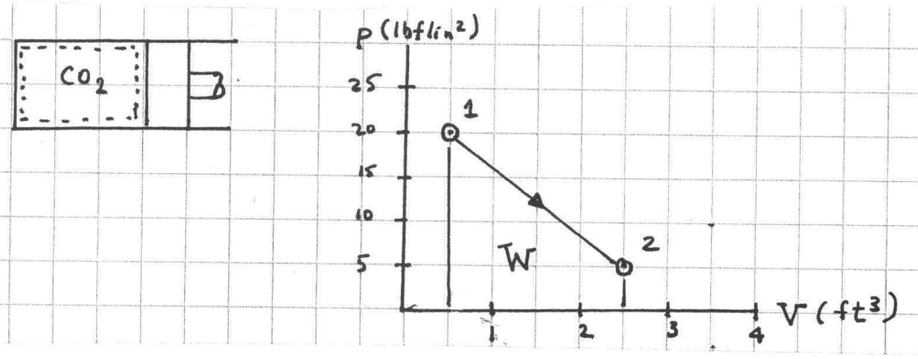
PROBLEM 2.26

KNOWN: CO_2 gas within a piston-cylinder assembly undergoes an expansion where the p-V relation is $p = A + BV$.

FIND: Evaluate A and B and the work, all in specified units.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The CO_2 is the closed system. (2) The p-V relation during expansion is linear. (3) Volume change is the only work mode.

ANALYSIS:

Part (b)

$$p_1 = A + BV_1 \Rightarrow (20\ lbf/in^2) = B\,(0.5\ ft^3) + A$$

$$p_2 = A + BV_2 \Rightarrow (5\ lbf/in^2) = B\,(2.5\ ft^3) + A$$

Solving, $B = -7.5\ \dfrac{lbf/in^2}{ft^3}$, $A = 23.75\ lbf/in^2$ ←

observe that A and B have numerical values and units.

Part (a) Since volume change is the work mode here, Eq. 2.17 applies. Thus

$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} [A + BV]\,dV = \left[AV + \frac{BV^2}{2}\right]_{V_1}^{V_2}$$

$$= A[V_2 - V_1] + \frac{B}{2}[V_2^2 - V_1^2]$$

$$= 23.75\ \frac{lbf}{in^2}\left|\frac{144\ in^2}{1\ ft^2}\right|[2.5 - 0.5]\ ft^3 + \left(\frac{-7.5}{2}\right)\frac{lbf/in^2}{ft^3}\left|\frac{144\ in^2}{1\ ft^2}\right|\left[(2.5)^2 - (0.5)^2\right](ft^3)^2$$

$$= 6840\ ft\cdot lbf - 3240\ ft\cdot lbf = 3600\ ft\cdot lbf. \leftarrow$$

$$= (3600\ ft\cdot lbf)\left|\frac{1\ Btu}{778\ ft\cdot lbf}\right| = 4.63\ Btu \leftarrow$$

Alternative solution of part (a):

Since the p-V relation is linear, W can be easily evaluated geometrically as the area under the process line:

$$W = p_{ave}(V_2 - V_1) = \left(\frac{P_1 + P_2}{2}\right)(V_2 - V_1) = \left(\frac{20+5}{2}\right)\frac{lbf}{in^2}\left|\frac{144\ in^2}{1\ ft^2}\right|(2.5 - 0.5)\ ft^3$$

$$= 3600\ ft\cdot lbf \leftarrow$$