PROBLEM 2.43

KNOWN: A wire suspended vertically is stretched by an applied force.

FIND: Obtain an expression for the work done on the wire and evaluate the work for a given set of data.

SCHEMATIC & GIVEN DATA:

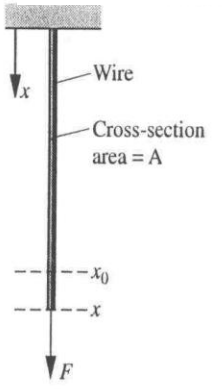

Fig. P2.39

$\sigma = C\varepsilon$ (normal stress, strain)

$\varepsilon = \frac{x - x_0}{x_0}$

C: Young's modulus

Data set:

$x_0 = 10\ ft$

$x = 10.01\ ft$

$C = 2.5 \times 10^7 \frac{lbf}{in^2}$

$A = 0.1\ in^2$

ENGR. MODEL:

1. The wire is the closed system.
2. The moving boundary is the only work mode.
3. The change in area A is negligible.
4. The normal stress and thus the applied force varies linearly with strain.

ANALYSIS: (a) The work done on the wire is given by Eq. 2.18

$$W = -\int_{x_0}^{x} \sigma A\, dx$$

From the given stress-strain relation

$$\sigma = C\varepsilon = C\left(\frac{x - x_0}{x_0}\right)$$

where C is a constant (Young's modulus). From this expression

$$d\varepsilon = \frac{dx}{x_0} \Rightarrow dx = x_0\, d\varepsilon$$

Substituting into the work expression

$$W = -\int_0^{\varepsilon} (C\varepsilon) A (x_0\, d\varepsilon) = -CAx_0 \int_0^{\varepsilon} \varepsilon\, d\varepsilon$$

Finally

$$W = -\frac{CAx_0\varepsilon^2}{2}$$

← work expression

PROBLEM 2.43 (Continued)

(b) Substituting given data into the work expression

$$W = -\frac{\left(2.5\times10^{7}\ \frac{lbf}{in^2}\right)(0.1\ in^2)(10\ ft)\left[\frac{0.01}{10}\right]^2}{2} = -12.5\ ft\cdot lbf$$

(10^{-6})

The downward force varies with strain according to

$$F = \sigma A = C\epsilon A = AC\left[\frac{x - x_0}{x_0}\right]$$

When $x = x_0$, $F = 0$. When $x - x_0 = 0.01\ ft$,

$$F = (0.1\ in^2)\left(2.5\times10^{7}\ \frac{lbf}{in^2}\right)\left(\frac{0.01\ ft}{10\ ft}\right) = 2500\ lbf$$