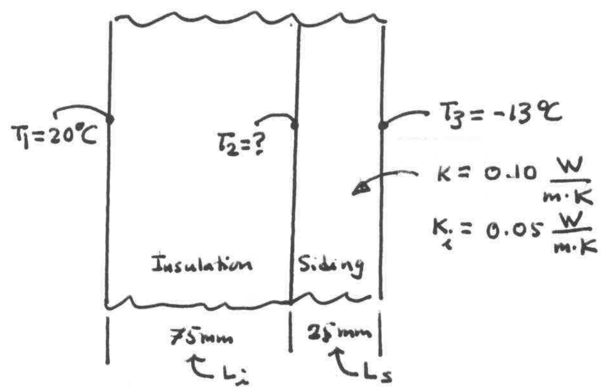
PROBLEM 2.47

KNOWN: Energy transfer occurs by conduction through a composite plane wall consisting of two layers.

FIND: Determine the temperature at the interface between the two layers and the rate of heat transfer through the wall.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The wall is at steady state.
2. The temperature varies linearly through each layer.

ANALYSIS: With Eq. 2.31 and recognizing that at steady state rate at which energy is conducted must be the same in each layer,

$$\frac{\dot{Q}}{A} = -k_i\left[\frac{T_2 - T_1}{L_i}\right] = -k\left[\frac{T_3 - T_2}{L_s}\right] \qquad (*)$$

Solving for T_2,

$$T_2 = \frac{\left(\frac{k_i}{L_i}\right)T_1 + \left(\frac{k}{L_s}\right)T_3}{(k_i/L_i) + (k/L_s)} \quad , \quad \frac{k_i}{L_i} = \frac{0.05\,W/m\cdot K}{.075m} = 2/3\,\frac{W}{m^2\cdot K}$$

$$\frac{k}{L_s} = \frac{0.10\,W/m\cdot K}{0.025\,m} = 4\,\frac{W}{m^2\cdot K}$$

$$= \frac{\left(\frac{2}{3}\frac{W}{m^2\cdot K}\right)(293K) + \left(4\frac{W}{m^2 K}\right)(260K)}{\left(\frac{2}{3}+4\right)\frac{W}{m^2 K}} = 264.7\,K \; (-8°C) \longleftarrow$$

Then, Eq(*) gives

$$\frac{\dot{Q}}{A} = -\frac{k_i}{L}\left[T_2 - T_1\right] = -\frac{2}{3}\frac{W}{m^2\cdot K}\left[264.7 - 293\right]K = 18.9\,\frac{W}{m^2}$$

Alternatively

$$\frac{\dot{Q}}{A} = -\frac{k}{L_s}\left[T_3 - T_2\right] = -4\frac{W}{m^2\cdot K}\left[260 - 264.7\right]K = 18.8\,\frac{W}{m^2}$$

which checks to within round-off.