**PROBLEM 2.70**

Air expands adiabatically in a piston-cylinder assembly from an initial state where $p_1$ = 100 lbf/in.$^2$, $v_1$ = 3.704 ft$^3$/lb, and $T_1$ = 1000 $^o$R, to a final state where $p_2$ = 50 lbf/in.$^2$ The process is polytropic with $n$ = 1.4. The change in specific internal energy, in Btu/lb, can be expressed in terms of temperature change as $\Delta u = (0.171)(T_2 - T_1)$. Determine the final temperature, in $^o$R. Kinetic and potential energy effects can be neglected.

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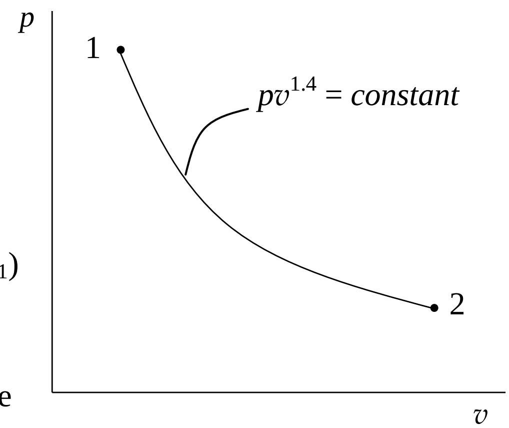
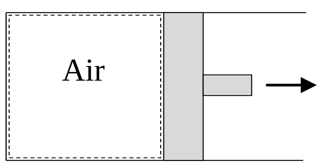
**KNOWN**: Air undergoes a polytropic process with known $n$ in a piston-cylinder assembly. Data are known at the initial and final states, and the change in specific internal energy is expressed as a function of temperature change.

**FIND**: Determine the final temperature.




**SCHEMATIC AND GIVEN DATA**:




$Q$ = 0

$p_1$ = 100 lbf/in.$^2$
$v_1$ = 3.704 ft$^3$/lb
$T_1$ = 1000 $^o$R
$p_2$ = 50 lbf/in.$^2$
$\Delta u$ = (0.171Btu/lb·$^o$R)($T_2 - T_1$)

**ENGINEERING MODEL**: (1) The air is a closed system. (2) The process is polytropic with $n$ = 1.4 and volume change is the only work mode. (3) The process is adiabatic: $Q$ = 0. (4) Kinetic and potential energy effects are negligible.

**ANALYSIS**: To find the final temperature, we will use the energy balance with the given expression for change in specific internal energy as a function of temperature change. First, determine the work using Eq. 2.17

$$W/m = \int_{v_1}^{v_2} p\,dv = \int_{v_1}^{v_2} \frac{const}{v^{1.4}}\,dv = \frac{(p_2v_2 - p_1v_1)}{1-1.4}$$

For the polytropic proess, $p_1v_1^{1.4} = p_2v_2^{1.4}$. Thus

$$v_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{1.4}} v_1 = \left(\frac{100\ \text{lbf/in.}^2}{50\ \text{lbf/in.}^2}\right)^{\frac{1}{1.4}} (3.704\ \text{ft}^3/\text{lb}) = 6.077\ \text{ft}^3/\text{lb}$$

So, the work is

$$W/m = \frac{(50\ \text{lbf/in.}^2)(6.077\ \text{ft}^3/\text{lb}) - (100\ )(3.704)}{1-1.4}\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft·lbf}}\right| = 30.794\ \text{Btu/lb}$$

The energy balance is: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W$. With $\Delta U = m(u_2 - u_1)$

**PROBLEM 2.70 (CONTINUED)**

$$(u_2 - u_1) = - W/m$$

Inserting values

$$(0.171 \text{ Btu/lb}\cdot{}^{o}\text{R})(T_2 - 1000\ {}^{o}\text{R}) = -\ (30.794 \text{ Btu/lb})$$

Solving; $T_2 = (-30.794)/(0.171) + 1000 = 819.9\ {}^{o}\text{R}$ ←