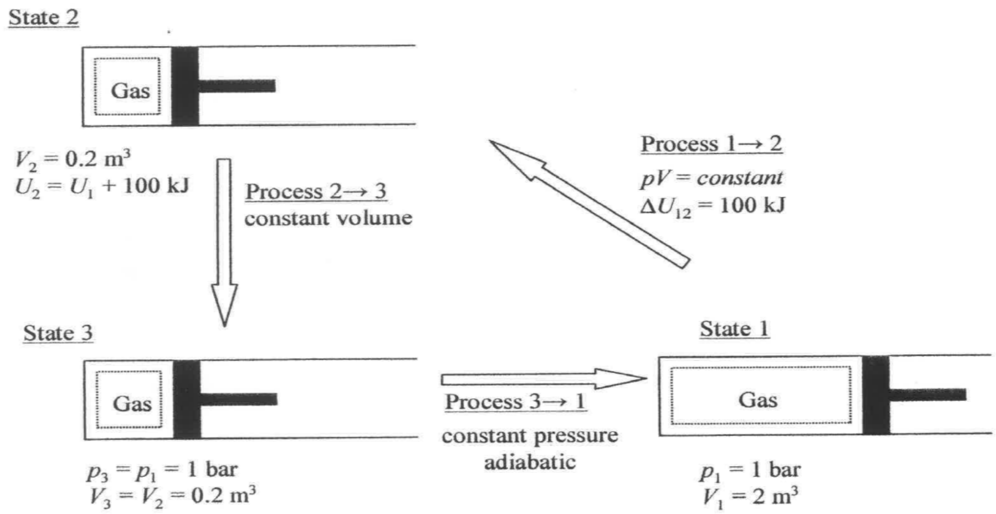
PROBLEM 2.78

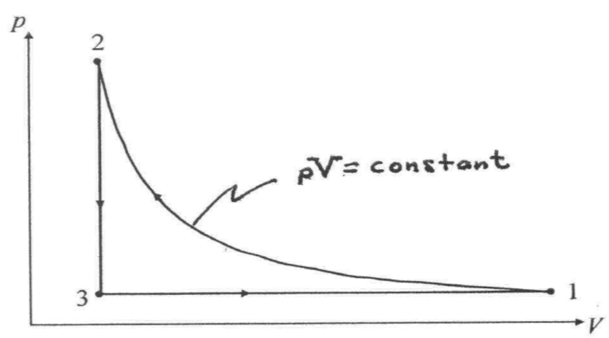
KNOWN: A closed system undergoes a cycle consisting of three processes.

FIND: Determine the net work of the cycle and the heat transfer for process 2-3. Identify whether the cycle is a power cycle or a refrigeration cycle and explain.

SCHEMATIC AND GIVEN DATA:



p-V Diagram



ENGINEERING MODEL:
1. The gas is a closed system.
2. Kinetic and potential energy effects are negligible
3. Process 1-2 is polytropic in which $pV = constant$.
4. Process 2-3 is constant volume.
5. Process 3-1 is constant pressure and adiabatic.

ANALYSIS:
Cycle work is the sum of work associated with each process in the cycle

$$W_{cycle} = W_{12} + W_{23} + W_{31}$$

Process 1-2 is a polytropic process with $pV = constant$. Therefore, $p = constant/V$.

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$$W_{12} = \int_{V_1}^{V_2} pdV = \int_{V_1}^{V_2} \frac{(\text{constant})dV}{V} = (\text{constant})\int_{V_1}^{V_2} \frac{dV}{V} = (\text{constant})\ln\left(\frac{V_2}{V_1}\right) = p_1V_1\ln\left(\frac{V_2}{V_1}\right)$$

$$W_{12} = (1\text{ bar})(2\text{ m}^3)\left|\frac{10^5\ \frac{\text{N}}{\text{m}^2}}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|\ln\left(\frac{0.2\text{ m}^3}{2\text{ m}^3}\right) = -460.5\text{ kJ}$$

Process 2-3 is constant volume; thus $W_{23} = \int_{V_2}^{V_3} pdV = 0\text{ kJ}$

Process 3-1 is constant pressure; thus $W_{31} = \int_{V_3}^{V_1} pdV = p_1(V_1 - V_3)$

$$W_{31} = (1\text{ bar})(2\text{ m}^3 - 0.2\text{ m}^3)\left|\frac{10^5\ \frac{\text{N}}{\text{m}^2}}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = 180\text{ kJ}$$

Substituting the work associated with each process yields the cycle work

$$W_{cycle} = (-460.5\text{ kJ}) + 0\text{ kJ} + 180\text{ kJ} = \underline{\mathbf{-280.5\text{ kJ}}}$$

Since the net cycle work is into the cycle (negative), the cycle is a <u>refrigeration cycle</u>.

For process 2-3, an energy balance is

$$\Delta\text{KE}_{23} + \Delta\text{PE}_{23} + (U_3 - U_2) = Q_{23} - W_{23}$$

Neglecting changes in kinetic energy ($\Delta\text{KE}_{23} = 0$) and potential energy ($\Delta\text{PE}_{23} = 0$), substituting $W_{23} = 0$ (determined above), and solving for Q_{23} yield

$$Q_{23} = U_3 - U_2$$

For the cycle,

$$(U_2 - U_1) + (U_3 - U_2) + (U_1 - U_3) = 0$$

Solving for $(U_3 - U_2)$ yields

$$(U_3 - U_2) = -(U_2 - U_1) - (U_1 - U_3)$$

From the problem statement, $(U_2 - U_1) = 100\text{ kJ}$

For process 3-1, an energy balance is

$$\Delta\text{KE}_{31} + \Delta\text{PE}_{31} + (U_1 - U_3) = Q_{31} - W_{31}$$

Neglecting changes in kinetic energy ($\Delta KE_{31} = 0$) and potential energy ($\Delta PE_{31} = 0$), substituting $Q_{31} = 0$ since process 3-1 is adiabatic, and solving for $(U_1 - U_3)$ give

$$(U_1 - U_3) = -W_{31} = -180 \text{ kJ}$$

Substituting for changes in internal energy gives

$$(U_3 - U_2) = -(100 \text{ kJ}) - (-180 \text{ kJ}) = 80 \text{ kJ}$$

Solving for Q_{23}

(1) $$Q_{23} = U_3 - U_2 = \underline{\mathbf{80\ kJ}}$$

The heat transfer is positive during process 2-3, denoting energy transfer by heat into the gas during this process.

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1. ***As an alternative solution***, for the overall cycle, $Q_{cycle} = Q_{12} + Q_{23} + Q_{31} = W_{cycle}$

Thus, $Q_{23} = W_{cycle} - Q_{12} - Q_{31}$

For process 1-2, an energy balance is

$$\Delta KE_{12} + \Delta PE_{12} + \Delta U_{12} = Q_{12} - W_{12}$$

Neglecting changes in kinetic energy ($\Delta KE_{12} = 0$) and potential energy ($\Delta PE_{12} = 0$) and solving for heat transfer give

$$Q_{12} = \Delta U_{12} + W_{12} = 100 \text{ kJ} + (-460.5 \text{ kJ}) = -360.5 \text{ kJ}$$

The heat transfer is negative during process 1-2, denoting energy transfer by heat from the gas during this process.

Since process 3-1 is adiabatic, $Q_{31} = 0$ kJ.

Substituting values for W_{cycle} and heat transfer associated with each process yields

$$Q_{23} = (-280.5 \text{ kJ}) - (-360.5 \text{ kJ}) - 0 \text{ kJ} = \underline{\mathbf{80\ kJ}}$$

The heat transfer is positive during process 2-3, denoting energy transfer by heat into the gas during this process.