PROBLEM 2.48

As shown in Fig. P2.48, an oven wall consists of a 0.635-cm-thick layer of steel (κ_s = 15.1 W/m·K) and a layer of brick (κ_b = 0.72 W/m·K). At steady state, a temperature decrease of 0.7°C occurs over the steel layer. The inner temperature of the steel layer is 300 K. If the temperature of the outer surface of the brick must be no greater than 40°C, determine the thickness of brick, in cm, that ensures this limit is met. What is the rate of conduction, in kW per m^2 of wall surface area?

KNOWN: Steady-state data are provided for a composite wall formed from a steel layer and a brick layer.

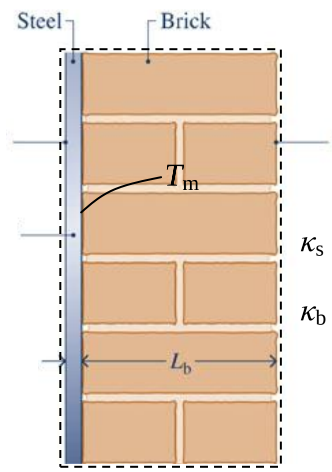
FIND: Determine the minimum thickness of the brick layer to keep the outer surface temperature of the brick at or below a specified value.

SCHEMATIC AND GIVEN DATA:

T_i = 300 K

ΔT = -0.7°C

L_s = 0.635 cm



$T_o \le$ 40°C

κ_s = 15.1 W/m·K

κ_b = 0.72 W/m·K

ENGINEERING MODEL: (1) The wall is the system at steady state. (2) The temperature varies linearly through each layer.

ANALYSIS: Using Eq. 2.31 together with model assumption 2

$$\left(\frac{\dot{Q}}{A}\right)_{\text{steel}} = -\kappa_s\left[\frac{T_m - T_i}{L_s}\right] \quad \text{and} \quad \left(\frac{\dot{Q}}{A}\right)_{\text{brick}} = -\kappa_b\left[\frac{T_o - T_m}{L_b}\right]$$

where T_m denotes the temperature at the steel-brick interface.

At steady state, the rate of conduction *to* the interface through the steel must equal the rate of conduction *from* the interface through the brick: $(\dot{Q}/A)_{\text{steel}} = (\dot{Q}/A)_{\text{brick}}$. Thus

$$-\kappa_s\left[\frac{T_m - T_i}{L_s}\right] = -\kappa_b\left[\frac{T_o - T_m}{L_b}\right]$$

And solving for L_b we get

(300 – 0.7) = 299.3 °C

$$L_b = \frac{\kappa_b}{\kappa_s}\left[\frac{T_o - T_m}{T_m - T_i}\right] L_s$$

= – 0.7 °C

PROBLEM 2.48 (Continued)

$$L_b = \left(\frac{0.72 \text{ W/m·K}}{15.1 \text{ W/m·K}}\right)\left[\frac{299.3 - T_o}{0.7}\right](0.635 \text{ cm})$$

Since $T_o \leq 40$ °C

$$L_b \geq \left(\frac{0.72}{15.1}\right)\left[\frac{299.3 - 40}{0.7}\right](0.635 \text{ cm})$$

$$L_b \geq 11.22 \text{ cm} \longleftarrow$$

The rate of conduction is

$$\left(\frac{\dot{Q}}{A}\right)_{steel} = -\kappa_s\left[\frac{T_m - T_i}{L_s}\right] = -(15.1 \text{ W/m} \cdot \text{K})\left[\frac{299.3 - 300}{0.635 \text{ cm}}\right]\left|\frac{100 \text{ cm}}{1 \text{ m}}\right|\left|\frac{1 \text{ kW}}{10^3 \text{W}}\right| = 1.665 \text{ kW/m}^2 \longleftarrow$$

or

$$\left(\frac{\dot{Q}}{A}\right)_{brick} = -\kappa_s\left[\frac{T_o - T_m}{L_b}\right] = -(0.72 \text{ W/m} \cdot \text{K})\left[\frac{40 - 299.3}{11.22 \text{ cm}}\right]\left|\frac{100 \text{ cm}}{1 \text{ m}}\right|\left|\frac{1 \text{ kW}}{10^3 \text{W}}\right| = 1.664 \text{ kW/m}^2 \longleftarrow$$

The slight difference is due to round-off.