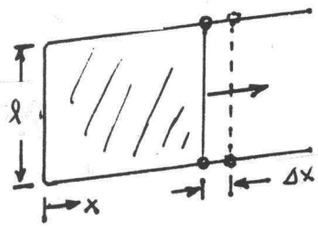
PROBLEM 2.41

KNOWN: A soap film on a wire frame is stretched.

FIND: Determine the work done.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The film is a closed system. (2) The moving boundary is the only work mode. (3) The surface tension is constant, acting on both sides of the film.

ANALYSIS: (a) The work is determined using Eq. 2.19

$$W = -\int_{A_1}^{A_2} \tau \, dA = -\int_{x_1}^{x_2} \tau \, 2\ell \, dx$$

For constant surface tension τ

$$W = -\tau 2\ell \Delta x \longleftarrow$$

(b) If $\ell = 5\text{cm}$, $\Delta x = 0.5\text{cm}$, $\tau = 25\times10^{-5}\ \text{N/cm}$,

$$W = -(25\times10^{-5}\text{N/cm})(2)(5\text{cm})(0.5\text{cm})\left|\frac{1\text{m}}{10^2\text{cm}}\right|\left|\frac{1\text{J}}{1\text{N}\cdot\text{m}}\right| = -1.25\times10^{-5}\ \text{J} \longleftarrow$$

The negative sign denotes work done on the film. Note the small magnitude of the work required to stretch the film.

PROBLEM 2.42

KNOWN: Data is provided for a spring stretched by a force applied at its end.

FIND: Obtain an expression for the work done in stretching the spring and evaluate the work using given data.

SCHEMATIC & GIVEN DATA:

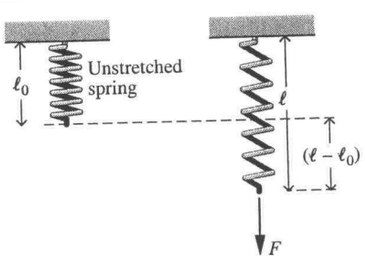


Fig. P2.41

$F = k(\ell - \ell_0)$

$\ell_0 = 3\text{cm}$

$\ell_1 = 6\text{cm}$

$\ell_2 = 10\text{cm}$

$k = 10^4\ \text{N/m}$

ENGR. MODEL:

1. The spring is the closed system.
2. The moving boundary is the only work mode.
3. Hooke's law applies.

ANALYSIS: (a) The work done in stretching the spring is given by

$$W = -\int_1^2 F\,d\ell$$

Letting $x = \ell - \ell_0$, this becomes

$$W = -\int_1^2 kx\,dx = -k\left[\frac{x_2^2 - x_1^2}{2}\right]$$

$$= -\frac{k}{2}\left[(\ell_2 - \ell_0)^2 - (\ell_1 - \ell_0)^2\right] \longleftarrow$$

(b) When $(\ell_1 - \ell_0) = 3\text{cm}$ and $(\ell_2 - \ell_0) = 7\text{cm}$,

$$W = \frac{(-10^4\ \text{N/m})}{2}\left[(7\text{cm})^2 - (3\text{cm})^2\right]\left|\frac{1\text{m}}{10^2\text{cm}}\right|^2\left|\frac{1\text{J}}{1\text{N}\cdot\text{m}}\right|$$

$$= -20\text{J} \longleftarrow$$