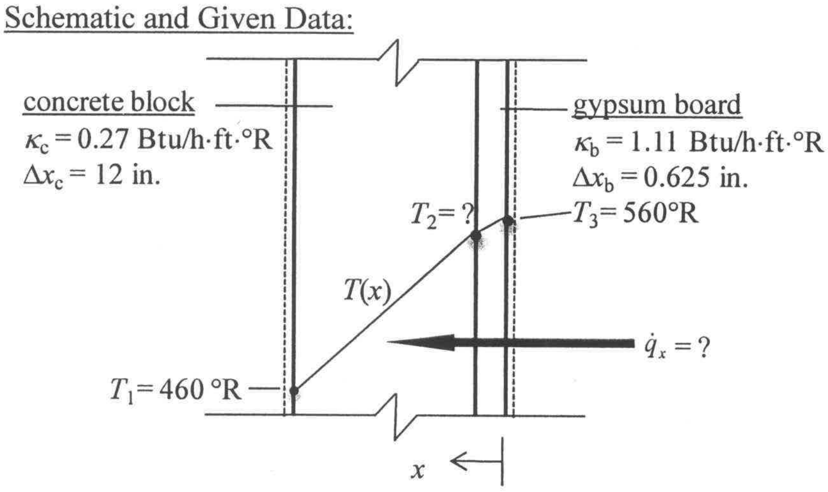
PROBLEM 2.46

Known: Insulated composite plane wall consists of insulated concrete block and gypsum board with known surface temperatures. There is perfect contact at the interface between the two layers.

Find: Determine at steady state the instantaneous rate of heat transfer, in Btu/h per ft^2 of surface area, and the temperature, in °R, at the interface between the concrete block and gypsum board.

Schematic and Given Data:



Engineering Model:
(1) The wall is a closed system at steady state.
(2) The temperature distributions are linear in both layers.
(3) The two layers are in perfect thermal contact.
(4) The thermal conductivity in each layer is uniform.

Analysis:

Begin with Eq. 2.31

$$\dot{Q}_x = -\kappa A \frac{dT}{dx}$$

$$\dot{q}_x = \frac{\dot{Q}_x}{A} = -\kappa \frac{dT}{dx} = -\kappa_c \left.\frac{dT}{dx}\right|_{\text{concrete}} = -\kappa_b \left.\frac{dT}{dx}\right|_{\text{gypsum board}}$$

PROBLEM 2.46 (Continued)

Using assumption (2)

$$\dot{q}_x = \kappa_c \frac{T_2 - T_1}{\Delta x_c} = \kappa_b \frac{T_3 - T_2}{\Delta x_b}$$

Use thermal resistance, $R = \Delta x/\kappa$, to simplify

$$R_c = \frac{\Delta x_c}{\kappa_c} \quad and \quad R_b = \frac{\Delta x_b}{\kappa_b}$$

$$\dot{q}_x = \frac{T_2 - T_1}{R_c} = \frac{T_3 - T_2}{R_b} \qquad (1)$$

Rearrange first part of Eq. (1) (2)

$$T_2 = T_1 + R_c \dot{q}_x$$

Substitute Eq. (2) into the second part of Eq. (1) and rearrange

#1
$$\dot{q}_x = \frac{T_3 - T_2}{R_b} = \frac{T_3 - (T_1 + R_c \dot{q}_x)}{R_b} = \frac{T_3 - T_1}{R_c + R_b} \qquad (3)$$

Solve for R values

$$R_c = \frac{\Delta x_c}{\kappa_E} = \frac{12 \text{ in.}}{0.27 \dfrac{\text{Btu}}{\text{h} \cdot \text{ft} \cdot {}^\circ\text{R}}} \left| \frac{1 \text{ ft}}{12 \text{ in.}} \right| = 3.7 \frac{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{R}}{\text{Btu}}$$

$$R_E = \frac{\Delta x_b}{\kappa_b} = \frac{0.625 \text{ in.}}{1.11 \dfrac{\text{Btu}}{\text{h} \cdot \text{ft} \cdot {}^\circ\text{R}}} \left| \frac{1 \text{ ft}}{12 \text{ in.}} \right| = 0.047 \frac{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{R}}{\text{Btu}}$$

Solve for $\dot{q}_x$ and T_2

$$\dot{q}_x = \frac{T_3 - T_1}{R_E + R_b} = \frac{560^\circ\text{R} - 460^\circ\text{R}}{(3.7 + 0.047) \dfrac{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{R}}{\text{Btu}}} = 26.69 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2} \longleftarrow$$

#2
$$T_2 = T_1 + R_c \dot{q}_x = 460^\circ\text{R} + \left(3.7 \frac{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{R}}{\text{Btu}}\right)\left(26.69 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2}\right) = 558.8^\circ\text{R} \longleftarrow$$

---

1. Eq. (3) illustrates the analogy between heat conduction through a composite wall and electric current flow through a series of resistances. The temperature difference in the numerator is analogous to a voltage difference, and the value R_c and R_b are "thermal resistances" analogous to electrical resistances.
2. Due to the relatively low value for R_b, the temperature change across the gypsum board is minimal compared to the concrete block.