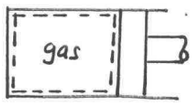
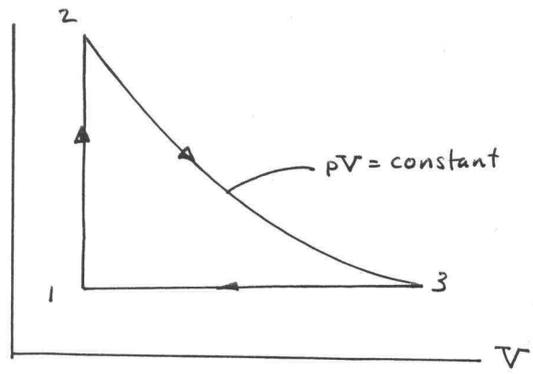
PROBLEM 2.76

KNOWN: A gas undergoes a thermodynamic cycle consisting of three processes.

FIND: Sketch the cycle on a p-V diagram. Calculate W_{cycle}, Q_{23}, Q_{31}, and determine whether the cycle is a power or refrigeration cycle.

SCHEMATIC & GIVEN DATA:





1-2: $V = 0.028\ m^3$, $U_2 - U_1 = 26.4\ kJ$

2-3: pV = constant, $U_3 = U_2$

3-1: $p = 1.4$ bar, $W_{31} = -10.5\ kJ$

ENGR. MODEL: 1. The gas is the closed system. 2. For the system, $\Delta KE = \Delta PE = 0$. 3. Volume change is the only work mode.

ANALYSIS: (b) $W_{cycle} = W_{12} + W_{23} + W_{31}$. Since volume is constant and volume change is the only work mode, $W_{12} = 0$. To find W_{23}

$$W_{23} = \int_{V_2}^{V_3} p\,dV = \int_{V_2}^{V_3} \frac{C}{V}\,dV = C \ln \frac{V_3}{V_2} = p_3 V_3 \ln \frac{V_3}{V_2}$$

$\leftarrow V_2 = V_1$

To evaluate V_3

$$W_{31} = \int_{V_3}^{V_1} p\,dV = p(V_1 - V_3) \Rightarrow V_3 = V_1 - W_{31}/p$$

or

$$V_3 = 0.028\ m^3 - \frac{(-10.5\ kJ)}{(1.4\ bar)} \left| \frac{1\ bar}{10^5\ N/m^2} \right| \left| \frac{10^3\ N \cdot m}{1\ kJ} \right| = 0.103\ m^3$$

Then

$$W_{23} = (1.4\ bar) \left| \frac{10^5\ N/m^2}{1\ bar} \right| (0.103\ m^3) \ln \left(\frac{0.103}{0.028} \right) \left| \frac{1\ kJ}{10^3\ N \cdot m} \right| = 18.78\ kJ$$

Finally,

$$W_{cycle} = 0 + 18.78 + (-10.5) = 8.28\ kJ$$

W_{cycle}. The cycle is a power cycle.

(c) An energy balance for 2-3 gives Q_{23}: $\cancel{\Delta U}^{0} + \cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0} = Q_{23} - W_{23} \Rightarrow Q_{23} = W_{23}$ ← Q_{23}

(d) To evaluate Q_{31} begin with an energy balance together with assumption 2

$$U_1 - U_3 = Q_{31} - W_{31} \Rightarrow Q_{31} = (U_1 - U_3) + W_{31}.$$

Since there is no overall change in internal energy for the cycle, $\Sigma(\Delta U) = 0$

$$(U_2 - U_1) + (U_3 - U_2) + (U_1 - U_3) = 0 \Rightarrow$$

$$(U_1 - U_3) = -26.4 - (0) = -26.4\ kJ$$

Then

$$Q_{31} = (-26.4) + (-10.5) = -36.9\ kJ$$ ← Q_{31}

① 1. As a check, note that $Q_{cycle} = W_{cycle}$, $Q_{cycle} = Q_{12} + Q_{23} + Q_{31}$. An energy balance for 1-2 gives $Q_{12} = 26.4$ kJ. So $Q_{cycle} = 26.4 + 18.78 + (-36.9) = 8.28$ kJ, which checks the result of part (b).