PROBLEM 1.30

Since the piston moves smoothly within the cylinder, the force exerted by the gas equals the resisting force composed of the piston weight, shaft weight, force exerted by the atmospheric pressure, and the force acting on the shaft, F. That is, the sum of the forces acting vertically is zero, giving

$$F_{gas} = \begin{bmatrix}\text{Piston}\\ \text{Weight}\end{bmatrix} + \begin{bmatrix}\text{Shaft}\\ \text{Weight}\end{bmatrix} + F_{atm} + F$$

Solving

$$F = F_{gas} - \begin{bmatrix}\text{Piston}\\ \text{weight}\end{bmatrix} - \begin{bmatrix}\text{shaft}\\ \text{weight}\end{bmatrix} - F_{atm} \qquad (*)$$

In this expression,

$F_{gas} = P_{gas} A_p$, where A_p is the piston face area:

$$A_p = \frac{\pi D^2}{4} = \frac{\pi (10\,cm)^2}{4} = 78.54\,cm^2$$

$$\therefore \quad F_{gas} = (3\,bar)\left|\frac{10^5\,N/m^2}{1\,bar}\right|(78.54\,cm^2)\left|\frac{1\,m}{10^2\,cm}\right|^2 = 2356.2\,N$$

The pressure of the atmosphere acts only on the net area at the top of the piston — namely, the piston face area less the area occupied by the shaft. The force is then

$$F_{atm} = P_{atm}[A_p - A]$$

$$= (1\,bar)(78.54 - 0.8)\,cm^2\left|\frac{10^5\,N/m^2}{1\,bar}\right|\left|\frac{1\,m}{10^2\,cm}\right|^2 = 777.4\,N$$

The total weight of the piston and shaft is

$$\text{Total weight} = (m_{piston} + m_{shaft})g$$

$$= (25\,kg)(9.81\,\frac{m}{s^2})\left|\frac{1\,N}{1\,kg\cdot m/s^2}\right| = 245.3\,N$$

Collecting results, Eq. (*) gives

$$F = 2356.2\,N - 245.3\,N - 777.4\,N$$

$$= 1333.5\,N \quad \longleftarrow$$