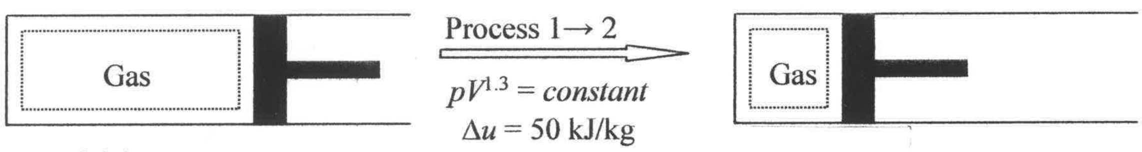
PROBLEM 2.63

KNOWN: A gas of known mass is compressed in a piston-cylinder assembly from a specified initial pressure to a known final state. The pressure-volume relationship for the process is given, and the specific internal energy change is known.

FIND: Determine the heat transfer for the process.

SCHEMATIC AND GIVEN DATA:

State 1 | State 2



$m_1 = 0.2$ kg
$p_1 = 2$ bar

$p_2 = 8$ bar
$V_2 = 0.02$ m^3
$u_2 = u_1 + 50$ kJ/kg

ENGINEERING MODEL:
1. The gas is a closed system.
2. The system undergoes a polytropic process in which $pV^{1.3} = constant$.
3. Kinetic and potential energy effects are negligible

ANALYSIS:
The heat transfer can be determined from the energy balance

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

Neglecting changes in kinetic energy ($\Delta KE = 0$) and potential energy ($\Delta PE = 0$) and solving for heat transfer give

$$Q = \Delta U + W$$

Internal energy change is determined from the given mass and change in specific internal energy

$$\Delta U = m\,\Delta u = (0.2 \text{ kg})(50 \text{ kJ/kg}) = 10 \text{ kJ}$$

PROBLEM 2.63 (Continued)

To solve for work

$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{(constant)dV}{V^{1.3}}$$

Integrating and simplifying

$$W = \frac{(constant)V_2^{1-1.3} - (constant)V_1^{1-1.3}}{1-1.3} = \frac{(p_2V_2^{1.3})V_2^{1-1.3} - (p_1V_1^{1.3})V_1^{1-1.3}}{1-1.3} = \frac{p_2V_2 - p_1V_1}{1-1.3}$$

The volume at state 1 is determined from

$$p_1V_1^{1.3} = p_2V_2^{1.3}$$

$$V_1 = V_2\left(\frac{p_2}{p_1}\right)^{\frac{1}{1.3}} = (0.02\ \text{m}^3)\left(\frac{8\ \text{bar}}{2\ \text{bar}}\right)^{\frac{1}{1.3}} = 0.058\ \text{m}^3$$

Substituting volume to solve for work yields

$$W = \frac{(8\ \text{bar})(0.02\ \text{m}^3) - (2\ \text{bar})(0.058\ \text{m}^3)}{1-1.3}\left|\frac{10^5\ \frac{\text{N}}{\text{m}^2}}{1\ \text{bar}}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right| = -14.6\ \text{kJ}$$

The work is negative, denoting energy transfer by work into the gas as it compresses.

Heat transfer is determined by substituting internal energy change and work values in the energy equation

$$Q = 10\ \text{kJ} + (-14.6\ \text{kJ}) = \underline{\mathbf{-4.6\ kJ}}$$

The heat transfer is negative, denoting energy transfer by heat from the gas as it compresses.