PROBLEM 2.70

KNOWN: A gas contained in a piston-cylinder assembly is slowly heated. State data and operating data are provided.

FIND: Determine the work done by the shaft mounted on the top of the piston and work done in displacing the atmosphere, each in kJ. Also, determine the heat transfer to the gas, in kJ, and develop an accounting of the heat transfer.

SCHEMATIC & GIVEN DATA:

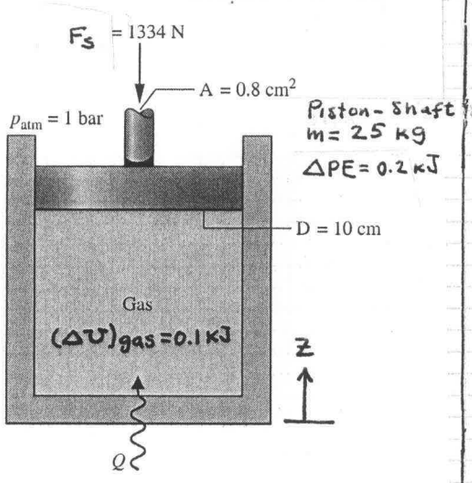

Fig. P2.70

ENGINEERING MODEL:

1. The closed system is the gas plus the piston and attached shaft.
2. There is no overall change in kinetic energy. For the piston-shaft, $\Delta U = 0$. For the gas, $\Delta PE = 0$.
3. $g = 9.81\ m/s^2$

ANALYSIS:

The work can be evaluated using $F\Delta Z$, where ΔZ is the change in elevation of the piston-shaft found as follows:

$$\Delta PE = mg\,\Delta Z$$

$$\Rightarrow \Delta Z = \frac{\Delta PE}{mg} = \frac{0.2\ kJ}{(25 kg)(9.81 m/s^2)} \left|\frac{10^3 N\cdot m}{1 kJ}\right| \left|\frac{1 kg\cdot m/s^2}{1 N}\right|$$

$$= 0.82\ m$$

Thus, the work done by the shaft is

$$W_S = F_S\,\Delta Z = (1334\ N)(0.82\ m)\left|\frac{1\ kJ}{10^3 N\cdot m}\right| = 1.094\ kJ \quad \leftarrow$$

The work done in displacing the atmosphere is $W_{atm} = (p_{atm} A_{net})\Delta Z$, where A_{net} is the net area: Area of piston face less area of the shaft. That is,

$$A_{net} = \left[\frac{\pi D^2}{4} - A\right] = \left[\frac{\pi (10 cm)^2}{4} - 0.8 cm^2\right] = 77.74\ cm^2. \text{ Thus}$$

$$W_{atm} = \left(10^5 \frac{N}{m^2}\right)(77.74 cm^2)\left|\frac{1 m}{100 cm}\right|^2 \left|\frac{1 kJ}{10^3 N\cdot m}\right| (0.82 m) = 0.637 kJ \quad \leftarrow$$

An energy balance for the system reads

$$\{[\cancel{\Delta KE} + \Delta PE + \cancel{\Delta U}]_{\substack{piston-\\shaft}} + [\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U]_{gas}\} = Q - W$$

$$\Rightarrow Q = (\Delta PE)_{\substack{piston-\\shaft}} + (\Delta U)_{gas} + W$$

$$= (0.2 kJ) + (0.1 kJ) + [1.094 kJ + 0.637 kJ] = 2.031\ kJ \quad \leftarrow$$

ENERGY "balance sheet":

ENERGY IN:

$Q = 2.031\ kJ$

DISPOSITION OF THE ENERGY IN:

ENERGY STORED: $(\Delta U)_{gas}$	0.10 kJ	(4.92%)
ENERGY STORED: $(\Delta PE)_{\substack{piston-\\shaft}}$	0.20 kJ	(9.85%)
ENERGY OUT BY WORK:		
✓ SHAFT	1.094 kJ	(53.87%)
✓ ATM	0.637 kJ	(31.36%)
	2.031 kJ	