

Instructor's Solutions Manual

to accompany

Design of Concrete Structures, 14e

Nilson/Darwin/Dolan

Chapters 1-4

The authors welcome feedback on the problem solutions and on the text in general. Please e-mail any comments to David Darwin at: daved@ku.edu

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1.1

$$A_s = 6.0 \text{ in}^2$$

$$f_y = 40,000 \text{ psi}$$

$$E_s = 29,000,000 \text{ psi}$$

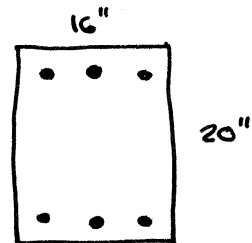
$$n = \frac{29,000,000}{3,600,000} = 8$$

$$A_g = 16 \times 20 = 320 \text{ in}^2$$

$$A_c = 320 - 6 = 314 \text{ in}^2$$

$$f_c' = 4,000 \text{ psi}$$

$$E_c = 3,600,000$$



a) $f_c = 1200 \text{ psi}$, $f_y = 40,000 \text{ psi}$

$$P = 1200(314 + 8 \times 6)$$

$$= 434,000 \text{ lbs}$$

$$P_s = 1200(8 \times 6) = 57,600 \text{ lbs}$$

$$P_s = 13.3\% \text{ of } P$$

b) $\epsilon_y = \frac{40,000}{29,000,000} = 0.00140$

for slow loading $f_c = 3000 \text{ psi}$

$$P = 3000(314) + 40,000(6) =$$

$$= 1,182,000 \text{ lbs}$$

$$P_s = 40,000(6) = 240,000 \text{ lb}$$

$$= 20.3\% P$$

c) $f_c = 3400 \text{ psi}$

$$P_u = 3400(314) + 40,000(6)$$

$$= 1,308,000 \text{ lb}$$

$$P_s = 240,000 \text{ lb} (18.3\% P_u)$$

a) $f_y = 60,000 \text{ psi}$
Same

$$\epsilon_y = \frac{60,000}{29,000,000} = 0.00207$$

$$f_c = 3300 \text{ psi}$$

$$P = 3300(314) + 60,000(6)$$

$$= 1,396,000 \text{ lbs}$$

$$P_s = 60,000(6) = 360,000 \text{ lb}$$

$$= 25.6\% P$$

$$P_u = 3400(314) + 60,000(6)$$

$$= 1,428,000 \text{ lb}$$

$$P_s = 360,000 \text{ lb} (25.2\% P_u)$$

Comments:

1. There is no difference in performance at $f_c = 1200 \text{ psi}$
2. As the strain increases, the steel with $f_y = 60,000 \text{ psi}$ contributes more to the total load and the column has a higher total load.
3. For the same cost, $f_y = 60,000 \text{ psi}$ provides a 9% increase in capacity.

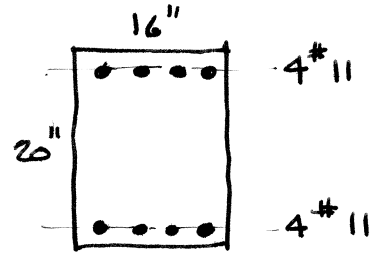
1.2

$$A_s = 8(1.56) = 12.48 \text{ in}^2$$

$$A_g = 320 \text{ in}^2 \quad A_c = 307.5 \text{ in}^2$$

$$n = 8 \quad E_s = 29,000,000 \text{ psi}$$

$$E_c = 3,600,000 \text{ psi}$$



$$f_y = 60,000 \text{ psi}$$

a)

$$P = 1200(307.5 + 8(12.48)) =$$

$$= 489,000 \text{ lb}$$

$$P_s = 1200(8)(12.48) = 120,000 \text{ lb}$$

$$(24.5\% P)$$

• Same as $f_y = 40,000$

b)

$$\epsilon_y = 0.00140, \quad f_c = 3000 \text{ psi}$$

$$P = 3000(307.5) + 40,000(12.48)$$

$$= 1,424,000 \text{ lb}$$

$$P_s = 40,000(12.48) = 500,000 \text{ lb}$$

$$(35.1\% \text{ of } P)$$

$$\epsilon_y = 0.00207 \quad f_c = 3300 \text{ psi}$$

$$P = 3300(307.5) + 60,000(12.48)$$

$$= 1,766,000 \text{ lb}$$

$$P_s = 60,000(12.48) = 750,000$$

$$(42.5\% \text{ of } P)$$

c)

$f_c = 3400 \text{ psi}$ both cases

$$P_0 = 3400(307.5) + 40,000(12.48)$$

$$= 1,547,000 \text{ lb}$$

$$P_s = 500,000 \text{ lb}$$

$$(32.3\% \text{ of } P_0)$$

$$P_0 = 3400(307.5) + 60,000(12.48)$$

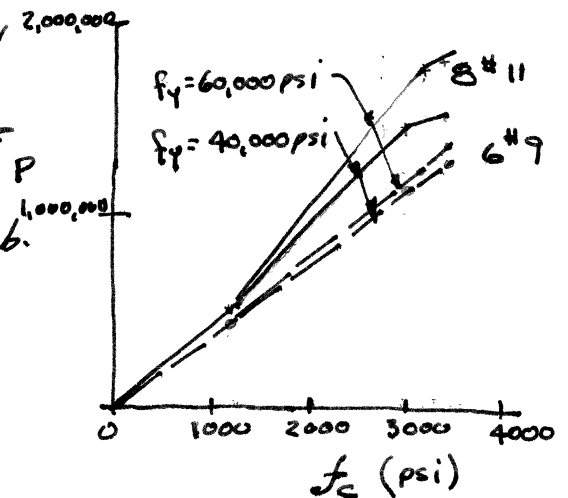
$$= 1,797,000 \text{ lb}$$

$$P_s = 750,000 \text{ lb}$$

$$(41.7\% \text{ of } P_0)$$

Comments

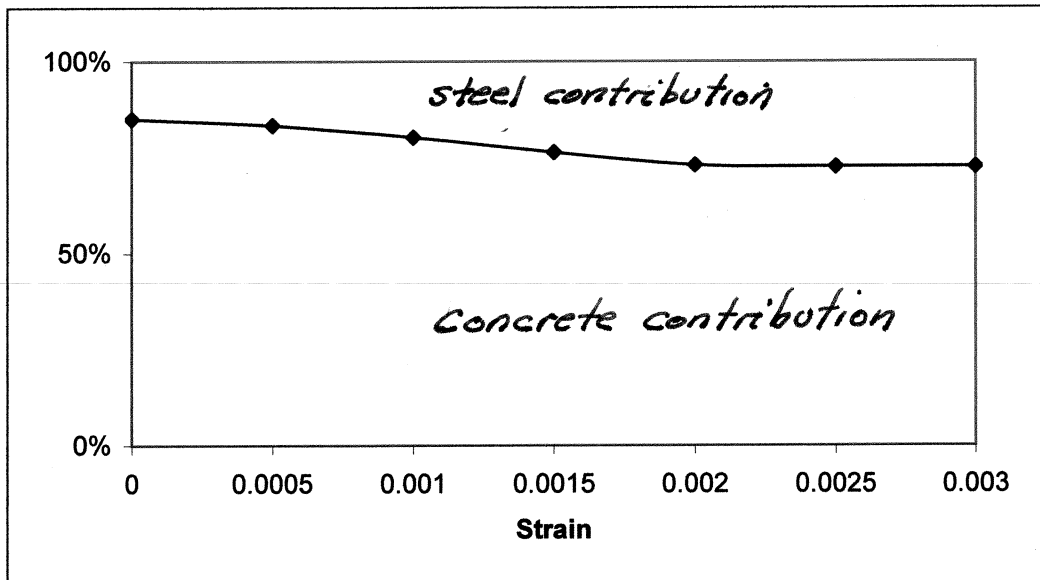
1. There is no strength difference at $f_c = 1200 \text{ psi}$
2. There is a 16% strength increase at ultimate using $f_y = 60,000 \text{ psi}$. This occurs at virtually no cost increase
3. The higher steel ratio produces a stronger column - compare to prob. 1.1



1.3

$A_s = 10.12 \text{ in}^2$
 $A_c = 474 \text{ in}^2$
 $f_y = 60000 \text{ psi}$
 $f'_c = 4000 \text{ psi}$

$\epsilon_c = \epsilon_s$	f_c (psi)	P_c (kips)	f_s (psi)	P_s (kips)	P_{total} (kips)	P_c/P_{total}	P_s/P_{total}
0	0	0	0	0	0	85.0%	15.0%
0.0005	1600	758.4	15000	151.8	910.2	83.3%	16.7%
0.001	2600	1232.4	30000	303.6	1536	80.2%	19.8%
0.0015	3100	1469.4	45000	455.4	1924.8	76.3%	23.7%
0.002	3300	1564.2	57000	576.84	2141.04	73.1%	26.9%
0.0025	3400	1611.6	60000	607.2	2218.8	72.6%	27.4%
0.003	3400	1611.6	60000	607.2	2218.8	72.6%	27.4%



1.4 A 20 x 24 in. column is made of the same concrete as Examples 1.1 and 1.2 but reinforced with six No. 11 (No. 36) bars with $f_y = 60$ ksi. For this column section, determine (a) the axial load the section will carry at a concrete stress of 1400 psi, (b) the load on the section when the steel begins to yield, (c) the maximum load if the section is loaded slowly and (d) the maximum load if the section is loaded rapidly. The area of one No. 11 (No. 36) bar is 1.56 in^2 . Determine the percent of the load carried by the steel and the concrete for each combination.

▣ Reinforcement Areas

Given Properties

$$f_c := 4000 \text{ psi} \quad f_y := 60000 \text{ psi} \quad f_c := 1400 \text{ psi} \quad n := 8 \quad E_s := 29000000 \text{ psi}$$

Column Properties

$$b := 20 \text{ in} \quad h := 24 \text{ in} \quad A_{st} := 6 \cdot A_{s11} = 9.36 \text{ in}^2 \quad \text{The total area of steel } A_{st} \text{ is six no. 11 bars}$$

Part (a) Compute the axial capacity of the section loaded below the elastic limit.

Solution: The axial capacity is based on the gross area of the column plus the effective area of the steel. Since we count the holes where the steel is removed, the additional effective area of the steel is $(n-1)A_{st}$.

$$A_g := b \cdot h \quad A_g = 480 \text{ in}^2 \quad A_{st} = 9.36 \text{ in}^2 \quad \text{Reinforcement ratio} = \frac{A_{st}}{A_g} = 0.0195$$

$$P := f_c \cdot [A_g + (n - 1) \cdot A_{st}] \quad P = 764 \text{ kip}$$

$$P_c := f_c \cdot (A_g - A_{st}) \quad P_c = 659 \text{ kip}$$

$$P_s := f_c \cdot n \cdot A_{st} \quad P_s = 105 \text{ kip} \quad 100 \cdot \frac{P_c}{P} = 86.3 \quad 100 \cdot \frac{P_s}{P} = 13.7$$

Part (b): Compute the capacity of the column when the steel begins to yield $\epsilon := 0.002069$ or 2/10 of one percent

Examining Figure 1.16, we are **beyond the elastic portion of the concrete** stress strain curve, but we are at the elastic limit of the steel.

$$f_s := \epsilon \cdot E_s \quad f_s = 60001 \text{ psi}$$

$$\text{From Figure 1.16} \quad f_c := 3100 \text{ psi} \quad \text{for slow loading}$$

Since the problem is nonlinear, we must break out the concrete and steel areas. We can no longer use the elastic equation from 1.1.

$$A_c := A_g - A_{st}$$

$$P := f_c \cdot A_c + f_s \cdot A_{st} \quad P = 2021 \text{ kip}$$

$$P_c := f_c \cdot A_c \quad P_c = 1459 \text{ kip}$$

$$P_s := f_s \cdot A_{st} \quad P_s = 562 \text{ kip} \quad 100 \cdot \frac{P_c}{P} = 72.2 \quad 100 \cdot \frac{P_s}{P} = 27.8$$

Part (c): Compute the maximum load capacity of the section

Examining Figure 1.16, we are **beyond the elastic portion of the concrete** stress strain curve, but we are in the plastic range of the steel.

$$f_s := f_y$$

$$f_s = 60000 \text{ psi}$$

From Figure 1.16

$$f_c := 3400 \text{ psi} \quad \text{for slow loading}$$

Since the problem is nonlinear, we must break out the concrete and steel areas. We can no longer use the elastic equation from 1.1.

$$A_c := A_g - A_{st}$$

$$P := f_c \cdot A_c + f_s \cdot A_{st} \quad P = 2162 \text{ kip}$$

$$P_c := f_c \cdot A_c \quad P_c = 1600 \text{ kip}$$

$$P_s := f_s \cdot A_{st} \quad P_s = 562 \text{ kip} \quad 100 \cdot \frac{P_c}{P} = 74.0 \quad 100 \cdot \frac{P_s}{P} = 26.0$$

If we reexamine the problem with a fast loading as would occur in a building, then the concrete stress would be

$$f_c := 4000 \text{ psi}$$

$$P := f_c \cdot A_c + f_s \cdot A_{st} \quad P = 2444 \text{ kip}$$

$$P_c := f_c \cdot A_c \quad P_c = 1883 \text{ kip} \quad 100 \cdot \frac{P_c}{P} = 77.0$$

$$P_s := f_s \cdot A_{st} \quad P_s = 562 \text{ kip} \quad 100 \cdot \frac{P_s}{P} = 23.0$$

Comments

1. As the concrete becomes non-linear, the steel picks up more load, but after the steel yields, the load goes to the concrete.
2. The slow loading is approximately 88% of the fast load scenario

1.5 A 24 in. diameter column is made of the same concrete as Examples 1.1 and 1.2. The area of reinforcement equals 2.1 percent of the gross cross section (i.e., $A_s = 0.021A_g$) and $f_y = 60$ ksi. For this column section, determine (a) the axial load the section will carry at a concrete stress of 1200 psi, (b) the load on the section when the steel begins to yield, (c) the maximum load if the section is loaded slowly, (d) the maximum load if the section is loaded rapidly and (e) the maximum load the reinforcement in the column is raised to 6.5 percent and the column is loaded slowly. Comment on your answer, especially the percent of the load carried by the steel and the concrete for each combination.

Given Properties

$$f_c := 4000\text{psi} \quad f_y := 60000\text{psi} \quad f_c := 1200\text{psi} \quad n := 8 \quad E_g := 29000000\text{psi}$$

Column Properties

$$d := 24\text{in} \quad A_g := \pi \frac{d^2}{4} \quad \rho := 0.021 \quad \rho \text{ is the reinforcement ratio or the fraction of the section that is steel}$$

$$A_{st} := \rho A_g \quad \text{The total area of steel } A_{st} \text{ is } A_{st} = 9.5 \text{ in}^2$$

Part (a) Compute the axial capacity of the section loaded below the elastic limit.

Solution: The axial capacity is based on the gross area of the column plus the effective area of the steel. Since we count the holes where the steel is removed, the additional effective area of the steel is $(n-1)A_{st}$.

$$A_c := A_g - A_{st} \quad A_g = 452 \text{ in}^2 \quad A_{st} = 9.50 \text{ in}^2 \quad A_c = 443 \text{ in}^2$$

$$P := f_c [A_g + (n-1)A_{st}] \quad P = 623 \text{ kip} \quad \text{Concrete and steel contribution}$$

$$P_c := f_c (A_g - A_{st}) \quad P_c = 531 \text{ kip} \quad 100 \cdot \frac{P_c}{P} = 85.4$$

$$P_s := f_y n A_{st} \quad P_s = 91 \text{ kip} \quad 100 \cdot \frac{P_s}{P} = 14.6$$

$$\epsilon_y := \frac{f_y}{E_s}$$

Part (b): Compute the capacity of the column when the steel begins to yield $\epsilon_y := \frac{f_y}{E_s}$

$$\epsilon_y = 0.00207 \quad \text{or } 2/10 \text{ of one percent}$$

Examining Figure 1.16, we are **beyond the elastic portion of the concrete stress strain curve**, but we are at the elastic limit of the steel.

$$f_s := \epsilon_y E_s \quad f_s = 60000 \text{ psi}$$

From Figure 1.16 $f_c := 3100\text{psi}$ for slow loading

Since the problem is nonlinear, we must break out the concrete and steel areas. We can no longer use the elastic equation from 1.1.

$$P := f_c A_c + f_s A_{st} \quad P = 1943 \text{ kip}$$

$$P_c := f_c A_c \quad P_c = 1373 \text{ kip} \quad 100 \cdot \frac{P_c}{P} = 27.4$$

$$P_s := f_s A_{st} \quad P_s = 570 \text{ kip} \quad 100 \cdot \frac{P_s}{P} = 29.3$$

Part (c): Compute the maximum load capacity of the section if loaded slowly

Examining Figure 1.16, we are **beyond the elastic portion of the concrete stress strain curve**

and we are in the plastic range of the steel.

$$f_s := f_y$$

$$f_s = 60000 \text{ psi}$$

From Figure 1.16

$$f_c := 3400 \text{ psi} \quad \text{for slow loading}$$

Since the problem is nonlinear, we must break out the concrete and steel areas. We can no longer use the elastic equation from 1.1:

$$P := f_c \cdot A_c + f_s \cdot A_{st}$$

$$P = 2076 \text{ kip}$$

$$P_c := f_c \cdot A_c$$

$$P_c = 1506 \text{ kip}$$

$$100 \cdot \frac{P_c}{P} = 72.5$$

$$P_s := f_s \cdot A_{st}$$

$$P_s = 570 \text{ kip}$$

$$100 \cdot \frac{P_s}{P} = 27.5$$

Part (d): If we reexamine the problem with a fast loading as would occur in a building, then the concrete stress would be

$$f_c := 4000 \text{ psi}$$

$$P := f_c \cdot A_c + f_s \cdot A_{st}$$

$$P = 2342 \text{ kip}$$

$$P_c := f_c \cdot A_c$$

$$P_c = 1772 \text{ kip}$$

$$100 \cdot \frac{P_c}{P} = 75.7 \quad 100 \cdot \frac{P_s}{P} = 24.3$$

$$P_s := f_s \cdot A_{st}$$

$$P_s = 570 \text{ kip}$$

Part (e): Determine the capacity for a slow loaded column with the steel changed to 6.5%

$$A_{st} := 0.065 \cdot A_g$$

$$A_{st} = 29.4 \text{ in}^2$$

$$f_s := f_y$$

$$f_s = 60000 \text{ psi}$$

From Figure 1.16

$$f_c := 3400 \text{ psi} \quad \text{for slow loading}$$

$$P := f_c \cdot A_c + f_s \cdot A_{st}$$

$$P = 3270 \text{ kip}$$

$$P_c := f_c \cdot A_c$$

$$P_c = 1506 \text{ kip}$$

$$100 \cdot \frac{P_c}{P} = 46.0$$

$$P_s := f_s \cdot A_{st}$$

$$P_s = 1764 \text{ kip}$$

$$100 \cdot \frac{P_s}{P} = 54.0$$

Comments

1. As the concrete becomes non-linear, the steel picks up more load, but after the steel yields, the load goes to the concrete.
2. The slow loading is approximately 88% of the fast load scenario - This is slightly higher than the 0.85 given in eq. 1.8.

1.5 A 24 in. diameter column is made of the same concrete as Examples 1.1 and 1.2. The area of reinforcement equals 2.1 percent of the gross cross section (i.e., $A_s = 0.021A_g$) and $f_y = 60$ ksi. For this column section, determine (a) the axial load the section will carry at a concrete stress of 1200 psi, (b) the load on the section when the steel begins to yield, (c) the maximum load if the section is loaded slowly, (d) the maximum load if the section is loaded rapidly and (e) the maximum load the reinforcement in the column is raised to 6.5 percent and the column is loaded slowly. Comment on your answer, especially the percent of the load carried by the steel and the concrete for each combination.

Given Properties

$$f_c := 4000\text{psi} \quad f_y := 60000\text{psi} \quad f_c := 1200\text{psi} \quad n := 8 \quad E_s := 29000000\text{psi}$$

Column Properties

$$d := 24\text{in} \quad A_g := \pi \frac{d^2}{4} \quad \rho := 0.021 \quad \rho \text{ is the reinforcement ratio or the fraction of the section that is steel}$$

$$A_{st} := \rho A_g \quad \text{The total area of steel } A_{st} \text{ is } A_{st} = 9.5 \text{ in}^2$$

Part (a) Compute the axial capacity of the section loaded below the elastic limit.

Solution: The axial capacity is based on the gross area of the column plus the effective area of the steel. Since we count the holes where the steel is removed, the additional effective area of the steel is $(n-1)A_{st}$.

$$A_c := A_g - A_{st} \quad A_g = 452 \text{ in}^2 \quad A_{st} = 9.50 \text{ in}^2 \quad A_c = 443 \text{ in}^2$$

$$P := f_c [A_g + (n-1)A_{st}] \quad P = 623 \text{ kip} \quad \text{Concrete and steel contribution}$$

$$P_c := f_c (A_g - A_{st}) \quad P_c = 531 \text{ kip} \quad 100 \cdot \frac{P_c}{P} = 85.4$$

$$P_s := f_y n A_{st} \quad P_s = 91 \text{ kip} \quad 100 \cdot \frac{P_s}{P} = 14.6$$

Part (b): Compute the capacity of the column when the steel begins to yield $\epsilon_y := \frac{f_y}{E_s}$

$$\epsilon_y = 0.00207 \quad \text{or } 2/10 \text{ of one percent}$$

Examining Figure 1.16, we are **beyond the elastic portion of the concrete stress strain curve**, but we are at the elastic limit of the steel.

$$f_s := \epsilon_y E_s \quad f_s = 60000 \text{ psi}$$

From Figure 1.16 $f_{max} := 3100\text{psi}$ for slow loading

Since the problem is nonlinear, we must break out the concrete and steel areas. We can no longer use the elastic equation from 1.1.

$$P_{max} := f_c A_c + f_s A_{st} \quad P = 1943 \text{ kip} \quad 100 \cdot \frac{P_c}{P} = 27.4$$

$$P_{con} := f_c A_c \quad P_c = 1373 \text{ kip}$$

$$P_{st} := f_s A_{st} \quad P_s = 570 \text{ kip} \quad 100 \cdot \frac{P_s}{P} = 29.3$$

Part (c): Compute the maximum load capacity of the section if loaded slowly

Examining Figure 1.16, we are **beyond the elastic portion of the concrete stress strain curve**

and we are in the plastic range of the steel.

$$f_{s,max} := f_y$$

$$f_s = 60000 \text{ psi}$$

From Figure 1.16

$$f_{s,max} := 3400 \text{ psi for slow loading}$$

Since the problem is nonlinear, we must break out the concrete and steel areas. We can no longer use the elastic equation from 1.1.

$$P_{max} := f_c \cdot A_c + f_s \cdot A_{st}$$

$$P = 2076 \text{ kip}$$

$$P_{max} := f_c \cdot A_c$$

$$P_c = 1506 \text{ kip}$$

$$100 \cdot \frac{P_c}{P} = 72.5$$

$$P_{max} := f_s \cdot A_{st}$$

$$P_s = 570 \text{ kip}$$

$$100 \cdot \frac{P_s}{P} = 27.5$$

Part (d): If we reexamine the problem with a fast loading as would occur in a building, then the concrete stress would be

$$f_{s,max} := 4000 \text{ psi}$$

$$P_{max} := f_c \cdot A_c + f_s \cdot A_{st}$$

$$P = 2342 \text{ kip}$$

$$P_{max} := f_c \cdot A_c$$

$$P_c = 1772 \text{ kip}$$

$$100 \cdot \frac{P_c}{P} = 75.7 \quad 100 \cdot \frac{P_s}{P} = 24.3$$

$$P_{max} := f_s \cdot A_{st}$$

$$P_s = 570 \text{ kip}$$

Part (e): Determine the capacity for a slow loaded column with the steel changed to 6.5%

$$A_{st,max} := 0.065 \cdot A_g$$

$$A_{st} = 29.4 \text{ in}^2$$

$$f_{s,max} := f_y$$

$$f_s = 60000 \text{ psi}$$

From Figure 1.16

$$f_{s,max} := 3400 \text{ psi for slow loading}$$

$$P_{max} := f_c \cdot A_c + f_s \cdot A_{st}$$

$$P = 3270 \text{ kip}$$

$$P_{max} := f_c \cdot A_c$$

$$P_c = 1506 \text{ kip}$$

$$100 \cdot \frac{P_c}{P} = 46.0$$

$$P_{max} := f_s \cdot A_{st}$$

$$P_s = 1764 \text{ kip}$$

$$100 \cdot \frac{P_s}{P} = 54.0$$

Comments

1. As the concrete becomes non-linear, the steel picks up more load, but after the steel yields, the load goes to the concrete.
2. The slow loading is approximately 88% of the fast load scenario - This is slightly higher than the 0.85 given in eq. 1.8.

2.1

$$f'_c = 6000 \text{ psi}$$

(a) No prior results

$$f'_{cr} = f'_c + 0.1 f'_c + 700 \text{ psi} = 6000 + 0.1 \times 6000 + 700 = 7300 \text{ psi}$$

(b) 20 prior tests for concrete with f'_c within 1000 psi of f'_c for project. $S_s = 580 \text{ psi}$ -

$$\text{From Table 1.1, } 1.08 \times 580 = 626 \text{ psi}$$

Because $f'_c > 5000 \text{ psi}$, use Eqs. (2.1) + (2.2b)

$$f'_{cr} = f'_c + 1.34 S_s = 6000 + 1.34 \times 626 = 6840 \text{ psi}$$

$$f'_{cr} = 0.9 f'_c + 2.33 S_s = 0.9 \times 6000 + 2.33 \times 626 = 6860 \text{ psi}$$

$$\text{Use } f'_{cr} = 6860 \text{ psi}$$

(c) 30 prior tests for concrete with f'_c within 1000 psi of f'_c for project. $S_s = 590 \text{ psi}$ -

$$f'_{cr} = f'_c + 1.34 S_s = 6000 + 1.34 \times 590 = 6790 \text{ psi}$$

$$f'_{cr} = 0.9 f'_c + 2.33 S_s = 0.9 \times 6000 + 2.33 \times 590 = 6770 \text{ psi}$$

$$\text{Use } f'_{cr} = 6790 \text{ psi}$$

2.2

(a) For $f'_c = 4000 \text{ psi}$, the strength results indicate satisfactory concrete quality because (1) no individual test is below $f'_c - 500 \text{ psi} = 3500 \text{ psi}$, and (2) every arithmetic average of any three consecutive tests equals or exceeds f'_c .

(b) For $S_s = 510 \text{ psi}$ for 30 consecutive tests,

calculate f'_{cr} using Eqs (2.1) and (2.2a)

$$f'_{cr} = f'_c + 1.34S_s = 4000 + 1.34 \times 510 = 4680 \text{ psi}$$

$$f'_{cr} = f'_c + 2.33S_s - 500 \text{ psi} = 4000 + 2.33 \times 510 - 500 \\ = 4690 \text{ psi}$$

Use 4690 psi

$$(4590 + 4750 + 5280 + 4210 + 4460 + 4170 + 3750 + 5110 + 4640 + 4170) / 10 \\ = 4510 < f'_{cr}$$

Because the average compressive strength is less than f'_{cr} , the water-cement ratio must be decreased, either by adding cement or reducing water, to increase strength. If the water is reduced, a water reducer must be added or the quantity of water reducer must be increased to maintain concrete workability.

Problem 3.1 A rectangular beam made using concrete with $f'_c = 6000$ psi and steel with $f_y = 60,000$ psi had a width $b = 20$ in., and an effective depth of $d = 17.5$ in and an $h = 20$ in. The Concrete modulus of rupture $f_r = 530$ psi. The elastic modulus of the steel and concrete are, respectively $E_c = 4,030,000$ psi and $E_s = 29,000,000$ psi. The area of steel is four No. 11 (No. 36) bars.

- Find the maximum service load that can be resisted without stressing the concrete above $0.45 f'_c$ or the steel above $0.40 f_y$.
- Determine if the beam will show cracking before reaching the service load
- Compute the nominal moment capacity of the beam
- Compute the ratio of the nominal capacity of the beam to the maximum service level capacity and compare your findings to the ACI load factors and strength reduction factor.

▣ Reinforcement sizes

Given data

Note: for all MathCAD based solutions, the area and diameter of reinforcement bars is in a common database. Hence the notation A_{s11} indicates the area of a single No. 11 (No. 36) bar.

$$A_s := 4 \cdot A_{s11} \quad A_s = 6.24 \text{ in}^2 \quad E_s := 29000000 \text{ psi}$$

$$b := 20 \text{ in} \quad d := 17.5 \text{ in} \quad h := 20 \text{ in}$$

$$f'_c := 6000 \text{ psi} \quad f_y := 60000 \text{ psi} \quad E_c := 57000 \sqrt{f'_c} \text{ psi} \quad E_c = 4415 \text{ ksi}$$

$$f_r := 7.5 \sqrt{f'_c} \text{ psi} \quad f_r = 581 \text{ psi} \quad n := \frac{E_s}{E_c} \quad n = 6.6$$

- Find the maximum service load that can be resisted without stressing the concrete above $0.45 f'_c$ or the steel above $0.40 f_y$.

$$f_c := 0.45 f'_c \quad f_c = 2700 \text{ psi}$$

$$f_s := 0.40 f_y \quad f_s = 36000 \text{ psi}$$

$$\rho := \frac{A_s}{b \cdot d} \quad \rho = 0.018$$

$$k := \sqrt{(\rho \cdot n)^2 + 2 \rho \cdot n} - \rho \cdot n \quad k = 0.381$$

$$j := 1 - \frac{k}{3} \quad j = 0.873$$

Moment due to concrete limits

$$M_{sc} := \frac{1}{2} \cdot f_c \cdot b \cdot k \cdot d \cdot \left(d - \frac{k \cdot d}{3} \right) \quad M_{sc} = 229 \text{ ft} \cdot \text{kip}$$

Moment due to steel limit

$$M_{ss} := A_s \cdot f_s \cdot j \cdot d \quad M_{ss} = 286 \text{ ft} \cdot \text{kip}$$

The maximum service moment is the minimum of the two values.

$$M_s := \min(M_{ss}, M_{sc}) \quad M_s = 229 \text{ ft} \cdot \text{kip}$$

(b) Determine if the beam will show cracking before reaching the service load

$$I_g := \frac{b \cdot h^3}{12} \quad I_g = 13333 \cdot \text{in}^4$$

$$M_{cr1} := \frac{f_r \cdot I_g}{\frac{h}{2}} \quad M_{cr1} = 64.5 \cdot \text{ft} \cdot \text{kip}$$

This is less than the service load so the section cracks. To demonstrate that the transformed section does not affect this conclusion, the following checks the cracked transformed section.

$$\Delta y := \frac{n \cdot A_s \cdot \left(d - \frac{h}{2} \right)}{n \cdot A_s + b \cdot h} \quad \Delta y = 0.697 \cdot \text{in}$$

$$I_{ut} := I_g + b \cdot d \cdot \Delta y^2 + n \cdot A_s \cdot \left(d - \frac{h}{2} - \Delta y \right)^2 \quad I_{ut} = 15400 \cdot \text{in}^4$$

$$M_{cr2} := \frac{f_r \cdot I_{ut}}{\frac{h}{2} - \Delta y} \quad M_{cr2} = 80.1 \cdot \text{ft} \cdot \text{kip} \quad \frac{M_{cr2}}{M_{cr1}} = 1.242$$

(c) Determine the nominal moment capacity of the section.

$$a := \frac{A_s \cdot f_y}{0.85 f_c \cdot b} \quad a = 3.67 \cdot \text{in}$$

$$M_n := A_s \cdot f_y \cdot \left(d - \frac{a}{2} \right) \quad M_n = 489 \cdot \text{ft} \cdot \text{kip}$$

(d) Compute the ratio of the nominal capacity of the beam to the maximum service level capacity and compare your findings to the ACI load factors and strength reduction factor.

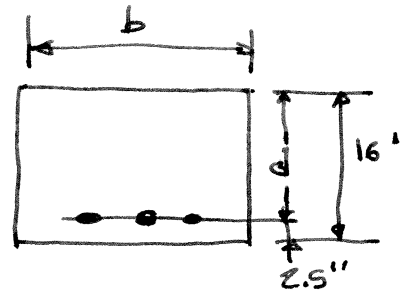
$$\text{Ratio} := \frac{M_n}{M_s} \quad \text{Ratio} = 2.13$$

First, the extra computation of the uncracked transformed area gives only a 18% increase in the cracking moment. Comparing the cracking moment to the service moment shows that the service moment is almost 3 times the cracking moment. Therefore, unless the service moment is very close to the service moment, you can be assured that the section will crack based on the gross section calculation.

Second, the margin of safety between the service moment and the nominal capacity is 2.11. This is greater than the ultimate load factors and phi factors from ASCE-7 and ACI ($1.8/0.9 = 2.00$ if the entire load is classified as live load) indicating that a service level design is more conservative than LRFD design.

3.2 $w_d = 500 \text{ plf}$
 $w_L = 1200 \text{ plf}$
 $L = 22'$

$f'_c = 3000 \text{ psi}$
 $f_y = 60,000 \text{ psi}$
 $\rho = 0.6 \rho_{max}$



Assume $w_o = w_d = 500 \text{ plf}$
 and check assumption at conclusion of
 the design

$$w_u = 1.2(500 + 500) + 1.6(1200) = 3120 \text{ plf}$$

$$M_u = \frac{wL^2}{8} = \frac{3.12(22)^2}{8} = 189 \text{ ft-kip}$$

$$\rho_{max} = 0.85 \beta_1 \frac{f'_c}{f_y} \left(\frac{\epsilon_u}{\epsilon_u + 0.004} \right) = 0.85(0.85) \frac{3}{60} \left(\frac{0.003}{0.003 + 0.004} \right)$$

$$= 0.0155 \quad (\text{or from Table A.4})$$

$$\rho = 0.6 \rho_{max} = 0.6(0.0155) = 0.0093$$

$$R = \rho f_y \left(1 - 0.588 \frac{\rho f_y}{f'_c} \right) = 0.0093(60000) \left(1 - 0.588 \frac{0.0093(60)}{3} \right)$$

$$R = 497 \quad (\text{or by interpolation from Table A.5b})$$

Since $\rho = 0.6 \rho_b$ assume $\phi = 0.9$ (Confirmed by Table A.4)

$$bd^2 = \frac{M_u}{\phi R} = \frac{189(12000)}{0.9(497)} = 5070$$

$$d = h - 2.5 = 16 - 2.5 = 13.5$$

Solve for b

$$b = \frac{5070}{13.5^2} = 27.8$$

Use $b = 28 \text{ in}$

Check weight assumption $w_o = \frac{16 \times 28}{144}(150) = 467 \text{ plf}$
 which is less than assumed.

$$A_s = \rho bd = 0.0093(27.8)(13.5) = 3.49 \text{ in}^2$$

$$3 \#10 (\#36) = 3.81 \text{ in}^2$$

3.3

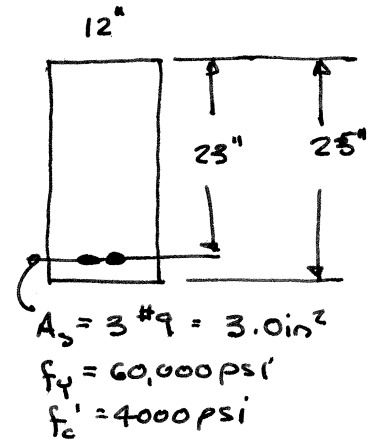
$$L = 20'$$

$$w_s = 2450 \text{ plf}$$

$$w_o = \frac{12 \times 25}{144} (150) = 313 \text{ plf}$$

$$M_s = \frac{wL^2}{8} = \frac{(2450 + 313)(20)^2}{8} \frac{1}{1000}$$

$$= 138 \text{ ft kip}$$



a)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{3.0 (60)}{0.85 (4) (12)} = 4.41 \text{ in}$$

$$M_u = A_s f_y \left(d - \frac{a}{2} \right) = 3.0 (60) \left(23 - \frac{4.41}{2} \right) \frac{1}{12} = 311 \text{ ft kips}$$

$$FS = \frac{M_u}{M_s} = \frac{311}{138} = 2.26$$

This exceeds the target value of 1.85

b)

$$n = 8 \quad \text{from } E_c = 57,000 \sqrt{f'_c} = 3,600,000$$

$$\rho = \frac{A_s}{bd} = \frac{3}{12 \times 23} = 0.0109$$

From Table A.6 $k = 0.339$, $j = 0.887$ by interpolation

$$\text{or } k = \sqrt{(8 \times 0.0109)^2 + 2.9 \times 0.0109} - 8(0.0109) = 0.339$$

$$f_s = \frac{M_s}{A_s j d} = \frac{138 (12000)}{3 (0.887) 23} = 27,600 \text{ psi}$$

$$f_c = \frac{M_s}{k j b d^2} = \frac{138 (12000)}{0.339 (0.887) 12 (23)^2} = 868 \text{ psi}$$

$$c) \quad f_r = 7.5 \sqrt{f'_c} = 7.5 \sqrt{4000} = 474 \text{ psi}$$

$$I_g = \frac{bh^3}{12} = \frac{12 (25)^3}{12} = 15,625 \text{ in}^4$$

$$M_{cr} = \frac{f_r I}{Y} = \frac{474 (15,625)}{25/2} \frac{1}{12000} = 49.4 \text{ ft k}$$

$M_{cr} \ll M_s \quad \therefore$ Beam will crack

Problem 3.4 A rectangular reinforced concrete section has dimension $b=14$ in., $d=25$ in, and $h = 28$ in., and is reinforced with 3 No. 10 (No. 32) bars. The material strengths are $f'_c = 5000$ psi, $f_y = 60,000$ psi.

- Find the moment that will produce first cracking at the bottom surface of the section basing your calculations on I_g , the moment of inertial of the gross section.
- Repeat the calculation using I_{ut} the uncracked transformed moment of inertia.
- Determine the maximum moment that can be carried without the concrete stress exceeding $0.45 f'_c$ or the steel stress exceeding $0.60 f_y$.
- Determine the nominal moment capacity of the section.
- Compute the ratio of nominal moment capacity from part (d) to the service level moment from part (c)
- Comment on your results with particular attention to comparing parts (a) and (b) and comparing part (e) to established load factors.

▣ Reinforcement sizes

Given data

$$\begin{aligned}
 A_s &:= 3 \cdot A_{s10} & A_s &= 3.81 \text{ in}^2 & E_s &:= 29000000 \text{ psi} \\
 b &:= 14 \text{ in} & d &:= 25 \text{ in} & h &:= 28 \text{ in} \\
 f'_c &:= 5000 \text{ psi} & f_y &:= 60000 \text{ psi} & E_c &:= 57000 \sqrt{f'_c} \text{ psi} & E_c &= 4031 \text{ ksi} \\
 f_r &:= 7.5 \sqrt{f'_c} \text{ psi} & f_r &= 530 \text{ psi} & n &:= \frac{E_s}{E_c} & n &= 7.2
 \end{aligned}$$

- Find the moment that will produce first cracking at the bottom surface of the section basing your calculations on I_g , the moment of inertial of the gross section.

$$I_g := \frac{b \cdot h^3}{12} \quad I_g = 25611 \text{ in}^4 \quad M_{cr1} := \frac{f_r \cdot I_g}{\frac{h}{2}} \quad M_{cr1} = 80.8 \text{ ft} \cdot \text{kip}$$

- Repeat the calculation using I_{ut} the uncracked transformed moment of inertia.

$$\begin{aligned}
 \Delta y &:= \frac{n \cdot A_s \cdot \left(d - \frac{h}{2}\right)}{n \cdot A_s + b \cdot h} & \Delta y &= 0.719 \text{ in} \\
 I_{ut} &:= I_g + b \cdot d \cdot \Delta y^2 + n \cdot A_s \cdot \left(d - \frac{h}{2} - \Delta y\right)^2 & I_{ut} &= 28689 \text{ in}^4 \\
 M_{cr2} &:= \frac{f_r \cdot I_{ut}}{\frac{h}{2} - \Delta y} & M_{cr2} &= 95.5 \text{ ft} \cdot \text{kip} & \frac{M_{cr2}}{M_{cr1}} &= 1.181
 \end{aligned}$$

- Determine the maximum moment that can be carried without the concrete stress exceeding $0.45 f'_c$ or the steel stress exceeding $0.60 f_y$.

$$f_c := 0.45 f'_c \quad f_c = 2250 \text{ psi}$$

$$f_s := 0.60f_y \quad f_s = 36000 \text{ psi}$$

$$\rho := \frac{A_s}{b \cdot d} \quad \rho = 0.011 \quad k := \sqrt{(\rho \cdot n)^2 + 2\rho \cdot n} - \rho \cdot n \quad k = 0.325 \quad j := 1 - \frac{k}{3} \quad j = 0.892$$

Moment due to concrete limits

Moment due to steel limit

$$M_{SC} := \frac{1}{2} \cdot f_c \cdot b \cdot k \cdot d \cdot \left(d - \frac{k \cdot d}{3} \right) \quad M_{SC} = 238 \text{ ft} \cdot \text{kip} \quad M_{SS} := A_s \cdot f_s \cdot j \cdot d \quad M_{SS} = 255 \text{ ft} \cdot \text{kip}$$

The maximum service moment is the minimum of the two values.

$$M_s := \min(M_{SS}, M_{SC}) \quad M_s = 238 \text{ ft} \cdot \text{kip}$$

(d) Determine the nominal moment capacity of the section.

$$a := \frac{A_s \cdot f_y}{0.85f_c \cdot b} \quad a = 3.84 \text{ in} \quad M_n := A_s \cdot f_y \cdot \left(d - \frac{a}{2} \right) \quad M_n = 440 \text{ ft} \cdot \text{kip}$$

(e) Compute the ratio of nominal moment capacity from part (d) to the service level moment from part (c)

$$\text{Ratio} := \frac{M_n}{M_s} \quad \text{Ratio} = 1.85 \quad 0.9 \text{ Ratio} = 1.66$$

$$\text{Ratio1} := \frac{M_s}{M_{cr1}} \quad \text{Ratio1} = 2.942$$

(f) Comment on your results with particular attention to comparing parts (a) and (b) and comparing part (e) to established load factors.

First, the extra computation of the uncracked transformed area gives an 18% increase in the cracking moment. Comparing the cracking moment to the service moment, Ratio1, shows that the service moment is almost 3 times the cracking moment. Therefore, unless the service moment is very close to the service moment, you can be assured that the section will crack.

Second, the margin of safety between the service moment and the nominal capacity is 1.8, 1.6 if a ϕ factor is included. This is greater than the ultimate load factors from ASCE-7 indicating that a service level design is far more conservative than LRFD design.

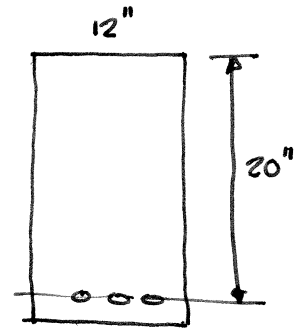
$$3.5 \quad f_y = 60,000 \text{ psi} \quad f'_c = 5000 \text{ psi}$$

$$a) \quad A_s = 2 \#8 = 2(0.79) = 1.58 \text{ in}^2$$

$$a = \frac{1.58(60)}{.85(5)12} = 1.86 \text{ in}$$

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) = 1.58(60) \left(20 - \frac{1.86}{2} \right) \frac{1}{12}$$

$$= 151 \text{ ft kips}$$



$$b) \quad A_s = 2 \#10 = 2(1.27) = 2.54 \text{ in}^2$$

$$a = \frac{A_s f_y}{.85 f'_c b} = \frac{2.54(60)}{.85(5)12} = 2.99 \text{ in}$$

$$M_n = 2.54(60) \left(20 - \frac{2.99}{2} \right) \frac{1}{12}$$

$$= 235 \text{ ft kips}$$

$$c) \quad A_s = 3 \#10 = 3(1.27) = 3.81 \text{ in}^2$$

$$a = \frac{3.81(60)}{.85(5)(12)} = 4.48 \text{ in}$$

$$M_n = 3.81(60) \left(20 - \frac{4.48}{2} \right) \frac{1}{12}$$

$$= 338 \text{ ft kips}$$

A check of net tensile strain shows $\epsilon_t = 0.0077$ so tensile steel has yielded.

Problem 3.6 A singly reinforced rectangular beam is to be designed, with an effective depth approximately 1.5 times the width, to carry a service load of 2000 lb/ft in addition to its own weight, on a 24 ft. simple span. The ACI Code factors are applied as usual. With $f_y = 60,000$ psi and $f'_c = 4000$ psi, determine the required concrete dimensions d , b , and h , and steel reinforcing bars for a) $\rho = 0.6 \rho_{max}$ and b) $\rho = \rho_{0.005}$. Include a sketch, to scale, of each cross section. Allow for No. 4 (No. 13) stirrups. Comment on your results.

Given properties

$$f_y := 60000 \text{ psi} \quad f'_c := 4000 \text{ psi} \quad l := 24 \text{ ft} \quad w_l := 2000 \cdot \frac{\text{lb}}{\text{ft}} \quad w_c := 150 \cdot \frac{\text{lb}}{\text{ft}^3}$$

Estimate beam dimensions for self weight determination

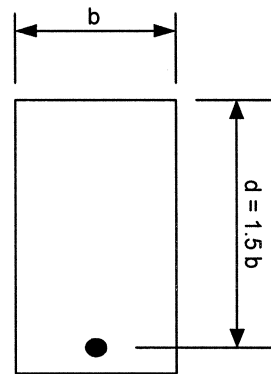
$$b := 12 \cdot \text{in} \quad d := 18 \cdot \text{in} \quad h := 21 \cdot \text{in}$$

$$w_o := b \cdot h \cdot w_c$$

$$w_u := 1.2 \cdot w_o + 1.6 \cdot w_l \quad w_u = 3.515 \frac{\text{kip}}{\text{ft}}$$

$$M_u := w_u \cdot \frac{l^2}{8} \quad M_u = 253.1 \text{ kip} \cdot \text{ft}$$

$$\beta_1 := 0.85 - \frac{0.05(f'_c - 4000 \text{ psi})}{1000 \text{ psi}} = 0.85$$



$$\rho_{max} := 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{0.003}{0.003 + 0.004} = 0.0206$$

a) For $\rho = 0.6 \rho_{max}$, find the section properties to carry M_u .

$$\rho := 0.6 \rho_{max} \quad \rho = 0.0124 \quad \text{From table} \quad \text{Resistance} := \rho \cdot f_y \cdot \left(1 - 0.588 \cdot \frac{\rho \cdot f_y}{f'_c} \right)$$

$$\text{Resistance} = 662 \text{ psi} \quad \text{Since } \rho < \rho_{0.005}, \text{ then } \phi := 0.90$$

$$b := \sqrt[3]{\frac{M_u}{2.25 \cdot \phi \cdot \text{Resistance}}} = 13.134 \text{ in} \quad d := 1.5 \cdot b \quad d = 19.7 \text{ in}$$

$$A_s := \rho \cdot b \cdot d \quad A_s = 3.2 \text{ in}^2$$

A solution is to use 4-#8, $b = 14$ ", $d = 20.5$ ", and $h = 22$ ".

$$A_s := 4 \cdot 0.79 \text{ in}^2 \quad b := 14 \text{ in} \quad d := 20.5 \text{ in} \quad \text{giving a design capacity of}$$

$$a := \frac{A_s \cdot f_y}{0.85 \cdot f'_c \cdot b} = 3.98 \text{ in}$$

$$\phi M_n := \phi \cdot A_s \cdot f_y \cdot \left(d - \frac{a}{2} \right) \quad \phi M_n = 263 \text{ ft} \cdot \text{kip} \quad > M_u \quad M_u = 253.1 \text{ ft} \cdot \text{kip} \quad \text{OK}$$

$$\begin{aligned}
 \text{b) } \rho_{005} &:= 0.85 \cdot \beta_1 \cdot \frac{f_c}{f_y} \cdot \frac{0.003}{0.003 + 0.005} \\
 \rho &:= \rho_{005} \quad \rho = 0.0181 \\
 \text{Resistance} &:= \rho \cdot f_y \cdot \left(1 - 0.588 \cdot \rho \cdot \frac{f_y}{f_c} \right) \quad \text{Resistance} = 911 \text{ psi} \\
 b &:= \sqrt[3]{\frac{M_u}{1.5^2 \cdot \phi \cdot \text{Resistance}}} = 11.8 \text{ in} \quad d := 1.5 \cdot b \quad d = 17.7 \text{ in} \\
 A_s &:= \rho \cdot b \cdot d \quad A_s = 3.78 \text{ in}^2
 \end{aligned}$$

This is satisfied by 4-#9 $A_s = 4.0 \text{ in}^2$. Beam dimensions would be $b = 12''$, $d = 18''$ and $h = 21''$ (or more when two layers of steel are used). The increase in depth accounts for the smaller area of steel.

Check:

$$\begin{aligned}
 b &:= 12 \text{ in} \quad d := 18 \text{ in} \quad A_s := 4 \cdot 1.0 \text{ in}^2 = 4.0 \text{ in}^2 \\
 a &:= \frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b} \quad a = 5.88 \text{ in} \\
 \phi M_n &:= \phi \cdot A_s \cdot f_y \cdot \left(d - \frac{a}{2} \right) \quad \phi M_n = 271.1 \text{ ft} \cdot \text{kip} > M_u \text{ OK}
 \end{aligned}$$

Comment: While there is a small savings in concrete using solution b, the final selection of reinforcement can often result in a reinforcement ratio greater than the code allows. In which case, the solution in a) is preferable.

3.7 A four span continuous beam of constant rectangular cross-section is supported at A, B, C, D, and E. The factored moments resulting from analysis are

At Supports, ft-kip	At midspan ft-kip
$M_a = 138$	$M_{ab} = 158$
$M_b = 220$	$M_{bc} = 138$
$M_c = 200$	$M_{cd} = 138$
$M_d = 220$	$M_{de} = 158$
$M_e = 138$	

Determine the required final concrete dimensions for this beam using $d=1.75 b$ and determine the reinforcement requirements at each critical moment section. Your final reinforcement ratio should not exceed $\rho = 0.6 \rho_{005}$, $f_y = 60,000$ psi and $f'_c = 6000$ psi

▣ Reinforcement Details

Given Properties

$$f'_c := 6000\text{psi} \quad f_y := 60000\text{psi} \quad \epsilon_u := 0.003$$

$$\beta_1 := 0.85 - 0.05 \cdot \frac{f'_c - 4000\text{psi}}{1000\text{psi}} = 0.75 \quad \phi := 0.90$$

Solution Approach:

Solve the first section using the Resistance Factors. Then, having selected a section, determine the reinforcement requirements for the remaining sections. Choose reinforcement bars to meet your criteria.

$$M_{n\text{reqd}} := \frac{220\text{ft}\cdot\text{kip}}{\phi} \quad M_{n\text{reqd}} = 244 \text{ft}\cdot\text{kip}$$

$$\rho_{005} := \frac{0.85 \cdot \beta_1 \cdot f'_c}{f_y} \cdot \left(\frac{\epsilon_u}{\epsilon_u + 0.005} \right) = 0.024 \quad \rho := 0.6 \rho_{005} = 0.014$$

$$R := \rho \cdot f_y \cdot \left(1 - .588 \frac{\rho \cdot f_y}{f'_c} \right) \quad R = 788 \text{psi}$$

$$b := \sqrt[3]{\frac{M_{n\text{reqd}}}{1.75^2 \cdot R}} = 10.7 \text{in} \quad d := 1.75 \cdot b = 18.7 \text{in}$$

$$A_s := b \cdot d \cdot \rho = 2.86 \text{in}^2 \quad n := \frac{A_s}{A_{s8}} = 3.6$$

Try the following dimensions and then check the solution

$$b := 10\text{in} \quad h := 22\text{in} \quad d := h - 4\text{in} \quad \text{Additional cover is required because the bars have to be in two layers, See Table A.7 in text.}$$

$$A_s := 4 \cdot A_{s8} \quad A_s = 3.16 \text{in}^2 \quad \text{Note: 3\#10 (No. 36) also works and allows the steel to be in one layer but require more steel.}$$

$$a := \frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b} = 3.72 \text{ in} \quad M_{n1} := A_s \cdot f_y \cdot \left(d - \frac{a}{2} \right) \quad M_{n1} = 255 \text{ ft} \cdot \text{kip} \quad \frac{M_{n1}}{M_{n\text{reqd}}} = 1.04 \quad \text{OK}$$

The next largest moment is 200 ft-kip. This is 10 percent less than the first interior joint so no adjustment of reinforcement is needed.

The largest positive moment is 158 ft-kip. This is 72 percent of the maximum moment so the steel requirements will be approximately:

$$A_{s2} := 0.72 A_s = 2.28 \text{ in}^2 \quad n_2 := \frac{A_{s2}}{A_{s8}} \quad n_2 = 2.88 \quad \text{use: } A_s := 3 A_{s8}$$

From Table A.7 use 3-#8 bars in one level

$$d_2 := h - 2.5 \text{ in} \quad a := \frac{A_s \cdot f_y}{0.85 f_c \cdot b} = 2.79 \text{ in} \quad M_{nab} := A_s \cdot f_y \cdot \left(d - \frac{a}{2} \right) = 197 \text{ ft} \cdot \text{kip}$$

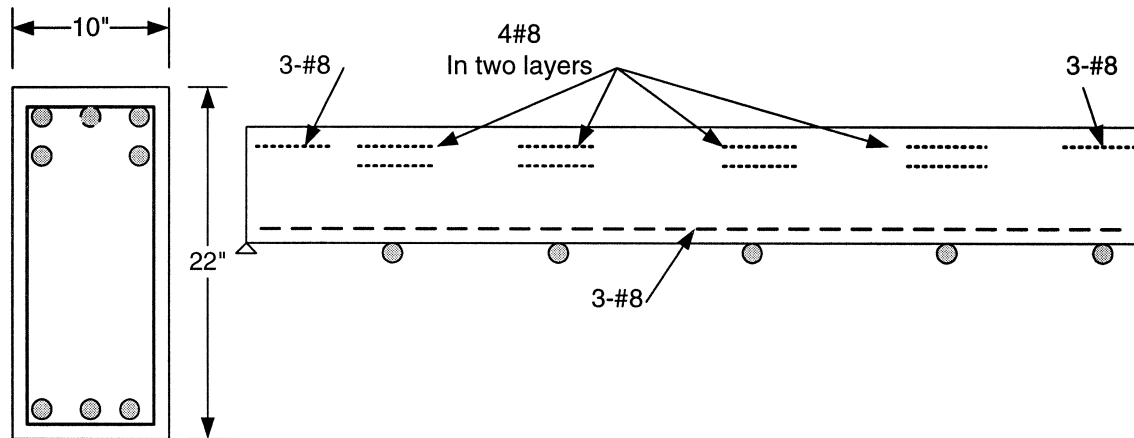
Check total Capacity $\frac{\phi \cdot M_{nab}}{158 \text{ ft} \cdot \text{kip}} = 1.12 \quad \text{OK}$

Design end negative moment at A and E

$$A_{s2} := \frac{138}{158} \cdot A_s = 2.07 \text{ in}^2 \quad \text{Use 3\#8} \quad A_s := 3 A_{s8} = 2.37 \text{ in}^2$$

$$a := \frac{A_s \cdot f_y}{0.85 f_c \cdot b} = 2.79 \text{ in} \quad M_{n2} := A_s \cdot f_y \cdot \left(d - \frac{a}{2} \right) \quad M_{n2} = 197 \text{ ft} \cdot \text{kip}$$

Check total Capacity $\frac{\phi \cdot M_{n2}}{158 \text{ ft} \cdot \text{kip}} = 1.12 \quad \text{OK}$



Problem 3.8 A two span continuous beam is supported on three concrete walls spaced 30 ft. on centers. A service live load of 1.5 kip/ft is to be carried in addition to the self weight of the beam and is to be applied in a pattern loading. The dimensions of the beam should be approximately $d=2b$, and the reinforcement is to be varied according to the demand. Determine concrete dimensions at all critical sections but select a constant section for the beam. Allow for No. 4 (No. 13) stirrups. Use a span to depth ratio of 15 for your first estimate of the depth. Adjust the depth if the reinforcement ratio is too high. Include sketches drawn to scale. Use $f'_c = 6000$ psi and $f_y = 60,000$ psi.

▣ Reinforcement Details

Given Properties

$$f'_c := 6000\text{psi} \quad f_y := 60000\text{psi} \quad \phi := 0.90 \quad \beta_1 := 0.85 - 0.05 \cdot \frac{f'_c - 4000\text{psi}}{1000\text{psi}} = 0.75$$

$$w_l := 1.5 \frac{\text{kip}}{\text{ft}} \quad L := 30\text{ft} \quad \gamma_c := 150\text{pcf}$$

Solution: Begin by estimating a beam depth and width, then compute the girder load.. Compute the maximum negative moment due to both spans being loaded then the maximum positive moment with only one span loaded with live load. Try:

$$h := \frac{L}{15} = 24 \text{ in} \quad d := h - 2.5\text{in} = 21.5 \text{ in} \quad b := \frac{h}{2} = 12.0 \text{ in}$$

$$w_g := \gamma_c \cdot b \cdot h = 0.30 \frac{\text{kip}}{\text{ft}} \quad w_u := 1.2w_g + 1.6w_l = 2.76 \frac{\text{kip}}{\text{ft}}$$

$$M_{\text{neg}} := \frac{w_u \cdot L^2}{8} = 310 \text{ ft} \cdot \text{kip}$$

By trial estimate a, then compute A_s for the negative moment over the support, select final A_s and check M_n

$$a := 4.0\text{in} \quad A_s := \frac{M_{\text{neg}}}{\phi \cdot f_y \cdot \left(d - \frac{a}{2}\right)} = 3.54 \text{ in}^2 \quad \text{Try 3-}\#10 \quad A_s := 3 \cdot A_{s10} = 3.81 \text{ in}^2$$

$$a := \frac{A_s \cdot f_y}{0.85 \cdot f'_c \cdot b} = 3.74 \text{ in} \quad M_n := A_s \cdot f_y \cdot \left(d - \frac{a}{2}\right) = 374 \text{ ft} \cdot \text{kip} \quad \phi \cdot M_n = 337 \text{ ft} \cdot \text{kip OK}$$

$$c := \frac{a}{\beta_1} = 4.98 \text{ in} \quad \frac{c}{d} = 0.232 < 3/8 \text{ therefore } \phi = 0.9 \text{ is OK}$$

By trial, compute the required A_s for the positive moment condition.

The maximum positive moment occurs with dead load on both spans and live load on one span only. The maximum moment for the DL occurs at $3/8L$ in from the end and the maximum moment for live load occurs $7/16L$ in from the end. Conservatively, take the sum of the two conditions and apply them at the same location.

$$M_{\text{pos}} := \frac{49 \cdot 1.6w_l \cdot L^2}{512} + \frac{9 \cdot 1.2 \cdot w_g \cdot L^2}{128} = 229 \text{ ft} \cdot \text{kip}$$