PROBLEM 3-1

Statement: Define the following examples as path, motion, or function generation cases.

a. A telescope aiming (star tracking) mechanism
b. A backhoe bucket control mechanism
c. A thermostat adjusting mechanism
d. A computer printing head moving mechanism
e. An XY plotter pen control mechanism

Solution: See Mathcad file P0301.

a. **Path generation**. A star follows a 2D path in the sky.

b. **Motion generation**. To dig a trench, say, the position and orientation of the bucket must be controlled.

c. **Function generation**. The output is some desired function of the input over some range of the input.

d. **Path generation**. The head must be at some point on a path.

e. **Path generation**. The pen follows a straight line from point to point.

PROBLEM 3-2

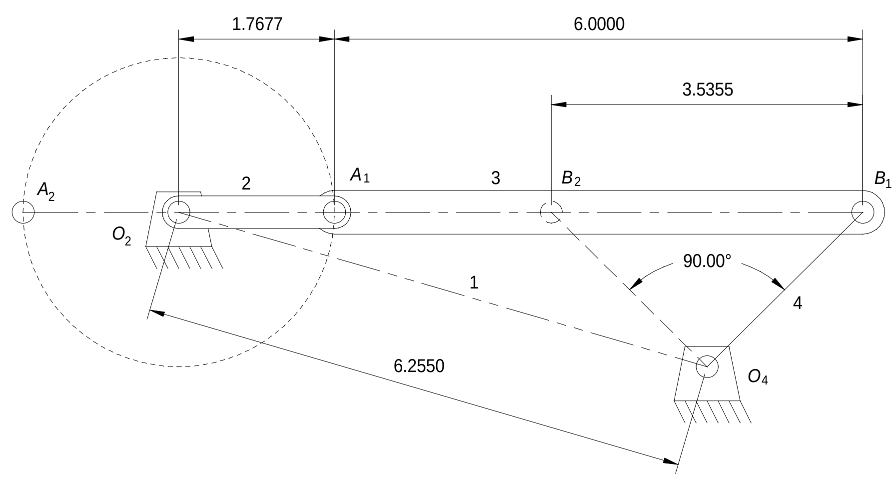
Statement: Design a fourbar Grashof crank-rocker for 90 deg of output rocker motion with no quick return. (See Example 3-1.) Build a cardboard model and determine the toggle positions and the minimum transmission angle.

Given: Output angle $\theta_4 := 90 \cdot deg$

Solution: See Example 3-1 and Mathcad file P0302.

Design choices: Link lengths: Link 3 $L_3 := 6.000$ Link 4 $L_4 := 2.500$

1. Draw the output link O_4B in both extreme positions, B_1 and B_2, in any convenient location such that the desired angle of motion θ_4 is subtended. In this solution, link 4 is drawn such that the two extreme positions each make an angle of 45 deg to the vertical.
2. Draw the chord B_1B_2 and extend it in any convenient direction. In this solution it was extended to the left.
3. Layout the distance A_1B_1 along extended line B_1B_2 equal to the length of link 3. Mark the point A_1.
4. Bisect the line segment B_1B_2 and layout the length of that radius from point A_1 along extended line B_1B_2. Mark the resulting point O_2 and draw a circle of radius O_2A_1 with center at O_2.
5. Label the other intersection of the circle and extended line B_1B_2, A_2.
6. Measure the length of the crank (link 2) as O_2A_1 or O_2A_2. From the graphical solution, $L_2 := 1.76775$
7. Measure the length of the ground link (link 1) as O_2O_4. From the graphical solution, $L_1 := 6.2550$



8. Find the Grashof condition.

$$Condition(a,b,c,d) := \begin{array}{|l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array}$$

$$Condition(L_1, L_2, L_3, L_4) = \text{"Grashof"}$$

PROBLEM 3-3

Statement: Design a fourbar mechanism to give the two positions shown in Figure P3-1 of output rocker motion with no quick-return. (See Example 3-2.) Build a cardboard model and determine the toggle positions and the minimum transmission angle.

Given: Coordinates of A_1, B_1, A_2, and B_2 (with respect to A_1):

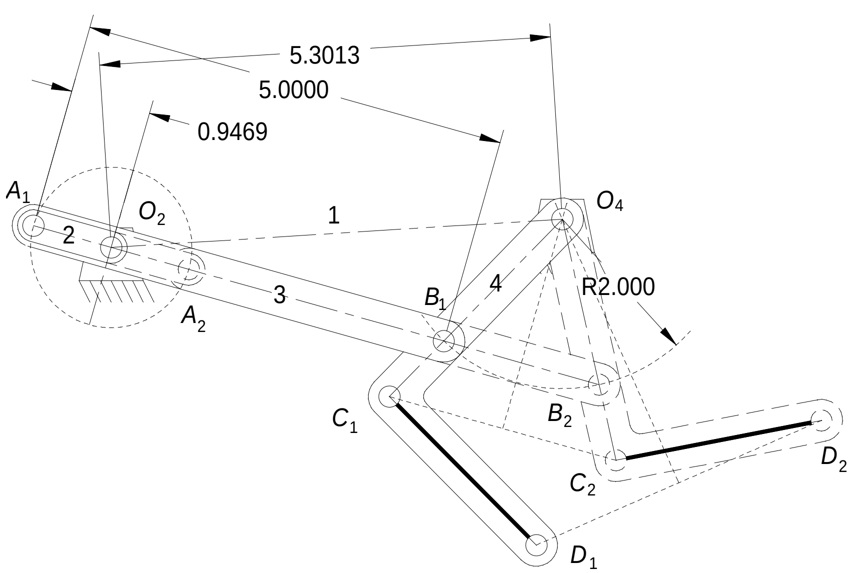
$x_{A1} := 0.00$ $x_{B1} := 1.721$ $x_{A2} := 2.656$ $x_{B2} := 5.065$

$y_{A1} := 0.00$ $y_{B1} := -1.750$ $y_{A2} := -0.751$ $y_{B2} := -0.281$

Solution: See Figure P3-1 and Mathcad file P0303.

Design choices: Link length: Link 3 $L_3 := 5.000$ Link 4 $L_4 := 2.000$

1. Following the notation used in Example 3-2 and Figure 3-5, change the labels on points *A* and *B* in Figure P3-1 to *C* and *D*, respectively. Draw the link *CD* in its two desired positions, C_1D_1 and C_2D_2, using the given coordinates.
2. Draw construction lines from C_1 to C_2 and D_1 to D_2.
3. Bisect line C_1C_2 and line D_1D_2 and extend their perpendicular bisectors to intersect at O_4.
4. Using the length of link 4 (design choice) as a radius, draw an arc about O_4 to intersect both lines O_4C_1 and O_4C_2. Label the intersections B_1 and B_2.
5. Draw the chord B_1B_2 and extend it in any convenient direction. In this solution it was extended to the left.
6. Layout the distance A_1B_1 along extended line B_1B_2 equal to the length of link 3. Mark the point A_1.
7. Bisect the line segment B_1B_2 and layout the length of that radius from point A_1 along extended line B_1B_2. Mark the resulting point O_2 and draw a circle of radius O_2A_1 with center at O_2.
8. Label the other intersection of the circle and extended line B_1B_2, A_2.
9. Measure the length of the crank (link 2) as O_2A_1 or O_2A_2. From the graphical solution, $L_2 := 0.9469$
10. Measure the length of the ground link (link 1) as O_2O_4. From the graphical solution, $L_1 := 5.3013$



11. Find the Grashof condition.

$$Condition(a,b,c,d) := \left| \begin{array}{l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return \ \text{"Grashof"} \quad if \ \ SL < PQ \\ return \ \text{"Special Grashof"} \quad if \ \ SL = PQ \\ return \ \text{"non-Grashof"} \quad otherwise \end{array} \right.$$

$$Condition\left(L_1, L_2, L_3, L_4\right) = \text{"Grashof"}$$

PROBLEM 3-4

Statement: Design a fourbar mechanism to give the two positions shown in Figure P3-1 of coupler motion. (See Example 3-3.) Build a cardboard model and determine the toggle positions and the minimum transmission angle. Add a driver dyad. (See Example 3-4.)

Given: Position 1 offsets: $x_{A1B1} := 1.721 \cdot in$ $\quad y_{A1B1} := 1.750 \cdot in$

Solution: See figure below for one possible solution. Input file P0304.mcd from the solutions manual disk to the Mathcad program for this solution, file P03-04.4br to the program FOURBAR to see the fourbar solution linkage, and file P03-04.6br into program SIXBAR to see the complete sixbar with the driver dyad included.

1. Connect the end points of the two given positions of the line *AB* with construction lines, i.e., lines from A_1 to A_2 and B_1 to B_2.
2. Bisect these lines and extend their perpendicular bisectors in any convenient direction. In the solution below the bisector of A_1A_2 was extended downward and the bisector of B_1B_2 was extended upward.
3. Select one point on each bisector and label them O_4 and O_6, respectively. In the solution below the distances O_4A and O_6B were each selected to be 4.000 in. This resulted in a ground-link-length O_4O_6 for the fourbar of 6.457 in.
4. The fourbar stage is now defined as O_4ABO_6 with link lengths

Link 5 (coupler) $L_5 := \sqrt{x_{A1B1}^2 + y_{A1B1}^2}$ $\quad L_5 = 2.454\, in$

Link 4 (input) $L_4 := 4.000 \cdot in$ $\quad$ Link 6 (output) $L_6 := 4.000 \cdot in$

Ground link 1b $L_{1b} := 6.457 \cdot in$

5. Select a point on link 4 (O_4A) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it *D*. (Note that link 4 is now a ternary link with nodes at O_4, *D*, and *A*.) In the solution below the distance O_4D was selected to be 2.000 in.
6. Draw a construction line through D_1D_2 and extend it to the left.
7. Select a point on this line and call it O_2. In the solution below the distance *CD* was selected to be 4.000 in.
8. Draw a circle about O_2 with a radius of one-half the length D_1D_2 and label the intersections of the circle with the extended line as C_1 and C_2. In the solution below the radius was measured as 0.6895 in.
9. The driver fourbar is now defined as O_2CDO_4 with link lengths

Link 2 (crank) $L_2 := 0.6895 \cdot in$ $\quad$ Link 3 (coupler) $L_3 := 4.000 \cdot in$

Link 4a (rocker) $L_{4a} := 2.000 \cdot in$ $\quad$ Link 1a (ground) $L_{1a} := 4.418 \cdot in$

10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

Shortest link $S := L_2$ $\quad S = 0.6895\, in$

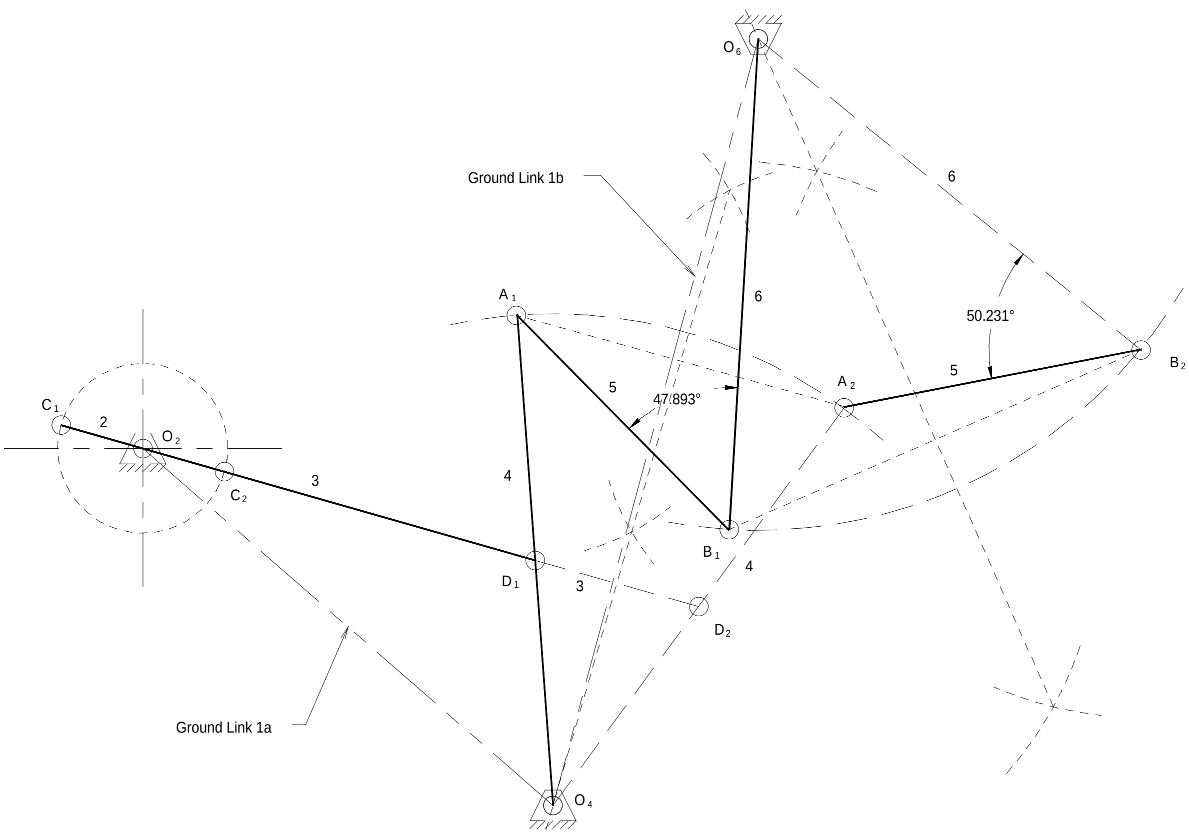
Longest link $L := L_{1a}$ $\quad L = 4.4180\, in$

Other links $P := L_3$ $\quad P = 4.0000\, in$

$Q := L_{4a}$ $\quad Q = 2.0000\, in$

$$Condition(a,b,c,d) := \begin{array}{|l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return \ \text{"Grashof"} \quad if \ \ SL < PQ \\ return \ \text{"Special Grashof"} \quad if \ \ SL = PQ \\ return \ \text{"non-Grashof"} \quad otherwise \end{array}$$

$$Condition(S,L,P,Q) = \text{"Grashof"}$$



11. Using the program FOURBAR and the link lengths given above, it was found that the fourbar O_4ABO_6 is non-Grashoff with toggle positions at θ_2 = -71.9 deg and +71.9 deg. The minimum transmission angle is 35.5 deg. The fourbar operates between θ_2 = +21.106 deg and -19.297 deg.

PROBLEM 3-5

Statement: Design a fourbar mechanism to give the three positions of coupler motion with no quick return shown in Figure P3-2. (See also Example 3-5.) Ignore the points O_2 and O_4 shown. Build a cardboard model and determine the toggle positions and the minimum transmission angle. Add a driver dyad.

Solution: See Figure P3-2 and Mathcad file P0305.

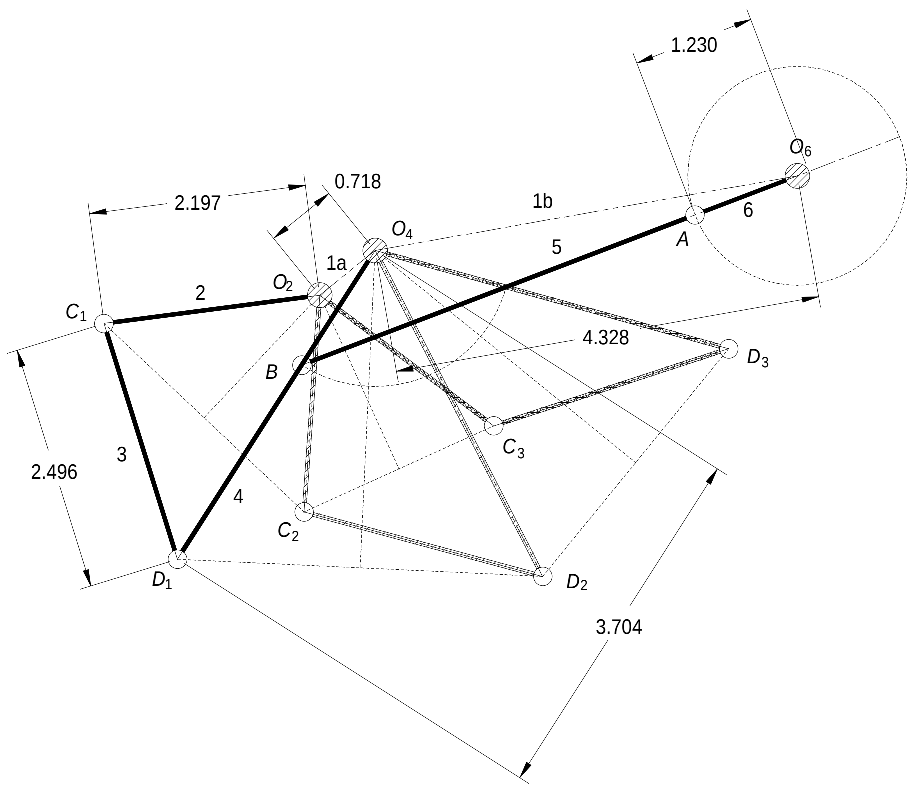
Design choices:

Length of link 5: $L_5 := 4.250$ Length of link 4b: $L_{4b} := 1.375$

1. Draw link *CD* in its three design positions C_1D_1, C_2D_2, C_3D_3 in the plane as shown.
2. Draw construction lines from point C_1 to C_2 and from point C_2 to C_3.
3. Bisect line C_1C_2 and line C_2C_3 and extend their perpendicular bisectors until they intersect. Label their intersection O_2.
4. Repeat steps 2 and 3 for lines D_1D_2 and D_2D_3. Label the intersection O_4.
5. Connect O_2 with C_1 and call it link 2. Connect O_4 with D_1 and call it link 4.
6. Line C_1D_1 is link 3. Line O_2O_4 is link 1 (ground link for the fourbar). The fourbar is now defined as O_2CDO_4 and has link lengths of

Ground link 1a $L_{1a} := 0.718$ Link 2 $L_2 := 2.197$

Link 3 $L_3 := 2.496$ Link 4 $L_4 := 3.704$



7. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

$$Condition(a,b,c,d) := \begin{array}{|l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"}\quad if\ \ SL < PQ \\ return\ \text{"Special Grashof"}\quad if\ \ SL = PQ \\ return\ \text{"non-Grashof"}\quad otherwise \end{array}$$

$$Condition\left(L_{1a},L_2,L_3,L_4\right) = \text{"Grashof"}$$

8. Select a point on link 4 (O_4D) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it B. (Note that link 4 is now a ternary link with nodes at O_4, C, and B.) In the solution above the distance O_4B was selected to be $L_{4b} = 1.375$.
9. Draw a construction line through B_1B_3 and extend it up to the right.
10. Layout the length of link 5 (design choice) along the extended line. Label the other end A.
11. Draw a circle about O_6 with a radius of one-half the length B_1B_3 and label the intersections of the circle with the extended line as A_1 and A_3. In the solution below the radius was measured as $L_6 := 1.230$.
12. The driver fourbar is now defined as O_4BAO_6 with link lengths

Link 6 (crank)	$L_6 = 1.230$
Link 5 (coupler)	$L_5 = 4.250$
Link 1b (ground)	$L_{1b} := 4.328$
Link 4b (rocker)	$L_{4b} = 1.375$

13. Use the link lengths in step 12 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$$Condition\left(L_6,L_{1b},L_{4b},L_5\right) = \text{"Grashof"}$$

PROBLEM 3-6

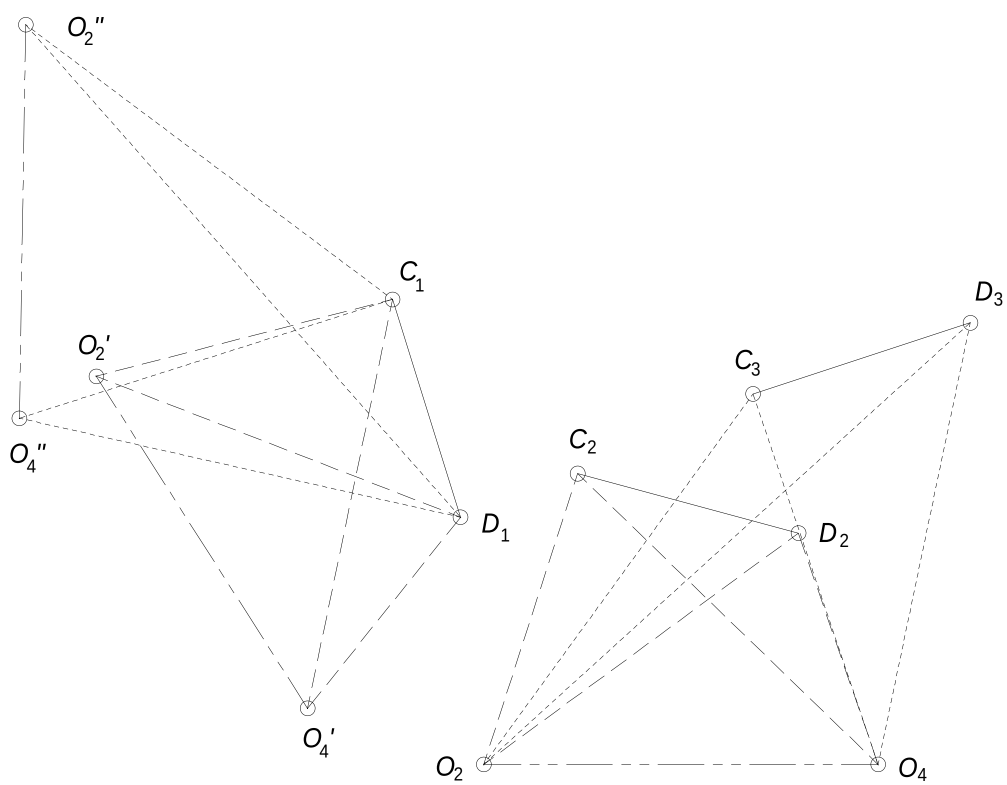
Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-2 using the fixed pivots O_2 and O_4 shown. Build a cardboard model and determine the toggle positions and the minimum transmission angle. Add a driver dyad.

Solution: See Figure P3-2 and Mathcad file P0306.

Design choices:

Length of link 5: $L_5 := 5.000$ Length of link 2b: $L_{2b} := 2.000$

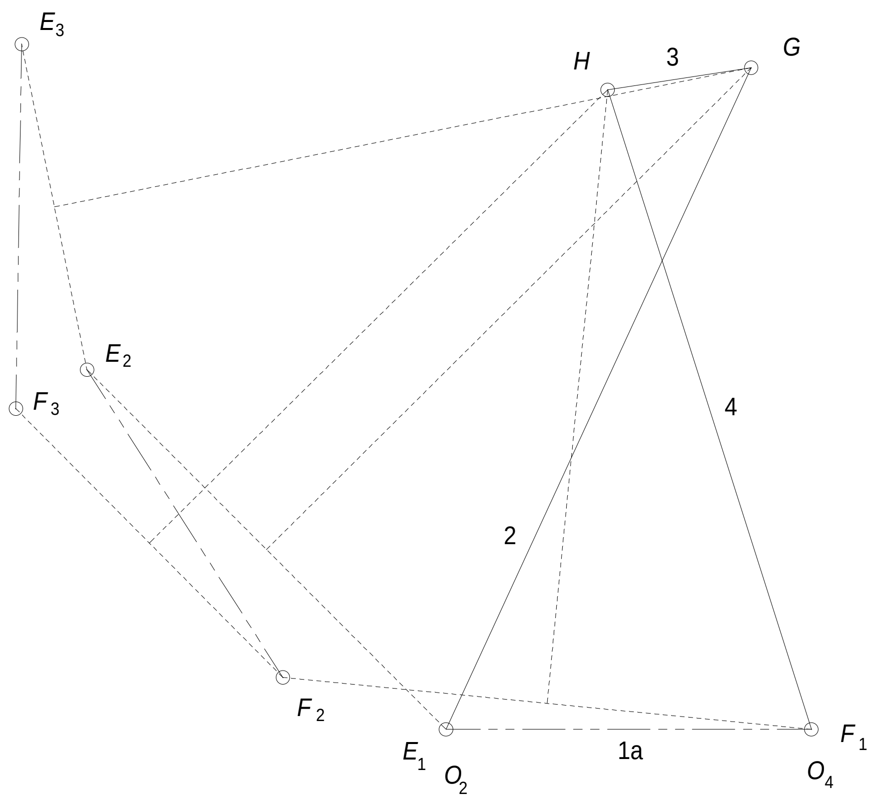
1. Draw link CD in its three design positions C_1D_1, C_2D_2, C_3D_3 in the plane as shown.
2. Draw the ground link O_2O_4 in its desired position in the plane with respect to the first coupler position C_1D_1.
3. Draw construction arcs from point C_2 to O_2 and from point D_2 to O_2 whose radii define the sides of triangle $C_2O_2D_2$. This defines the relationship of the fixed pivot O_2 to the coupler line CD in the second coupler position.
4. Draw construction arcs from point C_2 to O_4 and from point D_2 to O_4 whose radii define the sides of triangle $C_2O_4D_2$. This defines the relationship of the fixed pivot O_4 to the coupler line CD in the second coupler position.
5. Transfer this relationship back to the first coupler position C_1D_1 so that the ground plane position $O_2'O_4'$ bears the same relationship to C_1D_1 as O_2O_4 bore to the second coupler position C_2D_2.
6. Repeat the process for the third coupler position and transfer the third relative ground link position to the first, or reference, position.
7. The three inverted positions of the ground link that correspond to the three desired coupler positions are labeled O_2O_4, $O_2'O_4'$, and $O_2''O_4''$ in the first layout below and are renamed E_1F_1, E_2F_2, and E_3F_3, respectively, in the second layout, which is used to find the points G and H.



8. Draw construction lines from point E_1 to E_2 and from point E_2 to E_3.

9. Bisect line E_1E_2 and line E_2E_3 and extend their perpendicular bisectors until they intersect. Label their intersection *G*.
10. Repeat steps 2 and 3 for lines F_1F_2 and F_2F_3. Label the intersection *H*.
11. Connect E_1 with *G* and label it link 2. Connect F_1 with *H* and label it link 4. Reinverting, E_1 and F_1 are the original fixed pivots O_2 and O_4, respectively.
12. Line *GH* is link 3. Line O_2O_4 is link 1a (ground link for the fourbar). The fourbar is now defined as O_2GHO_4 and has link lengths of

Ground link 1a	$L_{1a} := 4.303$	Link 2	$L_2 := 8.597$
Link 3	$L_3 := 1.711$	Link 4	$L_4 := 7.921$



13. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

$$Condition(a,b,c,d) := \begin{array}{|l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"}\quad if\ SL < PQ \\ return\ \text{"Special Grashof"}\quad if\ SL = PQ \\ return\ \text{"non-Grashof"}\quad otherwise \end{array}$$

$$Condition(L_{1a}, L_2, L_3, L_4) = \text{"Grashof"}$$

The fourbar that will provide the desired motion is now defined as a Grashof double crank in the crossed configuration. It now remains to add the original points C_1 and D_1 to the coupler *GH* and to define the driving dyad.

14. Select a point on link 2 (O_2G) at a suitable distance from O_2 as the pivot point to which the driver dyad will be connected and label it B. (Note that link 2 is now a ternary link with nodes at O_2, B, and G.) In the solution below, the distance O_2B was selected to be $L_{2b} = 2.000$.
15. Draw a construction line through B_1B_3 and extend it up to the right.
16. Layout the length of link 5 (design choice) along the extended line. Label the other end A.
17. Draw a circle about O_6 with a radius of one-half the length B_1B_3 and label the intersections of the circle with the extended line as A_1 and A_3. In the solution below the radius was measured as $L_6 := 0.412$.
18. The driver fourbar is now defined as O_2BAO_6 with link lengths

Link 6 (crank) $L_6 = 0.412$

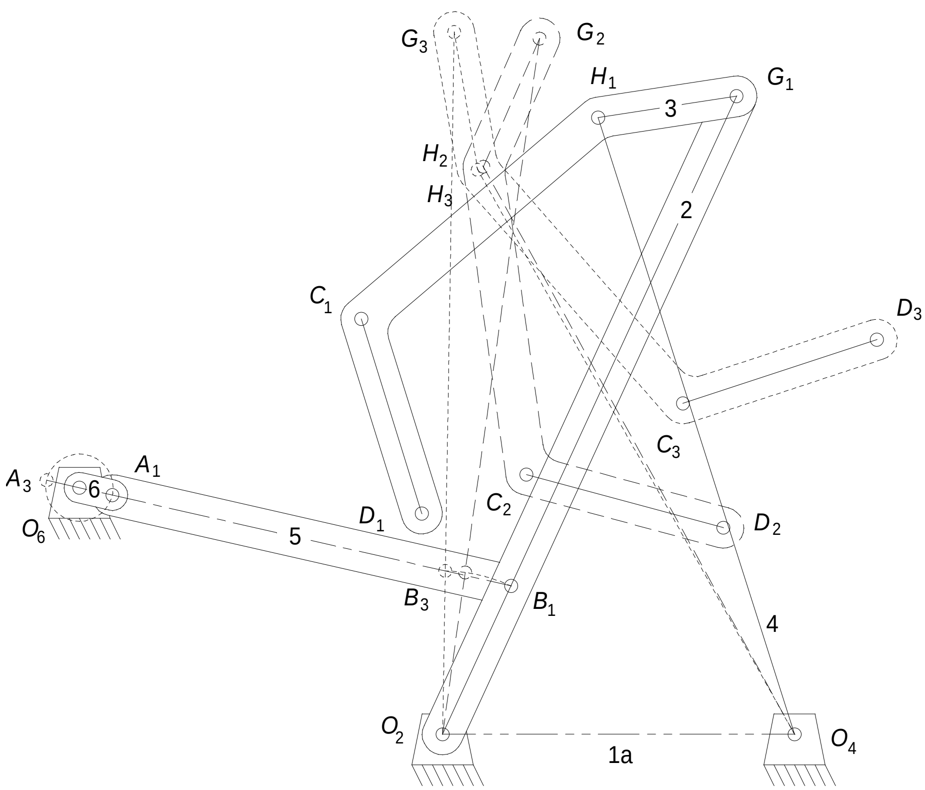
Link 5 (coupler) $L_5 = 5.000$

Link 1b (ground) $L_{1b} := 5.369$

Link 2b (rocker) $L_{2b} = 2.000$

19. Use the link lengths in step 18 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$$Condition\left(L_6, L_{1b}, L_{2b}, L_5\right) = \text{"Grashof"}$$



PROBLEM 3-7

Statement: Repeat Problem 3-2 with a quick-return time ratio of 1:1.4. (See Example 3.9). Design a fourbar Grashof crank-rocker for 90 degrees of output rocker motion with a quick-return time ratio of 1:1.4.

Given: Time ratio $T_r := \frac{1}{1.4}$

Solution: See figure below for one possible solution. Also see Mathcad file P0307.

1. Determine the crank rotation angles α and β, and the construction angle δ from equations 3.1 and 3.2.

$$T_r = \frac{\alpha}{\beta} \qquad \alpha + \beta = 360 \cdot deg$$

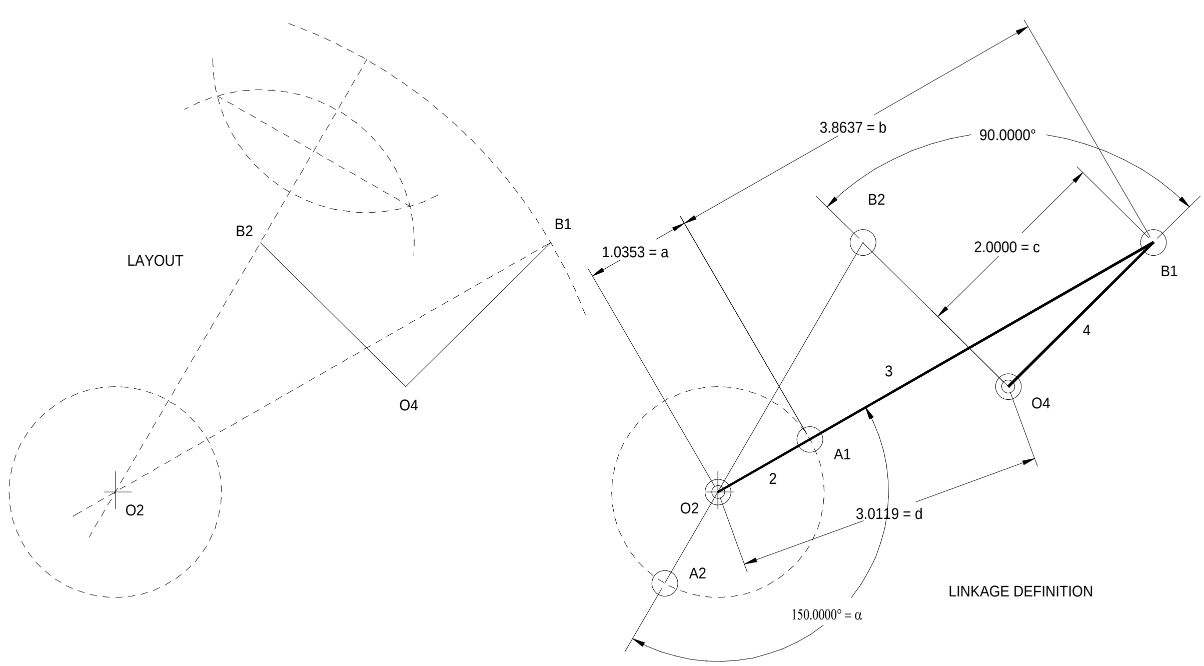
Solving for β, α, and δ

$$\beta := \frac{360 \cdot deg}{1 + T_r} \qquad \beta = 210\,deg$$

$$\alpha := 360 \cdot deg - \beta \qquad \alpha = 150\,deg$$

$$\delta := \beta - 180 \cdot deg \qquad \delta = 30\,deg$$

2. Start the layout by arbitrarily establishing the point O_4 and from it layoff two lines of equal length, 90 deg apart. Label one B_1 and the other B_2. In the solution below, each line makes an angle of 45 deg with the horizontal and has a length of 2.000 in.
3. Layoff a line through B_1 at an arbitrary angle (but not zero deg). In the solution below, the line is 30 deg to the horizontal.
4. Layoff a line through B_2 that makes an angle δ with the line in step 3 (60 deg to the horizontal in this case). The intersection of these two lines establishes the point O_2.
5. From O_2 draw an arc that goes through B_1. Extend O_2B_2 to meet this arc. Erect a perpendicular bisector to the extended portion of the line and transfer one half of the line to O_2 as the length of the input crank.



6. For this solution, the link lengths are:

Ground link (1)	$d := 3.0119 \cdot in$
Crank (2)	$a := 1.0353 \cdot in$
Coupler (3)	$b := 3.8637 \cdot in$
Rocker (4)	$c := 2.000 \cdot in$

PROBLEM 3-8

Statement: Design a sixbar drag link quick-return linkage for a time ratio of 1:2, and output rocker motion of 60 degrees. (See Example 3-10.)

Given: Time ratio $T_r := \frac{1}{2}$

Solution: See figure below for one possible solution. Also see Mathcad file P0308.

1. Determine the crank rotation angles α and β from equation 3.1.

$$T_r = \frac{\alpha}{\beta} \qquad \alpha + \beta = 360 \cdot deg$$

Solving for β and α $\quad \beta := \dfrac{360 \cdot deg}{1 + T_r} \qquad \beta = 240\, deg$

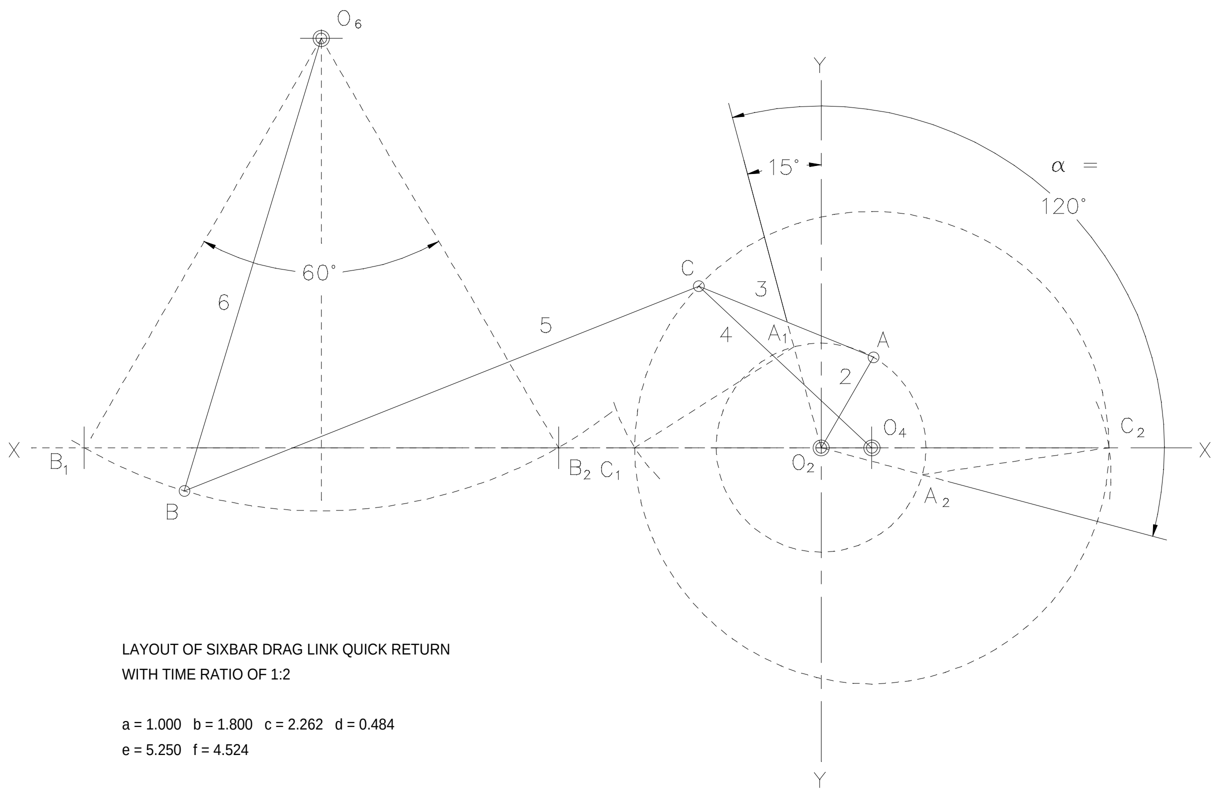
$$\alpha := 360 \cdot deg - \beta \qquad \alpha = 120\, deg$$

2. Draw a line of centers *XX* at any convenient location.
3. Choose a crank pivot location O_2 on line *XX* and draw an axis *YY* perpendicular to *XX* through O_2.
4. Draw a circle of convenient radius O_2A about center O_2. In the solution below, the length of O_2A is $a := 1.000 \cdot in$.
5. Lay out angle α with vertex at O_2, symmetrical about quadrant one.
6. Label points A_1 and A_2 at the intersections of the lines subtending angle α and the circle of radius O_2A.
7. Set the compass to a convenient radius *AC* long enough to cut *XX* in two places on either side of O_2 when swung from both A_1 and A_2. Label the intersections C_1 and C_2. In the solution below, the length of *AC* is $b := 1.800 \cdot in$.
8. The line O_2A is the driver crank, link 2, and the line *AC* is the coupler, link 3.
9. The distance C_1C_2 is twice the driven (dragged) crank length. Bisect it to locate the fixed pivot O_4.
10. The line O_2O_4 now defines the ground link. Line O_4C is the driven crank, link 4. In the solution below, O_4C measures $c := 2.262 \cdot in$ and O_2O_4 measures $d := 0.484 \cdot in$.
11. Calculate the Grashoff condition. If non-Grashoff, repeat steps 7 through 11 with a shorter radius in step 7.

$$Condition(a, b, c, d) := \left| \begin{array}{l} S \leftarrow min(a, b, c, d) \\ L \leftarrow max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"} \quad if\ SL < PQ \\ return\ \text{"Special Grashof"} \quad if\ SL = PQ \\ return\ \text{"non-Grashof"} \quad otherwise \end{array} \right.$$

$$Condition(a, b, c, d) = \text{"Grashof"}$$

12. Invert the method of Example 3-1 to create the output dyad using *XX* as the chord and O_4C_1 as the driving crank. The points B_1 and B_2 will lie on line *XX* and be spaced apart a distance that is twice the length of O_4C (link 4). The pivot point O_6 will lie on the perpendicular bisector of B_1B_2 at a distance from *XX* which subtends the specified output rocker angle, which is 60 degrees in this problem. In the solution below, the length *BC* was chosen to be $e := 5.250 \cdot in$.



LAYOUT OF SIXBAR DRAG LINK QUICK RETURN
WITH TIME RATIO OF 1:2

a = 1.000 b = 1.800 c = 2.262 d = 0.484
e = 5.250 f = 4.524

13. For the design choices made (lengths of links 2, 3 and 5), the length of the output rocker (link 6) was measured as $f := 4.524 \cdot in$.

PROBLEM 3-9

Statement: Design a crank-shaper quick-return mechanism for a time ratio of 1:3 (Figure 3-14, p. 112).

Given: Time ratio $T_R := \frac{1}{3}$

Solution: See Figure 3-14 and Mathcad file P0309.

Design choices:

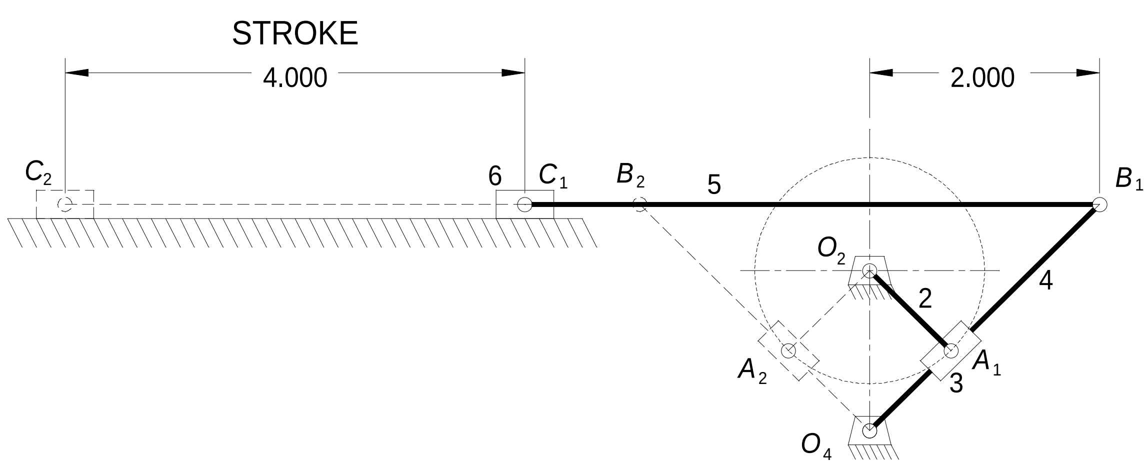
Length of link 2 (crank) $L_2 := 1.000$ Length of stroke $S := 4.000$

Length of link 5 (coupler) $L_5 := 5.000$

1. Calculate α from equations 3.1.

$$T_R := \frac{\alpha}{\beta} \qquad \alpha + \beta := 360 \cdot deg \qquad \alpha := \frac{360 \cdot deg}{1 + \frac{1}{T_R}} \qquad \alpha = 90.000\, deg$$

2. Draw a vertical line and mark the center of rotation of the crank, O_2, on it.
3. Layout two construction lines from O_2, each making an angle $\alpha/2$ to the vertical line through O_2.
4. Using the chosen crank length (see Design Choices), draw a circle with center at O_2 and radius equal to the crank length. Label the intersections of the circle and the two lines drawn in step 3 as A_1 and A_2.
5. Draw lines through points A1 and A2 that are also tangent to the crank circle (step 2). These two lines will simultaneously intersect the vertical line drawn in step 2. Label the point of intersection as the fixed pivot center O_4.
6. Draw a vertical construction line, parallel and to the right of O_2O_4, a distance S/2 (one-half of the output stroke length) from the line O_2O_4.
7. Extend line O_4A_1 until it intersects the construction line drawn in step 6. Label the intersection B_1.
8. Draw a horizontal construction line from point B_1, either to the left or right. Using point B_1 as center, draw an arc of radius equal to the length of link 5 (see Design Choices) to intersect the horizontal construction line. Label the intersection as C_1.
9. Draw the slider blocks at points A_1 and C_1 and finish by drawing the mechanism in its other extreme position.



PROBLEM 3-10

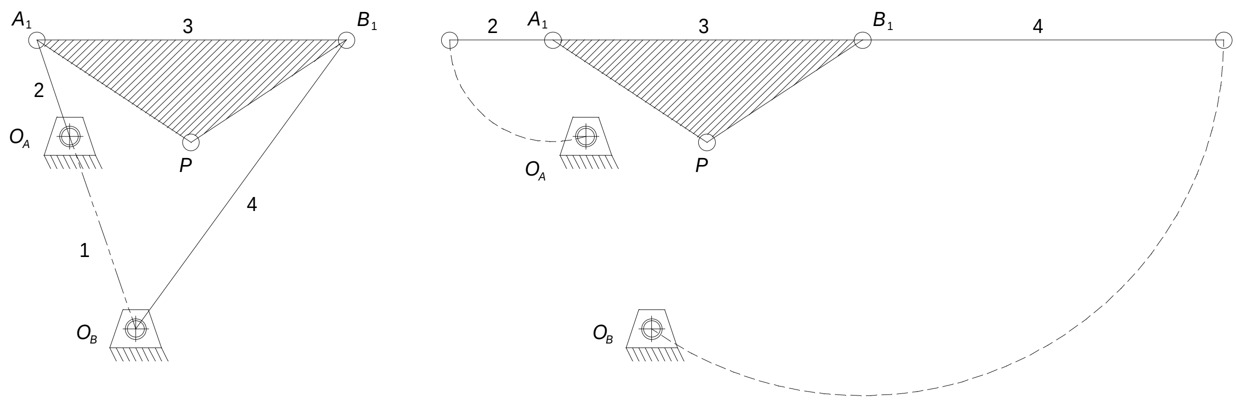
Statement: Find the two cognates of the linkage in Figure 3-17 (p. 116). Draw the Cayley and Roberts diagrams. Check your results with program FOURBAR.

Given: Link lengths: Coupler point data:

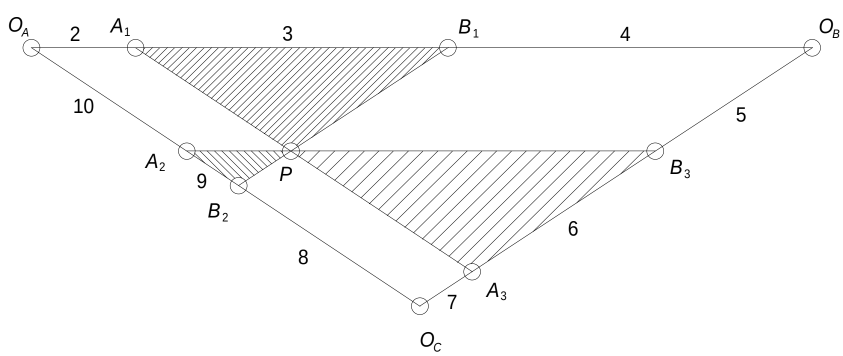
Ground link	$L_1 := 2$	Crank	$L_2 := 1$	$A1P := 1.800$	$\delta_1 := -34.000 \cdot deg$
Coupler	$L_3 := 3$	Rocker	$L_4 := 3.5$	$B1P := 1.813$	$\gamma_1 := -33.727 \cdot deg$

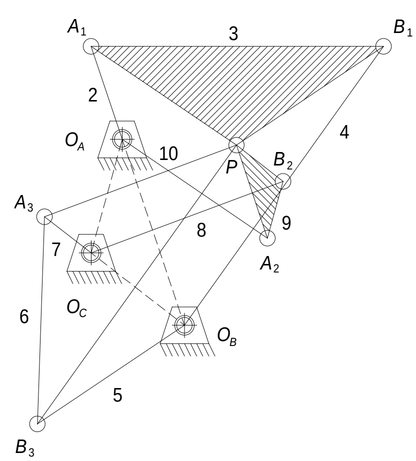
Solution: See Figure 3-17 and Mathcad file P0310.

1. Draw the original fourbar linkage, which will be cognate #1, and align links 2 and 4 with the coupler.



2. Construct lines parallel to all sides of the aligned fourbar linkage to create the Cayley diagram (see Figure 3-24

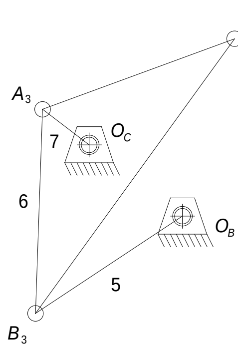




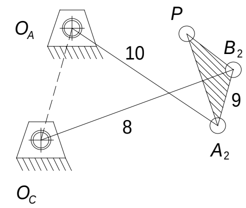
3. Return links 2 and 4 to their fixed pivots O_A and O_B and establish O_C as a fixed pivot by making triangle $O_AO_BO_C$ similar to A_1B_1P.
4. Separate the three cognates. Point P has the same path motion in each cognate.
5. Calculate the cognate link lengths based on the geometry of the Cayley diagram (Figure 3-24c, p. 114).

$L_5 := B1P \qquad L_5 = 1.813$

$$L_6 := \frac{L_4}{L_3} \cdot B1P \qquad L_6 = 2.115$$



Cognate #3



Cognate #2

$L_{10} := A1P$ $\quad L_{10} = 1.800$ $\quad L_9 := \frac{L_2}{L_3} \cdot A1P$ $\quad L_9 = 0.600$

$L_7 := L_9 \cdot \frac{B1P}{A1P}$ $\quad L_7 = 0.604$

$L_8 := L_6 \cdot \frac{A1P}{B1P}$ $\quad L_8 = 2.100$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

$L_{1BC} := \frac{L_1}{L_3} \cdot B1P$ $\quad L_{1BC} = 1.209$

$L_{1AC} := \frac{L_1}{L_3} \cdot A1P$ $\quad L_{1AC} = 1.200$

Calculate the coupler point data for cognates #2 and #3

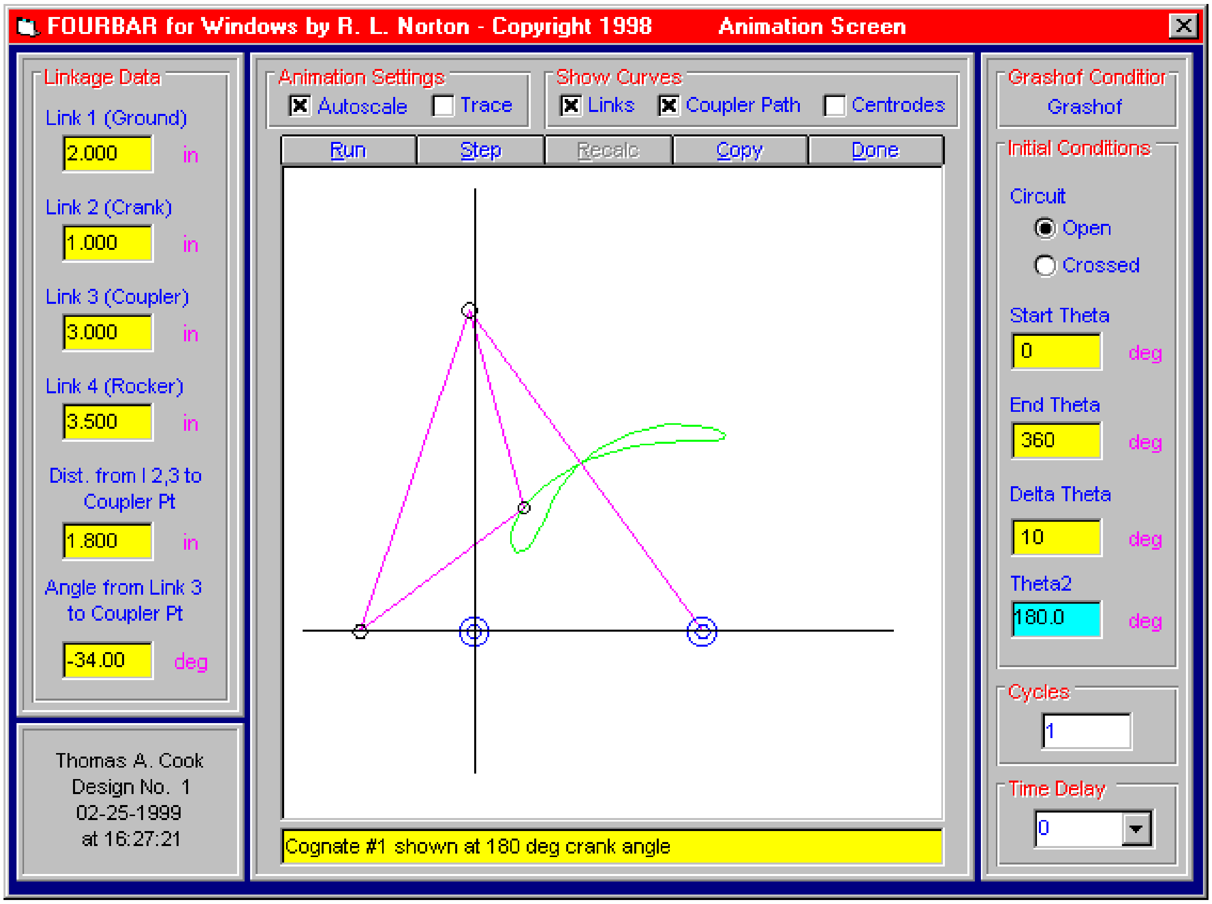
$A3P := L_8$ $\quad A3P = 2.100$ $\quad A2P := L_2$ $\quad A2P = 1.000$

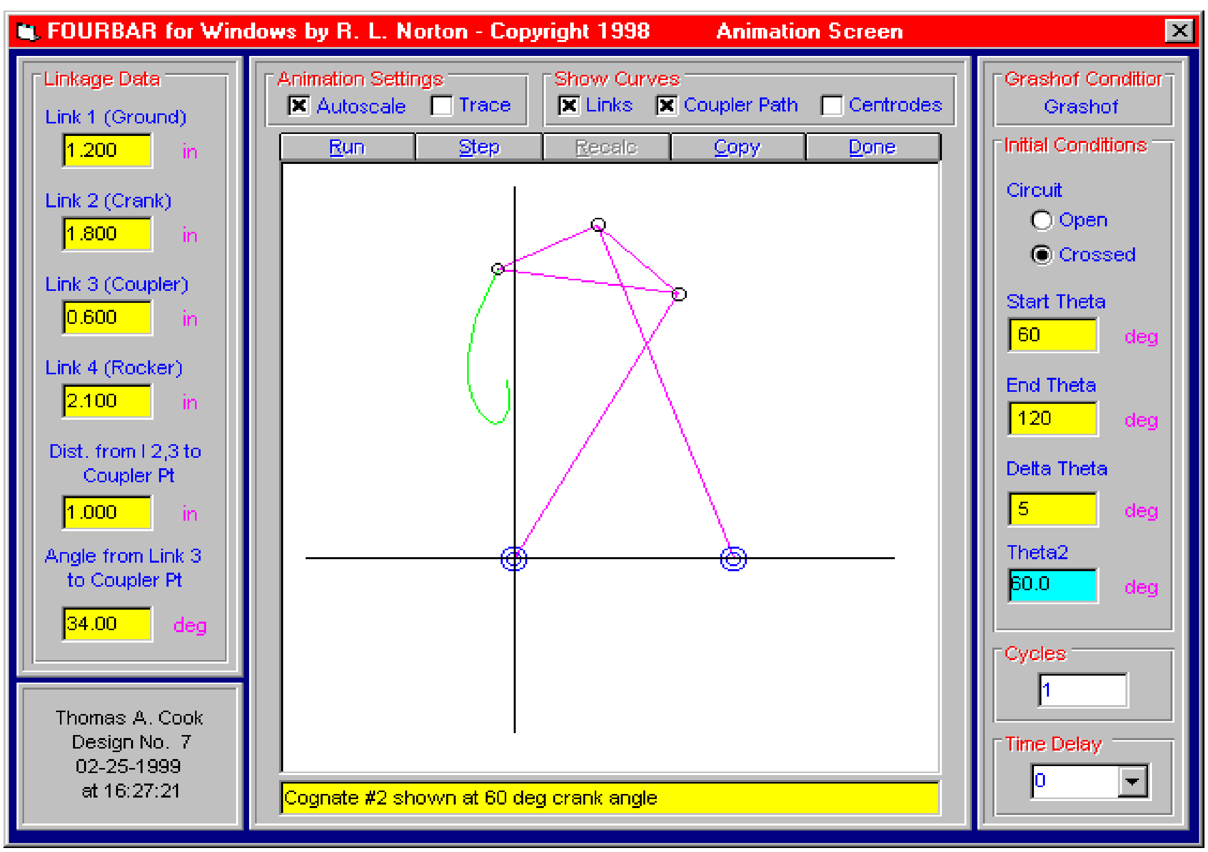
$\delta_3 := 180 \cdot deg - (\delta_1 + \gamma_1)$ $\quad \delta_3 = 247.727\, deg$ $\quad \delta_2 := -\delta_1$ $\quad \delta_2 = 34.000\, deg$

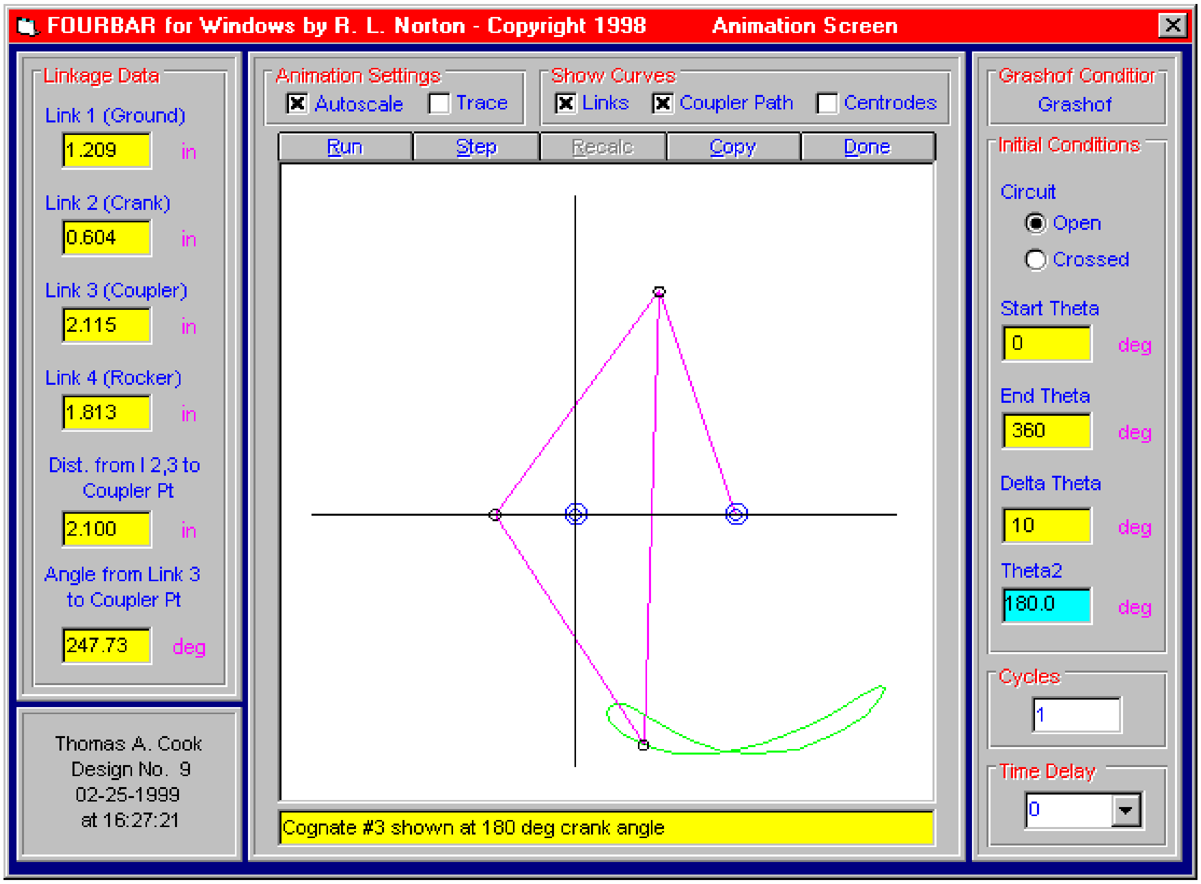
SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 2.000$	$L_{1AC} = 1.200$	$L_{1BC} = 1.209$
Crank length	$L_2 = 1.000$	$L_{10} = 1.800$	$L_7 = 0.604$
Coupler length	$L_3 = 3.000$	$L_9 = 0.600$	$L_6 = 2.115$
Rocker length	$L_4 = 3.500$	$L_8 = 2.100$	$L_5 = 1.813$
Coupler point	$A1P = 1.800$	$A2P = 1.000$	$A3P = 2.100$
Coupler angle	$\delta_1 = -34.000\, deg$	$\delta_2 = 34.000\, deg$	$\delta_3 = 247.727\, deg$

6. Verify that the three cognates yield the same coupler curve by entering the original link lengths in program FOURBAR and letting it calculate the cognates.


FOURBAR for Windows by R. L. Norton - Copyright 1998
Animation Screen
Linkage Data
Link 1 (Ground)
2.000 in
Link 2 (Crank)
1.000 in
Link 3 (Coupler)
3.000 in
Link 4 (Rocker)
3.500 in
Dist. from l 2,3 to Coupler Pt
1.800 in
Angle from Link 3 to Coupler Pt
-34.00 deg
Thomas A. Cook
Design No. 1
02-25-1999
at 16:27:21
Animation Settings
Autoscale
Trace
Show Curves
Links
Coupler Path
Centrodes
Run
Step
Recalc
Copy
Done
Cognate #1 shown at 180 deg crank angle
Grashof Condition
Grashof
Initial Conditions
Circuit
Open
Crossed
Start Theta
0 deg
End Theta
360 deg
Delta Theta
10 deg
Theta2
180.0 deg
Cycles
1
Time Delay
0


FOURBAR for Windows by R. L. Norton - Copyright 1998
Animation Screen
Linkage Data
Link 1 (Ground)
1.200 in
Link 2 (Crank)
1.800 in
Link 3 (Coupler)
0.600 in
Link 4 (Rocker)
2.100 in
Dist. from l 2,3 to Coupler Pt
1.000 in
Angle from Link 3 to Coupler Pt
34.00 deg
Thomas A. Cook
Design No. 7
02-25-1999
at 16:27:21
Animation Settings
Autoscale
Trace
Show Curves
Links
Coupler Path
Centrodes
Run
Step
Recalc
Copy
Done
Cognate #2 shown at 60 deg crank angle
Grashof Condition
Grashof
Initial Conditions
Circuit
Open
Crossed
Start Theta
60 deg
End Theta
120 deg
Delta Theta
5 deg
Theta2
60.0 deg
Cycles
1
Time Delay
0



Note that cognate #2 is a Grashof double rocker and, therefore, cannot trace out the entire coupler curve.

PROBLEM 3-11

Statement: Find the three equivalent geared fivebar linkages for the three fourbar cognates in Figure 3-25a (p. 125). Check your results by comparing the coupler curves with programs FOURBAR and FIVEBAR.

Given: Link lengths:

Ground link $L_1 := 39.5$ Crank $L_2 := 15.5$

Coupler $L_3 := 14.0$ Rocker $L_4 := 20.0$

Coupler point data: $A1P := 26.0$ $\delta_1 := 63.000 \cdot deg$

Solution: See Figure 3-25a and Mathcad file P0311.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle *APB*.

$$B1P := \left(L_3^{\,2} + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot cos(\delta_1)\right)^{0.5} \qquad B1P = 23.270$$

$$\gamma_1 := acos\left(\frac{L_3^{\,2} + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P}\right) \qquad \gamma_1 = 84.5843\, deg$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$L_5 := B1P$ $L_5 = 23.270$ $L_6 := \frac{L_4}{L_3} \cdot B1P$ $L_6 = 33.243$

$L_{10} := A1P$ $L_{10} = 26.000$ $L_9 := \frac{L_2}{L_3} \cdot A1P$ $L_9 = 28.786$

$L_7 := L_9 \cdot \frac{B1P}{A1P}$ $L_7 = 25.763$ $L_8 := L_6 \cdot \frac{A1P}{B1P}$ $L_8 = 37.143$

Calculate the coupler point data for cognates #2 and #3

$A3P := L_4$ $A3P = 20.000$ $A2P := L_2$ $A2P = 15.500$

$\delta_3 := \gamma_1$ $\delta_3 = 84.584\, deg$ $\delta_2 := -\delta_1$ $\delta_2 = -63.000\, deg$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

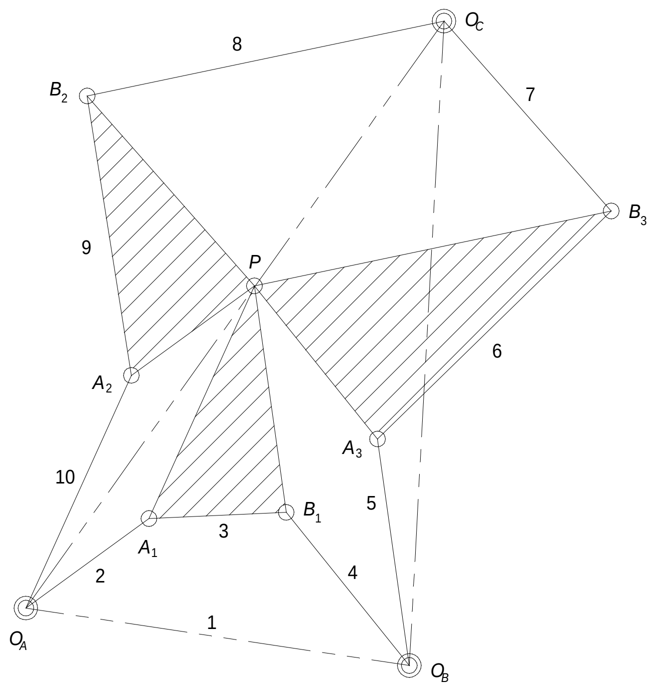
$L_{1BC} := \frac{L_1}{L_3} \cdot B1P$ $L_{1BC} = 65.6548$ $L_{1AC} := \frac{L_1}{L_3} \cdot A1P$ $L_{1AC} = 73.3571$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

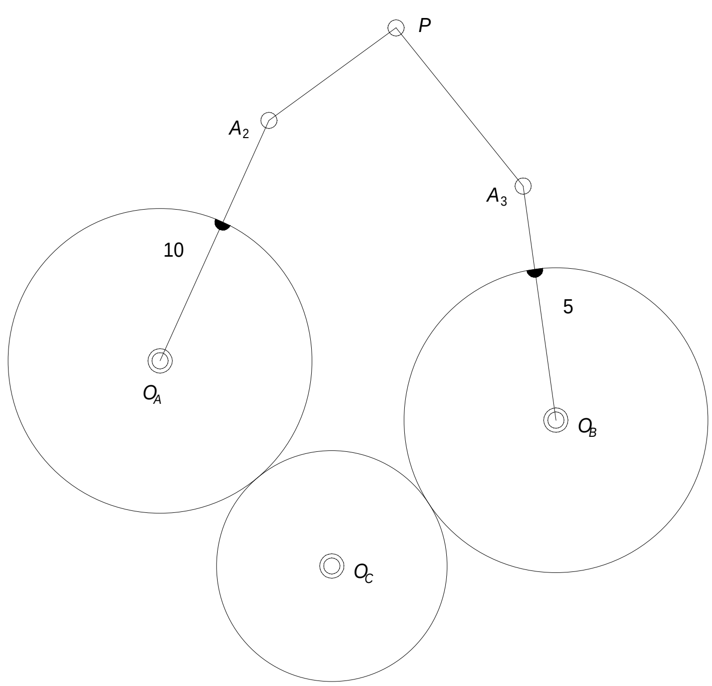
SUMMARY OF COGNATE SPECIFICATIONS:

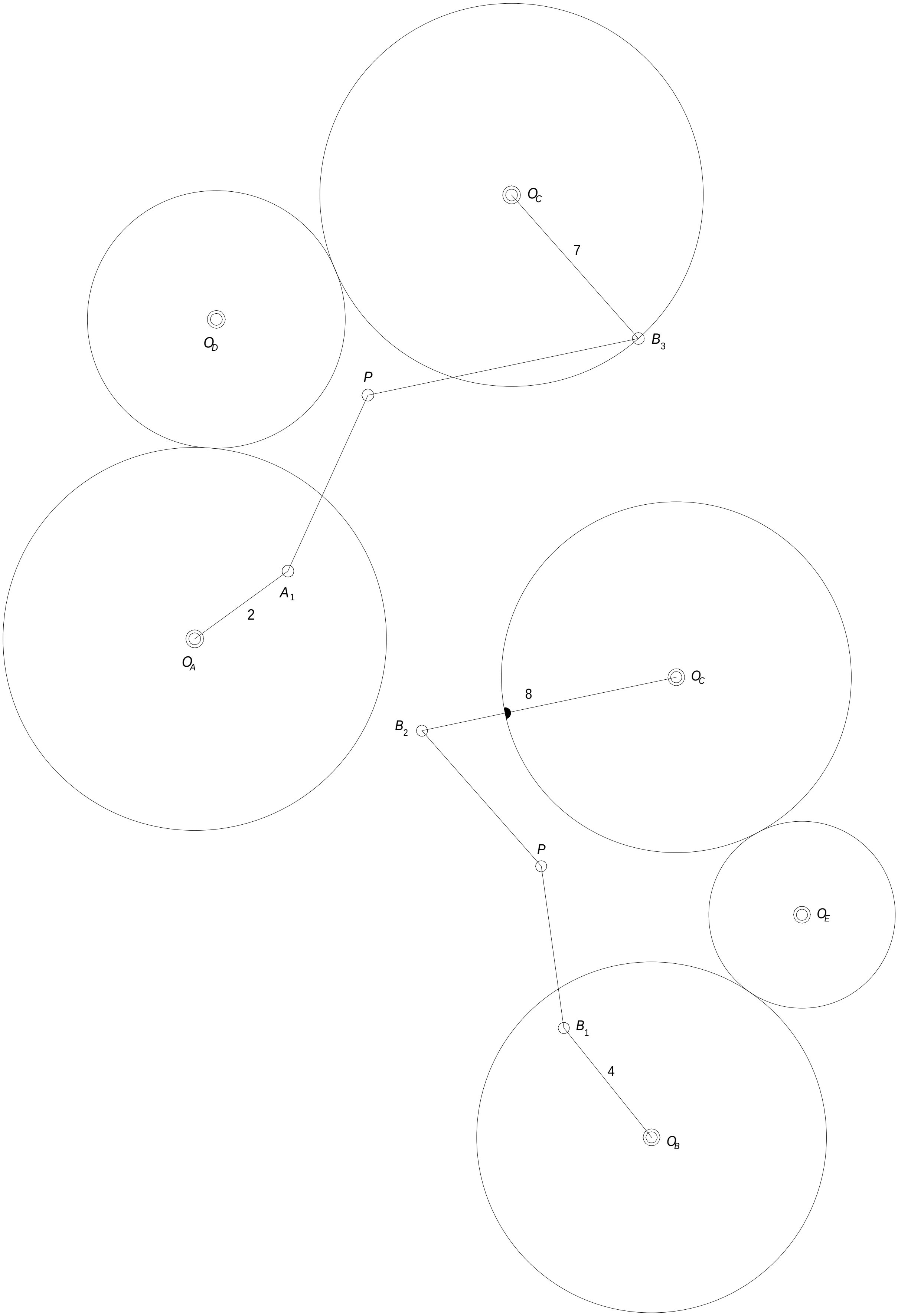
	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 39.500$	$L_{1AC} = 73.357$	$L_{1BC} = 65.655$
Crank length	$L_2 = 15.500$	$L_{10} = 26.000$	$L_7 = 25.763$
Coupler length	$L_3 = 14.000$	$L_9 = 28.786$	$L_6 = 33.243$

Rocker length	$L_4 = 20.000$	$L_8 = 37.143$	$L_5 = 23.270$
Coupler point	$A1P = 26.000$	$A2P = 15.500$	$A3P = 20.000$
Coupler angle	$\delta_1 = 63.000\,deg$	$\delta_2 = -63.000\,deg$	$\delta_3 = 84.584\,deg$



4. The three geared fivebar cognates can be seen in the Roberts diagram. They are: $O_AA_2PA_3O_B$, $O_AA_1PB_3O_C$, and $O_BB_1PB_2O_C$. They are shown individually below with their associated gears.



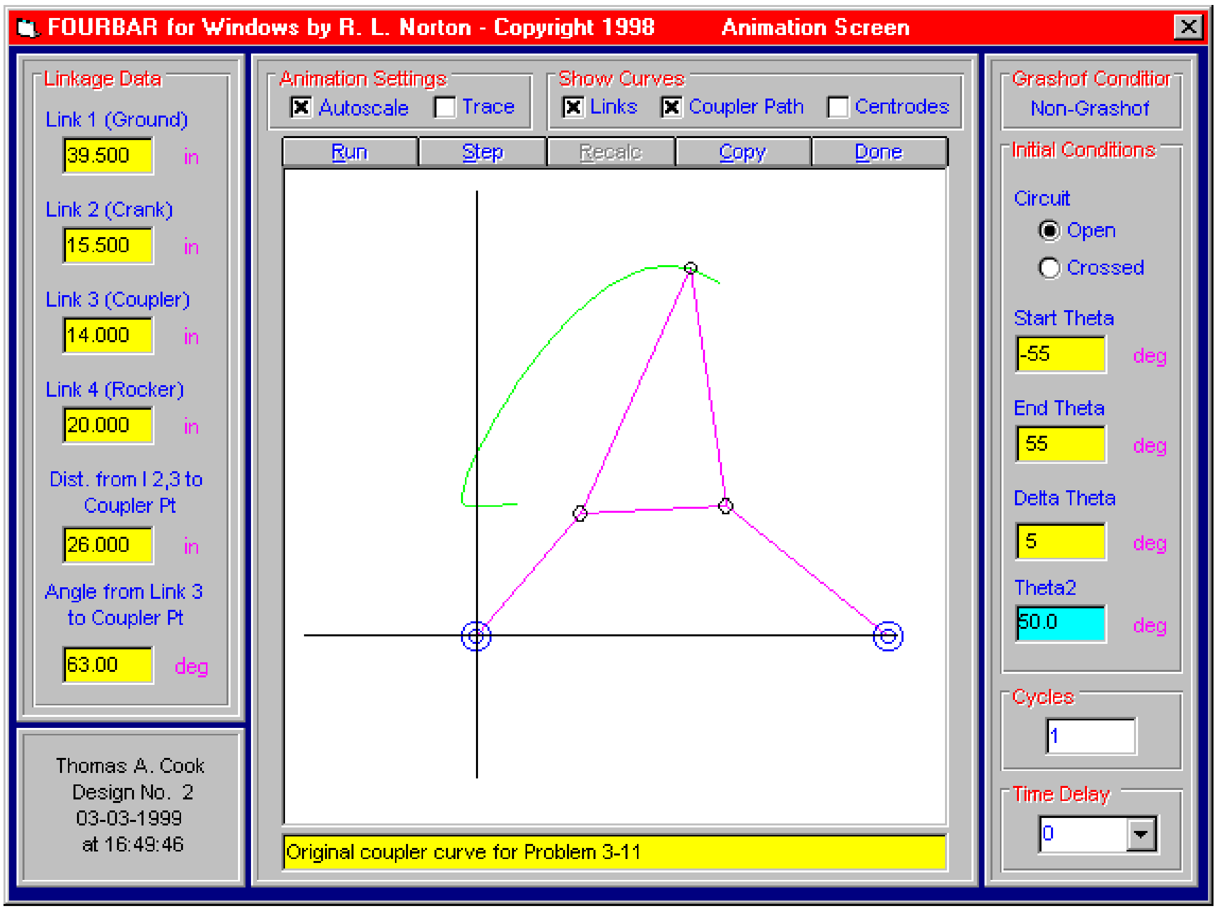


O_C
7
O_D
B_3
P
A_1
2
O_A
O_C
8
B_2
P
O_E
B_1
4
O_B

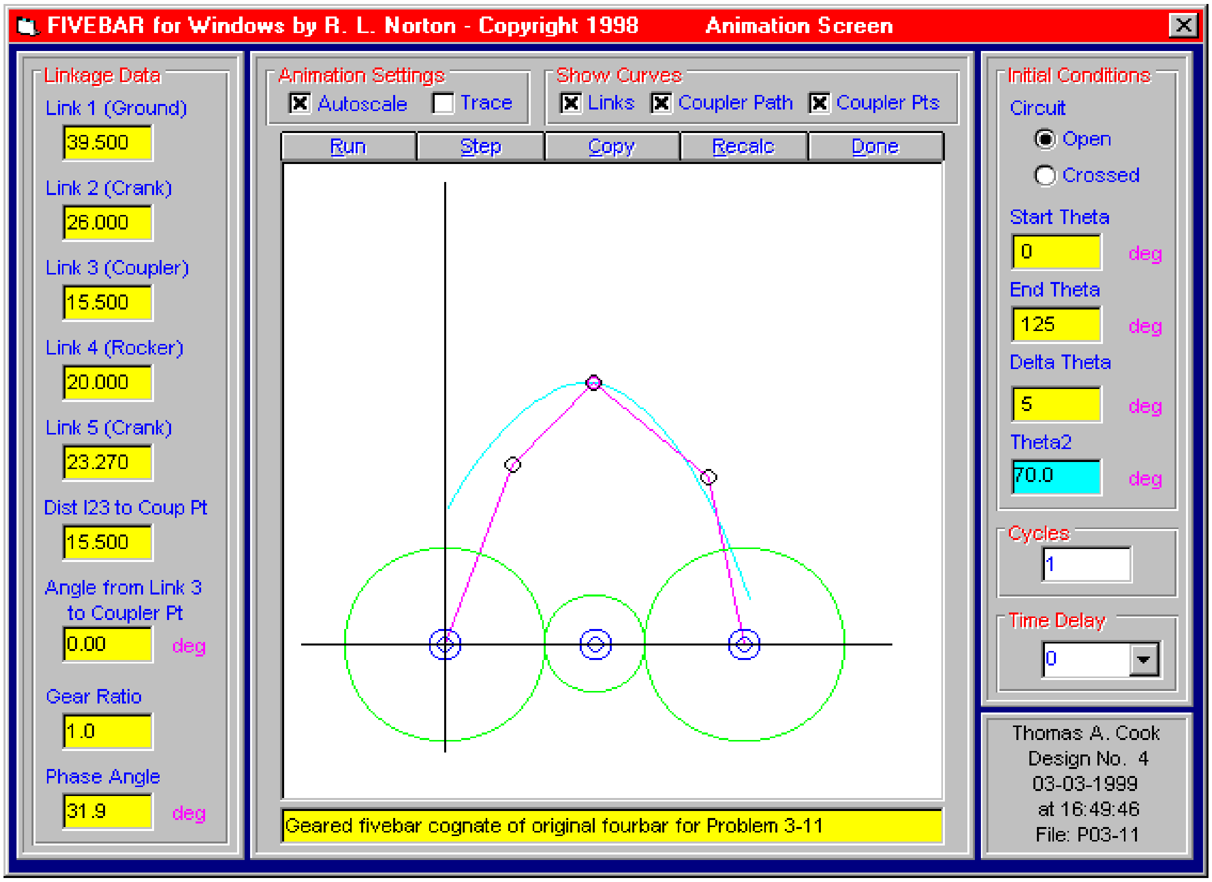
SUMMARY OF GEARED FIVEBAR COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 39.500$	$L_{1AC} = 73.357$	$L_{1BC} = 65.655$
Crank length	$L_{10} = 26.000$	$L_2 = 15.500$	$L_4 = 20.000$
Coupler length	$A2P = 15.500$	$A1P = 26.000$	$L_5 = 23.270$
Rocker length	$A3P = 20.000$	$L_8 = 37.143$	$L_7 = 25.763$
Crank length	$L_5 = 23.270$	$L_7 = 25.763$	$L_8 = 37.143$
Coupler point	$A2P = 15.500$	$A1P = 26.000$	$B1P = 23.270$
Coupler angle	$\delta_1 := 0.00 \cdot deg$	$\delta_2 := 0.00 \cdot deg$	$\delta_3 := 0.00 \cdot deg$

5. Enter the cognate #1 specifications into program FOURBAR to get a trace of the coupler path.



6. Enter the geared fivebar cognate #1 specifications into program FIVEBAR to get a trace of the coupler path for the geared fivebar (see next page).


FIVEBAR for Windows by R. L. Norton - Copyright 1998
Animation Screen
Linkage Data
Link 1 (Ground)
39.500
Link 2 (Crank)
26.000
Link 3 (Coupler)
15.500
Link 4 (Rocker)
20.000
Link 5 (Crank)
23.270
Dist I23 to Coup Pt
15.500
Angle from Link 3 to Coupler Pt
0.00
deg
Gear Ratio
1.0
Phase Angle
31.9
deg
Animation Settings
Autoscale
Trace
Show Curves
Links
Coupler Path
Coupler Pts
Run
Step
Copy
Recalc
Done
Geared fivebar cognate of original fourbar for Problem 3-11
Initial Conditions
Circuit
Open
Crossed
Start Theta
0
deg
End Theta
125
deg
Delta Theta
5
deg
Theta2
70.0
deg
Cycles
1
Time Delay
0
Thomas A. Cook
Design No. 4
03-03-1999
at 16:49:46
File: P03-11

PROBLEM 3-12

Statement: Design a sixbar, single-dwell linkage for a dwell of 90 deg of crank motion, with an output rocker motion of 45 deg.

Given: Crank dwell period: 90 deg.
Output rocker motion: 45 deg.

Solution: See Figures 3-20, 3-21, and Mathcad file P0312.

Design choices:

Ground link ratio, $L_1/L_2 = 2.0$: $GLR := 2.0$

Common link ratio, $L_3/L_2 = L_4/L_2 = BP/L_2 = 2.5$: $CLR := 2.5$

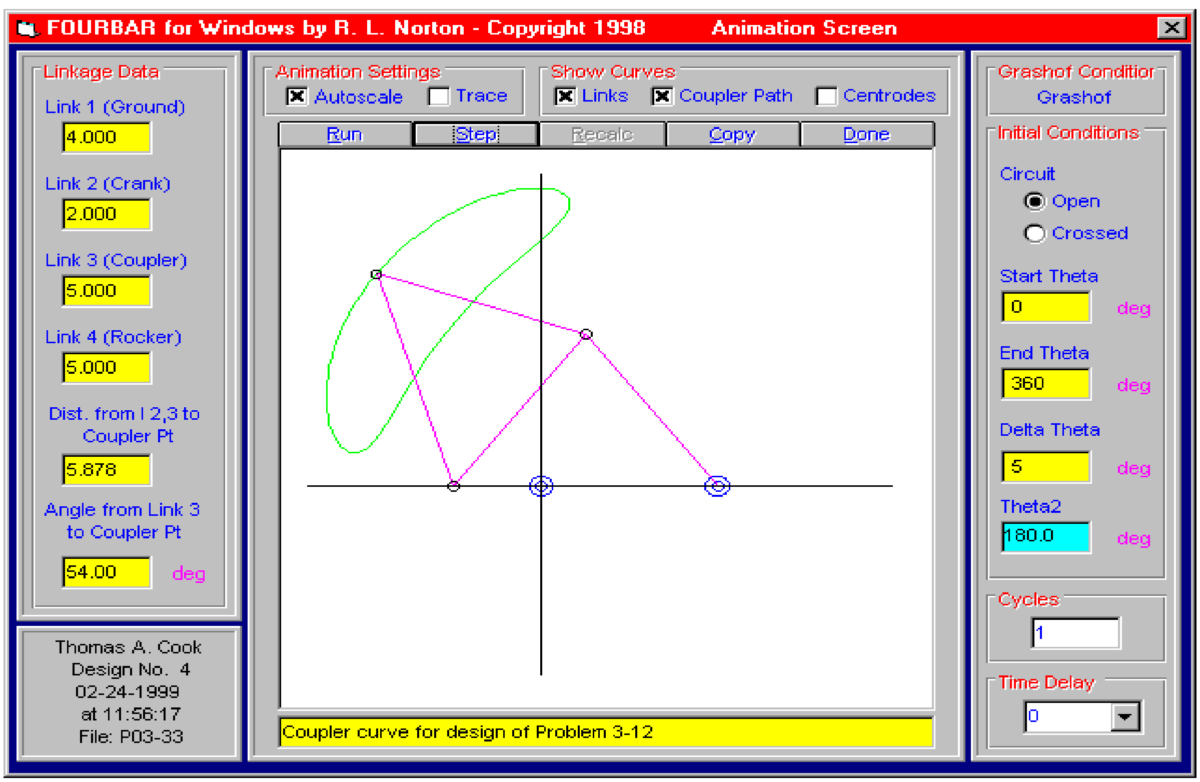
Coupler angle, $\gamma := 72 \cdot deg$

Crank length, $L_2 := 2.000$

1. For the given design choices, determine the remaining link lengths and coupler point specification.

Coupler link (3) length	$L_3 := CLR \cdot L_2$	$L_3 = 5.000$
Rocker link (4) length	$L_4 := CLR \cdot L_2$	$L_4 = 5.000$
Ground link (1) length	$L_1 := GLR \cdot L_2$	$L_1 = 4.000$
Angle *PAB*	$\delta := \dfrac{180 \cdot deg - \gamma}{2}$	$\delta = 54.000\,deg$
Length *AP* on coupler	$AP := 2 \cdot L_3 \cdot \cos(\delta)$	$AP = 5.878$

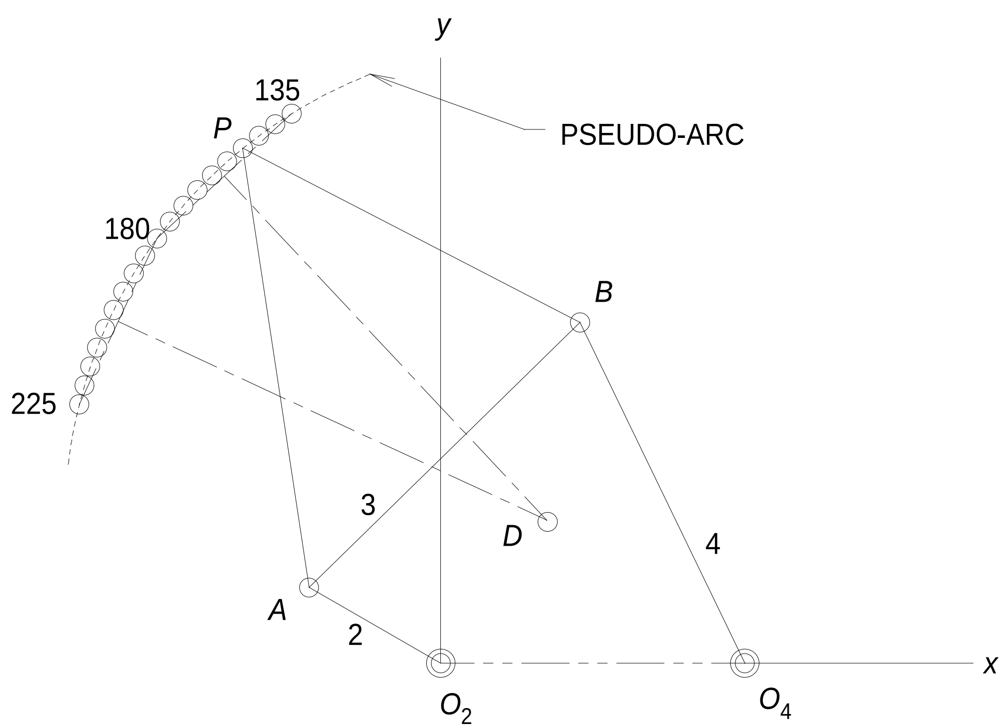
2. Enter the above data into program FOURBAR, plot the coupler curve, and determine the coordinates of the coupler curve in the selected range of crank motion, which in this case will be from 135 to 225 deg..



FOURBAR for Windows File P03-12.DAT

Angle Step Deg	Coupler Pt X	Coupler Pt Y	Coupler Pt Mag	Coupler Pt Ang
135	-1.961	7.267	7.527	105.099
140	-2.178	7.128	7.453	106.992
145	-2.393	6.977	7.375	108.930
150	-2.603	6.813	7.293	110.911
155	-2.809	6.638	7.208	112.933
160	-3.008	6.453	7.119	114.994
165	-3.201	6.257	7.028	117.093
170	-3.386	6.052	6.935	119.228
175	-3.563	5.839	6.840	121.396
180	-3.731	5.617	6.744	123.595
185	-3.890	5.389	6.646	125.822
190	-4.038	5.155	6.548	128.075
195	-4.176	4.915	6.450	130.351
200	-4.302	4.671	6.351	132.646
205	-4.417	4.424	6.252	134.955
210	-4.520	4.175	6.153	137.274
215	-4.610	3.924	6.054	139.598
220	-4.688	3.673	5.956	141.921
225	-4.753	3.424	5.858	144.235

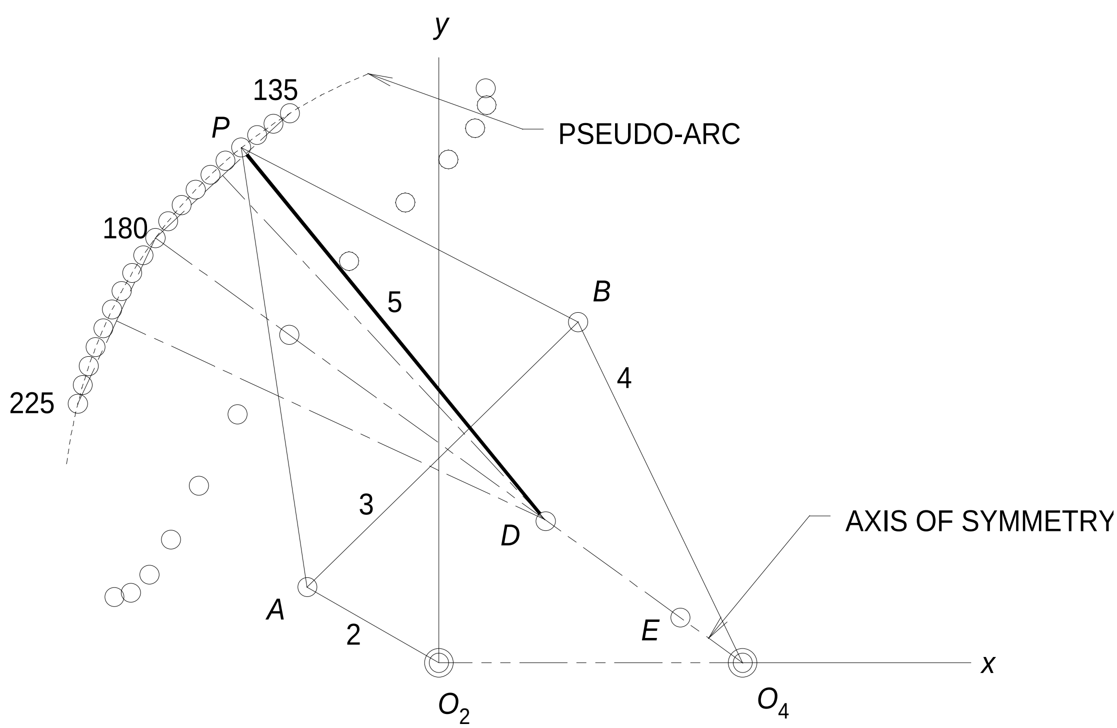
3. Layout this linkage to scale, including the coupler curve whose coordinates are in the table above. Use the points at crank angles of 135, 180, and 225 deg to define the pseudo-arc. Find the center of the pseudo-arc erecting perpendicular bisectors to the chords defined by the selected coupler curve points. The center will lie at the intersection of the perpendicular bisectors, label this point *D*. The radius of this circle is the length of link 5.



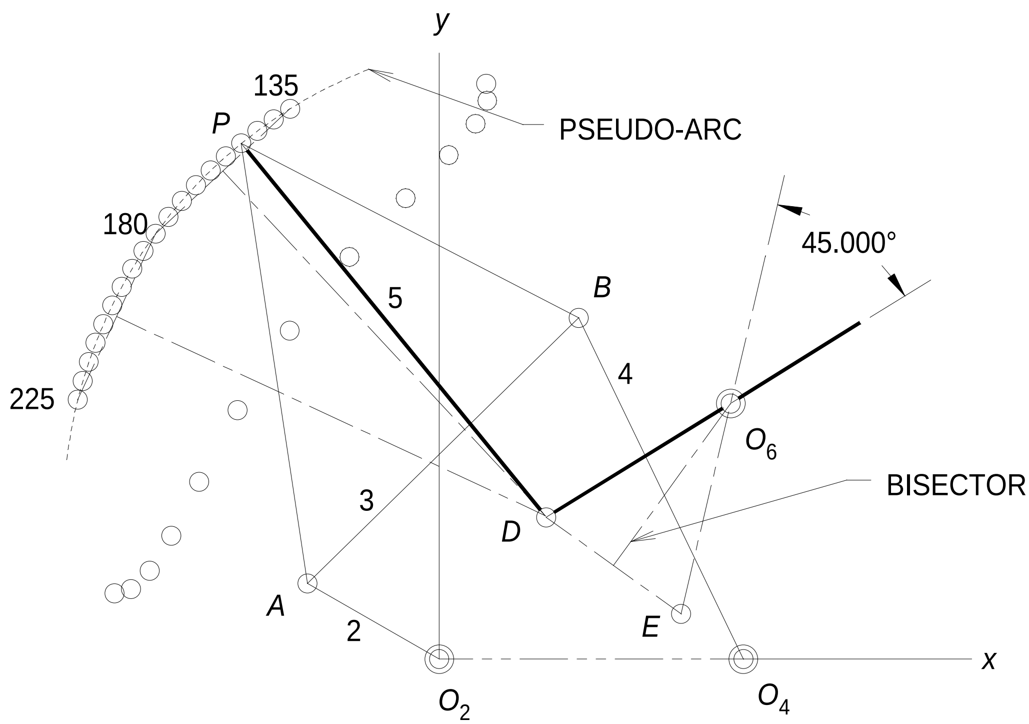
4. The position of the end of link 5 at point *D* will remain nearly stationary while the crank moves from 135 to 225 deg. As the crank motion causes the coupler point to move around the coupler curve there will be another extreme position of the end of link 5 that was originally at *D*. Since a symmetrical linkage was chosen, the other extreme position will be located along a line through the axis of symmetry (see Figure 3-20) a distance equal to the length of link 5 measured from the point where the axis of symmetry intersects the coupler curve near the 0 deg coupler point. Establish this point and label it *E*.

FOURBAR for Windows File P03-12.DAT

Angle Step Deg	Coupler Pt X	Coupler Pt Y	Coupler Pt Mag	Coupler Pt Ang
300	-4.271	0.869	4.359	168.495
310	-4.054	0.926	4.158	167.133
320	-3.811	1.165	3.985	162.998
330	-3.526	1.628	3.883	155.215
340	-3.159	2.343	3.933	143.437
350	-2.651	3.286	4.222	128.892
0	-1.968	4.336	4.762	114.414
10	-1.181	5.310	5.440	102.534
20	-0.441	6.085	6.101	94.142
30	0.126	6.654	6.656	88.914
40	0.478	7.068	7.085	86.129
50	0.631	7.373	7.400	85.111
60	0.617	7.598	7.623	85.354



5. The line segment *DE* represents the maximum displacement that a link of the length equal to link 5, attached at *P*, will reach along the axis of symmetry. Construct a perpendicular bisector of the line segment *DE* and extend it to the right (or left, which ever is convenient). Locate fixed pivot O_6 on the bisector of *DE* such that the lines O_6D and O_6E subtend the desired output angle, in this case 45 deg. Draw link 6 from *D* through O_6 and extend it to any convenient length. This is the output link that will dwell during the specified motion of the crank. See next page for the completed layout and further linkage specifications.



SUMMARY OF LINKAGE SPECIFICATIONS

Original fourbar:

Ground link	$L_1 = 4.000$	
Crank	$L_2 = 2.000$	
Coupler	$L_3 = 5.000$	
Rocker	$L_4 = 5.000$	
Coupler point	$AP = 5.878$	$\delta = 54.000\,deg$

Added dyad:

Coupler	$L_5 := 6.363$	
Output	$L_6 := 2.855$	
Pivot O_6	$x := 3.833$	$y := 3.375$

PROBLEM 3-13

Statement: Design a sixbar double-dwell linkage for a dwell of 90 deg of crank motion, with an output of rocker motion of 60 deg, followed by a second dwell of about 60 deg of crank motion.

Given: Initial crank dwell period: 90 deg
Final crank dwell period: 60 deg (approx.)
Output rocker motion between dwells: 60 deg

Solution: See Mathcad file P0313.

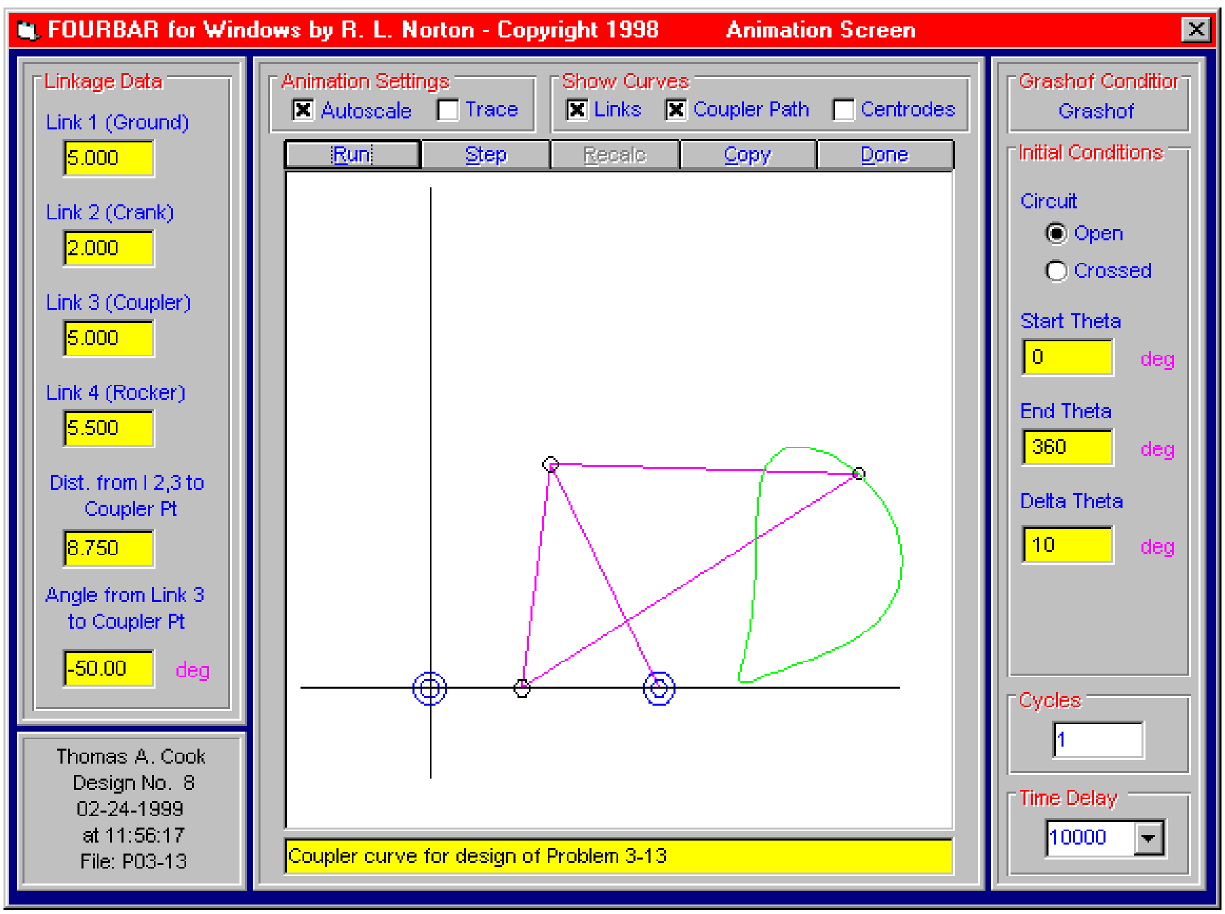
Design choices:

Ground link length $L_1 := 5.000$ Crank length $L_2 := 2.000$

Coupler link length $L_3 := 5.000$ Rocker length $L_2 := 5.500$

Coupler point data: $AP := 8.750$ $\delta := -50 \cdot deg$

1. In the absence of a linkage atlas it is difficult to find a coupler curve that meets the specifications. One approach is to start with a symmetrical linkage, using the data in Figure 3-21. Then, using program FOURBAR and by trial-and-error, adjust the link lengths and coupler point data until a satisfactory coupler curve is found. The link lengths and coupler point data given above were found this way. The resulting coupler curve is shown below and a printout of the coupler curve coordinates taken from FOURBAR is also printed below.



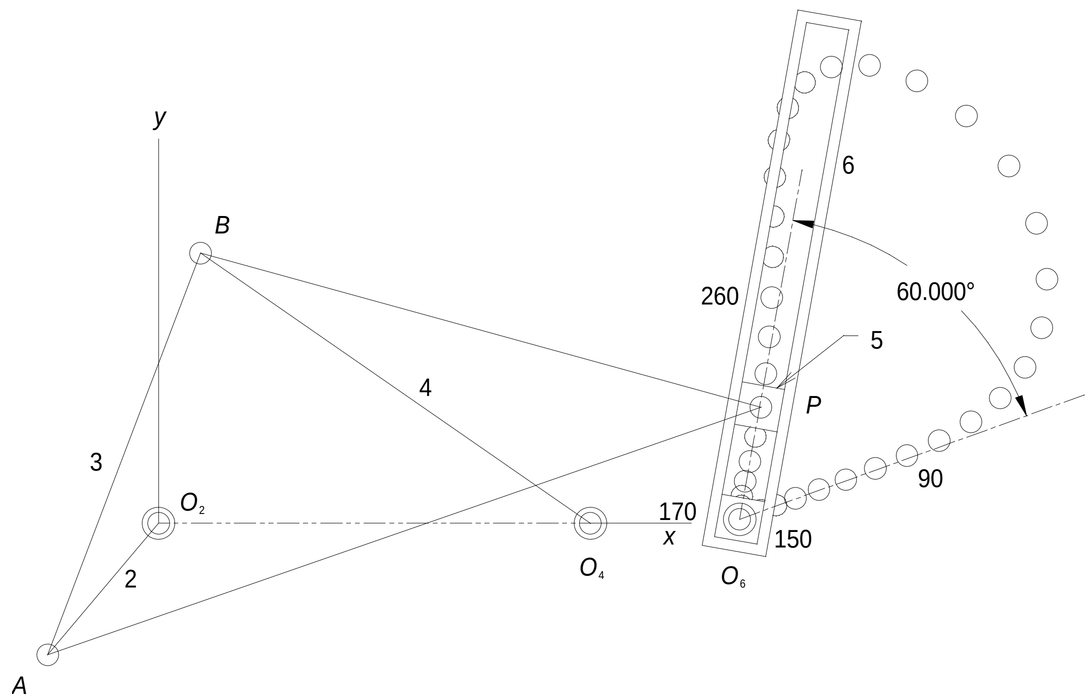
FOURBAR for Windows File P03-13.DAT

Angle Step Deg	Cpler Pt X	Cpler Pt Y	Cpler Pt Mag	Cpler Pt Ang
0.000	9.353	4.742	10.487	26.886
10.000	9.846	4.159	10.688	22.900
20.000	10.167	3.491	10.750	18.951
30.000	10.286	2.840	10.671	15.437
40.000	10.226	2.274	10.476	12.537
50.000	10.031	1.815	10.194	10.257
60.000	9.746	1.457	9.854	8.503
70.000	9.406	1.180	9.480	7.152
80.000	9.039	0.963	9.090	6.081
90.000	**8.665**	**0.787**	**8.701**	**5.187**
100.000	**8.301**	**0.637**	**8.325**	**4.391**
110.000	**7.958**	**0.507**	**7.974**	**3.644**
120.000	**7.647**	**0.391**	**7.657**	**2.928**
130.000	**7.376**	**0.291**	**7.382**	**2.256**
140.000	**7.151**	**0.209**	**7.154**	**1.671**
150.000	**6.977**	**0.151**	**6.978**	**1.242**
160.000	6.853	0.126	6.854	1.051
170.000	**6.778**	**0.140**	**6.779**	**1.182**
180.000	**6.748**	**0.201**	**6.751**	**1.708**
190.000	**6.755**	**0.316**	**6.763**	**2.678**
200.000	**6.792**	**0.488**	**6.809**	**4.110**
210.000	**6.847**	**0.719**	**6.885**	**5.996**
220.000	**6.912**	**1.008**	**6.985**	**8.300**
230.000	**6.976**	**1.351**	**7.105**	**10.963**
240.000	**7.031**	**1.741**	**7.243**	**13.911**
250.000	**7.073**	**2.170**	**7.398**	**17.057**
260.000	**7.099**	**2.626**	**7.569**	**20.302**
270.000	7.112	3.098	7.757	23.536
280.000	7.120	3.570	7.965	26.632
290.000	7.137	4.030	8.196	29.448
300.000	7.184	4.458	8.455	31.819
310.000	7.288	4.834	8.746	33.555
320.000	7.481	5.131	9.072	34.446
330.000	7.792	5.312	9.430	34.286
340.000	8.233	5.332	9.809	32.931
350.000	8.779	5.147	10.177	30.384
360.000	9.353	4.742	10.487	26.886

2. Layout this linkage to scale, including the coupler curve whose coordinates are in the table above. Fit tangent lines to the nearly straight portions of the curve. Label their intersection O_6. The coordinates of O_6 are (6.729, 0.046).

3. Design link 6 to lie along these straight tangents, pivoted at O_6. Provide a slot in link 6 to accommodate slider block 5, which pivots on the coupler point P. (See next page).

4. The beginning and ending crank angles for the dwell portions of the motion are indicated on the layout and in the table above by boldface entries.


y
B
6
260
60.000°
5
4
P
3
O2
170
90
x
150
2
O4
O6
A

PROBLEM 3-14

Statement: Figure P3-3 shows a treadle-operated grinding wheel driven by a fourbar linkage. Make a cardboard model of the linkage to any convenient scale. Determine its minimum transmission angles. Comment on its operation. Will it work? If so, explain how it does.

Given: Link lengths:

Link 2	$L_2 := 0.60 \cdot m$	Link 3	$L_3 := 0.75 \cdot m$
Link 4	$L_4 := 0.13 \cdot m$	Link 1	$L_1 := 0.90 \cdot m$

Grashof condition function:

$$Condition(a, b, c, d) := \left| \begin{array}{l} S \leftarrow min(a, b, c, d) \\ L \leftarrow max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return \text{ "Grashof" } \quad if \; SL < PQ \\ return \text{ "Special Grashof" } \quad if \; SL = PQ \\ return \text{ "non-Grashof" } \quad otherwise \end{array} \right.$$

Solution: See Mathcad file P0314.

1. Determine the Grashof condition of the mechanism from inequality 2.8 and its Barker classification from Table 2-4.

   Grashof condition: $Condition(L_1, L_2, L_3, L_4) = \text{"Grashof"}$

   Barker classification: Class I-4, Grashof rocker-rocker-crank, GRRC, since the shortest link is the output link.

2. As a Grashof rocker-crank, the minimum transmission angle will be 0 deg, twice per revolution of the output (link 4) crank.

3. Despite having transmission angles of 0 deg twice per revolution, the mechanism will work. That is, one will be able to drive the grinding wheel from the treadle (link 2). The reason is that the grinding wheel will act as a flywheel and will carry the linkage through the periods when the transmission angle is low. Typically, the operator will start the motion by rotating the wheel by hand.

PROBLEM 3-15

Statement: Figure P3-4 shows a non-Grashof fourbar linkage that is driven from link O_2A. All dimensions are in centimeters (cm).

(a) Find the transmission angle at the position shown.
(b) Find the toggle positions in terms of angle AO_2O_4.
(c) Find the maximum and minimum transmission angles over its range of motion.
(d) Draw the coupler curve of point P over its range of motion.

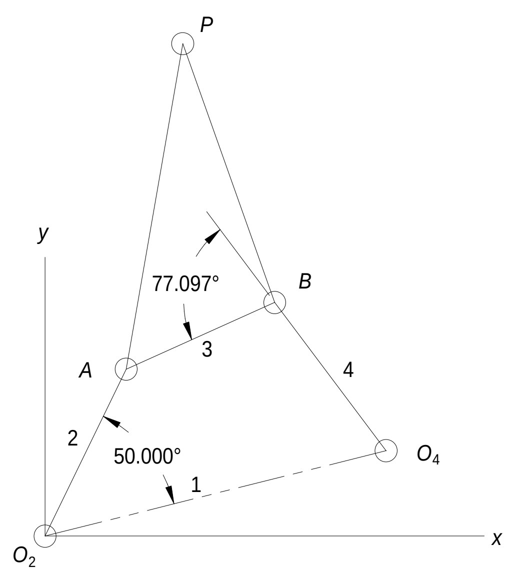
Given: Link lengths:

Link 1 (ground) $L_1 := 95 \cdot mm$ Link 2 (driver) $L_2 := 50 \cdot mm$

Link 3 (coupler) $L_3 := 44 \cdot mm$ Link 4 (driven) $L_4 := 50 \cdot mm$

Solution: See Figure P3-4 and Mathcad file P0315.

1. To find the transmission angle at the position shown, draw the linkage to scale in the position shown and measure the transmission angle ABO_4.



The measured transmission angle at the position shown is 77.097 deg.

2. The toggle positions will be symmetric with respect to the O_2O_4 axis and will occur when links 3 and 4 are colinear. Use the law of cosines to calculate the angle of link 2 when links 3 and 4 are in toggle.

$$(L_3 + L_4)^2 := L_1{}^2 + L_2{}^2 - 2 \cdot L_1 \cdot L_2 \cdot cos(\theta_2)$$

where θ_2 is the angle AO_2O_4. Solving for θ_2,

$$\theta_2 := acos\left[\frac{L_1{}^2 + L_2{}^2 - (L_3 + L_4)^2}{2 \cdot L_1 \cdot L_2}\right] \qquad \theta_2 = 73.558\, deg$$

The other toggle position occurs at $-\theta_2 = -73.558\, deg$

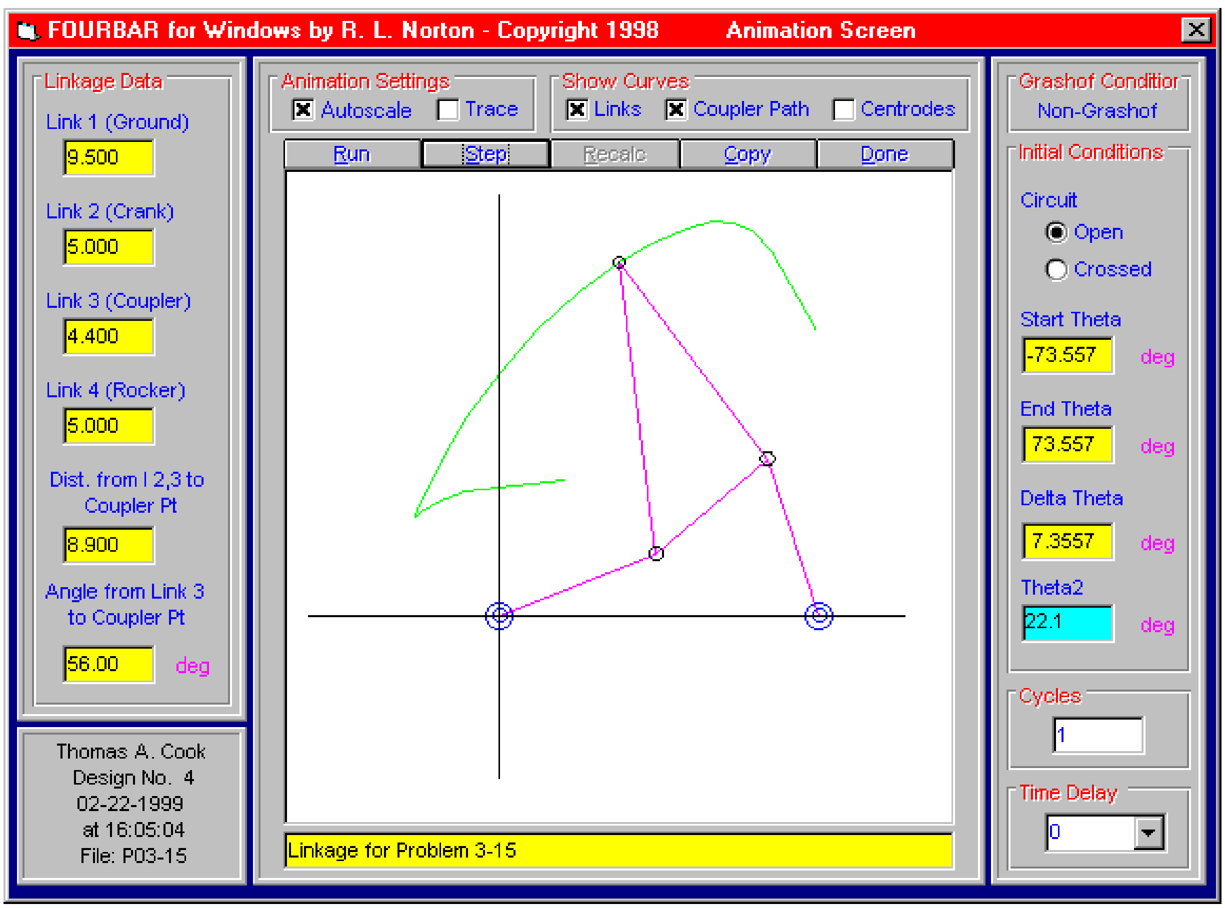
3. Use the program FOURBAR to find the maximum and minimum transmission angles.

FOURBAR for Windows File P03-15 Design # 1

Angle Step Deg	Theta2 Mag degrees	Theta3 Mag degrees	Theta4 Mag degrees	Trans Ang Mag degrees
-73.557	-73.557	30.861	-149.490	0.352
-58.846	-58.846	64.075	-176.312	60.387
-44.134	-44.134	77.168	170.696	86.472
-29.423	-29.423	83.147	157.514	74.367
-14.711	-14.711	80.604	142.103	61.499
0.000	0.000	68.350	125.123	56.773
14.711	14.711	50.145	111.644	61.499
29.423	29.423	32.106	106.473	74.367
44.134	44.134	16.173	109.701	86.472
58.846	58.846	0.566	120.179	60.387
73.557	73.557	-30.486	149.159	0.355

A partial output from FOURBAR is shown above. From it, we see that the maximum transmission angle is approximately 86.5 deg and the minimum is zero deg.

4. Use program FOURBAR to draw the coupler curve with respect to a coordinate frame through O_2O_4.



PROBLEM 3-16

Statement: Draw the Roberts diagram for the linkage in Figure P3-4 and find its two cognates. Are they Grashof or non-Grashof?

Given: Link lengths:

Ground link $L_1 := 9.5$ Crank $L_2 := 5$

Coupler $L_3 := 4.4$ Rocker $L_4 := 5$

Coupler point data: $A1P := 8.90$ $\delta_1 := 56.000 \cdot deg$

Solution: See Figure P3-4 and Mathcad file P0316.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle *APB*.

$$B1P := \left(L_3^2 + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1)\right)^{0.5} \qquad B1P = 7.401$$

$$\gamma_1 := acos\left(\frac{L_3^2 + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P}\right) \qquad \gamma_1 = 94.4701\,deg$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$L_5 := B1P$ $L_5 = 7.401$ $L_6 := \frac{L_4}{L_3} \cdot B1P$ $L_6 = 8.410$

$L_{10} := A1P$ $L_{10} = 8.900$ $L_9 := \frac{L_2}{L_3} \cdot A1P$ $L_9 = 10.114$

$L_7 := L_9 \cdot \frac{B1P}{A1P}$ $L_7 = 8.410$ $L_8 := L_6 \cdot \frac{A1P}{B1P}$ $L_8 = 10.114$

Calculate the coupler point data for cognates #2 and #3

$A3P := L_4$ $A3P = 5.000$ $A2P := L_2$ $A2P = 5.000$

$\delta_3 := \gamma_1$ $\delta_3 = 94.470\,deg$ $\delta_2 := -\delta_1$ $\delta_2 = -56.000\,deg$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

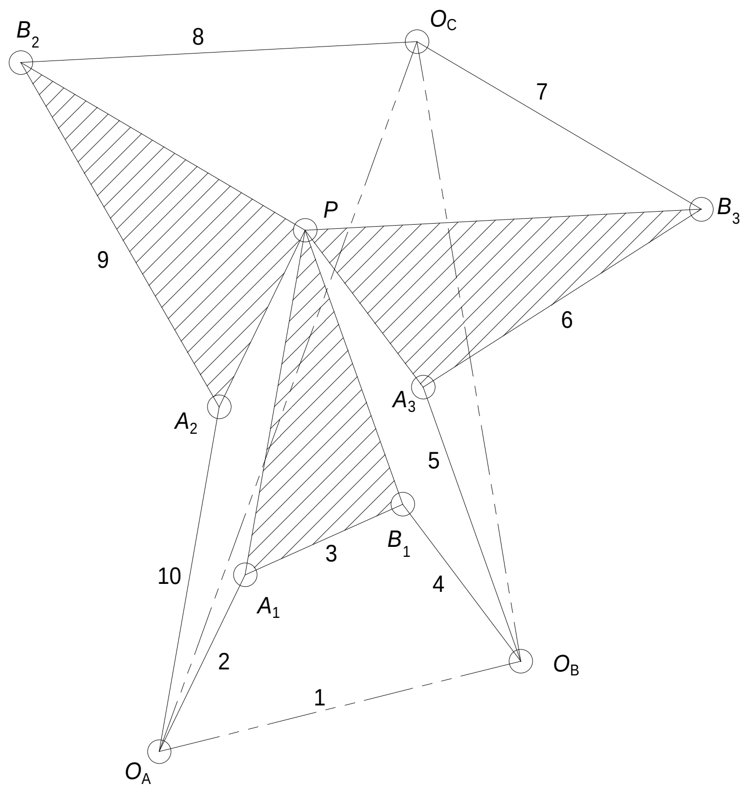
$L_{1BC} := \frac{L_1}{L_3} \cdot B1P$ $L_{1BC} = 15.9793$ $L_{1AC} := \frac{L_1}{L_3} \cdot A1P$ $L_{1AC} = 19.2159$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 9.500$	$L_{1AC} = 19.216$	$L_{1BC} = 15.979$
Crank length	$L_2 = 5.000$	$L_{10} = 8.900$	$L_7 = 8.410$
Coupler length	$L_3 = 4.400$	$L_9 = 10.114$	$L_6 = 8.410$

Rocker length	$L_4 = 5.000$	$L_8 = 10.114$	$L_5 = 7.401$
Coupler point	$A1P = 8.900$	$A2P = 5.000$	$A3P = 5.000$
Coupler angle	$\delta_1 = 56.000\,deg$	$\delta_2 = -56.000\,deg$	$\delta_3 = 94.470\,deg$



6. Determine the Grashof condition of each of the two additional cognates.

$$Condition(a,b,c,d) := \begin{array}{|l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"} \quad if\ SL < PQ \\ return\ \text{"Special Grashof"} \quad if\ SL = PQ \\ return\ \text{"non-Grashof"} \quad otherwise \end{array}$$

Cognate #2: $Condition(L_{10}, L_{1AC}, L_8, L_9) = \text{"non-Grashof"}$

Cognate #3: $Condition(L_5, L_{1BC}, L_6, L_7) = \text{"non-Grashof"}$

PROBLEM 3-17

Statement: Design a Watt-I sixbar to give parallel motion that follows the coupler path of point *P* of the linkage in Figure P3-4.

Given: Link lengths:

				Coupler point data:	
Ground link	$L_1 := 9.5$	Crank	$L_2 := 5$	$A1P := 8.90$	$\delta_1 := 56.000 \cdot deg$
Coupler	$L_3 := 4.4$	Rocker	$L_4 := 5$		

Solution: See Figure P3-4 and Mathcad file P0317.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle *APB*.

$$B1P := \left(L_3^{\,2} + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot cos(\delta_1)\right)^{0.5} \qquad B1P = 7.401$$

$$\gamma_1 := acos\left(\frac{L_3^{\,2} + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P}\right) \qquad \gamma_1 = 94.4701\ deg$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \qquad L_5 = 7.401 \qquad L_6 := \frac{L_4}{L_3} \cdot B1P \qquad L_6 = 8.410$$

$$L_{10} := A1P \qquad L_{10} = 8.900 \qquad L_9 := \frac{L_2}{L_3} \cdot A1P \qquad L_9 = 10.114$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \qquad L_7 = 8.410 \qquad L_8 := L_6 \cdot \frac{A1P}{B1P} \qquad L_8 = 10.114$$

Calculate the coupler point data for cognates #2 and #3

$$A3P := L_4 \qquad A3P = 5.000 \qquad A2P := L_2 \qquad A2P = 5.000$$

$$\delta_3 := \gamma_1 \qquad \delta_3 = 94.470\ deg \qquad \delta_2 := -\delta_1 \qquad \delta_2 = -56.000\ deg$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

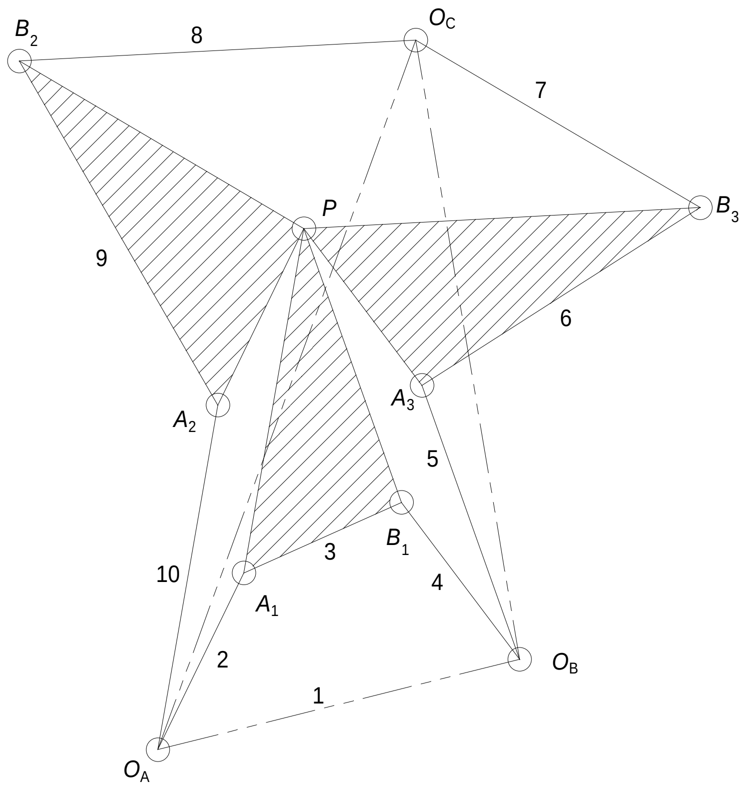
$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \qquad L_{1BC} = 15.9793 \qquad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \qquad L_{1AC} = 19.2159$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

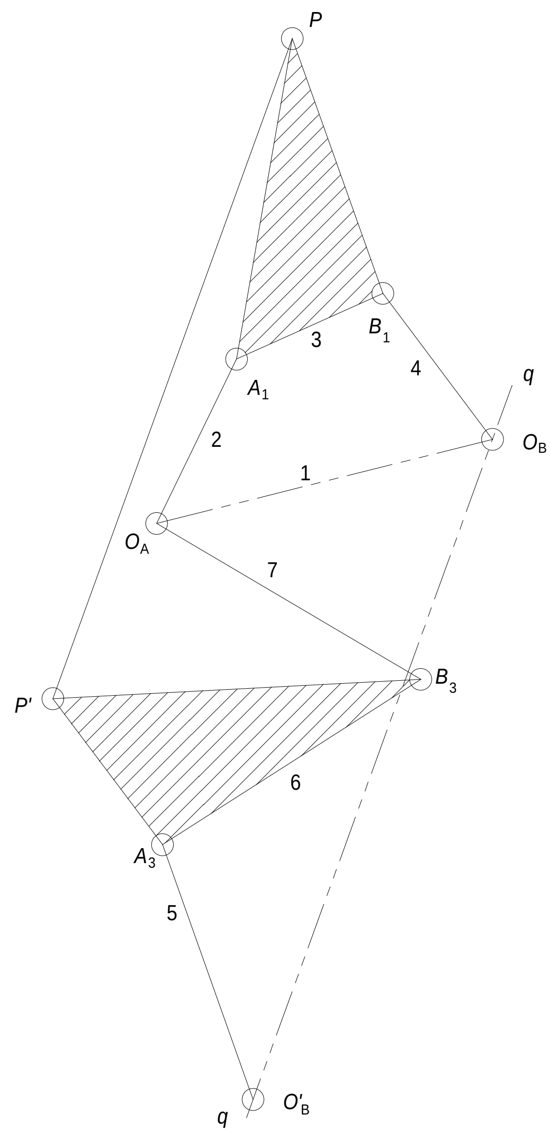
	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 9.500$	$L_{1AC} = 19.216$	$L_{1BC} = 15.979$
Crank length	$L_2 = 5.000$	$L_{10} = 8.900$	$L_7 = 8.410$
Coupler length	$L_3 = 4.400$	$L_9 = 10.114$	$L_6 = 8.410$

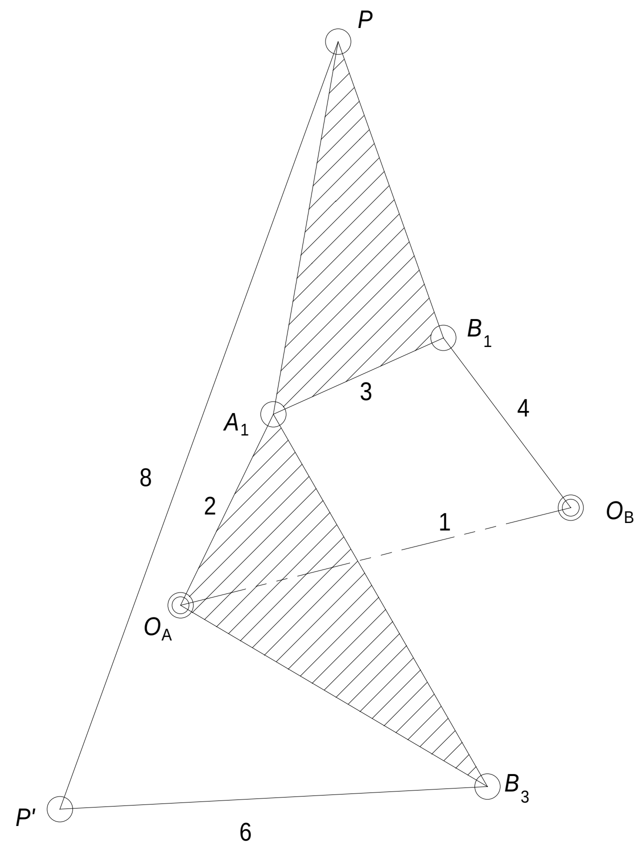
Rocker length	$L_4 = 5.000$	$L_8 = 10.114$	$L_5 = 7.401$
Coupler point	$A1P = 8.900$	$A2P = 5.000$	$A3P = 5.000$
Coupler angle	$\delta_1 = 56.000\,deg$	$\delta_2 = -56.000\,deg$	$\delta_3 = 94.470\,deg$



4. All three of these cognates are non-Grashof and will, therefore, have limited motion. However, following Example 3-11, discard cognate #2 and retain cognates #1 and #3. Draw line *qq* parallel to line O_AO_C and through point O_B. Without allowing links 5, 6, and 7 to rotate, slide them as an assembly along lines O_AO_C and *qq* until the free end of link 7 is at O_A. The free end of link 5 will then be at point O'_B and point *P* on link 6 will be at *P'*. Add a new link of length O_AO_C between *P* and *P'*. This is the new output link 8 and all points on it describe the original coupler curve.

5. Join links 2 and 7, making one ternary link. Remove link 5 and reduce link 6 to a binary link. The result is a Watt-I sixbar with links numbered 1, 2, 3, 4, 6, and 8 (see next page). Link 8 is in curvilinear translation and follows the coupler path of the original point *P*.




P
B_1
3
4
A_1
8
2
1
O_B
O_A
B_3
P'
6

PROBLEM 3-18

Statement: Design a Watt-I sixbar to give parallel motion that follows the coupler path of point *P* of the linkage in Figure P3-4 and add a driver dyad to drive it over its possible range of motion with no quick return. (The result will be an 8-bar linkage).

Given: Link lengths:

Ground link $L_1 := 9.5$ Crank $L_2 := 5$

Coupler $L_3 := 4.4$ Rocker $L_4 := 5$

Coupler point data: $A1P := 8.90$ $\delta_1 := 56.000 \cdot deg$

Solution: See Figure P3-4 and Mathcad file P0318.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle *APB*.

$$B1P := \left(L_3^{\,2} + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1)\right)^{0.5} \qquad B1P = 7.401$$

$$\gamma_1 := acos\left(\frac{L_3^{\,2} + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P}\right) \qquad \gamma_1 = 94.4701\,deg$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \qquad L_5 = 7.401 \qquad L_6 := \frac{L_4}{L_3} \cdot B1P \qquad L_6 = 8.410$$

$$L_{10} := A1P \qquad L_{10} = 8.900 \qquad L_9 := \frac{L_2}{L_3} \cdot A1P \qquad L_9 = 10.114$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \qquad L_7 = 8.410 \qquad L_8 := L_6 \cdot \frac{A1P}{B1P} \qquad L_8 = 10.114$$

Calculate the coupler point data for cognates #2 and #3

$$A3P := L_4 \qquad A3P = 5.000 \qquad A2P := L_2 \qquad A2P = 5.000$$

$$\delta_3 := \gamma_1 \qquad \delta_3 = 94.470\,deg \qquad \delta_2 := -\delta_1 \qquad \delta_2 = -56.000\,deg$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

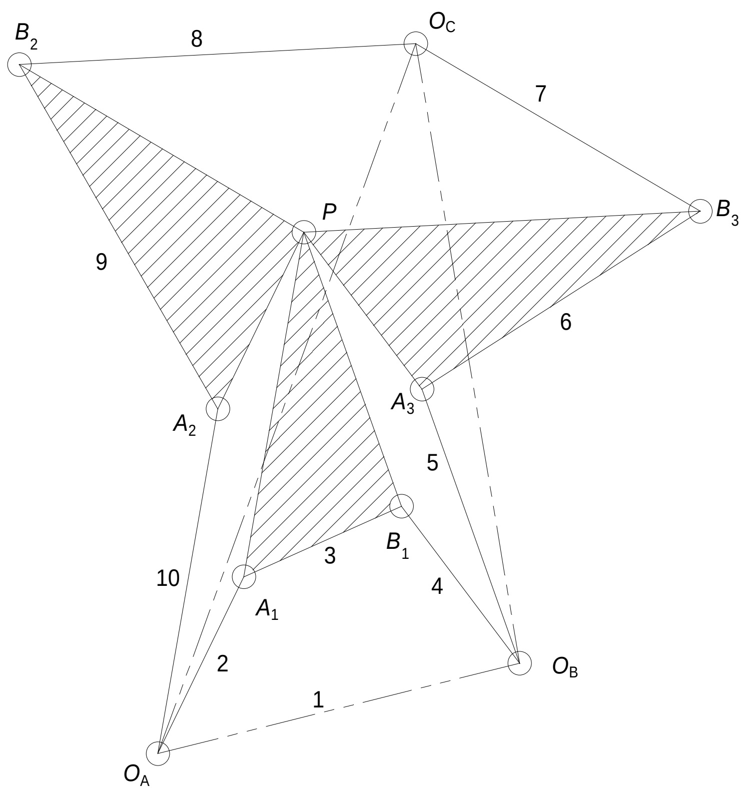
$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \qquad L_{1BC} = 15.9793 \qquad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \qquad L_{1AC} = 19.2159$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 9.500$	$L_{1AC} = 19.216$	$L_{1BC} = 15.979$
Crank length	$L_2 = 5.000$	$L_{10} = 8.900$	$L_7 = 8.410$
Coupler length	$L_3 = 4.400$	$L_9 = 10.114$	$L_6 = 8.410$

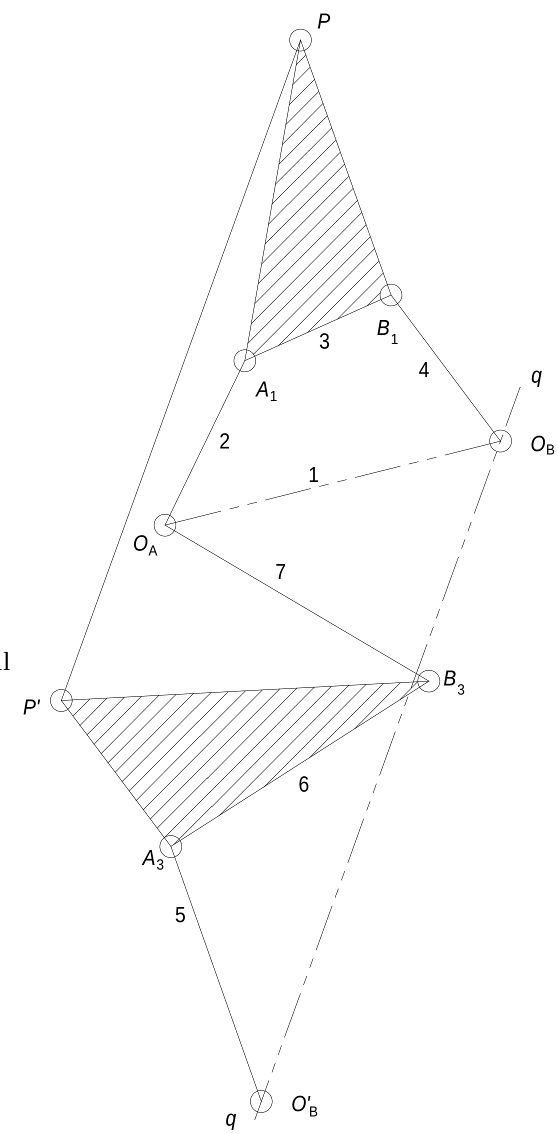
Rocker length	$L_4 = 5.000$	$L_8 = 10.114$	$L_5 = 7.401$
Coupler point	$A1P = 8.900$	$A2P = 5.000$	$A3P = 5.000$
Coupler angle	$\delta_1 = 56.000\ deg$	$\delta_2 = -56.000\ deg$	$\delta_3 = 94.470\ deg$

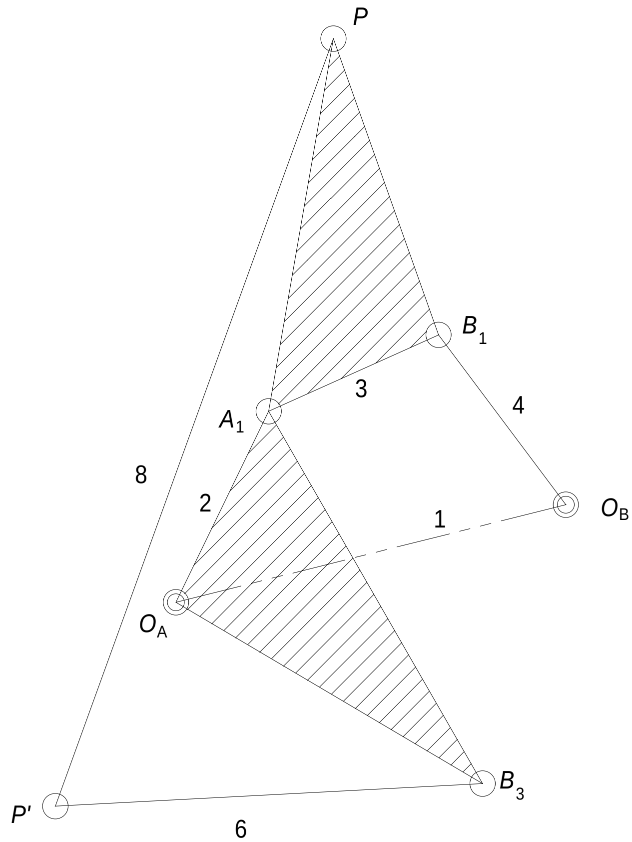


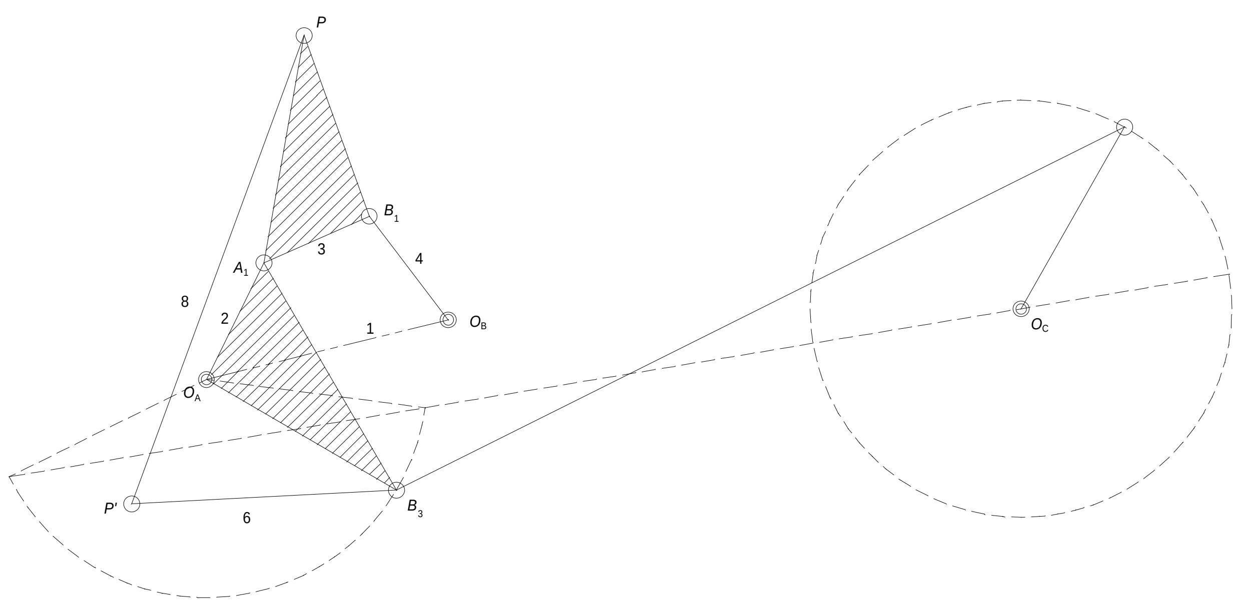
4. All three of these cognates are non-Grashof and will, therefore, have limited motion. However, following Example 3-11, discard cognate #2 and retain cognates #1 and #3. Draw line *qq* parallel to line O_AO_C and through point O_B. Without allowing links 5, 6, and 7 to rotate, slide them as an assembly along lines O_AO_C and *qq* until the free end of link 7 is at O_A. The free end of link 5 will then be at point O'_B and point *P* on link 6 will be at *P'*. Add a new link of length O_AO_C between *P* and *P'*. This is the new output link 8 and all points on it describe the original coupler curve.

5. Join links 2 and 7, making one ternary link. Remove link 5 and reduce link 6 to a binary link. The result is a Watt-I sixbar with links numbered 1, 2, 3, 4, 6, and 8 (see next page). Link 8 is in curvilinear translation and follows the coupler path of the original point *P*.

6. Add a driver dyad following Example 3-4.




P
B1
3
4
A1
8
2
1
OB
OA
B3
P'
6


P
B1
3
4
A1
8
2
1
OB
OA
OC
P'
6
B3

PROBLEM 3-19

Statement: Design a pin-jointed linkage that will guide the forks of the fork lift truck in Figure P3-5 up and down in an approximate straight line over the range of motion shown. Arrange the fixed pivots so they are close to some part of the existing frame or body of the truck.

Given: Length of straight line motion of the forks: $\Delta x := 1800 \cdot mm$

Solution: See Figure P3-5 and Mathcad file P0319.

Design choices:

Use a Hoeken-type straight line mechanism optimized for straightness.
Maximum allowable error in straightness of line: $\Delta C_y := 0.096 \cdot \%$

1. Using Table 3-1 and the required length of straight-line motion, determine the link lengths.

Link ratios from Table 3-1 for $\Delta C_y = 0.096\,\%$:

$L1overL2 := 2.200$ $L3overL2 := 2.800$ $\Delta xoverL2 := 4.181$

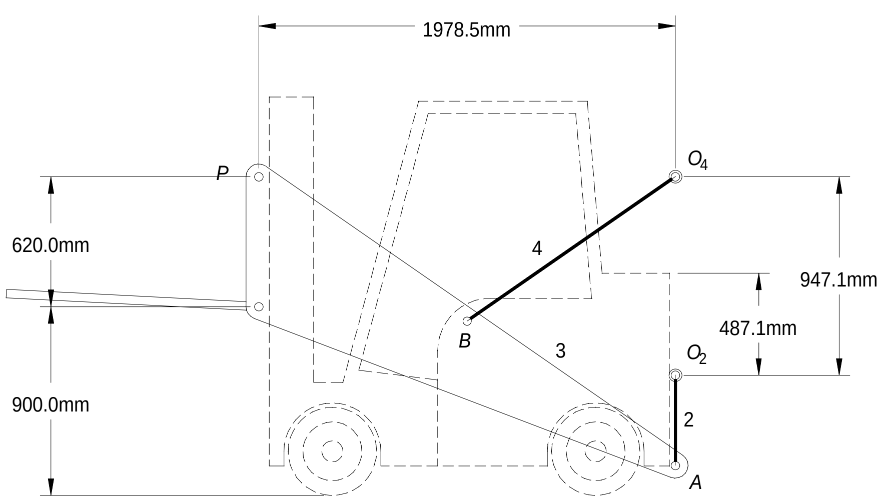
Link lengths:

Crank	$L_2 := \dfrac{\Delta x}{\Delta xoverL2}$	$L_2 = 430.5\,mm$
Coupler	$L_3 := L3overL2 \cdot L_2$	$L_3 = 1205.5\,mm$
Ground link	$L_1 := L1overL2 \cdot L_2$	$L_1 = 947.1\,mm$
Rocker	$L_4 := L_3$	$L_4 = 1205.5\,mm$
Coupler point	$BP := L_3$	$BP = 1205.5\,mm$

2. Calculate the distance from point P to pivot O_4 (C_y).

$$C_y := \sqrt{(2 \cdot L_3)^2 - (L_1 + L_2)^2} \qquad C_y = 1978.5\,mm$$

3. Draw the fork lift truck to scale with the mechanism defined in step 1 superimposed on it..



PROBLEM 3-20

Statement: Figure P3-6 shows a "V-link" off-loading mechanism for a paper roll conveyor. Design a pin-jointed linkage to replace the air cylinder driver that will rotate the rocker arm and V-link through the 90 deg motion shown. Keep the fixed pivots as close to the existing frame as possible. Your fourbar linkage should be Grashof and be in toggle at each extreme position of the rocker arm.

Given: Dimensions scaled from Figure P3-6:

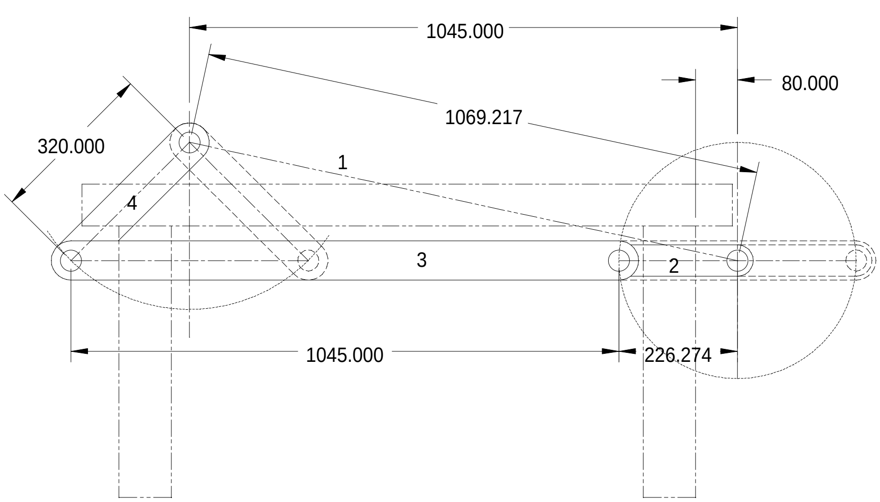
Rocker arm (link 4) distance between pin centers: $L_4 := 320 \cdot mm$

Solution: See Figure P3-6 and Mathcad file P0320.

Design choices:

1. Use the same rocker arm that was used with the air cylinder driver.
2. Place the pivot O_2 80 mm to the right of the right leg and on a horizontal line with the center of the pin on the rocker arm.
3. Design for two-position, 90 deg of output rocker motion with no quick return, similar to Example 3-2.

1. Draw the rocker arm (link 4) O_4B in both extreme positions, B_1 and B_2, in any convenient location such that the desired angle of motion θ_4 is subtended. In this solution, link 4 is drawn such that the two extreme positions each make an angle of 45 deg to the vertical.
2. Draw the chord B_1B_2 and extend it in any convenient direction. In this solution it was extended horizontally to the left.
3. Mark the center O_2 on the extended line such that it is 80 mm to the right of the right leg. This will allow sufficient space for a supporting pillow block bearing.
4. Bisect the line segment B_1B_2 and draw a circle of that radius about O_2.
5. Label the two intersections of the circle and extended line B_1B_2, A_1 and A_2.
6. Measure the length of the coupler (link 3) as A_1B_1 or A_2B_2. From the graphical solution, $L_3 := 1045 \cdot mm$
7. Measure the length of the crank (link 2) as O_2A_1 or O_2A_2. From the graphical solution, $L_2 := 226.274 \cdot mm$
8. Measure the length of the ground link (link 1) as O_2O_4. From the graphical solution, $L_1 := 1069.217 \cdot mm$



9. Find the Grashof condition.

$$Condition(a,b,c,d) := \left| \begin{array}{l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"} \quad if\ SL < PQ \\ return\ \text{"Special Grashof"} \quad if\ SL = PQ \\ return\ \text{"non-Grashof"} \quad otherwise \end{array} \right.$$

$$Condition\left(L_1, L_2, L_3, L_4\right) = \text{"Grashof"}$$

PROBLEM 3-21

Statement: Figure P3-7 shows a walking-beam transport mechanism that uses a fourbar coupler curve, replicated with a parallelogram linkage for parallel motion. Note the duplicate crank and coupler shown ghosted in the right half of the mechanism - they are redundant and have been removed from the duplicate fourbar linkage. Using the same fourbar driving stage (links 1, 2, 3, 4 with coupler point P), design a Watt-I sixbar linkage that will drive link 8 in the same parallel motion using two fewer links.

Given: Link lengths: Coupler point data:

Ground link $L_1 := 2.22$ Crank $L_2 := 1$ $A1P := 3.06$ $\delta_1 := 31.000 \cdot deg$

Coupler $L_3 := 2.06$ Rocker $L_4 := 2.33$

Solution: See Figure P3-7 and Mathcad file P0321.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle *APB*.

$$B1P := \left(L_3^{\,2} + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot cos(\delta_1)\right)^{0.5} \qquad B1P = 1.674$$

$$\gamma_1 := acos\left(\frac{L_3^{\,2} + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P}\right) \qquad \gamma_1 = 109.6560\,deg$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$L_5 := B1P$ $L_5 = 1.674$ $L_6 := \frac{L_4}{L_3} \cdot B1P$ $L_6 = 1.893$

$L_{10} := A1P$ $L_{10} = 3.060$ $L_9 := \frac{L_2}{L_3} \cdot A1P$ $L_9 = 1.485$

$L_7 := L_9 \cdot \frac{B1P}{A1P}$ $L_7 = 0.812$ $L_8 := L_6 \cdot \frac{A1P}{B1P}$ $L_8 = 3.461$

Calculate the coupler point data for cognates #2 and #3

$A3P := L_8$ $A3P = 3.461$ $A2P := L_2$ $A2P = 1.000$

$\delta_3 := -\left[180 \cdot deg - \left(\delta_1 + \gamma_1\right)\right]$ $\delta_2 := -\delta_1$ $\delta_2 = -31.000\,deg$

$\delta_3 = -39.344\,deg$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

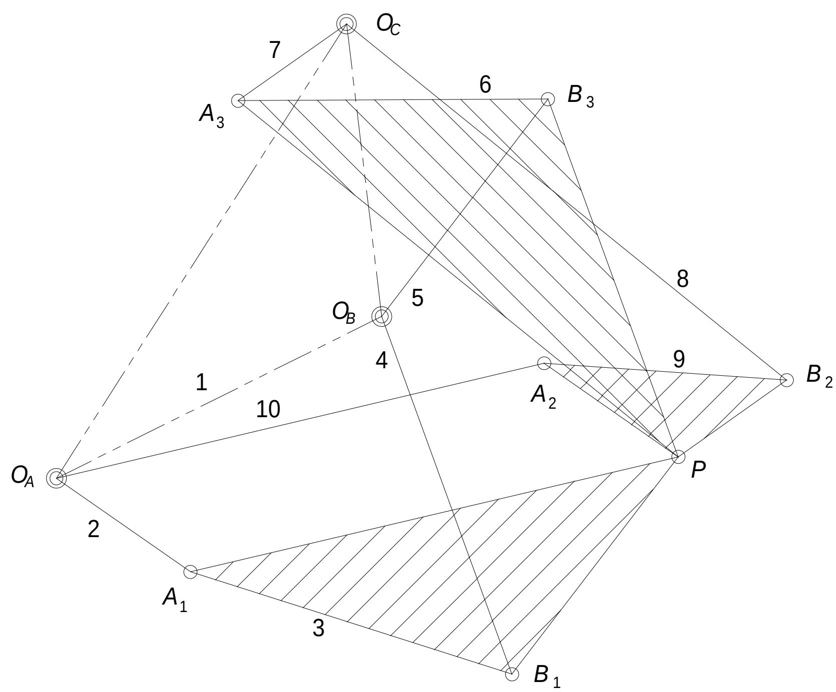
$L_{1BC} := \frac{L_1}{L_3} \cdot B1P$ $L_{1BC} = 1.8035$ $L_{1AC} := \frac{L_1}{L_3} \cdot A1P$ $L_{1AC} = 3.2977$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 2.220$	$L_{1AC} = 3.298$	$L_{1BC} = 1.804$

Crank length	$L_2 = 1.000$	$L_{10} = 3.060$	$L_7 = 0.812$
Coupler length	$L_3 = 2.060$	$L_9 = 1.485$	$L_6 = 1.893$
Rocker length	$L_4 = 2.330$	$L_8 = 3.461$	$L_5 = 1.674$
Coupler point	$A1P = 3.060$	$A2P = 1.000$	$A3P = 3.461$
Coupler angle	$\delta_1 = 31.000\,deg$	$\delta_2 = -31.000\,deg$	$\delta_3 = -39.344\,deg$



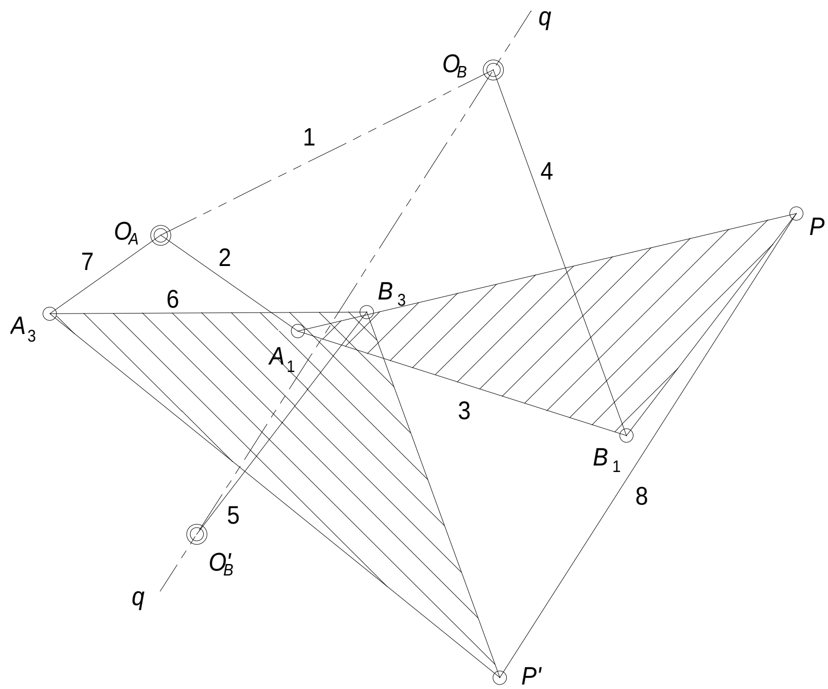
4. Determine the Grashof condition of each of the two additional cognates.

$$Condition(a,b,c,d) := \left| \begin{array}{l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"} \quad if\ SL < PQ \\ return\ \text{"Special Grashof"} \quad if\ SL = PQ \\ return\ \text{"non-Grashof"} \quad otherwise \end{array} \right.$$

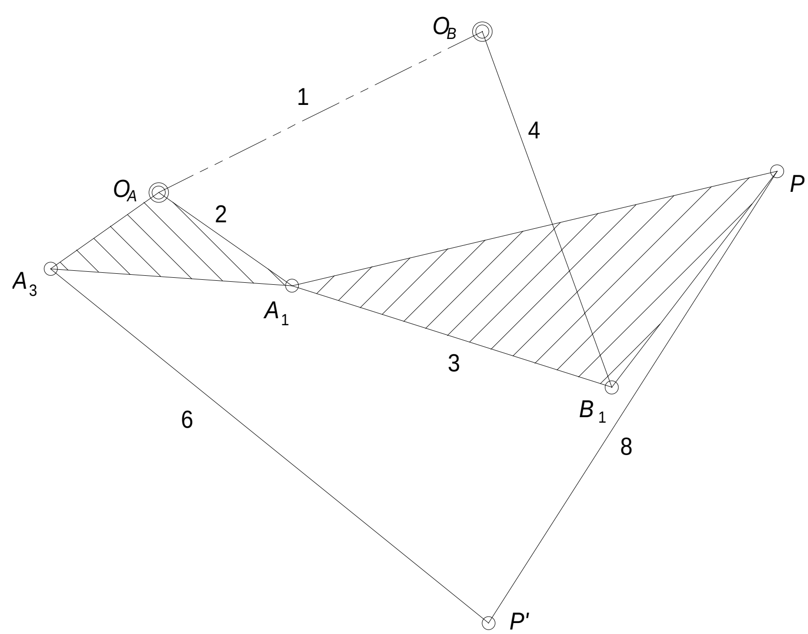
Cognate #2: $Condition\left(L_8, L_9, L_{10}, L_{1AC}\right) = \text{"Grashof"}$

Cognate #3: $Condition\left(L_5, L_6, L_7, L_{1BC}\right) = \text{"Grashof"}$

5. Both of these cognates are Grashof but cognate #3 is a crank rocker. Following Example 3-11, discard cognate #2 and retain cognates #1 and #3. Draw line *qq* parallel to line O_AO_C and through point O_B. Without allowing links 5, 6, and 7 to rotate, slide them as an assembly along lines O_AO_C and *qq* until the free end of link 7 is at O_A. The free end of link 5 will then be at point O'_B and point *P* on link 6 will be at *P'*. Add a new link of length O_AO_C between *P* and *P'*. This is the new output link 8 and all points on it describe the original coupler curve.



6. Join links 2 and 7, making one ternary link. Remove link 5 and reduce link 6 to a binary link. The result is a Watt-I sixbar with links numbered 1, 2, 3, 4, 6, and 8 (see next page). Link 8 is in curvilinear translation and follows the coupler path of the original point *P*. The walking-beam (link 8 in Figure P3-7) is rigidly attached to link 8 below.



PROBLEM 3-22

Statement: Find the maximum and minimum transmission angles of the fourbar driving stage (links L_1, L_2, L_3, L_4) in Figure P3-7 (to graphical accuracy).

Given: Link lengths: Link 2 $L_2 := 1.00$ Link 3 $L_3 := 2.06$

Link 4 $L_4 := 2.33$ Link 1 $L_1 := 2.22$

Grashof condition function:

$$Condition(a,b,c,d) := \left| \begin{array}{l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.$$

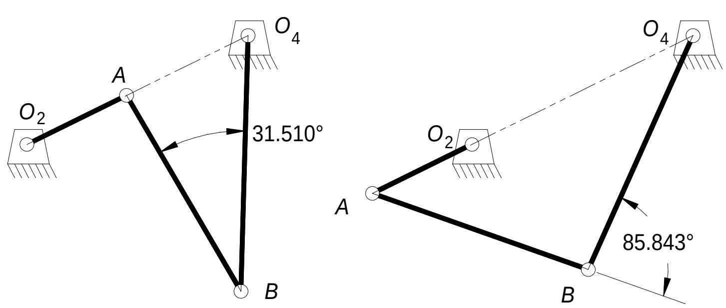
Solution: See Figure P3-7 and Mathcad file P0322.

1. Determine the Grashof condition of the mechanism from inequality 2.8 and its Barker classification from Table 2-4.

Grashof condition: $Condition(L_1, L_2, L_3, L_4) = \text{"Grashof"}$

Barker classification: Class I-2, Grashof crank-rocker-rocker, GCRR, since the shortest link is the input link.

2. It can be shown (see Section 4.10) that the minimum transmission angle for a fourbar GCRR linkage occurs when links 2 and 1 (ground link) are colinear. Draw the linkage in these two positions and measure the transmission angles.



3. As measured from the layout, the minimum transmission angle is 31.5 deg. The maximum is 90 deg.

PROBLEM 3-23

Statement: Figure P3-8 shows a fourbar linkage used in a power loom to drive a comb-like reed against the thread, "beating it up" into the cloth. Determine its Grashof condition and its minimum and maximum transmission angles to graphical accuracy.

Given: Link lengths:

Link 2 $L_2 := 2.00 \cdot in$ Link 3 $L_3 := 8.375 \cdot in$

Link 4 $L_4 := 7.187 \cdot in$ Link 1 $L_1 := 9.625 \cdot in$

Grashof condition function:

$$Condition(a,b,c,d) := \begin{array}{|l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array}$$

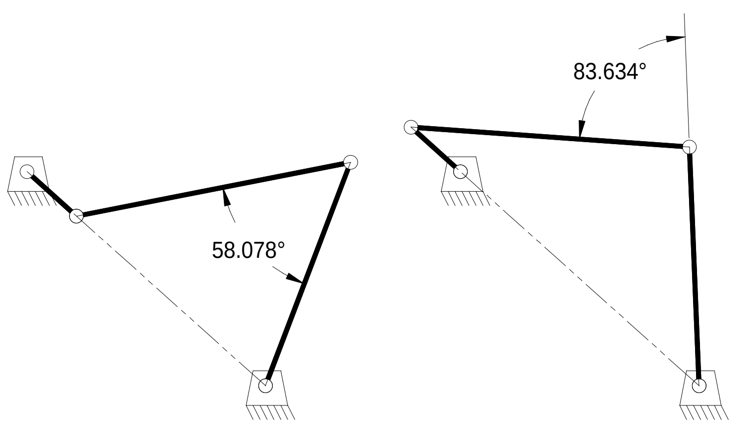
Solution: See Figure P3-8 and Mathcad file P0323.

1. Determine the Grashof condition of the mechanism from inequality 2.8 and its Barker classification from Table 2-4.

Grashof condition: $Condition(L_1, L_2, L_3, L_4) = \text{"Grashof"}$

Barker classification: Class I-2, Grashof crank-rocker-rocker, GCRR, since the shortest link is the input link.

2. It can be shown (see Section 4.10) that the minimum transmission angle for a fourbar GCRR linkage occurs when links 2 and 1 (ground link) are colinear. Draw the linkage in these two positions and measure the transmission angles.



3. As measured from the layout, the minimum transmission angle is 58.1 deg. The maximum is 90.0 deg.

PROBLEM 3-24

Statement: Draw the Roberts diagram and find the cognates for the linkage in Figure P3-9.

Given: Link lengths:

				Coupler point data:	
Ground link	$L_1 := 2.22$	Crank	$L_2 := 1.0$	$A1P := 3.06$	$\delta_1 := -31.00 \cdot deg$
Coupler	$L_3 := 2.06$	Rocker	$L_4 := 2.33$		

Solution: See Figure P3-9 and Mathcad file P0324.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle *APB*.

$$B1P := \left(L_3^{\,2} + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot cos(\delta_1)\right)^{0.5} \qquad B1P = 1.674$$

$$\gamma_1 := -acos\left(\frac{L_3^{\,2} + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P}\right) \qquad \gamma_1 = -109.6560\,deg$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \qquad L_5 = 1.674 \qquad L_6 := \frac{L_4}{L_3} \cdot B1P \qquad L_6 = 1.893$$

$$L_{10} := A1P \qquad L_{10} = 3.060 \qquad L_9 := \frac{L_2}{L_3} \cdot A1P \qquad L_9 = 1.485$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \qquad L_7 = 0.812 \qquad L_8 := L_6 \cdot \frac{A1P}{B1P} \qquad L_8 = 3.461$$

Calculate the coupler point data for cognates #2 and #3

$$A3P := L_8 \qquad A3P = 3.461 \qquad A2P := L_2 \qquad A2P = 1.000$$

$$\delta_3 := 180 \cdot deg - \left|\delta_1 + \gamma_1\right| \qquad \delta_3 = 39.344\,deg \qquad \delta_2 := -\delta_1 \qquad \delta_2 = 31.000\,deg$$

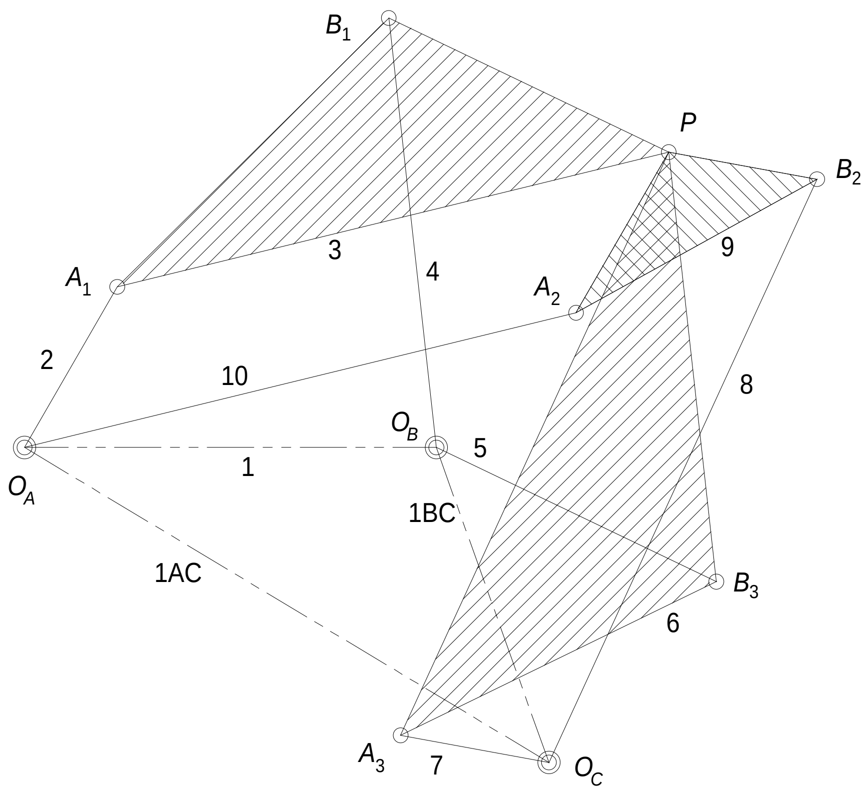
From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \qquad L_{1BC} = 1.8035 \qquad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \qquad L_{1AC} = 3.2977$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 2.220$	$L_{1AC} = 3.298$	$L_{1BC} = 1.804$
Crank length	$L_2 = 1.000$	$L_{10} = 3.060$	$L_7 = 0.812$
Coupler length	$L_3 = 2.060$	$L_9 = 1.485$	$L_6 = 1.893$
Rocker length	$L_4 = 2.330$	$L_8 = 3.461$	$L_5 = 1.674$
Coupler point	$A1P = 3.060$	$A2P = 1.000$	$A3P = 3.461$
Coupler angle	$\delta_1 = -31.000\,deg$	$\delta_2 = 31.000\,deg$	$\delta_3 = 39.344\,deg$


B1
P
B2
3
4
9
A1
A2
2
10
8
OB
5
1
OA
1BC
1AC
B3
6
A3
7
OC

PROBLEM 3-25

Statement: Find the equivalent geared fivebar mechanism cognate of the linkage in Figure P3-9.

Given: Link lengths: Coupler point data:

Ground link	$L_1 := 2.22$	Crank	$L_2 := 1.0$	$A1P := 3.06$	$\delta_1 := -31.00 \cdot deg$
Coupler	$L_3 := 2.06$	Rocker	$L_4 := 2.33$		

Solution: See Figure P3-9 and Mathcad file P0325.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle *APB*.

$$B1P := \left(L_3^{\,2} + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot cos(\delta_1)\right)^{0.5} \qquad B1P = 1.674$$

$$\gamma_1 := -acos\left(\frac{L_3^{\,2} + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P}\right) \qquad \gamma_1 = -109.6560\, deg$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \qquad L_5 = 1.674 \qquad L_6 := \frac{L_4}{L_3} \cdot B1P \qquad L_6 = 1.893$$

$$L_{10} := A1P \qquad L_{10} = 3.060 \qquad L_9 := \frac{L_2}{L_3} \cdot A1P \qquad L_9 = 1.485$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \qquad L_7 = 0.812 \qquad L_8 := L_6 \cdot \frac{A1P}{B1P} \qquad L_8 = 3.461$$

Calculate the coupler point data for cognates #2 and #3

$$A3P := L_8 \qquad A3P = 3.461 \qquad A2P := L_2 \qquad A2P = 1.000$$

$$\delta_3 := 180 \cdot deg - \left|\delta_1 + \gamma_1\right| \qquad \delta_3 = 39.344\, deg \qquad \delta_2 := -\delta_1 \qquad \delta_2 = 31.000\, deg$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

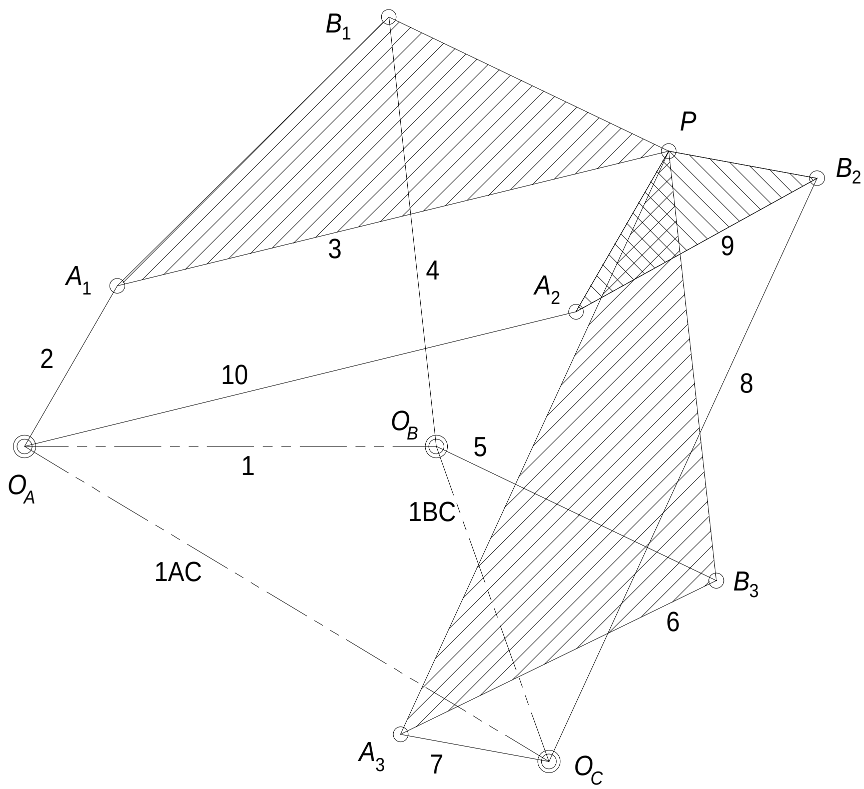
$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \qquad L_{1BC} = 1.8035 \qquad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \qquad L_{1AC} = 3.2977$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 2.220$	$L_{1AC} = 3.298$	$L_{1BC} = 1.804$
Crank length	$L_2 = 1.000$	$L_{10} = 3.060$	$L_7 = 0.812$
Coupler length	$L_3 = 2.060$	$L_9 = 1.485$	$L_6 = 1.893$
Rocker length	$L_4 = 2.330$	$L_8 = 3.461$	$L_5 = 1.674$

Coupler point	$A1P = 3.060$	$A2P = 1.000$	$A3P = 3.461$
Coupler angle	$\delta_1 = -31.000\,deg$	$\delta_2 = 31.000\,deg$	$\delta_3 = 39.344\,deg$



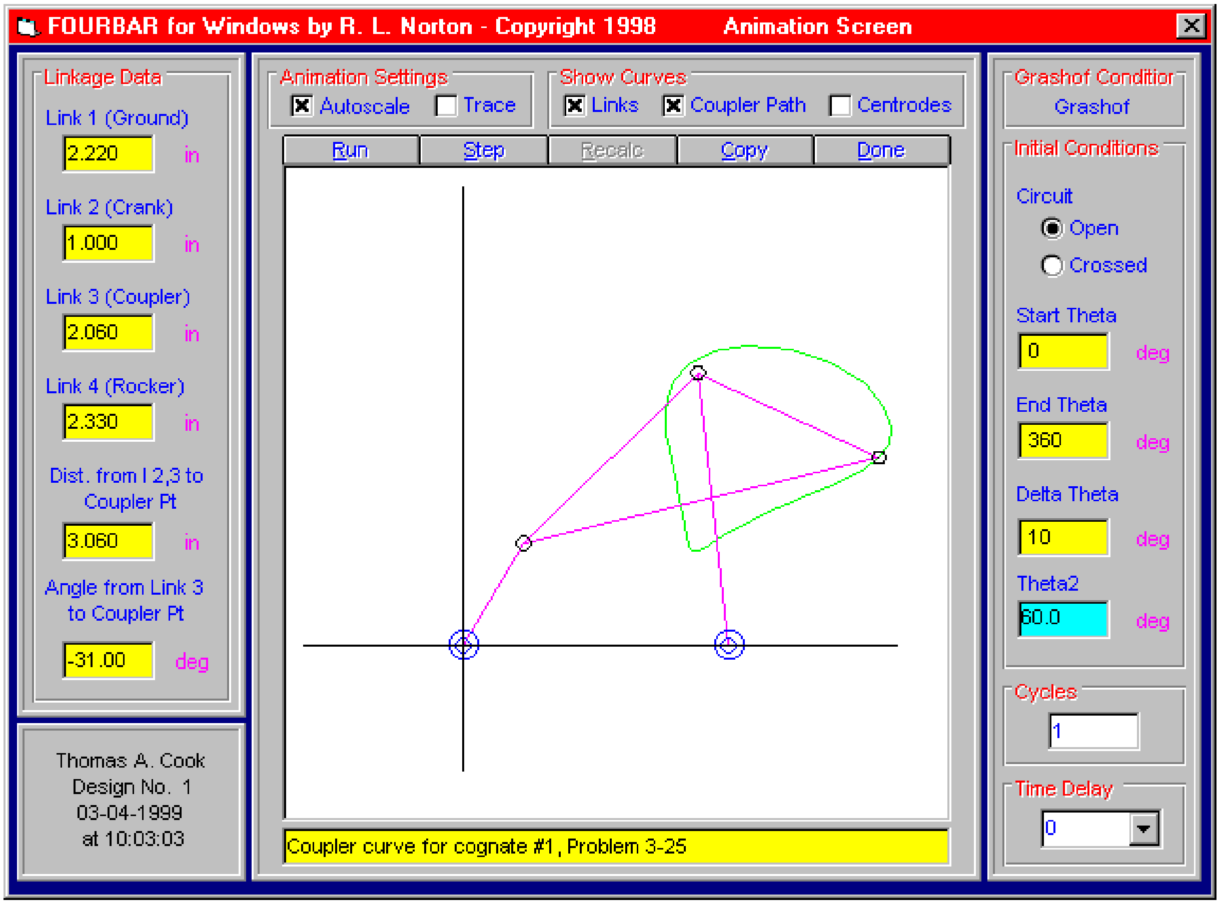
4. The three geared fivebar cognates can be seen in the Roberts diagram. They are: $O_AA_2PB_3O_B$, $O_AA_1PA_3O_C$, and $O_BB_1PB_2O_C$. The three geared fivebar cognates are summarized in the table below.

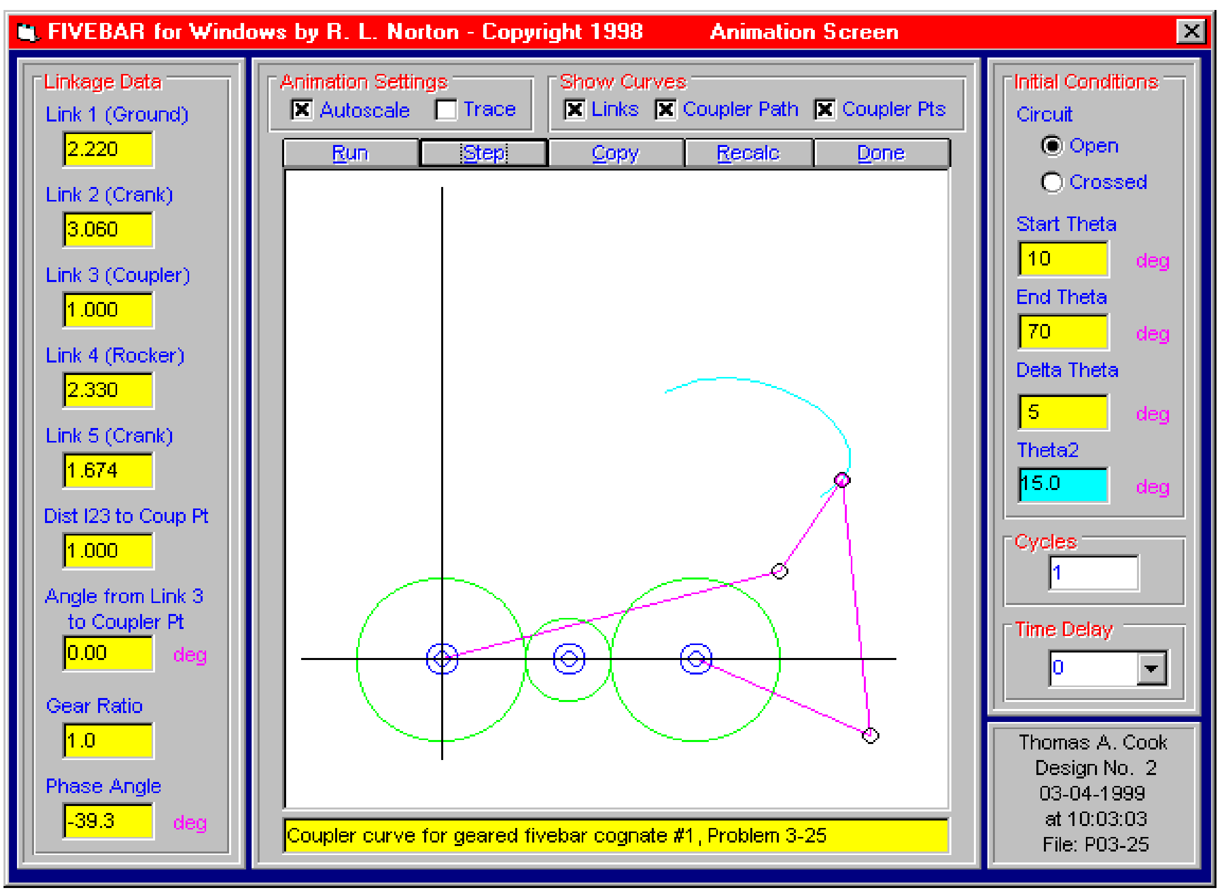
SUMMARY OF GEARED FIVEBAR COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 2.220$	$L_{1AC} = 3.298$	$L_{1BC} = 1.804$
Crank length	$L_{10} = 3.060$	$L_2 = 1.000$	$L_4 = 2.330$
Coupler length	$A2P = 1.000$	$A1P = 3.060$	$L_5 = 1.674$
Rocker length	$L_4 = 2.330$	$L_8 = 3.461$	$L_7 = 0.812$
Crank length	$L_5 = 1.674$	$L_7 = 0.812$	$L_8 = 3.461$
Coupler point	$A2P = 1.000$	$A1P = 3.060$	$B1P = 1.674$
Coupler angle	$\delta_1 := 0.00 \cdot deg$	$\delta_2 := 0.00 \cdot deg$	$\delta_3 := 0.00 \cdot deg$

5. Enter the cognate #1 specifications into program FOURBAR to get a trace of the coupler path (see next page)

6. Enter the geared fivebar cognate #1 specifications into program FIVEBAR to get a trace of the coupler path for the geared fivebar (see next page).


FOURBAR for Windows by R. L. Norton - Copyright 1998 Animation Screen
Linkage Data
Link 1 (Ground)
2.220 in
Link 2 (Crank)
1.000 in
Link 3 (Coupler)
2.060 in
Link 4 (Rocker)
2.330 in
Dist. from l 2,3 to Coupler Pt
3.060 in
Angle from Link 3 to Coupler Pt
-31.00 deg
Thomas A. Cook
Design No. 1
03-04-1999
at 10:03:03
Animation Settings
Autoscale
Trace
Show Curves
Links
Coupler Path
Centrodes
Run
Step
Recalc
Copy
Done
Coupler curve for cognate #1, Problem 3-25
Grashof Condition
Grashof
Initial Conditions
Circuit
Open
Crossed
Start Theta
0 deg
End Theta
360 deg
Delta Theta
10 deg
Theta2
60.0 deg
Cycles
1
Time Delay
0


FIVEBAR for Windows by R. L. Norton - Copyright 1998 Animation Screen
Linkage Data
Link 1 (Ground)
2.220
Link 2 (Crank)
3.060
Link 3 (Coupler)
1.000
Link 4 (Rocker)
2.330
Link 5 (Crank)
1.674
Dist l23 to Coup Pt
1.000
Angle from Link 3 to Coupler Pt
0.00 deg
Gear Ratio
1.0
Phase Angle
-39.3 deg
Animation Settings
Autoscale
Trace
Show Curves
Links
Coupler Path
Coupler Pts
Run
Step
Copy
Recalc
Done
Coupler curve for geared fivebar cognate #1, Problem 3-25
Initial Conditions
Circuit
Open
Crossed
Start Theta
10 deg
End Theta
70 deg
Delta Theta
5 deg
Theta2
15.0 deg
Cycles
1
Time Delay
0
Thomas A. Cook
Design No. 2
03-04-1999
at 10:03:03
File: P03-25

PROBLEM 3-26

Statement: Use the linkage in Figure P3-9 to design an eightbar double-dwell mechanism that has a rocker output through 45 deg.

Given: Link lengths: Coupler point data:

Ground link $L_1 := 2.22$ Crank $L_2 := 1.0$ $A1P := 3.06$ $\delta_1 := -31.00 \cdot deg$

Coupler $L_3 := 2.06$ Rocker $L_4 := 2.33$

Solution: See Figure P3-9 and Mathcad file P0326.

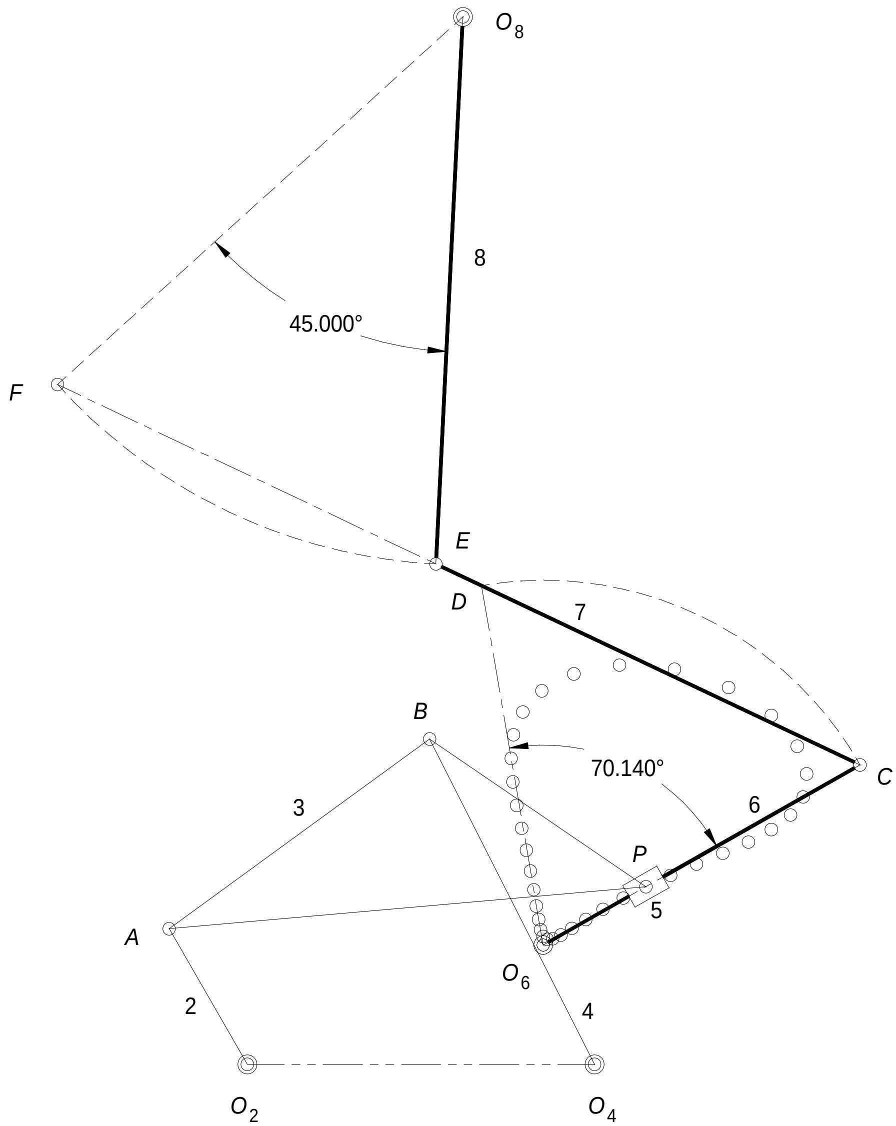
1. Enter the given data into program FOURBAR and print out the resulting coupler point coordinates (see table below).

FOURBAR for Windows File P03-26.DAT

Angle Step Deg	Cpler Pt X	Cpler Pt Y	Cpler Pt Mag	Cpler Pt Ang
0.000	2.731	2.523	3.718	42.731
10.000	3.077	2.407	3.906	38.029
20.000	3.350	2.228	4.023	33.626
30.000	3.515	2.032	4.060	30.035
40.000	3.576	1.855	4.028	27.412
50.000	3.554	1.708	3.943	25.672
60.000	3.473	1.592	3.820	24.635
70.000	3.350	1.499	3.671	24.107
80.000	3.203	1.420	3.503	23.915
90.000	3.040	1.348	3.326	23.915
100.000	2.872	1.278	3.144	23.988
110.000	2.706	1.207	2.963	24.039
120.000	2.548	1.135	2.789	24.001
130.000	2.403	1.062	2.627	23.834
140.000	2.274	0.990	2.480	23.533
150.000	2.164	0.925	2.354	23.134
160.000	2.075	0.869	2.249	22.719
170.000	2.005	0.826	2.168	22.404
180.000	1.953	0.802	2.111	22.326
190.000	1.917	0.798	2.076	22.614
200.000	1.892	0.817	2.061	23.365
210.000	1.875	0.860	2.063	24.632
220.000	1.862	0.925	2.079	26.417
230.000	1.848	1.011	2.107	28.678
240.000	1.832	1.115	2.145	31.340
250.000	1.810	1.235	2.192	34.306
260.000	1.784	1.367	2.248	37.463
270.000	1.754	1.508	2.313	40.683
280.000	1.723	1.654	2.388	43.826
290.000	1.698	1.804	2.477	46.730
300.000	1.687	1.955	2.582	49.207
310.000	1.702	2.105	2.707	51.038
320.000	1.761	2.251	2.858	51.965
330.000	1.883	2.386	3.040	51.715
340.000	2.088	2.494	3.253	50.064
350.000	2.380	2.550	3.488	46.967
360.000	2.731	2.523	3.718	42.731

2. Layout this linkage to scale, including the coupler curve whose coordinates are in the table above. Fit tangent lines to the nearly straight portions of the curve. Label their intersection O_6.

3. Design link 6 to lie along these straight tangents, pivoted at O_6. Provide a guide on link 6 to accommodate slider block 5, which pivots on the coupler point *P*.



4. Extend link 6 a convenient distance to point *C*. Draw an arc through point *C* with center at O_6. Label the intersection of the arc with the other tangent line as point *D*. Attach link 7 to the pivot at *C*. The length of link 7 is *CE*, a design choice. Extend line *CDE* from point *E* a distance equal to *CD*. Label the end point *F*. Layout two intersecting lines through *E* and *F* such that they subtend an angle of 45 deg. Label their intersection O_8. The link joining O_8 and point *E* is link 8. The link lengths and locations of O_6 and O_8 are:

Link 6 $L_6 := 2.330$ Link 7 $L_7 := 3.000$ Link 8 $L_8 := 3.498$

Fixed pivot O_6: $x := 1.892$ $y := 0.762$

Fixed pivot O_8: $x := 1.379$ $y := 6.690$

PROBLEM 3-27

Statement: Use the linkage in Figure P3-9 to design an eightbar double-dwell mechanism that has a slider output stroke of 5 crank units.

Given: Link lengths:

Ground link $L_1 := 2.22$ Crank $L_2 := 1.0$

Coupler $L_3 := 2.06$ Rocker $L_4 := 2.33$

Coupler point data: $A1P := 3.06$ $\delta_1 := -31.00 \cdot deg$

Solution: See Figure P3-9 and Mathcad file P0327.

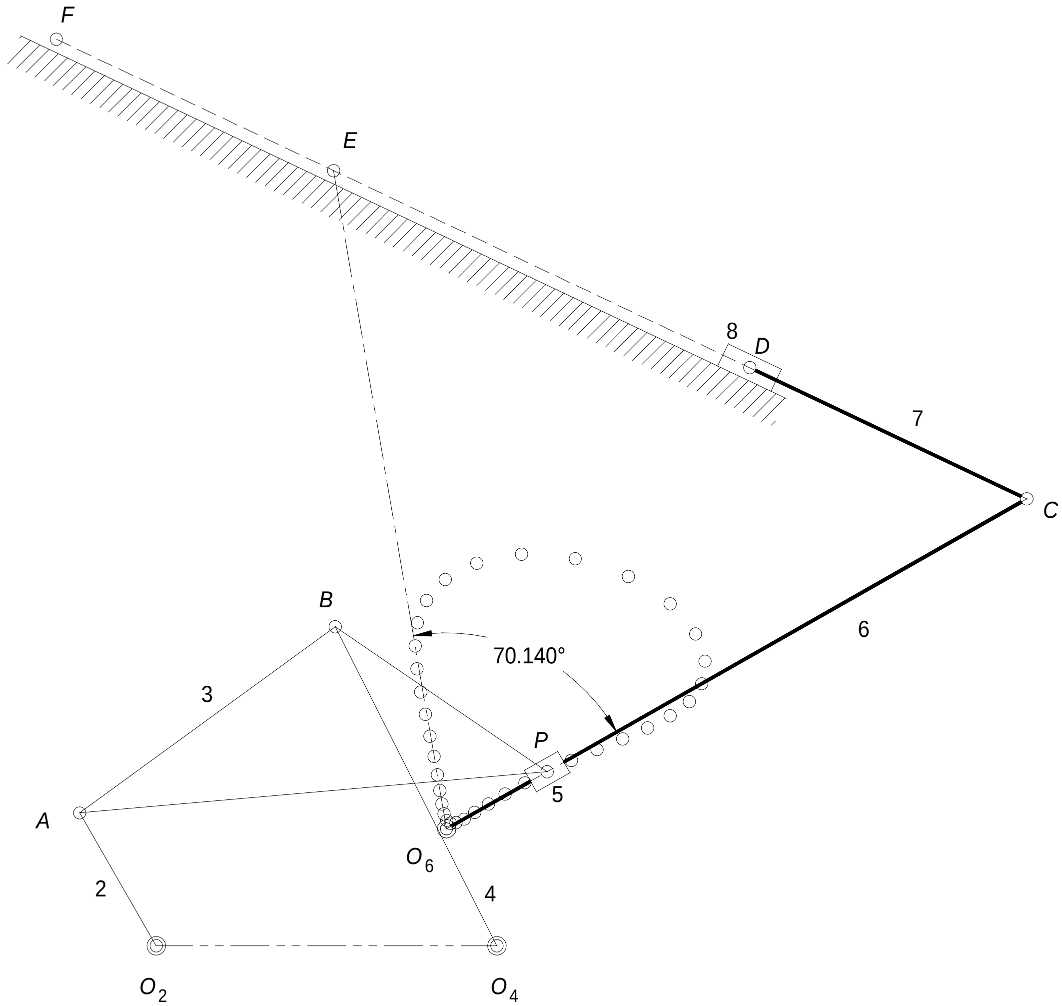
1. Enter the given data into program FOURBAR and print out the resulting coupler point coordinates (see table below).

FOURBAR for Windows File P03-26.DAT

Angle Step Deg	Cpler Pt X	Cpler Pt Y	Cpler Pt Mag	Cpler Pt Ang
0.000	2.731	2.523	3.718	42.731
10.000	3.077	2.407	3.906	38.029
20.000	3.350	2.228	4.023	33.626
30.000	3.515	2.032	4.060	30.035
40.000	3.576	1.855	4.028	27.412
50.000	3.554	1.708	3.943	25.672
60.000	3.473	1.592	3.820	24.635
70.000	3.350	1.499	3.671	24.107
80.000	3.203	1.420	3.503	23.915
90.000	3.040	1.348	3.326	23.915
100.000	2.872	1.278	3.144	23.988
110.000	2.706	1.207	2.963	24.039
120.000	2.548	1.135	2.789	24.001
130.000	2.403	1.062	2.627	23.834
140.000	2.274	0.990	2.480	23.533
150.000	2.164	0.925	2.354	23.134
160.000	2.075	0.869	2.249	22.719
170.000	2.005	0.826	2.168	22.404
180.000	1.953	0.802	2.111	22.326
190.000	1.917	0.798	2.076	22.614
200.000	1.892	0.817	2.061	23.365
210.000	1.875	0.860	2.063	24.632
220.000	1.862	0.925	2.079	26.417
230.000	1.848	1.011	2.107	28.678
240.000	1.832	1.115	2.145	31.340
250.000	1.810	1.235	2.192	34.306
260.000	1.784	1.367	2.248	37.463
270.000	1.754	1.508	2.313	40.683
280.000	1.723	1.654	2.388	43.826
290.000	1.698	1.804	2.477	46.730
300.000	1.687	1.955	2.582	49.207
310.000	1.702	2.105	2.707	51.038
320.000	1.761	2.251	2.858	51.965
330.000	1.883	2.386	3.040	51.715
340.000	2.088	2.494	3.253	50.064
350.000	2.380	2.550	3.488	46.967
360.000	2.731	2.523	3.718	42.731

2. Layout this linkage to scale, including the coupler curve whose coordinates are in the table above. Fit tangent lines to the nearly straight portions of the curve. Label their intersection O_6.

3. Design link 6 to lie along these straight tangents, pivoted at O_6. Provide a guide on link 6 to accommodate slider block 5, which pivots on the coupler point *P*.



4. Extend link 6 and the other tangent line until points *C* and *E* are 5 units apart. Attach link 7 to the pivot at *C*. The length of link 7 is *CD*, a design choice. Extend line *CDE* from point *D* a distance equal to *CE*. Label the end point *F*. As link 6 travels from C to E, slider block 8 will travel from D to F, a distance of 5 units. The link lengths and location of O_6:

Link 6 $L_6 := 4.351$ Link 7 $L_7 := 2.000$

Fixed pivot O_6: $x := 1.892$

$y := 0.762$

PROBLEM 3-28

Statement: Use two of the cognates in Figure 3-26 (p. 126) to design a Watt-I sixbar parallel motion mechanism that carries a link through the same coupler curve at all points. Comment on its similarities to the original Roberts diagram.

Given: Link lengths:

				Coupler point data:
Ground link	$L_1 := 45$	Crank	$L_2 := 56$	$A1P := 11.25 \quad \delta_1 := 0.000 \cdot deg$
Coupler	$L_3 := 22.5$	Rocker	$L_4 := 56$	

Solution: See Figure 3-26 and Mathcad file P0328.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle *APB*.

$$B1P := \left(L_3^{\,2} + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1)\right)^{0.5} \qquad B1P = 11.250$$

$$\gamma_1 := acos\left(\frac{L_3^{\,2} + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P}\right) \qquad \gamma_1 = 0.0000\, deg$$

2. Use the Cayley diagram (see Figure 3-26) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \qquad L_5 = 11.250 \qquad L_6 := \frac{L_4}{L_3} \cdot B1P \qquad L_6 = 28.000$$

$$L_{10} := A1P \qquad L_{10} = 11.250 \qquad L_9 := \frac{L_2}{L_3} \cdot A1P \qquad L_9 = 28.000$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \qquad L_7 = 28.000 \qquad L_8 := L_6 \cdot \frac{A1P}{B1P} \qquad L_8 = 28.000$$

Calculate the coupler point data for cognates #2 and #3

$$A3P := L_4 \qquad A3P = 56.000 \qquad A2P := L_2 \qquad A2P = 56.000$$

$$\delta_3 := \delta_1 \qquad \delta_3 = 0.000\, deg \qquad \delta_2 := \delta_1 \qquad \delta_2 = 0.000\, deg$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

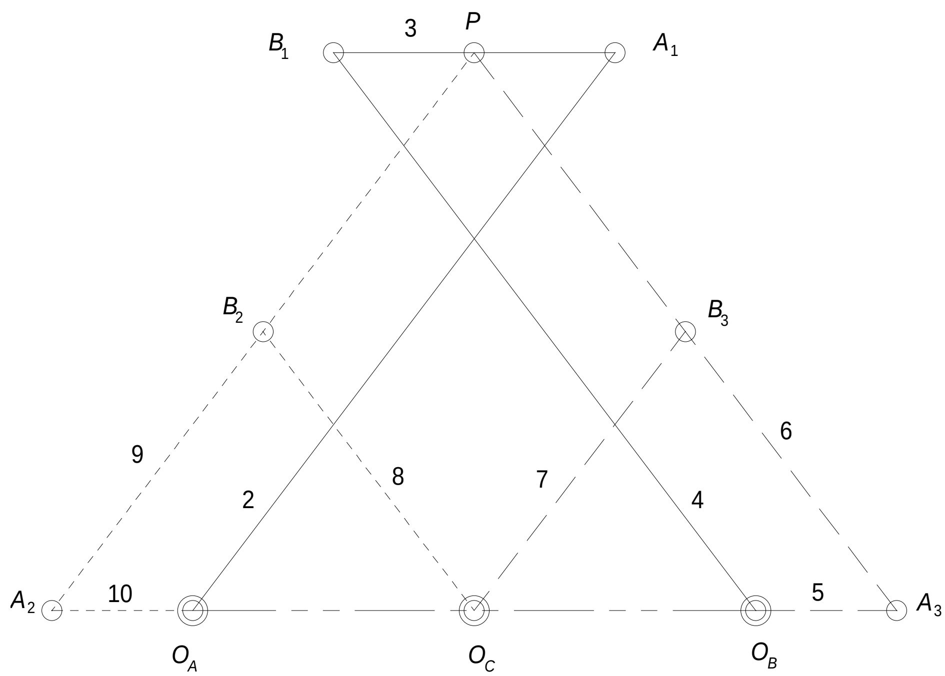
$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \qquad L_{1BC} = 22.5000 \qquad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \qquad L_{1AC} = 22.5000$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

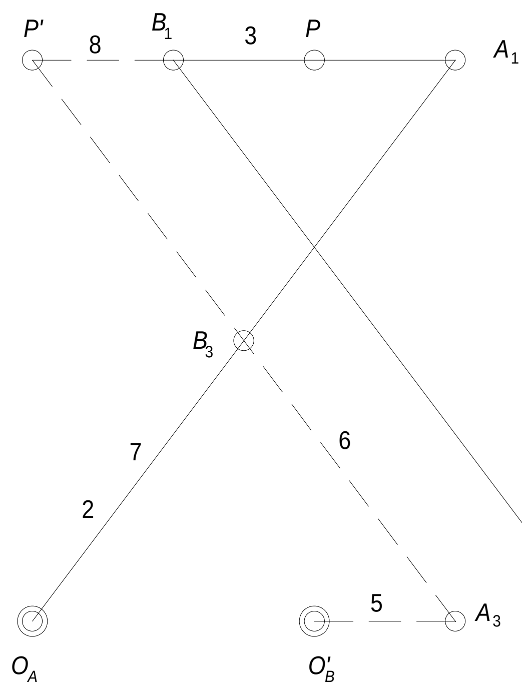
SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 45.000$	$L_{1AC} = 22.500$	$L_{1BC} = 22.500$
Crank length	$L_2 = 56.000$	$L_{10} = 11.250$	$L_7 = 28.000$
Coupler length	$L_3 = 22.500$	$L_9 = 28.000$	$L_6 = 28.000$

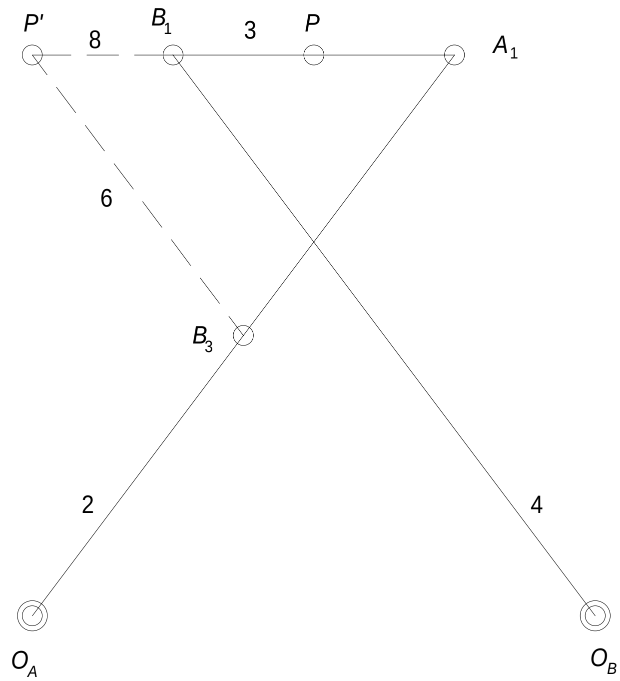
Rocker length	$L_4 = 56.000$	$L_8 = 28.000$	$L_5 = 11.250$
Coupler point	$A1P = 11.250$	$A2P = 56.000$	$A3P = 56.000$
Coupler angle	$\delta_1 = 0.000\ deg$	$\delta_2 = 0.000\ deg$	$\delta_3 = 0.000\ deg$



4. Both of these cognates are identical. Following Example 3-11, discard cognate #2 and retain cognates #1 and #3. Without allowing links 5, 6, and 7 to rotate, slide them as an assembly along line O_AO_C until the free end of link 7 is at O_A. The free end of link 5 will then be at point O'_B and point P on link 6 will be at P'. Add a new link of length O_AO_C between P and P'. This is the new output link 8 and all points on it describe the original coupler curve.



5. Join links 2 and 7, making one ternary link. Remove link 5 and reduce link 6 to a binary link. The result is a Watt-I sixbar with links numbered 1, 2, 3, 4, 6, and 8. Link 8 is in curvilinear translation and follows the coupler path of the original point *P*. Link 8 is a binary link with nodes at *P* and *P'*. It does not attach to link 4 at B_1.



PROBLEM 3-29

Statement: Find the cognates of the Watt straight-line mechanism in Figure 3-29a (p. 131).

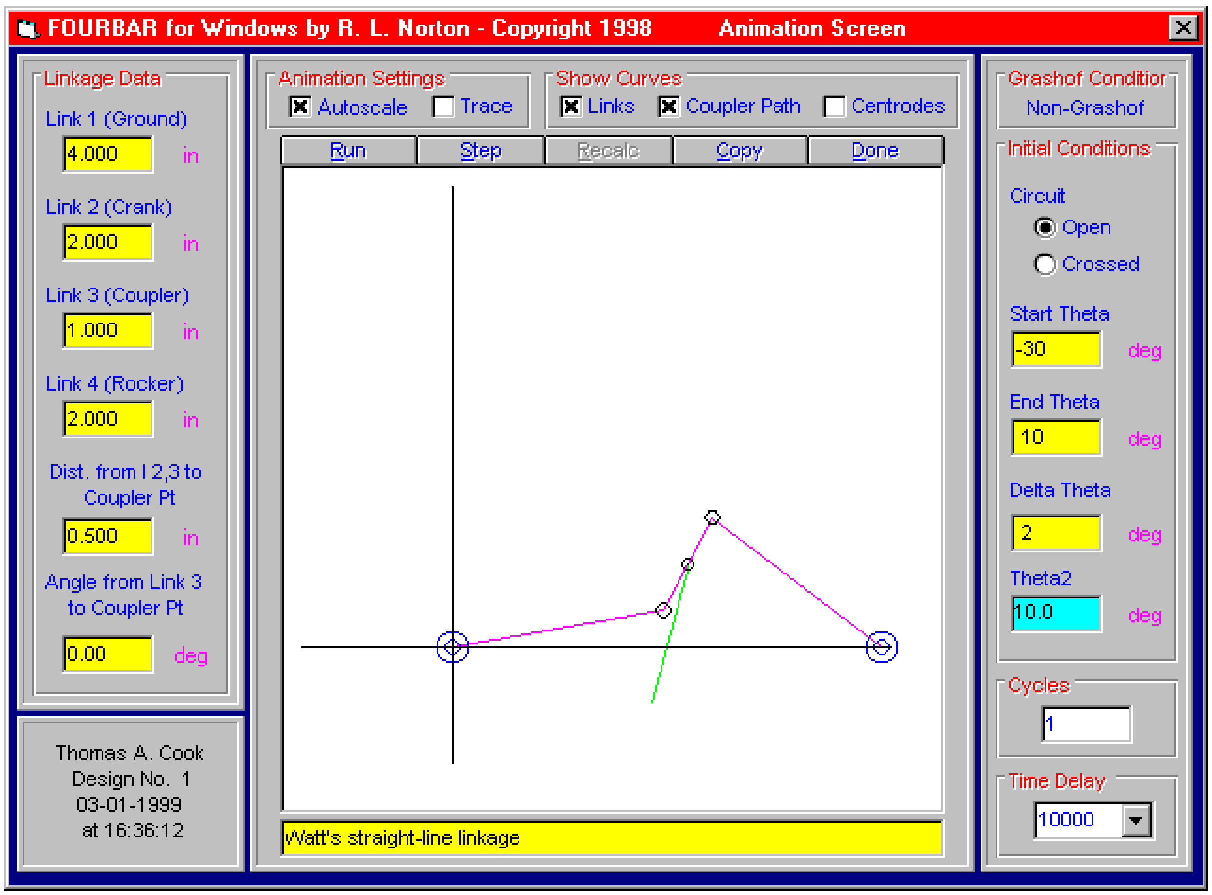
Given: Link lengths:

Ground link	$L_1 := 4$	Crank	$L_2 := 2$	$A1P := 0.500$	$\delta_1 := 0.00 \cdot deg$
Coupler	$L_3 := 1$	Rocker	$L_4 := 2$	$B1P := 0.500$	$\gamma_1 := 0.00 \cdot deg$

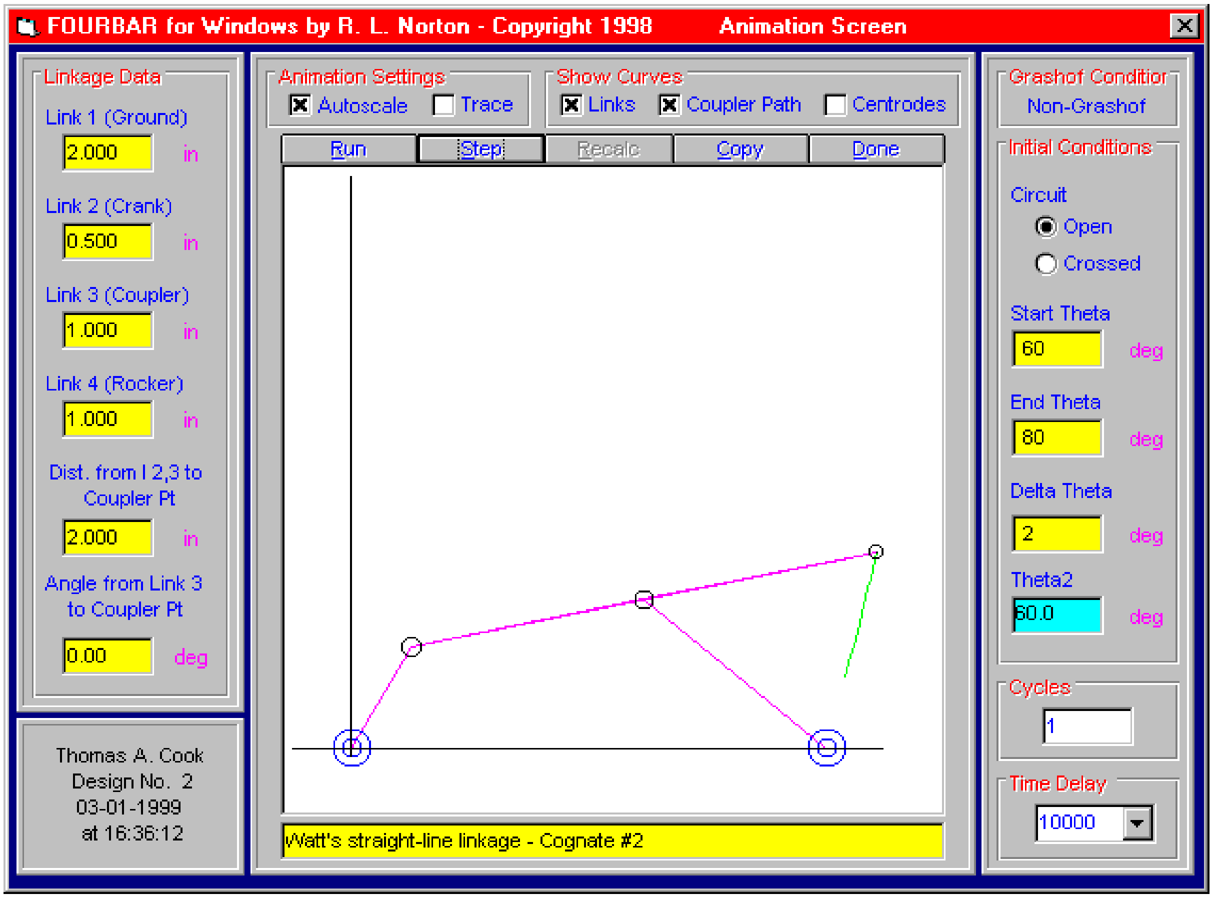
Coupler point data (last two columns).

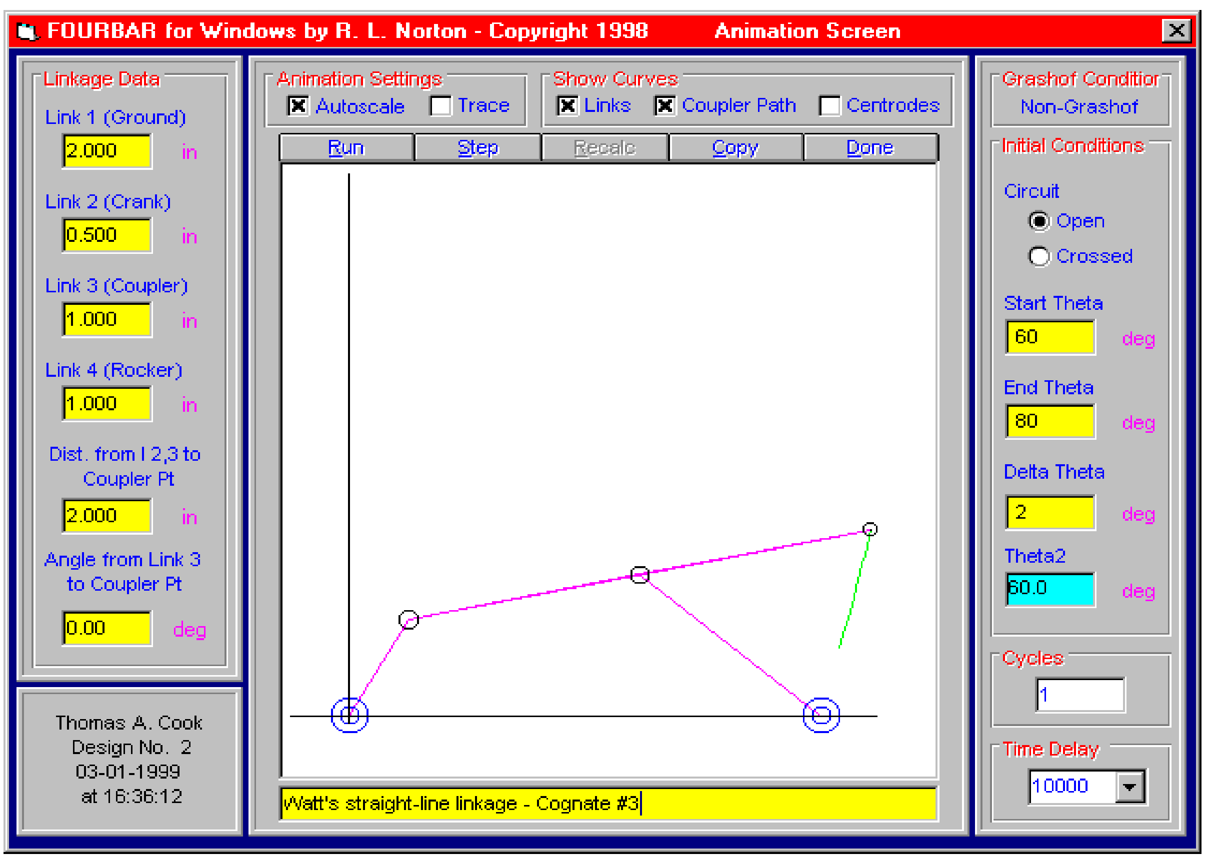
Solution: See Figure 3-29a and Mathcad file P0329.

1. Input the link dimensions and coupler point data into program FOURBAR.



2. Use the Cognate pull-down menu to get the link lengths for cognates #2 and #3 (see next page). Note that, for this mechanism, cognates #2 and #3 are identical. All three mechanisms are non-Grashof with limited crank angles.


FOURBAR for Windows by R. L. Norton - Copyright 1998 Animation Screen
Linkage Data
Link 1 (Ground)
2.000 in
Link 2 (Crank)
0.500 in
Link 3 (Coupler)
1.000 in
Link 4 (Rocker)
1.000 in
Dist. from l 2,3 to Coupler Pt
2.000 in
Angle from Link 3 to Coupler Pt
0.00 deg
Thomas A. Cook
Design No. 2
03-01-1999
at 16:36:12
Animation Settings
Autoscale
Trace
Show Curves
Links
Coupler Path
Centrodes
Run
Step
Recalc
Copy
Done
Watt's straight-line linkage - Cognate #2
Grashof Condition
Non-Grashof
Initial Conditions
Circuit
Open
Crossed
Start Theta
60 deg
End Theta
80 deg
Delta Theta
2 deg
Theta2
60.0 deg
Cycles
1
Time Delay
10000


FOURBAR for Windows by R. L. Norton - Copyright 1998 Animation Screen
Linkage Data
Link 1 (Ground)
2.000 in
Link 2 (Crank)
0.500 in
Link 3 (Coupler)
1.000 in
Link 4 (Rocker)
1.000 in
Dist. from l 2,3 to Coupler Pt
2.000 in
Angle from Link 3 to Coupler Pt
0.00 deg
Thomas A. Cook
Design No. 2
03-01-1999
at 16:36:12
Animation Settings
Autoscale
Trace
Show Curves
Links
Coupler Path
Centrodes
Run
Step
Recalc
Copy
Done
Watt's straight-line linkage - Cognate #3
Grashof Condition
Non-Grashof
Initial Conditions
Circuit
Open
Crossed
Start Theta
60 deg
End Theta
80 deg
Delta Theta
2 deg
Theta2
60.0 deg
Cycles
1
Time Delay
10000

PROBLEM 3-30

Statement: Find the cognates of the Roberts straight-line mechanism in Figure 3-29b.

Given: Link lengths:

Ground link $L_1 := 2$ Crank $L_2 := 1$

Coupler $L_3 := 1$ Rocker $L_4 := 1$

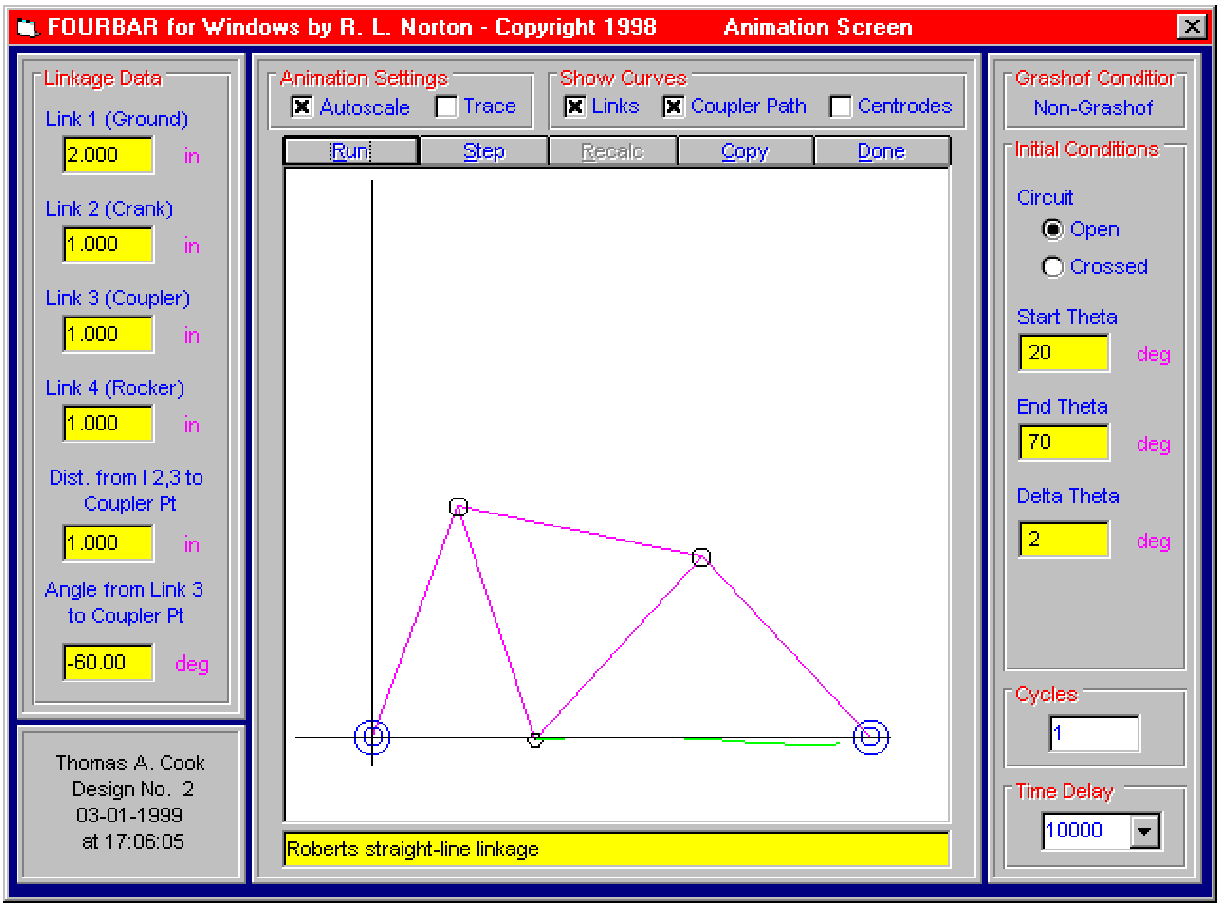
Coupler point data:

$A1P := 1.000$ $\delta_1 := -60.0 \cdot deg$

$B1P := 1.000$ $\gamma_1 := -60.0 \cdot deg$

Solution: See Figure 3-29b and Mathcad file P0330.

1. Input the link dimensions and coupler point data into program FOURBAR.



2. Note that, for this mechanism, cognates #2 and #3 are identical with cognate #1 because of the symmetry of the linkage (draw the Cayley diagram to see this). All three mechanisms are non-Grashof with limited crank angles.

PROBLEM 3-31

Statement: Design a Hoeken straight-line linkage to give minimum error in velocity over 22% of the cycle for a 15-cm-long straight line motion. Specify all linkage parameters.

Given: Length of straight line motion: $\Delta x := 150 \cdot mm$

Percentage of cycle over which straight line motion takes place: 22%

Solution: See Figure 3-30 and Mathcad file P0331.

1. Using Table 3-1 and the required length of straight-line motion, determine the link lengths.

Link ratios from Table 3-1 for 22% cycle:

$L1overL2 := 1.975 \qquad L3overL2 := 2.463 \qquad \Delta xoverL2 := 1.845$

Link lengths:

Crank	$L_2 := \dfrac{\Delta x}{\Delta xoverL2}$	$L_2 = 81.30\,mm$
Coupler	$L_3 := L3overL2 \cdot L_2$	$L_3 = 200.24\,mm$
Ground link	$L_1 := L1overL2 \cdot L_2$	$L_1 = 160.57\,mm$
Rocker	$L_4 := L_3$	$L_4 = 200.24\,mm$
Coupler point	$AP := 2 \cdot L_3$	$AP = 400.49\,mm$

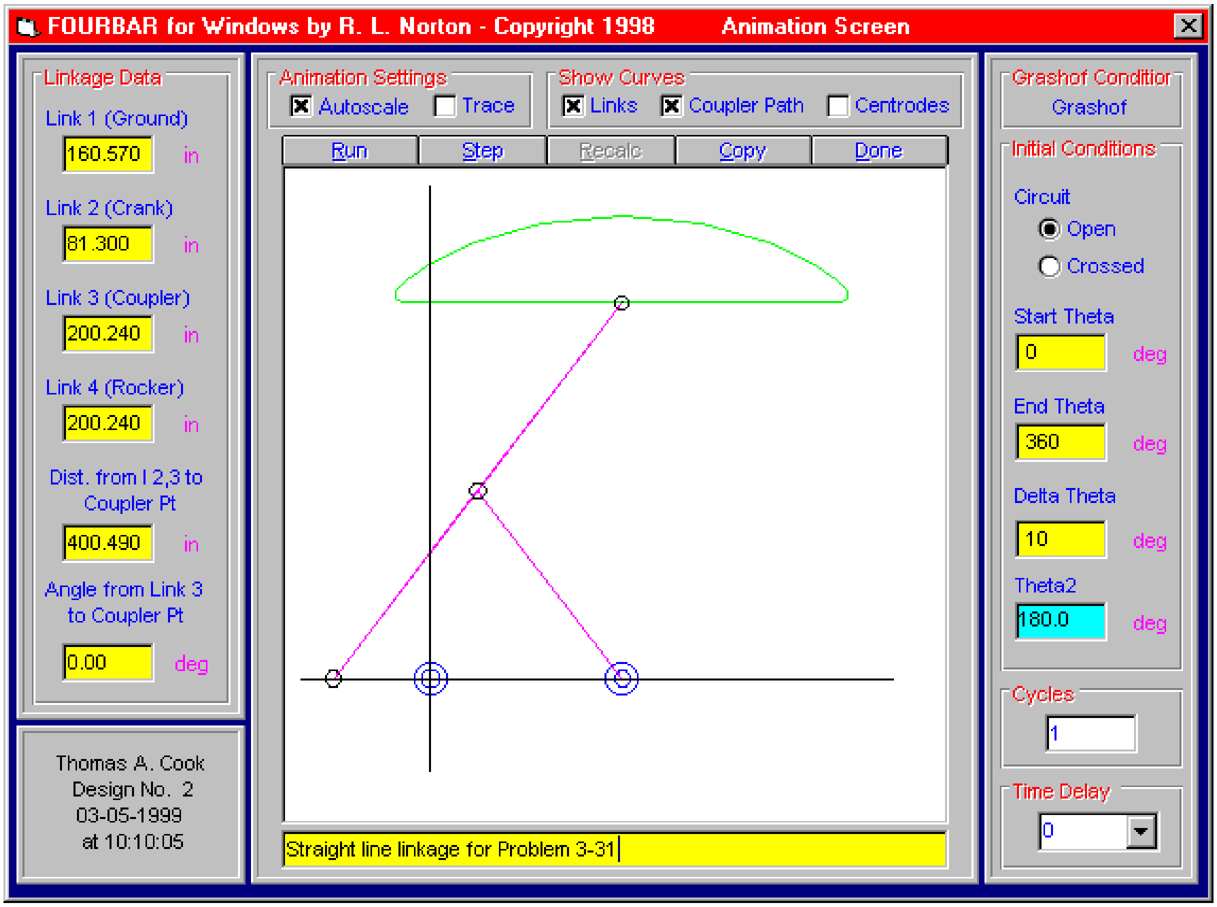
2. Calculate the distance from point *P* to pivot O_4 (C_y) when crank angle is 180 deg.

$$C_y := \sqrt{\left(2 \cdot L_3\right)^2 - \left(L_1 + L_2\right)^2} \qquad C_y = 319.20\,mm$$

3. Enter the link lengths into program FOURBAR to verify the design (see next page for coupler point curve). Using the PRINT facility, determine the x,y coordinates of the coupler curve and the x,y components of the coupler point velocity in the straight line region. A table of these values is printed below. Notice the small deviations over the range of crank angles from the y-coordinate and the x-velocity at a crank angle of 180 deg.

FOURBAR for Windows File P03-31.DOC

Angle Step Deg	Cpler Pt X mm	Cpler Pt Y mm	Veloc CP X mm/sec	Veloc CP Y mm/sec
140	235.60	319.95	-1,072.61	-10.73
150	216.84	319.72	-1,076.20	-14.74
160	198.06	319.46	-1,075.51	-13.54
170	179.31	319.27	-1,073.75	-7.99
180	160.58	319.20	-1,072.93	0.02
190	141.85	319.27	-1,073.75	8.03
200	123.09	319.47	-1,075.52	13.58
210	104.31	319.72	-1,076.22	14.78
220	85.55	319.95	-1,072.63	10.76


FOURBAR for Windows by R. L. Norton - Copyright 1998
Animation Screen
Linkage Data
Link 1 (Ground)
160.570 in
Link 2 (Crank)
81.300 in
Link 3 (Coupler)
200.240 in
Link 4 (Rocker)
200.240 in
Dist. from I 2,3 to Coupler Pt
400.490 in
Angle from Link 3 to Coupler Pt
0.00 deg
Thomas A. Cook
Design No. 2
03-05-1999
at 10:10:05
Animation Settings
Autoscale
Trace
Show Curves
Links
Coupler Path
Centrodes
Run
Step
Recalc
Copy
Done
Straight line linkage for Problem 3-31
Grashof Condition
Grashof
Initial Conditions
Circuit
Open
Crossed
Start Theta
0 deg
End Theta
360 deg
Delta Theta
10 deg
Theta2
180.0 deg
Cycles
1
Time Delay
0

PROBLEM 3-32

Statement: Design a Hoeken straight-line linkage to give minimum error in straightness over 39% of the cycle for a 20-cm-long straight line motion. Specify all linkage parameters.

Given: Length of straight line motion: $\Delta x := 200 \cdot mm$

Percentage of cycle over which straight line motion takes place: 39%

Solution: See Figure 3-30 and Mathcad file P0332.

1. Using Table 3-1 and the required length of straight-line motion, determine the link lengths.

Link ratios from Table 3-1 for 39% cycle:

$L1overL2 := 2.500$ $L3overL2 := 3.250$ $\Delta xoverL2 := 3.623$

Link lengths:

Crank $L_2 := \dfrac{\Delta x}{\Delta xoverL2}$ $L_2 = 55.20\ mm$

Coupler $L_3 := L3overL2 \cdot L_2$ $L_3 = 179.41\ mm$

Ground link $L_1 := L1overL2 \cdot L_2$ $L_1 = 138.01\ mm$

Rocker $L_4 := L_3$ $L_4 = 179.41\ mm$

Coupler point $AP := 2 \cdot L_3$ $AP = 358.82\ mm$

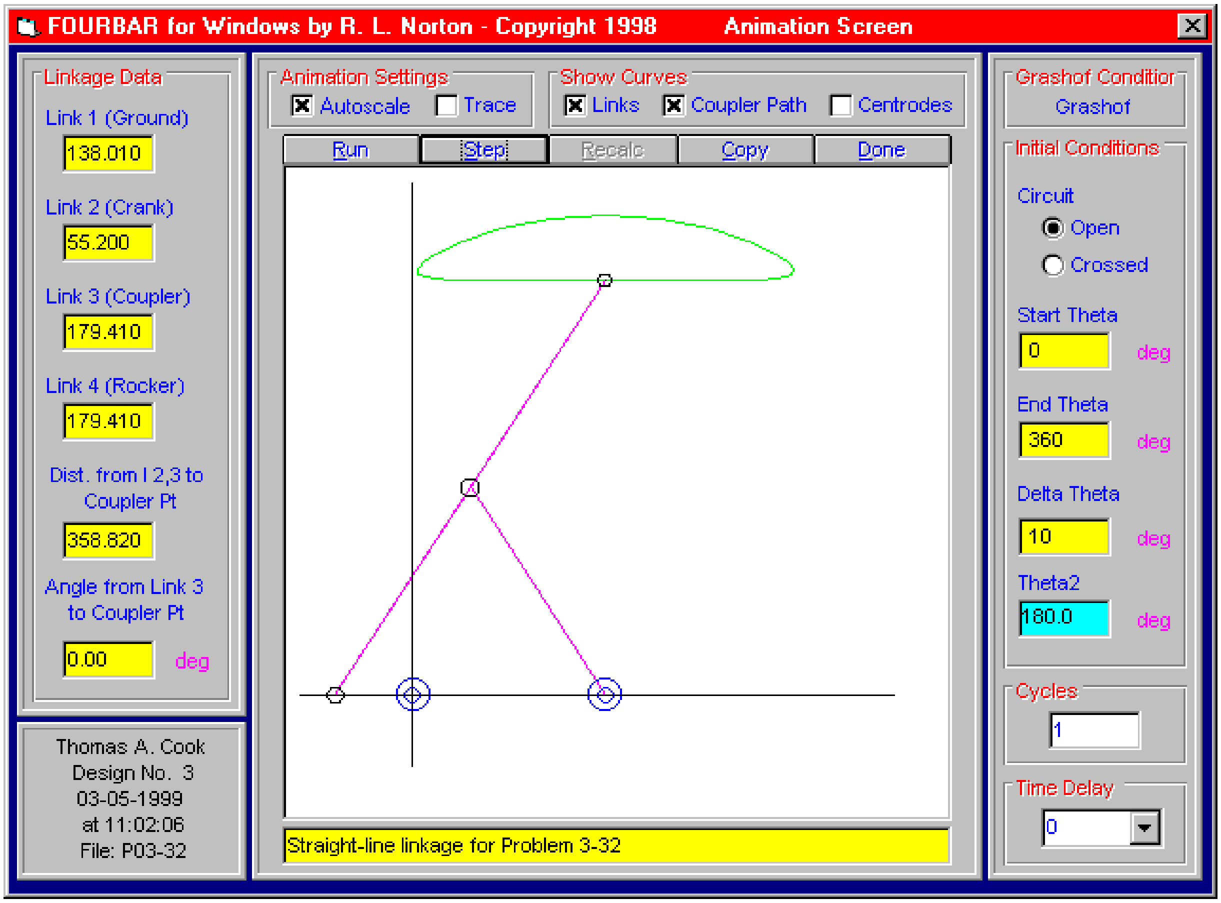
2. Calculate the distance from point P to pivot O_4 (C_y) when crank angle is 180 deg.

$$C_y := \sqrt{\left(2 \cdot L_3\right)^2 - \left(L_1 + L_2\right)^2} \qquad C_y = 302.36\ mm$$

3. Enter the link lengths into program FOURBAR to verify the design (see next page for coupler point curve). Using the PRINT facility, determine the x,y coordinates of the coupler curve and the x,y components of the coupler point velocity in the straight line region. A table of these values is printed below. Notice the small deviations over the range of crank angles from the y-coordinate and the x-velocity from a crank angle of 180 deg.

FOURBAR for Windows File P03-32.DAT

Angle Step Deg	Coupler Pt X mm	Coupler Pt Y mm	Veloc CP X mm/sec	Veloc CP Y mm/sec
110	237.992	302.408	-696.591	-6.416
120	225.289	302.361	-755.847	-0.019
130	211.710	302.378	-797.695	1.426
140	197.521	302.398	-826.217	0.664
150	182.927	302.399	-844.774	-0.483
160	168.076	302.385	-856.043	-1.052
170	153.076	302.368	-861.994	-0.800
180	138.010	302.360	-863.841	0.000
190	122.944	302.368	-861.994	0.800
200	107.944	302.385	-856.043	1.052
210	93.093	302.399	-844.774	0.483
220	78.499	302.398	-826.217	-0.664
230	64.311	302.378	-797.695	-1.426
240	50.731	302.361	-755.847	0.019
250	38.028	302.408	-696.591	6.416



FOURBAR for Windows by R. L. Norton - Copyright 1998 Animation Screen
Linkage Data
Link 1 (Ground)
138.010
Link 2 (Crank)
55.200
Link 3 (Coupler)
179.410
Link 4 (Rocker)
179.410
Dist. from I 2,3 to Coupler Pt
358.820
Angle from Link 3 to Coupler Pt
0.00
deg
Thomas A. Cook
Design No. 3
03-05-1999
at 11:02:06
File: P03-32
Animation Settings
Autoscale
Trace
Show Curves
Links
Coupler Path
Centrodes
Run
Step
Recalc
Copy
Done
Straight-line linkage for Problem 3-32
Grashof Condition
Grashof
Initial Conditions
Circuit
Open
Crossed
Start Theta
0
deg
End Theta
360
deg
Delta Theta
10
deg
Theta2
180.0
deg
Cycles
1
Time Delay
0

PROBLEM 3-33

Statement: Design a linkage that will give a symmetrical "kidney bean" shaped coupler curve as shown in Figure 3-16 (p. 114 and 115). Use the data in Figure 3-21 (p. 120) to determine the required link ratios and generate the coupler curve with program FOURBAR.

Solution: See Figures 3-16, 3-21, and Mathcad file P0333.

Design choices:

Ground link ratio, $L_1/L_2 = 2.0$: $GLR := 2.0$

Common link ratio, $L_3/L_2 = L_4/L_2 = BP/L_2 = 2.5$: $CLR := 2.5$

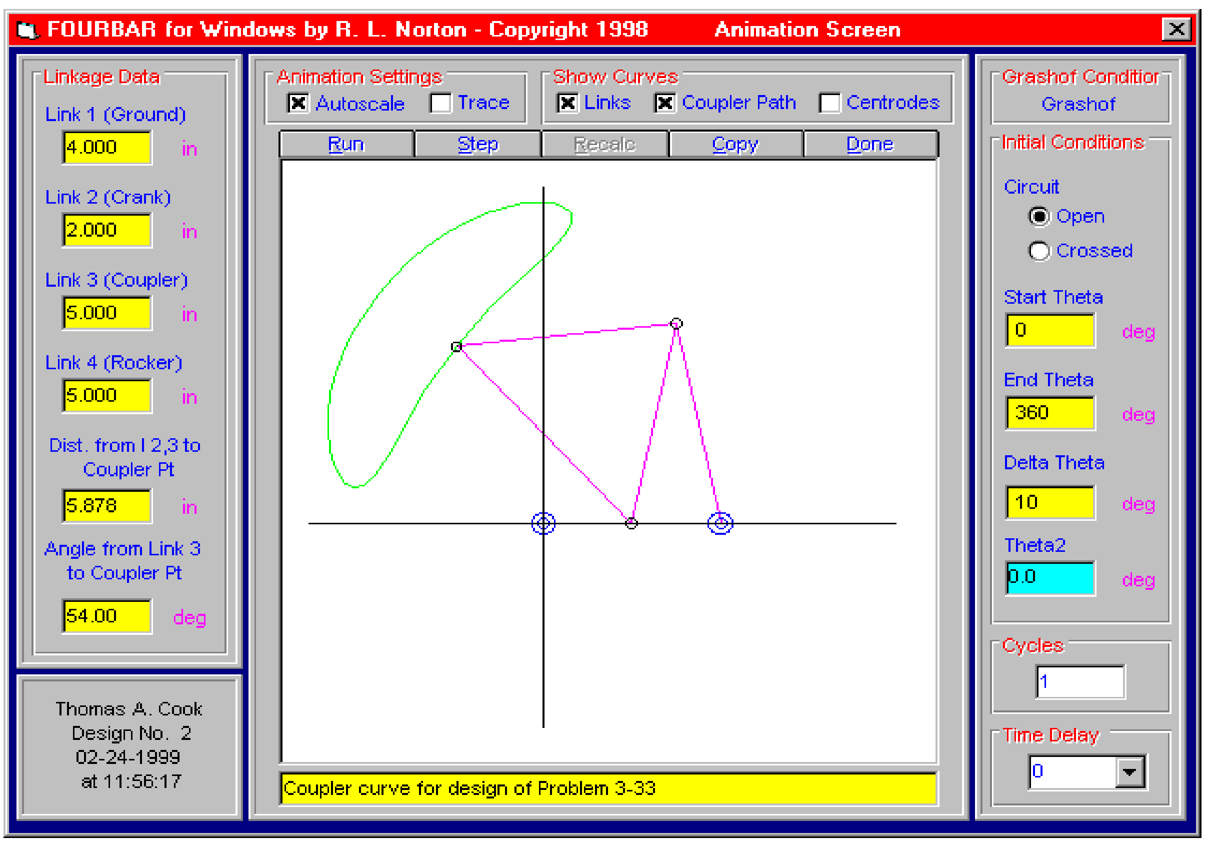
Coupler angle, $\gamma := 72 \cdot deg$

Crank length, $L_2 := 2.000$

1. For the given design choices, determine the remaining link lengths and coupler point specification.

Coupler link (3) length	$L_3 := CLR \cdot L_2$	$L_3 = 5.000$
Rocker link (4) length	$L_4 := CLR \cdot L_2$	$L_4 = 5.000$
Ground link (1) length	$L_1 := GLR \cdot L_2$	$L_1 = 4.000$
Angle *PAB*	$\delta := \dfrac{180 \cdot deg - \gamma}{2}$	$\delta = 54.000\, deg$
Length *AP* on coupler	$AP := 2 \cdot L_3 \cdot cos(\delta)$	$AP = 5.878$

2. Enter the above data into program FOURBAR and plot the coupler curve.



PROBLEM 3-34

Statement: Design a linkage that will give a symmetrical "double straight" shaped coupler curve as shown in Figure 3-16. Use the data in Figure 3-21 to determine the required link ratios and generate the coupler curve with program FOURBAR.

Solution: See Figures 3-16, 3-21, and Mathcad file P0334.

Design choices:

Ground link ratio, $L_1/L_2 = 2.5$: $GLR := 2.5$

Common link ratio, $L_3/L_2 = L_4/L_2 = BP/L_2 = 2.5$: $CLR := 2.5$

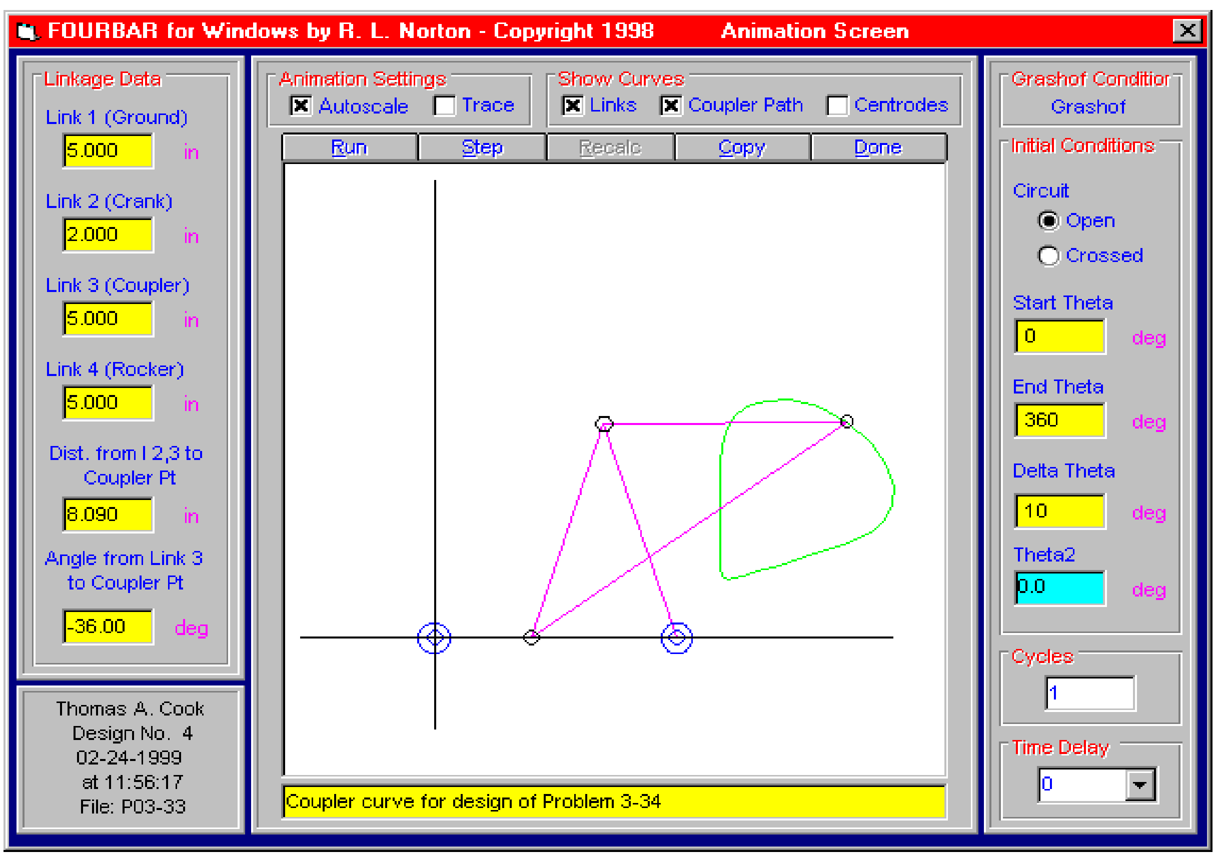
Coupler angle, $\gamma := 252 \cdot deg$

Crank length, $L_2 := 2.000$

1. For the given design choices, determine the remaining link lengths and coupler point specification.

Coupler link (3) length	$L_3 := CLR \cdot L_2$	$L_3 = 5.000$
Rocker link (4) length	$L_4 := CLR \cdot L_2$	$L_4 = 5.000$
Ground link (1) length	$L_1 := GLR \cdot L_2$	$L_1 = 5.000$
Angle *PAB*	$\delta := \dfrac{180 \cdot deg - \gamma}{2}$	$\delta = -36.000\, deg$
Length *AP* on coupler	$AP := 2 \cdot L_3 \cdot cos(\delta)$	$AP = 8.090$

2. Enter the above data into program FOURBAR and plot the coupler curve.



PROBLEM 3-35

Statement: Design a linkage that will give a symmetrical "scimitar" shaped coupler curve as shown in Figure 3-16. Use the data in Figure 3-21 to determine the required link ratios and generate the coupler curve with program FOURBAR. Show that there are (or are not) true cusps on the curve.

Solution: See Figures 3-16, 3-21, and Mathcad file P0334.

Design choices:

Ground link ratio, $L_1/L_2 = 2.0$: $GLR := 2.0$

Common link ratio, $L_3/L_2 = L_4/L_2 = BP/L_2 = 2.5$: $CLR := 2.5$

Coupler angle, $\gamma := 144 \cdot deg$

Crank length, $L_2 := 2.000$

1. For the given design choices, determine the remaining link lengths and coupler point specification.

Coupler link (3) length $\quad L_3 := CLR \cdot L_2 \quad L_3 = 5.000$

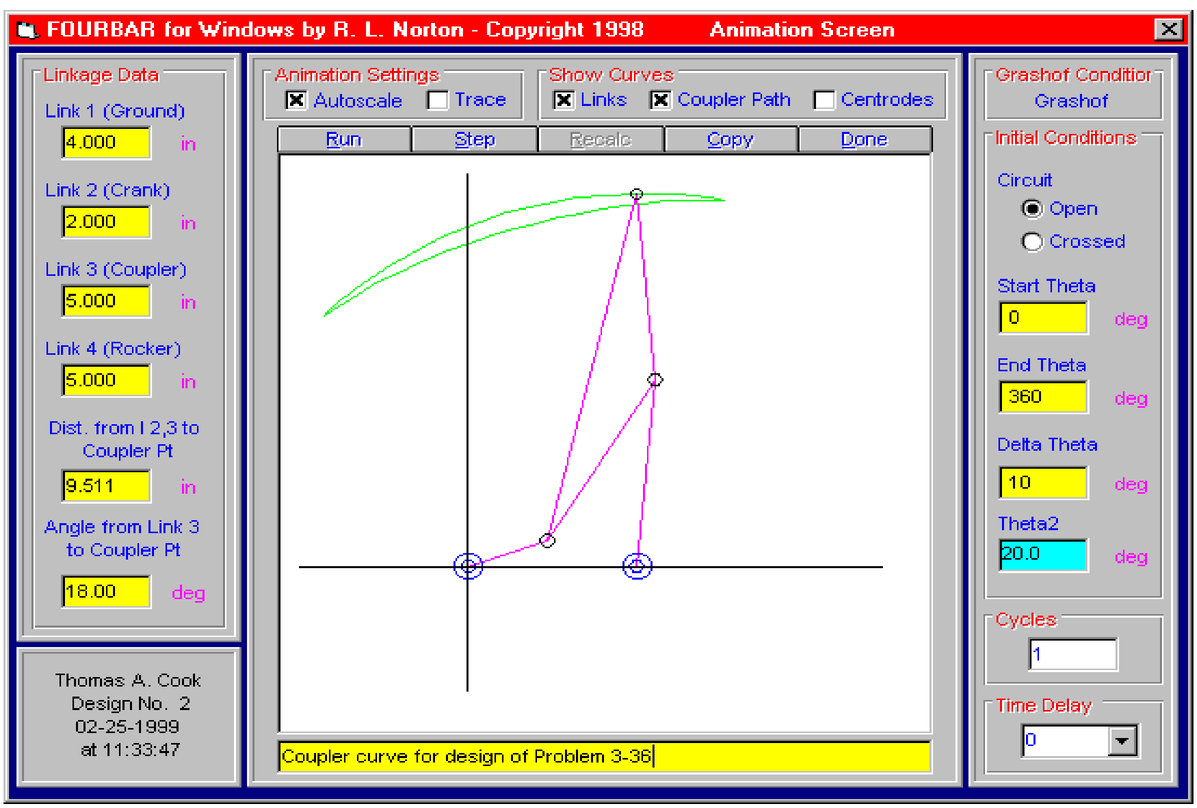
Rocker link (4) length $\quad L_4 := CLR \cdot L_2 \quad L_4 = 5.000$

Ground link (1) length $\quad L_1 := GLR \cdot L_2 \quad L_1 = 4.000$

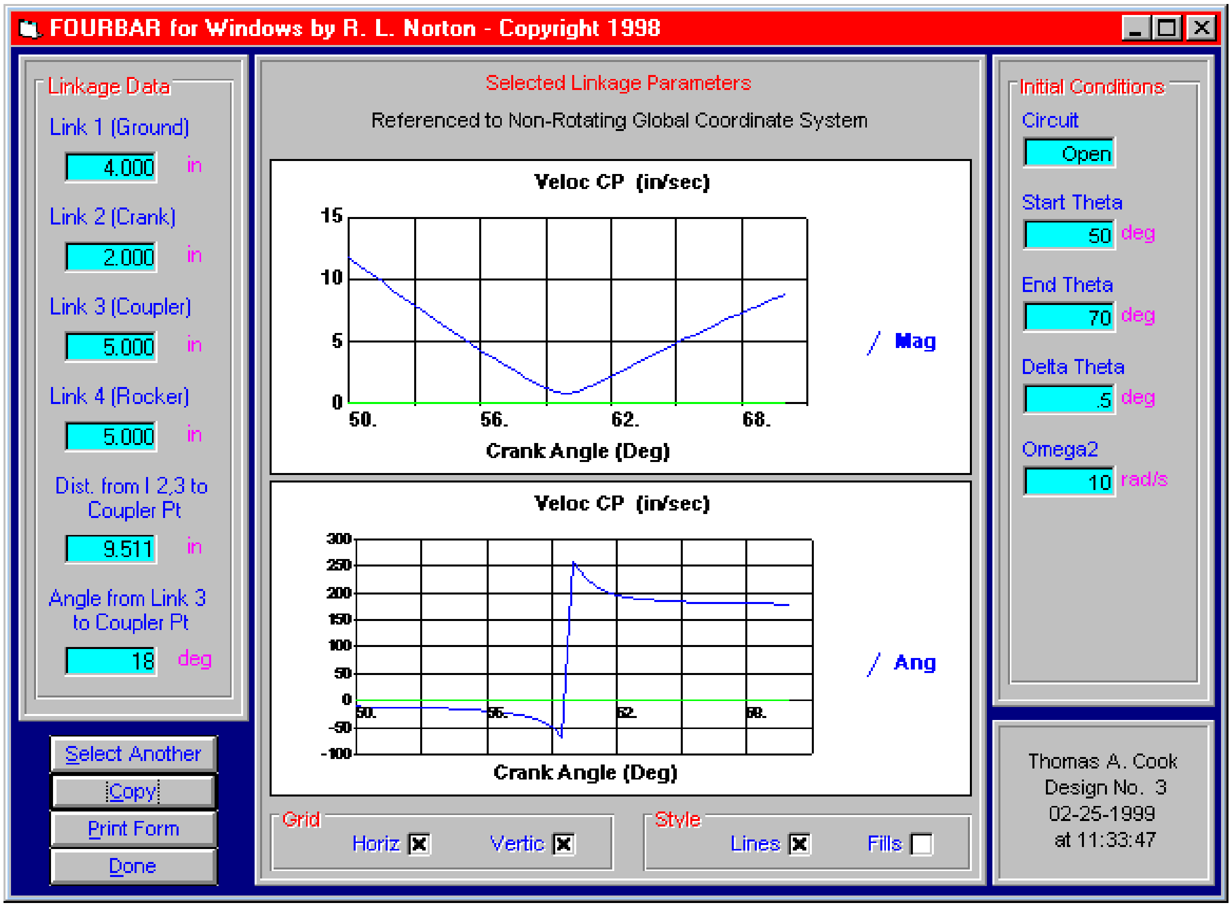
Angle *PAB* $\quad \delta := \dfrac{180 \cdot deg - \gamma}{2} \quad \delta = 18.000\, deg$

Length *AP* on coupler $\quad AP := 2 \cdot L_3 \cdot cos(\delta) \quad AP = 9.511$

2. Enter the above data into program FOURBAR and plot the coupler curve.



3. The points at the ends of the "scimitar" will be true cusps if the velocity of the coupler point is zero at these points. Using FOURBAR's plotting utility, plot the magnitude and angle of the coupler point velocity vector. As seen below for the range of crank angle from 50 to 70 degrees, the magnitude of the velocity does not quite reach zero. Therefore, these are not true cusps.



PROBLEM 3-36

Statement: Find the Grashof condition, inversion, any limit positions, and the extreme values of the transmission angle (to graphical accuracy) of the linkage in Figure P3-10.

Given: Link lengths: Link 2 $L_2 := 0.785$ Link 3 $L_3 := 0.356$

Link 4 $L_4 := 0.950$ Link 1 $L_1 := 0.544$

Grashof condition function:

$$Condition(a,b,c,d) := \begin{array}{|l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return \text{ "Grashof" } \; if \; SL < PQ \\ return \text{ "Special Grashof" } \; if \; SL = PQ \\ return \text{ "non-Grashof" } \; otherwise \end{array}$$

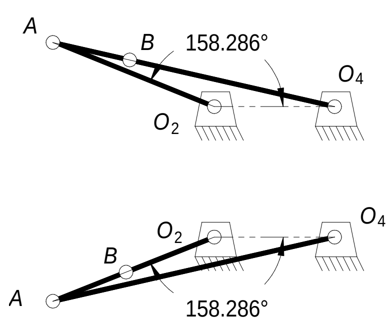
Solution: See Figure P3-10 and Mathcad file P0336.

1. Determine the Grashof condition of the mechanism from inequality 2.8 and its Barker classification from Table 2-4.

Grashof condition: $Condition\left(L_1, L_2, L_3, L_4\right) = \text{"Grashof"}$

Barker classification: Class I-3, Grashof rocker-crank-rocker, GRCR, since the shortest link is the coupler link.

2. A GRCR linkage will have two toggle positions. Draw the linkage in these two positions and measure the input link angles.



3. As measured from the layout, the input link angles at the toggle positions are: +158.3 and -158.3 deg.

4. Since the coupler link in a GRCR linkage can make a full rotation with respect to the input and output rockers, the minimum transmission angle is 0 deg and the maximum is 90 deg.

PROBLEM 3-37

Statement: Draw the Roberts diagram and find the cognates for the linkage in Figure P3-10.

Given: Link lengths: Coupler point data:

Ground link $L_1 := 0.544$ Crank $L_2 := 0.785$ $A1P := 1.09$ $\delta_1 := 0.00 \cdot deg$

Coupler $L_3 := 0.356$ Rocker $L_4 := 0.950$

Solution: See Figure P3-10 and Mathcad file P0337.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle *APB*.

$$B1P := \left(L_3^{\,2} + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1)\right)^{0.5} \qquad B1P = 0.734$$

$$\gamma_1 := acos\left(\frac{L_3^{\,2} + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P}\right) \qquad \gamma_1 = 180.0000\,deg$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \qquad L_5 = 0.734 \qquad L_6 := \frac{L_4}{L_3} \cdot B1P \qquad L_6 = 1.959$$

$$L_{10} := A1P \qquad L_{10} = 1.090 \qquad L_9 := \frac{L_2}{L_3} \cdot A1P \qquad L_9 = 2.404$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \qquad L_7 = 1.619 \qquad L_8 := L_6 \cdot \frac{A1P}{B1P} \qquad L_8 = 2.909$$

Calculate the coupler point data for cognates #2 and #3

$$A3P := L_4 \qquad A3P = 0.950 \qquad A2P := L_2 \qquad A2P = 0.785$$

$$\delta_3 := 180 \cdot deg - \delta_1 \qquad \delta_3 = 180.000\,deg \qquad \delta_2 := -\delta_1 \qquad \delta_2 = 0.000\,deg$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

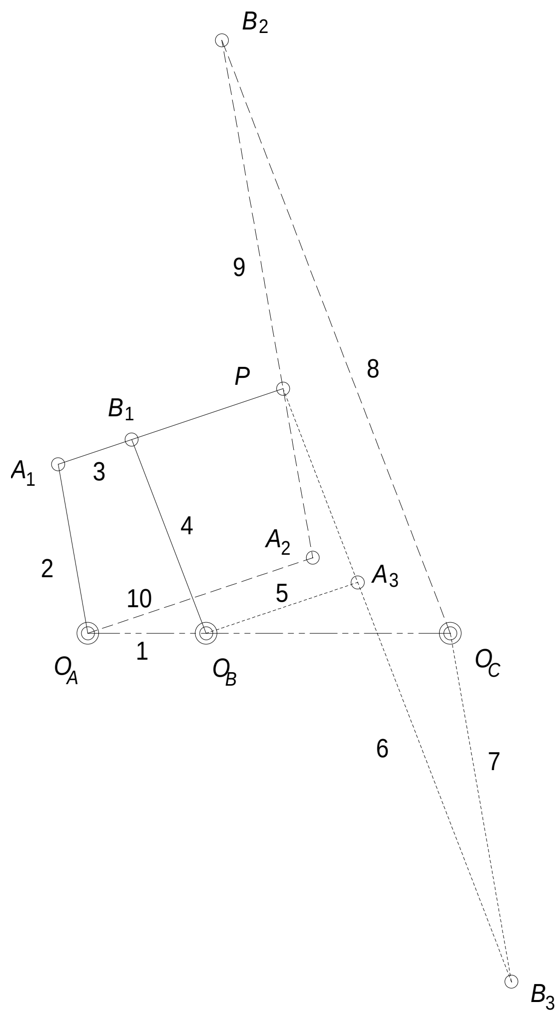
$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \qquad L_{1BC} = 1.1216 \qquad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \qquad L_{1AC} = 1.6656$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 0.544$	$L_{1AC} = 1.666$	$L_{1BC} = 1.122$
Crank length	$L_2 = 0.785$	$L_{10} = 1.090$	$L_7 = 1.619$
Coupler length	$L_3 = 0.356$	$L_9 = 2.404$	$L_6 = 1.959$

Rocker length	$L_4 = 0.950$	$L_8 = 2.909$	$L_5 = 0.734$
Coupler point	$A1P = 1.090$	$A2P = 0.785$	$A3P = 0.950$
Coupler angle	$\delta_1 = 0.000\ deg$	$\delta_2 = 0.000\ deg$	$\delta_3 = 180.000\ deg$



PROBLEM 3-38

Statement: Find the three geared fivebar cognates of the linkage in Figure P3-10.

Given: Link lengths:

Ground link $L_1 := 0.544$ Crank $L_2 := 0.785$

Coupler $L_3 := 0.356$ Rocker $L_4 := 0.950$

Coupler point data:

$A1P := 1.09$ $\delta_1 := 0.00 \cdot deg$

Solution: See Figure P3-10 and Mathcad file P0338.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle *APB*.

$$B1P := \left(L_3^{\,2} + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1)\right)^{0.5} \qquad B1P = 0.734$$

$$\gamma_1 := acos\left(\frac{L_3^{\,2} + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P}\right) \qquad \gamma_1 = 180.0000\,deg$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$L_5 := B1P$ $\quad L_5 = 0.734$ $\qquad L_6 := \frac{L_4}{L_3} \cdot B1P$ $\quad L_6 = 1.959$

$L_{10} := A1P$ $\quad L_{10} = 1.090$ $\qquad L_9 := \frac{L_2}{L_3} \cdot A1P$ $\quad L_9 = 2.404$

$L_7 := L_9 \cdot \frac{B1P}{A1P}$ $\quad L_7 = 1.619$ $\qquad L_8 := L_6 \cdot \frac{A1P}{B1P}$ $\quad L_8 = 2.909$

Calculate the coupler point data for cognates #2 and #3

$A3P := L_4$ $\quad A3P = 0.950$ $\qquad A2P := L_2$ $\quad A2P = 0.785$

$\delta_3 := 180 \cdot deg - \delta_1$ $\quad \delta_3 = 180.000\,deg$ $\qquad \delta_2 := -\delta_1$ $\quad \delta_2 = 0.000\,deg$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

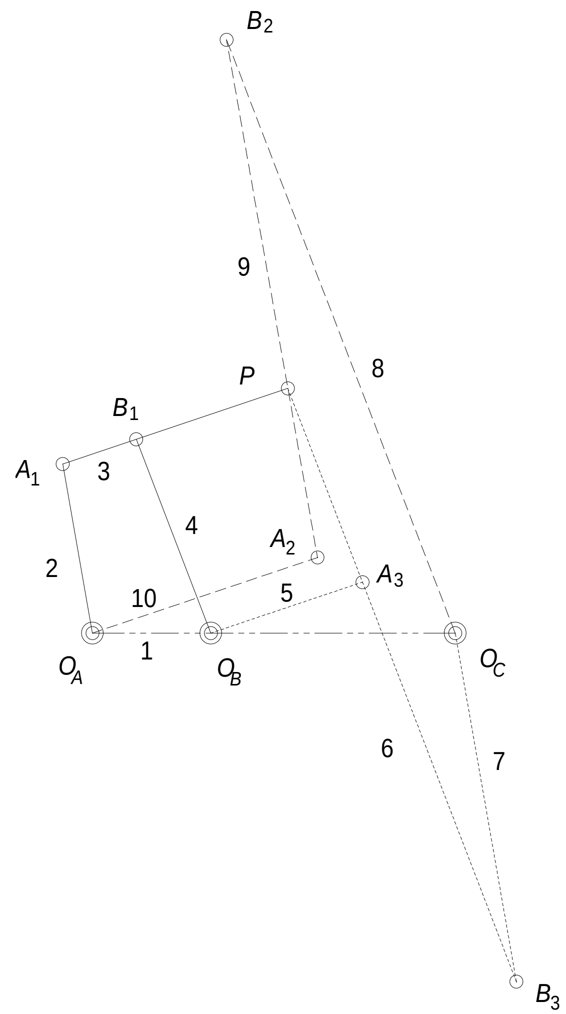
$L_{1BC} := \frac{L_1}{L_3} \cdot B1P$ $\quad L_{1BC} = 1.1216$ $\qquad L_{1AC} := \frac{L_1}{L_3} \cdot A1P$ $\quad L_{1AC} = 1.6656$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 0.544$	$L_{1AC} = 1.666$	$L_{1BC} = 1.122$
Crank length	$L_2 = 0.785$	$L_{10} = 1.090$	$L_7 = 1.619$
Coupler length	$L_3 = 0.356$	$L_9 = 2.404$	$L_6 = 1.959$
Rocker length	$L_4 = 0.950$	$L_8 = 2.909$	$L_5 = 0.734$
Coupler point	$A1P = 1.090$	$A2P = 0.785$	$A3P = 0.950$

Coupler angle $\delta_1 = 0.000\ deg$ $\delta_2 = 0.000\ deg$ $\delta_3 = 180.000\ deg$



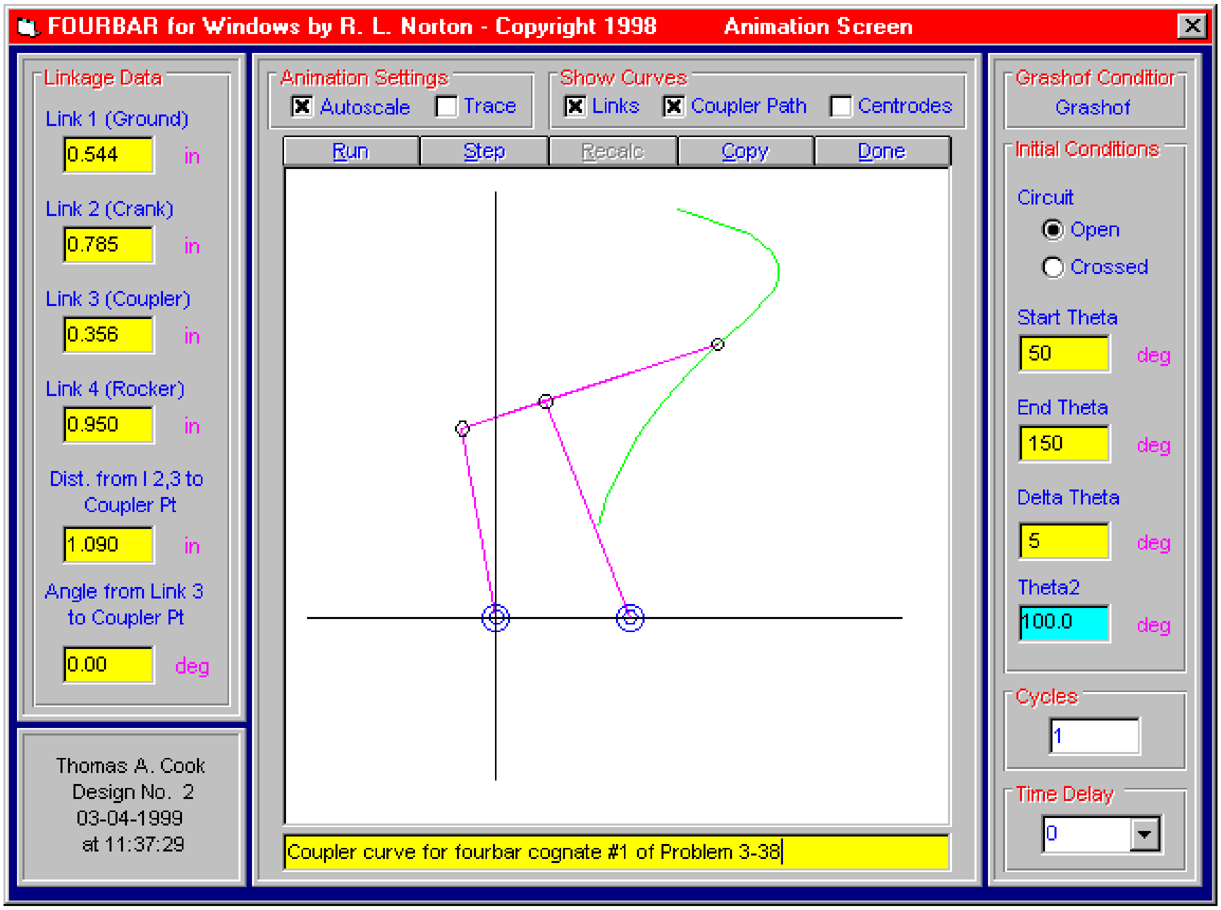
4. The three geared fivebar cognates can be seen in the Roberts diagram. They are: $O_AA_2PA_3O_B$, $O_AA_1PB_3O_C$, and $O_BB_1PB_2O_C$. They are specified in the summary table below.

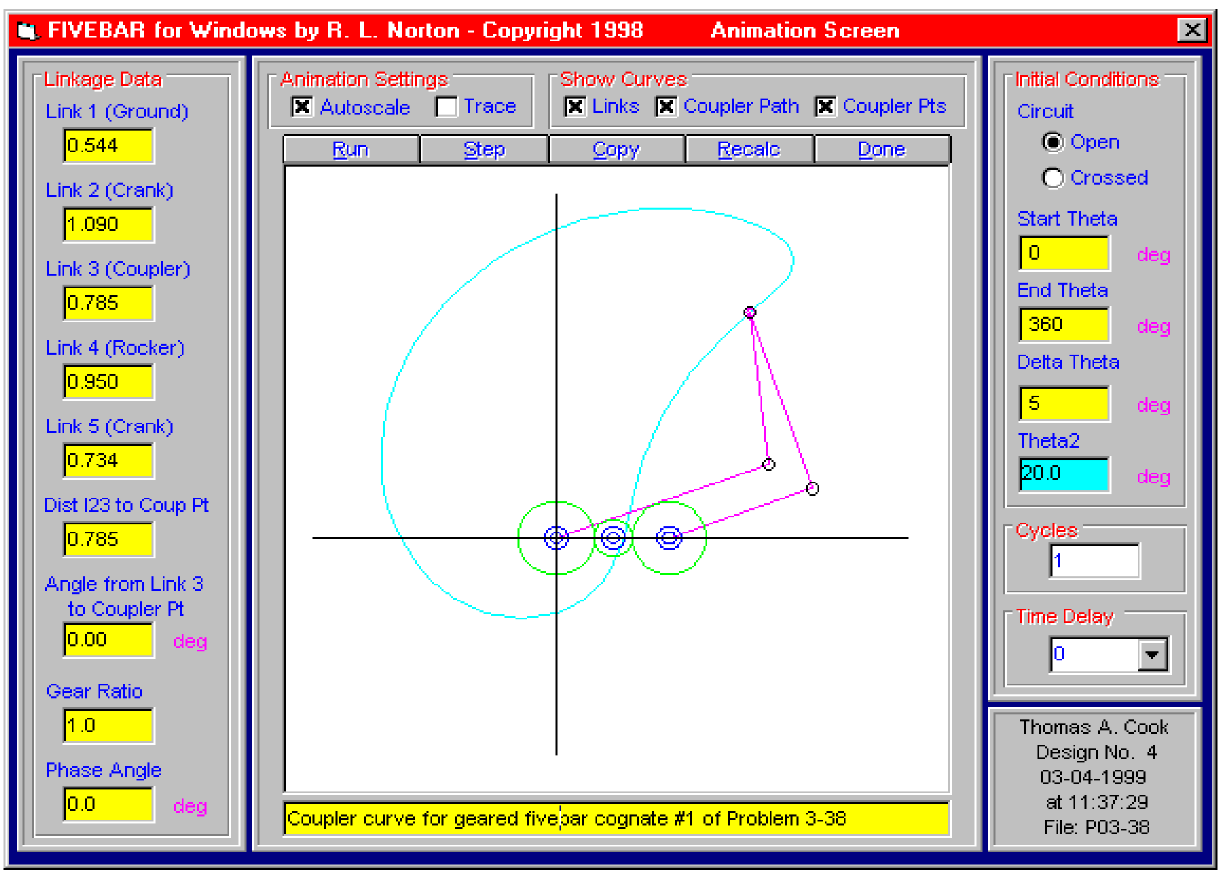
SUMMARY OF GEARED FIVEBAR COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 0.544$	$L_{1AC} = 1.666$	$L_{1BC} = 1.122$
Crank length	$L_{10} = 1.090$	$L_2 = 0.785$	$L_4 = 0.950$
Coupler length	$A2P = 0.785$	$A1P = 1.090$	$L_5 = 0.734$
Rocker length	$A3P = 0.950$	$L_8 = 2.909$	$L_7 = 1.619$
Crank length	$L_5 = 0.734$	$L_7 = 1.619$	$L_8 = 2.909$
Coupler point	$A2P = 0.785$	$A1P = 1.090$	$B1P = 0.734$
Coupler angle	$\delta_1 := 0.00 \cdot deg$	$\delta_2 := 0.00 \cdot deg$	$\delta_3 := 0.00 \cdot deg$

5. Enter the cognate #1 specifications into program FOURBAR to get a trace of the coupler path (see next page).

6. Enter the geared fivebar cognate #1 specifications into program FIVEBAR to get a trace of the coupler path for the geared fivebar (see next page).


FOURBAR for Windows by R. L. Norton - Copyright 1998 Animation Screen
Linkage Data
Link 1 (Ground)
0.544 in
Link 2 (Crank)
0.785 in
Link 3 (Coupler)
0.356 in
Link 4 (Rocker)
0.950 in
Dist. from I 2,3 to Coupler Pt
1.090 in
Angle from Link 3 to Coupler Pt
0.00 deg
Thomas A. Cook
Design No. 2
03-04-1999
at 11:37:29
Animation Settings
Autoscale
Trace
Show Curves
Links
Coupler Path
Centrodes
Run
Step
Recalc
Copy
Done
Coupler curve for fourbar cognate #1 of Problem 3-38
Grashof Condition
Grashof
Initial Conditions
Circuit
Open
Crossed
Start Theta
50 deg
End Theta
150 deg
Delta Theta
5 deg
Theta2
100.0 deg
Cycles
1
Time Delay
0


FIVEBAR for Windows by R. L. Norton - Copyright 1998 Animation Screen
Linkage Data
Link 1 (Ground)
0.544
Link 2 (Crank)
1.090
Link 3 (Coupler)
0.785
Link 4 (Rocker)
0.950
Link 5 (Crank)
0.734
Dist I23 to Coup Pt
0.785
Angle from Link 3 to Coupler Pt
0.00 deg
Gear Ratio
1.0
Phase Angle
0.0 deg
Animation Settings
Autoscale
Trace
Show Curves
Links
Coupler Path
Coupler Pts
Run
Step
Copy
Recalc
Done
Coupler curve for geared fivebar cognate #1 of Problem 3-38
Initial Conditions
Circuit
Open
Crossed
Start Theta
0 deg
End Theta
360 deg
Delta Theta
5 deg
Theta2
20.0 deg
Cycles
1
Time Delay
0
Thomas A. Cook
Design No. 4
03-04-1999
at 11:37:29
File: P03-38

PROBLEM 3-39

Statement: Find the Grashof condition, any limit positions, and the extreme values of the transmission angle (to graphical accuracy) of the linkage in Figure P3-11.

Given: Link lengths: Link 2 $L_2 := 0.86$ Link 3 $L_3 := 1.85$

Link 4 $L_4 := 0.86$ Link 1 $L_1 := 2.22$

Grashof condition function:

$$
Condition(a,b,c,d) := \begin{array}{|l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array}
$$

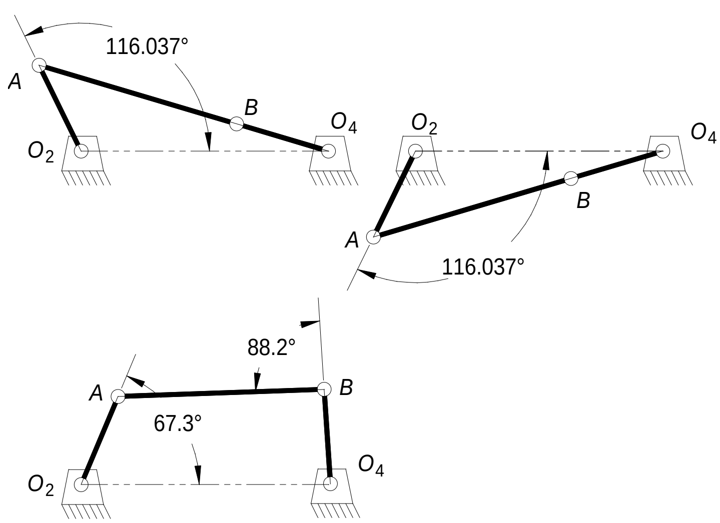
Solution: See Figure P3-11 and Mathcad file P0339.

1. Determine the Grashof condition of the mechanism from inequality 2.8 and its Barker classification from Table 2-4.

   Grashof condition: $Condition(L_1, L_2, L_3, L_4) = $ "non-Grashof"

   Barker classification: Class II-1, non-Grashof triple rocker, RRR1, since the longest link is the ground link.

2. An RRR1 linkage will have two toggle positions. Draw the linkage in these two positions and measure the input link angles.



3. As measured from the layout, the input link angles at the toggle positions are: +116 and -116 deg.

4. Since the coupler link in an RRR1 linkage cannot make a full rotation with respect to the input and output rockers, the minimum transmission angle is 0 deg and the maximum is 88 deg.

PROBLEM 3-40

Statement: Draw the Roberts diagram and find the cognates for the linkage in Figure P3-11.

Given: Link lengths: Coupler point data:

Ground link $L_1 := 2.22$ Crank $L_2 := 0.86$ $A1P := 1.33$ $\delta_1 := 0.00 \cdot deg$

Coupler $L_3 := 1.85$ Rocker $L_4 := 0.86$

Solution: See Figure P3-11 and Mathcad file P0340.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle *APB*.

$$B1P := \left(L_3^2 + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1)\right)^{0.5} \qquad B1P = 0.520$$

$$\gamma_1 := acos\left(\frac{L_3^2 + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P}\right) \qquad \gamma_1 = 0.0000\, deg$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$L_5 := B1P$ $L_5 = 0.520$ $L_6 := \frac{L_4}{L_3} \cdot B1P$ $L_6 = 0.242$

$L_{10} := A1P$ $L_{10} = 1.330$ $L_9 := \frac{L_2}{L_3} \cdot A1P$ $L_9 = 0.618$

$L_7 := L_9 \cdot \frac{B1P}{A1P}$ $L_7 = 0.242$ $L_8 := L_6 \cdot \frac{A1P}{B1P}$ $L_8 = 0.618$

Calculate the coupler point data for cognates #2 and #3

$A3P := L_8$ $A3P = 0.618$ $A2P := L_7$ $A2P = 0.242$

$\delta_3 := 180 \cdot deg$ $\delta_3 = 180.000\, deg$ $\delta_2 := 180 \cdot deg$ $\delta_2 = 180.000\, deg$

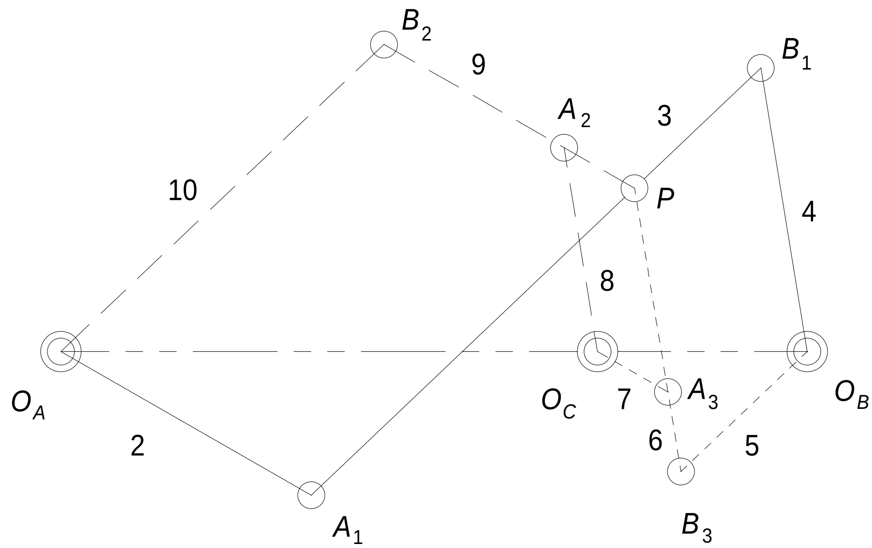
From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

$L_{1BC} := \frac{L_1}{L_3} \cdot B1P$ $L_{1BC} = 0.6240$ $L_{1AC} := \frac{L_1}{L_3} \cdot A1P$ $L_{1AC} = 1.5960$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 2.220$	$L_{1AC} = 1.596$	$L_{1BC} = 0.624$
Crank length	$L_2 = 0.860$	$L_{10} = 1.330$	$L_7 = 0.242$
Coupler length	$L_3 = 1.850$	$L_9 = 0.618$	$L_6 = 0.242$
Rocker length	$L_4 = 0.860$	$L_8 = 0.618$	$L_5 = 0.520$
Coupler point	$A1P = 1.330$	$A2P = 0.242$	$A3P = 0.618$
Coupler angle	$\delta_1 = 0.000\, deg$	$\delta_2 = 180.000\, deg$	$\delta_3 = 180.000\, deg$


B2
9
A2
3
B1
10
P
4
8
OA
OC
7
A3
OB
2
6
5
A1
B3

PROBLEM 3-41

Statement: Find the three geared fivebar cognates of the linkage in Figure P3-11.

Given: Link lengths:

Ground link $L_1 := 2.22$ Crank $L_2 := 0.86$

Coupler $L_3 := 1.85$ Rocker $L_4 := 0.86$

Coupler point data: $A1P := 1.33$ $\delta_1 := 0.00 \cdot deg$

Solution: See Figure P3-11 and Mathcad file P0341.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle *APB*.

$$B1P := \left(L_3^{\,2} + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot cos(\delta_1)\right)^{0.5} \qquad B1P = 0.520$$

$$\gamma_1 := acos\left(\frac{L_3^{\,2} + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P}\right) \qquad \gamma_1 = 0.0000\,deg$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \qquad L_5 = 0.520 \qquad L_6 := \frac{L_4}{L_3} \cdot B1P \qquad L_6 = 0.242$$

$$L_{10} := A1P \qquad L_{10} = 1.330 \qquad L_9 := \frac{L_2}{L_3} \cdot A1P \qquad L_9 = 0.618$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \qquad L_7 = 0.242 \qquad L_8 := L_6 \cdot \frac{A1P}{B1P} \qquad L_8 = 0.618$$

Calculate the coupler point data for cognates #2 and #3

$$A3P := L_8 \qquad A3P = 0.618 \qquad A2P := L_7 \qquad A2P = 0.242$$

$$\delta_3 := 180 \cdot deg \qquad \delta_3 = 180.000\,deg \qquad \delta_2 := 180 \cdot deg \qquad \delta_2 = 180.000\,deg$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

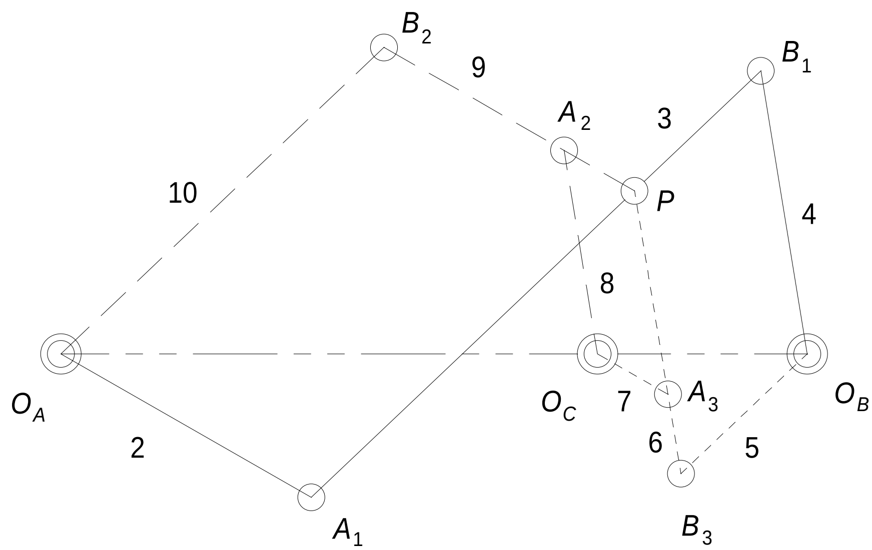
$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \qquad L_{1BC} = 0.6240 \qquad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \qquad L_{1AC} = 1.5960$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 2.220$	$L_{1AC} = 1.596$	$L_{1BC} = 0.624$
Crank length	$L_2 = 0.860$	$L_{10} = 1.330$	$L_7 = 0.242$
Coupler length	$L_3 = 1.850$	$L_9 = 0.618$	$L_6 = 0.242$
Rocker length	$L_4 = 0.860$	$L_8 = 0.618$	$L_5 = 0.520$

Coupler point	$A1P = 1.330$	$A2P = 0.242$	$A3P = 0.618$
Coupler angle	$\delta_1 = 0.000\ deg$	$\delta_2 = 180.000\ deg$	$\delta_3 = 180.000\ deg$



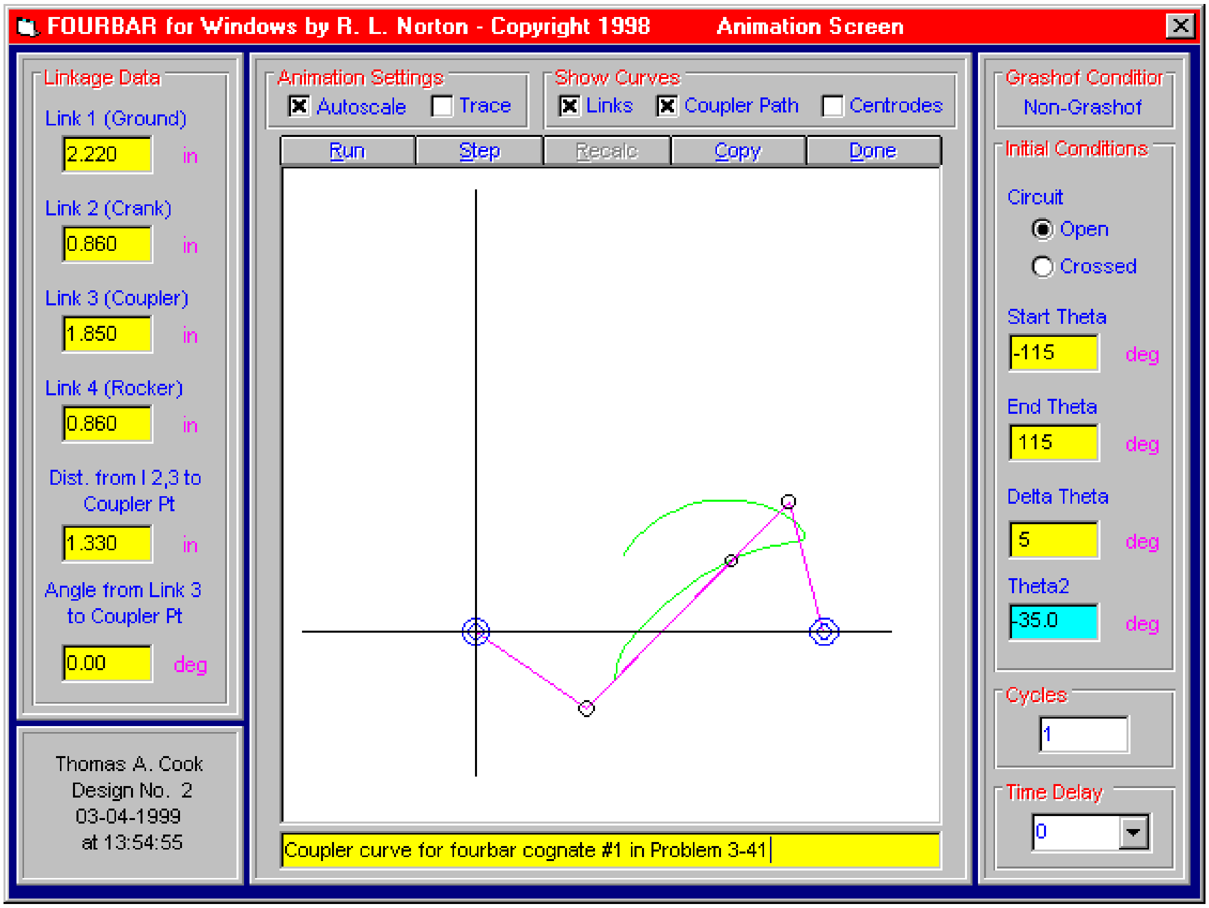
4. The three geared fivebar cognates can be seen in the Roberts diagram. They are: $O_AB_2PB_3O_B$, $O_AA_1PA_3O_C$, and $O_BB_1PA_2O_C$. The three geared fivebar cognates are summarized in the table below.

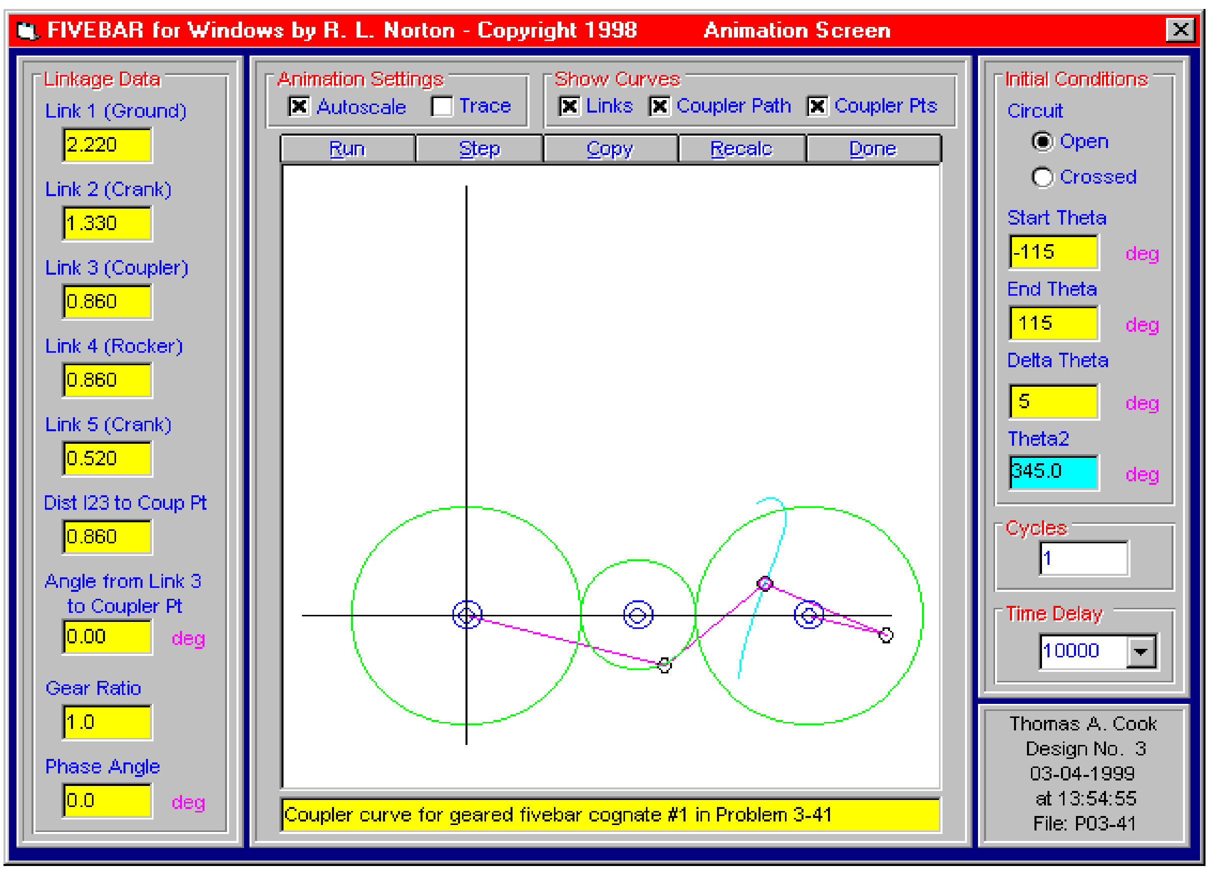
SUMMARY OF GEARED FIVEBAR COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 2.220$	$L_{1AC} = 1.596$	$L_{1BC} = 0.624$
Crank length	$L_{10} = 1.330$	$L_2 = 0.860$	$L_4 = 0.860$
Coupler length	$L_2 = 0.860$	$A1P = 1.330$	$L_5 = 0.520$
Rocker length	$L_4 = 0.860$	$L_8 = 0.618$	$L_7 = 0.242$
Crank length	$L_5 = 0.520$	$L_7 = 0.242$	$L_8 = 0.618$
Coupler point	$L_2 = 0.860$	$A1P = 1.330$	$B1P = 0.520$
Coupler angle	$\delta_1 := 0.00 \cdot deg$	$\delta_2 := 0.00 \cdot deg$	$\delta_3 := 0.00 \cdot deg$

5. Enter the cognate #1 specifications into program FOURBAR to get a trace of the coupler path (see next page)

6. Enter the geared fivebar cognate #1 specifications into program FIVEBAR to get a trace of the coupler path for the geared fivebar (see next page).


FOURBAR for Windows by R. L. Norton - Copyright 1998 Animation Screen
Linkage Data
Link 1 (Ground)
2.220 in
Link 2 (Crank)
0.860 in
Link 3 (Coupler)
1.850 in
Link 4 (Rocker)
0.860 in
Dist. from I 2,3 to Coupler Pt
1.330 in
Angle from Link 3 to Coupler Pt
0.00 deg
Thomas A. Cook
Design No. 2
03-04-1999
at 13:54:55
Animation Settings
Autoscale
Trace
Show Curves
Links
Coupler Path
Centrodes
Run
Step
Recalc
Copy
Done
Coupler curve for fourbar cognate #1 in Problem 3-41
Grashof Condition
Non-Grashof
Initial Conditions
Circuit
Open
Crossed
Start Theta
-115 deg
End Theta
115 deg
Delta Theta
5 deg
Theta2
-35.0 deg
Cycles
1
Time Delay
0


FIVEBAR for Windows by R. L. Norton - Copyright 1998 Animation Screen
Linkage Data
Link 1 (Ground)
2.220
Link 2 (Crank)
1.330
Link 3 (Coupler)
0.860
Link 4 (Rocker)
0.860
Link 5 (Crank)
0.520
Dist I23 to Coup Pt
0.860
Angle from Link 3 to Coupler Pt
0.00 deg
Gear Ratio
1.0
Phase Angle
0.0 deg
Animation Settings
Autoscale
Trace
Show Curves
Links
Coupler Path
Coupler Pts
Run
Step
Copy
Recalc
Done
Coupler curve for geared fivebar cognate #1 in Problem 3-41
Initial Conditions
Circuit
Open
Crossed
Start Theta
-115 deg
End Theta
115 deg
Delta Theta
5 deg
Theta2
345.0 deg
Cycles
1
Time Delay
10000
Thomas A. Cook
Design No. 3
03-04-1999
at 13:54:55
File: P03-41

PROBLEM 3-42

Statement: Find the Grashof condition, any limit positions, and the extreme values of the transmission angle (to graphical accuracy) of the linkage in Figure P3-12.

Given: Link lengths: Link 2 $L_2 := 0.72$ Link 3 $L_3 := 0.68$

Link 4 $L_4 := 0.85$ Link 1 $L_1 := 1.82$

Grashof condition function:

$$Condition(a,b,c,d) := \begin{array}{|l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return \text{ "Grashof" } \; if \; SL < PQ \\ return \text{ "Special Grashof" } \; if \; SL = PQ \\ return \text{ "non-Grashof" } \; otherwise \end{array}$$

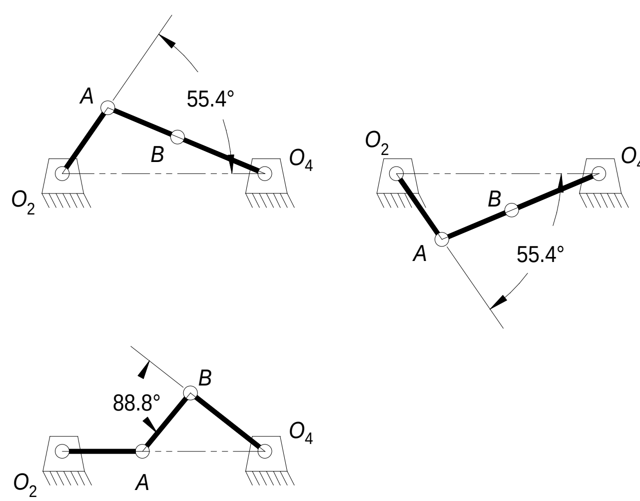
Solution: See Figure P3-12 and Mathcad file P0342.

1. Determine the Grashof condition of the mechanism from inequality 2.8 and its Barker classification from Table 2-4.

Grashof condition: $Condition(L_1, L_2, L_3, L_4) = \text{"non-Grashof"}$

Barker classification: Class II-1, non-Grashof triple rocker, RRR1, since the longest link is the ground link.

2. An RRR1 linkage will have two toggle positions. Draw the linkage in these two positions and measure the input link angles.



3. As measured from the layout, the input link angles at the toggle positions are: +55.4 and -55.4 deg.

4. Since the coupler link in an RRR1 linkage it cannot make a full rotation with respect to the input and output rockers, the minimum transmission angle is 0 deg and the maximum is 88.8 deg.

PROBLEM 3-43

Statement: Draw the Roberts diagram and find the cognates for the linkage in Figure P3-12.

Given: Link lengths:

Ground link $L_1 := 1.82$ Crank $L_2 := 0.72$

Coupler $L_3 := 0.68$ Rocker $L_4 := 0.85$

Coupler point data: $A1P := 0.97$ $\delta_1 := 54.0 \cdot deg$

Solution: See Figure P3-12 and Mathcad file P0343.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle *APB*.

$$B1P := \left(L_3{}^2 + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot cos(\delta_1)\right)^{0.5} \qquad B1P = 0.792$$

$$\gamma_1 := acos\left(\frac{L_3{}^2 + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P}\right) \qquad \gamma_1 = 82.0315\, deg$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \qquad L_5 = 0.792 \qquad L_6 := \frac{L_4}{L_3} \cdot B1P \qquad L_6 = 0.990$$

$$L_{10} := A1P \qquad L_{10} = 0.970 \qquad L_9 := \frac{L_2}{L_3} \cdot A1P \qquad L_9 = 1.027$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \qquad L_7 = 0.839 \qquad L_8 := L_6 \cdot \frac{A1P}{B1P} \qquad L_8 = 1.212$$

Calculate the coupler point data for cognates #2 and #3

$$A3P := L_4 \qquad A3P = 0.850 \qquad A2P := L_2 \qquad A2P = 0.720$$

$$\delta_3 := \gamma_1 \qquad \delta_3 = 82.032\, deg \qquad \delta_2 := -\delta_1 \qquad \delta_2 = -54.000\, deg$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

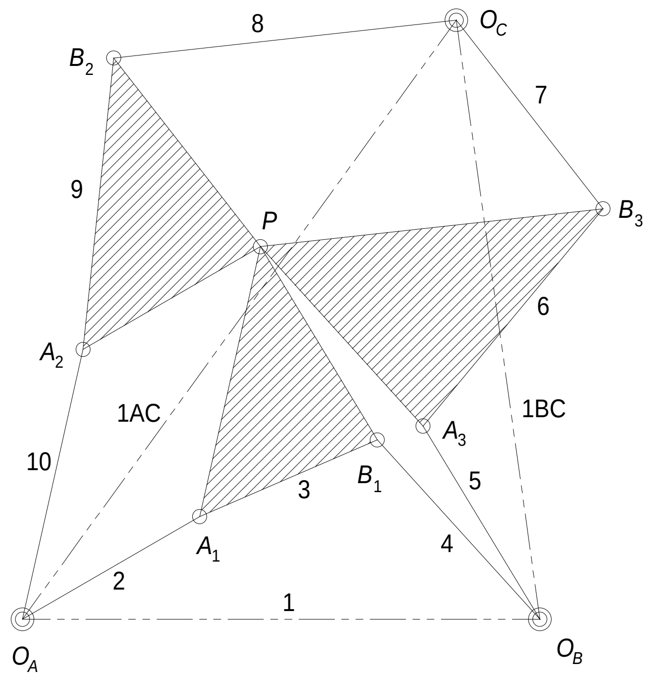
$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \qquad L_{1BC} = 2.1208 \qquad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \qquad L_{1AC} = 2.5962$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 1.820$	$L_{1AC} = 2.596$	$L_{1BC} = 2.121$
Crank length	$L_2 = 0.720$	$L_{10} = 0.970$	$L_7 = 0.839$
Coupler length	$L_3 = 0.680$	$L_9 = 1.027$	$L_6 = 0.990$
Rocker length	$L_4 = 0.850$	$L_8 = 1.212$	$L_5 = 0.792$
Coupler point	$A1P = 0.970$	$A2P = 0.720$	$A3P = 0.850$

Coupler angle $\delta_1 = 54.000\ deg$ $\delta_2 = -54.000\ deg$ $\delta_3 = 82.032\ deg$



PROBLEM 3-44

Statement: Find the three geared fivebar cognates of the linkage in Figure P3-12.

Given: Link lengths:

				Coupler point data:	
Ground link	$L_1 := 1.82$	Crank	$L_2 := 0.72$	$A1P := 0.97$	$\delta_1 := 54.0 \cdot deg$
Coupler	$L_3 := 0.68$	Rocker	$L_4 := 0.85$		

Solution: See Figure P3-12 and Mathcad file P0344.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle *APB*.

$$B1P := \left(L_3{}^2 + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot cos(\delta_1)\right)^{0.5} \qquad B1P = 0.792$$

$$\gamma_1 := acos\left(\frac{L_3{}^2 + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P}\right) \qquad \gamma_1 = 82.0315\, deg$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$L_5 := B1P$ $\quad L_5 = 0.792$ $\qquad L_6 := \frac{L_4}{L_3} \cdot B1P$ $\quad L_6 = 0.990$

$L_{10} := A1P$ $\quad L_{10} = 0.970$ $\qquad L_9 := \frac{L_2}{L_3} \cdot A1P$ $\quad L_9 = 1.027$

$L_7 := L_9 \cdot \frac{B1P}{A1P}$ $\quad L_7 = 0.839$ $\qquad L_8 := L_6 \cdot \frac{A1P}{B1P}$ $\quad L_8 = 1.212$

Calculate the coupler point data for cognates #2 and #3

$A3P := L_4$ $\quad A3P = 0.850$ $\qquad A2P := L_2$ $\quad A2P = 0.720$

$\delta_3 := \gamma_1$ $\quad \delta_3 = 82.032\, deg$ $\qquad \delta_2 := -\delta_1$ $\quad \delta_2 = -54.000\, deg$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

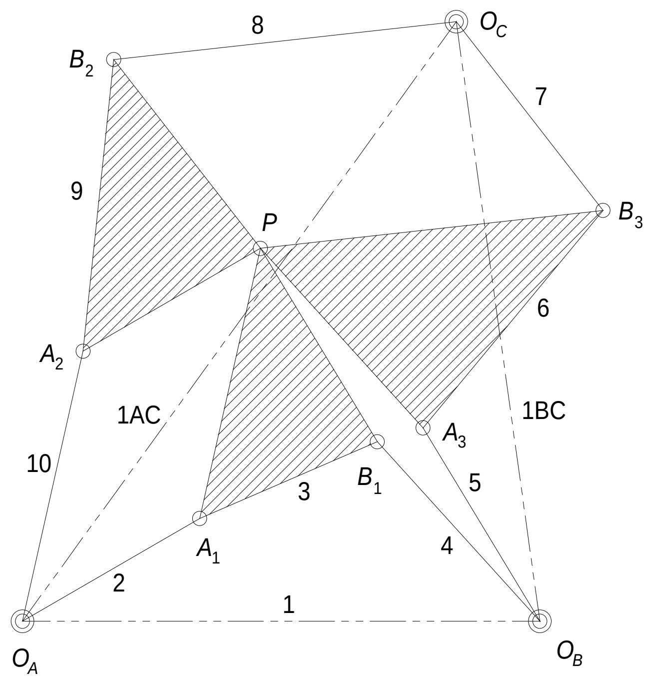
$L_{1BC} := \frac{L_1}{L_3} \cdot B1P$ $\quad L_{1BC} = 2.1208$ $\qquad L_{1AC} := \frac{L_1}{L_3} \cdot A1P$ $\quad L_{1AC} = 2.5962$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 1.820$	$L_{1AC} = 2.596$	$L_{1BC} = 2.121$
Crank length	$L_2 = 0.720$	$L_{10} = 0.970$	$L_7 = 0.839$
Coupler length	$L_3 = 0.680$	$L_9 = 1.027$	$L_6 = 0.990$
Rocker length	$L_4 = 0.850$	$L_8 = 1.212$	$L_5 = 0.792$
Coupler point	$A1P = 0.970$	$A2P = 0.720$	$A3P = 0.850$

Coupler angle $\delta_1 = 54.000\ deg$ $\delta_2 = -54.000\ deg$ $\delta_3 = 82.032\ deg$



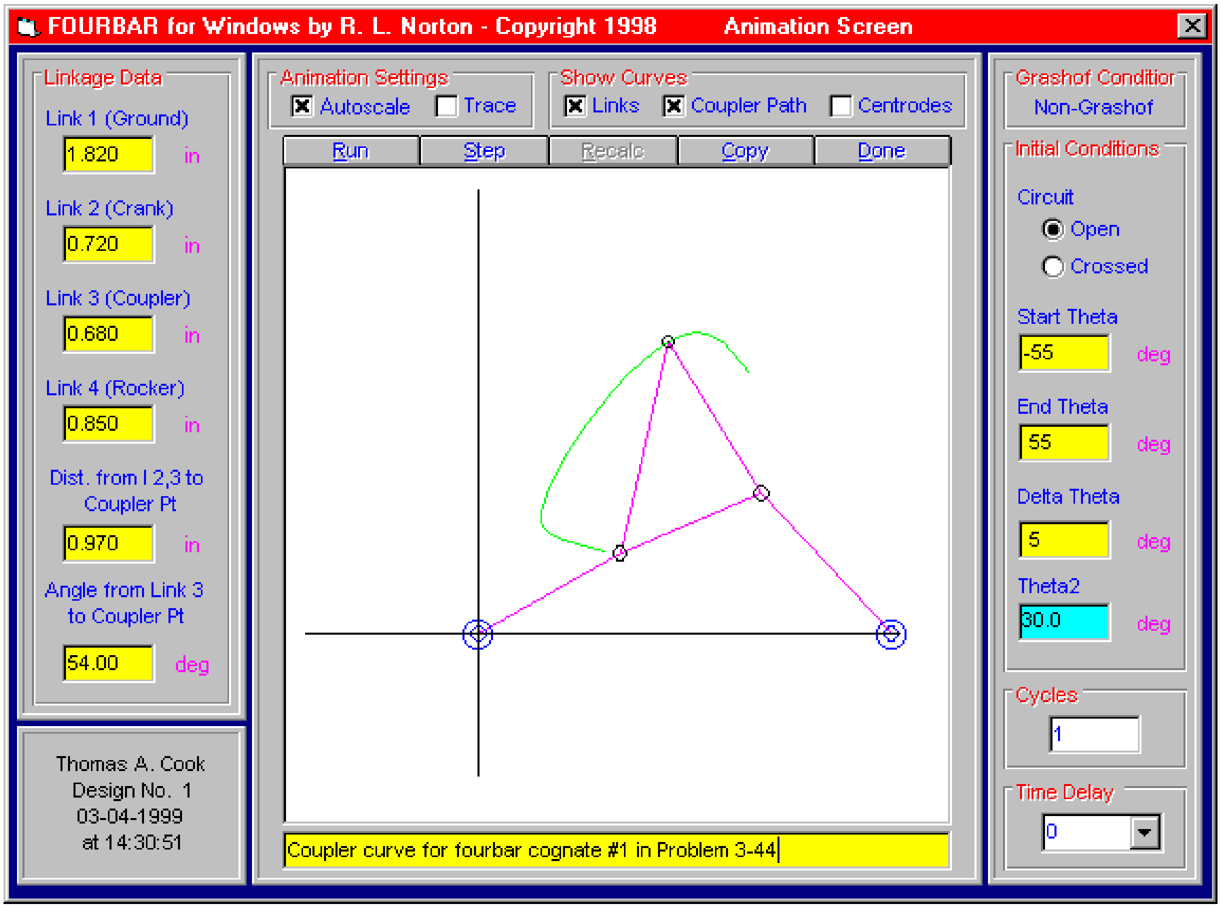
4. The three geared fivebar cognates can be seen in the Roberts diagram. They are: $O_AA_2PA_3O_B$, $O_AA_1PB_3O_C$, and $O_BB_1PB_2O_C$.

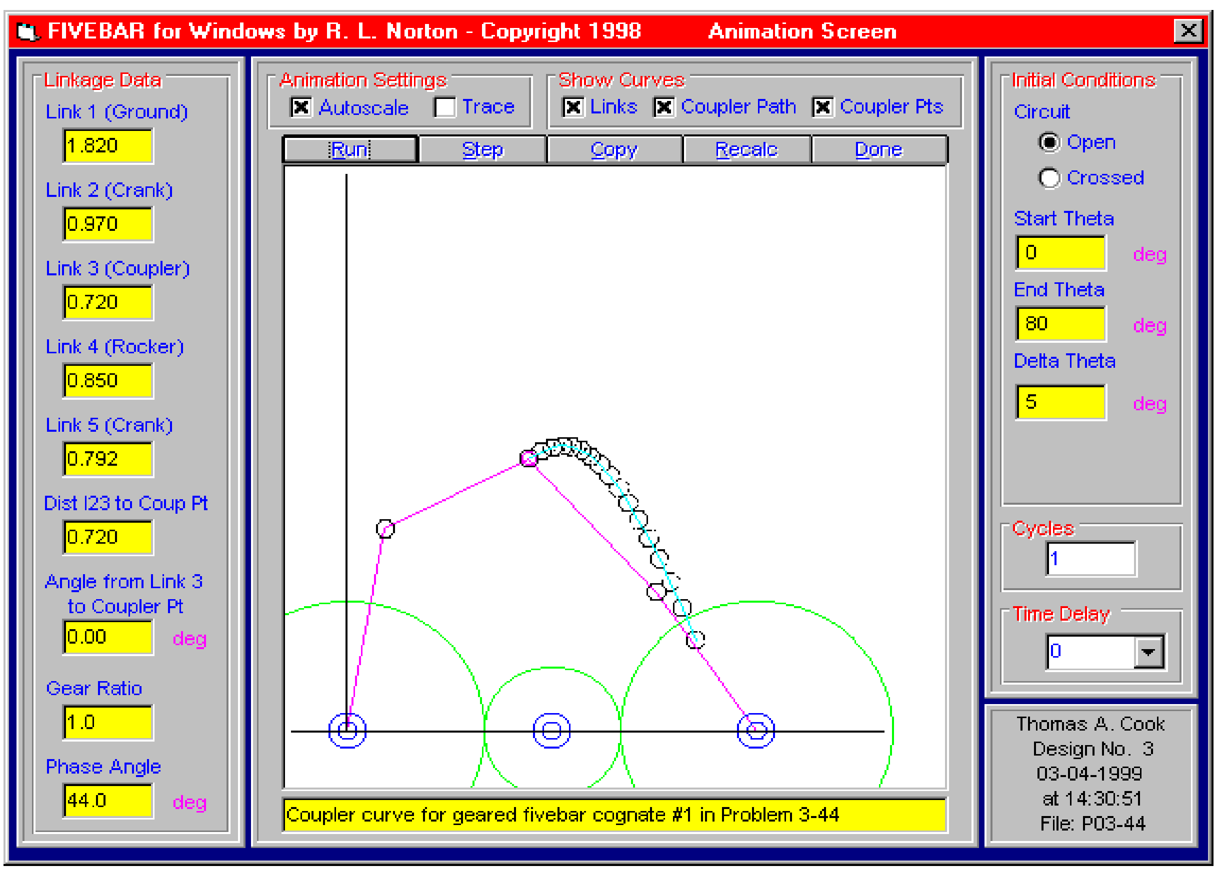
SUMMARY OF GEARED FIVEBAR COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 1.820$	$L_{1AC} = 2.596$	$L_{1BC} = 2.121$
Crank length	$L_{10} = 0.970$	$L_2 = 0.720$	$L_4 = 0.850$
Coupler length	$A2P = 0.720$	$A1P = 0.970$	$L_5 = 0.792$
Rocker length	$A3P = 0.850$	$L_8 = 1.212$	$L_7 = 0.839$
Crank length	$L_5 = 0.792$	$L_7 = 0.839$	$L_8 = 1.212$
Coupler point	$A2P = 0.720$	$A1P = 0.970$	$B1P = 0.792$
Coupler angle	$\delta_1 := 0.00 \cdot deg$	$\delta_2 := 0.00 \cdot deg$	$\delta_3 := 0.00 \cdot deg$

5. Enter the cognate #1 specifications into program FOURBAR to get a trace of the coupler path (see next page).

6. Enter the geared fivebar cognate #1 specifications into program FIVEBAR to get a trace of the coupler path for the geared fivebar (see next page).


FOURBAR for Windows by R. L. Norton - Copyright 1998 Animation Screen
Linkage Data
Link 1 (Ground) 1.820 in
Link 2 (Crank) 0.720 in
Link 3 (Coupler) 0.680 in
Link 4 (Rocker) 0.850 in
Dist. from I 2,3 to Coupler Pt 0.970 in
Angle from Link 3 to Coupler Pt 54.00 deg
Thomas A. Cook
Design No. 1
03-04-1999
at 14:30:51
Animation Settings
Autoscale
Trace
Show Curves
Links
Coupler Path
Centrodes
Run Step Recalc Copy Done
Coupler curve for fourbar cognate #1 in Problem 3-44
Grashof Condition
Non-Grashof
Initial Conditions
Circuit
Open
Crossed
Start Theta -55 deg
End Theta 55 deg
Delta Theta 5 deg
Theta2 30.0 deg
Cycles 1
Time Delay 0


FIVEBAR for Windows by R. L. Norton - Copyright 1998 Animation Screen
Linkage Data
Link 1 (Ground) 1.820
Link 2 (Crank) 0.970
Link 3 (Coupler) 0.720
Link 4 (Rocker) 0.850
Link 5 (Crank) 0.792
Dist I23 to Coup Pt 0.720
Angle from Link 3 to Coupler Pt 0.00 deg
Gear Ratio 1.0
Phase Angle 44.0 deg
Animation Settings
Autoscale
Trace
Show Curves
Links
Coupler Path
Coupler Pts
Run Step Copy Recalc Done
Coupler curve for geared fivebar cognate #1 in Problem 3-44
Initial Conditions
Circuit
Open
Crossed
Start Theta 0 deg
End Theta 80 deg
Delta Theta 5 deg
Cycles 1
Time Delay 0
Thomas A. Cook
Design No. 3
03-04-1999
at 14:30:51
File: P03-44

PROBLEM 3-45

Statement: Prove that the relationships between the angular velocities of various links in the Roberts diagram as shown in Figure 3-25 (p. 125) are true.

Given: $O_AA_1PA_2$, $O_CB_2PB_3$, and $O_BB_1PA_3$ are parallelograms for any position of link 2..

Proof:

1. O_AA_1 and A_2P are opposite sides of a parallelogram and are, therefore, always parallel.
2. Any change in the angle of O_AA_1 (link 2) will result in an identical change in the angle of A_2P.
3. Angular velocity is the change in angle per unit time.
4. Since O_AA_1 and A_2P have identical changes in angle, their angular velocities are identical.
5. A_2P is a line on link 9 and all lines on a rigid body have the same angular velocity. Therefore, link 9 has the same angular velocity as link 2.
6. O_CB_3 (link 7) and B_2P are opposite sides of a parallelogram and are, therefore, always parallel.
7. B_2P is a line on link 9 and all lines on a rigid body have the same angular velocity. Therefore, link 7 has the same angular velocity as links 9 and 2.
8. The same argument holds for links 3, 5, and 10; and links 4, 6, and 8.

PROBLEM 3-46

Statement: Design a fourbar linkage to move the object in Figure P3-13 from position 1 to 2 using points *A* and *B* for attachment. Add a driver dyad to limit its motion to the range of positions shown, making it a sixbar. All fixed pivots should be on the base.

Given: Length of coupler link: $L_3 := 52.000$

Solution: See Figure P3-13 and Mathcad file P0346.

Design choices:

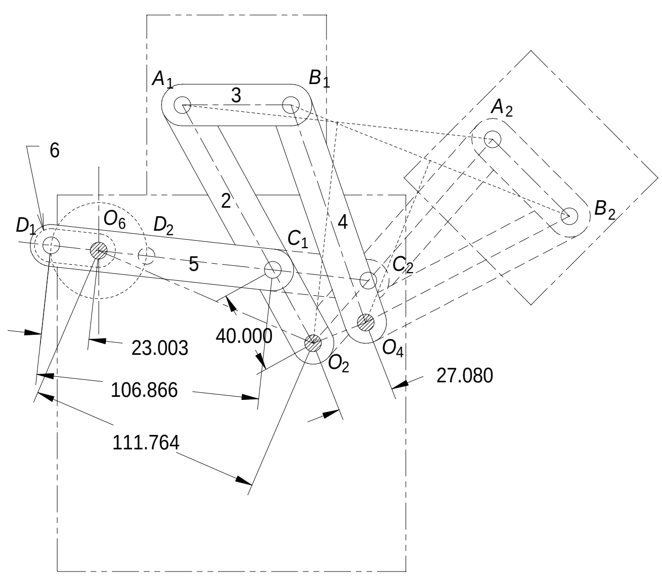
Length of link 2 $L_2 := 130$ Length of link 4 $L_4 := 110$

Length of link 2b $L_{2b} := 40$

1. Connect the end points of the two given positions of the line *AB* with construction limes, i.e., lines from A_1 to A_2 and B_1 to B_2.
2. Bisect these lines and extend their perpendicular bisectors into the base.
3. Select one point on each bisector and label them O_2 and O_4, respectively. In the solution below the distances O_2A was selected to be $L_2 = 130.000$ and O_4B to be $L_4 = 110.000$. This resulted in a ground-link-length O_2O_4 for the fourbar of 27.080.
4. The fourbar stage is now defined as O_2ABO_4 with link lengths

Ground link 1a	$L_{1a} := 27.080$	Link 2 (input)	$L_2 = 130.000$
Link 3 (coupler)	$L_3 = 52.000$	Link 4 (output)	$L_4 = 110.000$

5. Select a point on link 2 (O_2A) at a suitable distance from O_2 as the pivot point to which the driver dyad will be connected and label it *C*. (Note that link 2 is now a ternary link with nodes at O_2, *C*, and *A*.) In the solution below the distance O_2C was selected to be $L_{2b} = 40.000$.
6. Draw a construction line through C_1C_2 and extend it to the left.
7. Select a point on this line and call it O_6. In the solution below O_6 was placed 20 units from the left edge of the base.
8. Draw a circle about O_6 with a radius of one-half the length C_1C_2 and label the intersections of the circle with the extended line as D_1 and D_2. In the solution below the radius was measured as 23.003 units.



9. The driver fourbar is now defined as O_2CDO_6 with link lengths

Link 6 (crank)	$L_6 := 23.003$
Link 5 (coupler)	$L_5 := 106.866$
Link 1b (ground)	$L_{1b} := 111.764$
Link 2b (rocker)	$L_{2b} = 40.000$

10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$$Condition(a,b,c,d) := \left| \begin{array}{l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return \text{ "Grashof" } \quad if \ SL < PQ \\ return \text{ "Special Grashof" } \quad if \ SL = PQ \\ return \text{ "non-Grashof" } \quad otherwise \end{array} \right.$$

$$Condition\left(L_{1b}, L_{2b}, L_5, L_6\right) = \text{"Grashof"}$$

$$min\left(L_{1b}, L_{2b}, L_5, L_6\right) = 23.003$$

PROBLEM 3-47

Statement: Design a fourbar linkage to move the object in Figure P3-13 from position 2 to 3 using points *A* and *B* for attachment. Add a driver dyad to limit its motion to the range of positions shown, making it a sixbar. All fixed pivots should be on the base.

Given: Length of coupler link: $L_3 := 52.000$

Solution: See Figure P3-13 and Mathcad file P0347.

Design choices:

Length of link 2 $L_2 := 130$ Length of link 4 $L_4 := 225$

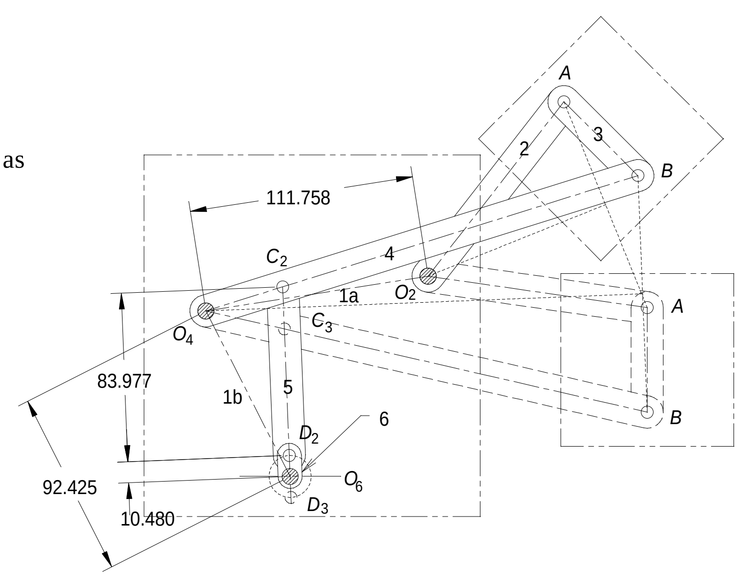
Length of link 4b $L_{4b} := 40$

1. Connect the end points of the two given positions of the line *AB* with construction limes, i.e., lines from A_2 to A_3 and B_2 to B_3.
2. Bisect these lines and extend their perpendicular bisectors into the base.
3. Select one point on each bisector and label them O_2 and O_4, respectively. In the solution below the distances O_2A was selected to be $L_2 = 130.000$ and O_4B to be $L_4 = 225.000$. This resulted in a ground-link-length O_2O_4 for the fourbar of 111.758.
4. The fourbar stage is now defined as O_2ABO_4 with link lengths

Ground link 1a $L_{1a} := 111.758$ Link 2 (input) $L_2 = 130.000$

Link 3 (coupler) $L_3 = 52.000$ Link 4 (output) $L_4 = 225.000$

5. Select a point on link 4 (O_4B) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it *C*. (Note that link 4 is now a ternary link with nodes at O_4, *C*, and *B*.) In the solution below the distance O_4C was selected to be $L_{4b} = 40.000$.
6. Draw a construction line through C_2C_3 and extend it downward.
7. Select a point on this line and call it O_6. In the solution below O_6 was placed 20 units from the bottom of the base.
8. Draw a circle about O_6 with a radius of one-half the length C_1C_2 and label the intersections of the circle with the extended line as D_2 and D_3. In the solution below the radius was measured as 10.480 units.



9. The driver fourbar is now defined as O_4CDO_6 with link lengths

Link 6 (crank) $L_6 := 10.480$

Link 5 (coupler) $L_5 := 83.977$

Link 1b (ground) $L_{1b} := 92.425$

Link 4b (rocker) $L_{4b} = 40.000$

10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$$Condition(a,b,c,d) := \left| \begin{array}{l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"} \quad if\ SL < PQ \\ return\ \text{"Special Grashof"} \quad if\ SL = PQ \\ return\ \text{"non-Grashof"} \quad otherwise \end{array} \right.$$

$$Condition\left(L_{1b}, L_{4b}, L_5, L_6\right) = \text{"Grashof"}$$

$$min\left(L_{1b}, L_{4b}, L_5, L_6\right) = 10.480$$

PROBLEM 3-48

Statement: Design a fourbar linkage to move the object in Figure P3-13 through the three positions shown using points *A* and *B* for attachment. Add a driver dyad to limit its motion to the range of positions shown, making it a sixbar. All fixed pivots should be on the base.

Given: Length of coupler link: $L_3 := 52.000$

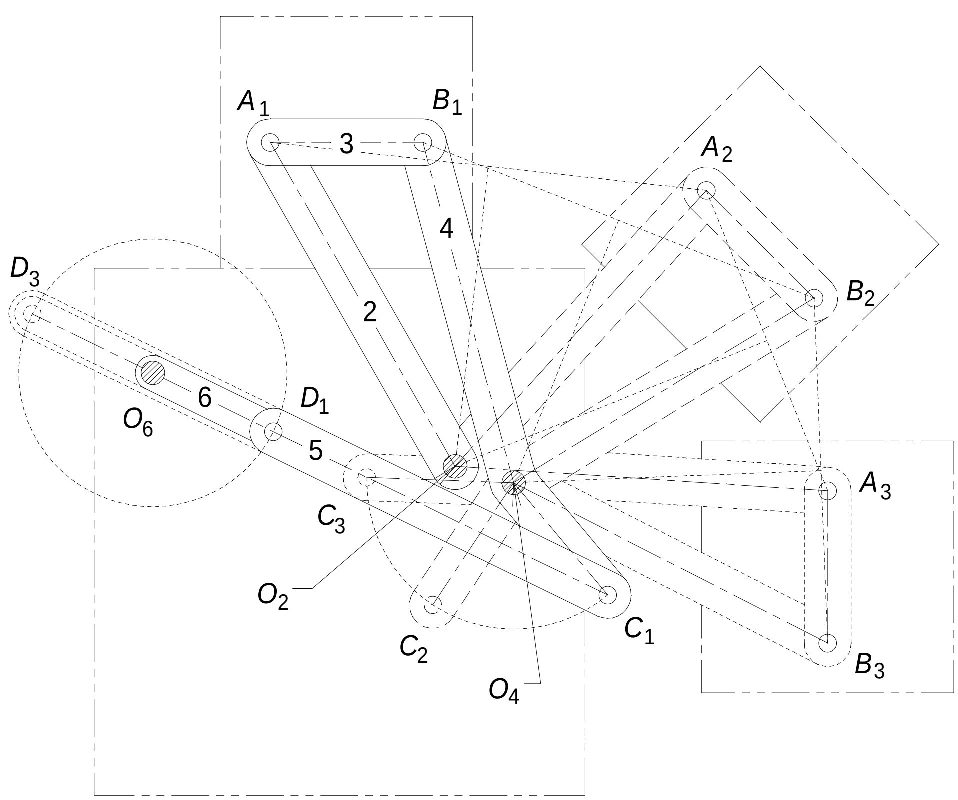
Solution: See Figure P3-13 and Mathcad file P0348.

Design choices:

Length of link 4b $L_{4b} := 50$

1. Draw link *AB* in its three design positions A_1B_1, A_2B_2, A_3B_3 in the plane as shown.
2. Draw construction lines from point A_1 to A_2 and from point A_2 to A_3.
3. Bisect line A_1A_2 and line A_2A_3 and extend their perpendicular bisectors until they intersect. Label their intersection O_2.
4. Repeat steps 2 and 3 for lines B_1B_2 and B_2B_3. Label the intersection O_4.
5. Connect O_2 with A_1 and call it link 2. Connect O_4 with B_1 and call it link 4.
6. Line A_1B_1 is link 3. Line O_2O_4 is link 1 (ground link for the fourbar). The fourbar is now defined as O_2ABO_4 and has link lengths of

Ground link 1a	$L_{1a} := 20.736$	Link 2	$L_2 := 127.287$
Link 3	$L_3 = 52.000$	Link 4	$L_4 := 120.254$



7. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

$$Condition(a,b,c,d) := \begin{array}{|l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"} \quad if\ \ SL < PQ \\ return\ \text{"Special Grashof"} \quad if\ \ SL = PQ \\ return\ \text{"non-Grashof"} \quad otherwise \end{array}$$

$$Condition\left(L_{1a}, L_2, L_3, L_4\right) = \text{"Grashof"}$$

8. Select a point on link 4 (O_4B) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it C. (Note that link 4 is now a ternary link with nodes at O_4, C, and B.) In the solution above the distance O_4C was selected to be $L_{4b} = 50.000$.

9. Draw a construction line through C_1C_3 and extend it to the left.

10. Select a point on this line and call it O_6. In the solution above O_6 was placed 20 units from the left edge of the base.

11. Draw a circle about O_6 with a radius of one-half the length C_1C_3 and label the intersections of the circle with the extended line as D_1 and D_3. In the solution below the radius was measured as $L_6 := 45.719$.

12. The driver fourbar is now defined as O_4CDO_6 with link lengths

Link 6 (crank)	$L_6 = 45.719$
Link 5 (coupler)	$L_5 := 126.875$
Link 1b (ground)	$L_{1b} := 128.545$
Link 4b (rocker)	$L_{4b} = 50.000$

13. Use the link lengths in step 12 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$$Condition\left(L_6, L_{1b}, L_{4b}, L_5\right) = \text{"Grashof"}$$

$$min\left(L_6, L_{1b}, L_{4b}, L_5\right) = 45.719$$

PROBLEM 3-49

Statement: Design a fourbar linkage to move the object in Figure P3-14 from position 1 to 2 using points *A* and *B* for attachment. Add a driver dyad to limit its motion to the range of positions shown, making it a sixbar. All fixed pivots should be on the base.

Given: Length of coupler link: $L_3 := 86.000$

Solution: See Figure P3-14 and Mathcad file P0349.

Design choices:

Length of link 2 $L_2 := 125$ Length of link 4 $L_4 := 140$

Length of link 2b $L_{4b} := 50$

1. Connect the end points of the two given positions of the line *AB* with construction limes, i.e., lines from A_1 to A_2 and B_1 to B_2.
2. Bisect these lines and extend their perpendicular bisectors into the base.
3. Select one point on each bisector and label them O_2 and O_4, respectively. In the solution below the distances O_2A was selected to be $L_2 = 125.000$ and O_4B to be $L_4 = 140.000$. This resulted in a ground-link-length O_2O_4 for the fourbar of 97.195.
4. The fourbar stage is now defined as O_2ABO_4 with link lengths

Ground link 1a $L_{1a} := 97.195$ Link 2 (input) $L_2 = 125.000$

Link 3 (coupler) $L_3 = 86.000$ Link 4 (output) $L_4 = 140.000$

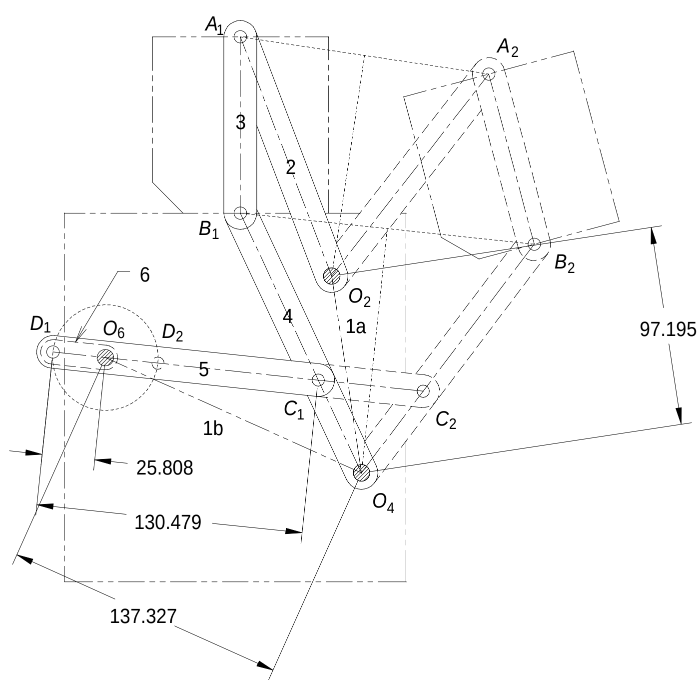
5. Select a point on link 4 (O_4B) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it *C*. (Note that link 4 is now a ternary link with nodes at O_4, *C*, and *B*.) In the solution below the distance O_4C was selected to be $L_{4b} = 50.000$.
6. Draw a construction line through C_1C_2 and extend it to the left.
7. Select a point on this line and call it O_6. In the solution below O_6 was placed 20 units from the left edge of the base.
8. Draw a circle about O_6 with a radius of one-half the length C_1C_2 and label the intersections of the circle with the extended line as D_1 and D_2. In the solution below the radius was measured as 25.808 units.
9. The driver fourbar is now defined as O_4CDO_6 with link lengths

Link 6 (crank) $L_6 := 25.808$

Link 5 (coupler) $L_5 := 130.479$

Link 1b (ground) $L_{1b} := 137.327$

Link 4b (rocker) $L_{4b} = 50.000$



10. Use the link lengths in step 9 to find the Grashof condition of the driving fourbar (it must be Grashof and the shortest link must be link 6).

$$Condition(a,b,c,d) := \left| \begin{array}{l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"} \quad if\ SL < PQ \\ return\ \text{"Special Grashof"} \quad if\ SL = PQ \\ return\ \text{"non-Grashof"} \quad otherwise \end{array} \right.$$

$$Condition\left(L_{1b}, L_{4b}, L_5, L_6\right) = \text{"Grashof"}$$

$$min\left(L_{1b}, L_{4b}, L_5, L_6\right) = 25.808$$

PROBLEM 3-50

Statement: Design a fourbar linkage to move the object in Figure P3-14 from position 2 to 3 using points *A* and *B* for attachment. Add a driver dyad to limit its motion to the range of positions shown, making it a sixbar. All fixed pivots should be on the base.

Given: Length of coupler link: $L_3 := 86.000$

Solution: See Figure P3-14 and Mathcad file P0350.

Design choices:

Length of link 2 $L_2 := 130$ Length of link 4 $L_4 := 130$

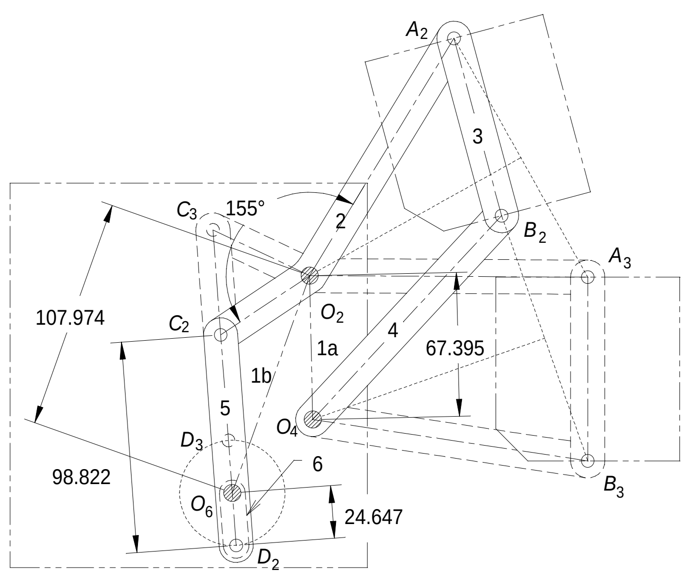
Length of link 2b $L_{2b} := 50$

1. Connect the end points of the two given positions of the line *AB* with construction limes, i.e., lines from A_2 to A_3 and B_2 to B_3.
2. Bisect these lines and extend their perpendicular bisectors into the base.
3. Select one point on each bisector and label them O_2 and O_4, respectively. In the solution below the distances O_2A was selected to be $L_2 = 130.000$ and O_4B to be $L_4 = 130.000$. This resulted in a ground-link-length O_2O_4 for the fourbar of 67.395.
4. The fourbar stage is now defined as O_2ABO_4 with link lengths

Ground link 1a	$L_{1a} := 67.395$	Link 2 (input)	$L_2 = 130.000$
Link 3 (coupler)	$L_3 = 86.000$	Link 4 (output)	$L_4 = 130.000$

5. Select a point on link 2 (O_2A) at a suitable distance from O_2 as the pivot point to which the driver dyad will be connected and label it *C*. (Note that link 4 is now a ternary link with nodes at O_2, *C*, and *A*.) In the solution below the distance O_2C was selected to be $L_{2b} = 50.000$ and the link was extended away from *A* to give a better position for the driving dyad.
6. Draw a construction line through C_2C_3 and extend it downward.
7. Select a point on this line and call it O_6. In the solution below O_6 was placed 35 units from the bottom of the base.
8. Draw a circle about O_6 with a radius of one-half the length C_1C_2 and label the intersections of the circle with the extended line as D_2 and D_3. In the solution below the radius was measured as 24.647 units.
9. The driver fourbar is now defined as O_2CDO_6 with link lengths

Link 6 (crank)	$L_6 := 24.647$
Link 5 (coupler)	$L_5 := 98.822$
Link 1b (ground)	$L_{1b} := 107.974$
Link 2b (rocker)	$L_{2b} = 50.000$



10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$$Condition(a,b,c,d) := \begin{array}{|l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return \text{ "Grashof" } \quad if \;\; SL < PQ \\ return \text{ "Special Grashof" } \quad if \;\; SL = PQ \\ return \text{ "non-Grashof" } \quad otherwise \end{array}$$

$$Condition(L_{1b}, L_{2b}, L_5, L_6) = \text{"Grashof"}$$

$$min(L_{1b}, L_{2b}, L_5, L_6) = 24.647$$

PROBLEM 3-51

Statement: Design a fourbar linkage to move the object in Figure P3-14 through the three positions shown using points *A* and *B* for attachment. Add a driver dyad to limit its motion to the range of positions shown, making it a sixbar. All fixed pivots should be on the base.

Given: Length of coupler link: $L_3 := 86.000$

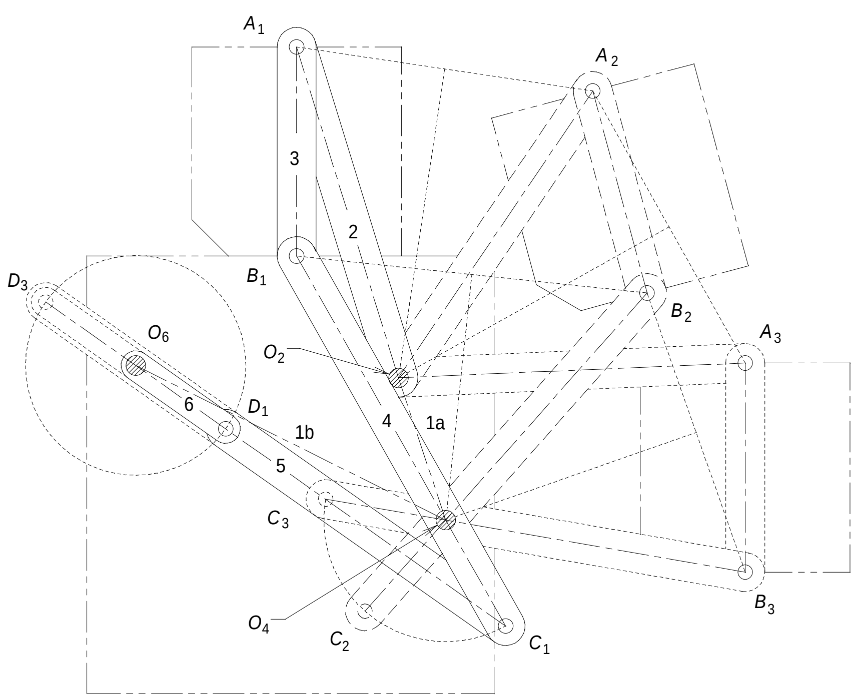
Solution: See Figure P3-14 and Mathcad file P0351.

Design choices:

Length of link 4b $L_{4b} := 50$

1. Draw link *AB* in its three design positions A_1B_1, A_2B_2, A_3B_3 in the plane as shown.
2. Draw construction lines from point A_1 to A_2 and from point A_2 to A_3.
3. Bisect line A_1A_2 and line A_2A_3 and extend their perpendicular bisectors until they intersect. Label their intersection O_2.
4. Repeat steps 2 and 3 for lines B_1B_2 and B_2B_3. Label the intersection O_4.
5. Connect O_2 with A_1 and call it link 2. Connect O_4 with B_1 and call it link 4.
6. Line A_1B_1 is link 3. Line O_2O_4 is link 1 (ground link for the fourbar). The fourbar is now defined as O_2ABO_4 and has link lengths of

Ground link 1a	$L_{1a} := 61.667$	Link 2	$L_2 := 142.357$
Link 3	$L_3 = 86.000$	Link 4	$L_4 := 124.668$



7. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

$$Condition(a,b,c,d) := \left| \begin{array}{l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"} \quad if\ \ SL < PQ \\ return\ \text{"Special Grashof"} \quad if\ \ SL = PQ \\ return\ \text{"non-Grashof"} \quad otherwise \end{array} \right.$$

$$Condition\left(L_{1a}, L_2, L_3, L_4\right) = \text{"Grashof"}$$

8. Select a point on link 4 (O_4B) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it C. (Note that link 4 is now a ternary link with nodes at O_4, C, and B.) In the solution above the distance O_4C was selected to be $L_{4b} = 50.000$.

9. Draw a construction line through C_1C_3 and extend it to the left.

10. Select a point on this line and call it O_6. In the solution above O_6 was placed 20 units from the left edge of the base.

11. Draw a circle about O_6 with a radius of one-half the length C_1C_3 and label the intersections of the circle with the extended line as D_1 and D_3. In the solution below the radius was measured as $L_6 := 45.178$.

12. The driver fourbar is now defined as O_4CDO_6 with link lengths

Link 6 (crank)	$L_6 = 45.178$
Link 5 (coupler)	$L_5 := 140.583$
Link 1b (ground)	$L_{1b} := 142.205$
Link 4b (rocker)	$L_{4b} = 50.000$

13. Use the link lengths in step 12 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$$Condition\left(L_6, L_{1b}, L_{4b}, L_5\right) = \text{"Grashof"}$$

14. Unfortunately, although the solution presented appears to meet the design specification, a simple cardboard model will quickly demonstrate that it has a branch defect. That is, in the first position shown, the linkage is in the "open" configuration, but in the 2nd and 3rd positions it is in the "crossed" configuration. The linkage cannot get from one circuit to the other without removing a pin and reassembling after moving the linkage. The remedy is to attach the points A and B to the coupler, but not at the joints between links 2 and 3 and links 3 and 4.

PROBLEM 3-52

Statement: Design a fourbar linkage to move the object in Figure P3-15 from position 1 to 2 using points *A* and *B* for attachment. Add a driver dyad to limit its motion to the range of positions shown, making it a sixbar. All fixed pivots should be on the base.

Given: Length of coupler link: $L_3 := 52.000$

Solution: See Figure P3-15 and Mathcad file P0352.

Design choices:

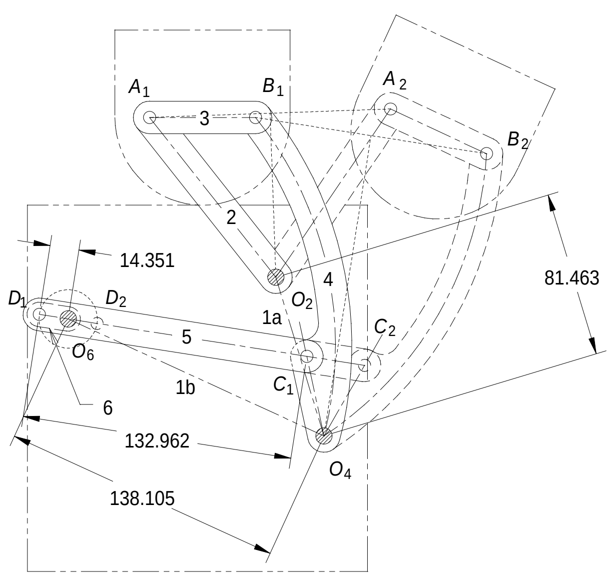
Length of link 2 $L_2 := 100$ Length of link 4 $L_4 := 160$

Length of link 4b $L_{4b} := 40$

1. Connect the end points of the two given positions of the line *AB* with construction limes, i.e., lines from A_1 to A_2 and B_1 to B_2.
2. Bisect these lines and extend their perpendicular bisectors into the base.
3. Select one point on each bisector and label them O_2 and O_4, respectively. In the solution below the distances O_2A was selected to be $L_2 = 100.000$ and O_4B to be $L_4 = 160.000$. This resulted in a ground-link-length O_2O_4 for the fourbar of 81.463.
4. The fourbar stage is now defined as O_2ABO_4 with link lengths

Ground link 1a	$L_{1a} := 81.463$	Link 2 (input)	$L_2 = 100.000$
Link 3 (coupler)	$L_3 = 52.000$	Link 4 (output)	$L_4 = 160.000$

5. Select a point on link 4 (O_4B) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it *C*. (Note that link 4 is now a ternary link with nodes at O_4, *C*, and *B*.) In the solution below the distance O_4C was selected to be $L_{4b} = 40.000$.
6. Draw a construction line through C_1C_2 and extend it to the left.
7. Select a point on this line and call it O_6. In the solution below O_6 was placed 20 units from the left edge of the base.
8. Draw a circle about O_6 with a radius of one-half the length C_1C_2 and label the intersections of the circle with the extended line as D_1 and D_2. In the solution below the radius was measured as 14.351 units.



9. The driver fourbar is now defined as O_4CDO_6 with link lengths

Link 6 (crank) $L_6 := 14.351$

Link 5 (coupler) $L_5 := 132.962$

Link 1b (ground) $L_{1b} := 138.105$

Link 4b (rocker) $L_{4b} = 40.000$

10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$$Condition(a,b,c,d) := \left| \begin{array}{l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"} \quad if\ \ SL < PQ \\ return\ \text{"Special Grashof"} \quad if\ \ SL = PQ \\ return\ \text{"non-Grashof"} \quad otherwise \end{array} \right.$$

$$Condition\left(L_{1b}, L_{4b}, L_5, L_6\right) = \text{"Grashof"}$$

$$min\left(L_{1b}, L_{4b}, L_5, L_6\right) = 14.351$$

PROBLEM 3-53

Statement: Design a fourbar linkage to move the object in Figure P3-15 from position 2 to 3 using points *A* and *B* for attachment. Add a driver dyad to limit its motion to the range of positions shown, making it a sixbar. All fixed pivots should be on the base.

Given: Length of coupler link: $L_3 := 52.000$

Solution: See Figure P3-15 and Mathcad file P0353.

Design choices:

Length of link 2 $L_2 := 150$ Length of link 4 $L_4 := 200$

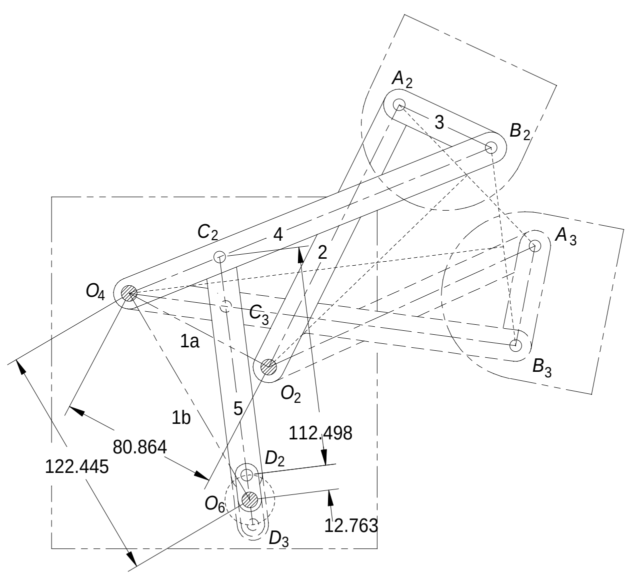
Length of link 4b $L_{4b} := 50$

1. Connect the end points of the two given positions of the line *AB* with construction limes, i.e., lines from A_2 to A_3 and B_2 to B_3.
2. Bisect these lines and extend their perpendicular bisectors into the base.
3. Select one point on each bisector and label them O_2 and O_4, respectively. In the solution below the distances O_2A was selected to be $L_2 = 150.000$ and O_4B to be $L_4 = 200.000$. This resulted in a ground-link-length O_2O_4 for the fourbar of $L_{1a} := 80.864$.
4. The fourbar stage is now defined as O_2ABO_4 with link lengths

Ground link 1a	$L_{1a} = 80.864$	Link 2 (input)	$L_2 = 150.000$
Link 3 (coupler)	$L_3 = 52.000$	Link 4 (output)	$L_4 = 200.000$

5. Select a point on link 4 (O_4B) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it *C*. (Note that link 4 is now a ternary link with nodes at O_4, *C*, and *B*.) In the solution below the distance O_4C was selected to be $L_{4b} = 50.000$.
6. Draw a construction line through C_2C_3 and extend it downward.
7. Select a point on this line and call it O_6. In the solution below O_6 was placed 25 units from the bottom of the base.
8. Draw a circle about O_6 with a radius of one-half the length C_1C_2 and label the intersections of the circle with the extended line as D_2 and D_3. In the solution below the radius was measured as $L_6 := 12.763$.
9. The driver fourbar is now defined as O_4CDO_6 with link lengths

Link 6 (crank)	$L_6 = 12.763$
Link 5 (coupler)	$L_5 := 112.498$
Link 1b (ground)	$L_{1b} := 122.445$
Link 4b (rocker)	$L_{4b} = 50.000$



10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$$Condition(a,b,c,d) := \left| \begin{array}{l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"}\quad if\ \ SL < PQ \\ return\ \text{"Special Grashof"}\quad if\ \ SL = PQ \\ return\ \text{"non-Grashof"}\quad otherwise \end{array} \right.$$

$$Condition\left(L_{1b}, L_{4b}, L_5, L_6\right) = \text{"Grashof"}$$

$$min\left(L_{1b}, L_{4b}, L_5, L_6\right) = 12.763$$

PROBLEM 3-54

Statement: Design a fourbar linkage to move the object in Figure P3-15 through the three positions shown using points *A* and *B* for attachment. Add a driver dyad to limit its motion to the range of positions shown, making it a sixbar. All fixed pivots should be on the base.

Given: Length of coupler link: $L_3 := 52.000$

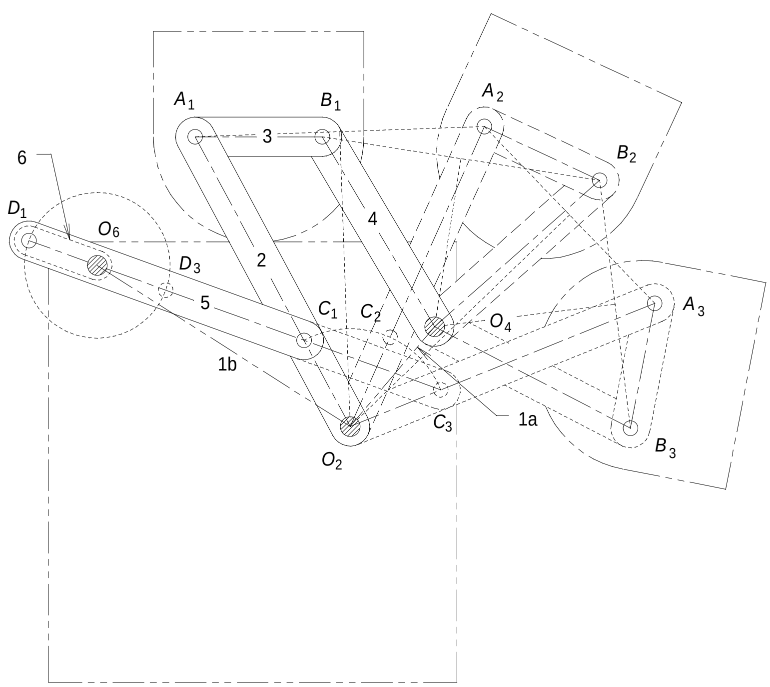
Solution: See Figure P3-15 and Mathcad file P0354.

Design choices:

Length of link 2b $L_{2b} := 40$

1. Draw link *AB* in its three design positions A_1B_1, A_2B_2, A_3B_3 in the plane as shown.
2. Draw construction lines from point A_1 to A_2 and from point A_2 to A_3.
3. Bisect line A_1A_2 and line A_2A_3 and extend their perpendicular bisectors until they intersect. Label their intersection O_2.
4. Repeat steps 2 and 3 for lines B_1B_2 and B_2B_3. Label the intersection O_4.
5. Connect O_2 with A_1 and call it link 2. Connect O_4 with B_1 and call it link 4.
6. Line A_1B_1 is link 3. Line O_2O_4 is link 1 (ground link for the fourbar). The fourbar is now defined as O_2ABO_4 and has link lengths of

Ground link 1a	$L_{1a} := 53.439$	Link 2	$L_2 := 134.341$
Link 3	$L_3 = 52.000$	Link 4	$L_4 := 90.203$



7. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

$$Condition(a,b,c,d) := \begin{array}{|l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"} \quad if\ \ SL < PQ \\ return\ \text{"Special Grashof"} \quad if\ \ SL = PQ \\ return\ \text{"non-Grashof"} \quad otherwise \end{array}$$

$$Condition\left(L_{1a}, L_2, L_3, L_4\right) = \text{"non-Grashof"}$$

Although this fourbar is non-Grashof, there are no toggle points within the required range of motion.

8. Select a point on link 2 (O_2A) at a suitable distance from O_2 as the pivot point to which the driver dyad will be connected and label it C. (Note that link 2 is now a ternary link with nodes at O_2, C, and A.) In the solution above the distance O_2C was selected to be $L_{2b} = 40.000$.
9. Draw a construction line through C_1C_3 and extend it to the left.
10. Select a point on this line and call it O_6. In the solution above O_6 was placed 20 units from the left edge of the base.
11. Draw a circle about O_6 with a radius of one-half the length C_1C_3 and label the intersections of the circle with the extended line as D_1 and D_3. In the solution below the radius was measured as $L_6 := 29.760$.
12. The driver fourbar is now defined as O_2CDO_6 with link lengths

Link 6 (crank)	$L_6 = 29.760$
Link 5 (coupler)	$L_5 := 119.665$
Link 1b (ground)	$L_{1b} := 122.613$
Link 2b (rocker)	$L_{2b} = 40.000$

13. Use the link lengths in step 12 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$$Condition\left(L_6, L_{1b}, L_{2b}, L_5\right) = \text{"Grashof"}$$

PROBLEM 3-55

Statement: Design a fourbar mechanism to move the link shown in Figure P3-16 from position 1 to position 2. Ignore the third position and the fixed pivots O_2 and O_4 shown. Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Given: Position 1 offsets: $x_{C1D1} := 3.744 \cdot in$ $\quad y_{C1D1} := 2.497 \cdot in$

Solution: See figure below for one possible solution. Input file P0355.mcd from the solutions manual disk to the Mathcad program for this solution, file P03-55.4br to the program FOURBAR to see the fourbar solution linkage, and file P03-55.6br into program SIXBAR to see the complete sixbar with the driver dyad included.

1. Connect the end points of the two given positions of the line CD with construction lines, i.e., lines from C_1 to C_2 and D_1 to D_2.
2. Bisect these lines and extend their perpendicular bisectors in any convenient direction. In the solution below the bisector of C_1C_2 was extended downward and the bisector of D_1D_2 was extended upward.
3. Select one point on each bisector and label them O_4 and O_6, respectively. In the solution below the distances O_4D and O_6C were each selected to be 7.500 in. This resulted in a ground-link-length O_4O_6 for the fourbar of 15.366 in.
4. The fourbar stage is now defined as O_4CDO_6 with link lengths

Link 5 (coupler) $L_5 := \sqrt{x_{C1D1}^2 + y_{C1D1}^2}$ $\qquad L_5 = 4.500\ in$

Link 4 (input) $L_4 := 7.500 \cdot in$ $\qquad$ Link 6 (output) $L_6 := 7.500 \cdot in$

Ground link 1b $L_{1b} := 15.366 \cdot in$

5. Select a point on link 4 (O_4D) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it B. (Note that link 4 is now a ternary link with nodes at O_4, B, and D.) In the solution below the distance O_4B was selected to be 4.000 in.
6. Draw a construction line through B_1B_2 and extend it to the right.
7. Select a point on this line and call it O_2. In the solution below the distance AB was selected to be 6.000 in.
8. Draw a circle about O_2 with a radius of one-half the length B_1B_2 and label the intersections of the circle with the extended line as A_1 and A_2. In the solution below the radius was measured as 1.370 in.
9. The driver fourbar is now defined as O_2ABO_4 with link lengths

Link 2 (crank) $L_2 := 1.370 \cdot in$ $\qquad$ Link 3 (coupler) $L_3 := 6.000 \cdot in$

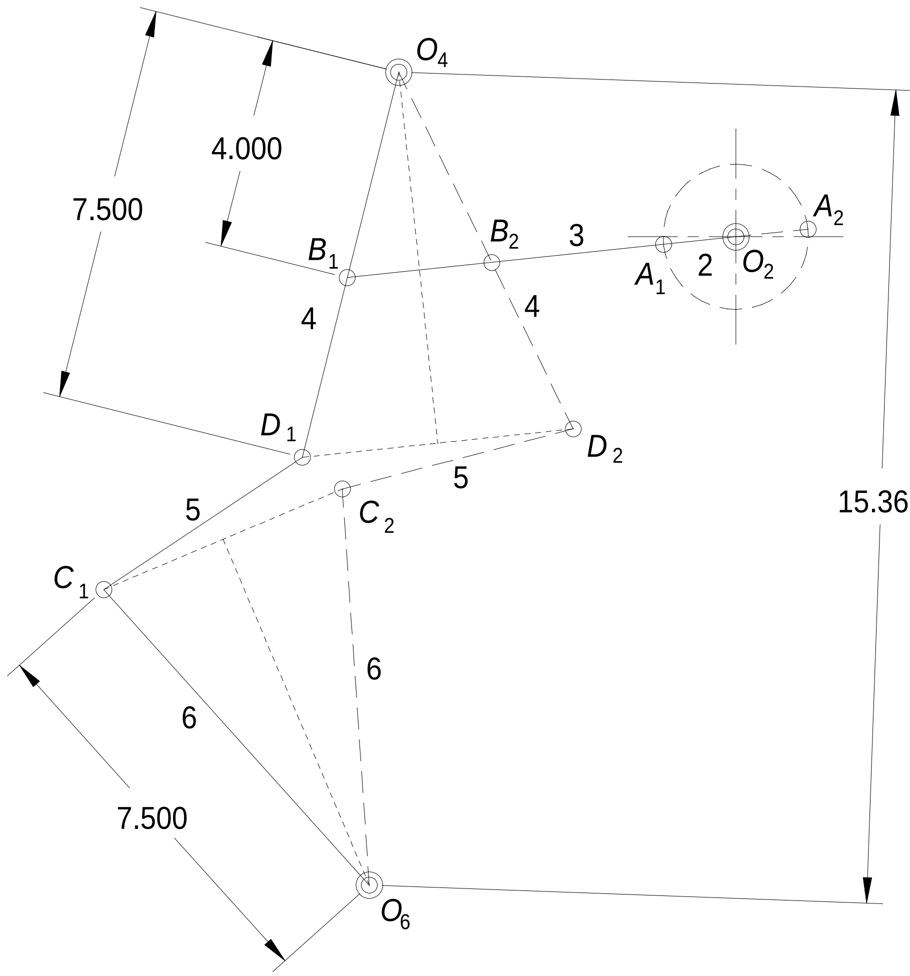
Link 4a (rocker) $L_{4a} := 4.000 \cdot in$ $\qquad$ Link 1a (ground) $L_{1a} := 7.080 \cdot in$

10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

$$Condition(a,b,c,d) := \left| \begin{array}{l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"}\quad if\ SL < PQ \\ return\ \text{"Special Grashof"}\quad if\ SL = PQ \\ return\ \text{"non-Grashof"}\quad otherwise \end{array} \right.$$

$Condition(L_{1a}, L_2, L_3, L_{4a}) =$ "Grashof"

$min(L_{1a}, L_2, L_3, L_{4a}) = 1.370\ in$



11. Using the program FOURBAR and the link lengths given above, it was found that the fourbar O_4DCO_6 is non-Grashoff with toggle positions at θ_4 = -49.9 deg and +49.9 deg. The fourbar operates between θ_4 = +28.104 deg and -11.968 deg.

PROBLEM 3-56

Statement: Design a fourbar mechanism to move the link shown in Figure P3-16 from position 2 to position 3. Ignore the third position and the fixed pivots O_2 and O_4 shown. Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Given: Position 2 offsets: $x_{C2D2} := 4.355 \cdot in$ $\quad y_{C2D2} := 1.134 \cdot in$

Solution: See figure below for one possible solution. Input file P0356.mcd from the solutions manual disk to the Mathcad program for this solution, file P03-56.4br to the program FOURBAR to see the fourbar solution linkage, and file P03-56.6br into program SIXBAR to see the complete sixbar with the driver dyad included.

1. Connect the end points of the two given positions of the line CD with construction lines, i.e., lines from C_2 to C_3 and D_2 to D_3.
2. Bisect these lines and extend their perpendicular bisectors in any convenient direction. In the solution below the bisector of C_2C_3 was extended downward and the bisector of D_2D_3 was extended upward.
3. Select one point on each bisector and label them O_4 and O_6, respectively. In the solution below the distances O_4D and O_6C were each selected to be 6.000 in. This resulted in a ground-link-length O_4O_6 for the fourbar of 14.200 in.
4. The fourbar stage is now defined as O_4DCO_6 with link lengths

Link 5 (coupler) $L_5 := \sqrt{x_{C2D2}^2 + y_{C2D2}^2}$ $\quad L_5 = 4.500\ in$

Link 4 (input) $L_4 := 6.000 \cdot in$ $\quad$ Link 6 (output) $L_6 := 6.000 \cdot in$

Ground link 1b $L_{1b} := 14.200 \cdot in$

5. Select a point on link 4 (O_4D) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it B. (Note that link 4 is now a ternary link with nodes at O_4, B, and D.) In the solution below the distance O_4B was selected to be 4.000 in.
6. Draw a construction line through B_1B_2 and extend it to the right.
7. Select a point on this line and call it O_2. In the solution below the distance AB was selected to be 6.000 in.
8. Draw a circle about O_2 with a radius of one-half the length B_1B_2 and label the intersections of the circle with the extended line as A_1 and A_2. In the solution below the radius was measured as 1.271 in.
9. The driver fourbar is now defined as O_2ABO_4 with link lengths

Link 2 (crank) $L_2 := 1.271 \cdot in$ $\quad$ Link 3 (coupler) $L_3 := 6.000 \cdot in$

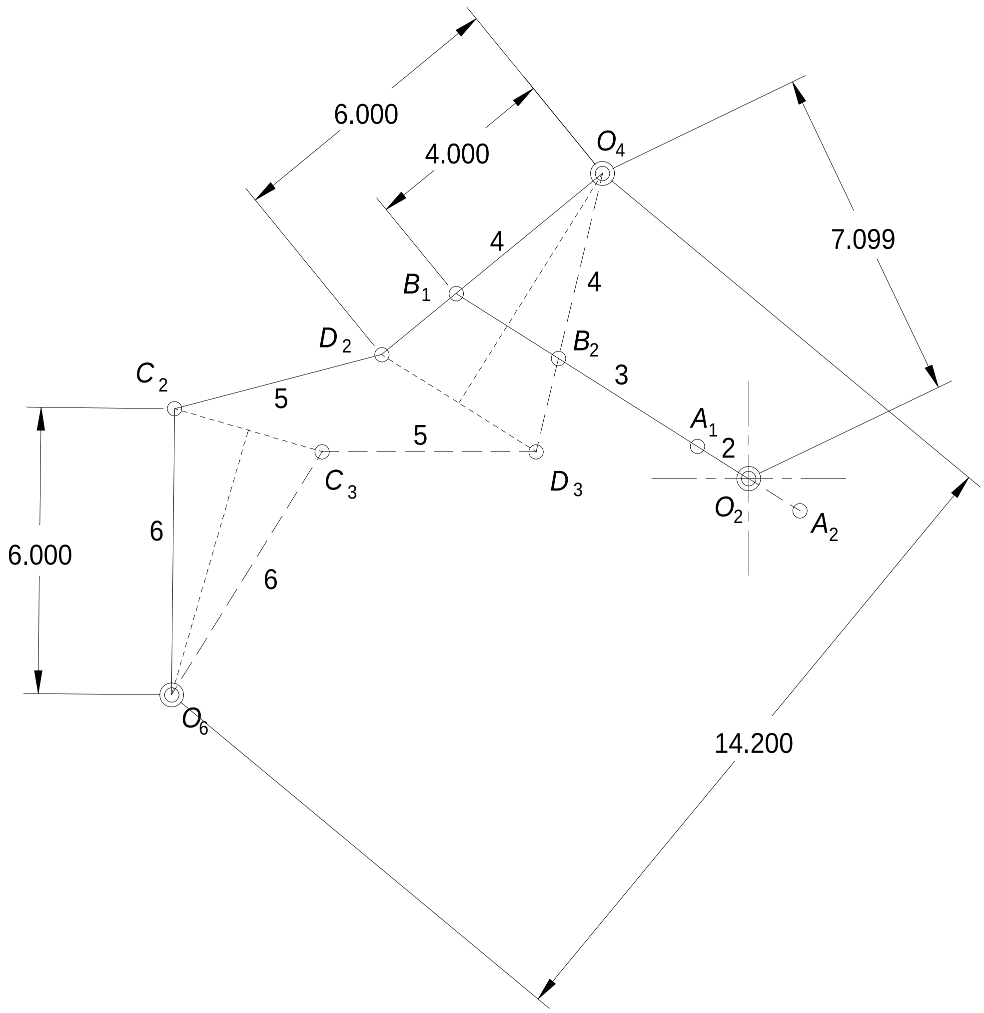
Link 4a (rocker) $L_{4a} := 4.000 \cdot in$ $\quad$ Link 1a (ground) $L_{1a} := 7.099 \cdot in$

10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

```
Condition(a, b, c, d) := | S ← min(a, b, c, d)
                         | L ← max(a, b, c, d)
                         | SL ← S + L
                         | PQ ← a + b + c + d − SL
                         | return "Grashof"  if SL < PQ
                         | return "Special Grashof"  if SL = PQ
                         | return "non-Grashof"  otherwise
```

$$Condition(L_{1a}, L_2, L_3, L_{4a}) = \text{"Grashof"}$$

$$min(L_{1a}, L_2, L_3, L_{4a}) = 1.271\ in$$



11. Using the program FOURBAR and the link lengths given above, it was found that the fourbar O_4DCO_6 is non-Grashoff with toggle positions at θ_4 = -41.6 deg and +41.6 deg. The fourbar operates between θ_4 = +26.171 deg and -11.052 deg.

PROBLEM 3-57

Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-16. Ignore the points O_2 and O_4 shown. Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Solution: See Figure P3-16 and Mathcad file P0357.

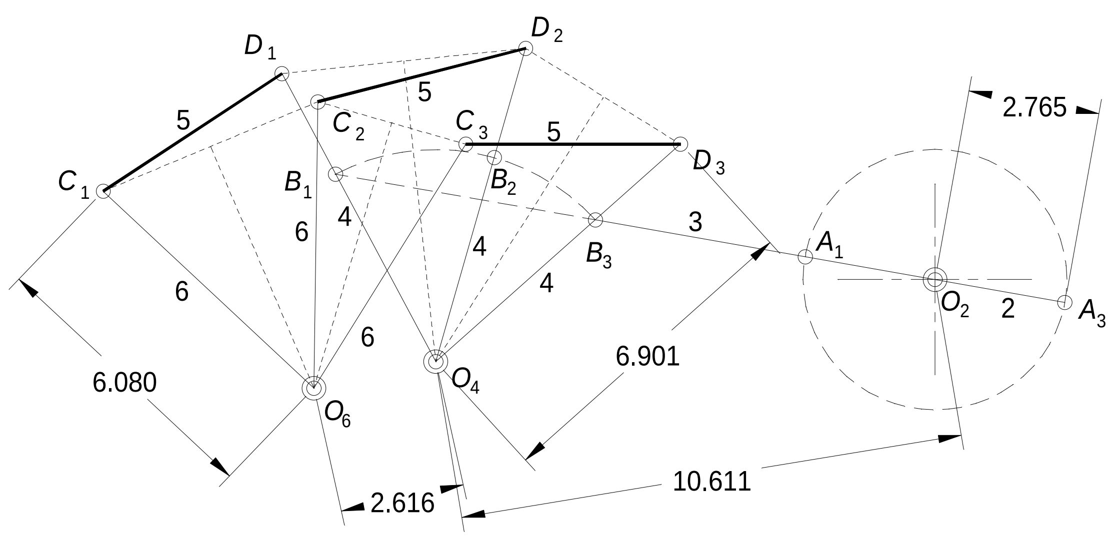
Design choices:

Length of link 3: $L_3 := 10.000$ Length of link 4b: $L_{4b} := 4.500$

1. Draw link *CD* in its three design positions C_1D_1, C_2D_2, C_3D_3 in the plane as shown.
2. Draw construction lines from point C_1 to C_2 and from point C_2 to C_3.
3. Bisect line C_1C_2 and line C_2C_3 and extend their perpendicular bisectors until they intersect. Label their intersection O_6.
4. Repeat steps 2 and 3 for lines D_1D_2 and D_2D_3. Label the intersection O_4.
5. Connect O_6 with C_1 and call it link 6. Connect O_4 with D_1 and call it link 4.
6. Line C_1D_1 is link 5. Line O_6O_4 is link 1*a* (ground link for the fourbar). The fourbar is now defined as O_6CDO_4 and has link lengths of

Ground link 1a $L_{1a} := 2.616$ Link 6 $L_6 := 6.080$

Link 5 $L_5 := 4.500$ Link 4 $L_4 := 6.901$



7. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

$$Condition(a,b,c,d) := \begin{array}{|l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"} \quad if\ SL < PQ \\ return\ \text{"Special Grashof"} \quad if\ SL = PQ \\ return\ \text{"non-Grashof"} \quad otherwise \end{array}$$

$Condition(L_{1a}, L_4, L_5, L_6) =$ "Grashof"

8. Select a point on link 4 (O_4D) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it B. (Note that link 4 is now a ternary link with nodes at O_4, D, and B.) In the solution above the distance O_4B was selected to be $L_{4b} = 4.500$.
9. Draw a construction line through B_1B_3 and extend it up to the right.
10. Layout the length of link 3 (design choice) along the extended line. Label the other end A.
11. Draw a circle about O_2 with a radius of one-half the length B_1B_3 and label the intersections of the circle with the extended line as A_1 and A_3. In the solution below the radius was measured as $L_2 := 2.765$.
12. The driver fourbar is now defined as O_4BAO_2 with link lengths

Link 2 (crank)	$L_2 = 2.765$
Link 3 (coupler)	$L_3 = 10.000$
Link 1b (ground)	$L_{1b} := 10.611$
Link 4b (rocker)	$L_{4b} = 4.500$

13. Use the link lengths in step 12 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

$Condition(L_2, L_3, L_{1b}, L_{4b}) =$ "Grashof"

$min(L_2, L_3, L_{1b}, L_{4b}) = 2.765$

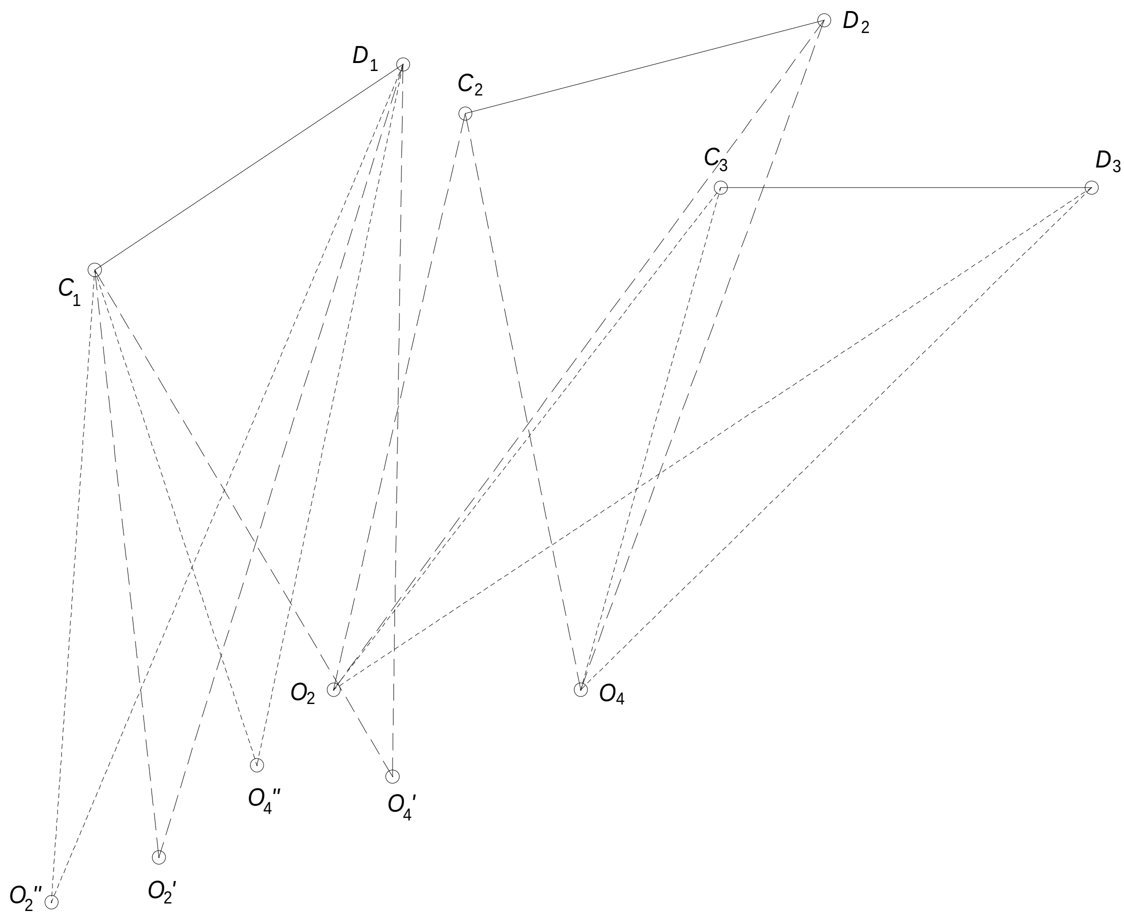
PROBLEM 3-58

Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-16 using the fixed pivots O_2 and O_4 shown. (See Example 3-7.) Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Solution: See Figure P3-16 and Mathcad file P0358.

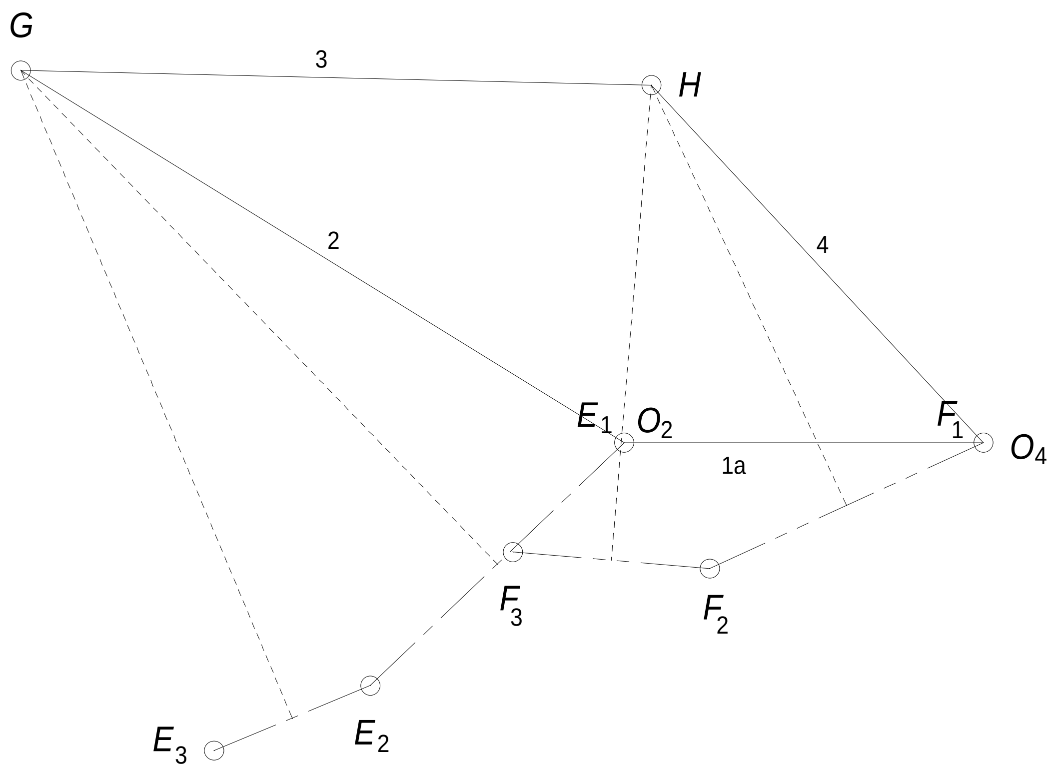
Design choices: Length of link 5: $L_5 := 5.000$ Length of link 2b: $L_{2b} := 2.500$

1. Draw link *CD* in its three design positions C_1D_1, C_2D_2, C_3D_3 in the plane as shown.
2. Draw the ground link O_2O_4 in its desired position in the plane with respect to the first coupler position C_1D_1.
3. Draw construction arcs from point C_2 to O_2 and from point D_2 to O_2 whose radii define the sides of triangle $C_2O_2D_2$. This defines the relationship of the fixed pivot O_2 to the coupler line *CD* in the second coupler position.
4. Draw construction arcs from point C_2 to O_4 and from point D_2 to O_4 whose radii define the sides of triangle $C_2O_4D_2$. This defines the relationship of the fixed pivot O_4 to the coupler line *CD* in the second coupler position.
5. Transfer this relationship back to the first coupler position C_1D_1 so that the ground plane position $O_2'O_4'$ bears the same relationship to C_1D_1 as O_2O_4 bore to the second coupler position C_2D_2.
6. Repeat the process for the third coupler position and transfer the third relative ground link position to the first, or reference, position.
7. The three inverted positions of the ground link that correspond to the three desired coupler positions are labeled O_2O_4, $O_2'O_4'$, and $O_2''O_4''$ in the first layout below and are renamed E_1F_1, E_2F_2, and E_3F_3, respectively, in the second layout, which is used to find the points *G* and *H*.



8. Draw construction lines from point E_1 to E_2 and from point E_2 to E_3.
9. Bisect line E_1E_2 and line E_2E_3 and extend their perpendicular bisectors until they intersect. Label their intersection *G*.
10. Repeat steps 2 and 3 for lines F_1F_2 and F_2F_3. Label the intersection *H*.
11. Connect E_1 with *G* and label it link 2. Connect F_1 with *H* and label it link 4. Reinverting, E_1 and F_1 are the original fixed pivots O_2 and O_4, respectively.
12. Line *GH* is link 3. Line O_2O_4 is link 1a (ground link for the fourbar). The fourbar is now defined as O_2GHO_4 and has link lengths of

Ground link 1a	$L_{1a} := 3.000$	Link 2	$L_2 := 8.597$
Link 3	$L_3 := 1.711$	Link 4	$L_4 := 7.921$



13. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

$$Condition(a,b,c,d) := \left| \begin{array}{l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.$$

$$Condition\left(L_{1a}, L_2, L_3, L_4\right) = \text{"Grashof"}$$

The fourbar that will provide the desired motion is now defined as a Grashof double crank in the crossed configuration. It now remains to add the original points C_1 and D_1 to the coupler *GH* and to define the driving dyad.

14. Select a point on link 2 (O_2G) at a suitable distance from O_2 as the pivot point to which the driver dyad will be connected and label it *B*. (Note that link 2 is now a ternary link with nodes at O_2, *B*, and *G*.) In the solution below, the distance O_2B was selected to be $L_{2b} = 2.500$.
15. Draw a construction line through B_1B_3 and extend it up to the left.
16. Layout the length of link 5 (design choice) along the extended line. Label the other end *A*.
17. Draw a circle about O_6 with a radius of one-half the length B_1B_3 and label the intersections of the circle with the extended line as A_1 and A_3. In the solution below the radius was measured as $L_6 := 1.541$.
18. The driver fourbar is now defined as O_2BAO_6 with link lengths

Link 6 (crank) $L_6 = 1.541$

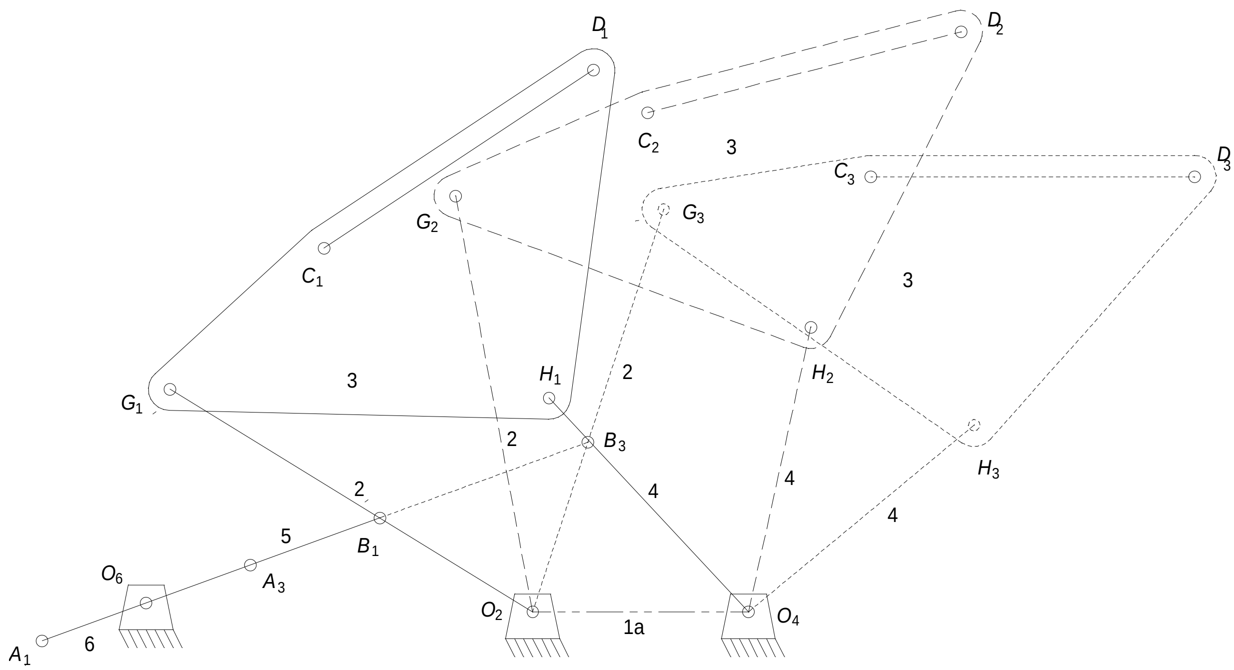
Link 5 (coupler) $L_5 = 5.000$

Link 1b (ground) $L_{1b} := 5.374$

Link 2b (rocker) $L_{2b} = 2.500$

19. Use the link lengths in step 18 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$$Condition(L_6, L_5, L_{1b}, L_{2b}) = \text{"Grashof"}$$



PROBLEM 3-59

Statement: Design a fourbar mechanism to move the link shown in Figure P3-17 from position 1 to position 2. Ignore the third position and the fixed pivots O_2 and O_4 shown. Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Given: Position 1 offsets: $x_{C1D1} := 1.896 \cdot in$ $\quad y_{C1D1} := 1.212 \cdot in$

Solution: See figure below for one possible solution. Input file P0359.mcd from the solutions manual disk to the Mathcad program for this solution, file P03-59.4br to the program FOURBAR to see the fourbar solution linkage, and file P03-59.6br into program SIXBAR to see the complete sixbar with the driver dyad included.

1. Connect the end points of the two given positions of the line *CD* with construction lines, i.e., lines from C_1 to C_2 and D_1 to D_2.
2. Bisect these lines and extend their perpendicular bisectors in any convenient direction. In the solution below the bisector of C_1C_2 was extended downward and the bisector of D_1D_2 was extended upward.
3. Select one point on each bisector and label them O_4 and O_6, respectively. In the solution below the distances O_6C and O_4D were each selected to be 6.500 in. This resulted in a ground-link-length O_4O_6 for the fourbar of 14.722 in.
4. The fourbar stage is now defined as O_4DCO_6 with link lengths

Link 5 (coupler) $L_5 := \sqrt{x_{C1D1}^2 + y_{C1D1}^2}$ $\quad L_5 = 2.250\ in$

Link 4 (input) $L_4 := 6.500 \cdot in$ $\quad$ Link 6 (output) $L_6 := 6.500 \cdot in$

Ground link 1b $L_{1b} := 14.722 \cdot in$

5. Select a point on link 4 (O_4D) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it *B*. (Note that link 4 is now a ternary link with nodes at O_4, *B*, and *D*.) In the solution below the distance O_4B was selected to be 4.500 in.
6. Draw a construction line through B_1B_2 and extend it to the right.
7. Select a point on this line and call it O_2. In the solution below the distance *AB* was selected to be 6.000 in.
8. Draw a circle about O_2 with a radius of one-half the length B_1B_2 and label the intersections of the circle with the extended line as A_1 and A_2. In the solution below the radius was measured as 1.037 in.
9. The driver fourbar is now defined as O_2ABO_4 with link lengths

Link 2 (crank) $L_2 := 0.645 \cdot in$ $\quad$ Link 3 (coupler) $L_3 := 6.000 \cdot in$

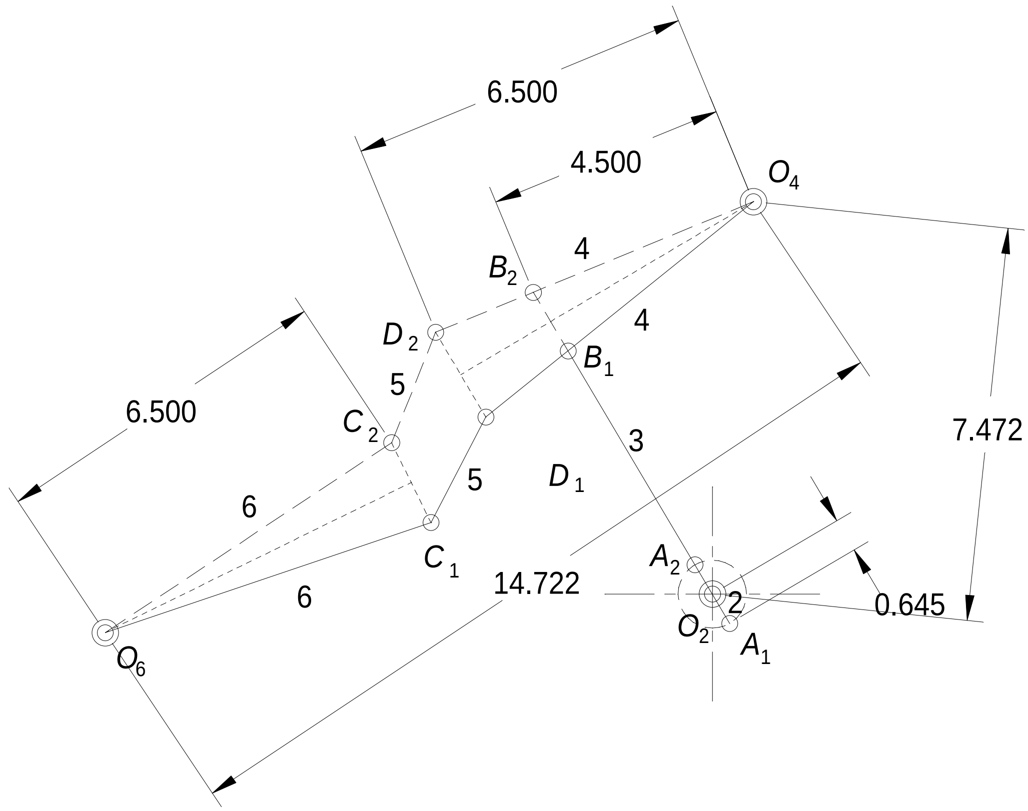
Link 4a (rocker) $L_{4a} := 4.500 \cdot in$ $\quad$ Link 1a (ground) $L_{1a} := 7.472 \cdot in$

10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

$$Condition(a,b,c,d) := \begin{array}{|l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array}$$

$Condition(L_{1a}, L_2, L_3, L_{4a}) = \text{"Grashof"}$

$min(L_{1a}, L_2, L_3, L_{4a}) = 0.645\ in$



11. Using the program FOURBAR and the link lengths given above, it was found that the fourbar O_4CDO_6 is non-Grashoff with toggle positions at θ_4 = -17.1 deg and +17.1 deg. The fourbar operates between θ_4 = +5.216 deg and -11.273 deg.

PROBLEM 3-60

Statement: Design a fourbar mechanism to move the link shown in Figure P3-17 from position 2 to position 3. Ignore the third position and the fixed pivots O_2 and O_4 shown. Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Given: Position 2 offsets: $x_{C2D2} := 0.834 \cdot in$ $\quad y_{C2D2} := 2.090 \cdot in$

Solution: See figure below for one possible solution. Input file P0360.mcd from the solutions manual disk to the Mathcad program for this solution, file P03-60.4br to the program FOURBAR to see the fourbar solution linkage, and file P03-60.6br into program SIXBAR to see the complete sixbar with the driver dyad included.

1. Connect the end points of the two given positions of the line CD with construction lines, i.e., lines from C_2 to C_3 and D_2 to D_3.
2. Bisect these lines and extend their perpendicular bisectors in any convenient direction. In the solution below the bisector of C_2C_3 was extended downward and the bisector of D_2D_3 was extended upward.
3. Select one point on each bisector and label them O_4 and O_6, respectively. In the solution below the distances O_4D and O_6C were each selected to be 6.000 in. This resulted in a ground-link-length O_4O_6 for the fourbar of 12.933 in.
4. The fourbar stage is now defined as O_4DCO_6 with link lengths

Link 5 (coupler) $L_5 := \sqrt{x_{C2D2}{}^2 + y_{C2D2}{}^2}$ $\quad L_5 = 2.250\, in$

Link 4 (input) $L_4 := 5.000 \cdot in$ $\quad$ Link 6 (output) $L_6 := 5.000 \cdot in$

Ground link 1b $L_{1b} := 12.933 \cdot in$

5. Select a point on link 4 (O_4D) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it B. (Note that link 4 is now a ternary link with nodes at O_4, B, and D.) In the solution below the distance O_4B was selected to be 4.000 in.
6. Draw a construction line through B_1B_2 and extend it to the right.
7. Select a point on this line and call it O_2. In the solution below the distance AB was selected to be 6.000 in.
8. Draw a circle about O_2 with a radius of one-half the length B_1B_2 and label the intersections of the circle with the extended line as A_1 and A_2. In the solution below the radius was measured as 0.741 in.
9. The driver fourbar is now defined as O_2ABO_4 with link lengths

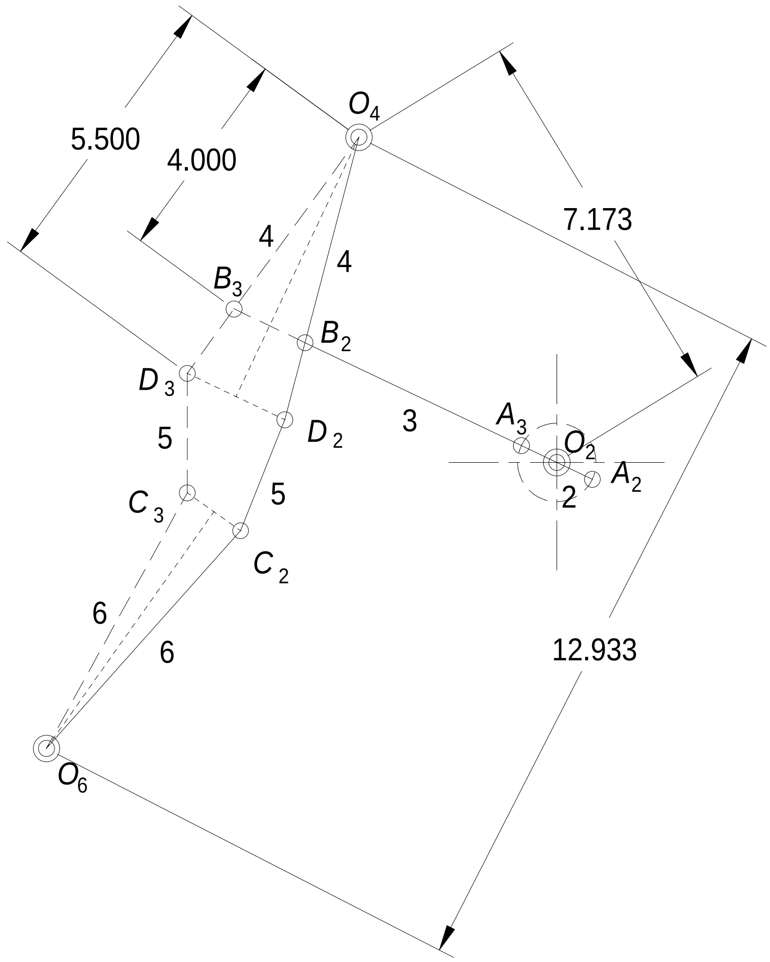
Link 2 (crank) $L_2 := 0.741 \cdot in$ $\quad$ Link 3 (coupler) $L_3 := 6.000 \cdot in$

Link 4a (rocker) $L_{4a} := 4.000 \cdot in$ $\quad$ Link 1a (ground) $L_{1a} := 7.173 \cdot in$

10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

$$
Condition(a,b,c,d) := \left|
\begin{array}{l}
S \leftarrow min(a,b,c,d) \\
L \leftarrow max(a,b,c,d) \\
SL \leftarrow S + L \\
PQ \leftarrow a + b + c + d - SL \\
return\ \text{"Grashof"}\quad if\ \ SL < PQ \\
return\ \text{"Special Grashof"}\quad if\ \ SL = PQ \\
return\ \text{"non-Grashof"}\quad otherwise
\end{array}
\right.
$$

$Condition(L_{1a}, L_2, L_3, L_4) = $ "Grashof"



11. Using the program FOURBAR and the link lengths given above, it was found that the fourbar O_4DCO_6 is non-Grashoff with toggle positions at θ_4 = -14.9 deg and +14.9 deg. The fourbar operates between θ_4 = +12.403 deg and -8.950 deg.

PROBLEM 3-61

Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-17. Ignore the points O_2 and O_4 shown. Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Solution: See Figure P3-17 and Mathcad file P0361.

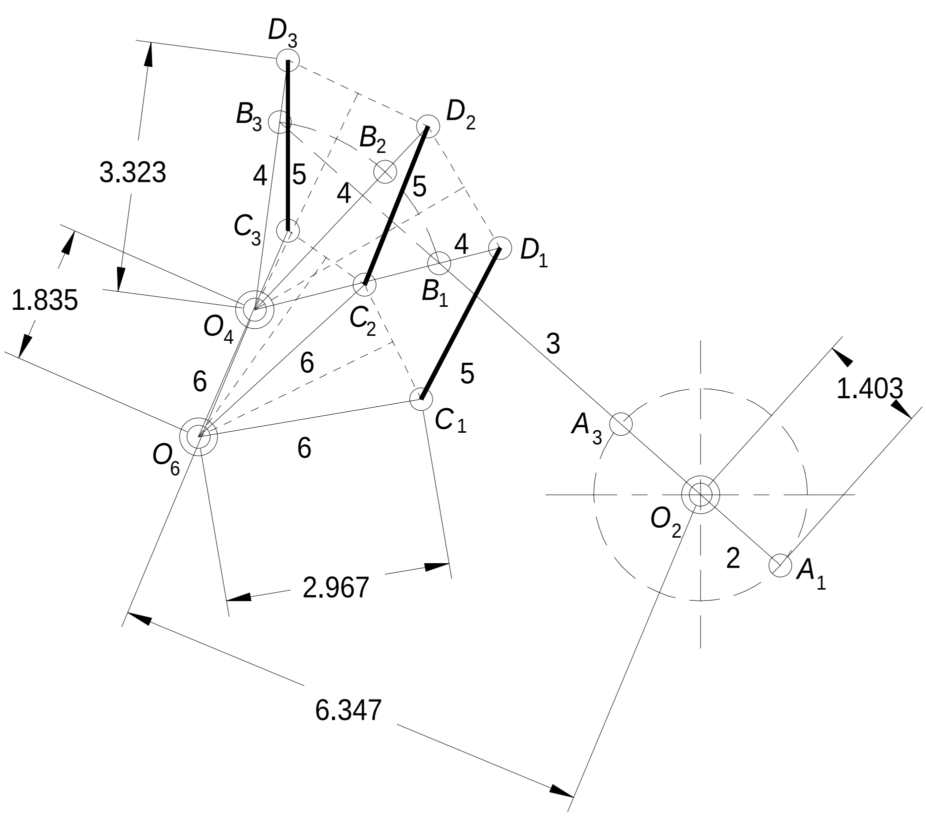
Design choices:

Length of link 3: $L_3 := 6.000$ Length of link 4b: $L_{4b} := 2.500$

1. Draw link *CD* in its three design positions C_1D_1, C_2D_2, C_3D_3 in the plane as shown.
2. Draw construction lines from point C_1 to C_2 and from point C_2 to C_3.
3. Bisect line C_1C_2 and line C_2C_3 and extend their perpendicular bisectors until they intersect. Label their intersection O_6.
4. Repeat steps 2 and 3 for lines D_1D_2 and D_2D_3. Label the intersection O_4.
5. Connect O_2 with C_1 and call it link 2. Connect O_4 with D_1 and call it link 4.
6. Line C_1D_1 is link 5. Line O_2O_4 is link 1*a* (ground link for the fourbar). The fourbar is now defined as O_6CDO_4 and has link lengths of

   Ground link 1a $L_{1a} := 1.835$ Link 6 $L_6 := 2.967$

   Link 5 $L_5 := 2.250$ Link 4 $L_4 := 3.323$



7. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

$$Condition(a,b,c,d) := \left| \begin{array}{l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"} \quad if\ SL < PQ \\ return\ \text{"Special Grashof"} \quad if\ SL = PQ \\ return\ \text{"non-Grashof"} \quad otherwise \end{array} \right.$$

$$Condition(L_{1a}, L_4, L_5, L_6) = \text{"Grashof"}$$

8. Select a point on link 4 (O_4D) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it B. (Note that link 4 is now a ternary link with nodes at O_4, D, and B.) In the solution above the distance O_4B was selected to be $L_{4b} = 2.500$.
9. Draw a construction line through B_1B_3 and extend it up to the right.
10. Layout the length of link 3 (design choice) along the extended line. Label the other end A.
11. Draw a circle about O_2 with a radius of one-half the length B_1B_3 and label the intersections of the circle with the extended line as A_1 and A_3. In the solution below the radius was measured as $L_2 := 1.403$.
12. The driver fourbar is now defined as O_2ABO_4 with link lengths

Link 2 (crank)	$L_2 = 1.403$
Link 3 (coupler)	$L_3 = 6.000$
Link 1b (ground)	$L_{1b} := 6.347$
Link 4b (rocker)	$L_{4b} = 2.500$

13. Use the link lengths in step 12 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

$$Condition(L_{1b}, L_2, L_3, L_{4b}) = \text{"Grashof"}$$

$$min(L_{1b}, L_2, L_3, L_{4b}) = 1.403$$

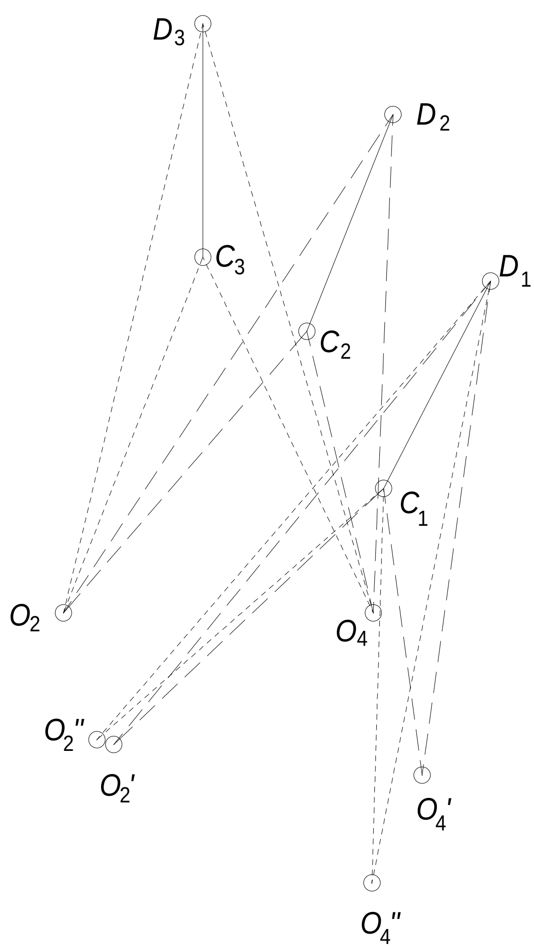
PROBLEM 3-62

Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-17 using the fixed pivots O_2 and O_4 shown. (See Example 3-7.) Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Solution: See Figure P3-17 and Mathcad file P0362.

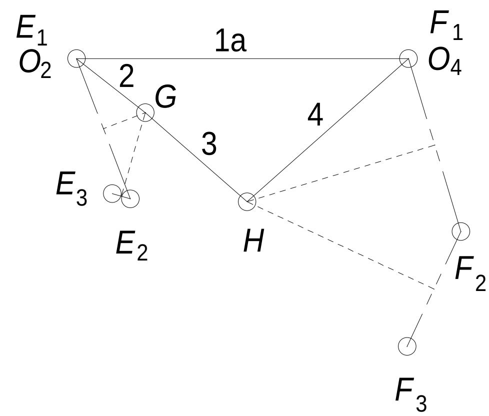
Design choices: Length of link 5: $L_5 := 4.000$ Length of link 2b: $L_{2b} := 0.791$

1. Draw link *CD* in its three design positions C_1D_1, C_2D_2, C_3D_3 in the plane as shown.
2. Draw the ground link O_2O_4 in its desired position in the plane with respect to the first coupler position C_1D_1.
3. Draw construction arcs from point C_2 to O_2 and from point D_2 to O_2 whose radii define the sides of triangle $C_2O_2D_2$. This defines the relationship of the fixed pivot O_2 to the coupler line *CD* in the second coupler position.
4. Draw construction arcs from point C_2 to O_4 and from point D_2 to O_4 whose radii define the sides of triangle $C_2O_4D_2$. This defines the relationship of the fixed pivot O_4 to the coupler line *CD* in the second coupler position.
5. Transfer this relationship back to the first coupler position C_1D_1 so that the ground plane position $O_2'O_4'$ bears the same relationship to C_1D_1 as O_2O_4 bore to the second coupler position C_2D_2.
6. Repeat the process for the third coupler position and transfer the third relative ground link position to the first, or reference, position.
7. The three inverted positions of the ground link that correspond to the three desired coupler positions are labeled O_2O_4, $O_2'O_4'$, and $O_2''O_4''$ in the first layout below and are renamed E_1F_1, E_2F_2, and E_3F_3, respectively, in the second layout, which is used to find the points *G* and *H*.



8. Draw construction lines from point E_1 to E_2 and from point E_2 to E_3.
9. Bisect line E_1E_2 and line E_2E_3 and extend their perpendicular bisectors until they intersect. Label their intersection *G*.
10. Repeat steps 2 and 3 for lines F_1F_2 and F_2F_3. Label the intersection *H*.
11. Connect E_1 with *G* and label it link 2. Connect F_1 with *H* and label it link 4. Reinverting, E_1 and F_1 are the original fixed pivots O_2 and O_4, respectively.
12. Line *GH* is link 3. Line O_2O_4 is link 1a (ground link for the fourbar). The fourbar is now defined as O_2GHO_4 and has link lengths of

Ground link 1a	$L_{1a} := 3.000$	Link 2	$L_2 := 0.791$
Link 3	$L_3 := 1.222$	Link 4	$L_4 := 1.950$



13. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

$$Condition(a,b,c,d) := \begin{array}{|l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"}\quad if\ SL < PQ \\ return\ \text{"Special Grashof"}\quad if\ SL = PQ \\ return\ \text{"non-Grashof"}\quad otherwise \end{array}$$

$$Condition(L_{1a}, L_2, L_3, L_4) = \text{"non-Grashof"}$$

The fourbar that will provide the desired motion is now defined as a non-Grashof double rocker in the crossed configuration. It now remains to add the original points C_1 and D_1 to the coupler *GH* and to define the driving dyad, which in this case will drive link 4 rather than link 2.

14. Select a point on link 2 (O_2G) at a suitable distance from O_2 as the pivot point to which the driver dyad will be connected and label it *B*. (Note that link 2 is now a ternary link with nodes at O_2, *B*, and *G*.) In the solution below, the distance O_2B was selected to be $L_{2b} = 0.791$. Thus, in this case *B* and *G* coincide.
15. Draw a construction line through B_1B_3 and extend it up to the left.
16. Layout the length of link 5 (design choice) along the extended line. Label the other end *A*.
17. Draw a circle about O_6 with a radius of one-half the length B_1B_3 and label the intersections of the circle

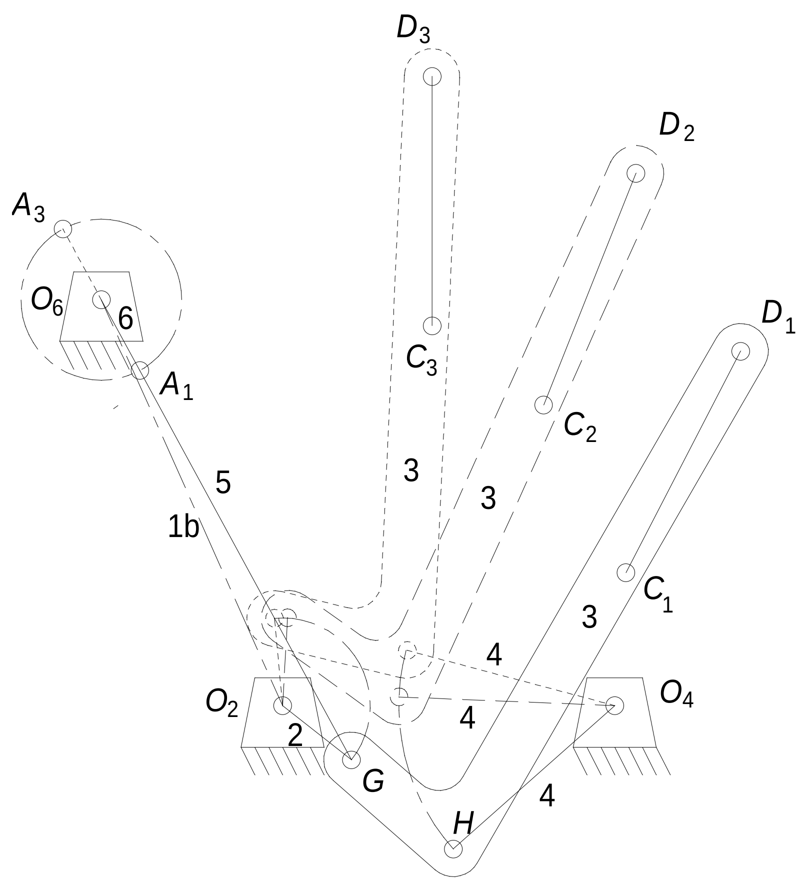
with the extended line as A_1 and A_3. In the solution below the radius was measured as $L_6 := 0.727$.

18. The driver fourbar is now defined as O_2BAO_6 with link lengths

Link 6 (crank)	$L_6 = 0.727$
Link 5 (coupler)	$L_5 = 4.000$
Link 1b (ground)	$L_{1b} := 4.012$
Link 2b (rocker)	$L_{2b} = 0.791$

19. Use the link lengths in step 18 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$$Condition(L_6, L_5, L_{1b}, L_{2b}) = \text{"Grashof"}$$



PROBLEM 3-63

Statement: Design a fourbar mechanism to move the link shown in Figure P3-18 from position 1 to position 2. Ignore the third position and the fixed pivots O_2 and O_4 shown. Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Given: Position 1 offsets: $x_{C1D1} := 1.591 \cdot in$ $\quad y_{C1D1} := 1.591 \cdot in$

Solution: See figure below for one possible solution. Input file P0363.mcd from the solutions manual disk to the Mathcad program for this solution, file P03-63.4br to the program FOURBAR to see the fourbar solution linkage, and file P03-63.6br into program SIXBAR to see the complete sixbar with the driver dyad included.

1. Connect the end points of the two given positions of the line *CD* with construction lines, i.e., lines from C_1 to C_2 and D_1 to D_2.
2. Bisect these lines and extend their perpendicular bisectors in any convenient direction. In the solution below the bisector of C_1C_2 was extended downward and the bisector of D_1D_2 was extended upward.
3. Select one point on each bisector and label them O_4 and O_6, respectively. In the solution below the distances O_4C and O_6D were each selected to be 5.000 in. This resulted in a ground-link-length O_4O_6 for the fourbar of 10.457 in.
4. The fourbar stage is now defined as O_4CDO_6 with link lengths

Link 5 (coupler) $L_5 := \sqrt{x_{C1D1}^2 + y_{C1D1}^2}$ $\quad L_5 = 2.250\, in$

Link 4 (input) $L_4 := 5.000 \cdot in$ $\quad$ Link 6 (output) $L_6 := 5.000 \cdot in$

Ground link 1b $L_{1b} := 10.457 \cdot in$

5. Select a point on link 4 (O_4C) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it *B*. (Note that link 4 is now a ternary link with nodes at O_4, *B*, and *C*.) In the solution below the distance O_4B was selected to be 3.750 in.
6. Draw a construction line through B_1B_2 and extend it to the right.
7. Select a point on this line and call it O_2. In the solution below the distance *AB* was selected to be 6.000 in.
8. Draw a circle about O_2 with a radius of one-half the length B_1B_2 and label the intersections of the circle with the extended line as A_1 and A_2. In the solution below the radius was measured as 0.882 in.
9. The driver fourbar is now defined as O_2ABO_4 with link lengths

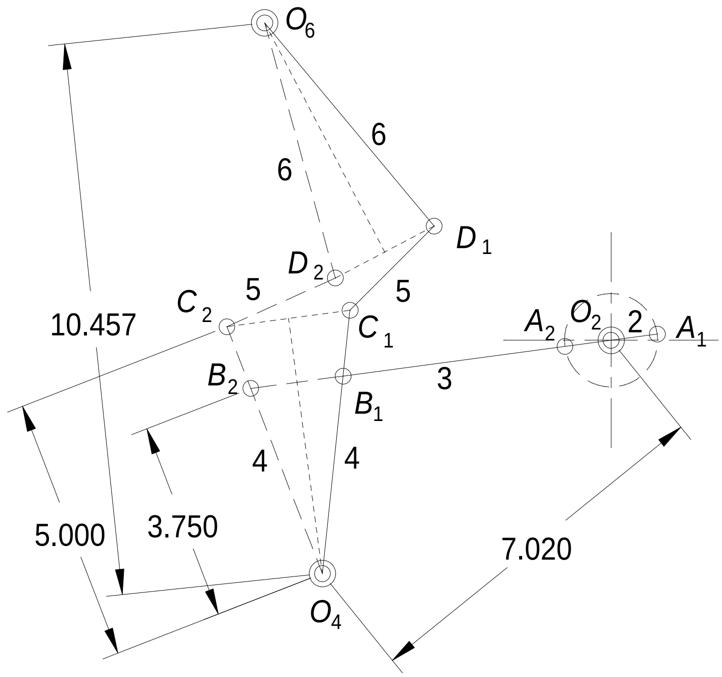
Link 2 (crank) $L_2 := 0.882 \cdot in$ $\quad$ Link 3 (coupler) $L_3 := 6.000 \cdot in$

Link 4a (rocker) $L_{4a} := 3.750 \cdot in$ $\quad$ Link 1a (ground) $L_{1a} := 7.020 \cdot in$

10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

$$Condition(a,b,c,d) := \left| \begin{array}{l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"} \quad if\ SL < PQ \\ return\ \text{"Special Grashof"} \quad if\ SL = PQ \\ return\ \text{"non-Grashof"} \quad otherwise \end{array} \right.$$

$$Condition(L_{1a}, L_2, L_3, L_{4a}) = \text{"Grashof"}$$



11. Using the program FOURBAR and the link lengths given above, it was found that the fourbar O_4CDO_6 is non-Grashoff with toggle positions at θ_4 = -38.5 deg and +38.5 deg. The fourbar operates between θ_4 = +15.206 deg and -12.009 deg.

PROBLEM 3-64

Statement: Design a fourbar mechanism to move the link shown in Figure P3-18 from position 2 to position 3. Ignore the third position and the fixed pivots O_2 and O_4 shown. Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Given: Position 2 offsets: $x_{C2D2} := 2.053 \cdot in$ $\quad y_{C2D2} := 0.920 \cdot in$

Solution: See figure below for one possible solution. Input file P0360.mcd from the solutions manual disk to the Mathcad program for this solution, file P03-60.4br to the program FOURBAR to see the fourbar solution linkage, and file P03-60.6br into program SIXBAR to see the complete sixbar with the driver dyad included.

1. Connect the end points of the two given positions of the line *CD* with construction lines, i.e., lines from C_2 to C_3 and D_2 to D_3.
2. Bisect these lines and extend their perpendicular bisectors in any convenient direction. In the solution below the bisector of C_2C_3 was extended downward and the bisector of D_2D_3 was extended upward.
3. Select one point on each bisector and label them O_4 and O_6, respectively. In the solution below the distances O_4D and O_6C were each selected to be 5.000 in. This resulted in a ground-link-length O_4O_6 for the fourbar of 8.773 in.
4. The fourbar stage is now defined as O_4DCO_6 with link lengths

Link 5 (coupler) $L_5 := \sqrt{x_{C2D2}^2 + y_{C2D2}^2}$ $\qquad L_5 = 2.250\, in$

Link 4 (input) $L_4 := 5.000 \cdot in$ $\qquad$ Link 6 (output) $L_6 := 5.000 \cdot in$

Ground link 1b $L_{1b} := 8.773 \cdot in$

5. Select a point on link 4 (O_4D) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it *B*. (Note that link 4 is now a ternary link with nodes at O_4, *B*, and *D*.) In the solution below the distance O_4B was selected to be 3.750 in.
6. Draw a construction line through B_1B_2 and extend it to the right.
7. Select a point on this line and call it O_2. In the solution below the distance *AB* was selected to be 6.000 in.
8. Draw a circle about O_2 with a radius of one-half the length B_1B_2 and label the intersections of the circle with the extended line as A_1 and A_2. In the solution below the radius was measured as 0.892 in.
9. The driver fourbar is now defined as O_2ABO_4 with link lengths

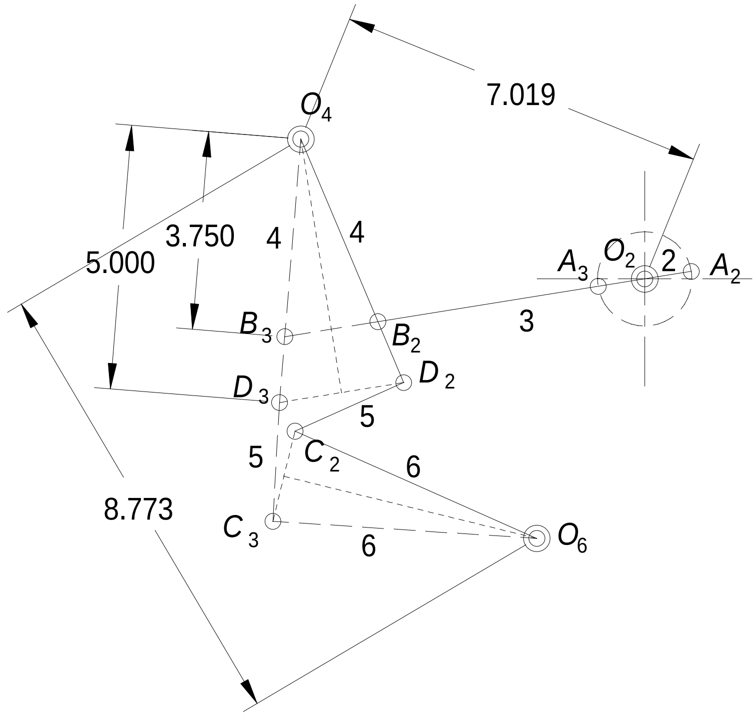
Link 2 (crank) $L_2 := 0.892 \cdot in$ $\qquad$ Link 3 (coupler) $L_3 := 6.000 \cdot in$

Link 4a (rocker) $L_{4a} := 3.750 \cdot in$ $\qquad$ Link 1a (ground) $L_{1a} := 7.019 \cdot in$

10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

$$Condition(a,b,c,d) := \left| \begin{array}{l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"} \quad if\ SL < PQ \\ return\ \text{"Special Grashof"} \quad if\ SL = PQ \\ return\ \text{"non-Grashof"} \quad otherwise \end{array} \right.$$

$Condition(L_{1a}, L_2, L_3, L_{4a}) =$ "Grashof"



11. Using the program FOURBAR and the link lengths given above, it was found that the fourbar O_4DCO_6 is non-Grashoff with toggle positions at θ_4 = -55.7 deg and +55.7 deg. The fourbar operates between θ_4 = -7.688 deg and -35.202 deg.

PROBLEM 3-65

Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-18. Ignore the points O_2 and O_4 shown. Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

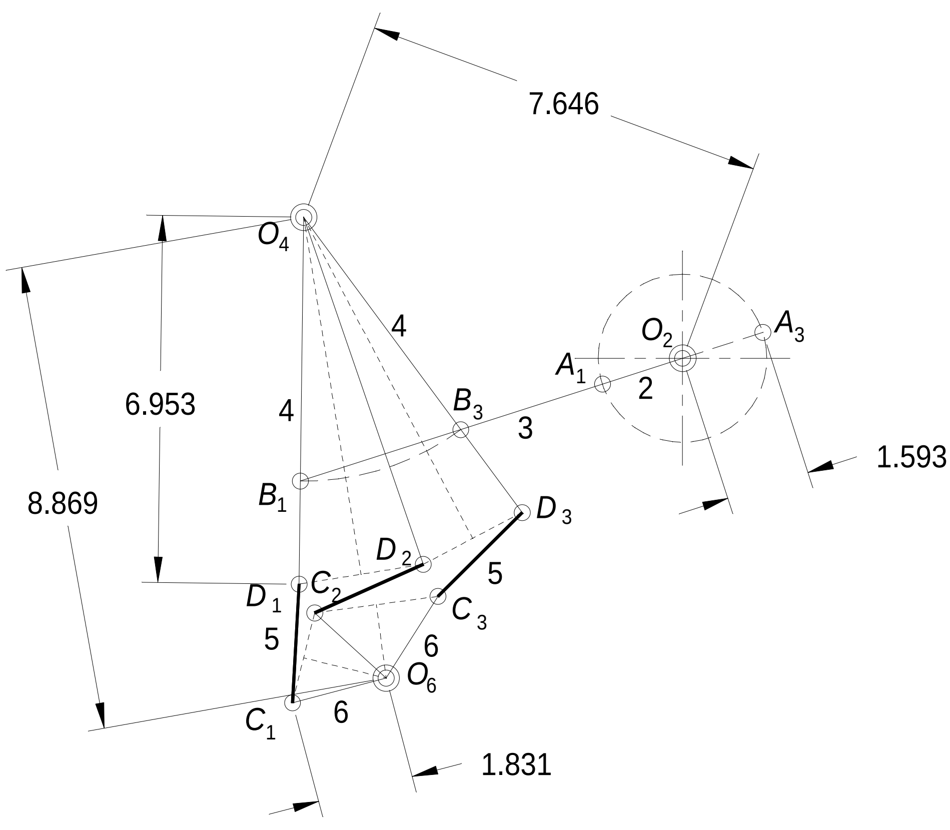
Solution: See Figure P3-18 and Mathcad file P0365.

Design choices:

Length of link 3: $L_3 := 6.000$ Length of link 4b: $L_{4b} := 5.000$

1. Draw link CD in its three design positions C_1D_1, C_2D_2, C_3D_3 in the plane as shown.
2. Draw construction lines from point C_1 to C_2 and from point C_2 to C_3.
3. Bisect line C_1C_2 and line C_2C_3 and extend their perpendicular bisectors until they intersect. Label their intersection O_6.
4. Repeat steps 2 and 3 for lines D_1D_2 and D_2D_3. Label the intersection O_4.
5. Connect O_6 with C_1 and call it link 6. Connect O_4 with D_1 and call it link 4.
6. Line C_1D_1 is link 5. Line O_6O_4 is link 1*a* (ground link for the fourbar). The fourbar is now defined as O_6CDO_4 and has link lengths of

Ground link 1a	$L_{1a} := 8.869$	Link 6	$L_6 := 1.831$
Link 5	$L_5 := 2.250$	Link 4	$L_4 := 6.953$



7. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

$$Condition(a,b,c,d) := \left| \begin{array}{l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"}\quad if\ \ SL < PQ \\ return\ \text{"Special Grashof"}\quad if\ \ SL = PQ \\ return\ \text{"non-Grashof"}\quad otherwise \end{array} \right.$$

$$Condition(L_6, L_{1a}, L_4, L_5) = \text{"non-Grashof"}$$

8. Select a point on link 4 (O_4D) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it B. (Note that link 4 is now a ternary link with nodes at O_4, D, and B.) In the solution above the distance O_4B was selected to be $L_{4b} = 5.000$.
9. Draw a construction line through B_1B_3 and extend it up to the right.
10. Layout the length of link 3 (design choice) along the extended line. Label the other end A.
11. Draw a circle about O_2 with a radius of one-half the length B_1B_3 and label the intersections of the circle with the extended line as A_1 and A_3. In the solution below the radius was measured as $L_2 := 1.593$.
12. The driver fourbar is now defined as O_2ABO_4 with link lengths

Link 2 (crank)	$L_2 = 1.593$
Link 3 (coupler)	$L_3 = 6.000$
Link 1b (ground)	$L_{1b} := 7.646$
Link 4b (rocker)	$L_{4b} = 5.000$

13. Use the link lengths in step 12 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

$$Condition(L_{1b}, L_2, L_3, L_{4b}) = \text{"Grashof"}$$

$$min(L_{1b}, L_2, L_3, L_{4b}) = 1.593$$

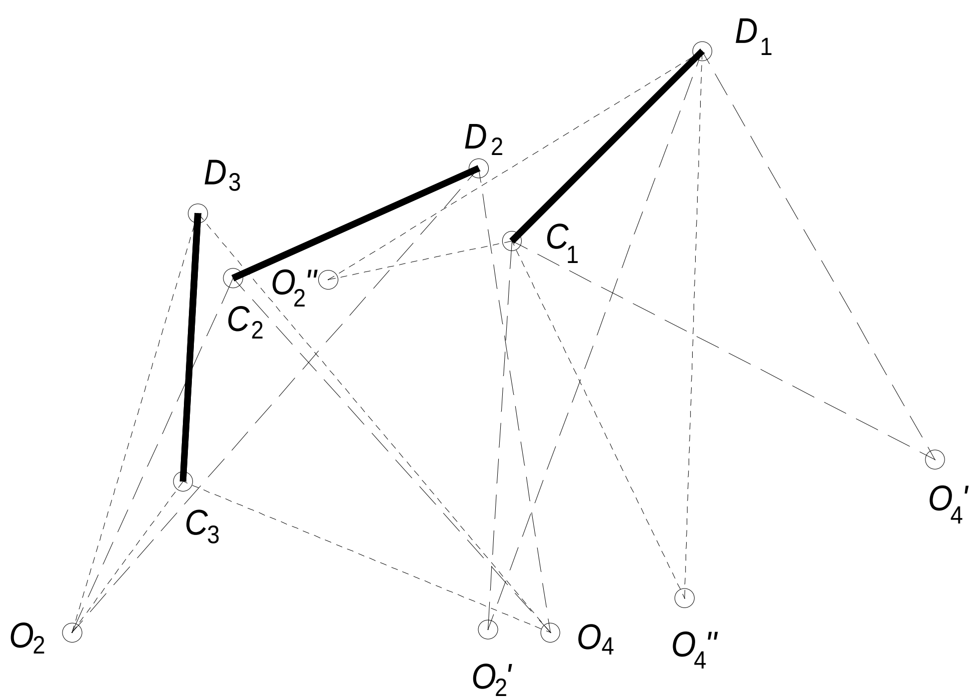
PROBLEM 3-66

Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-18 using the fixed pivots O_2 and O_4 shown. (See Example 3-7.) Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Solution: See Figure P3-18 and Mathcad file P0366.

Design choices: Length of link 5: $L_5 := 4.000$ Length of link 2b: $L_{2b} := 2.000$

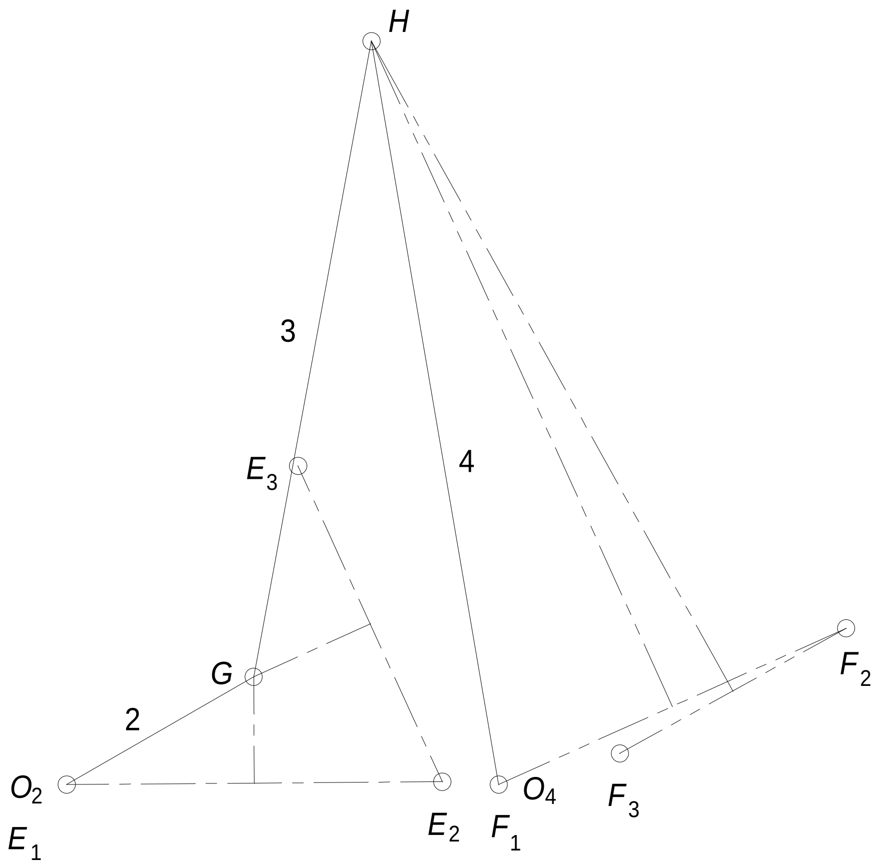
1. Draw link *CD* in its three design positions C_1D_1, C_2D_2, C_3D_3 in the plane as shown.
2. Draw the ground link O_2O_4 in its desired position in the plane with respect to the first coupler position C_1D_1.
3. Draw construction arcs from point C_2 to O_2 and from point D_2 to O_2 whose radii define the sides of triangle $C_2O_2D_2$. This defines the relationship of the fixed pivot O_2 to the coupler line *CD* in the second coupler position.
4. Draw construction arcs from point C_2 to O_4 and from point D_2 to O_4 whose radii define the sides of triangle $C_2O_4D_2$. This defines the relationship of the fixed pivot O_4 to the coupler line *CD* in the second coupler position.
5. Transfer this relationship back to the first coupler position C_1D_1 so that the ground plane position $O_2'O_4'$ bears the same relationship to C_1D_1 as O_2O_4 bore to the second coupler position C_2D_2.
6. Repeat the process for the third coupler position and transfer the third relative ground link position to the first, or reference, position.
7. The three inverted positions of the ground link that correspond to the three desired coupler positions are labeled O_2O_4, $O_2'O_4'$, and $O_2''O_4''$ in the first layout below and are renamed E_1F_1, E_2F_2, and E_3F_3, respectively, in the second layout, which is used to find the points *G* and *H*.



8. Draw construction lines from point E_1 to E_2 and from point E_2 to E_3.
9. Bisect line E_1E_2 and line E_2E_3 and extend their perpendicular bisectors until they intersect. Label their intersection *G*.
10. Repeat steps 2 and 3 for lines F_1F_2 and F_2F_3. Label the intersection *H*.

11. Connect E_1 with G and label it link 2. Connect F_1 with H and label it link 4. Reinverting, E_1 and F_1 are the original fixed pivots O_2 and O_4, respectively.
12. Line GH is link 3. Line O_2O_4 is link 1a (ground link for the fourbar). The fourbar is now defined as O_2GHO_4 and has link lengths of

Ground link 1a	$L_{1a} := 4.000$	Link 2	$L_2 := 2.000$
Link 3	$L_3 := 6.002$	Link 4	$L_4 := 7.002$



13. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

$$Condition(a,b,c,d) := \left| \begin{array}{l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return \text{ "Grashof" } \; if \; SL < PQ \\ return \text{ "Special Grashof" } \; if \; SL = PQ \\ return \text{ "non-Grashof" } \; otherwise \end{array} \right.$$

$$Condition(L_{1a}, L_2, L_3, L_4) = \text{"Grashof"}$$

The fourbar that will provide the desired motion is now defined as a non-Grashof crank rocker in the open configuration. It now remains to add the original points C_1 and D_1 to the coupler GH and to define the driving dyad, which in this case will drive link 4 rather than link 2.

14. Select a point on link 2 (O_2G) at a suitable distance from O_2 as the pivot point to which the driver dyad will be connected and label it *B*. (Note that link 2 is now a ternary link with nodes at O_2, *B*, and *G*.) In the solution below, the distance O_2B was selected to be $L_{2b} = 2.000$. Thus, in this case *B* and *G* coincide.
15. Draw a construction line through B_1B_3 and extend it up to the left.
16. Layout the length of link 5 (design choice) along the extended line. Label the other end *A*.
17. Draw a circle about O_6 with a radius of one-half the length B_1B_3 and label the intersections of the circle with the extended line as A_1 and A_3. In the solution below the radius was measured as $L_6 := 1.399$.
18. The driver fourbar is now defined as O_2BAO_6 with link lengths

Link 6 (crank) $L_6 = 1.399$

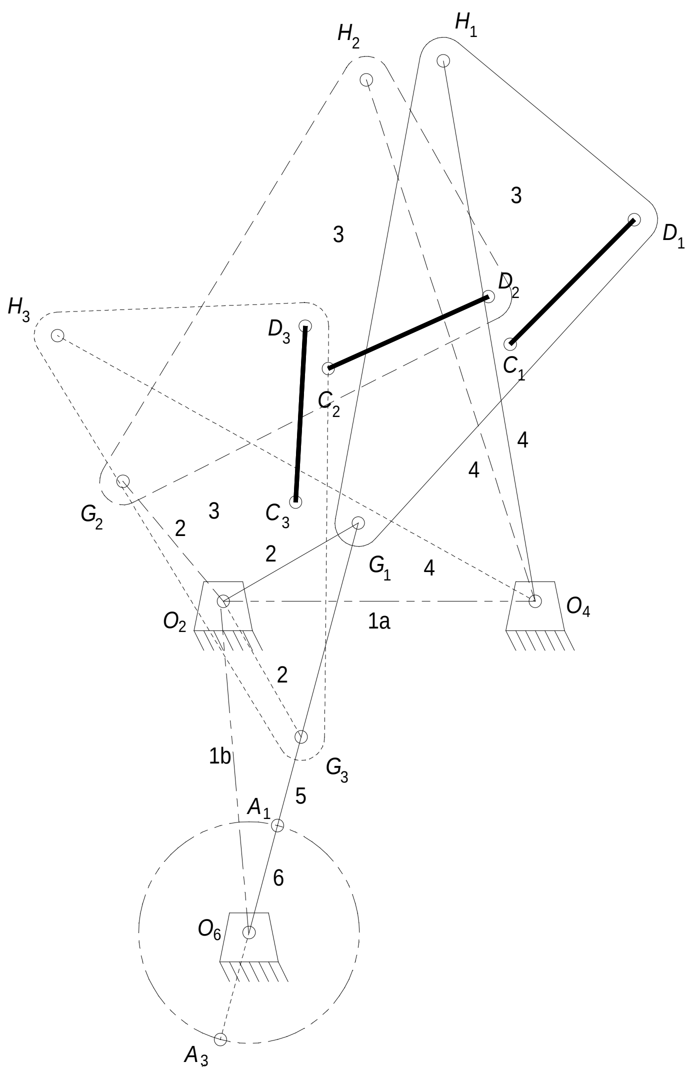
Link 5 (coupler) $L_5 = 4.000$

Link 1b (ground) $L_{1b} := 4.257$

Link 2b (rocker) $L_{2b} = 2.000$

19. Use the link lengths in step 18 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$Condition(L_6, L_{1b}, L_{2b}, L_5) = \text{"Grashof"}$



PROBLEM 3-67

Statement: Design a fourbar Grashof crank-rocker for 120 degrees of output rocker motion with a quick-return time ratio of 1:1.2. (See Example 3-9.)

Given: Time ratio $T_r := \frac{1}{1.2}$

Solution: See figure below for one possible solution. Also see Mathcad file P0367.

1. Determine the crank rotation angles α and β, and the construction angle δ from equations 3.1 and 3.2.

$$T_r = \frac{\alpha}{\beta} \qquad \alpha + \beta = 360 \cdot deg$$

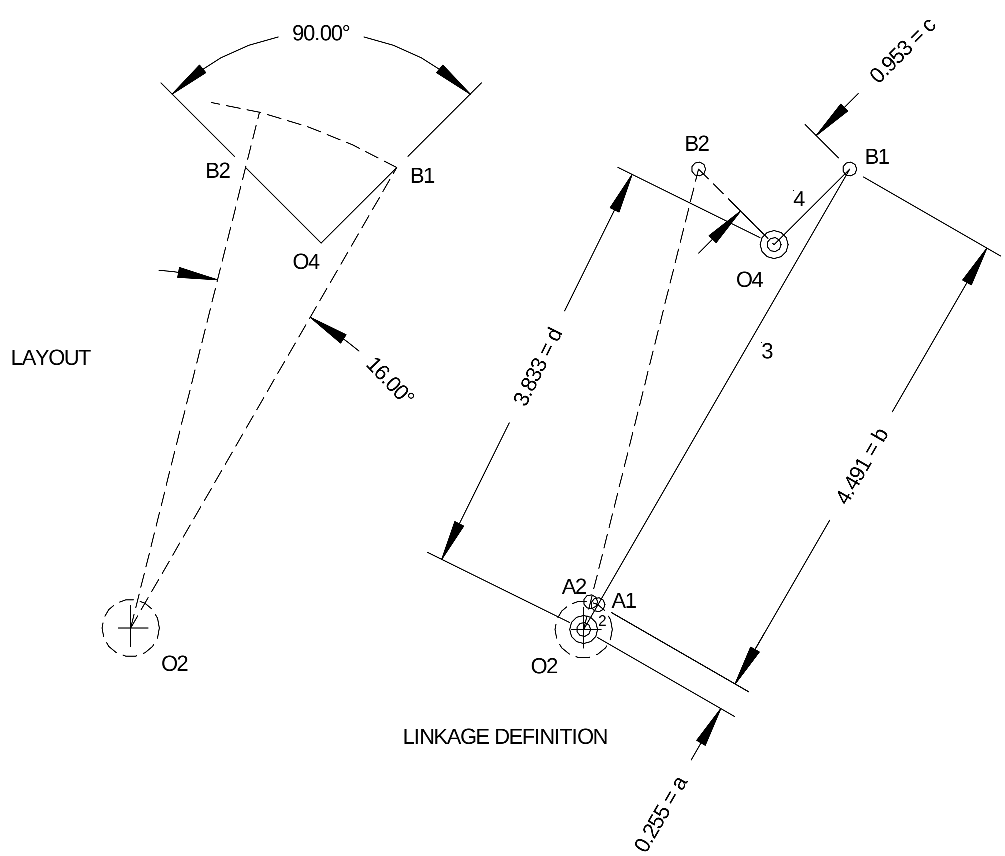
Solving for β, α, and δ

$$\beta := \frac{360 \cdot deg}{1 + T_r} \qquad \beta = 196 \cdot deg$$

$$\alpha := 360 \cdot deg - \beta \qquad \alpha = 164 \cdot deg$$

$$\delta := \beta - 180 \cdot deg \qquad \delta = 16 \cdot deg$$

2. Start the layout by arbitrarily establishing the point O_4 and from it layoff two lines of equal length, 90 deg apart. Label one B_1 and the other B_2. In the solution below, each line makes an angle of 45 deg with the horizontal and has a length of 1.000 in.
3. Layoff a line through B_1 at an arbitrary angle (but not zero deg). In the solution below the line is 60 deg to the horizontal.



4. Layoff a line through B_2 that makes an angle δ with the line in step 3 (76 deg to the horizontal in this case). The intersection of these two lines establishes the point O_2.

5. From O_2 draw an arc that goes through B_1. Extend O_2B_2 to meet this arc. Erect a perpendicular bisector to the extended portion of the line and transfer one half of the line to O_2 as the length of the input crank.

6. For this solution, the link lengths are:

Ground link (1)	$d := 3.833 \cdot in$	Coupler (3)	$b := 4.491 \cdot in$
Crank (2)	$a := 0.255 \cdot in$	Rocker (4)	$c := 0.953 \cdot in$

PROBLEM 3-68

Statement: Design a fourbar Grashof crank-rocker for 100 degrees of output rocker motion with a quick-return time ratio of 1:1.5. (See Example 3-9.)

Given: Time ratio $T_r := \frac{1}{1.5}$

Solution: See figure below for one possible solution. Also see Mathcad file P0368.

1. Determine the crank rotation angles α and β, and the construction angle δ from equations 3.1 and 3.2.

$$T_r = \frac{\alpha}{\beta} \qquad \alpha + \beta = 360 \cdot deg$$

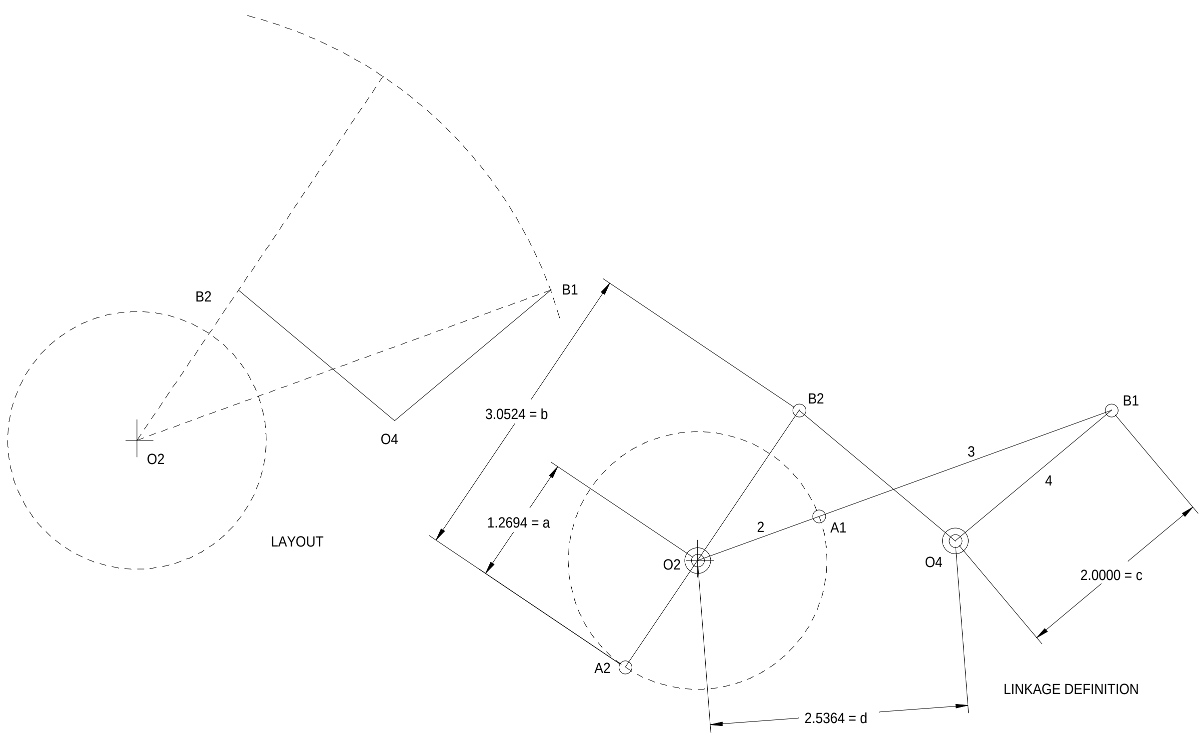
Solving for β, α, and δ

$$\beta := \frac{360 \cdot deg}{1 + T_r} \qquad \beta = 216\,deg$$

$$\alpha := 360 \cdot deg - \beta \qquad \alpha = 144\,deg$$

$$\delta := \beta - 180 \cdot deg \qquad \delta = 36\,deg$$

2. Start the layout by arbitrarily establishing the point O_4 and from it layoff two lines of equal length, 100 deg apart. Label one B_1 and the other B_2. In the solution below, each line makes an angle of 40 deg with the horizontal and has a length of 2.000 in.
3. Layoff a line through B_1 at an arbitrary angle (but not zero deg). In the solution below the line is 20 deg to the horizontal.
4. Layoff a line through B_2 that makes an angle δ with the line in step 3 (56 deg to the horizontal in this case). The intersection of these two lines establishes the point O_2.
5. From O_2 draw an arc that goes through B_1. Extend O_2B_2 to meet this arc. Erect a perpendicular bisector to the extended portion of the line and transfer one half of the line to O_2 as the length of the input crank.



6. For this solution, the link lengths are:

Ground link (1)	$d := 2.5364 \cdot in$	Coupler (3)	$b := 3.0524 \cdot in$
Crank (2)	$a := 1.2694 \cdot in$	Rocker (4)	$c := 2.000 \cdot in$

PROBLEM 3-69

Statement: Design a fourbar Grashof crank-rocker for 80 degrees of output rocker motion with a quick-return time ratio of 1:1.33. (See Example 3-9.)

Given: Time ratio $T_r := \frac{1}{1.33}$

Solution: See figure below for one possible solution. Also see Mathcad file P0369.

1. Determine the crank rotation angles α and β, and the construction angle δ from equations 3.1 and 3.2.

$$T_r = \frac{\alpha}{\beta} \qquad \alpha + \beta = 360 \cdot deg$$

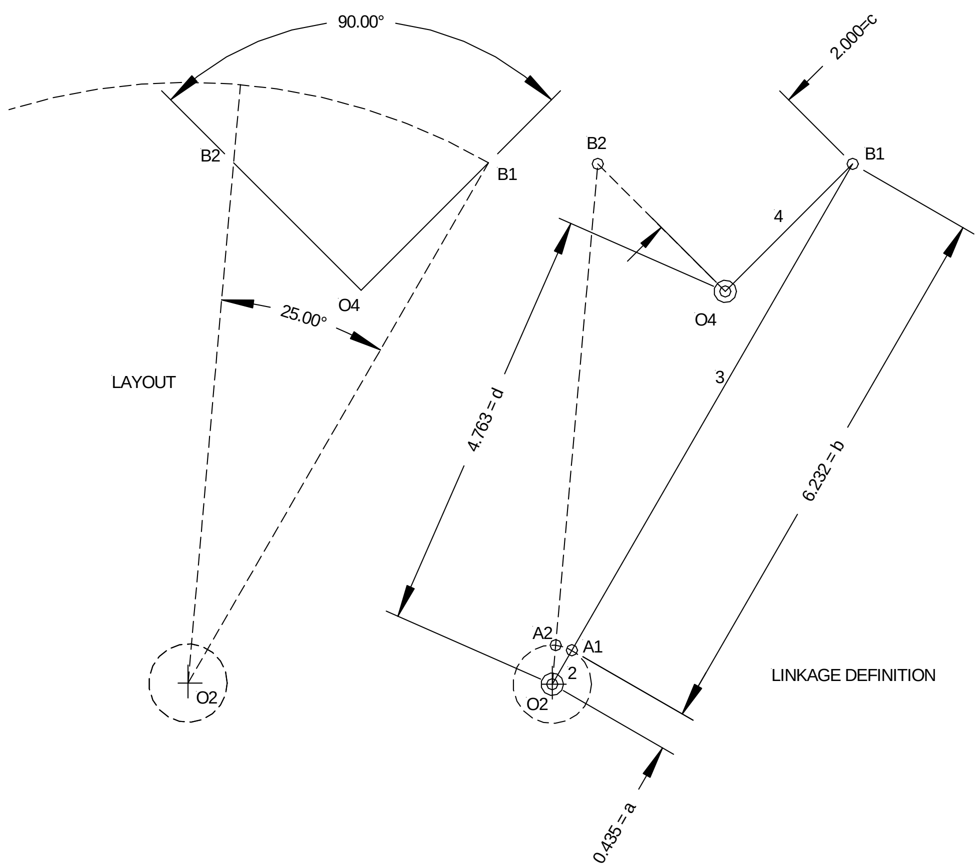
Solving for β, α, and δ

$$\beta := \frac{360 \cdot deg}{1 + T_r} \qquad \beta = 205 \cdot deg$$

$$\alpha := 360 \cdot deg - \beta \qquad \alpha = 155 \cdot deg$$

$$\delta := \beta - 180 \cdot deg \qquad \delta = 25 \cdot deg$$

2. Start the layout by arbitrarily establishing the point O_4 and from it layoff two lines of equal length, 100 deg apart. Label one B_1 and the other B_2. In the solution below, each line makes an angle of 40 deg with the horizontal and has a length of 2.000 in.
3. Layoff a line through B_1 at an arbitrary angle (but not zero deg). In the solution below the line is 150 deg to the horizontal.



4. Layoff a line through B_2 that makes an angle δ with the line in step 3 (73 deg to the horizontal in this case). The intersection of these two lines establishes the point O_2.
5. From O_2 draw an arc that goes through B_1. Extend O_2B_2 to meet this arc. Erect a perpendicular bisector to the extended portion of the line and transfer one half of the line to O_2 as the length of the input crank.
6. For this solution, the link lengths are:

Ground link (1)	$d := 4.763 \cdot in$	Coupler (3)	$b := 6.232 \cdot in$
Crank (2)	$a := 0.435 \cdot in$	Rocker (4)	$c := 2.000 \cdot in$

PROBLEM 3-70

Statement: Design a sixbar drag link quick-return linkage for a time ratio of 1:4 and output rocker motion of 50 degrees. (See Example 3-10.)

Given: Time ratio $T_r := \frac{1}{4}$

Solution: See figure below for one possible solution. Also see Mathcad file P0370.

1. Determine the crank rotation angles α and β from equation 3.1.

$$T_r = \frac{\alpha}{\beta} \qquad \alpha + \beta = 360 \cdot deg$$

Solving for β and α

$$\beta := \frac{360 \cdot deg}{1 + T_r} \qquad \beta = 288\, deg$$

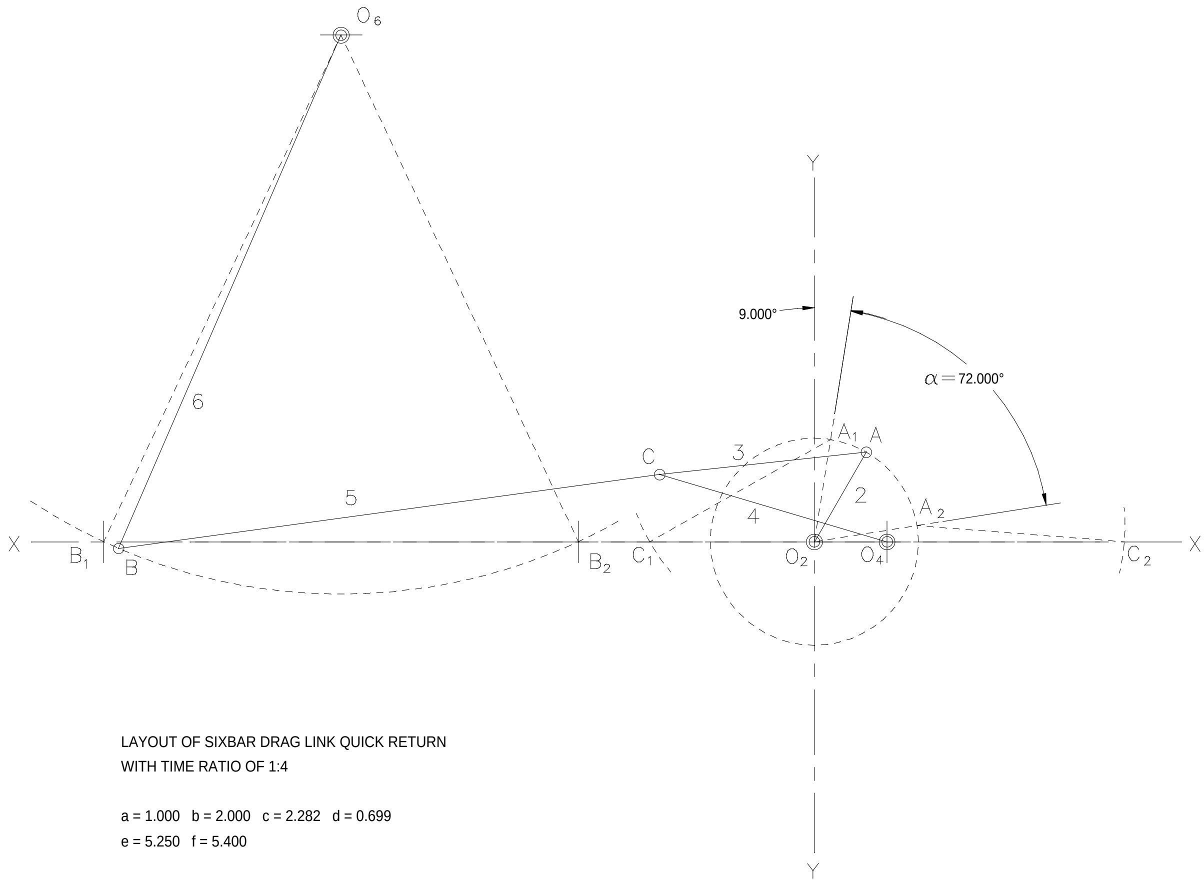
$$\alpha := 360 \cdot deg - \beta \qquad \alpha = 72\, deg$$

2. Draw a line of centers *XX* at any convenient location.
3. Choose a crank pivot location O_2 on line *XX* and draw an axis *YY* perpendicular to *XX* through O_2.
4. Draw a circle of convenient radius O_2A about center O_2. In the solution below, the length of O_2A is $a := 1.000 \cdot in$.
5. Lay out angle α with vertex at O_2, symmetrical about quadrant one.
6. Label points A_1 and A_2 at the intersections of the lines subtending angle α and the circle of radius O_2A.
7. Set the compass to a convenient radius *AC* long enough to cut *XX* in two places on either side of O_2 when swung from both A_1 and A_2. Label the intersections C_1 and C_2. In the solution below, the length of *AC* is $b := 2.000 \cdot in$.
8. The line O_2A is the driver crank, link 2, and the line *AC* is the coupler, link 3.
9. The distance C_1C_2 is twice the driven (dragged) crank length. Bisect it to locate the fixed pivot O_4.
10. The line O_2O_4 now defines the ground link. Line O_4C is the driven crank, link 4. In the solution below, O_4C measures $c := 2.282 \cdot in$ and O_2O_4 measures $d := 0.699 \cdot in$.
11. Calculate the Grashoff condition. If non-Grashoff, repeat steps 7 through 11 with a shorter radius in step 7.

$$Condition(a, b, c, d) := \left| \begin{array}{l} S \leftarrow min(a, b, c, d) \\ L \leftarrow max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"} \quad if\ SL < PQ \\ return\ \text{"Special Grashof"} \quad if\ SL = PQ \\ return\ \text{"non-Grashof"} \quad otherwise \end{array} \right.$$

$$Condition(a, b, c, d) = \text{"Grashof"}$$

12. Invert the method of Example 3-1 to create the output dyad using *XX* as the chord and O_4C_1 as the driving crank. The points B_1 and B_2 will lie on line *XX* and be spaced apart a distance that is twice the length of O_4C (link 4). The pivot point O_6 will lie on the perpendicular bisector of B_1B_2 at a distance from *XX* which subtends the specified output rocker angle, which is 50 degrees in this problem. In the solution below, the length *BC* was chosen to be $e := 5.250 \cdot in$.



13. For the design choices made (lengths of links 2, 3 and 5), the length of the output rocker (link 6) was measured as $f := 5.400 \cdot in$.

PROBLEM 3-71

Statement: Design a crank-shaper quick-return mechanism for a time ratio of 1:2.5 (Figure 3-14, p. 112).

Given: Time ratio $T_R := \dfrac{1}{2.5}$

Solution: See Figure 3-14 and Mathcad file P0371.

Design choices:

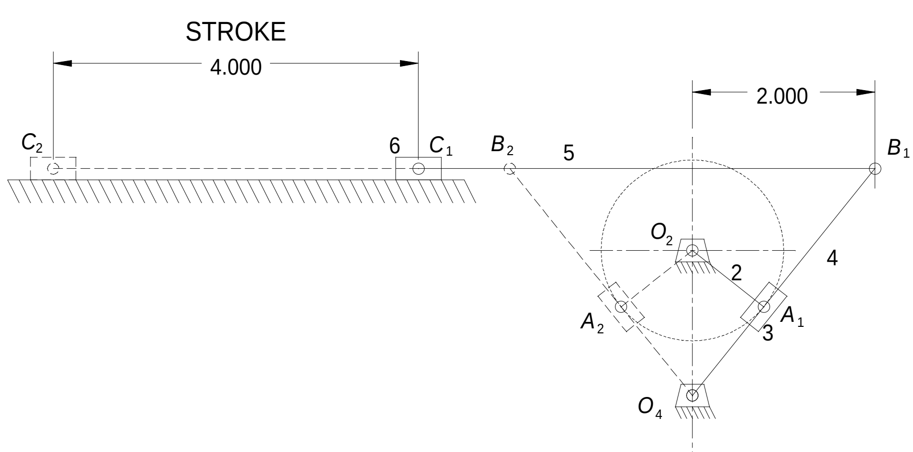
Length of link 2 (crank) $L_2 := 1.000$ Length of stroke $S := 4.000$

Length of link 5 (coupler) $L_5 := 5.000$

1. Calculate α from equations 3.1.

$$T_R := \frac{\alpha}{\beta} \qquad \alpha + \beta := 360 \cdot deg \qquad \alpha := \frac{360 \cdot deg}{1 + \dfrac{1}{T_R}} \qquad \alpha = 102.86\,deg$$

2. Draw a vertical line and mark the center of rotation of the crank, O_2, on it.
3. Layout two construction lines from O_2, each making an angle $\alpha/2$ to the vertical line through O_2.
4. Using the chosen crank length (see Design Choices), draw a circle with center at O_2 and radius equal to the crank length. Label the intersections of the circle and the two lines drawn in step 3 as A_1 and A_2.
5. Draw lines through points A1 and A2 that are also tangent to the crank circle (step 2). These two lines will simultaneously intersect the vertical line drawn in step 2. Label the point of intersection as the fixed pivot center O_4.
6. Draw a vertical construction line, parallel and to the right of O_2O_4, a distance S/2 (one-half of the output stroke length) from the line O_2O_4.
7. Extend line O_4A_1 until it intersects the construction line drawn in step 6. Label the intersection B_1.
8. Draw a horizontal construction line from point B_1, either to the left or right. Using point B_1 as center, draw an arc of radius equal to the length of link 5 (see Design Choices) to intersect the horizontal construction line. Label the intersection as C_1.
9. Draw the slider blocks at points A_1 and C_1 and finish by drawing the mechanism in its other extreme position.



PROBLEM 3-72

Statement: Design a sixbar, single-dwell linkage for a dwell of 70 deg of crank motion, with an output rocker motion of 30 deg using a symmetrical fourbar linkage with the following parameter values: ground link ratio = 2.0, common link ratio = 2.0, and coupler angle γ = 40 deg. (See Example 3-13.)

Given: Crank dwell period: 70 deg.
Output rocker motion: 30 deg.
Ground link ratio, $L_1/L_2 = 2.0$: $GLR := 2.0$
Common link ratio, $L_3/L_2 = L_4/L_2 = BP/L_2 = 2.0$: $CLR := 2.0$
Coupler angle, $\gamma := 40 \cdot deg$

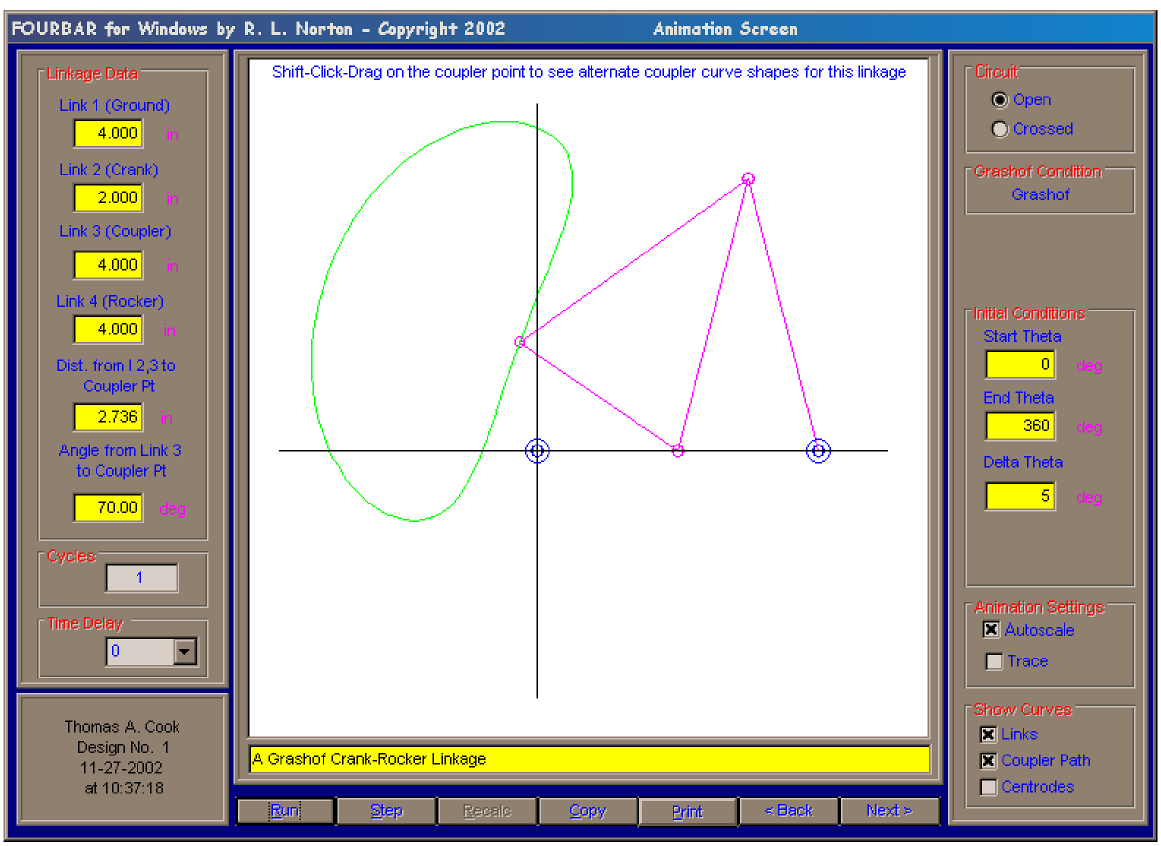
Design choice: Crank length, $L_2 := 2.000$

Solution: See Figures 3-20 and 3-21 and Mathcad file P0372.

1. For the given design choice, determine the remaining link lengths and coupler point specification.

Coupler link (3) length	$L_3 := CLR \cdot L_2$	$L_3 = 4.000$
Rocker link (4) length	$L_4 := CLR \cdot L_2$	$L_4 = 4.000$
Ground link (1) length	$L_1 := GLR \cdot L_2$	$L_1 = 4.000$
Angle *PAB*	$\delta := \dfrac{180 \cdot deg - \gamma}{2}$	$\delta = 70.000\, deg$
Length *AP* on coupler	$AP := 2 \cdot L_3 \cdot cos(\delta)$	$AP = 2.736$

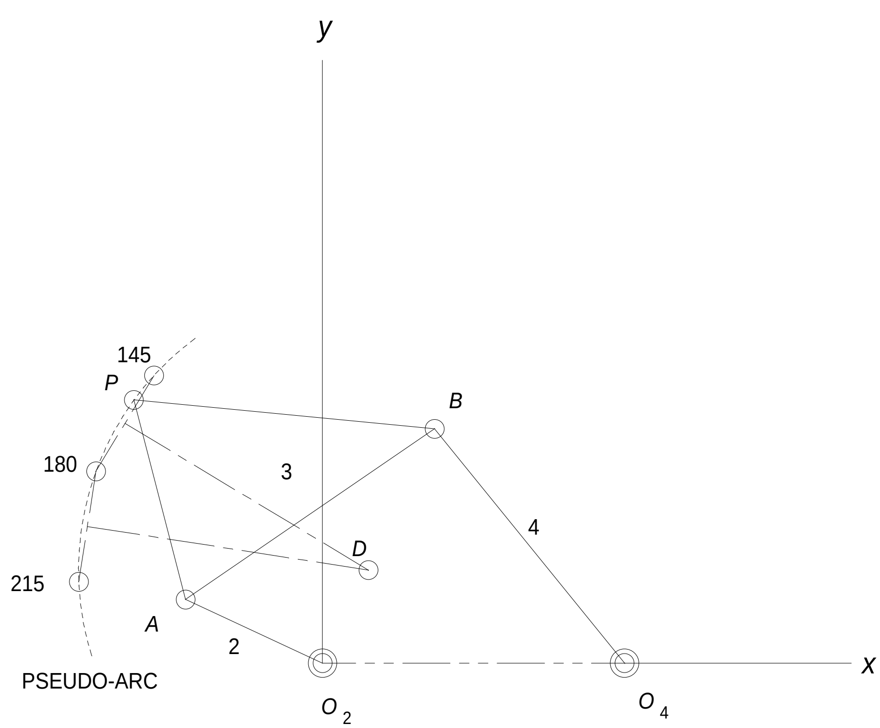
2. Enter the above data into program FOURBAR, plot the coupler curve, and determine the coordinates of the coupler curve in the selected range of crank motion, which in this case will be from 145 to 215 deg.



FOURBAR for Windows File P03-72

Angle Step Deg	Coupler Pt X in	Coupler Pt Y in	Coupler Pt Mag in	Coupler Pt Ang in
145	-2.231	3.818	4.422	120.297
150	-2.368	3.661	4.360	122.895
155	-2.497	3.494	4.295	125.549
160	-2.617	3.319	4.226	128.259
165	-2.728	3.135	4.156	131.025
170	-2.829	2.945	4.083	133.846
175	-2.919	2.749	4.009	136.723
180	-2.999	2.547	3.935	139.655
185	-3.067	2.342	3.859	142.639
190	-3.124	2.133	3.783	145.674
195	-3.169	1.923	3.707	148.757
200	-3.202	1.711	3.631	151.886
205	-3.223	1.499	3.555	155.055
210	-3.232	1.289	3.479	158.261
215	-3.227	1.080	3.403	161.498

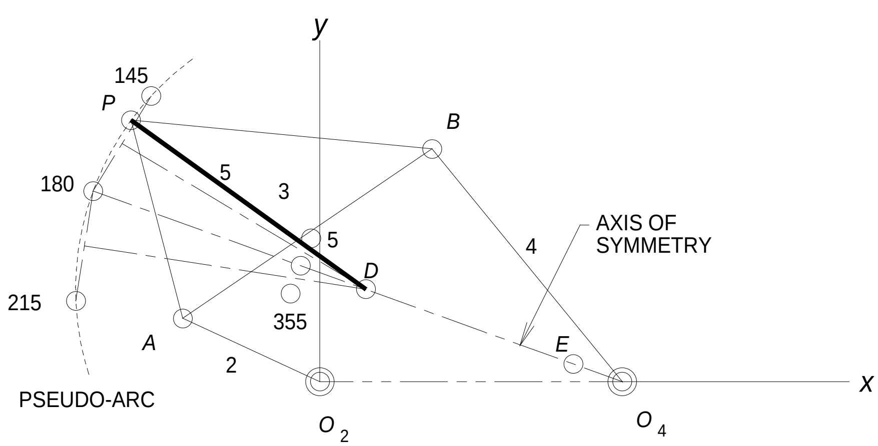
3. Layout this linkage to scale, including the coupler curve whose coordinates are in the table above. Use the points at crank angles of 145, 180, and 215 deg to define the pseudo-arc. Find the center of the pseudo-arc erecting perpendicular bisectors to the chords defined by the selected coupler curve points. The center will lie at the intersection of the perpendicular bisectors, label this point *D*. The radius of this circle is the length of link 5.



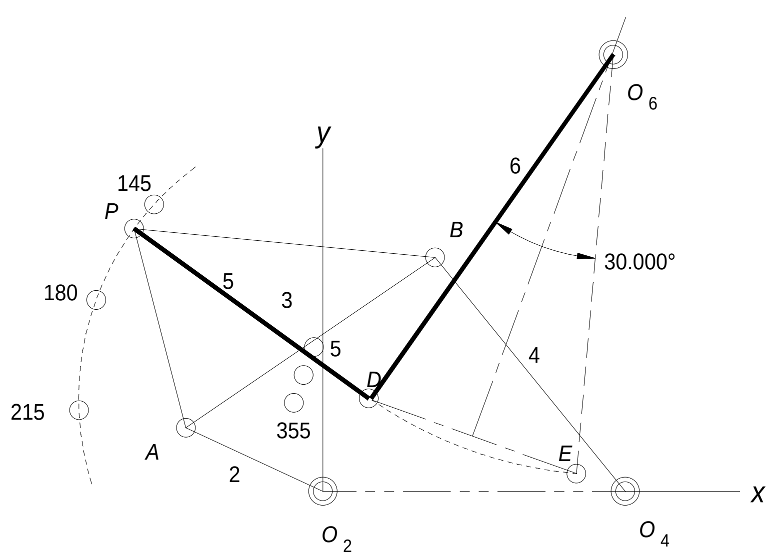
4. The position of the end of link 5 at point *D* will remain nearly stationary while the crank moves from 145 to 215 deg. As the crank motion causes the coupler point to move around the coupler curve there will be another extreme position of the end of link 5 that was originally at *D*. Since a symmetrical linkage was chosen, the other extreme position will be located along a line through the axis of symmetry (see Figure 3-20) a distance equal to the length of link 5 measured from the point where the axis of symmetry intersects the coupler curve near the 0 deg coupler point. Establish this point and label it *E*.

FOURBAR for Windows File P03-72

Angle Step Deg	Coupler Pt X in	Coupler Pt Y in	Coupler Pt Mag n	Coupler Pt Ang in
340.000	-0.718	0.175	0.739	166.325
345.000	-0.615	0.481	0.781	142.001
350.000	-0.506	0.818	0.962	121.717
355.000	-0.386	1.178	1.240	108.135
0.000	-0.255	1.549	1.570	99.365
5.000	-0.117	1.917	1.920	93.499
10.000	0.022	2.269	2.269	89.434
15.000	0.155	2.598	2.603	86.581



5. The line segment *DE* represents the maximum displacement that a link of the length equal to link 5, attached at *P*, will reach along the axis of symmetry. Construct a perpendicular bisector of the line segment *DE* and extend it to the right (or left, which ever is convenient). Locate fixed pivot O_6 on the bisector of *DE* such that the lines O_6D and O_6E subtend the desired output angle, in this case 30 deg. Draw link 6 from *D* through O_6 and extend it to any convenient length. This is the output link that will dwell during the specified motion of the crank.



SUMMARY OF LINKAGE SPECIFICATIONS

Original fourbar:

Ground link $L_1 = 4.000$

Crank $L_2 = 2.000$

Coupler $L_3 = 4.000$

Rocker $L_4 = 4.000$

Coupler point $AP = 2.736$

$\delta = 70.000\ deg$

Added dyad:

Coupler $L_5 := 3.840$

Output $L_6 := 5.595$

Pivot O_6 $x := 3.841$

$y := 5.809$

PROBLEM 3-73

Statement: Design a sixbar, single-dwell linkage for a dwell of 100 deg of crank motion, with an output rocker motion of 50 deg using a symmetrical fourbar linkage with the following parameter values: ground link ratio = 2.0, common link ratio = 2.5, and coupler angle γ = 60 deg. (See Example 3-13.)

Given: Crank dwell period: 100 deg.
Output rocker motion: 50 deg.
Ground link ratio, $L_1/L_2 = 2.0$: $GLR := 2.0$

Common link ratio, $L_3/L_2 = L_4/L_2 = BP/L_2 = 2.0$: $CLR := 2.5$

Coupler angle, $\gamma := 60 \cdot deg$

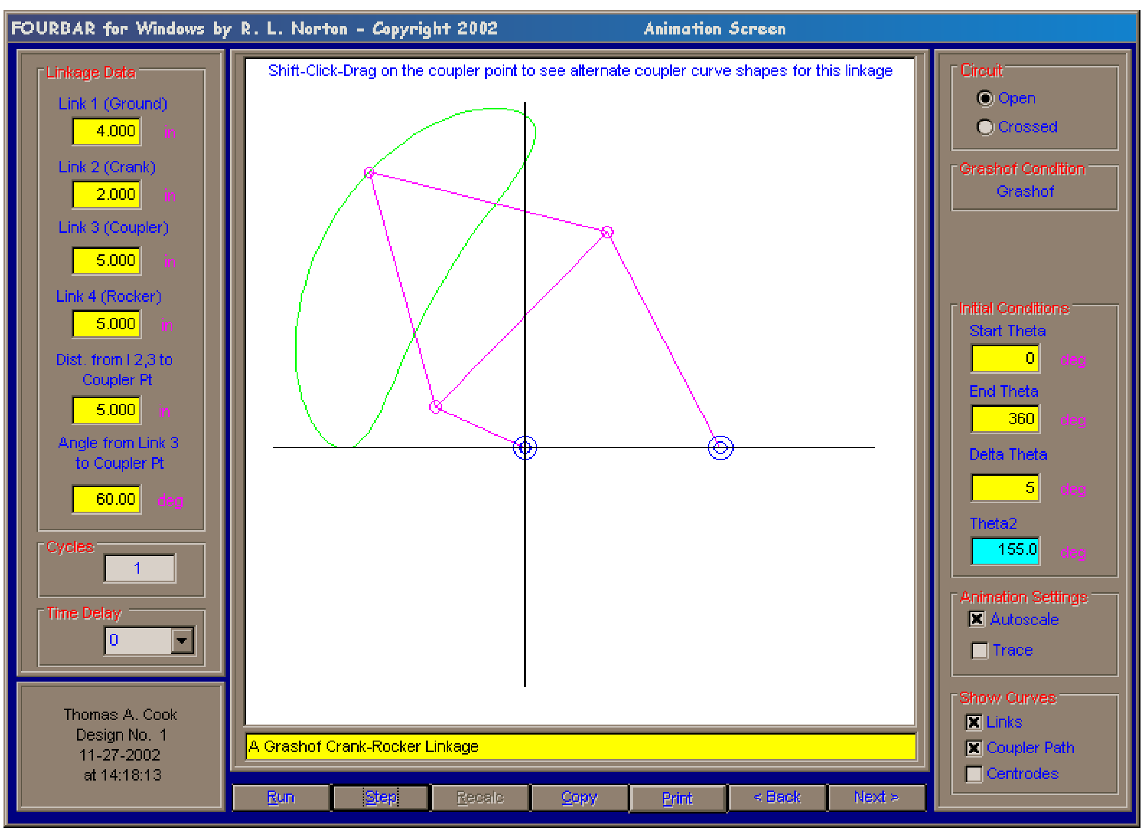
Design choice: Crank length, $L_2 := 2.000$

Solution: See Figures 3-20 and 3-21 and Mathcad file P0373.

1. For the given design choice, determine the remaining link lengths and coupler point specification.

Coupler link (3) length	$L_3 := CLR \cdot L_2$	$L_3 = 5.000$
Rocker link (4) length	$L_4 := CLR \cdot L_2$	$L_4 = 5.000$
Ground link (1) length	$L_1 := GLR \cdot L_2$	$L_1 = 4.000$
Angle *PAB*	$\delta := \dfrac{180 \cdot deg - \gamma}{2}$	$\delta = 60.000\, deg$
Length *AP* on coupler	$AP := 2 \cdot L_3 \cdot cos(\delta)$	$AP = 5.000$

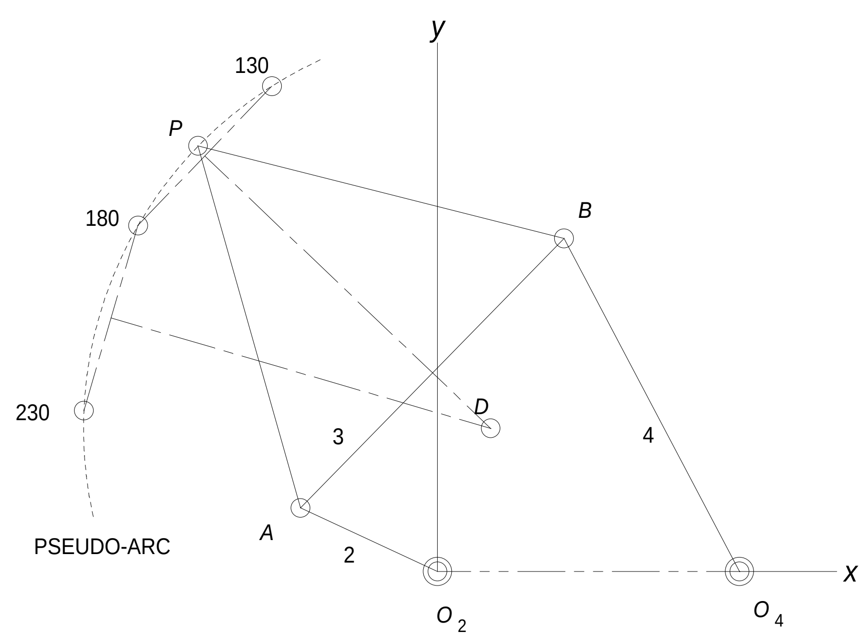
2. Enter the above data into program FOURBAR, plot the coupler curve, and determine the coordinates of the coupler curve in the selected range of crank motion, which in this case will be from 130 to 230 deg.



FOURBAR for Windows File P03-73

Angle Step Deg	Coupler Pt X in	Coupler Pt Y in	Coupler Pt Mag in	Coupler Pt Ang in
130	-2.192	6.449	6.812	108.774
140	-2.598	6.171	6.695	112.833
150	-2.986	5.840	6.559	117.078
160	-3.347	5.464	6.408	121.493
170	-3.675	5.047	6.244	126.060
180	-3.964	4.598	6.071	130.765
190	-4.209	4.123	5.892	135.588
200	-4.405	3.631	5.709	140.504
210	-4.551	3.130	5.523	145.482
220	-4.643	2.629	5.336	150.482
230	-4.681	2.138	5.146	155.454

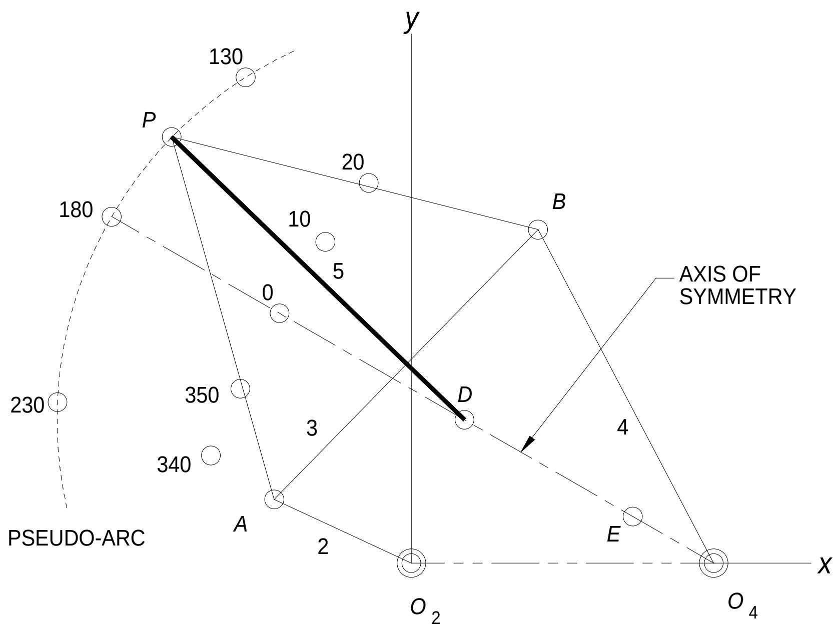
3. Layout this linkage to scale, including the coupler curve whose coordinates are in the table above. Use the points at crank angles of 130, 180, and 230 deg to define the pseudo-arc. Find the center of the pseudo-arc erecting perpendicular bisectors to the chords defined by the selected coupler curve points. The center will lie at the intersection of the perpendicular bisectors, label this point *D*. The radius of this circle is the length of link 5.



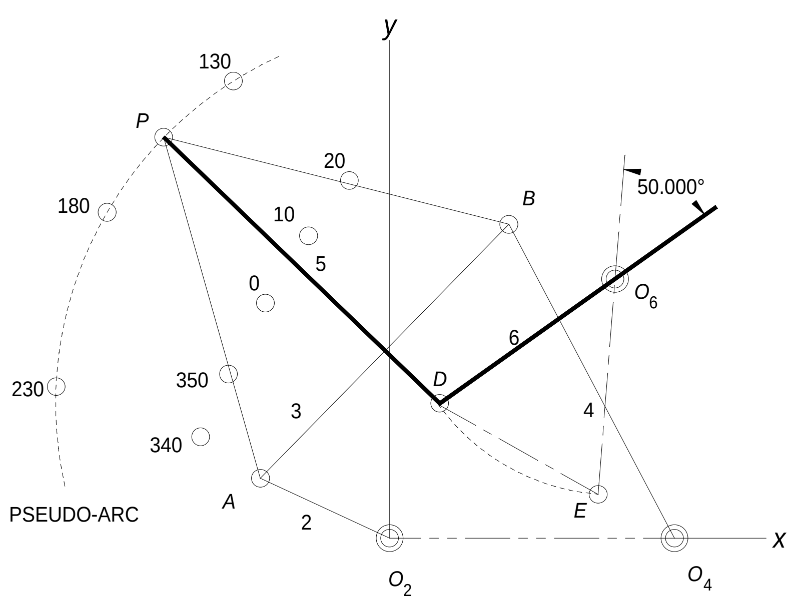
4. The position of the end of link 5 at point *D* will remain nearly stationary while the crank moves from 130 to 230 deg. As the crank motion causes the coupler point to move around the coupler curve there will be another extreme position of the end of link 5 that was originally at *D*. Since a symmetrical linkage was chosen, the other extreme position will be located along a line through the axis of symmetry (see Figure 3-20) a distance equal to the length of link 5 measured from the point where the axis of symmetry intersects the coupler curve near the 0 deg coupler point. Establish this point and label it *E*.

FOURBAR for Windows File P03-73

Angle Step Deg	Coupler Pt X in	Coupler Pt Y in	Coupler Pt Mag in	Coupler Pt Ang in
340	-2.652	1.429	3.013	151.688
350	-2.262	2.316	3.237	134.332
0	-1.743	3.316	3.746	117.727
10	-1.137	4.265	4.414	104.920
20	-0.564	5.047	5.078	96.371



5. The line segment *DE* represents the maximum displacement that a link of the length equal to link 5, attached at *P*, will reach along the axis of symmetry. Construct a perpendicular bisector of the line segment *DE* and extend it to the right (or left, which ever is convenient). Locate fixed pivot O_6 on the bisector of *DE* such that the lines O_6D and O_6E subtend the desired output angle, in this case 30 deg. Draw link 6 from *D* through O_6 and extend it to any convenient length. This is the output link that will dwell during the specified motion of the crank.



SUMMARY OF LINKAGE SPECIFICATIONS

Original fourbar:

Ground link	$L_1 = 4.000$
Crank	$L_2 = 2.000$
Coupler	$L_3 = 5.000$
Rocker	$L_4 = 5.000$
Coupler point	$AP = 5.000$
	$\delta = 60.000\, deg$

Added dyad:

Coupler	$L_5 := 5.395$
Output	$L_6 := 2.998$
Pivot O_6	$x := 3.166$
	$y := 3.656$

PROBLEM 3-74

Statement: Design a sixbar, single-dwell linkage for a dwell of 80 deg of crank motion, with an output rocker motion of 45 deg using a symmetrical fourbar linkage with the following parameter values: ground link ratio = 2.0, common link ratio = 1.75, and coupler angle $\gamma = 70$ deg. (See Example 3-13.)

Given: Crank dwell period: 80 deg.
Output rocker motion: 45 deg.
Ground link ratio, $L_1/L_2 = 2.0$: $GLR := 2.0$
Common link ratio, $L_3/L_2 = L_4/L_2 = BP/L_2 = 2.0$: $CLR := 1.75$
Coupler angle, $\gamma := 70 \cdot deg$

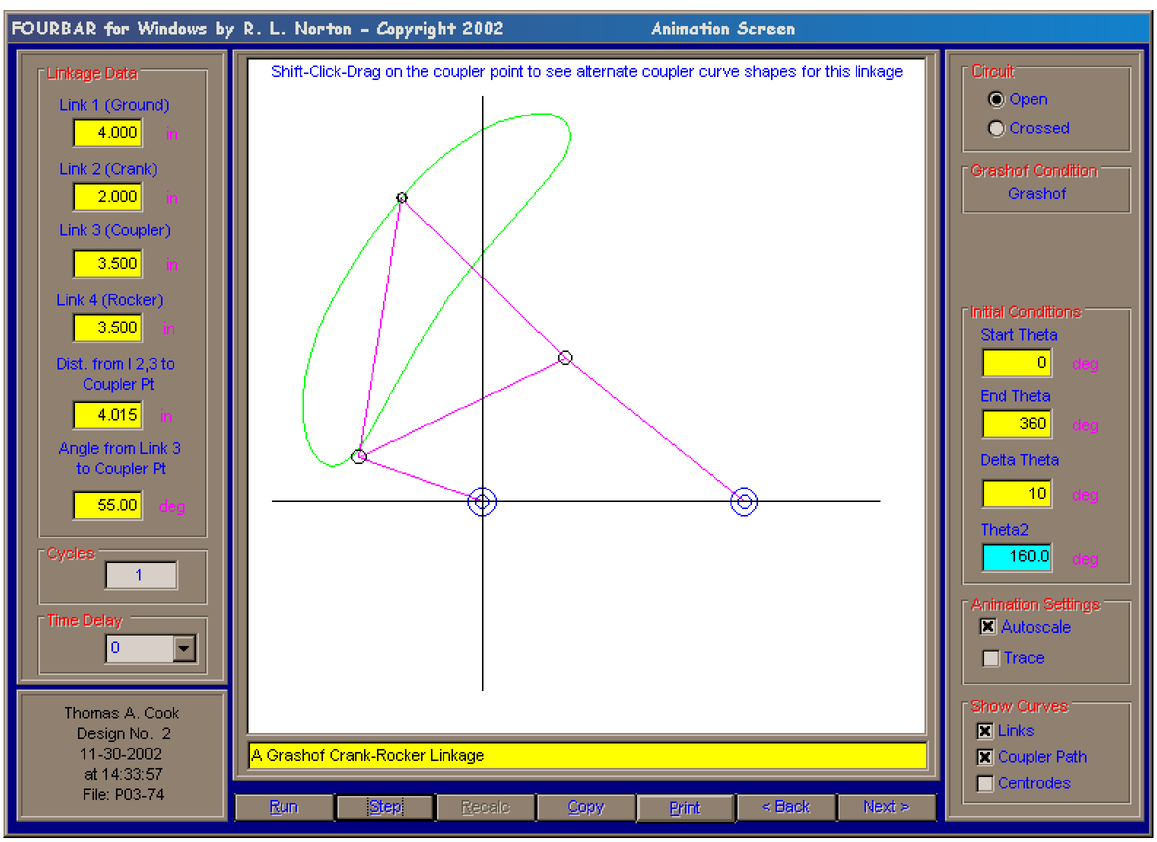
Design choice: Crank length, $L_2 := 2.000$

Solution: See Figures 3-20 and 3-21 and Mathcad file P0374.

1. For the given design choice, determine the remaining link lengths and coupler point specification.

Coupler link (3) length	$L_3 := CLR \cdot L_2$	$L_3 = 3.500$
Rocker link (4) length	$L_4 := CLR \cdot L_2$	$L_4 = 3.500$
Ground link (1) length	$L_1 := GLR \cdot L_2$	$L_1 = 4.000$
Angle *PAB*	$\delta := \dfrac{180 \cdot deg - \gamma}{2}$	$\delta = 55.000\, deg$
Length *AP* on coupler	$AP := 2 \cdot L_3 \cdot cos(\delta)$	$AP = 4.015$

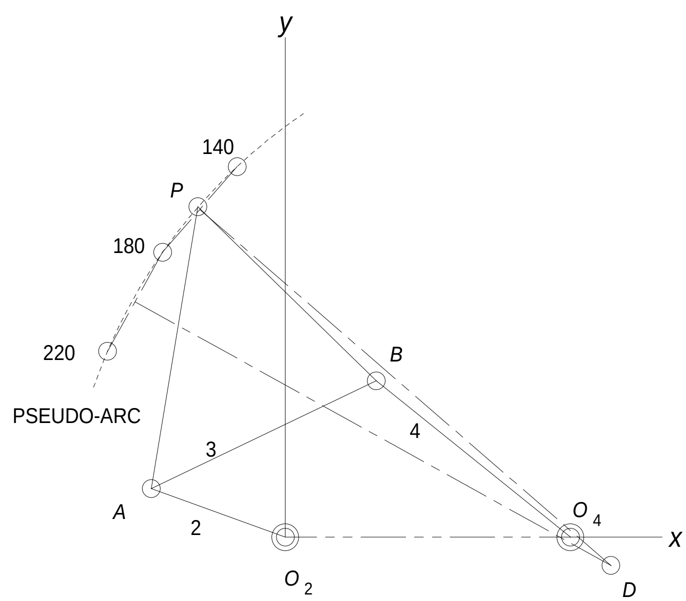
2. Enter the above data into program FOURBAR, plot the coupler curve, and determine the coordinates of the coupler curve in the selected range of crank motion, which in this case will be from 140 to 220 deg.



FOURBAR for Windows File P03-74

Angle Step Deg	Coupler Pt X in	Coupler Pt Y in	Coupler Pt Mag in	Coupler Pt Ang in
140	-0.676	5.208	5.252	97.395
150	-0.958	4.940	5.032	100.971
160	-1.226	4.645	4.804	104.781
170	-1.480	4.332	4.578	108.860
180	-1.720	4.005	4.359	113.242
190	-1.945	3.668	4.152	117.942
200	-2.153	3.322	3.958	122.946
210	-2.337	2.969	3.779	128.210
220	-2.493	2.613	3.612	133.663

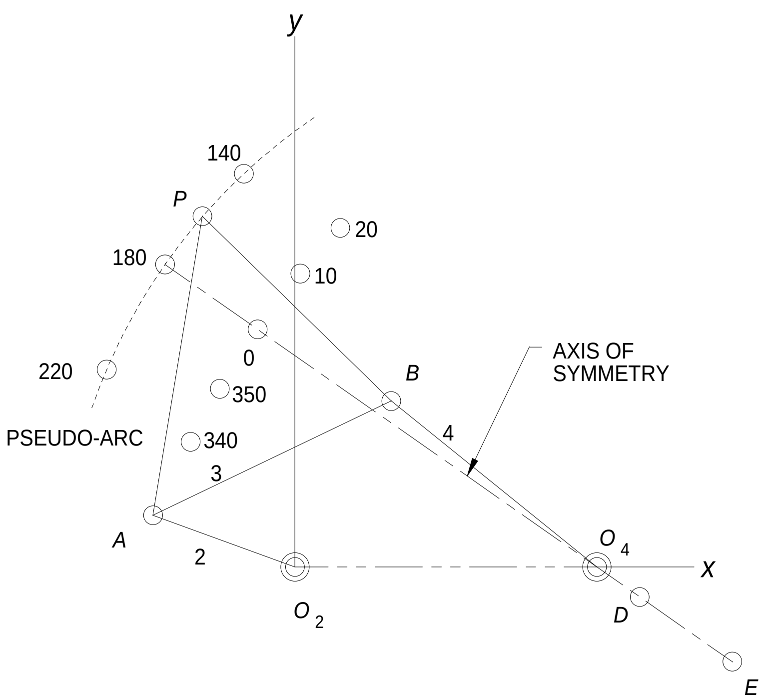
3. Layout this linkage to scale, including the coupler curve whose coordinates are in the table above. Use the points at crank angles of 140, 180, and 220 deg to define the pseudo-arc. Find the center of the pseudo-arc erecting perpendicular bisectors to the chords defined by the selected coupler curve points. The center will lie at the intersection of the perpendicular bisectors, label this point *D*. The radius of this circle is the length of link 5.



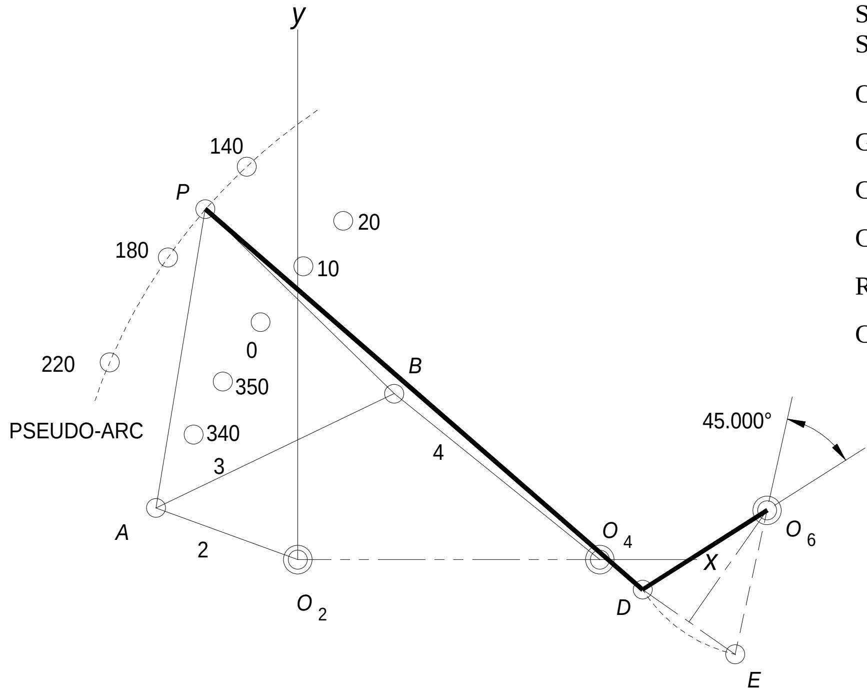
4. The position of the end of link 5 at point *D* will remain nearly stationary while the crank moves from 140 to 220 deg. As the crank motion causes the coupler point to move around the coupler curve there will be another extreme position of the end of link 5 that was originally at *D*. Since a symmetrical linkage was chosen, the other extreme position will be located along a line through the axis of symmetry (see Figure 3-20) a distance equal to the length of link 5 measured from the point where the axis of symmetry intersects the coupler curve near the 0 deg coupler point. Establish this point and label it *E*.

FOURBAR for Windows File P03-74

Angle Step Deg	Coupler Pt X in	Coupler Pt Y in	Coupler Pt Mag in	Coupler Pt Ang in
340	-1.382	1.658	2.158	129.810
350	-0.995	2.360	2.562	112.856
0	-0.494	3.147	3.185	98.919
10	0.074	3.886	3.887	88.916
20	0.601	4.490	4.530	82.372



5. The line segment *DE* represents the maximum displacement that a link of the length equal to link 5, attached at *P*, will reach along the axis of symmetry. Construct a perpendicular bisector of the line segment *DE* and extend it to the right (or left, which ever is convenient). Locate fixed pivot O_6 on the bisector of *DE* such that the lines O_6D and O_6E subtend the desired output angle, in this case 30 deg. Draw link 6 from *D* through O_6 and extend it to any convenient length. This is the output link that will dwell during the specified motion of the crank.



SUMMARY OF LINKAGE SPECIFICATIONS

Original fourbar:

Ground link	$L_1 = 4.000$
Crank	$L_2 = 2.000$
Coupler	$L_3 = 3.500$
Rocker	$L_4 = 3.500$
Coupler point	$AP = 4.015$
	$\delta = 55.000\,deg$

Added dyad:

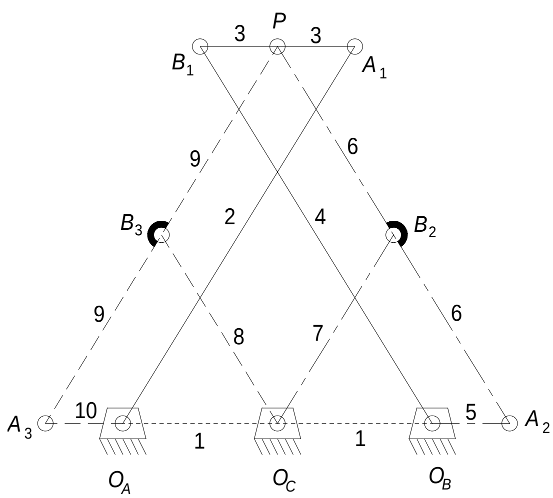
Coupler	$L_5 := 7.676$
Output	$L_6 := 1.979$
Pivot O_6	$x := 6.217$
	$y := 0.653$

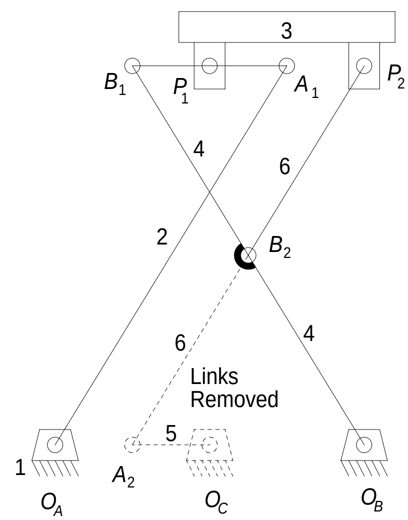
PROBLEM 3-75

Statement: Using the method of Example 3-11, show that the sixbar Chebychev straight-line linkage of Figure P2-5 is a combination of the fourbar Chebychev straight-line linkage of Figure 3-29d and its Hoeken's cognate of Figure 3-29e. See also Figure 3-26 for additional information useful to this solution. Graphically construct the Chebychev sixbar parallel motion linkage of Figure P2-5a from its two fourbar linkage constituents and build a physical or computer model of the result.

Solution: See Figures P2-5, 3-29d, 3-29e, and 3-26 and Mathcad file P0375.

1. Following Example 3-11and Figure 3-26 for the Chebyschev linkage of Figure 3-29d, the fixed pivot O_C is found by laying out the triangle $O_AO_BO_C$, which is similar to A_1B_1P. In this case, A_1B_1P is a striaght line with P halfway between A_1 and B_1 and therefore $O_AO_BO_C$ is also a straightline with O_C halfway between O_A and O_B. As shown below and in Figure 3-26, cognate #1 is made up of links numbered 1, 2, 3, and 4. Cognate #2 is links numbered 1, 5, 6, and 7. Cognate #3 is links numbered 1, 8, 9, and 10.





2. Discard cognate #3 and shift link 5 from the fixed pivot O_B to O_C and shift link 7 from O_C to O_B. Note that due to the symmetry of the figure above, $L_5 = 0.5\ L_3$, $L_6 = L_2$, $L_7 = 0.5\ L_2$ and $O_CO_B = 0.5\ O_AO_B$. Thus, cognate #2 is, in fact, the Hoeken straight-line linkage. The original Chebyschev linkage with the Hoeken linkage superimposed is shown above right with the link 5 rotated to 180 deg. Links 2 and 6 will now have the same velocity as will 7 and 4. Thus, link 5 can be removed and link 6 can be reduced to a binary link supported and constrained by link 4. The resulting sixbar is the linkage shown in Figure P2-5.

PROBLEM 3-76

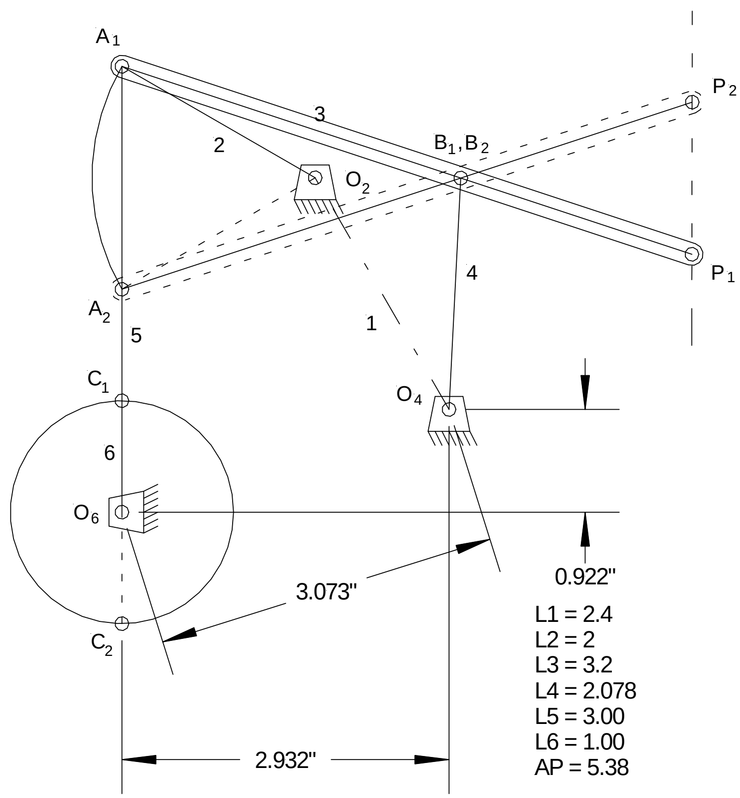
Statement: Design a driver dyad to drive link 2 of the Evans straigh-line linkage in Figure 3-29f from 150 deg to 210 deg. Make a model of the resulting sixbar linkage and trace the couple curve.

Given: Output angle $\theta_2 := 60 \cdot deg$

Solution: See Figjre 3-29f, Example 3-1, and Mathcad file P0376.

Design choices: Link lengths: Link 2 $L_2 := 2.000$ Link 5 $L_5 := 3.000$

1. Draw the input link O_2A in both extreme positions, A_1 and A_2, at the specified angles such that the desired angle of motion θ_2 is subtended.
2. Draw the chord A_1A_2 and extend it in any convenient direction. In this solution it was extended downward.
3. Layout the distance A_1C_1 along extended line A_1A_2 equal to the length of link 5. Mark the point C_1.
4. Bisect the line segment A_1A_2 and layout the length of that radius from point C_1 along extended line A_1A_2. Mark the resulting point O_6 and draw a circle of radius O_6C_1 with center at O_6.
5. Label the other intersection of the circle and extended line A_1A_2, C_2.
6. Measure the length of the crank (link 6) as O_6C_1 or O_6C_2. From the graphical solution, $L_6 := 1.000$
7. Measure the length of the ground link (link 1) as O_2O_6. From the graphical solution, $L_1 := 3.073$



8. Find the Grashof condition.

$$
Condition(a,b,c,d) := \begin{array}{|l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"}\quad if\ SL < PQ \\ return\ \text{"Special Grashof"}\quad if\ SL = PQ \\ return\ \text{"non-Grashof"}\quad otherwise \end{array}
$$

$$
Condition(L_1, L_2, L_5, L_6) = \text{"Grashof"}
$$

PROBLEM 3-77

Statement: Design a driver dyad to drive link 2 of the Evans straigh-line linkage in Figure 3-29g from -40 deg to 40 deg. Make a model of the resulting sixbar linkage and trace the couple curve.

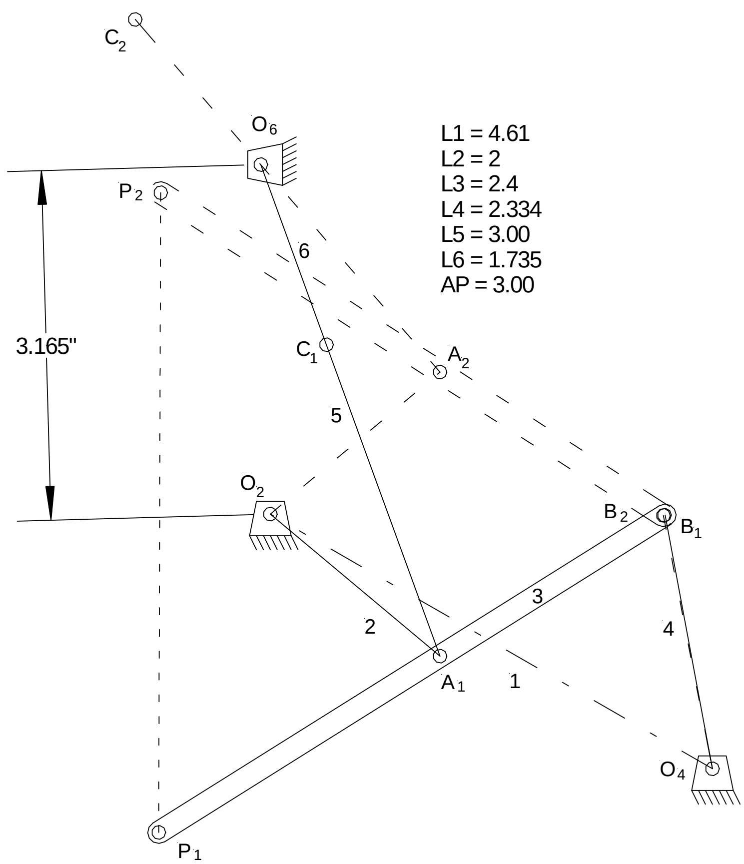
Given: Output angle $\theta_2 := 80 \cdot deg$

Solution: See Figjre 3-29G, Example 3-1, and Mathcad file P0377.

Design choices: Link lengths: Link 2 $L_2 := 2.000$ Link 5 $L_5 := 3.000$

1. Draw the input link O_2A in both extreme positions, A_1 and A_2, at the specified angles such that the desired angle of motion θ_2 is subtended.
2. Draw the line A_1C_1 and extend it in any convenient direction. In this solution it was extended at a 30-deg angle from A_1O_2 (see note below).
3. Layout the distance A_1C_1 along extended line A_1C_1 equal to the length of link 5. Mark the point C_1.
4. Bisect the line segment A_1A_2 and layout the length of that radius from point C_1 along extended line A_1C_1. Mark the resulting point O_6 and draw a circle of radius O_6C_1 with center at O_6.
5. Extend a line from A_2 through O_6. Label the other intersection of the circle and extended line A_2O_6, C_2.
6. Measure the length of the crank (link 6) as O_6C_1 or O_6C_2. From the graphical solution, $L_6 := 1.735$
7. Measure the length of the ground link (link 1) as O_2O_6. From the graphical solution, $L_1 := 3.165$

Note: If the angle between link 2 and link 5 is zero the resulting driving fourbar will be a special Grashof. For angles greater than zero but less than 33.68 degrees it is a Grashof crank-rocker. For angles greater than 33.68 it is a non-Grashof double rocker.



8. Find the Grashof condition.

$$Condition(a,b,c,d) := \begin{array}{|l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"}\quad if\ SL < PQ \\ return\ \text{"Special Grashof"}\quad if\ SL = PQ \\ return\ \text{"non-Grashof"}\quad otherwise \end{array}$$

$$Condition(L_1, L_2, L_5, L_6) = \text{"Grashof"}$$

PROBLEM 3-78

Statement: Figure 6 on page *ix* of the Hrones and Nelson atlas of fourbar coupler curves (on the book DVD) shows a 50-point coupler that was used to generate the curves in the atlas. Using the definition of the vector **R** given in Figure 3-17b of the text, determine the 10 possible pairs of values of ϕ and R for the first row of points above the horizontal axis if the gridpoint spacing is one half the length of the unit crank.

Given: Grid module $g := 0.5$

Solution: See Figure 6 H&N Atlas, Figure 3-17b, and Mathcad file P0378.

1. The moving pivot point is located on the 3rd grid line from the bottom and the third grid line from the left when the crank angle is π radians. Let the number of horizontal grid spaces from the left end of the coupler to the coupler point be $n := -2, -1 .. 7$ and the number of vertical grid spaces from the coupler to the coupler point be $m := -2, -1 .. 2$
2. For the first row of points above the horizontal axis shown in Figure 6, $n := -2, -1 .. 7$ and $m := 1$.
3. The angle, ϕ, between the coupler and the line from the coupler/crank pivot to the coupler point is

$$\phi(m,n) := if\left(n \neq 0, atan2(n,m), if\left(m = 0, 0, if\left(m > 0, \frac{\pi}{2}, \frac{-\pi}{2}\right)\right)\right)$$

4. The distance, R, from the pivot to the coupler point along the same line is

$$R(m,n) := g \cdot \sqrt{m^2 + n^2}$$

$n =$	$\frac{\phi(m,n)}{deg} =$	$R(m,n) =$
-2.000	153.435	1.118
-1.000	135.000	0.707
0.000	90.000	0.500
1.000	45.000	0.707
2.000	26.565	1.118
3.000	18.435	1.581
4.000	14.036	2.062
5.000	11.310	2.550
6.000	9.462	3.041
7.000	8.130	3.536

5. The coupler point distance, R, like the link lengths A, B, and C is a ratio of the given length to the the length of the driving crank.

PROBLEM 3-79

Statement: The set of coupler curves in the Hrones and Nelson atlas of fourbar coupler curves (on the book DVD, page 16 of the PDF file) has A = B = C = 1.5. Model this linkage with program FOURBAR using the coupler point fartherest to the left in the row shown on page 1 and plot the resulting coupler curve.

Given: $A := 1.5$ $B := 1.5$ $C := 1.5$

Solution: See Figure on page 1 H&N Atlas, Figure 3-17b, and Mathcad file P0379.

1. The moving pivot point is located on the 3rd grid line from the bottom and the third grid line from the left when the crank angle is π radians. Let the number of horizontal grid spaces from the left end of the coupler to the coupler point be $n := -2, -1 .. 7$ and the number of vertical grid spaces from the coupler to the coupler point be $m := -2, -1 .. 2$
2. For the second column of points to the left of the coupler pivot and the second row of points above the horizontal axis $n := -2$ and $m := 2$. The grid spacing is $g := 0.5$
3. The angle, ϕ, between the coupler and the line from the coupler/crank pivot to the coupler point is

$$\phi(m,n) := if\left(n \neq 0, atan2(n,m), if\left(m = 0, 0, if\left(m > 0, \frac{\pi}{2}, \frac{-\pi}{2}\right)\right)\right) \qquad \phi(m,n) = 135.000\ deg$$

4. The distance from the pivot to the coupler point, *R*, along the same line is

$$R(m,n) := g \cdot \sqrt{m^2 + n^2} \qquad R(m,n) = 1.414$$

5. Determine the values needed for input to FOURBAR.

Link 2 (Crank)	$a := 1$	
Link 3 (Coupler)	$b := A \cdot a$	$b = 1.500$
Link 4 (Rocker)	$c := B \cdot a$	$c = 1.500$
Link 1 (Ground)	$d := C \cdot a$	$d = 1.500$
Distance to coupler point		$R(m,n) = 1.414$
Angle from link 3 to coupler point		$\phi(m,n) = 135.000\ deg$

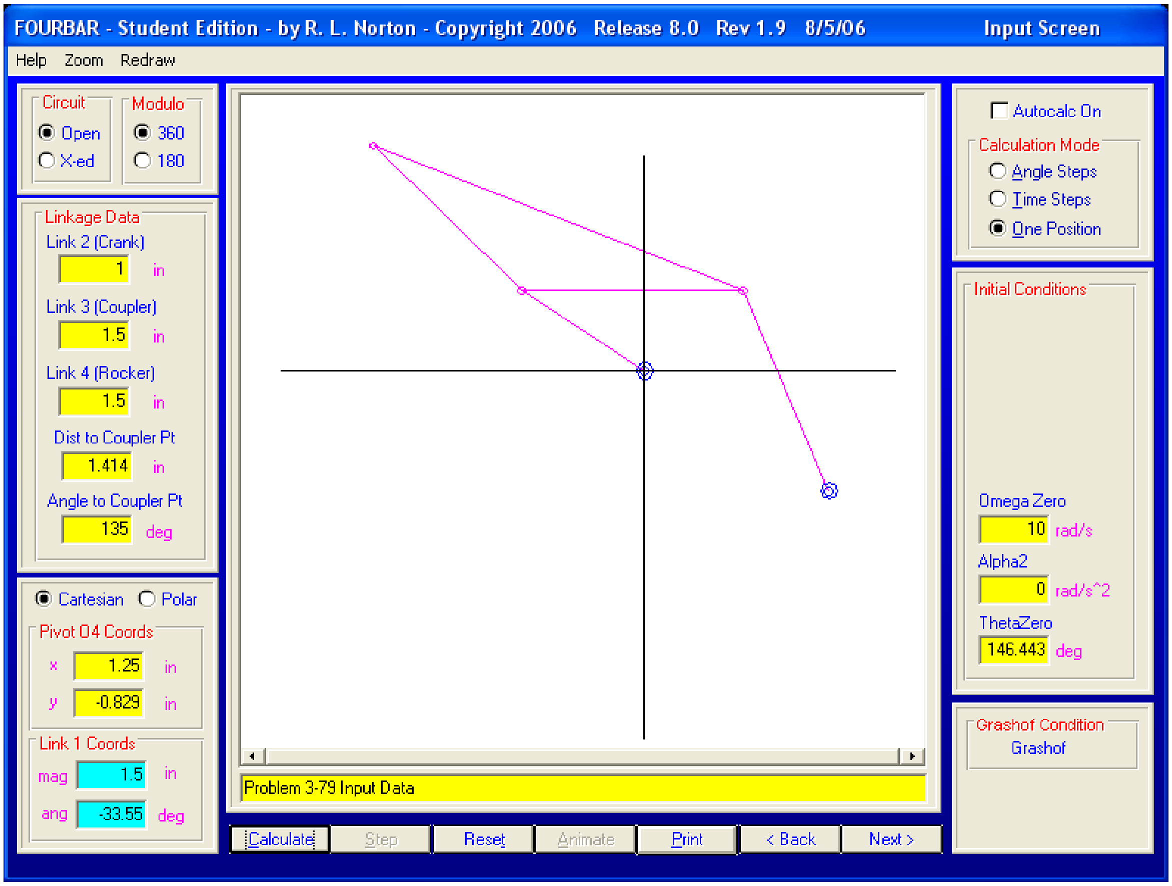
6. Calculate the coordinates of O_4. Let the angle between links 2 and 3 be α , then

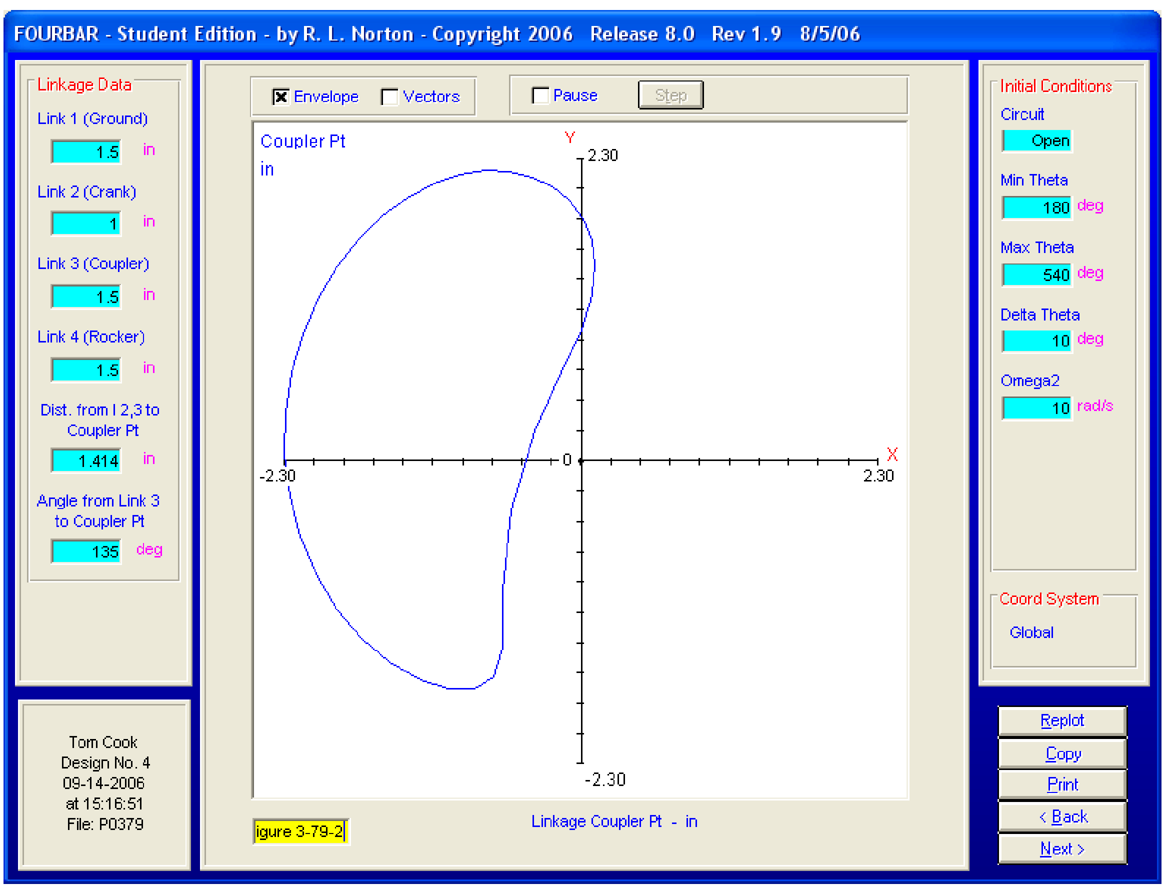
$$\alpha := acos\left[\frac{A^2 + (1 + C)^2 - B^2}{2 \cdot A \cdot (1 + C)}\right] \qquad \alpha = 33.557\ deg$$

$$x_{O4} := C \cdot cos(\alpha) \qquad x_{O4} = 1.250$$

$$y_{O4} := -C \cdot sin(\alpha) \qquad y_{O4} = -0.829$$

7. Enter this data into FOURBAR and then plot the coupler curve. (See next page)


FOURBAR - Student Edition - by R. L. Norton - Copyright 2006 Release 8.0 Rev 1.9 8/5/06 Input Screen
Help Zoom Redraw
Circuit
Open
X-ed
Modulo
360
180
Linkage Data
Link 2 (Crank)
1 in
Link 3 (Coupler)
1.5 in
Link 4 (Rocker)
1.5 in
Dist to Coupler Pt
1.414 in
Angle to Coupler Pt
135 deg
Cartesian Polar
Pivot O4 Coords
x 1.25 in
y -0.829 in
Link 1 Coords
mag 1.5 in
ang -33.55 deg
Problem 3-79 Input Data
Calculate Step Reset Animate Print < Back Next >
Autocalc On
Calculation Mode
Angle Steps
Time Steps
One Position
Initial Conditions
Omega Zero
10 rad/s
Alpha2
0 rad/s^2
ThetaZero
146.443 deg
Grashof Condition
Grashof


FOURBAR - Student Edition - by R. L. Norton - Copyright 2006 Release 8.0 Rev 1.9 8/5/06
Linkage Data
Link 1 (Ground)
1.5 in
Link 2 (Crank)
1 in
Link 3 (Coupler)
1.5 in
Link 4 (Rocker)
1.5 in
Dist. from I 2,3 to Coupler Pt
1.414 in
Angle from Link 3 to Coupler Pt
135 deg
Tom Cook
Design No. 4
09-14-2006
at 15:16:51
File: P0379
Envelope Vectors Pause Step
Coupler Pt
in
Y
2.30
-2.30
0
2.30
X
-2.30
igure 3-79-2
Linkage Coupler Pt - in
Initial Conditions
Circuit
Open
Min Theta
180 deg
Max Theta
540 deg
Delta Theta
10 deg
Omega2
10 rad/s
Coord System
Global
Replot
Copy
Print
< Back
Next >

PROBLEM 3-80

Statement: The set of coupler curves on page 17 in the Hrones and Nelson atlas of fourbar coupler curves (on the book DVD, page 32 of the PDF file) has A = 1.5, B = C = 3.0. Model this linkage with program FOURBAR using the coupler point fartherest to the right in the row shown and plot the resulting coupler curve.

Given: $A := 1.5$ $B := 3.0$ $C := 3.0$

Solution: See Figure on page 17 H&N Atlas, Figure 3-17b, and Mathcad file P0380.

1. The moving pivot point is located on the 3rd grid line from the bottom and the third grid line from the left when the crank angle is π radians. Let the number of horizontal grid spaces from the left end of the coupler to the coupler point be $n := -2, -1 .. 7$ and the number of vertical grid spaces from the coupler to the coupler point be $m := -2, -1 .. 2$
2. For the fifth column of points to the right of the coupler pivot and the first row of points above the horizontal axis $n := 5$ and $m := 1$. The grid spacing is $g := 0.5$
3. The angle, ϕ, between the coupler and the line from the coupler/crank pivot to the coupler point is

$$\phi(m,n) := if\left(n \neq 0, atan2(n,m), if\left(m = 0, 0, if\left(m > 0, \frac{\pi}{2}, \frac{-\pi}{2}\right)\right)\right) \qquad \phi(m,n) = 11.310\,deg$$

4. The distance from the pivot to the coupler point, R, along the same line is

$$R(m,n) := g\cdot\sqrt{m^2 + n^2} \qquad R(m,n) = 2.550$$

5. Determine the values needed for input to FOURBAR.

Link 2 (Crank)	$a := 1$	
Link 3 (Coupler)	$b := A\cdot a$	$b = 1.500$
Link 4 (Rocker)	$c := B\cdot a$	$c = 3.000$
Link 1 (Ground)	$d := C\cdot a$	$d = 3.000$
Distance to coupler point		$R(m,n) = 2.550$
Angle from link 3 to coupler point		$\phi(m,n) = 11.310\,deg$

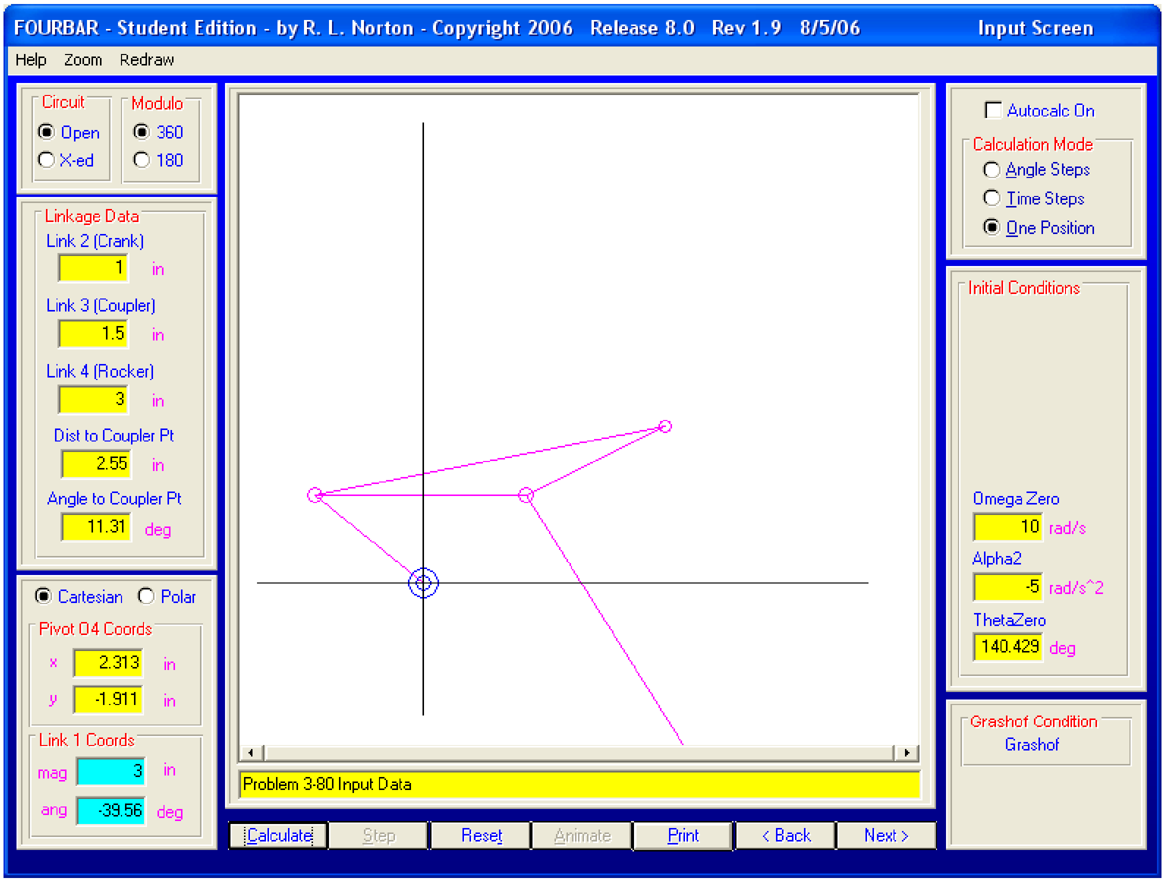
6. Calculate the coordinates of O_4. Let the angle between links 2 and 3 be α , then

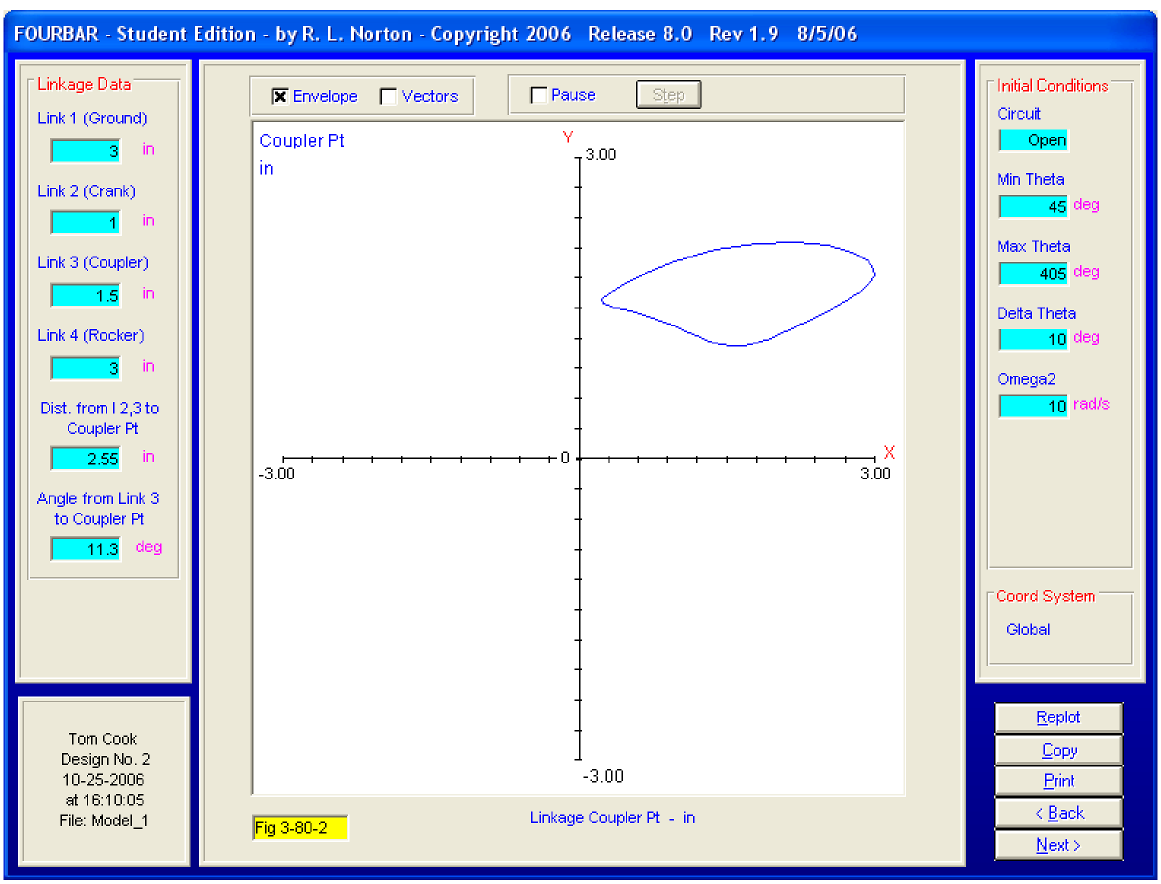
$$\alpha := acos\left[\frac{A^2 + (1 + C)^2 - B^2}{2\cdot A\cdot(1 + C)}\right] \qquad \alpha = 39.571\,deg$$

$$x_{O4} := C\cdot cos(\alpha) \qquad x_{O4} = 2.313$$

$$y_{O4} := -C\cdot sin(\alpha) \qquad y_{O4} = -1.911$$

7. Enter this data into FOURBAR and then plot the coupler curve. (See next page)


FOURBAR - Student Edition - by R. L. Norton - Copyright 2006 Release 8.0 Rev 1.9 8/5/06 Input Screen
Help Zoom Redraw
Circuit
Open
X-ed
Modulo
360
180
Linkage Data
Link 2 (Crank) 1 in
Link 3 (Coupler) 1.5 in
Link 4 (Rocker) 3 in
Dist to Coupler Pt 2.55 in
Angle to Coupler Pt 11.31 deg
Cartesian Polar
Pivot O4 Coords
x 2.313 in
y -1.911 in
Link 1 Coords
mag 3 in
ang -39.56 deg
Problem 3-80 Input Data
Calculate Step Reset Animate Print < Back Next >
Autocalc On
Calculation Mode
Angle Steps
Time Steps
One Position
Initial Conditions
Omega Zero 10 rad/s
Alpha2 -5 rad/s^2
ThetaZero 140.429 deg
Grashof Condition
Grashof


FOURBAR - Student Edition - by R. L. Norton - Copyright 2006 Release 8.0 Rev 1.9 8/5/06
Linkage Data
Link 1 (Ground) 3 in
Link 2 (Crank) 1 in
Link 3 (Coupler) 1.5 in
Link 4 (Rocker) 3 in
Dist. from I 2,3 to Coupler Pt 2.55 in
Angle from Link 3 to Coupler Pt 11.3 deg
Envelope Vectors
Pause Step
Coupler Pt
in
Y
3.00
X
-3.00
0
3.00
-3.00
Fig 3-80-2
Linkage Coupler Pt - in
Initial Conditions
Circuit
Open
Min Theta 45 deg
Max Theta 405 deg
Delta Theta 10 deg
Omega2 10 rad/s
Coord System
Global
Replot
Copy
Print
< Back
Next >
Tom Cook
Design No. 2
10-25-2006
at 16:10:05
File: Model_1

PROBLEM 3-81

Statement: The set of coupler curves on page 21 in the Hrones and Nelson atlas of fourbar coupler curves (on the book DVD, page 36 of the PDF file) has A = 1.5, B = C = 3.5. Model this linkage with program FOURBAR using the coupler point fartherest to the right in the row shown and plot the resulting coupler curve.

Given: $A := 1.5$ $B := 3.5$ $C := 3.5$

Solution: See Figure on page 21 H&N Atlas, Figure 3-17b, and Mathcad file P0381.

1. The moving pivot point is located on the 3rd grid line from the bottom and the third grid line from the left when the crank angle is π radians. Let the number of horizontal grid spaces from the left end of the coupler to the coupler point be $n := -2, -1 .. 7$ and the number of vertical grid spaces from the coupler to the coupler point be $m := -2, -1 .. 2$
2. For the fourth column of points to the right of the coupler pivot and the second row of points above the horizontal axis $n := 4$ and $m := 2$. The grid spacing is $g := 0.5$
3. The angle, ϕ, between the coupler and the line from the coupler/crank pivot to the coupler point is

$$\phi(m,n) := if\left(n \neq 0, atan2(n,m), if\left(m = 0, 0, if\left(m > 0, \frac{\pi}{2}, \frac{-\pi}{2}\right)\right)\right) \qquad \phi(m,n) = 26.565\,deg$$

4. The distance from the pivot to the coupler point, R, along the same line is

$$R(m,n) := g\cdot\sqrt{m^2 + n^2} \qquad R(m,n) = 2.236$$

5. Determine the values needed for input to FOURBAR.

Link 2 (Crank)	$a := 1$	
Link 3 (Coupler)	$b := A\cdot a$	$b = 1.500$
Link 4 (Rocker)	$c := B\cdot a$	$c = 3.500$
Link 1 (Ground)	$d := C\cdot a$	$d = 3.500$
Distance to coupler point		$R(m,n) = 2.236$
Angle from link 3 to coupler point		$\phi(m,n) = 26.565\,deg$

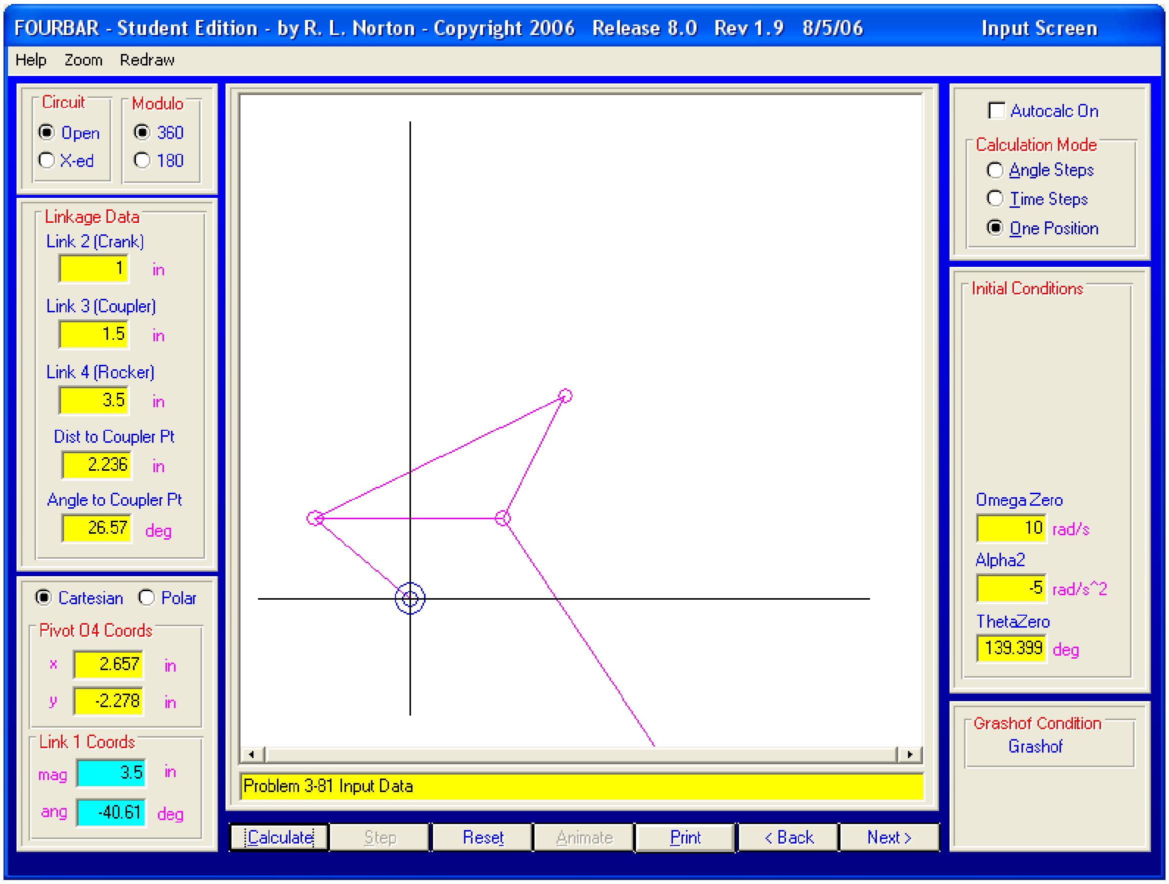
6. Calculate the coordinates of O_4. Let the angle between links 2 and 3 be α, then

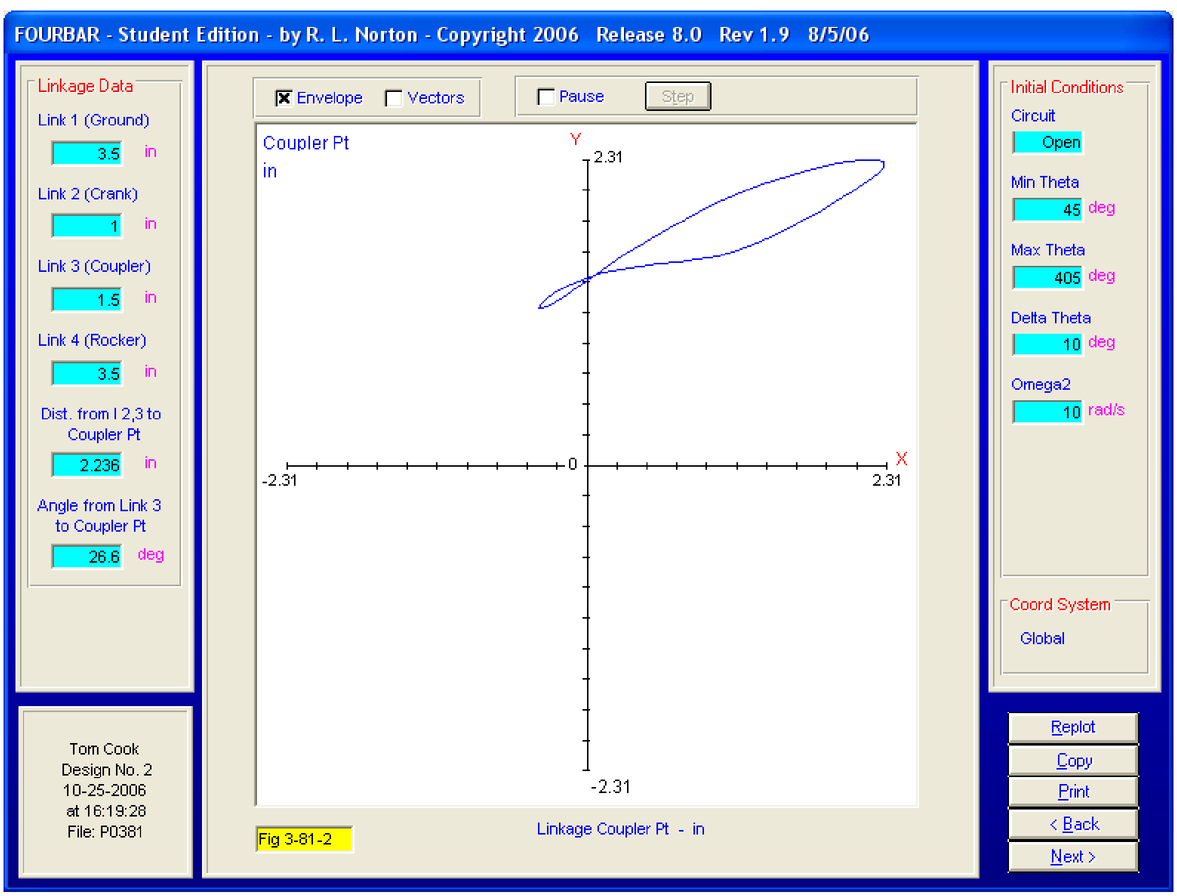
$$\alpha := acos\left[\frac{A^2 + (1 + C)^2 - B^2}{2\cdot A\cdot(1 + C)}\right] \qquad \alpha = 40.601\,deg$$

$$x_{O4} := C\cdot cos(\alpha) \qquad x_{O4} = 2.657$$

$$y_{O4} := -C\cdot sin(\alpha) \qquad y_{O4} = -2.278$$

7. Enter this data into FOURBAR and then plot the coupler curve. (See next page)


FOURBAR - Student Edition - by R. L. Norton - Copyright 2006 Release 8.0 Rev 1.9 8/5/06 Input Screen
Help Zoom Redraw
Circuit
Open
X-ed
Modulo
360
180
Linkage Data
Link 2 (Crank)
1 in
Link 3 (Coupler)
1.5 in
Link 4 (Rocker)
3.5 in
Dist to Coupler Pt
2.236 in
Angle to Coupler Pt
26.57 deg
Cartesian Polar
Pivot O4 Coords
x 2.657 in
y -2.278 in
Link 1 Coords
mag 3.5 in
ang -40.61 deg
Problem 3-81 Input Data
Calculate Step Reset Animate Print < Back Next >
Autocalc On
Calculation Mode
Angle Steps
Time Steps
One Position
Initial Conditions
Omega Zero
10 rad/s
Alpha2
-5 rad/s^2
ThetaZero
139.399 deg
Grashof Condition
Grashof


FOURBAR - Student Edition - by R. L. Norton - Copyright 2006 Release 8.0 Rev 1.9 8/5/06
Linkage Data
Link 1 (Ground)
3.5 in
Link 2 (Crank)
1 in
Link 3 (Coupler)
1.5 in
Link 4 (Rocker)
3.5 in
Dist. from I 2,3 to Coupler Pt
2.236 in
Angle from Link 3 to Coupler Pt
26.6 deg
Tom Cook
Design No. 2
10-25-2006
at 16:19:28
File: P0381
Envelope Vectors Pause Step
Coupler Pt
in
Y
2.31
X
-2.31
0
2.31
-2.31
Fig 3-81-2
Linkage Coupler Pt - in
Initial Conditions
Circuit
Open
Min Theta
45 deg
Max Theta
405 deg
Delta Theta
10 deg
Omega2
10 rad/s
Coord System
Global
Replot
Copy
Print
< Back
Next >

PROBLEM 3-82

Statement: The set of coupler curves on page 34 in the Hrones and Nelson atlas of fourbar coupler curves (on the book DVD, page 49 of the PDF file) has A = 2.0, B = 1.5, C = 2.0. Model this linkage with program FOURBAR using the coupler point fartherest to the right in the row shown and plot the resulting coupler curve.

Given: $A := 2.0$ $B := 1.5$ $C := 2.0$

Solution: See Figure on page 34 H&N Atlas, Figure 3-17b, and Mathcad file P0382.

1. The moving pivot point is located on the 3rd grid line from the bottom and the third grid line from the left when the crank angle is π radians. Let the number of horizontal grid spaces from the left end of the coupler to the coupler point be $n := -2, -1 .. 7$ and the number of vertical grid spaces from the coupler to the coupler point be $m := -2, -1 .. 2$
2. For the sixth column of points to the right of the coupler pivot and the first row of points below the horizontal axis $n := 6$ and $m := -1$. The grid spacing is $g := 0.5$
3. The angle, ϕ, between the coupler and the line from the coupler/crank pivot to the coupler point is

$$\phi(m,n) := if\left(n \neq 0, atan2(n,m), if\left(m = 0, 0, if\left(m > 0, \frac{\pi}{2}, \frac{-\pi}{2}\right)\right)\right) \qquad \phi(m,n) = -9.462\,deg$$

4. The distance from the pivot to the coupler point, R, along the same line is

$$R(m,n) := g\cdot\sqrt{m^2 + n^2} \qquad R(m,n) = 3.041$$

5. Determine the values needed for input to FOURBAR.

Link 2 (Crank)	$a := 1$	
Link 3 (Coupler)	$b := A\cdot a$	$b = 2.000$
Link 4 (Rocker)	$c := B\cdot a$	$c = 1.500$
Link 1 (Ground)	$d := C\cdot a$	$d = 2.000$
Distance to coupler point		$R(m,n) = 3.041$
Angle from link 3 to coupler point		$\phi(m,n) = -9.462\,deg$

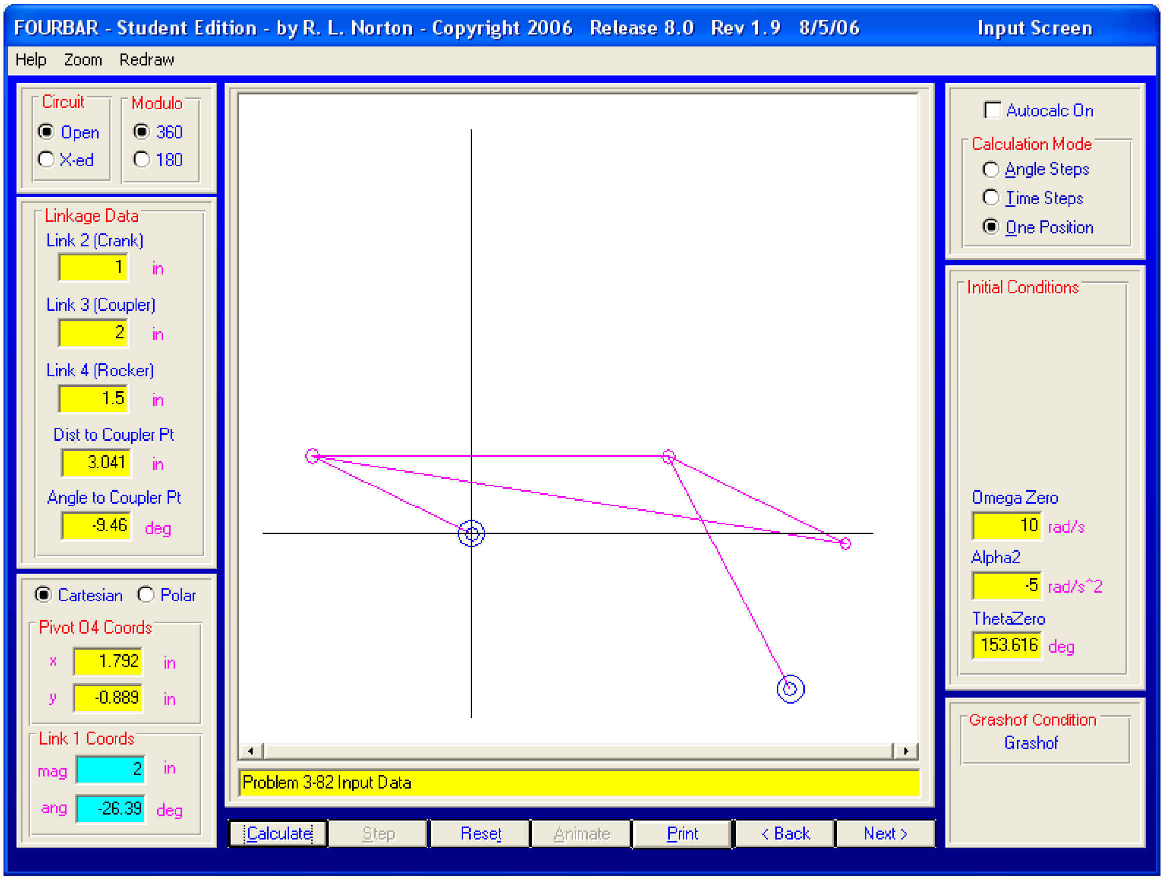
6. Calculate the coordinates of O_4. Let the angle between links 2 and 3 be α , then

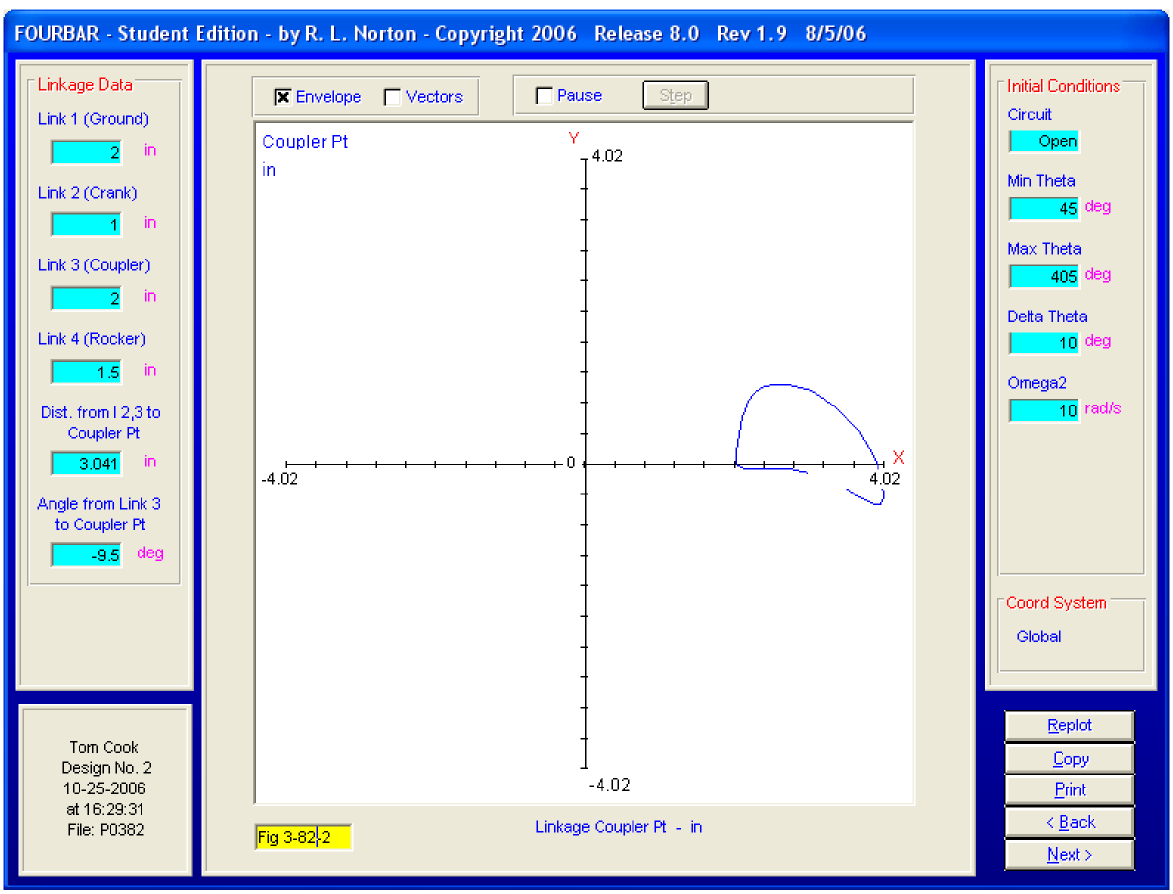
$$\alpha := acos\left[\frac{A^2 + (1 + C)^2 - B^2}{2\cdot A\cdot(1 + C)}\right] \qquad \alpha = 26.384\,deg$$

$$x_{O4} := C\cdot cos(\alpha) \qquad x_{O4} = 1.792$$

$$y_{O4} := -C\cdot sin(\alpha) \qquad y_{O4} = -0.889$$

7. Enter this data into FOURBAR and then plot the coupler curve. (See next page)


FOURBAR - Student Edition - by R. L. Norton - Copyright 2006 Release 8.0 Rev 1.9 8/5/06 Input Screen
Help Zoom Redraw
Circuit
Open
X-ed
Modulo
360
180
Linkage Data
Link 2 (Crank)
1 in
Link 3 (Coupler)
2 in
Link 4 (Rocker)
1.5 in
Dist to Coupler Pt
3.041 in
Angle to Coupler Pt
-9.46 deg
Cartesian Polar
Pivot O4 Coords
x 1.792 in
y -0.889 in
Link 1 Coords
mag 2 in
ang -26.39 deg
Problem 3-82 Input Data
Calculate Step Reset Animate Print < Back Next >
Autocalc On
Calculation Mode
Angle Steps
Time Steps
One Position
Initial Conditions
Omega Zero
10 rad/s
Alpha2
-5 rad/s^2
ThetaZero
153.616 deg
Grashof Condition
Grashof


FOURBAR - Student Edition - by R. L. Norton - Copyright 2006 Release 8.0 Rev 1.9 8/5/06
Linkage Data
Link 1 (Ground)
2 in
Link 2 (Crank)
1 in
Link 3 (Coupler)
2 in
Link 4 (Rocker)
1.5 in
Dist. from I 2,3 to Coupler Pt
3.041 in
Angle from Link 3 to Coupler Pt
-9.5 deg
Tom Cook
Design No. 2
10-25-2006
at 16:29:31
File: P0382
Envelope Vectors Pause Step
Coupler Pt
in
Y
4.02
-4.02
0
X
4.02
-4.02
Fig 3-82-2
Linkage Coupler Pt - in
Initial Conditions
Circuit
Open
Min Theta
45 deg
Max Theta
405 deg
Delta Theta
10 deg
Omega2
10 rad/s
Coord System
Global
Replot
Copy
Print
< Back
Next >

PROBLEM 3-83

Statement: The set of coupler curves on page 115 in the Hrones and Nelson atlas of fourbar coupler curves (on the book DVD, page 130 of the PDF file) has A = 2.5, B = 1.5, C = 2.5. Model this linkage with program FOURBAR using the coupler point fartherest to the right in the row shown and plot the resulting coupler curve.

Given: $A := 2.5$ $\quad B := 1.5$ $\quad C := 2.5$

Solution: See Figure on page 115 H&N Atlas, Figure 3-17b, and Mathcad file P0383.

1. The moving pivot point is located on the 3rd grid line from the bottom and the third grid line from the left when the crank angle is π radians. Let the number of horizontal grid spaces from the left end of the coupler to the coupler point be $n := -2, -1 .. 7$ and the number of vertical grid spaces from the coupler to the coupler point be $m := -2, -1 .. 2$

2. For the second column of points to the right of the coupler pivot and the second row of points below the horizontal axis $n := 2$ and $m := -2$. The grid spacing is $g := 0.5$

3. The angle, ϕ, between the coupler and the line from the coupler/crank pivot to the coupler point is

$$\phi(m,n) := if\left(n \neq 0, atan2(n,m), if\left(m = 0, 0, if\left(m > 0, \frac{\pi}{2}, \frac{-\pi}{2}\right)\right)\right) \qquad \phi(m,n) = -45.000\,deg$$

4. The distance from the pivot to the coupler point, R, along the same line is

$$R(m,n) := g \cdot \sqrt{m^2 + n^2} \qquad R(m,n) = 1.414$$

5. Determine the values needed for input to FOURBAR.

Link 2 (Crank)	$a := 1$	
Link 3 (Coupler)	$b := A \cdot a$	$b = 2.500$
Link 4 (Rocker)	$c := B \cdot a$	$c = 1.500$
Link 1 (Ground)	$d := C \cdot a$	$d = 2.500$
Distance to coupler point		$R(m,n) = 1.414$
Angle from link 3 to coupler point		$\phi(m,n) = -45.000\,deg$

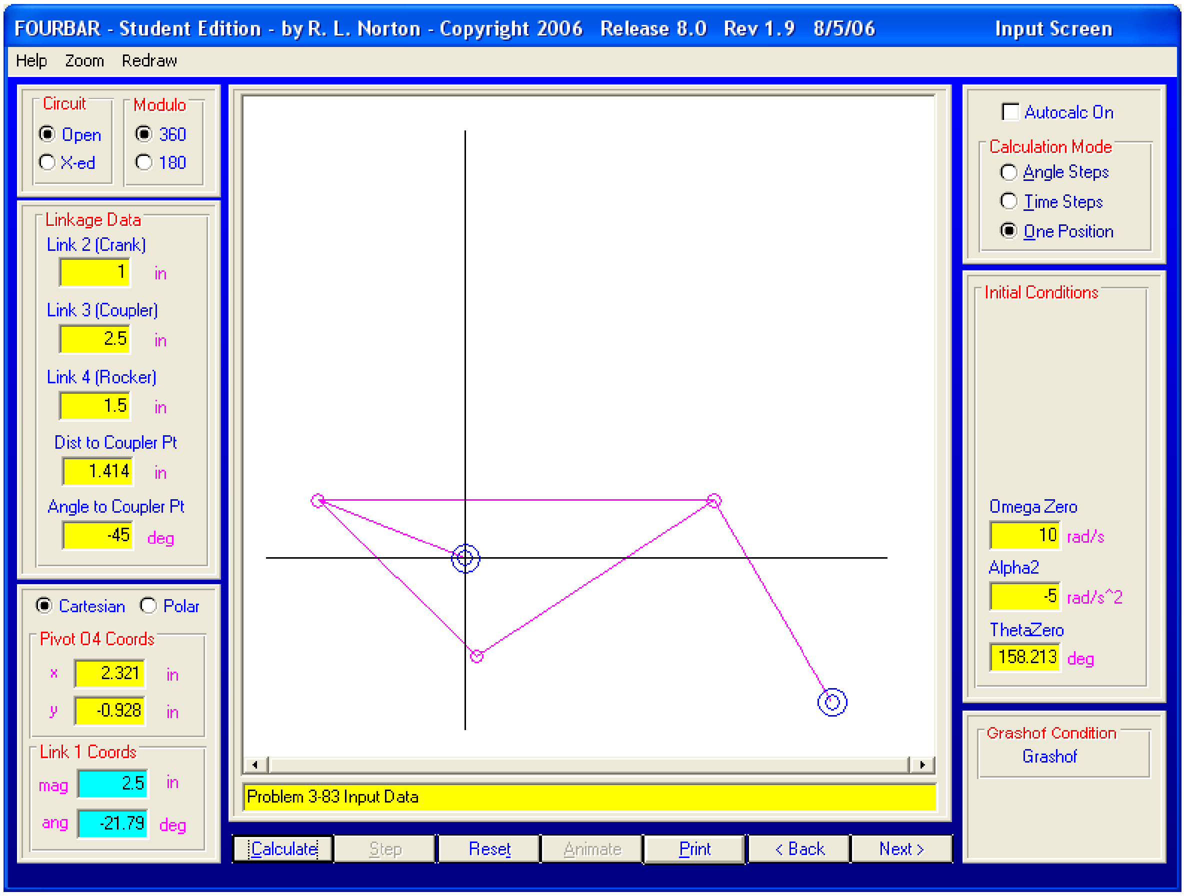
6. Calculate the coordinates of O_4. Let the angle between links 2 and 3 be α , then

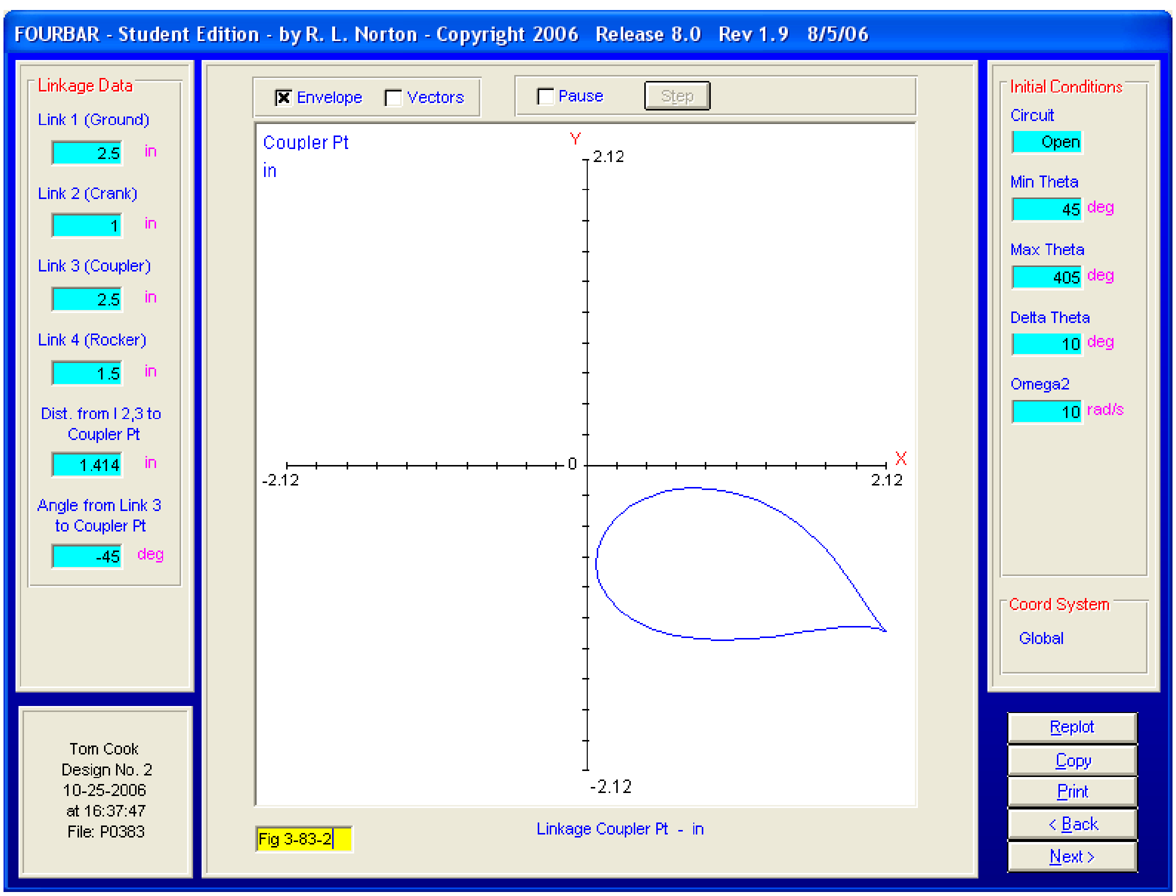
$$\alpha := acos\left[\frac{A^2 + (1+C)^2 - B^2}{2 \cdot A \cdot (1+C)}\right] \qquad \alpha = 21.787\,deg$$

$$x_{O4} := C \cdot cos(\alpha) \qquad x_{O4} = 2.321$$

$$y_{O4} := -C \cdot sin(\alpha) \qquad y_{O4} = -0.928$$

7. Enter this data into FOURBAR and then plot the coupler curve. (See next page)


FOURBAR - Student Edition - by R. L. Norton - Copyright 2006 Release 8.0 Rev 1.9 8/5/06 Input Screen
Help Zoom Redraw
Circuit
Open
X-ed
Modulo
360
180
Linkage Data
Link 2 (Crank)
1 in
Link 3 (Coupler)
2.5 in
Link 4 (Rocker)
1.5 in
Dist to Coupler Pt
1.414 in
Angle to Coupler Pt
-45 deg
Cartesian Polar
Pivot O4 Coords
x 2.321 in
y -0.928 in
Link 1 Coords
mag 2.5 in
ang -21.79 deg
Problem 3-83 Input Data
Calculate Step Reset Animate Print < Back Next >
Autocalc On
Calculation Mode
Angle Steps
Time Steps
One Position
Initial Conditions
Omega Zero
10 rad/s
Alpha2
-5 rad/s^2
ThetaZero
158.213 deg
Grashof Condition
Grashof


FOURBAR - Student Edition - by R. L. Norton - Copyright 2006 Release 8.0 Rev 1.9 8/5/06
Linkage Data
Link 1 (Ground)
2.5 in
Link 2 (Crank)
1 in
Link 3 (Coupler)
2.5 in
Link 4 (Rocker)
1.5 in
Dist. from I 2,3 to Coupler Pt
1.414 in
Angle from Link 3 to Coupler Pt
-45 deg
Tom Cook
Design No. 2
10-25-2006
at 16:37:47
File: P0383
Envelope Vectors Pause Step
Coupler Pt
in
Y
2.12
X
-2.12
0
2.12
-2.12
Fig 3-83-2
Linkage Coupler Pt - in
Initial Conditions
Circuit
Open
Min Theta
45 deg
Max Theta
405 deg
Delta Theta
10 deg
Omega2
10 rad/s
Coord System
Global
Replot
Copy
Print
< Back
Next >

PROBLEM 3-84

Statement: Design a fourbar mechanism to move the link shown in Figure P3-19 from position 1 to position 2. Ignore the third position and the fixed pivots O_2 and O_4 shown. Build a cardboard model that demonstrates the required movement.

Given: Position 1 offsets: $x_{C1D1} := 17.186 \cdot in$ $y_{C1D1} := 0.604 \cdot in$

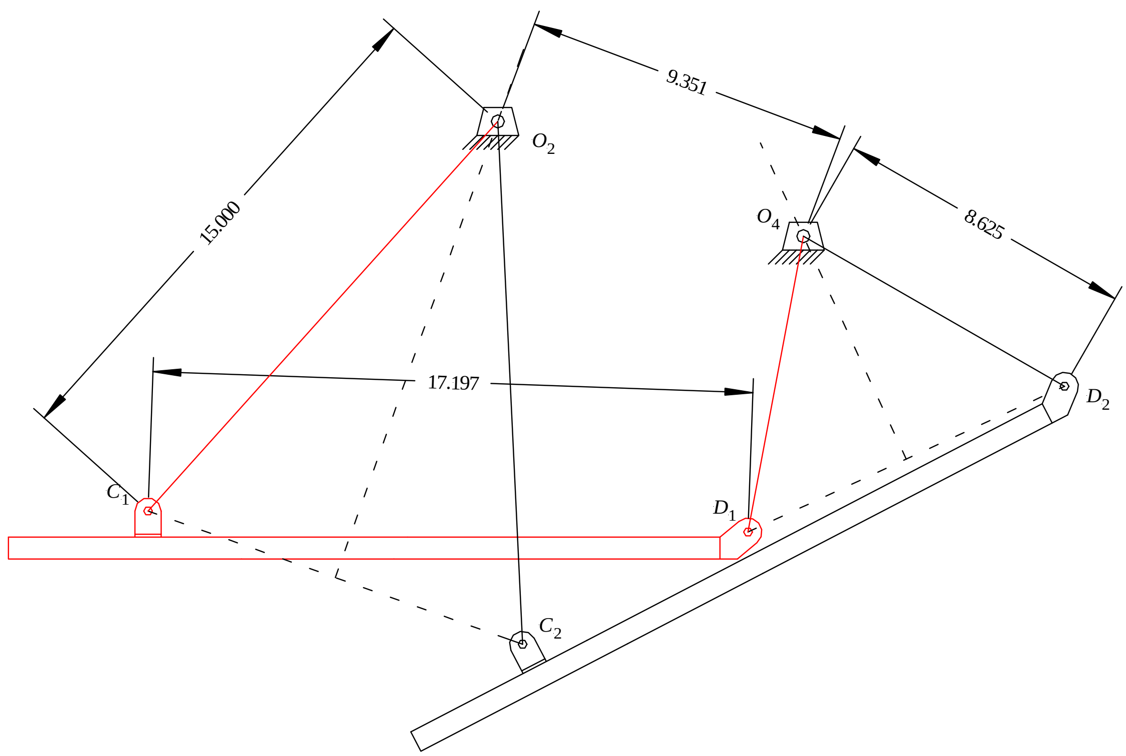
Solution: See figure below and Mathcad file P0384 for one possible solution.

1. Connect the end points of the two given positions of the line *CD* with construction lines, i.e., lines from C_1 to C_2 and D_1 to D_2.
2. Bisect these lines and extend their perpendicular bisectors in any convenient direction. In the solution below the bisector of C_1C_2 was extended upward and the bisector of D_1D_2 was also extended upward.
3. Select one point on each bisector and label them O_2 and O_4, respectively. In the solution below the distances O_2C and O_4D were selected to be 15.000 in. and 8.625 in, respectively. This resulted in a ground-link-length O_2O_4 for the fourbar of 9.351 in.
4. The fourbar is now defined as O_2CDO_4 with link lengths

Link 3 (coupler) $L_3 := \sqrt{x_{C1D1}^{\ 2} + y_{C1D1}^{\ 2}}$ $L_3 = 17.197 \cdot in$

Link 2 (input) $L_2 := 14.000 \cdot in$ Link 4 (output) $L_4 := 7.000 \cdot in$

Ground link 1 $L_1 := 9.351 \cdot in$



PROBLEM 3-85

Statement: Design a fourbar mechanism to move the link shown in Figure P3-19 from position 2 to position 3. Ignore the first position and the fixed pivots O_2 and O_4 shown. Build a cardboard model that demonstrates the required movement.

Given: Position 2 offsets: $x_{C2D2} := 15.524 \cdot in$ $\quad y_{C2D2} := 7.397 \cdot in$

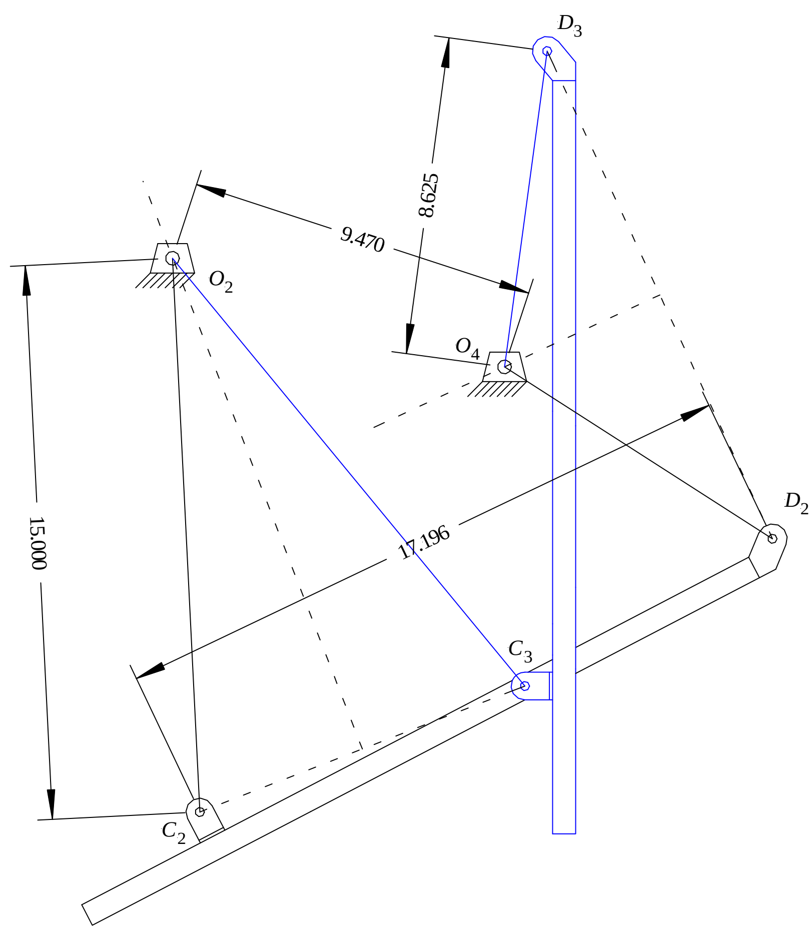
Solution: See figure below and Mathcad file P0385 for one possible solution.

1. Connect the end points of the two given positions of the line *CD* with construction lines, i.e., lines from C_2 to C_3 and D_2 to D_3.
2. Bisect these lines and extend their perpendicular bisectors in any convenient direction. In the solution below the bisector of C_2C_3 was extended upward and the bisector of D_2D_3 was also extended upward.
3. Select one point on each bisector and label them O_2 and O_4, respectively. In the solution below the distances O_2C and O_4D were selected to be 15.000 in and 8.625 in, respectively. This resulted in a ground-link-length O_2O_4 for the fourbar of 9.470 in.
4. The fourbar stage is now defined as O_2CDO_4 with link lengths

Link 3 (coupler) $L_3 := \sqrt{x_{C2D2}^{\ 2} + y_{C2D2}^{\ 2}}$ $\quad L_3 = 17.196 \cdot in$

Link 2 (input) $L_2 := 15.000 \cdot in$ $\quad$ Link 4 (output) $L_6 := 8.625 \cdot in$

Ground link 1b $L_{1b} := 9.470 \cdot in$



11. Using the program FOURBAR and the link lengths given above, it was found that the fourbar O_4DCO_6 is non-Grashoff with toggle positions at θ_4 = -14.9 deg and +14.9 deg. The fourbar operates between θ_4 = +12.403 deg and -8.950 deg.

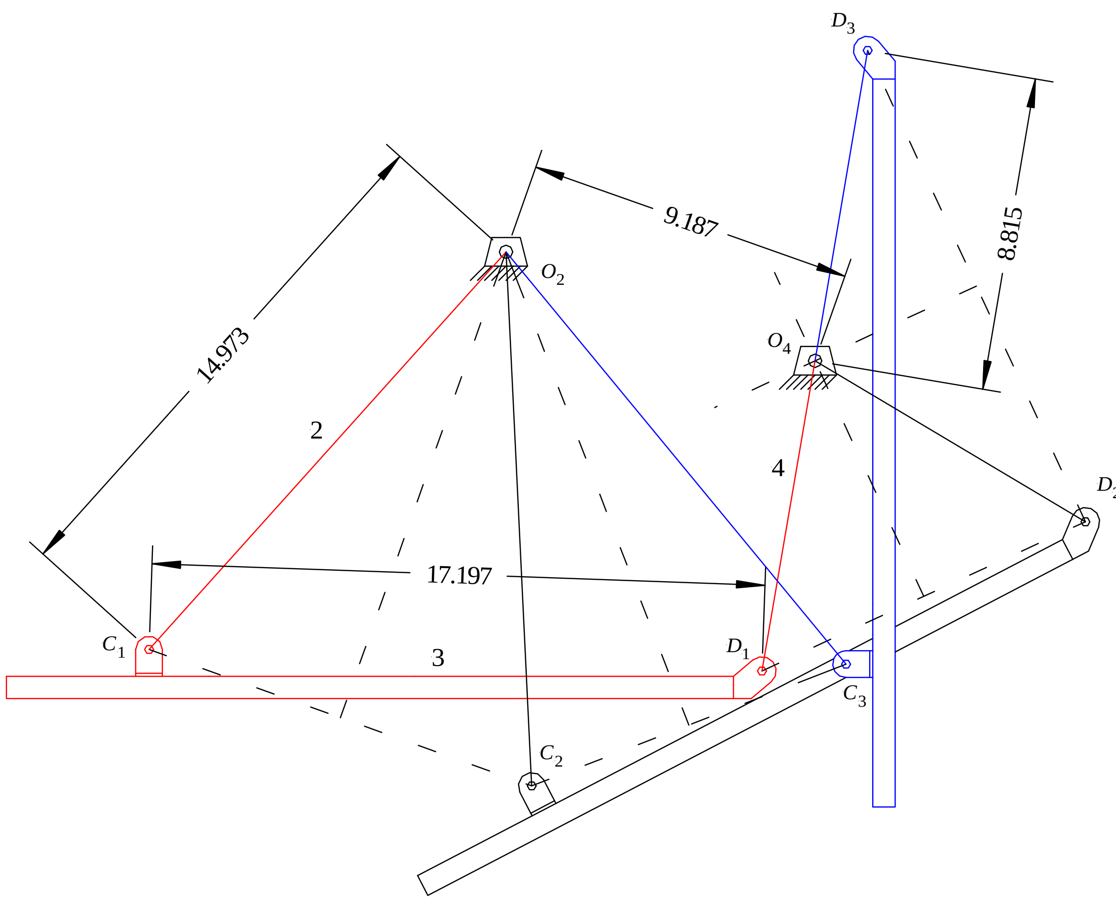
PROBLEM 3-86

Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-19. Ignore the points O_2 and O_4 shown. Build a cardboard model that has stops to limit its motion to the range of positions designed.

Solution: See Figure P3-19 and Mathcad file P0386.

1. Draw link *CD* in its three design positions C_1D_1, C_2D_2, C_3D_3 in the plane as shown.
2. Draw construction lines from point C_1 to C_2 and from point C_2 to C_3.
3. Bisect line C_1C_2 and line C_2C_3 and extend their perpendicular bisectors until they intersect. Label their intersection O_2.
4. Repeat steps 2 and 3 for lines D_1D_2 and D_2D_3. Label the intersection O_4.
5. Connect O_2 with C_1 and call it link 2. Connect O_4 with D_1 and call it link 4.
6. Line C_1D_1 is link 3. Line O_2O_4 is link 1 (ground link for the fourbar). The fourbar is now defined as O_2CDO_4 ar has link lengths of

Ground link 1	$L_1 := 9.187$	Link 2	$L_2 := 14.973$
Link 3	$L_3 := 17.197$	Link 4	$L_4 := 8.815$

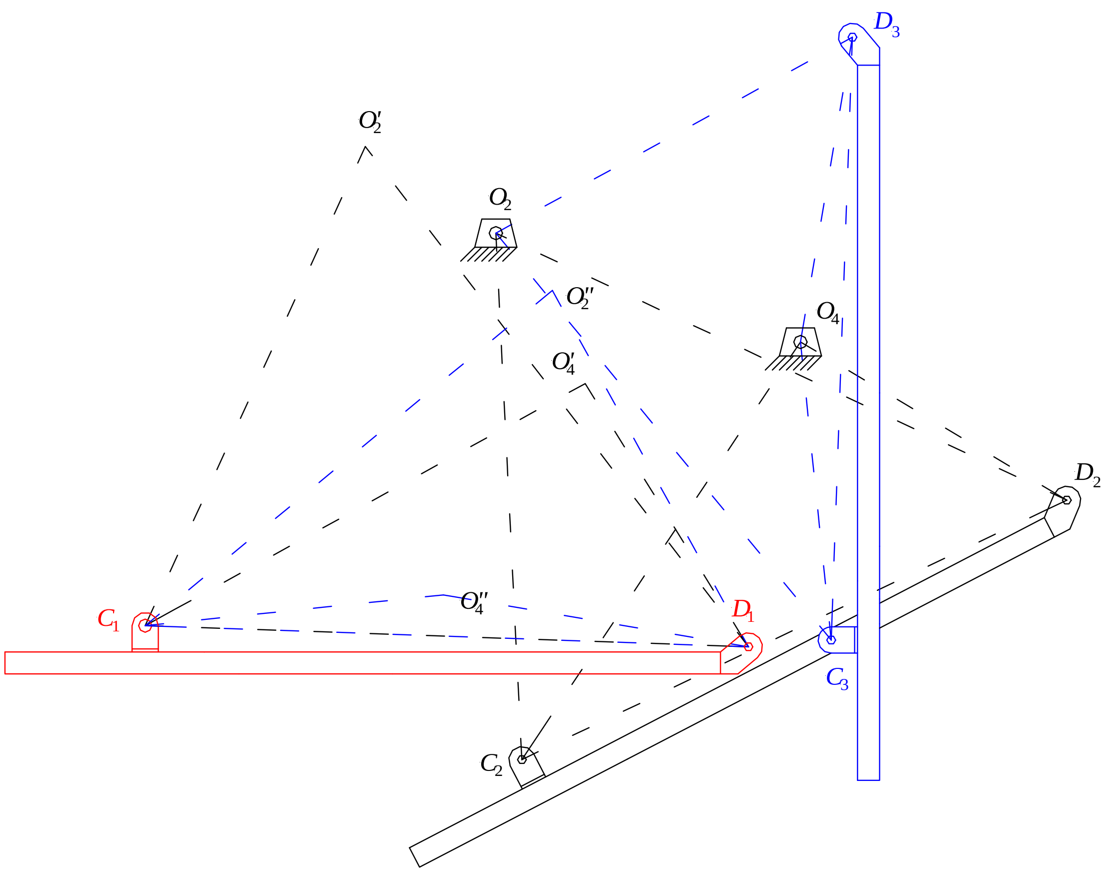


PROBLEM 3-87

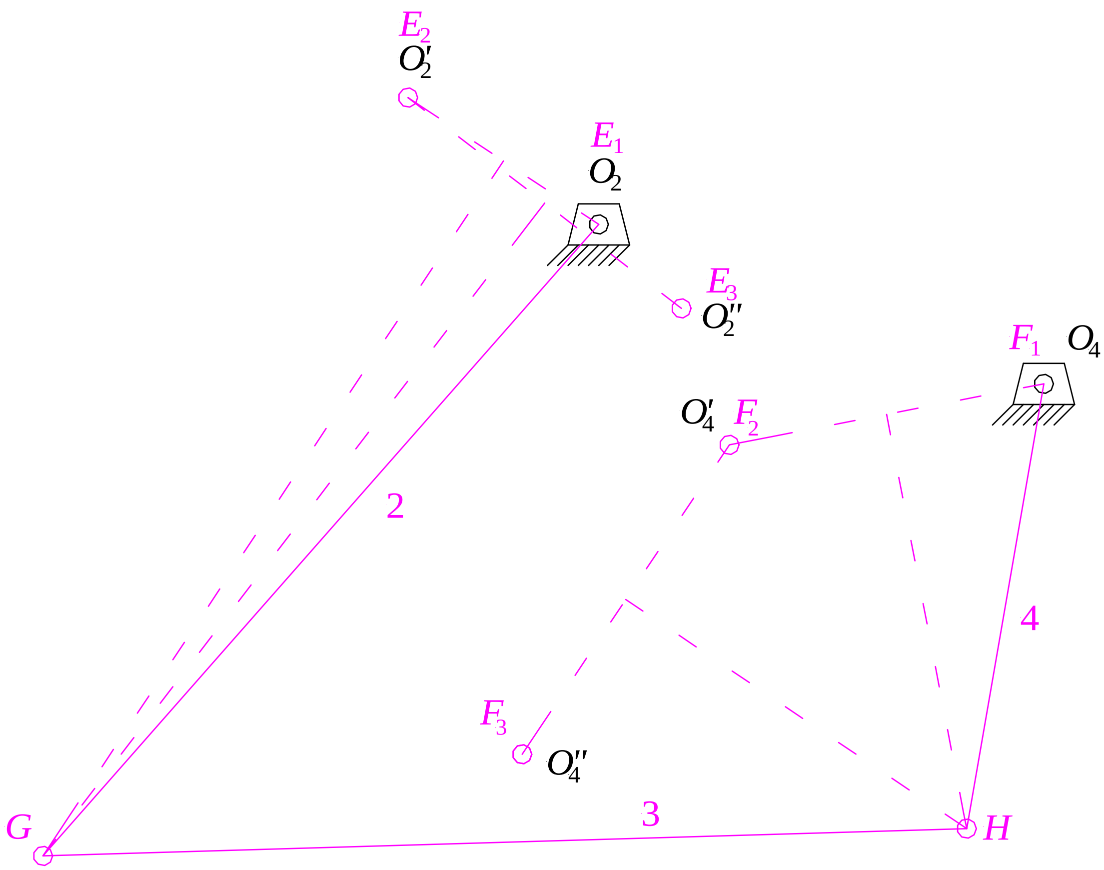
Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-17 using the fixed pivots O_2 and O_4 shown. (See Example 3-7.) Build a cardboard model that has stops to limit its motion to the range of positions designed.

Solution: See Figure P3-19 and Mathcad file P0387.

1. Draw link *CD* in its three design positions C_1D_1, C_2D_2, C_3D_3 in the plane as shown.
2. Draw the ground link O_2O_4 in its desired position in the plane with respect to the first coupler position C_1D_1.
3. Draw construction arcs from point C_2 to O_2 and from point D_2 to O_2 whose radii define the sides of triangle $C_2O_2D_2$. This defines the relationship of the fixed pivot O_2 to the coupler line *CD* in the second coupler position.
4. Draw construction arcs from point C_2 to O_4 and from point D_2 to O_4 whose radii define the sides of triangle $C_2O_4D_2$. This defines the relationship of the fixed pivot O_4 to the coupler line *CD* in the second coupler position.
5. Transfer this relationship back to the first coupler position C_1D_1 so that the ground plane position $O_2'O_4'$ bears the same relationship to C_1D_1 as O_2O_4 bore to the second coupler position C_2D_2.
6. Repeat the process for the third coupler position and transfer the third relative ground link position to the first, or reference, position.
7. The three inverted positions of the ground link that correspond to the three desired coupler positions are labeled O_2O_4, $O_2'O_4'$, and $O_2''O_4''$ in the first layout below and are renamed E_1F_1, E_2F_2, and E_3F_3, respectively, in the second layout, which is used to find the points *G* and *H*.



First layout for steps 1 through 7



Second layout for steps 8 through 12

8. Draw construction lines from point E_1 to E_2 and from point E_2 to E_3.

9. Bisect line E_1E_2 and line E_2E_3 and extend their perpendicular bisectors until they intersect. Label their intersection G.

10. Repeat steps 2 and 3 for lines F_1F_2 and F_2F_3. Label the intersection H.

11. Connect E_1 with G and label it link 2. Connect F_1 with H and label it link 4. Reinverting, E_1 and F_1 are the original fixed pivots O_2 and O_4, respectively.

12. Line GH is link 3. Line O_2O_4 is link 1a (ground link for the fourbar). The fourbar is now defined as O_2GHO_4 and has link lengths of

Ground link 1a	$L_{1a} := 9.216$	Link 2	$L_2 := 16.385$
Link 3	$L_3 := 18.017$	Link 4	$L_4 := 8.786$

13. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

$$
Condition(a,b,c,d) := \left| \begin{array}{l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.
$$

$$Condition\left(L_{1a}, L_2, L_3, L_4\right) = \text{"non-Grashof"}$$

The fourbar that will provide the desired motion is now defined as a non-Grashof double rocker in the open configuration. It now remains to add the original points C_1 and D_1 to the coupler *GH*.