Chapter 2: Concept Review Solutions

2.1 Consider steady-state heat conduction in a semi-infinite plate or slab of thickness L, a very long hollow cylinder, and a hollow sphere of inner radius r_i and outer radius r_o. Assuming uniform conductivity k in the plate, write the conduction equation and express the respective thermal resistance for each of the three geometries.

Semi-infinite plate or slab of thickness L: $\dfrac{d^2T}{dx^2} = 0$

For heat flow from the hot side at T_H to the cold side at T_C the heat transfer rate is

$$q_k = -kA\frac{\Delta T}{L} = \frac{(T_H - T_C)}{R_{th}},$$

where the thermal resistance is $R_{th} = \dfrac{L}{kA}$ and A is the surface area of the plate.

Long hollow cylinder of radii r_i and r_o: $\dfrac{d}{dr}\left(r\dfrac{dT}{dr}\right) = 0$

For heat flow from the inside of the cylinder to the outside ($T_i > T_o$), the heat transfer rate is

$$q_k = 2\pi\, Lk\frac{(T_i - T_o)}{\ln(r_o/r_i)} = \frac{(T_i - T_o)}{R_{th}},$$

where the thermal resistance is $R_{th} = \dfrac{\ln(r_o/r_i)}{2\pi\, Lk}$ and L is the length of cylinder.

Hollow sphere of radii r_i and r_o: $\dfrac{1}{r^2}\dfrac{d}{dr}\left(r^2\dfrac{dT}{dr}\right) = 0$

For heat flow from the inside of the sphere to the outside ($T_i > T_o$), the heat transfer rate is

$$q_k = 4\pi\, r_o r_i k\frac{(T_i - T_o)}{(r_o - r_i)} = \frac{(T_i - T_o)}{R_{th}},$$

where the thermal resistance is $R_{th} = \dfrac{r_o - r_i}{4\pi\, r_o r_i k}$.

2.2 What is the primary purpose of adding fins to a heat transfer surface? Consider a plate separating two fluids, A and B, with respective convection heat transfer

coefficients $\bar{h}_{cA}$ and $\bar{h}_{cB}$ such that $\bar{h}_{cA} >> \bar{h}_{cB}$. To what side of the plate surface should fins be added and why? In choosing the size of these fins, would you make them as long as the available space would permit? Why or why not?

The primary purpose of adding fins to a heat transfer surface is to increase its surface area so that it can increase the rate of heating or cooling, or *heat transfer enhancement*. In explanation, consider a surface of area A that exchanges heat by convection with its surroundings (e.g. cooling air flow over a heat dissipating microelectronic processor) for which the heat transfer rate can be expressed as

$$q_c = \bar{h}_c A \Delta T$$

Thus, for fixed ΔT and $\bar{h}_c$, a higher heat transfer rate q_c can be sustained by increasing the surface area A by adding fins. Another way to look at this problem is that by increasing A, the same heat transfer rate q_c can be accommodated with a much smaller temperature difference ΔT. Such considerations are important for the design of many heat exchangers in chemical processing plants, waste-heat recovery systems, micro-electronics cooling, and solar thermal energy conversion.

The overall thermal resistance of the two fluids, A and B, and the plate separating them is given by

$$R_o = R_{cA} + R_{plate} + R_{cB} = \left(\frac{1}{\bar{h}_{cA} A}\right) + R_{plate} + \left(\frac{1}{\bar{h}_{cB} A}\right)$$

Thus, if $\bar{h}_{cA} >> \bar{h}_{cB}$ then $R_{cB} >> R_{cA}$ and if the plate thermal resistance is much smaller than either of the two convection resistances, fins should be added to side B of the plate so that R_{cB} could be reduced.

In choosing the size of fins, optimization based on length of fin is not very useful because fin efficiency decreases with increasing fin length (fin efficiency is 100% for the trivial case of zero fin length). What is more important is the relative increase in the heat transfer surface area with the addition of fins, which have a reasonably high efficiency (relatively short fins). For this, the number of fins becomes more important and generally thin, slender, closely space fins provide the most benefit (compared to fewer and thicker fins).

2.3 Define the Biot number and briefly explain its physical interpretation. What would be the primary difference between transient heat conduction from a solid to a convective environment when (a) Bi is very small and (b) Bi is large? What value of Bi is generally taken to separate the two regimes in engineering practice?

The Biot number is usually defined as (see Eq. 2.81):

$$Bi = \frac{\bar{h}_c L}{k_s} = \frac{R_{internal}(conduction)}{R_{external}(convection)}$$

and as indicated on the right-hand side of the equation, it represents a balance between the internal thermal resistance of the solid body and the external thermal resistance due to convection to/from the surrounding fluid medium.

When Bi is very small (a), the internal thermal resistance of the solid body is negligible compared to the external convective resistance, and consequently the solid body can be treated as a *lumped capacitance* in which temperature within the solid body is uniform at any time instant. On the other hand, when Bi is large (b), the internal thermal resistance of the solid body is much greater than the external convective resistance and it therefore controls the transient heat transfer between the solid and the surrounding fluid medium. As a result, there would be significant internal temperature variation in the solid body.

In engineering practice, $\text{Bi} < 0.1$ is usually considered for lumped-heat-capacity analysis or to model the solid body as a *lumped capacitance* for transient heat transfer calculations.

2.4 When a cold can of soda is left on a table it warms up. Briefly describe the modes of heat transfer involved in this process and outline how you would model the problem.

The primary modes of heat transfer are (i) convection (natural convection) between the soda can and surrounding air, and (ii) conduction through the walls of the can and the bulk of soda liquid contained in the can.

To model the transient warming up of the cold soda can when left on a table, the student should consider the following:

(1) Assume that the radius r_o of the cylindrical can is much smaller than its height or length H, or $r_o < H$. Also, neglect the thickness of the aluminum can (which in any case is very small and has a very high thermal conductivity) and model it simply as a cylindrical "soda block" which has the thermal conductivity of the soda liquid (one can assume $k_{soda} \approx k_{water}$ in the absence of specific values for soda).

(2) Calculate the Bio number, Bi, where the characteristic length is $L = V/A_s$ (V is the volume of the can and A_s is the surface area of the can). The convective heat transfer coefficient can be assumed to be a nominal value for natural convection in air to/from a cylindrical body.

(3) If $\text{Bi} < 0.1$, consider the soda can as a *lumped capacitance* with uniform temperature distribution in the body of the soda. The change in its temperature with time can be determined from Eq. (2.89).

(4) If $\text{Bi} > 0.1$, then the soda temperature cannot be considered as uniform and it would have a distribution $T(r)$. The center-line or mid-point temperature in the soda can and its change with time can then be determined from the charts for transient heat conduction given in Fig. 2.43. Note that in these charts the characteristic length for the Biot number, Bi, and the Fourier number, Fo, is given as $L = r_o$.