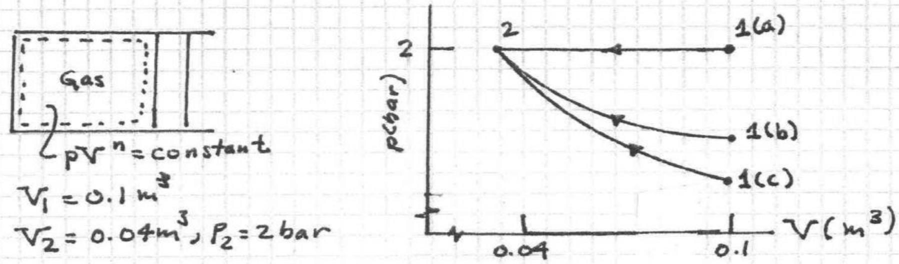
PROBLEM 2.28

KNOWN: A gas in a piston-cylinder assembly undergoes a compression process for which pV^n = constant. State data is provided.

FIND: For each of $n=0$, $n=1$, and $n=1.3$, determine the initial pressure, in bar, and the work, in kJ.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The gas within the piston-cylinder is the closed system.
2. Volume change is the only work mode.
3. The process of the gas obeys pV^n = constant, where (a) $n=0$, (b) $n=1$, (c) $n=1.3$.

ANALYSIS:

(a) $n=0$: Thus, pV^0 = constant $\Rightarrow$ p = constant. So, $P_1 = 2$ bar

Using Eq. 2.17 with p = constant,

(1) $$W = \int_1^2 p\,dV = p[V_2 - V_1] = 2\text{bar}[0.04 - 0.1]\,m^3 \left|\frac{10^5 N/m^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3 N\cdot m}\right| = -12\text{ kJ}$$

(b) $n=1$: Thus, pV = constant $\Rightarrow P_1V_1 = P_2V_2 \Rightarrow P_1 = P_2\left[\frac{V_2}{V_1}\right] = 2\text{bar}\left[\frac{0.04}{0.10}\right] = 0.8\text{bar}$

$$W = \int_1^2 p\,dV = \int_1^2 \frac{c}{V}\,dV = c\ln\frac{V_2}{V_1} = P_2V_2\ln\frac{V_2}{V_1} = \left(2\times10^5\frac{N}{m^2}\right)(0.04\,m^3)\left|\frac{1\text{ kJ}}{10^3 N\cdot m}\right|\ln\frac{0.04}{0.10}$$

$c = P_1V_1 = P_2V_2$

$$= -7.33\text{ kJ}$$

(c) $n=1.3$: Thus, $P_1V_1^n = P_2V_2^n$, where $n=1.3$. $\Rightarrow P_1 = P_2\left[\frac{V_2}{V_1}\right]^{1.3} = 2\text{bar}\left[\frac{0.04}{0.10}\right]^{1.3}$

$$= 0.608\text{ bar}$$

$$W = \int_1^2 p\,dV = \int_1^2 \frac{c}{V^n}\,dV = \frac{P_2V_2 - P_1V_1}{(1-n)}$$. See Example 2.1(a) for the integration.

$$\therefore W = \frac{(2\times10^5 N/m^2)(0.04m^3) - (0.608\times10^5 N/m^2)(0.1\,m^3)}{(1-1.3)}\left|\frac{1\text{ kJ}}{10^3 N\,m}\right|$$

$$= -6.4\text{ kJ}$$

1. The negative sign for W denotes work done on the gas during compression.