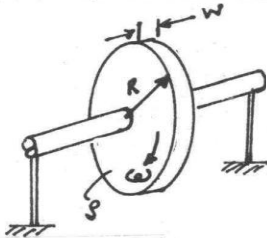


PROBLEM 2.11

KNOWN: Data are provided for a disk-shaped flywheel.

FIND: (a) Obtain appropriate expressions for the moment of inertia and the kinetic energy. (b) Using given data, determine the kinetic energy and mass for a steel flywheel. (c) Using results from part (b), determine the radius and mass of an aluminum flywheel.

SCHEMATIC & GIVEN DATA:



Steel flywheel:

$$\omega = 3000 \text{ RPM}$$

$$R = 0.38 \text{ m}$$

$$w = 0.025 \text{ m}$$

Aluminum flywheel:

$$\omega = 3000 \text{ RPM}$$

$$w = 0.025 \text{ m}$$

ENGR. MODEL:

1. The flywheel is the closed system. 2. Motion is relative to the flywheel support structure.

ANALYSIS:

(a) Evaluating the moment of inertia

$$I = \int_{\text{vol}} \rho r^2 dV$$

For the disk, $dV = (2\pi r dr)w$. Thus, since ρ is constant

$$I = \rho (2\pi) w \int_0^R r^3 dr = \rho \pi w R^4 / 2 \quad \leftarrow I$$

The kinetic energy is

$$KE = \int_{\text{vol}} \left(\frac{1}{2} \rho v^2 \right) dV$$

and $v = r\omega$, so

$$KE = \int_0^R \left(\frac{1}{2} \rho r^2 \omega^2 \right) (2\pi r dr) w$$

$$= \frac{1}{2} \rho \omega^2 (2\pi) w \int_0^R r^3 dr$$

$$= \frac{1}{2} \underbrace{\left(\rho \pi \frac{R^4}{2} w \right)}_I \omega^2$$

$$= \frac{1}{2} I \omega^2 \quad \leftarrow KE$$

(b) From Table A-19, the density of steel is $\rho = 8060 \text{ kg/m}^3$. Thus, the mass is

$$m = \rho V = \rho [w \cdot \pi R^2] \\ = (8060 \frac{\text{kg}}{\text{m}^3}) [(0.025 \text{ m}) \cdot \pi \cdot (0.38 \text{ m})^2] = 91.41 \text{ kg} \quad \leftarrow m$$

Using the result of part (a), $KE = \frac{1}{2} I \omega^2$, where

$$I = \pi \rho w \frac{R^4}{2} = \frac{\pi}{2} (8060 \frac{\text{kg}}{\text{m}^3}) (0.025 \text{ m}) (0.38 \text{ m})^4 = 6.6 \text{ kg} \cdot \text{m}^2$$

PROBLEM 2.11 (Contd.)

Accordingly,

$$KE = \frac{1}{2} I \omega^2 = \frac{1}{2} (6.6 \text{ kg}\cdot\text{m}^2) \left(3000 \frac{\text{REV}}{\text{min}} \left| \frac{2\pi \text{ rad}}{\text{REV}} \right| \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \right)^2 \left| \frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2} \right|$$
$$= 32.57 \times 10^4 \text{ N}\cdot\text{m} \quad \leftarrow KE$$

- (c) If ω , w , and KE are the same for the aluminum flywheel as for the steel flywheel

$$(KE)_{AL} = (KE)_{ST}$$

$$\left(\frac{1}{2} I \omega^2 \right)_{AL} = \left(\frac{1}{2} I \omega^2 \right)_{ST} \Rightarrow I_{AL} = I_{ST}$$

or

$$\left(\pi \rho w \frac{R^4}{2} \right)_{AL} = \left(\pi \rho w \frac{R^4}{2} \right)_{ST}$$

$$\Rightarrow (\rho R^4)_{AL} = (\rho R^4)_{ST}$$

$$R_{AL} = \left(\frac{\rho_{ST}}{\rho_{AL}} \right)^{1/4} R_{ST}$$

With ρ_{AL} from Table A-19, $\rho_{AL} = 2700 \text{ kg/m}^3$

$$R_{AL} = \left(\frac{8060}{2700} \right)^{1/4} (0.38 \text{ m})$$
$$= 0.5 \text{ m} \quad \leftarrow R_{AL}$$

Then, the mass of the aluminum flywheel is

$$m = \rho V = \rho [w \pi R^2]$$

$$= (2700 \frac{\text{kg}}{\text{m}^3}) (0.025 \text{ m}) (\pi) (0.5 \text{ m})^2 = 53.01 \text{ kg} \quad \leftarrow m$$