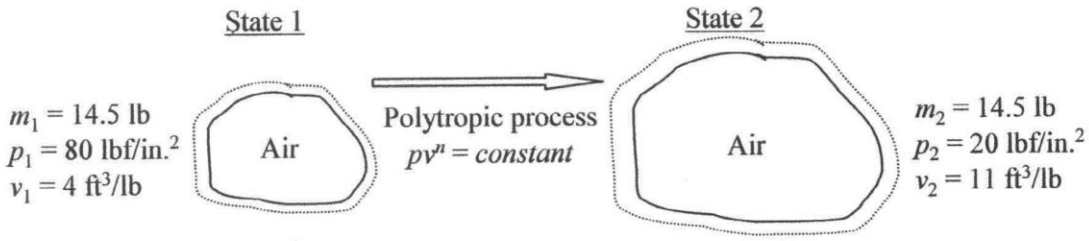
PROBLEM 2.31

KNOWN: Air undergoes a polytropic process between two specified states.

FIND: Determine the work.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The air is a closed system.
2. The system undergoes a polytropic process.

ANALYSIS:
The pressure-volume relationship for a polytropic process is

$$pV^n = constant$$
or
$$pv^n = constant$$

Thus

$$p_1 v_1^{\,n} = p_2 v_2^{\,n}$$

Solving for n yields

$$n = \frac{\ln\left(\frac{p_1}{p_2}\right)}{\ln\left(\frac{v_2}{v_1}\right)} = \frac{\ln\left(\frac{80\,\frac{\text{lbf}}{\text{in}^2}}{20\,\frac{\text{lbf}}{\text{in}^2}}\right)}{\ln\left(\frac{11\,\frac{\text{ft}^3}{\text{lb}}}{4\,\frac{\text{ft}^3}{\text{lb}}}\right)}$$

Problem 2.31 (Cont'd)

To solve for work

$$W = \int_{V_1}^{V_2} pdV = m\int_{v_1}^{v_2} pdv = m\int_{v_1}^{v_2} \frac{(constant)dv}{v^n}$$

$$W = m\frac{(constant)v_2^{1-n} - (constant)v_1^{1-n}}{1-n} = m\frac{(p_2v_2^n)v_2^{1-n} - (p_1v_1^n)v_1^{1-n}}{1-n} = m\frac{p_2v_2 - p_1v_1}{1-n}$$

$$W = (14.5\,\text{lb})\frac{\left(20\frac{\text{lbf}}{\text{in}^2}\right)\left(11\frac{\text{ft}^3}{\text{lb}}\right) - \left(80\frac{\text{lbf}}{\text{in}^2}\right)\left(4\frac{\text{ft}^3}{\text{lb}}\right)}{1-1.37}\left|\frac{144\,\text{in}^2}{1\,\text{ft}^2}\right|\left|\frac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right| = \underline{\mathbf{725.4\ Btu}}$$

The positive sign for work denotes energy transfer out of the system.