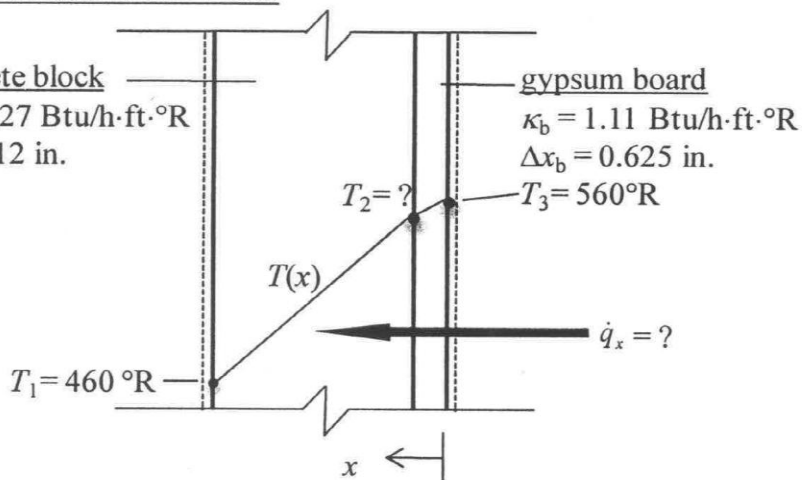


### PROBLEM 2.49

Known: Insulated composite plane wall consists of insulated concrete block and gypsum board with known surface temperatures. There is perfect contact at the interface between the two layers.

Find: Determine at steady state the instantaneous rate of heat transfer, in Btu/h per ft<sup>2</sup> of surface area, and the temperature, in °R, at the interface between the concrete block and gypsum board.

Schematic and Given Data:



Engineering Model:

- (1) The wall is a closed system at steady state.
- (2) The temperature distributions are linear in both layers.
- (3) The two layers are in perfect thermal contact.
- (4) The thermal conductivity in each layer is uniform.

Analysis:

Begin with Eq. 2.31

$$\dot{Q}_x = -\kappa A \frac{dT}{dx}$$

$$\dot{q}_x = \frac{\dot{Q}_x}{A} = -\kappa \frac{dT}{dx} = -\kappa_c \left. \frac{dT}{dx} \right|_{\text{concrete}} = -\kappa_b \left. \frac{dT}{dx} \right|_{\text{gypsum board}}$$

### PROBLEM 2.49 (CONTINUED)

Using assumption (2)

$$\dot{q}_x = \kappa_c \frac{T_2 - T_1}{\Delta x_c} = \kappa_b \frac{T_3 - T_2}{\Delta x_b}$$

Use thermal resistance,  $R = \Delta x / \kappa$ , to simplify

$$R_c = \frac{\Delta x_c}{\kappa_c} \quad \text{and} \quad R_b = \frac{\Delta x_b}{\kappa_b}$$

$$\dot{q}_x = \frac{T_2 - T_1}{R_c} = \frac{T_3 - T_2}{R_b} \quad (1)$$

Rearrange first part of Eq. (1) (2)

$$T_2 = T_1 + R_c \dot{q}_x$$

Substitute Eq. (2) into the second part of Eq. (1) and rearrange

#1 
$$\dot{q}_x = \frac{T_3 - T_2}{R_b} = \frac{T_3 - (T_1 + R_c \dot{q}_x)}{R_b} = \frac{T_3 - T_1}{R_c + R_b} \quad (3)$$

Solve for  $R$  values

$$R_c = \frac{\Delta x_c}{\kappa_c} = \frac{12 \text{ in.}}{0.27 \frac{\text{Btu}}{\text{h} \cdot \text{ft} \cdot ^\circ\text{R}}} \left| \frac{1 \text{ ft}}{12 \text{ in.}} \right| = 3.7 \frac{\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{R}}{\text{Btu}}$$

$$R_b = \frac{\Delta x_b}{\kappa_b} = \frac{0.625 \text{ in.}}{1.11 \frac{\text{Btu}}{\text{h} \cdot \text{ft} \cdot ^\circ\text{R}}} \left| \frac{1 \text{ ft}}{12 \text{ in.}} \right| = 0.047 \frac{\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{R}}{\text{Btu}}$$

Solve for  $\dot{q}_x$  and  $T_2$

$$\dot{q}_x = \frac{T_3 - T_1}{R_c + R_b} = \frac{560^\circ\text{R} - 460^\circ\text{R}}{(3.7 + 0.047) \frac{\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{R}}{\text{Btu}}} = 26.69 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2} \quad \leftarrow$$

#2 
$$T_2 = T_1 + R_c \dot{q}_x = 460^\circ\text{R} + \left( 3.7 \frac{\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{R}}{\text{Btu}} \right) \left( 26.69 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2} \right) = 558.8^\circ\text{R} \quad \leftarrow$$

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1. Eq. (3) illustrates the analogy between heat conduction through a composite wall and electric current flow through a series of resistances. The temperature difference in the numerator is analogous to a voltage difference, and the value  $R_c$  and  $R_b$  are "thermal resistances" analogous to electrical resistances.
  2. Due to the relatively low value for  $R_b$ , the temperature change across the gypsum board is minimal compared to the concrete block.