PROBLEM 2.76

KNOWN: Data are provided for a power cycle executed by a gas in a piston-cylinder assembly.

FIND: For each process evaluate W. Find Q for processes 1-2, 2-3. Evaluate the thermal efficiency.

SCHEMATIC & GIVEN DATA:

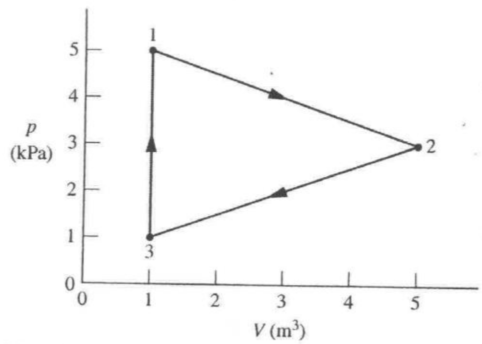
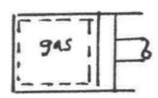


Fig. P2.73



$U_2 - U_1 = 15\text{ kJ}$

$Q_{31} = 10\text{ kJ}$

ENGR. MODEL

1. The gas is the closed system.
2. Volume change is the only work mode.
3. For each process, $\Delta KE = \Delta PE = 0$

ANALYSIS: (a) The work can be evaluated from Eq. 2.17. For Process 3-1, the piston does not move (volume is constant). Thus, $W_{31} = 0$. ← W_{31}

For Processes 1-2 and 2-3, the work can be evaluated geometrically. That is,

$$W_{12} = p_{ave}[V_2 - V_1] = \left(\frac{p_1+p_2}{2}\right)(V_2 - V_1) = \left[\left(\frac{5+3}{2}\right)\text{kPa}\right]\left[5-1\right]\text{m}^3\left|\frac{10^3\text{ N/m}^2}{1\text{ kPa}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$ ← W_{12}

$$= 16\text{ kJ}$$

$$W_{23} = p_{ave}[V_3 - V_2] = \left(\frac{p_3+p_2}{2}\right)(V_3 - V_2) = \left[\left(\frac{3+1}{2}\right)\text{kPa}\right]\left[1-5\right]\text{m}^3\left|\frac{10^3\text{ N/m}^2}{1\text{ kPa}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$ ← W_{23}

$$= -8\text{ kJ}$$

(b) Q_{31} is given. For Process 1-2, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{12} - W_{12}$

$$\Rightarrow Q_{12} = \Delta U + W_{12} = 15\text{ kJ} + 16\text{ kJ} = 31\text{ kJ}$$ ← Q_{12}

For Process 2-3, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{23} - W_{23}$

$$\Rightarrow Q_{23} = (U_3 - U_2) + W_{23}$$

To find $(U_3 - U_2)$, note that since internal energy is a property

$$(U_2 - U_1) + (U_3 - U_2) + (U_1 - U_3) = 0$$

$$\Rightarrow (U_3 - U_2) = -\underbrace{(U_2 - U_1)}_{15\text{ kJ}} - (U_1 - U_3)$$

↑ Energy balance for Process 3-1:

$$(U_1 - U_3) = Q_{31} - \cancel{W_{31}}^{\,0}$$
$$= 10\text{ kJ}$$

$$\therefore (U_3 - U_2) = -15\text{ kJ} - 10\text{ kJ} = -25\text{ kJ}$$

(1) $$\therefore Q_{23} = -25\text{ kJ} + (-8\text{ kJ}) = -33\text{ kJ}$$ ← Q_{23}

(c) For any power cycle, the thermal efficiency is $\eta = \dfrac{W_{cycle}}{Q_{in}}$

Here, $W_{cycle} = W_{12} + W_{23} + W_{31} = 16 - 8 + 0 = 8\text{ kJ}$

$Q_{in} = Q_{12} + Q_{31} = 31 + 10 = 41\text{ kJ}$

$$\therefore \eta = \frac{8\text{ kJ}}{41\text{ kJ}} = 0.195\ (19.5\%)$$ ← η

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1. Also, note that for any cycle, $W_{cycle} = Q_{cycle}$ (Eq. 2.40). Thus
$W_{12} + W_{23} + W_{31} = Q_{12} + Q_{23} + Q_{31} \Rightarrow Q_{23} = W_{12} + W_{23} + W_{31} - Q_{12} - Q_{31}$, or
$Q_{23} = 16 + (-8) + 0 - 31 - 10 = -33\text{ kJ}$.