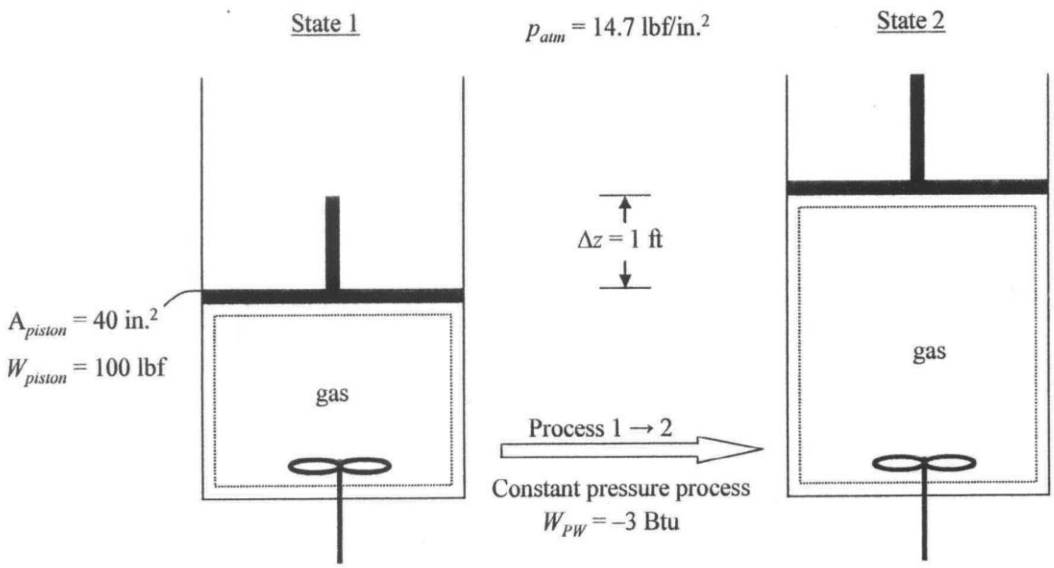
PROBLEM 2.65

KNOWN: A rotating shaft transfers energy to a gas contained in a vertical piston-cylinder assembly during a constant-pressure process.

FIND: Determine the change in internal energy of the gas.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The gas is the system.
2. The system undergoes an adiabatic process.
3. Kinetic and potential energy effects are negligible.
4. Thermal conduction and friction between the piston and cylinder can be neglected.

ANALYSIS:
The change in internal energy can be determined from the energy balance

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

Neglecting changes in kinetic energy ($\Delta KE = 0$) and potential energy ($\Delta PE = 0$), assuming an adiabatic process ($Q = 0$), and solving for internal energy give

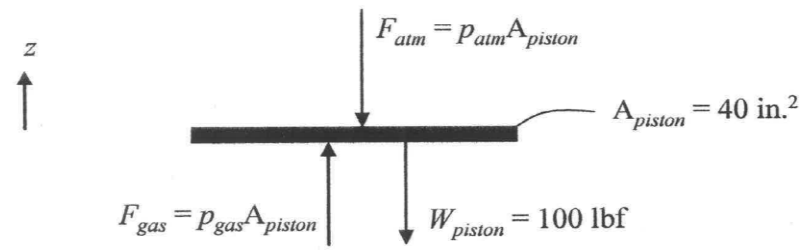
$$\Delta U = -W = -(W_{PW} + W_{exp})$$

PROBLEM 2.65 (Continued)

where W_{PW} = -3 Btu and W_{exp} is determined next:

$$W_{exp} = \int_{V_1}^{V_2} p_{gas} dV = \int_{x_1}^{x_2} p_{gas} A_{piston} dx = A_{piston} \int_{x_1}^{x_2} p_{gas} dx$$

To determine the gas pressure, draw a free body diagram for the piston. For equilibrium, sum of the forces in the z-direction equals zero.



$$\uparrow \Sigma F_z = 0$$

$$p_{gas}A_{piston} - p_{atm}A_{piston} - W_{piston} = 0$$

Solving for p_{gas} yields

$$p_{gas} = p_{atm} + \frac{W_{piston}}{A_{piston}} = 14.7\frac{\text{lbf}}{\text{in.}^2} + \frac{100\text{ lbf}}{40\text{ in.}^2} = 17.2\text{ lbf/in.}^2$$

Since p_{gas} is constant, the expression for W_{exp} can be integrated as follows:

$$W_{exp} = p_{gas} A_{piston} \int_{x_1}^{x_2} dx = p_{gas}A_{piston}(x_2 - x_1)$$

Substitute the value for p_{gas} to solve for work of expansion.

$$W_{exp} = \left(17.2\frac{\text{lbf}}{\text{in.}^2}\right)\left(40\text{ in.}^2\right)\left(1\text{ ft}\right)\left|\frac{\text{Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = 0.88\text{ Btu}$$

Substituting values for expansion work and shaft work yields change in internal energy.

$$\Delta U = -W = -(W_{PW} + W_{exp}) = -[(0.88\text{ Btu}) - (-3\text{ Btu})] = \underline{\mathbf{2.12\text{ Btu}}}$$