Problem 2.35

Air contained within a piston-cylinder assembly undergoes three processes in series:

Process 1-2: Compression during which the pressure-volume relationship is $pV = constant$ from $p_1 = 10$ lbf/in.^{2}, $V_1 = 4$ ft^3 to $p_2 = 50$ lbf/in.^{2}

Process 2-3: Constant volume from state 2 to state 3 where $p = 10$ lbf/in.^{2}

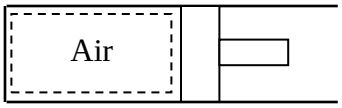
Process 3-1: Constant pressure expansion to the initial state.

Sketch the processes in series on a p-V diagram. Evaluate (a) the volume at state 2, in ft^3, and (b) the work for each process, in Btu.

KNOWN: Air within a piston-cylinder assembly undergoes three processes in series.

FIND: Sketch the processes in series on a p-V diagram. Evaluate (a) the volume at state 2, and (b) the work for each process.

SCHEMATIC AND GIVEN DATA:



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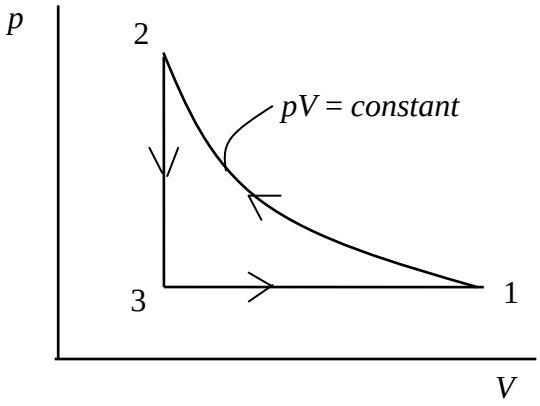

ENGINEERING MODEL: (1) The gas is the closed system. (2) Volume change is the only work mode. (3) Each of the three processes is specified.

ANALYSIS: (a) For process 1-2; $pV = constant$. Thus $p_1V_1 = p_2V_2$, and

$$V_2 = \left(\frac{p_1}{p_2}\right)V_1 = \left(\frac{10\text{lbf/in.}^2}{50\text{lbf/in.}^2}\right)(4\text{ ft}^3) = 0.8\text{ ft}^3 \longleftarrow$$

(b) Since volume change is the only work mode, Eq. 2.17 applies.

Process 1-2: For process 1-2, $pV = constant = p_1V_1$. Thus

$$W_{12} = \int_{V_1}^{V_2} pdV = \int_{V_1}^{V_2} \frac{C}{V}dV = C\ln\left(\frac{V_2}{V_1}\right) = (p_1V_1)\ln\left(\frac{V_2}{V_1}\right)$$

Problem 2.35 (Continued)

Inserting values and converting units

$$W_{12} = \left(10\ \frac{\text{lbf}}{\text{in.}^2}\right)(4\ \text{ft}^3)\ \ln\left(\frac{0.8\ \text{ft}^3}{4\ \text{ft}^3}\right)\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft·lbf}}\right| = -11.92\ \text{Btu (in)} \longleftarrow$$

Process 2-3: Constant volume (piston does not move). Thus $W_{23} = 0$ ⟵

Process 3-1: Constant pressure processes ($p_3 = p_1$): $W_{31} = \int_{V_3}^{V_1} p\,dV = p_1(V_1 - V_3)$

Noting that $V_3 = V_2$

$$W_{31} = \left(10\ \frac{\text{lbf}}{\text{in}^2.}\right)(4 - 0.8)\text{ft}^3\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft·lbf}}\right| = 5.92\ \text{Btu (out)} \longleftarrow$$

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1. The *net* work for the three process is

$W_{net} = W_{12} + W_{23} + W_{31} = (-11.92) + 0 + (5.92)) = -6$ kJ (*net* work is negative - in)