Problem 2.79

A gas undergoes a cycle in a piston-cylinder assembly consisting of the following three processes:

Process 1-2: Constant pressure, $p = 1.4$ bar, $V_1 = 0.028$ m^3, $W_{12} = 10.5$ kJ

Process 2-3: Compression with $pV = constant$, $U_3 = U_2$

Process 3-1: Constant volume, $U_1 - U_3 = -26.4$ kJ

There are no significant changes in kinetic or potential energy.

(a) Sketch the cycle on a p-V diagram.
(b) Calculate the net work for the cycle, in kJ.
(c) Calculate the heat transfer for process 1-2, in kJ

KNOWN: A gas undergoes a cycle consisting of three processes.

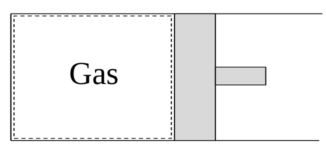
FIND: Sketch the cycle on a p-V diagram and determine the net work for the cycle and the heat transfer for process 1-2.

SCHEMATIC AND GIVEN DATA:

Process 1-2: Constant pressure, $p = 1.4$ bar, $V_1 = 0.028$ m^3, $W_{12} = 10.5$ kJ

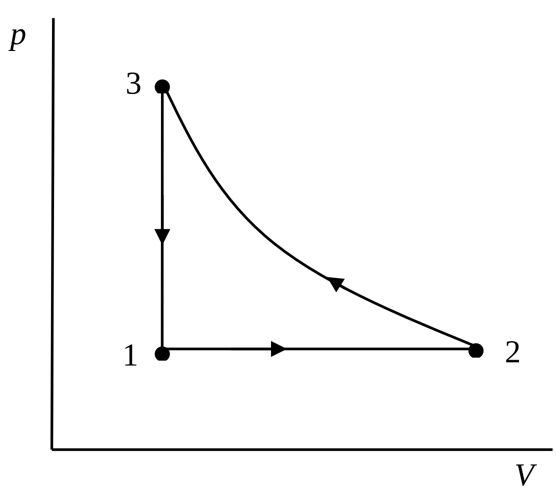
Process 2-3: Compression with $pV = constant$, $U_3 = U_2$

Process 3-1: Constant volume, $U_1 - U_3 = -26.4$ kJ



ENGINEERING MODEL: (1) The gas is a closed system. (2) Kinetic and potential energy effects are negligible. (3) The compression from state 2 to 3 is a polytropic process.

ANALYSIS: (a) Since $W_{12} > 0$, the process is an expansion. Thus



Problem 2.79 (Continued)

(b) The net work for the cycle is $W_{cycle} = W_{12} + W_{23} + \cancelto{0}{W_{31}}$. $W_{12} = 10.5$ kJ, so we need W_{23}.

$$W_{23} = \int_{V_2}^{V_3} p dV = \int_{V_2}^{V_3} \frac{const}{V} dV = (p_2V_2)\ln\left(\frac{V_3}{V_2}\right) = (p_2V_2)\ln\left(\frac{V_1}{V_2}\right) \qquad (*)$$

where $V_3 = V_1$ has been incorporated. But, we still need to evaluate V_2. For Process 1-2 at constant pressure

$$W_{12} = \int_{V_1}^{V_2} p dV = p(V_2 - V_1)$$

or

$$V_2 = \frac{W_{12}}{p} + V_1 = \frac{(10.5\text{ kJ})}{(1.4\text{ bar})}\left|\frac{10^3\text{N}\cdot\text{m}}{1\text{ kJ}}\right|\left|\frac{1\text{ bar}}{10^5\text{N/m}^2}\right| + 0.028\text{ m}^3 = 0.103\text{ m}^3$$

Thus, with Eq. (*)

$$W_{23} = (1.4\text{ bar})(0.103\text{ m}^3)\ln\left(\frac{0.028}{0.103}\right)\left|\frac{10^5\text{N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{N}\cdot\text{m}}\right| = -18.78\text{ kJ}$$

Thus

$$W_{cycle} = 10.5\text{ kJ} + (-18.78\text{ kJ}) + 0 = -8.28\text{ kJ} \longleftarrow$$

(c) To get Q_{12}, we apply the energy balance to process 1-2: $\Delta \cancelto{0}{KE} + \Delta \cancelto{0}{PE} + (U_2 - U_1) = Q_{12} - W_{12}$

With $U_2 = U_3$,

$$Q_{12} = (U_3 - U_1) + W_{12} = (+26.4\text{ kJ}) + (10.5\text{ kJ}) = 36.9\text{ kJ} \longleftarrow$$