Problem 2.27

Carbon dioxide (CO_2) gas within a piston-cylinder assembly undergoes a process from a state where p_1 = 5 lbf/in.^{2}, V_1 = 2.5 ft^3 to a state where p_2 = 20 lbf/in.^{2}, V_2 = 0.5 ft^3. The relationship between pressure and volume during the process is given by $p = 23.75 - 7.5V$, where V is in ft^3 and p is in lbf/in.^{2} Determine the work for the process, in Btu.

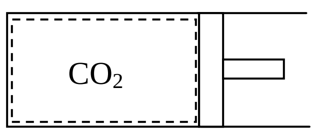
KNOWN: CO_2 gas within a piston-cylinder assembly undergoes a process where the *p*-*V* relation is given. The initial and final states are specified.

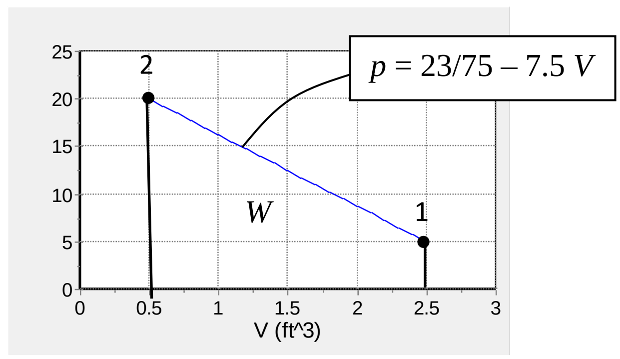
FIND: Determine the work for the process.

SCHEMATIC AND GIVEN DATA:

p_1 = 5 lbf/in.^{2}
V_1 = 2.5 ft^3

p_2 = 20 lbf/in.^{2}
V_2 = 0.5 ft^3





ENGINEERING MODEL: (1) The CO_2 is the closed system. (2) The *p*-*V* relation during the process is linear. (3) Volume change is the only work mode.

ANALYSIS: The given *p*-*V* relation can be used with Eq. 2.17 as follows:

$$W = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} [23.75 - 7.5V] dV = \left[23.75V - \frac{7.5V^2}{2}\right]_{V_1}^{V_2}$$

$$= 23.75[V_2 - V_1] - \frac{7.5}{2}[V_2^2 - V_1^2]$$

$$W = \left(23.75 \frac{\text{lbf}}{\text{in.}^2}\right)\left|\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right|[0.5 - 2.5]\text{ft}^3 - \left(\frac{7.5}{2}\frac{\text{lbf/in.}^2}{\text{ft}^3}\right)\left|\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right|[0.5^2 - 2.5^2](\text{ft}^3)^2$$

= -3600 ft·lbf

$= (-3600 \text{ ft} \cdot \text{lbf})\left|\frac{1 \text{ Btu}}{778 \text{ ft·lbf}}\right| = -4.63$ Btu (negative sign denotes energy transfer *in*.) ←

Alternative Solution

Since the *p*-*V* relation is linear, *W* can also be evaluated geometrically as the area under the process line:

$$W = p_{\text{ave}}(V_2 - V_1) = \left(\frac{p_1 + p_2}{2}\right)(V_2 - V_1) = \left(\frac{20 + 5}{2}\right)\frac{\text{lbf}}{\text{in}^2}\left|\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right|(0.5 - 2.5)\text{ft}^3\left|\frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}}\right|$$

= -4.63 Btu