Chapter 2

Density, Specific Gravity, Specific Weight

1. What is the specific gravity of 38°API oil?

$$38°\text{API oil} \quad \text{sp. gr.} = \frac{141.5}{131.5 + °\text{API}} = \frac{141.5}{131.5 + 38}$$

$$\boxed{\text{sp. gr.} = \frac{141.5}{169.5} = 0.835}$$

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2. The specific gravity of manometer gage oil is 0.826. What is its density and its °API rating?

sp. gr. = 0.826; $\boxed{\rho = 1000(0.826) = 826\ \text{kg/m}^3}$

$$\boxed{\rho = 62.4(0.826) = 51.54\ \text{lbm/ft}^3}$$

$$\text{sp. gr.} = \frac{141.5}{131.5 + °\text{API}} \qquad 131.5 + °\text{API} = \frac{141.5}{0.826}$$

°API = 171.3 - 131.5; $\boxed{°\text{API} = 39.8°\text{API} \approx 40°\text{API}}$

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3. What is the difference in density between a 50°API oil and a 40°API oil?

$$\text{sp. gr.} = \frac{141.5}{131.5 + °\text{API}} = \frac{141.5}{131.5 + 50} = 0.7796 \text{ for a } 50° \text{ oil}$$

$$\text{sp. gr.} = \frac{141.5}{131.5 + °\text{API}} = \frac{141.5}{131.5 + 40} = 0.826 \text{ for a } 40° \text{ oil}$$

$0.825 - 0.7796 = 0.0455$ density difference

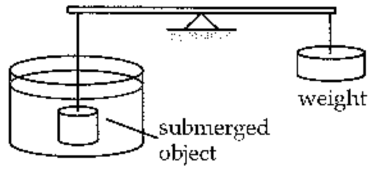


FIGURE P2.7.

Buoyant force = $mg_{in\ air} - mg_{submerged} = mg - 0.8$ $\qquad g_c = 1$ in this system

$$\frac{\text{buoyant force}}{\text{volume}} = \frac{mg - 0.8}{V} = \rho g \qquad V = \frac{\pi D^2}{4} h = \frac{\pi}{4}(1/12)^2(4/12) = 1.818 \times 10^{-3}\ \text{ft}^3$$

$$mg = \rho_b V g = 8.5(1.94)(1.818 \times 10^{-3})(32.2) = 0.965\ \text{lbf}$$

$$\rho = \frac{mg - 0.8}{gV} = \frac{0.965 - 0.8}{32.2(1.818 \times 10^{-3})}$$

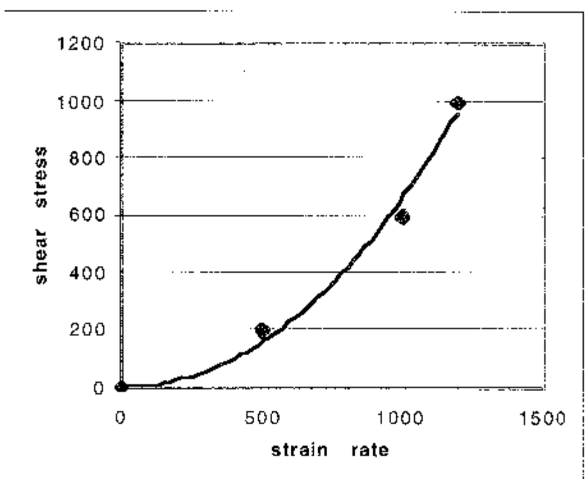
$$\boxed{\rho = 2.82\ \text{slug/ft}^3}$$

Viscosity

8. Actual tests on vaseline yielded the following data:

τ in N/m^2	0	200	600	1 000
dV/dy in 1/s	0	500	1 000	1 200

Determine the fluid type and the proper descriptive equation.



$$\tau = K\left(\frac{dV}{dy}\right)^n$$

Can be done instantly with spreadsheet; hand calculations follow for comparison purposes:

dV/dy	ln(dV/dy)	τ	ln τ	ln(τ)·ln(dV/dy)
0	—	0	—	·
500	6.215	200	5.298	32.93
1000	6.908	600	6.397	44.19
1200	7.090	1000	6.908	48.98
Sum	20.21		18.60	126.1

m=3   Summation $(\ln(dV/dy))^2 = 136.6$

$$b_1 = \frac{3(126.1) - 20.21(18.60)}{3(136.6) - 20.21^2} = 1.766$$

$$b_0 = \frac{18.60}{3} - 1.766\frac{20.21}{3} = -5.697$$

$K = \exp(b_0) = 0.00336$; $n = b_1 = 1.766$

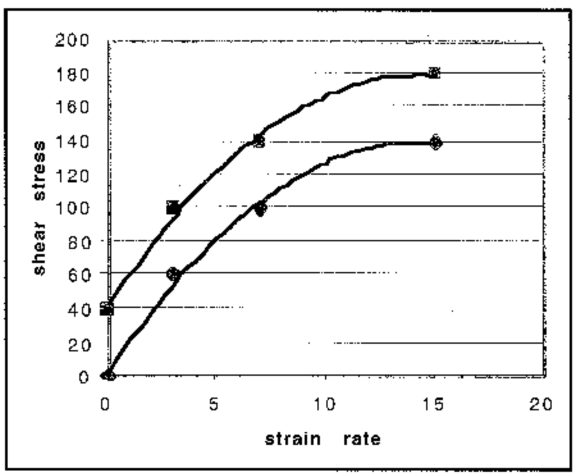
$$\tau = \tau_o + K\left(\frac{dV}{dy}\right)^n = 0.00336\left(\frac{dV}{dy}\right)^{1.766}$$

9. A popular mayonnaise is tested with a viscometer and the following data were obtained:

τ in g/cm^2	40	100	140	180
dV/dy in rev/s	0	3	7	15

Determine the fluid type and the proper descriptive equation.

The topmost line is the given data, but to curve fit, we subtract 40 from all shear stress readings.



$$\tau = \tau_o + K\left(\frac{dV}{dy}\right)^n \quad \text{which becomes } \tau' = \tau - \tau_o = K\left(\frac{dV}{dy}\right)^n$$

Can be done instantly with spreadsheet; hand calculations:

dV/dy	ln(dV/dy)	τ	τ'	ln τ'	$\ln(\tau')\cdot\ln(dV/dy)$
0	—	40	0	—	—
3	1.099	100	60	4.094	4.499
7	1.946	140	100	4.605	8.961
15	2.708	180	140	4.942	13.38
Sum	5.753			13.64	26.84

m = 3 Summation $(\ln(dV/dy))^2 = 12.33$

$$b_1 = \frac{3(26.84) - 5.753(13.64)}{3(12.33) - 5.753^2} = 0.526$$

$$b_0 = \frac{13.64}{3} - 0.526\frac{5.753}{3} = 3.537$$

$K = \exp(b_0) = 34.37$; $n = b_1 = 0.526$

$$\tau = \tau_o + K\left(\frac{dV}{dy}\right)^n = 40 + 34.37\left(\frac{dV}{dy}\right)^{0.526}$$

where dV/dy is in rev/s and τ in g/cm^2; these are not standard units.

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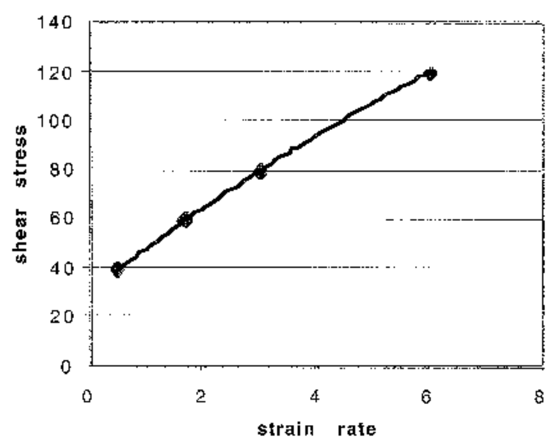
10. A cod-liver oil emulsion is tested with a viscometer and the following data were obtained:

τ in lbf/ft^2	0	40	60	80	120
dV/dy in rev/s	0	0.5	1.7	3	6

Graph the data and determine the fluid type. Derive the descriptive equation.

Cod liver oil; graph excludes the first data point.



$$\tau = K\left(\frac{dV}{dy}\right)^n$$

Can be done instantly with spreadsheet; hand calculations:

dV/dy	ln(dV/dy)	τ	ln τ	ln(τ)·ln(dV/dy)
0.5	-0.6931	40	3.689	-2.557
1.7	0.5306	60	4.094	2.172
3	1.099	80	4.382	4.816
6	1.792	120	4.787	8.578
Sum	2.729		16.95	13.01

m = 4 Summation $(\ln(dV/dy))^2 = 5.181$

$$b_1 = \frac{4(13.01) - 2.729(16.95)}{4(5.181) - 2.729^2} = 0.4356$$

$$b_0 = \frac{16.95}{4} - 0.4356\frac{2.729}{4} = 3.537$$

$K = \exp(b_0) = 51.43$; $n = b_1 = 0.4356$

$$\tau = \tau_o + K\left(\frac{dV}{dy}\right)^n = 51.43\left(\frac{dV}{dy}\right)^{0.4356}$$

where dV/dy is in rev/s and τ in lbf/ft²; these are not standard units.

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11. A rotating cup viscometer has an inner cylinder diameter of 2.00 in. and the gap between cups is 0.2 in. The inner cylinder length is 2.50 in. The viscometer is used to obtain viscosity data on a Newtonian liquid. When the inner cylinder rotates at 10 rev/min, the torque on the inner cylinder is measured to be 0.00011 in-lbf. Calculate the viscosity of the fluid. If the fluid density is 850 kg/m³, calculate the kinematic viscosity.

Rotating cup viscometer $R = 2/2 = 1$ inch = 0.0833 ft
$\delta = 0.2$ in = 0.01667 ft $L = 2.5$ in = 0.2083 ft

$$\omega = (10 \text{ rev/min})\cdot(2\pi \text{ rad/rev})(1 \text{ min}/60 \text{ s}) = 1.047 \text{ rad/s} = \frac{dV}{dy}$$

$$T = \frac{1.1 \text{ in·lbf}}{10^4}\frac{1 \text{ ft}}{12 \text{ in}} = 0.09167 \times 10^{-4} \text{ ft·lbf}$$

$\rho = 850$ kg/m³ sp. gr. = 0.850 $\rho = 62.4(0.850) = 53.04$ lbm/ft³

$$\mu = \frac{T\delta}{2\pi R^2(R + \delta)L\omega}$$

$$\mu = \frac{0.09167 \times 10^{-4}(0.01667)}{2\pi(0.0833)^2(0.0833 + 0.01667)(0.2083)(1.047)}$$

$\mu = 1.608 \times 10^{-4}\ \text{lbf·s/ft}^2$

$$\nu = \frac{\mu g_c}{\rho} = \frac{1.608 \times 10^{-4}(32.2)}{53.04}$$

$\nu = 9.762 \times 10^{-5}\ \text{ft}^2/\text{s}$

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12. A rotating cup viscometer has an inner cylinder whose diameter is 3.8 cm and whose length is 8 cm. The outer cylinder has a diameter of 4.2 cm. The viscometer is used to measure the viscosity of a liquid. When the outer cylinder rotates at 12 rev/min, the torque on the inner cylinder is measured to be 4 x 10⁻⁶ N·m. Determine the kinematic viscosity of the fluid if its density is 1 000 kg/m³.

$R = 3.8/2 = 1.9\ \text{cm} = 0.019\ \text{m}$; $L = 0.08\ \text{m}$
$R_{outside} = 4.2/2 = 2.1\ \text{cm}$

$\delta = 2.1 - 1.9 = 0.2\ \text{cm} = 0.002\ \text{m}$
$\omega = (12\ \text{rev/min})(2\pi/60) = 1.26\ \text{rad/s}$

$T = 3.8 \times 10^{-6}\ \text{N·m}$ $\rho = 1\,000\ \text{kg/m}^3 = 62.4\ \text{lbm/ft}^3$

$$\mu = \frac{T\delta}{2\pi R^2(R + \delta)(L\omega)} = \frac{3.8 \times 10^{-6}(0.002)}{2\pi(0.019)^2(0.019 + 0.002)(0.08)(1.26)}$$

$\mu = 1.58 \times 10^{-3}\ \text{N·s/m}^2$

$$\nu = \frac{\mu g_c}{\rho} = \frac{1.58 \times 10^{-3}}{1\,000} = 1.58 \times 10^{-6}\ \text{m}^2/\text{s}$$

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13. A rotating cup viscometer has an inner cylinder diameter of 2.25 in. and an outer cylinder diameter of 2.45 in. The inner cylinder length is 3.00 in. When the inner cylinder rotates at 15 rev/min, what is the expected torque reading if the fluid is propylene glycol?

$D = 2.25\ \text{in.}$ $R = 0.09375\ \text{ft}$ $2(R + \delta) = 2.45\ \text{in}$
$R + \delta = 1.225\ \text{in.} = 0.1021\ \text{ft}$

$$\delta = \frac{1.225}{12} - 0.09375 = 0.00833\ \text{ft} \qquad \rho = 0.968(1.94); \qquad \mu = 88 \times 10^{-5}\ \text{lbf·s/ft}^2$$

$\omega = (15\ \text{rev/min})(2\pi/60) = 1.572\ \text{rad/s}$

$$T = \frac{2\pi R^2(R + \delta)(L\omega)\mu}{\delta} = \frac{2\pi(0.09375)^2(0.1021)(3/12)(1.571)(88 \times 10^{-5})}{0.00833}$$

$T = 2.34 \times 10^{-4}\ \text{ft·lbf}$

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14. A capillary tube viscometer is used to measure the viscosity of water (density is 62.4 lbm/ft^3, viscosity is 0.89 x 10^{-3} N·s/m^2) for calibration purposes. The capillary tube inside diameter must be selected so that laminar flow conditions (i.e., VD/ν < 2 100) exist during the test. For values of L = 3 in. and z = 10 in., determine the maximum tube size permissible.

Capillary tube viscometer $\quad \frac{V\!\!\!\!-}{t} = \frac{\rho g\, z}{g_c\, L} \frac{\pi R^4}{8\mu} \quad \rho = 62.4 \text{ lbm/ft}^3$

$$\mu = 0.89 \times 10^{-3} \text{ N·s/m}^2 = \frac{0.89 \times 10^{-3}}{47.88} = 1.859 \times 10^{-5} \text{ lbf·s/ft}^2$$

$z = 10/12 = 0.8333$ ft $\qquad L = 3/12 = 0.25$ ft

$\frac{V\!\!\!\!-}{t}$ = Volume flow rate = AV = $\pi R^2 V$; substituting into the equation,

$\pi R^2 V = \frac{\rho g\, z}{g_c\, L} \frac{\pi R^4}{8\mu}$ Rearrange and solve for V $\quad V = \frac{\rho g\, z}{g_c\, L} \frac{R^2}{8\mu}$

The limiting value is Re < 2100; using equality,

$\frac{V(2R)}{\nu} = 2100; \quad \frac{\rho V(2R)}{\mu g_c} = 2100$ or

$V = \frac{2100 \mu g_c}{2\rho R} = \frac{\rho g\, z}{g_c\, L} \frac{R^2}{8\mu}$ Rearrange and solve for R^3

$$R^3 = \frac{2100\mu^2 g_c^2 (8)(L)}{2\rho^2 g z} = \frac{2100(1.859 \times 10^{-5})^2(32.2)^2(8)(0.25)}{2(62.4)^2(32.2)(0.8333)}$$

$R^3 = 7.202 \times 10^{-9}$ or

$\boxed{R = 1.931 \times 10^{-3} \text{ ft} = 0.02317 \text{ in}}$ Any larger, flow no longer laminar

15. A Saybolt viscometer is used to measure oil viscosity and the time required for 60 ml of oil to pass through a standard orifice is 180 SUS. The specific gravity of the oil is found as 44°API. Determine the absolute viscosity of the oil.

For 180 SUS,

$$\nu = 0.223 \times 10^{-6}(180) - \frac{155 \times 10^{-6}}{180} = \boxed{3.928 \times 10^{-5} \text{ m}^2\text{/s}}$$

44°API oil; $\quad \text{sp.gr.} = \frac{141.5}{131.5 + 44} = 0.8063; \; \rho = 806.3 \text{ kg/m}^3$

$$\mu = \frac{\rho \nu}{g_c} = 806.3(3.928 \times 10^{-2}) = \boxed{3.167 \times 10^{-2} \text{ N·s/m}^2}$$

16. A 10 cm^3 capillary tube viscometer is used to measure the viscosity of a liquid. For values of L = 4 cm, z = 25 cm, and D = 0.8 mm, determine the viscosity of the liquid. The time recorded for the experiment is 12 seconds.

$$\nu = \left(\frac{z\pi R^4 g}{8L\mathcal{V}}\right)t = \left(\frac{0.25\pi(0.000\,8/2)^4(9.81)}{8(0.04)(10 \times 10^{-6})}\right)(12)$$

$$\boxed{\nu = 7.39 \times 10^{-7}\ m^2/s}$$

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17. A Saybolt viscometer is used to obtain oil viscosity data. The time required for 60 ml of oil to pass through the orifice is 70 SUS. Calculate the kinematic viscosity of the oil. If the specific gravity of the oil is 35°API, find also its absolute viscosity.

For 70 SUS,

$$\nu = 0.224 \times 10^{-6}(70) - \frac{185 \times 10^{-6}}{70}$$

$$\boxed{\nu = 1.304 \times 10^{-5}\ m^2/s}$$

35°API oil

$$\text{sp. gr.} = \frac{141.5}{131.5 + 35} = 0.8498 \quad \rho = 849.8\ kg/m^3$$

$$\mu = \frac{\rho\nu}{g_c} = 849.8(1.304 \times 10^{-5})$$

$$\boxed{\mu = 1.108 \times 10^{-2}\ N\cdot s/m^2}$$

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18. A 2-mm diameter ball bearing is dropped into a container of glycerine. How long will it take the bearing to fall a distance of 1 m?

$$\mu = \left(\frac{\rho_s}{\rho} - 1\right)\frac{\rho g}{g_c}\frac{D^2}{18V} \qquad V = \frac{L}{t} \qquad L = 1\ m \qquad D = 2\ mm = 0.002\ m$$

$$\rho_s = 7.9(1\,000) \quad \rho = 1\,263 \quad \mu = 950 \times 10^{-3} \text{ glycerine}$$

$$V = \left(\frac{\rho_s}{\rho} - 1\right)\frac{\rho g}{g_c}\frac{D^2}{18\mu} = \left(\frac{7.9}{1.263} - 1\right)(1\,263)(9.81)(0.002^2)\frac{1}{18(950 \times 10^{-3})}$$

$$V = 0.015\,2\ m/s$$

$$\text{Check on Re} = \frac{\rho V D}{\mu g_c} = \frac{1\,263(0.015\,2)(0.002)}{950 \times 10^{-3}} = 0.04 < 1 \qquad \text{OK}$$

$$\frac{L}{t} = 0.015\,2; \qquad t = \frac{1}{0.015\,2}$$

$$\boxed{t = 65.8 \text{ s}}$$

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19. A 1/8-in. diameter ball bearing is dropped into a viscous oil. The terminal velocity of the sphere is measured as 2 ft/15 s. What is the kinematic viscosity of the oil if its specific gravity is 0.8?

$$\mu = \left(\frac{\rho_s}{\rho} - 1\right)\frac{\rho g}{g_c}\frac{D^2}{18V} \qquad V = \frac{L}{t} = \frac{2}{15} = 0.133 \text{ ft/s} \qquad D = \frac{1}{8}\frac{1}{12} = 0.01042 \text{ ft}$$

$$\rho_s = 7.9(1.94) \text{ slug/ft}^3$$

$$\nu = \frac{\mu g_c}{\rho} = \left(\frac{\rho_s}{\rho} - 1\right)\frac{gD^2}{18V} = \left(\frac{7.9}{0.8} - 1\right)\frac{(32.2)(0.01042^2)}{18(0.133)}$$

$$\boxed{\nu = 1.296 \times 10^{-2} \text{ ft}^2/\text{s}}$$

$$\text{Check on Re} = \frac{VD}{\nu} = \frac{0.133(0.01042)}{1.296 \times 10^{-2}} = 0.107 < 1 \quad \text{OK}$$

Pressure and Its Measurement

20. A mercury manometer is used to measure pressure at the bottom of a tank containing acetone, as shown in Figure P2.20. The manometer is to be replaced with a gage. What is the expected reading in psig if $\Delta h = 5$ in. and $x = 2$ in?

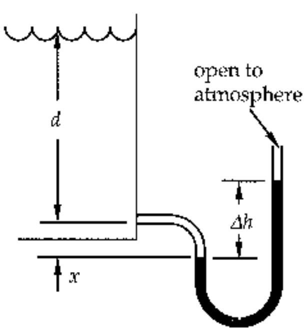

FIGURE P2.20.

Acetone $\rho_a = 0.787(1.94) = 1.527$

Hg $\rho = 13.6(1.94) = 26.38$

$$p_A + \frac{\rho_a g}{g_c}x = p_{atm} + \frac{\rho g}{g_c}\Delta h$$

$$p_A + 1.527(32.2)(2/12) = 14.7(144) + 26.38(32.2)(5/12)$$

$$p_A + 8.195 = 14.7(144) + 353.9$$

$$\boxed{p_A = 2463 \text{ psfa} = 17.1 \text{ psia} = 2.4 \text{ psig}}$$

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21. Referring to Figure P2.21, determine the pressure of the water at the point where the manometer attaches to the vessel. All dimensions are in inches and the problem is to be worked using Engineering or British Gravitational units.

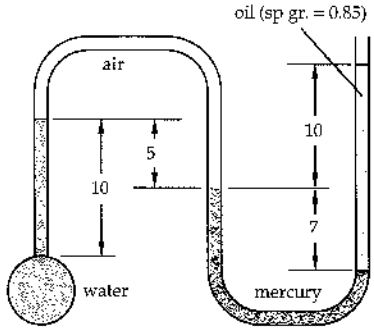

FIGURE P2.21.

$$p_W - \frac{\rho_w g}{g_c}\frac{10}{12} + \frac{\rho_{air} g}{g_c}\frac{5}{12} + \frac{\rho_{Hg} g}{g_c}\frac{7}{12} - \frac{\rho_o g}{g_c}\frac{17}{12} = p_{atm}$$

$$p_W - 1.94(32.2)(10/12) + 13.6(1.94)(32.2)(7/12) - 0.85(1.94)(32.2)(17/12) = 14.7(144)$$

$$p_W - 52.06 + 495.6 - 75.22 = 2117$$

$$\boxed{p_W = 1749 \text{ psf} = 12.14 \text{ psia}}$$

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22. Figure P2.22 shows a portion of a pipeline that conveys benzene. A gage attached to the line reads 150 kPa. It is desired to check the gage reading with a benzene-over-mercury U-tube manometer. Determine the expected reading Δh on the manometer.

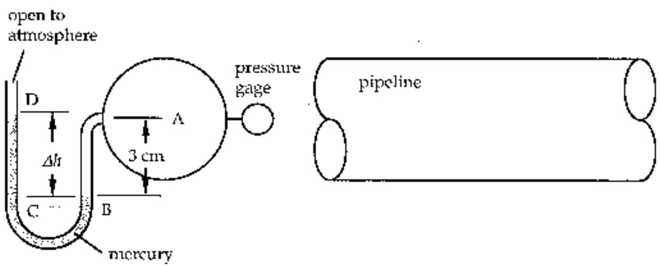

FIGURE P2.22.

$$p_D + \frac{\rho_{Hg} g}{g_c}\Delta h - \frac{\rho_B g}{g_c}(0.03) = p_A \qquad p_D = p_{atm} = 0$$

$$0 + 13.6(1\,000)(9.81)\Delta h - 876(9.81)(0.03) = 150\,000 \text{ (which is a gage reading)}$$

$$0 + 133\,400\Delta h - 257.8 = 150\,000$$

$$\Delta h = \frac{150\,000 + 257.8}{133\,400}$$

$$\boxed{\Delta h = 1.126 \text{ m}}$$

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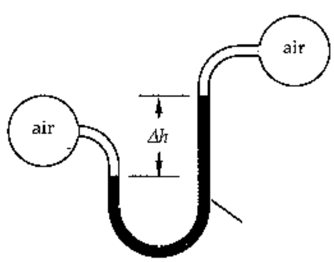
23. An unknown fluid is in the manometer of Figure P2.23. The pressure difference between the two air chambers is 700 kPa and the manometer reading Δh is 6 cm. Determine the density and specific gravity of the unknown fluid.



Because $\rho_{air} << \rho_{liquid}$, then

$p_A - p_B = \rho g \Delta h$; $\qquad \Delta h = 6 \text{ cm} = 0.06 \text{ m}$, and

$p_A - p_B = 700 \text{ N/m}^2$ given; so

$$\rho = \frac{p_A - p_B}{g\,\Delta h} = \frac{700}{9.81(0.06)} = \boxed{1\,190 \text{ kg/m}^3}$$

FIGURE P2.23.

24. A U-tube manometer is used to measure the pressure difference between two air chambers, as shown in Figure P2.24. If the reading Δh is 6 inches, determine the pressure difference.

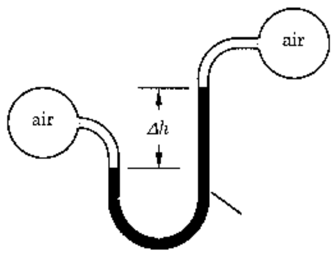


FIGURE P2.24

Because $\rho_{air} << \rho_{liquid}$, then

$p_A - p_B = \rho g \Delta h$; Δh = 6 inches = 0.5 ft

$p_A - p_B$ = 1.94 slug/ft³(32.2 ft/s²)(0.5 ft)

$$\boxed{p_A - p_B = 32.23 \text{ lbf/ft}^2}$$

25. A manometer containing mercury is used to measure the pressure increase experienced by a water pump as shown in Figure P2.25. Calculate the pressure rise if Δh is 7 cm of mercury (as shown). All dimensions are in cm.

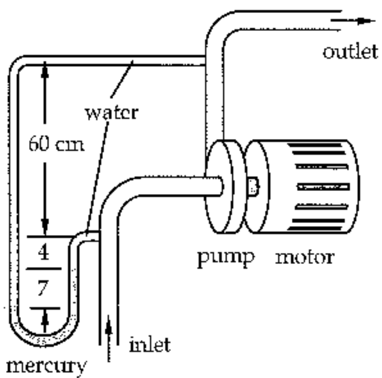


FIGURE P2.25.

$$p_{outlet} + \frac{\rho g}{g_c}\left(\frac{60+4+7}{100}\right) - \frac{\rho_{Hg} g}{g_c}(0.07) - \frac{\rho g}{g_c}(0.04) = p_{inlet}$$

$$p_{outlet} + 1\,000(9.81)(0.71) - 13.6(1\,000)(9.81)(0.07) - 1\,000(9.81)(0.04) = p_{inlet}$$

$$p_{outlet} + 6965 - 9339 - 392.4 = p_{inlet}$$

$$\boxed{p_{outlet} - p_{inlet} = 2\,7\,66 \text{ Pa} = 2.77 \text{ kPa}}$$

26. Determine the pressure difference between the linseed and castor oils of Figure P2.26. (All dimensions in inches.)

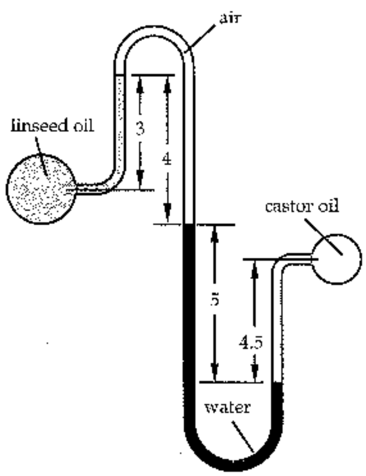


FIGURE P2.26.

$$p_A - \rho_{LO} g(3/12) + \rho_{air} g(4/12) + \rho_{H2O} g(5/12) - \rho_{CO} g(4.5/12) = p_B$$

$\rho_{LO} = 0.93(1.94)$; $\rho_{CO} = 0.96(1.94)$

$\rho_{H2O} = 1.94$ ρ_{air} negligible

$$p_A - p_B = \rho_{LO} g(3/12) + \rho_{H2O} g(5/12) - \rho_{CO} g(4.5/12)$$

$$p_A - p_B = 0.93(1.94)(32.2)(3/12) - 1.94(32.2)(5/12) - 0.96(1.94)(32.2)(4.5/12)$$

$$p_A - p_B = 14.52 - 26.03 + 22.49$$

$$\boxed{p_A - p_B = 10.98 \text{ lbf/ft}^2}$$

27. For the system of Figure P2.27, determine the pressure of the air in the tank.

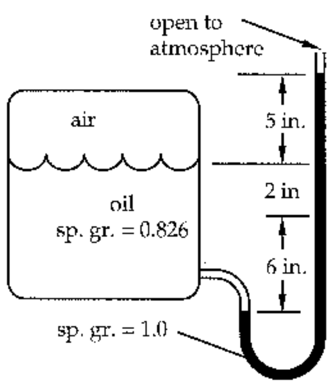


FIGURE P2.27.

$$p_{air} + \frac{\rho_{oil} g}{g_c}(2+6)/12 - \frac{\rho g}{g_c}(13/12) = p_{atm}$$

$$p_{air} + 0.826(1.94)(32.2)(8/12) - 1.94(32.2)(13/12) = 14.7(144)$$

$$p_{air} + 34.40 - 67.67 = 2117$$

$$\boxed{p_{air} = 2150 \text{ psf} = 14.93 \text{ psi}}$$

Continuity Equation

28. Figure P2.28 shows a reducing bushing. A liquid leaves the bushing at a velocity of 4 m/s. Calculate the inlet velocity. What effect does the fluid density have?

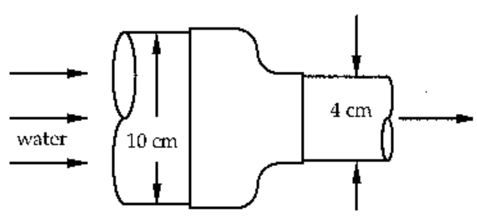


FIGURE P2.28, P2.29.

$D_1 = 10$ cm $= 0.1$ m; $D_2 = 4$ cm $= 0.04$ m $V_2 = 4$ m/s

Density has no effect

$$Q = A_1 V_1 = A_2 V_2$$

$$\frac{\pi D_1^2}{4} V_1 = \frac{\pi D_2^2}{4} V_2$$

$$V_1 = V_2 \frac{D_2^2}{D_1^2} = 4\left(\frac{0.04^2}{0.1^2}\right)$$

$$\boxed{V_1 = 0.64 \text{ m/s}}$$

29. Figure P2.29 shows a reducing bushing. Liquid enters the bushing at a velocity of 0.5 m/s. Calculate the outlet velocity.

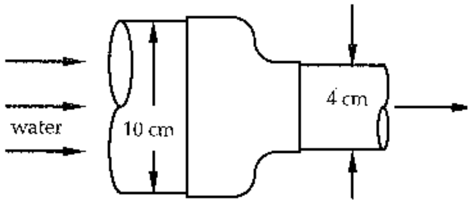


FIGURE P2.28, P2.29.

$D_1 = 10 \text{ cm} = 0.1 \text{ m}; \quad D_2 = 4 \text{ cm} = 0.04 \text{ m}$
$V_1 = 0.5 \text{ m/s}$

$$Q = A_1V_1 = A_2V_2$$

$$\frac{\pi D_1^2}{4} V_1 = \frac{\pi D_2^2}{4} V_2$$

$$V_2 = V_1 \frac{D_1^2}{D_2^2} = 0.5\left(\frac{0.1^2}{0.04^2}\right)$$

$$\boxed{V_2 = 3.13 \text{ m/s}}$$

---

30. Three gallons per minute of water enters the tank of Figure P2.30. The inlet line is 6.35 cm in diameter. The air vent is 3.8 cm in diameter. Determine the air exit velocity at the instant shown.

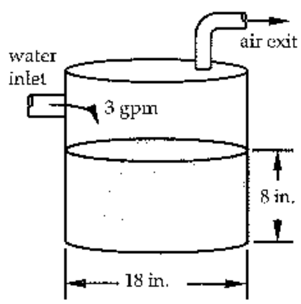


FIGURE P2.30.

For low pressures & temperatures, air can be treated as incompressible.

$$Q_{H20\text{ in}} = Q_{air\text{ out}}$$

$$Q_{H2O\text{ in}} = (3 \text{ gal/min})\frac{3.785 \times 10^{-3}}{60} = 1.262 \times 10^{-4} \text{ m}^3\text{/s}$$

$$\rho_{H2O} = 1\,000 \text{ kg/m}^3 \quad \rho_{air} = 1.19 \text{ kg/m}^3$$

$$Q_{air\text{ out}} = AV = \frac{\pi D^2}{4} V = \frac{\pi}{4}[(0.038]^2 = 1.14 \times 10^{-3}V$$

So $1.262 \times 10^{-4} = 1.14 \times 10^{-3}V$

$$\boxed{V_{air} = 0.111 \text{ m/s}}$$

---

31. An air compressor is used to pressurize a tank of volume 3 m^3. Simultaneously, air leaves the tank and is used for some process downstream. At the inlet, the pressure is 350 kPa, the temperature is 20°C, and the velocity is 2 m/s. At the outlet, the temperature is 20°C, the velocity is 0.5 m/s, and the pressure is the same as that in the tank. Both flow lines (inlet and outlet) have internal diameters of 2.7 cm. The temperature of the air in the tank is a constant at 20°C. If the initial tank pressure is 200 kPa, what is the pressure in the tank after 5 minutes?

$$0 = \frac{\partial m}{\partial t} + (\rho AV)_{out} - (\rho AV)_{in} \quad m = \frac{p\mathcal{V}}{RT} \qquad \frac{\partial m}{\partial t} = \frac{\mathcal{V}}{RT}\frac{dp}{dt}$$

$$(\rho AV)_{out} - (\rho AV)_{in} = \frac{p_{out}}{RT_{out}} A_{out} V_{out} - \frac{p_{in}}{RT_{in}} A_{in} V_{in}$$

Substituting,

$$0 = \frac{\forall}{RT}\frac{dp}{dt} + \frac{p_{out}}{RT_{out}} A_{out} V_{out} - \frac{p_{in}}{RT_{in}} A_{in} V_{in}$$

For constant T, all RT products cancel

$$\forall \frac{dp}{dt} = -\ p_{out} A_{out} V_{out} + p_{in} A_{in} V_{in} \quad p_{out} = p$$

$$A_{in} = \frac{\pi(0.027)^2}{4} = 5.726 \times 10^{-4}\ \text{m}^2 = A_{out}$$ Areas are equal

$$3\frac{dp}{dt} = -p(5.726 \times 10^{-4})(0.5) + 350\ 000(5.726 \times 10^{-4})(2)$$

$$3\frac{dp}{dt} = 400.8 - 2.863 \times 10^{-4} p \quad \text{or} \quad \frac{dp}{dt} = 133.6 - 9.543 \times 10^{-5} p$$

Separating variables,

$$\int_{p}^{200\ 000} \frac{dp}{133.6 - 9.543 \times 10^{-5} p} = \int_0^{300} dt$$

$$\left.\frac{\ln(133.6 - 9.543 \times 10^{-5} p)}{-9.543 \times 10^{-5}}\right|_{200\ 000}^{p} = 300 - 0$$

$$\ln(133.6 - 9.543 \times 10^{-5} p) - \ln(133.6 - 9.543 \times 10^{-5}(200\ 000)) = 300(-9.543 \times 10^{-5})$$

$$\ln(133.6 - 9.543 \times 10^{-5} p) - 4.741 = -2.863 \times 10^{-2}$$

$$\ln(133.6 - 9.543 \times 10^{-5} p) = 4.712$$

Exponentiating,

$$133.6 - 9.543 \times 10^{-5} p = 1.113 \times 10^2 \quad \text{or} \quad -9.543 \times 10^{-5} p = -22.3$$

$$\boxed{p = 2.34\ \text{kPa}}$$

---

32. Figure P2.32 shows a cross-flow heat exchanger used to condense Freon-12. Freon-12 vapor enters the unit at a flow rate of 0.065 kg/s. Freon-12 leaves the exchanger as a liquid (Sp. Gr. = 1.915) at room temperature and pressure. Determine the exit velocity of the liquid.

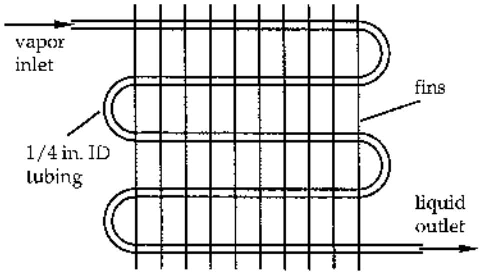


FIGURE P2.32.

$\dot{m}_{in} = \rho_{out} A_{out} V_{out}$

$\dot{m}_{in} = 0.065\ \text{kg/s}$

$\rho = 1.915(1\,000)\ \text{kg/m}^3$

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.25/12)^2}{4} = 3.41 \times 10^{-4}\ \text{ft}^2$$

$A = 3.41 \times 10^{-4}(9.29 \times 10^{-2}) = 3.17 \times 10^{-5}\ \text{m}^2$

Substituting,

$0.065 = 1.915(1\,000)(3.17 \times 10^{-5})V_{out}$

$\boxed{V_{out} = 1.07\ \text{m/s}}$

---

33. Nitrogen enters a pipe at a flow rate of 0.2 lbm/s. The pipe has an inside diameter of 4 in. At the inlet, the nitrogen temperature is 540°R (ρ = 0.073 lbm/ft^3) and at the outlet, the nitrogen temperature is 1800°R (ρ = 0.0213 lbm/ft^3). Calculate the inlet and outlet velocities of the nitrogen. Are they equal? Should they be?

$\dot{m} = 0.2\ \text{lbm/s}$ $\quad D = 4/12 = 0.333\ \text{ft}$ $\quad \rho_1 = 0.073\ \text{lbm/ft}^3$

$\rho_2 = 0.0213\ \text{lbm/ft}^3$

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.333)^2}{4} = 0.08727\ \text{ft}^2 \qquad \dot{m} = \rho A V$$

$$V_1 = \frac{\dot{m}}{\rho_1 A} = \frac{0.2}{0.073(0.08727)}$$

$\boxed{V_1 = 31.4\ \text{ft/s}}$

$$V_2 = \frac{0.2}{0.0213(0.08727)}$$

$\boxed{V_2 = 107.6\ \text{ft/s}}$

Momentum Equation

34. A garden hose is used to squirt water at someone who is protecting herself with a garbage can lid. Figure P2.34 shows the jet in the vicinity of the lid. Determine the restraining force F for the conditions shown.

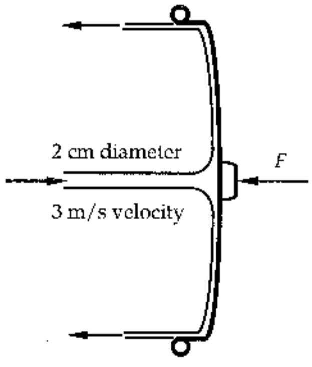


FIGURE P2.34

$$\Sigma F = \frac{\dot{m}}{g_c}(V_{out} - V_{in}) \qquad \dot{m}_{in} = \dot{m}_{out} \text{ frictionless flow}$$

magnitude of V_{in} = magnitude of V_{out}

$$F = \frac{[\rho AV]_{inlet}}{g_c}(-V_{in} - V_{in}) \qquad g_c = 1 \text{ in SI units}$$

$$F = 2\rho AV^2 \quad \rho = 1\,000 \text{ kg/m}^3$$

$$A = \frac{\pi(0.02)^2}{4} = 3.14 \times 10^{-4} \text{ m}^2 \qquad V = 3 \text{ m/s}$$

$$F = 2(1\,000)(3.14 \times 10^{-4})(3)^2$$

$$\boxed{F = 5.65 \text{ N}}$$

---

35. A two-dimensional, liquid jet strikes a concave semicircular object, as shown in Figure P2.35. Calculate the restraining force F.

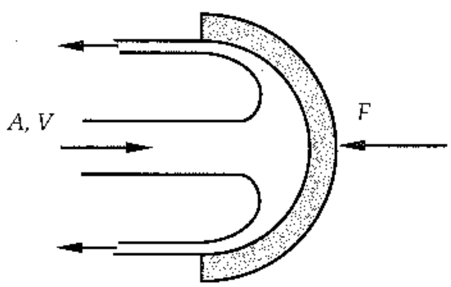


FIGURE P2.35.

$$\Sigma F = \frac{\dot{m}}{g_c}(V_{out} - V_{in})$$

$\dot{m}_{in} = \dot{m}_{out}$ frictionless flow

magnitude of V_{in} = magnitude of V_{out}

$$F = \frac{[\rho AV]_{inlet}}{g_c}(-V_{in} - V_{in})$$

$g_c = 1$ in SI units

$$\boxed{F = \frac{2\rho AV^2}{g_c}}$$

---

36. A two-dimensional, liquid jet strikes a concave semicircular object, as shown in Figure P2.36. Calculate the restraining force F.

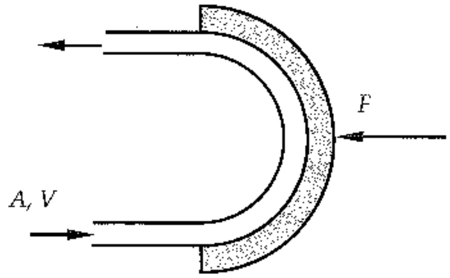


FIGURE P2.36.

$$\Sigma F = \frac{\dot{m}}{g_c}(V_{out} - V_{in})$$

$\dot{m}_{in} = \dot{m}_{out}$ frictionless flow

magnitude of V_{in} = magnitude of V_{out}

$$F = \frac{[\rho A V]_{\text{inlet}}}{g_c}(-V_{\text{in}} - V_{in})$$

$g_c = 1$ in SI units

$$\boxed{F = \frac{2\rho A V^2}{g_c}}$$

---

37. A two-dimensional liquid jet is turned through an angle θ ($0° < \theta < 90°$) by a curved vane, as shown in Figure P2.37. The forces are related by $F_2 = 3F_1$. Determine the angle θ through which the liquid jet is turned.

$$\Sigma F = \frac{\dot{m}}{g_c}(V_{out} - V_{in})\ ;\ \dot{m}_{in} = \dot{m}_{out} \text{ frictionless flow}$$

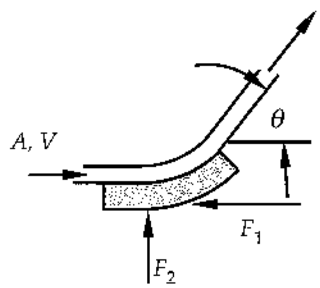


FIGURE P2.37.

magnitude of V_{in} = magnitude of V_{out}

$$-F_1 = \frac{[\rho A V]_{\text{inlet}}}{g_c}(V_{\text{out}x} - V_{inx})$$

$V_{\text{out}x} = V\cos\theta$; $\qquad V_{inx} = V$

$$-F_1 = \frac{[\rho A V]_{\text{inlet}}}{g_c}(V\cos\theta - V) = \frac{\rho A V^2}{g_c}(\cos\theta - 1)$$

$$F_1 = \frac{\rho A V^2}{g_c}(1 - \cos\theta) \qquad F_2 = \frac{[\rho A V]_{\text{inlet}}}{g_c}(V_{\text{out}y} - V_{iny})$$

$V_{outy} = V\sin\theta$; $\qquad V_{iny} = 0$

$$F_2 = \frac{[\rho A V]_{\text{inlet}}}{g_c}(V\sin\theta) = \frac{\rho A V^2}{g_c}(\sin\theta) \qquad F_2 = 3F_1; \qquad \sin\theta = 3(1 - \cos\theta)$$

$\frac{1}{3}\sin\theta = 1 - \cos\theta$ $\quad$ T & E solution is quickest

θ	$(1/3)\sin\theta$	$1 - \cos\theta$
45°	0.2357	0.2929
50°	0.2553	0.3572
40°	0.2143	0.234
35°	0.1912	0.1808
37°	0.2006	0.2014
36.8°	0.1997	0.1993

$$\boxed{\theta = 36.8°}$$

38. A two-dimensional liquid jet is turned through an angle $\theta\,(0^\circ < \theta < 90^\circ)$ by a curved vane as shown in Figure P2.38. The forces are related by $F_1 = 2F_2$. Determine the angle θ through which the liquid jet is turned.

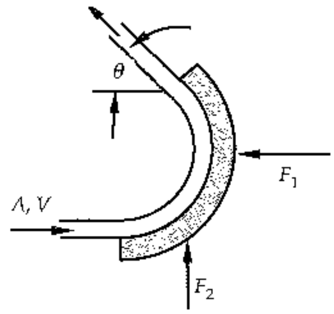


FIGURE 2.39.

$$\Sigma F = \frac{\dot{m}}{g_c}\left(V_{out} - V_{in}\right);\ \dot{m}_{in} = \dot{m}_{out} \text{ frictionless flow}$$

magnitude of V_{in} = magnitude of V_{out}

$$-F_1 = \frac{[\rho AV]_{\text{inlet}}}{g_c}(V_{\text{outx}} - V_{inx})$$

$$V_{\text{outx}} = -V\cos\theta; \qquad V_{inx} = V$$

$$-F_1 = \frac{[\rho AV]_{\text{inlet}}}{g_c}(-V\cos\theta - V) = -\frac{\rho AV^2}{g_c}(\cos\theta + 1)$$

$$F_1 = \frac{\rho AV^2}{g_c}(1 + \cos\theta)$$

$$F_2 = \frac{[\rho AV]_{\text{inlet}}}{g_c}(V_{\text{outy}} - V_{iny})$$

$$V_{outy} = V\sin\theta; \qquad V_{iny} = 0$$

$$F_2 = \frac{[\rho AV]_{\text{inlet}}}{g_c}(V\sin\theta) = = \frac{\rho AV^2}{g_c}(\sin\theta)$$

$$F_1 = 2F_2; \quad 1 + \cos\theta = 2\sin\theta$$

T & E solution is quickest

θ	$1 - \cos\theta$	$2\sin\theta$
45°	1.707	1.414
50°	1.643	1.532
55°	1.574	1.638
53°	1.602	1.597
54°	1.588	1.618
53.5°	1.595	1.608
53.4°	1.596	1.606
53.2°	1.599	1.601
53.1°	1.600	1.599

$\boxed{\theta = 53.1^\circ}$

Energy Equation

39. Figure P2.39 shows a water turbine located in a dam. The volume flow rate through the system is 5000 gpm. The exit pipe diameter is 4 ft. Calculate the work done by (or power received from) the water as it flows through the dam. (Compare to the results of the example problem in this chapter.)

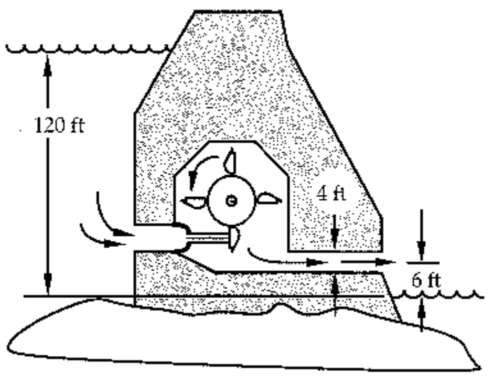


FIGURE P2.39.

We apply the energy equation between any two sections. Section 1 = the free surface upstream, and Section 2 = the outlet downstream.

$p_2 = p_1 = p_{atm}$

$V_1 \approx 0$ (reservoir surface velocity)

$z_2 = 6$ ft; $z_1 = 120$ ft

$$A_2 = \frac{\pi D^2}{4} = \frac{\pi(4)^2}{4} = 12.6 \text{ ft}^2$$

$$Q = 5000 \text{ gpm}\frac{3.785 \times 10^{-3}}{2.831 \times 10^{-2}}\frac{1 \text{ min}}{60 \text{ s}} = 11.14 \text{ ft}^3/\text{s}$$

$$V_2 = \frac{Q}{A} = \frac{11.14}{12.6} = 0.88 \text{ ft/s} \qquad \rho = 62.4 \text{ lbm/ft}^3$$

$\rho VA = \dot{m} = 62.4(12.6)(0.88) = 695$ lbm/s evaluated at outlet

Substituting,

$$-\frac{\partial W}{\partial t} = \left\{\left(\frac{p}{\rho} + \frac{V^2}{2g_c} + \frac{gz}{g_c}\right)\Big|_2 - \left(\frac{p}{\rho} + \frac{V^2}{2g_c} + \frac{gz}{g_c}\right)\Big|_1\right\}\rho VA$$

$$-\frac{\partial W}{\partial t} = \left\{\left(0 + \frac{0.88^2}{2(32.2)} + \frac{32.2(6)}{32.2}\right) - \left(0 + 0 + \frac{32.2(120)}{32.2}\right)\right\} 695$$

$$-\frac{\partial W}{\partial t} = \left\{0 + 0.01202 + 6 - 0 - 0 - 120\right\}695$$

$$\boxed{+\frac{\partial W}{\partial t} = 7.92 \times 10^4 \text{ ft·lbf/s} = 144 \text{ HP}}$$

---

40. Air flows through a compressor at a mass flow rate of 0.003 slug/s. At the inlet, the air velocity is negligible. At the outlet, air leaves through an exit pipe of diameter 2 in. The inlet properties are 14.7 psia and 75°F. The outlet pressure is 120 psia. For an isentropic (reversible and adiabatic) compression process, we have

$$\frac{T_2}{T_1} = \left\{\frac{p_2}{p_1}\right\}^{(\gamma - 1)/\gamma}$$

Determine the outlet temperature of the air and the power required. Assume that air behaves as an ideal gas ($dh = c_p\, dT$, $du = c_v\, dT$, and $\rho = p/RT$).

$$\frac{T_2}{T_1} = \left\{\frac{p_2}{p_1}\right\}^{(\gamma - 1)/\gamma}$$

Determine the outlet temperature of the air and the power required. Assume that air behaves as an ideal gas ($dh = c_p\, dT$, $du = c_v\, dT$ and $\rho = p/RT$).

Solution:

$\dot{m} = 0.003$ slug/s $\quad V_{in} = 0 \quad V_{out} =$ unknown

$p_{in} = 14.7$ psig $= 2117$ psfa $\quad p_{out} = 120$ psia $= 17280$ psfa

$D_{out} = 2$ in $= 0.1667$ ft $\quad A_{out} = \pi D^2/4 = 0.2182 \quad \gamma = 1.4$

$R_{air} = 1710$ ft·lbf/(slug·°R) $\quad c_{pair} = 7.72$ BTU/(slug·°R)

$$\frac{T_{out}}{T_{in}} = \left\{\frac{p_{out}}{p_{in}}\right\}^{(\gamma - 1)/\gamma}$$

$$\frac{T_{out}}{535} = \left\{\frac{17280}{2117}\right\}^{(1.4 - 1)/1.4} = 1.822 \qquad T_{out} = 535(1.822)$$

$$\boxed{T_{out} = 974.7°\text{R} = 514.7°\text{F}}$$

$$\rho_{out} = \frac{p}{RT} = \frac{17280}{1710(974.7)} = 0.01037 \text{ slug/ft}^3$$

$$V_{out} = \frac{\dot{m}}{\rho A_{out}} = \frac{0.003}{0.01037(0.02182)} = 13.3 \text{ ft/s} \qquad \frac{V_{out}^2}{2g_c} = \frac{13.3^2}{2} = 87.9$$

$$-\frac{\partial W}{\partial t} = \left\{\left(h + \frac{V^2}{2g_c} + \frac{gz}{g_c}\right)\Big|_{out} - \left(h + \frac{V^2}{2g_c} + \frac{gz}{g_c}\right)\Big|_{in}\right\} \rho VA$$

$$-\frac{\partial W}{\partial t} = \left(h_{out} - h_{in} + \frac{V_{out}^2}{2g_c}\right) \rho VA \qquad \Delta PE = 0$$

$(h_{out} - h_{in}) = c_p(T_{out} - T_{in}) = 7.72(514.7 - 75) = 3394$ BTU/slug

$(h_{out} - h_{in}) = (3394$ BTU/slug$)\cdot(778$ ft·lbf/BTU$) = 2.641 \times 10^6$ ft·lbf/slug

$$-\frac{\partial W}{\partial t} = (2.641 \times 10^6 + 87.9)(0.003) = 7293 \text{ ft·lbf/s} \quad 550 \text{ ft·lbf/HP}$$

$$\boxed{-\frac{\partial W}{\partial t} = 14.4 \text{ HP}}$$ Assuming no losses

---

41. An air turbine is used with a generator to generate electricity. Air at the turbine inlet is at 700 kPa and 25°C. The turbine discharges air to the atmosphere at a temperature of 11°C. Inlet and outlet air velocities are 100 m/s and 2 m/s, respectively. Determine the work per unit mass delivered to the turbine from the air.

$p_{in} = 700$ kPa $\quad p_{out} = 101.3$ kPa
$T_{in} = 25°C \quad T_{out} = 11°C$
$V_{in} = 100$ m/s $\quad V_{out} = 2$ m/s
$c_p = 1\,005.7$ J/(kg·K)

$$-\frac{\partial W}{\partial t} = \left\{\left(h + \frac{V^2}{2g_c} + \frac{gz}{g_c}\right)\Bigg|_{out} - \left(h + \frac{V^2}{2g_c} + \frac{gz}{g_c}\right)\Bigg|_{in}\right\}\rho VA$$

$$-\frac{\partial W/\partial t}{\dot{m}} = (h_{out} - h_{in}) + \left(\frac{V_{out}^2}{23_c} + \frac{V_{in}^2}{2g_c}\right) + \frac{g}{g_c}(z_{out} - z_{in})$$

$$(h_{out} - h_{in}) = c_p(T_{out} - T_{in}) \qquad z_{out} = z_{in}$$

$$-\frac{\partial W/\partial t}{\dot{m}} = 1\,005.7(25 - 11) + \left(\frac{2^2}{2} - \frac{100^2}{2}\right) = 1.4 \times 10^4 - 5 \times 10^3$$

$$\boxed{-\frac{\partial W/\partial t}{\dot{m}} = 9 \times 10^3 \text{ J/kg}}$$

---

42. A pump moving hexane is illustrated in Figure P2.42. The flow rate is 0.02 m^3/s; inlet and outlet gage pressure readings are –4 kPa and 190 kPa, respectively. Determine the required power input to the fluid as it flows through the pump.

We apply the energy equation between any two sections. Section 1 = inlet pressure gage (actually the centerline of the pipe where the pressure gage is attached), and Section 2 = outlet pressure gage.

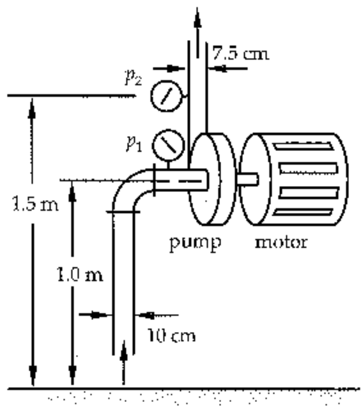


FIGURE P2.42.

p_2= 190 000 Pa z_2 = 1.5 m
p_1 = – 4000 Pa z_1 = 1.0 m

$AV = 0.02\ m^3/s$

$$A_1 = \frac{\pi D_1^2}{4} = \frac{\pi(0.10)^2}{4} = 7.854 \times 10^{-3}\ m^2$$

$$A_2 = \frac{\pi D_2^2}{4} = \frac{\pi(0.075)^2}{4} = 4.42 \times 10^{-3}\ m^2$$

$$V_1 = \frac{Q}{A_1} = \frac{0.02}{7.854 \times 10^{-3}} = 2.55\ m/s \qquad V_2 = \frac{Q}{A_2} = \frac{0.02}{4.42 \times 10^{-3}} = 4.52\ m/s$$

$\rho = 0.657(1\,000)$ for hexane

$$-\frac{\partial W}{\partial t} = \left\{\left(\frac{p}{\rho} + \frac{V^2}{2g_c} + \frac{gz}{g_c}\right)\Big|_2 - \left(\frac{p}{\rho} + \frac{V^2}{2g_c} + \frac{gz}{g_c}\right)\Big|_1\right\}\rho VA$$

$$-\frac{\partial W}{\partial t} = \left\{\frac{190\,000}{657} + \frac{4.52^2}{2} + 1.5(9.81) - \left(\frac{-4\,000}{657} + \frac{2.55^2}{2} + 1.0(9.81)\right)\right\} 657(0.02)$$

$$-\frac{\partial W}{\partial t} = \left\{289.2 + 10.22 + 14.72 + 6.088 - 3.25 - 9.81\right\}(13.14)$$

$$-\frac{\partial W}{\partial t} = 4.04 \times 10^3\ N{\cdot}m/s = 4.0\ kW$$

Bernoulli Equation

43. Figure 2.15 shows a venturi meter. Show that the Bernoulli and continuity equations when applied combine to become

$$Q = A_2 \sqrt{\frac{2g\Delta h}{1 - (D_2^4/D_1^4)}}$$

Hydrostatic equation for manometer; all measurements are from the centerline

$$p_1 - \frac{\rho_1 g x}{g_c} - \frac{\rho_1 g \Delta h}{g_c} = p_2 - \frac{\rho_1 g x}{g_c} - \frac{\rho_2 g \Delta h}{g_c} \quad \text{or} \quad p_1 - p_2 = -\frac{\rho_1 g \Delta h}{g_c}$$

$$\dot m_1 = \dot m_2 \quad \rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad \text{or} \quad A_1 V_1 = A_2 V_2$$

In terms of diameter, $\frac{\pi D_1^2}{4} V_1 = \frac{\pi D_2^2}{4} V_2 = Q$

Bernoulli Equation

$$\frac{p_1 g_c}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho_1 g} + \frac{V_2^2}{2g} + z_2 \qquad \text{With } z_1 = z_2,$$

$$\frac{(p_1 - p_2)g_c}{\rho_1 g} = \frac{1}{2g}(V_2^2 - V_1^2) \quad \text{Substitute for } V \text{ in terms of } Q$$

$$\frac{(p_1 - p_2)g_c}{\rho_1 g}(2g) = Q^2(1/A_2^2 - 1/A_1^2)$$

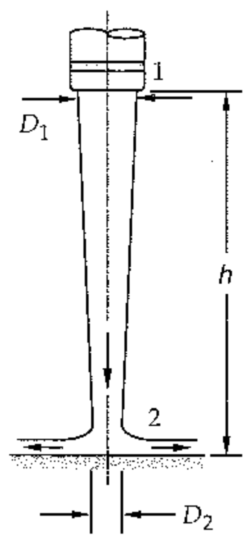
$$\frac{2\rho_1 g \Delta h g_c}{\rho_1 g_c} = \frac{Q^2}{A_2^2}(1 - \frac{A_2^2}{A_1^2}) = \frac{Q^2}{A_2^2}(1 - \frac{D_2^4}{D_1^4})$$

$$A_2\sqrt{2g\Delta h} = Q\sqrt{1 - D_2^4/D_1^4} \qquad \text{or finally,}$$

$$\boxed{Q = A_2\sqrt{\frac{2g\Delta h}{1 - D_2^4/D_1^4}}}$$

---

44. A jet of water issues from a kitchen faucet and falls vertically downward at a flow rate of 1.5 fluid ounces per second. At the faucet, which is 14 inches above the sink bottom, the jet diameter is 5/8 in. Determine the diameter of the jet where it strikes the sink.



$$Q = (1.5 \text{ ounces/s})\frac{2.957 \times 10^{-5}}{2.831 \times 10^{-2}} = 1.567 \times 10^{-3} \text{ ft}^3/\text{s}$$

$$D_1 = \frac{5}{8}\frac{1}{12} = 5.208 \times 10^{-2} \text{ ft} \qquad A_1 = 2.13 \times 10^{-3} \text{ m}^2$$

$$V_1 = \frac{Q}{A_1} = 0.735 \text{ ft/s} \qquad h = z_1 = 14 \text{ in.}$$

Bernoulli Equation

$$\frac{p_1 g_c}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho_1 g} + \frac{V_2^2}{2g} + z_2$$

$$p_1 = p_2 \qquad z_1 = 14/12 = 1.167 \text{ ft} \qquad z_2 = 0$$

Substituting,

$$0 + \frac{(0.735)^2}{2(32.2)} + \frac{32.2}{32.2}1.167 = 0 + \frac{V_2^2}{2(32.2)} + 0$$

which becomes

$(8.37 \times 10^{-3} + 1.167)(2(32.2)) = V_2^2$ or

$V_2 = 8.7$ ft/s

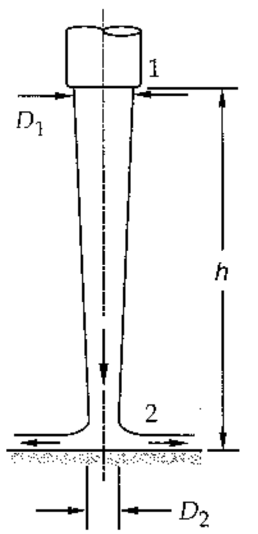
$$A_2 = \frac{Q}{V_2} = \frac{1.57 \times 10^{-3}}{8.7} = 1.8 \times 10^{-4} \text{ ft}^2$$

$$\frac{\pi D_2^2}{4} = 1.8 \times 10^{-4}; \quad D_2 = \sqrt{\frac{4}{\pi}(1.8 \times 10^{-4})} \quad \text{or}$$

$$\boxed{D_2 = 1.51 \times 10^{-2} \text{ ft} = 0.182 \text{ in.}}$$

---

45. A jet of water issues from a valve and falls vertically downward at a flow rate of 30 cm^3/s. The valve exit is 5 cm above the ground; the jet diameter at the ground is 5 mm. Determine the diameter of the jet at the valve exit.



Section 1 is the exit; section 2 is the ground.

$$\frac{p_1 g_c}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho_1 g} + \frac{V_2^2}{2g} + z_2$$

$Q = 30$ cm^3/s; $p_1 = p_2 = p_{atm}$; $z_2 = 0$; $z_1 = 0.05$ m

$$D_2 = 5 \text{ mm}; \; A_2 = \frac{\pi(0.005)^2}{4} = 1.963 \times 10^{-5} \text{ m}^2$$

$$V_2 = \frac{Q}{A_2} = \frac{30 \times 10^{-6}}{1.963 \times 10^{-5}} = 1.53 \text{ m/s};$$

Bernoulli Equation becomes

$$\frac{V_1^2}{2g} + z_1 = \frac{V_2^2}{2g}$$

$$\frac{V_1^2}{2(9.81)} = \frac{1.53^2}{2(9.81)} - 0.05 = 0.06931$$

$V_1^2 = 1.36$; $V_1 = 1.17$ m/s

$$Q = A_1 V_1 = \frac{\pi D_1^2}{4} V_1 \qquad D_1 = \sqrt{\frac{4Q}{\pi V_1}}$$

$$D_1 = \sqrt{\frac{4(30 \times 10^{-6})}{\pi(1.17)}} = 5.7 \times 10^{-3} \text{ m}$$

$$\boxed{D_1 = 5.7 \text{ mm}}$$

46. A garden hose is used as a siphon to drain a pool, as shown in Figure P2.46. The garden hose has a 3/4-in. inside diameter. Assuming no friction, calculate the flow rate of water through the hose if the hose is 25 ft long.

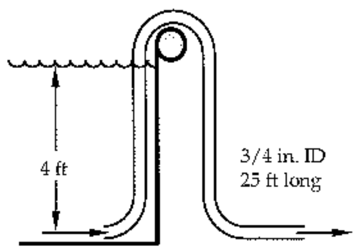


FIGURE P2.46.

Section 1 is the free surface; section 2 is the hose outlet.

$$\frac{p_1 g_c}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho_1 g} + \frac{V_2^2}{2g} + z_2 \qquad p_1 = p_2 = p_{atm}; \quad V_1 = 0; \; z_1 = 4 \text{ ft};$$

Substituting,

$$0 + 0 + \frac{32.2}{32.2}(4) = 0 + \frac{V_2^2}{2(32.2)} + 0$$

$$V_2 = \sqrt{2(32.2)(4)} = 16.05 \text{ ft/s}$$

$$D = 3/4 \text{ in.} = 0.0625 \text{ ft}; \qquad A = \frac{\pi D^2}{4} = 3.608 \times 10^{-3} \text{ ft}^2$$

$$Q = AV = 3.608 \times 10^{-3}(16.05);$$

$$\boxed{Q = 0.0492 \text{ ft}^3/\text{s}}$$

Miscellaneous Problems

47. A pump draws castor oil from a tank, as shown in Figure P2.47. A venturi meter with a throat diameter of 2 in. is located in the discharge line. For the conditions shown, calculate the expected reading on the manometer of the meter. Assume that frictional effects are negligible and that the pump delivers 0.25 HP to the liquid. If all that is available is a 6-ft tall manometer, can it be used in the configuration shown? If not, suggest an alternative way to measure pressure difference. (All measurements in inches.)

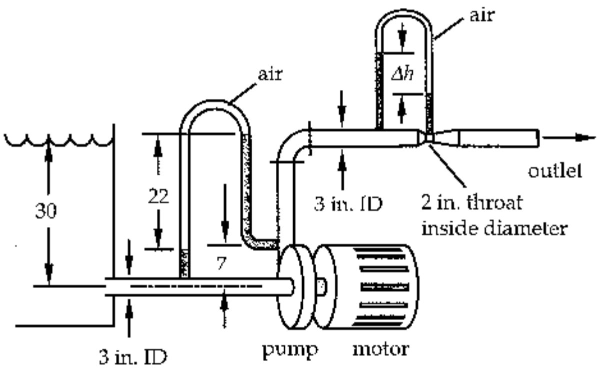


FIGURE P2.47.

$$p_1 - \frac{\rho g x}{g_c} - \frac{\rho_{air} g}{g_c}(22/12) + \frac{\rho g}{g_c}(22 + x - 7)/12 = p_2$$

ρ_{air} is negligible $\quad$ x terms cancel; $\quad$ $\rho = 0.960(1.94) = 1.862$ slug/ft^3

$$p_2 - p_1 = \frac{\rho g}{g_c}(15/12) = 1.862(32.2)(15/12) = 74.96 \text{ psf}$$

Energy equation, 1 to 2:

$$-\frac{\partial W}{\partial t} = \left\{\left(\frac{p}{\rho} + \frac{V^2}{2g_c} + \frac{gz}{g_c}\right)\Big|_2 - \left(\frac{p}{\rho} + \frac{V^2}{2g_c} + \frac{gz}{g_c}\right)\Big|_1\right\}\rho V A$$

$D_1 = D_2 = 3$ in. $\quad A_1V_1 = A_2V_2 \quad$ so $\quad V_1 = V_2$

$z_1 = 0 \quad z_2 = 7$ in $= 0.583$ ft $\quad \rho A V = \rho Q$

The power was given as

$$-\frac{\partial W}{\partial t} = 0.25(550) = 137.5 \text{ ft·lbf/s} \quad \text{Substituting,}$$

$$137.5 = \rho Q\left(\frac{(p_2 - p_1)}{\rho} + \frac{g z_2}{g_c}\right) = 1.862Q\left(\frac{74.96}{1.862} + 32.2(0.583)\right)$$

Solving for Q

$$Q = \frac{137.5}{109.9} = 1.251 \text{ ft}^3/\text{s}$$

Now for the venturi meter, the throat diameter is $D_{th} = 2/12 = 0.1667$ ft

$D = 3/12 = 0.25$ ft $\quad A_{th} = \frac{\pi D_{th}^2}{4} = 0.0218$ ft^2

$$Q = A_{th}\sqrt{\frac{2g\,\Delta h}{1 - D_{th}^4/D^4}}$$

$$1.251 = 0.0218\sqrt{\frac{2(32.2)\Delta h}{1 - (2/3)^4}} = 1.953\Delta h^{1/2}$$

$\boxed{\Delta h = 41.03 \text{ ft of castor oil}}$

A 6 ft tall air-over-oil manometer is not tall enough. A Hg manometer will work; pressure transducers will also work.

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48. A 4.2-cm ID pipe is used to drain a tank, as shown in Figure P2.48. Simultaneously, a 5.2-cm ID inlet line fills the tank. The velocity in the inlet line is 1.5 m/s. Determine the equilibrium height h of the liquid in the tank if it is octane. How does the height change if the liquid is ethyl alcohol? Assume in both cases that frictional effects are negligible, and that z is 4 cm.

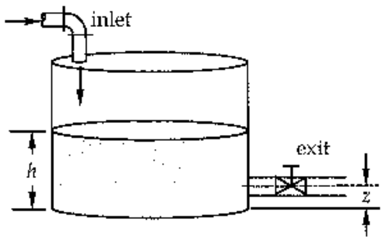

FIGURE P2.48.

$Q_{in} = AV$ $\qquad\qquad$ $A = \frac{\pi(0.052)^2}{4} = 2.124 \times 10^{-3}\text{ m}^2$

$Q_{in} = 2.124 \times 10^{-3}(1.5) = 3.19 \times 10^{-3}\text{ m}^3/\text{s}$

Section 1 is the free surface in the tank, and 2 is at the exit of the pipe. Apply the Bernoulli equation, 1 to 2:

$$\frac{p_1 g_c}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho_1 g} + \frac{V_2^2}{2g} + z_2$$

$p_1 = p_2 = p_{atm}$; $\quad V_1 = 0$; $z_1 = h$; $z_2 = 0.04$ m; the Bernoulli equation becomes

$h = \frac{V_2^2}{2g} + z_2$; $\qquad$ At equilibrium, $Q_{out} = Q_{in} = 3.19 \times 10^{-3}\text{ m}^3/\text{s}$

$A_{out} = \frac{\pi(0.042)^2}{4} = 1.39 \times 10^{-3}\text{ m}^2$; $\qquad$ and $\qquad V_2 = \frac{Q}{A_{out}} = \frac{3.19 \times 10^{-3}}{1.39 \times 10^{-3}} = 2.3\text{ m/s}$

$$h = \frac{V_2^2}{2g} + z_2 = \frac{2.3^2}{2(9.81)} + 0.04$$

$\boxed{h = 0.309\text{ m}}$ which is independent of fluid properties, and with no friction

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Computer Problems

49. One of the examples in this chapter dealt with the following impact problem, with the result that the ratio of forces is given by:

$$\frac{F_x}{F_y} = \frac{(\cos\theta_1 - \cos\theta_2)}{(\sin\theta_2 + \sin\theta_1)}$$

For an angle of $\theta_1 = 0$, produce a graph of the force ratio as a function of the angle θ_2.

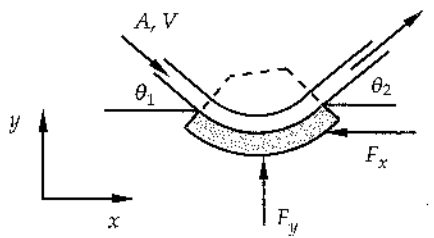
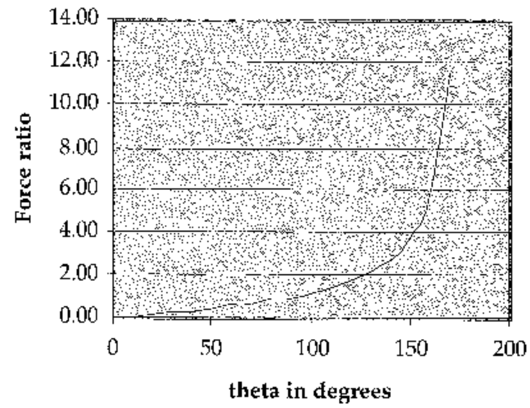


FIGURE P2.49



50. One of the examples in this chapter involved calculations made to determine the power output of a turbine in a dam (see Figure P2.50). When the flow through the turbine was 50,000 gpm, and the upstream height is 120 ft, the power was found to be 1427 hp. The relationship between the flow through the turbine and the upstream height is linear. Calculate the work done by (or power received from) the water as it flows through the dam for upstream heights that range from 60 to 120 ft.

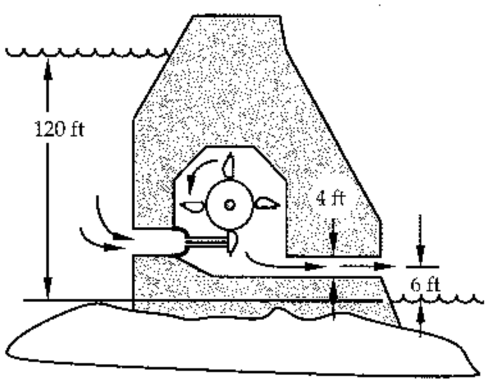


FIGURE P2.50

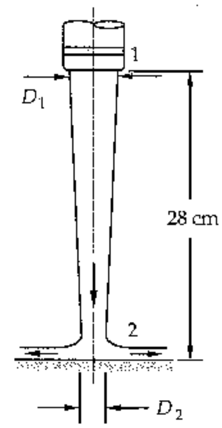
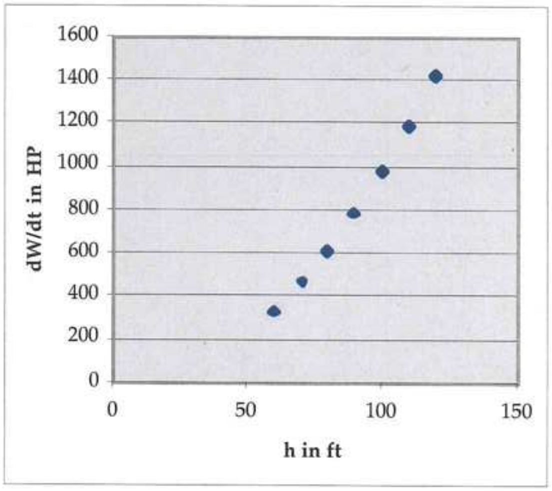
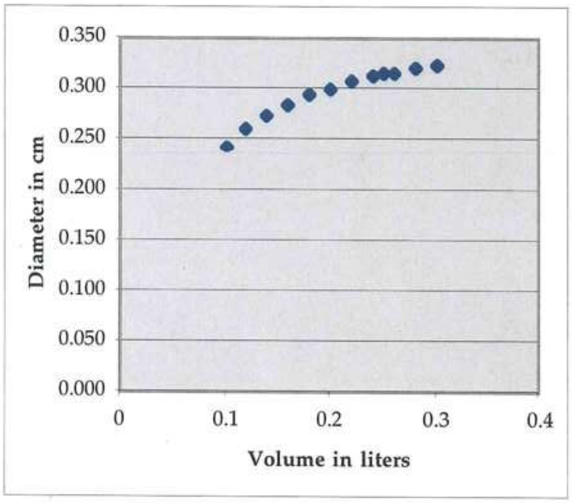


FIGURE P2.51





51. One of the examples in this chapter dealt with a water jet issuing from a faucet. The water flow rate was 250 ml per 8 seconds, the jet diameter at faucet exit is 0.35 cm, and the faucet is 28 cm above the sink. Calculations were made to find the jet diameter at impact on the sink surface. Repeat the calculations for volumes per time that range from 0.1 liters/8 seconds to 0.5 liters/8 seconds, and graph jet diameter at 2 as a function of the volume flow rate.