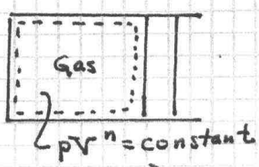
PROBLEM 2.27

KNOWN: A gas in a piston-cylinder assembly undergoes a compression process for which pV^n = constant. State data is provided.

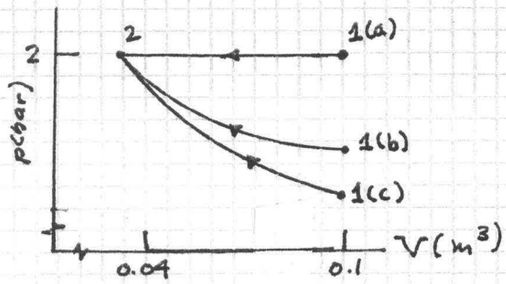
FIND: For each of $n = 0$, $n = 1$, and $n = 1.3$, determine the initial pressure, in bar, and the work, in kJ.

SCHEMATIC & GIVEN DATA:



$V_1 = 0.1\ m^3$

$V_2 = 0.04 m^3$, $P_2 = 2$ bar



ENGINEERING MODEL:

1. The gas within the piston-cylinder is the closed system.
2. Volume change is the only work mode.
3. The process of the gas obeys pV^n = constant, where (a) $n = 0$, (b) $n = 1$, (c) $n = 1.3$.

ANALYSIS:

(a) $n = 0$: Thus, pV^0 = constant $\Rightarrow$ p = constant. So, $P_1 = 2$ bar ←

Using Eq. 2.17 with p = constant,

(1) $$W = \int_1^2 p\,dV = p[V_2 - V_1] = 2\text{bar}\left[0.04 - 0.1\right] m^3 \left|\frac{10^5 N/m^2}{1\ \text{bar}}\right| \left|\frac{1\ kJ}{10^3 N \cdot m}\right| = -12\ kJ$$ ←

(b) $n = 1$: Thus, pV = constant $\Rightarrow$ $P_1V_1 = P_2V_2 \Rightarrow P_1 = P_2\left[\frac{V_2}{V_1}\right] = 2\text{bar}\left[\frac{0.04}{0.10}\right] = 0.8\text{bar}$ ←

$$W = \int_1^2 p\,dV = \int_1^2 \frac{C}{V}\,dV = C \ln\frac{V_2}{V_1} = P_2V_2 \ln\frac{V_2}{V_1} = \left(2\times10^5 \frac{N}{m^2}\right)(0.04\ m^3)\left|\frac{1\ kJ}{10^3 N\cdot m}\right| \ln\frac{0.04}{0.10}$$

($C = P_1V_1 = P_2V_2$)

$$= -7.33\ kJ$$ ←

(c) $n = 1.3$: Thus, $P_1V_1^n = P_2V_2^n$, where $n = 1.3$. $\Rightarrow$ $P_1 = P_2\left[\frac{V_2}{V_1}\right]^{1.3} = 2\text{bar}\left[\frac{0.04}{0.10}\right]^{1.3}$

$$= 0.608\ \text{bar}$$ ←

$$W = \int_1^2 p\,dV = \int_1^2 \frac{C}{V^n}\,dV = \frac{P_2V_2 - P_1V_1}{(1-n)}$$. See Example 2.1(a) for the integration.

$$\therefore W = \frac{\left(2\times10^5 N/m^2\right)(0.04 m^3) - \left(0.608\times10^5 N/m^2\right)(0.1\ m^3)}{(1-1.3)} \left|\frac{1\ kJ}{10^3 N\,m}\right|$$

$$= -6.4\ kJ$$ ←

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1. The negative sign for W denotes work done <u>on</u> the gas during compression.