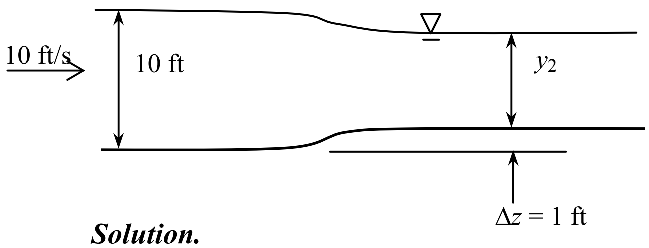
CHAPTER 2. Specific Energy

2.1. Water is flowing at a depth of 10 ft with a velocity of 10 ft/s in a channel of rectangular section. Find the depth and change in water surface elevation caused by a smooth upward step in the channel bottom of 1 ft. What is the maximum allowable step size so that choking is prevented? (Use a head loss coefficient = 0.)



Solution.

$$q = V_1 y_1 = (10)(10) = 100 \text{ ft}^2/\text{s}$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{100^2}{32.2}\right)^{1/3} = 6.77 \text{ ft} \Rightarrow \text{subcritical approach flow}$$

$$E_c = \frac{3}{2} y_c = (1.5)(6.77) = 10.16 \text{ ft}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 10 + \frac{10^2}{64.4} = 11.55 \text{ ft}$$

Now because $E_1 - \Delta z > E_c$, there is no choking. So write the energy equation from 1 to 2 and find a subcritical solution for y_2:

$$E_1 = y_2 + \frac{q^2}{2gy_2^2} + \Delta z$$

$$11.55 = y_2 + \frac{100^2}{64.4 y_2^2} + 1.0$$

$$y_2 + \frac{155.3}{y_2^2} = 10.55$$

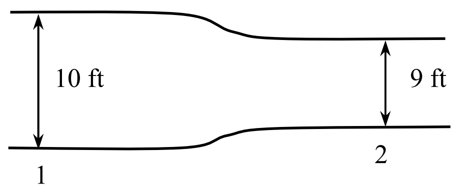
which can be solved by trial and error or an equation solver to give $\boldsymbol{y_2} \mathbf{= 8.29}$ **ft**. The water surface elevation drops by 10 – (8.29 +1) = **0.71 ft**.

For the limiting choking case, set the specific energy at section 2 equal to E_c:

$$E_1 = 11.55 = E_c + \Delta z_c = 10.16 + \Delta z_c$$

so Δz_c = 11.55 – 10.16 = **1.39 ft**.

2.2. The upstream conditions are the same as in Exercise 2.1, but there is a smooth contraction in width from 10 ft to 9 ft and a horizontal bottom. Find the depth of flow and change in water surface elevation in the contracted section. What is the greatest allowable contraction in width so that choking is prevented? (Head loss coefficient = 0.)



Solution.

From Exercise 2.1, q_1 = 100 cfs/ft; y_{c1} = 6.77 ft; and E_1 = 11.55 ft. Then from continuity, we have q_2 = (10/9) q_1 = (10/9)(100) = 111.1 cfs/ft and $y_{c2} = (111.1^2/32.2)^{1/3}$ = 7.26 ft. Writing the energy equation from 1 to 2:

$$E_1 = y_2 + \frac{q_2^2}{2gy_2^2}$$

$$11.55 = y_2 + \frac{(111.1)^2}{64.4y_2^2} = y_2 + \frac{191.7}{y_2^2}$$

from which the subcritical solution is $\boldsymbol{y_2}$ **= 9.36 ft** with a water surface elevation drop of (10 – 9.36) = **0.64 ft**.

For the limiting choking case, $E_1 = E_{c2} = 1.5\, y_{c2}$, so that

$$11.55 = 1.5\left(\frac{q_2^2}{g}\right)^{1/3}$$

$$q_2 = [(2/3)(11.55)]^{3/2}(32.2)^{1/2} = 121.2 \text{ cfs/ft}$$

But $b_2 = Q/q_2$ = 1000/121.2 = **8.25 ft**.

2.3. The upstream conditions in a rectangular channel are the same as in Exercise 2.1 with a smooth contraction in width from 10 ft to 8 ft. How much should the channel bottom drop to maintain a constant water surface elevation through the transition? (Head loss coefficient = 0)

Solution.

From Exercise 2.1, q_1 = 100 cfs/ft; y_{c1} = 6.77 ft; and E_1 = 11.55 ft for y_1 = 10 ft. Then from continuity, we have q_2 = (10/8) q_1 = (10/8)×(100) = 125 cfs/ft and $y_{c2} = (125^2/32.2)^{1/3}$ = 7.86 ft. Writing the energy equation from 1 to 2 and solving for Δz with $y_2 = y_1 + \Delta z$:

$$E_1 = y_2 + \frac{q_2^2}{2gy_2^2} - \Delta z$$

$$11.55 = y_1 + \Delta z + \frac{(125)^2}{64.4(y_1 + \Delta z)^2} - \Delta z = 10 + \frac{242.6}{(10 + \Delta z)^2}$$

from which the solution is **Δz = 2.51 ft** and **y_2 = 12.51 ft,** which is subcritical with a water surface elevation drop of zero.

2.4. Determine the downstream depth in the transition and the change in water surface elevation if the channel bottom rises 0.15 m and the upstream conditions are a velocity of 4.5 m/s and a depth of 0.6 m.

Solution.

Assuming a rectangular channel of constant width with negligible head loss, as in Exercise 2.1, check the approach conditions:

$$q = (4.5)(0.6) = 2.7 \text{ m}^2/\text{s}$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{2.7^2}{9.81}\right)^{1/3} = 0.906 \text{ m}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 0.6 + \frac{4.5^2}{19.62} = 1.63 \text{ m}$$

The approach flow is supercritical, and the minimum specific energy, $E_c = 1.5y_c = (1.5)(0.906) =$ 1.36 m. Because $E_1 - \Delta z = 1.63 - 0.15 = 1.48 \text{ m} > E_c$, choking is not expected. Solve the energy equation for y_2 in the supercritical flow regime:

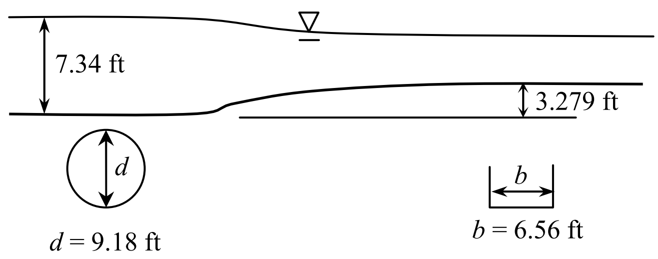
$$E_1 = 1.63 = y_2 + \frac{q^2}{2gy_2^2} + \Delta z = y_2 + \frac{2.7^2}{19.62y_2^2} + 0.15$$

$$y_2 + \frac{0.3716}{y_2^2} = 1.48$$

and the solution is **y_2 = 0.683 m**. The water surface elevation rises by 0.683 + 0.15 – 0.60 = 0.233 m. The limiting choking case is given by $\Delta z = E_1 - E_c = 1.63 - 1.36 = 0.27$ m.

As in Figure 2.9 for a width contraction with a supercritical approach flow, there is a second mode of choking in this example with a hydraulic jump upstream of the transition and critical depth in the transition. As discussed in Chapter 3, the sequent depth for an upstream hydraulic jump can be calculated to be 1.30 m corresponding to a specific energy after the jump of E_1 = 1.52 m; however, in this event, $E_1 - \Delta z = 1.37$ m, which is still greater than E_c , so no choking occurs by this mode either.

2.5. Determine the downstream depth in a subcritical transition if Q=262 cfs and the channel bottom rises 3.279 ft in going from an upstream circular channel to a downstream rectangular channel. The upstream circular channel has a diameter of 9.18 ft and a depth of flow of 7.34 ft. The downstream rectangular channel has a width of 6.56 ft. Neglect the head loss.



Solution.

For the upstream circular channel, calculate the flow area and top width for the given depth of 7.34 ft:

$$\theta = 2\cos^{-1}(1-2\frac{y}{d}) = 2\cos^{-1}(1-2\frac{7.34}{9.18}) = 4.4264 \text{ rad}$$
$$A = (\theta - \sin\theta)\frac{d^2}{8} = [4.4264 - \sin(4.4264)]\frac{9.18^2}{8} = 56.73 \text{ ft}^2$$
$$B = d\sin(\theta/2) = 9.18\sin(4.4264/2) = 7.35 \text{ ft}$$

Then the approach specific energy and Froude number are given by

$$E_1 = y_1 + \frac{Q^2}{2gA^2} = 7.34 + \frac{262^2}{64.4 \times 56.73^2} = 7.67 \text{ ft}$$
$$\mathbf{F}_1 = \frac{QB^{1/2}}{g^{1/2}A^{3/2}} = \frac{262 \times 7.35^{1/2}}{32.2^{1/2} \times 56.73^{3/2}} = 0.29$$

The approach flow is subcritical. Check for choking with critical depth occurring in the downstream rectangular section:

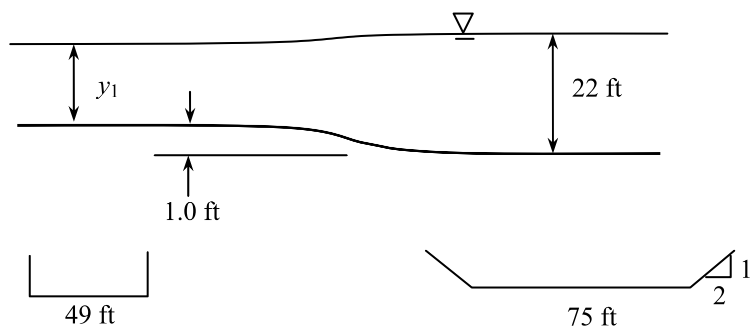
$$y_{c2} = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(262/6.56)^2}{32.2}\right)^{1/3} = 3.67 \text{ ft}$$
$$E_{c2} = 1.5y_c = 1.5 \times 3.67 = 5.51 \text{ ft}$$

Note that from the energy equation, $E_2 = E_1 - \Delta z = 7.67 - 3.279 = 4.39$ ft $< E_{c2}$. Thus, choking will occur with $\mathbf{y_2 = 3.67}$ **ft**, and y_1 has to be recalculated from

$$E_1 = E_{c2} + \Delta z = 5.51 + 3.279 = 8.79 \text{ ft}$$
$$y_1 + \frac{Q^2}{2gA_1^2} = y_1 + \frac{1066}{A_1^2} = 8.79$$

which can be solved by assuming a value of y_1; calculating for the circular cross-section, $\theta = 2\cos^{-1}(1 - 2y_1/d)$, and the flow area, $A_1 = (\theta - \sin\theta)d^2/8$; substituting to obtain E_1; and repeating until $E_1 = 8.79$ ft. The result is $\boldsymbol{y_1} = \mathbf{8.53}$ **ft**; $\theta = 5.2058$ rad; $A_1 = 64.12$ ft^2; and $E_1 = 8.789$ ft. Choking causes the upstream depth to increase by (8.53 – 7.34) = 1.19 ft.

2.6. Determine the upstream depth of flow in a subcritical transition from an upstream rectangular flume that is 49 ft wide to a downstream trapezoidal channel with a width of 75 ft and side slopes of 2:1. The transition bottom drops 1 ft from the upstream flume to the downstream trapezoidal channel. The flow rate is 12,600 cfs, and the depth in the downstream trapezoidal channel is 22 ft. Use a head loss coefficient of 0.5.



Solution.

First, calculate the downstream conditions:

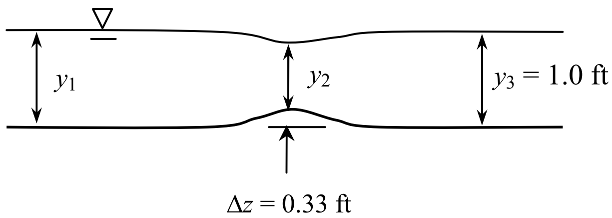
$$A_2 = y_2(b_2 + my_2) = (22.0)(75 + 2\times 22.0) = 2618\ \text{ft}^2$$
$$B_2 = b_2 + 2my_2 = 75 + 2\times 2\times 22.0 = 163\ \text{ft}$$
$$V_2 = \frac{Q}{A_2} = \frac{12{,}600}{2618} = 4.81\ \text{ft/s}$$
$$\mathbf{F}_2 = \frac{V_2}{[gA_2/B_2]^{1/2}} = \frac{4.81}{[32.2\times 2618/163]^{1/2}} = 0.21$$
$$E_2 = y_2 + \frac{Q^2}{2gA_2^2} = 22.0 + \frac{12{,}600^2}{64.4\times 2618^2} = 22.36\ \text{ft}$$

The downstream flow is subcritical, and we are looking for a subcritical depth upstream. Writing the energy equation including the head loss, and assuming that $V_1 > V_2$, we have

$$E_1 + \Delta z = E_2 + h_L$$
$$y_1 + \frac{Q^2}{2gA_1^2} + 1.0 = 22.36 + 0.5\left|\frac{Q^2}{2gA_1^2} - \frac{Q^2}{2gA_2^2}\right|$$
$$y_1 + (1.0 - 0.5)\frac{12{,}600^2}{64.4\times(49y_1)^2} = 21.36 - 0.5\frac{4.81^2}{64.4}$$
$$y_1 + \frac{513.4}{y_1^2} = 21.18$$

Solving, the result is $\boldsymbol{y_1} = \mathbf{19.88\ ft}$ where $y_{c1} = (q^2/g)^{1/3} = [(12{,}600/49)^2/32.2]^{1/3} = 12.7$ ft. The corresponding head loss is 1.12 ft, where $E_1 = 22.48$ ft and $V_1 = 12.94$ ft/s.

2.7. In a horizontal rectangular flume, suppose that a smooth "bump" with a height of 0.33 ft has been placed on the channel bottom. The discharge per unit width in the flume is 0.4 cfs/ft. Determine the depth at the obstruction for a tailwater depth of 1.0 ft and negligible head losses. Sketch the results on a specific energy diagram.



Solution.

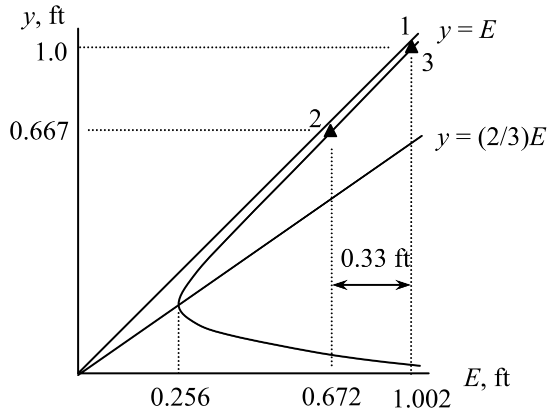
Calculate critical conditions and downstream specific energy:

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{0.4^2}{32.2}\right)^{1/3} = 0.171\,\text{ft}$$
$$E_c = 1.5y_c = 1.5\times 0.171 = 0.256\,\text{ft}$$
$$E_3 = y_3 + \frac{q^2}{2gy_3^2} = 1.0 + \frac{0.4^2}{64.4\times 1.0^2} = 1.0025\,\text{ft}$$

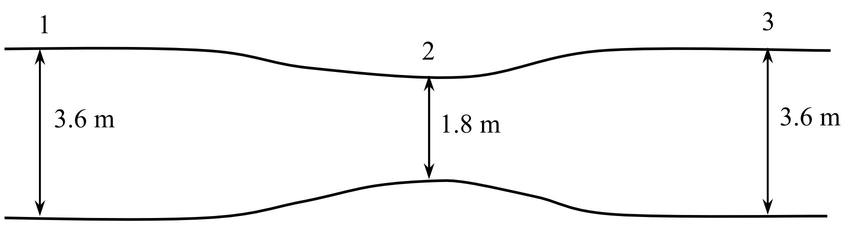
Because $E_3 - \Delta z = 1.0025 - 0.33 = 0.67$ ft $> E_c$, there is no choking, and the depth at point 2 is subcritical. Writing the energy equation from point 2 to point 3, we have

$$y_2 + \frac{q^2}{2gy_2^2} = E_3 - \Delta z = 1.0025 - 0.33 = 0.6725$$
$$y_2 + \frac{0.4^2}{64.4\times y_2^2} = y_2 + \frac{0.002484}{y_2^2} = 0.6725$$

Solving for y_2, we obtain $\boldsymbol{y_2} = \mathbf{0.667\ ft}$. As a consequence, the dip in the water surface over the bump is barely perceptible because we are so high up on the upper limb of the specific energy curve. Furthermore, because there is no energy loss, the depth at point 1 is identical to that at point 3. If the tailwater (y_3) is dropped to 0.6 ft, we get $y_2 = 0.23$ ft and a dip in the water surface elevation of 0.04 ft over the bump. Continuing to drop the tailwater elevation results in choking with passage through critical depth at the bump to supercritical flow followed by a hydraulic jump downstream of the bump. The specific energy diagram follows.



2.8. A rectangular channel 3.6 m wide contracts to a 1.8-m wide rectangular channel and then expands back to the 3.6-m width. The contraction is gradual enough that head losses can be neglected, but the expansion loss coefficient is 0.5. The discharge through the transition is 10 m^3/s. If the downstream depth at the re-expanded section is 2.4 m, calculate the depths at the approach section and the contracted section. Show the positions of the depth and specific energy for all three sections on a specific energy diagram.



Solution.

From continuity, $q_3 = q_1 = 10/3.6 = 2.778$ m^2/s and $q_2 = 10/1.8 = 5.556$ m^2/s. Check critical conditions and calculate downstream specific energy:

$$y_{c3} = \left(\frac{q_3^2}{g}\right)^{1/3} = \left(\frac{2.778^2}{9.81}\right)^{1/3} = 0.923\text{ m}$$

$$y_{c2} = \left(\frac{q_2^2}{g}\right)^{1/3} = \left(\frac{5.556^2}{9.81}\right)^{1/3} = 1.465\text{ m}$$

$$E_{c2} = 1.5 y_{c2} = 1.5 \times 1.465 = 2.198\text{ m}$$

$$E_3 = y_3 + \frac{q_3^2}{2gy_3^2} = 2.4 + \frac{2.778^2}{19.62 \times 2.4^2} = 2.468\text{ m}$$

Now because $E_3 > E_{c2}$, choking cannot occur, and we are seeking a subcritical depth at section 2. The energy equation from 2 to 3, including the head loss, is

$$y_2 + \frac{q_2^2}{2gy_2^2} = E_3 + h_L$$

$$y_2 + \frac{5.556^2}{19.62 \times y_2^2} = 2.468 + 0.5\left(\frac{q_2^2}{2gy_2^2} - \frac{q_3^2}{2gy_3^2}\right)$$

$$y_2 + 0.5 \times \frac{1.573}{y_2^2} = 2.468 - 0.5 \times \frac{2.778^2}{19.62 \times 2.4^2}$$

$$y_2 + \frac{0.786}{y_2^2} = 2.434$$

from which $\mathbf{y_2 = 2.28}$ **m** and $E_2 = y_2 + (q_2)^2/(2gy_2^2) = 2.28 + 5.556^2/[(19.62)(2.28)^2] =$ 2.583 m. The head loss is $(E_2 - E_3) = 0.12$ m. The energy equation from point 1 to point 2, neglecting head loss, is

$$y_1 + \frac{q_1^2}{2gy_1^2} = E_2 = 2.583$$

$$y_1 + \frac{2.778^2}{19.62 \times y_1^2} = y_1 + \frac{0.3933}{y_1^2} = 2.583$$

The solution is $\mathbf{y_1 = 2.52}$ **m**, which is 0.12 m higher than the downstream depth of 2.4 m. The increase in water surface elevation is approximately equal to the head loss because of the small difference in the velocity heads. See Figure 2.11 in the text for the specific energy diagram.

2.9. The head upstream of a circular culvert having a diameter of 6.0 ft is 5.0 ft above the culvert invert. If critical depth occurs at the culvert entrance, what is the discharge if the approach velocity head is negligible? Suppose that an impervious plug of mud and debris blocks the lower 2.0 ft of the culvert entrance above the invert in the form of a horizontal sill, what will the discharge be for the same head of 5.0 ft above the invert? Neglect entrance energy losses.

Solution.

One approach is to solve Equation 2.20 (nonrectangular channel) for y_c using the geometric relationships for a circular channel:

$$E_c = y_c + \frac{A_c}{2B_c}$$

$$5.0 = \frac{d}{2}[1 - \cos(\theta_c/2)] + \frac{(\theta_c - \sin\theta_c)d}{16\sin(\theta_c/2)}$$

Substituting d = 6.0 ft and solving by trial and error for θ_c, the result is θ_c =3.499 rad and y_c = 3.53 ft. The area and top width are given by

$$A_c = (\theta - \sin\theta)\frac{d^2}{8} = (3.499 - \sin(3.499))\frac{6^2}{8} = 17.32\,\text{ft}^2$$
$$B_c = d\sin(\theta/2) = 6.0\sin(3.499/2) = 5.904\,\text{ft}$$

Then it must be true that $\mathbf{F}^2$ = 1.0 at $y = y_c$ which can be solved for Q:

$$Q = \frac{\sqrt{g}A_c^{3/2}}{B_c^{1/2}} = \frac{\sqrt{32.2}(17.32)^{3/2}}{5.904^{1/2}} = 168\,\text{cfs}$$

Alternatively, Figure 2.14 can be used with E_c/d = 5/6 = 0.833 from which $Q/[g^{1/2}d^{5/2}]$ = 0.33 and Q = 165 cfs.

For the second part of the problem, the entrance geometry is different. First, the area of the sill is determined from

$$\theta_s = 2\cos^{-1}(1.0 - 2\frac{\Delta z}{d}) = 2\cos^{-1}(1.0 - 2\times\frac{2}{6}) = 2.462\,\text{rad}$$
$$\text{A}_\text{s} = (\theta - \sin\theta)\frac{d^2}{8} = (2.462 - \sin 2.462)\times\frac{36}{8} = 8.25\,\text{ft}^2$$

Then the energy equation written in the form of (2.20) can be written as

$$E_1 - \Delta z = E_c = y_c + \frac{A - A_s}{2B_c}$$
$$5.0 - 2.0 = 3.0 = \frac{d}{2}[1 - \cos(\theta/2)] - 2.0 + \frac{(\theta - \sin\theta)(d^2/8) - 8.25}{2d\sin(\theta/2)}$$

in which θ is the angular measure for the depth measured from the culvert invert rather than the sill, and A is the sill area plus the flow area. The result is θ = 3.804 rad and $A = (\theta - \sin\theta)(d^2/8)$ = 19.87 ft^2 so that $A_c = (A - A_s)$ = 11.62 ft^2. Also, we have $B_c = d\sin(\theta/2)$ = 5.674 ft, and y_c = (6/2)[1 − cos (3.804/2)] − 2.0 = 1.976 ft. Finally, the discharge comes from setting the Froude number equal to one as before:

$$Q = \frac{\sqrt{g}A_c^{3/2}}{B_c^{1/2}} = \frac{\sqrt{32.2}(11.62)^{3/2}}{5.674^{1/2}} = 94.4\,\text{cfs}$$

2.10. Determine the discharge in a circular culvert on a steep slope if the diameter is 1.0 m and the upstream head is 1.3 m with an unsubmerged entrance. Also calculate the critical depth. Neglect entrance losses. Repeat for a box culvert that is 1.0-m square.

Solution.

The entrance depth for a steep culvert is critical depth. Write the energy equation from a point just upstream of the entrance to the culvert entrance, and set the Froude number squared equal to one for critical conditions:

$$1.3 = y_c + \frac{Q^2}{2gA_c^2}$$

$$\frac{Q^2 B_c}{gA_c^3} = 1.0$$

Solve by trial by assuming a value of y_c; calculating θ_c, A_c, and B_c from the circular channel geometry; solving for Q from the second equation; and substituting back into the first equation to determine if the upstream head is equal to 1.3 m. The equations needed for the circular channel geometry are

$$\theta_c = 2\cos^{-1}(1.0 - 2\frac{y_c}{d})$$

$$A_c = (\theta - \sin\theta)\frac{d^2}{8}$$

$$B_c = d\sin(\theta/2)$$

in which d = diameter = 1.0 m. Organizing the computations in a table or a spreadsheet, we have

y_c, m	θ_c	A_c, m^2	B_c, m	Q, m^3/s	H, m
0.8	4.4286	0.674	0.800	1.94	1.22
0.9	4.9962	0.744	0.600	2.60	1.52
0.833	4.5993	0.699	0.746	2.12	1.30

The final iteration gives a discharge of **2.12 m^3/s** with critical depth equal to **0.833 m**. The solution can be obtained approximately from Figure 2.14. In this case, E_c/d = 1.3, and from Figure 2.14, we read $Z = Q/(g^{1/2} d^{5/2})$ = 0.7; therefore, $Q = (0.7)(9.81)^{1/2}$ = 2.2 m^3/s, which is acceptable considering the graphical error.

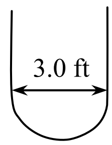
For a 1.0-m square box culvert, the channel shape is rectangular. Then for this case, we have

$$y_c = \frac{2}{3}H = \frac{2}{3}(1.30) = 0.867 \text{ m}$$

$$q = (gy_c^3)^{1/2} = (9.81 \times 0.867^3)^{1/2} = 2.53 \text{ m}^2/\text{s}$$

and $Q = bq$ = **2.53 m^3/s**.

2.11. An open channel has a semicircular bottom and vertical, parallel walls. If the diameter, *d*, is 3 ft, calculate the critical depth and the minimum specific energy for two discharges, 10 cfs and 30 cfs.



Solution.

First, calculate the critical discharge for the water level just filling the semicircular portion of the cross section. For this case, $y_c = 1.5$ ft; $A_c = \pi d^2/8 = \pi(3)^2/8 = 3.53$ ft^2; and $B_c = 3.0$ ft. Then from the Froude number criterion, we have

$$Q_c = \frac{g^{1/2} A_c^{3/2}}{B_c^{1/2}} = \frac{32.2^{1/2} \times 3.53^{3/2}}{3.0^{1/2}} = 21.7\ \text{cfs}$$

Hence, the lower discharge is for a circular geometry, and the higher discharge has a critical depth above the semicircular portion of the cross-section. For the discharge of 10 cfs, set the Froude number squared for a circular section to unity to obtain

$$\frac{A_c^3}{B_c} = \frac{Q^2}{g} = \frac{10^2}{32.2} = 3.106$$

$$\frac{[(\theta - \sin\theta) d^2/8]^3}{d \sin(\theta/2)} = 3.106$$

$$\frac{(\theta - \sin\theta)^3}{\sin(\theta/2)} = 6.544$$

from which we find $\theta = 2.462$ rad and $y_c = (d/2)[1 - \cos(\theta/2)] =$ **1.00 ft**. The corresponding values of area and top width are $A_c = 2.063$ ft^2 and $B_c = 2.828$ ft. Then $E_c = y_c + D_c/2 = 1.00 + 2.063/[(2)(2.828)] =$ **1.365 ft**.

For $Q = 30$ cfs, we add the value of flow area for the semicircular section (3.53 ft^2) to the area for a rectangular section stacked on top. The top width remains constant at 3.0 ft. Setting the Froude number squared to unity results in

$$\frac{A_c^3}{B_c} = \frac{Q^2}{g} = \frac{30^2}{32.2} = 27.95$$

$$\frac{[3.53 + 3 \times (y_c - 1.5)]^3}{3.0} = 27.95$$

and solving we obtain $\boldsymbol{y_c}$ **= 1.78 ft** for which $A_c = 4.37$ ft^2 and $B_c = 3.0$ ft. Then $E_c = 1.78 + 4.37/[(2)(3.0)] =$ **2.508 ft**.

2.12. Derive an exact solution for critical depth in a parabolic channel and place it in dimensionless form. Repeat for a triangular channel.

Solution.

Referring to Table 2-1 for the parabolic channel, the bank-full depth and width are designated by y_1 and B_1, respectively. Setting the Froude number squared to a value of unity, and substituting the expressions for flow area and top width from the table into the equation, the result for critical depth in a parabolic channel is obtained:

$$\frac{A_c^3}{B_c} = \frac{Q^2}{g}$$

$$\frac{[(2/3)(B_1/y_1^{1/2})y_c^{3/2}]^3}{(B_1/y_1^{1/2})y_c^{1/2}} = \frac{Q^2}{g}$$

$$\left(\frac{y_c}{y_1}\right)^4 = \left(\frac{3}{2}\right)^3 \frac{Q^2}{gB_1^2 y_1^3}$$

$$\frac{y_c}{y_1} = 1.355\left[\frac{Q}{g^{1/2}B_1 y_1^{3/2}}\right]^{1/2} \quad \text{(parabolic)}$$

For the triangular channel, refer again to Table 2-1, and repeat the procedure above using the triangular channel geometric expressions in which m = side-slope ratio:

$$\frac{A_c^3}{B_c} = \frac{Q^2}{g}$$

$$\frac{(my_c^2)^3}{2my_c} = \frac{Q^2}{g}$$

$$y_c = 1.149\left[\frac{Q}{g^{1/2}m}\right]^{2/5} \quad \text{(triangular)}$$

While y_c could be nondimensionalized by the bank-full top width, for example, the only geometric factor needed is the side-slope ratio, so the expression is left in this form.

2.13. Show that the ratio of critical depth to minimum specific energy, y_c/E_c, is 0.80 for a triangular channel and 0.75 for a parabolic channel.

Solution.

Substituting the geometric expressions for a triangular channel into Equation 2.20 for a nonrectangular channel results in

$$E_c = y_c + \frac{A_c}{2B_c} = y_c + \frac{my_c^2}{4my_c} = \frac{5}{4} y_c$$

Then it follows directly that $y_c/E_c = 0.80$. Repeating the substitution into (2.20) for parabolic geometry results in

$$E_c = y_c + \frac{A_c}{2B_c} = y_c + \frac{(2/3)(B_1/y_1)^{1/2} y_c^{3/2}}{2(B_1/y_1)^{1/2} y_c^{1/2}} = \frac{4}{3} y_c$$

from which it follows that $y_c/E_c = 0.75$.

2.14. A parabolic-shaped irrigation canal has a top width of 10 m at a bank-full depth of 2 m. Calculate the critical discharge, Q_c (i.e., the discharge for which the depth of uniform flow is equal to critical depth) for a uniform flow depth of 1.0 m. If $Q < Q_c$ for the uniform flow depth of 1.0 m, will the uniform flow be supercritical or subcritical?

Solution.

Calculate the flow area and top width of flow for $y = 1.0$ m using the geometric expressions from Table 2.1.

$$B = \frac{B_1}{y_1^{1/2}} y^{1/2} = \frac{10}{2^{1/2}} (1.0)^{1/2} = 7.07 \text{ m}$$
$$A = (2/3)By = (2/3)(7.07)(1.0) = 4.71 \text{ m}^2$$

Then from the definition of the Froude number, the critical discharge is defined for the Froude number equal to unity and $y = y_c$ to give

$$Q_c = \frac{g^{1/2} A^{3/2}}{B^{1/2}} = \frac{(9.81)^{1/2} \times (4.71)^{3/2}}{7.07^{1/2}} = 12.04 \text{ m}^3\text{/s}$$

If the uniform flow depth is equal to 1.0 m for $Q < Q_c$, then the Froude number is less than one, and the flow is subcritical.

2.15. A USGS study of natural channel shapes in the western United States reports an average ratio of maximum depth to hydraulic depth in the main channel (with no overflow) of $y/D = 1.55$ for 761 measurements.

(*a*) Calculate the ratio of maximum depth to hydraulic depth for a (1) triangular channel; (2) parabolic channel; (3) rectangular channel. What do you conclude?

(*b*) Calculate the discharge for a bank-full Froude number of $\mathbf{F}_1 = 1.0$ if $y/D = 1.55$ and $B_1 = 100$ ft for $y_1 = 10$ ft. What is the significance of this discharge?

Solution.

(*a*) For each geometric shape, substitute the expressions for flow area and top width into the definition of hydraulic depth:

Triangular:

$$D = \frac{A}{B} = \frac{my^2}{2my} = \frac{y}{2} \quad \Rightarrow \frac{y}{D} = 2.0$$

Parabolic:

$$D = \frac{A}{B} = \frac{(2/3)By}{B} = (2/3)y \quad \Rightarrow \frac{y}{D} = 1.5$$

Rectangular:

$$D = \frac{A}{B} = \frac{by}{b} = y \quad \Rightarrow \frac{y}{D} = 1.0$$

The implication is that natural channels from this data set are more nearly parabolic in shape.

(*b*) For $y/D = 1.55$ at bank-full flow, we can show that

$$\frac{y_1}{A_1/B_1} = 1.55$$

$$A_1 = \frac{B_1 y_1}{1.55} = \frac{(100)(10)}{1.55} = 645 \text{ ft}^2$$

Set the Froude number equal to unity and solve for Q_{c1} = critical bank-full discharge:

$$Q_{c1} = \frac{g^{1/2} A_1^{3/2}}{B_1^{1/2}} = \frac{(32.2)^{1/2}(645)^{3/2}}{100^{1/2}} = \mathbf{9295\ cfs}$$

If the actual $Q > Q_{c1}$, which is referred to as the upper limiting discharge Q_U in the text, then there is only one critical depth that occurs in overbank flow. In this case, any flows in the main channel alone would be supercritical.

2.16. A natural channel cross-section has a bank-full cross-sectional area of 45 m^2 and a top width of 37.5 m. The maximum value of $\mathbf{F}_c/\mathbf{F}_1$ has been calculated to be 1.236. Find the discharge range, if any, within which multiple critical depths could be expected.

Solution.

From the data given, A_1 = 45 m^2 and B_1 = 37.5 m. First calculate the upper limiting discharge, Q_U, as

$$Q_U = \frac{g^{1/2} A_1^{1/2}}{B_1^{1/2}} = \frac{(9.81)^{1/2}(45)^{3/2}}{37.5^{1/2}} = 154\ \text{m}^3/\text{s}$$

Then the lower limiting discharge is calculated by

$$\frac{Q_U}{Q_L} = \frac{\mathbf{F}_{c\max}}{\mathbf{F}_1} = 1.236$$

$$Q_L = \frac{154}{1.236} = 125\ \text{m}^3/s$$

So for Q in the range of **125 to 154 m^3/s**, there are two values of critical depth, one in main channel flow alone, and one in overbank flow for this cross section.

2.17. The main channel of North Fork, Peachtree Creek in Atlanta can be approximated as a parabolic channel with a bank-full depth of 8.0 ft and a bank-full top width of 50 ft. There are symmetric floodplains on either side of the main channel that are perfectly flat each with a width of 150 ft. If the flow rate is 3500 cfs, is it possible for there to be multiple critical depths for this cross-section? Use the computer program **Ycomp** on the book website to calculate the critical depth(s) for Q = 3500 cfs and Q = 3000 cfs.

Solution.

First check the upper limiting discharge for multiple critical depths from Equation 2.35:

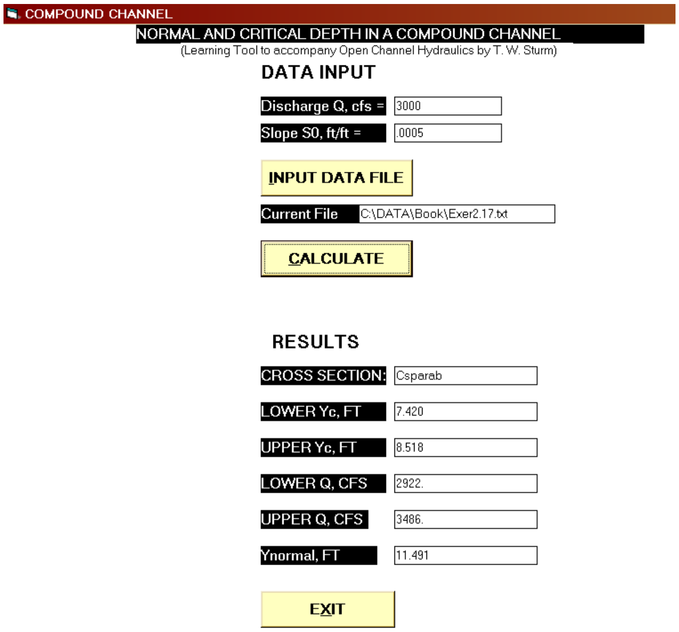
$$Q_U = \frac{\sqrt{g} A_1^{3/2}}{\sqrt{B_1}} = \frac{\sqrt{32.2}\,[(2/3)(50\times 8)]^{3/2}}{\sqrt{50}} = 3495\,\text{cfs}$$

So for Q = 3000 cfs, we have $Q < Q_U$ and multiple critical depths are possible, but only if it is also true that $Q > Q_L$. An analysis using the computer program **Ycomp** on the book website is necessary. The input geometry file is generated using a finite discretization of the parabolic main channel and the floodplains. It is given by

```
"Csparab"	31	3
0	15
0	8
150	8
152	6.771
154	5.645
156	4.621
158	3.699
160	2.880
162	2.163
164	1.549
166	1.037
168	0.627
170	0.320
172	0.115
174	0.013
175	0.000
176	0.013
178	0.115
180	0.320
182	0.627
184	1.037
186	1.549
188	2.163
190	2.880
192	3.699
194	4.621
196	5.645
198	6.771
200	8.000
350	8.000
350	15
150	0.08	200	0.03	350	0.08
```

The initial line gives the name of the cross-section, the number of boundary points, and the number of subsections. The following data in two columns represent the transverse station and the ground elevation. The right boundary station of the subsections and the Manning's n values to the left of the boundary station are given in the last line of the data file. The data were entered into an Excel spreadsheet and then saved as a tab delimited text file which was read by **Ycomp**.

The screen output is shown below. The calculated upper limiting Q is 3486 cfs instead of 3495 cfs because of the discretization of the parabolic portion of the cross-section. Note that the lower limiting discharge exists and is 2922 cfs so that discharges between these limits will have two critical depths. The critical depths for 3000 cfs are given in the output as 7.42 ft and 8.52 ft. At 3500 cfs there is only one upper critical depth and it is 9.02 ft (program output not shown).



2.18. Design a broad-crested weir for a laboratory flume with a width of 15 in. The discharge range is 0.1 to 1.0 cfs. The maximum approach flow depth is 18 in. Determine the height of the weir and the weir length in the flow direction. Plot the expected head-discharge relationship.

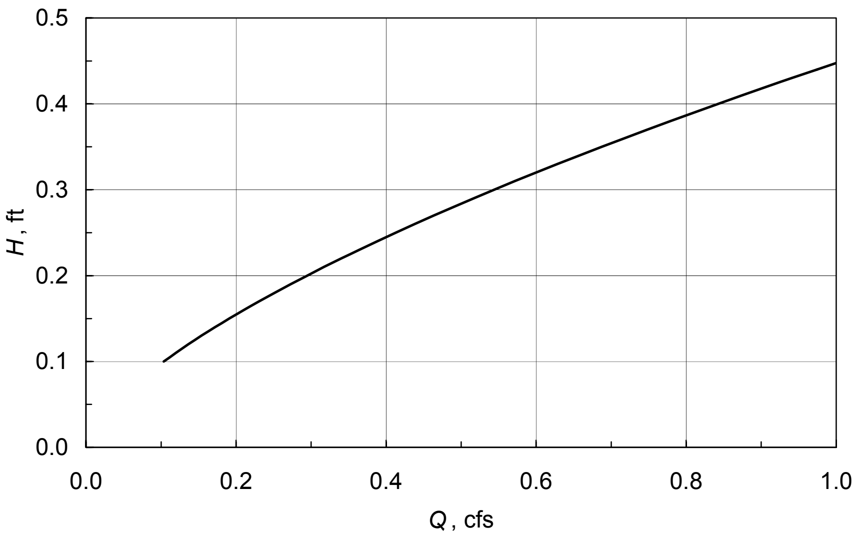
Solution.

With reference to Figure 2.25, we require $H/(H + P) \le 0.35$ and $0.08 \le H/l \le 0.33$ for broad-crested behavior. The head-discharge relationship is solved for H_{max} when $Q = Q_{max} = 1.0$ cfs to produce

$$H_{\max} = \left[\frac{Q_{\max}}{C_d(2/3)(2g/3)^{1/2}L}\right]^{2/3} = \left[\frac{1.0}{(0.848)(2/3)(2\times 32.2/3)^{1/2}(1.25)}\right]^{2/3} = 0.454 \text{ ft}$$

in which the coefficient of discharge has been taken to be 0.848, and the crest length is 1.25 ft. Similarly, the minimum head is determined to be 0.098 ft for $Q = 0.1$ cfs. For broad-crested behavior, set $H_{max}/l = 0.33$, and calculate $l = 0.454/0.33 = 1.38$ ft, which is the length of the weir in the flow direction. For the minimum head this gives $H_{min}/l = 0.071$, which is only slightly less than the allowable value. In addition, set $H/(H + P) = 0.35$ for $H = H_{max}$ and solve for the weir height, $P = (H_{max}/0.35) - H_{max} = (0.454/0.35) - 0.454 = 0.843$ ft. So the weir should have a length in the flow direction, $\boldsymbol{l} = \mathbf{1.4}$ **ft**, and a height, $\boldsymbol{P} = \mathbf{0.84}$ **ft**.

To plot the head-discharge relationship, use Equation 2.47 with $C_d = 0.848$; $L = 1.25$ ft; and C_v from Equation 2.48, which has to be solved for a calculated value of C_dA^*/A_1 for each head. The result is shown in the following figure:



2.19. Plot and compare the head-discharge relationships for a rectangular sharp-crested weir having a crest length of 1.0 ft in a 5-ft wide channel with that for a 90^o V-notch, sharp-crested weir if both weir crests are 1 ft above the channel bottom. Consider a head range of 0–0.5 ft.

Solution.

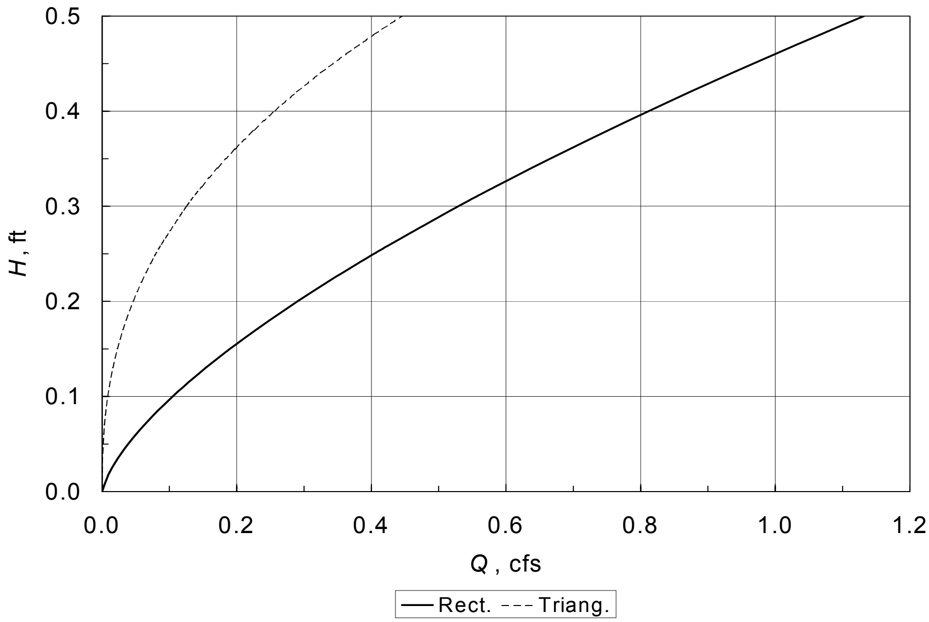
For the rectangular, sharp-crested weir, P = 1.0 ft; L = 1.0 ft; and b = 5 ft. Then L/b = 0.2 and from Table 2-3, C_{de} = 0.589 – 0.0018 H/P. In addition, k_L = 0.0082 ft (0.0025 m) from Figure 2.23 and k_h = 0.003 ft. The head-discharge relationship is given by

$$Q = \frac{2}{3}(2g)^{1/2} C_{de} L_e H_e^{3/2}$$
$$Q = 5.35 \times (0.589 - 0.0018 H / P) \times 1.0082 \times (H + 0.003)^{3/2}$$

The triangular weir has θ = 90^o and P = 1.0 ft. From Figure 2.24, C_{de} = 0.58 and k_h = 0.0033 ft (1 mm). The head-discharge relationship is given by

$$Q = C_{de} \frac{8}{15}(2g)^{1/2} \tan(\theta/2)(H + k_h)^{5/2}$$
$$Q = 0.58 \times \frac{8}{15} \times 64.4^{1/2} \times \tan(45°) \times (H + 0.0033)^{5/2}$$
$$Q = 2.48(H + 0.0033)^{5/2}$$

The head-discharge relationships are compared in the following figure.



2.20. Derive the head-discharge relationship for a triangular, broad-crested weir and a corresponding relationship for C_v analogous to Equation 2.48.

Solution.

The shape of the weir cross section is triangular with a notch angle of θ as shown in Figure 2.24. (The head H is measured from the crest to the free surface upstream of the weir plate.) As a broad-crested weir, however, the crest width is long enough in the flow direction that the theoretical value of critical depth occurs on the crest. The energy equation is written from the approach section to the critical section at the crest to yield

$$H + \frac{V_1^2}{2g} = H_e = y_c + \frac{D_c}{2}$$

$$H_e = y_c + \frac{A_c}{2B_c} = y_c + \frac{y_c^2 \tan(\theta/2)}{2[2y_c \tan(\theta/2)]} = \frac{5}{4} y_c$$

in which H_e = total energy head; V_1 = approach velocity; D_c = hydraulic depth; A_c = flow area at the crest; and B_c = flow top width at the crest. In Exercise 2.12, it was shown that critical depth for the triangular cross section is given by

$$y_c = 2^{1/5} \left[\frac{Q}{g^{1/2} \tan(\theta/2)} \right]^{2/5}$$

in which the side-slope ratio, $m = \tan(\theta/2)$. This expression for critical depth is substituted back into the equation, $H_e = (5/4)y_c$; and the approach velocity coefficient, $C_v = (H_e/H)^{5/2}$, and discharge coefficient, C_d, are incorporated to obtain the solution for Q:

$$Q = 0.405\, C_v C_d\; g^{1/2} \tan(\theta/2) H^{5/2}$$

Write the definition of the total energy head, H_e; substitute the solution for Q into the equation; and solve to give

$$H_e = H + \frac{Q^2}{2gA_1^2}$$

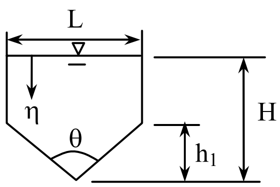
$$H_e = H + \frac{[0.405\, C_v C_d g^{1/2} \tan(\theta/2) H^{5/2}]^2}{2gA_1^2}$$

$$\left(\frac{H_e}{H}\right)^{5/2} = C_v = \left[1 + 0.0820\left(\frac{C_d A^*}{A_1}\right)^2 C_v^2\right]^{5/2}$$

$$\frac{C_d A^*}{A_1} = \frac{\left(C_v^{2/5} - 1\right)^{1/2}}{0.286\, C_v}$$

in which A^* = flow area in the control section for a depth equal to H, and A_1 = flow area in the approach cross section.

2.21. Derive the head-discharge relationship for a truncated, triangular, sharp-crested weir with notch angle θ and vertical walls that begin at a height of h_1 above the triangular crest. Assume that $H > h_1$.



Solution.

To obtain the theoretical discharge , Q_t, integrate the velocity distribution over the uncontracted flow area:

$$Q_t = \int_0^{H-h_1} \upsilon L\, d\eta + \int_{H-h_1}^{H} \upsilon[2(H-\eta)\tan(\theta/2)]d\eta$$

Neglecting the approach velocity head and assuming atmospheric pressure across the uncontracted nappe at the weir section, the velocity, $\upsilon = (2g\eta)^{1/2}$. Substituting into the previous equation, we have

$$Q_t = \int_0^{H-h_1} \sqrt{2g} L \eta^{1/2}\, d\eta + \int_{H-h_1}^{H} 2\sqrt{2g}\tan(\theta/2)[(H\eta^{1/2} - \eta^{3/2}]d\eta$$

Integrating, the result is

$$Q_t = \sqrt{2g}L \left.\frac{\eta^{3/2}}{3/2}\right|_0^{H-h_1} + 2\sqrt{2g}\tan(\theta/2)\left[\left.\frac{H\eta^{3/2}}{3/2}\right|_{H-h_1}^{H} - \left.\frac{\eta^{5/2}}{5/2}\right|_{H-h_1}^{H}\right]$$

Evaluating the limits and collecting terms, and introducing the coefficient of discharge, C_d, to compensate for the earlier assumptions, the final result for the actual discharge, Q, is

$$Q = \frac{2}{3}\sqrt{2g}\, C_d L(H-h_1)^{3/2} + 2\sqrt{2g}\tan(\theta/2)\, C_d\left[\frac{4}{15}H^{5/2} - \frac{2}{3}H(H-h_1)^{3/2} + \frac{2}{5}(H-h_1)^{5/2}\right]$$

2.22. A trapezoidal flume has a bottom width of 1.0 m and side slopes of 1:1. A sill with a height of 0.5 m is placed in the flume forming a trapezoidal critical control section. The length of the sill is 1.5 m in the flow direction. Calculate the discharge if the approach flow head is measured to be 0.60 m above the sill.

Solution.

The coefficient of discharge is determined first from Equation 2.51:

$$C_d = \left(\frac{H_e}{l} - 0.07\right)^{0.018} = \left(\frac{0.60}{1.5} - 0.07\right)^{0.018} = 0.980$$

in which the approach velocity head has been ignored for now. The discharge Q is given by Equation 2.50, but the value of y_c in that equation depends on the unknown Q. An additional equation is obtained by using $\mathbf{F}^2 = 1.0$, or equivalently, Equation 2.20, which incorporates $\mathbf{F}^2 = 1.0$. Then the solution involves a trial and error procedure to obtain y_c after which Equation 2.50 is solved for Q. If we begin with Equation 2.20, the bottom width of the trapezoidal crest is (1.0+2×0.5) = 2.0 m, and the value of E_c = 0.6 m so that (2.20) becomes

$$E_c = y_c + \frac{A_c}{2B_c}$$

$$0.6 = y_c + \frac{y_c(2+y_c)}{2(2+2y_c)}$$

from which y_c = 0.421 m and A_c = 0.421(2.0+1×0.421) = 1.019 m². Then substituting into (2.50), we have

$$Q = C_d A_c\sqrt{2g(H_e - y_c)} = 0.98\times 1.019\sqrt{19.62(0.6-0.421)} = 1.87\,\text{m}^3/s$$

Alternatively, from Figure 2.28, mH_e/b = 1.0×0.6/2.0 = 0.3 and y_c/H_e = 0.7 to give y_c = 0.42 m which can be substituted into Equation 2.50 to obtain the discharge. The approach flow area for a depth of y_1 = (0.5 + 0.6) = 1.1 m is $A_1 = y_1(1.0+my_1)$ = 1.1(1.0+1.0×1.1) =2.31 m². The approach flow velocity is $V_1 = Q/A_1$ = 1.87/2.31 = 0.810 m/s and the velocity head is 0.033 m. Iterating twice more produces H_e = 0.642 m and y_c = 0.451 m, and a final value of Q = **2.10 m³/s.**

2.23. Find the upstream head in the long-throated rectangular flume of Example 2.5 for the minimum discharge of 0.02 m^3/s which has a tailwater depth of 0.225 m. The sill length in the flow direction is 0.54 m and the height is 0.15 m as in Example 2.5. Also check if submergence will occur.

Solution.

Assume that C_d = 0.96, so the head becomes

$$H_e = \left[\frac{Q}{C_d \frac{2}{3}\sqrt{\frac{2}{3}g}L} \right]^{2/3} = \left[\frac{0.02}{(0.96)\times\frac{2}{3}\sqrt{\frac{2}{3}g}\times(0.3)} \right]^{2/3} = 0.118\text{ m}$$

Checking Equation 2.51 for H_e/l = 0.118/0.54 = 0.218, the result is C_d = 0.966. Repeating one more time with C_d = 0.966, the value of $\boldsymbol{H_e}$ **= 0.118 m** as before so this is the final answer. The approach flow energy head E_0 = (0.118+0.15) = 0.226 m, so the approach flow depth is obtained from:

$$y_0 + \frac{Q^2}{2gA_0^2} = y_0 + \frac{0.02^2}{19.62\times[y_0(0.75+y_0)]^2} = 0.226$$

The result is y_0 = 0.2256 m which means that the approach velocity head is negligible, and $\boldsymbol{H}$ **=** $\boldsymbol{H_e}$ **= 0.118 m**.

From Example 2.5, l_d = 0.9 m and l_t = 3.3 m. The velocity in the critical section on the sill is $V_c = Q/A_c$ = 0.02/(0.3 y_c) = 0.847 m/s in which y_c = (2/3) H_e = 0.0787 m. For the downstream tailwater section, assume that the maximum tailwater is equal to the actual tailwater for the first iteration. Then the velocity for a depth of 0.225 m is $V_t = Q/A_t$ = 0.02/[0.225(0.75+ 0.225)] = 0.0912 m/s. The hydraulic radius in the critical section is

$$R_c = \frac{A_c}{P_c} = \frac{0.3(0.0787)}{[0.3+2(0.0787)]} = 0.052\text{ m}$$

while the hydraulic radius of the tailwater section is given by

$$R_t = \frac{A_t}{P_t} = \frac{0.225(0.75+0.225)}{0.75+2\sqrt{2}(0.225)} = 0.158\text{ m}$$

Now for the friction loss, we have by substituting into Equation 2.54 with $f \cong 0.018$ for a smooth surface over this Reynolds number range:

$$\Delta H_f = 0.018\times\frac{0.90}{19.62}\times\frac{1}{2}\left(\frac{0.847^2}{4\times0.052}+\frac{0.0912^2}{4\times0.158}\right)+0.018\times\frac{3.3}{4\times0.158}\times\frac{0.0912^2}{19.62} = 0.0015\text{ m}$$

The expansion loss comes from substituting into Equation 2.57 to produce

$$\Delta H_{ex} = 0.66 \times \frac{(0.847 - 0.0912)^2}{19.62} = 0.019 \text{ m}$$

Finally, we check the possibility of submergence from Equation 2.58 to give

$$H_e - \Delta H_a - \Delta H_f - \Delta H_{ex} = 0.118 - (1 - 0.966^{1/1.5})(0.118) - 0.0015 - 0.019 = 0.095$$

The energy loss through the structure gives a tailwater energy head of 0.095 m, which is acceptable because it is greater than the actual tailwater head of 0.075 m relative to the sill (not including the velocity head, which is very small for this example).

2.24. A rectangular canal has a bottom width of 6.0 ft. A circular broad-crested weir is placed in the canal by constructing a headwall across the canal through which a 3.0 ft diameter circular pipe is placed. The pipe is horizontal with the invert located 0.5 ft above the bottom of the canal, and it has a length of 7.5 ft. If the upstream head on the weir is measured to be 1.5 ft relative to the invert of the pipe, calculate the discharge in the canal.

Solution.

The coefficient of discharge is determined first from Equation 2.51:

$$C_d = \left(\frac{H_e}{l} - 0.07\right)^{0.018} = \left(\frac{1.5}{7.5} - 0.07\right)^{0.018} = 0.964$$

in which the approach velocity head has been ignored for now. The discharge Q is given by Equation 2.50, but the value of y_c in that equation depends on the unknown Q. Use the graphical solution method of Figure 2.28 as suggested in Exercise 2.22. The value of $H_e/d = 1.5/3.0 = 0.5$ and Figure 2.28 gives $y_c/H_e = 0.73$. Then $y_c = 0.73 \times 1.5 = 1.095$ ft. The corresponding circular area comes from $\theta_c = 2\cos^{-1}(1 - 2(y_c/d)) = 2.595$ rad and $A_c = (\theta_c - \sin\theta_c)d^2/8 = 2.335$ ft^2. Substituting into Equation 2.50 we have

$$Q = C_d A_c \sqrt{2g(H_e - y_c)} = 0.964 \times 2.335\sqrt{64.4(1.5 - 1.095)} = 11.5 \text{ft}^3/s$$

The approach flow velocity is $Q/A_1 = 11.5/(6\times2) = 0.96$ ft/s and the velocity head is 0.014 ft which is negligible compared with the head of 1.5 ft so $H_e = 1.5$ ft is satisfactory and ***Q* = 11.5 cfs.**

2.25. Modify the computer program Y0YC in Appendix B to calculate the critical depth in a circular channel.

Solution.

The primary change to be made to Y0YC is the reformulation of the geometric elements of area, top width, and hydraulic radius for a circular section as given in Table 2-1. However, the relationship between θ and y/d requires the $\cos^{-1}$ function, which is not available in Visual Basic, but the $\tan^{-1}$ function is available. Hence, the following trigonometric identity is utilized:

$$\cos^{-1}(x) = \frac{\pi}{2} - \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

in which $x = (1 - 2y/d)$. This is shown in the following program listing in the function procedure F. In addition, upper limits must be placed on the root search for both critical and normal depth in the circular cross section. In the case of critical depth, the top width approaches zero as y/d approaches 1.0, so Q approaches infinity. In addition, the $\tan^{-1}$ argument goes to infinity as y/d approaches 1.0, so the upper limit Y2 is set equal to 0.9999d. For the normal depth function, it is clear from Figure 4.9 that there is a maximum in $AR^{2/3}/d^{8/3}$ above which there are two possible solutions, so the upper limit Y2 is set equal to 0.938d at which this maximum occurs. Finally, the initial test for the existence of a root in Sub BISECTION requires an indication of when the normal depth exceeds the maximum, and this is indicated by setting it equal to a very large number such 1.0E6. The program listing for the Sub procedure follows. The form module is similar to that shown for the program Y0YC in Appendix B except that the diameter d is entered for the circular section instead of b and m for the trapezoidal section.

```
Option Explicit
Dim Q As Single, S As Single, d As Single
Dim n As Single, Y0 As Single, YC As Single
Sub Y0YC(Q, S, d, n, Y0, YC)
Dim Y1 As Single, Y2 As Single, ER As Single
Dim NF As Integer
Y1 = 0.0001
Y2 = 0.9999 * d
ER = 0.0001
NF = 1
Call BISECTION(Y1, Y2, NF, ER, Q, S, d, n, YC)
Y1 = 0.0001
Y2 = 0.938 * d
NF = 2
Call BISECTION(Y1, Y2, NF, ER, Q, S, d, n, Y0)
End Sub

Sub BISECTION(Y1, Y2, NFUNC, ER, Q, S, d, n, Y3)
Dim FY1 As Single, FY2 As Single, FY3 As Single, FZ As Single
Dim I As Integer
        FY1 = F(Y1, NFUNC, Q, S, d, n)
        FY2 = F(Y2, NFUNC, Q, S, d, n)
        If FY1 * FY2 > 0 Then
            Y3 = 1000000#
            Exit Sub
            End If
For I = 1 To 50
        Y3 = (Y1 + Y2) / 2
        FY3 = F(Y3, NFUNC, Q, S, d, n)
        FZ = FY1 * FY3
        If FZ = 0 Then Exit Sub
        If FZ < 0 Then Y2 = Y3 Else Y1 = Y3
```

```
        If Abs((Y2 - Y1) / Y3) < ER Then Exit Sub
Next I
End Sub

Function F(Y, NFUNC, Q, S, d, n) As Single
Dim A As Single, P As Single, R As Single, T As Single
Dim PI As Double, THETA As Double, X As Double
        X = 1 - 2 * Y / d
        PI = 3.141592654
        THETA = 2 * (PI / 2 - Atn(X / (1 - X ^ 2) ^ 0.5))
        A = (THETA - Sin(THETA)) * d ^ 2 / 8
        P = THETA * d / 2
        R = A / P
        T = d * Sin(THETA / 2)
        If NFUNC = 1 Then
             F = Q - Sqr(32.2) * A ^ 1.5 / T ^ 0.5
        Else
             F = Q - (1.486 / n) * A * R ^ (2 / 3) * S ^ (1 / 2)
        End If
End Function
```

2.26. Write a computer program that computes the depth in a width contraction and the upstream depth given a subcritical tailwater depth as in Figure 2.11. Assume that the channel is rectangular at all three sections and make provision for a head-loss coefficient that is nonzero; include a check for possible choking.

Solution.

Referring to Fig. 2.11, the downstream depth, y_3; channel widths, b_3 and b_2; discharge, Q; and the expansion and contraction loss coefficients, K_{Lexp} and K_{Lcont}, respectively, must be entered in the form module. It is assumed that $b_1 = b_3$. In the program listing that follows, the Sub procedure YTRANS is called from the form module. First in YTRANS, the possibility of choking is checked from the following inequality:

$$E_{c2} < E_3 + h_{L(2c-3)}$$

in which E_3 = the known specific energy at the downstream section; E_{c2} = minimum specific energy at the contracted section 2; and $h_{L(2c\text{-}3)}$ = head loss from section 2 to 3 assuming that critical depth occurs in the contracted section. If this inequality is true, then choking does not occur and computations for the depth at section 2 can proceed by calling the BISECTION procedure. If it is true, on the other hand, then choking occurs and the depth at section 2 is set equal to the critical depth for the contracted section. For the nonchoking case, the equation to be solved is the energy equation written from section 2 to 3 as

$$y_2 + \frac{Q^2}{2gA_2^2} = E_3 + K_L \left| \frac{Q^2}{2gA_2^2} - \frac{Q^2}{2gA_3^2} \right|$$

in which y_2 is unknown while E_3 and A_3 are known from the specified downstream depth, y_3. The solution to this equation is obtained from Sub BISECTION used with the function subprocedure F, which is defined by the right hand side of the equation above minus the left hand side as shown in the program listing. The correspondence between the variables in the equation and the variable names in the program listing is given by: Y = y_2; Eds = E_3; Yds = y_3; and HL = the head loss term. Once this equation has been solved for y_2, control returns to the calling procedure, YTRANS, which calls BISECTION again with section 2 as the downstream section and section 1 as the upstream section where the depth is unknown. Then a solution is obtained for y_1. The program listing is given below.

```
Option Explicit
Dim Q As Single, KLEXP As Single, KLCONT As Single
Dim B3 As Single, B2 As Single, G As Single, YC3 As Single
Dim Y3 As Single, Y2 As Single, Y1 As Single, YC2 As Single
Private Sub cmdCalculate_Click()
Q = Val(txtQ.Text)
KLEXP = Val(txtHlexp.Text)
KLCONT = Val(txtHlcont.Text)
B3 = Val(txtB3.Text)
B2 = Val(txtB2.Text)
Y3 = Val(txtY3.Text)
G = 32.2
YC3 = ((Q / B3) ^ 2 / G) ^ (1 / 3)
YC2 = ((Q / B2) ^ 2 / G) ^ (1 / 3)
txtYC3.Text = Format(YC3, "###.000")
txtYC2.Text = Format(YC2, "###.000")
Call YTRANS(Q, KLEXP, KLCONT, B3, B2, Y3, Y2, Y1)
If Y2 = 0 Then
    txtY2.Text = "Enter y3 > YC3"
    txtY1.Text = "0"
Else
    txtY2.Text = Format(Y2, "###.000")
    txtY1.Text = Format(Y1, "###.000")
End If
End Sub
Private Sub cmdExit_Click()
End
End Sub

Option Explicit
Dim Q As Single, KLEXP As Single, KLCONT As Single
Dim BE As Single, BC As Single
Dim Y3 As Single, Y2 As Single, Y1 As Single
Sub YTRANS(Q, KLEXP, KLCONT, BE, BC, Y3, Y2, Y1)
Dim YL As Single, YR As Single, ER As Single
Dim YC3 As Single, YC2 As Single, EC2 As Single
Dim G As Single, HLC As Single, E3 As Single
```

```
G = 32.2
ER = 0.0001
YC3 = ((Q / BE) ^ 2 / G) ^ (1 / 3)
If Y3 <= YC3 Then Y2 = 0: Exit Sub
YC2 = ((Q / BC) ^ 2 / G) ^ (1 / 3)
EC2 = 1.5 * YC2
E3 = Y3 + Q ^ 2 / (2 * G * BE ^ 2 * Y3 ^ 2)
HLC = KLEXP * Abs(Q ^ 2 / (2 * G * (BE * Y3) ^ 2) _
 - Q ^ 2 / (2 * G * (BC * YC2) ^ 2))
If EC2 < (E3 + HLC) Then
    YL = YC2
    YR = 50
    Call BISECTION(YL, YR, ER, Q, KLEXP, BE, BC, Y3, Y2)
Else
    Y2 = YC2
End If
YL = YC3
YR = 50
Call BISECTION(YL, YR, ER, Q, KLCONT, BC, BE, Y2, Y1)
End Sub

Sub BISECTION(YL, YR, ER, Q, KL, Bds, Bus, Yds, Y)
Dim FYL As Single, FYR As Single, FY As Single, FZ As Single
Dim I As Integer
        FYL = F(YL, Q, KL, Bds, Bus, Yds)
        FYR = F(YR, Q, KL, Bds, Bus, Yds)
        If FYL * FYR > 0 Then
            Y = 1000000#
            Exit Sub
            End If
For I = 1 To 50
        Y = (YL + YR) / 2
        FY = F(Y, Q, KL, Bds, Bus, Yds)
        FZ = FYL * FY
        If FZ = 0 Then Exit Sub
        If FZ < 0 Then YR = Y Else YL = Y
        If Abs((YR - YL) / Y) < ER Then Exit Sub
Next I
End Sub

Function F(Y, Q, KL, Bds, Bus, Yds) As Single
Dim Eds As Single, CQ As Single, HL As Single, G As Single
        G = 32.2
        CQ = Q ^ 2 / (2 * G)
        Eds = Yds + CQ / (Bds * Yds) ^ 2
        HL = KL * Abs(CQ / (Bus * Y) ^ 2 - CQ / (Bds * Yds) ^ 2)
        F = Eds + HL - (Y + CQ / (Bus * Y) ^ 2)
End Function
```

2.27. A laboratory experiment has been conducted in a horizontal flume in which a sharp-crested weir plate has been installed to determine the head-discharge relationship for a rectangular, sharp-crested weir. With reference to Figure 2.23, $P = 0.506$ ft, $L = 0.25$ ft, and $b = 1.25$ ft. The discharge was measured by a bend meter for which the calibration is given by $Q = 0.075\ \Delta h^{0.523}$, in which Q = discharge in cubic feet per second; Δh = manometer deflection in inches of water; and the uncertainty in the calibration is ± 0.003 cfs. The head on the crest of the weir was measured by a point gauge and is given in the data table that follows. An upstream view of the weir nappe can be seen in Figure 2.28.

Δh, in.	H, ft
13.2	0.498
11.5	0.476
11.2	0.474
8.3	0.425
8.0	0.421
6.2	0.384
6.1	0.386
4.3	0.333
4.2	0.334
2.4	0.272
2.0	0.257

(*a*) Plot the head on the vertical scale and the discharge on the horizontal scale of log-log axes and obtain a least-squares regression fit forcing the inverse slope to be the theoretical value of 3/2. What are the single best-fit value of C_d and the standard error in C_d? Compare the standard error of the "Q estimate" with the uncertainty in the bend-meter calibration.

(*b*) Calculate the discharge using the Kindsvater-Carter relationship and using the single best-fit value of C_d. Compare both sets of results with the measured discharges by calculating the percent differences and also plotting the measured vs. calculated discharges.

Solution.

(*a*) The spreadsheet calculations are given next in which the Q values are calculated from the values of Δh using the calibration equation for the bend meter. Linear regression analysis between the Q values and $H^{3/2}$ is performed to force the exponent on the head to be equal to its theoretical value of 3/2. The best fit value of $C = Q/H^{3/2} = 0.819 \pm 0.002$. The standard error of estimate of the Q values is $\pm$ 0.0019 cfs, which is slightly smaller than the estimate of uncertainty in the bend meter calibration. The coefficient of determination, $R^2 = 0.999$. The coefficient $C = (2/3)(2g)^{1/2} C_d L$ from which we can solve for C_d to obtain $C_d = 0.612 \pm 0.003$. (The standard error also has to be converted from its value on C to the value on C_d.)

Δh, in.	H,ft	Q, cfs	H^3/2	Q, Best Fit	Q, K.-C.	% Dif.: K.-C.
13.2	0.498	0.289	0.351	0.288	0.288	-0.52%
11.5	0.476	0.269	0.328	0.269	0.269	-0.03%
11.2	0.474	0.265	0.326	0.267	0.267	0.73%
8.3	0.425	0.227	0.277	0.227	0.227	0.17%
8.0	0.421	0.223	0.273	0.224	0.224	0.69%
6.2	0.384	0.195	0.238	0.195	0.195	0.35%
6.1	0.386	0.193	0.240	0.196	0.197	1.99%
4.3	0.333	0.161	0.192	0.157	0.158	-1.67%
4.2	0.334	0.159	0.193	0.158	0.159	-0.01%
2.4	0.272	0.119	0.142	0.116	0.117	-1.19%
2.0	0.257	0.108	0.130	0.107	0.108	-0.07%

SUMMARY OUTPUT

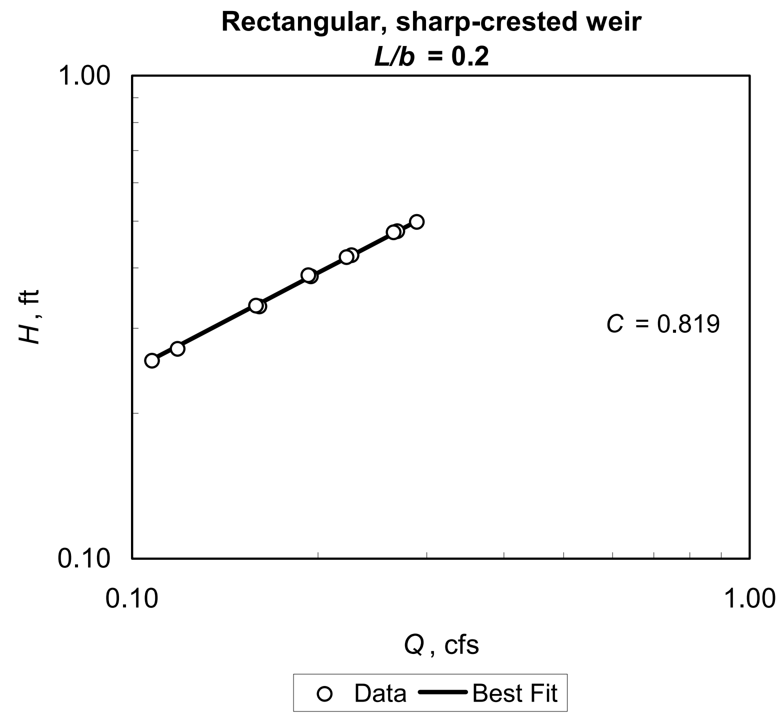
Regression Statistics	
Multiple R	0.99949325
R Square	0.99898676
Adjusted R Sq.	0.89898676
Standard Error	0.00192701
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.03661108	0.03661108	9859.31359	5.4068E-15
Residual	10	3.71335E-05	3.7133E-06		
Total	11	0.036648214			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0	#N/A	#N/A	#N/A	#N/A	#N/A
X Variable 1	0.8189643	0.002279472	359.278001	6.865E-22	0.81388531	0.82404328

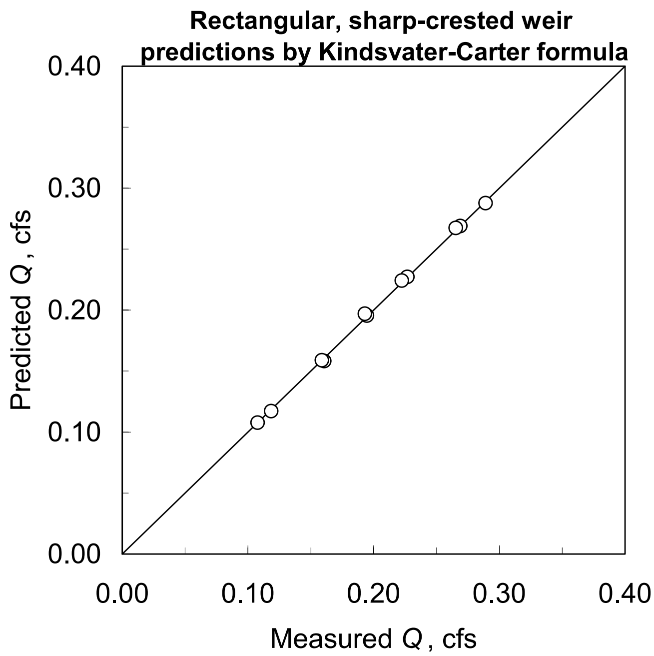
The plot of H vs. Q on log-log scales is shown in the following figure.



(b) The values of Q predicted by the Kindsvater-Carter relationship are determined from a coefficient of discharge obtained from Table 2-3 for L/b = 0.2 and given by

$$C_{de} = 0.589 - 0.0018\frac{H}{P}$$

The correction on H is k_H = 0.003 ft, and the crest length correction from Figure 2.23c is k_L = 0.0082 ft. The Kindsvater-Carter values of Q along with the Q values obtained from a single best-fit C_d value of 0.612 are shown in the spreadsheet given previously. The best-fit relationship and the Kindsvater-Carter formula give virtually identical results. The percent difference between the Q values predicted by the Kindsvater-Carter formula and the measured Q values are also shown in the spreadsheet. The maximum difference is 2.0 percent. The same comparison is shown graphically in the following figure.


Rectangular, sharp-crested weir
predictions by Kindsvater-Carter formula
0.40
0.30
0.20
0.10
0.00
Predicted Q, cfs
0.00
0.10
0.20
0.30
0.40
Measured Q, cfs