:= [seq(f(x), B=[1,3,2*Pi,5*Pi])];
plot(f_list, x=-4*Pi..4*Pi, scaling=constrained,
      color=[red,blue,green,cyan], linestyle=[1,3,4,7],
      legend=["B=1", "B=3", "B=2*Pi", "B=3*Pi"],
      title="#69 (Section 1.3)");
```

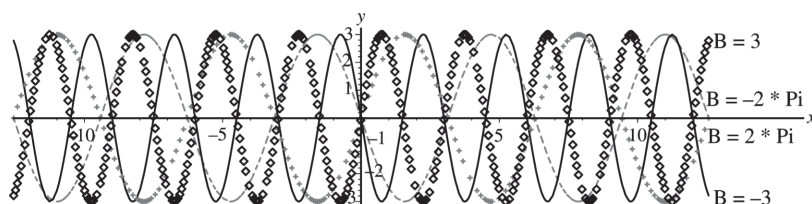
Mathematica:

```
Clear[a, b, c, d, f, x]
f[x_]:=a Sin[2Pi/b (x - c)]+d
Plot[f[x]/. {a -> 3, b -> 1, c -> 0, d -> 0}, {x, -4Pi, 4Pi}]
```

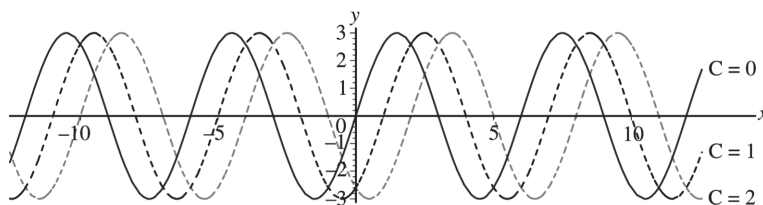
69. (a) The graph stretches horizontally.



- (b) The period remains the same: $\text{period} = |B|$. The graph has a horizontal shift of $\frac{1}{2}$ period.



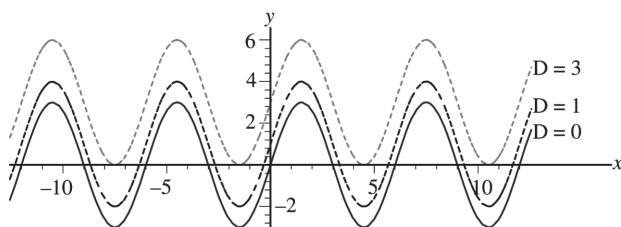
70. (a) The graph is shifted right C units.



- (b) The graph is shifted left C units.
 (c) A shift of \pm one period will produce no apparent shift. $|C| = 6$

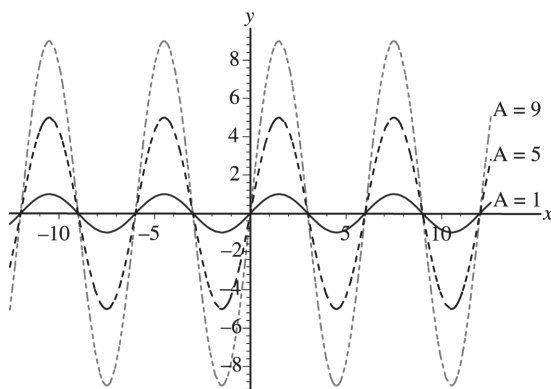
71. (a) The graph shifts upwards $|D|$ units for $D > 0$

- (b) The graph shifts down $|D|$ units for $D < 0$.



72. (a) The graph stretches $|A|$ units.

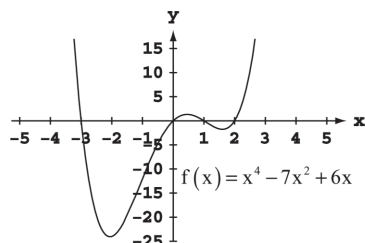
- (b) For $A < 0$, the graph is inverted.



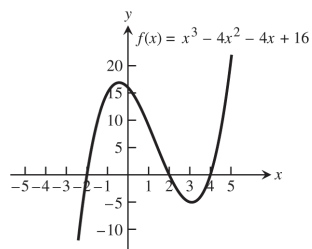
1.4 GRAPHING WITH SOFTWARE

1–4. The most appropriate viewing window displays the maxima, minima, intercepts, and end behavior of the graphs and has little unused space.

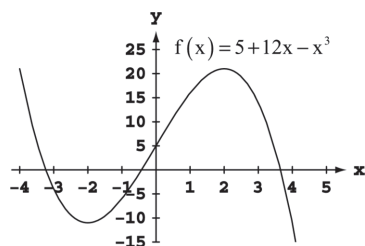
1. d.



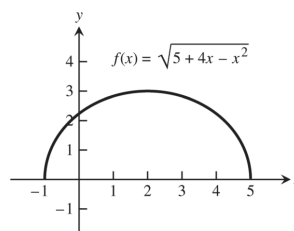
2. c.



3. d.

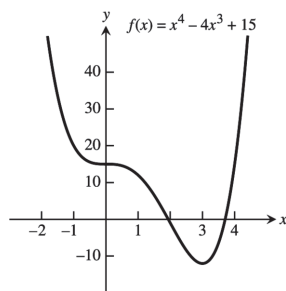


4. b.

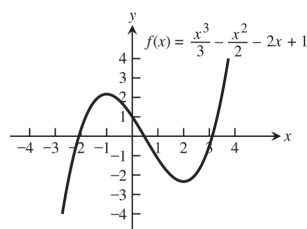


5–30. For any display there are many appropriate display windows. The graphs given as answers in Exercises 5–30 are not unique in appearance.

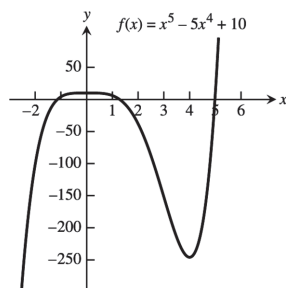
5. $[-2, 5]$ by $[-15, 40]$



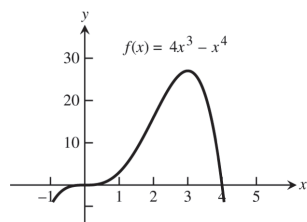
6. $[-4, 4]$ by $[-4, 4]$

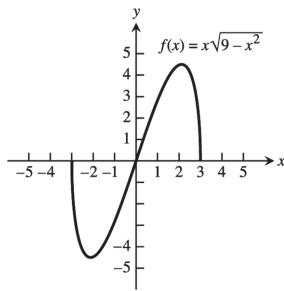
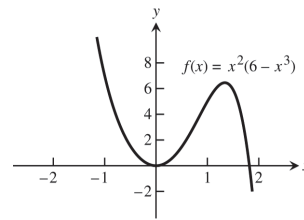
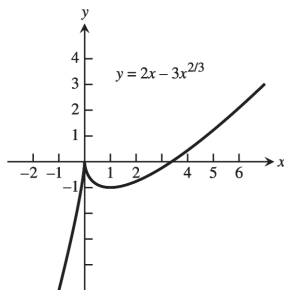
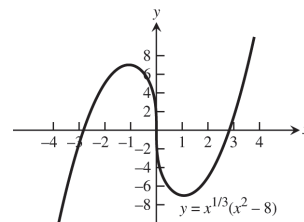
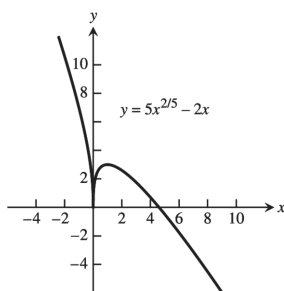
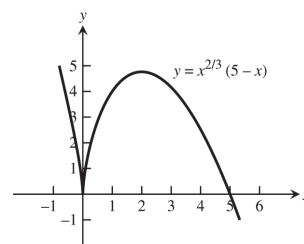
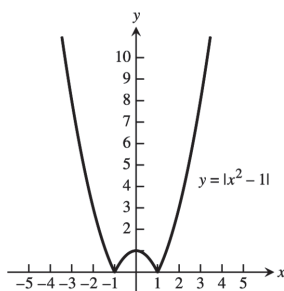
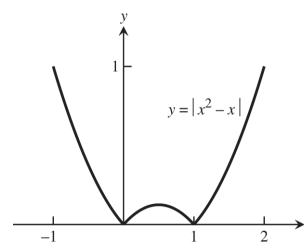


7. $[-2, 6]$ by $[-250, 50]$

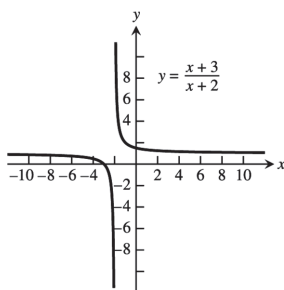


8. $[-1, 5]$ by $[-5, 30]$

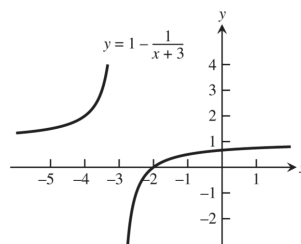


9. $[-4, 4]$ by $[-5, 5]$ 10. $[-2, 2]$ by $[-2, 8]$ 11. $[-2, 6]$ by $[-5, 4]$ 12. $[-4, 4]$ by $[-8, 8]$ 13. $[-1, 6]$ by $[-1, 4]$ 14. $[-1, 6]$ by $[-1, 5]$ 15. $[-3, 3]$ by $[0, 10]$ 16. $[-1, 2]$ by $[0, 1]$ 

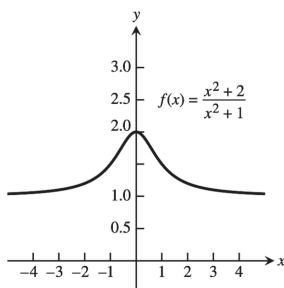
17. $[-5, 1]$ by $[-5, 5]$



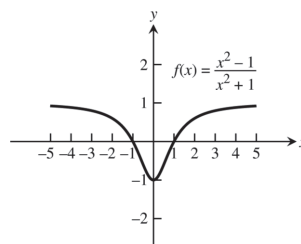
18. $[-5, 1]$ by $[-2, 4]$



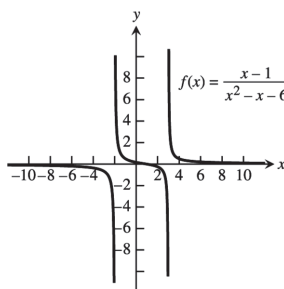
19. $[-4, 4]$ by $[0, 3]$



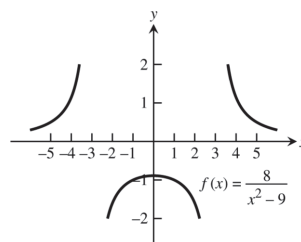
20. $[-5, 5]$ by $[-2, 2]$



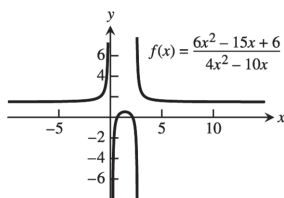
21. $[-10, 10]$ by $[-6, 6]$



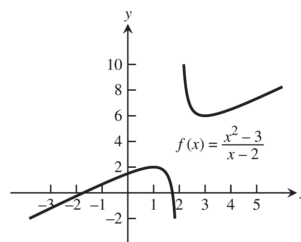
22. $[-5, 5]$ by $[-2, 2]$



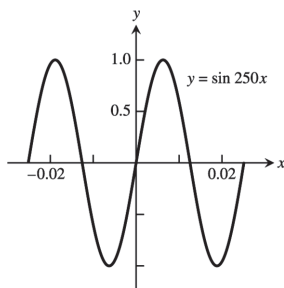
23. $[-6, 10]$ by $[-6, 6]$



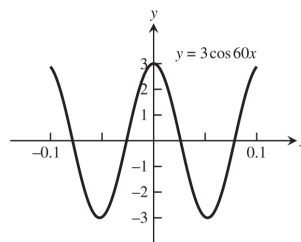
24. $[-3, 5]$ by $[-2, 10]$



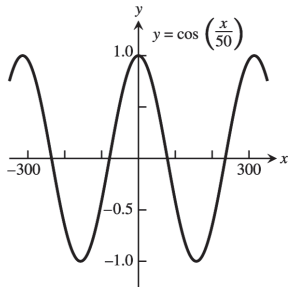
25. $[-0.03, 0.03]$ by $[-1.25, 1.25]$



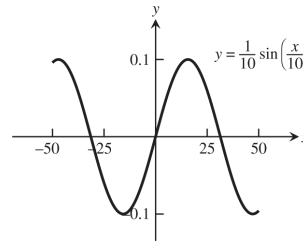
26. $[-0.1, 0.1]$ by $[-3, 3]$



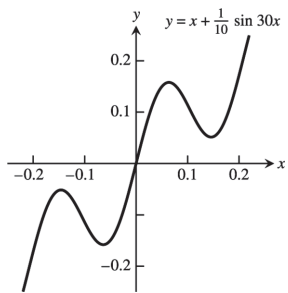
27. $[-300, 300]$ by $[-1.25, 1.25]$



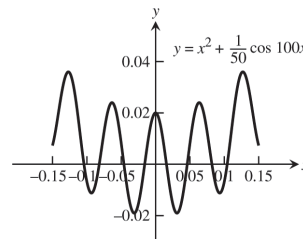
28. $[-50, 50]$ by $[-0.1, 0.1]$



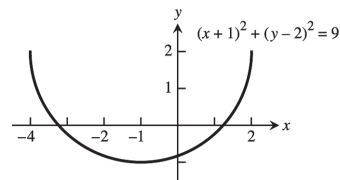
29. $[-0.25, 0.25]$ by $[-0.3, 0.3]$



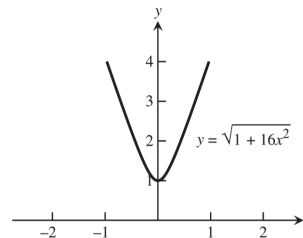
30. $[-0.15, 0.15]$ by $[-0.02, 0.05]$



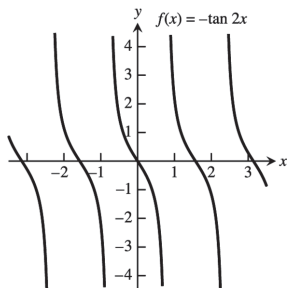
31. $x^2 + 2x = 4 + 4y - y^2 \Rightarrow y = 2 \pm \sqrt{-x^2 - 2x + 8}$.
The lower half is produced by graphing
 $y = 2 - \sqrt{-x^2 - 2x + 8}$.



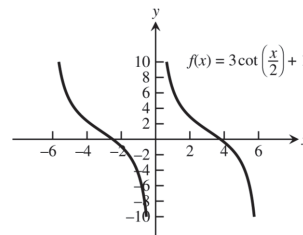
32. $y^2 - 16x^2 = 1 \Rightarrow y = \pm \sqrt{1 + 16x^2}$. The upper branch
is produced by graphing $y = \sqrt{1 + 16x^2}$.



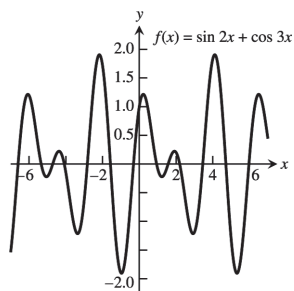
33.



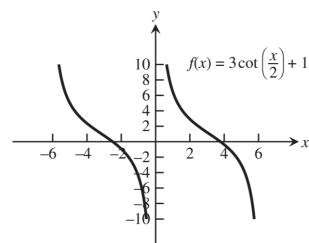
34.



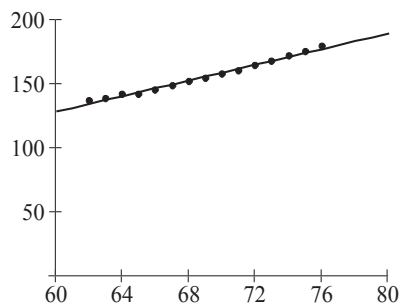
35.



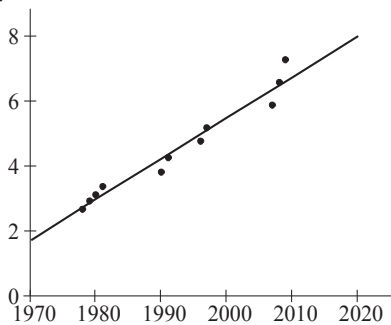
36.



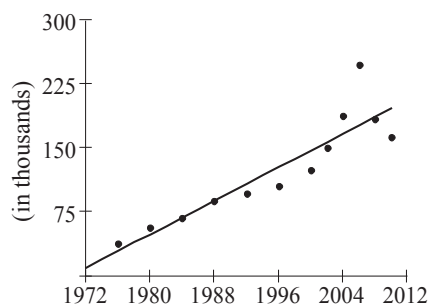
37.



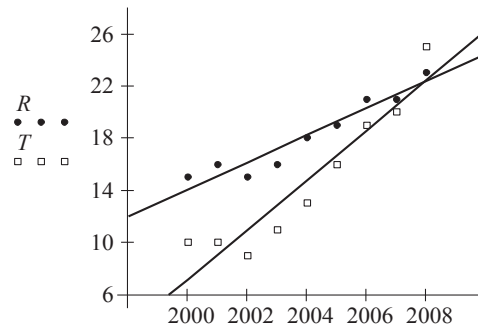
38.



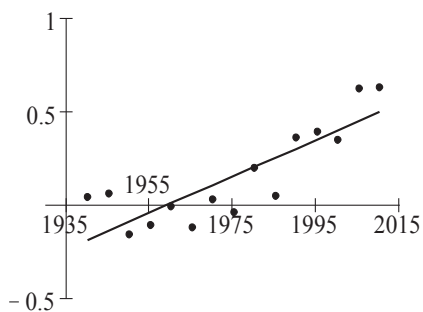
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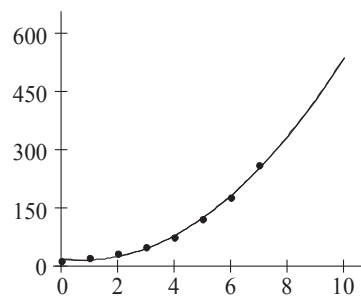
40.



41.

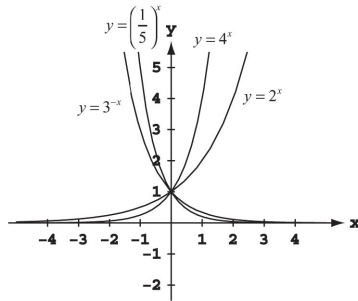


42.

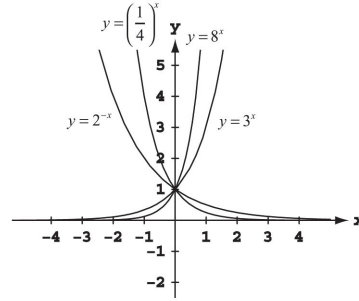


1.5 EXPONENTIAL FUNCTIONS

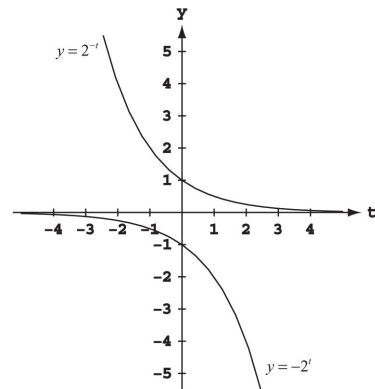
1.



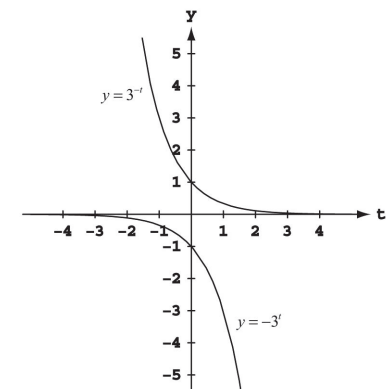
2.



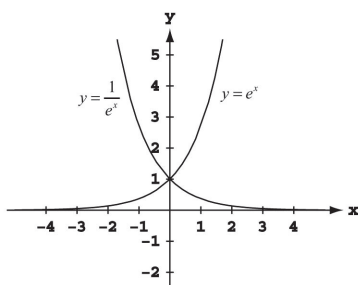
3.



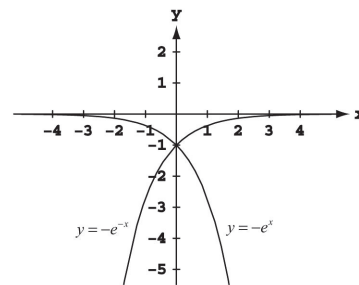
4.



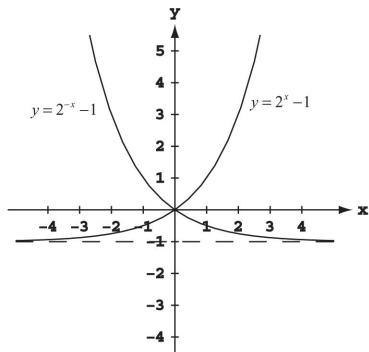
5.



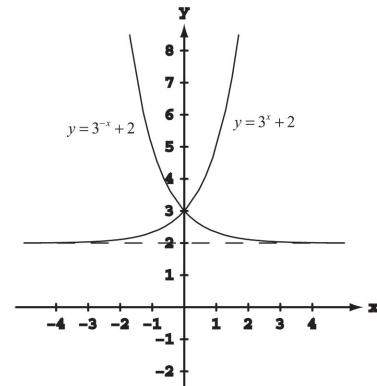
6.



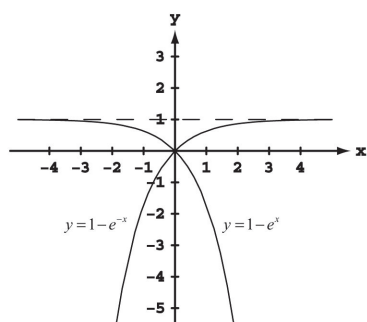
7.



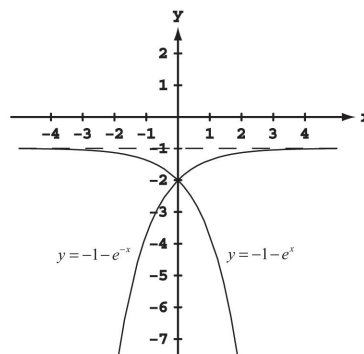
8.



9.



10.



$$11. \quad 16^2 \cdot 16^{-1.75} = 16^{2+(-1.75)} = 16^{0.25} = 16^{1/4} = 2$$

$$12. \quad 9^{1/3} \cdot 9^{1/6} = 9^{\frac{1}{3} + \frac{1}{6}} = 9^{1/2} = 3$$

$$13. \quad \frac{4^{4.2}}{4^{3.7}} = 4^{4.2-3.7} = 4^{0.5} = 4^{1/2} = 2$$

$$14. \quad \frac{3^{5/3}}{3^{2/3}} = 3^{\frac{5}{3} - \frac{2}{3}} = 3^1 = 3$$

$$15. \quad (25^{1/8})^4 = 25^{4/8} = 25^{1/2} = 5$$

$$16. \quad (13^{\sqrt{2}})^{\sqrt{2}/2} = 13^{2/2} = 13$$

$$17. \quad 2^{\sqrt{3}} \cdot 7^{\sqrt{3}} = (2 \cdot 7)^{\sqrt{3}} = 14^{\sqrt{3}}$$

$$18. \quad (\sqrt{3})^{1/2} (\sqrt{12})^{1/2} = (\sqrt{3} \cdot \sqrt{12})^{1/2} = (\sqrt{36})^{1/2} = 6^{1/2}$$

$$19. \quad \left(\frac{2}{\sqrt{2}}\right)^4 = \frac{2^4}{(2^{1/2})^4} = \frac{16}{2^2} = 4$$

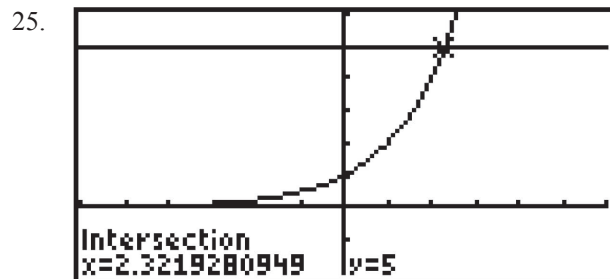
$$20. \quad \left(\frac{\sqrt{6}}{3}\right)^2 = \frac{(6^{1/2})^2}{3^2} = \frac{6}{9} = \frac{2}{3}$$

21. Domain: $(-\infty, \infty)$; y in range $\Rightarrow y = \frac{1}{2+e^x}$. As x increases, e^x becomes infinitely large and y becomes a smaller and smaller positive real number. As x decreases, e^x becomes a smaller and smaller positive real number, $y < \frac{1}{2}$, and y gets arbitrarily close to $\frac{1}{2} \Rightarrow$ Range: $(0, \frac{1}{2})$.

22. Domain: $(-\infty, \infty)$; y in range $\Rightarrow y = \cos(e^{-t})$. Since the values of e^{-t} are $(0, \infty)$ and $-1 \leq \cos x \leq 1 \Rightarrow$ Range: $[-1, 1]$.

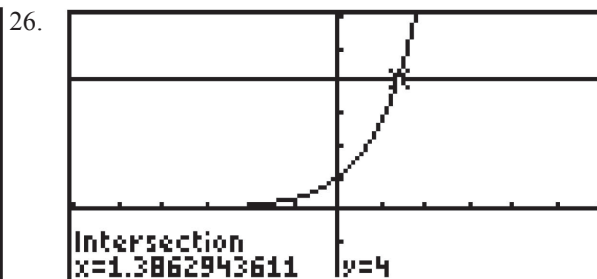
23. Domain: $(-\infty, \infty)$; y in range $\Rightarrow y = \sqrt{1+3^{-t}}$. Since the values of 3^{-t} are $(0, \infty) \Rightarrow$ Range: $(1, \infty)$.

24. If $e^{2x} = 1$, then $x = 0 \Rightarrow$ Domain: $(-\infty, 0) \cup (0, \infty)$; y in range $\Rightarrow y = \frac{3}{1-e^{2x}}$. If $x > 0$, then $1 < e^{2x} < \infty \Rightarrow -\infty < y < 0$. If $x < 0$, then $0 < e^{2x} < 1 \Rightarrow 3 < y < \infty \Rightarrow$ Range: $(-\infty, 0) \cup (3, \infty)$.



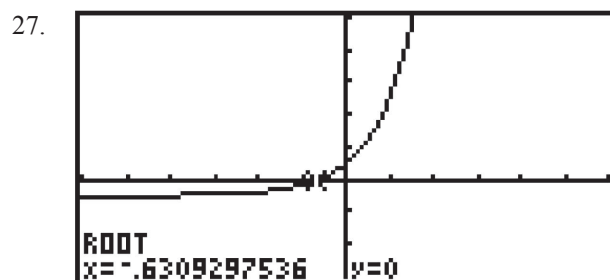
$[-6, 6]$ by $[-2, 6]$

$x \approx 2.3219$



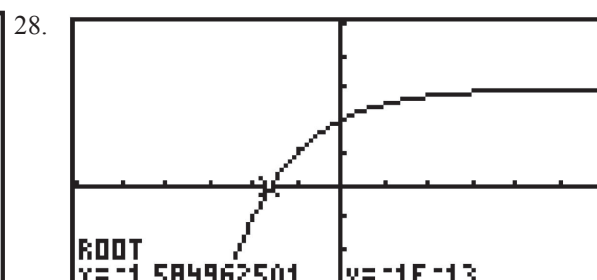
$[-6, 6]$ by $[-2, 6]$

$x \approx 1.3863$



$[-6, 6]$ by $[-3, 5]$

$x \approx -0.6309$



$[-6, 6]$ by $[-3, 5]$

$x \approx -1.5850$

29. Let t be the number of years. Solving $500,000(1.0375)^t = 1,000,000$ graphically, we find that $t \approx 18.828$. The population will reach 1 million in about 19 years.

30. (a) The population is given by $P(t) = 6250(1.0275)^t$, where t is the number of years after 1890.

Population in 1915: $P(25) \approx 12,315$

Population in 1940: $P(50) \approx 24,265$

- (b) Solving $P(t) = 50,000$ graphically, we find that $t \approx 76.651$. The population reached 50,000 about 77 years after 1890, in 1967.

31. (a) $A(t) = 6.6\left(\frac{1}{2}\right)^{t/14}$

- (b) Solving $A(t) = 1$ graphically, we find that $t \approx 38$. There will be 1 gram remaining after about 38.1145 days.

32. Let t be the number of years. Solving $2300(1.60)^t = 4150$ graphically, we find that $t \approx 10.129$. It will take about 10.129 years. (If the interest is not credited to the account until the end of each year, it will take 11 years.)

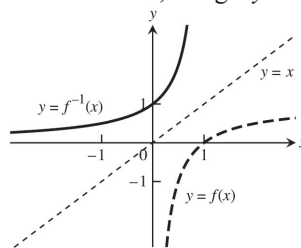
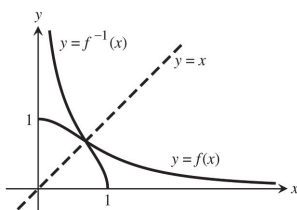
33. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A(1.0625)^t = 2A$, which is equivalent to $1.0625^t = 2$. Solving graphically, we find that $t \approx 11.433$. It will take about 11.433 years. (If the interest is credited at the end of each year, it will take 12 years.)

34. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $Ae^{0.0575t} = 3A$, which is equivalent to $e^{0.0575t} = 3$. Solving graphically, we find that $t \approx 19.106$. It will take about 19.106 years.

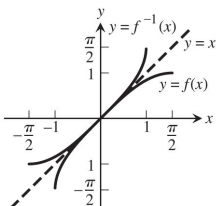
35. After t hours, the population is $P(t) = 2^{t/0.5}$, or equivalently, $P(t) = 2^{2t}$. After 24 hours, the population is $P(24) = 2^{48} \approx 2.815 \times 10^{14}$ bacteria.
36. (a) Each year, the number of cases is $100\% - 20\% = 80\%$ of the previous year's number of cases. After t years, the number of cases will be $C(t) = 10,000(0.8)^t$. Solving $C(t) = 1000$ graphically, we find that $t \approx 10.319$. It will take 10.319 years.
- (b) Solving $C(t) = 1$ graphically, we find that $t \approx 41.275$. It will take about 41.275 years.

1.6 INVERSE FUNCTIONS AND LOGARITHMS

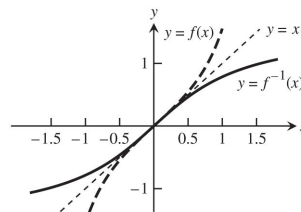
1. Yes one-to-one, the graph passes the horizontal line test.
2. Not one-to-one, the graph fails the horizontal line test.
3. Not one-to-one since (for example) the horizontal line $y = 2$ intersects the graph twice.
4. Not one-to-one, the graph fails the horizontal line test.
5. Yes one-to-one, the graph passes the horizontal line test.
6. Yes one-to-one, the graph passes the horizontal line test.
7. Not one-to-one since the horizontal line $y = 3$ intersects the graph an infinite number of times.
8. Yes one-to-one, the graph passes the horizontal line test.
9. Yes one-to one, graph passes the horizontal line test.
10. Not one-to-one since (for example) the horizontal line $y = 1$ intersects the graph twice.
11. Domain: $0 < x \leq 1$, Range: $0 \leq y$
12. Domain: $x < 1$, Range: $y > 0$



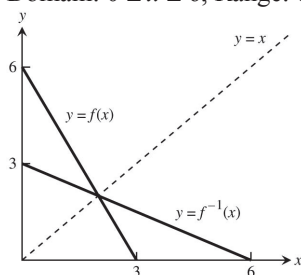
13. Domain: $-1 \leq x \leq 1$, Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$



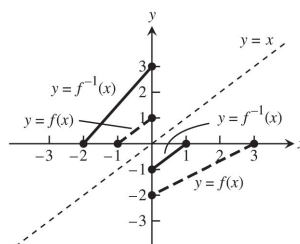
14. Domain: $-\infty < x < \infty$, Range: $-\frac{\pi}{2} < y \leq \frac{\pi}{2}$



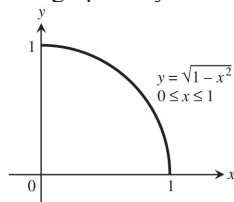
15. Domain:
- $0 \leq x \leq 6$
- , Range:
- $0 \leq y \leq 3$



16. Domain:
- $-2 \leq x \leq 1$
- , Range:
- $-1 \leq y < 3$



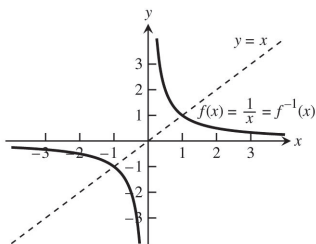
17. The graph is symmetric about
- $y = x$
- .



$$(b) \quad y = \sqrt{1-x^2} \Rightarrow y^2 = 1-x^2 \Rightarrow x^2 = 1-y^2 \Rightarrow x = \sqrt{1-y^2} \Rightarrow y = \sqrt{1-x^2} = f^{-1}(x)$$

18. (a) The graph is symmetric about
- $y = x$
- .

$$(b) \quad y = \frac{1}{x} \Rightarrow x = \frac{1}{y} \Rightarrow y = \frac{1}{x} = f^{-1}(x)$$



19. Step 1:
- $y = x^2 + 1 \Rightarrow x^2 = y - 1 \Rightarrow x = \sqrt{y-1}$

$$\text{Step 2: } y = \sqrt{x-1} = f^{-1}(x)$$

20. Step 1:
- $y = x^2 \Rightarrow x = -\sqrt{y}$
- , since
- $x \leq 0$
- .

$$\text{Step 2: } y = -\sqrt{x} = f^{-1}(x)$$

21. Step 1:
- $y = x^3 - 1 \Rightarrow x^3 = y + 1 \Rightarrow x = (y+1)^{1/3}$

$$\text{Step 2: } y = \sqrt[3]{x+1} = f^{-1}(x)$$

22. Step 1:
- $y = x^2 - 2x + 1 \Rightarrow y = (x-1)^2 \Rightarrow \sqrt{y} = x-1$
- , since
- $x \geq 1 \Rightarrow x = 1 + \sqrt{y}$

$$\text{Step 2: } y = 1 + \sqrt{x} = f^{-1}(x)$$

23. Step 1:
- $y = (x+1)^2 \Rightarrow \sqrt{y} = x+1$
- , since
- $x \geq -1 \Rightarrow x = \sqrt{y} - 1$

$$\text{Step 2: } y = \sqrt{x} - 1 = f^{-1}(x)$$

24. Step 1: $y = x^{2/3} \Rightarrow x = y^{3/2}$

Step 2: $y = x^{3/2} = f^{-1}(x)$

25. Step 1: $y = x^5 \Rightarrow x = y^{1/5}$

Step 2: $y = \sqrt[5]{x} = f^{-1}(x)$;

Domain and Range of f^{-1} : all reals;

$f(f^{-1}(x)) = (x^{1/5})^5 = x$ and $f^{-1}(f(x)) = (x^5)^{1/5} = x$

26. Step 1: $y = x^4 \Rightarrow x = y^{1/4}$

Step 2: $y = \sqrt[4]{x} = f^{-1}(x)$;

Domain of f^{-1} : $x \geq 0$, Range of f^{-1} : $y \geq 0$;

$f(f^{-1}(x)) = (x^{1/4})^4 = x$ and $f^{-1}(f(x)) = (x^4)^{1/4} = x$

27. Step 1: $y = x^3 + 1 \Rightarrow x^3 = y - 1 \Rightarrow x = (y - 1)^{1/3}$

Step 2: $y = \sqrt[3]{x - 1} = f^{-1}(x)$;

Domain and Range of f^{-1} : all reals;

$f(f^{-1}(x)) = ((x - 1)^{1/3})^3 + 1 = (x - 1) + 1 = x$ and $f^{-1}(f(x)) = ((x^3 + 1) - 1)^{1/3} = (x^3)^{1/3} = x$

28. Step 1: $y = \frac{1}{2}x - \frac{7}{2} \Rightarrow \frac{1}{2}x = y + \frac{7}{2} \Rightarrow x = 2y + 7$

Step 2: $y = 2x + 7 = f^{-1}(x)$;

Domain and Range of f^{-1} : all reals;

$f(f^{-1}(x)) = \frac{1}{2}(2x + 7) - \frac{7}{2} = \left(x + \frac{7}{2}\right) - \frac{7}{2} = x$ and $f^{-1}(f(x)) = 2\left(\frac{1}{2}x - \frac{7}{2}\right) + 7 = (x - 7) + 7 = x$

29. Step 1: $y = \frac{1}{x^2} \Rightarrow x^2 = \frac{1}{y} \Rightarrow x = \frac{1}{\sqrt{y}}$

Step 2: $y = \frac{1}{\sqrt{x}} = f^{-1}(x)$

Domain of f^{-1} : $x > 0$, Range of f^{-1} : $y > 0$;

$f(f^{-1}(x)) = \frac{1}{\left(\frac{1}{\sqrt{x}}\right)^2} = \frac{1}{\left(\frac{1}{x}\right)} = x$ and $f^{-1}(f(x)) = \frac{1}{\sqrt{\frac{1}{x^2}}} = \frac{1}{\left(\frac{1}{x}\right)} = x$ since $x > 0$.

30. Step 1: $y = \frac{1}{x^3} \Rightarrow x^3 = \frac{1}{y} \Rightarrow x = \frac{1}{y^{1/3}}$

Step 2: $y = \frac{1}{x^{1/3}} = \sqrt[3]{\frac{1}{x}} = f^{-1}(x)$;

Domain of f^{-1} : $x \neq 0$, Range of f^{-1} : $y \neq 0$;

$f(f^{-1}(x)) = \frac{1}{(x^{-1/3})^3} = \frac{1}{x^{-1}} = x$ and $f^{-1}(f(x)) = \left(\frac{1}{x^3}\right)^{-1/3} = \left(\frac{1}{x}\right)^{-1} = x$

31. Step 1: $y = \frac{x+3}{x-2} \Rightarrow y(x-2) = x+3 \Rightarrow xy - 2y = x+3 \Rightarrow xy - x = 2y+3 \Rightarrow x = \frac{2y+3}{y-1}$

Step 2: $y = \frac{2x+3}{x-1} = f^{-1}(x)$;

Domain of f^{-1} : $x \neq 1$, Range of f^{-1} : $y \neq 2$;

$$f(f^{-1}(x)) = \frac{\left(\frac{2x+3}{x-1}\right)+3}{\left(\frac{2x+3}{x-1}\right)-2} = \frac{(2x+3)+3(x-1)}{(2x+3)-2(x-1)} = \frac{5x}{5} = x \text{ and } f^{-1}(f(x)) = \frac{2\left(\frac{x+3}{x-2}\right)+3}{\left(\frac{x+3}{x-2}\right)-1} = \frac{2(x+3)+3(x-2)}{(x+3)-(x-2)} = \frac{5x}{5} = x$$

32. Step 1: $y = \frac{\sqrt{x}}{\sqrt{x}-3} \Rightarrow y(\sqrt{x}-3) = \sqrt{x} \Rightarrow y\sqrt{x}-3y = \sqrt{x} \Rightarrow y\sqrt{x}-\sqrt{x} = 3y \Rightarrow x = \left(\frac{3y}{y-1}\right)^2$

Step 2: $y = \left(\frac{3x}{x-1}\right)^2 = f^{-1}(x)$;

Domain of f^{-1} : $(-\infty, 0] \cup (1, \infty)$, Range of f^{-1} : $[0, 9) \cup (9, \infty)$;

$$f(f^{-1}(x)) = \frac{\sqrt{\left(\frac{3x}{x-1}\right)^2}}{\sqrt{\left(\frac{3x}{x-1}\right)^2}-3}; \text{ If } x > 1 \text{ or } x \leq 0 \Rightarrow \frac{3x}{x-1} \geq 0 \Rightarrow \frac{\sqrt{\left(\frac{3x}{x-1}\right)^2}}{\sqrt{\left(\frac{3x}{x-1}\right)^2}-3} = \frac{\frac{3x}{x-1}}{\frac{3x}{x-1}-3} = \frac{3x}{3x-3(x-1)} = \frac{3x}{3} = x \text{ and}$$

$$f^{-1}(f(x)) = \left(\frac{3\left(\frac{\sqrt{x}}{\sqrt{x}-3}\right)}{\frac{\sqrt{x}}{\sqrt{x}-3}-1}\right)^2 = \frac{9x}{(\sqrt{x}-(\sqrt{x}-3))^2} = \frac{9x}{9} = x$$

33. Step 1: $y = x^2 - 2x, x \leq 1 \Rightarrow y+1 = (x-1)^2, x \leq 1 \Rightarrow -\sqrt{y+1} = x-1, x \leq 1 \Rightarrow x = 1 - \sqrt{y+1}$

Step 2: $y = 1 - \sqrt{x+1} = f^{-1}(x)$;

Domain of f^{-1} : $[-1, \infty)$, Range of f^{-1} : $(-\infty, 1]$;

$$f(f^{-1}(x)) = (1 - \sqrt{x+1})^2 - 2(1 - \sqrt{x+1}) = 1 - 2\sqrt{x+1} + x + 1 - 2 + 2\sqrt{x+1} = x \text{ and}$$

$$f^{-1}(f(x)) = 1 - \sqrt{(x^2 - 2x) + 1}, x \leq 1 = 1 - \sqrt{(x-1)^2}, x \leq 1 = 1 - |x-1| = 1 - (1-x) = x$$

34. Step 1: $y = (2x^3 + 1)^{1/5} \Rightarrow y^5 = 2x^3 + 1 \Rightarrow y^5 - 1 = 2x^3 \Rightarrow \frac{y^5 - 1}{2} = x^3 \Rightarrow x = \sqrt[3]{\frac{y^5 - 1}{2}}$

Step 2: $y = \sqrt[3]{\frac{x^5 - 1}{2}} = f^{-1}(x)$;

Domain of f^{-1} : $(-\infty, \infty)$, Range of f^{-1} : $(-\infty, \infty)$;

$$f(f^{-1}(x)) = \left(2\left(\sqrt[3]{\frac{x^5 - 1}{2}}\right)^3 + 1\right)^{1/5} = \left(2\left(\frac{x^5 - 1}{2}\right) + 1\right)^{1/5} = ((x^5 - 1) + 1)^{1/5} = (x^5)^{1/5} = x \text{ and}$$

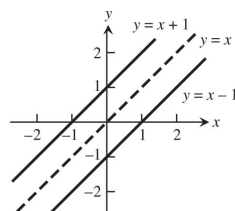
$$f^{-1}(f(x)) = \sqrt[3]{\frac{[(2x^3 + 1)^{1/5}]^5 - 1}{2}} = \sqrt[3]{\frac{(2x^3 + 1) - 1}{2}} = \sqrt[3]{\frac{2x^3}{2}} = x$$

35. (a) $y = mx \Rightarrow x = \frac{1}{m}y \Rightarrow f^{-1}(x) = \frac{1}{m}x$

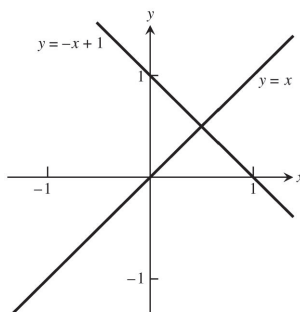
(b) The graph of $y = f^{-1}(x)$ is a line through the origin with slope $\frac{1}{m}$.

36. $y = mx + b \Rightarrow x = \frac{y}{m} - \frac{b}{m} \Rightarrow f^{-1}(x) = \frac{1}{m}x - \frac{b}{m}$; the graph of $f^{-1}(x)$ is a line with slope $\frac{1}{m}$ and y-intercept $-\frac{b}{m}$.

37. (a) $y = x + 1 \Rightarrow x = y - 1 \Rightarrow f^{-1}(x) = x - 1$
 (b) $y = x + b \Rightarrow x = y - b \Rightarrow f^{-1}(x) = x - b$
 (c) Their graphs will be parallel to one another and lie on opposite sides of the line $y = x$ equidistant from that line.



38. (a) $y = -x + 1 \Rightarrow x = -y + 1 \Rightarrow f^{-1}(x) = 1 - x$;
 the lines intersect at a right angle
 (b) $y = -x + b \Rightarrow x = -y + b \Rightarrow f^{-1}(x) = b - x$;
 the lines intersect at a right angle
 (c) Such a function is its own inverse



39. (a) $\ln 0.75 = \ln \frac{3}{4} = \ln 3 - \ln 4 = \ln 3 - \ln 2^2 = \ln 3 - 2 \ln 2$
 (b) $\ln \frac{4}{9} = \ln 4 - \ln 9 = \ln 2^2 - \ln 3^2 = 2 \ln 2 - 2 \ln 3$
 (c) $\ln \frac{1}{2} = \ln 1 - \ln 2 = -\ln 2$ (d) $\ln \sqrt[3]{9} = \frac{1}{3} \ln 9 = \frac{1}{3} \ln 3^2 = \frac{2}{3} \ln 3$
 (e) $\ln 3\sqrt{2} = \ln 3 + \ln 2^{1/2} = \ln 3 + \frac{1}{2} \ln 2$
 (f) $\ln \sqrt{13.5} = \frac{1}{2} \ln 13.5 = \frac{1}{2} \ln \frac{27}{2} = \frac{1}{2} (\ln 3^3 - \ln 2) = \frac{1}{2} (3 \ln 3 - \ln 2)$
40. (a) $\ln \frac{1}{125} = \ln 1 - 3 \ln 5 = -3 \ln 5$ (b) $\ln 9.8 = \ln \frac{49}{5} = \ln 7^2 - \ln 5 = 2 \ln 7 - \ln 5$
 (c) $\ln 7\sqrt{7} = \ln 7^{3/2} = \frac{3}{2} \ln 7$ (d) $\ln 1225 = \ln 35^2 = 2 \ln 35 = 2 \ln 5 + 2 \ln 7$
 (e) $\ln 0.056 - \ln \frac{7}{125} = \ln 7 - \ln 5^3 = \ln 7 - 3 \ln 5$
 (f) $\frac{\ln 35 + \ln \frac{1}{7}}{\ln 25} = \frac{\ln 5 + \ln 7 - \ln 7}{2 \ln 5} = \frac{1}{2}$

41. (a) $\ln \sin \theta - \ln \left(\frac{\sin \theta}{5} \right) = \ln \left(\frac{\sin \theta}{\frac{\sin \theta}{5}} \right) = \ln 5$ (b) $\ln(3x^2 - 9x) + \ln \left(\frac{1}{3x} \right) = \ln \left(\frac{3x^2 - 9x}{3x} \right) = \ln(x - 3)$
 (c) $\frac{1}{2} \ln(4t^4) - \ln 2 = \ln \sqrt{4t^4} - \ln 2 = \ln 2t^2 - \ln 2 = \ln \left(\frac{2t^2}{2} \right) = \ln(t^2)$

42. (a) $\ln \sec \theta + \ln \cos \theta = \ln[(\sec \theta)(\cos \theta)] = \ln 1 = 0$
 (b) $\ln(8x + 4) - \ln 2^2 = \ln(8x + 4) - \ln 4 = \ln \left(\frac{8x + 4}{4} \right) = \ln(2x + 1)$
 (c) $3 \ln \sqrt[3]{t^2 - 1} - \ln(t + 1) = 3 \ln(t^2 - 1)^{1/3} - \ln(t + 1) = 3 \left(\frac{1}{3} \right) \ln(t^2 - 1) - \ln(t + 1) = \ln \left(\frac{(t+1)(t-1)}{(t+1)} \right) = \ln(t - 1)$

43. (a) $e^{\ln 7.2} = 7.2$ (b) $e^{-\ln x^2} = \frac{1}{e^{\ln x^2}} = \frac{1}{x^2}$ (c) $e^{\ln x - \ln y} = e^{\ln(x/y)} = \frac{x}{y}$
44. (a) $e^{\ln(x^2 + y^2)} = x^2 + y^2$ (b) $e^{-\ln 0.3} = \frac{1}{e^{\ln 0.3}} = \frac{1}{0.3}$ (c) $e^{\ln \pi x - \ln 2} = e^{\ln(\pi x/2)} = \frac{\pi x}{2}$
45. (a) $2 \ln \sqrt{e} = 2 \ln e^{1/2} = (2)(\frac{1}{2}) \ln e = 1$ (b) $\ln(\ln e^e) = \ln(e \ln e) = \ln e = 1$
 (c) $\ln e^{(-x^2 - y^2)} = (-x^2 - y^2) \ln e = -x^2 - y^2$
46. (a) $\ln(e^{\sec \theta}) = (\sec \theta)(\ln e) = \sec \theta$ (b) $\ln e^{(e^x)} = (e^x)(\ln e) = e^x$
 (c) $\ln(e^{2 \ln x}) = \ln(e^{\ln x^2}) = \ln x^2 = 2 \ln x$
47. $\ln y = 2t + 4 \Rightarrow e^{\ln y} = e^{2t+4} \Rightarrow y = e^{2t+4}$ 48. $\ln y = -t + 5 \Rightarrow e^{\ln y} = e^{-t+5} \Rightarrow y = e^{-t+5}$
49. $\ln(y - 40) = 5t \Rightarrow e^{\ln(y-40)} = e^{5t} \Rightarrow y - 40 = e^{5t} \Rightarrow y = e^{5t} + 40$
50. $\ln(1 - 2y) = t \Rightarrow e^{\ln(1-2y)} = e^t \Rightarrow 1 - 2y = e^t \Rightarrow -2y = e^t - 1 \Rightarrow y = -\left(\frac{e^t - 1}{2}\right)$
51. $\ln(y - 1) - \ln 2 = x + \ln x \Rightarrow \ln(y - 1) - \ln 2 - \ln x = x \Rightarrow \ln\left(\frac{y-1}{2x}\right) = x \Rightarrow e^{\ln\left(\frac{y-1}{2x}\right)} = e^x \Rightarrow \frac{y-1}{2x} = e^x \Rightarrow y - 1 = 2xe^x \Rightarrow y = 2xe^x + 1$
52. $\ln(y^2 - 1) - \ln(y + 1) = \ln(\sin x) \Rightarrow \ln\left(\frac{y^2 - 1}{y + 1}\right) = \ln(\sin x) \Rightarrow \ln(y - 1) = \ln(\sin x) \Rightarrow e^{\ln(y-1)} = e^{\ln(\sin x)} \Rightarrow y - 1 = \sin x \Rightarrow y = \sin x + 1$
53. (a) $e^{2k} = 4 \Rightarrow \ln e^{2k} = \ln 4 \Rightarrow 2k \ln e = \ln 4 \Rightarrow 2k = \ln 4 \Rightarrow k = \ln 2$
 (b) $100e^{10k} = 200 \Rightarrow e^{10k} = 2 \Rightarrow \ln e^{10k} = \ln 2 \Rightarrow 10k \ln e = \ln 2 \Rightarrow 10k = \ln 2 \Rightarrow k = \frac{\ln 2}{10}$
 (c) $e^{k/1000} = a \Rightarrow \ln e^{k/1000} = \ln a \Rightarrow \frac{k}{1000} \ln e = \ln a \Rightarrow \frac{k}{1000} = \ln a \Rightarrow k = 1000 \ln a$
54. (a) $e^{5k} = \frac{1}{4} \Rightarrow \ln e^{5k} = \ln 4^{-1} \Rightarrow 5k \ln e = -\ln 4 \Rightarrow 5k = -\ln 4 \Rightarrow k = -\frac{\ln 4}{5}$
 (b) $80e^k = 1 \Rightarrow e^k = 80^{-1} \Rightarrow \ln e^k = \ln 80^{-1} \Rightarrow k \ln e = -\ln 80 \Rightarrow k = -\ln 80$
 (c) $e^{(\ln 0.8)k} = 0.8 \Rightarrow (e^{\ln 0.8})^k = 0.8 \Rightarrow (0.8)^k = 0.8 \Rightarrow k = 1$
55. (a) $e^{-0.3t} = 27 \Rightarrow \ln e^{-0.3t} = \ln 3^3 \Rightarrow (-0.3t) \ln e = 3 \ln 3 \Rightarrow -0.3t = 3 \ln 3 \Rightarrow t = -10 \ln 3$
 (b) $e^{kt} = \frac{1}{2} \Rightarrow \ln e^{kt} = \ln 2^{-1} = kt \ln e = -\ln 2 \Rightarrow t = -\frac{\ln 2}{k}$
 (c) $e^{(\ln 0.2)t} = 0.4 \Rightarrow (e^{\ln 0.2})^t = 0.4 \Rightarrow 0.2^t = 0.4 \Rightarrow \ln 0.2^t = \ln 0.4 \Rightarrow t \ln 0.2 = \ln 0.4 \Rightarrow t = \frac{\ln 0.4}{\ln 0.2}$

56. (a) $e^{-0.01t} = 1000 \Rightarrow \ln e^{-0.01t} = \ln 1000 \Rightarrow (-0.01t) \ln e = \ln 1000 \Rightarrow -0.01t = \ln 1000 \Rightarrow t = -100 \ln 1000$
 (b) $e^{kt} = \frac{1}{10} \Rightarrow \ln e^{kt} = \ln 10^{-1} = kt \ln e = -\ln 10 \Rightarrow kt = -\ln 10 \Rightarrow t = -\frac{\ln 10}{k}$
 (c) $e^{(\ln 2)t} = \frac{1}{2} \Rightarrow (e^{\ln 2})^t = 2^{-1} \Rightarrow 2^t = 2^{-1} \Rightarrow t = -1$

57. $e^{\sqrt{t}} = x^2 \Rightarrow \ln e^{\sqrt{t}} = \ln x^2 \Rightarrow \sqrt{t} = 2 \ln x \Rightarrow t = 4(\ln x)^2$

58. $e^{x^2} e^{2x+1} = e^t \Rightarrow e^{x^2+2x+1} = e^t \Rightarrow \ln e^{x^2+2x+1} = \ln e^t \Rightarrow t = x^2 + 2x + 1$

59. (a) $5^{\log_5 7} = 7$ (b) $8^{\log_8 \sqrt{2}} = \sqrt{2}$ (c) $1.3^{\log_3 75} = 75$
 (d) $\log_4 16 = \log_4 4^2 = 2 \log_4 4 = 2 \cdot 1 = 2$
 (e) $\log_3 \sqrt{3} = \log_3 3^{1/2} = \frac{1}{2} \log_3 3 = \frac{1}{2} \cdot 1 = \frac{1}{2} = 0.5$
 (f) $\log_4 \left(\frac{1}{4}\right) = \log_4 4^{-1} = -1 \log_4 4 = -1 \cdot 1 = -1$

60. (a) $2^{\log_2 3} = 3$ (b) $10^{\log_{10} (1/2)} = \frac{1}{2}$ (c) $\pi^{\log_\pi 7} = 7$
 (d) $\log_{11} 121 = \log_{11} 11^2 = 2 \log_{11} 11 = 2 \cdot 1 = 2$
 (e) $\log_{121} 11 = \log_{121} 121^{1/2} = \left(\frac{1}{2}\right) \log_{121} 121 = \left(\frac{1}{2}\right) \cdot 1 = \frac{1}{2}$
 (f) $\log_3 \left(\frac{1}{9}\right) = \log_3 3^{-2} = -2 \log_3 3 = -2 \cdot 1 = -2$

61. (a) Let $z = \log_4 x \Rightarrow 4^z = x \Rightarrow 2^{2z} = x \Rightarrow (2^z)^2 = x \Rightarrow 2^z = \sqrt{x}$
 (b) Let $z = \log_3 x \Rightarrow 3^z = x \Rightarrow (3^z)^2 = x^2 \Rightarrow 3^{2z} = x^2 \Rightarrow 9^z = x^2$
 (c) $\log_2 (e^{(\ln 2) \sin x}) = \log_2 2^{\sin x} = \sin x$

62. (a) Let $z = \log_5 (3x^2) \Rightarrow 5^z = 3x^2 \Rightarrow 25^z = 9x^4$
 (b) $\log_e (e^x) = x$
 (c) $\log_4 (2^{e^x \sin x}) = \log_4 4^{(e^x \sin x)/2} = \frac{e^x \sin x}{2}$

63. (a) $\frac{\log_2 x}{\log_3 x} = \frac{\ln x}{\ln 2} \div \frac{\ln x}{\ln 3} = \frac{\ln x}{\ln 2} \cdot \frac{\ln 3}{\ln x} = \frac{\ln 3}{\ln 2}$ (b) $\frac{\log_2 x}{\log_8 x} = \frac{\ln x}{\ln 2} \div \frac{\ln x}{\ln 8} = \frac{\ln x}{\ln 2} \cdot \frac{\ln 8}{\ln x} = \frac{3 \ln 2}{\ln 2} = 3$
 (c) $\frac{\ln_x a}{\ln_{x^2} a} = \frac{\ln a}{\ln x} \div \frac{\ln a}{\ln x^2} = \frac{\ln a}{\ln x} \cdot \frac{\ln x^2}{\ln a} = \frac{2 \ln x}{\ln x} = 2$

64. (a) $\frac{\log_9 x}{\log_3 x} = \frac{\ln x}{\ln 9} \div \frac{\ln x}{\ln 3} = \frac{\ln x}{2 \ln 3} \cdot \frac{\ln 3}{\ln x} = \frac{1}{2}$
 (b) $\frac{\log_{\sqrt{10}} x}{\log_{\sqrt{2}} x} = \frac{\ln x}{\ln \sqrt{10}} \div \frac{\ln x}{\ln \sqrt{2}} = \frac{\ln x}{\left(\frac{1}{2}\right) \ln 10} \cdot \frac{\left(\frac{1}{2}\right) \ln 2}{\ln x} = \frac{\ln 2}{\ln 10}$
 (c) $\frac{\log_a b}{\log_b a} = \frac{\ln b}{\ln a} \div \frac{\ln a}{\ln b} = \frac{\ln b}{\ln a} \cdot \frac{\ln b}{\ln a} = \left(\frac{\ln b}{\ln a}\right)^2$

65. (a) $-\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $-\frac{\pi}{3}$

66. (a) $\frac{\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{6}$
67. (a) $\arccos(-1) = \pi$ since $\cos(\pi) = -1$ and $0 \leq \pi \leq \pi$.
 (b) $\arccos(0) = \frac{\pi}{2}$ since $\cos\left(\frac{\pi}{2}\right) = 0$ and $0 \leq \frac{\pi}{2} \leq \pi$.
68. (a) $\arcsin(-1) = -\frac{\pi}{2}$ since $\sin\left(-\frac{\pi}{2}\right) = -1$ and $-\frac{\pi}{2} \leq -\frac{\pi}{2} \leq \frac{\pi}{2}$.
 (b) $\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$ since $\sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ and $-\frac{\pi}{2} \leq -\frac{\pi}{4} \leq \frac{\pi}{2}$.
69. The function $g(x)$ is also one-to-one. The reasoning: $f(x)$ is one-to-one means that if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$, so $-f(x_1) \neq -f(x_2)$ and therefore $g(x_1) \neq g(x_2)$. Therefore $g(x)$ is one-to-one as well.
70. The function $h(x)$ is also one-to-one. The reasoning: $f(x)$ is one-to-one means that if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$, so $\frac{1}{f(x_1)} \neq \frac{1}{f(x_2)}$, and therefore $h(x_1) \neq h(x_2)$.
71. The composite is one-to-one also. The reasoning: If $x_1 \neq x_2$ then $g(x_1) \neq g(x_2)$ because g is one-to-one. Since $g(x_1) \neq g(x_2)$, we also have $f(g(x_1)) \neq f(g(x_2))$ because f is one-to-one; thus, $f \circ g$ is one-to-one because $x_1 \neq x_2 \Rightarrow f(g(x_1)) \neq f(g(x_2))$.
72. Yes, g must be one-to-one. If g were not one-to-one, there would exist numbers $x_1 \neq x_2$ in the domain of g with $g(x_1) = g(x_2)$. For these numbers we would also have $f(g(x_1)) = f(g(x_2))$, contradicting the assumption that $f \circ g$ is one-to-one.
73. (a) $y = \frac{100}{1+2^{-x}} \rightarrow 1+2^{-x} = \frac{100}{y} \rightarrow 2^{-x} = \frac{100}{y} - 1 \rightarrow \log_2(2^{-x}) = \log_2\left(\frac{100}{y} - 1\right) \rightarrow -x = \log_2\left(\frac{100}{y} - 1\right)$
 $x = -\log_2\left(\frac{100}{y} - 1\right) = -\log_2\left(\frac{100-y}{y}\right) = \log_2\left(\frac{y}{100-y}\right)$.
 Interchange x and y : $y = \log_2\left(\frac{x}{100-x}\right) \rightarrow f^{-1}(x) = \log_2\left(\frac{x}{100-x}\right)$
 Verify.
 $(f \circ f^{-1})(x) = f\left(\log_2\left(\frac{x}{100-x}\right)\right) = \frac{100}{1+2^{\log_2\left(\frac{x}{100-x}\right)}} = \frac{100}{1+2^{\log_2\left(\frac{100-x}{x}\right)}} = \frac{100}{1+\frac{100-x}{x}} = \frac{100}{\frac{x+100-x}{x}} = \frac{100x}{100} = x$
 $(f^{-1} \circ f)(x) = f^{-1}\left(\frac{100}{1+2^{-x}}\right) = \log_2\left(\frac{\frac{100}{1+2^{-x}}}{100-\frac{100}{1+2^{-x}}}\right) = \log_2\left(\frac{100}{100(1+2^{-x})-100}\right) = \log_2\left(\frac{1}{2^{-x}}\right) = \log_2(2^x) = x$
- (b) $y = \frac{50}{1+1.1^{-x}} \rightarrow 1+1.1^{-x} = \frac{50}{y} \rightarrow 1.1^{-x} = \frac{50}{y} - 1 \rightarrow \log_{1.1}(1.1^{-x}) = \log_{1.1}\left(\frac{50}{y} - 1\right) \rightarrow -x = \log_{1.1}\left(\frac{50}{y} - 1\right)$
 $x = -\log_{1.1}\left(\frac{50}{y} - 1\right) = -\log_{1.1}\left(\frac{50-y}{y}\right) = \log_{1.1}\left(\frac{y}{50-y}\right)$.
 Interchange x and y : $y = \log_{1.1}\left(\frac{x}{50-x}\right) \rightarrow f^{-1}(x) = \log_{1.1}\left(\frac{x}{50-x}\right)$
 Verify.

$$(f \circ f^{-1})(x) = f\left(\log_{1.1}\left(\frac{x}{50-x}\right)\right) = \frac{50}{1+1.1^{\log_{1.1}\left(\frac{x}{50-x}\right)}} = \frac{50}{1+1.1^{\log_{1.1}\left(\frac{50-x}{x}\right)}} = \frac{50}{1+\frac{50-x}{x}} = \frac{50x}{x+50-x} = \frac{50x}{50} = x$$

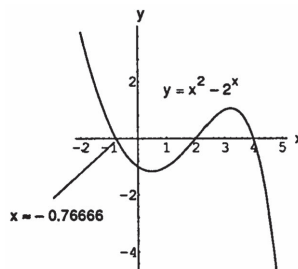
$$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{50}{1+1.1^{-x}}\right) = \log_{1.1}\left(\frac{\frac{50}{1+1.1^{-x}}}{50-\frac{50}{1+1.1^{-x}}}\right) = \log_{1.1}\left(\frac{50}{50(1+1.1^{-x})-50}\right) = \log_{1.1}\left(\frac{1}{1.1^{-x}}\right) = \log_{1.1}(1.1^x) = x$$

74. $\sin^{-1}(1) + \cos^{-1}(1) = \frac{\pi}{2} + 0 = \frac{\pi}{2}$; $\sin^{-1}(0) + \cos^{-1}(0) = 0 + \frac{\pi}{2} = \frac{\pi}{2}$; and $\sin^{-1}(-1) + \cos^{-1}(-1) = -\frac{\pi}{2} + \pi = \frac{\pi}{2}$.

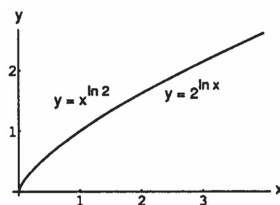
If $x \in (-1, 0)$ and $x = -a$, then $\sin^{-1}(x) + \cos^{-1}(x) = \sin^{-1}(-a) + \cos^{-1}(-a) = -\sin^{-1}a + (\pi - \cos^{-1}a)$
 $= \pi - (\sin^{-1}a + \cos^{-1}a) = \pi - \frac{\pi}{2} = \frac{\pi}{2}$ from Equations (3) and (4) in the text.

75. (a) Begin with $y = \ln x$ and reduce the y -value by 3 $\Rightarrow y = \ln x - 3$.
 (b) Begin with $y = \ln x$ and replace x with $x - 1 \Rightarrow y = \ln(x - 1)$.
 (c) Begin with $y = \ln x$, replace x with $x + 1$, and increase the y -value by 3 $\Rightarrow y = \ln(x + 1) + 3$.
 (d) Begin with $y = \ln x$, reduce the y -value by 4, and replace x with $x - 2 \Rightarrow y = \ln(x - 2) - 4$.
 (e) Begin with $y = \ln x$ and replace x with $-x \Rightarrow y = \ln(-x)$.
 (f) Begin with $y = \ln x$ and switch x and $y \Rightarrow x = \ln y$ or $y = e^x$.
76. (a) Begin with $y = \ln x$ and multiply the y -value by 2 $\Rightarrow y = 2 \ln x$.
 (b) Begin with $y = \ln x$ and replace x with $\frac{x}{3} \Rightarrow y = \ln\left(\frac{x}{3}\right)$.
 (c) Begin with $y = \ln x$ and multiply the y -value by $\frac{1}{4} \Rightarrow y = \frac{1}{4} \ln x$.
 (d) Begin with $y = \ln x$ and replace x with $2x \Rightarrow y = \ln 2x$.

77. From zooming in on the graph at the right, we estimate the third root to be $x \approx -0.76666$.



78. The functions $f(x) = x^{\ln 2}$ and $g(x) = 2^{\ln x}$ appear to have identical graphs for $x > 0$. This is no accident, because
 $x^{\ln 2} = e^{\ln 2 \cdot \ln x} = (e^{\ln 2})^{\ln x} = 2^{\ln x}$.



79. (a) Amount $= 8\left(\frac{1}{2}\right)^{t/12}$
 (b) $8\left(\frac{1}{2}\right)^{t/12} = 1 \rightarrow \left(\frac{1}{2}\right)^{t/12} = \frac{1}{8} \rightarrow \left(\frac{1}{2}\right)^{t/12} = \left(\frac{1}{2}\right)^3 \rightarrow \frac{t}{12} = 3 \rightarrow t = 36$
 There will be 1 gram remaining after 36 hours.

$$80. \quad 500(1.0475)^t = 1000 \rightarrow 1.0475^t = 2 \rightarrow \ln(1.0475^t) = \ln(2) \rightarrow t \ln(1.0475) = \ln(2) \rightarrow t = \frac{\ln(2)}{\ln(1.0475)} \approx 14.936$$

It will take about 14.936 years. (If the interest is paid at the end of each year, it will take 15 years.)

$$81. \quad 375,000(1.0225)^t = 1,000,000 \rightarrow 1.0225^t = \frac{8}{3} \rightarrow \ln(1.0225^t) = \ln\left(\frac{8}{3}\right) \rightarrow t \ln(1.0225) = \ln\left(\frac{8}{3}\right)$$

$$\rightarrow t = \frac{\ln\left(\frac{8}{3}\right)}{\ln(1.0225)} \approx 44.081$$

It will take about 44.081 years.

$$82. \quad y = y_0 e^{-0.18t} \text{ represents the decay equation; solving } (0.9)y_0 = y_0 e^{-0.18t} \Rightarrow t = \frac{\ln(0.9)}{-0.18} \approx 0.585 \text{ days}$$

CHAPTER 1 PRACTICE EXERCISES

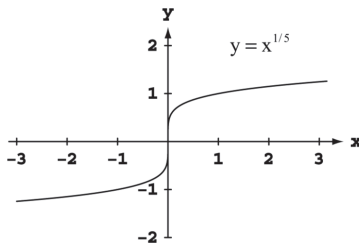
$$1. \quad \text{The area is } A = \pi r^2 \text{ and the circumference is } C = 2\pi r. \text{ Thus, } r = \frac{C}{2\pi} \Rightarrow A = \pi \left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{4\pi}.$$

$$2. \quad \text{The surface area is } S = 4\pi r^2 \Rightarrow r = \left(\frac{S}{4\pi}\right)^{1/2}. \text{ The volume is } V = \frac{4}{3}\pi r^3 \Rightarrow r = \sqrt[3]{\frac{3V}{4\pi}}. \text{ Substitution into the formula for surface area gives } S = 4\pi r^2 = 4\pi \left(\frac{3V}{4\pi}\right)^{2/3}.$$

$$3. \quad \text{The coordinates of a point on the parabola are } (x, x^2). \text{ The angle of inclination } \theta \text{ joining this point to the origin satisfies the equation } \tan \theta = \frac{x^2}{x} = x. \text{ Thus the point has coordinates } (x, x^2) = (\tan \theta, \tan^2 \theta).$$

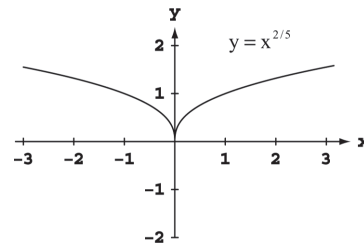
$$4. \quad \tan \theta = \frac{\text{rise}}{\text{run}} = \frac{h}{500} \Rightarrow h = 500 \tan \theta \text{ ft.}$$

5.



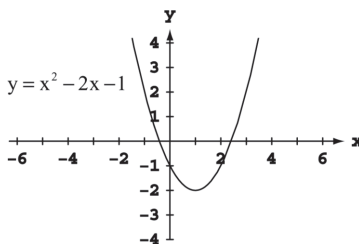
Symmetric about the origin.

6.



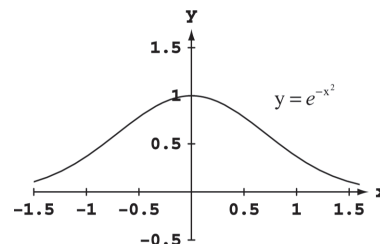
Symmetric about the y-axis.

7.



Neither

8.



Symmetric about the y-axis.

$$9. \quad y(-x) = (-x)^2 + 1 = x^2 + 1 = y(x). \text{ Even.}$$

10. $y(-x) = (-x)^5 - (-x)^3 - (-x) = -x^5 + x^3 + x = -y(x)$. Odd.
11. $y(-x) = 1 - \cos(-x) = 1 - \cos x = y(x)$. Even.
12. $y(-x) = \sec(-x) \tan(-x) = \frac{\sin(-x)}{\cos^2(-x)} = \frac{-\sin x}{\cos^2 x} = -\sec x \tan x = -y(x)$. Odd.
13. $y(-x) = \frac{(-x)^4 + 1}{(-x)^3 - 2(-x)} = \frac{x^4 + 1}{-x^3 + 2x} = -\frac{x^4 + 1}{x^3 - 2x} = -y(x)$. Odd.
14. $y(-x) = (-x) - \sin(-x) = (-x) + \sin x = -(x - \sin x) = -y(x)$. Odd.
15. $y(-x) = -x + \cos(-x) = -x + \cos x$. Neither even nor odd.
16. $y(-x) = (-x) \cos(-x) = -x \cos x = -y(x)$. Odd.
17. Since f and g are odd $\Rightarrow f(-x) = -f(x)$ and $g(-x) = -g(x)$.
 (a) $(f \cdot g)(-x) = f(-x)g(-x) = [-f(x)][-g(x)] = f(x)g(x) = (f \cdot g)(x) \Rightarrow f \cdot g$ is even.
 (b) $f^3(-x) = f(-x)f(-x)f(-x) = [-f(x)][-f(x)][-f(x)] = -f(x) \cdot f(x) \cdot f(x) = -f^3(x) \Rightarrow f^3$ is odd.
 (c) $f(\sin(-x)) = f(-\sin(x)) = -f(\sin(x)) \Rightarrow f(\sin(x))$ is odd.
 (d) $g(\sec(-x)) = g(\sec(x)) \Rightarrow g(\sec(x))$ is even.
 (e) $|g(-x)| = |-g(x)| = |g(x)| \Rightarrow |g|$ is even.
18. Let $f(a-x) = f(a+x)$ and define $g(x) = f(x+a)$. Then $g(-x) = f((-x)+a) = f(a-x) = f(a+x) = f(x+a) = g(x) \Rightarrow g(x) = f(x+a)$ is even.
19. (a) The function is defined for all values of x , so the domain is $(-\infty, \infty)$.
 (b) Since $|x|$ attains all nonnegative values, the range is $[-2, \infty)$.
20. (a) Since the square root requires $1-x \geq 0$, the domain is $(-\infty, 1]$.
 (b) Since $\sqrt{1-x}$ attains all nonnegative values, the range is $[-2, \infty)$.
21. (a) Since the square root requires $16-x^2 \geq 0$, the domain is $[-4, 4]$.
 (b) For values of x in the domain, $0 \leq 16-x^2 \leq 16$, so $0 \leq \sqrt{16-x^2} \leq 4$. The range is $[0, 4]$.
22. (a) The function is defined for all values of x , so the domain is $(-\infty, \infty)$.
 (b) Since 3^{2-x} attains all positive values, the range is $(1, \infty)$.
23. (a) The function is defined for all values of x , so the domain is $(-\infty, \infty)$.
 (b) Since $2e^{-x}$ attains all positive values, the range is $(-3, \infty)$.
24. (a) The function is equivalent to $y = \tan 2x$, so we require $2x \neq \frac{k\pi}{2}$ for odd integers k . The domain is given by $x \neq \frac{k\pi}{4}$ for odd integers k .
 (b) Since the tangent function attains all values, the range is $(-\infty, \infty)$.
25. (a) The function is defined for all values of x , so the domain is $(-\infty, \infty)$.
 (b) The sine function attains values from -1 to 1 , so $-2 \leq 2 \sin(3x + \pi) \leq 2$ and hence $-3 \leq 2 \sin(3x + \pi) - 1 \leq 1$. The range is $[-3, 1]$.

26. (a) The function is defined for all values of x , so the domain is $(-\infty, \infty)$.
 (b) The function is equivalent to $y = \sqrt[5]{x^2}$, which attains all nonnegative values. The range is $[0, \infty)$.
27. (a) The logarithm requires $x - 3 > 0$, so the domain is $(3, \infty)$.
 (b) The logarithm attains all real values, so the range is $(-\infty, \infty)$.
28. (a) The function is defined for all values of x , so the domain is $(-\infty, \infty)$.
 (b) The cube root attains all real values, so the range is $(-\infty, \infty)$.
29. (a) Increasing because volume increases as radius increases.
 (b) Neither, since the greatest integer function is composed of horizontal (constant) line segments.
 (c) Decreasing because as the height increases, the atmospheric pressure decreases.
 (d) Increasing because the kinetic (motion) energy increases as the particles velocity increases.
30. (a) Increasing on $[2, \infty)$ (b) Increasing on $[-1, \infty)$
 (c) Increasing on $(-\infty, \infty)$ (d) Increasing on $[\frac{1}{2}, \infty)$
31. (a) The function is defined for $-4 \leq x \leq 4$, so the domain is $[-4, 4]$.
 (b) The function is equivalent to $y = \sqrt{|x|}$, $-4 \leq x \leq 4$, which attains values from 0 to 2 for x in the domain. The range is $[0, 2]$.
32. (a) The function is defined for $-2 \leq x \leq 2$, so the domain is $[-2, 2]$.
 (b) The range is $[-1, 1]$.
33. First piece: Line through $(0, 1)$ and $(1, 0)$. $m = \frac{0-1}{1-0} = \frac{-1}{1} = -1 \Rightarrow y = -x + 1 = 1 - x$
 Second piece: Line through $(1, 1)$ and $(2, 0)$. $m = \frac{0-1}{2-1} = \frac{-1}{1} = -1 \Rightarrow y = -(x-1) + 1 = -x + 2 = 2 - x$

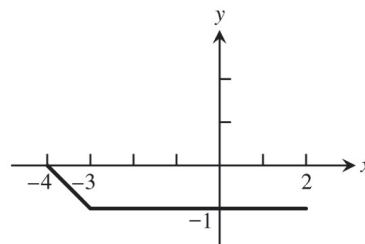
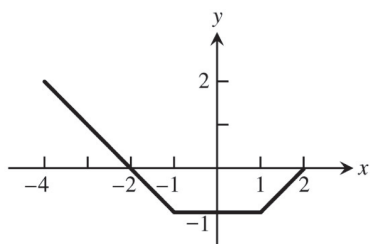
$$f(x) = \begin{cases} 1-x, & 0 \leq x < 1 \\ 2-x, & 1 \leq x \leq 2 \end{cases}$$
34. First piece: Line through $(0, 0)$ and $(2, 5)$. $m = \frac{5-0}{2-0} = \frac{5}{2} \Rightarrow y = \frac{5}{2}x$
 Second piece: Line through $(2, 5)$ and $(4, 0)$. $m = \frac{0-5}{4-2} = \frac{-5}{2} = -\frac{5}{2} \Rightarrow y = -\frac{5}{2}(x-2) + 5 = -\frac{5}{2}x + 10 = 10 - \frac{5x}{2}$

$$f(x) = \begin{cases} \frac{5}{2}x, & 0 \leq x < 2 \\ 10 - \frac{5x}{2}, & 2 \leq x \leq 4 \end{cases} \quad (\text{Note: } x = 2 \text{ can be included on either piece.})$$
35. (a) $(f \circ g)(-1) = f(g(-1)) = f\left(\frac{1}{\sqrt{-1+2}}\right) = f(1) = \frac{1}{1} = 1$
 (b) $(g \circ f)(2) = g(f(2)) = g\left(\frac{1}{2}\right) = \frac{1}{\sqrt{\frac{1}{2}+2}} = \frac{1}{\sqrt{2.5}}$ or $\sqrt{\frac{2}{5}}$
 (c) $(f \circ f)(x) = f(f(x)) = f\left(\frac{1}{x}\right) = \frac{1}{1/x} = x, x \neq 0$
 (d) $(g \circ g)(x) = g(g(x)) = g\left(\frac{1}{\sqrt{x+2}}\right) = \frac{1}{\sqrt{\frac{1}{\sqrt{x+2}}+2}} = \frac{\sqrt[4]{x+2}}{\sqrt{1+2\sqrt{x+2}}}$
36. (a) $(f \circ g)(-1) = f(g(-1)) = f\left(\sqrt[3]{-1+1}\right) = f(0) = 2 - 0 = 2$
 (b) $(g \circ f)(2) = f(g(2)) = g(2-2) = g(0) = \sqrt[3]{0+1} = 1$
 (c) $(f \circ f)(x) = f(f(x)) = f(2-x) = 2 - (2-x) = x$
 (d) $(g \circ g)(x) = g(g(x)) = g\left(\sqrt[3]{x+1}\right) = \sqrt[3]{\sqrt[3]{x+1}+1}$

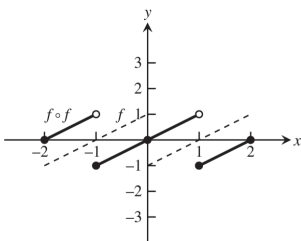
37. (a) $(f \circ g)(x) = f(g(x)) = f(\sqrt{x+2}) = 2 - (\sqrt{x+2})^2 = -x, x \geq -2.$
 $(g \circ f)(x) = g(f(x)) = g(2 - x^2) = \sqrt{(2 - x^2) + 2} = \sqrt{4 - x^2}$
 (b) Domain of $f \circ g$: $[-2, \infty).$
 Domain of $g \circ f$: $[-2, 2].$
 (c) Range of $f \circ g$: $(-\infty, 2].$
 Range of $g \circ f$: $[0, 2].$

38. (a) $(f \circ g)(x) = f(g(x)) = f(\sqrt{1-x}) = \sqrt{\sqrt{1-x}} = \sqrt[4]{1-x}.$
 $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{1 - \sqrt{x}}$
 (b) Domain of $f \circ g$: $(-\infty, 1].$
 Domain of $g \circ f$: $[0, 1].$
 (c) Range of $f \circ g$: $[0, \infty).$
 Range of $g \circ f$: $[0, 1].$

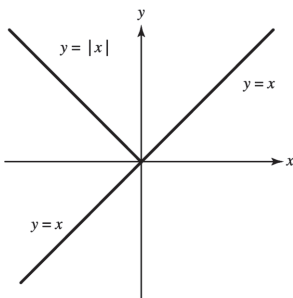
39. $y = f(x)$



40.

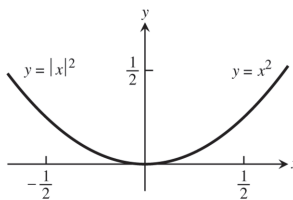


41.



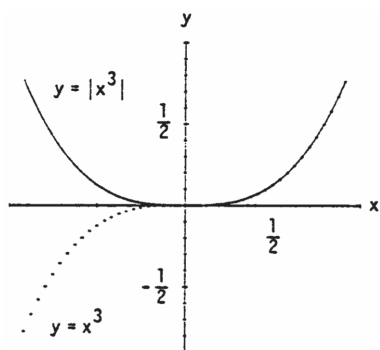
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y -axis. The graph of $f_2(x)$ to the left of the y -axis is the reflection of $y = f_1(x), x \geq 0$ across the y -axis.

42.



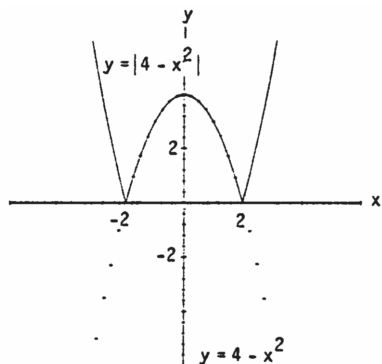
It does not change the graph.

43.



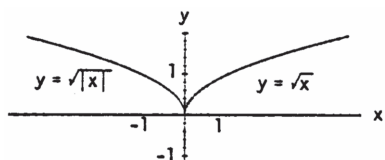
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as the graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x -axis.

45.



Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x -axis.

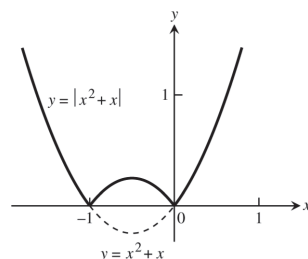
47.



The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y -axis. The graph of $f_2(x)$ to the left of the y -axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y -axis.

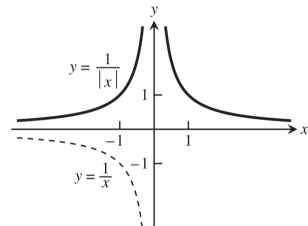
49. (a) $y = g(x-3) + \frac{1}{2}$
 (c) $y = g(-x)$
 (e) $y = 5 \cdot g(x)$

44.



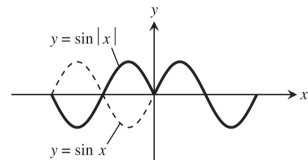
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as the graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x -axis.

46.



The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y -axis. The graph of $f_2(x)$ to the left of the y -axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y -axis.

48.

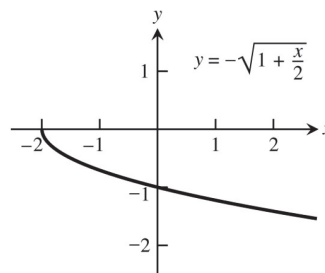


The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y -axis. The graph of $f_2(x)$ to the left of the y -axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y -axis.

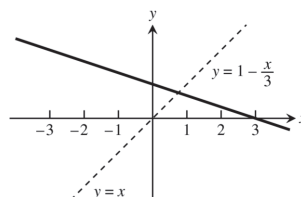
- (b) $y = g\left(x + \frac{2}{3}\right) - 2$
 (d) $y = -g(x)$
 (f) $y = g(5x)$

50. (a) Shift the graph of f right 5 units
 (b) Horizontally compress the graph of f by a factor of 4
 (c) Horizontally compress the graph of f by a factor of 3 and then reflect the graph about the y -axis
 (d) Horizontally compress the graph of f by a factor of 2 and then shift the graph left $\frac{1}{2}$ unit.
 (e) Horizontally stretch the graph of f by a factor of 3 and then shift the graph down 4 units.
 (f) Vertically stretch the graph of f by a factor of 3, then reflect the graph about the x -axis, and finally shift the graph up $\frac{1}{4}$ unit.

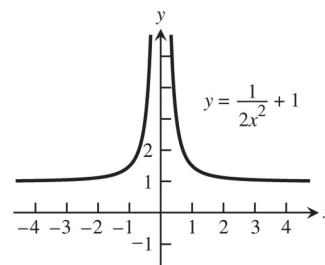
51. Reflection of the graph of $y = \sqrt{x}$ about the x -axis followed by a horizontal compression by a factor of $\frac{1}{2}$ then a shift left 2 units.



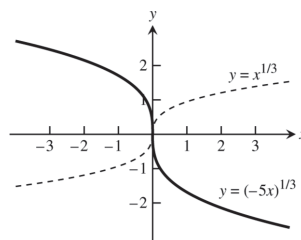
52. Reflect the graph of $y = x$ about the x -axis, followed by a vertical compression of the graph by a factor of 3, then shift the graph up 1 unit.



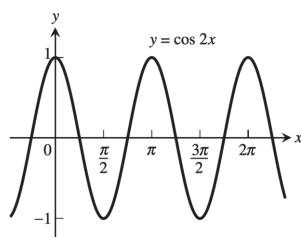
53. Vertical compression of the graph of $y = \frac{1}{x^2}$ by a factor of 2, then shift the graph up 1 unit.



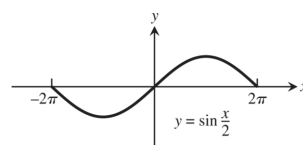
54. Reflect the graph of $y = x^{1/3}$ about the y -axis, then compress the graph horizontally by a factor of 5.



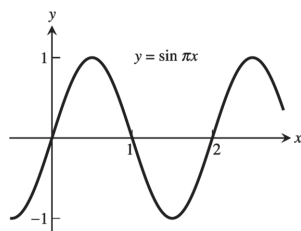
55.

period = π

56.

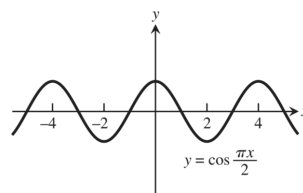
period = 4π

57.



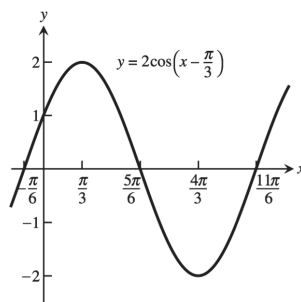
period = 2

58.

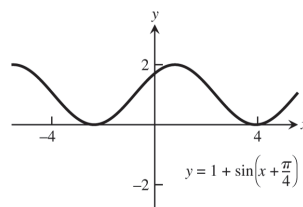


period = 4

59.

period = 2π

60.

period = 2π

61. (a) $\sin B = \sin \frac{\pi}{3} = \frac{b}{c} = \frac{b}{2} \Rightarrow b = 2 \sin \frac{\pi}{3} = 2 \left(\frac{\sqrt{3}}{2} \right) = \sqrt{3}$. By the theorem of Pythagoras,
 $a^2 + b^2 = c^2 \Rightarrow a = \sqrt{c^2 - b^2} = \sqrt{4 - 3} = 1$.

(b) $\sin B = \sin \frac{\pi}{3} = \frac{b}{c} = \frac{2}{c} \Rightarrow c = \frac{2}{\sin \frac{\pi}{3}} = \frac{2}{(\frac{\sqrt{3}}{2})} = \frac{4}{\sqrt{3}}$. Thus, $a = \sqrt{c^2 - b^2} = \sqrt{\left(\frac{4}{\sqrt{3}}\right)^2 - (2)^2} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$.

62. (a) $\sin A = \frac{a}{c} \Rightarrow a = c \sin A$

(b) $\tan A = \frac{a}{b} \Rightarrow a = b \tan A$

63. (a) $\tan B = \frac{b}{a} \Rightarrow a = \frac{b}{\tan B}$

(b) $\sin A = \frac{a}{c} \Rightarrow c = \frac{a}{\sin A}$

64. (a) $\sin A = \frac{a}{c}$

(b) $\sin A = \frac{a}{c} = \frac{\sqrt{c^2 - b^2}}{c}$

65. Let h = height of vertical pole, and let b and c denote the distances of points B and C from the base of the pole, measured along the flat ground, respectively.

Then, $\tan 50^\circ = \frac{h}{c}$, $\tan 35^\circ = \frac{h}{b}$, and $b - c = 10$.

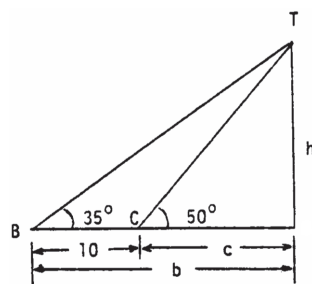
Thus, $h = c \tan 50^\circ$ and $h = b \tan 35^\circ = (c + 10) \tan 35^\circ$

$$\Rightarrow c \tan 50^\circ = (c + 10) \tan 35^\circ$$

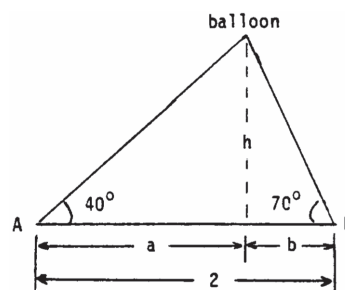
$$\Rightarrow c(\tan 50^\circ - \tan 35^\circ) = 10 \tan 35^\circ$$

$$\Rightarrow c = \frac{10 \tan 35^\circ}{\tan 50^\circ - \tan 35^\circ} \Rightarrow h = c \tan 50^\circ$$

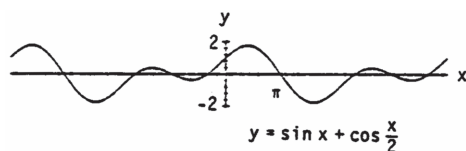
$$= \frac{10 \tan 35^\circ \tan 50^\circ}{\tan 50^\circ - \tan 35^\circ} \approx 16.98 \text{ m.}$$



66. Let h = height of balloon above ground. From the figure at the right, $\tan 40^\circ = \frac{h}{a}$, $\tan 70^\circ = \frac{h}{b}$, and $a + b = 2$. Thus, $h = b \tan 70^\circ \Rightarrow h = (2 - a) \tan 70^\circ$ and $h = a \tan 40^\circ \Rightarrow (2 - a) \tan 70^\circ = a \tan 40^\circ$
- $$\Rightarrow a(\tan 40^\circ + \tan 70^\circ) = 2 \tan 70^\circ$$
- $$\Rightarrow a = \frac{2 \tan 70^\circ}{\tan 40^\circ + \tan 70^\circ} \Rightarrow h = a \tan 40^\circ$$
- $$= \frac{2 \tan 70^\circ \tan 40^\circ}{\tan 40^\circ + \tan 70^\circ} \approx 1.3 \text{ km.}$$



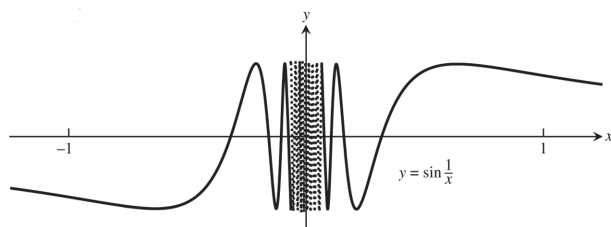
67. (a)



- (b) The period appears to be 4π .

- (c) $f(x + 4\pi) = \sin(x + 4\pi) + \cos\left(\frac{x + 4\pi}{2}\right) = \sin(x + 2\pi) + \cos\left(\frac{x}{2} + 2\pi\right) = \sin x + \cos \frac{x}{2}$
since the period of sine and cosine is 2π . Thus, $f(x)$ has period 4π .

68. (a)



- (b) $D = (-\infty, 0) \cup (0, \infty)$; $R = [-1, 1]$

- (c) f is not periodic. For suppose f has period p . Then $f\left(\frac{1}{2\pi} + kp\right) = f\left(\frac{1}{2\pi}\right) = \sin 2\pi = 0$ for all integers k .

Choose k so large that $\frac{1}{2\pi} + kp > \frac{1}{\pi} \Rightarrow 0 < \frac{1}{(1/(2\pi)) + kp} < \pi$. But then $f\left(\frac{1}{2\pi} + kp\right) = \sin\left(\frac{1}{(1/(2\pi)) + kp}\right) > 0$ which is a contradiction. Thus f has no period, as claimed.

69. (a) $D: -\infty < x < \infty$

- (b) $D: x > 0$

70. (a) $D = (-\infty, 0) \cup (0, \infty)$

(b) $D = (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

71. (a) $D: -3 \leq x \leq 3$

(b) $D: 0 \leq x \leq 4$

72. (a) $D = [-1, 1]$

(b) $D = [-1, 1]$

73. $(f \circ g)(x) = \ln(4 - x^2)$ and domain: $-2 < x < 2$;

$(g \circ f)(x) = 4 - (\ln x)^2$ and domain: $x > 0$;

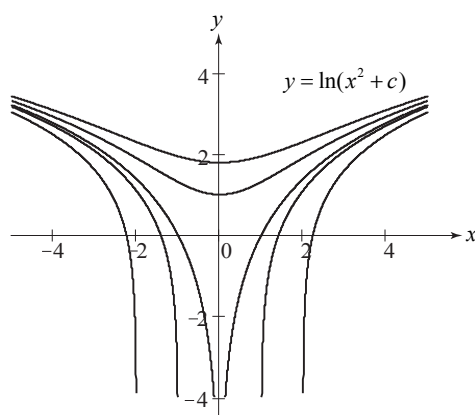
$(f \circ f)(x) = \ln(\ln x)$ and domain: $x > 1$;

$(g \circ g)(x) = -x^4 + 8x^2 - 12$ and domain: $-\infty < x < \infty$.

74. (a) Even (b) Neither even nor odd (c) Neither even nor odd (d) Even

76. For $c > 0$, $D = (-\infty, \infty)$

For $c \leq 0$, $D = (-\infty, -\sqrt{c}) \cup (\sqrt{c}, \infty)$



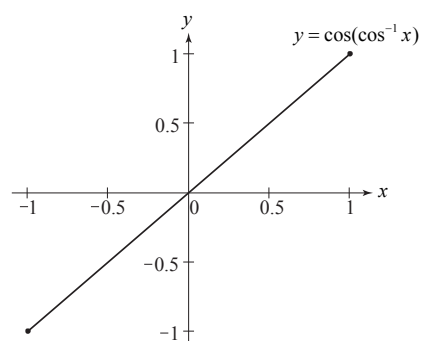
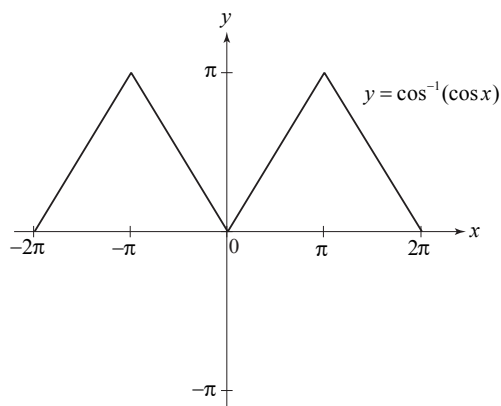
78. For large values of x , $y = a^x$ has the largest values; $y = \log_a x$ has the smallest.

79. (a) $D: (-\infty, \infty)$ $R: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(b) $D: [-1, 1]$ $R: [-1, 1]$

80. (a) $D: -\infty < x < \infty$; $R: 0 \leq y \leq \pi$

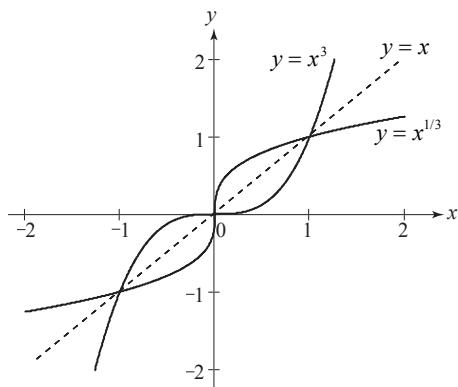
(b) $D: -1 \leq x \leq 1$; $R: -1 \leq y \leq 1$



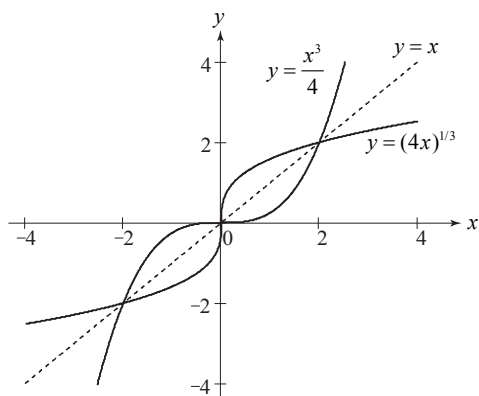
81. (a) No (b) Yes

82. Answers depend on the view screen used. For $[15, 17] \times [5 \cdot 10^6, 10^7]$ it appears that $e^x > 10^7$ for $x \geq 16.128$.83. (a) $f(g(x)) = (\sqrt[3]{x})^3 = x$, $g(f(x)) = \sqrt[3]{x^3} = x$

(b)

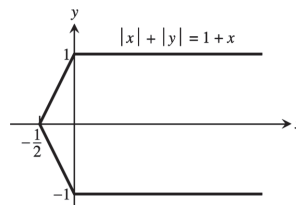
84. (a) $h(k(x)) = \frac{1}{4}((4x)^{1/3})^3 = x$, $k(h(x)) = \left(4 \cdot \frac{x^3}{4}\right)^{1/3} = x$

(b)

**CHAPTER 1 ADDITIONAL AND ADVANCED EXERCISES**

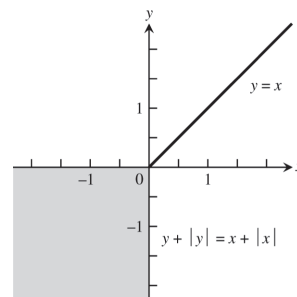
1. There are (infinitely) many such function pairs. For example, $f(x) = 3x$ and $g(x) = 4x$ satisfy $f(g(x)) = f(4x) = 3(4x) = 12x = 4(3x) = g(3x) = g(f(x))$.
2. Yes, there are many such function pairs. For example, if $g(x) = (2x + 3)^3$ and $f(x) = x^{1/3}$, then $(f \circ g)(x) = f(g(x)) = f((2x + 3)^3) = ((2x + 3)^3)^{1/3} = 2x + 3$.
3. If f is odd and defined at x , then $f(-x) = -f(x)$. Thus $g(-x) = f(-x) - 2 = -f(x) - 2$ whereas $-g(x) = -(f(x) - 2) = -f(x) + 2$. Then g cannot be odd because $g(-x) = -g(x) \Rightarrow -f(x) - 2 = -f(x) + 2 \Rightarrow 4 = 0$, which is a contradiction. Also, $g(x)$ is not even unless $f(x) = 0$ for all x . On the other hand, if f is even, then $g(x) = f(x) - 2$ is also even: $g(-x) = f(-x) - 2 = f(x) - 2 = g(x)$.
4. If g is odd and $g(0)$ is defined, then $g(0) = g(-0) = -g(0)$. Therefore, $2g(0) = 0 \Rightarrow g(0) = 0$.

5. For (x, y) in the 1st quadrant, $|x| + |y| = 1 + x$
 $\Leftrightarrow x + y = 1 + x \Leftrightarrow y = 1$. For (x, y) in the 2nd quadrant, $|x| + |y| = x + 1 \Leftrightarrow -x + y = x + 1$
 $\Leftrightarrow y = 2x + 1$. In the 3rd quadrant, $|x| + |y| = x + 1$
 $\Leftrightarrow -x - y = x + 1 \Leftrightarrow y = -2x - 1$. In the 4th quadrant, $|x| + |y| = x + 1 \Leftrightarrow x + (-y) = x + 1$
 $\Leftrightarrow y = -1$. The graph is given at the right.



6. We use reasoning similar to Exercise 5.

- (1) 1st quadrant: $y + |y| = x + |x|$
 $\Leftrightarrow 2y = 2x \Leftrightarrow y = x$.
 (2) 2nd quadrant: $y + |y| = x + |x|$
 $\Leftrightarrow 2y = x + (-x) = 0 \Leftrightarrow y = 0$.
 (3) 3rd quadrant: $y + |y| = x + |x|$
 $\Leftrightarrow y + (-y) = x + (-x) \Leftrightarrow 0 = 0$
 \Rightarrow all points in the 3rd quadrant satisfy the equation.
 (4) 4th quadrant: $y + |y| = x + |x|$
 $\Leftrightarrow y + (-y) = 2x \Leftrightarrow 0 = x$. Combining these results we have the graph given at the right:

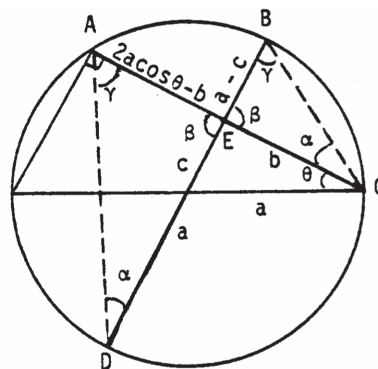


7. (a) $\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x = 1 - \cos^2 x = (1 - \cos x)(1 + \cos x) \Rightarrow (1 - \cos x) = \frac{\sin^2 x}{1 + \cos x} \Rightarrow \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$
 (b) Using the definition of the tangent function and the double angle formulas, we have

$$\tan^2\left(\frac{x}{2}\right) = \frac{\sin^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)} = \frac{\frac{1 - \cos\left(2\left(\frac{x}{2}\right)\right)}{2}}{\frac{1 + \cos\left(2\left(\frac{x}{2}\right)\right)}{2}} = \frac{1 - \cos x}{1 + \cos x}.$$

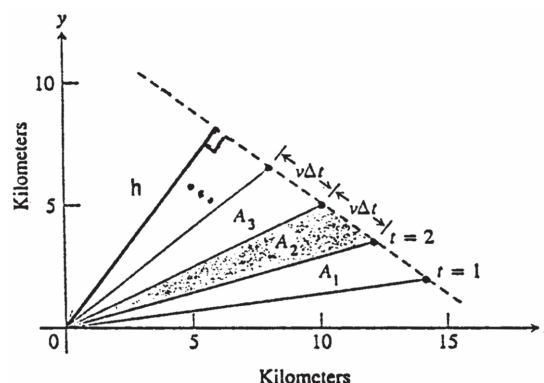
8. The angles labeled γ in the accompanying figure are equal since both angles subtend arc CD . Similarly, the two angles labeled α are equal since they both subtend arc AB . Thus, triangles AED and BEC are similar which implies $\frac{a - c}{b} = \frac{2a \cos \theta - b}{a + c}$

$$\begin{aligned} \Rightarrow (a - c)(a + c) &= b(2a \cos \theta - b) \\ \Rightarrow a^2 - c^2 &= 2ab \cos \theta - b^2 \\ \Rightarrow c^2 - a^2 + b^2 &= 2ab \cos \theta. \end{aligned}$$



9. As in the proof of the law of sines of Section 1.3, Exercise 61, $ah = bc \sin A = ab \sin C = ac \sin B$
 \Rightarrow the area of $ABC = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}ah = \frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B$.
10. As in Section 1.3, Exercise 61, $(\text{Area of } ABC)^2 = \frac{1}{4}(\text{base})^2(\text{height})^2 = \frac{1}{4}a^2h^2 = \frac{1}{4}a^2b^2 \sin^2 C$
 $= \frac{1}{4}a^2b^2(1 - \cos^2 C)$. By the law of cosines, $c^2 = a^2 + b^2 - 2ab \cos C \Rightarrow \cos C = \frac{a^2 + b^2 - c^2}{2ab}$. Thus,
 $(\text{area of } ABC)^2 = \frac{1}{4}a^2b^2(1 - \cos^2 C) = \frac{1}{4}a^2b^2 \left(1 - \left(\frac{a^2 + b^2 - c^2}{2ab}\right)^2\right) = \frac{a^2b^2}{4} \left(1 - \frac{(a^2 + b^2 - c^2)^2}{4a^2b^2}\right)$
 $= \frac{1}{16} \left(4a^2b^2 - (a^2 + b^2 - c^2)^2\right) = \frac{1}{16} [(2ab + (a^2 + b^2 - c^2))(2ab - (a^2 + b^2 - c^2))]$
 $= \frac{1}{16} [((a + b)^2 - c^2)(c^2 - (a - b)^2)] = \frac{1}{16} [(a + b + c)((a + b) - c)(c + (a - b))(c - (a - b))]$
 $= \left[\left(\frac{a + b + c}{2}\right)\left(\frac{-a + b + c}{2}\right)\left(\frac{a - b + c}{2}\right)\left(\frac{a + b - c}{2}\right)\right] = s(s - a)(s - b)(s - c), \text{ where } s = \frac{a + b + c}{2}.$
 Therefore, the area of ABC equals $\sqrt{s(s - a)(s - b)(s - c)}$.

11. If f is even and odd, then $f(-x) = -f(x)$ and $f(-x) = f(x) \Rightarrow f(x) = -f(x)$ for all x in the domain of f . Thus $2f(x) = 0 \Rightarrow f(x) = 0$.
12. (a) As suggested, let $E(x) = \frac{f(x) + f(-x)}{2} \Rightarrow E(-x) = \frac{f(-x) + f(-(-x))}{2} = \frac{f(x) + f(-x)}{2} = E(x) \Rightarrow E$ is an even function. Define $O(x) = f(x) - E(x) = f(x) - \frac{f(x) + f(-x)}{2} = \frac{f(x) - f(-x)}{2}$. Then $O(-x) = \frac{f(-x) - f(-(-x))}{2} = \frac{f(-x) - f(x)}{2} = -\left(\frac{f(x) - f(-x)}{2}\right) = -O(x) \Rightarrow O$ is an odd function $\Rightarrow f(x) = E(x) + O(x)$ is the sum of an even and an odd function.
- (b) Part (a) shows that $f(x) = E(x) + O(x)$ is the sum of an even and an odd function. If also $f(x) = E_1(x) + O_1(x)$, where E_1 is even and O_1 is odd, then $f(x) - f(x) = 0$
 $= (E_1(x) + O_1(x)) - (E(x) + O(x))$. Thus, $E(x) - E_1(x) = O_1(x) - O(x)$ for all x in the domain of f (which is the same as the domain of $E - E_1$ and $O - O_1$). Now $(E - E_1)(-x) = E(-x) - E_1(-x) = E(x) - E_1(x)$ (since E and E_1 are even) $= (E - E_1)(x) \Rightarrow E - E_1$ is even. Likewise, $(O_1 - O)(-x) = O_1(-x) - O(-x) = -O_1(x) - (-O(x)) = -(O_1(x) - O(x)) = -(O_1 - O)(x) \Rightarrow O_1 - O$ is odd. Therefore, $E - E_1$ and $O_1 - O$ are both even and odd so they must be zero at each x in the domain of f by Exercise 11. That is, $E_1 = E$ and $O_1 = O$, so the decomposition of f found in part (a) is unique.
13. $y = ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a} + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$
- (a) If $a > 0$ the graph is a parabola that opens upward. Increasing a causes a vertical stretching and a shift of the vertex toward the y -axis and upward. If $a < 0$ the graph is a parabola that opens downward. Decreasing a causes a vertical stretching and a shift of the vertex toward the y -axis and downward.
- (b) If $a > 0$ the graph is a parabola that opens upward. If also $b > 0$, then increasing b causes a shift of the graph downward to the left; if $b < 0$, then decreasing b causes a shift of the graph downward and to the right. If $a < 0$ the graph is a parabola that opens downward. If $b > 0$, increasing b shifts the graph upward to the right. If $b < 0$, decreasing b shifts the graph upward to the left.
- (c) Changing c (for fixed a and b) by Δc shifts the graph upward Δc units if $\Delta c > 0$, and downward $-\Delta c$ units if $\Delta c < 0$.
14. (a) If $a > 0$, the graph rises to the right of the vertical line $x = -b$ and falls to the left. If $a < 0$, the graph falls to the right of the line $x = -b$ and rises to the left. If $a = 0$, the graph reduces to the horizontal line $y = c$. As $|a|$ increases, the slope at any given point $x = x_0$ increases in magnitude and the graph becomes steeper. As $|a|$ decreases, the slope at x_0 decreases in magnitude and the graph rises or falls more gradually.
- (b) Increasing b shifts the graph to the left; decreasing b shifts it to the right.
- (c) Increasing c shifts the graph upward; decreasing c shifts it downward.
15. Each of the triangles pictured has the same base $b = v\Delta t = v(1 \text{ sec})$. Moreover, the height of each triangle is the same value h . Thus $\frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}bh = A_1 = A_2 = A_3 = \dots$. In conclusion, the object sweeps out equal areas in each one second interval.



16. (a) Using the midpoint formula, the coordinates of P are $\left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$. Thus the slope of $\overline{OP} = \frac{\Delta y}{\Delta x} = \frac{b/2}{a/2} = \frac{b}{a}$.
- (b) The slope of $\overline{AB} = \frac{b-0}{0-a} = -\frac{b}{a}$. The line segments \overline{AB} and \overline{OP} are perpendicular when the product of their slopes is $-1 = \left(\frac{b}{a}\right)\left(-\frac{b}{a}\right) = -\frac{b^2}{a^2}$. Thus, $b^2 = a^2 \Rightarrow a = b$ (since both are positive). Therefore, \overline{AB} is perpendicular to \overline{OP} when $a = b$.
17. From the figure we see that $0 \leq \theta \leq \frac{\pi}{2}$ and $AB = AD = 1$. From trigonometry we have the following:
 $\sin \theta = \frac{EB}{AB} = EB$, $\cos \theta = \frac{AE}{AB} = AE$, $\tan \theta = \frac{CD}{AD} = CD$, and $\tan \theta = \frac{EB}{AE} = \frac{\sin \theta}{\cos \theta}$. We can see that:
 $\text{area } \triangle AEB < \text{area sector } \widehat{DB} < \text{area } \triangle ADC \Rightarrow \frac{1}{2}(AE)(EB) < \frac{1}{2}(AD)^2 \theta < \frac{1}{2}(AD)(CD)$
 $\Rightarrow \frac{1}{2} \sin \theta \cos \theta < \frac{1}{2}(1)^2 \theta < \frac{1}{2}(1)(\tan \theta) \Rightarrow \frac{1}{2} \sin \theta \cos \theta < \frac{1}{2} \theta < \frac{1}{2} \frac{\sin \theta}{\cos \theta}$
18. $(f \circ g)(x) = f(g(x)) = a(cx + d) + b = acx + ad + b$ and $(g \circ f)(x) = g(f(x)) = c(ax + b) + d = acx + cb + d$
 Thus $(f \circ g)(x) = (g \circ f)(x) \Rightarrow acx + ad + b = acx + bc + d \Rightarrow ad + b = bc + d$. Note that $f(d) = ad + b$ and $g(b) = cb + d$, thus $(f \circ g)(x) = (g \circ f)(x)$ if $f(d) = g(b)$.
19. (a) The expression $a(b^{c-x}) + d$ is defined for all values of x , so the domain is $(-\infty, \infty)$. Since b^{c-x} attains all positive values, the range is (d, ∞) if $a > 0$ and the range is $(-\infty, d)$ if $a < 0$.
- (b) The expression $a \log_b(x - c) + d$ is defined when $x - c > 0$, so the domain is (c, ∞) . Since $a \log_b(x - c) + d$ attains every real value for some value of x , the range is $(-\infty, \infty)$.
20. (a) Suppose $f(x_1) = f(x_2)$. Then:

$$\frac{ax_1 + b}{cx_1 + d} = \frac{ax_2 + b}{cx_2 + d}$$

$$(ax_1 + b)(cx_2 + d) = (ax_2 + b)(cx_1 + d)$$

$$acx_1x_2 + adx_1 + bcx_2 + bd = acx_1x_2 + adx_2 + bcx_1 + bd$$

$$adx_1 + bcx_2 = adx_2 + bcx_1$$

$$(ad - bc)x_1 = (ad - bc)x_2$$
 Since $ad - bc \neq 0$, this means that $x_1 = x_2$.
- (b) $y = \frac{ax+b}{cx+d}$
 $cx + dy = ax + b$
 $(cy - a)x = -dy + b$
 $x = \frac{-dy + b}{cy - a}$
 Interchange x and y .
 $y = \frac{-dx + b}{cx - a}$
 $f^{-1}(x) = \frac{-dx + b}{cx - a}$

21. (a) $y = 100,000 - 10,000x, 0 \leq x \leq 10$

(b) $y = 55,000$

$100,000 - 10,000x = 55,000$

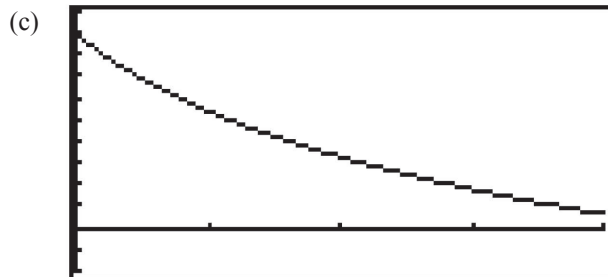
$-10,000x = 55,000$

$x = 4.5$

The value is \$55,000 after 4.5 years.

22. (a) $f(0) = 90$ units

(b) $f(2) = 90 - 52 \ln 3 \approx 32.8722$ units



[0, 4] by [-20, 100]

23. $1500(1.08)^t = 5000 \rightarrow 1.08^t = \frac{5000}{1500} = \frac{10}{3} \rightarrow \ln(1.08)^t = \ln \frac{10}{3} \rightarrow t \ln 1.08 = \ln \frac{10}{3} \rightarrow t = \frac{\ln(10/3)}{\ln 1.08} \approx 15.6439$

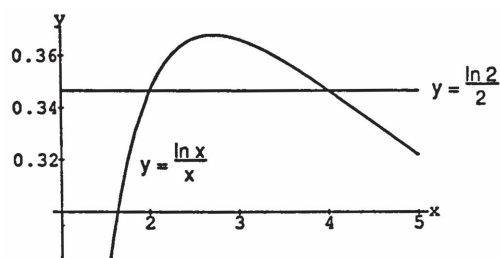
It will take about 15.6439 years. (If the bank only pays interest at the end of the year, it will take 16 years.)

24. $A(t) = A_0 e^{rt}; A(t) = 2A_0 \Rightarrow 2A_0 = A_0 e^{rt} \Rightarrow e^{rt} = 2 \Rightarrow rt = \ln 2 \Rightarrow t = \frac{\ln 2}{r} \Rightarrow t \approx \frac{0.7}{r} = \frac{70}{100r} = \frac{70}{(r\%)}$

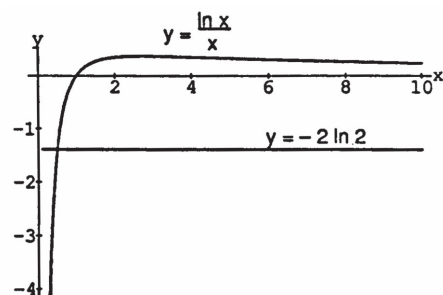
25. $\ln x^{(x^x)} = x^x \ln x$ and $\ln(x^x)^x = x \ln(x^x) = x^2 \ln x$; then, $x^x \ln x = x^2 \ln x \Rightarrow x^x = x^2 \Rightarrow x \ln x = 2 \ln x \Rightarrow x = 2$.

Therefore, $x^{(x^x)} = (x^x)^x$ when $x = 2$.

26. (a) No, there are two intersections: one at $x = 2$ and the other at $x = 4$.



(b) Yes, because there is only one intersection.



$$27. \frac{\log_4 x}{\log_2 x} = \frac{\left(\frac{\ln x}{\ln 4}\right)}{\left(\frac{\ln x}{\ln 2}\right)} = \frac{\ln x}{\ln 4} \cdot \frac{\ln 2}{\ln x} = \frac{\ln 2}{\ln 4} = \frac{\ln 2}{2 \ln 2} = \frac{1}{2}$$

$$28. (a) f(x) = \frac{\ln 2}{\ln x}, g(x) = \frac{\ln x}{\ln 2}$$

- (b) f is negative when g is negative, positive when g is positive, and undefined when $g = 0$; the values of f decrease as those of g increase.

$$(c) \frac{\ln 2}{\ln x} = \frac{\ln x}{\ln 2} \Rightarrow (\ln 2)^2 = (\ln x)^2$$

$$\Rightarrow (\ln 2 \ln x)(\ln 2 + \ln x) = 0 \Rightarrow \ln x = \ln 2$$

or $\ln x = -\ln 2 \Rightarrow e^{\ln x} = e^{\ln 2}$ or

$$e^{\ln x} = e^{\ln(1/2)} \Rightarrow x = 2 \text{ or } x = \frac{1}{2}.$$

Therefore, the two curves cross at the two

points $\left(\frac{1}{2}, \frac{\ln(1/2)}{\ln 2}\right) = \left(\frac{1}{2}, -1\right)$ and

$\left(2, \frac{\ln 2}{\ln 2}\right) = (2, 1).$

