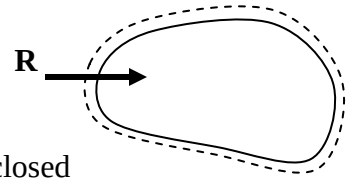
Problem 2.10

An object whose mass is 300 lb experiences changes in kinetic and potential energies owing to the action of a resultant force **R**. The work done on the object by the resultant force is 140 lbf. There are no other interactions between the object and its surroundings. It the object's elevation increases by 100 ft and its final velocity is 200 ft/s, what is the initial velocity, in ft/s? Let g = 32.2 ft/s^2.

KNOWN: An object of known mass experiences changes in kinetic and potential energy due to the action of a resultant force. The final velocity, the change in elevation, and the work done by the force are given.

FIND: Determine the final velocity.

SCHEMATIC AND GIVEN DATA:



m = 300 lb
V_2 = 200 ft/s
$z_2 - z_1$ = +100 ft
g = 32.2 ft/s^2

Work done by resultant force = 140 Btu

ENGINEEIRNG MODEL: (1) The object is a closed system. (2) The force of gravity acts on the object, and g = 32.2 ft/s^2. (3) The resultant force accounts for all interactions between the system and its surroundings.

ANALYSIS: By modeling assumption (3), the work of the resultant force must equal the sum of the changes in kinetic and gravitational potential energies. Thus, with Eq. 2.9

$$\text{Work} = \tfrac{1}{2}\, m(V_2^2 - V_1^2) + mg(z_2 - z_1)$$

Solving for V_1^2 and inserting values

$$V_1^2 = \frac{2[mg(z_2 - z_1) - \text{Work}]}{m} + V_2^2$$

First

$$mg(z_2 - z_1) = (300\text{ lb})(32.2\text{ ft/s}^2)(100\text{ ft})\left|\frac{1\text{ lbf}}{32.2\text{ lb}\cdot\text{ft/s}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = 38.6\text{ Btu}$$

So

$$V_1^2 = \frac{2[38.6 - 140]\text{BTU}}{(300\text{ lb})}\left|\frac{778\text{ ft}\cdot\text{lbf}}{1\text{ Btu}}\right|\left|\frac{32.2\text{ lb}\cdot\text{ft/s}^2}{1\text{ lbf}}\right| + 200^2\text{ ft}^2/\text{s}^2 = 23065\text{ ft}^2/\text{s}^2$$

or

① V_1 = 151.9 ft/s ←

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1. The increase in velocity reflects the increase in kinetic energy of the object as a result of energy transferred to it by the work of the resultant force. Carefully observe that in Eq. 2.9 the work of the resultant force acting *on* the body is positive.