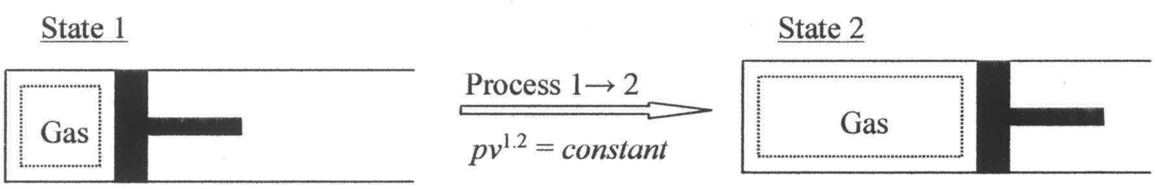
PROBLEM 2.67

KNOWN: A gas undergoes a polytropic process between two specified states. The relationship between pressure, volume, and internal energy is known for the gas.

FIND: Determine the heat transfer per unit mass of gas.

SCHEMATIC AND GIVEN DATA:



$p_1 = 40$ lbf/in^2
$v_1 = 4.0$ ft^3/lb

$p_2 = 14$ lbf/in^2

ENGINEERING MODEL:
1. The gas is a closed system.
2. The system undergoes a polytropic process in which $pv^{1.2} = constant$.
3. Kinetic and potential energy effects are negligible.

ANALYSIS:
The heat transfer can be determined from an energy balance.

$$\Delta \text{KE} + \Delta \text{PE} + \Delta U = Q - W$$

Neglecting changes in kinetic energy (ΔKE = 0) and potential energy (ΔPE = 0), and solving for heat transfer give

$$Q = W + \Delta U$$

Dividing by mass for an energy balance on a per unit mass basis

$$Q/m = W/m + \Delta u$$

To solve for work per unit mass

PROBLEM 2.67 (Continued)

$$W/m = \int_{v_1}^{v_2} p\,dv = \int_{v_1}^{v_2} \frac{(constant)dv}{v^{1.2}}$$

Integrating and simplifying

$$W/m = \frac{(constant)v_2^{1-1.2} - (constant)v_1^{1-1.2}}{1-1.2} = \frac{(p_2 v_2^{1.2})v_2^{1-1.2} - (p_1 v_1^{1.2})v_1^{1-1.2}}{1-1.2} = \frac{p_2 v_2 - p_1 v_1}{1-1.2}$$

The specific volume at state 2 is determined from

$$p_1 {v_1}^{1.2} = p_2 {v_2}^{1.2}$$

$$v_2 = v_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{1.2}} = \left(4\frac{\text{ft}^3}{\text{lb}}\right)\left(\frac{40\frac{\text{lbf}}{\text{in.}^2}}{14\frac{\text{lbf}}{\text{in.}^2}}\right)^{\frac{1}{1.2}} = 9.59\ \text{m}^3/\text{lb}$$

Substituting specific volume to solve for work yields

$$W/m = \frac{\left(14\frac{\text{lbf}}{\text{in.}^2}\right)\left(9.59\frac{\text{ft}^3}{\text{lb}}\right) - \left(40\frac{\text{lbf}}{\text{in.}^2}\right)\left(4.0\frac{\text{ft}^3}{\text{lb}}\right)}{1-1.2}\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| = 23.8\ \text{Btu/lb}$$

The work is positive, denoting energy transfer by work out of the gas as it expands.

Evaluating $(u_2 - u_1)$

$$(u_2 - u_1) = \left[\left(0.464\frac{\text{Btu}\cdot\text{in.}^2}{\text{lbf}\cdot\text{ft}^3}\right)p_2 v_2 - 0.7095\frac{\text{Btu}}{\text{lb}}\right] - \left[\left(0.464\frac{\text{Btu}\cdot\text{in.}^2}{\text{lbf}\cdot\text{ft}^3}\right)p_1 v_1 - 0.7095\frac{\text{Btu}}{\text{lb}}\right]$$

$$(u_2 - u_1) = \left(0.464\frac{\text{Btu}\cdot\text{in.}^2}{\text{lbf}\cdot\text{ft}^3}\right)(p_2 v_2 - p_1 v_1)$$

$$(u_2 - u_1) = \left(0.464\frac{\text{Btu}\cdot\text{in.}^2}{\text{lbf}\cdot\text{ft}^3}\right)\left(\left(14\frac{\text{lbf}}{\text{in.}^2}\right)\left(9.59\frac{\text{ft}^3}{\text{lb}}\right) - \left(40\frac{\text{lbf}}{\text{in.}^2}\right)\left(4.0\frac{\text{ft}^3}{\text{lb}}\right)\right) = -11.9\ \text{Btu/lb}$$

Substituting into the energy equation to solve for heat transfer per unit mass

$$Q/m = 23.8\ \text{Btu/lb} + (-11.9\ \text{Btu/lb}) = \underline{\mathbf{11.9\ Btu/lb}}$$

The positive sign for heat transfer indicates energy into the system by heat.