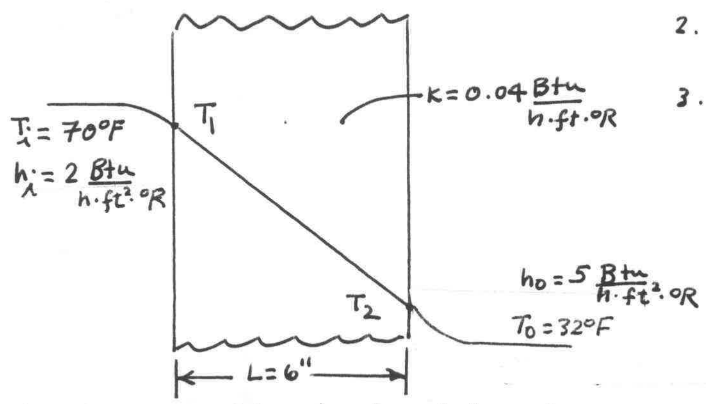
PROBLEM 2.48

KNOWN: Steady-state data are provided for the outer wall of a house.

FIND: Determine the rate of heat transfer through the wall.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The wall is at steady state.
2. Temperature varies linearly through the wall.
3. At the inner and outer surfaces, convection is the only heat transfer mode.

ANALYSIS: At steady state, the rates of energy transfer to the wall by convection, through the wall by conduction, and from the wall by convection are all equal. That is,

$$\frac{\dot{Q}}{A} = \underbrace{h_i (T_i - T_1)}_{\text{convection to wall}} = \underbrace{-k\left[\frac{T_2 - T_1}{L}\right]}_{\text{conduction through wall}} = \underbrace{h_o[T_2 - T_o]}_{\text{convection from wall}} \qquad (*)$$

Solving the first and third of Eqs.(*),

$$T_1 = T_i - \frac{\dot{Q}/A}{h_i}, \qquad T_2 = T_o + \frac{\dot{Q}/A}{h_o}$$

Substituting into the second of Eqs.(*) and simplifying

(1)
$$\frac{\dot{Q}}{A} = -k\left[\frac{\left(T_o + \frac{\dot{Q}/A}{h_o}\right) - \left(T_i - \frac{\dot{Q}/A}{h_i}\right)}{L}\right] = \frac{T_i - T_o}{\underbrace{\left(\frac{1}{h_i} + \frac{L}{k} + \frac{1}{h_o}\right)}_{\text{thermal "resistances"}}} \qquad (**)$$

Inserting values into Eq.(**)

$$\frac{\dot{Q}}{A} = \frac{[530°R - 492°R] \quad (Btu/h \cdot ft^2 \cdot °R)}{\left[\underbrace{\frac{1}{2}}_{0.5} + \underbrace{\frac{0.5}{0.04}}_{12.5} + \underbrace{\frac{1}{5}}_{0.2}\right]}$$

$$= 2.88 \frac{Btu/h}{ft^2} \longleftarrow$$

---

1. The form of Eq.(**) illustrates the analogy between heat transfer and electric current flow through resistances in series. The temperature difference in the numerator is analogous to the voltage difference, and the terms in the denominator account for "thermal resistances" analogous to electrical resistances. In this case, the insulated wall is the greater of the three resistances, as shown by the calculation below.