PROBLEM 2.58

KNOWN: A gas within a piston-cylinder assembly undergoes two different processes between the same end states.

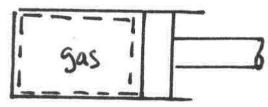
FIND: Sketch the processes on p-V coordinates. For each process evaluate W and Q.

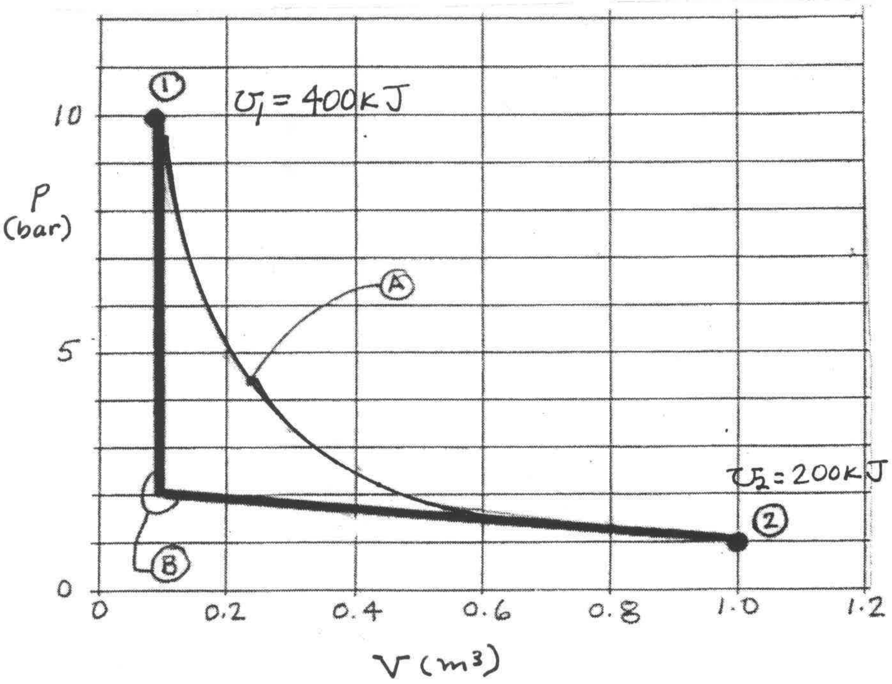
SCHEMATIC & GIVEN DATA:

Data: $p_1 = 10$ bar, $V_1 = 0.1\ m^3$, $U_1 = 400$ kJ and $p_2 = 1$ bar, $V_2 = 1.0\ m^3$, $U_2 = 200$ kJ:

Process A: Process from 1 to 2 during which the pressure-volume relation is pV = *constant*

Process B: Constant-volume process from state 1 to a pressure of 2 bar, followed by a linear pressure-volume process to state 2.





ENGR. MODEL:

1. The gas is the closed system.
2. Kinetic and potential energy effects can be ignored.
3. The p-V relation is specified for each process.
4. The moving boundary is the only work mode.

ANALYSIS: For Process A, $W_A = \int_1^2 p\,dV = \int_{V_1}^{V_2} \frac{C}{V}\,dV = C \ln \frac{V_2}{V_1} = P_1V_1 \ln \frac{V_2}{V_1}$ ($C = P_1V_1 = P_2V_2$)

Thus, $W_A = (10\text{ bar})(0.1\text{ m}^3) \ln\left[\frac{1.0\text{ m}^3}{0.1\text{ m}^3}\right] \left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right| \left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = +230.26\text{ kJ}$ ← W_A

$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_A - W_A \Rightarrow Q_A = \Delta U + W_A = (200 - 400)\text{ kJ} + 230.26\text{ kJ} = +30.26\text{ kJ}$ ← Q_A

For Process B, the piston does not move for the first step (constant volume) and thus there is no work. The work can be evaluated geometrically for the second step, during which the p-V relation is linear.

$W_B = P_{ave}[V_2 - V_1] = \left[\frac{2\text{ bar} + 1\text{ bar}}{2}\right]\left[1.0\text{ m}^3 - 0.1\text{ m}^3\right] \left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right| \left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = 135\text{ kJ}$ ← W_B

$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_B - W_B \Rightarrow Q_B = \Delta U + W$

Note: Since U is a property, ΔU is the same for each process

$Q_B = (200 - 400)\text{ kJ} + 135\text{ kJ} = -65\text{ kJ}$ ← Q_B