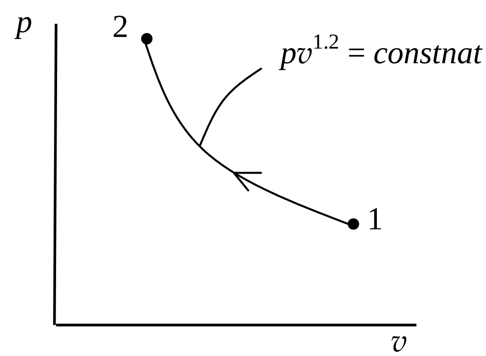
Problem 2.66

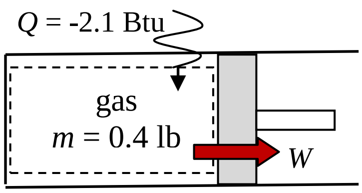
A gas undergoes a process in a piston-cylinder assembly during which the pressure-specific volume relation is $pv^{1.2} = constant$. The mass of the gas is 0.4 lb and the following data are known: p_1 = 160 lbf/in.^{2}, V_1 = 1 ft^3, and p_2 = 390 lbf/in.^{2} During the process, heat transfer *from* the gas is 2.1 Btu. Kinetic and potential energy effects are negligible. Determine the change in specific internal energy of the gas, in Btu/lb.

KNOWN: A gas is compressed in a piston-cylinder assembly. The pressure-specific volume relation is specified.

FIND: Determine the change in specific internal energy.



SCHEMATIC AND GIVEN DATA:



p_1 = 160 lbf/in.^{2}
V_1 = 1 ft^3
p_2 = 390 lbf/in.^{2}

ENGINEERING MODEL: (1) The gas is a closed system. (2) The process follows $pv^{1.2} = constant$. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: The change in specific internal energy will be found from an energy balance. First, determine the work. Since volume change is the only work mode, Eq. 2.17 applies:

$$W = \int_{V_1}^{V_2} p dV == \int_{V_1}^{V_2} \frac{const}{V^{1.2}} dV = \frac{(p_2V_2 - p_1V_1)}{1-1.2}$$

Evaluating V_2

$$V_2 = \left(\frac{p_1}{p_2}\right)^{1/1.2} V_1 = \left(\frac{160 \text{ lbf/in.}^2}{390 \text{ lbf/in.}^2}\right)^{1/1.2} (1 \text{ ft}^3) = 0.4759 \text{ ft}^3$$

Thus

$$W = \left[\frac{(390 \text{ lbf/in.}^2)(0.4759 \text{ ft}^3) - (160)(1)}{1-1.2}\right] \left|\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right| \left|\frac{1 \text{ Btu}}{778 \text{ ft·lbf}}\right| = -23.69 \text{ Btu (in)}$$

Now, writing the energy balance: $\Delta KE + \Delta PE + \Delta U = Q - W$

With $\Delta U = m\Delta u$

(1) $$\Delta u = \frac{Q-W}{m} = \frac{(-2.1 \text{ Btu}) - (-23.69 \text{ Btu})}{0.4 \text{ lb}} = 54.0 \text{ Btu}$$ ←

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1. The amount of energy transfer in by work exceeds the amount of energy transfer out by heat, resulting in a net increase in internal energy.