Chapter 2

PROBLEM 2.1

The heat conduction equation in cylindrical coordinates is

$$\rho c \frac{\partial T}{\partial t} = k \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

(a) Simplify this equation by eliminating terms equal to zero for the case of steady-state heat flow without sources or sinks around a right-angle corner such as the one in the accompanying sketch. It may be assumed that the corner extends to infinity in the direction perpendicular to the page. (b) Solve the resulting equation for the temperature distribution by substituting the boundary condition. (c) Determine the rate of heat flow from T_1 to T_2. Assume k = 1 W/(m K) and unit depth perpendicular to the page.

GIVEN

- Steady state conditions
- Right-angle corner as shown below
- No sources or sinks
- Thermal conductivity (k) = 1 W/(m K)

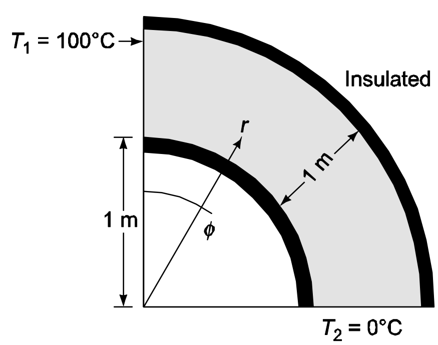
FIND

(a) Simplified heat conduction equation
(b) Solution for the temperature distribution
(c) Rate of heat flow from T_1 to T_2

ASSUMPTIONS

- Corner extends to infinity perpendicular to the paper
- No heat transfer in the z direction
- Heat transfer through the insulation is negligible

SKETCH



SOLUTION

The boundaries of the region are given by

$$1 \text{ m} \le r \ge 2 \text{ m}$$

$$0 \le \phi \ge \frac{\pi}{2}$$

Assuming there is no heat transfer through the insulation, the boundary condition is

$$\frac{\partial T}{\partial r} = 0 \text{ at } r = 1 \text{ m}$$

$$\frac{\partial T}{\partial r} = 0 \text{ at } r = 2 \text{ m}$$

$$T_1 = 100°\text{C at } \phi = 0$$

$$T_2 = 0°\text{C at } \phi = \frac{\pi}{2}$$

(a) The conduction equation is simplified by the following

Steady state

$$\frac{\partial T}{\partial t} = 0$$

No sources or sinks

$$q_k = 0$$

No heat transfer in the z direction

$$\frac{\partial^2 T}{\partial z^2} = 0$$

Since $\frac{\partial T}{\partial r} = 0$ over both boundaries, $\frac{\partial T}{\partial r} = 0$ throughout the region

(Maximum principle); therefore, $\frac{\partial^2 T}{\partial r^2} = 0$ throughout the region also.

Substituting these simplifications into the conduction equation

$$0 = k\left(0 + 0 + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + 0\right)$$

$$\frac{\partial^2 T}{\partial \phi^2} = 0$$

(b) Integrating twice

$$T = c_1\,\phi + c_2$$

The boundary condition can be used to evaluate the constants

$$\text{At } \phi = 0,\ T = 100°\text{C} : 100°\text{C} = c_2$$

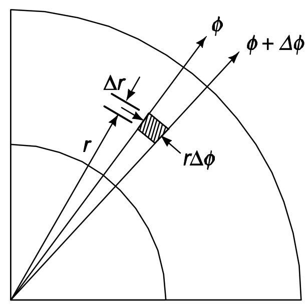
$$\text{At } \phi = \frac{\pi}{2},\ T = 0°\text{C} : 0°\text{C} = c_1\,(\pi/2) + 100°\text{C}$$

$$\therefore c_1 = -\frac{200\,^{\circ}\text{C}}{\pi}$$

Therefore, the temperature distribution is

$$T(\phi) = 100 - \frac{200\,°\mathrm{C}}{\pi}\,\phi\ °\mathrm{C}$$

(c) Consider a slice of the corner as follow



The heat transfer flux through the shaded element in the ϕ direction is

$$q'' = \frac{-k\,\Delta T}{\text{thickness}} = \frac{-k(T_{\phi} - T_{\phi+\Delta\phi})}{r\,\Delta\phi}$$

In the limit as $\Delta\phi \to 0$, $q'' = -k\,\dfrac{dT}{r\,d\phi}$

Multiplying by the surface area $drdz$ and integrating along the radius

$$q = \int_{r_1}^{r_o} q''\,drdz = \frac{200°\mathrm{C}\ k}{\pi}\int_{r_1}^{r_o}\frac{dr}{r} = \frac{200°\mathrm{C}\ k}{\pi}\ \ln\frac{r_o}{r_1}$$

$$q = \frac{200°\mathrm{C}\ k}{\pi}\ \ [1\ \mathrm{W/(m\ K)}]\ \ln(2\ \mathrm{m}/1\ \mathrm{m}) = 44.1\ \mathrm{W/m}$$ 44.1W per meter in the z direction

COMMENTS

Due to the boundary conditions, the heat flux direction is normal to radial lines.

PROBLEM 2.2

Write Equation (2.20) in a dimensionless form similar to Equation (2.17).

GIVEN

- Equation (2.20)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{q}_G}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

FIND

- Dimensionless form of the equation

SOLUTION

Let

$$\tau = \frac{t}{t_r} \Rightarrow t = \tau\, t_r$$

$$\theta = \frac{T}{T_r} \Rightarrow T = \theta\, T_r$$

$$\zeta = \frac{r}{R_r} \Rightarrow r = \zeta\, R_r$$

Where T_r, R_r, and t_r are reference temperature, reference radius, and reference time, respectively. Substituting these into Equation (2.20)

$$\frac{1}{\zeta R_r}\frac{\partial}{\partial(\zeta R_r)}\left(\zeta R_r \frac{\partial(\theta T_r)}{\partial(\zeta R_r)}\right) + \frac{\dot{q}_G}{k} = \frac{1}{\alpha}\frac{\partial(T_r\theta)}{\partial(\tau\, t_r)}$$

$$\frac{1}{\zeta R_r^2}\frac{\partial}{\partial\zeta}\left(\zeta T_r \frac{\partial\theta}{\partial\zeta}\right) + \frac{\dot{q}_G}{k} = \frac{1}{\alpha}\frac{T_r}{t_r}\frac{\partial\theta}{\partial\tau}$$

$$\frac{1}{\zeta}\frac{\partial}{\partial\zeta}\left(\zeta \frac{\partial\theta}{\partial\zeta}\right) + \frac{R_r^2\,\dot{q}_G}{T_r\,k} = \frac{R_r^2}{t_r\,\alpha}\frac{\partial\theta}{\partial\tau}$$

$$\text{let } \dot{Q}_G = \frac{R_r^2\,\dot{q}_G}{T_r\,k} \text{ and } F_o = \frac{t_r\,\alpha}{R_r^2}$$

$$\frac{1}{\zeta}\frac{\partial}{\partial\zeta}\left(\zeta \frac{\partial\theta}{\partial\zeta}\right) + \dot{Q}_G = \frac{1}{F_o}\frac{\partial\theta}{\partial\tau}$$

PROBLEM 2.3

Calculate the rate of heat loss per foot and the thermal resistance for a 6 in. schedule 40 steel pipe covered with a 3 in. thick layer of 85% magnesia. Superheated steam at 300°F flows inside the pipe [$\bar{h}_c$ = 30 Btu/(h ft^2 °F)] and still air at 60°F is on the outside [$\bar{h}_c$ = 5 Btu/(h ft^2 °F)].

GIVENS

- A 6 in. standard steel pipe covered with 85% magnesia
- Magnesia thickness = 3 in.
- Superheated steam at 300°F flows inside the pipe
- Surrounding air temperature (T_∞) = 60°F
- Heat transfer coefficients
- Inside ($\bar{h}_{ci}$) = 30 Btu/(h ft^2 °F)
- Outside ($\bar{h}_{co}$) = 5 Btu/(h ft^2 °F)

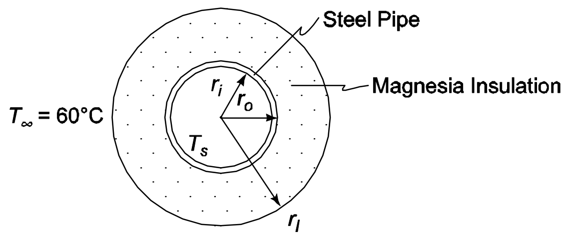
FIND

(a) The thermal resistance (R)
(b) The rate of heat loss per foot (q/L)

ASSUMPTIONS

- Constant thermal conductivity
- The pipe is made of 1% carbon steel

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Tables 10, 11, and 41

For a 6 in. schedule 40 pipe

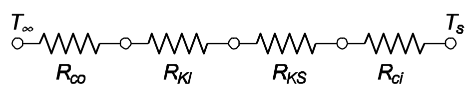
- Inside diameter (D_i) = 6.065 in.
- Outside diameter (D_o) = 6.625 in.

Thermal Conductivities

- 85% Magnesia (k_I) = 0.034 Btu/(h ft °F) at 68°F
- 1% Carbon steel (k_s) = 24.8 Btu/(h ft °F) at 68°F

SOLUTION

The thermal circuit for the insulated pipe is shown below



(a) The values of the individual resistances can be calculated using Equations (1.14) and (2.39)

$$R_{co} = \frac{1}{\bar{h}_{co}\,A_o} = \frac{1}{\bar{h}_{co}\,\pi\,D_i L} = \frac{1}{[5\,\text{Btu/(h ft}^2\,°\text{F)}]\,\pi\left(\frac{6.625+3}{12}\,\text{ft}\right)L} = \frac{1}{L}\ 0.0794\ \text{(h ft °F)/Btu}$$

$$R_{kI} = \frac{\ln\left(\frac{r_i}{r_o}\right)}{2\pi\,L\,k_i} = \frac{\ln\left(\frac{(6.625+3)\,\text{in}}{6.625\,\text{in}}\right)}{2\pi\ 0.035\ \text{Btu/(h ft}^2\,°\text{F)}} = \frac{1}{L}\ 1.748\ \text{(h ft °F)/Btu}$$

$$R_{ks} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi\,L\,k_s} = \frac{\ln\left(\frac{6.625\,\text{in}}{6.625\,\text{in}}\right)}{2\pi\ 24.8\,\text{Btu/(h ft °F)}} = \frac{1}{L}\ 0.000567\ \text{(h ft °F)/Btu}$$

$$R_{ci} = \frac{1}{\bar{h}_{ci}\,A_i} = \frac{1}{\bar{h}_{ci}\,\pi\,D_i\,L} = \frac{1}{[30\,\text{Btu/(hft}^2\,°\text{F)}]\,\pi\left(\frac{6.065}{12}\,\text{ft}\right)L} = \frac{1}{L}\ 0.0210\ \text{(h ft °F)/Btu}$$

The total resistance is

$$R_{\text{total}} = R_{co} + R_{kI} + R_{ks} + R_{ci}$$

$$R_{\text{total}} = \frac{1}{L}\ (0.0794 + 1.748 + 0.000567 + 0.021)\ \text{(h ft °F)/Btu}$$

$$R_{\text{total}} = \frac{1}{L}\ 1.85\ \text{(h ft °F)/Btu}$$

(b) The rate of heat transfer is given by

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{300°\text{F} - 60°\text{F}}{\frac{1}{L} 1.85 \text{ (h ft °F)/Btu}}$$

$$\therefore \frac{q}{L} = 130 \text{ Btu/(hft)}$$

COMMENTS

Note that almost all of the thermal resistance is due to the insulation and that the thermal resistance of the steel pipe is negligible.

PROBLEM 2.4

> **Suppose that a pipe carrying a hot fluid with an external temperature of T_i and outer radius r_i is to be insulated with an insulation material of thermal conductivity k and outer radius r_o. Show that if the convective heat transfer coefficient on the outside of the insulation is h and the environmental temperature is T_∞, the addition of insulation can actually increase the rate of heat loss if $r_o < k/\bar{h}$ and that maximum heat loss occurs when $r_o = k/\bar{h}$. This radius, r_c, is often called the critical radius.**

GIVEN

- An insulated pipe
- External temperature of the pipe = T_i
- Outer radius of the pipe = r_i
- Outer radius of insulation = r_o
- Thermal conductivity = k
- Ambient temperature = T_∞
- Convective heat transfer coefficient = $\bar{h}$

FIND

Show that

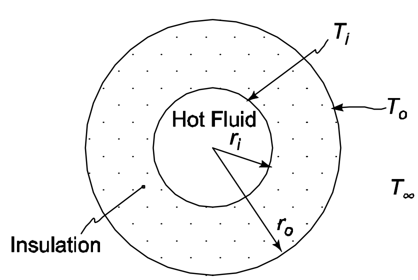
(a) The insulation can increase the heat loss if $r_o < k/\bar{h}$

(b) Maximum heat loss occurs when $r_o = k/\bar{h}$

ASSUMPTIONS

- The system has reached steady state
- The thermal conductivity does not vary appreciably with temperature
- Conduction occurs in the radial direction only

SKETCH



SOLUTION

Radial conduction for a cylinder of length L is given by Equation (2.37)

$$q_k = 2\,\pi L\,k\,\frac{T_i - T_o}{\ln\dfrac{r_o}{r_i}}$$

Convection from the outer surface of the cylinder is given by equation (1.10)

$$q_c = \bar{h}_c\;A\,\Delta T = \bar{h}\;2\,\pi\,r_o\,L\,(T_o - T_\infty)$$

For steady state

$$q_k = q_c$$

$$2\,\pi L\,k\,\frac{T_i - T_o}{\ln\dfrac{r_o}{r_i}} = \bar{h}\;2\,\pi\,r_o\,L\,(T_o - T_\infty)$$

The outer wall temperature, T_o, is an unknown and must be eliminated from the equation

Solving for $T_i - T_o$

$$T_i - T_o = \frac{\bar{h}\,r_o}{k}\;\ln\frac{r_o}{r_i}\;(T_o - T_\infty)$$

$$T_i - T_\infty = (T_i - T_o) + (T_o - T_\infty) = \frac{\bar{h}\,r_o}{k}\;\ln\frac{r_o}{r_i}\;(T_o - T_\infty) + (T_o - T_\infty)$$

$$T_i - T_\infty = \left(\frac{\bar{h}\,r_o}{k}\ln\frac{r_o}{r_i} + 1\right)\;(T_o - T_\infty)$$

or $$T_o - T_\infty = \frac{T_i - T_\infty}{1 + \dfrac{\bar{h}\,r_o}{k}\ln\dfrac{r_o}{r_i}}$$

Substituting this into the convection equation

$$q = q_c = \bar{h}\;2\,\pi\,r_o\,L\left[\frac{T_i - T_\infty}{1 + \dfrac{\bar{h}\,r_o}{k}\ln\dfrac{r_o}{r_i}}\right]$$

$$q = \frac{T_i - T_\infty}{\left(\dfrac{1}{2\pi\,r_o\,L\,\bar{h}} + \dfrac{\ln\frac{r_o}{r_i}}{2\pi\,Lk}\right)}$$

Examining the above equation, the heat transfer rate is a maximum when the term

$\left(\dfrac{1}{2\pi\,r_o\,L\,\bar{h}} + \dfrac{\ln\frac{r_o}{r_i}}{2\pi\,Lk}\right)$ is a minimum, which occurs when its differential with respect to r_o is zero

$$\frac{1}{2\pi\,k\,L}\,\frac{d}{dr_o}\left(\frac{k}{r_o\,\bar{h}} + \ln\frac{r_o}{r_i}\right) = 0$$

$$\frac{k}{\bar{h}}\frac{d}{dr_o}\left(\frac{1}{r_o}\right) + \frac{d}{dr_o}\left(\ln\frac{r_o}{r_i}\right) = 0$$

$$\frac{k}{\bar{h}}\left(-\frac{1}{r_o^2}\right) + \frac{1}{r_o} = 0$$

$$r_o = \frac{k}{\bar{h}}$$

The second derivative of the denominator is

$$\frac{k}{\bar{h}}\frac{2}{r_o^3} - \frac{1}{r_o^2}$$

which is greater than zero at $r_o = k/h$, therefore $r_o = k/h$ is a true minimum and the maximum heat loss occurs when the diameter is $r_o = k/h$. Adding insulation to a pipe with a radius less than k/h will increase the heat loss until the radius of k/h is reached.

COMMENTS

A more detailed solution taking into account the dependence of h_c on the temperature has been obtained by Sparrow and Kang, Int. J. Heat Mass Transf., 28: 2049–2060, 1985.

PROBLEM 2.5

> **A solution with a boiling point of 180°F boils on the outside of a 1-in. tube with a No. 14 BWG gauge wall. On the inside of the tube flows saturated steam at 60 psia. The convective heat transfer coefficients are 1500 Btu/(h ft^2 °F) on the steam side and 1100 Btu/(h ft^2 °F) on the exterior surface. Calculate the increase in the rate of heat transfer for a copper over a steel tube.**

GIVEN

- Tube with saturated steam on the inside and solution boiling at 180°F outside
- Tube specification: 1 in. No. 14 BWG gauge wall
- Saturated steam in the pipe is at 60 psia
- Convective heat transfer coefficients
  - Steam side ($\bar{h}_{ci}$) : 1500 Btu/(h ft^2 °F)
  - Exterior surface ($\bar{h}_{co}$) : 1100 Btu/(h ft^2 °F)

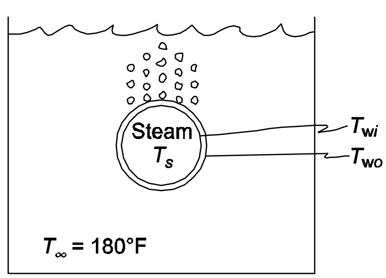
FIND

- The increase in the rate of heat transfer for a copper over a steel tube

ASSUMPTIONS

- The system is in steady state
- Constant thermal conductivities

SKETCH



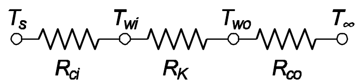
PROPERTIES AND CONSTANTS:

From Appendix 2, Tables 10, 12, 13 and 42

- Temperature of saturated steam at 60 psia (T_s) = 291°F
- Thermal conductivities
- Copper (k_c) = 226 Btu/(h ft °F) at 261°F
- 1% Carbon steel (k_s) = 25 Btu/(h ft °F) at 68°F
- Tube inside diameter (D_i) = 0.834 in.

SOLUTION

The thermal circuit for the tube is shown below



The individual resistances are:

$$R_{ci} = \frac{1}{\overline{h}_{ci} A_i} = \frac{1}{\overline{h}_{ci}\,\pi D_i L} = \frac{1}{[1500\ \text{Btu/(h ft}^2\ \text{°F)}]\,\pi \left(\frac{0.834}{12}\ \text{ft}\right)}\ L = \frac{1}{L}\ 0.00305\ \text{(h ft °F)/Btu}$$

$$R_{co} = \frac{1}{\overline{h}_{co} A_o} = \frac{1}{\overline{h}_{co}\,\pi D_i L} = \frac{1}{[1100\ \text{Btu/(h ft}^2\ \text{°F)}]\,\pi \left(\frac{1}{12}\ \text{ft}\right)}\ L = \frac{1}{L}\ 0.00347\ \text{(h ft °F)/Btu}$$

$$R_{kc} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi L k_c} = \frac{\ln\left(\frac{(1)\ \text{in}}{0.834\ \text{in}}\right)}{2\pi\ 226\ \text{Btu/h ft °F}} = \frac{1}{L}\ 0.000128\ \text{(h ft °F)/Btu}$$

$$R_{ks} = \frac{\ln\left(\frac{r_o}{r_s}\right)}{2\pi L k_s} = \frac{\ln\left(\frac{(1)\ \text{in}}{0.834\ \text{in}}\right)}{2\pi\ 25\ \text{Btu/h ft °F}} = \frac{1}{L}\ 0.00116\ \text{(h ft °F)/Btu}$$

The rate of heat transfer is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_s - T_\infty}{R_{\text{ci}} + R_{\text{k}} + R_{\text{co}}}$$

For the copper tube

$$\frac{q_c}{L} = \frac{291\text{°F} - 180\text{°F}}{(0.00305 + 0.00128 + 0.00347)\ \text{(h ft°F)/Btu}} = 16{,}700\ \text{Btu/h}$$

For the steel tube

$$\frac{q_s}{L} = \frac{291\text{°F} - 180\text{°F}}{(0.00305 + 0.00116 + 0.00347)\ \text{(h ft°F)/Btu}} = 14{,}450\ \text{Btu/h}$$

The increase in the rate of heat transfer per unit length with the copper tube is

$$\text{Increase} = \frac{q_c}{L} - \frac{q_s}{L} = 2250\ \text{Btu/ft}$$

$$\text{Percent increase} = \frac{2250}{14{,}450} \times 100 = 16\%$$

COMMENTS

The choice of tubing material is significant in this case because the convective heat transfer resistances are small making the conductive resistant a significant portion of the overall thermal resistance.

PROBLEM 2.6

> **Steam having a quality of 98% at a pressure of 1.37×10^5 N/m^2 is flowing at a velocity of 1 m/s through a steel pipe of 2.7 cm OD and 2.1 cm ID. The heat transfer coefficient at the inner surface, where condensation occurs, is 567 W/(m^2 K). A dirt film at the inner surface adds a unit thermal resistance of 0.18 (m^2 K)/W. Estimate the rate of heat loss per meter length of pipe if; (a) the pipe is bare, (b) the pipe is covered with a 5 cm layer of 85% magnesia insulation. For both cases assume that the convective heat transfer coefficient at the outer surface is 11 W/(m^2 K) and that the environmental temperature is 21°C. Also estimate the quality of the steam after a 3-m length of pipe in both cases.**

GIVEN

- A steel pipe with steam condensing on the inside
- Diameters
  - Outside (D_o) = 2.7 cm = 0.027 m
  - Inside (D_i) = 2.1 cm = 0.021 m
- Velocity of the steam (V) = 1 m/s
- Initial steam quality (X_i) = 98%
- Steam pressure = 1.37×10^5 N/m^2
- Heat transfer coefficients
  - Inside (h_{ci}) = 567 W/(m^2 K)
  - Outside (h_{co}) = 11 W/(m^2 K)
- Thermal resistance of dirt film on inside surface (R_f) = 0.18 (m^2 K)/W
- Ambient temperature (T_∞) = 21°C

FIND

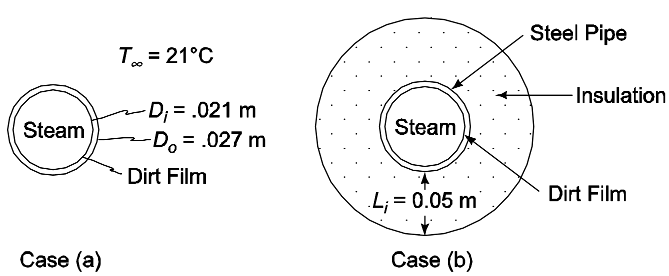
The heat loss per meter (q/L) and the change in the quality of the steam per 3 m length for

(a) A bare pipe
(b) A pipe insulated with 85% Magnesia: thickness (L_i) = 0.05 m

ASSUMPTIONS

- Steady state conditions exist
- Constant thermal conductivity
- Steel is 1% carbon steel
- Radiative heat transfer from the pipe is negligible
- Neglect the pressure drop of the steam

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Tables 10, 11, and 13

The thermal conductivities are: 1% carbon steel (k_s) = 43 W/(m K) at 20°C

85% Magnesia (k_i) = 0.059 W/(m K) at 20°C

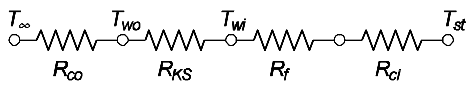
For saturated steam at 1.37×10^5 N/m^2: Temperature (T_{st}) = 107°C

Heat of vaporization (h_{fg}) = 2237 kJ/kg

Specific volume (υ_s) = 1.39 m^3/kg

SOLUTION

(a) The thermal circuit for the uninsulated pipe is shown below



Evaluating the individual resistances

$$R_{co} = \frac{1}{\overline{h}_{co} A_o} = \frac{1}{\overline{h}_{co}\, \pi D_o L} = \frac{1}{[11\,\text{W/(m}^2\text{K)}]\,\pi\,(0.027\,\text{m})\,L} = \frac{1}{L}\;1.072\ \text{(mK)/W}$$

$$R_{ks} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi\, L k_i} = \frac{\ln\left(\frac{0.027}{0.021}\right)}{2\pi\,[43\,\text{W/(mK)}]} = \frac{1}{L}\;0.00093\ \text{(mK)/W}$$

$$R_f = \frac{r_f}{A} = \frac{r_f}{2\pi D_i L} = \frac{1}{L}\,\frac{0.18\,\text{m}^2\text{K/W}}{\pi\,(0.021\text{m})} = \frac{1}{L}\;2.728\ \text{(mK)/W}$$

$$R_{ci} = \frac{1}{\overline{h}_{ci} A_i} = \frac{1}{\overline{h}_{ci}\,\pi D_i L} = \frac{1}{[567\,\text{W/(m}^2\text{K)}]\,\pi\,(0.021\text{m})L} = \frac{1}{L}\;0.0267\ \text{(mK)/W}$$

The rate of heat transfer through the pipe is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_{st} - T_\infty}{R_\infty + R_{ks} + R_i + R_{ci}}$$

$$\frac{q}{L} = \frac{107\,°\text{C} - 21°\text{C}}{(1.072 + 0.00093 + 2.728 + 0.267)\,\text{(mK)/W}} = 22.5\ \text{W/m}$$

The total rate of transfer of a three meter section of the pipe is

$$q = 22.5\ \text{W/m (3 m)} = 67.4\ \text{W}$$

The mass flow rate of the steam in the pipe is

$$\dot{m}_s = \frac{A_i V}{\upsilon_s} = \frac{\pi D_i^2 V}{4\upsilon_s} = \frac{\pi\,(0.021\text{m})^2\,(1\text{m/s})}{4\,(1.39\,\text{m}^3\text{/kg})\,(1\text{kg/1000g})} = 0.249\ \text{g/s}$$

The mass rate of steam condensed in a 3 meter section of the pipe is equal to the rate of heat transfer divided by the heat of vaporization of the steam

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{67.4\,\text{W}}{2237\,\text{J/g(Ws/J)}} = 0.030\ \text{g/s}$$

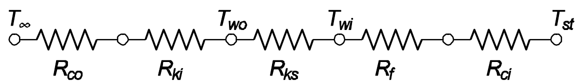
The quality of the saturated steam is the fraction of the steam which is vapor. The quality of the steam after a 3 meter section, therefore, is

$$X_i = \frac{\text{(original vapor mass)} - \text{(mass of vapor condensed)}}{\text{total mass of steam}} = \frac{X_i\,\dot{m}_s - \dot{m}_c}{\dot{m}_s}$$

$$X_i = \frac{0.98(0.249\,\text{g/s}) - 0.030\,\text{g/s}}{0.249\,\text{g/s}} = 0.86 = 86\%$$

The quality of the steam changed by 12%.

The thermal circuit for the pipe with insulation is shown below



The convective resistance on the outside of the pipe is different than that in part (a) because it is based on the outer area of the insulation

$$R_{co} = \frac{1}{\overline{h}_{co} A_o} = \frac{1}{\overline{h}_{co}\,\pi (D_o + 2L_i)L} = \frac{1}{[11\,\text{W/(m}^2\text{K)}]\,\pi\,(0.027\,\text{m}+0.1\,\text{m})L} = \frac{1}{L}\;0.228\ \text{(mK)/W}$$

The thermal resistance of the insulation is

$$R_{ki} = \frac{\ln\left(\dfrac{D_o + 2L_i}{r_i}\right)}{2\pi L k_i} = \frac{\ln\left(\dfrac{0.027 + 0.1}{0.027}\right)}{2\pi\,[0.059\,\text{W/(mK)}]} = \frac{1}{L}\;4.18\ \text{(mK)/W}$$

The rate of heat transfer is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_{si} - T_\infty}{R_\infty + R_{ki} + R_{ks} + R_f + R_{ci}}$$

$$\therefore\ \frac{q}{L} = \frac{107°\text{C} - 21°\text{C}}{(0.228 + 4.18 + 0.00093 + 2.728 + 0.0267)\text{(mK)/W}} = 12.0\ \text{W/m}$$

Therefore, the rate of steam condensed in 3 meters is

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{12.0\,\text{W}}{2237\,\text{J/g}\,\text{(Ws/J)}} = 0.016\ \text{g/s}$$

The quality of the steam after 3 meters of pipe is

$$X_f = \frac{0.98(0.249\,\text{g/s}) - 0.016\,\text{g/s}}{0.249\,\text{g/s}} = 0.92 = 92\%$$

The change in the quality of the steam is 6%.

COMMENTS

Notice that the resistance of the steel pipe and the convective resistance on the inside of the pipe are negligible compared to the other resistances.

The resistance of the dirt film is the dominant resistance for the uninsulated pipe.

PROBLEM 2.7

> **Estimate the rate of heat loss per unit length from a 2 in. ID, $2^3/8$ in. OD steel pipe covered with high temperature insulation having a thermal conductivity of 0.065 Btu/(h ft) and a thickness of 0.5 in. Steam flows in the pipe. It has a quality of 99% and is at 300°F. The unit thermal resistance at the inner wall is 0.015 (h ft^2 °F)/Btu, the heat transfer coefficient at the outer surface is 3.0 Btu/(h ft^2 °F), and the ambient temperature is 60°F.**

GIVEN

- Insulated, steam filled steel pipe
- Diameters
  - *ID* of pipe (D_i) = 2 in.
  - *OD* of pipe (D_o) = 2.375 in.
- Thickness of insulation (L_i) = 0.5 in.
- Steam quality = 99%
- Steam temperature (T_s) = 330°F
- Unit thermal resistance at inner wall ($A\,R_i$) = 0.015 (h ft^2 °F)/Btu
- Heat transfer coefficient at outer wall (h_o) = 3.0 Btu /(h ft^2 °F)
- Ambient temperature (T_∞) = 60°F
- Thermal conductivity of the insulation (k_I) = 0.065 Btu /(h ft °F)

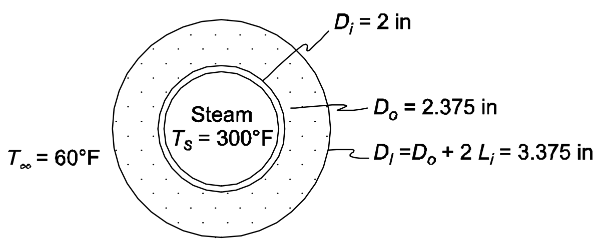
FIND

- Rate of heat loss per unit length (q/L)

ASSUMPTIONS

- 1% carbon steel
- Constant thermal conductivities
- Steady state conditions

SKETCH



PROPERTIES AND CONSTANTS

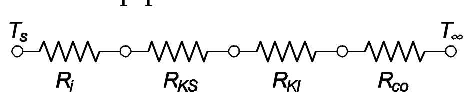
From Appendix 2, Table 10

The thermal conductivity of 1% carbon steel (k_s) = 24.8 Btu/(h ft^2 °F) at 68 °F

SOLUTION

The outer diameter of the insulation (D_I) = 2.375 in. + 2(0.5 in) = 3.375 in.

The thermal circuit of the insulated pipe is shown below



The values of the individual resistances are

$$R_i = \frac{AR_i}{A_i} = \frac{AR_i}{\pi D_i L} = \frac{0.015(\text{hft}^2\ °\text{F})/\text{Btu}}{\pi L\left(\frac{2}{12}\text{ft}\right)} = \frac{1}{L}\ 0.02865\ (\text{h ft °F})/\text{Btu}$$

$$R_{ks} = \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi L k_s} = \frac{\ln\left(\frac{3.375\,\text{in}}{2\ \text{in}}\right)}{2\pi\ 24.8\,\text{Btu}/(\text{h ft °F})} = \frac{1}{L}\ 0.001103\ (\text{h ft °F})/\text{Btu}$$

$$R_{kI} = \frac{\ln\left(\frac{D_I}{D_o}\right)}{2\pi L k_i} = \frac{\ln\left(\frac{3.375\,\text{in}}{2.375\,\text{in}}\right)}{2\pi\ 0.065\,\text{Btu}/(\text{h ft °F})} = \frac{1}{L}\ 0.8604\ (\text{h ft °F})/\text{Btu}$$

$$R_{co} = \frac{1}{\bar{h}_{co} A_o} = \frac{1}{\bar{h}_{co}\, \pi\, D_I L} = \frac{1}{[3\,\text{Btu/(h ft}^2\,°\text{F)}]\,\pi \left(\frac{3.375}{12}\,\text{ft}\right)}\, L = \frac{1}{L}\ 0.3773\ (\text{h ft °F)/Btu}$$

The rate of heat transfer is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_s - T_\infty}{R_i + R_{ks} + R_{kI} + R_{co}}$$

$$\therefore\ \frac{q}{L} = \frac{300°\text{F} - 60°\text{F}}{(0.02865 + 0.8604 + 0.001103 + 0.3773)(\text{h ft °F})/\text{Btu}} = 189\ \text{Btu/ft}$$

PROBLEM 2.8

The rate of heat flow per unit length *q*/*L* through a hollow cylinder of inside radius r_i and outside radius r_o is

$$q/L = (\bar{A}\, k\, \Delta T)/(r_o - r_i)$$

where $A = 2\pi (r_o - r_i)/\ln(r_o/r_i)$. Determine the percent error in the rate of heat flow if the arithmetic mean area $\pi (r_o + r_i)$ is used instead of the logarithmic mean area *A* for ratios of outside to inside diameters (D_o/D_i) of 1.5, 2.0, and 3.0. Plot the results.

GIVEN

- A hollow cylinder
- Inside radius = r_i
- Outside radius = r_o
- Heat flow per unit length as given above

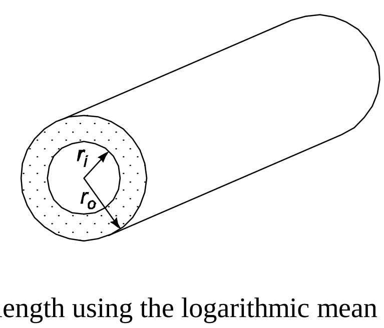
FIND

(a) Percent error in the rate of heat flow if the arithmetic rather than the logarithmic mean area is used for ratios of outside to inside diameters of 1.5, 2.0, and 3.0.
(b) Plot the results

ASSUMPTIONS

- Radial conduction only
- Constant thermal conductivity
- Steady state prevails

SKETCH



SOLUTION

The rate of heat transfer per unit length using the logarithmic mean area is

$$\left(\frac{q}{L}\right)_{\log} = \frac{2\pi (r_o - r_i)}{\ln\left(\frac{r_o}{r_i}\right)} \frac{k\Delta T}{r_o - r_i} = \frac{2\pi\, k\Delta T}{\ln\left(\frac{r_o}{r_i}\right)}$$

The rate of heat transfer per unit length using the arithmetic mean area is

$$\left(\frac{q}{L}\right)_{\text{arith}} = \pi\,(r_o + r_i)\,\frac{k\,\Delta T}{r_o - r_i} = \pi\,k\,\Delta T\,\frac{r_o + r_i}{r_o - r_i}$$

The percent error is

$$\%\ \text{error} = \frac{\left(\frac{q}{L}\right)_{\text{log}} - \left(\frac{q}{L}\right)_{\text{arith}}}{\left(\frac{q}{L}\right)_{\text{log}}} \times 100 = \frac{\dfrac{2\pi\,k\,\Delta T}{\ln\left(\frac{r_o}{r_i}\right)} - \pi\,k\,\Delta T\,\dfrac{r_o + r_i}{r_o - r_i}}{\dfrac{2\pi\,k\,\Delta T}{\ln\left(\frac{r_o}{r_i}\right)}} \times 100$$

$$\%\ \text{error} = \left[1 - \frac{1}{2}\ln\left(\frac{r_o}{r_i}\right)\frac{\left(\frac{r_o}{r_i} + 1\right)}{\left(\frac{r_o}{r_i} - 1\right)}\right] \times 100$$

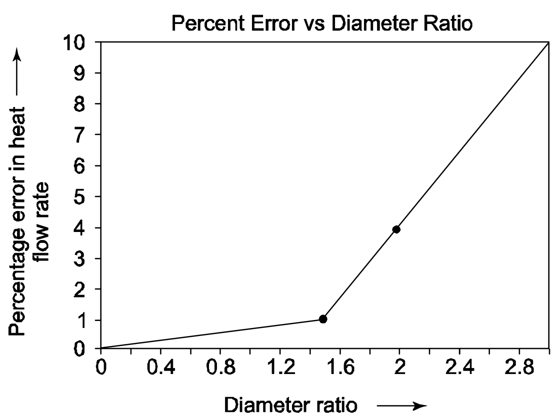
For a ratio of outside to inside diameters of 1.5

$$\%\ \text{error} = \left[1 - \frac{1}{2}\ln(1.5)\left(\frac{1.5 + 1}{1.5 - 1}\right)\right] \times 100 = -1.37\%$$

The percent errors for the other diameter ratios can be calculated in a similar manner with the following results

Diameter ratio	% Error
1.5	–1.37
2.0	–3.97
3.0	–9.86

(b)



COMMENTS

For diameter ratios less than 2, use of the arithmetic mean area will not introduce more than a 4% error.

PROBLEM 2.9

A 2.5-cm-*OD*, 2-cm-*ID* copper pipe carriers liquid oxygen to the storage site of a space shuttle at –183°C and 0.04 m^3/min. The ambient air is at 21°C and has a dew point of 10°C. How much insulation with a thermal conductivity of 0.02 W/(m K) is needed to prevent condensation on the exterior of the insulation if $h_c + h_r$ = 17 W/(m^2 K) on the outside?

GIVEN

- Insulated copper pipe carrying liquid oxygen
- Inside diameter (D_i) = 2 cm = 0.02 m
- Outside diameter (D_o) = 2.5 cm = 0.025 m
- LOX temperature (T_{ox}) = – 183°C
- LOX flow rate (m_{ox}) = 0.04 m^3/min
- Thermal conductivity of insulation (k_i) = 0.02 W/(m K)
- Exterior heat transfer coefficients ($h_o = h_c + h_r$) = 17 W/(m^2 K)
- Ambient air temperature (T_∞) = 21°C
- Ambient air dew point (T_{dp}) = 10°C

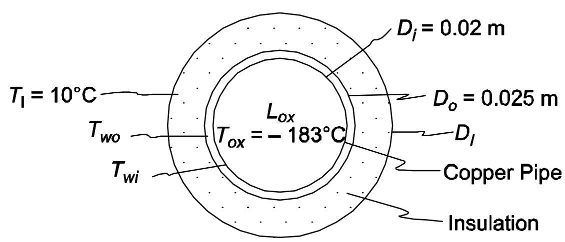
FIND

- Thickness of insulation (L) needed to prevent condensation

ASSUMPTIONS

- Steady-state conditions have been reached
- The thermal conductivity of the insulation does not vary appreciably with temperature
- Radial conduction only
- The thermal resistance between the inner surface of the pipe and the liquid oxygen is negligible, therefore $T_{wi} = T_{ox}$

SKETCH

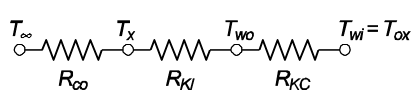


PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, thermal conductivity of copper (k_c) = 401 W/(m K) at 0°C

SOLUTION

The thermal circuit for the pipe is shown below



The rate of heat transfer from the pipe is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_\infty - T_{ox}}{\dfrac{1}{\bar{h}_o A_I} \quad \dfrac{\ln\left(\frac{D_I}{D_o}\right)}{2\pi L k_I} + \dfrac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi L k_c}}$$

The rate of heat transfer by convection and radiation from the outer surface of the pipe is

$$q = \frac{\Delta T}{R_o} = \frac{T_\infty - T_I}{\dfrac{1}{h_o A_i}}$$

Equating these two expressions

$$\frac{T_\infty - T_{ox}}{\dfrac{1}{\bar{h}_o A_I} + \dfrac{\ln\left(\frac{D_I}{D_o}\right)}{2\pi L k_I} + \dfrac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi L k_c}} = \frac{T_\infty - T_I}{\dfrac{1}{h_o A_I}}$$

$$\frac{T_\infty - T_{ox}}{T_\infty - T_I} = \frac{\dfrac{1}{\bar{h}_o \pi D_I L} + \dfrac{\ln\left(\frac{D_I}{D_o}\right)}{2\pi L k_I} + \dfrac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi L k_c}}{\dfrac{1}{h_o \pi D_I L}}$$

$$\frac{T_\infty - T_{ox}}{T_\infty - T_I} = 1 + \frac{h_o}{2} D_I \left(\frac{\ln\left(\frac{D_I}{D_o}\right)}{k_I} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{k_c} \right)$$

$$D_I \left(\frac{\ln D_I}{k_I} + \frac{\ln D_o}{k_I} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{k_c} \right) = \frac{2}{h_o} \left(\frac{T_\infty - T_{ox}}{T_\infty - T_I} - 1 \right)$$

$$D_I \left(\frac{\ln D_I}{0.02\,\text{W/(m K)}} + \frac{\ln(0.025)}{0.02\,\text{W/(m K)}} + \frac{\ln\dfrac{0.025}{0.02}}{401\,\text{W/(m K)}} \right) = \frac{2}{17\,\text{W/(m}^2\text{ K)}}$$

$$\left(\frac{21^\circ\text{C} - (183^\circ\text{C})}{21^\circ\text{C} - 10^\circ\text{C}} \right) - D_I \left(\frac{\ln D_I}{0.02} + 184.4 + 0.00056 \right) = 2.064\ (\text{m}^2\ \text{K})/\text{W}$$

Solving this by trial and error

$$D_I = 0.054\text{ m} = 5.4\text{ cm}$$

Therefore, the thickness of the insulation is

$$L = \frac{D_I - D_o}{2} = \frac{5.4\,\text{cm} - 2.5\,\text{cm}}{2} = 1.5\text{ cm}$$

COMMENTS

Note that the thermal resistance of the copper pipe is negligible compared to that of the insulation.

PROBLEM 2.10

A salesman for insulation material claims that insulating exposed steam pipes in the basement of a large hotel will be cost effective. Suppose saturated steam at 5.7 bars flows through a 30 cm *OD* steel pipe with a 3 cm wall thickness. The pipe is surrounded by air at 20°C. The convective heat transfer coefficient on the outer surface of the pipe is estimated to be 25 W/(m^2 K). The cost of generating steam is estimated to be \$5 per 10^9 J and the salesman offers to install a 5 cm thick layer of 85% magnesia insulation on the pipes for \$200/m or a 10 cm thick layer for \$300/m. Estimate the payback time for these two alternatives assuming that the steam line operates all year long and make a recommendation to the hotel owner. Assume that the surface of the pipe as well as the insulation have a low emissivity and radiative heat transfer is negligible.

GIVEN

- Steam pipe in a hotel basement
- Pipe outside diameter (D_o) = 30 cm = 0.3 m
- Pipe wall thickness (L_s) = 3 cm = 0.03 m
- Surrounding air temperature (T_∞) = 20°C

- Convective heat transfer coefficient (h_c) = 25 W/(m^2 K)
- Cost of steam = \$5/10^9 J
- Insulation is 85% magnesia

FIND

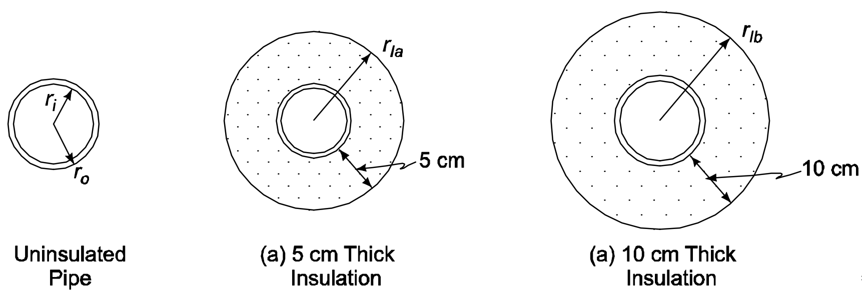
Payback time for

(a) Insulation thickness (L_{Ia}) = 5 cm = 0.05 m; Cost = \$200/m
(b) Insulation thickness (L_{Ib}) = 10 cm = 0.10 m; Cost = \$300/m
Make a recommendation to the hotel owner.

ASSUMPTIONS

- The pipe and insulation are black (ε = 1.0)
- The convective resistance on the inside of the pipe is negligible, therefore the inside pipe surface temperature is equal to the steam temperature
- The pipe is made of 1% carbon steel
- Constant thermal conductivities

SKETCH



**

PROPERTIES AND CONSTANTS

From Appendix 1, Table 5: The Stefan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)
From Appendix 2, Table 10 and 11

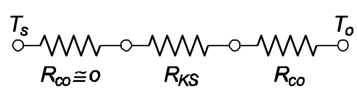
Thermal conductivities: 1% Carbon Steel (k_s) = 43 W/(m K) at 20°C
85% Magnesia (k_I) = 0.059 W/(m K) at 20°C

From Appendix 2, Table 13

The temperature of saturated steam at 5.7 bars (T_s) = 156°C

SOLUTION

The rate of heat loss and cost of the uninsulated pipe will be calculated first.
The thermal circuit for the uninsulated pipe is shown below



Evaluating the individual resistances

$$R_{ks} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi L k_s} = \frac{\ln\left(\frac{0.15}{0.12}\right)}{2\pi\,[43\,\mathrm{W/(m\,K)}]} = \frac{1}{L}\ 0.000826\ \mathrm{(m\,K)/W}$$

$$R_{co} = \frac{1}{\bar{h}_c A_o} = \frac{1}{\bar{h}_c 2\pi r_o L} = \frac{1}{[25\,\mathrm{W/(m^2\,K)}]\,2\pi\,(0.15\,\mathrm{m})\,L} = \frac{1}{L}\ 0.0424\ \mathrm{(m\,K)/W}$$

The rate of heat transfer for the uninsulated pipe is

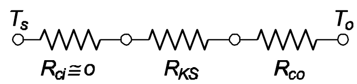
$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_s - T_\infty}{R_{\text{ks}} + R_{\text{co}}}$$

$$\therefore \ \frac{q}{L} = \frac{156^{\circ}\text{C} - 20^{\circ}\text{C}}{(0.000826 + 0.0424)\,(\text{K m})/\text{W}} = 3148 \text{ W/m}$$

The cost to supply this heat loss is

cost = (3148 w/m) (J/W s) (3600 s/h) (24 h/day) (365 days/yr) ($5/109J) = $496/(yr m)

For the insulated pipe the thermal circuit is



The resistance of the insulation is given by:

$$R_{kIa} = \frac{\ln\left(\frac{r_{Ia}}{r_o}\right)}{2\pi\, L k_I} = \frac{\ln\left(\frac{0.2}{0.15}\right)}{2\pi\,[0.059\,\text{W/(mK)}]} = \frac{1}{L}\ 0.776 \text{ (m K)/W}$$

$$R_{kIb} = \frac{\ln\left(\frac{r_{Io}}{r_o}\right)}{2\pi\, L k_I} = \frac{\ln\left(\frac{0.25}{0.15}\right)}{2\pi\,[0.059\,\text{W/(mK)}]} = \frac{1}{L}\ 1.378 \text{ (m K)/W}$$

(a) The rate of heat transfer for the pipe with 5 cm of insulation is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_s - T_\infty}{R_{ks} + R_{kIa} + R_{co}}$$

$$\therefore \ \frac{q}{L} = \frac{156^{\circ}\text{C} - 20^{\circ}\text{C}}{(0.000826 + 0.776 + 0.0424)\,(\text{Km})/\text{W}} = 166 \text{ W/m}$$

The cost of this heat loss is

cost = (166 w/m) (J/W s) (3600 s/h) (24 h/day) (365 days/yr) ($5/$10^9$J) = $26/yr m

Comparing this cost to that of the uninsulated pipe we can calculate the payback period

$$\text{Payback period} = \frac{\text{Cost of installation}}{\text{uninsulated cost} - \text{insulated cost}} = \frac{\$200/\text{m}}{\$496/(\text{yr m}) - \$26/(\text{yr m})}$$

Payback period = 0.43 yr = 5 months

(b) The rate of heat loss for the pipe with 10 cm of insulation is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_s - T_\infty}{R_{ks} + R_{kIb} + R_{co}}$$

$$\therefore \ \frac{q}{L} = \frac{156^{\circ}\text{C} - 20^{\circ}\text{C}}{(0.000826 + 1.378 + 0.0424)\,(\text{Km})/\text{W}} = 95.7 \text{ W/m}$$

The cost of this heat loss

cost = (95.7 w/m) (J/W s) (3600 s/h) (24 h/day) (365 days/yr) ($5/$10^9$ J) = $15/yr m

Comparing this cost to that of the uninsulated pipe we can calculate the payback period

$$\text{Payback period} = \frac{\$300/\text{m}}{\$496/\text{yr m} - \$15/\text{yr m}} = 0.62 \text{ yr} = 7.5 \text{ months}$$

COMMENTS

The 5 cm insulation is a better economic investment. The 10 cm insulation still has a short payback period and is the superior environmental investment since it is a more energy efficient design. Moreover, energy costs are likely to increase in the future and justify the investment in thicker insulation.

PROBLEM 2.11

A hollow sphere with inner and outer radii of R_1 and R_2, respectively, is covered with a layer of insulation having an outer radius of R_3. Derive an expression for the rate of heat transfer through the insulated sphere in terms of the radii, the thermal conductivities, the heat transfer coefficients, and the temperatures of the interior and the surrounding medium of the sphere.

GIVEN

- An insulated hollow sphere
- Radii
  - Inner surface of the sphere = R_1
  - Outer surface of the sphere = R_2
  - Outer surface of the insulation = R_3

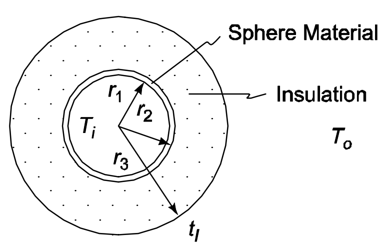
FIND

- Expression for the rate of heat transfer

ASSUMPTIONS

- Steady state heat transfer
- Conduction in the radial direction only
- Constant thermal conductivities

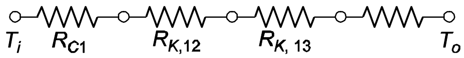
SKETCH



SOLUTION

Let k_{12} = the thermal conductivity of the sphere
k_{23} = the thermal conductivity of the insulation
h_1 = the interior heat transfer coefficient
h_3 = the exterior heat transfer coefficient
T_i = the temperature of the interior medium
T_o = the temperature of the exterior medium

The thermal circuit for the sphere is shown below



The individual resistances are

$$R_{c1} = \frac{1}{\bar{h}_1 A_1} = \frac{1}{\bar{h}_1 4\pi R_1^2 L}$$

From Equation (2.48)

$$R_{k12} = \frac{R_2 - R_1}{4\pi k_{12} R_2 R_1}$$

$$R_{k23} = \frac{R_3 - R_2}{4\pi k_{23} R_3 R_2}$$

$$R_{c3} = \frac{1}{\bar{h}_3 A_3} = \frac{1}{\bar{h}_3 4\pi R_3^2 L}$$

The rate of heat transfer is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{\Delta T}{R_{c1} + R_{k12} + R_{k23} + R_{c3}}$$

$$q = \frac{\Delta T}{\frac{1}{4\pi}\left(\frac{1}{R_1^2 h_1} + \frac{R_2 - R_1}{k_{12} R_2 R_1} + \frac{R_3 - R_2}{k_{23} R_3 R_2} + \frac{1}{R_3^2 h_3}\right)}$$

$$q = \frac{4\pi \Delta T}{\frac{1}{R_1^2 h_1} + \frac{R_2 - R_1}{k_{12} R_2 R_1} + \frac{R_3 - R_2}{k_{23} R_3 R_2} + \frac{1}{R_3^2 h_3}}$$

PROBLEM 2.12

The thermal conductivity of a material may be determined in the following manner. Saturated steam 2.41×10^5 N/m^2 is condensed at the rate of 0.68 kg/h inside a hollow iron sphere that is 1.3 cm thick and has an internal diameter of 51 cm. The sphere is coated with the material whose thermal conductivity is to be evaluated. The thickness of the material to be tested is 10 cm and there are two thermocouples embedded in it, one 1.3 cm from the surface of the iron sphere and one 1.3 cm from the exterior surface of the system. If the inner thermocouple indicates a temperature of 110°C and the outer themocouple a temperature of 57°C, calculate (a) the thermal conductivity of the material surrounding the metal sphere, (b) the temperatures at the interior and exterior surfaces of the test material, and (c) the overall heat transfer coefficient based on the interior surface of the iron sphere, assuming the thermal resistances at the surfaces, as well as the interface between the two spherical shells, are negligible.

GIVEN

- Hollow iron sphere with saturated steam inside and coated with material outside
- Steam pressure = 2.41×10^5 N/m^2
- Steam condensation rate ($\dot{m}_s$) = 0.68 kg/h
- Inside diameter (D_i) = 51 cm = 0.51 m
- Thickness of the iron sphere (Ls) = 1.3 cm = 0.013 m
- Thickness of material layer (L_m) = 10 cm = 0.1 m
- Two thermocouples are located 1.3 cm from the inner and outer surface of the material layer

- Inner thermocouple temperature (T_1) = 110°C
- Outer thermocouple temperature (T_2) = 57°C

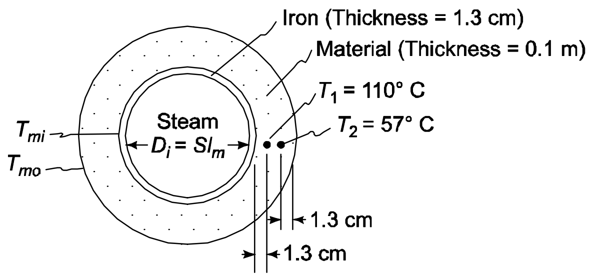
FIND

(a) Thermal conductivity of the material (k_m)
(b) Temperatures at the interior and exterior surfaces of the test material (T_{mi}, T_{mo})
(c) Overall heat transfer coefficient based on the inside area of the iron sphere (U)

ASSUMPTIONS

- Thermal resistance at the surface is negligible
- Thermal resistance at the interface is negligible
- The system has reached steady-state
- The thermal conductivities are constant
- One dimensional conduction radially

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13: For saturated steam at 2.41×10^5 N/m^2,
Saturation temperature (T_s) = 125°C
Heat of vaporization (h_{fg}) = 2187 kJ/kg

SOLUTION

(a) The rate of heat transfer through the sphere must equal the energy released by the condensing steam:

$$q = \dot{m}_s h_{fg} = 0.68\ \text{kg/h}\,(2187\ \text{kJ/kg})(1000\ \text{J/kJ})\left(\frac{\text{h}}{3600\text{s}}\right)((\text{Ws})/\text{J}) = 413.1\ \text{W}$$

The thermal conductivity of the material can be calculated by examining the heat transfer between the thermocouple radii

$$q = \frac{\Delta T}{R_{k12}} = \frac{T_2 - T_1}{\left(\dfrac{r_2 - r_1}{4\pi\, k_m\, r_2\, r_1}\right)}$$

Solving for the thermal conductivity

$$k_m = \frac{q(r_2 - r_1)}{4\pi\, r_2\, r_1 (T_2 - T_1)}$$

$$r_1 = \frac{D_i}{2} + L_s + 0.013\ \text{m} = \frac{0.51\text{m}}{2} + 0.013\ \text{m} + 0.013\ \text{m} = 0.281\ \text{m}$$

$$r_2 = \frac{D_i}{2} + L_s + L_m - 0.013\text{ m} = \frac{0.51\text{ m}}{2} + 0.013\text{ m} + 0.1\text{ m} - 0.013\text{ m} = 0.355$$

$$k_m = \frac{413.1\text{W}(0.355\text{ m} - 0.281\text{ m})}{4\pi\,(0.355\text{m})(0.281\text{m})(110^{o}C - 57^{o}C)} = 0.46\text{ W/(m K)}$$

(b) The temperature at the inside of the material can be calculated from the equation for conduction through the material from the inner radius, the radius of the inside thermocouple

$$q = \frac{\Delta T}{R_{ki1}} = \frac{T_{mi} - T_i}{\left(\dfrac{r_1 - r_i}{4\pi\, k_m\, r_1\, r_i}\right)}$$

Solving for the temperature of the inside of the material

$$T_{mi} = T_1 + \frac{q\,(r_1 = r_i)}{4\pi\, k_m\, r_1\, r_i}$$

$$r_i = \frac{D_i}{2} + L_m = \frac{0.51\text{ m}}{2} + 0.013\text{ m} = 0.268\text{ m}$$

$$T_{mi} = 110°\text{C} + \frac{413.1\text{W}\,(0.013\text{m})}{4\pi\,[0.46\text{ W/(mK)}](0.281\text{ m})(0.268\text{m})} = 122°\text{C}$$

The temperature at the outside radius of the material can be calculated from the equation for conduction through the material from the radius of the outer thermocouple to the outer radius

$$q = \frac{\Delta T}{R_{k2o}} = \frac{T_2 - T_{mo}}{\left(\dfrac{r_o - r_2}{4\pi\, k_m\, r_o\, r_2}\right)}$$

Solving for the temperature of the outer surface of the material

$$T_{mo} = T_2 - \frac{q\,(r_o - r_2)}{4\pi\, k_m\, r_o\, r_2}$$

$$r_o = \frac{D_i}{2} + L_s + L_m = \frac{0.51\text{ m}}{2} + 0.013\text{ m} + 0.01\text{ m} = 0.368\text{ m}$$

$$T_{mo} = 57°\text{C} - \frac{413.1\text{W}\,(0.013\text{m})}{4\pi\,[0.46\text{ W/(mK)}](0.368\text{ m})(0.355\text{m})} = 50°\text{C}$$

(c) The heat transfer through the sphere can be expressed as

$$q = U\,A_i\,\Delta T = U\,\pi\,D_1^2\,(T_s - T_{mo})$$

$$\therefore\ U = \frac{q}{\pi\,D_i^2(T_s - T_{mo})} = \frac{413.1\text{ W}}{\pi\,(0.51\text{m})^2(125°\text{C} - 50°\text{C})} = 6.74\text{ W/(m}^2\text{ K)}$$

PROBLEM 2.13

A cylindrical liquid oxygen (LOX) tank has a diameter of 4 ft, a length of 20 ft, and hemispherical ends. The boiling point of LOX is – 297°F. An insulation is sought which will reduce the boil-off rate in the steady state to no more than 25 lb/h. The heat of

vaporization of LOX is 92 Btu/lb. If the thickness of this insulation is to be no more than 3 in., what would the value of it's thermal conductivity have to be?

GIVEN

- Insulated cylindrical tank with hemispherical ends filled with LOX
- Diameter of tank (D_t) = 4 ft
- Length of tank (L_t) = 20 ft
- Boiling point of LOX (T_{bp}) = –297°F
- Heat of vaporization of LOX (h_{fg}) = 92 Btu/lb
- Steady state boil-off rate ($\dot{m}$) = 25 lb/h
- Maximum thickness of insulation (L) = 3 in. = 0.25 ft

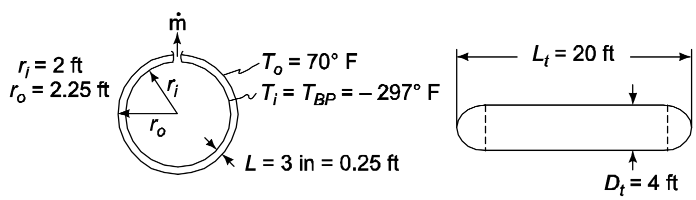
FIND

- The thermal conductivity (k) of the insulation necessary to maintain the boil-off rate below 25 lb/h.

ASSUMPTIONS

- The length given includes the hemispherical ends
- The thermal resistance of the tank is negligible compared to the insulation
- The thermal resistance at the interior surface of the tank is negligible

SKETCH



SOLUTION

The tank can be thought of as a sphere (the ends) separated by a cylindrical section, therefore the total heat transfer is the sum of that through the spherical and cylindrical sections. The steady state conduction through a spherical shell with constant thermal conductivity, from Equation (2.47), is

$$q_s = \frac{4\pi\, K\, r_o r_i\,(T_o - T_i)}{r_o - r_i}$$

The rate of steady state conduction through a cylindrical shell, from Equation (2.37), is

$$q_c = 2\,\pi\, L_c\, k\, \frac{T_o - T_i}{\ln\left(\frac{r_o}{r_i}\right)} \qquad (L_c = L_t - 4\text{ ft} = 16\text{ ft})$$

The total heat transfer through the tank is the sum of these

$$q = q_s + q_c = \frac{4\pi\, k\, r_o r_i\,(T_o - T_i)}{r_o - r_i} + 2\,\pi\, L_c\, k\, \frac{(T_o - T_i)}{\ln\left(\frac{r_o}{r_i}\right)} = 2\,\pi\, k\,(T_o - T_i)\left[\frac{2\, r_o\, r_i}{r_o - r_i} + \frac{L_c}{\ln\left(\frac{r_o}{r_i}\right)}\right]$$

The rate of heat transfer required to evaporate the liquid oxygen at m is $m\, h_{fg}$, therefore

$$\dot{m}_s\, h_{fg} = 2\,\pi\, k\,(T_o - T_i)\left[\frac{2\, r_o\, r_i}{r_o - r_i} + \frac{L_c}{\ln\left(\frac{r_o}{r_i}\right)}\right]$$

$$\therefore\ k = \frac{\dot{m}\, h_{fg}}{2\pi\, k(T_o - T_i)\left[\dfrac{2\, r_o r_i}{r_o - r_i} + \dfrac{L_c}{\ln\left(\dfrac{r_o}{r_i}\right)}\right]}$$

$$k = \frac{25\ \text{lb/h}\ (92\ \text{Btu/lb})}{2\pi\,[70°\text{F} - (-297°\text{F})]\left[\dfrac{2(2.25\ \text{ft})(2.0\ \text{ft})}{0.25\ \text{ft}} + \dfrac{16\ \text{ft}}{\ln\left(\dfrac{2.25}{2}\right)}\right]}$$

$$k = 0.0058\ \text{Btu/(h ft °F)}$$

COMMENTS

Based on data given in Appendix 2, Table 11, no common insulation has such low value of thermal conductivity. However, *Marks Standard Handbook for Mechanical Engineers* lists the thermal conductivity of expanded rubber board, ‘Rubatex’, at –330°F to be 0.004 Btu/(h ft °F).

PROBLEM 2.14

The addition of insulation to a cylindrical surface, such as a wire, may increase the rate of heat dissipation to the surroundings (see Problem 2.4). (a) For a No. 10 wire (0.26 cm in diameter), what is the thickness of rubber insulation [k = 0.16 W/(m K)] that will maximize the rate of heat loss if the heat transfer coefficient is 10 W/(m^2 K)? (b) If the current-carrying capacity of this wire is considered to be limited by the insulation temperature, what percent increase in capacity is realized by addition of the insulation? State your assumptions.

GIVEN

- An insulated cylindrical wire
- Diameter of wire (D_w) = 0.26 cm = 0.0026 m
- Thermal conductivity of rubber (k) = 0.16 W/(m K)
- Heat transfer coefficient ($\bar{h}_c$) = 10 W/(m^2 K)

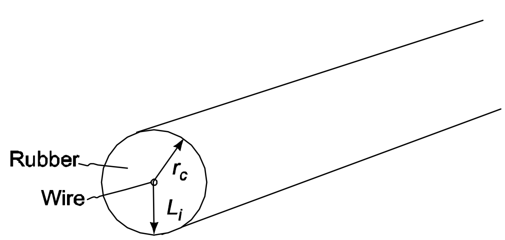
FIND

(a) Thickness of insulation (L_i) to maximize heat loss
(b) Percent increase in current carrying capacity

ASSUMPTIONS

- The system is in steady state
- The thermal conductivity of the rubber does not vary with temperature

SKETCH



SOLUTION

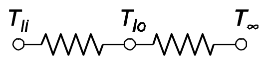
(a) From Problem 2.4, the radius that will maximize the rate of heat transfer (r_c) is:

$$r_c = \frac{k}{h} = \frac{0.16\,\text{W/(mK)}}{10\,\text{W/(m}^2\,\text{K)}} = 0.016\text{ m}$$

The thickness of insulation needed to make this radius is

$$L_i = r_c - r_w = 0.016\text{ m} - \frac{0.0026\,\text{m}}{2} = 0.015\text{ m} = 1.5\text{ cm}$$

(b) The thermal circuit for the insulated wire is shown below



where $$R_{kI} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi Lk} \quad\text{and}\quad R_c = \frac{1}{\bar{h}_c A} = \frac{1}{\bar{h}_c 2\pi r_o L}$$

The rate of heat transfer from the wire is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_{Ii} - T_\infty}{R_{kI} + R_c} = \frac{2\pi L(T_{Ii} - T_\infty)}{\frac{\ln\left(\frac{r_o}{r_i}\right)}{k} + \frac{1}{\bar{h}_c r_o}}$$

If only a very thin coat of insulation is put on the wire to insulate it electrically then $r_o = r_i = D_w/2 = 0.0013$ m. The rate of heat transfer from the wire is

$$\frac{q}{L} = \frac{2\pi(T_{Ii} - T_\infty)}{0 + \frac{1}{10\,\text{W/(m}^2\text{K)}(0.0013\,\text{m})}} = 0.082\,(T_{Ii} - T_\infty)$$

For the wire with the critical insulation thickness

$$\frac{q}{L} = \frac{2\pi(T_{Ii} - T_\infty)}{\frac{\ln\left(\frac{0.016}{0.0013}\right)}{10\,\text{W/(m K)}} + \frac{1}{10\,\text{W/(m}^2\text{K)}(0.016\,\text{m})}} = 0.286\,(T_{Ii} - T_\infty)$$

The current carrying capacity of the wire is directly related to the rate of heat transfer from the wire. For a given maximum allowable insulation temperature, the increase in current carrying capacity of the wire with the critical thickness of insulation over that of the wire with a very thin coating of insulation is

$$\%\text{ increase} = \frac{\left(\frac{q}{L}\right)_{r_a} - \left(\frac{q}{L}\right)_{\text{thin coat}}}{\left(\frac{q}{L}\right)_{\text{thin coat}}} \times 100 = \frac{0.286 - 0.082}{0.082} \times 100 = 250\%$$

COMMENTS

This would be an enormous amount of insulation to add to the wire changing a thin wire into a rubber cable over an inch in diameter and would not be economically justifiable. Thinner coatings of rubber will achieve smaller increases in current carrying capacity.

PROBLEM 2.15

For the system outlined in Problem 2.11, determine an expression for the critical radius of the insulation in terms of the thermal conductivity of the insulation and the surface coefficient between the exterior surface of the insulation and the surrounding fluid. Assume that the temperature difference, R_1, R_2, the heat transfer coefficient on the interior, and the thermal conductivity of the material of the sphere between R_1 and R_2 are constant.

GIVEN

- An insulated hollow sphere
- Radii
  - Inner surface of the sphere = R_1
  - Outer surface of the sphere = R_2
  - Outer surface of the insulation = R_3

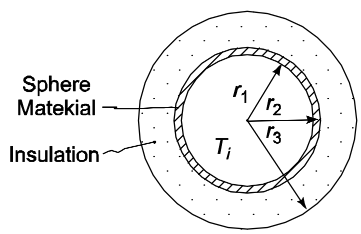
FIND

- An expression for the critical radius of the insulation

ASSUMPTIONS

- Constant temperature difference, radii, heat transfer coefficients, and thermal conductivities
- Steady state prevails

SKETCH



SOLUTION

Let k_{12} = the thermal conductivity of the sphere
k_{23} = the thermal conductivity of the insulation
h_1 = the interior heat transfer coefficient
h_3 = the exterior heat transfer coefficient
T_i = the temperature of the interior medium
T_o = the temperature of the exterior medium

From Problem 2.11, the rate of heat transfer through the sphere is

$$q = \frac{4\pi \Delta T}{\frac{1}{R_1^2 h_1} + \frac{R_2 - R_1}{k_{12} R_2 R_1} + \frac{R_3 - R_2}{k_{23} R_3 R_2} + \frac{1}{R_3^2 h_3}}$$

The rate of heat transfer is a maximum when the denominator of the above equation is a minimum. This occurs when the derivative of the denominator with respect to R_3 is zero

$$\frac{d}{dR_3} = \left(\frac{1}{R_1^2 h_1} + \frac{R_2 - R_1}{k_{12} R_2 R_1} + \frac{R_3 - R_2}{k_{23} R_3 R_2} + \frac{1}{R_3^2 h_3} \right) = 0$$

$$-\frac{2}{h_3 R_3} + \frac{1}{k_{23}} = 0$$

$$R_3 = \frac{2\,k_{23}}{h_3}$$

The maximum heat transfer will occur when the outer insulation radius is equal to 2 k_{23}/h_3.

COMMENTS

A more realistic analysis should take the dependence of h_c on temperature into account. Such an analysis was made for a pipe by Sparrow and Kang, Int. J. Heat Mass Transf., 28: 2049-2060, 1985.

PROBLEM 2.16

A standard 4 in. steel pipe (*ID* = 4.026 in., *OD* = 4.500 in.) carries superheated steam at 1200°F in an enclosed space where a fire hazard exists, limiting the outer surface temperature to 100°F. In order to minimize the insulation cost, two materials are to be used; first a high temperature insulation (relatively expensive) applied to the pipe and then magnesia (a less expensive material) on the outside. The maximum temperature of the magnesia is to be 600°F. The following constants are known.

Steam-side coefficient	h **= 100 Btu/(h ft^2 °F)**
High-temperature insulation conductivity	k **= 0.06 Btu/(h ft °F)**
Magnesia conductivity	k **= 0.045 Btu/(h ft °F)**
Outside heat transfer coefficient	h **= 2.0 Btu/(h ft^2 °F)**
Steel conductivity	k **= 25 Btu/(h ft °F)**
Ambient temperature	T_a **= 70°F**

(a) Specify the thickness for each insulating material.
(b) Calculate the overall heat transfer coefficient based on the pipe *OD*.
(c) What fraction of the total resistance is due to (1) steam-side resistance, (2) steel pipe resistance, (3) insulation (combination of the two), and (4) outside resistance?
(d) How much heat is transferred per hour, per foot length of pipe?

GIVEN

- Steam filled steel pipe with two layers of insulation
- Pipe inside diameter (D_i) = 4.026 in.
- Pipe outside diameter (D_o) = 4.500 in.
- Superheated steam temperature (T_s) = 1200°F
- Maximum outer surface temperature (T_{so}) = 100°F
- Maximum temperature of the Magnesia (T_m) = 600°F
- Thermal conductivities
  - High-temperature insulation (k_h) = 0.06 Btu/(h ft °F)
  - Magnesia (k_m) = 0.045 Btu/(h ft °F)
  - Steel (k_s) = 25 Btu/(h ft °F)
- Heat transfer coefficients
  - Steam side ($\bar{h}_{ci}$) = 100 Btu/(h ft^2 °F)
  - Outside ($\bar{h}_{co}$) = 2.0 Btu/(h ft^2 °F)
- Ambient temperature (T_a) = 70°F

FIND

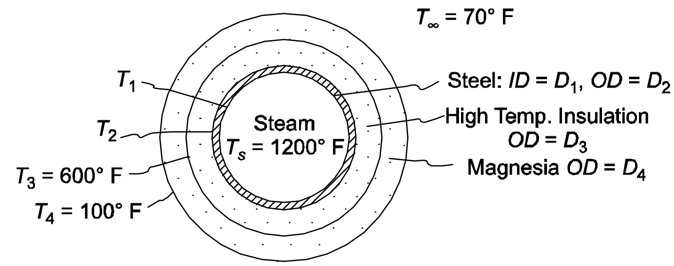
(a) Thickness for each insulation material

(b) Overall heat transfer coefficient based on the pipe *OD*
(c) Fraction of the total resistance due to
  - Steam-side resistance
  - Steel pipe resistance
  - Insulation
  - Outside resistance
(d) The rate of heat transfer per unit length of pipe (q/L)

ASSUMPTIONS

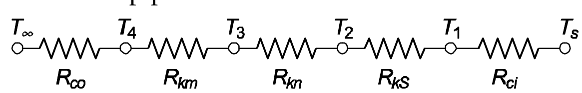
- The system is in steady state
- Constant thermal conductivities
- Contact resistance is negligible

SKETCH



SOLUTION

The thermal circuit for the insulated pipe is shown below



The values of the individual resistances can be evaluated with Equations (1.14) and (2.39)

$$R_{co} = \frac{1}{\bar{h}_{co} A_o} = \frac{1}{\bar{h}_{co}\, 2\pi\, r_4 L}$$

$$R_{km} = \frac{\ln\left(\dfrac{r_4}{r_3}\right)}{2\pi\, L\, k_m}$$

$$R_{kh} = \frac{\ln\left(\dfrac{r_3}{r_2}\right)}{2\pi\, L\, k_h}$$

$$R_{ks} = \frac{\ln\left(\dfrac{r_2}{r_1}\right)}{2\pi\, L\, k_s}$$

$$R_{ci} = \frac{1}{\bar{h}_{ci} A_i} = \frac{1}{\bar{h}_{ci}\, 2\pi\, r_1 L}$$

The variables in the above equations are

$$r_1 = 2.013 \text{ in}$$

$$r_2 = 2.25 \text{ in}$$

$$r_3 = ?$$

$$r_4 = ?$$

$$k_m = 0.045\ \text{Btu/(h ft °F)}$$

$$k_s = 25\ \text{Btu/(h ft °F)}$$

$$k_h = 0.06\ \text{Btu/(h ft °F)}$$

$$\bar{h}_{co} = 2\ \text{Btu/(h ft}^2\ \text{°F)}$$

$$\bar{h}_{ci} = 100\ \text{Btu/(h ft}^2\ \text{°F)}$$

The temperatures for this problem are

$$T_s = 1200\ \text{°F}$$

$$T_1 = ?$$

$$T_2 = ?$$

$$T_3 = 600\ \text{°F}$$

$$T_4 = 100\ \text{°F}$$

$$T_a = 70\ \text{°F}$$

There are five unknowns in this problem: q/L, T_1, T_2, r_3, and r_4. These can be solved for by writing the equation for the heat transfer through each of the five resistances and solving them simultaneously.

1. Steam side convective heat transfer

$$q = \frac{\Delta T}{R_{ci}} = 2\,\pi\,\bar{h}_{ci}\,r_1\,L\,(T_s - T_1) = 2\,\pi L\,[100\ \text{Btu/(h ft}^2\,\text{°F)}]\left(\frac{2.013}{12}\ \text{ft}\right)(1200\ \text{°F} - T_1)$$

$$\frac{q}{L} = 126{,}480 - 105.4T_1\ \text{Btu/(h ft)} \qquad [1]$$

2. Conduction through the pipe wall

$$q = \frac{\Delta T}{R_{ks}} = \frac{2\pi\,k_s L}{\ln\left(\frac{r_2}{r_1}\right)}(T_1 - T_2) = \frac{2\pi\,L[25\ \text{Btu/(h ft}^2\,\text{°F)}]}{\ln\left(\frac{2.25}{2.013}\right)}(T_1 - T_2)$$

$$\frac{q}{L} = 1411\,(T_1 - T_2)\ \text{Btu/(h ft)} \qquad [2]$$

3. Conduction through the high temperature insulation

$$q = \frac{\Delta T}{R_{kh}} = \frac{2\pi\,k_h L}{\ln\left(\frac{r_1}{r_2}\right)}(T_2 - T_3) = \frac{2\pi\,L[0.06\ \text{Btu/(h ft}^2\,\text{°F)}]}{\ln\left(\frac{r_3}{r_2}\right) - \ln\left(\frac{2.25}{12}\right)}(T_2 - 600\text{°F})$$

$$\frac{q}{L} = \frac{0.377}{\ln r_3 + 1.674}(T_2 - 600\ \text{°F})\ \text{Btu/(h ft)} \qquad [3]$$

4. Conduction through the magnesia insulation

$$q = \frac{\Delta T}{R_{km}} = \frac{2\pi\,k_m L}{\ln\left(\frac{r_4}{r_3}\right)}(T_3 - T_4) = \frac{2\pi\,L[0.045\ \text{Btu/(h ft}^2\,\text{°F)}]}{\ln(r_4) - \ln(r_3)}(600\ \text{°F} - 100\ \text{°F})$$

$$\frac{q}{L} = \frac{141.4}{\ln(r_4) + \ln(r_3)}\ \text{Btu/(h ft)} \qquad [4]$$

5. Air side convective heat transfer

$$q = \frac{\Delta T}{R_{co}} = 2\pi\,\overline{h_{co}}\;r_4\,L\,(T_4 - T_a) = 2\,\pi\,L\,r_4\;(2.0\text{ Btu/s(h ft}^2\,°\text{F))}\;\;(100\text{ °F} - 70\text{ °F})$$

$$\frac{q}{L} = 377\,r_4\text{ Btu/(h ft)} \qquad [5]$$

To maintain steady state, the heat transfer rate through each resistance must be equal. Equations [1] through [5] are a set of five equations with five unknowns, they may be solved through numerical iterations using a simple program or may be combined algebraically as follows

Substituting Equation [1] into Equation [2] yields

$$T_2 = 1.075\,T_1 - 89.64$$

Substituting this into Equation [3] and combining the result with Equation [1]

$$\ln r_3 = \frac{0.405T_1 - 260.0}{126{,}480 - 105.4T_1} - 1.674$$

Substituting this into Equation [4] and combining the result with Equation [1]

$$r_4 = \exp\left[\frac{0.405T_1 - 118.6}{126{,}480 - 105.4T_1} - 1.674\right]$$

Finally, substituting this into Equation [5] and combining the result with Equation [1]

$$126{,}480 - 105.4\,T_1 = 377\,\exp\left[\frac{0.405T_1 - 118.6}{126{,}480 - 105.4T_1} - 1.674\right]$$

Solving this by trial and error: T_1 = 1197°F

This result can be substituted into the equations above to find the unknown radii

$$r_3 = 0.382\text{ ft} = 4.6\text{ in}\quad r_4 = 0.597\text{ ft} = 7.2\text{ in}$$

The thickness of the high temperature insulation = $r_3 - r_2$ = 2.3 in

The thickness of the magnesia insulation = $r_4 - r_3$ = 2.6 in

(b) Substituting T_1 = 1197°F into [1] yields a heat transfer rate of 316.2 Btu/(h ft). The overall heat transfer coefficient based on the pipe outside area must satisfy the following equation

$$q = U\,A_2\,(T_s - T_a) = U\,\pi\,D_2\,L\,(T_s - T_a)$$

$$\therefore\; U = \frac{q}{L}\,\frac{1}{\pi\,D_2(T_s - T_a)} = 316.2\text{ Btu/(h ft}^2\,°\text{F)}\;\frac{1}{\pi\left(\frac{4.5}{12}\text{ ft}\right)(1200\,°\text{F} - 70\,°\text{F})}$$

$$U = 0.238\text{ Btu/(h ft}^2\,°\text{F)}$$

(c) The overall resistance for the insulated pipe is

$$R_{\text{total}} = \frac{1}{UA_2} = \frac{1}{[0.238\text{ Btu/(h ft}^2\,°\text{F)}]\,\pi\left(\frac{4.5}{12}\text{ ft}\right)L} = \frac{1}{L}\;3.57\text{ (h ft)/Btu}$$

(4) The convective thermal resistance on the air side is

$$R_{co} = \frac{1}{\overline{h_{co}}A_o} = \frac{1}{\overline{h_{co}}\,2\pi\,r_4 L} = \frac{1}{(2\text{ Btu/(h ft °F))}\,2\pi\,(0.597\text{ ft})L} = \frac{1}{L}\;0.133\text{ (h ft)/Btu}$$

The fraction of the resistance due to air side convection = 0.133/3.57 = 0.04.

(3) The thermal resistance of the magnesia insulation is

$$R_{km} = \frac{\ln\left(\frac{r_4}{r_3}\right)}{2\pi\, L k_m} = \frac{\ln\left(\frac{0.597}{0.382}\right)}{2\pi\, L[0.045\,\text{Btu/(h ft °F)}]} = \frac{1}{L}\;1.58\ \text{(h ft)/Btu}$$

The thermal resistance of the high temperature insulation is

$$R_{kh} = \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi\, L k_h} = \frac{\ln\left(\frac{4.6}{2.25}\right)}{2\pi\, L[0.06\,\text{Btu/(h ft °F)}]} = \frac{1}{L}\;1.90\ \text{(h ft)/Btu}$$

The fraction of the resistance due to the insulation = 3.48/3.57 = 0.97.

(2) The thermal resistance of the steel pipe is

$$R_{ks} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\, L k_s} = \frac{\ln\left(\frac{2.25}{2.013}\right)}{2\pi\, L[25\,\text{Btu/(h ft °F)}]} = \frac{1}{L}\;0.0007\ \text{(h ft)/Btu}$$

The fraction of the resistance due to the steel pipe = 0.0007/3.57 = 0.00.

(1) The thermal resistance of the steam side convection is

$$R_{ci} = \frac{1}{\overline{h_{ci}} A_i} = \frac{1}{\overline{h_{ci}}\, 2\pi\, r_1 L} = \frac{1}{(100\ \text{Btu/(h ft}^2\ \text{°F))}\, 2\pi \left(\frac{2.013}{12\,\text{ft}}\right) L} = \frac{1}{L}\;0.0095\ \text{(h ft)/Btu}$$

The fraction of the resistance due to steam side convection = 0.0095/3.57 = 0.00.

(d) The rate of heat transfer is

$$q = U A_2\,(T_s - T_a) = U\,\pi\,D_2\,L\,(T_s - T_a)$$

$$\frac{q}{L} = 2.38\ \text{Btu/(h ft}^2\ \text{°F)}\ 2\,\pi \left(\frac{2.25}{12\ \text{ft}}\right)\ (1200\text{°F} - 70\text{°F}) = 317\ \text{(h ft)/Btu}$$

COMMENTS

Notice that the insulation accounts for 97% of the total thermal resistance and that the thermal resistance of the steel pipe and the steam side convection are negligible.

PROBLEM 2.17

Show that the rate of heat conduction per unit length through a long hollow cylinder of inner radius r_i and outer radius r_o, made of a material whose thermal conductivity varies linearly with temperature, is given by

$$\frac{q_k}{L} = \frac{T_i - T_o}{(r_o - r_i)/k_m \bar{A}}$$

where T_i **= temperature at the inner surface**

T_o **= temperature at the outer surface**

A **=** $2\,\pi\,(r_o - r_i)/\ln\left(\frac{r_o}{r_i}\right)$

k_m **=** $k_o\,[1 + \beta_k\,(T_i + T_o)/2]$

L **= length of cylinder**

GIVEN

- A long hollow cylinder
- The thermal conductivity varies linearly with temperature

- Inner radius = r_i
- Outer radius = r_o

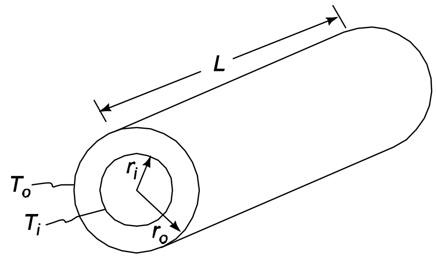
FIND

- Show that the rate of heat conduction per unit length is given by the above equation

ASSUMPTIONS

- Conduction occurs in the radial direction only
- Steady state prevails

SKETCH



SOLUTION

The rate of radial heat transfer through a cylindrical element of radius r is

$$\frac{q}{L} = k\,A\,\frac{dT}{dr} = k\,2\,\pi\,r\,\frac{dT}{dr} = a \text{ constant}$$

But the thermal conductivity varies linearly with the temperature

$$k = k_o\,(1 + \beta\,T)$$

$$\therefore\ \frac{q}{L} = 2\pi\,r\,k_o\,(1 + \beta\,T)\,\frac{dT}{dr}$$

$$\frac{q}{L}\,\frac{1}{r}\,dr = 2\pi\,k_o\,(1 + \beta\,T)\,dT$$

Integrating between the inner and outer radii:

$$\frac{q}{L}\int_{r_i}^{r_o}\frac{1}{r}\,dr = 2\pi\,k_o\int_{T_i}^{T_o}(1 + \beta T)\,dt$$

$$\frac{q}{L}\,(\ln r_o - \ln r_i) = 2\pi\,k_o\left[T_o + \frac{\beta}{2}T_o^2 - T_i - \frac{\beta}{2}T_i^2\right]$$

$$\frac{q}{L}\left(\ln\frac{r_o}{r_i}\right) = 2\pi\,k_o\left[(T_o - T_i) + \frac{\beta}{2}\,(T_o^2 - T_i^2)\right]$$

$$\frac{q}{L} = \left[\frac{2\pi\,(r_o - r_i)}{\ln\frac{r_o}{r_i}\,(r_o - r_i)}\right] k_o\,(T_o - T_i)\left[1 + \frac{\beta}{2}\,(T_o - T_i)\right]$$

$$\frac{q}{L} = \frac{\bar{A}}{(r_o - r_i)}\,k_m\,(T_o - T_i)$$

$$\frac{q}{L} = \frac{T_o - T_i}{\left(\frac{r_o - r_i}{k_m\bar{A}}\right)}$$

PROBLEM 2.18

A long, hollow cylinder is constructed from a material whose thermal conductivity is a function of temperature according to k = 0.060 + 0.00060 T, where T is in °F and k is in Btu/h °F. The inner and outer radii of the cylinder are 5 and 10 in., respectively. Under steady-state conditions, the temperature at the interior surface of the cylinder is 800°F and the temperature at the exterior surface is 200°F.

(a) Calculate the rate of heat transfer per foot length, taking into account the variation in thermal conductivity with temperature. (b) If the heat transfer coefficient on the exterior surface of the cylinder is 3 Btu/(h ft^2 °F), calculate the temperature of the air on the outside of the cylinder.

GIVEN

- A long hollow cylinder
- Thermal conductivity (k) = 0.060 + 0.00060 T [T in °F, k in Btu/(h ft °F)]
- Inner radius (r_i) = 5 in.
- Outer radius (r_o) = 10 in.
- Interior surface temperature (T_{wi}) = 800°F
- Exterior surface temperature (T_{wo}) = 200°F
- Exterior heat transfer coefficient ($\bar{h}_o$) = 3 Btu/(h ft^2 °F)
- Steady-state conditions

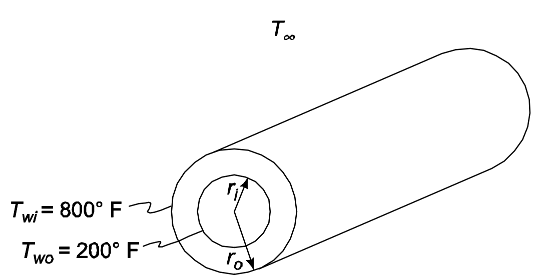
FIND

(a) The rate of heat transfer per foot length (q/L)
(b) The temperature of the air on the outside (T_∞)

ASSUMPTIONS

- Steady state heat transfer
- Conduction occurs in the radial direction only

SKETCH



SOLUTION

(a) The rate of radial conduction is given by Equation (2.37)

$$q = -k\,A\,\frac{dT}{dr}$$

$$q = -(0.06 + 0.0006\,T)\,2\pi\,r\,L\,\frac{dT}{dr}$$

$$\frac{1}{r}\,dr = \frac{2\pi L}{q}\,(0.06 + 0.006\,T)\,dT$$

Integrating this from the inside radius to the outside radius

$$\int_{r_i}^{r_o}\frac{1}{r}\,dr = -\frac{2\pi L}{q}\int_{T_{wi}}^{T_{wo}}(0.06 + 0.0006\,T)\,dt$$

$$\text{In}\; r_o - \ln r_i = -\frac{2\pi L}{q}\,[0.06\,(T_{wo} - T_{wi}) + 0.0003\,(T_{wo}^2 - T_{wi}^2)]$$

$$\ln\frac{r_o}{r_i} = 2\pi\frac{L}{q}\,[0.06\,(T_{wo} - T_{wi}) + 0.0003\,(T_{wo}^2 - T_{wi}^2)]$$

$$\frac{q}{L} = \frac{2\pi}{\ln\left(\frac{r_o}{r_i}\right)}\,[0.06\,(T_{wo} - T_{wi}) + 0.0003\,(T_{wo}^2 - T_{wi}^2)]$$

$$\frac{q}{L} = \frac{2\pi}{\ln\left(\frac{10}{5}\right)}\,[0.06\,(800 - 200) + 0.0003\,(800^2 - 200^2)]\ \text{Btu/(h ft)}$$

$$\frac{q}{L} = 1958\ \text{Btu/(h ft)}$$

(b) The conduction through the hollow cylinder must equal the convection from the outer surface in steady state

$$\frac{q}{L} = \overline{h_o}\,A_o\,\Delta T = \overline{h_o}\,2\pi\,r_o\,(T_{wo} - T_\infty)$$

Solving for the air temperature

$$T_\infty = T_{wo} - \frac{q}{L}\,\frac{1}{\overline{h_o}\,2\pi\,r_o} = 200\ ^\circ\text{F} - 1958\ \text{Btu/(h ft)}\ \frac{1}{\text{Btu/(h ft}^2\ ^\circ\text{F)}\ 2\pi\left(\frac{2}{10}\ \text{ft}\right)} = 75^\circ\text{F}$$

PROBLEM 2.19

A plane wall 15 cm thick has a thermal conductivity given by the relation

$$\boldsymbol{k = 2.0 + 0.0005\ T\ \text{W/(m K)}}$$

where *T* is in degrees Kelvin. If one surface of this wall is maintained at 150 °C and the other at 50 °C, determine the rate of heat transfer per square meter. Sketch the temperature distribution through the wall.

GIVEN

- A plane wall
- Thickness (L) = 15 cm = 0.15 m
- Thermal conductivity (k) = 2.0 + 0.0005 T W/(m K) (with T in Kelvin)
- Surface temperatures: T_h = 150 °C T_c = 50 °C

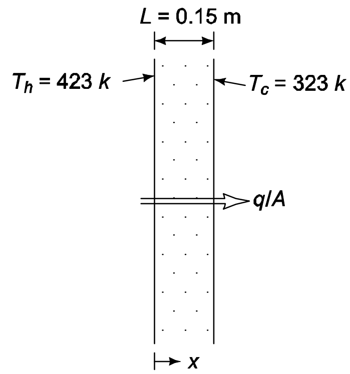
FIND

(a) The rate of heat transfer per square meter (q/A)
(b) The temperature distribution through the wall

ASSUMPTIONS

- The wall has reached steady state
- Conduction occurs in one dimension

SKETCH



SOLUTION

Simplifying Equation (2.2) for steady state conduction with no internal heat generation but allowing for the variation of thermal conductivity with temperature yields

$$\frac{d}{dx} k \frac{dT}{dx} = 0$$

with boundary conditions: T = 423 K at x = 0

T = 323 K at x = 0.15 m

The rate of heat transfer does not vary with x

$$-k \frac{dT}{dx} = \frac{q}{A} = \text{constant}$$

$$-(2.0 + 0.0005T)\, dT = \frac{q}{A}\, dx$$

Integrating

$$2.0T + 0.00025\, T^2 = -\frac{q}{A}\, x + C$$

The constant can be evaluated using the first boundary condition

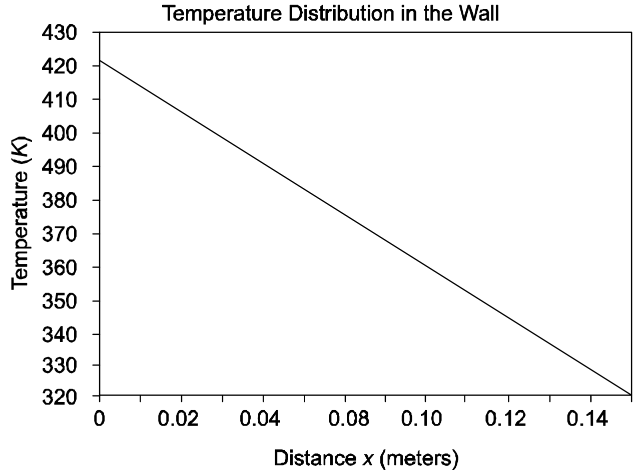
$$2.0\,(423) + 0.00025\,(423)^2 = C - \frac{q}{A}\,(0) \Rightarrow C = 890.7$$

(a) The rate of heat transfer can be evaluated using the second boundary condition:

$$2.0\,(323) + 0.00025\,(323)^2 = 890.7 - \frac{q}{A}\,(0.15\text{ m}) \Rightarrow q_k = 1457\text{ W/m}^2$$

(b) Therefore, the temperature distribution is

$$0.00025\, T^2 + 2.0\, T = 890.7 - 1458\, x$$



COMMENTS

Notice that although the temperature distribution is not linear due to the variation of the thermal conductivity with temperature, it is nearly linear because this variation is small compared to the value of the thermal conductivity.

If the variation of thermal conductivity with temperature had been neglected, the rate of heat transfer would have been 1333 W/m^2, an error of 8.5%.

PROBLEM 2.20

> **A plane wall 7.5 cm thick, generates heat internally at the rate of 10^5 W/m^3. One side of the wall is insulated, and the other side is exposed to an environment at 90°C. The convective heat transfer coefficient between the wall and the environment is 500 W/(m^2 K). If the thermal conductivity of the wall is 12 W/(m K), calculate the maximum temperature in the wall.**

GIVEN

- Plane wall with internal heat generation
- Thickness (L) = 7.5 cm = 0.075 m
- Internal heat generation rate ($\dot{q}_G$) = 10^5 W/m^3
- One side is insulated
- Ambient temperature on the other side (T_∞) = 90 °C
- Convective heat transfer coefficient ($\bar{h}_c$) = 500 W/(m^2 K)
- Thermal conductivity (k) = 12 W/(m K)

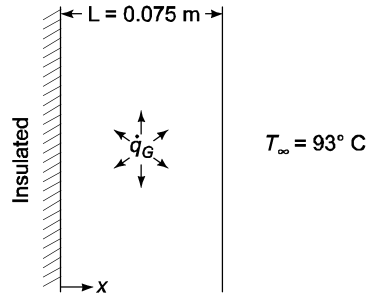
FIND

- The maximum temperature in the wall (T_{max})

ASSUMPTIONS

- The heat loss through the insulation is negligible
- The system has reached steady state
- One dimensional conduction through the wall

SKETCH



SOLUTION

The one dimensional conduction equation, given in Equation (2.5), is

$$k\,\frac{\partial^2 T}{\partial x^2} + \dot{q}_G = \rho\, c\,\frac{\partial T}{\partial t}$$

For steady state, $\frac{\partial T}{\partial t} = 0$ therefore

$$k\,\frac{d^2 T}{d x^2} + \dot{q}_G = 0$$

$$\frac{d^2 T}{d x^2} = -\frac{\dot{q}_G}{k}$$

This is subject to the following boundary conditions

No heat loss through the insulation

$$\frac{dT}{dx} = 0 \text{ at } x = 0$$

Convection at the other surface

$$-k\,\frac{dT}{dx} = \overline{h}_c\,(T - T_\infty) \qquad \text{at} \quad x = L$$

Integrating the conduction equation once

$$\frac{dT}{dx} = \frac{\dot{q}_G}{k} + C_1$$

C_1 can be evaluated using the first boundary condition

$$0 = -\frac{\dot{q}_G}{k}\,(0) + C_1 \Rightarrow \quad C_1 = 0$$

Integrating again

$$T = -\frac{\dot{q}_G}{2k}\,x^2 + C_2$$

The expression for T and its first derivative can be substituted into the second boundary condition to evaluate the constant C_2

$$-k\left(\frac{\dot{q}_G L}{k}\right) = \overline{h}_c\left(-\frac{\dot{q}_G L^2}{2k} + C_2 - T_\infty\right) \Rightarrow C_2 = \dot{q}_G L\left(\frac{1}{\overline{h}_c} + \frac{L}{2k}\right) + T_\infty$$

Substituting this into the expression for T yields the temperature distribution in the wall

$$T(x) = \frac{\dot{q}_G}{2k}\, x^2 + \dot{q}_G L \left(\frac{1}{\bar{h}_c} + \frac{L}{2k} \right) + T_\infty$$

$$T(x) = T_\infty + \frac{\dot{q}_G}{2k} \left(L^2 + \frac{2kL}{\bar{h}_c} - x^2 \right)$$

Examination of this expression reveals that the maximum temperature occurs at $x = 0$

$$T_{\max} = T_\infty + \frac{\dot{q}_G}{2k} \left(L^2 + \frac{2kL}{\bar{h}_c} \right)$$

$$T_{\max} = 90°\text{C} + \frac{10^5\,\text{W/m}^3}{2[12\,\text{W/(mK)}]} \left((0.075\,\text{m})^2 + \frac{2[12\,\text{W/(mK)}](0.075\,\text{m})}{500\,\text{W/(m}^2\text{K)}} \right) = 128°\text{C}$$

PROBLEM 2.21

> **A small dam, which may be idealized by a large slab 1.2 m thick, is to be completely poured in a short period of time. The hydration of the concrete results in the equivalent of a distributed source of constant strength of 100 W/m^3. If both dam surfaces are at 16°C, determine the maximum temperature to which the concrete will be subjected, assuming steady-state condition. The thermal conductivity of the wet concrete may be taken as 0.84 W/(m K).**

GIVEN

- Large slab with internal heat generation
- Internal heat generation rate ($\dot{q}_G$) = 100 W/m^3
- Both surface temperatures (T_s) = 16°C
- Thermal conductivity (k) = 0.84 W/(m K)

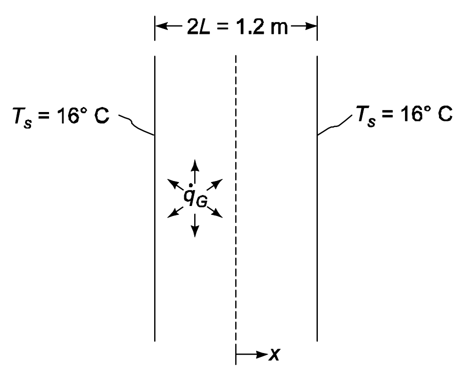
FIND

- The maximum temperature ($T_{\max}$)

ASSUMPTIONS

- Steady state conditions prevail

SKETCH



SOLUTION

The dam is symmetric, therefore x will be measured from the centerline of the dam. The equation for one dimensional conduction is given by Equation (2.5)

$$k\,\frac{\partial^2 T}{\partial x^2} + \dot{q}_G = \rho\, c\,\frac{\partial T}{\partial t}$$

For steady state, $\frac{\partial T}{\partial t} = 0$ therefore

$$k\,\frac{d^2 T}{d x^2} + \dot{q}_G = 0$$

This is subject to the following boundary conditions

1. By symmetry, $dT/dx = 0$ at $x = o$
2. $T = T_s$ at $x = L$

Also note that for this problem $\dot{q}_G$ is a constant.

Integrating the conduction equation

$$\frac{dT}{dx} = -\frac{\dot{q}_G}{k}\,x + C_1$$

The constant C_1 can be evaluated using the first boundary condition

$$0 = -\frac{\dot{q}_G}{k}\,(0) + C_1 \Rightarrow C_1 = 0$$

Integrating once again

$$T = \frac{\dot{q}_G}{2k}\,x^2 + C_2$$

The constant C_2 can be evaluated using the second boundary condition

$$T_s = \frac{\dot{q}_G}{2k}\,L^2 + C_2 \Rightarrow C_2 = T_s + \frac{\dot{q}_G}{2k}\,L^2$$

Therefore, the temperature distribution in the dam is

$$T = T_s + \frac{\dot{q}_G}{2k}\,(L^2 - x^2)$$

The maximum temperature occurs at $x = 0$

$$T_{\max} = T_s + \frac{\dot{q}_G}{2k}\,(L^2 - (0)^2) = 16°\mathrm{C} + \frac{100\,\mathrm{W/m^3}}{2[0.84\,\mathrm{W/(m\,K)}]}\,(0.6\ \mathrm{m})^2 = 37°\mathrm{C}$$

COMMENTS

This problem is simplified significantly by choosing $x = 0$ at the centerline and taking advantage of the problem's symmetry.

For a more complete analysis, the change in thermal conductivity with temperature and moisture content should be measured. The system could then be analyzed by numerical methods discussed in chapter 3.

PROBLEM 2.22

Two large steel plates at temperatures of 90° and 70°C are separated by a steel rod 0.3 m long and 2.5 cm in diameter. The rod is welded to each plate. The space between the plates is filled with insulation, which also insulates the circumference of the rod. Because of a voltage difference between the two plates, current flows through the rod, dissipating electrical energy at a rate of 12 W. Determine the maximum temperature in the rod and the heat flow rate at each end. Check your results by comparing the net heat flow rate at the two ends with the total rate of heat generation.

GIVEN

- Insulated steel rod with internal heat generation
- Length (L) = 0.3 m
- Diameter (D) = 2.5 cm = 0.025 m
- Internal heat generation rate ($\dot{q}_G V$) = 12 W
- End temperature of the rod: T_1 = 90°C T_2 = 70°C

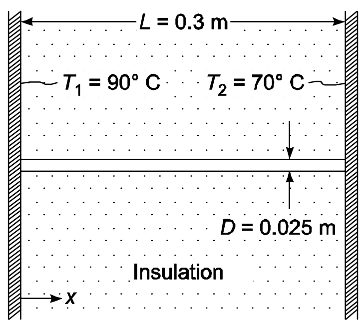
FIND

(a) Maximum temperature in the rod (T_{max})
(b) Heat flow rate at each end (q_0 and q_L)
(c) Check the results by comparing with the heat generation

ASSUMPTIONS

- The system has reached steady state
- The heat loss through the insulation is negligible
- The steel is 1% carbon steel
- Constant thermal conductivity
- The plate temperatures are constant
- Heat is generated uniformly throughout the rod

SKETCH



PROPERTIES AND CONSTANTS:

From Appendix 2, Table 10
Thermal conductivity of 1% carbon steel (k) = 43 W/(m K) at 20°C

SOLUTION

The heat generation per unit volume of the rod is

$$\dot{q}_G = \frac{\dot{q}_G V}{V} = \frac{\dot{q}_G V}{\frac{\pi}{4} D^2 L} = \frac{12\,\text{W}}{\frac{\pi}{4}(0.025\,\text{m})^2\,(0.3\text{ m})} = 81{,}487\ \text{W/m}^3$$

(a) The temperature distribution in the rod will be evaluated from the conduction equation, Equation (2.5), and the boundary conditions. The one dimensional conduction equation is

$$k\,\frac{\partial^2 T}{\partial x^2} + \dot{q}_G = \rho\, c\, \frac{\partial T}{\partial t}$$

For steady state, $\frac{\partial T}{\partial t}$ = 0 therefore

$$\frac{d^2T}{dx^2} = \frac{\dot{q}_G}{k} = 0$$

This is subject to the following boundary conditions

$$T = T_1 \text{ at } x = 0 \text{ and } T = T_2 \text{ at } x = L$$

Integrating the conduction equation yields

$$\frac{dT}{dx} = \frac{\dot{q}_G}{k}\, x + C_1$$

Integrating a second time

$$T = -\frac{\dot{q}_G}{2k}\, x^2 + C_1\, x + C_2$$

The constant C_2 can be evaluated using the first boundary condition

$$T_1 = \frac{\dot{q}_G}{2k}\, (0)^2 + C_1\,(0) + C_2 \Rightarrow C_2 = T_1$$

Therefore, the temperature distribution becomes

$$T = -\frac{\dot{q}_G}{2k}\, x^2 + C_1\, x + T_1$$

The second boundary condition can be used to evaluate the constant C_1

$$T_2 = -\frac{\dot{q}_G}{2k}\, L^2 + C_1\, L + T_1 \quad y \Rightarrow C_1 = \frac{1}{L}\,(T_2 - T_1) + \frac{\dot{q}_G L}{2k}$$

The temperature distribution in the rod is

$$T = -\frac{\dot{q}_G}{2k}\, x^2 + \left[\frac{1}{L}(T_2 - T_1) + \frac{\dot{q}_G L}{2k}\right] x + T_1$$

The maximum temperature in the rod occurs where the first derivative of the temperature distribution is zero

$$\frac{dT}{dx} = -\frac{\dot{q}_G}{k}\, x_m + \frac{1}{L}\,(T_2 - T_1) + \frac{\dot{q}_G L}{2k} = 0$$

$$x_m = \frac{k}{L\, q_G}\,(T_2 - T_1) + \frac{L}{2} = \frac{43\,\text{W/(m K)}}{0.3\,\text{m}\,(81{,}487\,\text{W/m}^3)}\,(70°\text{C} - 90°\text{C}) + \frac{0.3\,\text{m}}{2} = 0.1148\ \text{m}$$

Evaluating the temperature at this value of x

$$T_{\max} = \frac{\dot{q}_G}{2k}\, {x_m}^2 + \left[\frac{1}{L}(T_2 - T_1) + \frac{\dot{q}_G L}{2k}\right] x_m + T_1$$

$$T_{\max} = \frac{81{,}847\ \text{W/m}^3}{2\left(43\ \text{W/(mK)}\right)}\,(0.1148\ \text{m})^2 + \left[\frac{90°\text{C} - 70°\text{C}}{0.3\ \text{m}} + \frac{81{,}847\ \text{W/m}^3\ (0.3\,\text{m})}{2\left(43\ \text{W/(mK)}\right)}\right](0.1148\text{m}) + 90°$$

$$T_{\max} = 102°\text{C}$$

(b) The heat flow from the rod at $x = 0$ can be calculated from Equation (1.1)

$$q_0 = -kA\,\frac{dT}{dx}\Big|_{x=0} = -kA\left[\frac{\dot{q}_G}{k}x + \frac{1}{L}(T_2 - T_1) + \frac{\dot{q}_G L}{2k}\right]_{x=0}$$

$$q_0 = -k\left(\frac{\pi}{4}D^2\right)\left[\frac{1}{L}(T_2 - T_1) + \frac{\dot{q}_G L}{2k}\right]$$

$$q_0 = -43\ \text{W/(m K)}\left(\frac{\pi}{4}(0.025\,\text{m})^2\right)\left[\frac{1}{0.3\,\text{m}}(70°\text{C} - 90°\text{C}) + \frac{81{,}847\ \text{W/m}^3\,(0.3\,\text{m})}{2(43\ \text{W/(m K)})}\right] = -4.6\ \text{W}$$

(The negative sign indicates that heat is flowing to the left, out of the rod)

The heat flow from the rod at $x = L$ is

$$q_L = -kA\,\frac{dT}{dx}\Big|_{x=L} = -kA\left[-\frac{\dot{q}_G}{k}L + \frac{1}{L}(T_2 - T_1) + \frac{\dot{q}_G L}{2k}\right]$$

$$q_L = \frac{\pi}{4}D^2\left[\frac{\dot{q}_G}{2}L - \frac{k(T_2 - T_1)}{L}\right]$$

$$q_L = -\frac{\pi}{4}(0.025\text{m})^2\left[\frac{\left(81{,}847\ \text{W/m}^3\right)(0.3\,\text{m})}{2} - \frac{\left(43\,\text{W/(m K)}\right)(70°\text{C} - 90°\text{C})}{0.3\,\text{m}}\right] = 7.4\ \text{W}$$

(The positive value indicates that heat is flowing to the right, out of the rod)

(c) The total heat loss is the sum of the loss from each end

$$q_{\text{total}} = |\,q_0\,| + |\,q_L\,| = 4.6\text{W} + 7.4\text{W} = 12.0\text{W}$$

The total rate of heat loss is equal to the rate of heat generation within the rod.

PROBLEM 2.23

The shield of a nuclear reactor can be idealized by a large 10 in. thick flat plate having a thermal conductivity of 2 Btu/(h ft °F). Radiation from the interior of the reactor penetrates the shield and produces heat generation in the shield which decreases exponentially from a value of 10 Btu/(h in^3). at the inner surface to a value of 1.0 Btu/(h in^3) at a distance of 5 in. from the interior surface. If the exterior surface is kept at 100°F by forced convection, determine the temperature at the inner surface of the shield. Hint: First set up the differential equation for a system in which the heat generation rate varies according to $\dot{q}(x) = \dot{q}(0)e^{-Cx}$.

GIVEN

- Large flat plate with non-uniform internal heat generation
- Thickness (L) = 10 in.
- Thermal conductivity (k) = 2 Btu/(h ft °F)
- Exterior surface temperature (T_o) = 100°F
- Heat generation is exponential with values of
- 10 Btu/(h in^3) at the inner surface
- 1.0 Btu/(h in^3) at 5 in. from the inner surface

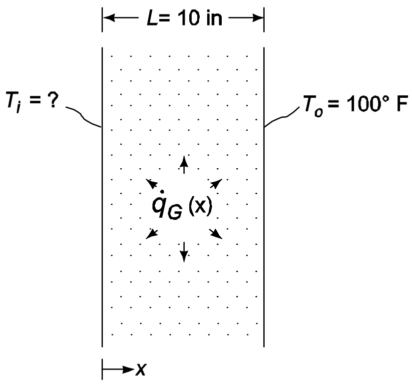
FIND

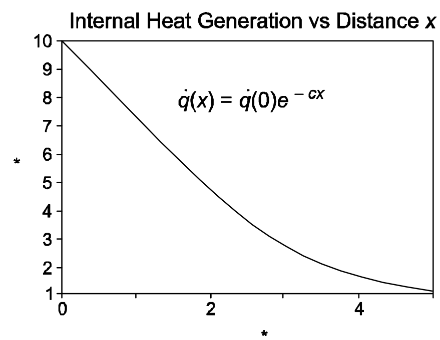
- The inner surface temperature (T_i)

ASSUMPTIONS

- One dimensional, steady state conduction
- The thermal conductivity is constant
- No heat transfer at the inner surface of the shield

SKETCH





SOLUTION

From the hint, the internal heat generation is

$$\dot{q}\,(x) \;=\; \dot{q}\,(0)\; e^{-cx} \text{ where } \dot{q}\,(0) = 10 \text{ Btu/(h in}^3\text{)}$$

Solving for the constant c using the fact that $q(x) = 1$ Btu/h in^3 at $x = 5$ in = 0.417 ft

$$c \;= -\frac{1}{x}\ln\left(\frac{\dot{q}(x)}{\dot{q}(0)}\right) = -\frac{1}{0.417\,\text{ft}}\ln\left(\frac{1\,\text{Btu/(h in}^3)}{10\,\text{Btu/(h in}^3)}\right) = 5.52\;\frac{1}{\text{ft}}$$

The one dimensional conduction equation is given by Equation (2.5)

$$k\;\frac{\partial^2 T}{\partial x^2} \;+\; \dot{q}_G \;= \rho\, c\;\frac{\partial T}{\partial t} \;= 0 \text{ (steady state)}$$

$$\frac{d^2T}{dx^2} \;= -\;\frac{\dot{q}_G(x)}{k} \;=\; \frac{\dot{q}\,(0)}{k}\; e^{-cx}$$

The boundary conditions are

$$\frac{dT}{dx} \;= 0 \text{ at } x = 0$$

$$T(L) \;= T_o = 100°\text{F at } x = L$$

Integrating the conduction equation

$$\frac{dT}{dx} \;= -\;\frac{\dot{q}\,(0)}{ck}\;\, e^{-cx} + C_1$$

The constant C_1 can be evaluated by applying the first boundary condition

$$0 \;= -\;\frac{\dot{q}\,(0)}{ck}\;\, e^{-c(0)} + C_1 \qquad \Rightarrow C_1 = \;\frac{-\,\dot{q}\,(0)}{ck}$$

Integrating again

$$T(x) = \frac{-\dot{q}(0)}{c^2 k}\, e^{-cx} - \frac{\dot{q}(0)}{c^2 k}\, x + C_2$$

The constant C_2 can be evaluated by applying the second boundary condition

$$T(L) = T_o = \frac{-\dot{q}(0)}{c^2 k}\, e^{-cL} - \frac{\dot{q}(0)}{c k}\, L + C_2 \quad \Rightarrow C_2 = T_o + \frac{\dot{q}(0)}{c k}\left(L + \frac{1}{c} e^{-cL}\right)$$

Therefore, the temperature distribution is

$$T(x) = T_o + \frac{-\dot{q}(0)}{c^2 k}\, [e^{-cL} - e^{-cx} + c(L - x)]$$

Solving for the temperature at the inside surface ($x = 0$)

$$T_i = T(0) = T_o + \frac{\dot{q}(0)}{c^2 k}\, [e^{-cL} - 1 + cL]$$

$$T_i = 100°\text{F} + \frac{17{,}280\ \text{Btu/(h ft}^3)}{\frac{5.52}{\text{ft}^2}(2\ \text{Btu/(h ft °F)})}\left[e^{-(5.52/\text{ft})\frac{10}{12}\text{ft}} - 1 + 5.52\frac{1}{\text{ft}}\left(\frac{10}{12}\ \text{ft}\right)\right] = 1124°\text{F}$$

PROBLEM 2.24

> **Derive an expression for the temperature distribution in an infinitely long rod of uniform cross section within which there is uniform heat generation at the rate of 1 W/m. Assume that the rod is attached to a surface at T_s and is exposed through a convective heat transfer coefficient h to a fluid at T_f.**

GIVEN

- An infinitely long rod with internal heat generation
- Temperature at one end = T_s
- Heat generation rate ($\dot{q}_G A$) = 1 W/m
- Convective heat transfer coefficient = h_c
- Ambient fluid temperature = T_f

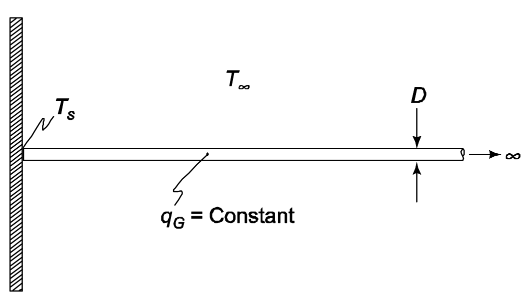
FIND

- Expression for the temperature distribution

ASSUMPTIONS

- The rod is in steady state
- The thermal conductivity (k) is constant

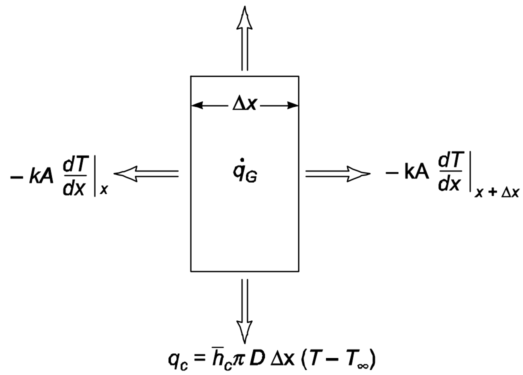
SKETCH



SOLUTION

Let A = the cross sectional area of the rod = $\pi D^2/4$

An element of the rod with heat flows is shown at the right



Conservation of energy requires that

Energy entering the element + Heat generation = Energy leaving the element

$$-kA \left.\frac{dT}{dx}\right|_x + \dot{q}_G A \Delta x = -kA \left.\frac{dT}{dx}\right|_{x+\Delta x} + \overline{h_c}\, \pi D\, \Delta x\, [T(x) - T_f]$$

$$kA \left(\left.\frac{dT}{dx}\right|_{x+\Delta x} - \left.\frac{dT}{dx}\right|_x \right) = \overline{h_c}\, \pi D\, \Delta x\, (T - T_f) - \dot{q}_G\, A\, \Delta x$$

Dividing by Δx and letting $\Delta x \to 0$ yields

$$kA \frac{d^2T}{dx^2} = \overline{h_c}\, \pi D\, (T - T_f) - \dot{q}_G\, A$$

$$\frac{d^2T}{dx^2} = \frac{4\overline{h_c}}{D\,k} (T - T_f) - \frac{\dot{q}_G}{k}$$

Let $\qquad \theta = T - T_f$ and $m^2 = \dfrac{4\, h_c}{(D\, k)}$

$$\frac{d^2\theta}{dx^2} - m^2\, \theta = \frac{-\dot{q}_G}{k}$$

This is a second order, linear, nonhomogeneous differential equation with constant coefficients. Its solution is the addition of the homogeneous solution and a particular solution. The solution to the homogeneous equation

$$\frac{d^2\theta}{dx^2} - m^2\, \theta = 0$$

is determined by its characteristic equation. Substituting $\theta = e^{\lambda x}$ and its derivatives into the homogeneous equation yields the characteristic equation

$$\lambda^2 e^{\lambda x} - m^2 e^{\lambda x} = 0 \Rightarrow \lambda = \pm m$$

Therefore, the homogeneous solution has the form

$$\theta_h = C_1\, c^{mx} + C_2\, e^{-mx}$$

A particular solution for this problem is simply a constant

$$\theta = a_o$$

Substituting this into the differential equation

$$0 - m^2\, a_o = \frac{-\dot{q}_G}{k} \Rightarrow a_o = \frac{\dot{q}_G}{m^2 k}$$

Therefore, the general solution is

$$q = C_1\, e^{mx} + C_2\, e^{-mx} + \frac{\dot{q}_G}{m^2 k}$$

With the boundary conditions

$$\theta = a \text{ finite number as } x \to \infty$$

$$\theta = T_s - T_f \text{ at } x = 0$$

From the first boundary condition, as $x \to \infty$ $e^{mx} \to \infty$, therefore $C_1 = 0$
From the second boundary condition

$$T_s - T_f = C_2 + \frac{\dot{q}_G}{m^2 k} \Rightarrow C_2 = T_s - T_f - \frac{\dot{q}_G}{m^2 k}$$

The temperature distribution in the rod is

$$q = T(x) - T_f = \left(T_s - T_f - \frac{\dot{q}_G}{m^2 k}\right) e^{-mx} + \frac{\dot{q}_G}{m^2 k}$$

$$T(x) = T_f + \left(T_s - T_f - \frac{\dot{q}_G}{m^2 k}\right) e^{-mx} + \frac{\dot{q}_G}{m^2 k}$$

PROBLEM 2.25

Derive an expression for the temperature distribution in a plane wall in which there are uniformly distributed heat sources which vary according to the linear relation

$$\dot{q}_G = \dot{q}_w\, [1 - \beta(T - T_w)]$$

where q_w is a constant equal to the heat generation per unit volume at the wall temperature T_w. Both sides of the plate are maintained at T_w and the plate thickness is 2L.

GIVEN

- A plane wall with uniformly distributed heat sources as in the above equation
- Both surface temperatures = T_w
- Thickness = 2L

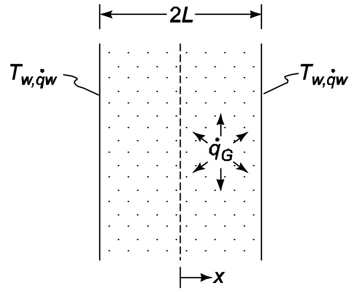
FIND

- An expression for the temperature distribution

ASSUMPTIONS

- Constant thermal conductivity (k)

SKETCH



SOLUTION

The equation for one dimensional, steady state (dT/dt = 0) conduction from Equation (2.5) is

$$\frac{d^2T}{dx^2} = \frac{-\dot{q}_G}{k} = \frac{-\dot{q}_w}{k}\,[1 - \beta\,(T - T_w)] = \frac{\dot{q}_w \text{b}}{k}\,(T - T_w) - \frac{\dot{q}_w}{k}$$

With the boundary conditions

$$\frac{dT}{dx} = 0 \text{ at } x = 0$$

$$T = T_w \text{ at } x = L$$

Let $\theta = T - T_w$ and $m^2 = (\dot{q}_w\,\beta)/k$ then

$$\frac{d^2\text{q}}{dx^2} - m^2\,\theta = \frac{-\dot{q}_w}{k}$$

This is a second order, linear, nonhomogeneous differential equation with constant coefficients. Its solution is the addition of the homogeneous solution and a particular solution. The solution to the homogeneous equation

$$\frac{d^2\text{q}}{dx^2} - m^2\,\theta = 0$$

is determined by its characteristics equation. Substituting $\theta = e^{\lambda x}$ and its derivatives into the homogeneous equation yields the characteristics equation

$$\lambda^2\,e^{\lambda x} - m^2\,e^{\lambda x} = 0 \Rightarrow \lambda = m$$

Therefore, the homogeneous solution has the form

$$\theta_h = C_1\,c^{mx} + C_2\,e^{-mx}$$

A particular solution for this problem is simply a constant: $\theta = a_o$

Substituting this into the differential equation

$$0 - m^2\,a_o = \frac{-\dot{q}_w}{k} \Rightarrow a_o = \frac{\dot{q}_w}{m^2 k}$$

Therefore, the general solution is

$$\theta = C_1\,e^{mx} + C_2\,e^{-mx} + \frac{\dot{q}_w}{m^2 k}$$

With the boundary condition

$$\frac{d\theta}{dx} = 0 \text{ at } x = 0$$

$$\theta = 0 \text{ at } x = L$$

Applying the first boundary condition:

$$\frac{d\theta}{dx} = C_1\, m e^{(0)} - C_2\, m e^{(0)} = 0 \Rightarrow C_1 = C_2 = C$$

From the second boundary condition

$$0 = C\,(e^{mL} + e^{-mL}) + \frac{\dot{q}_w}{m^2 k} \Rightarrow C = \frac{-\dot{q}_w}{m^2 k\,(e^{mL} + e^{-mL})}$$

The temperature distribution in the wall is

$$\theta = T(x) - T_w = \frac{-\dot{q}_w}{m^2 k\,(e^{mL} + e^{-mL})}\,(e^{mx} + e^{-mx}) + \frac{\dot{q}_w}{m^2 k}$$

$$T(x) = T_w + \frac{\dot{q}_w}{m^2 k}\left(1 - \frac{e^{mx} + e^{-mx}}{e^{mL} + e^{-mL}}\right)$$

$$T(x) = T_w + \frac{\dot{q}_w}{m^2 k}\left(1 - \frac{\cosh(mx)}{\cosh(mL)}\right)$$

PROBLEM 2.26

A plane wall of thickness 2L has internal heat sources whose strength varies according to

$$\dot{q}_G = \dot{q}_0 \cos(ax)$$

where $\dot{q}_0$ is the heat generated per unit volume at the center of the wall (x = 0) and a is a constant. If both sides of the wall are maintained at a constant temperature of T_w, derive an expression for the total heat loss from the wall per unit surface area.

GIVEN

- A plane wall with internal heat sources
- Heat source strength: $\dot{q}_G = \dot{q}_0 \cos(ax)$
- Wall surface temperatures = T_w
- Wall thickness = 2L

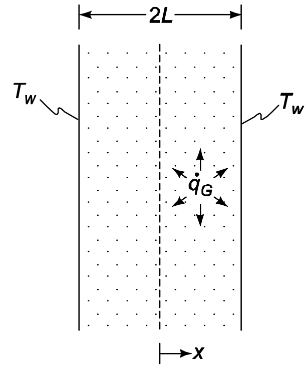
FIND

An expression for the total heat loss per unit area (q/A)

ASSUMPTIONS

- Steady state conditions prevail
- The thermal conductivity of the wall (k) is constant
- One dimensional conduction within the wall

SKETCH



SOLUTION

Equation (2.5) gives the equation for one dimensional conduction. For steady state, dT/dt = 0, therefore

$$k\,\frac{\partial^2 T}{\partial x^2} + \dot{q}_G = \rho c\,\frac{\partial T}{\partial t} = 0$$

$$\frac{d^2 T}{d x^2} = \frac{-\dot{q}_G}{k} = \frac{-\dot{q}_0\cos(ax)}{k}$$

With boundary conditions:

$$\frac{dT}{dx} = 0 \text{ at } x = 0 \text{ (by symmetry)}$$

$$T = T_w \text{ at } x = L \text{ (given)}$$

Integrating the conduction equation once

$$\frac{dT}{dx} = \frac{\dot{q}_o}{a\,k}\sin(ax) + C_1$$

Applying the first boundary condition yields: $C_1 = 0$

The rate of heat transfer from one side of the wall is

$$q_k = -kA\,\frac{dT}{dx}\bigg|_{x=L} = -kA\left[-\frac{\dot{q}_G}{a\,k}\sin(aL)\right] = \frac{\dot{q}_o A}{a}\sin(aL)$$

The total rate of heat transfer is twice the rate of heat transfer from one side of the wall

$$\left(\frac{q_k}{A}\right)_{\text{total}} = \frac{2\dot{q}_o}{a}\sin(aL)$$

An alternative method of solution for this problem involves recognizing that at steady state the rate of heat generation within the entire wall must equal the rate of heat transfer from the wall surfaces

$$A\int_{-L}^{L}\dot{q}_G(x)\,dx = q$$

$$\dot{q}_o\int_{-L}^{L}\cos(ax)\,dx = \frac{q}{A}$$

$$\frac{\dot{q}_o}{a}\left[\sin(aL) - \sin(-aL)\right] = \frac{q}{A}$$

$$\frac{q}{A} = \frac{2\dot{q}_o}{a}\sin(aL)$$

COMMENTS

The heat loss can be determined by solving for the temperature distribution and then the rate of heat transfer or via the conservation of energy which allows us to equate the heat generation rate with the rate of heat loss.

PROBLEM 2.27

Heat is generated uniformly in the fuel rod of a nuclear reactor. The rod has a long, hollow cylindrical shape with its inner and outer surfaces at temperatures of T_i and T_o, respectively. Derive an expression for the temperature distribution.

GIVEN

- A long, hollow cylinder with uniform internal generation
- Inner surface temperature = T_i
- Outer surface temperature = T_o

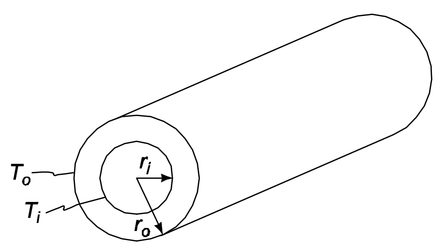
FIND

- The temperature distribution

ASSUMPTIONS

- Conduction occurs only in the radial direction
- Steady state prevails

SKETCH



SOLUTION

Let r_i = the inner radius

r_o = the outer radius

$\dot{q}_G$ = the rate of internal heat generation per unit volume

k = the thermal conductivity of the fuel rod

The one dimensional, steady state conduction equation in cylindrical coordinates is given in Equation (2.21)

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{q}_G}{k} = 0$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = \frac{-r\dot{q}_G}{k}$$

With boundary conditions

$$T = T_i \text{ at } r = r_i$$

$$T = T_o \text{ at } r = r_o$$

Integrating the conduction equation once

$$r\frac{dT}{dr} = \frac{-r^2\dot{q}_G}{2k} + C_1$$

$$dT = \left(\frac{-r^2\dot{q}_G}{2k} + \frac{C_1}{r}\right)dr$$

Integrating again

$$T = \frac{-r^2\dot{q}_G}{4k} + C_1 \ln(r) + C_2$$

Applying the first boundary condition

$$T_i = \frac{-r_i^2\dot{q}_G}{4k} + C_1 \ln(r_i) + C_2$$

$$C_2 = T_i + \frac{r_i^2\dot{q}_G}{4k} - C_i \ln(r_i)$$

Applying the second boundary condition

$$T_o = \frac{-r_o^2\dot{q}_G}{4k} + C_1 \ln(r_o) + C_2$$

$$T_o = \frac{-r_o^2\dot{q}_G}{4k} + C_1 \ln(r_o) + T_i + \frac{r_i^2\dot{q}_G}{4k} - C_1 \ln(r_i)$$

$$C_1 = \frac{T_o - T_i + \frac{\dot{q}_G}{4k}(r_o^2 - r_i^2)}{\ln\left(\frac{r_o}{r_i}\right)}$$

Substituting the constants into the temperature distribution

$$T = \frac{-r^2\dot{q}_G}{4k} + \left(\frac{T_o - T_i + \frac{\dot{q}_G}{4k}(r_o^2 - r_i^2)}{\ln\left(\frac{r_o}{r_i}\right)}\right)\ln(r) + T_i + \frac{r_i^2\dot{q}_G}{4k} - \left(\frac{T_o - T_i + \frac{\dot{q}_G}{4k}(r_o^2 - r_i^2)}{\ln\left(\frac{r_o}{r_i}\right)}\right)$$

$$T = \frac{\dot{q}_G}{4k}\left(\frac{(r_o^2 - r_i^2)\ln\left(\frac{r}{r_i}\right)}{\ln\left(\frac{r_o}{r_i}\right)} + (r_i^2 - r^2)\right) + \frac{(T_o - T_i)\ln\left(\frac{r}{r_i}\right)}{\ln\left(\frac{r_o}{r_i}\right)} + T_i$$

PROBLEM 2.28

Show that the temperature distribution in a sphere of radius r_o, made of a homogeneous material in which energy is released at a uniform rate per unit volume $\dot{q}_G$, is

$$T(r) = T_o + \frac{\dot{q}_G\, r_o^2}{6\,k}\left[1 - \left(\frac{r}{r_o}\right)^2\right]$$

GIVEN

- A homogeneous sphere with energy generation
- Radius = r_o

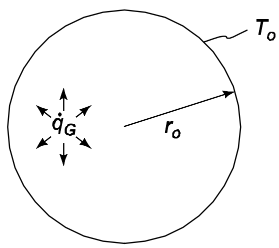
FIND

- Show that the temperature distribution is as shown above.

ASSUMPTIONS

- Steady state conditions persist
- The thermal conductivity of the sphere material is constant
- Conduction occurs in the radial direction only

SKETCH



SOLUTION

Let k = the thermal conductivity of the material

T_o= the surface temperature of the sphere

Equation (2.23) can be simplified to the following equation by the assumptions of steady state and radial conduction only

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \frac{\dot{q}_G}{k} = 0$$

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = \frac{-\,r^2\,\dot{q}_G}{k}$$

With the following boundary conditions

$$\frac{dT}{dr} = 0 \text{ at } r = 0$$

$$T = T_o \text{ at } r = r_o$$

Integrating the differential equation once

$$r^2 \frac{dT}{dr} = \frac{-r^3 \dot{q}_G}{3k} + C_1$$

From the first boundary condition

$$C_1 = 0$$

Integrating once again

$$T = \frac{-r^2 \dot{q}_G}{6k} + C_2$$

Applying the second boundary condition

$$T_o = \frac{-r_o^2 \dot{q}_G}{6k} + C_2 \Rightarrow C_2 = T_o + \frac{-r_o^2 \dot{q}_G}{6k}$$

Therefore, the temperature distribution in the sphere is

$$T = \frac{-r^2 \dot{q}_G}{6k} + T_o + \frac{-r_o^2 \dot{q}_G}{6k}$$

$$T(r) = T_o + \frac{\dot{q}_G r_o^2}{6k} \left[1 - \left(\frac{r}{r_o} \right)^2 \right]$$

PROBLEM 2.29

In a cylindrical fuel rod of a nuclear reactor, heat is generated internally according to the equation

$$\dot{q}_G = \dot{q}_1 \left[1 - \left(\frac{r}{r_o} \right)^2 \right]$$

where $\dot{q}_g$ **= local rate of heat generation per unit volume at** r

r_o **= outside radius**

$\dot{q}_1$ **= rate of heat generation per unit volume at the centerline**

Calculate the temperature drop from the center line to the surface for a 1 in. OD rod having a thermal conductivity of 15 Btu/(h ft °F) if the rate of heat removal from its surface is 500,000 Btu/(h ft^2).

GIVEN

- A cylindrical rod with internal generation and heat removal from its surface
- Outside diameter (D_o) = 1 in
- Rate of heat generation is as given above
- Thermal conductivity (k) = 15 Btu/(h ft °F)
- Heat removal rate (q/A) = 500,000 Btu/(h ft^2)

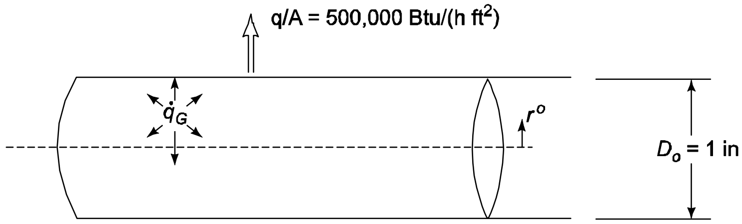
FIND

- The temperature drop from the center line to the surface (ΔT)

ASSUMPTIONS

- The heat flow has reached steady state
- The thermal conductivity of the fuel rod is constant
- One dimensional conduction in the radial direction

SKETCH



SOLUTION

The equation for one dimensional conduction in cylindrical coordinates is given in Equation (2.21)

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)\frac{\dot{q}_G}{k} = 0$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = \frac{-r}{k}\ \dot{q}_1\ \left[1-\left(\frac{r}{r_o}\right)^2\right]$$

With the boundary conditions

$$\frac{dT}{dr} = 0 \text{ at } r = 0$$

$$T = T_s \text{ at } r = r_o$$

Integrating once

$$r\ \frac{dT}{dr} = \frac{-r^2 q_1}{2k} + \frac{r^4 q_1}{4k\, r_o^2} + C_1$$

From the first boundary condition: $C_1 = 0$, therefore

$$\frac{dT}{dr} = \frac{q_1}{2k}\left(\frac{r^3}{2r_o^2} - r\right)$$

Integrating again

$$T = \frac{q_1}{2k}\left(\frac{r^4}{8r_o^2} - \frac{r^2}{2}\right) + C_2$$

Evaluate this expression at the surface of the cylinder and at the centerline of the sphere and subtracting the results gives us the temperature drop in the cylinder

$$\Delta T = T_0 - T_{r_o} = \frac{q_1}{2k}\left(\frac{(0)^4}{8r_o^2} - \frac{(0)^2}{2} - \frac{r_o^4}{8r_o^2} + \frac{r_o^2}{2}\right) = \frac{3q_1 r_o^2}{16k}$$

The rate of heat generation at the centerline (q_1) can be evaluated using the conservation of energy. The total rate of heat transfer from the cylinder must equal the total rate of heat generation within the cylinder

$$\left(\frac{q}{A}\right) A = L \int_{r=0}^{r=r_o} q_1\ \left[1-\frac{r^4}{r_o^2}\right]\ 2\pi r\, dr$$

$$\left(\frac{q}{A}\right)\ 2\pi r_o L = 2\pi L\ q_1\ \left[\frac{r^2}{2} - \frac{r^4}{4r_o^2}\right]_0^{r_o}$$

$$\left(\frac{q}{A}\right)\ r_o = q_1\ \left[\frac{r_o^2}{2} - \frac{r_o^2}{4}\right] = q_1\ \frac{r_o^2}{4}$$

$$\therefore\ q_1 = \frac{4}{r_o}\left(\frac{q}{A}\right) = \frac{4}{\frac{0.05}{12}\,\text{ft}}\ [500{,}000\ \text{Btu/(h ft}^2)] = 4.8 \times 10^7\ \text{Btu/(h ft}^2)$$

Therefore, the temperature drop within the cylinder is

$$\Delta T = \frac{3[4.8 \times 10^7\ \text{Btu/(h ft}^3)]\left(\frac{0.5}{12}\,\text{ft}\right)^2}{16[15\ \text{Btu/(h ft °F)}]} = 1042\text{°F}$$

PROBLEM 2.30

An electrical heater capable of generating 10,000 W is to be designed. The heating element is to be a stainless steel wire, having an electrical resistivity of 80 × 10^{-6} ohm-centimeter. The operating temperature of the stainless steel is to be no more than 1260°C. The heat transfer coefficient at the outer surface is expected to be no less than 1720 W/(m^2 K) in a medium whose maximum temperature is 93°C. A transformer capable of delivering current at 9 and 12 V is available. Determine a suitable size for the wire, the current required, and discuss what effect a reduction in the heat transfer coefficient would have. Hint: Demonstrate first that the temperature drop between the center and the surface of the wire is independent of the wire diameter, and determine its value.

GIVEN

- A stainless steel wire with electrical heat generation
- Heat generation rate ($\dot{Q}_G$) = 10,000 W
- Electrical resistivity (ρ) = 80 × 10^{-6} ohms-cm
- Maximum temperature of stainless steel (T_{max}) = 1260°C
- Heat transfer coefficient ($\bar{h}_c$) = 1700 W/(m^2 K)
- Maximum temperature of medium (T_∞) = 93°C
- Voltage (V) = 9 or 12 V

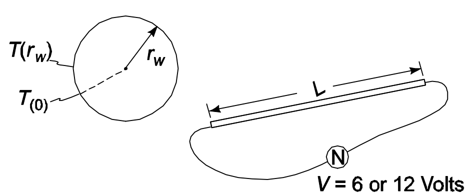
FIND

(a) A suitable wire size: diameter (d_w) and length (L)
(b) The current required (I)
(c) Discuss the effect of reduction in the heat transfer coefficient

ASSUMPTIONS

- Variation in the thermal conductivity of stainless steel is negligible
- The system is in steady-state
- Conduction occurs in the radial direction only

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10, Thermal conductivity of stainless steel (k) = 14.4 W/(m^2 K)

SOLUTION

The rate of heat generation per unit volume is

$$\dot{q}_G = \frac{\dot{Q}_G}{\text{volume}} = \frac{\dot{Q}_G}{\pi r_w^2 L}$$

The temperature distribution in a long cylinder with internal heat generation is given in Section 2.3.3

$$T(r) = C_2 - \frac{\dot{q}_G r^2}{4k}$$

where C_2 is a constant determined by boundary conditions. Therefore

$$T(0) - T(r_w) - = [C_2 - 0] - \left[C_2 - \frac{\dot{q}_G r_w^2}{4k}\right] = \frac{\dot{q}_G r_w^2}{4k} = \frac{\dot{Q}_G}{4\pi kL}$$

The convective heat transfer from the outer surface must equal the internal heat generation

$$q_c = \bar{h}_c A\,[T(r_w) - T_\infty] = \dot{Q}_G$$

$$\therefore\ T(r_w) - T_\infty = \frac{\dot{Q}_G}{2\pi r_w L \bar{h}_c}$$

Adding the two temperature differences calculated above yields

$$[T(0) - T(r_w)] + [T(r_w) - T_\infty] = \frac{\dot{Q}_G}{4\pi kL} + \frac{\dot{Q}_G}{2\pi r_w L \bar{h}_c}$$

$$T(0) - T_\infty = \frac{\dot{Q}_G}{2\pi}\left(\frac{1}{2kL} + \frac{1}{r_w L \bar{h}_c}\right)$$

The wire length and its radius are related through an expression for the electric power dissipation

$$\dot{Q}_G = P_e = \frac{V^2}{R_e} = \frac{V^2}{\frac{\rho L}{A}} = \frac{V^2 \pi r_w^2}{\rho L} \Rightarrow L = \frac{\pi V^2 \pi r_w^2}{\rho \dot{Q}_G}$$

$$\therefore\ T(0) - T_\infty = \frac{\dot{Q}_G^2 \rho}{2\pi^2 V^2}\left(\frac{1}{2k r_w^2} + \frac{1}{r_w^3 \bar{h}_c}\right)$$

$$r_w^2\,[T(0) - T_\infty] - \frac{\dot{Q}_G^2 \rho}{2\pi^2 V^2}\left(\frac{r_w}{2k} + \frac{1}{\bar{h}_c}\right) = 0$$

For the 12 volt case

$$r_w^3\,(1260°\text{C} - 90°\text{C}) - \frac{(10{,}000\,\text{W})^2 (80 \times 10^{-6}\ \text{ohm-cm})}{2\pi^2 (12V^2)(100\,\text{cm/m})}\left(\frac{r_w}{2(14.4\ \text{W/(mK)})} + \frac{1}{1700\ (\text{W/(m}^2\text{K))}}\right) = 0$$

After checking the units, they are dropped for clarity

$$1167\, r_w^3 - 0.0281(0.0347\, r^2 + 0.000581) = 0$$

Solving by trial and error

$$r_w = 0.0025\ \text{m} = 2.5\ \text{mm}$$

For the 12 volt case, the suitable wire diameter is

$$d_w = 2(r_w) = 5\ \text{mm}$$

The length of the wire required is

$$L = \frac{\pi\,(12\,\text{V})^2 (0.0025\,\text{m})^2 - (100\ \text{cm/m})}{80 \times 10^{-6}\ \text{ohm-cm}\,(10{,}000\,\text{W})} = 0.353\ \text{m}$$

The electrical resistance of this wire is

$$R_e = \frac{\rho L}{\pi r_w^2} = \frac{80 \times 10^{-6}\ \text{ohm-cm}\,(0.353\,\text{m})}{\pi\,(0.0025\,\text{m})^2 \times (100\ \text{cm/m})} = 0.0144\ \text{ohm}$$

Therefore, the current required for the 12 volt case is

$$I = \frac{V}{R_e} = \frac{12\,\text{V}}{0.0144\,\text{ohm}} = 833\ \text{amps}$$

This same procedure can be used for the 9 volt case yielding

$$d_w = 6.3\ \text{mm}$$
$$L = 0.306\ \text{m}$$
$$R_e = 0.0081\ \text{ohm}$$
$$I = 1111\ \text{amps}$$

COMMENTS

The 5 mm diameter wire would be a better choice since the amperage is less. However 833 amps is still extremely high.

The effect of a lower heat transfer coefficient would be an increase in the diameter and length of the wire as well as an increase in the surface temperature of the wire.

PROBLEM 2.31

> **The addition of aluminum fins has been suggested to increase the rate of heat dissipation from one side of an electronic device 1 m wide and 1 m tall. The fins are to be rectangular in cross section, 2.5 cm long and 0.25 cm thick. There are to be 100 fins per meter. The convective heat transfer coefficient, both for the wall and the fins, is estimated at 35 W/(m^2 K). With this information, determine the percent increase in the rate of heat transfer of the finned wall compared to the bare wall.**

GIVEN

- Aluminum fins with a rectangular cross section
- Dimensions: 2.5 cm long and 0.25 mm thick
- Number of fins per meter = 100
- The convective heat transfer coefficient ($\overline{h_c}$) = 35 W/(m^2 K)

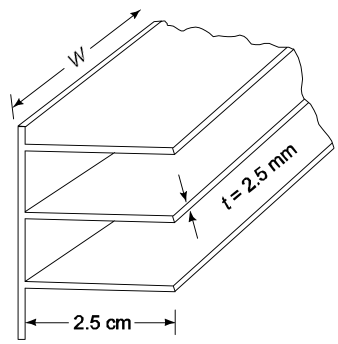
FIND

- The percent increase in the rate of heat transfer of the finned wall compared to the bare wall

ASSUMPTIONS

- Steady state heat transfer

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12

The thermal conductivity of aluminum (k) = 240 W/(m K) at 127°C

SOLUTION

Since the fins are of uniform cross section, Table 2.1 can be used to calculate the heat transfer rate from a single fin with convection at the tip

$$q_f = M \frac{\sinh(mL) + \dfrac{\bar{h}_c}{(mk)}\cosh(mL)}{\cosh(mL) + \left(\dfrac{\bar{h}_c}{mk}\right)\sinh(mL)}$$

where
$$M = \sqrt{\bar{h}_c P\, kA}\ \theta_s = \sqrt{\bar{h}_c\ 2(t+w)\,k\,(tw)}\ \theta_s$$

$$\theta_s = T_s - T_\infty$$

For a 1 m width (w = 1 m)

$$M = \sqrt{\left(35\,\text{W/(m}^2\text{K)}\right)2\,(1.0025\,\text{m})\,(240\text{W/(m K)})(0.025\ \text{m}^2)}\ \ \theta_s = 6.49\ \theta_s\ \text{W/K}$$

$$m\,L = \sqrt{\frac{\bar{h}_c P}{kA}} = L\sqrt{\frac{\bar{h}_c 2(t+w)}{k\,(tw)}} = 0.025\ \text{m}\sqrt{\frac{\left(35\ \text{W/(m}^2\ \text{K)}\right)2\,(1.0025\text{m})}{(240\text{W/(m K)})(0.0025\,\text{m}^2)}}$$

$$L\ m = 0.025\ \text{m}\left(10.81\frac{1}{\text{m}}\right) = 0.270$$

$$\frac{\bar{h}_c}{\text{m K}} = \frac{35\,\text{W/(m}^2\text{K)}}{\left(10.81\,\dfrac{1}{\text{m}}\right)(240\ \text{W/(m K)})} = 0.0135$$

Therefore, the rate of heat transfer from one fin, 1 meter wide is:

$$q_f = 6.49\ \theta_s\ \text{W/K}\,\frac{\sin\text{h}\,(0.27) + 0.0135\,\cos\text{h}\ (0.27)}{\cos\text{h}\,(0.27) + +0.0135\sin\text{h}\,(0.27)}$$

$$q_f = 1.792\ \theta_s\ \text{W/K}$$

In 1 m^2 of wall area there are 100 fins covering 100 tw = 100 (0.0025 m) (1 m) = 0.25 m^2 of wall area leaving 0.75 m^2 of bare wall. The total rate of heat transfer from the wall with fins is the sum of the heat transfer from the bare wall and the heat transfer from 100 fins.

$$q_{\text{tot}} = q_{\text{bare}} + 100\ q_{\text{fin}} = \bar{h}\,A_{\text{bare}}\ \theta_s + 100\ q_{\text{fin}}$$

$$q_{\text{tot}} = \left(35\ \text{W/(m}^2\text{K)}\right)\ (0.75\ \text{m}^2)\ \theta_s + 100\ (1.792)\ \theta_s\ \text{W/K} = 205.3\ \theta_s\ \text{W/K}$$

The rate of heat transfer from the wall without fins is

$$q_{\text{bare}} = \bar{h}_c\,A\ \theta_s = \left(35\ \text{W/(m}^2\text{K)}\right)\ (1\ \text{m}^2)\ \theta_s = 35.0\ \text{W/K}$$

The percent increase due to the addition of fins is

$$\%\ \text{increase} = \frac{205.3 - 35}{35} \times 100 = 486\%$$

COMMENTS

This problem illustrates the dramatic increase in the rate of heat transfer that can be achieved with properly designed fins.

The assumption that the convective heat transfer coefficient is the same for the fins and the wall is an oversimplification of the real situation, but does not affect the final results appreciably. In later chapters, we will learn how to evaluate the heat transfer coefficient from physical parameters and the geometry of the system.

PROBLEM 2.32

> **The tip of a soldering iron consists of a 0.6-cm-OD copper rod, 7.6 cm long. If the tip must be 204°C, what is the required minimum temperature of the base and the heat flow, in Btu's per hour and in watts, into the base? Assume that $\bar{h}$ = 22.7 W/(m^2 K) and T_{air} = 21°C.**

GIVEN

- Tip of soldering iron consists of copper rod
- Outside diameter (D) = 0.6 cm = 0.006 m
- Length (L) = 7.6 cm = 0.076 m
- Temperature of the tip (T_L) = 204°C
- Heat transfer coefficient ($\bar{h}$) = 22.7 W/(m^2 K)
- Ambient temperature (T_∞) = 21°C

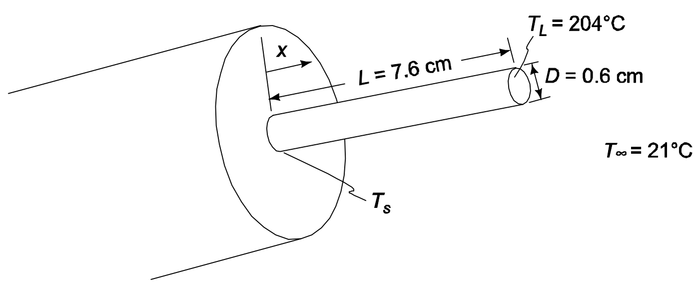
FIND

(a) Minimum temperature of the base (T_s)
(b) Heat flow into the base (q) in Btu/h and W

ASSUMPTIONS

- The tip is in steady state
- The thermal conductivity of copper is uniform and constant, i.e., not a function of temperature
- The copper tip can be treated as a fin

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12
The thermal conductivity of copper (K) = 388 W/(m K) at 227°C

SOLUTION

(a) From Table 2.1, the temperature distribution for a fin with a uniform cross section and convection from the tip is

$$\frac{\theta}{\theta_s} = \frac{\cosh[m(L-x)] + \left(\frac{\bar{h}}{mk}\right)\sinh[m(L-x)]}{\cosh(mL) + \left(\frac{\bar{h}}{mk}\right)\sinh(mL)}$$

where $\theta = T - T_\infty$ and $\theta_s = \theta(0) = T_s - T_\infty$

$$L\,m = L\sqrt{\frac{\bar{h}\,P}{kA}} = L\sqrt{\frac{\bar{h}\,\pi\,D}{k\,\frac{\pi}{4}D^2}} = \sqrt{\frac{4\,\bar{h}}{kD}} = 0.076\text{ m}\sqrt{\frac{4\left(22.7\text{W/(m}^2\text{K)}\right)}{\left(388\text{W/(m K)}\right)(0.006\,\text{m})}}$$

$$L\,m = 0.076\text{ m}\left(6.25\frac{1}{\text{m}}\right) = 0.475$$

$$\frac{\bar{h}}{mK} = \frac{22.7\text{ W/(m}^2\text{K)}}{\left(6.25\frac{1}{\text{m}}\right)\left(388\text{W/(mK)}\right)} = 0.00936$$

Evaluating the temperature at $x = L$

$$\frac{\theta_L}{\theta_s} = \frac{T_L - T_\infty}{T_s - T_\infty} = \frac{\cosh(0) + 0.00936\sinh(0)}{\cosh(0.475) + 0.00936\sinh(0.475)} = 0.8932$$

Solving for the base temperature

$$T_s = T_\infty + \frac{T_L - T_\infty}{0.8932} = 21°\text{C} + \frac{204^\circ\text{C} - 21^\circ\text{C}}{0.8932} = 226°\text{C}$$

(b) To maintain steady state conditions, the rate of heat transfer into the base must be equal to the rate of heat loss from the rod. From Table 2.1, the rate of heat loss is

$$q_f = M\,\frac{\sinh(mL) + \left(\frac{\bar{h}}{mk}\right)\cosh(mL)}{\cosh(mL) + \left(\frac{\bar{h}}{mk}\right)\sinh(mL)} \quad \text{where } M = \sqrt{\bar{h}PkA}\,\theta_s = \sqrt{\bar{h}\,k\,\frac{\pi^2}{4}D^3}\;(T_s - T_\infty)$$

$$M = \sqrt{\left(22.7\text{W/(m}^2\text{K)}\right)\left(388\text{W/(m K)}\right)\frac{\pi^2}{4}(0.006\,\text{m})^3}\;(226°\text{C} - 21°\text{C}) = 14.045\text{ W}$$

$$q_f = 14.045\text{ W}\;\frac{\sinh(0.475) + .00936\cosh(0.475)}{\cosh(0.475) + .00936\sinh(0.475)} = 6.3\text{ W}$$

$$6.3\text{ W}\left(\frac{3.412\,\text{Btu/h}}{\text{W}}\right) = 21.5\text{ Btu/h}$$

COMMENTS

A small soldering iron such as this will typically be rated at 30 W to allow for radiation heat losses and more rapid heat-up.

PROBLEM 2.33

One end of a 0.3 m long steel rod is connected to a wall at 204°C. The other end is connected to a wall which is maintained at 93°C. Air is blown across the rod so that a heat transfer coefficient of 17 W/(m^2 K) is maintained over the entire surface. If the diameter of the rod is 5 cm and the temperature of the air is 38°C, what is the net rate of heat loss to the air?

GIVEN

- A steel rod connected to walls at both ends
- Length of rod (L) = 0.3 m
- Diameter of the rod (D) = 5 cm = 0.05 m

- Wall temperatures: T_s = 204°C T_L = 93°C
- Heat transfer coefficient ($\bar{h}_c$) = 17 W/(m^2 K)
- Air temperature (T_∞) = 38°C

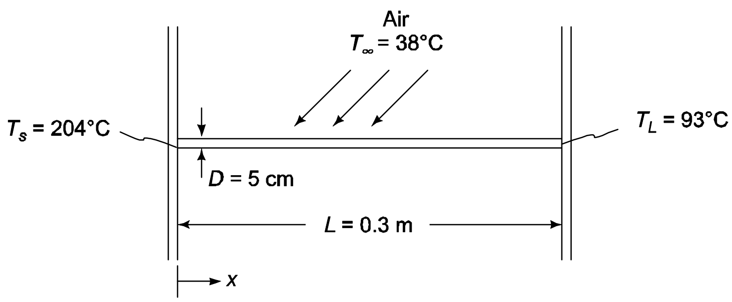
FIND

The net rate of heat loss to the air (q_f)

ASSUMPTIONS

- The wall temperatures are constant
- The system is in steady state
- The rod is 1% carbon steel
- The thermal conductivity of the rod is uniform and not dependent on temperature
- One dimensional conduction along the rod

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10
The thermal conductivity of 1% carbon steel (k) = 43 W/(m K) (at 20°C)

SOLUTION

The rod can be idealized as a fin of uniform cross section with fixed temperatures at both ends. From Table 2.1 the rate of heat loss is

$$q_f = M \frac{\cosh(mL) - \left(\dfrac{\theta_L}{\theta_s}\right)}{\sinh(mL)}$$

where $\theta_L = T_L - T_\infty$ = 93°C – 38°C = 55°C and $\theta_s = T_s - T_\infty$ = 204°C – 38°C = 166°C

$$L\,m = L\sqrt{\frac{\bar{h}_c P}{kA}} = L\sqrt{\frac{\bar{h}_c \pi D}{k\dfrac{\pi}{4}D^2}} = L\sqrt{\frac{4\,\bar{h}_c}{kD}} = 0.3\text{ m}\sqrt{\frac{4\left(17\text{ W/(m}^2\text{ K)}\right)}{4\left(17\text{ W/(m K)}\right)(0.05\text{ m})}} = 1.687$$

$$M = \sqrt{\bar{h}PkA}\;\theta_s = \sqrt{h\frac{\pi^2}{4}D^3k}\;\theta_s = \sqrt{\left(17\text{W/(m}^2\text{K)}\right)\frac{\pi^2}{4}(0.05\text{ m})^3\left(43\text{W/(m K)}\right)}\;(166°\text{C}) = 78.82\text{ W}$$

$$q_f = 78.82\text{ W}\;\frac{\cosh(1.687) - \dfrac{55}{166}}{\sinh(1.687)} = 74.4\text{ W}$$

COMMENTS

In a real situation the convective heat transfer coefficient will not be uniform over the circumference. It will be higher over the side facing the air stream. But because of the high thermal conductivity, the temperature at any given section will be nearly uniform.

PROBLEM 2.34

Both ends of a 0.6 cm copper U-shaped rod, as shown in the accompanying sketch, are rigidly affixed to a vertical wall, the temperature of which is maintained at 93°C. The developed length of the rod is 0.6 m and it is exposed to air at 38°C. The combined radiative and convective heat transfer coefficient for this system is 34 W/(m^2 K). (a) Calculate the temperature of the midpoint of the rod. (b) What will the rate of heat transfer from the rod be?

GIVEN

- U-shaped copper rod rigidly affixed to a wall
- Diameter (D) = 0.6 cm = 0.006 m
- Developed length (L) = 0.6 m
- Wall temperature is constant at (T_s) = 93°C
- Air temperature (T_∞) = 38°C
- Heat transfer coefficient ($\bar{h}$) = 34 W/(m^2 K)

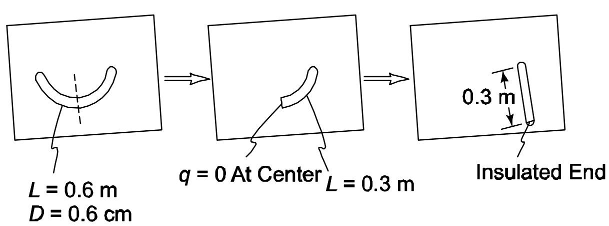
FIND

(a) Temperature of the midpoint (T_{Lf})
(b) Rate of heat transfer from the rod (M)

ASSUMPTIONS

- The system is in steady state
- Variation in the thermal conductivity of copper is negligible
- The U-shaped rod can be approximated by a straight rod of equal length
- Uniform temperature across any section of the rod

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, thermal conductivity of copper (k) = 396 W/(m^2 K) at 64°C

SOLUTION

By symmetry, the conduction through the rod at the center must be zero. Therefore, the rod can be thought of as two pin fins with insulated ends as shown in the sketch above.

(a) From Table 2.1, the temperature distribution for a fin of uniform cross section with an adiabatic tip is

$$\frac{q}{q_s} = \frac{\cosh[m(L_f - x)]}{\cosh(mL)}$$

where $\theta = T - T_\infty$, $\theta_s = T_s - T_\infty$ and L_f = length of the fin

$$m = \sqrt{\frac{\bar{h}P}{kA}} = \sqrt{\frac{\bar{h}\pi D}{k\left(\frac{\pi}{4}D^2\right)}} = \sqrt{\frac{4\bar{h}}{kD}} = \sqrt{\frac{4\left(34\,\text{W/(m}^2\text{K)}\right)}{\left(396\,\text{W/(m K)}\right)(0.006\,\text{m})}} = 7.57\ \frac{1}{\text{m}}$$

Evaluating the temperature of the tip of the pin fin

$$\frac{\theta(L_f)}{\theta_s} = \frac{\cosh[m(L_f - L_f)]}{\cosh(m L_f)} = \frac{1}{\cosh(m L_f)}$$

The length of the fin is half of the wire length (L_f = 0.3 m)

$$\frac{\theta(L_f)}{\theta_s} = \frac{T(Lf) - T_\infty}{T_s - T_\infty} = \frac{1}{\cosh\left[7.57\,\frac{1}{\text{m}}(0.3\text{m})\right]} = 0.205$$

$$T(L_f) = 0.205\,(T_s - T_\infty) + T_\infty = 0.205\,(93°\text{C} - 38°\text{C}) + 38°\text{C} = 49.2°\text{C}$$

The temperature at the tip of the fin is the temperature at the midpoint of the curved rod (49.2°C).

(b) From Table 2.1, the heat transfer from the fin is

$$q_{\text{fin}} = M \tanh(m L_f)$$

$$\text{where } M = \sqrt{\bar{h}PkA}\ \theta_s = \sqrt{\bar{h}(\pi D)k\left(\frac{\pi}{4}D^2\right)}\ (T_s - T_\infty)$$

$$M = \sqrt{\frac{\pi}{4}\left(34\,\text{W/(m}^2\text{K)}\right)\left(396\,\text{W/(m K)}\right)(0.006\,\text{m})^3}\ (93°\text{C} - 38°\text{C}) = 4.653\ \text{W}$$

$$\therefore\ q_{\text{fin}} = 4.653\ \text{W}\tanh\left(7.57\,\frac{1}{\text{m}}\right)(0.3\ \text{m}) = 4.56\ \text{W}$$

The rate of heat transfer from the curved rod is approximately twice the heat transfer of the pin fin

$$q_{\text{rod}} = 2\,q_{\text{fin}} = 2(4.56\ \text{W}) = 9.12\ \text{W}$$

PROBLEM 2.35

A circumferential fin of rectangular cross section, 3.7 cm *OD* and 0.3 cm thick surrounds a 2.5 cm diameter tube. The fin is constructed of mild steel. Air blowing over the fin produces a heat transfer coefficient of 28.4 W/(m^2 K). If the temperatures of the base of the fin and the air are 260°C and 38°C, respectively, calculate the heat transfer rate from the fin.

GIVEN

- A mild steel circumferential fin of a rectangular cross section on a tube
- Tube diameter (D_t) = 2.5 cm = 0.025 m
- Fin outside diameter (D_f) = 3.7 cm = 0.037 m
- Fin thickness (t) = 0.3 cm = 0.003 m
- Heat transfer coefficient ($\bar{h}_c$) = 28.4 W/(m^2 K)
- Fin base temperature (T_s) = 260°C
- Air temperature (T_∞) = 38°C

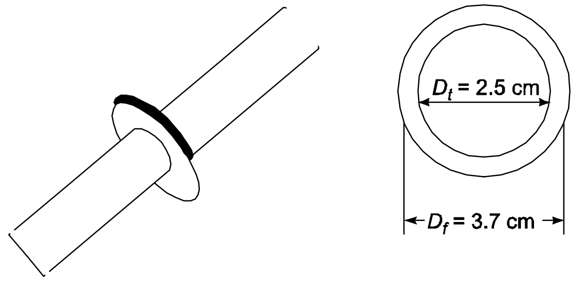
FIND

- The rate of heat transfer from the fin (q_{fin})

ASSUMPTIONS

- The system has reached steady state
- The mild steel is 1% carbon steel
- The thermal conductivity of the steel is uniform
- Radial conduction only (temperature is uniform across the cross section of the fin)
- The heat transfer from the end of the fin can be accounted for by increasing the length by half the thickness and assuming the end is insulated

SKETCH



PROPERTIES AND CONSTANTS

Thermal conductivity of 1% carbon steel (k) = 43 W/(m K) at 20°C

SOLUTION

The rate of heat transfer for the fin can be calculated using the fin efficiency determined from the efficiency graph for this geometry, Figure 2.17.

The length of a fin (L) = ($D_f - D_t$)/2 = 0.006 m

The parameters needed are

$$r_i = \frac{D_t}{2} = 0.125 \text{ m} \qquad r_o = \frac{D_t}{2} + L = 0.125 \text{ m} + 0.006 \text{ m} = 0.0185 \text{ m}$$

$$\left(r_o + \frac{t}{2} - r_i\right)^{\frac{3}{2}} \left(\frac{2\bar{h}_c}{k\,t(r_o - r_i)}\right)^{\frac{1}{2}} = \left(0.0815\,\text{m} + \frac{0.003\,\text{m}}{2} - 0.0125\text{ m}\right)^{\frac{3}{2}}$$

$$\left(\frac{2\left(28.4\text{w/(m}^2\text{K)}\right)}{(43\text{W/(m K)})(0.003\text{ m})(0.0185\text{m} - 0.0125\text{ m})}\right)^{\frac{1}{2}} = 0.176$$

$$\frac{\left(r_o + \frac{t}{2}\right)}{r_i} = \frac{0.0185\,\text{m} + 0.0015\,\text{m}}{0.0125\text{ m}} = 1.6$$

From Figure 2.17, the fin efficiency for these parameters is:

$$\eta_f = 98\%$$

The rate of heat transfer from the fin is

$$q_{\text{fin}} = \eta_f\, \bar{h}_c\, A_{\text{fin}}\,(T_s - T_\infty) = \eta_f\, \bar{h}_c\, 2\pi \left[\left(r_o + \frac{t}{2}\right)^2 - r_i^2\right](T_s - T_\infty)$$

$$q_{\text{fin}} = (0.98)\left(28.4\text{W/(m}^2\text{K)}\right)2\pi\,[(0.085\text{ m} + 0.0015\text{ m})^2 - (0.0125\text{ m})^2]\,(260°\text{C} - 38°\text{C}) = 9.46\text{ W}$$

PROBLEM 2.36

A turbine blade 6.3 cm long (see sketch on p. 156), with cross-sectional area A = 4.6 × 10^{-4} m^2 and perimeter P = 0.12 m, is made of stainless steel (k = 18 W/(m K). The temperature of the root, T_s, is 428°C. The blade is exposed to a hot gas at 871°C, and the heat transfer coefficient h is 454 W/(m^2 K). Determine the temperature of the blade tip and the rate of heat flow at the root of the blade. Assume that the tip is insulated.

GIVEN

- Stainless steel turbine blade
- Length of blade (L) = 6.3 cm = 0.063 m
- Cross-sectional area (A) = 4.6 × 10^{-4} m^2
- Perimeter (P) = 0.12 m
- Thermal conductivity (k) = 18 W/(m K)
- Temperature of the root (T_s) = 482°C
- Temperature of the hot gas (T_∞) = 871°C
- Heat transfer coefficient ($\bar{h}_c$) = 454 W/(m^2 K)

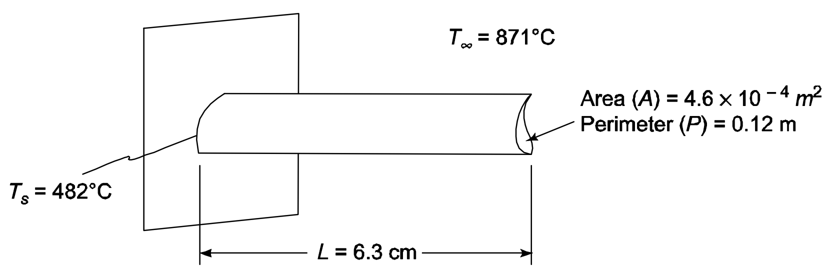
FIND

(a) The temperature of the blade tip (T_L)
(b) The rate of heat flow (q) at the roof of the blade

ASSUMPTIONS

- Steady state conditions prevail
- The thermal conductivity is uniform
- The tip is insulated
- The cross-section of the blade is uniform
- One dimensional conduction

SKETCH



SOLUTION

(a) The temperature distribution in a fin of uniform cross-section with an insulated tip, from Table 2.1, is

$$\frac{\text{q}}{\text{q}_s} = \frac{\cosh[m(L - x)]}{\cosh(mL)}$$

$$\text{where } \text{m} = \sqrt{\frac{\bar{h}\,P}{k\,A}} = \sqrt{\frac{454\text{W/(m}^2\text{K)}(0.12\text{ m})}{18\text{W/(m K)}(4.6\ ´\ 10^{-4}\text{ m}^2)}} = 81.1\,\frac{1}{\text{m}}$$

$$\theta = T - T_\infty$$

At the blade tip, $x = L$, therefore

$$\frac{\theta_L}{\theta_s} = \frac{T_L - T_\infty}{T_s - T_\infty} = \frac{\cosh[m(0)]}{\cosh(mL)} = \frac{1}{\cosh(mL)}$$

$$T_L = T_\infty + \frac{T_s - T_\infty}{\cosh(mL)} = 871°\text{C} + \frac{482^\text{o}\,\text{C} - 871^\text{o}\,\text{C}}{\cosh\left[\left(81.1\frac{1}{\text{m}}\right)(0.063\,\text{m})\right]} = 866°\text{C}$$

(b) The rate of heat transfer from the fin is given by Table 2.1 to be

$$q = M \tanh\,(m\,L)$$

where $M = \sqrt{\bar{h}_c P\,k\,A}\;\;\theta_s$

$$M = \sqrt{454\text{W/(m}^2\text{K)}(0.12\text{ m})\left(18\text{W/(m K)}\right)(4.6 \times 10^{-4}\,\text{m}^2)}\;\;(482°\text{C} – 871°\text{C}) = – 261\text{ W}$$

$$\therefore\; q = (– 261\text{ W})\tanh\left[81.1\frac{1}{\text{m}}(0.063\text{ m})\right] = – 261\text{ W (out of the blade)}$$

COMMENTS

In a real situation, the heat transfer coefficient will vary over the surface with the highest value near the leading edge. But because of the high thermal conductivity of the blade, the temperature at any section will be esentially uniform.

PROBLEM 2.37

To determine the thermal conductivity of a long, solid 2.5 cm diameter rod, one half of the rod was inserted into a furnace while the other half was projecting into air at 27°C. After steady state had been reached, the temperatures at two points 7.6 cm apart were measured and found to be 126°C and 91°C, respectively. The heat transfer coefficient over the surface of the rod exposed to the air was estimated to be 22.7 W/(m^2 K). What is the thermal conductivity of the rod?

GIVEN

- A solid rod, one half inserted into a furnace
- Diameter of rod (D) = 2.5 cm = 0.25 m
- Air temperature (T_∞) = 27°C
- Steady state has been reached
- Temperatures at two points 7.6 cm apart
- ▪ T_1 = 126°C
- ▪ T_2 = 91°C
- The heat transfer coefficient ($\bar{h}_c$) = 22.7 W/(m^2 K)

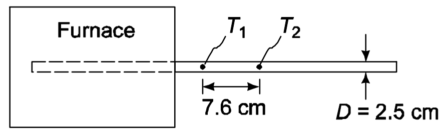
FIND

- The thermal conductivity (k) of the rod

ASSUMPTIONS

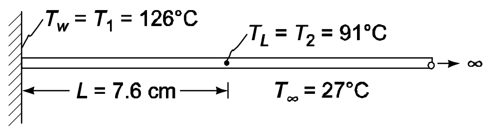
- Uniform thermal conductivity
- One dimensional conduction along the rod
- The rod approximates a fin of infinite length protruding out of the furnace

SKETCH



SOLUTION

This problem can be visualized as the following pin fin problem shown below



The fin is of uniform cross section, therefore Table 2.1 can be used. The temperature distribution for a fin of infinite length, from Table 2.1, is

$$\frac{\theta}{\theta_s} = e^{-mx}$$

$$\text{where } m = \sqrt{\frac{\bar{h}_c P}{kA}} = \sqrt{\frac{\bar{h}_c \pi D}{k \frac{\pi}{2} D^2}} = \sqrt{\frac{4\bar{h}_c}{kD}}$$

Substituting this into the temperature distribution and solving for k

$$\frac{\theta}{\theta_s} = \exp\left(-\sqrt{\frac{4\bar{h}_c}{kD}}\,x\right) \Rightarrow k = \frac{4\bar{h}_c}{D\left(\dfrac{\ln\left(\frac{\theta}{\theta_s}\right)}{x}\right)^2}$$

at $x = L$

$$\theta_L = T_L - T_\infty = 91°\text{C} - 27°\text{C} = 64°\text{C}$$

$$\theta_s = T_W - T_\infty = 126°\text{C} - 27°\text{C} = 99°\text{C}$$

$$\frac{\theta_L}{\theta_s} = \frac{64}{99} = 0.6465$$

Therefore

$$k = \frac{4\left(22.7\,\text{w/(m}^2\text{K)}\right)}{0.025\left[\dfrac{\ln(0.6465)}{0.076\,\text{m}}\right]^2} = 110\ \text{W/(m K)}$$

COMMENTS

Note that this procedure can only be used if the assumption of an infinite length fin is valid. Otherwise, the location of the temperature measurements along the fin must be specified to determine the thermal conductivity.

PROBLEM 2.38

Heat is transferred from water to air through a brass wall (k = 54 W/(m K)). The addition of rectangular brass fins, 0.08 cm thick and 2.5 cm long, spaced 1.25 cm apart, is contemplated. Assuming a water-side heat transfer coefficient of 170 W/(m^2 K) and an air-side heat transfer coefficient of 17 W/(m^2 K), compare the gain in heat transfer rate achieved by adding fins to: (a) the water side, (b) the air side, and (c) both sides. (Neglect temperature drop through the wall.)

GIVEN

- A brass wall with brass fins between air and water
- Thermal conductivity of the brass (k) = 54 W/(m K)
- Fin thickness (t) = 0.08 cm = 0.0008 m
- Fin length (L) = 2.5 cm = 0.025 m
- Fin spacing (d) = 1.25 cm = 0.125 m
- Water-side heat transfer coefficient ($\bar{h}_{cw}$) = 170 W/(m^2 K)
- Air-side heat transfer coefficient ($\bar{h}_{ca}$) = 17 W/(m^2 K)

FIND

Compare the heat transfer rate with fins added to

(a) the water side, $q_{(a)}$

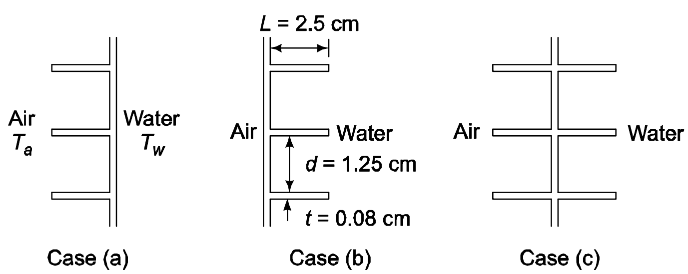
(b) the air side, $q_{(b)}$

(c) both sides, $q_{(c)}$

ASSUMPTIONS

- The thermal resistance of the wall is negligible
- Steady state conditions prevail
- Constant thermal conductivity
- One dimensional conduction
- Heat transfer from the tip of the fins is negligible

SKETCH



SOLUTION

The fins are of uniform cross-section, therefore Table 2.1 may be used. To simplify the analysis, the heat transfer from the end of the fin will be neglected. For a fin with adiabatic tip, the rate of heat transfer is

$$q_f = M \tanh (m L)$$

where $$M = \sqrt{\bar{h}_c PkA}\ \theta_s = \sqrt{\bar{h}_c (2w)k(wt)}\ \theta_s = w\ \sqrt{2\bar{h}_c kt}\ \ \theta_s$$

$$m = \sqrt{\frac{\bar{h}_c P}{kA}} = \sqrt{\frac{\bar{h}_c (2w)}{kwt}} = \sqrt{\frac{2\bar{h}_c}{kt}}$$

The number of fins per square meter of wall is

$$\frac{\text{number of fins}}{\text{m}^2} = \frac{1}{(0.0133\,\text{m/fin})1\text{m width})} = 75.2 \text{ fins/m}^2$$

Fraction of the wall area not covered by fins is

$$\frac{A_{\text{bare}}}{A_n} = \frac{1\text{m}^2 - 75.2\,(1\text{m width})\,(0.008\,\text{m})}{\text{m}^2} = 0.939 \approx 0.94$$

The rate of heat transfer from the wall with fins is equal to the sum of the heat transfer from the bare wall and from the fins

$$q = \bar{h}_c \; A_{\text{bare}}\; \theta_s + (\text{number of fins})\;[\text{M tanh}\;(m\,L)]$$

$$q = \left[\bar{h}_c A_{\text{bare}} + 75.2\, A_w \frac{M}{\theta_s} \tanh(mL)\right]\theta_s = \frac{\theta_s}{R_c}$$

where A_w is the total base area, i.e., with fins removed.

Therefore, the thermal resistance of a wall with fins based on a unit of base area is

$$R_c = \frac{1}{A_W \left[\bar{h}_c \dfrac{A_{\text{bare}}}{A_w} + 75.2 \dfrac{M}{\theta_s} \tanh(mL)\right]}$$

For fins on the water side

$$\frac{M_w}{\theta_s} = 1\text{ m width } \sqrt{170\text{W/(m}^2\text{K)}(2)\left(54\text{W/(m K)}\right)(0.0008\text{ m})} = 3.832\text{ W/K}$$

$$m_{\dot{w}} = \sqrt{\frac{2\left(170\text{W/(m}^2\text{K)}\right)}{54\text{W/(m K)}(.0008\text{ m})}} = 88.72\;\frac{1}{\text{m}}$$

$$\tan\text{h}\;(m_a\,L) = \tanh\left(88.72\frac{1}{\text{m}}\right)\;(0.025\text{ m}) = 0.977$$

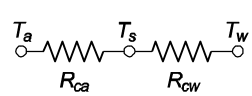
For fins on the air side

$$\frac{M_a}{\theta_s} = 1\text{ m width } \sqrt{\left(17\text{W(m}^2\text{K)}\right)(2)\left(54\text{W/(m K)}\right)(0.0008)} = 1.212\text{ W/K}$$

$$m_a = \sqrt{\frac{2\left(17\text{W/(m}^2\text{K)}\right)}{\left(54\text{W/(m K)}\right)(0.0008\text{ m})}} = 28.05\;\frac{1}{\text{m}}$$

$$\tan\text{h}\;m_a\,L = \tan\text{h}\left(28.05\frac{1}{\text{m}}\right)\;(0.025\text{ m}) = 0.605$$

The thermal circuit for the problem is



The values of thermal resistances with and without fins are

$$(R_{ca})_{\text{nofins}} = \frac{1}{A_w \bar{h}_{ca}} = \frac{1}{A_w\left(17\text{W/(m}^2\text{K)}\right)} = \frac{1}{A_w}\;\;0.0588\;(\text{m}^2\,\text{K})/\text{W}$$

$$(R_{cw})_{\text{nofins}} = \frac{1}{A_w \bar{h}_{cw}} = \frac{1}{A_w\left(170\,\text{W/(m}^2\text{K)}\right)} = \frac{1}{A_w}\ 0.00588\ (\text{m}^2\,\text{K})/\text{W}$$

$$(R_{ca})_{\text{fins}} = \frac{1}{A_w\left[17\ \text{W/(m}^2\text{K)}(0.94) + 75.2\ \text{m}^{-2}\left(1.212\text{W/K}\right)(0.605)\right]} = \frac{1}{A_w}\ 0.0141\ (\text{m}^2\,\text{K})/\text{W}$$

$$(R_{cw})_{\text{fins}} = \frac{1}{A_w\left[170\ \text{W/(m}^2\text{K)}(0.94) + 75.2\ \text{m}^{-2}\left(3.832\ \text{W/K}\right)(0.977)\right]} = \frac{1}{A_w}\ 0.00227\ (\text{m}^2\,\text{K})/\text{W}$$

(a) The rate of heat transfer with fins on the water side only is

$$q_{(a)} = \frac{\Delta T}{(R_{ca})_{\text{no fins}} + (R_{cw})_{\text{fins}}}$$

$$\frac{q_{(a)}}{A_w} = \frac{\Delta T}{(0.0588 + 0.00227)(\text{m}^2\ \text{K})/\text{W}} = 16.4\ \Delta T\ \ \text{W/(m}^2\,\text{K)}$$

(b) The rate of heat transfer with fins on the air side only is

$$q_{(b)} = \frac{\Delta T}{(R_{ca})_{\text{fins}} + (R_{cw})_{\text{no fins}}}$$

$$\frac{q_{(b)}}{A_w} = \frac{\Delta T}{(0.0141 + 0.00588)(\text{m}^2\ \text{K})/\text{W}} = 50.1\ \Delta T\ \text{W/(m}^2\,\text{K)}$$

(c) With fins on both sides, the rate of heat transfer is

$$q_{(c)} = \frac{\Delta T}{(R_{ca})_{\text{fins}} + (R_{cw})_{\text{no fins}}}$$

$$\frac{q_{(c)}}{A_w} = \frac{\Delta T}{(0.0141 + 0.00227)(\text{m}^2\text{K})/\text{W}} = 61.1\ \Delta T\ \text{W/(m}^2\,\text{K)}$$

As a basis of comparison, the rate of heat transfer without fins on either side is:

$$\frac{q}{A_w} = \frac{\Delta T}{(0.0588 + 0.00588)(\text{m}^2\text{W})/\text{K}} = 15.5\ \Delta T\ \text{W/(m}^2\,\text{K)}$$

The following percent increase over the no fins case occurs

Case	% Increase
(a) fins on water side	5.8
(b) fins on air side	223
(c) fins on both sides	294

COMMENTS

Placing the fins on the side with the larger thermal resistance, i.e., the air side, has a much greater effect on the rate of heat transfer.

The small gain in heat transfer rate achieved by placing fins on the water side only would most likely not be justified due to the high cost of attaching the fins.

PROBLEM 2.39

The wall of a liquid-to-gas heat exchanger has a surface area on the liquid side of 1.8 m^2 (0.6m × 3m) with a heat transfer coefficient of 255 W/(m^2 K). On the other side of the heat exchanger wall flows a gas, and the wall has 96 thin rectangular steel fins 0.5 cm thick and 1.25 cm high [k = 3 W/(m K)]. The fins are 3 m long and the heat transfer coefficient on the gas side is 57 W/(m^2 K). Assuming that the thermal resistance of the wall is negligible, determine the rate of heat transfer if the overall temperature difference is 38°C.

GIVEN

- The wall of a heat exchanger has 96 fins on the gas side
- Surface area on the liquid side (A_L) = 1.8 m^2 (0.6 m × 3 m)
- Heat transfer coefficient on the liquid side (h_{cL}) = 255 W/(m^2 K)
- The wall has 96 thin steel fins 0.5 cm thick and 1.25 cm high

- Thermal conductivity of the steel (k) = 3 W/(m K)
- Fin length (w) = 3 m, Fin height (L) = 1.25 cm = 0.0125 m
- Fin thickness (t) = 0.5 cm = 0.005 m
- Heat transfer coefficient on the gas side (h_{cg}) = 57 W/(m^2 K)
- The overall temperature difference (ΔT) = 38°C

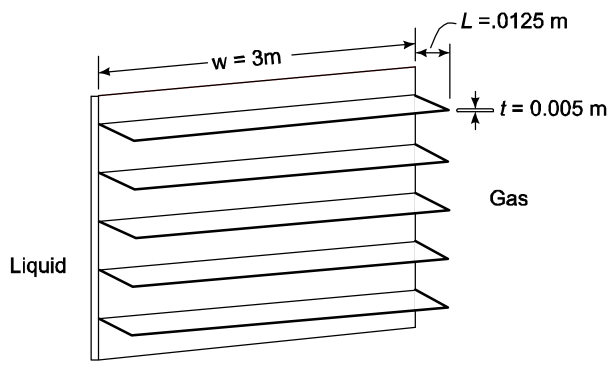
FIND

- The rate of heat transfer (q)

ASSUMPTIONS

- The thermal resistance of the wall is negligible
- The heat transfer through the wall is steady state
- The thermal conductivity of the steel is constant

SKETCH



A Section of the Wall

SOLUTION

The heat transfer from a single fin can be calculated from Table 2.1 for a fin with convection from the tip

$$q_f = M \; \frac{\sinh(mL) + \left(\dfrac{h_c}{mk}\right)\cosh(mL)}{\cosh(mL) + \left(\dfrac{h_c}{mk}\right)\sinh(mL)}$$

where $$m = \sqrt{\frac{\bar{h}_c P}{kA}} = \sqrt{\frac{\bar{h}_c(2t+2w)}{k(wt)}} = \sqrt{\frac{57\,\text{W/(m}^2\text{K)}(6\,\text{m}+0.01\,\text{m})}{3\,\text{W/(m K)}(3\,\text{m})(0.005\,\text{m})}} = 87.25\ \frac{1}{\text{m}}$$

$$mL = 87.25\ \frac{1}{\text{m}}\ (0.0125\ \text{m}) = 1.091 \text{ and } \frac{\bar{h}_c}{mk} = \frac{57\,\text{W/(m}^2\text{K)}}{87.25\frac{1}{\text{m}}(3\,\text{W/(m K)})} = 0.2178$$

$$M = \sqrt{\bar{h}_c PkA}\ \theta_s = \sqrt{(57\ \text{W/(m}^2\text{K)})(6.01\,\text{m})(3\text{W/(m K)})(3\,\text{m})(0.005\,\text{m})}\ (T_s - T_g) = 3.926\ (T_s - T_g)\text{W/K}$$

$$q_f = (3.926(T_s - T_g)\text{W/K})\ \frac{\sinh(1.091) + 0.2178\cosh(1.091)}{\cosh(1.091) + 0.2178\sinh(1.091)} = 3.395\ (T_s - T_g)\ \text{W/K}$$

The rate of heat transfer on the gas side is the sum of the convection from the fins and the convection from the bare wall between the fins. The bare area is

$$A_{\text{bare}} = A_{\text{wall}} - (\text{number of fins})\ (\text{Area of one fin})$$
$$= 1.8\ \text{m}^2 - (96\ \text{fins})\ [(3\ \text{m})\ (0.005\ \text{m})/\text{fin}] = 0.36\ \text{m}^2$$

The total rate of heat transfer to the gas is

$$q_g = q_{\text{bare}} + (\text{number of fins})\ q_f = \bar{h}_{cg}\ A_{\text{bare}}\ (T_s - T_g) + 96(3.395)\ (T_s - T_g)\ \ \text{W/K}$$

$$q_g = \left[57\ \text{W/(m}^2\text{K)}(0.36\,\text{m}^2) + 96(3.395)\right]\ (T_s - T_g)\ \text{W/K} = 346.4\ (T_s - T_g)\ \text{W/K} = \frac{T_s - T_g}{R_g}$$

The thermal resistance on the gas side is

$$R_g = \frac{1}{346.4\ \text{K/W}} = 0.002887\ \text{K/W}$$

The thermal resistance on the liquid side is

$$R_L = \frac{1}{\bar{h}_{cL} A_w} = \frac{1}{255\,\text{W/(m}^2\text{K)}(1.8\ \text{m}^2)} = 0.002179\ \text{K/W}$$

The rate of heat transfer is

$$q = \frac{\Delta T}{R_{\text{tot}}} = \frac{\Delta T}{R_g + R_L} = \frac{38^\circ\text{C}}{(0.002887 + 0.002179)\text{K/W}} = 7500\ \text{W}$$

COMMENTS

Note that despite the much lower heat transfer coefficient on the gas side, the thermal resistance is no larger than on the liquid side. This is the result of balancing the fin geometries which is a desirable situation from the thermal design perspective. Adding fins on the liquid side would not increase the rate of heat transfer appreciably.

PROBLEM 2.40

The top of a 12 in. I-beam is maintained at a temperature of 500°F, while the bottom is at 200°F. The thickness of the web is 1/2 in. Air at 500°F is blowing along the side of the beam so that $\bar{h}$ = 7 Btu/(h ft^2 °F). The thermal conductivity of the steel may be assumed constant and equal to 25 Btu/(h ft °F). Find the temperature distribution along the web from top to bottom and plot the results.

GIVEN

- A steel 12 in. I-beam
- Temperature of the top (T_L) = 500°F
- Temperature of the bottom (T_s) = 200°F
- Thickness of the web (t) = 0.5 in.
- Air temperature (T_∞) = 500°F
- Heat transfer coefficient ($\bar{h}_c$) = 7 Btu/(h ft^2 °F)
- Thermal conductivity of the steel (k) = 25 Btu/(h ft °F)

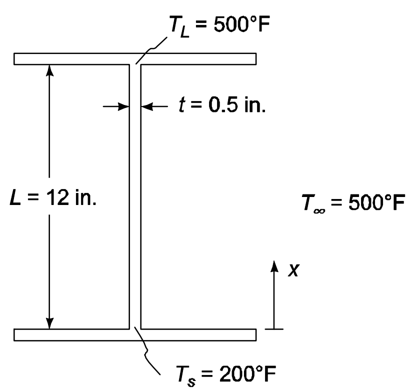
FIND

- The temperature distribution along the web and the plot the results

ASSUMPTIONS

- The thermal conductivity of the steel is uniform
- The beam has reached steady state conditions
- One dimensional through the web
- The beam is very long compared to the web thickness

SKETCH



SOLUTION

The web of the I beam can be thought of as a fin with a uniform rectangular cross section and a fixed tip temperature. From Table 2.1, the temperature distribution along the web is

$$\frac{\theta}{\theta_s} = \frac{\left(\frac{\theta_L}{\theta_s}\right)\sinh(m\,x) + \sinh[m(L - x)]}{\sinh(mL)}$$

where $\theta = T - T_\infty$

$$m = \sqrt{\frac{\bar{h}_c P}{kA}} = \sqrt{\frac{\bar{h}_c 2(w+t)}{kwt}} = \sqrt{\frac{2\bar{h}_c}{kt}} = \sqrt{\frac{2\left(7\,\text{Btu/(h ft}^2\,{}^\circ\text{F)}\right)}{25\,\text{Btu/(h ft }{}^\circ\text{F)}\left(\frac{0.5}{12}\,\text{ft}\right)}} = 3.66\ \frac{1}{\text{ft}}$$

mL = 3.666 sinh (m L) = 19.54

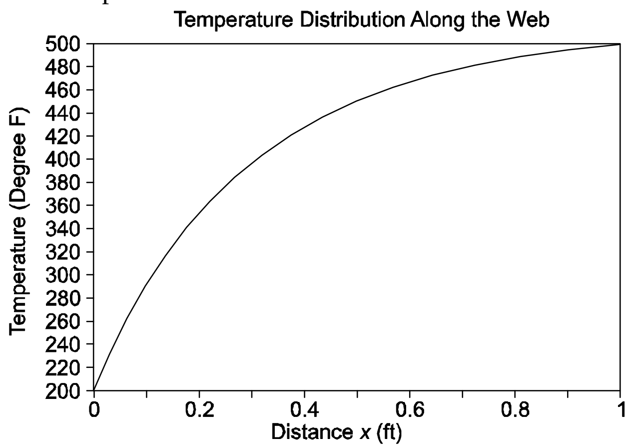
θ_s = $T_s - T_\infty$ = 200°F – 500°F = – 300°F $\qquad$ $\theta_L - T_L - T_\infty = 0$

Substitute these into the temperature distribution

$$\frac{T(x) - T_\infty}{\theta_s} = 0.0512 \sinh[3.666(1 - x)]$$

$$T_{(x)} = 500^\circ\text{F} - 15.353 \sinh[3.66(1 - x)]$$

This temperature distribution is plotted below



COMMENTS

In a real situation, the heat transfer coefficient is likely to vary with distance and this would require a numerical solution.

PROBLEM 2.41

The handle of a ladle used for pouring molten lead is 30 cm long. Originally the handle was made of 1.9 × 1.25 cm mild steel bar stock. To reduce the grip temperature, it is proposed to form the handle of tubing 0.15 cm thick to the same rectangular shape. If the average heat transfer coefficient over the handle surface is 14 W/(m^2 K), estimate the reduction of the temperature at the grip in air at 21°C.

GIVEN

- A steel handle of a ladle used for pouring molten lead
- Handle length (L) = 30 cm = 0.3 m
- Original handle: 1.9 by 1.25 cm mild steel bar stock
- New handle: tubing 0.15 cm thick with the same shape
- The average heat transfer coefficient ($\bar{h}_c$) = 14 W/(m^2 K)
- Air temperature (T_∞) = 21°C

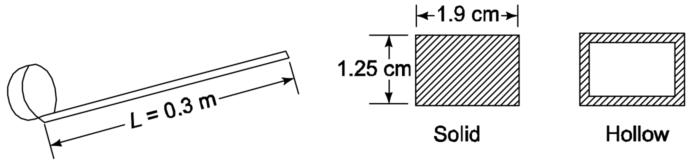
FIND

- The reduction of the temperature at the grip

ASSUMPTIONS

- The lead is at the melting temperature
- The handle is made of 1% carbon steel
- The ladle is normally in steady state during use
- The variation of the thermal conductivity is negligible
- One dimensional conduction
- Heat transfer from the end of the handle can be neglected

SKETCH

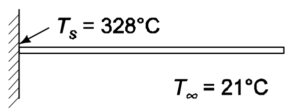


PROPERTIES

From Appendix 2, Tables 10 and 12
Thermal conductivity of 1% carbon steel = 43 W/(m K) at 20°C
Melting temperature of lead (T_s) = 601 K = 328°C

SOLUTION

The ladle handle can be treated as a fin with an adiabatic end as shown below



The temperature distribution in the handle, from Table 2.1 is

$$\frac{q}{q_s} = \frac{\cosh[m(L - x)]}{\cosh(mL)}$$

where $\theta = T(x) - T_\infty$ $\qquad \theta_s = T_s - T_\infty = 328°C - 21°C = 307°C$

$$m = \sqrt{\frac{\bar{h}_c P}{kA}}$$

where $P = 2w + 2t = 2(0.019\text{ m}) + 2(0.0125\text{ m}) = 0.063\text{ m}$

The only difference in the two handles is the cross-sectional area
Solid handle

$$A_s = wt = (0.019\text{ m})\,(0.0125\text{ m}) = 0.0002375\text{ m}^2$$

$$m\,L = 0.3\text{ m}\sqrt{\frac{14\text{ W/(m}^2\text{K)}(0.063\text{ m})}{43\text{ W/(m K)}(0.0002375\text{ m}^2)}} = 2.788$$

$$\frac{q_L}{q_s} = \frac{\cosh(0)}{\cosh(2.788)} = 0.1266 \Rightarrow \theta_L = T_L - T_\infty = 0.1226\ \theta_s$$

$$\therefore\ T_L = T_\infty + 0.1266\ \theta_s = 21°C + 0.1266\ (307°C) = 60°C$$

Hollow handle

$$A_H = wt - [w - 2(0.0015\text{ m})]\,[t - 2(0.0015\text{ m})]$$

$$= (0.019\text{ m})\,(0.0125\text{ m}) - (0.016)\,(0.0095\text{ m}) = 0.0000855\text{ m}^2$$

$$m\,L = 0.3\text{ m}\sqrt{\frac{14\text{ W/(m}^2\text{K)}(0.063\text{ m})}{43\text{ W/(m K)}(0.0000855\text{ m}^2)}} = 4.65$$

$$\frac{q_L}{q_s} = \frac{\cosh(0)}{\cosh(4.647)} = 0.0192$$

$$T_L = T_\infty + 0.01919\ \theta_s = 21°C + 0.0192\ (307°C) = 27°C$$

The temperature of the grip is reduced 33°C by using the hollow handle.

COMMENTS

The temperature of the hollow handle would be comfortable to the bare hand, therefore no insulation is required. This will reduce the cost of the item without reducing utility.

PROBLEM 2.42

A 0.3-cm thick aluminum plate has rectangular fins on one side, 0.16 × 0.6 cm, spaced 0.6 cm apart. The finned side is in contact with low pressure air at 38°C and the average heat transfer coefficient is 28.4 W/(m^2 K). On the unfinned side water flows at 93°C and the heat transfer coefficient is 283.7 W/(m^2 K). (a) Calculate the efficiency of the fins (b) calculate the rate of heat transfer per unit area of wall and (c) comment on the design if the water and air were interchanged.

GIVEN

- Aluminum plate with rectangular fins on one side
- Plate thickness (D) = 0.3 cm = 0.003 m
- Fin dimensions ($t \times L$) = 0.0016 m × 0.006 m
- Fin spacing (s) = 0.006 m apart
- Finned side
  - Air temperature (T_a) = 38°C
  - Heat transfer coefficient ($\bar{h}_a$) = 28.4 W/(m^2 K)
- Unfinned side
  - Water temperature (T_w) = 93°C
  - Heat transfer coefficient ($\bar{h}_w$) = 283.7 W/(m^2 K)

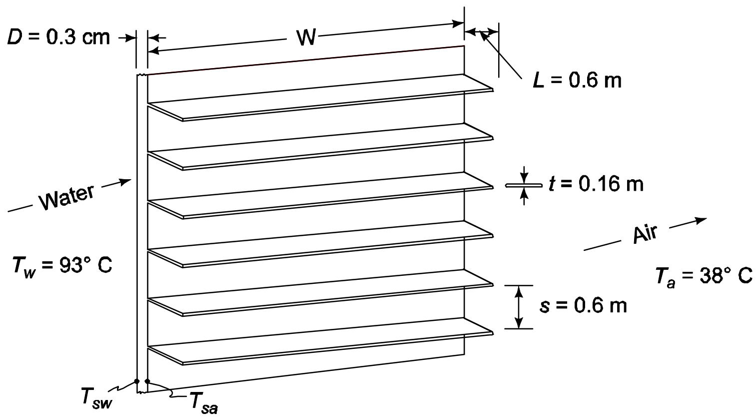
FIND

(a) The fin efficiency (η_f)
(b) Rate of heat transfer per unit wall area (q/A_w)
(c) Comment on the design if the water and air were interchanged

ASSUMPTIONS

- The aluminum is pure
- Width of fins is much longer than their thickness
- The system has reached steady state
- The thermal conductivity of the aluminum is constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12

The thermal conductivity of aluminum (k) = 238 W/(m K) at 65°C

SOLUTION

(a) The fin efficiency is defined as the actual heat transfer rate divided by the rate of heat transfer if the entire fin were at the wall temperature. Since the fin is of uniform cross section, Table 2.1 can be used to find an expression for the heat transfer from a fin with a convection from the tip

$$q_f = M\,\frac{\sinh(mL) + \left(\frac{\bar{h}_a}{mk}\right)\cosh(mL)}{\cosh(mL) + (\bar{h}_a mk)\sinh(mL)}$$

where $$m^2 = \frac{\bar{h}_a P}{kA} = \frac{\bar{h}_a 2w}{k(wt)} = \frac{2\bar{h}_a}{kt}$$

$$M = \sqrt{\bar{h}_a PkA}\;\theta_s = w\sqrt{2\bar{h}_a tk}\;\theta_s$$

where $$\theta_s = T_{sa} - T_a$$

If the entire fin were at the wall temperature (T_{sa}) the rate of heat transfer would be

$$q'_f = \bar{h}_a A_f (T_{sa} - T_a) = \bar{h}_a w(2L + t)(T_{sa} - T_a)$$

The fin efficiency is

$$\eta_f = \frac{q_f}{q'_f} = \frac{\left[M\,\dfrac{\sinh(mL) + \left(\frac{\bar{h}_a}{mk}\right)\cosh(mL)}{\cosh(mL) + \left(\frac{\bar{h}_a}{mk}\right)\sinh(mL)}\right]}{\bar{h}_a w(2L + t)(T_{sa} + T_a)}$$

$$m = \sqrt{\frac{2\bar{h}_a}{kt}} = \sqrt{\frac{2\left(28.4\ \text{W/(m}^2\text{K)}\right)}{238\ \text{W/(m K)}(0.0016\ \text{m})}} = 12.2\ \frac{1}{\text{m}}$$

$$mL = 12.2\ \frac{1}{\text{m}}\ (0.006\ \text{m}) = 0.0733$$

$$M = w\,(T_{sa} - T_a)\sqrt{2\left(28.4\,\text{W/(m}^2\text{K)}\right)(0.0016\ \text{m})\left(238\,\text{W/(m}^2\text{K)}\right)} = 4.65\ w\,(T_{sa} - T_w)\text{s W/(mK)}$$

$$\frac{\bar{h}_a}{mk} = \frac{28.4\ \text{W/(m}^2\text{K)}}{12.2\,\frac{1}{\text{m}}\left(238\text{W/(m K)}\right)} = 0.0098$$

$$\eta_f = \frac{4.65\text{W/(m}^2\text{K)}\left(\dfrac{\sinh(0.0733) + 0.00977\cosh(0.0733)}{\cosh(0.0733) + 0.00977\sinh(0.0733)}\right)}{28.4\ \text{W/(m}^2\text{K)}\ [(2)0.006\ \text{m} + 0.0016\,\text{m}]} = 0.998$$

(b) The heat transfer to the air is equal to the sum of heat transfer from the fins and the heat transfer from the wall area not covered by fins.

The number of fins per meter height is

$$\frac{1\text{m}}{0.076\ \text{m/fin}} = 131.6\ \text{fins}$$

The wall area not covered by fins per m^2 of total wall area is

$$A_{\text{bare}} = 1\ \text{m}^2 - (131.\ 6\ \text{fins})\ (0.0016\ \text{m/fin})\ (1\ \text{m width}) = 0.789\ \text{m}^2$$

The surface area of the fins per m^2 of wall area is

$$A_{\text{fins}} = 131.6\ \text{fins}\ (2(0.006\ \text{m}) + 0.0016\ \text{m})\ (1\ \text{m width}) = 1.79\ \text{m}^2$$

The rate of heat transfer to the air is

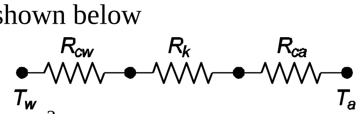
$$q_a = \bar{h}_a\ A_{\text{bare}}\ (T_{sa} - T_a) + \bar{h}_a\ \eta_{\text{f}}\, A_{\text{fins}}\ (T_{sa} - T_a)$$

$$q_a = \bar{h}_a\ (A_{\text{bare}} + \eta_f\, A_{\text{fins}})\ (T_{sa} - T_a) = \frac{T_{sa} - T_a}{R_{ca}}$$

Therefore, the resistance to heat transfer on the air side (R_a) is

$$R_{ca} = \frac{1}{\bar{h}_a(A_{\text{bare}} + \text{h}_f\, A_{\text{fins}})} \approx \frac{1}{\bar{h}_a A_{\text{total}}}$$

The thermal circuit for the wall is shown below



The individual resistance based on 1 m^2 of wall area are

$$R_{cw} = \frac{1}{\bar{h}_w A_w} = \frac{1}{238.7\ \text{W/(m}^2\text{K)(1m}^2)} = 0.00419\ \text{K/W}$$

$$R_k = \frac{D}{kA_w} = \frac{0.003\,\text{m}}{238.7\ \text{W/m K(1m}^2)} = 0.0000126\ \text{K/W}$$

$$R_{ca} = \frac{1}{\bar{h}_a(A_{\text{bare}} + \text{h}_f\, A_{\text{fins}})} = \frac{0.003\,\text{m}}{28.4\ \text{W/(m}^2\text{K)}\ [0.789\,\text{m}^2 + (0.998)(1.79\,\text{m}^2)]} = 0.0137\ \text{K/W}$$

The rate of heat transfer through the wall is

$$q = \frac{\text{D}T}{R_{\text{tot}}} = \frac{T_w - T_a}{R_{cw} + R_k + R_{ca}} = \frac{93^oC - 38^oC}{(0.00419 + 0.0000126 + 0.0137)\text{K/W}} = 3072\ \text{W (per m}^2\text{ of wall)}$$

(c) Note that the air side convective resistance is by far the dominant resistance in the problem. Therefore, the fins will enhance the overall heat transfer much less on the water side.

For fins on the water side

$$m = \sqrt{\frac{2\left(283.7\ \text{W/(m}^2\text{K)}\right)}{238\,\text{W/(m K)}\,(0.0016\ \text{m})}} = 38.6\ \frac{1}{\text{m}}\ \text{ and } m\,L = 38.6\ \frac{1}{\text{m}}\ (0.006\ \text{m}) = 0.2316$$

$$M = w\,(T_{sw} - T_w)\ \sqrt{2\left(283.7\ \text{W/(m}^2\text{K)}\right)(0.0016\ \text{m})2\left(238\ \text{W/(m K)}\right)} = 14.70\ w\,(T_{sw} - T_{\text{w}})\text{W/m K}$$

$$\frac{\bar{h}_w}{mk}\ \frac{283.7\ \text{W/(m}^2\text{K)}}{38.6\,\frac{1}{\text{m}}\left(238\ \text{W/(m K)}\right)} = 0.0309$$

$$\eta_f = \frac{14.70\ \text{W/(m K)}\left[\frac{\sinh(0.2316) + 0.0309\cosh(0.2316)}{\cosh(0.2316) + 0.0309\sinh(0.2316)}\right]}{283.7\ \text{W/(m}^2\text{K)}\,[2\,(0.006\ \text{m}) + 0.0016\ \text{m}} = 0.978$$

$$q = \frac{T_w - T_a}{\frac{1}{\bar{h}_{ca}} + \frac{D}{k} + \frac{1}{\bar{h}_{cw}(0.089 + \text{h}1.79)}} = \frac{93^{\text{o}}\text{C} - 38^{\text{o}}\text{C}}{(0.0352 + 0.0000126 + 0.00139)\ (\text{m}^2\text{K})/\text{W}}$$

$$= 1502\ \text{W/m}^2$$

COMMENTS

The fins are most effective in the medium with the lowest heat transfer coefficient.
With no fins, the rate of heat transfer would be 1419 W/m^2. Fins on the water side increase the rate of heat transfer 6%. Fins on the air side increase the rate of heat transfer 116%. Therefore, installing fins on the water side would be a poor design.

PROBLEM 2.43

Compare the rate of heat flow from the bottom to the top in the aluminum structure shown in the sketch with the rate of heat flow through a solid slab. The top is at –10°C, the bottom at 0°C. The holes are filled with insulation which does not conduct heat appreciably.

GIVEN

- The aluminum structure shown in the sketch below
- Temperature of the top (T_T) = – 10°C
- Temperature of the bottom (T_B) = 0°C
- The holes are filled with insulation which does not conduct heat appreciably

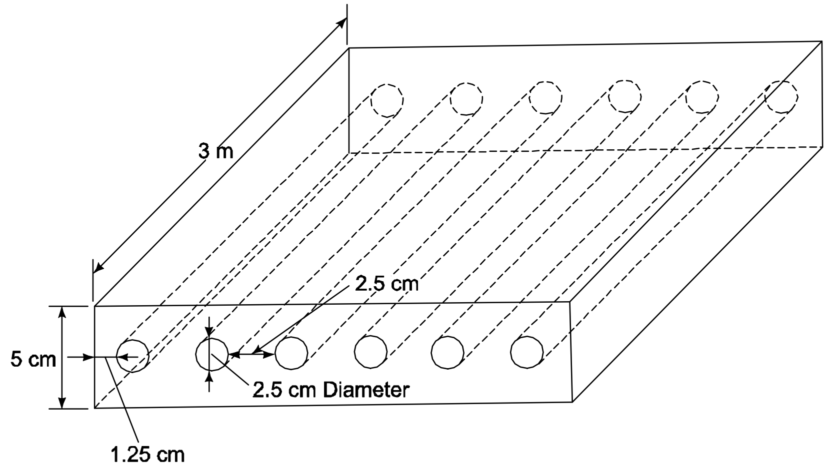
FIND

- Compare the rate of heat flow from the bottom to the top with the rate of heat flow through a solid slab

ASSUMPTIONS

- The structure is in steady state
- Heat transfer through the insulation is negligible
- The thermal conductivity of the aluminum is uniform
- The edges of the structure are insulated
- Two dimensional conduction through the structure

SKETCH

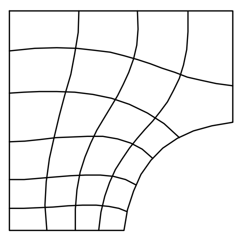


PROPERTIES AND CONSTANTS

The thermal conductivity of aluminum (k) = 236 W/(m K) at 0°C

SOLUTION

Because of the symmetry of the structure, we can draw the flux plot for just one of the twenty-four equivalent sections



(a) The total number of flow lanes in the structure, (M) = (12) (4) = (48). Each flow lane consists of 12 curvilinear squares (6 on top as shown, and 6 on bottom. Therefore, the shape factor is

$$S = \frac{M}{N} = \frac{48}{12} = 4$$

The heat flow per meter, from Equation (2.80), is

$$q = kS\Delta T_{\text{overall}} = 236 \text{ W/m K } (4)\ (0°\text{C} - (-10°\text{C})) = 9440 \text{ W/m}$$

The total rate of heat flow is

$$q_{\tau o\tau} = q \text{ (length of structure)} = (9440 \text{ W/m})\ (3 \text{ m}) = 28{,}320 \text{ W}$$

(b) For a solid aluminum plate, the total heat flow from Equation (1.2), is

$$q_{\text{TOT}} = \frac{Ak}{t}\,\Delta T = \frac{(3\text{m})(0.3\text{m})[236\text{ W/(m K)}]}{0.05}\ (10\text{ C}) = 42{,}500 \text{ W}$$

Therefore, the insulation filled tubes reduce the heat transfer rate by 33%.

COMMENTS

The shape factor was determined graphically and can easily be in error by 10%.

Also, the surface temperature will not be uniform in the insulated structure.

PROBLEM 2.44

Determine by means of a flux plot the temperatures and heat flow per unit depth in the ribbed insulation shown in the accompanying sketch.

GIVEN

- The sketch below

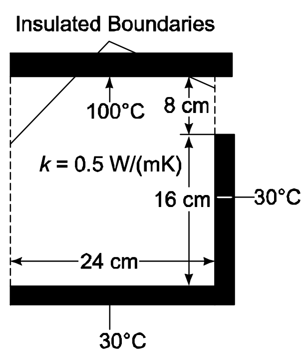
FIND

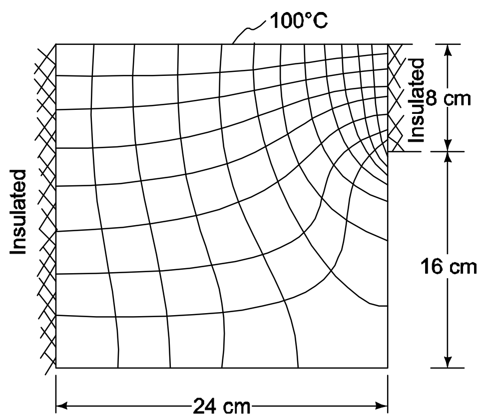
(a) The temperatures
(b) The heat flow per unit depth

ASSUMPTIONS

- Steady state conditions
- Two dimensional heat flow
- The heat loss through the insulation is negligible
- The thermal conductivity of the material is uniform

SKETCH





SOLUTION

The total number of heat flow lanes (M) = 11
The number of curvilinear squares per lane (N) = 8
Therefore, the shape factor is

$$S = \frac{M}{N} = \frac{11}{8} = 1.38$$

The rate of heat transfer for unit depth is given by Equation 2.80

$$q = kS\Delta T = (0.5 \text{ W/(m K)})\ (1.38)\ (100°\text{C} - 30°\text{C}) = 48.3 \text{ W/m}$$

PROBLEM 2.45

Use a flux plot to estimate the rate of heat flow through the object shown in the sketch. The thermal conductivity of the material is 15 W/(m K). Assume no heat is lost from the sides.

GIVEN

- The shape of object shown in the sketch
- The thermal conductivity of the material (k) = 15 W/(m K)
- The temperatures at the upper and lower surfaces (30°C & 10°C)

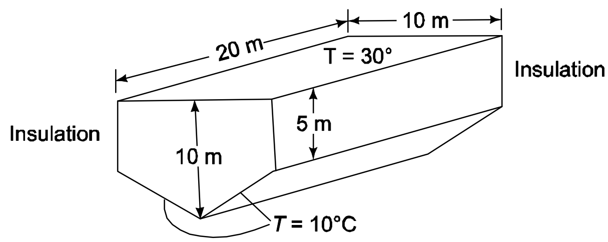
FIND

- The rate of heat flow through the object (By means of a flux plot)

ASSUMPTIONS

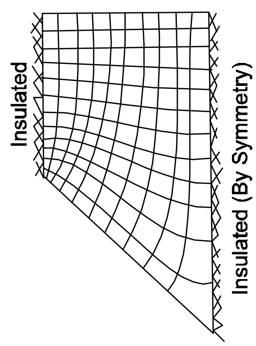
- No heat is lost from the sides and ends
- Uniform thermal conductivity
- Two dimensional conduction
- Steady state

SKETCH



SOLUTION

The flux plot is shown below



The number of heat flow lanes (M) = 2 × 10 = 20
The number of curvilinear squares in each lane (N) = 12
Therefore, the shape factor for this object is

$$S = \frac{M}{N} = \frac{20}{12} = 1.67$$

The rate of heat transfer per unit length from Equation (2.80) is

$$q = kS\Delta T_{\text{overall}} = [15\text{ W/(m K)}]\ (1.67)\ (20°\text{C}) = 500\text{ W/m}$$

The total rate of heat transfer is

$$q_{\text{tot}} = q\text{L} = (500\text{ W/m})\ (20\text{ m}) = 10{,}000\text{ W}$$

PROBLEM 2.46

Determine the rate of heat transfer per unit length from a 5-cm-*OD* pipe at 150°C placed eccentrically within a larger cylinder of 85% Magnesia wool as shown in the sketch. The outside diameter of the larger cylinder is 15 cm and the surface temperature is 50°C.

GIVEN

- A pipe placed eccentrically within a larger cylinder of 85% Magnesia wool as shown in the sketch
- Outside diameter of the pipe (D_p) = 5 cm = 0.05 m
- Temperature of the pipe (T_s) = 150°C
- Outside diameter of the larger cylinder (D_o) = 15 cm = 0.15 m
- Temperature of outer pipe (T_o) = 50°C

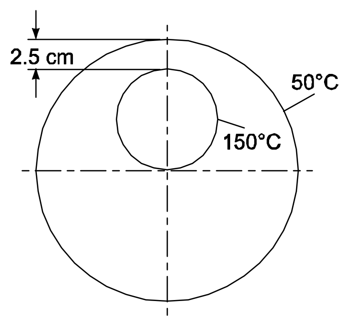
FIND

- The rate of heat transfer per meter length (q)

ASSUMPTIONS

- Two dimensional heat flow (no end effects)
- The system is in steady state
- Uniform thermal conductivity

SKETCH

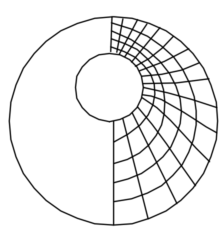


PROPERTIES AND CONSTANTS

From Appendix 2, Table 11
The thermal conductivity of 85% Magnesia wool (k) = 0.059 W/(m K) (at 20°C).

SOLUTION

The rate of heat transfer can be estimated from a flux plot



The number of flow lanes (M) = 2 × 15 = 30
The number of squares per lane (N) = 5

Therefore, the shape factor is

$$S = \frac{M}{N} = \frac{30}{5} = 6$$

Equation (2.80) can be used to find the rate of heat transfer per unit length

$$q = kS\Delta T = kS(T_s - T_o) = [0.059 \text{ W/(m K)}]\,(6)\,(150°\text{C} - 50°\text{C}) = 35.4 \text{ W/m}$$

COMMENTS

This problem can also be solved analytically (see Table 2.2)

$$S = \frac{2\pi}{\cosh^{-1}\left(\frac{D^2 + d - 4z^2}{2Dd}\right)} = 6.53$$

(z = the distance between the centers of the circular cross sections)

$$\therefore\ q = kS\Delta T = 38.5 \text{ W/m}$$

The answer from the graphical solution is 8% less than the analytical value.

PROBLEM 2.47

Determine the rate of heat flow per foot length from the inner to the outer surface of the molded insulation in the accompanying sketch. Use k = 0.1 Btu/(h ft °F).

GIVEN

- The object with a cross section as shown in the sketch below
- The thermal conductivity (k) = 0.1 Btu/(h ft °F)

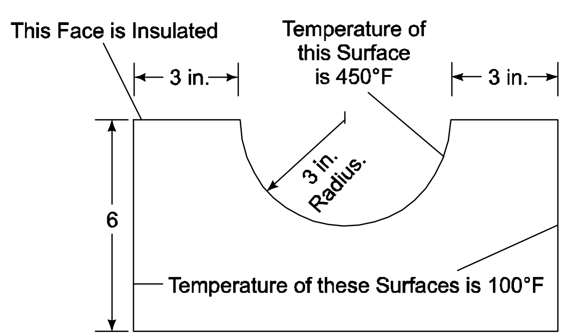
FIND

- The rate of heat flow per foot length from the inner to the outer surface (q)

ASSUMPTIONS

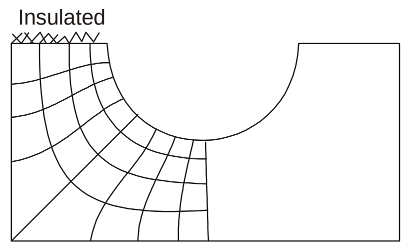
- The system has reached steady state
- The thermal conductivity does not vary with temperature
- Two dimensional conduction

SKETCH



SOLUTION

A flux plot for the object is shown below



The number of heat flow lanes (M) = 2 × 8 = 16
The number of curvilinear squares per lane (N) = 4
Therefore, the shape factor is

$$S = \frac{16}{4} = 4$$

The heat flow per unit length, from equation (2.80) is

$$q = kS\Delta T_{\text{overall}} = [0.1 \text{ Btu/(h ft °F)}]\,(4)\,(350\text{°F}) = 140 \text{ Btu/(h ft)}$$

COMMENTS

The problem can also be solved analytically. From Table 2.2

$$S = \frac{\pi}{\ln(1.08\,\mathrm{W/D})} = \frac{\pi}{\ln\left(1.08\dfrac{12}{6}\right)} = 4.08$$

$$q = kS\Delta T = 143 \text{ Btu/(h ft)}$$

The analytical solution yields a rate of heat flow that is about 2% larger than the value obtained from the flux plot.

PROBLEM 2.48

> **A long 1-cm-diameter electric copper cable is embedded in the center of a 25 cm square concrete block. If the outside temperature of the concrete is 25°C and the rate of electrical energy dissipation in the cable is 150 W per meter length, determine the temperatures at the outer surface and at the center of the cable.**

GIVEN

- A long electric copper cable embedded in the center of a square concrete block
- Diameter of the pipe (D_p) = 1 cm = 0.01 m
- Length of a side of the block = 25 cm = 0.25 m
- The outside temperature of the concrete (T_o) = 25°C
- The rate of electrical energy dissipation ($\dot{Q}_G / L$) = 150 W/m

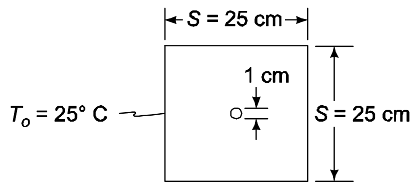
FIND

- The temperatures at the outer surface (T_s) and at the center of the cable (T_c)

ASSUMPTIONS

- Two dimensional, steady state heat transfer
- Uniform thermal conductivities

SKETCH

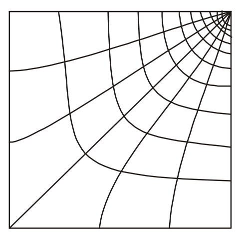


PROPERTIES AND CONSTANTS

From Appendix 2, Table 11
The thermal conductivity of concrete (k_b) = 0.128 W/(m K) at 20°C
From Appendix 2, Table 12
The thermal conductivity of copper (k_c) = 396 W/(m K) at 63°C

SOLUTION

For steady state, the rate of heat transfer through the concrete block must equal the rate of electrical energy dissipation. The heat transfer rate can be estimated with a flux plot of one quarter of the block:



The number of flow lanes (M) = 4 × 6 = 24
The number of squares per lane (N) = 10
Therefore, the shape factor is

$$S = \frac{\mathrm{M}}{\mathrm{N}} = \frac{24}{10} = 2.4$$

The rate of heat flow per unit length is given by Equation (2.80)

$$q = k_b S \Delta T = k_b S (T_s - T_o) = \frac{\dot{Q}_G}{L}$$

Solving for the surface temperature of the cable

$$T_s = T_o + \frac{\left(\frac{\dot{Q}_G}{L}\right)}{k_b S} = 25°\mathrm{C} + \frac{150\mathrm{W/m}}{[0.128\mathrm{W/(m\ K)}](2.4)} = 513°\mathrm{C}$$

From Equation (2.51) the temperature in the center of the cable is

$$T_c = T_s + \frac{\dot{q}_G r_0^2}{4 k_C}$$

Where $\dot{q}_G$ = heat generation per unit volume $\frac{\dot{q}_G}{\pi\, r_0^2 L}$

$$T_C = T_s + \frac{\left(\frac{\dot{Q}_G}{L}\right)}{4\pi\, k_C} = 513°\text{C} + \frac{150\text{W/m}}{4\pi\,(396\text{W/(m K)})} = 513°\text{C} + 0.03°\text{C} \approx 513°\text{C}$$

COMMENT

The thermal conductivity of the cable is quite large and therefore its temperature is essentially uniform.

The analytical solution for this geometry, given in Table 2.2, is

$$S = \frac{2\,\pi}{\text{In}\left(0.8\dfrac{\text{W}}{\text{D}}\right)} = \frac{2\,\pi}{\text{In}\left(1.08\dfrac{25\,\text{cm}}{1\,\text{cm}}\right)} = 1.91$$

This would lead to a cable temperature of 639°C, 20% higher than the flux plot estimate. The high error is probably due to the difficulty in drawing the flux plot close to the cable and may be improved by drawing a larger scale flux plot is geometries that involve tight curves.

PROBLEM 2.49

> **A large number of 1.5-in.-*OD* pipes carrying hot and cold liquids are embedded in concrete in an equilateral staggered arrangement with center line 4.5 in. apart as shown in the sketch. If the pipes in rows A and C are at 60°F while the pipes in rows B and D are at 150°F, determine the rate of heat transfer per foot length from pipe *X* in row *B*.**

GIVEN

- A large number of pipes embedded in concrete as shown below
- Outside diameter of pipes (D) = 1.5 in.
- The temperature of the pipes in rows A and C = 60°F
- The temperature of the pipes in rows B and D = 150°F

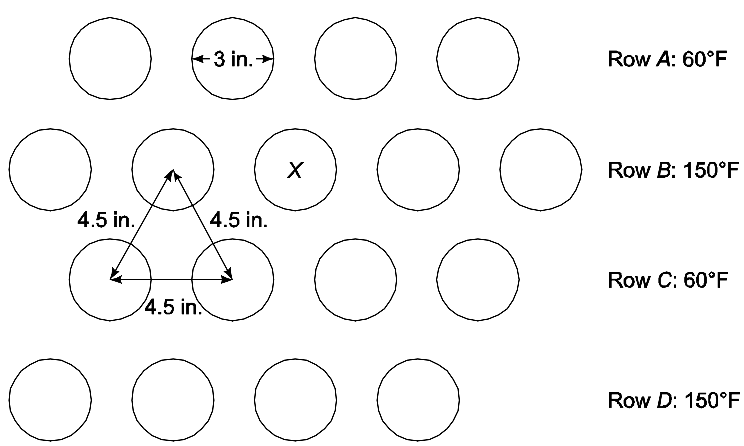
FIND

- The rate of hat transfer per foot length from pipe X in row B

ASSUMPTIONS

- Steady state, two dimensional heat transfer
- Uniform thermal conductivity in the concrete

SKETCH

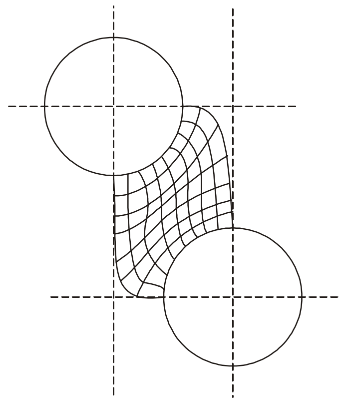


PROPERTIES AND CONSTANTS

From Appendix 2, Table 11
The thermal conductivity of concrete (k_b) = 0.128 W/(m K) at 20°C

SOLUTION

A flux diagram for this problem is shown below



By symmetry, the total heat transfer from the tube X is four times that shown in the flux diagram.
The number of heat flow lanes (M) = 8 × 4 = 32
The number of curvilinear squares per lane (N) = 7
Therefore, the shape factor is

$$S = \frac{\text{M}}{\text{N}} = \frac{32}{7} = 4.6$$

The heat transfer per unit length from Table 2.2, from Equation (2.80) is

$$q\ KS\Delta T_{\text{overall}} = [0.074 \text{ Btu/(h ft °F)}]\ (4.6)\ (150\text{°F} - 60\text{°F}) = 30.4 \text{ Btu/(h ft)}$$

PROBLEM 2.50

A long 1-cm-diameter electric cable is imbedded in a concrete wall (*k* = 0.13 W/(m K)) which is 1 m by 1 m, as shown in the sketch below. If the lower surface is insulated, the surface of the cable is 100°C and the exposed surface of the concrete is 25°C, estimate the rate of energy dissipation per meter of cable.

GIVEN

- A long electric cable imbedded in a concrete wall
- Cable diameter (D) = 1 cm = 0.01 m
- Thermal conductivity of the wall (k) = 0.13 W/(m K)
- Wall dimensions are 1 m by 1 m, as shown in the sketch below
- The lower surface is insulated
- The surface temperature of the cable (T_s) = 100°C
- The temperature of the exposed concrete surfaces (T_o) = 25°C

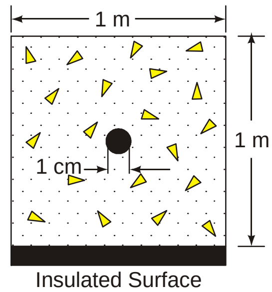
FIND

- The rate of energy dissipation per meter of cable (q/L)

ASSUMPTIONS

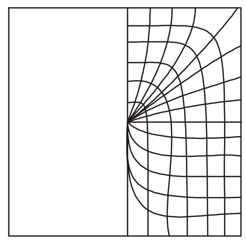
- The system is in steady state
- The thermal conductivity of the wall is uniform
- Two dimensional heat transfer

SKETCH



SOLUTION

By symmetry, only half of the flux plot needs to be drawn



The number of heat flow lanes (M) = 2 × 14 = 28

The number of curvilinear squares per lane (N) = 6

Therefore, the shape factor is

$$S = \frac{M}{N} = \frac{28}{6} = 4.7$$

For steady state, the rate of energy dissipation per unit length in the cable must equal the rate of heat transfer per unit length from the cable which, from Equation (2.80), is

$$q = kS(T_s - T_o) = \big(0.13\ \text{W/(m K)}\ (4.7)\big)\ (100°\text{C} - 25°\text{C}) = 46\ \text{W/m}$$

PROBLEM 2.51

> **Determine the temperature distribution and heat flow rate per meter length in a long concrete block having the shape shown below. The cross-sectional area of the block is square and the hole is centered.**

GIVEN

- A long concrete block having the shape shown below
- The cross-sectional area of the block is square
- The hole is centered

FIND

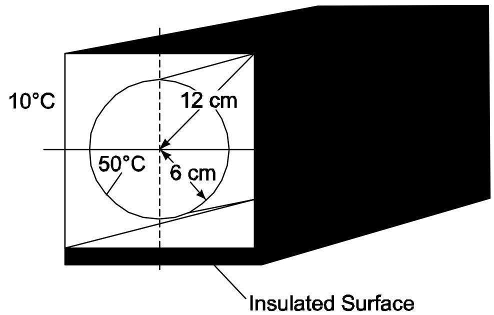
(a) The temperature distribution in the block

(b) The heat flow rate per meter length

ASSUMPTIONS

- The heat flow is two dimensional and in steady state
- The thermal conductivity in the block is uniform

SKETCH

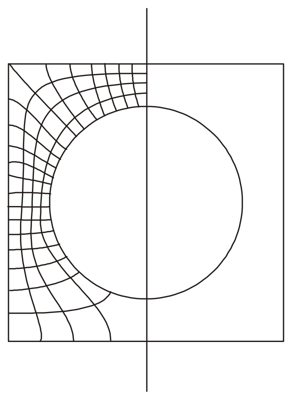


PROPERTIES AND CONSTANTS

From Appendix 2, Table 11
The thermal conductivity of concrete (k_b) = 0.128 W/(m K) at 20°C

SOLUTION

The temperature distribution and heat flow rate may be estimated with a flux plot



(a) The temperature distribution is given by the isotherms in the flux plot.
(b) The number of flow lanes (M) = 2 × 21 = 42
The number of squares per lane (N) = 4

Therefore, the shape factor is

$$S = \frac{M}{N} = \frac{42}{4} = 10.5$$

From Equation (2.80), the rate of heat flow per unit length is

$$q = kS\Delta T = [0.128 \text{ W/(m K)}]\ (10.5)\ (40°\text{C}) = 54 \text{ W/m}$$

COMMENTS

If the lower surface were not insulated, the shape factor from Table 2.2, would be

$$S = \frac{2\text{p}}{\text{In}\left(1.08\frac{\text{W}}{\text{D}}\right)} = 14.8 \Rightarrow q = 75.6 \text{ W/m}$$

The rate of heat transfer with the insulation as calculated with the flux plot is about 29% less than the analytical result without insulation. We would expect a reduction of slightly less than 25%.

PROBLEM 2.52

A 30-cm-*OD* pipe with a surface temperature of 90°C carries steam over a distance of 100 m. The pipe is buried with its center line at a depth of 1 m, the ground surface is –6°C, and the mean thermal conductivity of the soil is 0.7 W/(m K). Calculate the heat loss per day, and the cost, if steam heat is worth \$3.00 per 10^6 kJ. Also, estimate the thickness of 85% magnesia insulation necessary to achieve the same insulation as provided by the soil with a total heat transfer coefficient of 23 W/(m^2 K) on the outside of the pipe.

GIVEN

- A buried steam pipe
- Outside diameter of the pipe (D) = 30 cm = 0.3 m
- Surface temperature (T_s) = 90°C
- Length of pipe (L) = 100 m
- Depth of its center line (Z) = 1 m
- The ground surface temperature (T_g) = –6°C
- The mean thermal conductivity of the soil (k) = 0.7 W/(m K)
- Steam heat is worth \$3.00 per 10^6 kJ
- The heat transfer coefficient (h_c) = 23 W/(m^2 K) for the insulated pipe

FIND

(a) The heat loss per 24 hour day

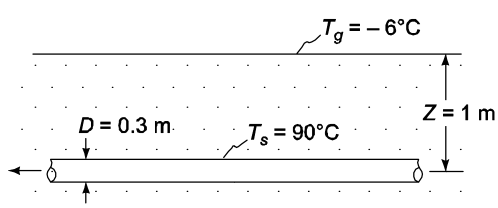
(b) The value of the lost heat

(c) The thickness of 85% magnesia insulation necessary to achieve the same insulation

ASSUMPTIONS

- Steady state conditions
- Uniform thermal conductivity
- Two dimensional heat transfer from the pipe

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 11

The thermal conductivity of 85% magnesia (k_i) = 0.059 W/(m K) (at 20°C)

SOLUTION

(a) The shape factor for this problem, from Table 2.2, is

$$S = \frac{2\pi L}{\cosh^{-1}\left(\frac{2z}{D}\right)} \quad \text{If} \quad z/L < 1$$

Note that the condition $Z/L << 1$ is satisfied in this problem.

$$S = \frac{2\pi\,(100\,\text{m})}{\cosh^{-1}\left(\dfrac{2(1\,\text{m})}{0.3\,\text{m}}\right)} = 243\ \text{m}$$

From Equation (2.80), the rate of heat transfer is

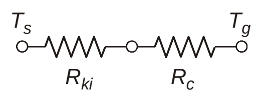
$$q = kS\Delta T = 0.7\ \text{W/(m K)}\ (243\ \text{m})\ (90°\text{C} - (-6°\text{C}))$$

$$q = 16{,}300\ \text{W}\ \ (\text{J/Ws})\left(\frac{\text{kJ}}{1000\,\text{J}}\right)\left(\frac{3600\,\text{s}}{\text{h}}\right)\left(\frac{24\,\text{h}}{\text{day}}\right) = 1.41 \times 10^6\ \text{kJ/Day}$$

(b) The cost of this heat loss is

$$\text{Cost} = \left(1.41\times 10^6\,\text{kJ/day}\right)\left(\frac{\$3.00}{10^6\,\text{kJ}}\right) = \$4.23/\text{day}$$

(c) The thermal circuit for the pipe covered with insulation is



The rate of heat loss from the pipe is

$$q = \frac{T_s - T_g}{R_{ki} + R_c} = \frac{T_s - T_g}{\dfrac{1}{2\pi Lk_i}\ln\left(\dfrac{r_o}{r_1}\right) + \dfrac{1}{2\pi\, Lr_oh_c}} = 16{,}300\ \text{W}$$

$$16{,}300\ \text{W} = \frac{2\pi L(T_s - T_g)}{\dfrac{1}{k_i}\ln\left(\dfrac{r_o}{r_i}\right) + \dfrac{1}{r_oh_c}} = \frac{2\pi L(100\,\text{m})[90°C - (-6°)]}{\dfrac{1}{0.059\,\text{W/(m K)}}\ln\left(\dfrac{r_o}{0.15\,\text{m}}\right) + \dfrac{1}{r_o\left(23\,\text{W/(m}^2\text{K)}\right)}}$$

$$\ln\ \frac{r_o}{0.15} + 0.00257\ \frac{1}{r_o} = 0.2183$$

By trial and error: $r_o = 0.184$ m

Insulation thickness = $r_o - r_i$ = 0.184 m – 0.15 m = 0.034 m = 3.4 cm

COMMENTS

The value of the heat loss per year is 365 × \$4.23 = \$1544. Hence insulation will pay for itself quite rapidly.

PROBLEM 2.53

Two long pipes, one having a 10-cm-*OD* and a surface temperature of 300°C, the other having a 5-cm-*OD* and a surface temperature of 100°C, are buried deeply in dry sand with their centerlines 15 cm apart. Determine the rate of heat flow from the larger to the smaller pipe per meter length.

GIVEN

- Two long pipes buried deeply in dry sand
- Pipe 1
  - Diameter (D_1) = 10 cm = 0.1 m,
  - Surface temperature (T_1) = 300°C
- Pipe 2

- Diameter (D_2) = 5 cm = 0.05 m,
- Surface temperature (T_2) = 100°C
- Spacing between their centerlines (s) = 15 cm = 0.15 m

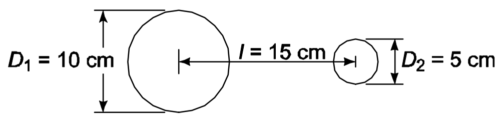
FIND

- The rate of heat flow per meter length (q/L)

ASSUMPTIONS

- The heat flow between the pipes is two dimensional
- The system has reached steady state
- The thermal conductivity of the sand is uniform

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 11
Thermal conductivity of dry sand (k) = 0.582 W/(m K) at 20°C

SOLUTION

The shape factor for this geometry, from Table 2.2, is

$$S = \frac{2\pi}{\cosh^{-1}\left(\frac{L^2 - 1 - r^2}{2r}\right)}$$

where $$r = \frac{r_1}{r_2} = \frac{5\,\text{cm}}{2.5\,\text{cm}} = 2 \text{ and } L = \frac{l}{r_2} = \frac{15\,\text{cm}}{2.5\,\text{cm}} = 6$$

$$\therefore\ S = \frac{2\pi}{\cosh^{-1}\left(\frac{36 - 1 - 4}{4}\right)} = 2.296$$

The rate of heat transfer per unit length is

$$q = Sk\Delta T = (2.296)\ 0.582\ \text{W/(m K)}\ (300°\text{C} - 100°\text{C}) = 267\ \text{W/m}$$

PROBLEM 2.54

> **A radioactive sample is to be stored in a protective box with 4 cm thick walls having interior dimensions 4 by 4 by 12 cm. The radiation emitted by the sample is completely absorbed at the inner surface of the box, which is made of concrete. If the outside temperature of the box is 25°C, but the inside temperature is not to exceed 50°C, determine the maximum permissible radiation rate from the sample, in watts.**

GIVEN

- A radioactive sample in a protective concrete box
- Wall thickness (t) = 4 cm = 0.4 m
- Box interior dimensions: 4 × 4 × 12 cm
- All radiation emitted is completely absorbed at the inner surface of the box
- The outside temperature of the box (T_o) = 25°C
- The maximum inside temperature (T_i) = 50°C

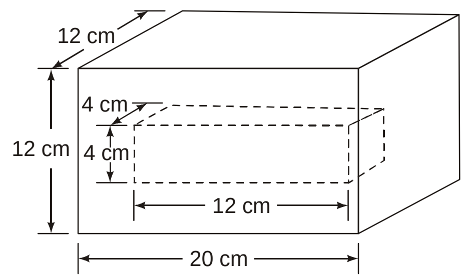
FIND

- The maximum permissible radiation rate from the sample, q (in watts)

ASSUMPTIONS

- The system is in steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 11
The thermal conductivity of concrete (k_b) = 0.128 W/(m K) at 20°C

SOLUTION

The box consists of 4 wall sections: A = 4 cm × 12 cm
2 wall sections: A = 4 cm × 4 cm
4 edge sections: D = 12 cm long
8 edge sections: D = 4 cm long
8 corner sections: L = 4 cm thick

The shape factors for this geometry (when all interior dimensions are greater than one-fifth of the wall thickness, as in this case) is given on Section 2.5.2 of the text
For the wall sections

$$S_1 = \frac{A}{L} = \frac{(4\,\text{cm})(12\,\text{cm})}{4\,\text{cm}} = 12\text{ m} \quad \text{and} \quad S_2 = \frac{A}{L} = \frac{(4\,\text{cm})(4\,\text{cm})}{4\,\text{cm}} = 4\text{ cm}$$

For the edge sections

$$S_3 = 0.54\,D = 0.54\ (12\text{ cm}) = 6.48\text{ cm} \quad \text{and} \quad S_4 = 0.54\,D = 0.54\ (4\text{ cm}) = 2.16\text{ cm}$$

For the corner sections

$$S_5 = 0.15\,L = 0.15\ (4\text{ cm}) = 0.6\text{ cm}$$

Multiplying each shape factor by the number of elements having that shape factor and summing them

$$S = 4\,S_1 + 2\,S_2 + 4\,S_3 + 8\,S_4 + 8\,S_5$$

$$S = 4\ (12\text{ cm}) + 2(4\text{ cm}) + 4(6.48\text{ cm}) + 8(2.16\text{ cm}) + 8(0.6\text{ cm}) = 104\text{ cm}$$

The rate of heat transfer is

$$q = kS\Delta T = 0.128\text{ W/(m K)}\ (104\text{ cm})\ (1\text{m}/100\text{ cm})\ (50°\text{C} - 25°\text{C}) = 3.3\text{ W}$$

COMMENTS

The conductivity of the concrete was evaluated at 20°C while the actual temperature is between 50°C and 25°C. Therefore, the actual rate of heat flow may be slightly different than that calculated, but no better property value is available in the text.

PROBLEM 2.55

A 6-in.-*OD* pipe is buried with its centerline 50 in. below the surface of the ground [k of soil is 0.20 Btu/(h ft °F)]. An oil having a density of 6.7 lb/gal and a specific heat of 0.5 Btu/(lb °F) flows in the pipe at 100 gpm. Assuming a ground surface temperature of 40°F and a pipe wall temperature of 200°F, estimate the length of pipe in which the oil temperature decreases by 10°F.

GIVEN

- An oil filled pipe buried below the surface of the ground
- Pipe outside diameter (D) = 6 in. = 0.5 ft
- Depth of centerline (z) = 50 in.
- Thermal conductivity of the soil (k) = 0.20 Btu/h ft °F
- Specific gravity of oil (Sp. Gr.) = 0.8
- Specific heat of oil (c_p) = 0.5 Btu/lb °F
- Flows rate of oil $\dot{m}$ = 100 gpm
- The ground surface temperature (T_s) = 40°F
- The pipe wall temperature (T_p) = 200°F

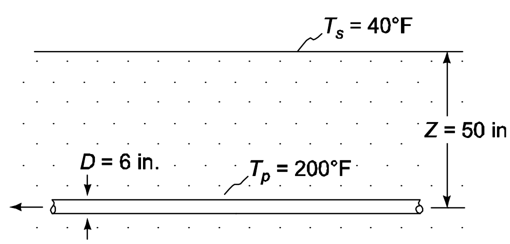
FIND

- The length of pipe (L) in which the oil temperature decreases by 10°F

ASSUMPTIONS

- Steady state condition
- Two dimensional heat transfer

SKETCH



SOLUTION

The rate of heat flow from the pipe can be calculated using the shape factor from Table 2.2 for an infinitely long cylinder

$$S = \frac{2p}{\cosh^{-1}\left(\frac{2Z}{D}\right)} = \frac{2\pi}{\cosh^{-1}\left(\frac{2(50\,\text{in})}{6\,\text{in}}\right)} = 1.79$$

The rate of heat transfer per unit length is given by Equation (2.80)

$$q = kS\Delta T_{\text{overall}} = (0.20\ \text{Btu/(h ft °F)})\ (1.79)\ (200\ \text{F} - 40\ \text{F}) = 57.4\ \text{Btu/(h ft)}$$

The total heat loss required to decrease the oil by 10°F is

$$q_t = \dot{m}\ c_p\ \Delta T = 100\ [\text{g/(min)}]\ (6.7\ \text{lb/gallon})(0.5\ \text{Btu/lb °F})\ (10\text{°F})\ (60\ \text{min/hr}) = 200{,}000\ \text{Btu/hr}$$

We can estimate the length of pipe in which the oil temperature drops 10°F by assuming the rate of heat loss from the pipe per unit length is constant, then:

$$q_t = qL \Rightarrow L = \frac{q_t}{q} = \frac{200{,}000\ \text{Btu/h}}{57.4\ \text{Btu/(h ft)}} = 3481\ \text{ft}$$

COMMENTS

The heat loss from the pipe will actually be less because as the oil temperature and therefore also the pipe temperature decreases with distance from the inlet. This means the length will be slightly longer than the estimate above. If the calculation is based on an arithmetic mean pipe temperature of 195°F, the estimated length is 3604 ft, about 4% more.

PROBLEM 2.56

> **A 2.5-cm-*OD* hot steam line at 100°C runs parallel to a 5.0 cm *OD* cold water line at 15°C. The pipes are 5 cm center to center and deeply buried in concrete with a thermal conductivity of 0.87 W/(m K). What is the heat transfer per meter of pipe between the two pipes?**

GIVEN

- A hot steam line runs parallel to a cold water line buried in concrete
- Hot pipe outside diameter (D_h) = 2.5 cm = 0.025 m
- Hot pipe temperature (T_h) = 100°C
- Cold pipe outside diameter (D_c) = 5.0 cm = 0.05 m
- Cold pipe temperature (T_c) = 15°C
- Center to center distance between pipes (l) = 5 cm = 0.05 m
- Thermal conductivity of concrete (k) = 0.87 W/(m K)

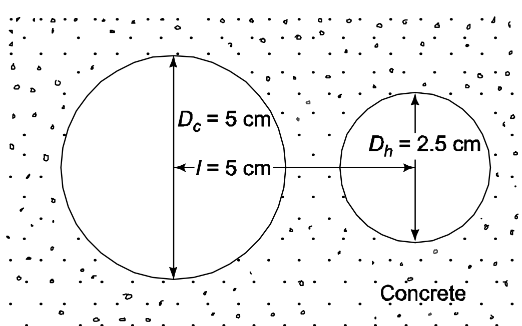
FIND

- The heat transfer per meter of pipe (q/L)

ASSUMPTIONS

- Two dimensional heat transfer between the pipes
- Steady state conditions
- Uniform thermal conductivity

SKETCH



PROPERTIES AND CONSTANTS

Specific heat of water (c_p) = 1 Btu/(lb °F) = 4187 J/(kg K)

SOLUTION

The shape factor for this geometry is in Table 2.2

$$S = \frac{2\pi}{\cosh^{-1}\left(\dfrac{L^2 - 1 - r^2}{2r}\right)}$$

Where $$L = \frac{1}{D_h} = \frac{0.05\,\text{m}}{\cosh^{-1}\left(\dfrac{0.025\,\text{m}}{2}\right)} = 4 \text{ and } r = \frac{r_c}{r_h} = \frac{D_c}{D_h} = \frac{0.05}{0.025} = 2$$

$$\therefore S = \frac{2\pi}{\cosh^{-1}\left(\dfrac{16 - 1 - 4}{4}\right)} = 3.763$$

The rate of heat transfer per unit length, from Equation (2.80), is

$$q = kS\Delta T_{\text{overall}} = 0.87 \text{ W/(m K)} (3.763) (100°\text{C} – 15°\text{C}) = 278 \text{ W/m}$$

COMMENTS

Normally, the temperature of both fluids will change as heat is transferred between them. Hence, for any appreciable length of pipe, an average temperature difference must be used.

PROBLEM 2.57

> **Calculate the rate of heat transfer between a 15-cm-*OD* pipe at 120°C and a 10-cm-*OD* pipe at 40°C. The two pipes are 330 m long and are buried in sand [k = 0.33 W/(m K)] 12 m below the surface (T_s = 25°C). The pipes are parallel and are separated by 23 cm (center to center) distance.**

GIVEN

- Two parallel pipes buried in sand
- Pipe 1
  - Outside diameter (D_1) = 15 cm = 0.15 m
  - Temperature (T_1) = 120°C
- Pipe 2
  - Outside diameter (D_2) = 10 cm = 0.1 m
  - Temperature (T_2) = 40°C
- Length of pipes (L) = 330 m
- Thermal conductivity of the sand (k) = 0.33 W/(m K)
- Depth below surface (d) = 1.2 m
- Surface temperature (T_s) = 25°C
- Center to center distance between pipes (s) = 23 cm = 0.23 m

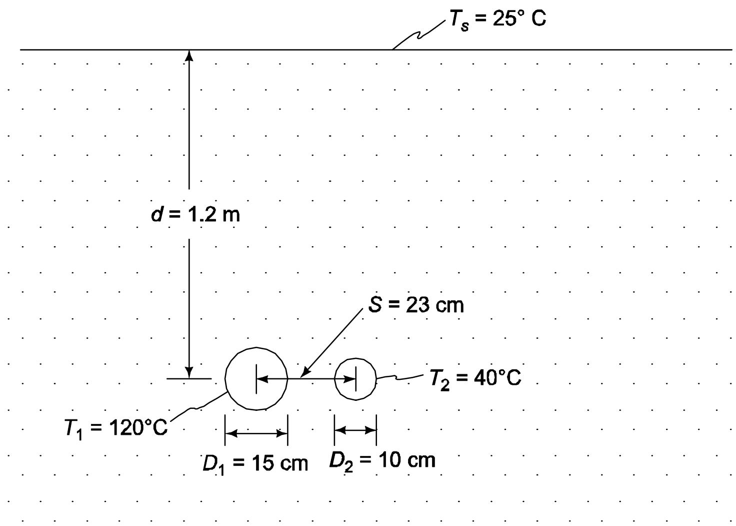
FIND

- The rate of heat transfer between the pipes (q)

ASSUMPTIONS

- The thermal conductivity of the sand is uniform
- Two dimensional, steady state heat transfer

SKETCH



SOLUTION

For the pipe-to-pipe heat transfer, the surface is not important since $Z >> D$. The shape factor for this geometry, from Table 2.2, is

$$S = \frac{2\pi}{\cosh^{-1}\left(\frac{L^2 - 1 - r^2}{2r}\right)}$$

$$\text{where} \quad L = \frac{1}{r_2} = \frac{0.23\,\text{m}}{0.05\,\text{m}} = 4.6 \quad \text{and} \quad r = \frac{r_1}{r_2} = \frac{D_1}{D_2} = \frac{15\,\text{m}}{0.1\,\text{m}} = 1.5$$

$$\therefore\ S = \frac{2\pi}{\cosh^{-1}\left(\frac{(4.6)^2 - 1 - (1.5)^2}{2(1.5)}\right)} = 2.541$$

The rate of heat transfer per unit length is

$$\frac{q}{L} = kS\Delta T = 0.33\ \text{W/(m K)}\ (2.541)\ (120°\text{C} - 40°\text{C}) = 67\ \text{W/m}$$

$$\text{For } L = 330\ \text{m: } q = 67\ \text{W/m}\ (330\ \text{m}) = 22{,}100\ \text{W}$$

COMMENTS

Normally, the temperature of both fluids will change as heat is transferred between them. Hence, for any appreciable length of pipe, an average temperature difference must be used.

PROBLEM 2.58

> **A 0.6-cm-diameter mild steel rod at 38°C is suddenly immersed in a liquid at 93°C with $\bar{h}_c$ = 110 W/(m^2 K). Determine the time required for the rod to warm to 88°C.**

GIVEN

- A mild steel rod is suddenly immersed in a liquid
- Rod diameter (D) = 0.6 cm = 0.006 m
- Initial temperature of the rod (T_o) = 38°C
- Liquid temperature (T_∞) = 93°C
- Heat transfer coefficient ($\bar{h}_c$) = 113.5 W/(m^2 K)

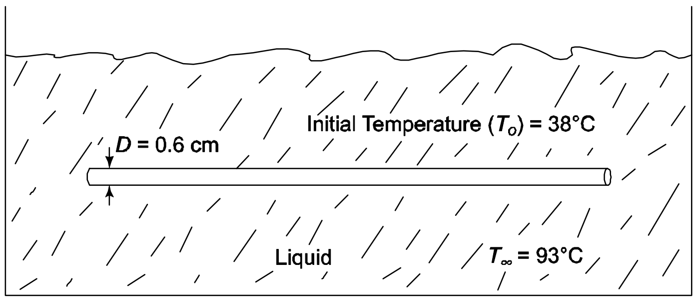
FIND

- The time required for the rod to warm to 88°C

ASSUMPTIONS

- The rod is 1% carbon steel
- Constant thermal conductivity
- End effects are negligible
- The rod is very long compared to its diameter
- There is radial conduction only in the rod

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10: For 1% carbon steel at 20°C:
  Thermal conductivity (k) = 43 W/(m K)
  Specific heat (c) = 473 J/(kg K)
  Density (ρ) = 7801 kg/m^3
  Thermal diffusivity (α) = 1.172 × 10^{-5} m^2/s. [$\alpha = k/\rho c$].

SOLUTION

The Biot number is calculated first to check if the internal resistance is negligible

$$B_i = \frac{\bar{h}_c D}{4k} = \frac{\left(110\,\text{W/(m}^2\text{K)}\right)(0.006\,\text{m})}{4\left(43\,\text{W/(m}^2\text{ K)}\right)} = 0.0038 << 0.1$$

Therefore, the internal resistance of the rod is negligible.
The temperature-time history of the rod, from Equation (2.84) is

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(\frac{\bar{h}_c A_s}{c\rho V} t\right)$$

$$\frac{\bar{h}_c A_s}{c\rho V} = \frac{\bar{h}_c \pi D L}{c\rho \frac{\pi}{4} D^2 L} = \frac{4\bar{h}_c}{c\rho D} = \frac{4\left(100\,\text{W/(m}^2\text{K)}\right)(\text{J/Ws})}{(473\,\text{W/kg K})\left(7801\,\text{Kg/m}^3\right)(0.006\,\text{m})} = 0.020\ 1/S$$

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left[-\left(0.020\frac{1}{S}\right)t\right]$$

Solving for the time

$$t = -(50.3\text{ s})\ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

The time required to reach 88°C is

$$t = -(50.3\text{ s})\ln\left(\frac{88 - 93}{38 - 93}\right) = 121\text{s}$$

COMMENTS

The analysis has assumed that the heat capacity of the liquid is much larger than that of the rod and thus the liquid temperature remains constant.

PROBLEM 2.59

A spherical shell satellite (3-m-*OD*, 1.25-cm-wall thickness, made of stainless steel) reenters the atmosphere from outer space. If its original temperature is 38°C, the effective average temperature of the atmosphere is 1093°C, and the effective heat transfer coefficient is 115 W/(m^2 °C), estimate the temperature of the shell after reentry, assuming the time of reentry is 10 min and the interior of the shell is evacuated.

GIVEN

- A spherical stainless steel satellite reentering the atmosphere
- Outside diameter (D) = 3 m
- Wall thickness (L) = 1.25 cm = 0.0125 m
- Its original temperature (T_o) = 38°C
- The effective temperature of the atmosphere (T_∞) = 1093°C
- The effective heat transfer coefficient $\bar{h}_c$ = 115 W/(m^2 °C)
- The time of reentry (t_r) = 10 min = 600 s
- The interior of the shell is evacuated

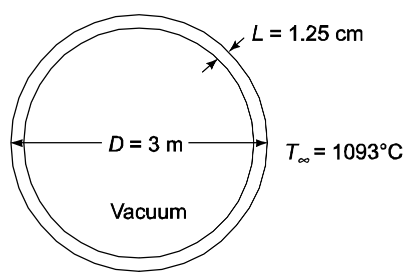
FIND

- The temperature of the shell after reentry (T_f)

ASSUMPTIONS

- Exterior heat transfer is uniform over the shell
- Assume radiation heat transfer is allowed for in the heat transfer coefficient

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10, for stainless steel at 20°C

Thermal conductivity (k) = 14.4 W/(m K)
Density (ρ) = 7817 kg/m^3
Specific heat(c) = 461 J/(kg K)

SOLUTION

Since the thickness of the shell is much smaller than the shell radius, the wall can be treated as a plane wall. To estimate the importance of internal thermal resistance, the Biot number is calculated first

$$B_i = \frac{\bar{h}_c L}{k_s} = \frac{[115\,\text{W/(m}^2\,°\text{C)}](0.0125\,\text{m})}{14.4\,\text{W/(m K)}} = 0.099 < 0.1$$

Therefore, the internal resistance is less than 10% of the external resistance and may be neglected. The temperature-time history of the satellite is given by Equation (2.84):

$$\frac{T - T_\infty}{T_s - T_\infty} = \exp\left(-\frac{\bar{h}_c A_s}{c\rho V}t\right) = \exp\,(-\,Bi\,Fo)$$

$$Bi\,Fo = \left(\frac{\bar{h}_c L}{k_s}\right)\left(\frac{\alpha t}{L^2}\right) = \frac{\bar{h}_c A_s}{c\rho V}t = \frac{\bar{h}_c \pi D^2 t}{c\rho\frac{4}{3}\pi\left[\left(\frac{D}{2}\right)^3 - \left(\frac{D}{2} - L\right)^3\right]}$$

$$Bi\,Fo = \frac{[115\,\text{W/(m}^2\text{K)}](3\,\text{m})^2\,(\text{J/(W s)})t}{[461\,\text{J/(kg K)}]\left(7817\,\text{kg/m}^3\right)\frac{4}{3}[(1.5\,\text{m})^3 - (1.5\,\text{m} - 0.0125\,\text{m})^3]}$$

$$= 0.0025t \text{ (t in seconds)}$$

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-0.0025t}$$

$$T = T_\infty + (T_o - T_\infty)e^{-0.0025t}$$

$$T_f = 1093°\text{C} + (38°\text{C} - 1093°\text{C})\,e^{-0.0025(600)} = 868°\text{C}$$

COMMENTS

The analysis has neglected thermodynamic heating during reentry.

PROBLEM 2.60

> **A thin-wall cylindrical vessel (1 m in diameter) is filled to a depth of 1.2 m with water at an initial temperature of 15°C. The water is well stirred by a mechanical agitator. Estimate the time required to heat the water to 50°C if the tank is suddenly immersed into oil at 105°C. The overall heat transfer coefficient between the oil and the water is 284 W/(m^2 K), and the effective heat transfer surface area is 4.2 m^2.**

GIVEN

- A thin wall cylindrical vessel filled with water is suddenly immersed into oil
- Diameter of vessel (D) = 1 m
- Depth of water is vessel = 1.2 m
- Initial temperature (T_o) = 15°C
- Final temperature (T_f) = 50°C
- Oil temperature (T_∞) = 105°C
- The overall heat transfer coefficient between the oil and water $(\bar{h})$ = 284 W/(m^2 K)
- The effective heat transfer surface area (A) = 4.2 m^2

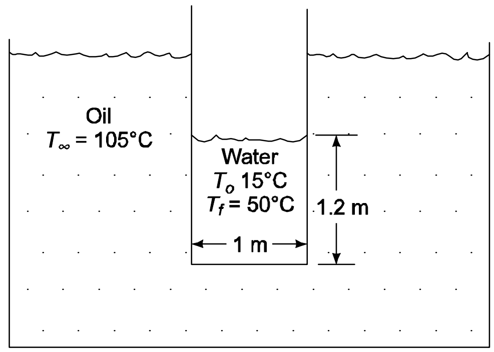
FIND

- The time required to heat the water to 50°C

ASSUMPTIONS

- The thermal capacitance of the cylindrical vessel is negligible
- The temperature of the water is uniform
- The oil temperature remains constant

SKETCH



PROPERTIES AND CONSTANTS

Specific heat of water (c) = 1 Btu/lb = 4187 J/(kg K)
Density of water (ρ) = 1000 kg/m^3

SOLUTION

From Equation (2.83), the temperature-time relationship is

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(-\frac{\bar{h} A_s}{c \rho V} t\right)$$

Solving for the time

$$t = \frac{-c \rho V}{\bar{h} A_s} \ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

$$t = \frac{-(4187\,\text{J/(kg K)})\left(1000\,\text{kg/m}^3\right)[\pi\,(0.5\,\text{m})^2\,(1.2\,\text{m})]}{[284\,\text{W/(m}^2\text{K)}](4.2\,\text{m}^2)(\text{J/(W s)})} \ln\left(\frac{50°\text{C} - 105°\text{C}}{15°\text{C} - 105°\text{C}}\right)$$

$$= 1629 \text{ s} = 27 \text{ min}$$

PROBLEM 2.61

A thin-wall jacketed tank, heated by condensing steam at one atmosphere contains 91 kg of agitated water. The heat transfer area of the jacket is 0.9 m^2 and the overall heat transfer coefficient U = 227 W/(m^2 K) based on that area. Determine the heating time required for an increase in temperature from 16°C to 60°C.

GIVEN

- A thin wall jacketed tank, heated by condensing steam
- Steam pressure = one atmosphere
- Mass of water in the tank = 91 kg
- The heat transfer area (A) = 0.9 m^2
- The overall heat transfer coefficient (U) = 227 W/(m^2 K) based on that area
- Temperature increases from 16°C to 60°C

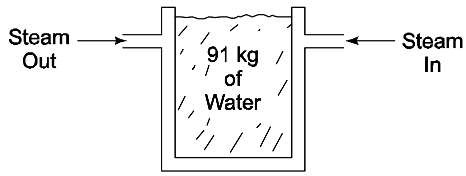
FIND

- Determine the heating time required

ASSUMPTIONS

- Uniform water temperature due to agitation
- Thermal capacitance of the tank wall is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13

The specific heat of water (c) = 1 Btu/lb = 4187 J/(kg K)

Temperature of saturated steam at 1 atmosphere (1.01×10^5 ρa) = 100°C

SOLUTION

The temperature-time history for this system is given by Equation (2.83).

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(-\frac{UA_s}{c\rho V}t\right) = \exp\left(-\frac{UA_s}{c\rho\left(\frac{m}{\rho}\right)}t\right) = \exp\left(-\frac{UA_s}{cm}t\right)$$

Solving this expression for the time

$$t = -\frac{cm}{UA_s}\ln\left(\frac{T_f - T_\infty}{T_o - T_\infty}\right) = -\frac{[4187\,\mathrm{J/(kg\,K)}](91\,\mathrm{kg})((\mathrm{Ws/J}))}{[227\,\mathrm{W/(m^2K)}](0.9\,\mathrm{m^2})}\ln\left(\frac{60°\mathrm{C} - 100°\mathrm{C}}{16°\mathrm{C} - 100°\mathrm{C}}\right) = 1384\ \mathrm{s} = 23$$

min.

PROBLEM 2.62

The heat transfer coefficients for the flow of 26.6°C air over a 1.25 cm diameter sphere are measured by observing the temperature-time history of a copper ball of the same dimension. The temperature of the copper ball (c = 376 J/(kg K), ρ = 8928 kg/m^3) was measured by two thermocouples, one located in the center, and the other near the surface. Both of the thermocouples registered, within the accuracy of the recording instruments, the same temperature at a given instant. In one test run, the initial temperature of the ball was 66°C and in 1.15 min, the temperature decreased by 7°C. Calculate the heat transfer coefficient for this case.

GIVEN

- A copper ball with air flowing over it
- Ball diameter (D) = 1.25 cm = 0.0125 m
- Air temperature (T_∞) = 26.6°C
- Specific heat of ball (c) = 376 J/(kg K)
- Density of the ball (P) = 8928 kg/m^3
- Thermocouples in the center and the surface registered the same temperature
- Initial temperature of the ball (T_o) = 66°C
- Lapse time = 1.15 min = 69 s
- The temperature decrease ($T_o - T_f$) = 7°C

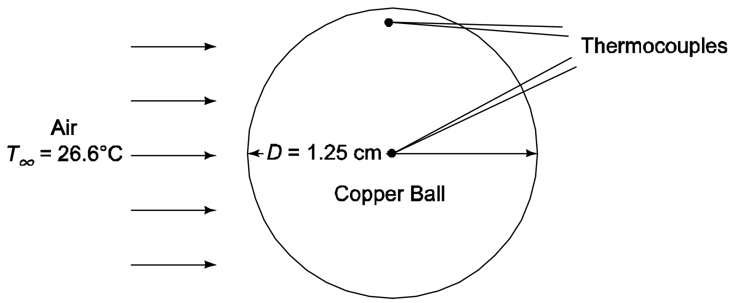
FIND

- The heat transfer coefficient $\left(\bar{h}_c\right)$

ASSUMPTIONS

- The heat transfer coefficient remains constant during the cooling period.

SKETCH



SOLUTION

Since the thermocouples register essentially the same temperature, the internal resistance of the ball is small compared to the external resistance and the ball can be treated with the lumped heat capacity method.

From Equation (2.84) the temperature-time history is

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(-\frac{\bar{h}_c A}{c\rho V} t\right) = \exp\left(-\frac{\bar{h}_c (\pi D^2)}{c\rho\left(\frac{\pi}{6} - D^3\right)} t\right) = \exp\left(\frac{-6\bar{h}_c}{c\rho D} t\right)$$

Solving for the heat transfer coefficient

$$\bar{h}_c = \frac{c\rho D}{6t} \ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

$$\bar{h}_c = -\frac{[376\text{J/(kg K)}]\left(8928\,\text{kg/m}^3\right)(0.0125\,\text{m})}{6(69\,\text{s})(\text{J/(Ws)})} \ln\left(\frac{(66°\text{C} - 7°\text{C}) - 26.6°\text{C}}{66°\text{C} - 26.6°\text{C}}\right)$$

$$= 19.8\ \text{W/(m}^2\,\text{K)}$$

COMMENTS

The value is an average over the cooling period.

The procedure described by this problem can be used to evaluate heat transfer coefficients for odd shaped object experimentally.

PROBLEM 2.63

> **A spherical stainless steel vessel at 93°C contains 45 kg of water initially at the same temperature. If the entire system is suddenly immersed in ice water, determine (a) the time required for the water in the vessel to cool to 16°C, and (b) the temperature of the walls of the vessel at that time. Assume that the heat transfer coefficient at the inner surface is 17 W/(m^2 K), the heat transfer coefficient at the outer surface is 22.7 W/(m^2 K), and the wall of the vessel is 2.5 cm thick.**

GIVEN

- A spherical stainless steel vessel of water is suddenly immersed in ice water
- Initial temperature of vessel and water (T_i) = 93°C

- Mass of water in the vessel (m) = 45 kg
- The inner heat transfer coefficient $\bar{h}_{ci}$ = 17 W/(m^2 K)
- The outer heat transfer coefficient $\bar{h}_{co}$ = 22.7 W/(m^2 K)
- The vessel wall thickness (L) = 2.5 cm = 0.025 m

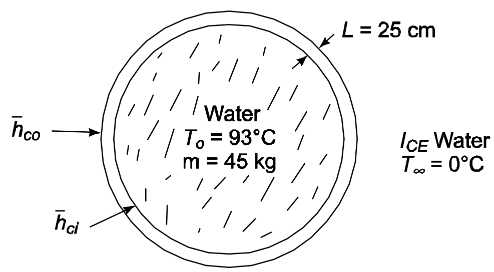
FIND

(a) The time required for the water in the vessel to cool to 16°C
(b) The temperature of the walls of the vessel at that time (T_{sf})

ASSUMPTIONS

- The water in the vessel is well mixed, therefore its temperature is uniform
- The vessel is completely filled with water

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10
For stainless steel: The thermal conductivity (k_s) = 14.4 W/(m K)
Density (ρ) = 7817 kg/m^3
Specific heat (c) = 461 J/(kg K)

SOLUTION

If the vessel is completely filled with water

$$V = \frac{m_w}{\rho} = \frac{\pi}{6} D_1^{\,3}$$

$$D_i = \left(\frac{6\, m_w}{\pi\, \rho}\right)^{\frac{1}{3}} = \left(\frac{6\,(45\,\mathrm{kg})}{\pi\,(1000\,\mathrm{kg/m^3})}\right)^{\frac{1}{3}} = 0.44\ \mathrm{m}$$

$$D_o = D_i + 2\,L = 0.44\ \mathrm{m} + 2\,(0.025\ \mathrm{m}) = 0.49\ \mathrm{m}$$

The internal resistance of the water can be neglected since the water is assumed to be well mixed. The importance of the internal resistance of the vessel wall is indicated by the Biot number of the vessel wall. The characteristic length for the vessel wall is

$$L = \frac{\text{volume}}{\text{Surface area}} = \frac{\frac{\pi}{6}(D_o^3 - D_i^3)}{\pi\,(D_o^2 + D_i^2)} = \frac{1}{6}\,\frac{(0.49\,\mathrm{m})^3 - (0.44\,\mathrm{m})^3}{(0.49\,\mathrm{m})^2 + (0.44\,\mathrm{m})^2} = 0.0125\ \mathrm{m}$$

$$\therefore\ Bi = \frac{\bar{h}\,L}{k_s} = \frac{\frac{1}{2}(\bar{h}_{ci} + \bar{h}_{\infty})\,L}{k_s} = \frac{\frac{1}{2}(17 + 22.7)\,[\mathrm{W/(m^2K)}]\,(0.0125\,\mathrm{m})}{14.4\,\mathrm{W/(m\,K)}} = 0.017 < 0.1$$

Therefore, the vessel and its contents can be treated as a lumped capacitance and the system approximated two lumped capacitances as covered in Section 2.6.1 of the text.

(a) The temperature-time history of the water in the vessel is given by Equation (2.87)

$$\frac{T_w - T_\infty}{T_0 - T_\infty} = \frac{m_2}{m_2 - m_1} e^{m_1 t} - \frac{m_1}{m_2 - m_1} e^{m_2 t}$$

where T_w = temperature of the water, a function of time

$m_1 = 0.5 \ \{-(k_1 + k_2 + k_3) + [(k_1 + k_2 + k_3)^2 - 4k_1 k_3]^{0.5}\}$

$m_2 = 0.5 \ \{-(k_1 + k_2 + k_3) - [(k_1 + k_2 + k_3)^2 - 4k_1 k_3]^{0.5}\}$

$$k_1 = \frac{\bar{h}_{ci} A_i}{\rho_w c_w V_i} = \frac{\bar{h}_{ci}\, \pi\, D_i^2}{\rho_w c_w \frac{\pi}{6} D_i^3} = \frac{6\bar{h}_{ci}}{\rho_w c_w\, D_i} = \frac{6\left(17\,\mathrm{W/(m^2K)}\right)}{1000\,\mathrm{kg/m^3}\left(4187\,\mathrm{J/(kg\,K)}\right)(0.44\,\mathrm{m})} = 5.53 \times 10^{-5}\ \mathrm{1/s}$$

$$k_2 = \frac{\bar{h}_{ci} A_i}{\rho_s c_s V_s} = \frac{\bar{h}_{ci}\, \pi\, D_i^2}{\rho_s c_s \frac{\pi}{6}(D_o^3 - D_i^3)} = \frac{\left(17\,\mathrm{W/(m^2K)}\right)(0.44\,\mathrm{m})^2}{7817\,\mathrm{kg/m^3}\left(461\,\mathrm{J/(kg\,K)}\right)(1/6)\,(0.49^3 - 0.044^3)\,\mathrm{m^3}} = 1.69 \times 10^{-4}\ \mathrm{1/s}$$

$$k_3 = \frac{\bar{h}_{co}\, A_o}{\rho_s c_s V_s} = \frac{\bar{h}_{co}\, \pi\, D_o^2}{\rho_s c_s \frac{\pi}{6}(D_o^3 - D_i^3)} = \frac{\left(22.7\,\mathrm{W/(m^2K)}\right)(0.49\,\mathrm{m})^2}{7817\,\mathrm{kg/m^3}\left(461\,\mathrm{J/(kg/K)}\right)(1/6)\,(0.49^3 - 0.044^3)\,\mathrm{m^3}} = 2.79 \times 10^{-4}\ \mathrm{1/s}$$

$$k_1 + k_2 + k_3 = 5.04 \times 10^{-4}\ \mathrm{s^{-1}}$$

$$4k_1 k_3 = 6.17 \times 10^{-8}\ \mathrm{s^{-1}}$$

$$m_1 = -3.28 \times 10^{-5}\ \mathrm{s^{-1}}$$

$$m_2 = -4.71 \times 10^{-4}\ \mathrm{s^{-1}}$$

$$m_2 - m_1 = 4.38 \times 10^{-4}\ \mathrm{s^{-1}}$$

The temperature-time history of the water is

$$\frac{T_w - T_\infty}{T_o - T_\infty} = \frac{-4.71 \times 10^{-4}}{-4.38 \times 10^{-4}} e^{\left(-3.28 \times 10^{-5}\frac{1}{s}\right)t} - \frac{-3.28 \times 10^{-5}}{-4.38 \times 10^{-4}} e^{\left(-4.71 \times 10^{-4}\frac{1}{s}\right)t}$$

For the water to cool to 16°C

$$\frac{16°\mathrm{C} - 0°\mathrm{C}}{93°\mathrm{C} - 0°\mathrm{C}} = 0.1720 = 1.075\ E^{\left(-3.28 \times 10^{-5}\frac{1}{s}\right)t} - 0.075\ E^{\left(-4.71 \times 10^{-4}\frac{1}{s}\right)t}$$

By trial and error: t = 55,870 s = 15.5 hours

(b) The energy balance for the fluid is given by Equation (2.86a)

$$-(c\,\rho\,V)_w \frac{dT_w}{dt} = \bar{h}_i\, A_i\, (T_w - T_s)$$

Differentiating the temperature-time history

$$\frac{dT_w}{dt} = (T_0 - T_\infty)\left[\frac{m_1\, m_2}{m_2 - m_1} e^{m_1 t} - \frac{m_1\, m_2}{m_2 - m_1} e^{m_1 t}\right] = (T_0 - T_\infty)\ \frac{m_1 - m_2}{m_2 - m_1}\left(e^{m_1 t} - e^{m_2 t}\right)$$

Substituting this into the energy balance for the fluid

$$-\ (c\ \rho\ V)_w\,(T_0 - T_\infty)\ \frac{m_1\ m_2}{m_2\ -\ m_1}\,(e^{m_1 t}\ -\ e^{m_2 t})\ =\ \bar{h}_{ci}\ A_{\text{i}}\ (T_w - T_s)$$

$$T_0 =\ T_w +\ \frac{(\text{cm})_w}{\bar{h}_{ci}\ A_i}\ (T_0 - T_\infty)\ \frac{m_1\ m_2}{m_2\ -\ m_1}\,(e^{m_1 t}\ -\ e^{m_2 t})$$

$$T_{\text{s}}\ = 16°\text{C} +\ \frac{[4187\text{J/(kg K)}](45\ \text{kg})}{[17\ \text{W/(m}^2\text{K)}]\pi\,(0.44\,\text{m})^2}\,(93°\text{C} - 0°\text{C})\ \frac{-\ 3.28\ \times\ 10^{-5}(1/\text{s})\left(-\ 4.71\ \times\ 10^{-4}(1/\text{s})\right)}{-\ 4.38\ \times\ 10^{-4}(1/\text{s})}$$

$$\times\ \left(e^{-3.28\ \times\ 10^{-5}(1/\text{s})\ (55870\text{a})}\ -\ e^{-4.71\ \times\ 10^{-4}\ (1/\text{s})\ (55870\text{a})}\right)$$

$$T_s =\ 6.4\ \text{s}$$

PROBLEM 2.64

A copper wire, 1/32-in.-*OD*, 2 in. long, is placed in an air stream whose temperature rises at T_{air} = (50 + 25*t*)°F, where t is the time in seconds. If the initial temperature of the wire is 50°F, determine its temperature after 2 s, 10 s and 1 min. The heat transfer coefficient between the air and the wire is 7 Btu/(h ft^2 °F).

GIVEN

- A copper wire is placed in an air stream
- Wire diameter (D) = 1/32 in.
- Wire length (L) = 2 in.
- Air stream temperature is: T_{air} = (50 + 25t)°F
- The initial temperature of the wire (T_{o}) = 50°F
- The heat transfer coefficient $(\bar{h}_c)$ = 7 Btu/(h ft^2 °F)

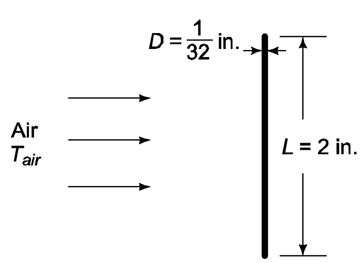
FIND

- The wire temperature after 2 s, 10 s and 1 min

ASSUMPTIONS

- Constant and uniform heat transfer coefficient

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12
For copper at 261°F

- Thermal conductivity (k) = 226 Btu/(h ft °F)
- Density (ρ) = 558 lb/ft^3
- Specific heat (c) = 0.0915 Btu/(lb °F)

SOLUTION

The Biot number for this problem is

$$Bi = \frac{\bar{h}_c D}{2\, k_c} = \frac{\left(7\ \text{Btu/(h ft}^2\,°\text{F)}\right)(1/32\,\text{in})\left(1\,\text{ft}/(12\,\text{in})\right)}{2\left(226\ \text{Btu/(h ft °F)}\right)} = 10^{-5} << 0.1$$

Therefore the internal resistance of the wire can be neglected.
The temperature-time history of the wire can be calculated from the energy balance, Equation (2.82)

$$-\,c\,\rho\,V\,dT = \bar{h}\ A_s\,(T - T_\infty)\,dt$$

but $T_\infty = T_{\text{air}} = 50 + 25t$

$$\therefore\ -\,c\,\rho\,V\,dT = \bar{h}\ A_s\,(T - 50 + 25t)\,dt$$

Rearranging

$$\frac{dT}{dt} = \frac{\bar{h}\,A_s}{c\,\rho\,V}\,(50 + 25t - T)$$

$$\text{let } m = \frac{\bar{h}\,A_s}{c\,\rho\,V} = \frac{\bar{h}\,(\pi\,DL)}{c\,\rho\left(\frac{\pi}{4}D^2L\right)} = \frac{4\,\bar{h}}{c\,\rho\,D} = \frac{4\left(7\text{Btu/(h ft}^2\,°\text{F)}\right)\left(1\,\text{hr}/(3600\,\text{s})\right)}{[0.0915\text{Btu/(lb °F)}]\left(558\text{lb/(ft}^2)\right)\left(\frac{1}{32}\,\text{in}\right)\left(1\,\text{ft}/(12\,\text{in})\right)} = 0.0585\ \text{s}^{-1}$$

$$\therefore\ \frac{dT}{dt} + m\,t = 25\,m\,(2 + t)$$

This is a linear, first order, non-homogeneous differential equation with a homogeneous solution of $T = c\,e^{-mt}$ and a particular solution $T = c_o + c_1\,t$. Therefore, the general solution has the form:

$$T = c_o + c_1\,t + c_2\,e^{-mt}$$

$$\frac{dT}{dt} = c_1 - c_2\,m\,e^{-mt}$$

$$\frac{dT}{dt} + m\,t = c_1 - c_2\,m\,e^{-mt} + c_o\,m + c_1\,m\,t + c_2\,m\,e^{-mt} = 25\,m\,(2 + t)$$

$$c_o\,m + c_1 + c_1\,m\,t + 25\,m\,(t + 2)$$

$$c_1\,m\,t = 25\,m\,t \Rightarrow c_1 = 25$$

$$c_o\,m + c_1 = c_o\,m + 25 = 50\ \text{m} \Rightarrow c_o = 50 - \frac{25}{m}$$

Substituting these back into the assumed solution yields

$$T = 50 - \frac{25}{m} + 25\,t + c_2\,e^{-mt}$$

Applying the initial condition: T = 50°F when t = 0

$$50 = 50 - \frac{25}{m} + c_2 \Rightarrow c_2 = \frac{25}{m}$$

Therefore, the temperature-time history of the wire is

$$T = 50 + 25t - \frac{25}{m}\,(e^{-mt} - 1)$$

Evaluating the wire temperature at the requested times

$$\text{At } t = 2 \text{ sec: } T = 50 + 50 - \frac{25}{0.0585}\, e^{-0.0585\,(2)} - 1) = 53°\text{F}$$

$$\text{At } t = 10 \text{ sec: } T = 50 + 250 - \frac{25}{0.0585}\,(e^{-0.0585\,(10)} - 1) = 111°\text{F}$$

$$\text{At } t = 1 \text{ min} = 60 \text{ sec: } T = 50 + (25)\,(60) - \frac{25}{0.0585}\,(e^{-0.0585\,(60)} - 1) = 1135°\text{F}$$

COMMENT

Radiation from the wire will become important well before 60 sec has elapsed.

PROBLEM 2.65

> **A large 2.54-cm.-thick copper plate is placed between two air streams. The heat transfer coefficient on the one side is 28 W/(m^2 K) and on the other side is 57 W/(m^2 K). If the temperature of both streams is suddenly changed from 38°C to 93°C, determine how long it will take for the copper plate to reach a temperature of 82°C.**

GIVEN

- A large copper plate between two air streams whose temperatures suddenly change
- Plate thickness (2L) = 2.54 cm = 0.0254 m
- The heat transfer coefficients are $\bar{h}_{c1}$ = 28 W/(m^2 K)
- $\bar{h}_{c2}$ = 57 W/(m^2 K)
- Air temperature changes from 38°C to 93°C

FIND

- How long it will take for the copper plate to reach a temperature of 82°C

ASSUMPTIONS

- The initial temperature of the plate is 38°C
- The plate can be treated as an infinite slab

SKETCH

PROPERTIES AND CONSTANTS

From Appendix 2, Table 12
For copper Thermal conductivity (k) = 396 W/(m K) at 63°C
Density (ρ) = 8933 kg/m^3
Specific heat (c) = 383 J/kg

SOLUTION

The Biot number for this case, using the larger of the heat transfer coefficients is

$$Bi = \frac{\bar{h}_c\ L}{k} = \frac{[57\text{W/(m}^2\text{K)}](0.0254/2\text{ m})}{396\text{W/(m K)}} = 0.002 << 0.1$$

Therefore, the internal resistance of the slab can be neglected (the temperature of the slab remains uniform) and the temperature-time history can be calculated from an energy balance

Change in internal energy = heat flow from both sides

$$-c\,\rho\,V\,dT = \bar{h}_{c1}\ A\ (T - T_\infty)\,dt + \bar{h}_{c2}\ A\ (T - T_\infty)\,dt$$

$$-c\,\rho\,V\,dT = (\ \bar{h}_{c1} + \bar{h}_{c2}\ )\,A\,(T - T_\infty)\,dt$$

Rearranging

$$\frac{dT}{T - T_\infty} = \frac{d\,(T - T_\infty)}{T - T_\infty} = -\frac{(\bar{h}_{c1} + \bar{h}_{c2})}{c\rho\,V}dt$$

Integrating between a temperature of T_o at time = 0 to a temperature of T at time = t yields

$$\ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right) = \frac{(\bar{h}_{c1} + h_{c2})\,A}{c\rho\,V}\ t = \frac{(\bar{h}_{c1} + h_{c2})\,A}{c\rho\,(2\,LA)}\ t$$

Solving this for the time

$$t = -\frac{2\,Lc\,\rho}{\bar{h}_{c1} + \bar{h}_{c2}}\ \ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

$$t = \frac{0.0254\text{ m}\,(383\text{J/(kg K)})((\text{Ws})/\text{J})(8933\text{kg/m}^3)}{(28+57)\text{W/(m}^2\text{K)}}\ \ln\left(\frac{82°\text{C} - 93°\text{C}}{30°\text{C} - 93°\text{C}}\right)$$

$$t = 1645\text{ s} = 27\text{ min}$$

COMMENTS

Because heat transfer is occurring at both sides of the slab, the characteristic length in the Biot number is approximately half of the slab's thickness. However, since the heat transfer coefficients on the two surfaces are not equal, the center plane is not equivalent to an insulated surface.

PROBLEM 2.66

A 1.4-kg aluminum household iron has a 500 W heating element. The surface area is 0.046 m^2. The ambient temperature is 21°C and the surface heat transfer coefficient is 11 W/(m^2 K). How long after the iron is plugged in will its temperature reach 104°C?

GIVEN

- An aluminum household iron
- Mass of the iron (M) = 1.4 kg
- Power output $(\dot{Q}_G)$ = 500 W
- Surface area (A_s) = 0.046 m^2

- The ambient temperature (T_∞) = 21°C
- The heat transfer coefficient $(\bar{h}_c)$ = 11 W/(m^2 K)

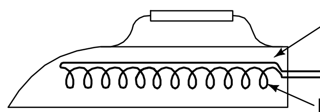
FIND

- How long after the iron is plugged in will its temperature reach 104°C

ASSUMPTIONS

- Constant heat transfer coefficient
- The mass given is for the heated aluminum portion only

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12
For aluminum
  Thermal conductivity (k) = 240 W/(m K) at 127°C
  Specific heat (c) = 896 J/(kg K)

SOLUTION

To calculate the Biot number for this problem, we must first calculate the characteristic length

$$L = \frac{\text{Volume}}{\text{Surface area}} = \frac{\frac{M}{\rho}}{A_s} = \frac{M}{\rho\, A_s} = \frac{1.4\ \text{kg}}{(2702\text{kg/m}^3)(0.046\ \text{m}^2)} = 0.0113\ \text{m}$$

The Biot number is

$$Bi = \frac{\bar{h}_c L}{k} = \frac{[11\ \text{W/(m}^2\ \text{K)}](0.0113\ \text{m})}{240\text{W/(m K)}} = 0.0005 < 0.1$$

Therefore, the lumped capacity method may be used. The energy balance for the iron is

Change in internal energy = heat generation – net heat flow from the iron.

$$c\,\rho\,V\,dT = \dot{Q}_G - \bar{h}_c\,A_s\,(T - T_\infty)\,dt$$

$$\text{Let } \Theta = T - T_\infty \text{ and } m = \frac{\bar{h}_c\ A_s}{c\,\rho\,V} = \frac{\bar{h}_c\ A_s}{c\,\rho\left(\frac{M}{\rho}\right)} = \frac{\bar{h}_c\ A_s}{c\,M}$$

Then the heat balance can be written

$$\frac{d\Theta}{dt} + m\,\Theta = \frac{\dot{Q}_G}{cM}$$

This is a linear, first order, non-homogeneous differential equation. The solution to the homogeneous equation is $\theta_h = c\ e^{-mt}$ and a particular solution is $\theta_p = c$. The general solution is the sum of the homogeneous and particular solutions

$$\Theta = c_1 + c_2\, e^{-mt}$$

Integrating

$$\frac{dQ}{dt} = -\, c_2\, m\, e^{-mt} = -\, m\, (\Theta - c_1) \text{ (From the previous equation)}$$

Substituting this into the heat balance

$$-\, m\, (\Theta - c_1) + m\, \Theta = \frac{\dot{Q}_G}{cM} \Rightarrow c_1 = \frac{\dot{Q}_G}{M\, cM}$$

Applying the initial condition, $\theta = 0$ at $t = 0$ yields

$$-\, c_2\, m = c_1\, m \Rightarrow c_2 = -\, c_1 = \frac{\dot{Q}_G}{M\, cM}$$

Therefore, the temperature-time history of the iron is given by

$$\Theta = \frac{\dot{Q}_G}{m\, cM}\ (1 - e^{-mt})$$

Solving for t

$$t = -\frac{1}{m} \ln \left(1 - \frac{Q m\, cM}{\dot{Q}_G}\right)$$

$$m = \frac{\bar{h}_c A_s}{cM} = \frac{[11\ \text{W/(m}^2\ \text{K)}](0.046\ \text{m}^2)}{[896\ \text{J/(kg K)}](1.4\ \text{kg})((\text{Ws})/\text{J})} = 4.034 \times 10^{-4}\ \text{s}^{-1}$$

$$t = -\frac{1}{4.034 \times 10^{-4}\text{s}^{-1}} \ln \left[1 - \frac{(104°\text{C} - 21°\text{C})\left(4.034 \times 10^{-4}\text{s}^{-1}\right)(896\ 1/(\text{kg K}))((\text{W s})/\text{J})\ (1.4\ \text{kg})}{500\ \text{W}}\right]$$

$$t = 217\ \text{s} = 3.6\ \text{min}$$

PROBLEM 2.67

Estimate the depth in moist soil at which the annual temperature variation will be 10% of that at the surface.

GIVEN

- Moist soil

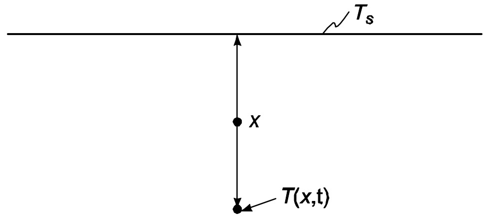
FIND

- The depth in moist soil at which the annual temperature variation will be 10 per cent of that at the surface

ASSUMPTIONS

- Conduction is one dimensional
- The soil has uniform and constant properties
- Annual temperature variation can be treated as a step change in surface temperature with a 6 month response time

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 11

For wet soil Thermal conductivity (k) = 2.60 W/(m K) at 20°C

Density (ρ) = 1500 kg/m^3

Thermal diffusivity (α) = 0.0414×10^{-5} m^2/s

SOLUTION

The geometry of this problem is a semi infinite solid as covered in Section 2.6.3. The transient temperature for a change in surface temperature is given by Equation (2.105)

$$\frac{T_{(x,t)} - T_s}{T_i - T_s} = \text{erf}\left(\frac{x}{2\sqrt{\alpha\, t}}\right)$$

Where T_i is the temperature of the soil until the surface temperature is increased to T_s. For an annual temperature variation of less than 10% of that of the surface

$$T(x,t) - T_i = 0.1\,(T_s - T_i) \text{ at } t = 6 \text{ months}$$

$$T(x,t) = 0.1T_s + 0.9T_i$$

Therefore $T(x,t) - Ts = 0.1T_s + 0.9T_i - T_s = 0.9\,(T_i - T_s)$

$$\frac{T_{(x,t)} - T_s}{T_i - T_s} = 0.9 = \text{erf}\left(\frac{x}{2\sqrt{\alpha\, t}}\right)$$

$$\text{erf}\left(\frac{x}{2\sqrt{[0.0414 \times 10^{-5}(\text{m}^2\text{s})](0.5\text{ year})(365\text{ (days/year)})(24\text{ (h/day)})(3600\text{(s/h)})}}\right) = 0.9$$

$$\text{erf}\left(\frac{x}{5.110\text{ m}}\right) = 0.9$$

From Appendix 2, Table 43

$$\text{erf}\,(1.16) = 0.9$$

$$\therefore \quad \frac{x}{5.110\text{ m}} = 1.16$$

$$x = 6\text{ m}$$

PROBLEM 2.68

A small aluminum sphere of diameter D, initially at a uniform temperature T_o, is immersed in a liquid whose temperature, T_∞, varies sinusoidally according to

$$T_\infty - T_m = A\sin(\omega t)$$

where: T_m = time-averaged temperature of the liquid

A **= amplitude of the temperature fluctuation**

ω **= frequency of the fluctuations**

If the heat transfer coefficient between the fluid in the sphere, $\bar{h}_a$, is constant and the system may be treated as a 'lumped capacity,' derive an expression for the sphere temperature as a function of time.

GIVEN

- A small aluminum sphere is immersed in a liquid whose temperature varies sinusoidally
- Diameter of sphere = D
- Liquid temperature variation: $T_\infty - T_m = A \sin(\omega t)$
- The heat transfer coefficient = $\bar{h}_a$ (constant)
- The system may be treated as a 'lumped capacity'

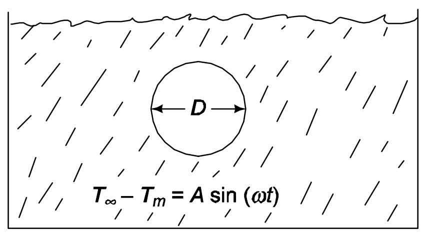
FIND

- An expression for the sphere temperature as a function of time

ASSUMPTIONS

- Constant thermal conductivity

SKETCH



SOLUTION

Let k = thermal conductivity of sphere

ρ = density of sphere

c = specific heat of sphere

An energy balance on the sphere yields

Change in internal energy = heat transfer to liquid

$$\rho c \frac{dT}{dt} = \bar{h}_a A_s (T - T_\infty)$$

$$\frac{dT}{dt} = \frac{h_s A_s}{r c V} [T - T_m - A \sin(\omega t)]$$

$$\text{Let } m = \frac{\bar{h}_s A_s}{r c V} = \frac{\bar{h}_s \, p \, d^2}{r \, c \frac{p}{6} d^3} = \frac{6 \bar{h}_s}{r \, c \, D} \text{ and } \Theta = T - T_m$$

$$\frac{dQ}{dt} + m \Theta = m A_s \sin(\omega t)$$

This is a first order, linear, non-homogeneous differential equation. The general solution is the sum of the homogeneous solution and a particular solution. The homogeneous solution is determined by the characteristic equation, found by substituting $\theta = e^{\lambda t}$ into the homogeneous equation

$$\lambda\, e^{\lambda t} + m\, e^{\lambda t} = 0 \qquad (\lambda = -m)$$

The homogeneous solution is $\theta_h = Ce^{-mt.}$

As a particular solution, try $\theta_p = K \cos(\omega t) + M \sin(\omega t)$, substituting θ_p and its derivative into the energy balance

$$-\omega K \sin(\omega t) + M\omega \cos(\omega t) + m K \cos(\omega t) + m\ M \sin(\omega t) = m A \sin(\omega t)$$

$$(M\omega + mK) \cos(\omega t) - (\omega K - mM) \sin(\omega t) = m A_s \sin(\omega t)$$

$$\therefore\ M\omega + m K = 0 \Rightarrow M = -\frac{m K}{\omega}$$

$$\text{and } \omega K - m M = -m A_s \quad \Rightarrow \omega K + \frac{m K}{\omega}\ m = -m A_s$$

$$\therefore\ K = \frac{M\ A_s\, \omega}{\omega^2 + m^2} \text{ and } M = \frac{m^2\ A_s}{\omega^2 + m^2}$$

Therefore, the general solution is

$$\Theta = C\, e^{-mt} + \frac{M\ A_s}{\omega^2 + m^2}\ [(-\omega \cos(\omega t) + m \sin(\omega t)]$$

At $t = 0$, $T = T_o$ and $\theta = \theta_o = T_o - T_m$

$$\Theta_o = C - \frac{m\ A_s\, \omega}{\omega^2 + m^2} \Rightarrow C = \Theta_o + \frac{m\ A_s\, \omega}{\omega^2 + m^2}$$

The dimensionless temperature distribution is

$$\frac{\Theta}{\Theta_o} = \left(1 + \frac{m\ A_s\, \omega}{\Theta_o(\omega^2 + m^2)}\right) e^{-mt} + \frac{m\ A_s}{\omega^2 + m^2}\ [(m \sin(\omega t) - \omega \cos(\omega t)]$$

PROBLEM 2.69

> **A wire of perimeter *P* and cross-sectional area A emerges from a die at a temperature *T* above ambient and with a velocity *U*. Determine the temperature distribution along the wire in the steady state if the exposed length downstream from the die is quite long. State clearly and try to justify all assumptions.**

GIVEN

- A wire emerging from a die at a temperature (T) above ambient
- Wire perimeter = P
- Cross-sectional area = A
- Wire emerges at a temperature T above ambient
- Wire velocity = U

FIND

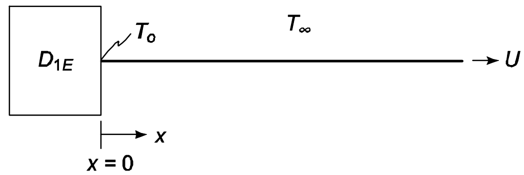
- The temperature distribution along the wire in the steady state if the exposed length downstream from the die is quite long. State clearly and try to justify all assumptions

ASSUMPTIONS

- Ambient temperature is constant at T_∞
- Heat transfer coefficient between the wire and the air is uniform and constant at h_c

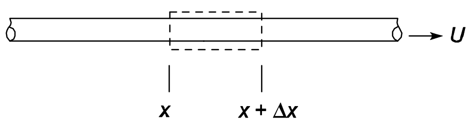
- The material properties of the wire are constant
    - Thermal conductivity = k
    - Thermal diffusivity = α
- Axial conduction only
- Wire temperature is uniform at a cross section (negligible internal thermal resistance)

SKETCH



SOLUTION

Consider a control volume around the wire



Performing an energy balance on the control volume

Conduction into volume + Energy carried into the volume by the moving wire = Conduction out of volume + Convection to the environment + Energy carried out of the volume by the moving wire.

$$-k A \left.\frac{dT}{dx}\right|_x + U A \rho c\, T(x) = -k A \left.\frac{dT}{dx}\right|_{x+\Delta x} + \bar{h}_c\ P\, \Delta_x (T - T_\infty) + U A \rho c\, T(x + \Delta x)$$

$$\frac{\left.\dfrac{dT}{dx}\right|_x - \left.\dfrac{dT}{dx}\right|_{x+\Delta x}}{\Delta x} = \frac{\dfrac{\rho c}{k} U[T(x+\Delta x) - T(x)]}{\Delta x} + \frac{\bar{h}_c P}{k A}\ (T - T_\infty)$$

letting $\Delta x \to 0$

$$\frac{d^2T}{dx^2} = \frac{U}{\alpha}\frac{dT}{dx} + \frac{\bar{h}_c P}{k\ A}\ (T - T_\infty)$$

$$\text{Let } \theta = T - T_\infty \text{ and } m = \frac{\bar{h}_c P}{k\ A} = \frac{\bar{h}_c\, \pi\, D}{k\, \frac{\pi}{4} D^2} = \frac{4\,\bar{h}_c}{k\, D}$$

Then

$$\frac{d^2\theta}{dx^2} - \frac{U}{\alpha}\frac{d\theta}{dx} - m\ \theta = 0$$

This is a linear, differential equation with constant coefficients. The solution has the following form

$$\theta = c_1\ e^{s_1 x} + c_2\ e^{s_2 x}$$

Substituting this solution and its derivatives into the differential equation:

$$s_1^{\,2}\, c_1\ e^{s_1 x} + s_2^{\,2}\, c_2\ e^{s_2 x} - \frac{U}{\alpha}\,(s_1\, c_1\, e^{s_1 x} + s_2\, c_2\, e^{s_2 x}) - m\,(c_1 e^{s_1 x} + c_2 e^{s_2 x}) = 0$$

$$s_1^2 - \frac{U}{\alpha}\, s_1 - m = 0 \Rightarrow s_1 = \frac{1}{2}\left(\frac{U}{\alpha} \pm \sqrt{\left(\frac{U}{\alpha}\right)^2 + 4m}\right)$$

$$s_2^2 - \frac{U}{\alpha}\, s_2 - m = 0 \Rightarrow s_2 = \frac{1}{2}\left(\frac{U}{\alpha} \pm \sqrt{\left(\frac{U}{\alpha}\right)^2 + 4m}\right)$$

$$\therefore \text{ Let } s_1 = \frac{1}{2}\left(\frac{U}{\alpha} + \sqrt{\left(\frac{U}{\alpha}\right)^2 + 4m}\right) \text{ and } s_2 = \frac{1}{2}\left(\frac{U}{\alpha} - \sqrt{\left(\frac{U}{\alpha}\right)^2 + 4m}\right)$$

The boundary conditions for the problem are

1. $\theta = \theta_o$ at $x = 0$
2. $\theta \to 0$ at $x \to \infty$

Applying the first boundary condition

$$\theta_o = c_1 + c_2$$

Since, by inspection, s_1 must be positive, for the second boundary condition to be satisfied, the constant c_1 must be zero. Therefore, the temperature distribution in the wire is

$$\theta = \theta_o\, e^{s_2 t}$$

or

$$T = T_\infty + (T_o - T_\infty)\exp\left[\frac{x}{2}\left(\frac{U}{\alpha} - \sqrt{\left(\frac{U}{\alpha}\right)^2 + 4m}\right)\right]$$

PROBLEM 2.70

Ball bearings are to be hardened by quenching them in a water bath at a temperature of 37°C. Suppose you are asked to devise a continuous process in which the balls could roll from a soaking oven at a uniform temperature of 870°C into the water, where they are carried away by a rubber conveyer belt. The rubber conveyor belt would, however, not be satisfactory if the surface temperature of the balls leaving the water is above 90°C. If the surface coefficient of heat transfer between the balls and the water may be assumed to be equal to 590 W/(m^2 K), (a) find an approximate relation giving the minimum allowable cooling time in the water as a function of the ball radius for balls up to 1.0-cm in diameter, (b) calculate the cooling time, in seconds, required for a ball having a 2.5-cm diameter, and (c) calculate the total amount of heat in watts which would have to be removed from the water bath in order to maintain its temperature uniform if 100,000 balls of 2.5-cm diameter are to be quenched per hour.

GIVEN

- Ball bearings quenched in a water bath
- Water bath temperature (T_∞) = 37°C
- Initial temperature of the balls (T_o) = 870°C
- Final surface temperature of the balls (T_f) = 90°C
- Heat transfer coefficie ($\bar{h}_c$) = 590 W/(m^2 K)

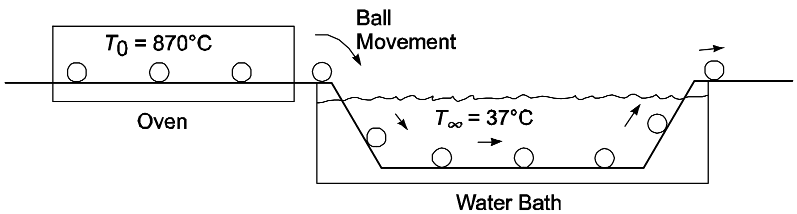
FIND

(a) An approximate relation giving the minimum allowable cooling time in the water as a function of the ball radius for balls upto 1.0 cm in diameter
(b) The cooling time, in seconds, required for a ball having a 2.5 cm diameter
(c) The total amount of heat in watts which would have to be removed from the water bath in order to maintain its temperature uniform if 100,000 balls of 2.5 cm diameter are to be quenched per hour

ASSUMPTIONS

- The ball bearings are 1% carbon steel

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10

For 1% carbon steel — Thermal conductivity (k) = 43 W/(m K)
Density (ρ) = 7.801 kg/m^3
Specific heat (c) = 473 J/(kg K)
Thermal diffusivity (α) = 1.172×10^{-5} m^2/s

SOLUTION

(a) For 1.0 cm diameter balls

$$Bi = \frac{\bar{h}_c\, r_o}{k} = \frac{[590\,\text{W/(m}^2\,\text{K)}]\,(0.005\,\text{m})}{43\,\text{W/(m K)}} = 0.07 < 0.1$$

Therefore, a lumped capacity method can be used for balls less than 1 cm in diameter. The time temperature history of the ball is given by Equation 2.84

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{h A_s}{c \rho V} t} = e^{-\frac{h\, 4 \pi r_o^2}{c \rho \frac{4}{3} \pi r_o^3} t} = e^{-\frac{3h}{c \rho r_o} t}$$

Solving for the minimum cooling time

$$t = -\frac{c \rho\, r_o}{3h} \ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

$$t = -\frac{[473\,\text{J/(kg K)}](\text{Ws/J})\left(7801\,(\text{kg/m}^3)\right) r_o}{3\left(590\,\text{W/(m}^2\,\text{K)}\right)} \ln\left(\frac{90°\text{C} - 37°\text{C}}{870°\text{C} - 37°\text{C}}\right) = 5743\, r_o \text{ s/m}$$

(b) For balls having a diameter of 2.5 cm

$$Bi = \frac{h_c L}{k} = \frac{[590\,\text{W/(m}^2\,\text{K)}](0.0125\,\text{m})}{43\,\text{W/(m K)}} = 0.172 > 0.1$$

Therefore, the internal resistance is significant and a chart solution will be used. From Figure 2.39 for $1/B_i$ = 5.8 and $r = r_o$

$$\frac{T(r_o, t) - T_\infty}{T(0, t) - T_\infty} = 0.92$$

For a final surface temperature ($T\,(r_o,\, t)$) of 90°C

$$T(0,\, t) = T_\infty + \frac{1}{0.92}\,(T(r_o,\, t) - T_\infty) = 37°\text{C} = \frac{1}{0.92}\,(90°\text{C} - 37°\text{C}) = 94.6°\text{C}$$

$$\frac{T_{o,t} - T_\infty}{T_o - T_\infty} = \frac{94.6°C - 37°C}{870°C - 37°C} = 0.069$$

From Figure 2.39, for $(T_o, t - T_\infty) / (T_o - T_\infty) = 0.069$ and $1/Bi = 5.3$: $Fo = 5.3 = \alpha\, t/r_o^2$

$$t = \frac{Fo\, r_o^2}{\alpha} = \frac{5.3\,(0.0125\text{ m})^2}{1.172 \times 10^{-5}(\text{m}^2/\text{s})} = 71 \text{ sec}$$

(c) Figure 2.39 can be used to calculate the heat transferred from one ball during the cooling time:

$$(Bi)^2\, Fo = (0.172)^2\,(5.3) – 0.157$$

From Figure 2.39 $Q(t)/Q_i = 0.93$
From Table 2.3

$$Q_i = \rho\, c \frac{4}{3} \pi r_o^3 (T_o - T_\infty) = [7801\,(\text{kg/m}^3)](473(\text{J/kg K}))\frac{4}{3}\pi\,(0.0125\text{ m})^3\,(870°C - 37°C) = 25{,}150 \text{ J}$$

$$\therefore\ Q(t) = 0.93\, Q_i = 0.93\,(25{,}159 \text{ J}) = 23{,}390 \text{ J}$$

The amount of heat needed to quench 100,000 balls per hour is

$$q = (\text{Balls/hr})\,(\text{Energy/ball}) = \frac{[100{,}000\,(1/\text{h})](23{,}390 \text{ J})}{3600\,(\text{s/h})} = 650{,}000 \text{ W}$$

PROBLEM 2.71

Estimate the time required to heat the center of a 1.5-kg roast in a 163°C over to 77°C. State your assumptions carefully and compare your results with cooking instructions in a standard cookbook.

GIVEN

- A roast in an oven
- Mass of the roast (m) = 1.5 kg
- Oven temperature (T_∞) = 163°C
- Final temperature of the roast's center (T_f) = 77°C

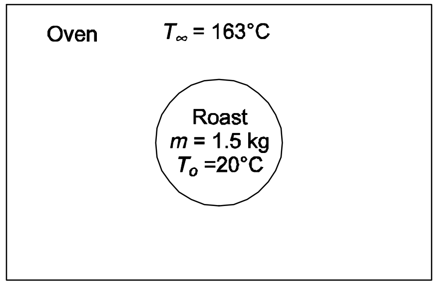
FIND

- The time required to heat the roast

ASSUMPTIONS

- The shape of the roast can be approximated by a sphere
- The roast temperature is initially uniform at (T_o) = 20°C
- The properties of the roast are approximately those of water
- Thermal conductivity (k) = 0.5 W/(m K)
- Density (ρ) = 1000 kg/m^3
- Specific heat = 4000 J/(kg K)
- A uniform heat transfer coefficient of $(\bar{h}_c)$ = 18 W/(m^2 K) exists between the roast and the oven air (midline of the range for free convection given in Table 1.4.)

SKETCH



SOLUTION

With the assumptions listed above, the radius of the spherical roast is given by

$$V = \frac{m}{\rho} = \frac{4}{3}\pi r_o^3 \Rightarrow r_o = \left(\frac{3m}{4\pi\rho}\right)^{\frac{1}{3}} = \left(\frac{3(1.5\text{ kg})}{4\pi\left(1000\text{ (kg/m}^3)\right)}\right)^{\frac{1}{3}} = 0.071\text{ m}$$

Figure 2.39 can be used to find the Fourier number. To use Figure 2.39, the following parameters (which are listed in Table 2.3) are needed

$$Bi = \frac{\bar{h}_c\, r_o}{k} = \frac{[18\text{ W/(m}^2\text{K)}](0.071\text{ m})}{0.5\text{ W/(m K)}} = 2.56 \Rightarrow \frac{1}{Bi} = 0.391$$

$$\frac{\theta(0,t)}{\theta_o} = \frac{T - T_\infty}{T_o - T_\infty} = \frac{77°\text{C} - 163°\text{C}}{20°\text{C} - 163°\text{C}} = 0.60$$

From Figure 2.39 $Fo = 0.2$

From Table 2.3 $Fo = \alpha\, t/r_o^2$

Solving for the time

$$t = \frac{r_o^2\; Fo}{\alpha} = \frac{r_o^2\; Fo\,\rho\, c}{k}$$

$$t = \frac{(0.071\text{m})^2(0.2)\left(1000\cdot\text{kg/m}^3\right)\left(4000\text{J/(kg K)}\right)}{0.5\text{W/m K}\left(\text{J/W s}\right)}$$

$$t = 8065\text{ s} = 134\text{ min}$$

The Better Homes and Gardens Cookbook recommends cooking a Standing Rib Roast with the oven set at 325°F (163°C) for 27-30 minutes per pound to achieve a center temperature of 170°F (77°C) which is considered well done.

This calculation yielded 134 minutes for 1.5 kg (3.3 lbs) or 40 minutes per pound. The discrepancy is probably due to inaccuracies in the assumed properties of the roast.

PROBLEM 2.72

A stainless steel cylindrical billet [k = 14.4 W/(m K), $\alpha = 3.9 \times 10^{-6}$ m²/s] is heated to 593°C preparatory to a forming process. If the minimum temperature permissible for forming is 482°C, how long may the billet be exposed to air at 38°C if the average heat transfer coefficient is 85 W/(m² K)? The shape of the billet is shown in the sketch.

GIVEN

- A stainless steel cylindrical billet exposed to air
- Thermal conductivity (k) = 14.4 W/(m K)
- Thermal diffusivity (α) = 3.9×10^{-6} m²/s
- Initial temperature (T_o) = 593°C

- The minimum temperature permissible for forming is 482°C
- Air temperature (T_∞) = 38°C
- Average heat transfer coefficient $(\bar{h}_c)$ = 85 W/(m^2 K)

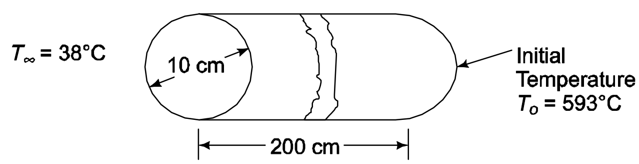
FIND

- How long may the billet be exposed to the air?

ASSUMPTIONS

- End effects are negligible
- Constant heat transfer coefficient
- Conduction in the radial direction only
- Uniform thermal properties

SKETCH



SOLUTION

The Biot number is calculated to determine if internal resistance is significant

$$Bi = \frac{\bar{h}_c\, r_o}{k} = \frac{[85\ \text{W/(m}^2\text{K)}](0.05\ \text{m})}{14.4\ \text{W/(m K)}} = 0.3 > 0.1$$

Therefore, internal resistance is important, and a chart solution is used.

The chart for this geometry is Figure 2.38. The approach will be as follows:

1. Use the Biot number and the minimum surface temperature given to find $(T_{o,t} - T_\infty)/(T_o - T_\infty)$ from Figure 2.38.
2. Apply $(T_{o,t} - T_\infty)/(T_o - T_\infty)$ and the Biot number to Figure 2.38 to find the Fourier number.
3. Use the Fourier number to find the time it takes for the surface to cool to the given minimum surface temperature.
1. From Figure 2.38, for $r = r_o$ (r/r_o = 1.0) and 1/Bi = 3.33

$$\frac{T(r_o,t) - T_\infty}{T_o - T_\infty} = 0.87$$

The surface temperature must not fall below 482°C

$$\frac{T(r_o,t) - T_\infty}{T_o - T_\infty} = \frac{482°\text{C} - 38°\text{C}}{593°\text{C} - 30°\text{C}} = 0.80$$

Combining these results

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = \frac{\left(\dfrac{T(r_o,t) - T_\infty}{T_o - T_\infty}\right)}{\left(\dfrac{T(r_o,t) - T_\infty}{T(0,t) - T_\infty}\right)} = \frac{0.80}{0.87} = 0.92$$

2. From Figure 2.38, for $1/Bi$ = 3.33 and $(T(0, t) - T_{oo})/(T_o - T_{oo})$ = 0.92

$$F_o = \frac{\text{a}\, t}{r_o^2} = 0.2$$

3. Solving for the time

$$t = \frac{Fo\, r_o^2}{\text{a}} = \frac{0.2\,(0.05\text{ m})^2}{3.9\ ´\ 10^{-6}(\text{m}^2/\text{s})} = 128\text{ s} = 2.1\text{ min}$$

PROBLEM 2.73

In the vulcanization of tires, the carcass is placed into a jig, and steam at 149°C is admitted suddenly to both sides. If the tire thickness is 2.5 cm, the initial temperature is 21°C, the heat transfer coefficient between the tire and the steam is 150 W/(m^2 K), and the specific heat of the rubber is 1650 J/(kg K), estimate the time required for the center of the rubber to reach 132°C.

GIVEN

- Tire suddenly exposed to steam on both sides
- Steam temperature (T_∞) = 149°C
- Tire thickness ($2L$) = 2.5 cm = 0.025 m
- Initial tire temperature (T_o) = 21°C
- The heat transfer coefficient (h_c) = 150 W/(m^2 K)
- The specific heat of the rubber (c) = 165 J/(kg K)

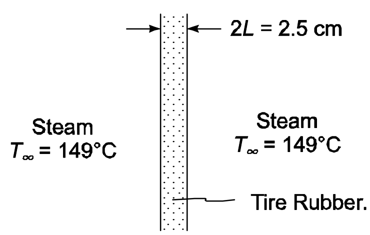
FIND

- The time required for the central layer to reach 132°C

ASSUMPTIONS

- Shape effects are negligible, tire can be treated as an infinite plate

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 11
For bana rubber

Thermal conductivity (k) = 0.465 W/(m K) at 20°C
Density (ρ) = 1250 g/m^3

SOLUTION

The significance of the internal resistance is determined from the Biot number

$$Bi = \frac{\bar{h}_c\, L}{k} = \frac{[150\,\text{W/(m}^2\text{K)}]\left(\dfrac{0.025}{2}\,\text{m}\right)}{0.465\,\text{W/(m K)}} = 4.0 >> 0.1$$

Therefore, the internal resistance is significant and a chart solution will be used. Figure 2.37 contains the charts for this geometry.

The time required can be calculated from the Fourier number which can be found from Figure 2.37. The centerline at time t must be 132°C, therefore

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = \frac{132°\text{C} - 149°\text{C}}{21°\text{C} - 149°\text{C}} = 0.13$$

From Figure 2.37, for $(T(0, t) - T_\infty)/(T_o - T_\infty) = 0.13$ and $1/Bi = 0.25$

$$Fo = \frac{\alpha\, t}{r_o^2} = \frac{k\, t}{\rho\, c\, L^2} = 1.32$$

Solving for the time

$$t = \frac{\rho\, c L^2 Fo}{k} = \frac{[1250\,(\text{kg/m}^3)](1650\ \text{J/(kg K)})((\text{W s})/\text{J})(0.025/2\ \text{m})^2\,(1.3)}{0.465\,\text{W/(m K)}}$$

$$t = 900\ \text{s} = 15\ \text{min}$$

PROBLEM 2.74

> **A long copper cylinder 0.6 m in diameter and initially at a uniform temperature of 38°C is placed in a water bath at 93°C. Assuming that the heat transfer coefficient between the copper and the water is 1248 W/(m^2 K), calculate the time required to heat the center of the cylinder to 66°C. As a first approximation, neglect the temperature gradient within the cylinder r/h, then repeat your calculation without this simplifying assumption and compare your results.**

GIVEN

- A long copper cylinder is placed in a water bath
- Diameter of cylinder (D) = 0.6 m
- Initial temperature (T_o) = 38°C
- Water bath temperature (T_∞) = 93°C
- The heat transfer coefficient $(\bar{h}_c)$ = 1248 W/(m^2 K)

FIND

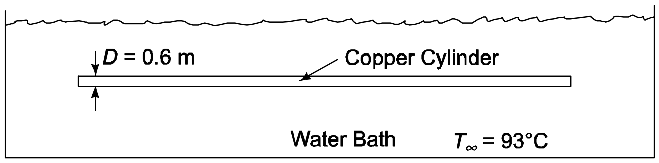
Calculate the time required to heat the center of the cylinder to 66°C assuming

(a) Negligible temperature gradient within the cylinder
(b) Without this simplification, then
(c) Compare your results

ASSUMPTIONS

- Neglect end effects
- Radial conduction only

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12
For copper

Thermal conductivity (k) = 396 W/(m K) at 63°C
Density (ρ) = 8933 kg/m^3
Specific heat (c) = 383 J/(kg K)
Thermal diffusivity (α) = 1.166 × 10^{-4} m^2/s

SOLUTION

(a) For a negligible temperature gradient within the cylinder, the temperature-time history is given by Equation (2.84)

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{\bar{h}_c A_s}{c \rho V} t} = e^{-\frac{\bar{h}_c \pi D L}{c \rho \frac{\pi}{4} D^2 L} t} = e^{-\frac{4 \bar{h}_c}{c \rho D} t}$$

Solving for the time

$$t = -\frac{c \rho D}{4 h_c} \ln \left(\frac{T - T_\infty}{T_o - T_\infty} \right)$$

$$t = \frac{[383\ \text{J/(kg K)}](\text{W s/J})\left(8933(\text{kg/m}^3)\right)(0.6\ \text{m})}{4\left(1248\ \text{W/(m}^2\ \text{K)}\right)} \ln \left(\frac{66°\text{C} - 93°\text{C}}{38°\text{C} - 93°\text{C}} \right)$$

$$t = 293\ \text{sec} = 4.9\ \text{min}$$

(b) The chart method can be used to take the temperature gradient within the cylinder into account. Figure 2.38 contains the charts for a long cylinder.

$$Bi = \frac{\bar{h}_c r_o}{k} = \frac{[1248\ \text{W/(m}^2\text{K)}](0.3\,\text{m})}{396\ \text{W/(m K)}} = 0.95 \Rightarrow \frac{1}{Bi} = 1.1$$

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = \left(\frac{66°\text{C} - 93°\text{C}}{38°\text{C} - 93°\text{C}} \right) = 0.49$$

From Figure 2.38, for 1/Bi = 1.1 and $T(0, t) - T_\infty)/(T_o - T_\infty)$ = 0.49

$$Fo = \frac{\alpha t}{r_o^2} = 0.5$$

Solving for the time

$$t = \frac{Fo\, r_o^2}{\alpha} = \frac{0.5(0.3\text{m})^2}{1.166 \times 10^{-4} \text{m}^2/\text{s}} = 386\ \text{s} = 6.4\ \text{min}$$

(c) The lumped capacity method (a) underestimates the required time by 24%.

COMMENTS

Since the Biot number is of the order of magnitude of unity, we could not expect that the lumped capacity assumption is valid.

PROBLEM 2.75

> **A steel sphere with a diameter of 7.6 cm is to be hardened by first heating it to a uniform temperature of 870°C and then quenching it in a large bath of water at a temperature of 38°C. The following data apply**
>
> **surface heat transfer coefficient $\bar{h}$ = 590 W/(m^2 K)**
>
> **thermal conductivity of steel = 43 W/(m K)**
>
> **specific heat of steel = 628 J/(kg K)**
>
> **density of steel = 7840 kg/m^3**
>
> **Calculate: (a) time elapsed in cooling the surface of the sphere to 204°C and (b) time elapsed in cooling the center of the sphere to 204°C.**

GIVEN

- A steel sphere is quenched in a large water bath
- Diameter (D) = 7.6 cm = 0.076 m
- Initial uniform temperature (T_o) = 870°C
- Water temperature (T_∞) = 38°C
- Surface heat transfer coefficient (h) = 590 W/(m^2 K)
- Thermal conductivity of steel (k) = 43 W/(m K)
- Specific heat of steel (c) = 628 J/(kg K)
- Density of steel (ρ) = 7840 kg/m^3

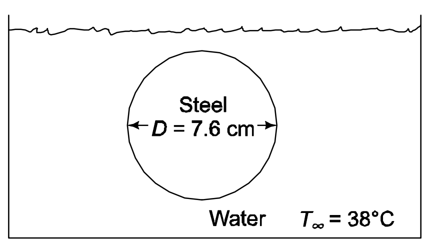
FIND

(a) Time elapsed in cooling the surface of the sphere to 204°C
(b) Time elapsed in cooling the center of the sphere to 204°C

ASSUMPTIONS

- Constant water bath temperature, thermal properties, and transfer coefficient

SKETCH



SOLUTION

The importance of the internal resistance can be determined from the Biot number

$$Bi = \frac{\bar{h}_c r_o}{k} = \frac{[590\ \text{W/(m}^2\ \text{K)}]\left(\frac{0.076}{2}\ \text{m}\right)}{43\ \text{W/(m K)}} = 0.52 > 0.1$$

Therefore, the internal resistance is significant and a chart solution will be used.

Figure 2.39 contains the charts for this geometry.

(a) From Figure 2.39, for $r = r_o$ and $1/Bi = 1.9$:

$$\frac{T(r_o,t) - T_{\infty}}{T(0,t) - T_{\infty}} = 0.78$$

Solving for the center temperature

$$T(0, t) = T_\infty + 1.28\ (T(r_o, t) - T_\infty) = 38°\text{C} + 1.28(204°\text{C} - 38°\text{C}) = 251°\text{C}$$

$$\therefore\ \frac{T(0,t) - T_\infty}{T_o - T_\infty} = \frac{251°\text{C} - 38°\text{C}}{870°\text{C} - 38°\text{C}} = 0.26$$

From Figure 2.39 for $(T(0, t) - T_\infty)/(T_o - T_\infty) = 0.26$, $1/Bi = 1.9$

$$Fo = \frac{\alpha t}{r_o^2} = \frac{k t}{\rho c r_o^2} = 0.8$$

Solving for the time

$$t = \frac{Fo\ \rho c r_o^2}{k} = \frac{0.8\left(7840\text{kg/m}^3\right)\left(628\text{J/(kg K)}\right)\left(\frac{0.076}{2}\ \text{m}\right)^2}{43\text{W/(m}^2\text{K)}} = 132\ \text{s} = 2.2\ \text{min}$$

(For the surface temperature to reach 204°C)

(b) For a center temperature of 204°C

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = \frac{204°\text{C} - 38°\text{C}}{870°\text{C} - 38°\text{C}} = 0.20$$

From Figure 2.39 for $(T(0, t) - T_\infty)/(T_o - T_\infty) = 0.2$, $1/Bi = 1.9$: $Fo = 1.1$, therefore

$$t = \frac{1.1\left(7840\text{kg/m}^3\right)\left(628\text{J/(kg K)}\right)\left(\frac{0.076}{2}\ \text{m}\right)^2}{43\text{W/(m}^2\ \text{K)}} = 182\ \text{s} = 3.0\ \text{min}$$

(For the center temperature to reach 204°C)

PROBLEM 2.76

A 2.5-cm-thick sheet of plastic initially at 21°C is placed between two heated steel plates that are maintained at 138°C. The plastic is to be heated just long enough for its midplane temperature to reach 132°C. If the thermal conductivity of the plastic is 1.1×10^{-3} W/(m K), the thermal diffusivity is 2.7×10^{-6} m²/s, and the thermal resistance at the interface between the plates and the plastic is negligible, calculate: (a) the required heating time, (b the temperature at a plane 0.6 cm from the steel plate at the moment the heating is discontinued, and (c) the time required for the plastic to reach a temperature of 132°C 0.6 cm from the steel plate.

GIVEN

- A sheet of plastic is placed between two heated steel plates

- Sheet thickness ($2L$) = 2.5 cm = 0.025 m
- Initial temperature (T_o) = 21°C
- Temperature of steel plates (T_s) = 138°C
- Heat until midplane temperature of sheet (T_c) = 132°C
- The thermal conductivity of the plastic (k) = 1.1×10^{-3} W/(m K)
- The thermal diffusivity (α) = 2.7×10^{-6} m^2/s
- The thermal resistance at the interface between the plates and the plastic is negligible

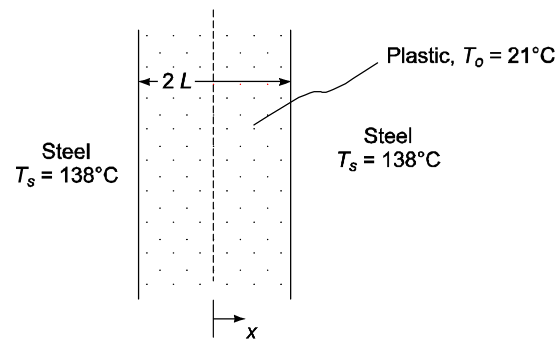
FIND

(a) The required heating time
(b) The temperature at a plane 0.6 cm from the steel plate at the moment the heating is discontinued
(c) The time required for the plastic to reach a temperature of 13°C 0.6 cm from the steel.

ASSUMPTIONS

- The initial temperature of the sheet is uniform
- The temperature of the steel plates is constant
- The thermal conductivity of the sheet is constant

SKETCH



SOLUTION

The chart solutions apply to convective boundary conditions but can be applied to this problem by letting $h_c \to \infty$. Therefore, $1/Bi = 0$.

(a) To find the time required to heat the midplane from 21°C to 132°C, first calculate the coordinate of Figure 2.37

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = \frac{132°\text{C} - 138°\text{C}}{21°\text{C} - 138°\text{C}} = 0.0513$$

From Figure 2.37

$$Fo = \frac{\alpha t}{L^2} = 1.3$$

Solving for the time:

$$t = \frac{Fo\,L^2}{\alpha} = \frac{1.3\left(\frac{0.025}{2}\,\text{m}\right)^2}{27 \times 10^{-6}\,\text{m}^2/\text{s}} = 75\ \text{sec}$$

(b) At 0.6 cm from the steel plate

$$x = L - 0.006\text{ m} = 0.0125\text{ m} - 0.006\text{ m} = 0.0065\text{ m} \Rightarrow \frac{x}{L} = \frac{0.0065\,\text{m}}{0.0125\,\text{m}} = 0.52$$

From Figure 2.37

$$\frac{T(0.0065\,\text{m}, t) - T}{T(0,t) - T_\infty} = 0.70$$

$$T(0.0065\ m,\ t) = 0.7\,(T(0,\ t) - T_\infty) + T_\infty = 0.7\,(132°\text{C} - 138°\text{C}) + 138°\text{C} = 133.8°\text{C}$$

(c) When the temperature 0.6 cm from the steel plate is 132°C, the center temperature

$$T(0,\ t) = T_\infty + \frac{1}{0.7}\,(T(0.0065\text{ m},\ t) - T_\infty) = 138°\text{C} + \frac{1}{0.7}\,(132°\text{C} - 130°\text{C}) = 129.4°\text{C}$$

$$\therefore\ \frac{T(0,t) - T_\infty}{T_o - T_\infty} = \frac{129.4^{\circ}\text{C} - 138^{\circ}\text{C}}{21^{\circ}\text{C} - 138^{\circ}\text{C}} = 0.0733$$

From Figure 2.37

$$t = \frac{FoL^2}{\alpha} = \frac{1.15\left(\frac{0.025}{2}\,\text{m}\right)^2}{2.7 \times 10^{-6}\,\text{m}^2/\text{s}} = 67\text{ sec}$$

PROBLEM 2.77

A monster turnip (assumed spherical) weighing in at 0.45 kg is dropped into a cauldron of water boiling at atmospheric pressure. If the initial temperature of the turnip is 17°C, how long does it take to reach 92°C at the center? Assume that

$$\bar{h}_c = 1700\text{ W/(m}^2\text{ K)} \qquad c_r = 3900\text{ J/(kg K)}$$

$$k = 0.52\text{ W/(m K)} \qquad \rho = 1040\text{ kg/m}^3$$

GIVEN

- A turnip is dropped into boiling water
- Mass of turnip (M) = 0.45 kg
- Water is boiling at atmospheric pressure
- Initial temperature of the turnip (T_o) = 17°C

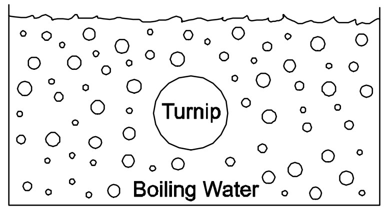
FIND

- Time needed to reach 92°C at the center

ASSUMPTIONS

- Heat transfer coefficient (h_c) = 1700 W/(m^2 K)
- Specific heat ($c\rho$) = 3900 J/(kg K)
- Thermal conductivity (k) = 0.52 W/(m K)
- Density (ρ) = 1040 kg/m^3
- The specific heat of the turnip is constant
- Altitude is sea level, therefore, temperature of boiling water (T_∞) = 100°C
- One dimensional conduction in the radial direction

SKETCH



SOLUTION

The radius of the turnip is given by

$$\text{Volume} = \frac{4}{3}\pi r_o^3 = \frac{M}{\rho} \Rightarrow r_o = \left(\frac{3M}{4\pi\rho}\right)^{\frac{1}{3}} = \left(\frac{3(0.45\,\text{kg})}{4\pi\left(1040\ \text{kg/m}^3\right)}\right)^{\frac{1}{3}} = 0.047\ \text{m}$$

The Biot number is

$$Bi = \frac{\overline{h}_c\, r_o}{k} = \frac{[1700\ \text{W/(m}^2\ \text{K)}]\,(0.047\ \text{m})}{0.52\ \text{W/(m K)}} = 153 > 0.1$$

Therefore, internal resistance is significant and the chart method will be used.

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = \frac{92^\circ\text{C} - 100^\circ\text{C}}{17^\circ\text{C} - 100^\circ\text{C}} = 0.096$$

From Figure 2.39, $(T(0, t) - T_\infty)/(T_o - T_\infty) = 0.096$ and $1/Bi = 0.0065$

$$Fo = \frac{\alpha\, t}{r_o^2} = \frac{k\, t}{\rho\, c r_o^2} = 0.25$$

Solving for the time

$$t = \frac{Fo\,\rho\, c r_o^2}{k} = \frac{0.25\left(1040\ \text{kg/m}^3\right)\left(3900\ \text{J/(kg K)}\right)\left(0.047\ \text{m}\right)^2}{0.52\ \text{W/(m}^2\ \text{K)}}$$

$$t = 4307\ \text{s} = 72\ \text{min} = 1.2\ \text{hours}$$

PROBLEM 2.78

An egg, which for the purposes of this problem can be assumed to be a 5-cm-diameter sphere having the thermal properties of water, is initially at a temperature of 4°C. It is immersed in boiling water at 100°C for 15 min. The heat transfer coefficient from the water to the egg may be assumed to be 1700 W/(m^2 K). What is the temperature of the egg center at the end of the cooking period?

GIVEN

- An egg is immersed in boiling water
- Initial temperature (T_o) = 4°C
- Temperature of boiling water (T_∞) = 100°C
- Time that the egg is in the water (t) = 15 min. = 900 s
- The heat transfer coefficient (h_c) = 1700 W/(m^2 K)

FIND

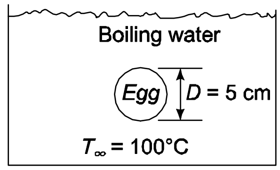
- The temperature of the egg center at the end of the cooking period

ASSUMPTIONS

- The egg is a sphere of diameter (D) = 5 cm = 0.05 m

- The egg has the thermal properties of water (From Appendix 2, Table 13)
  Thermal conductivity (k) = 0.682 W/(m K)
  Density (ρ) = 958.4 kg/m^3
  Specific Heat (c) = 4211 J/(kg K)

SKETCH



SOLUTION

The Biot number for the egg is

$$Bi = \frac{\overline{h_c}\, r_o}{k} = \frac{[1700\ \text{W/(m}^2\ \text{K)}]\,(0.025\ \text{m})}{0.682\ \text{W/(m K)}} = 62.3 > 0.1$$

Therefore, the internal resistance is significant. Figure 2.39 can be used to solve the problem. The Fourier number at t = 900 s is

$$Fo = \frac{\alpha\, t}{r_o^2} = \frac{k\, t}{\rho\, c r_o^2} = \frac{0.682\ \text{W/(m K)}\,(900s)}{[4211\ \text{J/(kg K)}]((\text{W s})/\text{J})\left(958.4\ \text{kg/m}^3\right)} = 0.24$$

From Figure 2.39 for Fo = 0.24 and $1/Bi$ = 0.016

$$\frac{T(0,t) - T_\infty}{T(0,t) - T_\infty} = 0.10 \Rightarrow T(0,\ t) = T_\infty + 0.1(T_o - T_\infty) = 100°\text{C} + 0.1\ (4°\text{C} - 100°\text{C})$$

$$T(0,\ t) = 90.4°\text{C}$$

PROBLEM 2.79

> **A long wooden rod at 38°C with a 2.5 cm diameter is placed into an airstream at 600°C. The heat transfer coefficient between the rod and air is 28.4 W/(m^2 K). If the ignition temperature of the wood is 427°C, ρ = 800 kg/m^3, k = 0.173 W/(m K), and c = 2500 J/(kg K), determine the time between initial exposure and ignition of the wood.**

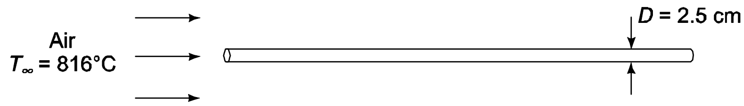
GIVEN

- A long wooden rod is placed into an airstream
- Rod outside diameter (D) = 2.5 cm = 0.025 m
- Initial temperature of the rod (T_o) = 38°C
- Temperature of the airstream (T_∞) = 816°C
- The heat transfer coefficient (h_c) = 28.4 W/(m^2 K)
- The ignition temperature of the wood (T_I) = 427°C
- Density of the rod (ρ) = 800 kg/m^3
- Thermal conductivity (k) = 0.173 W/(m K)
- Specific heat (c) = 2500 J/(kg K)

FIND

- The time between initial exposure and ignition of the wood

SKETCH



SOLUTION

The Biot number for the rod is

$$Bi = \frac{\bar{h}_c r_o}{2k} = \frac{[28.4\ \text{W/(m}^2\ \text{K)}]\left(\frac{0.025}{2}\ \text{m}\right)}{0.173\ \text{W/(m K)}} = 2.05 > 0.1$$

$$\frac{1}{Bi} = 0.49$$

Therefore, the internal thermal resistance of the rod is significant and the chart solution of Figure 2.38 will be used. From Figure 2.38 for $r/r_o = 1.0$ and $1/Bi = 0.49$

$$\frac{T(r_o,t) - T_\infty}{T(0,t) - T_\infty} = 0.52$$

Solving for the difference between the center and ambient temperatures

$$T(0, t) - T_\infty = \frac{1}{0.52}\ (T(r_o, t) - T_\infty)$$

When the surface temperature of the rod is 427°C

$$T(0, t) - T_\infty = \frac{1}{0.52}\ (427°\text{C} - 600°\text{C}) = -333°\text{C}$$

$$\therefore\ \frac{T(0,t) - T_\infty}{T_o - T_\infty} = \frac{-333°\text{C}}{38°\text{C} - 600°\text{C}} = 0.59$$

From Figure 2.38 for $(T(0, t) - T_\infty)/(T_o - T_\infty) = 0.59$ and $1/Bi = 0.49$

$$Fo = \frac{\alpha t}{r_o^2} = 0.2$$

Solving for the time

$$t = \frac{Fo\, r_o^2}{\alpha} = \frac{Fo\, \rho c r_o^2}{k} = \frac{0.2\left(800\ \text{kg/m}^3\right)\left(2500\ \text{J/(kg K)}\right)\left(\frac{0.025}{2}\ \text{m}\right)^2}{0.173\ \text{W/(m}^2\ \text{K)}} = 361\ \text{sec} = 6.0\ \text{min}$$

PROBLEM 2.80

> **In the inspection of a sample of meat intended for human consumption, it was found that certain undesirable organisms were present. In order to make the meat safe for consumption, it is ordered that the meat be kept at a temperature of at least 121°C for a period of at least 20 min during the preparation. Assume that a 2.5-cm.-thick slab of this meat is originally at a uniform temperature of 27°C; that it is to be heated from both sides in a constant temperature oven; and that the maximum temperature meat can withstand is 154°C. Assume furthermore that the surface coefficient of heat transfer**

remains constant and is 10 W/(m^2 K). The following data may be taken for the sample of meat: specific heat = 4184 J/(kg K); density = 1280 kg/m^3; thermal conductivity = 0.48 W/(m K). Calculate the oven temperature and the minimum total time of heating required to fulfill the safety regulation.

GIVEN

- A slab of meat is heated in constant temperature over
- Meat be kept at a temperature of at least 121°C for a period of at least 20 min during the preparation
- Slab thickness ($2L$) = 2.5 cm = 0.025 m
- Initial uniform temperature (T_o) = 27°C
- The maximum temperature meat can withstand is 154°C
- Specific heat (c) = 4184 J/(kg K)
- Density (ρ) = 1280 kg/m^3
- Thermal conductivity (k) = 0.48 W/(m K)

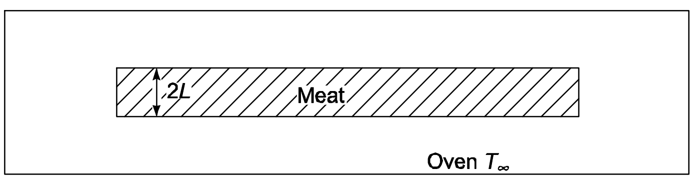
FIND

- The minimum total time of heating required to fulfill the safety regulation

ASSUMPTIONS

- The surface heat transfer coefficient ($\bar{h}_c$)= 10 W/(m K)
- Edge effects are negligible
- One dimensional conduction

SKETCH



SOLUTION

The Biot number for the meat is

$$Bi = \frac{\bar{h}_c L}{k} = \frac{[10\ \text{W/(m}^2\ \text{K)}]\left(\frac{0.025}{2}\ \text{m}\right)}{0.48\text{W/(m K)}} = 0.26 > 0.1$$

Therefore, the internal resistance is significant and the transient conduction charts will be used to find a solution.

The highest temperature will occur at the surface of the meat while the lowest will occur at the center of the meat. Therefore, the maximum possible oven temperature (T_∞) can be obtained from Figure 2.37 for $1/Bi = 3.8$; $X = L$

$$\frac{T(L,t) - T_\infty}{T(0,t) - T_\infty} = 0.88$$

$$T_\infty = \frac{0.88(T(0,t) - T_\infty)}{0.9 - 1.0} = \frac{0.88(121^\circ\text{C} - 154^\circ\text{C})}{-0.1} = 475°\text{C}$$

The actual oven temperature must be less than this so the center temperature can remain above 121°C without the surface temperature exceeding 154°C. The oven temperature and cooking time must be found by iterating the steps below

1. Pick an oven temperature.
2. Use Figure 2.37 to find the Fourier number which determines the time required for the center temperature to reach 121°C.
3. Add 20 min to the time and calculate a new Fourier number.
4. Use the new Fourier number and Figure 2.37 to find the center temperature at the end of the cooking period.
5. Use $(T(r_o, t) - T_\infty)/(T(0, 2t) - T_\infty) = 0.9$ to find the surface temperature at the end of the cooking period.

1. For the first iteration, let the oven temperature (T_∞) = 300°C.

2. $\dfrac{T(0,t) - T_\infty}{T_o - T_\infty} = \dfrac{121^\circ\text{C}}{27^\circ\text{C}} = 0.656$

From Figure 2.37

$$Fo = \frac{\alpha\, t}{r_o^2} = \frac{k\, t}{\rho\, c L^2} = 1.7$$

Solving for the time for the center to reach 121°C:

$$t = \frac{Fo\, \rho\, c L^2}{k} = \frac{1.7(4187\,\text{kg/m}^3)(1280\,\text{J/(kg K)})(0.0125\,\text{m})^2}{0.48\ \text{W/(m}^2\ \text{K)}} = 2963\ \text{sec}$$

3. After 20 min (1200s) cooking time: $t = 4163$, $F_o = 2.4$.
4. From Figure 2.37 for $F_o = 2.4$, $1/Bi = 3.8$

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = 0.55$$

$$T(0, t) = T_\infty + 0.55\,(T_o - T_\infty) = 300^\circ\text{C} + 0.55\,(27^\circ\text{C} - 300^\circ\text{C}) = 150^\circ\text{C}$$

5. $$\frac{T(L,t) - T_\infty}{T(0,t) - T_\infty} = 0.9$$

$$T(L, t) = T_\infty + 0.9\,(T(0, t) - T_\infty) = 300^\circ\text{C} + 0.9\,(150^\circ\text{C} - 300^\circ\text{C}) = 165^\circ\text{C}$$

Therefore, an oven temperature of 300°C is too high. The following iterations were performed using the same procedure

Oven Temperature	Fo	Time to Reach 121°C	Fo for 20 min	$\dfrac{T(0,t) - T_\infty}{T_\infty - T_\infty}$	T_o (°C)	T_L (°C)
300°C	1.7	2963 s	2.4	0.55	150	165
200°C	3.2	5578 s	3.9	0.37	136	142
150°C	2.4	4182 s	3.1	0.48	143	156

Therefore, the oven temperature should be set at 250°C and the meat should be heated for a total of 4184 s + 1200 s = 5384 s = 90 min.

PROBLEM 2.81

A frozen-food company freezes its spinach by first compressing it into large slabs and then exposing the slab of spinach to a low-temperature cooling medium. The large slab of compressed spinach is initially at a uniform temperature of 21°C; it must be reduced to an average temperature over the entire slab of –34°C. The temperature at any part of the slab, however, must never drop below –51°C. The cooling medium which passes across both sides of the slab is at a constant temperature of –90°C. The following data may be used for the spinach: density = 80 kg/m^3; thermal conductivity = 0.87 W/(m K); specific heat = 2100 J/(kg K). Present a detailed analysis outlining a method estimate the maximum thickness of the slab of spinach that can be safely cooled in 60 min.

GIVEN

- Large slabs of spinach are exposed to a low-temperature cooling medium
- Initial uniform temperature (T_o) = 21°C
- Average temperature must be reduced to –34°C
- The temperature at any part must never drop below –51°C
- Cooling medium temperature (T_∞) = –90°C
- Density of spinach (ρ) = 80 kg/m^3
- Thermal conductivity (k) = 0.87 W/(m K)
- Specific heat (c) = 2100 J/(kg K)

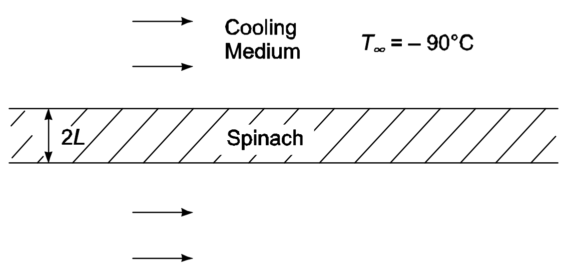
FIND

- Present a detailed analysis outlining a method to estimate the maximum thickness of the slab of spinach that can be safely cooled in 60 min

ASSUMPTIONS

- One dimensional conduction through the slab
- Constant and uniform thermal properties
- The average temperature within the slab is equal to the average of the center and surface temperatures

SKETCH



SOLUTION

For a final average temperature in the slab of –34°C, and a final surface temperature of –51°C, the final center temperature must be

$$T(0,\, t) = 2\, T_{\text{Ave}} - T(L,\, t) = 2(-\,34°\text{C}) + 51°\text{C} = -17°\text{C}$$

Figure 2.37 can be used to find the Biot number for the spinach slab

$$\frac{T(L,t) - T_\infty}{T(0,t) - T_\infty} = \frac{-51^{\text{o}}\text{C} - (-90^{\text{o}}\text{C})}{-17^{\text{o}}\text{C} - (-90^{\text{o}}\text{C})} = 0.53$$

From Figure 2.37 $1/Bi = 0.6$.

Figure 2.37 can be used to find the Fourier number

$$\frac{T(0,t) - T_\infty}{T_\infty - T_\infty} = \frac{-17^\circ\text{C} - (-90^\circ\text{C})}{-21^\circ\text{C} - (-90^\circ\text{C})} = 0.66$$

From Figure 2.37 $F_o = 0.4$

$$Fo = \frac{\alpha t}{r_o^2} = \frac{k t}{\rho c L^2}$$

Solving for L

$$L = \left(\frac{k t}{Fo\, \rho\, c}\right)^{0.5} = \left(\frac{[0.87\text{W/(m K)}](\text{J/(W s)})(60\text{ min})(60\text{ s/min})}{0.4\left(80\text{ kg/m}^3\right)(2100\text{ J/(kg K)})}\right)^{0.5} = 0.22\text{ m}$$

The thickness of the slab of spinach that can be cooled in 60 minutes is $2L = 0.44$ m = 44 cm.

The heat transfer coefficient needed to achieve this cooling can be calculated from the Biot number

$$Bi = \frac{\overline{h_c}\, L}{k} \Rightarrow \overline{h_c} = Bi\ \frac{k}{L} = \frac{1}{0.6}\ \frac{0.87\text{ W/(m K)}}{0.22\text{ m}} = 6.7\ \text{W/(m}^2\text{K)}$$

COMMENTS

The heat transfer coefficients is on the low side of the range for free convection in air (see Table 1.2).

Note that if the heat transfer coefficient is greater than 6.7 W/(m^2 K), the surface temperature of the spinach will drop below –51°C before the average temperature is lowered to –34°C.

PROBLEM 2.82

In the experimental determination of the hat transfer coefficient between a heated steel ball and crushed mineral solids, a series of 1.5% carbon steel balls were heated to a temperature of 700°C and the center temperature-time history of each was measured with a thermocouple while it was cooling in a bed of crushed iron ore, which was placed in a steel drum rotating horizontally at about 30 rpm. For a 5-cm-diameter ball, the time required for the temperature difference between the ball center and the surrounding ore to decrease from 500°C initially to 250°C was found to be 64, 67, and 72 s, respectively, in three different test runs. Determine the average heat transfer coefficient between the ball and the ore. Compare the results obtained by assuming the thermal conductivity to be infinite with those obtained by taking the internal thermal resistance of the ball into account.

GIVEN

- Heat steel balls are put in crushed iron ore
- Balls are 1.5% carbon steel balls
- Initial temperature of balls (T_o) = 700°C
- Ball diameter = 5 cm = 0.05 m

- Temperature difference between the ball center and the ore
- Center temperature of the balls decreases from 500°C to 250°C
- Time taken was found to be 64, 67, and 72 s, respectively, in three different test runs

FIND

The average heat transfer coefficient between the ball and the ore.

Compare the results obtained

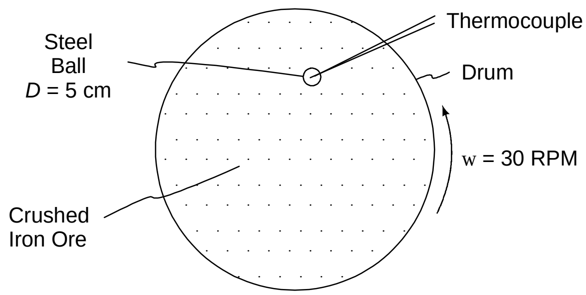
(a) by assuming the thermal conductivity to be infinite with

(b) those obtained by taking the internal thermal resistance of the ball into account

ASSUMPTIONS

- Temperature of the iron ore is uniform and constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10

For 1.5% carbon steel Thermal conductivity (k) = 36 W/(m K)

Density (ρ) = 7753 kg/m^3

Specific heat (c) = 486 J/(kg K)

Thermal diffusivity (α) = 0.97 × 10^{-5} m^2/s

SOLUTION

(a) Assuming the internal resistance of the balls is negligible. The temperature-time history is given by Equation (2.84)

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{h_c A_s}{c\rho V}t} = e^{-\frac{h_c \pi D^2}{c\rho \frac{\pi}{4} D^3}t} = e^{-\frac{6 h_c}{c\rho D}t}$$

Solving for the heat transfer coefficient

$$h_c = \frac{c\rho D}{6t} \ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

$$h_c = \frac{[486\ \text{J/(kg K)}]((\text{Ws})/\text{J})\left(7753\ \text{kg/m}^3\right)(0.05\ \text{m})}{6t} \ln\left(\frac{250^oC}{500^oC}\right) = \frac{21,765}{t}\ \text{Ws/(m}^2\ \text{K)}$$

For the three test runs: t = 64 s → h_c = 340 W/(m^2 K)

t = 67 s → h_c = 325 W/(m^2 K)

t = 72 s → h_c = 302 W/(m^2 K)

The average heat transfer coefficient is 322 W/(m^2 K)

(b) The chart method will be used to take the internal thermal resistance into account. Figure 2.39 can be used to determine the Biot number for the balls

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = \frac{250^\circ\text{C}}{500^\circ\text{C}} = 0.5$$

$$Fo = \frac{\alpha\, t}{r_o^2} = \frac{0.97 \times 10^{-5}\,\text{m}^2/\text{s}(t)}{(0.025\,\text{m})^2}$$

For the three test runs: $t = 64\ s \rightarrow$ $Fo = 0.99$

$t = 67\ s \rightarrow$ $Fo = 1.04$

$t = 72\ s \rightarrow$ $Fo = 1.12$

Figure 2.39 is not detailed enough to distinguish between the first two test runs

For the first two runs: $F_o = 1.0 \rightarrow 1/Bi = 4.0$ $Bi = 0.25$

For the third run: $F_o = 1.1 \rightarrow 1/Bi = 4.2$ $Bi = 0.238$

The average Bi number = [2(0.250) = 0.263]/3 = 0.246 = $(h_c\, r_o)/k$

Solving for the transfer coefficient

$$h_c = \frac{Bi\, k}{r_o} = \frac{0.246\big(36\text{W/(m K)}\big)}{0.025\text{ m}} = 354\ \text{W/(m}^2\text{ K)}$$

Neglecting the internal resistance resulted in a calculated heat transfer coefficient 9% lower than using the chart method.

PROBLEM 2.83

A mild-steel cylindrical billet, 25-cm in diameter, is to be raised to a minimum temperature of 760°C by passing it through a 6-m long strip type furnace. If the furnace gases are at 1538°C and the overall heat transfer coefficient on the outside of the billet is 68 W/(m^2 K), determine the maximum speed at which a continuous billet entering at 204°C can travel through the furnace.

GIVEN

- A mild-steel cylindrical billet is passed through a furnace
- Diameter of billet = 25 cm = 0.25 m
- Billet is to be raised to a minimum temperature of 760°C
- Length of furnace = 6 m
- Temperature of furnace gases (T_∞) = 1538°C
- The overall heat transfer coefficient $\left(\bar{h}_c\right)$ = 68 W/(m^2 K)
- Initial temperature of billet (T_o) = 204°C

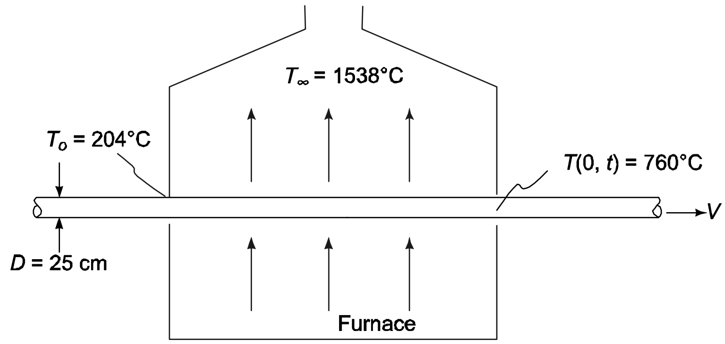
FIND

- The maximum speed at which a continuous billet can travel through the furnace

ASSUMPTIONS

- The heat transfer coefficient is constant
- Billet is 1% carbon steel
- Radial conduction only in the billet, neglect axial conduction

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10

For 1% carbon steel Thermal conductivity (k) = 43 W/(m K)
Thermal diffusivity (α) = 1.172×10^{-5} m^2/s

SOLUTION

The Biot number for the billet is

$$Bi = \frac{\bar{h}r_o}{K} = \frac{[68\ \text{W/(m}^2\ \text{K)}](0.125\,\text{m})}{43\ \text{W/(m K)}} = 0.198 > 0.1$$

Therefore, internal resistance is significant and we cannot use the lumped parameter method, a chart solution must be used.

The billet must obtain a centerline temperature of 760°C, therefore

$$\frac{T(0,t) - T_\infty}{T_i - T_\infty} = \frac{760°\text{C} - 1538°\text{C}}{204°\text{C} - 1538°\text{C}} = 0.583$$

The Fourier number from Figure 2.38 for $1/Bi = 1/0.198$ and $(T(0, t) - T_\infty)/(T_o - T_\infty) = 0.583$ is

$$Fo = \frac{\alpha t}{r_o^2} = 1.4$$

Solving for the time

$$t = \frac{For_o^2}{\alpha} = \frac{1.4(0.125\,\text{m})^2}{1.172 \times 10^{-5}\text{m}^2/\text{s}} = 1866\ \text{s}$$

The maximum speed of the billet is

$$V = \frac{\text{Length of furnace}}{\text{time needed}} = \frac{6\,\text{m}}{1866\,\text{s}} = 0.0032\ \text{m/s}$$

PROBLEM 2.84

A solid lead cylinder 0.6-m in diameter and 0.6-m long, initially at a uniform temperature of 121°C, is dropped into a 21°C liquid bath in which the heat transfer coefficient $\bar{h}_c$ is 1135 W/(m^2 K). Plot the temperature-time history of the center of this cylinder and compare it with the time histories of a 0.6 m diameter, infinitely long lead cylinder and a lead slab 0.6-m thick.

GIVEN

- A solid lead cylinder dropped into a liquid bath
- Cylinder diameter (D) = 0.6 m
- Cylinder (L) = 0.6 m
- Initial uniform temperature (T_o) = 121°C
- Liquid bath temperature (T_∞) = 21°C
- Heat transfer coefficient $\left(\bar{h}_c\right)$ = 1135 W/(m^2 K)

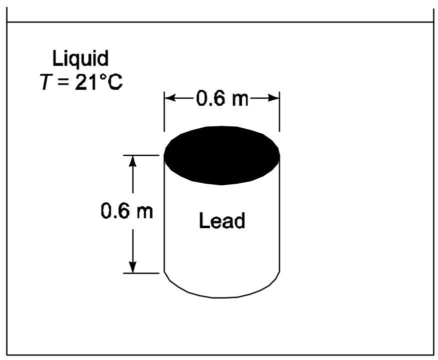
FIND

(a) Plot the temperature-time history of the cylinder center
(b) Compare it with the time history of a 0.6 m diameter, infinitely long lead cylinder
(c) Compare it with the time history of a lead slab 0.6 m thick

ASSUMPTIONS

- Two dimensional conduction within the cylinder
- Constant and uniform properties
- Constant liquid bath temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12

For lead Thermal conductivity (k) = 34.7 W/(m K) at 63°C
Density (ρ) = 11340 kg/m^3
Specific heat (c) = 129 J/(kg K)
Thermal diffusivity (α) = 24.1 × 10^{-6} m^2/s

SOLUTION

The Biot number based on radius is

$$Bi = \frac{\bar{h}_c r_o}{K} = \frac{[1135\text{W/(m}^2\text{ K)}](0.3\text{m})}{34.7\text{W/(m K)}} = 9.81 > 0.1$$

Therefore, internal resistance is significant.

(a) This two-dimensional system required a product solution. From Table 2.4 the product solution is

$$\frac{\theta_p(x,r)}{\theta_o} = P(x)\,C(r)$$

where

$$P(x) = \frac{\theta(x,t)}{\theta_o} \text{ for an infinite plate (Figure 2.37)}$$

$$C(r) = \frac{\theta(r,t)}{\theta_o} \text{ for a long cylinder (Figure 2.38)}$$

Since the length of the cylinder is the same as its diameter, the Biot number based on length is the same as that based on radius

$$\frac{1}{Bi} = \frac{1}{9.81} = 0.102$$

The Fourier number is

$$Fo = \frac{\alpha\, t}{\left(L/2 \text{ or } r_o\right)^2} = \frac{24.1 \times 10^{-6}\,\text{m}^2/\text{s}\,(t)}{(0.3\,\text{m})^2} = 0.000268\, t\ \text{s}^{-1}$$

The temperature of the center of the cylinder ($x = 0$, $r = 0$) is determined by calculating the Fourier number for that time, finding $P(0)$ on Figure 2.37, finding $C(0)$ on Figure 2.38, and applying

$$\frac{\theta_p(0,0)}{\theta_o} = \frac{T(0,0) - T_\infty}{T_o - T_\infty} = P(x)\,C(r)$$

$$T(0,0) = T_\infty + P(x)\,C(r)\,(T_o - T_\infty)$$

(b) The center temperature for a long cylinder is

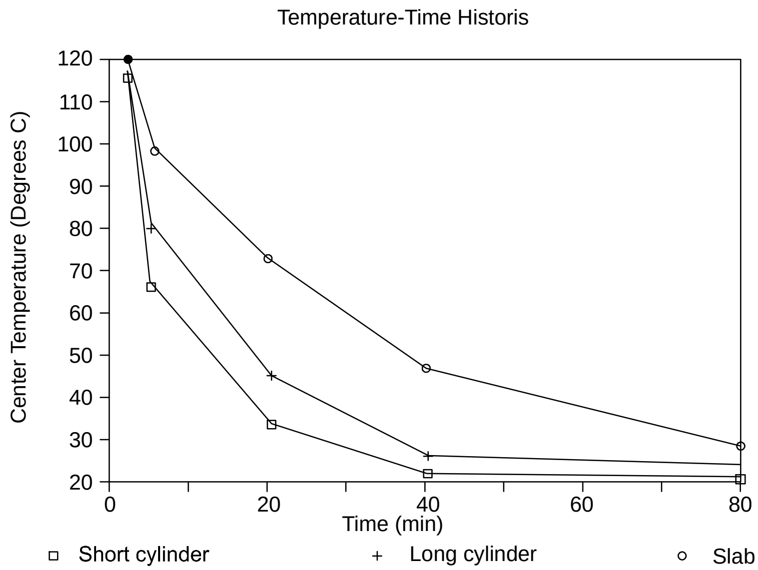
$$T(r = o, t) = T_\infty + C(o)\,(T_o - T_\infty)$$

(c) The center temperature for a slab is

$$T(x = o, t) = T_\infty + P(o)\,(T_o - T_\infty)$$

The temperature-time histories of these three cases are tabulated and plotted below

Time(s)	(min)	*Fo*	*P*(0)	*C*(0)	*T*(0, 0) (°C) (a) Short Cylinder	(b) Long Cylinder	(c) Slab
120	2	0.03	0.99	0.95	115	116	120
300	5	0.08	0.78	0.60	68	81	99
1200	20	0.32	0.52	0.24	33	45	73
4800	80	1.28	.075	.033	21	14	29



PROBLEM 2.85

A long 0.6-m-*OD* 347 stainless steel (*k* = 14 W/(m K) cylindrical billet at 16°C room temperature is placed in an oven where the temperature is 260°C. If the average heat transfer coefficient is 170 W/(m^2 K), (a) estimate the time required for the center temperature to increase to 323°C by using the appropriate chart and (b) determine the instantaneous surface heat flux when the center temperature is 232°C.

GIVEN

- A long cylindrical billet placed in an oven
- Billet outside diameter = 0.6 m
- Thermal conductivity (k) = 14 W/(m K)
- Initial temperature (T_i) = 16°C
- Oven temperature (T_∞) = 260°C
- The average heat transfer coefficient $\left(\bar{h}_c\right)$ = 170 W/(m^2 K)
- Center temperature increases to 232°C

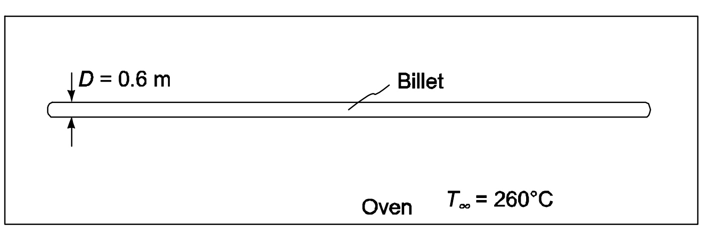
FIND

(a) The time required using the appropriate chart
(b) The instantaneous surface heat fluxes when the center temperature is 232°C

ASSUMPTIONS

- Radial conduction only in billet
- Uniform and constant properties

SKETCH



SOLUTION

(a) The Biot number for the billet is

$$Bi = \frac{\bar{h}_c r_o}{K} = \frac{[170\,\text{W/(m}^2\,\text{K)}](0.3\,\text{m})}{14\,\text{W/(m K)}} = 3.643 > 0.1$$

$$\frac{1}{Bi} = 0.275$$

$$\frac{T(0,t_f) - T_\infty}{T_o - T_\infty} = \frac{232°\text{C} - 260°\text{C}}{16°\text{C} - 260°\text{C}} = 0.115$$

From Figure 2.38

$$Fo = \frac{\alpha\, t}{r_o^{\,2}} = 0.65$$

Solving for the time

$$t = \frac{Fo\, r_o^{\,2}}{\alpha} = \frac{0.65(0.3\,\text{m})^2}{0.387 \times 10^{-5}\,\text{m}^2/\text{s}} = 15{,}116\ \text{s} = 252\ \text{min} = 4.2\ \text{hr}$$

(b) The surface temperature is needed to find the surface heat flux. For $1/Bi = 0.275$ and $r = r_o$, from Figure 2.38.

$$\frac{T(r_o,t) - T_\infty}{T(0,t) - T_\infty} = 0.3$$

$$T(r_o,\, t) = T_\infty + 0.3\,(T(0,\, t) - T_\infty) = 260°\text{C} + 0.3\,(232°\text{C} - 260°\text{C}) = 251.6°\text{C}$$

The instantaneous surface flux is

$$\frac{q}{A} = \bar{h}\ [T_\infty - T(r_o,\, t)] = 170\ \text{W/(m}^2\ \text{K)}\ (251°\text{C} - 260°\text{C}) = 1428\ \text{W/m}^2$$

PROBLEM 2.86

Repeat Problem 2.85(a), but assume that the billet is only 1.2-m long and the average heat transfer coefficient at both ends is 136 W/(m^2 K).

PROBLEM 2.85

A long, 0.6 m *OD* 347 stainless steel (*k* = 14 W/(m K)) cylindrical billet at 16°C room temperature is placed in an over where the temperature is 260°C. If the average heat transfer coefficient is 170 W/(m^2 K), estimate the time required for the center temperature to increase to 232°C by using the appropriate chart.

GIVEN

- A cylindrical billet placed in an over
- Billet outside diameter = 0.6 m
- Thermal conductivity (k) = 14 W/(m K)
- Initial temperature (T_o) = 16°C
- Oven temperature (T_∞) = 260°C
- The average heat transfer coefficient $\left(\bar{h}_{cs}\right)$ = 170 W/(m^2 K)
- Increase of the center temperature is 232°C
- Billet length ($2L$) = 1.2 m
- Heat transfer coefficient at the ends $\left(\bar{h}_{ce}\right)$ = 136 W/(m^2 K)

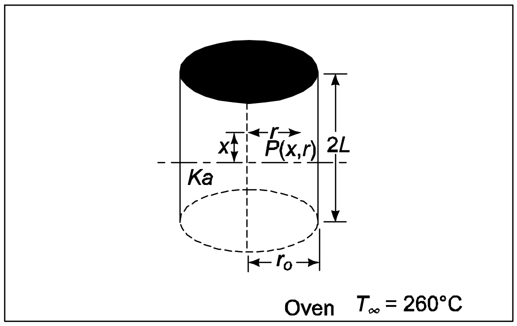
FIND

- The time required using the appropriate charts

ASSUMPTIONS

- Two dimensional conduction within the billet
- Constant and uniform thermal properties
- Constant oven temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10 For Type 304 stainless steel

Thermal diffusivity (α) = 0.387×10^{-5} m^2/s

SOLUTION

From Table 2.4, the solution for this geometry is

$$\frac{\theta_p(x,r)}{\theta_o} = P(x)\,C(r)$$

where

$$P(x) = \frac{\theta(x,t)}{\theta_o} \text{ for an infinite plate (Figure 2.37)}$$

$$C(r) = \frac{\theta(r,t)}{\theta_o} \text{ for a long cylinder (Figure 2.38)}$$

$$\frac{\theta_p(0,0)}{\theta_o} = \frac{T(0,0) - T_\infty}{T_o - T_\infty} = \frac{232°\text{C} - 260°\text{C}}{16°\text{C} - 260°\text{C}} = 0.11 = P(0)\,C(0)$$

For the infinite plate solution

$$(Bi)_x = \frac{\bar{h}_{ce} L}{k} = \frac{[136\ \text{W/(m}^2\ \text{K)}](0.6\,\text{m})}{14\ \text{W/(m K)}} = 5.83 \Rightarrow \frac{1}{Bi} = 0.17$$

$$Fo = \frac{\alpha\, t}{L^2} = \frac{0.387 \times 10^{-5}\text{m}^2/\text{s}}{(0.6\,\text{m})^2}\, t = 1.075 \times 10^{-5}\ t\ \text{s}^{-1}$$

For the long cylinder solution

$$(Bi)_r = \frac{\bar{h}_{cs} r_o}{k} = \frac{[170\ \text{W/(m}^2\ \text{K)}](0.3\,\text{m})}{14\ \text{W/(m K)}} = 3.54 \Rightarrow \frac{1}{Bi} = 0.28$$

$$Fo = \frac{\alpha\, t}{r_o^{\,2}} = \frac{0.387 \times 10^{-5}\,\text{m}^2/\text{s}}{(0.3\,\text{m})^2}\, t = 4.3 \times 10^{-5}\ t\ \text{s}^{-1}$$

The time required to reach a product solution of 0.115 is found through trial and error.

Time(s)	(min)	Fo_x	$P(0)$	Fo_r	$C(0)$	$P(0)C(0)$
6,000	100	0.065	0.99	0.26	0.37	0.0366
12,000	200	0.13	0.82	0.52	0.17	0.139
15,000	250	0.16	0.54	0.645	0.10	0.054
13,000	217	0.14	0.60	0.56	0.15	0.090
12,500	208	0.134	0.70	0.538	0.208	0.11

The time required is approximately 208 min or 3.4 hours.

COMMENTS

The uncertainty in the solution is high because of the difficulty reading Figure 2.37 at very low Fourier numbers. For higher accuracy, the differential equations that describe the problem would have to be solved.

PROBLEM 2.87

> **A large billet of steel initially at 260°C is placed in a radiant furnace where the surface temperature is held at 1200°C. Assuming the billet is infinite in extent, compute the temperature at point *P* shown in the accompanying sketch after 25 min has elapsed. The average properties of steel are: *k* = 28 W/(m K), ρ = 7360 kg/m^3, and *c* = 500 J/(kg K).**

GIVEN

- A large billet of steel is placed in a radiant furnace
- Initial temperature (T_o) = 260°C
- Surface temperature of billet in the oven (T_s) = 1200°C
- Lapse time (t) = 25 min = 1500 s
- Thermal conductivity (k) = 28 W/(m K)
- Density (ρ) = 7360 kg/m^3
- Specific heat (c) = 500 J/(kg K)

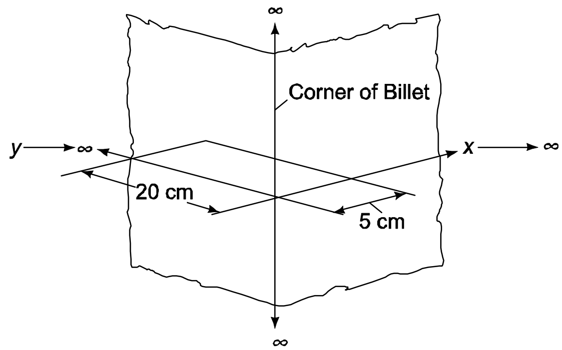
FIND

- The temperature at point *P* shown in the accompanying sketch

ASSUMPTIONS

- The billet infinite in extent

SKETCH



SOLUTION

From Table 2.4, the solution for a one quarter infinite solid is

$$\frac{\theta_p(x,y)}{\theta_o} = \frac{T(x,y,t)-T_s}{T_o-T_s} = S(x)\,S(y)$$

Where $S(x)$ and $S(y)$ are solutions for a semi-infinite solid, which are given for a constant surface temperature by Equation (2.105)

$$\frac{T(x,t)-T_\infty}{T_o-T_\infty} = \text{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Therefore, the solution to this problem is

$$\frac{T(x,y,t)-T_\infty}{T_o-T_\infty} = \text{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)\ \text{erf}\left(\frac{y}{2\sqrt{\alpha t}}\right)$$

$$T(x,y,t) = T_s + (T_o - T_s)\left[\text{erf}\left(\frac{x}{M}\right)\text{erf}\left(\frac{y}{M}\right)\right]$$

where

$$M = 2\sqrt{\alpha t} = 2\sqrt{\frac{k\,t}{\rho\,c}} = 2\sqrt{\frac{[28\text{ W/(m K)}](1500\text{ s})}{(7360\text{ kg/m}^2)(500\text{ J/(kg K)})}} = 0.2137$$

$$\therefore\quad T\,(0.05\text{ m}, 0.2\text{ m}, 1500\text{ s}) = 1200°\text{C} = (260°\text{C} - 1200°\text{C})\ \text{erf}\left(\frac{0.05\text{ m}}{0.2137\text{ m}}\right)\text{erf}\left(\frac{0.2\text{ m}}{0.2137\text{ m}}\right)$$

Using Appendix 2, Table 43 for the error function values

$$T\,(0.05\text{ m}, 0.2\text{ m}, 1500\text{ s}) = 1200°\text{C} - 940°\,(0.259)\,(0.814) = 1002°\text{C}$$