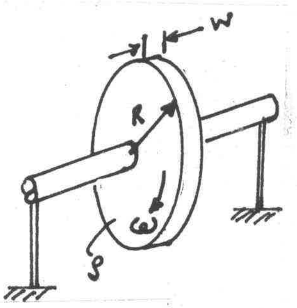
PROBLEM 2.11

KNOWN: Data are provided for a disk-shaped flywheel.

FIND: (a) Obtain appropriate expressions for the moment of inertia and the kinetic energy. (b) Using given data, determine the kinetic energy and mass for a steel flywheel. (c) Using results from part (b), determine the radius and mass of an aluminum flywheel.

SCHEMATIC & GIVEN DATA:



Steel flywheel:
$\omega = 3000$ RPM
$R = 0.38$ m
$w = 0.025$ m

Aluminum flywheel:
$\omega = 3000$ RPM
$w = 0.025$ m

ENGR. MODEL: 1. The flywheel is the closed system. 2. Motion is relative to the flywheel support structure.

ANALYSIS:

(a) Evaluating the moment of inertia

$$I = \int_{vol} \rho r^2 dV$$

For the disk, $dV = (2\pi r dr)w$. Thus, since ρ is constant

$$I = \rho(2\pi)w\int_0^R r^3 dr = \rho\pi w R^4/2 \longleftarrow I$$

The kinetic energy is

$$KE = \int_{vol}\left(\tfrac{1}{2}\rho V^2\right)dV$$

and $V = r\omega$, so

$$KE = \int_0^R \left(\tfrac{1}{2}\rho r^2\omega^2\right)(2\pi r dr)w$$

$$= \tfrac{1}{2}\rho\omega^2(2\pi)w\int_0^R r^3 dr$$

$$= \tfrac{1}{2}\underbrace{\left(\rho\pi\tfrac{R^4}{2}w\right)}_{I}\omega^2$$

$$= \tfrac{1}{2}I\omega^2 \longleftarrow KE$$

(b) From Table A-19, the density of steel is $\rho = 8060\ \text{kg/m}^3$. Thus, the mass is

$$m = \rho V = \rho[w\cdot\pi R^2]$$
$$= \left(8060\tfrac{kg}{m^3}\right)\left[(0.025m)\cdot\pi\cdot(0.38m)^2\right] = 91.41\,kg \longleftarrow m$$

Using the result of part (a), $KE = \tfrac{1}{2}I\omega^2$, where

$$I = \pi\rho w\frac{R^4}{2} = \frac{\pi}{2}\left(8060\tfrac{kg}{m^3}\right)(0.025m)(0.38m)^4 = 6.6\ kg\cdot m^2$$

PROBLEM 2.11 (Contd.)

Accordingly,

$$KE = \frac{1}{2} I \omega^2 = \frac{1}{2}(6.6\ kg\cdot m^2)\left(3000\ \frac{Rev}{min}\left|\frac{2\pi\ rad}{Rev}\right|\left|\frac{1\ min}{60s}\right|\right)^2\left|\frac{1N}{1kg\cdot m/s^2}\right|$$

$$= 32.57\times10^4\ N\cdot m \longleftarrow KE$$

(c) If ω, W, and KE are the same for the aluminum flywheel as for the steel flywheel

$$(KE)_{AL} = (KE)_{ST}$$

$$\left(\frac{1}{2} I\omega^2\right)_{AL} = \left(\frac{1}{2} I\omega^2\right)_{ST} \Rightarrow I_{AL} = I_{ST}$$

or

$$\left(\pi\rho W\frac{R^4}{2}\right)_{AL} = \left(\pi\rho W\frac{R^4}{2}\right)_{ST}$$

$$\Rightarrow (\rho R^4)_{AL} = (\rho R^4)_{ST}$$

$$R_{AL} = \left(\frac{\rho_{ST}}{\rho_{AL}}\right)^{1/4} R_{ST}$$

with ρ_{AL} from Table A-19, $\rho_{AL} = 2700\ kg/m^3$

$$R_{AL} = \left(\frac{8060}{2700}\right)^{\frac{1}{4}}(0.38m)$$

$$= 0.5m \longleftarrow R_{AL}$$

Then, the mass of the aluminum flywheel is

$$m = \rho V = \rho[w\pi R^2]$$

$$= \left(2700\ \frac{kg}{m^3}\right)(0.025m)(\pi)(0.5m)^2 = 53.01\ kg \longleftarrow m$$

PROBLEM 2.12

From Problem 2.11, $KE = \frac{1}{2} I\omega^2 \Rightarrow \omega = \left(\frac{2KE}{I}\right)^{1/2}$ where $I = 200\ lb\cdot ft^2$.

The change in potential energy of a 100 lb mass raised 30 ft is

$$\Delta PE = mg(z_2 - z_1) = (100\ lb)\left(32.2\ \frac{ft}{s^2}\right)(30ft)\left|\frac{1\ lbf}{32.2\ lb\cdot ft/s^2}\right| = 3000 ft\ lbf.$$

Thus, with KE = 3000 ft·lbf,

$$\omega = \left(\frac{2(3000ft\cdot lbf)}{200\ lb\cdot ft^2}\left|\frac{32.2\ lb\cdot ft/s^2}{1\ lbf}\right|\right)^{1/2} = 31.08\ \frac{1}{s}$$

In terms of RPM

$$\omega = \left(31.08\ \frac{1}{s}\right)\left|\frac{1\ rev}{2\pi}\right|\left|\frac{60s}{1\ min}\right| = 297\ rev/min \longleftarrow$$
