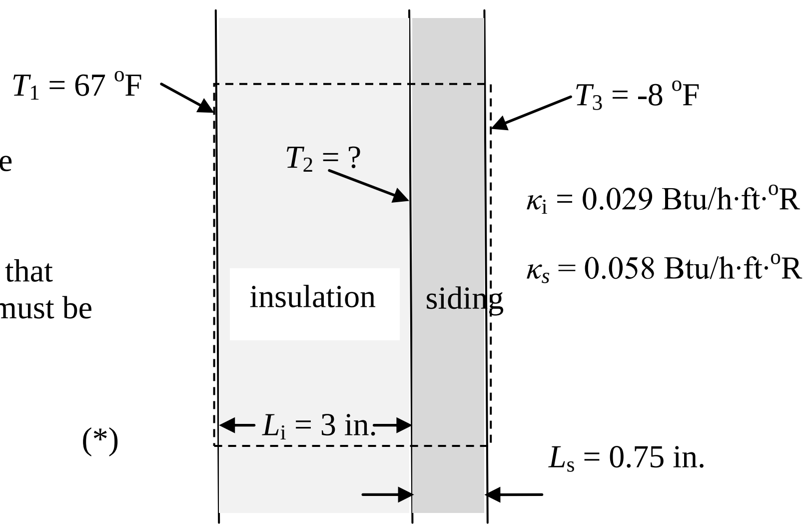
Problem 2.50

A composite plane wall consists of a 3-in.-thick layer of insulation (κ_i = 0.029 Btu/h·ft·^{o}R) and a 0.75-in.-thick layer of siding (κ_s = 0.058 Btu/h·ft·^{o}R). The inner temperature of the insulation is 67^{o}F. The outer temperature of the siding is -8^{o}F. Determine at steady state (a) the temperature at the interface of the two layers, in ^{o}F, and (b) the rate of heat transfer through the wall in Btu per ft^2 of surface area.

KNOWN: Energy transfer by conduction occurs through a composite wall consisting of two layers.

FIND: Determine the temperature at the interface between the two layers and the rate of heat transfer per unit area through the wall.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The wall is the system at steady state. (2) The temperature varies linearly through each layer.

ANALYSIS: With Eq. 2.17, and recognizing that at steady state the rates of energy conduction must be equal through each layer

$$\frac{\dot{Q}}{A} = -\kappa_i\left[\frac{T_2 - T_1}{L_i}\right] = -\kappa_s\left[\frac{T_3 - T_2}{L_s}\right] \qquad (*)$$

Solving for T_2

$$T_2 = \frac{\left(\frac{\kappa_i}{L_i}T_1 + \frac{\kappa_s}{L_s}T_3\right)}{(\kappa_i/L_i) + (\kappa_s/L_s)}$$

$$\frac{\kappa_i}{L_i} = \frac{0.029\ \text{Btu/h·ft·R}}{3\ \text{in.}}\left|\frac{12\ \text{in.}}{1\ \text{ft}}\right| = 0.116\ \text{Btu/h·}^o\text{R} \qquad \frac{\kappa_s}{L_s} = \frac{0.058\ \text{Btu/h·ft·R}}{.75\ \text{in.}}\left|\frac{12}{1}\right| = 0.928\ \text{Btu/h·}^o\text{R}$$

Thus

$$T_2 = \frac{(0.116)(527) + (0.928)(452)}{(0.116) + (0.928)} = 460.3\ ^o\text{R} = 0.33\ ^o\text{F} \longleftarrow$$

Thus, using Eq. (*)

$$\frac{\dot{Q}}{A} = -\kappa_i\left[\frac{T_2 - T_1}{L_i}\right] = \left(-0.029\,\frac{\text{Btu}}{\text{h·ft·R}}\right)\left[\frac{(0.33-67)\text{R}}{\frac{3}{12}\text{ft}}\right] = 7.73\ \text{Btu/ft}^2$$

$$\longleftarrow$$

$$\frac{\dot{Q}}{A} = -\kappa_s\left[\frac{T_3 - T_2}{L_2}\right] = \left(-0.058\,\frac{\text{Btu}}{\text{h·ft·R}}\right)\left[\frac{(-8-0.33)\text{R}}{\frac{0.75}{12}\text{ft}}\right] = 7.73\ \text{Btu/ft}^2$$