Eliminating the Impossible:

Solving a Game When Rationality Is Common Knowledge

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1. Derive the strategic form of the Mugging game in Figure 2.9 of Chapter 2 (page 30), and determine whether any strategies are either strictly dominated or weakly dominated.

ANSWER: The strategic form game is shown in the figure below. As the mugger has one information set (the initial node) and three actions, he has three strategies. Simon has two information sets; one is associated with the mugger showing a gun (which is a singleton) and one with the mugger not showing a gun (which comprises two nodes; one in which the mugger has a gun and one in which he does not). As Simon has two feasible actions at each information set, he has four feasible strategies. Let a strategy be represented by *x*/*y*, where Simon chooses action *x* at the information set in which a gun is shown and action *y* when a gun is not shown. *x* and *y* can either be *resist* (R) or *do not resist* (DNR).

	Simon				
		R/R	*R/DNR*	*DNR/R*	*DNR/DNR*
Mugger	*Use gun and show*	3,2	3,2	4,5	4,5
	Use gun and hide	3,2	5,4	3,2	5,4
	Do not use gun	2,6	6,3	2,6	6,3

For the mugger, none of his strategies is either strictly or weakly dominated. For Simon, none of his strategies is strictly dominated, but he does have some weakly dominated strategies: DNR/R weakly dominates R/R, and DNR/DNR weakly dominates R/DNR. If the mugger chooses to use a gun and show it, then Simon should definitely choose DNR. Weak dominance eliminates those strategies, which has him resist when he sees a gun.

2. In the Dr. Seuss story "The Zax," a North-Going Zax and a South-Going Zax on their treks soon find themselves facing each other. Each Zax must decide whether to continue in their current direction or move to the side so that the other may pass. As the story reveals, neither of them moves and that stalemate perpetuates for many years. Write down a strategic form game of this situation.

ANSWER: Each Zax has two strategies: *remain* (in its present position) and *move* (to the side). For each, *remain* is a dominant strategy, so one possible strategic form is that shown in the figure below.

	South-Going Zax		
		Remain	*Move*
North-Going Zax	*Remain*	1,1	3,0
	Move	0,3	2,2

3. For the Team-project game, suppose a jock is matched up with a sorority girl, as shown.

Sorority girl

Jock		*Low*	*Moderate*	*High*
	Low	3,0	4,1	5,2
	Moderate	2,2	3,4	4,3
	High	1,6	2,5	3,4

a. Assume that both are rational and that the jock knows that the sorority girl is rational. What happens?

ANSWER: For the jock, both *low* and *moderate* strictly dominate *high*, and *low* strictly dominates *moderate*. None of the sorority girl's strategies is strictly dominated, however. After eliminating the strictly dominated strategies, the reduced game is as shown in the figure below. As we don't know what the sorority girl believes about the jock, we cannot go any further. The answer is then that the jock chooses *low* and the sorority girl chooses *low*, *moderate*, or *high*.

Sorority girl

Jock		*Low*	*Moderate*	*High*
	Low	3,0	4,1	5,2

b. Assume that both are rational and that the sorority girl knows that the jock is rational. What happens?

ANSWER: With the game shown in the figure above, the sorority girl now knows the jock is rational and thus will play *low*. Hence, she should choose *high* as it strictly dominates both *low* and *moderate*. Hence, the jock chooses low effort and the sorority girl chooses high effort.

4. Consider the strategic form game shown.

Player 2

Player 1		*x*	*y*	*z*
	a	1,3	1,1	0,2
	b	3,1	2,2	1,0
	c	0,2	1,2	3,0

a. Assume that both players are rational. What happens?

ANSWER: For player 1, a is strictly dominated by b. Neither b nor c is strictly dominated. For player 2, z is strictly dominated by x. Player 1 plays either b or c and player 2 plays either x or y.

b. Assume that both players are rational and that each believes that the other is rational. What happens?

ANSWER: By the assumption, we can go two rounds of the iterative deletion of strictly dominated strategies (IDSDS). After eliminating the strictly dominated strategies, the game is as shown in the figure below. Now b strictly dominates c for player 1. Neither of player 2's strategies is strictly dominated. Thus, player 1 chooses b and player 2 chooses either x or y.

Player 2

Player 1		*x*	*y*
	b	3,1	2,2
	c	0,2	1,2

c. Find the strategies that survive the IDSDS.

ANSWER: After the first two rounds, the game is as shown in the figure below. Now y strictly dominates x for player 2. Thus, player 1 chooses b and player 2 chooses y.

Player 2

Player 1		x	y
	b	3,1	2,2

5. For the strategic form game shown, derive the strategies that survive the IDSDS.

Player 2

Player 1		x	y	z
	a	5,2	3,4	2,1
	b	4,4	3,2	3,3
	c	3,5	4,4	0,4
	d	2,3	1,5	3,0

ANSWER: For player 1, no strategy is strictly dominated. Between a and b, a is better when player 2 uses x, while b is better when player 2 uses z. Hence, a does not strictly dominate b, and b does not strictly dominate a. With a and c, a is better when player 2 uses x, while c is better when player 2 uses y. With a and d, a is better when player 2 uses x, while d is better when player 2 uses z. With b and c, b is better when player 2 uses x, while c is better when player 2 uses y. With b and d, b is better when player 2 uses x, while d is just as good when player 2 uses z. (Note that b weakly dominates d but does not strictly dominate it.) Finally, with c and d, c is better when player 2 uses x, while d is better when player 2 uses z. Hence, none of player 1's strategies is strictly dominated.

Turning to player 2, x strictly dominates z as it yields a strictly higher payoff for all strategies of player 1. We can then eliminate z. With x and y, x is better when player 1 uses b, but y is better when player 1 uses a. After the first round of the iterative deletion of strictly dominated strategies (IDSDS), we are left with $\{a,b,c,d\}$ and $\{x,y\}$. The remaining game is then as shown in the figure below.

Player 2

Player 1		x	y
	a	5,2	3,4
	b	4,4	3,2
	c	3,5	4,4
	d	2,3	1,5

Now d is strictly dominated by a (as well as by b and c). One can show that no other strategies are strictly dominated. The surviving game is then as shown in the figure below.

Player 2

Player 1		x	y
	a	5,2	3,4
	b	4,4	3,2
	c	3,5	4,4

No further strategies can be eliminated. All we can conclude is that player 1 will play either a, b, or c and player 2 will play either x or y.

6. Two Celtic clans—the Garbh Clan and the Conchubhair Clan—are set to battle. (Pronounce them as you'd like; I don't speak Gaelic.) According to tradition, the leader of each clan selects one warrior and the two warriors chosen engage in a fight to the death, the winner determining which will be the dominant clan. The three top warriors for Garbh are Bevan (which is Gaelic for "youthful warrior"), Cathal (strong in battle), and Duer (heroic). For Conchubhair, it is Fagan (fiery one), Guy (sensible), and Neal (champion). The leaders of the two clans know the following information about their warriors, and each knows that the other leader knows it, and furthermore, each leader knows that the other leader knows that the other leader knows it, and so forth (in other words, the game is common knowledge): Bevan is superior to Cathal against Guy and Neal, but Cathal is superior to Bevan against Fagan. Cathal is superior to Duer against Fagan, Guy, and Neale. Against Bevan, Guy is best. Against Cathal, Neal is best. Against Duer, Fagan is best. Against Bevan, Fagan is better than Neal. Against Cathal, Guy is better than Fagan. Against Duer, Guy and Neal are comparable. Assuming that each leader cares only about winning the battle, what can you say about who will be chosen to fight?

ANSWER: Let us use the iterative deletion of strictly dominated strategies to determine who will battle. Since Cathal is superior to Duer against all three of the warriors from the Conchubhair Clan, we can then eliminate Duer. Next, note that Bevan is best against Guy and Neal (since Bevan is superior to Cathal and Cathal is superior to Duer) and Cathal is best against Fagan. Therefore, neither Bevan nor Cathal is strictly dominated. Thus, in the first round, we can delete only Duer from the Garbh Clan. Since each of the warriors in the Conchubhair Clan is best against someone, none of them is strictly dominated. After one round of the IDSDS, we are left with Bevan and Cathal for the Garbh Clan and Fagan, Guy, and Neal for the Conchubhair Clan. Against Bevan and Cathal, Guy is superior to Fagan, so Fagan can be eliminated. Since Guy is better than Neal against Bevan and Neal is better than Guy against Cathal, neither Guy nor Neal can be eliminated. Note that we cannot eliminate anyone from the Garbh Clan in round 2 because the Conchubhair Clan has the same three warriors as in round 1. At the end of the second round, we are left with Bevan and Cathal for the Garbh Clan and Guy and Neal for the Conchubhair Clan. Against Guy and Neal, Bevan is superior to Cathal, so we can delete Cathal. Since the warriors for the Garbh Clan in round 3 are the same as in round 2, we cannot eliminate anyone from the Conchubhair Clan. After three rounds, we are left with Bevan for the Garbh Clan and Guy and Neal for the Conchubhair Clan. Since Guy is superior to Neal against Bevan, Neal is eliminated. We conclude that the leader of the Garbh Clan will choose Bevan and the leader of the Conchubhair Clan will select Guy. As an alternative approach to finding this pair, we could have used the information given to construct a strategic form game. One such game, which is compatible with the information given, is shown in the figure below.

Conchubhair Clan

Garbh Clan		*Fagan*	*Guy*	*Neal*
	Bevan	2,1	1,2	2,0
	Cathal	3,0	0,1	1,2
	Duer	1,2	−1,0	0,0

7. Consider the two-player strategic form game depicted.

Player 2

Player 1		*w*	*x*	*y*	*z*
	a	1,2	0,5	2,2	4,0
	b	1,3	5,2	5,3	2,0
	c	2,3	4,0	3,3	6,2
	d	3,4	2,1	4,0	7,5

a. Derive the strategies that survive the IDSDS.

ANSWER: Examining player 1's strategies, first note that d is optimal for player 1 when player 2 is expected to use w. Thus, d cannot be strictly dominated since to be strictly dominated requires that there is another strategy that yields a higher payoff for all strategies of the other player. Since b is best for player 1 when 2 uses x, then b is not strictly dominated either. c is not strictly dominated since it yields a higher payoff than a and b when player 2 uses w and a higher payoff than d when player 2 uses x. a is strictly dominated by c (and also by d) in that c yields a higher payoff than a for any strategy of player 2. We then find that the set of strategies for player 1 that survive the first round of the iterative deletion of strictly dominated strategies (IDSDS) is $\{b,c,d\}$.

Turning to player 2's strategies, x is best for player 2 when player 1 uses a, w and y are both optimal when player 1 uses b, and z is best when player 1 uses d. Thus, none of player 2's strategies is strictly dominated. After one round of IDSDS, the game is as shown in the figure below.

Player 2

Player 1		w	x	y	z
	b	1,3	5,2	5,3	2,0
	c	2,3	4,0	3,3	6,2
	d	3,4	2,1	4,0	7,5

Since we failed to eliminate any of player 2's strategies in the first round, we are unable to eliminate any of player 1's strategies in the second round (if you are unconvinced by this statement, check for yourself). Turning to player 2, w and y are best when player 1 uses b, and z is best when player 1 uses d. Thus, w, y, and z are not strictly dominated. However, x is strictly dominated by w.

After two rounds of IDSDS, the game is as shown in the figure below.

Player 2

Player 1		w	y	z
	b	1,3	5,3	2,0
	c	2,3	3,3	6,2
	d	3,4	4,0	7,5

For player 1, d is best when player 2 uses z and b is best when player 2 uses y. However, c is strictly dominated by d. Since none of player 1's strategies was eliminated in the second round, none of player 2's strategies can be eliminated in the third round.

After three rounds of IDSDS, the game is as shown in the figure below.

Player 2

Player 1		w	y	z
	b	1,3	5,3	2,0
	d	3,4	4,0	7,5

Since none of player 2's strategies was eliminated in the third round, none of player 1's strategies can be eliminated in the fourth round. Since w and y are both optimal when player 1 uses b, and z is optimal when player 1 uses d, then we cannot eliminate any of player 2's strategies. Given that no strategies are eliminated in this round, no strategies can be eliminated in any further rounds. We conclude that $\{b,d\}$ for player 1 and $\{w,y,z\}$ for player 2 are the strategies that survive the IDSDS.

b. Derive the strategies that survive the iterative deletion of weakly dominated strategies. (The procedure works the same as the IDSDS, except that you eliminate all *weakly* dominated strategies at each stage.)

ANSWER: Examining player 1's strategies, first note that *d* is the unique optimal strategy for player 1 when player 2 is expected to use *w*. Thus, *d* cannot be weakly dominated since to be weakly dominated requires that there is another strategy that yields at least as high a payoff for all strategies of the other player and a strictly higher payoff for some strategies of the other player. Since *b* is the unique optimal strategy for player 1 when player 2 uses *x*, then *b* is not weakly dominated either. *c* is not weakly dominated since it yields a strictly higher payoff than *a* and *b* when player 2 uses *w* and a strictly higher payoff than *d* when player 2 uses *x*. *a* is weakly (and strictly) dominated by *c* (and also by *d*). We then find that the set of strategies for player 1 which survive the first round of the iterative deletion of weakly dominated strategies (IDSDS) is {*b*,*c*,*d*}.

Turning to player 2's strategies, *x* is the unique optimal strategy for player 2 when player 1 uses *a*, and *z* is the unique optimal strategy when player 1 uses *d*. *w* weakly dominates *y* since it yields an identical payoff when player 1 uses *a*, *b*, or *c* and a strictly higher payoff when player 1 uses *d*. *w* is not weakly dominated by *x* since it yields a strictly higher payoff when player 1 uses *b*, and it is not weakly dominated by *z* since it yields a strictly higher payoff when player 1 uses *a*. Therefore, the set of strategies for player 1 that survive the first round of the IDSDS is {*w*,*x*,*z*}.

After one round of IDSDS, the game is as shown in the figure below.

Player 2

Player 1	w	x	z
b	1,3	5,2	2,0
c	2,3	4,0	6,2
d	3,4	2,1	7,5

d is not weakly dominated since it is the unique optimal strategy when player 2 uses *w*, and *b* is not weakly dominated since it is the unique optimal strategy when player 2 uses *x*. *c* is not weakly dominated by *b* since it yields a strictly higher payoff when player 2 uses *w*, and it is not weakly dominated by *d* since it yields a strictly higher payoff when player 2 uses *x*. None of player 1's strategies is eliminated in the second round of IDSDS.

Turning to player 2, *w* is the unique optimal strategy when player 1 uses *b* and *z* is the unique optimal strategy when player 1 uses *d*. *x* is strictly and thus weakly dominated by *w*.

After two rounds of IDSDS, the game is as shown in the figure below.

Player 2

Player 1	w	z
b	1,3	2,0
c	2,3	6,2
d	3,4	7,5

For player 1, *d* strictly and therefore weakly dominates both *b* and *c*. Since none of player 1's strategies was eliminated in the second round, none of player 2's strategies can be eliminated in the third round.

After three rounds of IDSDS, the game is as shown in the figure below.

Player 2

Player 1	w	z
d	3,4	7,5

There is nothing left for player 1 to do. Since *z* yields a strictly higher payoff than *w* when player 1 uses *d*, then *z* strictly and therefore weakly dominates *w*.

8. Consider the three-player game shown. Player 1 selects a row, either a_1, b_1 or c_1. Player 2 selects a column, either a_2 or b_2. Player 3 selects a matrix, either a_3 or b_3. The first number in a cell is player 1's payoff, the second number is player 2's payoff, and the last number is player 3's payoff. Derive the strategies that survive the IDSDS.

a_3

	a_2	b_2
a_1	3,1,0	2,3,1
b_1	0,3,1	1,1,0
c_1	1,0,2	1,2,1

b_3

	a_2	b_2
a_1	3,1,1	1,3,2
b_1	2,0,2	2,2,1
c_1	1,1,1	0,2,0

ANSWER: To begin, consider player 1. Neither a_1 nor b_1 is strictly dominated, as a_1 yields the highest payoff for player 1 when players 2 and 3 choose (a_2,a_3), while b_1 is best when players 2 and 3 choose (b_2,b_3). However, a_1 strictly dominates c_1. Thus, the surviving strategies for player 1 are a_1 and b_1. Turning to player 2, neither of her strategies is strictly dominated since a_2 is best when players 1 and 3 choose (b_1,a_3) and b_2 is best when players 1 and 3 choose (a_1, a_3). Finally, neither of player 3's strategies is strictly dominated as a_3 is best when players 1 and 2 choose (c_1,a_2) and b_3 is best when players 1 and 2 choose (a_1,a_2). After the first round, the reduced game is as shown in the figure below.

a_3

	a_2	b_2
a_1	3,1,0	2,3,1
b_1	0,3,1	1,1,0

b_3

	a_2	b_2
a_1	3,1,1	1,3,2
b_1	2,0,2	2,2,1

Since no strategies of players 2 and 3 were eliminated in the first round, no strategies of player 1 can be eliminated in the second round. Neither of player 2's strategies is strictly dominated, as a_2 is best when players 1 and 3 choose (b_1, a_3) and b_2 is best when players 1 and 3 choose (a_1, a_3). For player 3, b_3 strictly dominates a_3. After the second round, the reduced game is as shown in the figure below.

b_3

	a_2	b_2
a_1	3,1,1	1,3,2
b_1	2,0,2	2,2,1

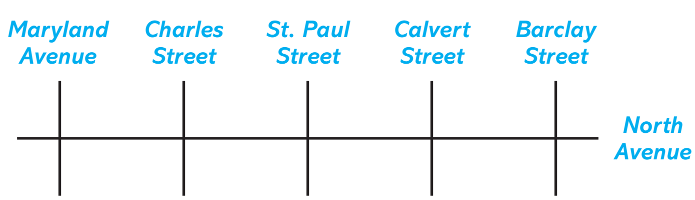
In round 3, neither of player 1's strategies is strictly dominated, but for player 2, b_2 strictly dominates a_2. After the third round, the reduced game is as shown in the figure below.

b_3

	b_2
a_1	1,3,2
b_1	2,2,1

In the fourth round, b_1 strictly dominates a_1 and this solves the game since each player has only one strategy remaining. Thus, by the iterative deletion of strictly dominated strategies, we conclude that the strategy profile that will be played is (b_1,b_2,b_3).

9. A gang controls the drug trade along North Avenue between Maryland Avenue and Barclay Street. The city grid is shown below.



The gang leader sets the price of the drug being sold and assigns two gang members to place themselves along North Avenue. He tells each of them that they'll be paid 20% of the money they collect. The only decision that each of the drug dealers has is whether to locate at the corner of North Avenue and either Maryland Avenue, Charles Street, St. Paul Street, Calvert Street, or Barclay Street. The strategy set of each drug dealer is then composed of the latter five streets. Since the price is fixed by the leader and the gang members care only about money, each member wants to locate so as to maximize the number of units he sells.

For simplicity, assume that the five streets are equidistant from each other. Drug customers live only along North Avenue and are evenly distributed between Maryland Avenue and Barclay Street (so no customers live to the left of Maryland Avenue or to the right of Barclay Street). Customers know that the two dealers set the same price, so they buy from the dealer that is closest to them. The total number of units sold on North Avenue is fixed. The only issue is whether a customer buys from drug dealer 1 or drug dealer 2. This means that a drug dealer will want to locate so as to maximize his share of customers. We can then think about a drug dealer's payoff as being his customer share. The figure below shows the customer shares or payoffs.

Drug Dealers' Payoffs Based on Location

Dealer 2's location

Dealer 1's location	Maryland	Charles	St. Paul	Calvert	Barclay
Maryland	$\frac{1}{2},\frac{1}{2}$	$\frac{1}{8},\frac{7}{8}$	$\frac{1}{4},\frac{3}{4}$	$\frac{3}{8},\frac{5}{8}$	$\frac{1}{2},\frac{1}{2}$
Charles	$\frac{7}{8},\frac{1}{8}$	$\frac{1}{2},\frac{1}{2}$	$\frac{3}{8},\frac{5}{8}$	$\frac{1}{2},\frac{1}{2}$	$\frac{5}{8},\frac{3}{8}$
St. Paul	$\frac{3}{4},\frac{1}{4}$	$\frac{5}{8},\frac{3}{8}$	$\frac{1}{2},\frac{1}{2}$	$\frac{5}{8},\frac{3}{8}$	$\frac{3}{4},\frac{1}{4}$
Calvert	$\frac{5}{8},\frac{3}{8}$	$\frac{1}{2},\frac{1}{2}$	$\frac{3}{8},\frac{5}{8}$	$\frac{1}{2},\frac{1}{2}$	$\frac{7}{8},\frac{1}{8}$
Barclay	$\frac{1}{2},\frac{1}{2}$	$\frac{3}{8},\frac{5}{8}$	$\frac{1}{4},\frac{3}{4}$	$\frac{1}{8},\frac{7}{8}$	$\frac{1}{2},\frac{1}{2}$

Let us go through a few so that you understand how they were derived. For example, suppose dealer 1 locates at the corner of Maryland and North and dealer 2 parks his wares at the corner of St. Paul and North. All customers who live between Maryland and Charles buy from dealer 1, as he is the closest to them, while the customers who live to the right of Charles buy from dealer 2. Hence, dealer 1 gets 25% of the market and dealer 2 gets 75%. Thus, we see that $(\frac{1}{4}, \frac{3}{4})$ are the payoffs for strategy pair (Maryland, St. Paul). Now, suppose instead that dealer 2 locates at Charles and dealer 1 at Maryland. The customer who lies exactly between Maryland and Charles will be indifferent as to whom to buy from. All those customers to his left will prefer the dealer at Maryland, and they make up one-eighth of the street. Thus, the payoffs are $(\frac{1}{8}, \frac{7}{8})$ for the strategy pair (Maryland, Charles). If two dealers locate at the same street corner, we'll suppose that customers divide themselves equally between the two dealers, so the payoffs are $(\frac{1}{2}, \frac{1}{2})$. Using the IDSDS, find where the drug dealers locate.

ANSWER: Consider what locations are optimal for dealer 1 for some location of dealer 2. If dealer 2 chooses to locate at Maryland, then dealer 1's optimal location is at Charles. At this location, he picks up all of the customers to the right of Charles and half of those between Maryland and Charles, which means getting an impressive market share of $\frac{7}{8}$. Thus, Charles cannot be strictly dominated as it is the best location for some location of dealer 2. If instead dealer 2 locates at Charles, then dealer 1's optimal location is to park it just to the right of dealer 2 at St. Paul and pick up $\frac{5}{8}$ of the market (all those customers to the right of St. Paul and those to the left of St. Paul up to the mid-point between Charles and St. Paul). Thus, St. Paul is not strictly dominated either. If dealer 2 locates in the middle of North Avenue at St. Paul, then dealer 1 wants to do the same. Hence, St. Paul is not strictly dominated. If dealer 2 locates at Calvert, then dealer 1 wants to locate just to his left at St. Paul, which we already know is not dominated. Finally, if dealer 2 locates at Barclay, then dealer 1 sets up shop at Calvert to grab $\frac{7}{8}$ of the market. Thus, locating at Calvert is not strictly dominated. In sum, Charles, St. Paul, and Calvert are not strictly dominated locations because each is optimal for some location of dealer 2 (and thus there is not any other location that produces a strictly higher payoff for every location of dealer 2). This leaves only Maryland and Barclay as candidates for elimination. In fact, Maryland is strictly dominated by Charles, while Barclay is strictly dominated by Calvert. It is then strictly dominated for dealer 1 to locate at either end of North Avenue. As the game is symmetric, the same answer is derived for dealer 2; the only strictly dominated locations are Maryland and Barclay. After one round of the IDSDS, the reduced game is as shown in the figure below.

Dealer 2's location

Dealers 1's location		Charles	St. Paul	Calvert
	Charles	$\frac{1}{2}, \frac{1}{2}$	$\frac{3}{8}, \frac{5}{8}$	$\frac{1}{2}, \frac{1}{2}$
	St. Paul	$\frac{5}{8}, \frac{3}{8}$	$\frac{1}{2}, \frac{1}{2}$	$\frac{5}{8}, \frac{3}{8}$
	Calvert	$\frac{1}{2}, \frac{1}{2}$	$\frac{3}{8}, \frac{5}{8}$	$\frac{1}{2}, \frac{1}{2}$

Inspection reveals that St. Paul dominates both Charles and Calvert for dealer 1, and similarly for dealer 2. Thus, after two rounds of the IDSDS, we are left with a single strategy for each dealer, which is to locate at St. Paul. If gang members are rational and each gang member believes the other is rational, then they'll both locate at St. Paul and each sell to half of the customers there.

10. Two students are to take an exam, and the professor has instructed them that the student with the higher score will receive a grade of A and the one with the lower score will receive a B. Student 1's score equals $x_1 + 1.5$, where x_1 is the amount of effort she invests in studying. (That is, I assume that the greater the effort, the higher is the score.) Student 2's score equals x_2, where x_2 is the amount of effort he exerts. It is implicitly assumed that student 1 is the smarter of the two, in that, if the amount of effort is held fixed, student 1 has a higher score by an amount of 1.5. Assume that x_1 and x_2 can take any value in {0,1,2,3,4,5}. The payoff to student i is $10 - x_i$ if she gets an A, and $8 - x_i$ if she gets a B, $i = 1, 2$.

a. Derive the strategies that survive the IDSDS.

ANSWER: Let us first show that for either player, zero effort strictly dominates effort levels of 3, 4, and 5. If $x_i = 0$ then i's payoff is at least 8, which occurs when she gets a B. By choosing effort x_i, the highest possible payoff is $10 - x_i$, which occurs when she gets an A. Since $8 > 10 - x_i$ when $x_i > 2$, then zero effort strictly dominates effort of 3, 4, or 5. Now consider player 1. We know that strategies 3, 4, and 5 are strictly dominated, but what about strategies 0, 1, and 2? A useful point to note is that if there is some strategy for player 2 such that a particular strategy for player 1 is best (that is, it yields the highest payoff for player 1), then that strategy for 1 is not strictly dominated since strict dominance means there is another strategy that yields a strictly higher payoff for all strategies of the other player. In considering strategy 0, note that $x_1 = 0$ is the uniquely best strategy for player 1 when x_2 is 0 or 1, as in both cases player 1 receives a payoff of 10, whereas when $x_1 > 0$ his payoff is $10 - x_1$, which is lower. Thus, there is no strategy that strictly dominates zero effort. A similar argument shows that $x_1 = 1$ is not strictly dominated. If $x_2 = 2$, then $x_1 = 1$ yields a payoff of 9, while $x_1 = 0$ yields a payoff of 8 (as player 2's score is higher in that case) and the payoff is $10 - x_1$ when $x_1 > 1$, which is lower than 9. Finally, note that $x_1 = 2$ is optimal when $x_2 = 3$, as the resulting payoff is 8, whereas the payoff is 8 from $x_1 = 0$, 7 from $x_1 = 1$, and $10 - x_1$ when $x_1 > 2$. Though $x_1 = 2$ generates the same payoff as $x_1 = 0$, we can still conclude that there is no strategy that strictly dominates $x_1 = 2$. It is concluded that the strategies for player 1 that survive the first round of deletion of strictly dominated strategies are {0,1,2}. Now consider player 2. $x_2 = 0$ is not strictly dominated, as it is the optimal strategy for player 2 when $x_1 \geq 4$, as in that case player 1 gets a B regardless of x_2. Since his payoff is then $8 - x_2$, it is clearly maximized at $x_2 = 0$. Now consider $x_2 = 1$. I want to show that this strategy is strictly dominated by $x_2 = 0$. Note that regardless of x_1, player 2 has the lower score whether he chooses $x_2 = 0$ or $x_2 = 1$. Since $x_2 = 0$ gives a payoff of 8 and $x_2 = 1$ gives a payoff of 7, zero effort strictly dominates effort of player 1. Now consider the remaining strategy of $x_2 = 2$. Suppose $x_1 = 0$. In that case, the payoff to player 2 from $x_2 \leq 1$ is $8 - x_2$ (since he gets a B) and from $x_2 \geq 2$ is $10 - x_2$ (since he gets an A). His payoff is maximized at $x_2 = 2$. Given there is some strategy for player 1 such that $x_2 = 2$ yields the maximum payoff, then $x_2 = 2$ cannot be strictly dominated. It is concluded that the strategies for player 2 that survive the first round of the IDSDS are {0,2}. Now move to round 2 of this iterative process. The payoff matrix in the figure below shows the surviving strategies. The first number in a cell is player 1's payoff.

		x_2: 0	2
x_1	0	10,8	8,8
	1	9,8	9,6
	2	8,8	8,6

$x_1 = 1$ strictly dominates $x_1 = 2$. Neither of the other two strategies for player 1 is dominated. None of player 2's strategies is strictly dominated since $x_2 = 0$ yields a higher payoff than does $x_2 = 2$ when $x_1 = 1$ or 2 and $x_2 = 2$ yields the same payoff as does $x_2 = 0$ when $x_1 = 0$. It is concluded that the strategies that survive the second round of the IDSDS for player 1 are {0,1} and for player 2 are {0,2}. Moving to the third round, the resulting payoff matrix is shown in the figure below.

		x_2: 0	2
x_1	0	10,8	8,8
	1	9,8	9,6

Inspection reveals that no strategies are strictly dominated. Thus, the IDSDS predicts that player 1 will exert effort of 0 or 1 and player 2 will exert effort of 0 or 2.

b. Derive the strategies that survive the iterative deletion of weakly dominated strategies. (The procedure works the same as the iterative deletion of strictly dominated strategies, except that you eliminate all weakly dominated strategies at each stage.)

ANSWER: Now let us repeat the analysis when we are instead iteratively deleting weakly dominated strategies. From part (a), we know that effort of 3, 4, and 5 are all strictly dominated and therefore weakly dominated. For player 1, efforts of 0 and 1 are not weakly dominated for the same reasons given in part (a): there is a strategy for player 2 such that each yields the strictly highest payoff. While $x_1 = 2$ is not strictly dominated, it is weakly dominated by $x_1 = 0$. The latter yields a payoff of at least 8 regardless what player 2 does, while $x_1 = 2$ yields a payoff of at most 8. Furthermore, zero effort does strictly better when player 2 chooses effort of 0 or 1. The strategies for player 1 that survive the first round of the IDSDS are {0,1}. With respect to player 2, it was previously shown that strategies 1, 3, 4, and 5 are strictly dominated and, therefore, weakly dominated. Zero effort is not weakly dominated by the same argument as in part (a). $x_2 = 2$ is weakly dominated for reasons analogous to those given for player 1. The lone strategy for player 2 that survives the first round of the IDSDS is $x_1 = 0$.

Moving to the second round, $x_1 = 0$ weakly dominates $x_1 = 1$ since both result in player 1's getting an A (as player 2 is choosing 0 effort). So the former yields a payoff of 10 and the latter a payoff of 9. The IDSDS predicts that both players will exert zero effort.

11. Groucho Marx once said, "I'll never join any club that would have me for a member." Well, Groucho is not interested in joining your investment club, but Julie is. Your club has 10 members, and the procedure for admitting a new member is simple: Each person receives a ballot that has two options: (1) admit Julie and (2) do not admit Julie. Each person can check one of those two options or abstain by not submitting a ballot. For Julie to be admitted, she must receive at least six votes in favor of admittance. Letting m be the number of ballots submitted with option 1 checked, assume that your payoff function is

$$\begin{cases} 1 & \text{if } m = 6, 7, 8, 9, 10 \\ 0 & \text{if } m = 0, 1, 2, 3, 4, 5 \end{cases}$$

a. Prove that checking option 1 (admit Julie) is not a dominant strategy.

ANSWER: If all the other players check option 2, then Julie is not admitted regardless whether you check option 1, check option 2, or abstain. Since all three strategies yield the highest payoff in that case, none of them is strictly dominated and thus there is no dominant strategy.

b. Prove that abstaining is a weakly dominated strategy.

ANSWER: If at least six of the other members submit ballots with option 1 checked, then Julie is admitted regardless of what you do; your payoff is 1 with all strategies. If four or fewer of the other members vote in favor of option 1, then Julie is denied admittance regardless of what you do; your payoff is 0 with all strategies. This leaves only the case when five of the other members submit ballots in favor of admitting Julie. Abstaining results in a payoff of 0, as Julie ends up with only five supporting votes. Voting and checking option 1 results in her admittance and thus a payoff of 1. Hence, abstaining is weakly dominated by voting in favor of Julie.

c. Now suppose you're tired at the end of the day, so that it is costly for you to attend the evening's meeting to vote. By not showing up, you abstain from the vote. This is reflected in your payoff function having the form

$$\begin{cases} 1 & \text{if } m = 6, 7, 8, 9, 10 \text{ and you abstained} \\ \frac{1}{2} & \text{if } m = 6, 7, 8, 9, 10 \text{ and you voted} \\ 0 & \text{if } m = 0, 1, 2, 3, 4, 5 \text{ and you abstained} \\ -\frac{1}{2} & \text{if } m = 0, 1, 2, 3, 4, 5 \text{ and you voted} \end{cases}$$

Prove that abstaining is not a weakly dominated strategy.

ANSWER: If the other members vote so that $m \geq 6$, then the payoff from showing up and voting is $\frac{1}{2}$, while it's 1 from not showing up and thus abstaining. Since there are strategies for the other players whereby abstention is the unique optimal strategy, then abstention cannot be weakly dominated.

12. Derive all of the rationalizable strategies for the game shown.

Player 2

Player 1		*x*	*y*	*z*
	a	0,4	1,1	2,3
	b	1,1	2,2	0,0
	c	3,2	0,0	1,4

ANSWER: All strategies are rationalizable. The following cycle of beliefs serves to show strategies a and c for player 1 and x and z for player 2 are rationalizable. (1) Player 1 using c is optimal if player 1 believes player 2 will use x. Hence, player 1 using c is consistent with player 1 being rational. (2) x is optimal for player 2 if player 2 believes player 1 will use a. Hence, player 1 believing that player 2 will use x is consistent with player 1 believing that player 2 is rational. (This "rationalizes" player 1's belief in step 1 that player 2 will use *x*.) (3) *a* is optimal for player 1 if player 1 believes player 2 will use *z*. Hence, player 1 believing that player 2 believes that player 1 will use *a* is consistent with player 1 believing that player 2 believes that player 1 is rational. (4) *z* is optimal for player 2 if player 2 believes player 1 will use *c*. Hence, player 1 believing that player 2 believes that player 1 believes that player 2 will use *z* is consistent with player 1 believing that player 2 believes that player 1 believes that player 2 is rational. (5) *c* is optimal for player 1 if player 1 believes player 2 will use *x*. Hence, player 1 believing that player 2 believes that player 1 believes that player 2 believes that player 1 will use *c* is consistent with player 1 believing that player 2 believes that player 1 believes that player 2 believes that player 1 is rational. One can then use the argument in step 2 to rationalize player 1's belief about player 2 using *x*. We then have a cycle in which there is always another layer of beliefs that rationalizes the current layer. All the strategies in this cycle are then rationalizable, which means *a* and *c* for player 1 and *x* and *z* for player 2. This doesn't necessarily mean that *b* and *y* are not rationalizable, as there may be beliefs, consistent with rationality being common knowledge, that support them. In fact, there are—in which case it turns out that all strategies in this game are rationalizable.

Here is a cycle that rationalizes *b* and *y*. (1) Player 1 using *b* is optimal if player 1 believes player 2 will use *y*. Hence, player 1 using *b* is consistent with player 1 being rational. (2) *y* is optimal for player 2 if player 2 believes player 1 will use *b*. Hence, player 1 believing that player 2 will use *y* is consistent with player 1 believing that player 2 is rational. (3) *b* is optimal for player 1 if player 1 believes player 1 will use *y*. Hence, player 1 believing that player 2 believes that player 1 will use *b* is consistent with player 1 believing that player 2 believes that player 1 is rational. (4) *y* is optimal for player 2 if player 2 believes player 1 will use *b*. Hence, player 1 believing that player 2 believes that player 1 believes that player 2 will use *y* is consistent with player 1 believing that player 2 believes that player 1 believes that player 2 is rational. We have a cycle!

13. Consider the two-player game:

Player 2

Player 1		*x*	*y*	*z*
	a	5,1	4,2	0,1
	b	1,2	0,4	6,3
	c	2,3	1,2	2,1

a. Find the strategies that are consistent with both players being rational and each player believing the other player is rational.

ANSWER: Round 1: *y* strictly dominates *z* for player 2.

Player 2

Player 1		*x*	*y*
	a	5,1	4,2
	b	1,2	0,4
	c	2,3	1,2

Round 2: *a* strictly dominates *b* and *c* for player 1. Neither strategy strictly dominates for player 2.

1 plays *a* and 2 plays *x* or *y*. As 2 does not know whether 1 knows that 2 is rational, then 2 does not know whether 1 knows that 2 does not play *z*.

b. In addition to that assumed in part (a), assume that player 2 knows player 1 knows player 2 is rational. Find strategies consistent with these beliefs.

ANSWER: Round 3: y strictly dominates x for player 2.

Player 2

Player 1		x	y
	a	5,1	4,2

Note that the information available to player 1 has not changed from part (a), therefore, Player 1 still plays a. However, now player 2 knows player 1 knows player 2 is rational.

Therefore, player 2 knows player 1 knows that player 2 will not play z from which it follows that player 2 knows player 1 plays a (because a strictly dominates b and c in the absence of z). Player 2 then plays y. Hence, player 1 plays a and player 2 plays y.

14. Len and Melanie are deciding what to do Saturday night. The options are to see Mozart's opera *Don Giovanni* or go to the local arena to watch Ultimate Fighter. Len prefers Ultimate Fighter, while Melanie prefers *Don Giovanni*. As a possible compromise, a friend suggests that they attend "*Rocky: The Ballet,*" which is a newly produced ballet about Rocky Balboa, the down-and-out boxer from the streets of Philadelphia who gets a shot at the title. Each would like to go to their most preferred performance, but each also cares about attending with the other person. Also, Len may feel guilty about spending a lot of money for a ticket to Ultimate Fighter when Melanie is not with him; *Rocky: The Ballet* is cheaper. *Don Giovanni* and Ultimate Fighter are both expensive tickets, but Melanie would not feel guilty about attending her first choice alone and spending a lot of money. Both Len and Melanie are flying back into town Saturday afternoon and each must independently decide which to attend. The strategic form of the game is shown below. Using the IDSDS, what will they do?

Melanie

Len		*Don Giovanni*	Ultimate Fighter	*Rocky: The Ballet*
	Don Giovanni	1,5	0,0	0,2
	Ultimate Fighter	3,3	6,1	3,2
	Rocky: The Ballet	4,3	2,0	5,4

ANSWER: Round 1: Ultimate Fighter (UF) and Rocky: The Ballet (R) strictly dominate Don Giovanni (DG) for Len. For Melanie, DG and R strictly dominate UF. After eliminating these strategies, the game is now:

Melanie

Len		*Don Giovanni*	*Rocky: The Ballet*
	Ultimate Fighter	3,3	3,2
	Rocky: The Ballet	4,3	5,4

Round 2: R strictly dominates UF for Len, while nothing is strictly dominated for Melanie. The game is now:

Melanie

		Don Giovanni	*Rocky: The Ballet*
Len	*Rocky: The Ballet*	4,3	5,4

Round 3: R strictly dominates DG for Melanie. Thus, they both go to see *Rocky: The Ballet*.

15. A total of 10 players are each choosing a number from {0,1,2,3,4,5,6,7,8}. If a player's number equals exactly half of the average of the numbers submitted by the other nine players, then she is paid \$100; otherwise, she is paid 0. Solve for the strategies that survive the IDSDS.

ANSWER: Round 1: The average is no higher than 8 so half the average is no higher than 4. Thus, 4 weakly dominates any number above 4. Compare 4 to a number above 4, call it x. If the average is 8 then half of the average is 4 in which case 4 pays off while x does not pay off. If the average is less than 8 then neither 4 nor x pays off. Hence, 4 weakly dominates 5,6,7, and 8. Next note that 0,1,2,3, and 4 are not weakly dominated. If all others choose 8 then 4 is uniquely best. If all others choose 6 then 3 uniquely is best. If all others choose 4 then 2 is uniquely best. If all others choose 2 then 1 is uniquely best. If all others choose 0 then 0 is uniquely best.

Round 2: The remaining strategies are 0,1,2,3, and 4. By the same argument as used in Round 1, 2 weakly dominates 3 and 4. 2 is not weakly dominated because it is uniquely best when all others choose 4; 1 is not weakly dominated because it is uniquely best when all others choose 2; 0 is not weakly dominated because it is uniquely best when all others choose 0.

Round 3: The remaining strategies are 0,1, and 2. By the same argument as used in Round 1, 1 weakly dominates 2. 1 is not weakly dominated because it is uniquely best when all others choose 2; 0 is not weakly dominated because it is uniquely best when all others choose 0.

Round 4: The remaining strategies are 0 and 1. 0 weakly dominates 1. If all others choose 0 then, because half of 0 is 0, it pays off, and 1 does not. If anything else is chosen then the average lies between 0 and 1 in which case neither 0 nor 1 pays off.

16. Monica and Isabel are roommates who, on this particular Saturday morning, are trying to decide what scarf to wear. Each has a Burberry scarf (which we'll denote B), a tan scarf (denoted T), and a mauve scarf (denoted M). They care about the scarf but also about whether they end up wearing the same or different scarves. The preference ordering (from best to least preferred outcome) for Monica is: (1) she wears B and Isabel wears T or M; (2) she wears T and Isabel wears B or M; (3) she wears B and Isabel wears B; (4) she wears T and Isabel wears T: (5) she wears M and Isabel wears M; and (6) she wears M and Isabel wears B or T. Isabel's preference ordering is: (1) she wears T and Monica wears B or M; (2) she wears M and Monica wears B or T; (3) she wears T and Monica wears T; (4) she wears M and Monica wears M; (5) she wears B and Monica wears B; and (6) she wears B and Monica wears T or M. Applying the IDSDS, which scarves will be worn?

ANSWER: The preferences of Isabel and Monica can be represented in the following strategic form game:

		Isabel		
		B	*T*	*M*
Monica	*B*	3,1	5,5	5,4
	T	4,0	2,3	4,4
	M	0,0	0,5	1,2

Round 1: If Monica expects Isabel to wear the Burberry (denoted B) or mauve (denoted M) scarf then Monica wants to wear the tan scarf (denoted T). If Monica expects Isabel to wear B or M then Monica wants to wear B. Thus, neither B nor T are strictly dominated. However, M is strictly dominated by both B and T for Monica. If Isabel expects Monica to wear B or T then Isabel wears M. If Isabel expects Monica to wear B or M then she wears T. Thus, neither M nor T are strictly dominated for Isabel. However, B is strictly dominated by T and M for Isabel.

Round 2: After Round 1, B and T are the surviving strategies for Monica, and T and M are the surviving strategies for Isabel. Let us consider Monica. If Isabel wears T then Monica prefers B and if Isabel wears M then Monica prefers B. B is Monica's favorite scarf and she knows that it is a strictly dominated strategy for Isabel to wear it. Turning to Isabel, if Monica wears B then Isabel prefers T; and if Monica wears T then Isabel prefers M. Thus, both scarves are not strictly dominated for Isabel.

Round 3: After Round 2, B survives for Monica, and T and M survive for Isabel. For Isabel, T strictly dominates M given Monica is choosing B.

The answer is that Monica wears the Burberry scarf and Isabel wears the tan scarf.

17. Consider the following game.

Player 2

Player 1	*w*	*x*	*y*	*z*
a	1,3	4,4	2,2	6,1
b	0,4	3,2	0,0	5,5
c	1,2	5,3	2,2	1,6
d	2,3	2,4	4,2	6,2

a. Find the strategies that survive the IDSDS.

ANSWER: The strategies that survive after round 1 of the iterative deletion of strictly dominated strategies are:

Player 2

Player 1	*w*	*x*	*z*
a	1,3	4,4	6,1
c	1,2	5,3	1,6
d	2,3	2,4	6,2

The strategies that survive after round 2 of the iterative deletion of strictly dominated strategies are:

Player 2

Player 1	*x*	*z*
a	4,4	6,1
c	5,3	1,6
d	2,4	6,2

In round 3, no strategies are strictly dominated. Therefore, strategies a, c, and d survive for player 1, and strategies x and z survive for player 2.

b. Find the rationalizable strategies.

ANSWER: We know that the strategies eliminated by the iterative deletion of strictly dominated strategies are not rationalizable. Thus, it follows from part (a) that we can focus on this reduced game in answering part (b):

Player 2

Player 1	*x*	*z*
a	4,4	6,1
c	5,3	1,6
d	2,4	6,2

Strategy a is optimal for player 1 if she believes player 2 will choose z; z is optimal for 2 if he believes 1 will choose c; c is optimal for 1 if she believes 2 will choose x; and x is optimal for 2 if he believes 1 will choose a. We then have a cycle which means strategies a and c for player 1 and strategies x and z for player 2 are rationalizable. Next consider: d is optimal for 1 if she believes 2 will choose z, and we've already shown that z is rationalizable. Thus, a, c, d are rationalizable for player 1, and x and z are rationalizable for player 2.

18. Consider the three-player game below. Player 1 selects a row, either a_1, b_1, or c_1. Player 2 selects a column, either a_2, b_2, or c_2. Player 3 selects a matrix, either a_3 or b_3 or c_3. The first number in a cell is player 1's payoff, the second number is player 2's payoff, and the last number is player 3's payoff. Derive the strategies that survive the IDSDS.

a_3

	a_2	b_2	c_2
a_1	3,1,4	2,2,2	3,1,4
b_1	2,4,1	5,3,3	1,2,2
c_1	5,4,5	4,1,6	5,0,1

b_3

	a_2	b_2	c_2
a_1	1,1,2	3,3,1	2,2,2
b_1	2,2,0	1,1,0	3,0,3
c_1	1,3,3	0,4,1	3,2,2

c_3

	a_2	b_2	c_2
a_1	4,0,1	3,1,1	3,5,2
b_1	2,5,0	2,4,2	3,2,1
c_1	2,6,3	6,1,3	0,0,0

ANSWER:

Round 1

Player 1: No strategies are strictly dominated because a_1 is a best reply to (a_2, c_3), b_1 is a best reply to (b_2, a_3), and c_1 is a best reply to (a_2, a_3)

Player 2: No strategies are strictly dominated because a_2 is a best reply to (b_1, a_3), b_2 is a best reply to (c_1, b_3), and c_2 is a best reply to (a_1, c_3)

Player 3: a_3 strictly dominates c_3. a_3 is a best reply to (a_1, a_2) and b_3 is a best reply to (b_1, c_2).

After eliminating the strictly dominated strategies, we have:

Player 3: a_3

Player 2

Player 1	a_2	b_2	c_2
a_1	3,1,4	2,2,2	3,1,4
b_1	2,4,1	5,3,3	1,2,2
c_1	5,4,5	4,1,6	5,0,1

Player 3: b_3

Player 2

Player 1	a_2	b_2	c_2
a_1	1,1,2	3,3,1	2,2,2
b_1	2,2,0	1,1,0	3,0,3
c_1	1,3,3	0,4,1	3,2,2

Round 2

Player 1: No strategies are strictly dominated.

Player 2: b_2 strictly dominates c_2.

Player 3: No strategies are strictly dominated.

After eliminating the strictly dominated strategies, we have:

Player 3: a_3

Player 2

Player 1	a_2	b_2
a_1	3,1,4	2,2,2
b_1	2,4,1	5,3,3
c_1	5,4,5	4,1,6

Player 3: b_3

Player 2

Player 1	a_2	b_2
a_1	1,1,2	3,3,1
b_1	2,2,0	1,1,0
c_1	1,3,3	0,4,1

Round 3

Player 1: No strategies are strictly dominated.

Player 2: No strategies are strictly dominated.

Player 3: a_3 strictly dominates b_3.

After eliminating the strictly dominated strategies, we have:

Player 3: a_3

Player 2

Player 1		a_2	b_2
	a_1	3,1,4	2,2,2
	b_1	2,4,1	5,3,3
	c_1	5,4,5	4,1,6

Round 4

Player 1: c_1 strictly dominates a_1.

Player 2: No strategies are strictly dominated.

Player 3: No strategies are strictly dominated.

After eliminating the strictly dominated strategies, we have:

Player 3: a_3

Player 2

Player 1		a_2	b_2
	b_1	2,4,1	5,3,3
	c_1	5,4,5	4,1,6

Round 5

Player 1: No strategies are strictly dominated.

Player 2: a_2 strictly dominates b_2.

Player 3: No strategies are strictly dominated.

After eliminating the strictly dominated strategies, we have:

Player 3: a_3

Player 2

Player 1		a_2
	b_1	2,4,1
	c_1	5,4,5

Round 6

Player 1: c_1 strictly dominates b_1.

Player 2: No strategies are strictly dominated.

Player 3: No strategies are strictly dominated.

The answer is (c_1,a_2,a_3).

19. Consider a four-player game in which each player chooses between two strategies: *a* and *b*. Their payoffs are shown in the accompanying table for the 16 possible strategy profiles. Find the strategies that survive the IDSDS.

Strategy Profiles				Payoffs			
Player 1	Player 2	Player 3	Player 4	Player 1	Player 2	Player 3	Player 4
a	a	a	a	3	1	2	1
a	a	a	b	2	5	3	3
a	a	b	a	4	2	4	4
a	a	b	b	3	2	5	2
a	b	a	a	2	3	1	0
a	b	a	b	4	4	0	3
a	b	b	a	3	5	2	6
a	b	b	b	2	0	3	5
b	a	a	a	1	5	3	3
b	a	a	b	5	2	1	2
b	a	b	a	1	6	4	5
b	a	b	b	1	3	5	1
b	b	a	a	2	3	2	4
b	b	a	b	2	3	1	0
b	b	b	a	2	7	4	3
b	b	b	b	4	5	3	1

ANSWER:

Round 1

Player 1: No strategy is strictly dominated as *a* is a best reply to (*a*,*a*,*a*) and *b* is a best reply to (*b*,*b*,*b*)

Player 2: No strategy is strictly dominated as *a* is a best reply to (*b*,*b*,*b*) and *b* is a best reply to (*b*,*a*,*b*)

Player 3: *b* strictly dominates *a*.

Player 4: No strategy is strictly dominated as *a* is a best reply to (*a*,*a*,*b*) and *b* is a best reply to (*a*,*a*,*a*)

We now have:

Players' Strategies				Players' Payoffs			
Player 1	Player 2	Player 3	Player 4	Player 1	Player 2	Player 3	Player 4
a	a	b	a	4	2	4	4
a	a	b	b	3	2	5	2
a	b	b	a	3	5	2	6
a	b	b	b	2	0	3	5
b	a	b	a	1	6	4	5
b	a	b	b	1	3	5	1
b	b	b	a	2	3	4	3
b	b	b	b	4	5	3	1

Round 2

Player 1: No strategy is strictly dominated as *a* is a best reply to (*a*,*b*,*a*) and *b* is a best reply to (*b*,*b*,*a*).

Player 2: No strategy is strictly dominated as *a* is a best reply to (*b*,*b*,*a*) and *b* is a best reply to (*b*,*b*,*a*)

Player 3: No strategy is strictly dominated.

Player 4: *a* strictly dominates *b*.

Players' Strategies				Players' Payoffs			
Player 1	Player 2	Player 3	Player 4	Player 1	Player 2	Player 3	Player 4
a	a	b	a	4	2	4	4
a	b	b	a	3	5	2	6
b	a	b	a	1	6	4	5
b	b	b	a	2	3	4	3

Round 3

Player 1: *a* strictly dominates *b*.

Player 2: No strategy is strictly dominated as *a* is a best reply to (*b*,*b*,*a*) and *b* is a best reply to (*a*,*b*,*a*).

Player 3: No strategy is strictly dominated.

Player 4: No strategy is strictly dominated.

Players' Strategies				Players' Payoffs			
Player 1	Player 2	Player 3	Player 4	Player 1	Player 2	Player 3	Player 4
a	a	b	a	4	2	4	4
a	b	b	a	3	5	2	6

Round 4: For player 2, *b* strictly dominates *a*.

The answer is (*a*,*b*,*b*,*a*).

20. For the game below, find the strategies that survive the IDSDS when mixed strategies can be used to eliminate a pure strategy as being strictly dominated.

		Player 2		
		x	y	z
Player 1	a	2,4	3,0	0,1
	b	0,0	1,5	4,2

ANSWER:

Round 1: For player 1, strategy *a* is best when player 2 chooses *x* or *y*; strategy *b* is best when 2 chooses *z*. Thus, none of player 1's strategies are strictly dominated. For player 2, strategies *x* and *y* are not strictly dominated because they are the best strategy when 1's strategy is *a* and *b*, respectively. While strategy *z* is not strictly dominated by either *x* or *y*, it is strictly dominated by a mixed strategy composed of x and y. Suppose x is chosen with probability p and y with probability $1 - p$. This mixed strategy strictly dominates pure strategy z when:

Player 1 chooses a: $p \times 4 + (1 - p) \times 0 > 1$ or $p > \frac{1}{4}$
Player 1 chooses b: $p \times 0 + (1 - p) \times 5 > 2$ or $\frac{3}{5} > p$

Thus, if $.25 < p < .6$ then choosing x with probability p and y with probability $1 - p$ yields a strictly higher payoff than pure strategy z.

Round 2: The game is now

		Player 2	
		x	y
Player 1	a	2,4	3,0
	b	0,0	1,5

For player 1, strategy a strictly dominates b. For player 2, neither x nor y are strictly dominated.

Round 3: Given that player 1's only surviving strategy is a, x strictly dominates y for player 2. Therefore, the strategy pair (a,x) is the lone survivor of the iterative deletion of strictly dominated strategies. Note that if mixed strategies are not allowed then all strategy pairs survive the iterative deletion of strictly dominated strategies.