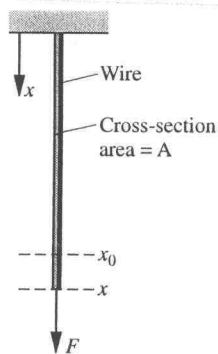


PROBLEM 2.40

KNOWN: A wire suspended vertically is stretched by an applied force.

FIND: Obtain an expression for the work done on the wire and evaluate the work for a given set of data.

SCHEMATIC & GIVEN DATA:



$$\sigma = C \epsilon$$

normal stress
strain

$$\epsilon = \frac{x - x_0}{x_0}$$

C: Young's modulus

Data set:

$$x_0 = 10 \text{ ft}$$

$$x = 10.01 \text{ ft}$$

$$C = 2.5 \times 10^7 \frac{\text{lb f}}{\text{in}^2}$$

$$A = 0.1 \text{ in}^2$$

ENGR. MODEL:

1. The wire is the closed system.
2. The moving boundary is the only work mode.
3. The change in area A is negligible.
4. The normal stress and thus the applied force varies linearly with strain.

Fig. P2.39

ANALYSIS: (a) The work done on the wire is given by Eq. 2.18

$$W = - \int_{x_0}^x \sigma A dx$$

From the given stress-strain relation

$$\sigma = C \epsilon = C \left(\frac{x - x_0}{x_0} \right)$$

Where C is a constant (Young's modulus). From this expression

$$d\epsilon = \frac{dx}{x_0} \Rightarrow dx = x_0 d\epsilon$$

Substituting into the work expression

$$W = - \int_0^{\epsilon} (C \epsilon) A (x_0 d\epsilon) = - C A x_0 \int_0^{\epsilon} \epsilon d\epsilon$$

Finally

$$W = - \frac{C A x_0 \epsilon^2}{2}$$

work expression

(b) Substituting given data into the work expression

$$W = - \frac{(2.5 \times 10^7 \frac{\text{lb f}}{\text{in}^2})(0.1 \text{ in}^2)(10 \text{ ft}) \left[\frac{0.01}{10} \right]^2}{2} = -12.5 \text{ ft} \cdot \text{lb f}$$

The downward force varies with strain according to

$$F = \sigma A = C \epsilon A = A C \left[\frac{x - x_0}{x_0} \right]$$

When $x = x_0$, $F = 0$. When $x - x_0 = 0.01 \text{ ft}$,

$$F = (0.1 \text{ in}^2)(2.5 \times 10^7 \frac{\text{lb f}}{\text{in}^2}) \left(\frac{0.01 \text{ ft}}{10 \text{ ft}} \right) = 2500 \text{ lb f}$$

