Chapter 2

Differentiation: Basic Concepts

2.1 The Derivative

1. If $f(x) = 4$, then $f(x + h) = 4$.
The difference quotient (DQ) is
$$\frac{f(x+h)-f(x)}{h}=\frac{4-4}{h}=0.$$
$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=0$$
The slope is $m = f'(0) = 0$.

2. $f(x) = -3$
The difference quotient is
$$\frac{f(x+h)-f(x)}{h}=\frac{-3-(-3)}{h}=\frac{0}{h}=0$$
Then $f'(x)=\lim_{h\to 0} 0 = 0.$
The slope of the line tangent to the graph of f at $x = 1$ is $f'(1) = 0$.

3. If $f(x) = 5x - 3$, then
$f(x + h) = 5(x + h) - 3$.
The difference quotient (DQ) is
$$\frac{f(x+h)-f(x)}{h}=\frac{[5(x+h)-3]-[5x-3]}{h}=\frac{5h}{h}=5$$
$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=5$$
The slope is $m = f'(2) = 5$.

4. $f(x) = 2 - 7x$
The difference quotient is
$$\frac{f(x+h)-f(x)}{h}=\frac{(2-7(x+h))-(2-7x)}{h}=\frac{2-7x-7h-2+7x}{h}=\frac{-7h}{h}=-7$$
Then $f'(x)=\lim_{h\to 0}(-7)=-7.$
The slope of the line tangent to the graph of f at $x = -1$ is $f'(-1) = -7$.

5. If $f(x)=2x^2-3x+5$, then
$f(x+h)=2(x+h)^2-3(x+h)+5.$
The difference quotient (DQ) is
$$\frac{f(x+h)-f(x)}{h}.$$
$$\frac{[2(x+h)^2-3(x+h)+5]}{h}-\frac{[2x^2-3x+5]}{h}$$
$$=\frac{4xh+2(h)^2-3h}{h}$$
$$=4x+2h-3$$
$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=4x-3$$
The slope is $m = f'(0) = -3$.

6. $f(x)=x^2-1$
The difference quotient is
$$\frac{f(x+h)-f(x)}{h}=\frac{((x+h)^2-1)-(x^2-1)}{h}=\frac{x^2+2hx+h^2-1-x^2+1}{h}=\frac{2hx+h^2}{h}=2x+h$$
Then $f'(x)=\lim_{h\to 0}(2x+h)=2x$.
The slope of the line tangent to the graph of f at $x=-1$ is $f'(-1)=-2$.

7. If $f(x) = x^3 - 1$, then

$$f(x+h) = (x+h)^3 - 1$$
$$= (x^2 + 2xh + h^2)(x+h) - 1$$
$$= x^3 + 3x^2h + 3xh^2 + h^3 - 1$$
$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 1 - (x^3 - 1)}{h}$$
$$= \frac{3x^2h + 3xh^2 + h^3}{h}$$
$$= \frac{h(3x^2 + 3xh + h^2)}{h}$$
$$= 3x^2 + 3xh + h^2$$
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} 3x^2 + 3xh + h^2$$
$$= 3x^2$$

The slope is $m = f'(2) = 3(2)^2 = 12$.

8. $f(x) = -x^3$

The difference quotient is

$$\frac{f(x+h) - f(x)}{h} = \frac{(-(x+h)^3) - (-x^3)}{h}$$
$$= \frac{-x^3 - 3x^2h - 3xh^2 - h^3 + x^3}{h}$$
$$= \frac{-3x^2h - 3xh^2 - h^3}{h}$$
$$= -3x^2 - 3xh - h^2$$

Then

$$f'(x) = \lim_{h \to 0} (-3x^2 - 3xh - h^2) = -3x^2.$$

The slope of the line tangent to the graph of f at $x = 1$ is $f'(1) = -3$.

9. If $g(t) = \frac{2}{t}$, then $g(t+h) = \frac{2}{t+h}$.

The difference quotient (DQ) is

$$\frac{g(t+h) - g(t)}{h} = \frac{\frac{2}{t+h} - \frac{2}{t}}{h}$$
$$= \frac{\frac{2}{t+h} - \frac{2}{t}}{h} \cdot \frac{t(t+h)}{t(t+h)}$$
$$= \frac{2t - 2(t+h)}{h(t)(t+h)}$$
$$= \frac{-2}{t(t+h)}$$
$$g'(t) = \lim_{h \to 0} \frac{g(t+h) - g(t)}{h} = -\frac{2}{t^2}$$

The slope is $m = g'\left(\frac{1}{2}\right) = -8$.

10. $f(x) = \frac{1}{x^2}$

The difference quotient is

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$
$$= \frac{\frac{1}{x^2+2hx+h^2} - \frac{1}{x^2}}{h}$$
$$= \frac{\frac{x^2-(x^2+2hx+h^2)}{(x^2+2hx+h^2)x^2}}{h}$$
$$= \frac{-2hx - h^2}{h(x^2 + 2hx + h^2)x^2}$$
$$= \frac{-2x - h}{(x^2 + 2hx + h^2)x^2}$$

Then $f'(x) = \lim_{h \to 0} \frac{-2x - h}{(x^2 + 2hx + h^2)x^2}$

$$= \frac{-2x}{x^4}$$
$$= -\frac{2}{x^3}.$$

The slope of the line tangent to the graph of f at $x = 2$ is $f'(2) = -\frac{1}{4}$.

11. If $H(u) = \frac{1}{\sqrt{u}}$, then $H(u+h) = \frac{1}{\sqrt{u+h}}$.

The difference quotient is

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\frac{1}{\sqrt{u+h}}-\frac{1}{\sqrt{u}}}{h}\cdot\frac{\sqrt{u}\sqrt{u+h}}{\sqrt{u}\sqrt{u+h}}$$
$$=\frac{\sqrt{u}-\sqrt{u+h}}{h\sqrt{u}\sqrt{u+h}}\cdot\frac{\left(\sqrt{u}+\sqrt{u+h}\right)}{\left(\sqrt{u}+\sqrt{u+h}\right)}$$
$$=\frac{u-(u+h)}{h\sqrt{u}\sqrt{u+h}\left(\sqrt{u}+\sqrt{u+h}\right)}$$
$$=\frac{-h}{h\sqrt{u}\sqrt{u+h}\left(\sqrt{u}+\sqrt{u+h}\right)}$$
$$=\frac{-1}{\sqrt{u}\sqrt{u+h}\left(\sqrt{u}+\sqrt{u+h}\right)}$$
$$H'(u)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{-1}{\sqrt{u}\sqrt{u+h}\left(\sqrt{u}+\sqrt{u+h}\right)}$$
$$=\frac{-1}{\sqrt{u}\cdot\sqrt{u}\left(\sqrt{u}+\sqrt{u}\right)}$$
$$=\frac{-1}{u\left(2\sqrt{u}\right)}$$
$$=-\frac{1}{2u\sqrt{u}}$$

The slope is $m=H'(4)=-\frac{1}{2(4)\sqrt{4}}=-\frac{1}{16}.$

12. $f(x)=\sqrt{x}$

The difference quotient is
$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\sqrt{x+h}-\sqrt{x}}{h}$$
$$=\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$
$$=\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$
$$=\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$
$$=\frac{1}{\sqrt{x+h}+\sqrt{x}}$$

Then $f'(x)=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$

The slope of the line tangent to the graph of f at $x = 9$ is $f'(9)=\frac{1}{6}.$

13. If $f(x) = 2$, then $f(x + h) = 2$.

The difference quotient (DQ) is
$$\frac{f(x+h)-f(x)}{h}=\frac{2-2}{h}=0.$$
$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}0=0$$

The slope of the tangent is zero for all values of x. Since $f(13) = 2$.

$y - 2 = 0(x - 13)$, or $y = 2$.

14. For $f(x)=3$,
$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{3-3}{h}=0$$

for all x. So at the point $c=-4$, the slope of the tangent line is $m=f'(-4)=0$. The point $(-4,3)$ is on the tangent line so by the point-slope formula the equation of the tangent line is $y-3=0[x-(-4)]$ or $y=3$.

15. If $f(x) = 7 - 2x$, then

$f(x + h) = 7 - 2(x + h)$.

The difference quotient (DQ) is
$$\frac{f(x+h)-f(x)}{h}=\frac{[7-2(x+h)]-[7-2x]}{h}$$
$$=-2$$
$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=-2$$

The slope of the line is $m=f'(5)=-2.$

Since $f(5) = -3$, $(5, -3)$ is a point on the curve and the equation of the tangent line is $y - (-3) = -2(x - 5)$ or $y = -2x + 7$.

16. For $f(x) = 3x$,
$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{3x+3h-3x}{h}$$
$$=3$$

for all x. So at the point $c = 1$, the slope of the tangent line is $m = f'(1) = 3$. The point (1, 3) is on the tangent line so by the point-slope formula the equation of the tangent line is $y - 3 = 3(x - 1)$ or $y = 3x$.

17. If $f(x) = x^2$, then $f(x+h) = (x+h)^2$.
The difference quotient (DQ) is

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2}{h} = \frac{2xh + h^2}{h} = 2x + h$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = 2x$$

The slope of the line is $m = f'(1) = 2$.
Since $f(1) = 1$, (1, 1) is a point on the curve and the equation of the tangent line is $y - 1 = 2(x - 1)$ or $y = 2x - 1$.

18. For $f(x) = 2 - 3x^2$,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(2 - 3(x+h)^2) - (2 - 3x^2)}{h} = \lim_{h\to 0}(-6x - 3h) = -6x$$

for all x. At the point $c = 1$, the slope of the tangent line is $m = f'(1) = -6$. The point $(1, -1)$ is on the tangent line so by the point-slope formula the equation of the tangent line is $y - (-1) = -6(x - 1)$ or $y = -6x + 5$.

19. If $f(x) = -\frac{2}{x}$, then $f(x+h) = \frac{-2}{x+h}$.
The difference quotient (DQ) is

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{-2}{x+h} - \frac{-2}{x}}{h} = \frac{\frac{-2}{x+h} + \frac{2}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)} = \frac{-2x + 2(x+h)}{h(x)(x+h)} = \frac{2}{x(x+h)}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \frac{2}{x^2}$$

The slope of the line is $m = f'(-1) = 2$.
Since $f(-1) = 2$, $(-1, 2)$ is a point on the curve and the equation of the tangent line is $y - 2 = 2(x - (-1))$

$$y = 2x + 4$$

20. For $f(x) = \frac{3}{x^2}$,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{3}{(x+h)^2} - \frac{3}{x^2}}{h} = \lim_{h\to 0} \frac{-6x - 3h}{(x+h)^2 x^2} = \frac{-6}{x^3}$$

At the point $c = \frac{1}{2}$, the slope of the tangent line is $m = f'\left(\frac{1}{2}\right) = -48$. The point $\left(\frac{1}{2}, 12\right)$ is on the tangent line so by the point-slope formula the equation of the tangent line is

$y - 12 = -48\left(x - \frac{1}{2}\right)$ or $y = -48x + 36$.

21. First we obtain the derivative of $g(x) = \sqrt{x}$.
The difference quotient is

$$\frac{g(x+h)-g(x)}{h}$$
$$=\frac{\sqrt{x+h}-\sqrt{x}}{h}$$
$$=\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$
$$=\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$
$$=\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$
$$=\frac{1}{\sqrt{x+h}+\sqrt{x}}$$

Then $g'(x)=\lim\limits_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$

Now since $\frac{d}{dx}k\cdot f\left(x\right)=k\cdot\frac{d}{dx}f(x)$,

$$f'(x)=2\left(\frac{1}{2\sqrt{x}}\right)=\frac{1}{\sqrt{x}}.$$

The slope is $m=f'(4)=\frac{1}{2}$, $f(4)=4$, the equation of the tangent line is

$$y-4=\frac{1}{2}(x-4),\text{ or } y=\frac{1}{2}x+2.$$

22. For $f(x)=\frac{1}{\sqrt{x}}$,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h}$$
$$=\lim_{h\to 0}\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x^2+xh}}$$
$$=\lim_{h\to 0}\frac{x-(x+h)}{h\sqrt{x^2+xh}\left(\sqrt{x}+\sqrt{x+h}\right)}$$
$$=\lim_{h\to 0}\frac{-1}{\sqrt{x^2+xh}\left(\sqrt{x}+\sqrt{x+h}\right)}$$
$$=\frac{-1}{\sqrt{x^2}\left(2\sqrt{x}\right)}$$
$$=\frac{-1}{2x^{3/2}}$$

So at the point $c=1$, the slope of the tangent line is $f'(1)=-\frac{1}{2}$. The point (1, 1) is on the tangent line so by the point-slope formula, the equation of the tangent line is $y-1=-\frac{1}{2}(x-1)$ or $y=-\frac{1}{2}x+\frac{3}{2}$.

23. If $f(x)=\frac{1}{x^3}$, then $f(x+h)=\frac{1}{(x+h)^3}$.

The difference quotient (DQ) is

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\frac{1}{(x+h)^3}-\frac{1}{x^3}}{h}\cdot\frac{x^3(x+h)^3}{x^3(x+h)^3}$$
$$=\frac{x^3-(x+h)^3}{hx^3(x+h)^3}$$
$$=\frac{x^3-(x^3+3x^2h+3xh^2+h^3)}{hx^3(x+h)^3}$$
$$=\frac{-3x^2h-3xh^2-h^3}{hx^3(x+h)^3}$$
$$=\frac{h(-3x^2-3xh-h^2)}{hx^3(x+h)^3}$$
$$=\frac{-3x^2-3xh-h^2}{x^3(x+h)^3}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{-3x^2-3xh-h^2}{x^3(x+h)^3}$$
$$=\frac{-3x^2}{x^3(x)^3}$$
$$=-\frac{3}{x^4}$$

The slope is $m=f'(1)=-\frac{3}{(1)^4}=-3$.

Further, $f(1)=1$ so the equation of the line is $y-1=-3(x-1)$, or $y=-3x+4$.

24. From Exercise 7 of this section $f'(x)=3x^2$. At the point $c=1$, the slope of the tangent line is $m = f'(1) = 3$. The point $(1,0)$ is on the tangent line so by the point-slope formula the equation of the tangent line is $y - 0 = 3(x-1)$ or $y = 3x - 3$.

25. If $y = f(x) = 3$, then $f(x+h) = 3$.
The difference quotient (DQ) is
$$\frac{f(x+h)-f(x)}{h} = \frac{3-3}{h} = 0.$$
$$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = 0$$
$$\frac{dy}{dx} = 0 \text{ when } x = 2.$$

26. For $f(x) = -17$, $\frac{dy}{dx}$ at $x_0 = 14$ is
$$\begin{aligned} f'(14) &= \lim_{h\to 0}\frac{f(14+h)-f(14)}{h} \\ &= \lim_{h\to 0}\frac{-17-(-17)}{h} \\ &= \lim_{h\to 0}\frac{0}{h} \\ &= 0 \end{aligned}$$

27. If $y = f(x) = 3x + 5$, then
$f(x+h) = 3(x+h) + 5 = 3x + 3h + 5$.
The difference quotient (DQ) is
$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{3x+3h+5-(3x+5)}{h} \\ &= \frac{3h}{h} \\ &= 3 \end{aligned}$$
$$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} 3 = 3$$
$$\frac{dy}{dx} = 3 \text{ when } x = -1.$$

28. For $f(x) = 6 - 2x$, $\frac{dy}{dx}$ at $x_0 = 3$ is
$$\begin{aligned} f'(3) &= \lim_{h\to 0}\frac{f(3+h)-f(3)}{h} \\ &= \lim_{h\to 0}\frac{(6-2(3+h))-(6-2(3))}{h} \\ &= \lim_{h\to 0}\frac{-2h}{h} \\ &= -2 \end{aligned}$$

29. If $y = f(x) = x(1-x)$, or $f(x) = x - x^2$, then $f(x+h) = (x+h) - (x+h)^2$.
The difference quotient (DQ) is
$$\begin{aligned} &\frac{f(x+h)-f(x)}{h} \\ &= \frac{[(x+h)-(x+h)^2]-[x-x^2]}{h} \\ &= \frac{h-2xh-h^2}{h} \\ &= 1-2x-h \end{aligned}$$
$$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = 1-2x$$
$$\frac{dy}{dx} = 3 \text{ when } x = -1.$$

30. For $f(x) = x^2 - 2x$, $\frac{dy}{dx}$ at $x_0 = 1$ is
$$\begin{aligned} f'(1) &= \lim_{h\to 0}\frac{f(1+h)-f(1)}{h} \\ &= \lim_{h\to 0}\frac{((1+h)^2-2(1+h))-(1^2-2(1))}{h} \\ &= \lim_{h\to 0}\frac{h^2}{h} \\ &= 0 \end{aligned}$$

31. If $y = f(x) = x - \frac{1}{x}$, then
$$f(x+h) = x + h - \frac{1}{x+h}.$$
The difference quotient (DQ) is
$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{x+h-\frac{1}{x+h}-\left(x-\frac{1}{x}\right)}{h} \\ &= \frac{h-\frac{1}{x+h}+\frac{1}{x}}{h}\cdot\frac{x(x+h)}{x(x+h)} \\ &= \frac{hx(x+h)-x+x+h}{h} \\ &= \frac{hx^2+h^2x+h}{h} \\ &= \frac{h(x^2+hx+1)}{h} \\ &= x^2+hx+1 \end{aligned}$$

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} x^2 + hx + 1$$
$$= x^2 + 1$$

When $x = 1$, $\frac{dy}{dx} = (1)^2 + 1 = 2.$

32. For $f(x) = \frac{1}{2-x}$, $\frac{dy}{dx}$ at $x_0 = -3$ is

$$f'(-3) = \lim_{h\to 0}\frac{f(-3+h)-f(-3)}{h}$$
$$= \lim_{h\to 0}\frac{\frac{1}{2-(-3+h)} - \frac{1}{2-(-3)}}{h}$$
$$= \lim_{h\to 0}\frac{1}{5(5-h)}$$
$$= \frac{1}{25}$$

33. **(a)** If $f(x) = x^2$, then $f(-2) = (-2)^2 = 4$ and $f(-1.9) = (-1.9)^2 = 3.61.$ The slope of the secant line joining the points (–2, 4) and (–1.9, 3.61) on the graph of *f* is

$$m_{\text{sec}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3.61 - 4}{-1.9-(-2)} = -3.9.$$

(b) If $f(x) = x^2$, then

$f(x+h) = (x+h)^2 = x^2 + 2xh + h^2.$

The difference quotient (DQ) is

$$\frac{f(x+h)-f(x)}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h}$$
$$= \frac{2xh + h^2}{h}$$
$$= \frac{h(2x+h)}{h}$$
$$= 2x + h$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} 2x + h$$
$$= 2x$$

The slope of the tangent line at the point (–2, 4) on the graph of *f* is $m_{\tan} = f'(-2) = 2(-2) = -4.$

34. **(a)** $m = \dfrac{f\left(\frac{1}{2}\right) - f(0)}{\frac{1}{2} - 0}$

$$= \frac{\left(2\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)^2\right) - (2(0) - 0^2)}{\frac{1}{2}}$$
$$= \frac{\frac{3}{4} - 0}{\frac{1}{2}}$$
$$= \frac{3}{2}$$

(b) $f'(0) = \lim_{h\to 0}\dfrac{f(0+h)-f(0)}{h}$

$$= \lim_{h\to 0}\frac{2h - h^2 - 0}{h}$$
$$= \lim_{h\to 0}(2-h)$$
$$= 2$$

The answer is part (a) is a relatively good approximation to the slope of the tangent line.

35. **(a)** If $f(x) = x^3$, then *f*(1) = 1, $f(1.1) = (1.1)^3 = 1.331.$

The slope of the secant line joining the points (1, 1) and (1.1, 1.331) on the graph of *f* is

$$m_{\text{sec}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.331 - 1}{1.1 - 1} = 3.31.$$

(b) If $f(x) = x^3$, then

$f(x+h) = (x+h)^3.$

The difference quotient (DQ) is

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^3 - x^3}{h}$$
$$= \frac{3x^2h + 3xh^2 + h^3}{h}$$
$$= 3x^2 + 3xh + h^2$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = 3x^2$$

The slope is $m_{\tan} = f'(1) = 3.$

Notice that this slope was approximated by the slope of the secant in part (a).

36. (a) $m = \dfrac{f\left(-\frac{1}{2}\right) - f(-1)}{-\frac{1}{2} - (-1)}$

$= \dfrac{\dfrac{-\frac{1}{2}}{-\frac{1}{2}-1} - \dfrac{-1}{-1-1}}{\frac{1}{2}}$

$= \dfrac{\frac{1}{3} - \frac{1}{2}}{\frac{1}{2}}$

$= -\dfrac{1}{3}$

(b) $f'(-1) = \lim\limits_{h\to 0} \dfrac{f(-1+h) - f(-1)}{h}$

$= \lim\limits_{h\to 0} \dfrac{\frac{-1+h}{-1+h-1} - \frac{-1}{-1-1}}{h}$

$= \lim\limits_{h\to 0} \dfrac{1}{2(h-2)}$

$= -\dfrac{1}{4}$

The answer in part (a) is a relatively good approximation to the slope of the tangent line.

37. (a) If $f(x) = 3x^2 - x$, the average rate of change of f is $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$.

Since $f(0) = 0$ and

$f\left(\dfrac{1}{16}\right) = 3\left(\dfrac{1}{16}\right)^2 - \dfrac{1}{16} = -\dfrac{13}{256}$,

$\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{-\frac{13}{256} - 0}{\frac{1}{16} - 0}$

$= -\dfrac{13}{16}$

$= -0.8125.$

(b) If $f(x) = 3x^2 - x$, then

$f(x+h) = 3(x+h)^2 - (x+h).$

The difference quotient (DQ) is

$\dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{3(x+h)^2 - (x+h) - (3x^2 - x)}{h}$

$= \dfrac{3x^2 + 6xh + 3h^2 - x - h - 3x^2 + x}{h}$

$= \dfrac{6xh + 3h^2 - h}{h}$

$= 6x + 3h - 1.$

$f'(x) = \lim\limits_{h\to 0} (6x + 3h - 1) = 6x - 1$

The instantaneous rate of change at $x = 0$ is $f'(0) = -1$. Notice that this rate is estimated by the average rate in part **(a)**.

38. (a) $f_{\text{ave}} = \dfrac{f\left(\frac{1}{2}\right) - f(0)}{\frac{1}{2} - 0}$

$= \dfrac{\frac{1}{2}\left(1 - 2\left(\frac{1}{2}\right)\right) - 0(1 - 2(0))}{\frac{1}{2}}$

$= \dfrac{0 - 0}{\frac{1}{2}}$

$= 0$

(b) $f'(0) = \lim\limits_{h\to 0} \dfrac{f(0+h) - f(0)}{h}$

$= \lim\limits_{h\to 0} \dfrac{h(1-2h) - 0}{h}$

$= \lim\limits_{h\to 0} (1 - 2h)$

$= 1$

The answer in part a is not a very good approximation to the average rate of change.

39. (a) If $s(t) = \dfrac{t-1}{t+1}$, the average rate of change of s is $\dfrac{s(t_2) - s(t_1)}{t_2 - t_1}$.

Since $s\left(-\dfrac{1}{2}\right) = \dfrac{\frac{1}{2} - 1}{-\frac{1}{2} + 1} = -3$ and

$s(0) = \dfrac{0-1}{0+1} = -1$, $\dfrac{-3+1}{-\frac{1}{2} - 0} = 4.$

(b) If $s(t)=\frac{t-1}{t+1}$, then

$$s(t+h)=\frac{(t+h)-1}{(t+h)+1}.$$

The difference quotient (DQ) is

$$\frac{s(t+h)-s(t)}{h}=\frac{\frac{t+h-1}{t+h+1}-\frac{t-1}{t+1}}{h}.$$

Multiplying numerator and denominator by $(t + h + 1)(t + 1)$.

$$=\frac{(t+h-1)(t+1)-(t-1)(t+h+1)}{h(t+h+1)(t+1)}$$

$$=\frac{t^2+th-t+t+h-1-t^2-th-t+t+h+1}{h(t+h+1)(t+1)}$$

$$=\frac{2h}{h(t+h+1)(t+1)}$$

$$=\frac{2}{(t+h+1)(t+1)}$$

$$s'(t)=\lim_{h\to 0}\frac{2}{(t+h+1)(t+1)}=\frac{2}{(t+1)^2}$$

The instantaneous rate of change when $t=-\frac{1}{2}$ is

$$s'\left(-\frac{1}{2}\right)=\frac{2}{\left(-\frac{1}{2}+1\right)^2}=8.$$

Notice that the estimate given by the average rate in part **(a)** differs significantly.

40. (a)

$$s_{\text{ave}}=\frac{s\left(\frac{1}{4}\right)-s(1)}{\frac{1}{4}-1}$$

$$=\frac{\sqrt{\frac{1}{4}}-\sqrt{1}}{-\frac{3}{4}}$$

$$=\frac{\frac{1}{2}-1}{-\frac{3}{4}}$$

$$=\frac{2}{3}$$

(b)

$$s'(1)=\lim_{h\to 0}\frac{s(1+h)-s(1)}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{1+h}-\sqrt{1}}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{1+h}-1}{h}\cdot\frac{\sqrt{1+h}+1}{\sqrt{1+h}+1}$$

$$=\lim_{h\to 0}\frac{1+h-1}{h\left(\sqrt{1+h}+1\right)}$$

$$=\lim_{h\to 0}\frac{1}{\sqrt{1+h}+1}$$

$$=\frac{1}{2}$$

The answer in part a is a relatively good approximation to the instantaneous rate of change.

41. (a) The average rate of temperature change between t_0 and t_0+h hours after midnight. The instantaneous rate of temperature change t_0 hours after midnight.

(b) The average rate of change in blood alcohol level between t_0 and t_0+h hours after consumption. The instantaneous rate of change in blood alcohol level t_0 hours after consumption.

(c) The average rate of change of the 30-year fixed mortgage rate between t_0 and t_0+h years after 2005. The instantaneous rate of change of 30-year fixed mortgage rate t_0 years after 2005.

42. (a) ... the average rate of change of revenue when the production level changes from x_0 to x_0+h units.
... the instantaneous rate of change of revenue when the production level is x_0 units.

(b) ... the average rate of change in the fuel level, in lb/ft, as the rocket travels between x_0 and x_0+h feet above the ground.

... the instantaneous rate in fuel level when the rocket is x_0 feet above the ground.

(c) ... the average rate of change in volume of the growth as the drug dosage changes from x_0 to $x_0 + h$ mg.
... the instantaneous rate in the growth's volume when x_0 mg of the drug have been injected.

43. $P(x) = 4{,}000(15 - x)(x - 2)$

(a) The difference quotient (DQ) is

$$\frac{P(x+h)-P(x)}{h}$$
$$= \frac{[4{,}000(15-(x+h))((x+h)-2)]}{h} - \frac{[4{,}000(15-x)(x-2)]}{h}$$
$$= \frac{4{,}000[(15-x-h)(x+h-2)-(15-x)(x-2)]}{h}$$
$$= \frac{4{,}000(17h-2xh-h^2)}{h}$$
$$= 4{,}000(17-2x-h)$$

$$P'(x) = \lim_{h\to 0}\frac{P(x+h)-P(x)}{h} = 4{,}000(17-2x)$$

(b) $P'(x) = 0$ when $4{,}000(17 - 2x) = 0$.
$x = \frac{17}{2} = 8.5,$ or 850 units.
When $P'(x) = 0,$ the line tangent to the graph of P is horizontal. Since the graph of P is a parabola which opens down, this horizontal tangent indicates a maximum profit.

44. (a) Profit = (number sold)(profit on each)
Profit on each
= selling price – cost to obtain
$P(p) = (120 - p)(p - 50)$
Since $q = 120 - p$, $p = 120 - q$.
$P(q) = q[(120 - q) - 50]$
or $P(q) = q(70 - q) = 70q - q^2$.

(b) The average rate as q increases from $q = 0$ to $q = 20$ is

$$\frac{P(20)-P(0)}{20} = \frac{[70(20)-(20)^2]-0}{20} = \$50 \text{ per recorder}$$

(c) The rate the profit is changing at $q = 20$ is $P'(20)$.
The difference quotient is

$$\frac{P(q+h)-P(q)}{h}$$
$$= \frac{[70(q+h)-(q+h)^2]-[70q-q^2]}{h}$$
$$= \frac{70q+70h-q^2-2qh-h^2-70q+q^2}{h}$$
$$= \frac{70h-2qh-h^2}{h}$$
$$= 70-2q-h$$

$$P'(q) = \lim_{h\to 0}\frac{P(q+h)-P(q)}{h} = 70-2q$$

$P'(20) = 70 - 2(20) = \$30$ per recorder.
Since $P'(20)$ is positive, profit is increasing.

45. $C(x) = 0.04x^2 + 2.1x + 60$

(a) As x increases from 10 to 11, the average rate of change is

$$\frac{C(11)-C(10)}{11-10}$$

$$C(11) = 0.04(11)^2 + 2.1(11) + 60 = 87.94$$
$$C(10) = 0.04(10)^2 + 2.1(10) + 60 = 85$$

$$\frac{87.94-85}{11-10} = 2.94$$

or \$2,940 per unit.

(b) $C(x+h)$

$= 0.04(x+h)^2 + 2.1(x+h) + 60$

So, the difference quotient (DQ) is

$$\frac{C(x+h)-C(x)}{h}$$

$$= \frac{\left[0.04(x+h)^2 + 2.1(x+h) + 60\right.}{h}$$

$$\frac{\left.-\left(0.04x^2 + 2.1x + 60\right)\right]}{h}$$

$$= \frac{\left[0.04x^2 + 0.08xh + 0.04h^2 + 2.1x\right.}{h}$$

$$\frac{\left.+2.1h + 60 - 0.04x^2 - 2.1x - 60\right]}{h}$$

$$= \frac{0.08xh + 0.04h^2 + 2.1h}{h}$$

$= 0.08x + 0.04h + 2.1$

$C'(x) = \lim_{h\to 0}(0.08x + 0.04h + 2.1)$

$= 0.08x + 2.1$

$C'(10) = 0.08(10) + 2.1 = 2.90$

or $2,900 per unit

The average rate of change is close to this value and is an estimate of this instantaneous rate of change.

Since $C'(10)$ is positive, the cost will increase.

46. (a) $Q_{\text{ave}} = \dfrac{Q(3{,}100) - Q(3{,}025)}{3{,}100 - 3{,}025}$

$$= \frac{3{,}100\sqrt{3{,}100} - 3{,}100\sqrt{3{,}025}}{75}$$

$$= \frac{3{,}100\left(10\sqrt{31} - 55\right)}{75}$$

≈ 28.01

The average rate of change in output is about 28 units per worker-hour.

(b) $$\begin{aligned} Q'(3{,}025) &= \lim_{h\to 0} \frac{Q(3{,}025+h)-Q(3{,}025)}{h} \\ &= \lim_{h\to 0} \frac{3{,}100\sqrt{3{,}025+h}-3{,}100\sqrt{3{,}025}}{h} \\ &= \lim_{h\to 0} \frac{3{,}100\left(\sqrt{3{,}025+h}-55\right)}{h} \cdot \frac{\sqrt{3{,}025+h}+55}{\sqrt{3{,}025+h}+55} \\ &= \lim_{h\to 0} \frac{3{,}100(3{,}025+h-3{,}025)}{h\left(\sqrt{3{,}025+h}+55\right)} \\ &= \lim_{h\to 0} \frac{3{,}100}{\sqrt{3{,}025+h}+55} \\ &= \frac{3{,}100}{110} \\ &\approx 28.2 \end{aligned}$$

The instantaneous rate of change is 28.2 units per worker-hour.

47. Writing Exercise—Answers will vary.

48. **(a)** $$\begin{aligned} E(x) &= x \cdot D(x) \\ &= x(-35x+200) \\ &= -35x^2+200x \end{aligned}$$

(b) $$\begin{aligned} E_{\text{ave}} &= \frac{E(5)-E(4)}{5-4} \\ &= -35(5)^2+200(5)-(-35(4)^2+200(4)) \\ &= 125-240 \\ &= -115 \end{aligned}$$

The average change in consumer expenditures is –$115 per unit.

(c) $$\begin{aligned} E'(4) &= \lim_{h\to 0} \frac{E(4+h)-E(4)}{h} \\ &= \lim_{h\to 0} \frac{-35(4+h)^2+200(4+h)-(-35(4)^2+200(4))}{h} \\ &= \lim_{h\to 0} \frac{-35h^2-80h}{h} \\ &= \lim_{h\to 0} (-35h-80) \\ &= -80 \end{aligned}$$

The instantaneous rate of change is –$80 per unit when $x = 4$. The expenditure is decreasing when $x = 4$.

49. When $t = 30$, $\frac{dV}{dt} \approx \frac{65-50}{50-30} = \frac{3}{4}$.

In the "long run," the rate at which V is changing with respect to time is getting smaller and smaller, decreasing to zero.

50. Answers will vary. Drawing a tangent line at each of the indicated points on the curve shows the population is growing at approximately 10/day after 20 days and 8/day after 36 days. The tangent line slope is steepest between 24 and 30 days at approximately 27 days.

51. When h = 1,000 meters,

$$\begin{aligned}\frac{dT}{dh} &\approx \frac{-6-0}{2{,}000-1{,}000}\\ &= \frac{-6}{1{,}000}\\ &= -0.006°\text{C/meter}\end{aligned}$$

When h = 2,000 meters, $\frac{dT}{dh} = 0°\text{C/meter.}$

Since the line tangent to the graph at h = 2,000 is horizontal, its slope is zero.

52. $P(t) = -6t^2 + 12t + 151$

(a) The average rate of change is $\frac{P(t_2)-P(t_1)}{t_2-t_1} = \frac{P(2)-P(0)}{2-0}$.

Since $P(2) = -6(2)^2 + 12(2) + 151 = 151$

and $P(0) = -6(0)^2 + 12(0) + 151 = 151$,

$\frac{P(2)-P(0)}{2-0} = \frac{151-151}{2} = 0.$

The population's average rate of change for 2010–2012 is zero.

(b) To find the instantaneous rate, calculate $P'(2)$.

$P(t+h) = -6(t+h)^2 + 12(t+h) + 151$ so the difference quotient (DQ) is

$$\begin{aligned}\text{DQ} &= \frac{P(t+h)-P(t)}{h}\\ &= \frac{-6(t+h)^2 + 12(t+h) + 151 - \left(-6t^2 + 12t + 151\right)}{h}\\ &= \frac{-6t^2 - 12ht - 6h^2 + 12t + 12h + 151 + 6t^2 - 12t - 151}{h}\\ &= \frac{-12ht - 6h^2 + 12h}{h}\\ &= -12t - 6h + 12\end{aligned}$$

$P'(x) = \lim_{h\to 0} \text{DQ} = \lim_{h\to 0}(-12t - 6h + 12) = -12t + 12$

For 2012, t = 2, so the instantaneous rate of change is $P'(2) = -12(2) + 12 = -12$, or a decrease of 12,000 people/year.

53. $H(t) = 4.4t - 4.9t^2$

(a) $H(t+h)$

$= 4.4(t+h) - 4.9(t+h)^2$

$= 4.4t + 4.4h - 4.9(t^2 + 2th + h^2)$

$= 4.4t + 4.4h - 4.9t^2 - 9.8th - 4.9h^2$

The difference quotient (DQ) is

$$\frac{H(t+h) - H(t)}{h}$$

$$= \frac{4.4t + 4.4h - 4.9t^2 - 9.8th - 4.9h^2 - (4.4t - 4.9t^2)}{h}$$

$$= \frac{4.4h - 9.8th - 4.9h^2}{h}$$

$$= \frac{h(4.4 - 9.8t - 4.9h)}{h}$$

$= 4.4 - 9.8t - 4.9h$

$$H'(t) = \lim_{h \to 0} \frac{H(t+h) - H(t)}{h}$$

$$= \lim_{h \to 0} 4.4 - 9.8t - 4.9h$$

$$= 4.4 - 9.8t$$

After 1 second, *H* is changing at a rate of $H'(1) = 4.4 - 9.8(1) = -5.4$ m/sec, where the negative represents that *H* is decreasing.

(b) $H'(t) = 0$ when $4.4 - 9.8t = 0$, or

$t \approx 0.449$ seconds.

This represents the time when the height is not changing (neither increasing nor decreasing). That is, this represents the highest point in the jump.

(c) When the flea lands, the height $H(t)$ will be zero (as it was when $t = 0$).

$4.4t - 4.9t^2 = 0$

$(4.4 - 4.9t)t = 0$

$4.4 - 4.9t = 0$

$$t = \frac{44}{49} \approx 0.898 \text{ seconds}$$

At this time, the rate of change is $H'\left(\frac{44}{49}\right) = 4.4 - 9.8\left(\frac{44}{49}\right)$

$= -4.4$ m/sec.

Again, the negative represents that *H* is decreasing.

54. (a) If $P(t)$ represents the blood pressure function then $P(0.7) \approx 80$, $P(0.75) \approx 77$, and $P(0.8) \approx 85$.

The average rate of change on [0.7,0.75] is approximately $\frac{77-80}{0.5} = -6$ mm/sec while on [0.75, 0.8] the average rate of change is about $\frac{85-77}{0.5} = 16$ mm/sec. The rate of change is greater in magnitude in the period following the burst of blood.

(b) Writing exercise—answers will vary.

55. $D(p) = -0.0009p^2 + 0.13p + 17.81$

(a) The average rate of change is $\frac{D(p_2) - D(p_1)}{p_2 - p_1}$.

Since
$D(60)$
$= -0.0009(60)^2 + 0.13(60) + 17.81$
$= 22.37$
and
$D(61)$
$= -0.0009(61)^2 + 0.13(61) + 17.81$
$= 22.3911,$
$= \frac{22.3911 - 22.37}{61 - 60}$
$= 0.0211$ mm per mm of mercury

(b) $D(p+h) = -0.0009(p+h)^2 + 0.13$
$(p + h) + 17.81$
So, the difference quotient (DQ) is

$$\frac{D(p+h) - D(p)}{h}$$

$$\frac{[-0.0009(p+h)^2 + 0.13(p+h) + 17.81 - (-0.0009p^2 + 0.13p + 17.81)]}{h}$$

$$= \frac{[-0.0009p^2 - 0.0018ph - 0.0009h^2 + 0.13p + 0.13h + 17.81 + 0.0009p^2 - 0.13p - 17.81]}{h}$$

$$= \frac{-0.0018ph - 0.0009h^2 + 0.13h}{h}$$

$= -0.0018p - 0.0009h + 0.13$

$D'(x)$
$= \lim_{h \to 0} (-0.0018p - 0.0009h + 0.13)$
$= -0.0018p + 0.13$

The instantaneous rate of change when p = 60 is
$D'(60) = -0.0018(60) + 0.13$
$= 0.022$ mm
per mm of mercury. Since $D'(60)$ is positive, the pressure is increasing when p = 60.

(c) $-0.0018p + 0.13 = 0$
$p \approx 72.22$ mm of mercury

At this pressure, the diameter is neither increasing nor decreasing.

56. (a) The rocket is $h(6) = -576 + 1200 = 624$ feet above ground.

(b) The average velocity between 0 and 40 seconds is given by
$\frac{h(6) - h(0)}{6} = \frac{624}{6} = 104$ feet/second.

(c) $h'(0) = 200$ ft/sec and $h'(40) = -1080$ ft/sec. The negative sign in the second velocity indicates the rocket is falling.

57. $s(t) = 4\sqrt{t+1} - 4$
$= 4(t+1)^{1/2} - 4$

(a) $s(t+h) = 4[(t+h)+1]^{1/2} - 4$

So, the difference quotient (DQ) is

$$\frac{4(t+h+1)^{1/2} - 4 - \left[4(t+1)^{1/2} - 4\right]}{h}$$

$$= \frac{4(t+h+1)^{1/2} - 4 - 4(t+1)^{1/2} + 4}{h}$$

$$= \frac{4(t+h+1)^{1/2} - 4(t+1)^{1/2}}{h}$$

Multiplying the numerator and denominator by $4(t+h+1)^{1/2} + 4(t+1)^{1/2}$ gives

$$\frac{16(t+h+1)-16(t+1)}{h\left[4(t+h+1)^{1/2}+4(t+1)^{1/2}\right]}$$
$$=\frac{16t+16h+16-16t-16}{4h\left[(t+h+1)^{1/2}+(t+1)^{1/2}\right]}$$
$$=\frac{16h}{4h\left[(t+h+1)^{1/2}+(t+1)^{1/2}\right]}$$
$$=\frac{4}{(t+h+1)^{1/2}+(t+1)^{1/2}}$$
$$s'(t)=\lim_{h\to 0}\frac{4}{(t+h+1)^{1/2}+(t+1)^{1/2}}$$
$$=\frac{4}{(t+1)^{1/2}+(t+1)^{1/2}}$$
$$=\frac{4}{2(t+1)^{1/2}}$$

$$v_{\text{ins}}(t)=\frac{2}{(t+1)^{1/2}}=\frac{2}{\sqrt{t+1}}$$

(b) $v_{\text{ins}}(0)=\frac{2}{(0+1)^{1/2}}=\frac{2}{\sqrt{1}}=2\text{ m/sec}\sqrt{2}$

(c) $s(3)=4\sqrt{3+1}-4=8-4=4\text{ m}$

$$v_{\text{ins}}(3)=\frac{2}{\sqrt{3+1}}=\frac{2}{2}=1\text{ m/sec}$$

58. (a) $f'(x)=\lim_{h\to 0}\frac{(3(x+h)-2)-(3x-2)}{h}$
$=\lim_{h\to 0}\frac{3h}{h}$
$=3$

(b) At $x=-1$, $y=3(-1)-2=-5$ and $(-1,-5)$ is a point on the tangent line. Using the point-slope formula with $m=3$ gives $y-(-5)=3(x-(-1))$ or $y=3x-2$.

(c) The line tangent to a straight line at any point is the line itself.

59. (a) For $y=f(x)=x^2$,
$f(x+h)=(x+h)^2$.
The difference quotient (DQ) is

$$\frac{f(x+h)-f(x)}{h}=\frac{(x+h)^2-x^2}{h}$$
$$=\frac{2xh+h^2}{h}$$
$$=2x+h$$

$$\frac{dy}{dx}=f'(x)$$
$$=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=2x$$

For $y=f(x)=x^2-3$,
$f(x+h)=(x+h)^2-3$.
The difference quotient (DQ) is

$$\frac{[(x+h)^2-3]-(x^2-3)}{h}=\frac{2xh+h^2}{h}$$
$$=2x+h$$

$$\frac{dy}{dx}=f'(x)$$
$$=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=2x$$

The graph of $y=x^2-3$ is the graph of $y=x^2$ shifted down 3 units. So the graphs are parallel and their tangent lines have the same slopes for any value of *x*. This accounts geometrically for the fact that their derivatives are identical.

(b) Since $y=x^2+5$ is the parabola $y=x^2$ shifted up 5 units and the constant appears to have no effect on the derivative, the derivative of the function $y=x^2+5$ is also 2*x*.

60. **(a)** For $f(x) = x^2 + 3x$, the derivative is

$$f'(x) = \lim_{h\to 0} \frac{[(x+h)^2 + 3(x+h)] - (x^2+3x)}{h}$$
$$= \lim_{h\to 0} \frac{x^2 + 2hx + h^2 + 3x + 3h - x^2 - 3x}{h}$$
$$= \lim_{h\to 0} (2x + h + 3) = 2x + 3$$

(b) For $g(x) = x^2$, the derivative is

$$g'(x) = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$$
$$= \lim_{h\to 0} \frac{x^2 + 2hx + h^2 - x^2}{h}$$
$$= \lim_{h\to 0} (2x + h)$$
$$= 2x$$

While for $h(x) = 3x$, the derivative is

$$h'(x) = \lim_{h\to 0} \frac{3(x+h) - 3x}{h} = \lim_{h\to 0} \frac{3h}{h} = 3$$

(c) The derivative of the sum is the sum of the derivatives.

(d) The derivative of $f(x)$ is the sum of the derivative of $g(x)$ and $h(x)$.

61. **(a)** For $y = f(x) = x^2$,

$f(x+h) = (x+h)^2$.

The difference quotient (DQ) is

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2}{h}$$
$$= \frac{2xh + h^2}{h}$$
$$= 2x + h$$

$$\frac{dy}{dx} = f'(x)$$
$$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= 2x$$

For $y = f(x) = x^3$,

$f(x+h) = (x+h)^3$.

The difference quotient (DQ) is

$$\frac{(x+h)^3 - x^3}{h} = \frac{3x^2h + 3xh^2 + h^3}{h}$$
$$= 3x^2 + 3xh + h^2$$

$$\frac{dy}{dx} = f'(x)$$
$$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= 3x^2$$

(b) The pattern seems to be that the derivative of x raised to a power (x^n) is that power times x raised to the power decreased by one (nx^{n-1}). So, the derivative of the function $y = x^4$ is $4x^3$ and the derivative of the function $y = x^{27}$ is $27x^{26}$.

62. If $y = mx + b$ then

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{[m(x+h) + b] - (mx + b)}{h}$$
$$= \lim_{h\to 0} \frac{mx + mh + b - mx - b}{h}$$
$$= \lim_{h\to 0} \frac{mh}{h}$$
$$= \lim_{h\to 0} m$$
$$= m, \text{ a constant.}$$

63. When $x < 0$, the difference quotient (DQ) is

$$\frac{f(x+h) - f(x)}{h} = \frac{-(x+h) - (-x)}{h}$$
$$= \frac{-h}{h}$$
$$= -1$$

So, $f'(x) = \lim_{h\to 0} -1 = -1.$

When $x > 0$, the difference quotient (DQ) is $\frac{f(x+h) - f(x)}{h} = \frac{(x+h) - x}{h} = 1.$

So, $f'(x) = \lim_{h\to 0} 1 = 1.$

Since there is a sharp corner at $x = 0$ (graph changes from $y = -x$ to $y = x$), the graph makes an abrupt change in direction at $x = 0$. So, f is not differentiable at $x = 0$.

64. (a) Write any number x as $x = c + h$. If the value of x is approaching c, then h is approaching 0 and vice versa. Thus the indicated limit is the same as the limit in the definition of the derivative. Less formally, note that if $x \neq c$ then $\frac{f(x) - f(c)}{x - c}$ is the slope of a secant line. s x approaches c the slopes of the secant lines approach the slope of the tangent at c.

(b) $\lim_{x \to c}[f(x) - f(c)]$
$= \lim_{x \to c}\left[\frac{f(x) - f(c)}{x - c}(x - c)\right]$
$\lim_{x \to c}[f(x) - f(c)]$
$= \lim_{x \to c}\left[\frac{f(x) - f(c)}{x - c}\right]\lim_{x \to c}(x - c)$
$= f'(c) \cdot 0$
$= 0$
using part (a) for the first limit on the right.

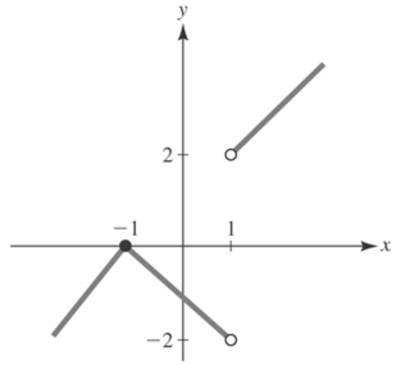
(c) Using the properties of limits and the result of part (b)

$$0 = \lim_{x \to c}[f(x) - f(c)]$$
$$= \lim_{x \to c} f(x) - \lim_{x \to c} f(c)$$
$$= \lim_{x \to c} f(x) - f(c)$$

so $\lim_{x \to c} f(x) = f(c)$ meaning $f(x)$ is continuous at $x = c$.

65. To show that $f(x) = \frac{|x^2 - 1|}{x - 1}$ is not differentiable at $x = 1$, press $\boxed{y =}$ and input $(\text{abs}(x^2 - 1)) / (x - 1)$ for $y_1 =$
The abs is under the NUM menu in the math application.
Use window dimensions [–4, 4]1 by [–4, 4]1
Press $\boxed{\text{Graph}}$
We see that f is not defined at $x = 1$. There can be no point of tangency.

$$\lim_{x \to 1^+} \frac{|x^2 - 1|}{x - 1} = \lim_{x \to 1^+} \frac{|(x-1)(x+1)|}{x - 1} = 2$$
$$\lim_{x \to 1^-} \frac{|x^2 - 1|}{x - 1} = \lim_{x \to 1^-} \frac{|(x-1)(x+1)|}{x - 1} = -2$$



66. Using the TRACE feature of a calculator with the graph of $y = 2x^3 - 0.8x^2 + 4$ shows a peak at $x = 0$ and a valley at $x = 0.2667$. Note the peaks and valleys are hard to see on the graph unless a small rectangle such as [–0.3, 0.5] × [3.8, 4.1] is used.

67. To find the slope of line tangent to the graph of $f(x) = \sqrt{x^2 + 2x - \sqrt{3x}}$ at $x = 3.85$, fill in the table below.
The $x + h$ row can be filled in manually.
For $f(x)$, press $\boxed{y =}$ and input $\sqrt{\ }\left(x \wedge 2 + 2x - \sqrt{\ }(3x)\right)$ for $y_1 =$
Use window dimensions [–1, 10]1 by [–1, 10]1.
Use the value function under the calc menu and enter $x = 3.85$ to find $f(x) = 4.37310$.
For $f(x + h)$, use the value function under the calc menu and enter $x = 3.83$ to find $f(x + h) = 4.35192$. Repeat this process for $x = 3.84, 3.849, 3.85, 3.851, 3.86$, and 3.87.
The $\frac{f(x+h) - f(x)}{h}$ can be filled in manually given that the rest of the table is now complete. So, slope $= f'(3.85) \approx 1.059$.

h	-0.02	-0.01	-0.001
$x+h$	3.83	3.84	3.849
$f(x)$	4.37310	4.37310	4.37310
$f(x+h)$	4.35192	4.36251	4.37204
$\frac{f(x+h)-f(x)}{h}$	1.059	1.059	1.059

0	0.001	0.01	0.02
3.85	3.851	3.86	3.87
4.37310	4.37310	4.37310	4.37310
4.37310	4.37415	4.38368	4.39426
undefined	1.058	1.058	1.058

2.2 Techniques of Differentiation

1. Since the derivative of any constant is zero,

$$y=-2$$
$$\frac{dy}{dx}=0$$

(Note: $y = -2$ is a horizontal line and all horizontal lines have a slope of zero, so $\frac{dy}{dx}$ must be zero.)

2. $y = 3$

$$\frac{dy}{dx}=0$$

3. $y=5x-3$

$$\frac{dy}{dx}=\frac{d}{dx}(5x)-\frac{d}{dx}(3)$$
$$\frac{dy}{dx}=5-0=5$$

4. $y = -2x + 7$

$$\frac{dy}{dx}=-2(1)+0=-2$$

5. $y=x^{-4}$

$$\frac{dy}{dx}=-4x^{-4-1}=-4x^{-5}=-\frac{4}{x^5}$$

6. $y=x^{7/3}$

$$\frac{dy}{dx}=\frac{7}{3}x^{7/3-1}=\frac{7}{3}x^{4/3}$$

7. $y=x^{3.7}$

$$\frac{dy}{dx}=3.7x^{3.7-1}=3.7x^{2.7}$$

8. $y=4-x^{-1.2}$

$$\frac{dy}{dx}=0-(-1.2)x^{-1.2-1}=1.2x^{-2.2}$$

9. $y=\pi r^2$

$$\frac{dy}{dx}=\pi(2r^{2-1})=2\pi r$$

10. $y=\frac{4}{3}\pi r^3$

$$\frac{dy}{dr}=\frac{4}{3}\pi(3r^2)=4\pi r^2$$

11. $y=\sqrt{2x}=\sqrt{2}\cdot x^{1/2}$

$$\frac{dy}{dx}=\sqrt{2}\left(\frac{1}{2}x^{1/2-1}\right)$$
$$=\sqrt{2}\left(\frac{1}{2}x^{-1/2}\right)$$
$$=\sqrt{2}\cdot\frac{1}{2x^{1/2}} \text{ or } \frac{\sqrt{2}}{2\sqrt{x}}$$

12. $y=2\sqrt[4]{x^3}=2x^{3/4}$

$$\frac{dy}{dx}=2\left(\frac{3}{4}\right)x^{3/4-1}=\frac{3}{2}x^{-1/4}=\frac{3}{2\sqrt[4]{x}}$$

13. $y=\frac{9}{\sqrt{t}}=9t^{-1/2}$

$$\frac{dy}{dx}=9\left(-\frac{1}{2}t^{-1/2-1}\right)$$
$$=9\left(-\frac{1}{2}t^{-3/2}\right)$$
$$=-\frac{9}{2t^{3/2}} \text{ or } -\frac{9}{2\sqrt{t^3}}$$

14. $y = \frac{3}{2t^2} = \frac{3}{2}t^{-2}$

$\frac{dy}{dt} = \frac{3}{2}(-2t^{-3}) = \frac{-3}{t^3}$

15. $y = x^2 + 2x + 3$

$\frac{dy}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dx}(2x) + \frac{d}{dx}(3)$

$\frac{dy}{dx} = 2x + 2$

16. $y = 3x^5 - 4x^3 + 9x - 6$

$\frac{dy}{dx} = 3(5x^4) - 4(3x^2) + 9(1) - 0$

$= 15x^4 - 12x^2 + 9$

17. $y = x^9 - 5x^8 + x + 12$

$\frac{dy}{dx} = \frac{d}{dx}(x^9) - \frac{d}{dx}(5x^8) + \frac{d}{dx}(x) + \frac{d}{dx}(12)$

$\frac{dy}{dx} = 9x^8 - 40x^7 + 1$

18. $f(x) = \frac{1}{4}x^8 - \frac{1}{2}x^6 - x + 2$

$f'(x) = 8 \cdot \frac{1}{4}x^7 - 6 \cdot \frac{1}{2}x^5 - 1 + 0$

$= 2x^7 - 3x^5 - 1$

19. $f(x) = -0.02x^3 + 0.3x$

$f'(x) = \frac{d}{dx}(-0.02x^3) + \frac{d}{dx}(0.3x)$

$f'(x) = -0.02(3x^2) + 0.3 = -0.06x^2 + 0.3$

20. $f(u) = 0.07u^4 - 1.21u^3 + 3u - 5.2$

$f'(u) = 4(0.07u^3) - 3(1.21u^2) + 3 - 0$

$= 0.28u^3 - 3.63u^2 + 3$

21. $y = \frac{1}{t} + \frac{1}{t^2} - \frac{1}{\sqrt{t}} = t^{-1} + t^{-2} - t^{-1/2}$

$\frac{dy}{dt} = \frac{d}{dt}(t^{-1}) + \frac{d}{dt}(t^{-2}) - \frac{d}{dt}(t^{-1/2})$

$= -1t^{-1-1} + -2t^{-2-1} - \left(-\frac{1}{2}t^{-1/2-1}\right)$

$= -1t^{-2} - 2t^{-3} + \frac{1}{2}t^{-3/2}$

$= -\frac{1}{t^2} - \frac{2}{t^3} + \frac{1}{2t^{3/2}}$,

or $-\frac{1}{t^2} - \frac{2}{t^3} + \frac{1}{2\sqrt{t^3}}$

22. $y = \frac{3}{x} - \frac{2}{x^2} + \frac{2}{3x^3} = 3x^{-1} - 2x^{-2} + \frac{2}{3}x^{-3}$

$\frac{dy}{dx} = (-1)(3x^{-2}) - (-2)(2x^{-3}) + (-3)\left(\frac{2}{3}x^{-4}\right)$

$= -3x^{-2} + 4x^{-3} - 2x^{-4}$

$= -\frac{3}{x^2} + \frac{4}{x^3} - \frac{2}{x^4}$

23. $f(x) = \sqrt{x^3} + \frac{1}{\sqrt{x^3}} = x^{3/2} + x^{-3/2}$,

$f'(x) = \frac{d}{dx}(x^{3/2}) + \frac{d}{dx}(x^{-3/2})$

$= \frac{3}{2}x^{3/2-1} + \frac{-3}{2}x^{-3/2-1}$

$= \frac{3}{2}x^{1/2} - \frac{3}{2}x^{-5/2}$

$= \frac{3}{2}x^{1/2} - \frac{3}{2x^{5/2}}$, or $\frac{3}{2}\sqrt{x} - \frac{3}{2\sqrt{x^5}}$

24. $f(t) = 2\sqrt{t^3} + \frac{4}{\sqrt{t}} - \sqrt{2}$

$= 2t^{3/2} + 4t^{-1/2} - \sqrt{2}$

$f'(t) = \frac{3}{2}(2t^{3/2-1}) + \frac{-1}{2}(4t^{-1/2-1}) - 0$

$= 3t^{1/2} - 2t^{-3/2}$

$= 3\sqrt{t} - \frac{2}{\sqrt{t^3}}$

25. $y = -\frac{x^2}{16} + \frac{2}{x} - x^{3/2} + \frac{1}{3x^2} + \frac{x}{3}$

$= -\frac{1}{16}x^2 + 2x^{-1} - x^{3/2} + \frac{1}{3}x^{-2} + \frac{1}{3}x,$

$\frac{dy}{dx} = \frac{d}{dx}\left(-\frac{1}{16}x^2\right) + \frac{d}{dx}(2x^{-1}) - \frac{d}{dx}(x^{3/2})$

$+ \frac{d}{dx}\left(\frac{1}{3}x^{-2}\right) + \frac{d}{dx}\left(\frac{1}{3}x\right)$

$= -\frac{1}{16}(2x) + 2(-1x^{-1-1}) - \frac{3}{2}x^{3/2-1}$

$+ \frac{1}{3}(-2x^{-2-1}) + \frac{1}{3}$

$= -\frac{1}{8}x - 2x^{-2} - \frac{3}{2}x^{1/2} - \frac{2}{3}x^{-3} + \frac{1}{3}$

$= -\frac{1}{8}x - \frac{2}{x^2} - \frac{3}{2}x^{1/2} - \frac{2}{3x^3} + \frac{1}{3},$

or $-\frac{1}{8}x - \frac{2}{x^2} - \frac{3}{2}\sqrt{x} - \frac{2}{3x^3} + \frac{1}{3}$

26. $y = \frac{-7}{x^{1.2}} + \frac{5}{x^{-2.1}} = -7x^{-1.2} + 5x^{2.1}$

$\frac{dy}{dx} = -1.2(-7x^{-1.2-1}) + 2.1(5x^{2.1-1})$

$= 8.4x^{-2.2} + 10.5x^{1.1}$

$= \frac{8.4}{x^{2.2}} + 10.5x^{1.1}$

27. $y = \frac{x^5 - 4x^2}{x^3}$

$= \frac{x^5}{x^3} - \frac{4x^2}{x^3}$

$= x^2 - \frac{4}{x}$

$= x^2 - 4x^{-1}$

$\frac{dy}{dx} = \frac{d}{dx}(x^2) - \frac{d}{dx}(4x^{-1})$

$= 2x - 4(-1x^{-1-1})$

$= 2x + 4x^{-2}$

$= 2x + \frac{4}{x^2}$

28. $y = x^2(x^3 - 6x + 7) = x^5 - 6x^3 + 7x^2$

$\frac{dy}{dx} = 5x^4 - 18x^2 + 14x$

29. $y = -x^3 - 5x^2 + 3x - 1$

$\frac{dy}{dx} = -3x^2 - 10x + 3$

At $x = -1$, $\frac{dy}{dx} = 10.$ The equation of the tangent line at $(-1, -8)$ is $y + 8 = 10(x + 1)$, or $y = 10x + 2$.

30. Given $y = x^5 - 3x^3 - 5x + 2$ and the point $(1, -5)$, then $\frac{dy}{dx} = 5x^4 - 9x^2 - 5$ and the slope of the tangent line at $x = 1$ is $m = 5(1^4) - 9(1^2) - 5 = -9$. The equation of the tangent line is then $y - (-5) = -9(x - 1)$ or $y = -9x + 4$.

31. $y = 1 - \frac{1}{x} + \frac{2}{\sqrt{x}} = 1 - x^{-1} + 2x^{-1/2}$

$\frac{dy}{dx} = x^{-2} - x^{-3/2} = \frac{1}{x^2} - \frac{1}{x^{3/2}}$

At $\left(4, \frac{7}{4}\right)$, $\frac{dy}{dx} = -\frac{1}{16}$. The equation of the tangent line is $y - \frac{7}{4} = -\frac{1}{16}(x - 4)$, or

$y = -\frac{1}{16}x + 2.$

32. Given $y = \sqrt{x^3} - x^2 + \frac{16}{x^2}$ and the point $(4, -7)$, then $\frac{dy}{dx} = \frac{3}{2}\sqrt{x} - 2x - \frac{32}{x^3}$ and the slope of the tangent line at $x = 4$ is $m = \frac{3}{2}\sqrt{4} - 2(4) - \frac{32}{4^3} = -\frac{11}{2}$. The equation of the tangent line is then $y - (-7) = -\frac{11}{2}(x - 4)$ or $y = -\frac{11}{2}x + 15$.

33. $y = (x^2 - x)(3 + 2x) = 2x^3 + x^2 - 3x$

$\frac{dy}{dx} = 6x^2 + 2x - 3$

At $x = -1$, $\frac{dy}{dx} = 1.$ The equation of the tangent line at (–1, 2) is $y - 2 = 1(x + 1)$, or $y = x + 3$.

34. Given $y = 2x^4 - \sqrt{x} + \frac{3}{x}$ and the point $(1, 4)$, then $\frac{dy}{dx} = 8x^3 - \frac{1}{2\sqrt{x}} - \frac{3}{x^2}$ and the slope of the tangent line at $x = 1$ is $m = 8(1^3) - \frac{1}{2(\sqrt{1})} - \frac{3}{1^2} = \frac{9}{2}$. The equation of the tangent line is then $y - 4 = \frac{9}{2}(x - 1)$ or $y = \frac{9}{2}x - \frac{1}{2}$.

35. $f(x) = -2x^3 + \frac{1}{x^2} = -2x^3 + x^{-2}$

$f'(x) = -6x^2 - \frac{2}{x^3}$

At $x = -1$, $f'(-1) = -4$. Further, $y = f(-1) = 3$. The equation of the tangent line at (–1, 3) is $y - 3 = -4(x + 1)$, or $y = -4x - 1$.

36. $f(x) = x^4 - 3x^3 + 2x^2 - 6;\ x = 2$

$f'(x) = 4x^3 - 9x^2 + 4x$

$f(2) = 16 - 24 + 8 - 6 = -6$ so $(2, -6)$ is a point on the tangent line. The slope is $m = f'(2) = 32 - 36 + 8 = 4$. The equation of the tangent line is $y - (-6) = 4(x - 2)$ or $y = 4x - 14$.

37. $f(x) = x - \frac{1}{x^2} = x - x^{-2}$

$f'(x) = 1 + \frac{2}{x^3}$

At $x = 1$, $f'(1) = 3$. Further, $y = f(1) = 0$. The equation of the tangent line at (1, 0) is $y - 0 = 3(x - 1)$, or $y = 3x - 3$.

38. $f(x) = x^3 + \sqrt{x};\ x = 4$

$f'(x) = 3x^2 + \frac{1}{2\sqrt{x}}$

$f(4) = 64 + 2 = 66$ so $(4, 66)$ is a point on the tangent line. The slope is $m = f'(2) = 48 + \frac{1}{4} = \frac{193}{4}$. The equation of the tangent line is $y - 66 = \frac{193}{4}(x - 4)$ or $y = \frac{193}{4}x - 127$.

39. $f(x) = -\frac{1}{3}x^3 + \sqrt{8x} = -\frac{1}{3}x^3 + \sqrt{8} \cdot x^{1/2}$

$f'(x) = -x^2 + \frac{\sqrt{8}}{2x^{1/2}}$

At $x = 2$,
$$\begin{aligned} f'(2) &= -4 + \frac{\sqrt{8}}{2\sqrt{2}} \\ &= -4 + \frac{1}{2}\sqrt{\frac{8}{2}} \\ &= -4 + \frac{1}{2} \cdot 2 \\ &= -3. \end{aligned}$$

Further, $y = f(2) = -\frac{8}{3} + 4 = \frac{4}{3}$. The equation of the tangent line at $\left(2, \frac{4}{3}\right)$ is $y - \frac{4}{3} = -3(x - 2)$, or $y = -3x + \frac{22}{3}$.

40. $f(x) = x(\sqrt{x} - 1) = x^{3/2} - x;\ x = 4$

$f'(x) = \frac{3}{2}\sqrt{x} - 1$

$f(4) = 8 - 4 = 4$ so (4, 4) is a point on the tangent line. The slope is $m = f'(4) = 3 - 1 = 2$. The equation of the tangent line is $y - 4 = 2(x - 4)$ or $y = 2x - 4$.

41. $f(x) = 2x^4 + 3x + 1$

$f'(x) = 8x^3 + 3$

The rate of change of f at $x = -1$ is $f'(-1) = -5$.

42. $f(x) = x^3 - 3x + 5;\ x = 2$

$f'(x) = 3x^2 - 3$

$f'(2) = 3(4) - 3 = 9$

43. $f(x) = x - \sqrt{x} + \frac{1}{x^2} = x - x^{1/2} + x^{-2}$

$f'(x) = 1 - \frac{1}{2x^{1/2}} - \frac{2}{x^3}$

The rate of change of f at $x = 1$ is

$f'(1) = -\frac{3}{2}.$

44. $f(x) = \sqrt{x} + 5x;\ x = 4$

$f'(x) = \frac{1}{2\sqrt{x}} + 5$

$f'(4) = \frac{1}{2(2)} + 5 = \frac{21}{4}$

45. $f(x) = \frac{x + \sqrt{x}}{\sqrt{x}}$

$= \frac{x}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{x}}$

$= \sqrt{x} + 1$

$= x^{1/2} + 1$

$f'(x) = \frac{1}{2x^{1/2}}$

The rate of change of f at $x = 1$ is

$f'(1) = \frac{1}{2}.$

46. $f(x) = \frac{2}{x} - x\sqrt{x};\ x = 1$

$f'(x) = \frac{-2}{x^2} - \frac{3}{2}\sqrt{x}$

$f'(1) = -2 - \frac{3}{2} = -\frac{7}{2}$

47. $f(x) = 2x^3 - 5x^2 + 4$

$f'(x) = 6x^2 - 10x$

The relative rate of change is

$\frac{f'(x)}{f(x)} = \frac{6x^2 - 10x}{2x^3 - 5x^2 + 4}.$

When $x = 1$, $\frac{f'(1)}{f(1)} = \frac{6 - 10}{2 - 5 + 4} = -4.$

48. $f(x) = x + \frac{1}{x} = x + x^{-1};\ f(1) = 1 + 1 = 2$

$f'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2};\ f'(1) = 1 - 1 = 0$

At $c = 1$, the relative rate of change is

$\frac{f'(1)}{f(1)} = \frac{0}{2} = 0$

49. $f(x) = x\sqrt{x} + x^2$

$= x \cdot x^{1/2} + x^2$

$= x^{3/2} + x^2$

$f'(x) = \frac{3}{2}x^{1/2} + 2x = \frac{3}{2}\sqrt{x} + 2x$

The relative rate of change is

$\frac{f'(x)}{f(x)} = \frac{\frac{3}{2}\sqrt{x} + 2x}{x\sqrt{x} + x^2} \cdot \frac{2}{2} = \frac{3\sqrt{x} + 4x}{2\left(x\sqrt{x} + x^2\right)}.$

When $x = 4$, $\frac{f'(x)}{f(4)} = \frac{3\sqrt{4} + 4(4)}{2\left(4\sqrt{4} + 4^2\right)} = \frac{11}{24}.$

50. $f(x) = (4 - x)x^{-1} = 4x^{-1} - 1;$

$f(3) = \frac{4}{3} - 1 = \frac{1}{3}$

$f'(x) = -4x^{-2};\ f'(3) = -\frac{4}{9}$

At $c = 3$, the relative rate of change is

$\frac{f'(3)}{f(3)} = \frac{-\frac{4}{9}}{\frac{1}{3}} = -\frac{4}{3}.$

51. (a) $A(t) = 0.1t^2 + 10t + 20$

$A'(t) = 0.2t + 10$

In the year 2008, the rate of change is $A'(4) = 0.8 + 10$ or \$10,800 per year.

(b) $A(4) = (0.1)(16) + 40 + 20 = 61.6$, so the percentage rate of change is

$\frac{(100)(10.8)}{61.6} = 17.53\%.$

52. (a) Since $f(x) = -x^3 + 6x^2 + 15x$ is the number of radios assembled x hours after 8:00 A.M., the rate of assembly after x hours is

$f'(x) = -3x^2 + 12x + 15$ radios per hour.

(b) The rate of assembly at 9:00 A.M. ($x = 1$) is
$f'(1) = -3 + 12 + 15 = 24$ radios per hour.

(c) At noon, $t = 4$.
$f'(4) = -3(4)^2 + 12(4) + 15 = 15$
and $f'(1) = 24$. So, Lupe is correct: the assembly rate is less at noon than at 9 A.M.

53. (a) $T(x) = 20x^2 + 40x + 600$ dollars
The rate of change of property tax is
$T'(x) = 40x + 40$ dollars/year.
In the year 2008, $x = 0$,
$T'(0) = 40$ dollars/year.

(b) In the year 2012, $x = 4$ and $T(4) = \$1,080$. In the year 2008, $x = 0$ and $T(0) = \$600$.
The change in property tax is
$T(4) - T(0) = \$480$.

54. $M(x) = 2,300 + \dfrac{125}{x} - \dfrac{517}{x^2}$

$M'(x) = -\dfrac{125}{x^2} + \dfrac{1034}{x^3}$

$M'(9) = -\dfrac{125}{9^2} + \dfrac{1034}{9^3} \approx -0.125$. Sales are decreasing at a rate of approximately 1/8 motorcycle per \$1,000 of advertising.

55. (a) Cost = cost driver + cost gasoline
cost driver $= 20(\#\text{hrs})$

$$= 20\left(\frac{250 \text{ mi}}{x}\right)$$

$$= \frac{5,000}{x}$$

cost gasoline
$= 4.0(\#\text{gals})$

$$= 4.0(250)\left[\frac{1}{250}\left(\frac{1,200}{x} + x\right)\right]$$

$$= \frac{4,800}{x} + 4.0x \text{ dollars}$$

So, the cost function is

$$C(x) = \frac{9,800}{x} + 4x.$$

(b) The rate of change of the cost is $C'(x)$.

$C(x) = 9,800x^{-1} + 4x$

$C'(x) = -\dfrac{9,800}{x^2} + 4$ dollars/mi per hr

When $x = 40$,
$C'(40) = -2.125$ dollars/mi per hr.
Since $C'(40)$ is negative, the cost is decreasing.

56. (a) Since $C(t) = 100t^2 + 400t + 5,000$ is the circulation t years from now, the rate of change of the circulation in t years is
$C'(t) = 200t + 400$ newspapers per year.

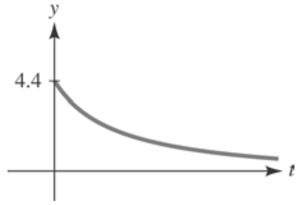
(b) The rate of change of the circulation 5 years from now is
$C'(5) = 200(5) + 400 = 1,400$ newspapers per year. The circulation is increasing.

(c) The actual change in the circulation during the 6th year is

$$C(6) - C(5) = 11,000 - 9,500$$
$$= 1,500 \text{ newspapers.}$$

57. (a) Since Gary's starting salary is \$45,000 and he gets a raise of \$2,000 per year, his salary t years from now will be
$S(t) = 45,000 + 2,000t$ dollars.
The percentage rate of change of this salary t years from now is

$$100\left[\frac{S'(t)}{S(t)}\right] = 100\left(\frac{2{,}000}{45{,}000 + 2{,}000t}\right)$$
$$= \frac{200}{45 + 2t} \text{ percent per year}$$



(b) The percentage rate of change after 1 year is
$\frac{200}{47} \approx 4.26\%$

(c) In the long run, $\frac{200}{45 + 2t}$ approaches 0.
That is, the percentage rate of Gary's salary will approach 0 (even though Gary's salary will continue to increase at a constant rate.)

58. Let $G(t)$ be the GDP in billions of dollars where t is years and $t = 0$ represents 1997. Since the GDP is growing at a constant rate, $G(t)$ is a linear function passing through the points $(0,125)$ and $(8,155)$.
Then
$$G(t) = \frac{155 - 125}{8 - 0}t + 125 = \frac{15}{4}t + 125.$$
In 2012, $t = 15$ and the model predicts a GDP of $G(15) = 181.25$ billion dollars.

59. (a) $f(x) = -6x + 582$
The rate of change of SAT scores is $f'(x) = -6.$

(b) The rate of change is constant, so the drop will not vary from year to year. The rate of change is negative, so the scores are declining.

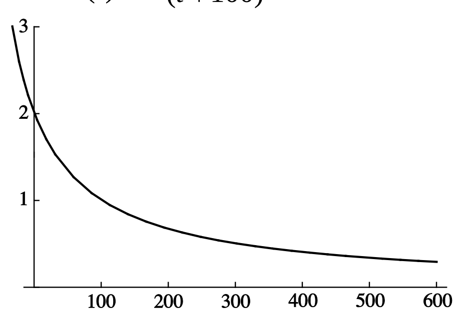
60. (a) Since $N(x) = 6x^3 + 500x + 8{,}000$ is the number of people using rapid transit after x weeks, the rate at which system use is changing after x weeks is
$N'(x) = 18x^2 + 500$ commuters per week. After 8 weeks this rate is
$N'(8) = 18(8^2) + 500 = 1652$ users per week.

(b) The actual change in usage during the 8th week is
$$N(8) - N(7) = 15{,}072 - 13{,}558$$
$$= 1{,}514 \text{ riders.}$$

61. (a) $P(x) = 2x + 4x^{3/2} + 5{,}000$ is the population x months from now. The rate of population growth is
$$P'(x) = 2 + 4\left(\frac{3x^{1/2}}{2}\right)$$
$$= 2 + 6x^{1/2} \text{ people per month.}$$
Nine months from now, the population will be changing at the rate of $P'(9) = 2 + 6(9^{1/2}) = 20$ people per month.

(b) The percentage rate at which the population will be changing 9 months from now is
$$100\frac{P'(9)}{P(9)} = \frac{100(20)}{2(9) + 4(9^{3/2}) + 5{,}000}$$
$$= \frac{2{,}000}{5{,}126}$$
$$= 0.39\%.$$

62. (a) $P(t) = t^2 + 200t + 10{,}000 = (t + 100)^2$
$P'(t) = 2t + 200 = 2(t + 100)$
The percentage rate of change is
$$100\frac{P'(t)}{P(t)} = \frac{200(t + 100)}{(t + 100)^2} = \frac{200}{t + 100}.$$



(b) The percentage rate of changes approaches 0 since $\lim_{t\to\infty} \frac{200}{t+100} = 0$.

63. $N(t) = 10t^3 + 5t + \sqrt{t} = 10t^3 + 5t + t^{1/2}$

The rate of change of the infected population is

$N'(t) = 30t^2 + 5 + \frac{1}{2t^{1/2}}$ people/day.

On the 9th day, $N'(9) = 2{,}435$ people/day.

64. (a) $N(t) = 5{,}175 - t^3(t-8)$

$= 5{,}175 - t^4 + 8t^3$

$N'(3) = -4(3^3) + 8\cdot 3(3^2) = 108$ people per week.

(b) The percentage rate of change of N is given by

$100\frac{N'(t)}{N(t)} = \frac{100(-4t^3 + 24t^2)}{5{,}175 - t^4 + 8t^3}$.

A graph of this function shows that it never exceeds 25%.

(c) Writing exercise—answers will vary.

65. (a) $T(t) = -68.07t^3 + 30.98t^2 + 12.52t + 37.1$

$T'(t) = -204.21t^2 + 61.96t + 12.52$

$T'(t)$ represents the rate at which the bird's temperature is changing after t days, measured in °C per day.

(b) $T'(0) = 12.52$°C/day since $T'(0)$ is positive, the bird's temperature is increasing.

$T'(0.713) \approx -47.12$°C/day

Since $T'(0.713)$ is negative, the bird's temperature is decreasing.

(c) Find t so that $T'(t) = 0$.

$0 = -204.21t^2 + 61.96t + 12.52$

$$t = \frac{-61.96 \pm \sqrt{(61.96)^2 - 4(-204.21)(12.52)}}{2(-204.21)}$$

$t \approx 0.442$ days

The bird's temperature when $t = 0.442$ is $T(0.442) \approx 42.8$°C.

The bird's temperature starts at $T(0) = 37.1$°C, increases to $T(0.442) = 42.8$°C, and then begins to decrease.

66. (a) Using the graph, the x-value (tax rate) that appears to correspond to a y-value (percentage reduction) of 50 is 150, or a tax rate of 150 dollars per ton carbon.

(b) Using the points (200, 60) and (300, 80), from the graph, the rate of change is approximately

$$\frac{dP}{dT} \approx \frac{80-60}{300-200} = \frac{20}{100} = 0.2\%$$

or increasing at approximately 0.2% per dollar. (Answers will vary depending on the choice of h.)

(c) Writing exercise – Answers will vary.

67. (a) $Q(t) = 0.05t^2 + 0.1t + 3.4$ PPM

$Q'(t) = 0.1t + 0.1$ PPM/year

The rate of change of Q at $t = 1$ is $Q'(1) = 0.2$ PPM/year.

(b) $Q(1) = 3.55$ PPM, $Q(0) = 3.40$, and $Q(1) - Q(0) = 0.15$ PPM.

(c) $Q(2) = 0.2 + 0.2 + 3.4 = 3.8$, $Q(0) = 3.4$, and $Q(2) - Q(0) = 0.4$ PPM.

68. $P = \frac{4}{3}\pi N\left(\frac{\mu^2}{3kT}\right) = \left(\frac{4\pi N\mu^2}{9k}\right)T^{-1}$

$$\frac{dP}{dt} = -\left(\frac{4\pi N\mu^2}{9k}\right)T^{-2} = -\frac{4\pi N\mu^2}{9kT^2}$$

69. Since g represents the acceleration due to gravity for the planet our spy is on, the formula for the rock's height is

$H(t) = -\frac{1}{2}gt^2 + V_0 t + H_0$

Since he throws the rock from ground

level, $H_0 = 0$. Also, since it returns to the ground after 5 seconds,

$$0 = -\frac{1}{2}g(5)^2 + V_0(5)$$
$$0 = -12.5g + 5V_0$$
$$V_0 = \frac{12.5g}{5} = 2.5g$$

The rock reaches its maximum height halfway through its trip, or when $t = 2.5$. So,

$$37.5 = -\frac{1}{2}g(2.5)^2 + V_0(2.5)$$
$$37.5 = -3.125g + 2.5V_0$$

Substituting $V_0 = 2.5g$

$$37.5 = -3.125g + 2.5(2.5g)$$
$$37.5 = -3.125g + 6.25g$$
$$37.5 = 3.125g$$
$$g = 12 \text{ ft/sec}^2$$

So, our spy is on Mars.

70. **(a)** $s(t) = t^2 - 2t + 6$ for $0 \le t \le 2$
$v(t) = 2t - 2$
$a(t) = 2$

(b) The particle is stationary when $v(t) = 2t - 2 = 0$ which is at time $t = 1$.

71. **(a)** $s(t) = 3t^2 + 2t - 5$ for $0 \le t \le 1$
$v(t) = 6t + 2$ and $a(t) = 6$

(b) $6t + 2 = 0$ at $t = -3$. The particle is not stationary between $t = 0$ and $t = 1$.

72. **(a)** $s(t) = t^3 - 9t^2 + 15t + 25$ for $0 \le t \le 6$
$v(t) = 3t^2 - 18t + 15 = 3(t-1)(t-5)$
$a(t) = 6t - 18 = 6(t-3)$

(b) The particle is stationary when $v(t) = 3(t-1)(t-5) = 0$ which is at times $t = 1$ and $t = 5$.

73. **(a)** $s(t) = t^4 - 4t^3 + 8t$ for $0 \le t \le 4$
$v(t) = 4t^3 - 12t^2 + 8$ and
$a(t) = 12t^2 - 24t$

(b) To find all time in given interval when stationary,

$$4t^3 - 12t^2 + 8 = 0$$
$$4(t^3 - 3t^2 + 2) = 0$$
$$t^3 - 3t^2 + 2 = 0$$
$$(t-1)(t^2 - 2t - 2) = 0$$
$$t = 1 \text{ or } t = \frac{2 \pm \sqrt{4 - 4\cdot 1\cdot(-2)}}{2}$$

Since $0 \le t \le 4$, $t = 1$ or $t = 1 + \sqrt{3}$.

74. **(a)** Since the initial velocity is $V_0 = 0$ feet per second, the initial height is $H_0 = 144$ feet and $g = 32$ feet per second per second, the height of the stone at time t is

$$H(t) = -\frac{1}{2}gt^2 + V_0t + H_0$$
$$= -16t^2 + 144.$$

The stone hits the ground when $H(t) = -16t^2 + 144 = 0$, that is when $t^2 = 9$ or after $t = 3$ seconds.

(b) The velocity at time t is given by $H(t) = -32t$. When the stone hits the ground, its velocity is $H'(3) = -96$ feet per second.

75. **(a)** If after 2 seconds the ball passes you on the way down, then $H(2) = H_0$, where $H(t) = -16t^2 + V_0t + H_0$.
So, $-16(2^2) + (V_0)(2) + H_0 = H_0$,
$-64 + 2V_0 = 0$, or $V_0 = 32\ \frac{\text{ft}}{\text{sec}}$.

(b) The height of the building is H_0 feet. From part **(a)** you know that $H(t) = -16t^2 + 32t + H_0$. Moreover, $H(4) = 0$ since the ball hits the ground after 4 seconds. So,

$-16(4)^2 + 32(4) + H_0 = 0$, or
$H_0 = 128$ feet.

(c) From parts **(a)** and **(b)** you know that $H(t) = -16t^2 + 32t + 128$ and so the speed of the ball is
$H'(t) = -32t + 32 \ \frac{\text{ft}}{\text{sec}}$.
After 2 seconds, the speed will be $H'(2) = -32$ feet per second, where the minus sign indicates that the direction of motion is down.

(d) The speed at which the ball hits the ground is $H'(4) = -96 \ \frac{\text{ft}}{\text{sec}}$.

76. Let (x, y) be a point on the curve where the tangent line goes through (0, 0). Then the slope of the tangent line is equal to $\frac{y-0}{x-0} = \frac{y}{x}$. The slope is also given by $f'(x) = 2x - 4$. Thus $\frac{y}{x} = 2x - 4$ or
$y = 2x^2 - 4x$.
Since (x, y) is a point on the curve, we must have $y = x^2 - 4x + 25$. Setting the two expressions for y equal to each other gives

$$x^2 - 4x + 25 = 2x^2 - 4x$$
$$x^2 = 25$$
$$x = \pm 5$$

If $x = -5$, then $y = 70$, the slope is -14 and the tangent line is $y = -14x$.
If $x = 5$, then $y = 30$, the slope is 6 and the tangent line is $y = 6x$.

77. $f(x) = ax^2 + bx + c$

Since $f(0) = 0$, $c = 0$ and $f(x) = ax^2 + bx$.
Since $f(5) = 0$, $0 = 25a + 5b$.
Further, since the slope of the tangent is 1 when $x = 2$, $f'(2) = 1$.

$$f'(x) = 2ax + b$$
$$1 = 2a(2) + b = 4a + b$$

Now, solve the system: $0 = 25a + 5b$ and $1 = 4a + b$. Since $1 - 4a = b$, using substitution

$$0 = 25a + 5(1 - 4a)$$
$$0 = 25a + 5 - 20a$$
$$0 = 5a + 5$$

or $a = -1$ and $b = 1 - 4(-1) = 5$.
So, $f(x) = -x^2 + 5x$.

78. **(a)** If $f(x) = x^4$ then

$$f(x+h) = (x+h)^4$$
$$= x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$
$$f(x+h) - f(x) = 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$

and

$$\frac{f(x+h) - f(x)}{h} = 4x^3 + 6x^2h + 4xh^2 + h^3$$

(b) If $f(x) = x^n$ then

$$f(x+h) = (x+h)^n = x^n + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \ldots + nxh^{n-1} + h^n$$

$$f(x+h) - f(x) = nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \ldots + nxh^{n-1} + h^n$$

and

$$\frac{f(x+h)-f(x)}{h}=nx^{n-1}+\frac{n(n-1)}{2}x^{n-2}h+\ldots+nxh^{n-2}+h^{n-1}$$

(c) From part (b)

$$\frac{f(x+h)-f(x)}{h}=nx^{n-1}+h\left[\frac{n(n-1)}{2}x^{n-2}+\ldots+h^{n-2}\right]$$

The first term on the right does not involve h while the second term approaches 0 as $h \to 0$.

Thus $\frac{d}{dx}[x^n]=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=nx^{n-1}.$

79. $(f+g)'(x)$

$$= \lim_{h\to 0}\frac{(f+g)(x+h)-(f+g)(x)}{h}$$
$$= \lim_{h\to 0}\frac{f(x+h)+g(x+h)-[f(x)+g(x)]}{h}$$
$$= \lim_{h\to 0}\frac{f(x+h)-f(x)+g(x+h)-g(x)}{h}$$
$$= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$$
$$= f'(x)+g'(x)$$

2.3 Product and Quotient Rules; Higher-Order Derivatives

1. $f(x) = (2x + 1)(3x - 2)$,

$$f'(x)=(2x+1)\frac{d}{dx}(3x-2)+(3x-2)\frac{d}{dx}(2x+1)$$
$$=(3x+1)(3)+(3x-2)(2)$$
$$=12x-1$$

2. $f(x)=(x-5)(1-2x)$

$$f(x)=(x-5)\frac{d}{dx}(1-2x)+(1-2x)\frac{d}{dx}(x-5)$$
$$=-2(x-5)+1(1-2x)$$
$$=11-4x$$

3. $y = 10(3u + 1)(1 - 5u)$,

$$\frac{dy}{du}$$

$$= 10\frac{d}{du}(3u+1)(1-5u)$$

$$= 10\left[(3u+1)\frac{d}{du}(1-5u)+(1-5u)\frac{d}{du}(3u+1)\right]$$

$$= 10[(3u+1)(-5)+(1-5u)(3)]$$

$$= -300u-20$$

4. $y = 400(15-x^2)(3x-2)$

$$\frac{dy}{dx} = 400\frac{d}{dx}\left[(15-x^2)(3x-2)\right]$$

$$= 400\left[(15-x^2)\frac{d}{dx}(3x-2)+(3x-2)\frac{d}{dx}(15-x^2)\right]$$

$$= 400\left[(15-x^2)(3)+(3x-2)(-2x)\right]$$

$$= 400(-9x^2+4x+45)$$

5. $$f'(x) = \frac{1}{3}\left[(x^5-2x^3+1)\frac{d}{dx}\left(x-\frac{1}{x}\right)\right.$$

$$\left.+\left(x-\frac{1}{x}\right)\frac{d}{dx}(x^5-2x^3+1)\right]$$

$$= \frac{1}{3}\left[(x^5-2x^3+1)\left(1+\frac{1}{x^2}\right)\right.$$

$$\left.+\left(x-\frac{1}{x}\right)(5x^4-6x^2)\right]$$

$$= 2x^5-4x^3+\frac{4}{3}x+\frac{1}{3x^2}+\frac{1}{3}$$

6. $f(x) = -3(5x^3-2x+5)(\sqrt{x}+2x)$

$$f'(x) = -3\left[(5x^3-2x+5)\left(\frac{1}{2\sqrt{x}}+2\right)+\left(\sqrt{x}+2x\right)(15x^2-2)\right]$$

$$= -\frac{105}{2}x^{5/2}-120x^3+9x^{1/2}+24x-\frac{15}{2x^{1/2}}-30$$

7. $y = \dfrac{x+1}{x-2}$,

$$\frac{dy}{dx} = \frac{(x-2)\frac{d}{dx}(x+1) - (x+1)\frac{d}{dx}(x-2)}{(x-2)^2}$$
$$= \frac{(x-2)(1) - (x+1)(1)}{(x-2)^2}$$
$$= -\frac{3}{(x-2)^2}$$

8. $y = \dfrac{2x-3}{5x+4}$

$$\frac{dy}{dx} = \frac{(5x+4)\dfrac{d}{dx}(2x-3) - (2x-3)\dfrac{d}{dx}(5x+4)}{(5x+4)^2}$$
$$= \frac{2(5x+4) - 5(2x-3)}{(5x+4)^2}$$
$$= \frac{23}{(5x+4)^2}$$

9. $f(t) = \dfrac{t}{t^2 - 2}$,

$$f'(t) = \frac{(t^2-2)\frac{d}{dt}(t) - t\frac{d}{dt}(t^2-2)}{(t^2-2)^2}$$
$$= \frac{(t^2-2)(1) - (t)(2t)}{(t^2-2)^2}$$
$$= \frac{-t^2-2}{(t^2-2)^2}$$

10. $f(x) = \dfrac{1}{x-2}$

$$f'(x) = \frac{(x-2)(0) - 1(1)}{(x-2)^2} = \frac{-1}{(x-2)^2}$$

11. $y = \dfrac{3}{x+5}$,

$$\frac{dy}{dx} = \frac{(x+5)\frac{d}{dx}(3) - 3\frac{d}{dx}(x+5)}{(x+5)^2}$$
$$= \frac{(x+5)(0) - 3(1)}{(x+5)^2}$$
$$= -\frac{3}{(x+5)^2}$$

12. $y = \dfrac{t^2+1}{1-t^2}$

$$\frac{dy}{dt} = \frac{(1-t^2)(2t)-(t^2+1)(-2t)}{(1-t^2)^2} = \frac{4t}{(1-t^2)^2}$$

13. $f(x) = \dfrac{x^2-3x+2}{2x^2+5x-1},$

$$f'(x) = \frac{(2x^2+5x-1)\frac{d}{dx}(x^2-3x+2)}{(2x^2+5x-1)^2} - \frac{(x^2-3x+2)\frac{d}{dx}(2x^2+5x-1)}{(2x^2+5x-1)^2}$$

$$= \frac{(2x^2+5x-1)(2x-3)}{(2x^2+5x-1)^2} - \frac{(x^2-3x+2)(4x+5)}{(2x^2+5x-1)^2}$$

$$= \frac{11x^2-10x-7}{(2x^2+5x-1)^2}$$

14. $g(x) = \dfrac{(x^2+x+1)(4-x)}{2x-1}$

$$g'(x) = \frac{\left[(2x-1)[-1(x^2+x+1)+(4-x)(2x+1)]-(x^2+x+1)(4-x)(2)\right]}{(2x-1)^2}$$

$$= \frac{-4x^3+9x^2-6x-11}{(2x-1)^2}$$

15. $f(x) = (2+5x)^2 = (2+5x)(2+5x)$

$$f'(x) = (2+5x)\frac{d}{dx}(2+5x) + (2+5x)\frac{d}{dx}(2+5x)$$

$$= 2(2+5x)\frac{d}{dx}(2+5x)$$

$$= 2(2+5x)(5)$$

$$= 20+50x$$

$$= 10(2+5x)$$

16. $f(x) = \left(x+\dfrac{1}{x}\right)^2 = x^2+2+\dfrac{1}{x^2}$

$$f'(x) = 2x - \frac{2}{x^3}$$

17. $g(t)=\dfrac{t^2+\sqrt{t}}{2t+5}=\dfrac{t^2+t^{1/2}}{2t+5}$

$$g'(t)=\frac{(2t+5)\frac{d}{dt}(t^2+t^{1/2})-(t^2+t^{1/2})\frac{d}{dt}(2t+5)}{(2t+5)^2}$$

$$=\frac{(2t+5)\left(2t+\frac{1}{2t^{1/2}}\right)-(t^2+t^{1/2})(2)}{(2t+5)^2}$$

$$=\frac{2t^2+10t-t^{1/2}+\frac{5}{2t^{1/2}}}{(2t+5)^2}\cdot\frac{2t^{1/2}}{2t^{1/2}}$$

$$=\frac{4t^{5/2}+20t^{3/2}-2t+5}{2t^{1/2}(2t+5)^2}$$

$$=\frac{4\sqrt{t^5}+20\sqrt{t^3}-2t+5}{2\sqrt{t}(2t+5)^2}$$

18. $h(x)=\dfrac{x}{x^2-1}+\dfrac{4-x}{x^2+1}$

$$h'(x)=\frac{(x^2-1)(1)-x(2x)}{(x^2-1)^2}+\frac{(x^2+1)(-1)-(4-x)(2x)}{(x^2+1)^2}$$

$$=\frac{-x^2-1}{(x^2-1)^2}+\frac{x^2-8x-1}{(x^2+1)^2}$$

19. $y=(5x-1)(4+3x)$

$\dfrac{dy}{dx}=30x+17$

When $x = 0$, $y = -4$ and $\dfrac{dy}{dx}=17.$ The equation of the tangent line at (0, –4) is $y + 4 = 17(x - 0)$, or $y = 17x - 4$.

20. $y=(x^2+3x-1)(2-x)$

$y'=(x^2+3x-1)(-1)+(2x+3)(2-x)$

At $x_0=1$, $y=(3)(1)=3$ and $y'=(3)(-1)+(5)(1)=2$. The equation of the tangent line is then $y-3=2(x-1)$ or $y=2x+1$.

21. $y=\dfrac{x}{2x+3}$

$\dfrac{dy}{dx}=\dfrac{3}{(2x+3)^2}$

When $x = -1$, $y = -1$ and $\dfrac{dy}{dx}=3.$ The equation of the tangent line at (–1, –1) is $y + 1 = 3(x + 1)$, or $y = 3x + 2$.

22. $y = \dfrac{x+7}{5-2x}$

$y' = \dfrac{(5-2x)(1)-(x+7)(-2)}{(5-2x)^2}$

At $x_0 = 0$, $y = \dfrac{7}{5}$ and $y' = \dfrac{5+14}{5^2} = \dfrac{19}{25}$.

The equation of the tangent line is then $y - \dfrac{7}{5} = \dfrac{19}{25}(x-0)$ or $y = \dfrac{19}{25}x + \dfrac{7}{5}$.

23. $y = \left(3\sqrt{x} + x\right)(2 - x^2)$

$= (3x^{1/2} + x)(2 - x^2)$

$\dfrac{dy}{dx} = -3x^2 - \dfrac{15}{2}x^{3/2} + \dfrac{3}{x^{1/2}} + 2$

When $x = 1$, $y = 4$ and $\dfrac{dy}{dx} = -\dfrac{11}{2}$.

The equation of the tangent line at (1, 4) is $y - 4 = -\dfrac{11}{2}(x-1)$, or $y = -\dfrac{11}{2}x + \dfrac{19}{2}$.

24. $f(x) = (x-1)(x^2 - 8x + 7)$

$f'(x) = 1\cdot(x^2 - 8x + 7) + (x-1)(2x-8)$

$= 3x^2 - 18x + 15$

$= 3(x-1)(x-5)$

$f'(x) = 0$ when $x = 1$ and $x = 5$.

$f(1) = (1-1)(1^2 - 8\cdot 1 + 7) = 0$

$f(5) = (5-1)(5^2 - 8\cdot 5 + 7) = -32$

The tangent lines at (1, 0) and (5, –32) are horizontal.

25. $f(x) = (x+1)(x^2 - x - 2)$

$f'(x) = (x+1)(2x-1) + (x^2 - x - 2)(1)$

$= 3x^2 - 3$

Since $f'(x)$ represents the slope of the tangent line and the slope of a horizontal line is zero, need to solve

$0 = 3x^2 - 3 = 3(x+1)(x-1)$ or $x = -1, 1$.

When $x = -1$, $f(-1) = 0$ and when $x = 1$, $f(1) = -4$. So, the tangent line is horizontal at the points (–1, 0) and (1, –4).

26. $f(x) = \dfrac{x^2 + x - 1}{x^2 - x + 1}$

$f'(x)$

$= \dfrac{(2x+1)(x^2 - x + 1) - (x^2 + x - 1)(2x-1)}{(x^2 - x + 1)^2}$

$= \dfrac{-2x^2 + 4x}{(x^2 - x + 1)^2}$

$= \dfrac{-2x(x-2)}{(x^2 - x + 1)^2}$

$f'(x) = 0$ when $x = 0$ and $x = 2$

$f(0) = \dfrac{0^2 + 0 - 1}{0^2 - 0 + 1} = -1$

$f(2) = \dfrac{2^2 + 2 - 1}{2^2 - 2 + 1} = \dfrac{5}{3}$

The tangent lines at (0, –1) and $\left(2, \frac{5}{3}\right)$ are horizontal.

27. $f(x) = \frac{x+1}{x^2+x+1}$

$f'(x) = \frac{-x^2-2x}{(x^2+x+1)^2}$

Since $f'(x)$ represents the slope of the tangent line and the slope of a horizontal line is zero, need to solve

$0 = \frac{-x^2-2x}{(x^2+x+1)^2}$

$0 = -x^2 - 2x = -x(x+2)$ or $x = 0, -2$.

When x = 0, f(0) = 1 and when x = –2, $f(-2) = -\frac{1}{3}$. So, the tangent line is horizontal at the points (0, 1) and $\left(-2, -\frac{1}{3}\right)$.

28. $y = (x^2+2)(x+\sqrt{x})$

$\frac{dy}{dx} = (x^2+2)\left(1+\frac{1}{2\sqrt{x}}\right) + 2x\left(x+\sqrt{x}\right)$

At $x_0 = 4$,

$\frac{dy}{dx} = (18)\left(1+\frac{1}{4}\right) + 8(6) = 70.5$

29. $y = (x^2+3)(5-2x^3)$

$\frac{dy}{dx} = (x^2+3)(-6x^2) + (5-2x^3)(2x)$

When x = 1,

$\frac{dy}{dx} = (1+3)(-6) + (5-2)(2) = -18.$

30. $y = \frac{2x-1}{3x+5}$

$\frac{dy}{dx} = \frac{2(3x+5)-3(2x-1)}{(3x+5)^2} = \frac{13}{(3x+5)^2}$

At $x_0 = 1$, $\frac{dy}{dx} = \frac{13}{8^2} = \frac{13}{64}$

31. $y = x + \frac{3}{2-4x}$

$\frac{dy}{dx} = 1 + \frac{(2-4x)(0)-3(-4)}{(2-4x)^2}$

When x = 0, $\frac{dy}{dx} = 1 + \frac{12}{(2)^2} = 4.$

32. $y = x^2 + 3x - 5$

$y' = 2x + 3$

At $x = 0$, $y' = 3$ so the slope of the perpendicular line is $m = -\frac{1}{3}$. The perpendicular line passes through the point (0, –5) and so has equation

$y = -\frac{1}{3}x - 5$.

33. $y = \frac{2}{x} - \sqrt{x} = 2x^{-1} - x^{1/2}$

$\frac{dy}{dx} = \frac{-2}{x^2} - \frac{1}{2x^{1/2}}$

When x = 1, $\frac{dy}{dx} = -2 - \frac{1}{2} = -\frac{5}{2}.$

The slope of a line perpendicular to the tangent line at x = 1 is $\frac{2}{5}$.

The equation of the normal line at (1, 1) is $y - 1 = \frac{2}{5}(x-1)$, or $y = \frac{2}{5}x + \frac{3}{5}$.

34. $y = (x+3)\left(1-\sqrt{x}\right)$

$y' = (x+3)\left(-\frac{1}{2\sqrt{x}}\right) + \left(1-\sqrt{x}\right)$

At $x = 1$, $y' = -2$ so the slope of the perpendicular line is $m = \frac{1}{2}$. The perpendicular line passes through the point $(1, 0)$ and so has equation

$y = \frac{1}{2}x - \frac{1}{2}$.

35. $y = \dfrac{5x+7}{2-3x}$

$\dfrac{dy}{dx} = \dfrac{(2-3x)(5)-(5x+7)(-3)}{(2-3x)^2}$

When $x = 1$,

$\dfrac{dy}{dx} = \dfrac{(2-3)(5)-(5+7)(-3)}{(2-3)^2} = 31.$

The slope of a line perpendicular to the tangent line at $x = 1$ is $-\dfrac{1}{31}$.

The equation of the normal line at $(1, -12)$ is $y+12 = -\dfrac{1}{31}(x-1)$, or

$y = -\dfrac{1}{31}x - \dfrac{371}{31}.$

36. $h(x) = (x^2+3)g(x)$

$$h'(x) = (2x)g(x) + g'(x)(x^2+3)$$
$$= (2\cdot 2)g(2) + g'(2)(2^2+3)$$
$$= 4\cdot 3 + (-2)(7) = -2$$

37. $h(x) = \left[3x^2 - 2g(x)\right]\left[g(x)+5x\right]$

Using the product rule,

$$h'(x) = \left[3x^2 - 2g(x)\right]\left[g'(x)+5\right] + \left[g(x)+5x\right]\left[6x - 2g'(x)\right]$$

Substituting $x = -3$,

$$h'(-3) = \left[3(-3)^2 - 2g(-3)\right]\left[g'(-3)+5\right] + \left[g(-3)+5(-3)\right]\left[6(-3) - 2g'(-3)\right]$$
$$= \left[27 - 2g(-3)\right]\left[g'(-3)+5\right] + \left[g(-3) - 15\right]\left[-18 - 2g'(-3)\right]$$

Since $g(-3) = 1$ and $g'(-3) = 2$,

$$h'(-3) = \left[27 - 2(1)\right]\left[2+5\right] + \left[1-15\right]\left[-18-2(2)\right]$$
$$= (25)(7) + (-14)(-22) = 483$$

38. $h(x) = \dfrac{3x^2 - 5g(x)}{g(x)+4}$

$$h'(x) = \frac{\left[g(x)+4\right]\left[6x - 5g'(x)\right] - \left[3x^2 - 5g(x)\right]\left[g'(x)\right]}{\left[g(x)+4\right]^2}$$
$$= \frac{\left[g(0)+4\right]\left[6(0) - 5g'(0)\right] - \left[3(0)^2 - 5g(0)\right]\left[g'(0)\right]}{\left[g(0)+4\right]^2}$$
$$= \frac{(2+4)\left[0 - 5(-3)\right] - \left[0 - 5\cdot 2\right](-3)}{(2+4)^2} = \frac{5}{2}$$

39. $h(x)=\dfrac{x^3+xg(x)}{3x-5}$

Using the quotient rule, and noting that the derivative of the numerator requires the product rule for the term $x\cdot g(x)$,

$$h'(x)=\frac{\left([3x-5]\left[3x^2+xg'(x)+g(x)\cdot 1\right]\right.}{(3x-5)^2}-\frac{\left.\left[x^3+xg(x)\right][3]\right)}{(3x-5)^2}$$

Substituting $x=-1$,

$$h'(-1)=\frac{[3(-1)-5]\left[3(-1)^2+(-1)g'(-1)+g(-1)\right]}{[3(-1)-5]^2}-\frac{\left[(-1)^3+(-1)g(-1)\right][3]}{[3(-1)-5]^2}$$

$$h'(-1)=\frac{([-8][3-g'(-1)+g(-1)]}{(-8)^2}-\frac{[-1-g(-1)][3])}{(-8)^2}$$

Since $g(-1)=0$ and $g'(-1)=1$,

$$h'(-1)=\frac{(-8)(3-1+0)-(-1-0)(3)}{64}=\frac{-16+3}{64}=-\frac{13}{64}$$

40. (a) $y=2x^2-5x-3$

$y'=4x-5$

(b) $y=(2x+1)(x-3)$

$y'=(2x+1)(1)+(2)(x-3)=4x-5$

41. (a) $y=\dfrac{2x-3}{x^3}$

$$\frac{dy}{dx}=\frac{(x^3)(2)-(2x-3)(3x^2)}{x^6}=\frac{-4x^3+9x^2}{x^6}=\frac{-4x+9}{x^4}$$

(b) $y=(2x-3)(x^{-3})$

$$\frac{dy}{dx}=(2x-3)(-3x^{-4})+(x^{-3})(2)=\frac{-3(2x-3)+2x}{x^4}=\frac{-4x+9}{x^4}$$

(c) $y=2x^{-2}-3x^{-3}$

$$\frac{dy}{dx}=-4x^{-3}+9x^{-4}=\frac{-4}{x^3}+\frac{9}{x^4}=\frac{-4x+9}{x^4}$$

42. $f(x)=5x^{10}-6x^5-27x+4$

$f'(x)=50x^9-30x^4-27$

$f''(x)=450x^8-120x^3$

43. $f(x)=\dfrac{2}{5}x^5-4x^3+9x^2-6x-2$

$f'(x)=2x^4-12x^2+18x-6$

$f''(x)=8x^3-24x+18$

44. $y=5\sqrt{x}+\dfrac{3}{x^2}+\dfrac{1}{3\sqrt{x}}+\dfrac{1}{2}$

$$\frac{dy}{dx}=\frac{5}{2}x^{-1/2}-6x^{-3}-\frac{1}{6}x^{-3/2}$$

$$\frac{d^2y}{dx^2}=-\frac{5}{4}x^{-3/2}+18x^{-4}+\frac{1}{4}x^{-5/2}=-\frac{5}{4x^{3/2}}+\frac{18}{x^4}+\frac{1}{4x^{5/2}}$$

45. $y = \frac{2}{3}x^{-1} - \sqrt{2}x^{1/2} + \sqrt{2}x - \frac{1}{6}x^{-1/2}$

$$\frac{dy}{dx} = y'$$
$$= -\frac{2}{3}x^{-2} - \frac{\sqrt{2}}{2}x^{-1/2} + \sqrt{2} + \frac{1}{12}x^{-3/2}$$
$$\frac{d^2y}{dx^2} = y'' = \frac{4}{3}x^{-3} + \frac{\sqrt{2}}{4}x^{-3/2} - \frac{1}{8}x^{-5/2}$$
$$= \frac{4}{3x^3} + \frac{\sqrt{2}}{4x^{3/2}} - \frac{1}{8x^{5/2}}$$

46. $y = (x^2 - x)\left(2x - \frac{1}{x}\right)$

$$\frac{dy}{dx} = (x^2 - x)\left(2 + \frac{1}{x^2}\right) + (2x-1)\left(2x - \frac{1}{x}\right)$$
$$= 6x^2 - 4x - 1$$
$$\frac{d^2y}{dx^2} = 12x - 4$$

47. $y = (x^3 + 2x - 1)(3x + 5)$

$$\frac{dy}{dx} = y'$$
$$= (x^3 + 2x - 1)(3) + (3x+5)(3x^2 + 2)$$
$$= 12x^3 + 15x^2 + 12x + 7$$
$$\frac{d^2y}{dx^2} = y'' = 36x^2 + 30x + 12$$

48. **(a)** $p(x) = \frac{1{,}000}{0.3x^2 + 8}$

$$p'(x) = \frac{(0.3x^2+8)(0) - 1{,}000(0.6x)}{(0.3x^2+8)^2}$$
$$= \frac{-600x}{(0.3x^2+8)^2}$$

when the level of production is 3,000 ($x = 3$) calculators, demand is changing at the rate of $p'(3) = -15.72$ dollars per thousand calculators.

(b) $R(x) = xp(x)$

$$R'(x) = xp'(x) + p(x)(1)$$
$$= x\left(\frac{-600x}{(0.3x^2+8)^2}\right) + \frac{1{,}000}{0.3x^2+8}$$
$$= \frac{-300x^2 + 8{,}000}{(0.3x^2+8)^2}$$

$R'(3) = 46.29$ so revenue is increasing at the rate of \$46.29 per thousand calculators.

49. $S(t) = \frac{2000t}{4 + 0.3t}$

(a) $S'(t) = \frac{(4+0.3t)(2000) - (2000t)(0.3)}{(4+0.3t)^2}$

The rate of change in the year 2010 is

$$S'(2) = \frac{(4+0.6)(2{,}000) - (4{,}000)(0.3)}{(4+0.6)^2}$$
$$\approx \$378{,}072 \text{ per year.}$$

(b) Rewrite the function as $S(t) = \frac{2{,}000}{\frac{4}{t} + 0.3}$.

Since $\frac{4}{t} \to 0$ as $t \to +\infty$, sales approach $\frac{2{,}000}{0.3} \approx 6{,}666.67$ thousand, or approximately \$6,666,667 in the long run.

50. **(a)** Since profit equals revenue minus cost and revenue equals price times the quantity sold, the profit function $P(p)$ is given by

$$P(p) = pB(p) - C(p)$$
$$= \frac{500p}{p+3} - (0.2p^2 + 3p + 200)$$

(b) $P'(p) = \frac{(p+3)500 - 500p(1)}{(p+3)^2} - 0.4p - 3$

$$= \frac{1500}{(p+3)^2} - 0.4p - 3$$

When the price is \$12 per bottle, $P'(12) = -1.133$. The profit is decreasing.

51. $P(t) = 100\left[\dfrac{t^2 + 5t + 5}{t^2 + 10t + 30}\right]$

(a) $P'(t) = 100\dfrac{(t^2 + 10t + 30)(2t + 5) - (t^2 + 5t + 5)(2t + 10)}{(t^2 + 10t + 30)^2}$

The rate of change after 5 weeks is

$P'(5) = 100\dfrac{(25 + 50 + 30)(10 + 5) - (25 + 25 + 5)(10 + 10)}{(25 + 50 + 30)^2}$

$P'(5) = 4.31\%$ per week.

Since $P'(5)$ is positive, the percentage is increasing.

(b) Rewrite the function as

$p(t) = 100\dfrac{1 + \frac{5}{t} + \frac{5}{t^2}}{1 + \frac{10}{t} + \frac{30}{t^2}}.$

Since $\dfrac{5}{t}, \dfrac{5}{t^2}, \dfrac{10}{t}$ and $\dfrac{30}{t^2}$ all go to zero as $t \to +\infty$, the percentage approaches 100% in the long run, so the rate of change approaches 0.

52. (a) $Q(t) = -t^3 + 8t^2 + 15t$

$R(t) = Q'(t) = -3t^2 + 16t + 15$

(b) The rate of change of the worker's rate is the second derivative

$R'(t) = Q''(t) = -6t + 16$

At 9:00 a.m., $t = 1$ and

$Q''(t) = -6(1) + 16 = 10$ units/hr^2

53. (a) Revenue = (# units)(selling price)
x = # units = 1000 – 4t, where t is measured in weeks
p = selling price = 5 + 0.05t
So,

$R(t) = x(t)\,p(t)$

To find $R(x)$, solve the first equation for t

$t = \dfrac{1000 - x}{4} = 250 - 0.25x$

Substitute this into the second equation to get

$p = 5 + 0.05(250 - 0.25x)$

$p = 5 + 12.5 - 0.0125x$
$\quad = 17.5 - 0.0125x$

Then,

$R(x) = x(17.5 - 0.0125x)$
$\quad = 17.5x - 0.0125x^2$

$R'(x) = 17.5 - 0.025x$

Currently, when $x = 0$,

$R'(0) = 17.5$

or $17.50 per unit

Since $R'(0)$ is positive, the revenue is currently increasing.

(b) $AR(x) = \dfrac{R(x)}{x}$

$AR(x) = 17.5 - 0.0125x$

$AR'(x) = -0.0125$

$AR'(0) = -0.0125$

Since $AR'(0)$ is negative, the average revenue is currently decreasing.

54. (a) revenue = quantity × price

$R(t) = (400 - 2t)(30 + 0.75t)$

Since x is the level of production at time t, $x = 400 - 2t$ or $t = 200 - 0.5x$
and

$R(t) = (400 - 2t)(30 + 0.75t)$

$R(x) = x\left[30 + 0.75(200 - 0.5x)\right]$
$\quad = x(180 - 0.375x) = 180x - 0.375x^2$

We know $C(x) = 10{,}000$. Therefore,

$$P(x) = R(x) - C(x)$$
$$= 180x - 0.375x^2 - 10{,}000$$

and

$$P'(x) = -0.75x + 180$$

When $t = 0$, $x = 400$

and $P'(400) = -0.75(400) + 180 = -120$

So the profit is decreasing at the rate of \$120/unit.

(b)

$$\frac{P(x)}{x} = \frac{180x - 0.375x^2 - 10{,}000}{x}$$
$$= 180 - 0.375x - \frac{10{,}000}{x}$$
$$\left[\frac{P(x)}{x}\right]' = -0.375 + \frac{10{,}000}{x^2}$$
$$\left[\frac{P(400)}{400}\right]' = -0.375 + \frac{10{,}000}{400^2} = -0.3125$$

The average profit will be decreasing at the rate of about 0.31 dollars per unit per unit.

55. $P(x) = \dfrac{100\sqrt{x}}{0.03x^2 + 9} = 100\dfrac{x^{1/2}}{0.03x^2 + 9}$

(a) $$P'(x) = 100\frac{(0.03x^2 + 9)\left(\frac{1}{2}x^{-1/2}\right) - (x^{1/2})(0.06x)}{(0.03x^2 + 9)^2}$$

The rate of change of percentage pollution when 16 million dollars are spent is $P'(16)$

$$= 100\left(\frac{[0.03(16)^2 + 9]\left[\frac{1}{2}(16)^{-1/2}\right]}{[0.03(16)^2 + 9]^2} - \frac{(16)^{1/2}[0.06(16)]}{[0.03(16)^2 + 9]^2}\right)$$

$= -0.63$ percent

Since $P'(16)$ is negative, the percentage is decreasing.

(b) $P'(x) = 0$ when $0 = (0.03x^2 + 9)\left(\dfrac{1}{2}x^{-1/2}\right) - (x^{1/2})(0.06x)$ or $x = 10$ million dollars.

Testing one value less than 10 and one value greater than 10 shows $P'(x)$ is increasing when $0 < x < 10$, and decreasing when $x > 10$.

56. (a) $P(t)=\dfrac{24t+10}{t^2+1}$

$$P'(t)=\frac{(t^2+1)(24)-(24t+10)(2t)}{(t^2+1)^2}$$
$$=\frac{-24t^2-20t+24}{(t^2+1)^2}$$
$$=\frac{-4(2t+3)(3t-2)}{(t^2+1)^2}$$

$P'(1)=-5$ so the population is decreasing at $t=1$.

(b) The population rate of change is 0 at $t=\frac{2}{3}$, positive for $t<\frac{2}{3}$ and negative for $t>\frac{2}{3}$. The population begins to decline after $t=\frac{2}{3}$ or 40 minutes after the introduction of the toxin.

57. $F=\dfrac{1}{3}(KM^2-M^3)$

(a) $S=\dfrac{dF}{dM}$

$$=\frac{1}{3}(2KM-3M^2)$$
$$=\frac{2}{3}KM-M^2$$

(b) $\dfrac{dS}{dM}=\dfrac{1}{3}(2K-6M)=\dfrac{2}{3}K-2M$ is the rate the sensitivity is changing.

58. (a) $C(t)=\dfrac{2t}{3t^2+16}$

$$R(t)=C'(t)=\frac{(3t^2+16)2-2t(6t)}{(3t^2+16)^2}=\frac{32-6t^2}{(3t^2+16)^2}$$

$R(t)$ is changing at the rate

$$R'(t)=\frac{(3t^2+16)^2(-12t)-(32-6t^2)(2)(3t^2+16)(6t)}{(3t^2+16)^4}=\frac{36t(t^2-16)}{(3t^2+16)^3}$$

(b) $C'(1)=\dfrac{26}{361}$, the concentration is increasing at this time.

(c) $R(t)$ is positive and the concentration is increasing until $R(t)=0$ or when $32-6t^2=0$. This occurs when $t=\dfrac{4}{\sqrt{3}}\approx 2.3$ hours (ignoring the negative solution.) The concentration begins to decline after roughly 2.3 hours.

(d) The concentration is changing at a declining rate when $R'(t)=\dfrac{36t(t^2-16)}{(3t^2+16)^3}<0$ or when $36t(t^2-16)<0$ (assuming $t>0$). This occurs when $0<t<4$.

59. $P(t)=20-\dfrac{6}{t+1}$

(a) $$P'(t)=0-\frac{(t+1)(0)-(6)(1)}{(t+1)^2}$$
$$=\frac{6}{(t+1)^2}$$

(b) $P'(1)=\dfrac{6}{(1+1)^2}=\dfrac{6}{4}=1.5$

or increasing at a rate of 1,500 people per year

(c) Actual change = $P(2)-P(1)$

$P(2)=20-\dfrac{6}{2+1}=20-2=18$

$P(1)=20-\dfrac{6}{1+1}=20-3=17$

So, the population will actually increase by 1 thousand people.

(d) $P'(9)=\dfrac{6}{(9+1)^2}=\dfrac{60}{100}=0.06$

or increasing at a rate of 60 people per year

(e) In the long run,

$$\lim_{t\to\infty}P'(t)=\lim_{t\to\infty}\frac{6}{(t+1)^2}=0$$

So, the population growth approaches zero.

60. (a) $p(x)=\dfrac{Ax}{B+x^m}$

$$p'(x)=\frac{(B+x^m)(A)-(Ax)(mx^{m-1})}{(B+x^m)^2}$$
$$=\frac{A(B+(1-m)x^m)}{(B+x^m)^2}$$

(b) $p''(x) = \dfrac{Amx^{m-1}\left[(m-1)x^m - B(1+m)\right]}{(B+x^m)^3}$

$p''(x) = 0$ when $x = \sqrt[m]{\dfrac{B(1+m)}{1-m}}$

61. (a) $s(t) = 3t^5 - 5t^3 - 7$

$v(t) = 15t^4 - 15t^2 = 15(t^4 - t^2)$

$a(t) = 15(4t^3 - 2t) = 30t(2t^2 - 1)$

(b) $a(t) = 0$ when $30t(2t^2 - 1) = 0$, or $t = 0$ and $t = \dfrac{\sqrt{2}}{2}$.

62. (a) $s(t) = 2t^4 - 5t^3 + t - 3$

$v(t) = s'(t) = 8t^3 - 15t^2 + 1$

$$\begin{aligned} a(t) = v'(t) = s''(t) &= 24t^2 - 30t \\ &= 6t(4t - 5) \end{aligned}$$

(b) $a(t) = 0$ at $t = 0$ and $t = \dfrac{5}{4}$.

63. $s(t) = -t^3 + 7t^2 + t + 2$

(a) $v(t) = -3t^2 + 14t + 1$
$a(t) = -6t + 14$

(b) $a(t) = 0$ when $-6t + 14 = 0$, or $t = \dfrac{7}{3}$.

64. (a) $s(t) = 4t^{5/2} - 15t^2 + t - 3$

$v(t) = s'(t) = 10t^{3/2} - 30t + 1$

$a(t) = v'(t) = s''(t) = 15t^{1/2} - 30$

(b) $a(t) = 0$ when $t = 4$.

65. $D(t) = 10t + \dfrac{5}{t+1} - 5$

(a) Speed = rate of change of distance with respect to time.

$$\frac{dD}{dt} = 10 + \frac{(t+1)(0)-(5)(1)}{(t+1)^2}$$

$$= 10 - \frac{5}{(t+1)^2}$$

When $t = 4$, $\frac{dD}{dt} = 10 - \frac{5}{25} = \frac{49}{5} = 9.8$ meters/minute.

(b) $D(5) = 10(5) + \frac{5}{5+1} - 5 = 45 + \frac{5}{6}$

$D(4) = 10(4) + \frac{5}{4+1} - 5 = 36$

$D(5) - D(4) = 9 + \frac{5}{6} \approx 9.83$ meters.

66. **(a)** $D(t) = 64t + \frac{10}{3}t^2 - \frac{2}{9}t^3$

$v(t) = D'(t) = 64 + \frac{20}{3}t - \frac{2}{3}t^2$

$a(t) = v'(t) = D''(t) = \frac{20}{3} - \frac{4}{3}t$

(b) $a(6) = \frac{20}{3} - \frac{24}{3} = -\frac{4}{3}$ indicating the velocity is decreasing at a rate of approximately 1.33 kilometers per hour.

(c) During the seventh hour, the velocity changes by $v(7) - v(6) = 78 - 80 = -2$ km/hr.

67. $H(t) = -16t^2 + V_0 t + H_0$

(a) $H'(t) = -32t + V_0$ and the acceleration is $H''(t) = -32$.

(b) Since the acceleration is a constant, it does not vary with time.

(c) The only acceleration acting on the object is due to gravity. The negative sign signifies that this acceleration is directed downward.

68. $f(x) = x^5 - 2x^4 + x^3 - 3x^2 + 5x - 6$

$f'(x) = 5x^4 - 8x^3 + 3x^2 - 6x + 5$

$f''(x) = 20x^3 - 24x^2 + 6x - 6$

$f'''(x) = 60x^2 - 48x + 6$

$f^{(4)}(x) = 120x - 48$

69. $y = x^{1/2} - \frac{1}{2}x^{-1} + \frac{1}{\sqrt{2}}x$

$$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-2} + \frac{1}{\sqrt{2}}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{4}x^{-3/2} - x^{-3}$$

$$\frac{d^3y}{dx^3} = \frac{3}{8}x^{-5/2} + 3x^{-4} = \frac{3}{8x^{5/2}} + \frac{3}{x^4}$$

70. **(a)** $\frac{d}{dx}[fgh] = \frac{d}{dx}[(fg)h]$

$$= (fg)\frac{dh}{dx} + h\frac{d}{dx}(fg)$$

$$= fg\frac{dh}{dx} + h\left[f\frac{dg}{dx} + g\frac{df}{dx}\right]$$

$$= fg\frac{dh}{dx} + fh\frac{dg}{dx} + gh\frac{df}{dx}$$

(b) $y = (2x+1)(x-3)(1-4x)$

$$y' = (2x+1)(x-3)(-4) + (2x+1)(1)(1-4x) + (2)(x-3)(1-4x)$$

$$= -24x^2 + 44x + 7$$

71. **(a)** $\frac{d}{dx}\left(\frac{fg}{h}\right) = \frac{h\frac{d}{dx}(fg) - (fg)\frac{d}{dx}h}{h^2}$

$$= \frac{h\left(f\frac{d}{dx}g + g\frac{d}{dx}f\right) - fg\frac{d}{dx}h}{h^2}$$

(b) $y = \frac{(2x+7)(x^2+3)}{3x+5}$

$$\frac{dy}{dx} = \frac{(3x+5)[(2x+7)(2x) + (x^2+3)(2)]}{(3x+5)^2} - \frac{(2x+7)(x^2+3)(3)}{(3x+5)^2}$$

$$= \frac{(3x+5)(6x^2+14x+6)}{(3x+5)^2} - \frac{3(2x^3+7x^2+6x+21)}{(3x+5)^2}$$

$$= \frac{12x^3+51x^2+70x-33}{(3x+5)^2}$$

72. $\frac{d}{dx}[cf] = c\frac{df}{dx} + f\frac{d}{dx}c$

$= c\frac{df}{dx} + f(0)$

$= c\frac{df}{dx}$

73. For *f*/*g* the difference quotient (DQ) is

$$= \frac{\left(\frac{f}{g}\right)(x+h) - \left(\frac{f}{g}\right)(x)}{h}$$

$$= \frac{1}{h}\left[\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}\right]$$

$$= \frac{1}{h}\left[\frac{f(x+h)g(x) - f(x)g(x+h)}{g(x+h)g(x)}\right]$$

$$= \frac{1}{h}\left[\frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x)}{g(x+h)g(x)} - \frac{f(x)g(x+h)}{g(x+h)g(x)}\right]$$

$$= \frac{1}{h}\left[\frac{g(x)[f(x+h) - f(x)]}{g(x+h)g(x)} - \frac{f(x)[g(x+h) - g(x)]}{g(x+h)g(x)}\right]$$

$$= \frac{1}{g(x+h)g(x)} \cdot \left[\frac{g(x)[f(x+h) - f(x)]}{h} - \frac{f(x)[g(x+h) - g(x)]}{h}\right]$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \lim_{h\to 0}\frac{1}{g(x+h)g(x)} \cdot \left[\frac{g(x)[f(x+h) - f(x)]}{h} - \frac{f(x)[g(x+h) - g(x)]}{h}\right]$$

$$= \frac{1}{g(x)g(x)}[g(x)f'(x) - f(x)g'(x)]$$

$$= \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

74. Suppose *n* is a negative integer so that $n = -p$ where *p* is a positive integer. Then $\frac{d}{dx}x^p = px^{p-1}$ since the power rule applies to positive integer powers. Now note

$$\frac{d}{dx}[x^n] = \frac{d}{dx}[x^{-p}] = \frac{d}{dx}\left[\frac{1}{x^p}\right]$$

$$= \frac{x^p(0) - 1(px^{p-1})}{x^{2p}}$$

$$= -p(x^{p-1})(x^{-2p})$$

$$= -px^{-p-1} = nx^{n-1}$$

proving the power rule for negative integer powers.

75. To use a graphing utility to sketch $f(x) = x^2(x-1)$ and find where $f'(x) = 0$,

Press [y =]

Input $x^2(x-1)$ for $y_1 =$

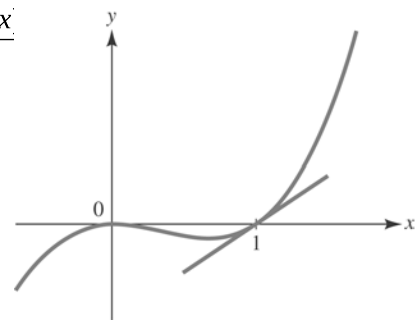
Use window dimensions [–2, 3].5 by [–2, 2].5

Press graph

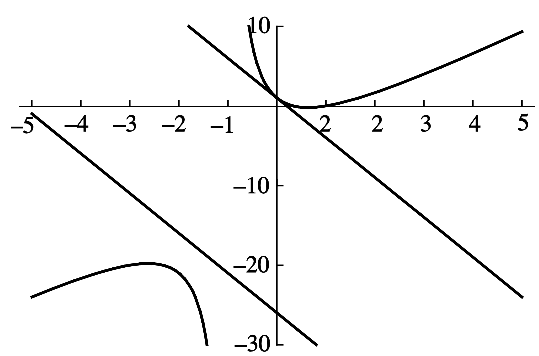
Press [2nd] [Draw] and enter the tangent function

Enter *x* = 1

The calculator draws the line tangent to the graph of *f* at *x* = 1 and gives *y* = 1.000001*x* – 1.000001 as the equation of that line. $f'(x) = 0$ when the slope of the line tangent to the graph of *f* is zero. This happens where the graph of *f* has a local high or low point. Use the trace button to move cross-hairs to the local low point on the graph of *f*. Use the zoom-in function under the zoom menu to find $f'(x) = 0$ when *x* ≈ 0.673. Repeat this process to find where the local high point occurs. We see $f'(x) = 0$ also for *x* = 0.

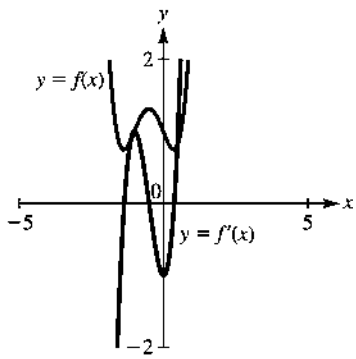


76.



$f'(x) = 0$ when $x = 0.633$ and $x = -2.633$.

77. To use a graphing utility to graph $f(x) = x^4 + 2x^3 - x + 1$ and to find minima and maxima,
Press $\boxed{y=}$ and input $x \wedge 4 + 2x \wedge 3 - x + 1$ for $y_1 =$
Use window dimensions [–5, 5]1 by [0, 2].5
Press $\boxed{\text{Graph}}$
We see from the graph that there are two minimums and one maximum.
To find the first minimum, use trace and zoom-in for a more accurate reading. Alternatively, use the minimum function under the calc menu. Using trace, enter a value to the left of (but close to) the minimum for the left bound. Enter a value to the right of (but close to) the minimum for the right bound. Finally, enter a guess in between the bounds and the minimum is displayed.
One minimum occurs at (–1.37, 0.75).
Repeat this process for the other minimum and find it to be at (0.366, 0.75).
Repeat again for the maximum (using the maximum function) to find it at (–0.5, 1.31).
$f'(x) = 4x^3 + 6x^2 - 1$
Press $\boxed{y=}$ and input $4x \wedge 3 + 6x^2 - 1$ for $y_2 =$
Change window dimensions to [–5, 5]1 by [–2, 2].5.
Use trace and zoom-in to find the x-intercepts of $f'(x)$ or use the zero function under the calc menu. The three x-intercepts of $f'(x)$ are $x \approx -1.37$, –0.5, and 0.366.
The x-values extrema occur at the x-intercepts of f' because the tangent line at the corresponding points on the curve are horizontal and so, the slopes are zero.



78. $f(x) = x^3(x-2)^2$
$f'(x) = x^3(2(x-2)) + 3x^2(x-2)^2$
$= x^2(5x-6)(x-2)$
The x intercepts of the graph of $f'(x)$ occur at $x = 0$, $x = \frac{6}{5}$, and $x = 2$. The function $f(x)$ has a maximum at $x = \frac{6}{5}$ and a minimum at $x = 2$. The maximum and minimum of $f(x)$ correspond to points where the tangent line is horizontal, that is, where $f'(x) = 0$.

2.4 The Chain Rule

1. $y = u^2 + 1,\ u = 3x - 2,$

$$\frac{dy}{du} = 2u,\ \frac{du}{dx} = 3,$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = (2u)(3) = 6(3x-2).$$

2. $y = 1 - 3u^2;\ u = 3 - 2x$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$
$$= (-6u)(-2)$$
$$= 12(3-2x)$$
$$= -24x + 36$$

3. $y=\sqrt{u}=u^{1/2}$, $u=x^2+2x-3$,

$$\frac{dy}{du}=\frac{1}{2}u^{-1/2}=\frac{1}{2u^{1/2}},$$

$$\frac{du}{dx}=2x+2=2(x+1),$$

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=\frac{x+1}{(x^2+2x-3)^{1/2}}.$$

4. $y=2u^2-u+5;\ \ u=1-x^2$

$$\begin{aligned}\frac{dy}{dx}&=\frac{dy}{du}\frac{du}{dx}\\&=(4u-1)(-2x^2)\\&=(4-4x^2-1)(-2x)\\&=-2x(-4x^2+3)\\&=8x^3-6x\end{aligned}$$

5. $y=\frac{1}{u^2}=u^{-2}$, $u=x^2+1$,

$$\frac{dy}{du}=-2u^{-3}=-\frac{2}{u^3},\ \ \frac{du}{dx}=2x,$$

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=-\frac{4x}{(x^2+1)^3}$$

6. $y=\frac{1}{u};\ \ u=3x^2+5$

$$\begin{aligned}\frac{dy}{dx}&=\frac{dy}{du}\frac{du}{dx}\\&=\frac{-1}{u^2}(6x)\\&=\frac{-6x}{(3x^2+5)^2}\end{aligned}$$

7. $y=\frac{1}{u-1}=(u-1)^{-1}$, $u=x^2$

$$\frac{dy}{du}=-(u-1)^{-2}=-\frac{1}{(u-1)^2},$$

$$\frac{du}{dx}=2x,\ \ \frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=-\frac{2x}{(x^2-1)^2}.$$

8. $y=\frac{1}{\sqrt{u}};\ u=x^2-9$

$$\begin{aligned}\frac{dy}{dx}&=\frac{dy}{du}\frac{du}{dx}\\&=\left(-\frac{1}{2u^{3/2}}\right)(2x)\\&=-\frac{x}{(x^2-9)^{3/2}}\end{aligned}$$

9. $y=u^2+2u-3,\ \ u=\sqrt{x}=x^{1/2}$

$$\frac{dy}{du}=2u+2,\ \frac{du}{dx}=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$$

$$\begin{aligned}\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}&=(2u+2)\cdot\frac{1}{2\sqrt{x}}\\&=\left(2\sqrt{x}+2\right)\cdot\frac{1}{2\sqrt{x}}\\&=1+\frac{1}{\sqrt{x}}\end{aligned}$$

10. $y=u^3+u;\ u=\frac{1}{\sqrt{x}}$

$$\begin{aligned}\frac{dy}{dx}&=\frac{dy}{du}\frac{du}{dx}\\&=(3u^2+1)\left(-\frac{1}{2x^{3/2}}\right)\\&=\left(\frac{3}{x}+1\right)\left(-\frac{1}{2x^{3/2}}\right)\\&=-\frac{3}{2x^{5/2}}-\frac{1}{2x^{3/2}}\end{aligned}$$

11. $y = u^2 + u - 2,\ u = \frac{1}{x} = x^{-1}$

$$\frac{dy}{du} = 2u + 1,\ \frac{du}{dx} = -x^{-2} = \frac{-1}{x^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = (2u + 1) \cdot \frac{-1}{x^2}$$

$$= \left(\frac{2}{x} + 1\right) \cdot \frac{-1}{x^2}$$

$$= -\frac{2}{x^3} - \frac{1}{x^2}$$

$$= \frac{-2}{x^3} + \frac{-1}{x^2} \cdot \frac{x}{x}$$

$$= \frac{-2}{x^3} + \frac{-x}{x^3}$$

$$= -\frac{2 + x}{x^3}$$

12. $y = u^2;\ u = \frac{1}{x - 1}$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

$$= 2u\left(\frac{-1}{(x-1)^2}\right) = \left(\frac{2}{x-1}\right)\left(\frac{-1}{(x-1)^2}\right)$$

$$= -\frac{2}{(x-1)^3}$$

13. $y = u^2 - u,\ u = 4x + 3$

$$\frac{dy}{du} = 2u - 1,\ \frac{du}{dx} = 4$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = (2u - 1) \cdot 4$$

When $x = 0$, $u = 4(0) + 3 = 3$, so

$$\frac{dy}{dx} = (2(3) - 1) \cdot 4 = 20.$$

14. $y = u + \frac{1}{u} = u + u^{-1};\ u = 5 - 2x$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (1 - u^{-2})(-2)$$

At $x = 0$, $u = 5 - 2(0) = 5$.

$$\left.\frac{dy}{dx}\right|_{x=0} = \left.\frac{dy}{du}\right|_{u=5}\left.\frac{du}{dx}\right|_{x=0}$$

$$= \left(1 - \frac{1}{25}\right)(-2)$$

$$= -\frac{48}{25}$$

15. $y = 3u^4 - 4u + 5,\ u = x^3 - 2x - 5$

$$\frac{dy}{du} = 12u^3 - 4,\ \frac{du}{dx} = 3x^2 - 2,$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = (12u^3 - 4)(3x^2 - 2).$$

When $x = 2$, $u = 2^3 - 2(2) - 5 = -1$, so

$$\frac{dy}{dx} = [12(-1)^3 - 4][3(2^2)) - 2] = -160$$

16. $y = u^5 - 3u^2 + 6u - 5;\ u = x^2 - 1$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (5u^4 - 6u + 6)(2x)$$

At $x = 1$, $u = 1^2 - 1 = 0$.

$$\left.\frac{dy}{dx}\right|_{x=1} = \left.\frac{dy}{du}\right|_{u=0}\left.\frac{du}{dx}\right|_{x=1} = (6)(2) = 12$$

17. $y = \sqrt{u} = u^{1/2},\ u = x^2 - 2x + 6,$

$$\frac{dy}{du} = \frac{1}{2}u^{-1/2} = \frac{1}{2u^{1/2}},$$

$$\frac{du}{dx} = 2x - 2,\quad \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{x - 1}{u^{1/2}}.$$

When $x = 3$, $u = 3^2 - 2(3) + 6 = 9$, so

$$\frac{dy}{dx} = \frac{3 - 1}{9^{1/2}} = \frac{2}{3}.$$

18. $y = 3u^2 - 6u + 2;\ u = \frac{1}{x^2}$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (6u - 6)\left(\frac{-2}{x^3}\right)$$

At $x = \frac{1}{3}$, $u = 9$.

$$\left.\frac{dy}{dx}\right|_{x=1/3} = \left.\frac{dy}{du}\right|_{u=9} \left.\frac{du}{dx}\right|_{x=1/3} = (48)(-54) = -2{,}592$$

19. $y = \frac{1}{u} = u^{-1}$, $u = 3 - \frac{1}{x^2} = 3 - x^{-2}$,

$$\frac{dy}{du} = -u^{-2} = -\frac{1}{u^2}, \quad \frac{du}{dx} = 2x^{-3} = \frac{2}{x^3}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = -\frac{1}{u^2} \cdot \frac{2}{x^3}$$

When $x = \frac{1}{2}$, $u = 3 - \frac{1}{\left(\frac{1}{2}\right)^2} = 3 - 4 = -1$,

$$\frac{dy}{dx} = \frac{-1}{(-1)^2} \cdot \frac{2}{\left(\frac{1}{2}\right)^3} = -16$$

20. $y = \frac{1}{u+1}$; $u = x^3 - 2x + 5$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{-1}{(u+1)^2}(3x^2 - 2)$$

At $x = 0$, $u = 5$.

$$\left.\frac{dy}{dx}\right|_{x=0} = \left.\frac{dy}{du}\right|_{u=5} \left.\frac{du}{dx}\right|_{x=0} = \frac{-1}{36}(-2) = \frac{1}{18}$$

21. $f(x) = (2x+3)^{1.4}$

$$f'(x) = 1.4(2x+3)^{0.4}\frac{d}{dx}(2x+3) = 1.4(2x+3)^{0.4} \cdot 2 = 2.8(2x+3)^{0.4}$$

22. $f(x) = \frac{1}{\sqrt{5} - 3x} = \left(\sqrt{5} - 3x\right)^{-1}$

$$f'(x) = -1\left(\sqrt{5} - 3x\right)^{-2}(-3) = \frac{3}{\left(\sqrt{5} - 3x\right)^2}$$

23. $f(x) = (2x+1)^4$,

$$f'(x) = 4(2x+1)^3\frac{d}{dx}(2+1) = 8(2x+1)^3$$

24. $f(x) = \sqrt{5x^6 - 12} = (5x^6 - 12)^{1/2}$

$$f'(x) = \frac{1}{2}(5x^6 - 12)^{-1/2}(30x^5) = \frac{15x^5}{\sqrt{5x^6 - 12}}$$

25. $f(x) = (x^5 - 4x^3 - 7)^8$

$$f'(x) = 8(x^5 - 4x^3 - 7)^7\frac{d}{dx}(x^5 - 4x^3 - 7) = 8(x^5 - 4x^3 - 7)^7(5x^4 - 12x^2) = 8x^2(x^5 - 4x^3 - 7)^7(5x^2 - 12)$$

26. $f(t) = (3t^4 - 7t^2 + 9)^5$

$$f'(t) = 5(3t^4 - 7t^2 + 9)^4(12t^3 - 14t) = 10t(6t^2 - 7)(3t^4 - 7t^2 + 9)^4$$

27. $f(t) = \frac{1}{5t^2 - 6t + 2} = (5t^2 - 6t + 2)^{-1}$,

$$f'(t) = -(5t^2 - 6t + 2)^{-2}\frac{d}{dt}(5t^2 - 6t + 2) = -\frac{10t - 6}{(5t^2 - 6t + 2)^2} = \frac{-2(5t - 3)}{(5t^2 - 6t + 2)^2}$$

28. $f(x) = \frac{2}{(6x^2 + 5x + 1)^2} = 2(6x^2 + 5x + 1)^{-2}$

$$f'(x) = -4(6x^2 + 5x + 1)^{-3}(12x + 5) = -\frac{48x + 20}{(6x^2 + 5x + 1)^3}$$

29. $g(x) = \dfrac{1}{\sqrt{4x^2+1}} = (4x^2+1)^{-1/2}$

$$g'(x) = -\frac{1}{2}(4x^2+1)^{-3/2}\frac{d}{dx}(4x^2+1)$$
$$= \frac{-8x}{2(4x^2+1)^{3/2}}$$
$$= \frac{-4x}{(4x^2+1)^{3/2}}$$

30. $f(s) = \dfrac{1}{\sqrt{5s^3+2}} = (5s^3+2)^{-1/2}$

$$f(s) = \left(\frac{-1}{2}\right)(5s^3+2)^{-3/2}(15s^2) = \frac{-15s^2}{2(5s^3+2)^{3/2}}$$

31. $f(x) = \dfrac{3}{(1-x^2)^4} = 3(1-x^2)^{-4},$

$$f'(x) = -12(1-x^2)^{-5}\frac{d}{dx}(1-x^2)$$
$$= \frac{24x}{(1-x^2)^5}$$

32. $f(x) = \dfrac{2}{3(5x^4+1)^2} = \dfrac{2}{3}(5x^4+1)^{-2}$

$$f'(x) = \frac{-4}{3}(5x^4+1)^{-3}(20x^3) = \frac{-80x^3}{3(5x^4+1)^3}$$

33. $h(s) = \left(1+\sqrt{3s}\right)^5$

$$h'(s) = 5\left(1+\sqrt{3s}\right)^4\frac{d}{ds}\left(1+\sqrt{3s}\right)$$
$$= 5\left(1+\sqrt{3s}\right)^4\frac{d}{ds}\left(1+\sqrt{3}s^{1/2}\right)$$
$$= 5\left(1+\sqrt{3s}\right)^4\cdot\frac{\sqrt{3}}{2s^{1/2}}$$
$$= \frac{5\sqrt{3}\left(1+\sqrt{3s}\right)^4}{2\sqrt{s}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$
$$= \frac{15(1+\sqrt{3s})^4}{2\sqrt{3s}}$$

34. $g(x)=\sqrt{1+\frac{1}{3x}}=\left(1+\frac{1}{3x}\right)^{1/2}$

$$g'(x)=\frac{1}{2}\left(1+\frac{1}{3x}\right)^{-1/2}\left(\frac{-1}{3x^2}\right)=\frac{-\sqrt{3x}}{6x^2\sqrt{3x+1}}$$

35. $f(x)=(x+2)^3(2x-1)^5$

$$f'(x)=(x+2)^3\frac{d}{dx}(2x-1)^5 + (2x-1)^5\frac{d}{dx}(x+2)^3$$

Now, $\frac{d}{dx}(2x-1)^5=5(2x-1)^4\frac{d}{dx}(2x-1)$
$=10(2x-1)^4$

and $\frac{d}{dx}(x+2)^3=3(x+2)^2\frac{d}{dx}(x+2)$
$=3(x+2)^2.$

So, $f'(x)=10(x+2)^3(2x-1)^4 + 3(2x-1)^5(x+2)^2$
$=(x+2)^2(2x-1)^4[10(x+2) + 3(2x-1)]$
$=(x+2)^2(2x-1)^4(16x+17)$

36. $f(x)=2(3x+1)^4(5x-3)^2$

$$f'(x)=2(3x+1)^4(2)(5x-3)(5)+2(4)(3x+1)^3(3)(5x-3)^2$$
$$=4(3x+1)^3(5x-3)(45x-13)$$

37. $f(x)=\frac{(x+1)^5}{(1-x)^4}$

$$f'(x)=\frac{(1-x)^4\frac{d}{dx}(x+1)^5-(x+1)^5\frac{d}{dx}(1-x)^4}{[(1-x)^4]^2}$$

Now, $\frac{d}{dx}(x+1)^5=5(x+1)^4\frac{d}{dx}(x+1)$
$=5(x+1)^4$

and $\frac{d}{dx}(1-x)^4 = 4(1-x)^3\frac{d}{dx}(1-x)$

$= -4(1-x)^3.$

So, $f'(x)$

$$= \frac{5(1-x)^4(x+1)^4 + 4(x+1)^5(1-x)^3}{(1-x)^8}$$

$$= \frac{(1-x)^3(x+1)^4[5(1-x)+4(x+1)]}{(1-x)^8}$$

$$= \frac{(x+1)^4(9-x)}{(1-x)^5}.$$

38. $f(x) = \frac{1-5x^2}{\sqrt[3]{3+2x}} = \frac{1-5x^2}{(3+2x)^{1/3}}$

$$f'(x) = \frac{(3+2x)^{1/3}(-10x) - (1-5x^2)\frac{1}{3}(3+2x)^{-2/3}(2)}{(3+2x)^{2/3}}$$

$$= \frac{-2(25x^2+45x+1)}{3(3+2x)^{4/3}}$$

39. $f(x) = \sqrt{3x+4} = (3x+4)^{1/2}$

$f'(x) = \frac{1}{2}(3x+4)^{-1/2}\cdot 3 = \frac{3}{2\sqrt{3x+4}}$

$m = f'(0) = \frac{3}{2\sqrt{3(0)+4}} = \frac{3}{4}$ and $f(0) = 2$

So, the equation of the tangent line at

(0, 2) is $y = \frac{3}{4}x + 2.$

40. $f(x) = (9x-1)^{-1/3}$

$f'(x) = -\frac{1}{3}(9x-1)^{-4/3}(9) = -3(9x-1)^{-4/3}$

At $x = 1$, $y = f(1) = \frac{1}{2}$, $f'(1) = -\frac{3}{16}$ and the equation of the tangent line is $y - \frac{1}{2} = -\frac{3}{16}(x-1)$ or $y = -\frac{3}{16}x + \frac{11}{16}.$

41. $f(x) = (3x^2+1)^2$

$f'(x) = 2(3x^2+1)(6x)$

$m = f'(-1) = -48$ and $f(-1) = 16$, so the equation of the tangent line at $(-1, 16)$ is $y - 16 = -48(x + 1)$, or $y = -48x - 32$.

42. $f(x)=(x^2-3)^5(2x-1)^3$

$f'(x)=(x^2-3)^5(3)(2x-1)^2(2)+(2x-1)^3(5)(x^2-3)^4(2x)$

At $x=2$, $y=f(2)=27$, $f'(2)=594$ and the equation of the tangent line is $y-27=594(x-2)$ or $y=594x-1161$.

43. $f(x)=\dfrac{1}{(2x-1)^6}=(2x-1)^{-6}$

$f'(x)=-6(2x-1)^{-5}(2)=-\dfrac{12}{(2x-1)^5}$

$m=f'(1)=-12$ and $f(1)=1$, so the equation of the tangent line at (1, 1) is $y-1=-12(x-1)$, or $y=-12x+13$.

44. $f(x)=\left(\dfrac{x+1}{x-1}\right)^3$

$f'(x)=3\left(\dfrac{x+1}{x-1}\right)^2\left(\dfrac{(x-1)(1)-(x+1)(1)}{(x-1)^2}\right)$

At $x=3$, $y=f(3)=8$, $f'(3)=-6$ and the equation of the tangent line is $y-8=-6(x-3)$ or $y=-6x+26$.

45. $f(x)=\sqrt[3]{\dfrac{x}{x+2}}=\left(\dfrac{x}{x+2}\right)^{1/3}$

$f'(x)=\dfrac{1}{3}\left(\dfrac{x}{x+2}\right)^{-2/3}\cdot\dfrac{(x+2)(1)(x)(1)}{(x+2)^2}$

$=\dfrac{(x+2)^{2/3}}{3x^{2/3}}\cdot\dfrac{2}{(x+2)^2}$

$=\dfrac{2}{3x^{2/3}(x+2)^{4/3}}$

$m=f'(-1)=\dfrac{2}{3}$ and $f(-1)=-1$, so the equation of the tangent line at $(-1,-1)$ is

$y+1=\dfrac{2}{3}(x+1)$, or $y=\dfrac{2}{3}x-\dfrac{1}{3}$.

46. $f(x)=x^2\sqrt{2x+3}$

$f'(x)=2x\sqrt{2x+3}+x^2\cdot\dfrac{1}{2}(2x+3)^{-1/2}(2)$

$=2x\sqrt{2x+3}+\dfrac{x^2}{\sqrt{2x+3}}$

At $x=-1$,

$y=f(-1)=(-1)^2\sqrt{2(-1)+3}=1$,

$f'(-1)=2(-1)\sqrt{2(-1)+3}+\dfrac{(-1)^2}{\sqrt{2(-1)+3}}=-1$,

and the equation of the tangent line is $y-1=-[x-(-1)]$ or $y=-x$.

47. $f(x)=(x^2+x)^2$

$f'(x)=2(x^2+x)(2x+1)$

$=2x(x+1)(2x+1)$

$=0$

when $x=-1$, $x=0$, and $x=-\dfrac{1}{2}$.

48. $f(x)=x^3(2x^2+x-3)^2$

$f'(x)=x^3(2)(2x^2+x-3)(4x+1)+3x^2(2x^2+x-3)^2$

$=x^2(x-1)(14x-9)(3+2x)(x+1)$

The tangent line to the graph of $f(x)$ is horizontal when $f'(x)=0$ or when

$x=0,1,\dfrac{9}{14},-\dfrac{3}{2},-1$.

49. $f(x)=\dfrac{x}{(3x-2)^2}$

$f'(x)=\dfrac{(3x-2)^2(1)-(x)[2(3x-2)(3)]^2}{[(3x-2)^2]^2}$

$=\dfrac{(3x-2)[(3x-2)-6x]}{(3x-2)^4}$

$=\dfrac{-3x-2}{(3x-2)^3}$

$0=\dfrac{-3x-2}{(3x-2)^3}$ when $-3x-2=0$, or

$x=-\dfrac{2}{3}$.

50. $f(x)=\frac{2x+5}{(1-2x)^3}$

$$f'(x)=\frac{(1-2x)^3(2)-(2x+5)(3)(1-2x)^2(-2)}{(1-2x)^6}$$
$$=\frac{8(4+x)}{(1-2x)^4}$$

The tangent line to the graph of $f(x)$ is horizontal when $f'(x)=0$ or when $x=-4$.

51. $f(x)=\sqrt{x^2-4x+5}=(x^2-4x+5)^{1/2}$

$$f'(x)=\frac{1}{2}(x^2-4x+5)^{-1/2}(2x-4)$$
$$=\frac{2x-4}{2(x^2-4x+5)^{1/2}}$$
$$=\frac{x-2}{(x^2-4x+5)^{1/2}}$$

$0=\frac{x-2}{(x^2-4x+5)^{1/2}}$ when $x-2=0$, or $x=2$.

52. $f(x)=(x-1)^2(2x+3)^3$

$$f'(x)=(x-1)^2(3)(2x+3)^2(2)+(2x+3)^3(2)(x-1)$$
$$=10x(x-1)(2x+3)^2$$

The tangent line to the graph of $f(x)$ is horizontal when $f'(x)=0$ or when $x=0,1,-\frac{3}{2}$.

53. $f(x)=(3x+5)^2$

(a) $f'(x)=2(3x+5)(3)=6(3x+5)$

(b) $f(x)=(3x+5)(3x+5)$
$$f'(x)=(3x+5)(3)+(3x+5)(3)$$
$$=6(3x+5)$$

54. $f(x)=(7-4x)^2=(7-4x)(7-4x)$

By the general power rule
$$f'(x)=2(7-4x)(-4)=32x-56$$
By the product rule
$$f'(x)=(7-4x)(-4)+(7-4x)(-4)$$
$$=-28+16x-28+16x=32x-56.$$

55. $f(x)=(3x+1)^5$

$$f'(x)=5(3x+1)^4(3)=15(3x+1)^4,$$
$$f''(x)=60(3x+1)(3)^3=180(3x+1)^3$$

56. $f(t)=\frac{2}{5t+1}=2(5t+1)^{-1}$

$$f'(t)=-2(5t+1)^{-2}(5)=\frac{-10}{(5t+1)^2}$$
$$f''(t)=(-2)(-10)(5t+1)^{-3}(5)=\frac{100}{(5t+1)^3}$$

57. $h=\left(t^2+5\right)^8$

$$\frac{dh}{dt}=8\left(t^2+5\right)^7\left(2t\right)=16t\left(t^2+5\right)^7,$$
$$\frac{d^2h}{dt^2}$$
$$=16t\left[7\left(t^2+5\right)^6\left(2t\right)\right]+\left(t^2+5\right)^7\left(16\right)$$
$$=16\left(t^2+5\right)^6\left[14t^2+\left(t^2+5\right)\right]$$
$$=16\left(t^2+5\right)^6\left(15t^2+5\right)$$
$$=80\left(t^2+5\right)^6\left(3t^2+1\right)$$

58. $y=(1-2x^3)^4$

$$y'=4(1-2x^3)^3(-6x^2)=-24x^2(1-2x^3)^3$$
$$y''=-24x^2(3)(1-2x^3)^2(-6x^2)-48x(1-2x^3)^3$$
$$=48x(1-2x^3)^2(11x^3-1)$$

59. $f(x)=\sqrt{1+x^2}=\left(1+x^2\right)^{1/2}$

$$f'(x)=\frac{1}{2}\left(1+x^2\right)^{-1/2}(2x)$$

$$=\frac{x}{\left(1+x^2\right)^{1/2}}$$

$$f''(x)=\frac{\left(1+x^2\right)^{1/2}(1)-(x)}{1+x}$$

$$\frac{\left[\frac{1}{2}\left(1+x^2\right)^{-1/2}(2x)\right]}{1+x}$$

$$=\frac{\left(1+x^2\right)^{1/2}-\dfrac{x^2}{\left(1+x^2\right)^{1/2}}}{1+x^2}\cdot\frac{\left(1+x^2\right)^{1/2}}{\left(1+x^2\right)^{1/2}}$$

$$=\frac{1+x^2-x^2}{\left(1+x^2\right)^{3/2}}=\frac{1}{\left(1+x^2\right)^{3/2}}$$

60. $f(u)=\dfrac{1}{(3u^2-1)^2}=(3u^2-1)^{-2}$

$$f'(u)=-2(3u^2-1)^{-3}(6u)=\frac{-12u}{(3u^2-1)^3}$$

$$f''(u)=\frac{(3u^2-1)^3(-12)-(-12u)(3)(3u^2-1)^2(6u)}{(3u^2-1)^6}$$

$$=\frac{12(15u^2+1)}{(3u^2-1)^4}$$

61. $h(x)=\sqrt{5x^2+g(x)}=\left[5x^2+g(x)\right]^{1/2}$

Using the general power rule,

$$h'(x)=\frac{1}{2}\left[5x^2+g(x)\right]^{-1/2}\left[10x+g'(x)\right]$$

$$=\frac{10x+g'(x)}{2\sqrt{5x^2+g(x)}}$$

Substituting $x = 0$, $g(0)=4$ and $g'(0)=2$,

$$h'(0)=\frac{0+2}{2\sqrt{0+4}}=\frac{2}{4}=\frac{1}{2}$$

62. $h(x)=\left[3g^2(x)+4g(x)+2\right]^5\left[g(x)+x\right]$

$$h'(x)=\left[3g^2(x)+4g(x)+2\right]^5\left[g(x)+x\right]'+\left[g(x)+x\right]\left(\left[3g^2(x)+4g(x)+2\right]^5\right)'$$

$$=\left[3g^2(x)+4g(x)+2\right]^5\left[g'(x)+1\right]$$
$$+\left[g(x)+x\right]\left(5\left[3g^2(x)+4g(x)+2\right]^4\left[3g^2(x)+4g(x)+2\right]'\right)$$

$$=\left[3g^2(x)+4g(x)+2\right]^5\left[g'(x)+1\right]$$
$$+\left[g(x)+x\right]\left(5\left[3g^2(x)+4g(x)+2\right]^4\left[6g(x)g'(x)+4g'(x)\right]\right)$$

$$h'(-1)=\left[3g^2(-1)+4g(-1)+2\right]^5\left[g'(-1)+1\right]$$
$$+\left[g(-1)+(-1)\right]\left(5\left[3g^2(-1)+4g(-1)+2\right]^4\left[6g(-1)g'(-1)+4g'(-1)\right]\right)$$

$$=\left[3(-1)^2+4(-1)+2\right]^5(1+1)$$
$$+(-1-1)\cdot 5\cdot\left[3(-1)^2+4(-1)+2\right]^4\left[6\cdot(-1)\cdot 1+4\cdot 1\right]$$
$$=(1)^5\cdot 2+(-2)\cdot 5\cdot 1^4\cdot(-2)=2+20=22$$

63. $h(x)=\left[3x+\dfrac{1}{g(x)}\right]^{3/2}$

Using the general power rule and noting that the derivative of the term $\dfrac{1}{g(x)}$ requires the quotient rule (or a second application of the general power rule),

$$h'(x)=\frac{3}{2}\left[3x+\frac{1}{g(x)}\right]^{1/2}\left[3+\frac{g(x)(0)-(1)g'(x)}{\left[g(x)\right]^2}\right]$$
$$=\frac{3}{2}\sqrt{3x+\frac{1}{g(x)}}\left(3-\frac{g'(x)}{\left[g(x)\right]^2}\right)$$

Substituting $x = 1$, $g(1)=g'(1)=1$,

$$h'(x)=\frac{3}{2}\sqrt{3(1)+\frac{1}{1}}\left(3-\frac{1}{1^2}\right)$$
$$=(3)(2)=6$$

64. $h(x)=\left[\dfrac{g(x)-x}{3+g(x)}\right]^2$

$$h'(x)=2\cdot\frac{g(x)-x}{3+g(x)}\left[\frac{g(x)-x}{3+g(x)}\right]'$$
$$=2\frac{g(x)-x}{3+g(x)}\cdot\frac{[3+g(x)][g'(x)-1]-[g(x)-x]g'(x)}{[3+g(x)]^2}$$
$$h'(0)=2\frac{g(0)-0}{3+g(0)}\cdot\frac{[3+g(0)][g'(0)-1]-[g(0)-0]g'(0)}{[3+g(0)]^2}$$
$$=2\frac{3}{3+3}\cdot\frac{(3+3)(-2-1)-3\cdot(-2)}{(3+3)^2}$$
$$=2\cdot\frac{1}{2}\cdot\frac{6\cdot(-3)-(-6)}{36}=-\frac{1}{3}$$

65. **(a)** $f(t)=\sqrt{10t^2+t+229}$
$=(10t^2+t+229)^{1/2}$

The rate at which the earnings are growing is

$$f'(t)=\frac{1}{2}(10t^2+t+229)^{-1/2}(20t+1)$$
$$=\frac{20t+1}{2(10t^2+t+229)^{1/2}}\text{ thousand}$$

dollars per year.

The rate of growth in 2015 (t = 5) is

$$f'(5)=\frac{20(5)+1}{2(10(5)^2+5+229)^{1/2}}\approx 2.295$$

or $2,295 per year.

(b) The percentage rate of the earnings increases in 2015 was

$$100\frac{f'(5)}{f(5)}=\frac{100(2.295)}{\sqrt{10(5^2)+5+229}}$$
$$\approx 10.4\%\text{ per year.}$$

66. $C(q)=0.2q^2+q+900$

$q(t)=t^2+100t$

$$\frac{dC}{dt}=\frac{dC}{dq}\frac{dq}{dt}=(0.4q+1)(2t+100)$$

At $t=1$, $q=101$.

$$\left.\frac{dC}{dt}\right|_{t=1}=\left.\frac{dC}{dq}\right|_{q=101}\left.\frac{dq}{dt}\right|_{t=1}$$
$$=(41.4)(102)$$
$$=4222.8$$

After 1 hour the manufacturing cost is changing at the rate of $4,222.80 per hour.

67. $D(p)=\dfrac{4,374}{p^2}=4,374p^{-2}$

(a) $\dfrac{dD}{dp}=-8,748p^{-3}=\dfrac{-8,784}{p^3}$

When the price is $9,

$\dfrac{dD}{dp}=\dfrac{-8,748}{(9)^3}=-12$ pounds per dollar.

(b) $\dfrac{dD}{dt}=\dfrac{dD}{dp}\cdot\dfrac{dp}{dt}$

Now, $p(t)=0.02t^2+0.1t+6$

$\frac{dp}{dt} = 0.04t + 0.1$ dollars per week

$\frac{dD}{dt} = \frac{-8,748}{p^3}(0.04t + 0.1)$ pounds per week

When $t = 10$,

$p(10) = 0.02(10)^2 = 0.1(10) + 6 = 9.$

So, $\frac{dD}{dt} = \frac{-8748}{9^3}[0.04(10) + 0.1]$

$= -6$ pounds per week

Since the rate is negative, demand will be decreasing.

68. $D(p) = \frac{40,000}{p}$

$p(t) = 0.4t^{3/2} + 6.8$

Need $100\frac{\frac{dD}{dt}}{d(t)}$ when $t = 4$.

When $t = 4$, $p(4) = 0.4(4)^{3/2} + 6.8 = 10.$

$D(10) = \frac{40,000}{10} = 4,000$

$\frac{dD}{dt} = \frac{dD}{dp} \cdot \frac{dp}{dt}$

Since $D(p) = \frac{40,000}{p} = 40,000p^{-1}$,

$\frac{dD}{dp} = -40,000p^{-2} = \frac{-40,000}{p^2}$ and

$\frac{dp}{dt} = 0.6t^{1/2} = 0.6\sqrt{t}.$

$\frac{dD}{dt} = \frac{-40,000}{p^2} \cdot 0.6\sqrt{t}$

When $t = 4$,

$\frac{dD}{dt} = \frac{-40,000}{(10)^2} \cdot 0.6\sqrt{4} = -480$

$100\frac{\frac{dD}{dt}}{D(t)} = 100\frac{-480}{4,000} = -12\%$

69. $N(t) = \sqrt{t^2 + 3t + 6} = \left(t^2 + 3t + 6\right)^{1/2}$

Using the general power rule,

$N'(t) = \frac{1}{2}\left(t^2 + 3t + 6\right)^{-1/2}(2t + 3)$

$= \frac{2t + 3}{2\sqrt{t^2 + 3t + 6}}$

When $t = 2$,

$N'(2) = \frac{2(2) + 3}{2\sqrt{(2)^2 + 3(2) + 6}} = \frac{7}{2\sqrt{16}} = 0.875$

or 875 units per month.

Since $N'(2)$ is positive, production is increasing at this time.

70. $N(t) = \frac{2t}{t^2 + 3t + 12}$

$N'(t) = \frac{\left(t^2 + 3t + 12\right) \cdot 2 - 2t(2t + 3)}{\left(t^2 + 3t + 12\right)^2}$

$= \frac{-2t^2 + 24}{\left(t^2 + 3t + 12\right)^2}$

$N'(4) = \frac{-2(4)^2 + 24}{\left(4^2 + 3 \cdot 4 + 12\right)^2} = -\frac{1}{200}$

And $-\frac{1}{200}$ thousand is the same as –5, so production is decreasing by 5 units per month.

71. $Q(K) = 500K^{2/3}$

$K(t) = \frac{2t^4 + 3t + 149}{t + 2}$

(a) $K(3) = \frac{2(3)^4 + 3(3) + 149}{3 + 2} = 64$ or $64,000.

$Q(64) = 500(64)^{2/3} = 8,000$ units

(b) $\frac{dQ}{dt} = \frac{dQ}{dK} \cdot \frac{dK}{dt}$

$\frac{dQ}{dK} = 500\left(\frac{2}{3}K^{-1/3}\right) = \frac{1000}{3K^{1/3}}$

$$\frac{dK}{dt} = \frac{(t+2)(8t^3+3)-(2t^4+3t+149)(1)}{(t+2)^2} = \frac{6t^4+16t^3-143}{(t+2)^2}$$

When $t = 5$,

$$K(5) = \frac{2(5)^4+3(5)+149}{5+2} = 202.$$

So,

$$\frac{dQ}{dt} = \frac{1000}{3(202)^{1/3}} \cdot \frac{6(5)^4+16(5)^3-143}{(5+2)^2} \approx 6{,}501 \text{ units per month}$$

Since $\frac{dQ}{dt}$ is positive when $t = 5$, production will be increasing.

72. (a) $L(5) = \sqrt{739+3(5)-5^2} = \sqrt{729} = 27$

$Q(27) = 300(27)^{1/3} = 300(3) = 900$

In 5 months, 27 worker-hours will be employed and 900 units will be produced.

(b) $Q'(L) = \frac{1}{3}(300)L^{\frac{1}{3}-1} = \frac{100}{L^{2/3}}$

$$L'(t) = \frac{1}{2}(739+3t-t^2)^{\frac{1}{2}-1}(3-2t) = \frac{3-2t}{2\sqrt{739+3t-t^2}}$$

$$L'(5) = \frac{3-2(5)}{2\sqrt{739+3(5)-5^2}} = -\frac{7}{2(27)} = -\frac{7}{54}$$

When $t = 5$, $L = 27$.

$$Q'(27) = \frac{100}{27^{2/3}} = \frac{100}{9}$$

$$\left.\frac{dQ}{dt}\right|_{t=5} = \left.\frac{dQ}{dL}\right|_{L=27} \left.\frac{dL}{dt}\right|_{t=5} = \left(\frac{100}{9}\right)\left(-\frac{7}{54}\right) \approx -1.44$$

Production will be decreasing at a rate of about 1.44 units per month.

73. $A = 10{,}000\left(1+\frac{0.01r}{12}\right)^{120} = 10{,}000\left(1+\frac{1}{1200}r\right)^{120}$

(a) $A' = 1{,}200{,}000\left(1+\frac{1}{1200}r\right)^{119}\left(\frac{1}{1200}\right)$

$$A' = 1{,}000\left(1+\frac{1}{1200}(5)\right)^{119}$$

$$A'(5) = 1{,}000\left[1+\frac{1}{1200}(5)\right]^{119} \approx \$1{,}640.18 \text{ per percent}$$

(b) When r goes from 5 to 6, the actual change in the amount is $A(6) - A(5)$.

$$A(6) = 10{,}000\left[1+\frac{0.01(6)}{12}\right]^{120} \approx 18{,}193.9673$$

$$A(5) = 10{,}000\left[1+\frac{0.01(5)}{12}\right]^{120} \approx 16{,}470.0950$$

The actual change is approximately \$1,723.87.

74. $V(N)=\left(\dfrac{3N+430}{N+1}\right)^{2/3}$

$N(t)=\sqrt{t^2-10t+45}=(t^2-10t+45)^{1/2}$

(a) $N(9)=\sqrt{(9)^2-10(9)+45}$

$=6$ hours per day

$V(6)=\left[\dfrac{3(6)+430}{6+1}\right]^{2/3}$

$=16$ or \$16,000.

(b) $\dfrac{dV}{dt}=\dfrac{dV}{dN}\cdot\dfrac{dN}{dt}$

$$\frac{dV}{dN}=\frac{2}{3}\left(\frac{3N+430}{N+1}\right)^{-1/3}\cdot\frac{(N+1)(3)-(3N+430)(1)}{(N+1)^2}$$

$$=\frac{2(N+1)^{1/3}}{3(3N+430)^{1/3}}\cdot\frac{-427}{(N+1)^2}$$

$$=-\frac{854}{3(3N+430)^{1/3}(N+1)^{5/3}}$$

$$\frac{dN}{dt}=\frac{1}{2}(t^2-10t+45)^{-1/2}(2t-10)$$

$$=\frac{t-5}{(t^2-10t+45)^{1/2}}$$

Using t = 9 and N = 6.

$$\frac{dV}{dt}=\frac{854}{3[3(6)+430]^{1/3}(6+1)^{5/3}}\cdot\frac{9-5}{[(9)^2-10(9)+45]^{1/2}}$$

≈ -0.968 thousand

or –968 dollars per month

Since $\dfrac{dV}{dt}$ is negative when t = 9, the value will be decreasing.

75. $p(t)=20-\dfrac{6}{t+1}=20-6(t+1)^{-1}$

$c(p)=0.5\sqrt{p^2+p+58}$

$=0.5(p^2+p+58)^{1/2}$

(a) $\dfrac{dc}{dp}=\dfrac{1}{4}(p^2+p+58)^{-1/2}(2p+1)$

$$=\frac{2p+1}{4\sqrt{p^2+p+58}}$$

When p = 18,

$$\frac{dc}{dp}=\frac{2(18)+1}{4\sqrt{18^2+18+58}}$$

$$=\frac{37}{80}$$

$=0.4625$ ppm/thous people

(b) $\frac{dc}{dt} = \frac{dc}{dp} \cdot \frac{dp}{dt}$

$\frac{dp}{dt} = 0 + 6(t+1)^{-2} \cdot 1 = \frac{6}{(t+1)^2}$

$\frac{dc}{dt} = \frac{2p+1}{4\sqrt{p^2+p+58}} \cdot \frac{6}{(t+1)^2}$

When $t = 2$, $p(2) = 20 - \frac{6}{2+1} = 18$ and

$\frac{dc}{dt} = (0.4625) \cdot \frac{6}{(2+1)^2}$

≈ 0.308 ppm/year

Since $\frac{dc}{dt}$ is positive, the level is increasing.

76. $E = \frac{1}{v}[0.074(v-35)^2 + 32]$

$E' = \frac{1}{v}(2)(0.074)(v-35) + \frac{-1}{v^2}[0.074(v-35)^2 + 32]$

$= \frac{0.074v^2 - 122.65}{v^2}$

77. $L = 0.25w^{2.6}$; $w = 3 + 0.21A$

(a) $\frac{dL}{dw} = 0.65w^{1.6}$ mm per kg

When $w = 60$,

$\frac{dL}{dw} = 0.65(60)^{1.6} \approx 455$ mm per kg.

(b) When $A = 100$,
$w = 3 + 0.21(100) = 24$ and

$L(24) = 0.25(24)^{2.6} \approx 969$ mm long.

$\frac{dL}{dA} = \frac{dL}{dw} \cdot \frac{dw}{dA}$

Since $\frac{dw}{dA} = 0.21$,

$\frac{dL}{dA} = (0.65w^{1.6})(0.21).$

When $A = 100$, since $w = 24$,

$\frac{dL}{dA} = 0.65(24)^{1.6}(0.21) \approx 22.1.$

The tiger's length is increasing at the rate of about 22.1 mm per day.

78. **(a)** $Q(4) = \frac{4^2 + 2(4) + 3}{2(4)+1} = \frac{27}{9} = 3$

$p(3) = 3(3)^2 + 4(3) + 200 = 239$

In 4 years the quality-of-life index is expected to be 3, and the population is expected to be 239,000.

(b)

$Q'(t) = \frac{(2t+1)(2t+2) - (t^2+2t+3)(2)}{(2t+1)^2}$

$= \frac{2t^2 + 2t - 4}{(2t+1)^2}$

$Q'(4) = \frac{2(4)^2 + 2(4) - 4}{(2(4)+1)^2} = \frac{36}{81} = \frac{4}{9}$

$p'(Q) = 6Q + 4$

$p'(3) = 6(3) + 4 = 22$

$\left.\frac{dp}{dt}\right|_{t=4} = \left.\frac{dp}{dQ}\right|_{Q=3} \left.\frac{dQ}{dt}\right|_{t=4}$

$= 22\left(\frac{4}{9}\right)$

≈ 9.778

In 4 years, the population is expected to be increasing at a rate of about 9,778 people per year.

79. $P(t) = 1 - \frac{12}{t+12} + \frac{144}{(t+12)^2}$

(a) $P(t) = 1 - 12(t+12)^{-1} + 144(t+12)^{-2}$

$P' = 0 + 12(t+12)^{-2} \cdot 1 - 288(t+12)^{-3} \cdot 1$

$= \frac{12}{(t+12)^2} - \frac{288}{(t+12)^3}$

When $t = 10$,

$P'(10) = \frac{12}{(10+12)^2} - \frac{288}{(10+12)^3}$

≈ -0.002254

$= -0.2254\%$ per day

where the negative sign indicates that the proportion is decreasing.

(b) $P'(15) = \dfrac{12}{(15+12)^2} - \dfrac{288}{(15+12)^3}$
≈ 0.001829

Since this value is positive, the proportion is increasing.

(c) $\lim\limits_{t\to+\infty} P(t) = \lim\limits_{t\to+\infty} 1 - \dfrac{12}{t+12} + \dfrac{144}{(t+12)^2}$
$= 1 - 0 + 0$
$= 1$

Since $P(0) = 1$, this is the normal level in the lake.

80. $T = aL\sqrt{L-b} = aL\left(L-b\right)^{1/2}$

(a)

$$\frac{dT}{dL} = aL\cdot\frac{1}{2}\left(L-b\right)^{-1/2}(1) + \left(L-b\right)^{1/2}(a)$$

$$= \frac{aL}{2\left(L-b\right)^{1/2}} + a\left(L-b\right)^{1/2}\frac{2\left(L-b\right)^{1/2}}{2\left(L-b\right)^{1/2}}$$

$$= \frac{aL + 2a\left(L-b\right)}{2\left(L-b\right)^{1/2}} = \frac{3aL-2ab}{2\sqrt{L-b}} = \frac{a\left(3L-2b\right)}{2\sqrt{L-b}}$$

$\dfrac{dT}{dL}$ is the rate of change in the time required with respect to the number of items in the list.

(b) Writing Exercise – Answers will vary.

81. $V(T) = 0.41(-0.01T^2 + 0.4T + 3.52)$

$m(V) = \dfrac{0.39V}{1+0.09V}$

(a) $\dfrac{dV}{dt} = 0.41(-0.02T + 0.4)$ cm^3 per °C

(b) $\dfrac{dm}{dV} = \dfrac{(1+0.09V)(0.39) - (0.39V)(0.09)}{(1+0.09V)^2}$

$= \dfrac{0.39}{(1+0.09V)^2}$ gm per cm^3

(c) When $T = 10$,

$V(10)$
$= 0.41[-0.01(10)^2 + 0.4(10) + 3.52]$
$= 2.6732$ cm^3

$\dfrac{dm}{dT} = \dfrac{dm}{dV}\cdot\dfrac{dV}{dt}$
$= \dfrac{0.39}{(1+0.09V)^2} \cdot 0.41(-0.02T + 0.4)$

When $T = 10$,

$\dfrac{dm}{dT} = \dfrac{0.39}{[1+0.09(2.6732)]^2} \cdot 0.41[-0.02(10) + 0.4]$
$= 0.02078$ gm per °C

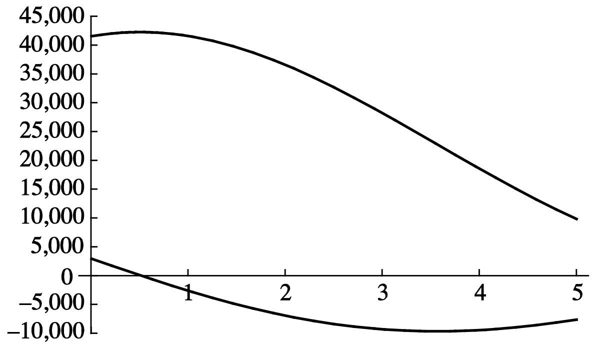
82. (a) $v(t) = (2t+9)^2(8-t)^3 \qquad 0 \le t \le 5$

$a(t) = (2t+9)^2(3)(8-t)^2(-1) + (8-t)^3(2)(2t+9)(2)$
$= 5(2t+9)(8-t)^2(1-2t)$

(b) The object is stationary when $v(t) = 0$. The function $(2t+9)^2(8-t)^3$ is 0 when $t = \dfrac{-9}{2}$ and $t = 8$. Since neither value falls in the interval $0 \le t \le 5$, the object is never stationary.

(c) $a(t) = 0$ when $t = \dfrac{1}{2}$. At this time $v\left(\dfrac{1}{2}\right) = 42{,}187.5$.

(d)



(e) The object is speeding up for $0 \le t \le \frac{1}{2}$.

83. $s(t) = (3+t-t^2)^{3/2},\ 0 \le t \le 2$

(a) $v(t) = s'(t) = \frac{3}{2}(3+t-t^2)^{1/2}(1-2t)$

$$\begin{aligned} a(t) &= v'(t) \\ &= \frac{3}{2}\left[(3+t-t^2)^{1/2}(-2) + (1-2t)\frac{1}{2}(3+t-t^2)^{-1/2}(1-2t)\right] \\ &= \frac{3}{2}\left[-2(3+t-t^2)^{1/2}\frac{2(3+t-t^2)^{1/2}}{2(3+t-t^2)^{1/2}} + \frac{(1-2t)^2}{2(3+t-t^2)^{1/2}}\right] \\ &= \frac{3}{2}\left[\frac{-4(3+t-t^2)+(1-2t)^2}{2(3+t-t^2)^{1/2}}\right] \\ &= \frac{3}{2}\left[\frac{-12-4t+4t^2+1-4t+4t^2}{2(3+t-t^2)^{1/2}}\right] \\ &= \frac{24t^2-24t-33}{4\sqrt{3+t-t^2}} \end{aligned}$$

(b) To find when object is stationary for $0 \le t \le 2,\ \frac{3}{2}\sqrt{3+t-t^2}(1-2t) = 0.$

Press $\boxed{y=}$ and input $1.5\sqrt{\ }(3+x-x^2)*(1-2x)$ for $y_1 =$

Use window dimensions [–5, 5]1 by [–5, 5]1

Use the zero function under calc menu to find the only x-intercept occurs at $x = \frac{1}{2}$.

(Note: algebraically, $\sqrt{3+t-t^2} = 0$ when $t = \frac{1+\sqrt{13}}{2}$, but this value is not in the domain.)

Object is stationary when $t = \frac{1}{2}$.

$$s\left(\frac{1}{2}\right) = \left[3 + \frac{1}{2} - \left(\frac{1}{2}\right)^2\right]^{3/2}$$
$$= \frac{\sqrt{2197}}{8}$$
$$\approx 5.859$$

$$a\left(\frac{1}{2}\right) = \frac{24\left(\frac{1}{2}\right)^2 - 24\left(\frac{1}{2}\right) - 33}{4\sqrt{3 + \frac{1}{2} - \left(\frac{1}{2}\right)^2}}$$
$$= \frac{-39}{2\sqrt{13}}$$
$$= \frac{-3\sqrt{13}}{2}$$
$$\approx -5.4083$$

For $a\left(\frac{1}{2}\right)$ you can use the $\frac{dy}{dx}$ function under the calc menu and enter $x = .5$ to find $v'\left(\frac{1}{2}\right) = a\left(\frac{1}{2}\right) \approx -5.4083.$

(c) To find when the acceleration is zero for $0 \le t \le 2$, $\frac{24t^2 - 24t - 33}{4\sqrt{3 + t - t^2}} = 0.$

Press [y =] and input $(24x^2 - 24x - 33)/(4\sqrt{\ }(3 + x - x^2))$ for $y_2 =$

Press [Graph]

You may wish to deactivate y_1 so only the graph of y_2 is shown.

Use zero function under the calc menu to find the x-intercepts are $x \approx -0.775$ and $x \approx 1.77$. (Disregard $x = -0.775$.)

The acceleration is zero for $t = 1.77$, $s(1.77) = (3 + 1.77 - (1.77)^2)^{3/2} \approx 2.09$

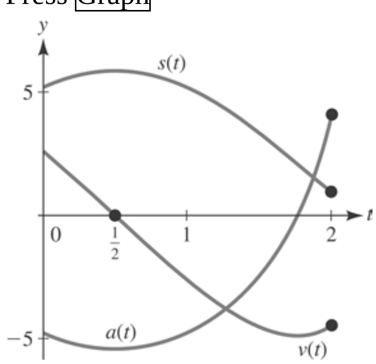
Reactivate y_1 and use the value function under the calc menu. Make sure that y_1 is displayed in the upper left corner and enter $x = 1.77$ to find $v(1.77) \approx -4.87$.

(d) We already have $v(t)$ inputted for $y_1 =$ and $a(t)$ inputted for $y_2 =$

Pres [y =] and input $(3 + x - x^2)\wedge(3/2)$ for $y_3 =$

Use window dimensions [0, 2]1 by [–5, 5]1

Press [Graph]



(e) To determine when $v(t)$ and $a(t)$ have opposite signs, press [y =] and deactivate $y_3 =$ so only $v(t)$ and $a(t)$ are shown. Press [graph]. We see from the graph, $v(t)$ and $a(t)$ have opposite signs in two intervals. We know the

t-intercept of $v(t)$ is $t = \frac{1}{2}$ and the

t-intercept of $a(t)$ is $t = 1.77$. The object is slowing down for $0 \le t < 0.5$ and $1.77 < t \le 2$.

84. (a) $f(x) = L(u(x));\ u(x) = x^2$

$$\begin{aligned} f'(x) &= L'(u(x))u'(x) \\ &= \frac{1}{u(x)}(2x) \\ &= \frac{1}{x^2}(2x) \\ &= \frac{2}{x} \end{aligned}$$

(b) $f(x) = L(u(x));\ u(x) = \frac{1}{x}$

$$\begin{aligned} f'(x) &= L'(u(x))u'(x) \\ &= \frac{1}{u(x)}\left(\frac{-1}{x^2}\right) \\ &= x\left(\frac{-1}{x^2}\right) \\ &= \frac{-1}{x} \end{aligned}$$

(c) $f(x) = L(u(x));\ u(x) = \frac{2}{3\sqrt{x}}$

$$\begin{aligned} f'(x) &= L'(u(x))u'(x) \\ &= \frac{1}{u(x)}\left(\frac{-1}{3x^{3/2}}\right) \\ &= \frac{3\sqrt{x}}{2}\left(\frac{-1}{3x^{3/2}}\right) \\ &= \frac{-1}{2x} \end{aligned}$$

(d) $f(x) = L(u(x));\ u(x) = \dfrac{2x+1}{1-x}$

$$f'(x) = L'(u(x))u'(x)$$
$$= \frac{1}{u(x)}\left(\frac{3}{(1-x)^2}\right)$$
$$= \frac{1-x}{2x+1}\left(\frac{3}{(1-x)^2}\right)$$
$$= \frac{3}{(2x+1)(1-x)}$$

85. To prove that $\dfrac{d}{dx}[h(x)]^2 = 2h(x)h'(x),$ use the product rule to get

$$\frac{d}{dx}[h(x)]^2 = \frac{d}{dx}[h(x)h(x)]$$
$$= h(x)h'(x) + h'(x)h(x)$$
$$= 2h(x)h'(x).$$

86. $\dfrac{d}{dx}[h(x)]^3$

$$= \frac{d}{dx}h(x)[h(x)]^2$$
$$= h(x)2(h(x))h'(x) + [h(x)]^2 h'(x)$$
$$= 2[h(x)]^2 h'(x) + [h(x)]^2 h'(x)$$
$$= 3[h(x)]^2 h'(x)$$

87. To use numeric differentiation to calculate $f'(1)$ and $f'(-3)$, press $\boxed{y =}$ and input

$(3.1x^2 + 19.4)$ ^ $(1/3)$ for $y_1 =$

Use the window dimensions [–5, 5]1 by

[–3, 8]1

Press $\boxed{\text{Graph}}$

Use the *dy*/*dx* function under the calc menu and enter $x = 1$ to find $f'(1) \approx 0.2593$.

Repeat this for $x = -3$ to find $f'(-3) \approx -0.474$

Since there is only one minimum, we can conclude the graph has only one horizontal tangent.

88. $f'(0)$ does not exist while $f'(4.3) \approx 16.63$. The graph of *f*(*x*) has one horizontal tangent when $x \approx 0.50938$.

89. $f'(x)=\dfrac{1}{1+x^2}$

$g(x)=f(2x+1)$

$g'(x)=f'(2x+1)\cdot(2x+1)'$

$g'(x)=\dfrac{1}{1+(2x+1)^2}\cdot 2=\dfrac{2}{1+(2x+1)^2}$

90. $f(3)=-1,\ f'(x)=\sqrt{x^2+3}$

$g(x)=x^3 f\left(\dfrac{x}{x-2}\right)$

$g'(x)=x^3 f'\left(\dfrac{x}{x-2}\right)+f\left(\dfrac{x}{x-2}\right)\cdot 3x^2$

$=x^3\sqrt{\left(\dfrac{x}{x-2}\right)^2+3}+f\left(\dfrac{x}{x-2}\right)\cdot 3x^2$

$g'(3)=3^3\sqrt{\left(\dfrac{3}{3-2}\right)^2+3}+f\left(\dfrac{3}{3-2}\right)\cdot 3(3)^2$

$=27\sqrt{3^2+3}+f(3)\cdot 3\cdot 9$

$=27\sqrt{12}+(-1)\cdot 27=27\left(2\sqrt{3}-1\right)$

2.5 Marginal Analysis and Approximations Using Increments

1. $C(x)=\dfrac{1}{5}x^2+4x+57;\ \ p(x)=\dfrac{1}{4}(48-x)=12-\dfrac{1}{4}x$

(a) Marginal cost $=C'(x)=\dfrac{2}{5}x+4$

Revenue = (# sold)(selling price)

$R(x)=x\left(12-\dfrac{1}{4}x\right)=12x-\dfrac{1}{4}x^2$

Marginal revenue $=R'(x)=12-\dfrac{1}{2}x$

(b) Estimated cost of 21st unit

$= C'(20) = \frac{2}{5}(20) + 4 = \$12$

Actual cost of 21st unit is

$C(21) - C(20)$

$C(21) = \frac{1}{5}(21)^2 + 4(21) + 57$

$= 229.20$

$C(20) = \frac{1}{5}(20)^2 + 4(20) + 57$

$= 217.00$

So, actual cost is \$12.20

(c) Estimated revenue from sale of 21st unit

$= R'(20) = 12 - \frac{1}{2}(20) = \$2$

Actual revenue from sale of 21st unit

$= R(21) - R(20)$

$R(21) = 12(21) - \frac{1}{4}(21)^2 = 141.75$

$R(20) = 12(20) - \frac{1}{4}(20)^2 = 140.00$

So, actual revenue is \$1.75

2. (a) $C(x) = \frac{1}{4}x^2 + 3x + 67$

$R(x) = x\ p(x)$

$= x\left[\frac{1}{5}(45 - x)\right]$

$= 9x - \frac{1}{5}x^2$

$C'(x) = \frac{1}{2}x + 3$

$R'(x) = 9 - \frac{2}{5}x$

(b) The marginal cost for producing the 21^{st} unit is

$C'(20) = \frac{1}{2}(20) + 3 = 13$, or \$13.

The actual cost is

$C(21) - C(20) = \left(\frac{1}{4}\cdot 21^2 + 3\cdot 21 + 67\right) - \left(\frac{1}{4}\cdot 20^2 + 3\cdot 20 + 67\right)$

$= 240.25 - 227 = 13.25,$

or \$13.25.

(c) The marginal revenue at $x = 20$ is

$R'(20) = 9 - \frac{2}{5}(20) = 1$, or \$1.

The actual revenue obtained from the sale of the 21st unit is

$$R(21) - R(20) = \left(9 \cdot 21 - \frac{1}{5} \cdot 21^2\right) - \left(9 \cdot 20 - \frac{1}{5} \cdot 20^2\right)$$
$$= 100.8 - 100 = 0.8,$$

or \$0.80.

3. $C(x) = \frac{1}{3}x^2 + 2x + 39$; $p(x) = -x^2 - 10x + 4{,}000$

(a) Marginal cost $= C'(x) = \frac{2}{3}x + 2$

Revenue = (# sold)(selling price)

$$R(x) = x\left(-x^2 - 10x + 4{,}000\right)$$
$$= -x^3 - 10x^2 + 4{,}000x$$

Marginal revenue

$$= R'(x) = -3x^2 - 20x + 4{,}000$$

(b) Estimated cost of 21st unit

$$= C'(20) = \frac{2}{3}(20) + 2 \approx \$15.33$$

Actual cost of 21st unit

$$= C(21) - C(20)$$
$$C(21) = \frac{1}{3}(21)^2 + 2(21) + 39 = 228.00$$
$$C(20) = \frac{1}{3}(20)^2 + 2(20) + 39 \approx 212.33$$

So, actual cost is \$15.67

(c) Estimated revenue from sale of 21st unit

$$= R'(20) = -3(20)^2 - 20(20) + 4{,}000$$
$$= \$2{,}400$$

Actual revenue from sale of 21st unit

$$= R(21) - R(20)$$
$$R(21) = -(21)^3 - 10(21)^2 + 4{,}000(21)$$
$$= 70{,}329$$

So, actual revenue is \$2,329.

$$R(20) = -(20)^3 - 10(20)^2 + 4{,}000(20)$$
$$= 68{,}000$$

4. (a) $C(x) = \frac{5}{9}x^2 + 5x + 73$

$$R(x) = xp(x) = x\left[-2x^2 - 15x + 6000\right] = -2x^3 - 15x^2 + 6000x$$

$$C'(x) = \frac{10}{9}x + 5$$

$$R'(x) = -6x^2 - 30x + 6000$$

(b) The marginal cost for producing the 21st unit is

$C'(20) = \frac{10}{9}(20) + 5 = 27.\overline{2}$, or about $27.22.

The actual cost is

$$C(21) - C(20) = \left(\frac{5}{9}\cdot 21^2 + 5\cdot 21 + 73\right) - \left(\frac{5}{9}\cdot 20^2 + 5\cdot 20 + 73\right) = 423 - 395.\overline{2} = 27.\overline{7},$$

or about $27.78.

(c) The marginal revenue at $x = 20$ is

$R'(20) = -6\cdot 20^2 - 30\cdot 20 + 6{,}000 = 3{,}000$, or $3,000.

The actual revenue obtained from the sale of the 21st unit is

$$R(21) - R(20) = \left(-2\cdot 21^3 - 15\cdot 21^2 + 6{,}000\cdot 21\right) - \left(-2\cdot 20^3 - 15\cdot 20^2 + 6{,}000\cdot 20\right) = 100{,}863 - 98{,}000 = 2{,}863,$$

or $2,863.

5. $C(x) = \frac{1}{4}x^2 + 43;\quad p(x) = \frac{3 + 2x}{1 + x}$

(a) Marginal cost = $C'(x) = \frac{1}{2}x$

Revenue = (# sold)(selling price)

$$R(x) = x\left(\frac{3 + 2x}{1 + x}\right) = \frac{3x + 2x^2}{1 + x}$$

Marginal revenue

$$R'(x) = \frac{(1 + x)(3 + 4x) - (3x + 2x^2)(1)}{(1 + x)^2} = \frac{3 + 7x + 4x^2 - 3x - 2x^2}{(1 + x)^2} = \frac{2x^2 + 4x + 3}{(1 + x)^2}$$

(b) Estimated cost of 21st unit

$= C'(20) = \frac{1}{2}(20) = \$10.00$

Actual cost of 21st unit

$= C(21) - C(20)$

$$C(21) = \frac{1}{4}(21)^2 + 43 = 153.25$$

$$C(20) = \frac{1}{4}(20)^2 + 43 = 143.00$$

So, the actual cost is \$10.25.

(c) Estimated revenue from sale of 21st unit

$$= R'(20) = \frac{2(20)^2 + 4(20) + 3}{(1+20)^2} \approx \$2.00$$

Actual revenue from sale of 21st unit

$= R(21) - R(20)$

$$R(21) = \frac{3(21) + 2(21)^2}{1+21} \approx 42.95$$

$$R(20) = \frac{3(20) + 2(20)^2}{1+20} \approx 40.95$$

So, the actual revenue is \$2.00.

6. **(a)** $C(x) = \frac{2}{7}x^2 + 65$

$$R(x) = xp(x) = x\left[\frac{12+2x}{3+x}\right] = \frac{12x + 2x^2}{3+x}$$

$$C'(x) = \frac{4}{7}x$$

$$R'(x) = \frac{2(x^2 + 6x + 18)}{(x+3)^2}$$

(b) The marginal cost for producing the 21st unit is

$C'(20) = \frac{4}{7}(20) \approx 11.43$, or about \$11.43.

The actual cost is

$$C(21) - C(20) = \left(\frac{2}{7} \cdot 21^2 + 65\right) - \left(\frac{2}{7} \cdot 20^2 + 65\right)$$

$$\approx 191 - 179.29 = 11.71,$$

or about \$11.71.

(c) The marginal revenue at $x = 20$ is

$$R'(20) = \frac{2\cdot 20^2 + 12\cdot 20 + 36}{(3+20)^2} = \frac{1{,}076}{529} \approx 2.03,$$

or about \$2.03.

The actual revenue obtained from the sale of the 21^{st} unit is

$$R(21) - R(20) = \frac{12\cdot 21 + 2\cdot 21^2}{3+21} - \frac{12\cdot 20 + 2\cdot 20^2}{3+20}$$
$$\approx 47.25 - 45.22 = 2.03,$$

or about \$2.03.

7. $f(x) = x^2 - 3x + 5;\ x$ increases from 5 to 5.3

$\Delta f \approx f'(x)\Delta x$

$f'(x) = 2x - 3$

$\Delta x = 5.3 - 5 = 0.3$

$\Delta f \approx [2(5) - 3](0.3) = 2.1$

8. $f(x) = \frac{x}{x+1} - 3$

$f'(x) = \frac{1}{(x+1)^2}$

$f'(4) = \frac{1}{25}$ and $\Delta x = 3.8 - 4 = -0.2$ so

$\Delta f \approx \frac{1}{25}(-0.2) = -0.008$

Thus $f(x)$ will decrease by about 0.008.

9. $f(x) = x^2 + 2x - 9;\ x$ increases from 4 to 4.3. Estimated percentage change is

$100\frac{\Delta f}{f}$ where $\Delta f \approx f'(x)\Delta x$

$f'(x) = 2x + 2,\ \Delta x = 4.3 - 4 = 0.3$

$\Delta f \approx [2(4) + 2](0.3) = 3$

$f(4) = (4)^2 + 2(4) - 9 = 15$

$100\frac{\Delta f}{f} = 100\frac{3}{15} = 20\%$

10. $f(x) = 3x + \frac{2}{x}$

$f'(x) = 3 - \frac{2}{x^2}$

$f'(5) = 3 - \frac{2}{25} = 2.92$ and $\Delta x = 5 - 4.6 = -0.4$ so

$\Delta f \approx 2.92(-0.4) = -1.168$

Thus $f(x)$ will decrease by about 1.168.

11. $R(q) = 240q - 0.05q^2$

(a) Estimated revenue from 81st unit

$= R'(80)$

Now, $R'(q) = 240 - 0.1q$

So, $R'(80) = 240 - 0.1(80) = \$232$

Yes, since $R'(80)$ is positive, revenue is increasing and she should recommend increasing production.

$$= R(81) - R(80)$$

(b) Actual revenue from 81st unit $R(81) = 240(81) - 0.05(81)^2 = 19{,}111.95$

$$R(80) = 240(80) - 0.05(80)^2 = 18{,}880.00$$

So, the actual revenue is \$231.95 and the estimate is quite accurate.

12. **(a)** $C'(q) = 0.003q^2 - 0.1q + 40$

The marginal cost at $q = 250$ is

$C'(250) = 0.003 \cdot 250^2 - 0.1 \cdot 250 + 40 = 202.5$, or \$202.50.

(b) The actual cost of producing the 251st unit is

$$C(251) - C(250) = \left(0.001 \cdot 251^3 - 0.05 \cdot 251^2 + 40 \cdot 251 + 4{,}000\right)$$
$$-\left(0.001 \cdot 250^3 - 0.05 \cdot 250^2 + 40 \cdot 250 + 4{,}000\right),$$
$$= 26{,}703.201 - 26{,}500 \approx 203.20,$$

or about \$203.20.

13. $C(q) = 3q^2 + q + 500$

(a) $C'(q) = 6q + 1$

$C'(40) = 6(40) + 1 = \$241$

(b) $C(41) - C(40)$

$= [3(41)^2 + 41 + 500]$

$-[3(40)^2 + 40 + 500]$

$= \$244$

14. $C(q) = 0.1q^3 - 0.5q^2 + 500q + 200$

$C'(q) = 0.3q^2 - q + 500$

$C'(4) = 500.8,\ \Delta q = 4.1 - 4 = 0.1$

The approximate change in cost will be

$\Delta C \approx C'(4)\Delta q = \$50.08$.

15. $R(q) = 240q - 0.05q^2$

$\Delta R \approx R'(q)\Delta q$

$R'(q) = 240 - 0.1q$

Since will decrease by 0.65 unit,

$\Delta q = -0.65$

$\Delta R \approx R'(80)(-0.65)$

$= [240 - 0.1(80)](-0.65)$

$= -150.8,$

or a decrease of approximately \$150.80.

16. $f(x) = -x^3 + 6x^2 + 15x$

$f'(x) = -3x^2 + 12x + 15$

At 9:00 A.M., $x = 1$

$f'(1) = 24$

Further, $\Delta x = 0.25$ (one quarter hour) so the approximate change in radio production from 9:00 to 9:15 A.M. will be $\Delta f \approx f'(1)\Delta x = 6$ radios.

17. $Q(K) = 600K^{1/2}$

$\Delta Q \approx Q'(K)\Delta K$

$Q'(K) = 300K^{-1/2} = \dfrac{300}{\sqrt{K}}$

Since K is measured in thousands of dollars, the current value of K is 900 and

$\Delta K = \dfrac{800}{1000} = 0.8$

$\Delta Q \approx Q'(900)(0.8)$

$= \left(\dfrac{300}{\sqrt{900}}\right)(0.8)$

$= 8,$

or an increase of approximately 8 units.

18. $Q(L) = 60{,}000L^{1/3}$

$Q'(L) = \dfrac{20{,}000}{L^{2/3}}$

$Q'(1{,}000) = 200,\ \Delta L = 940 - 1{,}000 = -60$

The approximate effect on output will be $\Delta Q \approx Q'(1{,}000)\Delta L = -12{,}000$,

that is, a decrease of about 12,000 units.

19. $Q = 3{,}000K^{1/2}L^{1/3}$

If the labor force is kept constant,

$Q = 3{,}000K^{1/2}(1{,}331)^{1/3}$

$= 33{,}000K^{1/2}$

Increasing the capital expenditure causes a change in weekly output that can be estimated using

$\Delta Q \approx Q'(K_0)\Delta K$

Now, substituting $K_0 = 400,\ \Delta K = 10$ and using

$Q'(K) = 16{,}500K^{-1/2} = \dfrac{16{,}500}{\sqrt{K}}$

$\Delta Q \approx \dfrac{16{,}500}{\sqrt{400}}(10) = 8{,}250$ units

By comparison, if the capital expenditure is kept constant,

$Q = 3{,}000(400)^{1/2}L^{1/3}$

$= 60{,}000L^{1/3}$

Increasing the labor force causes a change in weekly output that can be estimated using

$\Delta Q \approx Q'(L_0)\Delta L$

Now, substituting $L_0 = 1{,}331$, $\Delta L = 10$, and using

$Q' = 20{,}000L^{-2/3} = \dfrac{20{,}000}{L^{2/3}}$

$\Delta Q = \dfrac{20{,}000}{(1{,}331)^{2/3}}(10) \approx 1{,}653$ units

So, capital expenditure should be increased by \$10,000.

20. $Q(L) = 300L^{2/3}$

$Q'(L) = \dfrac{200}{L^{1/3}}$, $Q'(512) = 25$

We seek ΔL so that

$12.5 = \Delta Q \approx Q'(512)\Delta L = 25\Delta L$,

so $\Delta L = 0.5$ more worker-hours are needed.

21. $C(q) = \dfrac{1}{6}q^3 + 642q + 400$

$\Delta C \approx C'(q)\Delta q$

We want to approximate Δq, so

$\Delta q \approx \dfrac{\Delta C}{C'(q)}$

$C'(q) = \dfrac{1}{2}q^2 + 642,$

$C'(4) = \dfrac{1}{2}(4)^2 + 642 = 650,$ and

$\Delta C = -130$. So, $\Delta q \approx \dfrac{-130}{650} = -0.2,$ or increase production by 0.2 units.

22. $T(x) = 60x^{3/2} + 40x + 1{,}200$

Estimated percentage change is

$$100\frac{\Delta T}{T} \text{ where } \Delta T \approx T'(x)\Delta x$$

$T'(x) = 90x^{1/2} + 40 = 90\sqrt{x} + 40$

The beginning of the year 2013 is 8 years after the beginning of 2005, so the beginning value of t is 8.
Measured in years, 6 months $= \frac{1}{2}$ year $= \Delta T$.

$$\Delta T \approx T'(8)\left(\frac{1}{2}\right) = \left(90\sqrt{8} + 40\right)\left(\frac{1}{2}\right) = 147.279$$

$$T(8) = 60(8)^{3/2} + 40(8) + 1{,}200 = 2{,}877.645$$

$$100\frac{\Delta T}{T} = 100\frac{147.279}{2877.645} \approx 5.12\%$$

23. $C(t) = 100t^2 + 400t + 5{,}000$

$\Delta C \approx C'(t)\Delta t$

$C'(t) = 200t + 400$

Since t is measured in years, the next six months $= \frac{1}{2}$ year $= \Delta t$

$$\Delta C \approx C'(0)\left(\frac{1}{2}\right) = [200(0) + 400]\left(\frac{1}{2}\right) = 200,$$

or an increase of approximately 200 newspapers.

24. $Q(t) = 0.05t^2 + 0.1t + 3.4$

$Q'(t) = 0.1t + 0.1$

$Q'(0) = 0.1,\ \Delta t = 0.5$ (6 months)

The approximate change in carbon monoxide level will be

$\Delta Q = Q'(0)\Delta t = 0.05$ ppm.

25. $P(t) = -t^3 + 9t^2 + 48t + 200$

(a) $R(t) = P'(t) = -3t^2 + 18t + 48$

(b) $R'(t) = -6t + 18$

(c) $\Delta R \approx R'(t_0)\Delta t$

Substituting $t_0 = 3$ and $\Delta t = \frac{1}{12}$,

$$\Delta R \approx \left[-6(3) + 18\right]\left[\frac{1}{12}\right] = 0$$

or no change in the growth rate is expected during the 1st month of the 4th year. The actual change in the growth rate during the 1st month

$$= R\left(3\frac{1}{12}\right) - R(3)$$

$$R\left(\frac{37}{12}\right) = -3\left(\frac{37}{12}\right)^2 + 18\left(\frac{37}{12}\right) + 48 \approx 74.979$$

$$R(3) = -3(3)^2 + 18(3) + 48 = 75$$

So, the growth rate actually decreases by 0.021, or 21 people per year.

26. A 1% increase in r means $\Delta r = 0.01r$ or $\frac{\Delta r}{r} = 0.01$. For the surface area, $S = 4\pi r^2$,

$$\Delta S \approx \frac{dS}{dr}\Delta r = 8\pi r\Delta r = 8\pi r(0.01)r = 0.08\pi r^2 = 0.02(4\pi r^2) = 0.02S$$

So the surface area increases by approximately 2%.

27. The maximum percentage error in C is $100\frac{\Delta C}{C}$ where $\Delta C \approx C'(x)\Delta x.$

$$C'(x) = -a(x-b)^{-2}(1) = \frac{-a}{(x-b)^2}$$

$$\Delta C \approx C'(c)(\pm 0.03c) = \frac{-a}{(c-b)^2}(\pm 0.03c) = \frac{\pm 0.03ac}{(c-b)^2}$$

$C(c) = \frac{a}{c-b}$

So, $100\frac{\Delta C}{C} = 100\frac{\frac{\pm .03ac}{(c-b)^2}}{\frac{a}{c-b}} = \frac{\pm 3c}{|c-b|}\%.$

28. The inner volume of the balloon is given by $V = \frac{4}{3}\pi(0.01)^3 = 4.189\times10^{-6}$ cubic millimeters. The volume of the balloon skin can be approximated as

$\Delta V = V'(r)\Delta r = 4\pi(0.01)^2(0.0005)$

$= 6.283\times10^{-7}$ cubic millimeters.

The total volume inserted is

$V + \Delta V = 4.817\times10^{-6}$ mm^3.

29. $V = \pi R^2 L,$ where L is constant for a given artery. The percentage error in V is $100\frac{\Delta V}{V}$ where $\Delta V \approx V'(R)\Delta R.$

$V'(R) = 2\pi RL$ so, noting that the radius is decreased by the plaque,

$\Delta V \approx V'(0.3)(-0.07)$

$= 2\pi(0.3)L(-0.07)$

$= -0.042\pi L$

$V(0.3) = \pi(0.3)^2 L = 0.09\pi L,$ so

$100\frac{\Delta V}{V} = 100\frac{-0.042\pi L}{0.09\pi L} = -46.67\%,$ or a blockage in the volume of 46.67%.

30. **(a)** $\frac{\Delta V}{V} \approx \frac{V'\Delta R}{V} = \frac{4kR^3\Delta R}{kR^4} = 4\frac{\Delta R}{R}$

If $\frac{\Delta R}{R} = 0.05$ then

$\frac{\Delta V}{V} \approx 4(0.05) = 0.20$ or the volume increases by about 20%.

(b) Writing exercise—answers will vary.

31. $\Delta L \approx L'(T)\Delta T$

Since $\sigma = \frac{L'(T)}{L(T)},\ L'(T) = \sigma L(T).$

Also, $\Delta T = 35 - (-20) = 55.$

So, $\Delta L \approx \sigma L(T)\Delta T$

$\approx (1.4\times10^{-5})(50)(55)$

$\approx 3{,}850\times10^{-5}$

This is an increase in length of approximately 0.0385 m, or 3.85 cm.

32. $100\frac{\Delta R}{R} \approx 100\frac{R'\Delta T}{R} = 100\frac{4kT^3\Delta T}{kT^4}$

$= 400\frac{\Delta T}{T}$

If $\frac{\Delta T}{T} = 0.02$ then the percentage change in R is approximately 8%.

33. First application of Newton's method: The equation of the tangent line at $(x_0,\ f(x_0))$ is $y - f(x_0) = f'(x_0)(x - x_0).$

The x-intercept is when $y = 0$, or when $-f(x_0) = f'(x_0)(x - x_0).$

Solving for $x = x_1$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

Second application of Newton's method: Using the point $(x,\ f(x_1)),$

$y - f(x_1) = f'(x_1)(x - x_1)$

$-f(x_1) = f'(x_1)(x - x_1)$

Solving for $x = x_2$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

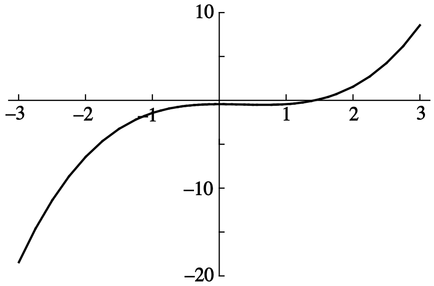
In general, using the point $(x_{n-1},\ f(x_{n-1})),$

$y - f(x_{n-1}) = f'(x_{n-1})(x - x_{n-1})$

$-f(x_{n-1}) = f'(x_{n-1})(x - x_{n-1})$

Solving for $x = x_n,\ \ x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}.$

34. **(a)**



The root is approximately 1.465571.

(b) For $f(x) = x^3 - x^2 - 1$, the iterative formula for Newton's method is

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

$$= x_{n-1} - \frac{x_{n-1}^3 - x_{n-1}^2 - 1}{3x_{n-1}^2 - 2x_{n-1}}$$

Beginning with $x_0 = 1$, the sequence of approximations to the root are

$x_0 = 1$ $\quad x_1 = 2$ $\quad x_2 = 1.625$
$x_3 = 1.485786$ $\quad x_4 = 1.465956$
$x_5 = 1.465571$ $\quad x_6 = 1.465571$

The sixth estimate agrees with the fifth to at least four decimal places.

(c) Answers will vary based on the accuracy of the estimate in part (a).

35. To use graphing utility to graph *f* and to estimate each root,
Press [y =] and input *x* ^ 4 – 4*x* ^ 3 + 10 for $y_1 =$
Use window dimensions [–10, 10]1 by [–20, 20]2
Press [Graph]
Use the zero function under the calc menu to find the zeros (*x*-intercepts) of *f* to be $x \approx 1.6$ and $x \approx 3.8$
To use Newton's method,

$f(x) = x^4 - 4x^3 + 10$ and

$f'(x) = 4x^3 - 12x^2$

$$x - \frac{f(x)}{f'(x)} = x - \frac{x^4 - 4x^3 + 10}{4x^3 - 12x^2}$$

$$= \frac{3x^4 - 8x^3 - 10}{4x^3 - 12x^2}$$

For *n* = 1, 2, 3, ...

$$x_n = \frac{3x_{n-1}^4 - 8x_{n-1}^3 - 10}{4x_{n-1}^3 - 12x_{n-1}^2}$$

Using the graph shown on the calculator, we see one *x*-intercept is between 1 and 2.
Let $x_0 = 1$, then

$$x_1 = \frac{3x_0^4 - 8x_0^3 - 10}{4x_0^3 - 12x_0^2} = \frac{-15}{-8} = 1.875$$

using $x_0 = 1$

$$x_2 = \frac{3x_1^4 - 8x_1^3 - 10}{4x_1^3 - 12x_1^2} \approx 1.622 \text{ using}$$

$x_1 = 1.875$

$$x_3 = \frac{3x_2^4 - 8x_2^3 - 10}{4x_2^3 - 12x_2^2} \approx 1.612 \text{ using}$$

$x_2 = 1.622$

$$x_4 = \frac{3x_3^4 - 8x_3^3 - 10}{4x_3^3 - 12x_3^2} \approx 1.612 \text{ using}$$

$x_3 = 1.612$

Thus, one *x*-intercept is $x \approx 1.612$.
The second *x*-intercept is between 3 and 4.
Let $x_0 = 4$, then

$$x_1 = \frac{3x_0^4 - 8x_0^3 - 10}{4x_0^3 - 12x_0^2} \approx 3.844 \text{ using } x_0 = 4$$

$$x_2 = \frac{3x_1^4 - 8x_1^3 - 10}{4x_1^3 - 12x_1^2} \approx 3.821 \text{ using}$$

$x_1 = 3.844$

$$x_3 = \frac{3x_2^4 - 8x_2^3 - 10}{4x_2^3 - 12x_2^2} \approx 3.821 \text{ using}$$

$x_2 = 3.821$

Thus, the second *x*-intercept is *x* = 3.821.
Note: Enter (3*x* ^ 4 – 8*x* ^ 3 – 10)/(4*x* ^ 3 – 12*x* ^ 2) for $y_2 =$ and use the value function under the calc menu to do all the calculations for Newton's method.

36. **(a)** Suppose N is a fixed number and let $f(x)=x^2-N$. Then $f'(x)=2x$ and Newton's method becomes

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}=x_n-\frac{x_n^2-N}{2x_n}$$
$$=x_n-\frac{x_n}{2}+\frac{N}{2x_n}=\frac{1}{2}\left(x_n+\frac{N}{x_n}\right)$$

(b) Writing exercise—answers will vary.

37. $f(x)=\sqrt[3]{x}=x^{1/3};\ \ f'(x)=\frac{1}{3}x^{-2/3}=\frac{1}{3x^{2/3}}$

(a) $x_{n+1}=x_n-\frac{(x_n)^{1/3}}{\frac{1}{3(x_n)^{2/3}}}$

$x_{n+1}=x_n-3x_n$, or $x_{n+1}=-2x_n$

So, if x_0 is first guess,

$x_1=-2x_0$,

$x_2=-2x_1=-2(-2x_0)=4x_0$,

$x_3=-2x_2=-2(4x_0)=-8x_0$, etc.

(b) To use the graphing utility to graph f and to draw the tangent lines,
Press [y =] and input x ^ (1/3) for y_1 =
Use window dimensions [–5, 5]1 by [–5, 5]1
Arbitrarily, let's use $x_0=1$. Then we will draw tangent lines to the graph of f for x = 1, –2, 4...
Press [2nd] [Draw] and use the tangent function. Enter x = 1 and the tangent line is drawn. Repeat for x = –2 and x = 4.
From the graph, we can see that x = 0 is the root of $\sqrt[3]{x}$. Any choice besides zero for the first estimate leads to successive approximations on opposite sides of the root, getting farther and farther from the root.

2.6 Implicit Differentiation and Related Rates

1. $2x + 3y = 7$

(a) $2+3\frac{dy}{dx}=0$

$$\frac{dy}{dx}=-\frac{2}{3}$$

(b) Solving for y, $y=-\frac{2}{3}x+\frac{7}{3}$

$$\frac{dy}{dx}=-\frac{2}{3}$$

2. **(a)** Differentiating both sides of $5x - 7y = 3$ with respect to x yields $5-7\frac{dy}{dx}=0.$ Solving for $\frac{dy}{dx}$ gives $\frac{dy}{dx}=\frac{5}{7}.$

(b) Solving for y gives $y=\frac{5}{7}x-\frac{3}{7}$ so

$\frac{dy}{dx}=\frac{5}{7}.$

3. $x^3-y^2=5$

(a) $3x^2-2y\frac{dy}{dx}=0$

$$\frac{dy}{dx}=\frac{3x^2}{2y}$$

(b) Solving for y,

$$y=\pm\sqrt{x^3-5}=\pm(x^3-5)^{1/2}$$
$$\frac{dy}{dx}=\pm\frac{1}{2}(x^3-5)^{-1/2}\cdot 3x^2$$
$$=\pm\frac{3x^2}{2\sqrt{x^3-5}}$$
$$=\frac{3x^2}{2y}$$

4. **(a)** Differentiating both sides of $x^2+y^3=12$ with respect to x yields

$2x + 3y^2 \frac{dy}{dx} = 0$.

Solving for $\frac{dy}{dx}$ gives $\frac{dy}{dx} = -\frac{2x}{3y^2}$.

(b) Solving for y gives $y = (12 - x^2)^{1/3}$ so
$\frac{dy}{dx} = \frac{-2x}{3(12 - x^2)^{2/3}} = -\frac{2x}{3y^2}$.

5. $xy = 4$

(a) $x \cdot \frac{dy}{dx} + y \cdot 1 = 0$

$$\frac{dy}{dx} = -\frac{y}{x}$$

(b) Solving for y, $y = \frac{4}{x} = 4x^{-1}$

$$\frac{dy}{dx} = -4x^{-2} = -\frac{4}{x^2} = -\frac{\frac{4}{x}}{x} = -\frac{y}{x}$$

6. **(a)** Differentiating both sides of $x + \frac{1}{y} = 5$ with respect to x yields

$1 - \frac{1}{y^2} \frac{dy}{dx} = 0$.

Solving for $\frac{dy}{dx}$ gives $\frac{dy}{dx} = y^2$.

(b) Solving for y gives $y = \frac{1}{5 - x}$ so
$\frac{dy}{dx} = \frac{1}{(5 - x)^2} = y^2$.

7. $xy + 2y = 3$

(a) $x\frac{dy}{dx} + y \cdot 1 + 2\frac{dy}{dx} = 0$

$$(x+2)\frac{dy}{dx} = -y$$

$$\frac{dy}{dx} = \frac{-y}{x+2}$$

(b) Solving for y, $y = \frac{3}{x+2} = 3(x+2)^{-1}$

$$\frac{dy}{dx} = -3(x+2)^{-2}(1)$$
$$= \frac{-3}{(x+2)^2}$$
$$= \frac{3}{x+2} \cdot \frac{-1}{x+2}$$
$$= y \cdot \frac{-1}{x+2}$$
$$= \frac{-y}{x+2}$$

8. **(a)** Differentiating both sides of $xy + 2y = x^2$ with respect to x yields $y + x\frac{dy}{dx} + 2\frac{dy}{dx} = 2x$. Solving for $\frac{dy}{dx}$ gives $\frac{dy}{dx} = \frac{2x - y}{x + 2}$.

(b) Solving for y gives $y = \frac{x^2}{x+2}$ so

$$\frac{dy}{dx} = \frac{2x(x+2) - x^2(1)}{(x+2)^2}$$
$$= \frac{2x - \frac{x^2}{x+2}}{x+2}$$
$$= \frac{2x - y}{x+2}.$$

9. $x^2 + y^2 = 25$

$$2x + 2y\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = -\frac{x}{y}$$

10. $x^2 + y = x^3 + y^2$

$$2x + \frac{dy}{dx} = 3x^2 + 2y\frac{dy}{dx}$$
$$(1 - 2y)\frac{dy}{dx} = 3x^2 - 2x$$
$$\frac{dy}{dx} = \frac{3x^2 - 2x}{1 - 2y}$$

11.
$$x^3+y^3=xy$$
$$3x^2+3y^2\frac{dy}{dx}=x\frac{dy}{dx}+y\cdot 1$$
$$(3y^2-x)\frac{dy}{dx}=y-3x^2$$
$$\frac{dy}{dx}=\frac{y-3x^2}{3y^2-x}$$

12.
$$5x-x^2y^3=2y$$
$$5-x^2(3y^2)\frac{dy}{dx}-2xy^3=2\frac{dy}{dx}$$
$$(2+3x^2y^2)\frac{dy}{dx}=5-2xy^3$$
$$\frac{dy}{dx}=\frac{5-2xy^3}{2+3x^2y^2}$$

13. $y^2+(2x)(y^2)-3x+1=0$
$$2y\frac{dy}{dx}+(2x)\left(2y\frac{dy}{dx}\right)+(y^2)(2)-3+0=0$$
$$(2y+4xy)\frac{dy}{dx}=3-2y^2$$
$$\frac{dy}{dx}=\frac{3-2y^2}{2y(1+2x)}$$

14.
$$\frac{1}{x}+\frac{1}{y}=1$$
$$-\frac{1}{x^2}-\frac{1}{y^2}\frac{dy}{dx}=0$$
$$\frac{dy}{dx}=-\frac{y^2}{x^2}$$

15. $\sqrt{x}+\sqrt{y}=1$, or $x^{1/2}+y^{1/2}=1$
$$\frac{1}{2}x^{-1/2}+\frac{1}{2}y^{-1/2}\frac{dy}{dx}=0$$
$$x^{-1/2}+y^{-1/2}\frac{dy}{dx}=0$$
$$\frac{dy}{dx}=\frac{-x^{-1/2}}{y^{-1/2}}=\frac{-\sqrt{y}}{\sqrt{x}}$$

16.
$$\sqrt{2x}+y^2=4$$
$$\frac{\sqrt{2}}{2\sqrt{x}}+2y\frac{dy}{dx}=0$$
$$\frac{dy}{dx}=-\frac{1}{2y\sqrt{2x}}$$

17. $xy-x=y+2$
$$x\frac{dy}{dx}+y\cdot 1-1=\frac{dy}{dx}+0$$
$$(x-1)\frac{dy}{dx}=1-y$$
$$\frac{dy}{dx}=\frac{1-y}{x-1}$$

18.
$$y^2+3xy-4x^2=9$$
$$2y\frac{dy}{dx}+3x\frac{dy}{dx}+3y-8x=0$$
$$(2y+3x)\frac{dy}{dx}=8x-3y$$
$$\frac{dy}{dx}=\frac{8x-3y}{2y+3x}$$

19. $(2x+y)^3=x,$
$$3(2x+y)^2\left(2+\frac{dy}{dx}\right)=1$$
$$2+\frac{dy}{dx}=\frac{1}{3(2x+y)^2}$$
$$\frac{dy}{dx}=\frac{1}{3(2x+y)^2}-2$$

20.
$$(x-2y)^2=y$$
$$2(x-2y)\left(1-2\frac{dy}{dx}\right)=\frac{dy}{dx}$$
$$2(x-2y)=\frac{dy}{dx}+4(x-2y)\frac{dy}{dx}$$
$$\frac{dy}{dx}=\frac{2(x-2y)}{1+4(x-2y)}$$
$$=\frac{2x-4y}{1+4x-8y}$$

21. $(x^2+3y^2)^5=(2x)(y)$

$$5(x^2+3y^2)^4\left(2x+6y\frac{dy}{dx}\right)=2x\frac{dy}{dx}+y\cdot 2$$

$$10x(x^2+3y^2)^4+30y(x^2+3y^2)^4\frac{dy}{dx}=2x\frac{dy}{dx}+2y$$

$$5x(x^2+3y^2)^4+15y(x^2+3y^2)^4\frac{dy}{dx}=x\frac{dy}{dx}+y$$

$$[15y(x^2+3y^2)^4-x]\frac{dy}{dx}=y-5x(x^2+3y^2)^4$$

$$\frac{dy}{dx}=\frac{y-5x(x^2+3y^2)^4}{15y(x^2+3y^2)^4-x}$$

22. $(3xy^2+1)^4=2x-3y$

$$4(3xy^2+1)^3\left(6xy\frac{dy}{dx}+3y^2\right)=2-3\frac{dy}{dx}$$

$$24xy(3xy^2+1)^3\frac{dy}{dx}+3\frac{dy}{dx}=2-12y^2(3xy^2+1)^3$$

$$\frac{dy}{dx}=\frac{2-12y^2(3xy^2+1)^3}{24xy(3xy^2+1)^3+3}$$

23. $x^2=y^3$

$$2x=3y^2\frac{dy}{dx}$$

$$\frac{2x}{3y^2}=\frac{dy}{dx}$$

The slope of the tangent line at (8, 4) is $\frac{dy}{dx}=\frac{2(8)}{3(4)^2}=\frac{1}{3}$ and the equation of the tangent line is

$y-4-\frac{1}{3}(x-8)$, or $y=\frac{1}{3}x+\frac{4}{3}$.

24. $x^2-y^3=2x$

$$2x-3y^2\frac{dy}{dx}=2$$

$$\frac{dy}{dx}=\frac{2-2x}{-3y^2}$$

At (1, –1) the slope is $\frac{2-2(1)}{-3(-1)^2}=0$ and the equation of the tangent line is $y=-1$.

25. $$xy = 2$$
$$x\frac{dy}{dx} + y \cdot 1 = 0$$
$$\frac{dy}{dx} = \frac{-y}{x}$$

The slope of the tangent line at (2, 1) is $\frac{dy}{dx} = \frac{-1}{2}$ and the equation of the tangent line is $y - 1 = -\frac{1}{2}(x-2)$, or $y = -\frac{1}{2}x + 2.$

26. $$\frac{1}{x} - \frac{1}{y} = 2$$
$$-\frac{1}{x^2} + \frac{1}{y^2}\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = \frac{y^2}{x^2}$$

At $\left(\frac{1}{4}, \frac{1}{2}\right)$ the slope is $\frac{\left(\frac{1}{2}\right)^2}{\left(\frac{1}{4}\right)^2} = 4$ and the equation of the tangent line is $y = 4x - \frac{1}{2}$.

27. $$xy^2 - x^2y = 6$$
$$x \cdot 2y\frac{dy}{dx} + y^2 \cdot 1 - x^2 \cdot \frac{dy}{dx} - y \cdot 2x = 0$$
$$2xy\frac{dy}{dx} - x^2\frac{dy}{dx} = 2xy - y^2$$ The slope of the tangent line at (2, –1) is
$$\frac{dy}{dx} = \frac{2xy - y^2}{2xy - x^2}$$

$\frac{dy}{dx} = \frac{2(2)(-1) - (-1)^2}{2(2)(-1) - (2)^2} = \frac{5}{8}$ and the equation of the tangent line is $y - (-1) = \frac{5}{8}(x-2)$, or $y = \frac{5}{8}x - \frac{9}{4}.$

28. $$x^2y^3 - 2xy = 6x + y + 1$$
$$3x^2y^2\frac{dy}{dx} + 2xy^3 - 2x\frac{dy}{dx} - 2y = 6 + \frac{dy}{dx}$$
$$\frac{dy}{dx} = \frac{2y - 2xy^3 + 6}{3x^2y^2 - 2x - 1}$$

At $(0, -1)$ the slope is $\frac{-2 - 0 + 6}{0 - 0 - 1} = -4$ and the equation of the tangent line is $y = -4x - 1$.

29. $(1-x+y)^3 = x+7$

$$3(1-x+y)^2\left(-1+\frac{dy}{dx}\right)=1+0$$

$$-1+\frac{dy}{dx}=\frac{1}{3(1-x+y)^2}$$

$$\frac{dy}{dx}=\frac{1}{3(1-x+y)^2}+1$$

When $x = 1$, $(1-1+y)^3 = 1+7$, so $y = 2$ and the slope of the tangent line is

$$\frac{dy}{dx}=\frac{1}{3(1-1+2)^2}+1=\frac{13}{12}.$$

The equation of the tangent line is $y-2=\frac{13}{12}(x-1)$, or $y=\frac{13}{12}x+\frac{11}{12}$.

30. $(x^2+2y)^3 = 2xy^2+64$

$$3(x^2+2y)^2\left(2x+2\frac{dy}{dx}\right)=4xy\frac{dy}{dx}+2y^2$$

At (0, 2) this equation becomes $96\frac{dy}{dx}=8$ so the slope at (0, 2) is $\frac{1}{12}$ and the equation of the tangent line is $y=\frac{1}{12}x+2$.

31. $x+y^2=9$

$$1+2y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-1}{2y}$$

(a) For horizontal tangent(s), need $\frac{dy}{dx}=0$, but $-\frac{1}{2y}\neq 0$ for any value of *y*, so there are no horizontal tangents.

(b) For vertical tangent(s), need the denominator of the slope $2y = 0$, or $y = 0$. When $y = 0$, $x + 0 = 9$, or $x = 9$. There is a vertical tangent at (9, 0).

32. **(a)** $x^2+xy+y=3$

$$2x+x\frac{dy}{dx}+y\cdot 1+\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-2x-y}{x+1}$$

$\frac{-2x-y}{x+1}=0$ when $-2x - y = 0$, or $y = -2x$.

Substituting in the original equation,

$$x^2 - 2x^2 - 2x = 3$$
$$0 = x^2 + 2x + 3$$

Since there are no real solutions, there are no horizontal tangents.

(b) $x + 1 = 0$ when $x = -1$.
When $x = -1$, $1 - y + y = 3$
So no such y exists and there are no vertical tangents.

33. $xy = 16y^2 + x$

$$x \cdot \frac{dy}{dx} + y \cdot 1 = 32y\frac{dy}{dx} + 1$$
$$x\frac{dy}{dx} - 32y\frac{dy}{dx} = 1 - y$$
$$\frac{dy}{dx} = \frac{1-y}{x-32y}$$

(a) $\frac{1-y}{x-32y} = 0$ when $1 - y = 0$, or $y = 1$.

Substituting into the original equation,
$$x \cdot 1 = 16(1)^2 + x$$
$$x = 16 + x$$
$$0 = 16$$

Since there is no solution to this equation, there are no points on the given curve where the tangent line is horizontal.

(b) For $\frac{dy}{dx}$ to be undefined, $x - 32y = 0$, or $x = 32y$. Substituting into the original equation,

$$(32y)y = 16y^2 + 32y$$
$$16y^2 - 32y = 0$$
$$16y(y-2) = 0$$
$$y = 0 \text{ or } y = 2$$

When $y = 0$, $x = 32(0) = 0$ and when
$y = 2$, $x = 32(2) = 64$. So, there are vertical tangents at (0, 0) and (64, 2).

34. $\frac{y}{x} - \frac{x}{y} = 5$

$$\frac{x\frac{dy}{dx} - y}{x^2} - \frac{y - x\frac{dy}{dx}}{y^2} = 0 \text{ which simplifies to } \frac{dy}{dx} = \frac{y}{x}.$$

From the equation of the curve, there can be no points on the curve having either $x = 0$ or $y = 0$. Thus there are no points where the numerator or denominator of the derivative is 0. There are no horizontal or vertical tangents to this curve.

35. $x^2 + xy + y^2 = 3$

$$2x + x\frac{dy}{dx} + y \cdot 1 + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$$

(a) $\frac{-2x - y}{x + 2y} = 0$ when $-2x - y = 0$, or

$y = -2x$.

Substituting in the original equation,

$$x^2 - 2x^2 + 4x^2 = 3$$

$$3x^2 = 3$$

$$x = \pm 1$$

When $x = -1$, $y = -2(-1) = 2$, and when $x = 1$, $y = -2(1) = -2$. So, there are horizontal tangents at (–1, 2) and (1, –2).

(b) $x + 2y = 0$ when $x = -2y$.

Substituting in the original equation,

$$4y^2 - 2y^2 + y^2 = 3$$

$$3y^2 = 3$$

$$y = \pm 1$$

When $y = -1$, $x = -2(-1) = 2$, and when $y = 1$, $x = -2(1) = -2$. So, there are vertical tangents at (−2, 1) and

(2, –1).

36. (a) $x^2 - xy + y^2 = 3$

$$2x - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{y - 2x}{2y - x}$$

For the tangent line to be horizontal we must have $\frac{dy}{dx} = 0$ and $y = 2x$ at such a point.

Substituting this expression into the equation of the curve gives $x^2 - x(2x) + (2x)^2 = 3x^2 = 3$ so $x = \pm 1$. Since $y = 2x$, the points where the tangent is horizontal are (1, 2) and (–1, –2).

(b) The tangent line will be vertical when the denominator in the derivative is 0 while the numerator is not 0. The denominator is 0 at points where $x = 2y$. Substitution into the original equation gives $3y^2 = 3$ and the points where the tangent line is vertical are (2, 1) and (–2, –1).

37. $x^2 + 3y^2 = 5$

$$2x + 6y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{3y}$$

$$\frac{d^2y}{dx^2} = \frac{(3y)(-1) - (-x)\left(3\frac{dy}{dx}\right)}{(3y)^2}$$

$$= \frac{-3y + 3x\frac{dy}{dx}}{9y^2}$$

$$= \frac{-3y + 3x\left(\frac{-x}{3y}\right)}{9y^2}$$

$$= \frac{-3y - \frac{x^2}{y}}{9y^2} \cdot \frac{y}{y}$$

$$= \frac{-3y^2 - x^2}{9y^3}$$

$$= \frac{-(x^2 + 3y^2)}{9y^3}$$

$$= -\frac{5}{9y^3}$$

38. $xy + y^2 = 1$

$$x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-y}{x + 2y}$$

$$\frac{d^2y}{dx^2} = \frac{(x + 2y)\left(-\frac{dy}{dx}\right) - (-y)\left(1 + 2\frac{dy}{dx}\right)}{(x + 2y)^2}$$

$$= \frac{(x + 2y)\left(\frac{y}{x + 2y}\right) - (-y)\left(1 + 2\left(\frac{-y}{x + 2y}\right)\right)}{(x + 2y)^2}$$

$$= \frac{2y(x + y)}{(x + 2y)^3}$$

39. Need to find $\Delta y \approx \frac{dy}{dx}$.

Since Q is to remain constant, let c be the constant value of Q. Then

$$c = 0.08x^2 + 0.12xy + 0.03y^2$$

$$0 = 0.16x + 0.12x\frac{dy}{dx} + y \cdot 0.12 + 0.06t\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-0.16x - 0.12y}{0.12x + 0.06y}$$

Since $x = 80$ and $y = 200$, $\frac{dy}{dx} \approx -1.704$, or a decrease of 1.704 hours of unskilled labor.

40. $Q = 0.06x^2 + 0.14xy + 0.05y^2$

The goal is keep Q constant hence upon differentiating

$$0 = \frac{dQ}{dx} = 0.12x + 0.14x\frac{dy}{dx} + 0.14y + 0.10y\frac{dy}{dx} \text{ or } \frac{dy}{dx} = \frac{-0.12x - 0.14y}{0.14x + 0.10y} = \frac{-6x - 7y}{7x + 5y}.$$

Use the approximation formula $\Delta y \approx \frac{dy}{dx}\Delta x$ with $x = 60$, $y = 300$ and $\Delta x = 1$.

$$\Delta y \approx \frac{-6(60) - 7(300)}{7(60) + 5(300)}(1) = -1.28125.$$

To maintain output at the current level decrease the unskilled labor by 1.28125 hours.

41. $3p^2 - x^2 = 12$

$$6p\frac{dp}{dt} - 2x\frac{dx}{dt} = 0$$

When $p = 4$, $48 - x^2 = 12$, $x^2 = 36$, or
$x = 6$.

Substituting, $6(4)(0.87) - 2(6)\frac{dx}{dt} = 0$,

$$20.88 - 12\frac{dx}{dt} = 0$$

$\frac{dx}{dt} = \frac{20.88}{12} = 1.74$ or increasing at a rate of 174 units/month.

42. $x^2 + 3px + p^2 = 79$

$$2x\frac{dx}{dt} + 3p\frac{dx}{dt} + 3x\frac{dp}{dt} + 2p\frac{dp}{dt} = 0 \text{ or } \frac{dx}{dt} = \frac{-3x - 2p}{2x + 3p}\frac{dp}{dt}$$

When $p = 5$, the demand x satisfies $p = 5$

$x^2 + 3(5)x + 5^2 = 79$ or $x^2 + 15x - 54 = (x + 18)(x - 3) = 0$ so $x = 3$. Give $\frac{dp}{dt} = 0.30$.

$\frac{dx}{dt} = \frac{-19}{21}(0.30) = -0.27143$ or demand is decreasing at the rate 27.143 units per month.

43. $D(p) = \frac{32,670}{2p + 1} = 32,670(2p + 1)^{-1}$

$$p(t) = 0.04t^{3/2} + 44$$

Need $\frac{dD}{dt}$ when $t = 25$.

$$\frac{dD}{dt}=\frac{dD}{dp}\cdot\frac{dp}{dt}$$

Now, $\dfrac{dD}{dp}=-32,670(2p+1)^{-2}(2)=-\dfrac{65,340}{(2p+1)^2}$

$$\frac{dp}{dt}=0.06t^{1/2}$$

When t = 25, $p=0.04(25)^{3/2}+44=49$, so $\dfrac{dD}{dt}=-\dfrac{65,340}{[2(49)+1]^2}\cdot 0.06(25)^{1/2}=-2$, or the demand will be decreasing at a rate of 2 toasters per month.

44. $Q=3u^2+\dfrac{2u+3v}{(u+v)^2}$

The goal is keep Q constant hence upon differentiating

$$0=\frac{dQ}{du}=6u+\frac{\left[(u+v)^2\left[2+3\frac{dv}{du}\right]-2(2u+3v)(u+v)\left[1+\frac{dv}{du}\right]\right]}{(u+v)^4}$$

When $u=10$ and $v=25$, solving the above for $\dfrac{dv}{du}$ gives $\dfrac{dv}{du}=\dfrac{514476}{17}$.

Use the approximation formula $\Delta v\approx\dfrac{dv}{du}\Delta u$ with $\Delta u=-0.7$.

$$\Delta v\approx\frac{514476}{17}(-0.7)=21{,}184.3.$$

To maintain output at the current level decrease the unskilled labor by 21,184 units.

45. Since Q is to remain constant, $C=60K^{1/3}L^{2/3}$.

$0=60K^{1/3}\cdot\dfrac{2}{3}L^{-1/3}\dfrac{dL}{dt}+L^{2/3}\cdot 20K^{-2/3}\dfrac{dK}{dt}$ Substituting,

$$0=\frac{40(8)^{1/3}}{(1{,}000)^{1/3}}(25)+\frac{20(1{,}000)^{2/3}}{(8)^{2/3}}\frac{dK}{dt}$$

$$0=200+500\frac{dK}{dt}$$

$$\frac{dK}{dt}=-0.4,$$

or decreasing at a rate of \$400 per week.

46. Need $\Delta y\approx\dfrac{dy}{dx}\Delta x$

Since Q is to remain constant, let C be the constant value of Q. Then

$$C = 2x^3 + 3x^2y^2 + (1+y)^3$$
$$0 = 6x^2 + (3x^2)\left(2y\frac{dy}{dx}\right) + (y^2)(6x) + 3(1+y)^2\frac{dy}{dx}$$

Substituting,

$$0 = 6(30)^2 + 6(30)^2(2\cdot 20)\frac{dy}{dx} + (20)^2(6\cdot 30) + 3(1+20)^2\frac{dy}{dx}$$
$$0 = 77{,}400 + 217{,}323\frac{dy}{dx}$$
$$\frac{dy}{dx} \approx -0.3562$$

Since $\Delta x = -0.8$,

$$\Delta y \approx (-0.3562)(-0.8) = 0.2849,$$

or an increase of 0.2849 units in input y.

47. $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$

Substituting, $0.002\pi = 4\pi(0.005)^2\frac{dr}{dt}$

$$\frac{dr}{dt} = \frac{0.002}{4(0.005)^2} = 20,$$

or increasing at a rate of 20 mm per min.

48. $Q(p) = p^2 + 4p + 900$

$Q' = (2p+4)p'$ where the derivatives are with respect to t. At the time in question $p = 50$ and $p' = 1.5$ so the pollution level is changing at the rate of $Q' = (2(50)+4)(1.5) = 156$ units per year.

49. $V = \frac{4}{3}\pi R^3$

$$\frac{dV}{dt} = 4\pi R^2\frac{dR}{dt}$$

Substituting, $\frac{dV}{dt} = 4\pi(0.54)^2(0.13) \approx 0.476$ or increasing at a rate of $0.476\ \text{cm}^3$ per month.

50. $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

When $r = 200$ and $\frac{dr}{dt} = 20$

$\frac{dA}{dt} = 2\pi(200)20 = 8{,}000\pi$ or roughly 25,133 square feet per hour.

51. $M = 70w^{3/4}$

(a) $\frac{dM}{dt} = 52.5w^{-1/4}\frac{dw}{dt}$

Substituting, $\frac{dM}{dt} = \frac{52.5}{(80)^{1/4}}(0.8) \approx 14.04,$ or increasing at a rate of 14.04 kcal per day^2.

(b) $\frac{dM}{dt} = \frac{52.5}{(50)^{1/4}}(-0.5) \approx -9.87,$ or decreasing at a rate of 9.87 kcal per day^2.

52. $s = 1.1w^{0.2}$

$\frac{ds}{dt} = 0.22w^{-0.8}\frac{dw}{dt}$

When $w = 11$ and $\frac{dw}{dt} = 0.02,$

$$\frac{ds}{dt} = 0.22(11)^{-0.8}(0.02)$$
$$= 0.000646 \text{ meters per second per day}$$

53. $v = \frac{K}{L}(R^2 - r^2)$

At the center of the vessel, $r = 0$ so $v = \frac{K}{L}R^2 = KL^{-1}R^2$.

Using implicit differentiation with t as the variable, $\frac{dv}{dt} = K\left[L^{-1}\left(2R\frac{dR}{dt}\right) + R^2\left(-L^{-2}\frac{dL}{dt}\right)\right]$.

Since the speed is unaffected, $\frac{dv}{dt} = 0$ and

$$0 = K\left[\frac{2R}{L}\cdot\frac{dR}{dt} - \frac{R^2}{L^2}\cdot\frac{dL}{dt}\right]$$
$$0 = \frac{2R}{L}\cdot\frac{dR}{dt} - \frac{R^2}{L^2}\cdot\frac{dL}{dt}$$

Solving for the relative rate of change of L,

$$\frac{R^2}{L^2}\cdot\frac{dL}{dt} = \frac{2R}{L}\cdot\frac{dR}{dL}$$
$$\frac{\frac{dL}{dt}}{L} = 2\frac{\frac{dR}{dL}}{R}$$

or double the relative rate of change of R.

54. The population is

$$p(t) = 10 - \frac{20}{(t+1)^2} = 10 - 20(t+1)^{-2}$$

and the carbon monoxide level is

$$c(p) = 0.8\sqrt{p^2 + p + 139}$$
$$= 0.8(p^2 + p + 139)^{1/2}$$

By the chain rule, the rate of change of the carbon monoxide level with respect to time is

$$\frac{dc}{dt} = \frac{dc}{dp}\frac{dp}{dt}$$
$$= 0.4(p^2 + p + 139)^{-1/2}(2p+1)\left[40(t+1)^{-3}\right]$$
$$= \frac{0.4(2p+1)}{\sqrt{p^2 + p + 139}}\frac{40}{(t+1)^3}$$

At $t = 1$, $p = p(1) = 10 - \frac{20}{4} = 5$, $c = c(5) = 0.8\sqrt{169} = 10.4$.

The percentage rate of change is

$$100\frac{dc/dt}{c} = 100\frac{0.4(10+1)}{\sqrt{169}}\frac{40}{(1+1)^3}\frac{1}{10.4} \approx 16.27\% \text{ per year.}$$

55. $F = kD^2\sqrt{A-C} = kD^2\left(A-C\right)^{1/2}$

(a) Treating *A* and *D* as constants,

$$\frac{dF}{dC} = \frac{1}{2}kD^2\left(A-C\right)^{-1/2}\left(-1\right)$$
$$= \frac{-kD^2}{2\sqrt{A-C}}$$

As *C* increases, the denominator increases, so *F* decreases.

(b) We need $100\frac{dF/dA}{F}$

Treating *C* and *D* as constants,

$$\frac{dF}{dA} = \frac{1}{2}kD^2\left(A-C\right)^{-1/2}\left(1\right)$$
$$= \frac{kD^2}{2\sqrt{A-C}}$$

$$100\frac{\frac{dF}{dA}}{F} = 100\frac{\frac{kD^2}{2\sqrt{A-C}}}{kD^2\sqrt{A-C}}$$
$$= \frac{50}{\left(A-C\right)}\%$$

56. $V = \frac{\pi}{3}H(R^2 + rR + r^2)$

$\frac{dV}{dt} = \frac{\pi}{3}\left[H\left(2R\frac{dR}{dt} + r\frac{dR}{dt} + R\frac{dr}{dt} + 2r\frac{dr}{dt}\right) + \frac{dH}{dt}(R^2 + rR + r^2)\right]$

Substituting *r* = 2, *R* = 3, *H* = 15 and $\frac{dr}{dt} = \frac{4}{12}$, $\frac{dR}{dt} = \frac{5}{12}$, $\frac{dH}{dt} = \frac{9}{12}$ yields

$\frac{dV}{dt} = \frac{397}{12}\pi \approx 103.93$ cubic feet per year.

57. Let *x* be the distance between the man and the base of the street light and *L* the length of the shadow. Because of similar triangles, $\frac{L}{6} = \frac{x+L}{12}$ or *L* = *x*.

So, $\frac{dL}{dt} = \frac{dx}{dt} = 4$, or increasing at a rate of 4 feet per second.

58. $PV^{1.4} = C$

Differentiating with respect to *t* yields

$P(1.4V^{-0.4})V' + P'V^{1.4} = 0$ so

$V' = \frac{-P'V^{1.8}}{1.4P}$

Given $V = 5$, $P = 0.6$ and $P' = 0.23$

$V' = \frac{-0.23(5^{1.8})}{1.4(0.6)} = -4.961$.

V is decreasing at roughly 4.96 m^3 per sec.

59. $PV = C$

Differentiating with respect to time,

$P \cdot \frac{dV}{dt} + V \cdot \frac{dP}{dt} = 0$

Solving for $\frac{dP}{dt}$ gives $\frac{dP}{dt} = \frac{-P \cdot \frac{dV}{dt}}{V}$

Substituting *P* = 70, *V* = 40 and $\frac{dV}{dt} = 12$, $\frac{dP}{dt} = \frac{-70(12)}{40} = -21 \text{ lb/in}^2 / \text{sec}$

Since $\frac{dP}{dt}$ s negative, the pressure is decreasing.

60. **(a)** Since $V = s^3$, $s^3 = 125{,}000$ and $s = 50$.

Differentiating with respect to *t* $V' = 3s^2 s'$

At the present time $V' = -1{,}000$ and $s = 50$ so $s' = \frac{V'}{3s^2} = \frac{-1{,}000}{3(50)^2} = -\frac{2}{15}$ cm per hour.

(b) $S = 6s^2$

$$S' = 12ss' = 12(50)\left(-\frac{2}{15}\right) = -80 \text{ cm}^2 \text{ per hour}$$

61. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{\frac{-2x}{a^2}}{\frac{2y}{b^2}} = \frac{-b^2x}{a^2y}$$

Substituting, $\frac{dy}{dx} = \frac{-b^2x_0}{a^2y_0}$ and the equation of the tangent line is

$$y - y_0 = \frac{-b^2x_0}{a^2y_0}(x - x_0)$$

$$\frac{y_0y}{b^2} - \frac{y_0^2}{b^2} = \frac{-x_0x}{a^2} + \frac{x_0^2}{a^2}$$

$$\frac{x_0x}{a^2} + \frac{y_0y}{b^2} = \frac{x_0^2}{a^2} + \frac{y_0^2}{b^2}$$

So, $\frac{x_0x}{a^2} + \frac{y_0y}{b^2} = 1.$

62. **(a)**

$$x^2 + y^2 = 6y - 10$$

$$x^2 + y^2 - 6y + 9 = -10 + 9$$

$$x^2 + (y-3)^2 = -1$$

Since the sum of two squares cannot be negative, there are no points (x, y) that satisfy this equation.

(b) $2x + 2y\frac{dy}{dx} = 6\frac{dy}{dx}$

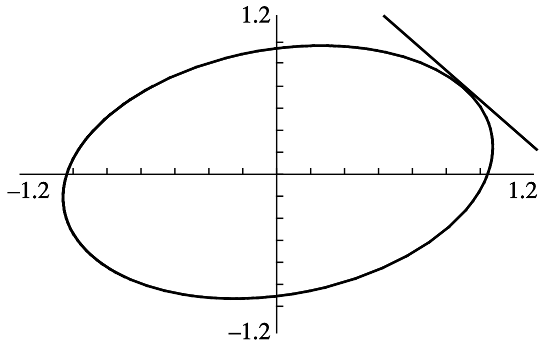
$$2(3 - y)\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{x}{3 - y}$$

63. $y = x^{r/s}$ or $y^s = x^r$

$$sy^{s-1}\frac{dy}{dx} = rx^{r-1}$$
$$\frac{dy}{dx} = \frac{rx^{r-1}}{sy^{s-1}}$$
$$= \frac{rx^{r-1}}{s(x^{r/s})^{s-1}}$$
$$= \frac{rx^{r-1}}{sx^{r-r/s}}$$
$$= \frac{r}{s}x^{(r-1)-(r-r/s)}$$
$$= \frac{r}{s}x^{r/s-1}$$

64.



$$5x^2 - 2xy + 5y^2 = 8$$
$$10x - 2x\frac{dy}{dx} - 2y + 10y\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = \frac{y-5x}{5y-x}$$

At (1, 1) the slope is $\frac{1-5}{5-1} = -1$ and the tangent line is $y = -x + 2$.

For the tangent line to be horizontal at a point, we must have $y = 5x$ so that $\frac{dy}{dx} = 0$. Substituting $y = 5x$ in $5x^2 - 2xy + 5y^2 = 8$ gives $5x^2 - 2x(5x) + 5(5x)^2 = 120x^2 = 8$ yielding $x = \pm\frac{1}{\sqrt{15}}$ and $y = \pm\frac{5}{\sqrt{15}}$. There are two horizontal tangents with equations $y = \frac{5}{\sqrt{15}}$ and $y = -\frac{1}{\sqrt{15}}$.

65. To use the graphing utility to graph $11x^2 + 4xy + 14y^2 = 21$, we must express *y* in terms of *x*.

$$11x^2 + 4xy + 14y^2 = 21$$
$$14y^2 + 4xy + 11x^2 - 21 = 0$$

Using the quadratic formula, $y = \dfrac{-4x \pm \sqrt{16x^2 - 4(14)(11x^2 - 21)}}{28} = \dfrac{-2x \pm \sqrt{294 - 150x^2}}{14}$

Press [y =] and input $(-2x + \sqrt{\ }(294 - 150x^2))/14$ for $y_1 =$

Input $(-2x - \sqrt{\ }(294 - 150x^2))/14$ for $y_2 =$

Use window dimensions [–1.5, 1.5].5 by [–1.5, 1.5].5

Press [Graph]

Press [2nd] [Draw] and select tangent function. Enter $x = -1$. We see the equation of the line tangent at (–1, 1) is approximately $y = .75x + 1.75$. To find the horizontal tangents.

$$22x + 4x\left(\frac{dy}{dx}\right) + 4y + 28y\left(\frac{dy}{dx}\right) = 0$$

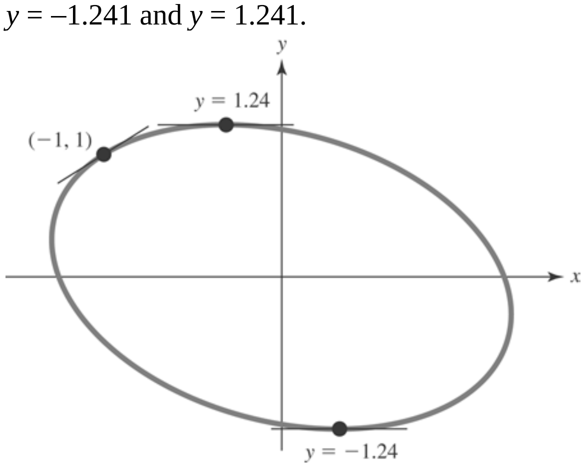
$$\frac{dy}{dx} = \frac{-22x - 4y}{4x + 28y}$$

$$\frac{dy}{dx} = 0 \text{ when } y = -\frac{11}{2}x$$

$$11x^2 + 4x11x^2 + 4x\left(-\frac{11}{2}x\right) + 14\left(-\frac{11}{2}x\right)^2 = 21$$

Solving yields $x = \pm 0.226$ and $y = \mp 1.241$.

The two horizontal tangents are at $y = -1.241$ and $y = 1.241$.



66. (a)

$$x^3 + y^3 = 3xy$$

$$3x^2 + 3y^2\frac{dy}{dx} = 3x\frac{dy}{dx} + 3y$$

$$\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$$

A point where the tangent is horizontal must satisfy $\frac{dy}{dx} = 0$ or $y = x^2$. Substituting into the equation of the curve gives

$$x^3+(x^2)^3=3x(x^2)$$
$$x^6-2x^3=x^3(x^3-2)=0$$

yielding $x=0$ and $x=\sqrt[3]{2}$. If $x=0$, then $y=0$ and the derivative is undefined. When $x=\sqrt[3]{2}$, then $y=\sqrt[3]{4}$ and so the equation of the horizontal tangent is $y=\sqrt[3]{4}$.

(b) Substituting $y=x$ into the equation of the curve gives

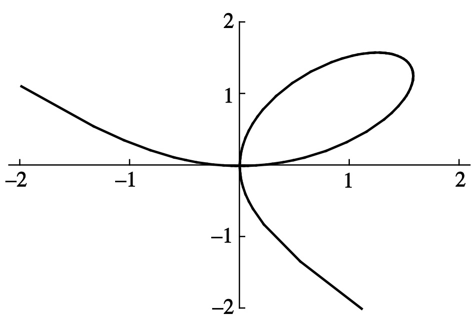
$$x^3+x^3=3x^2$$
$$2x^3-3x^2=x^2(2x-3)$$

so $x=\dfrac{3}{2}$ and $y=\dfrac{3}{2}$. The slope at this point is

$$\frac{\frac{3}{2}-\left(\frac{3}{2}\right)^2}{\left(\frac{3}{2}\right)^2-\frac{3}{2}}=-1$$

and the equation of the tangent line is $y=-x+3$.

(c)



67. To use the graphing utility to graph curve $x^2+y^2=\sqrt{x^2+y^2}+x$,

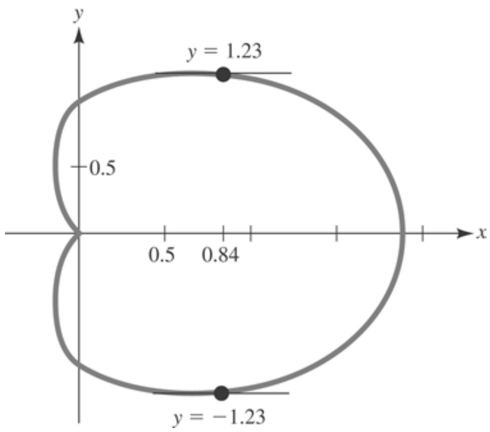
it is best to graph $x^2+y^2=\sqrt{x^2+y^2}+x$ using polar coordinates.

Given that $r^2=x^2+y^2$ and $r\cos\theta=x$, we change the equation to

$$r^2=r+r\cos\theta$$
$$r(r-1-\cos\theta)=0$$

$r=0$ gives the origin and thus, we graph
$r=1+\cos\theta$ using the graphing utility.
Press 2nd format and select Polar Gc
Press mode and select Pol
Pres $y=$ and input $1+\cos\theta$
In the viewing window, use θmin = 0,
θmax = 2π, θstep = $\pi/24$ and dimensions
[–1, 2]0.5 by [–1.5, 1.5]0.5
Using trace and zoom, it appears that a horizontal tangent is approximately

$y = \pm 1.23$.



Checkup for Chapter 2

1. (a) $y = 3x^4 - 4\sqrt{x} + \dfrac{5}{x^2} - 7$

$$y = 3x^4 - 4x^{1/2} + 5x^{-2} - 7$$
$$\frac{dy}{dx} = 12x^3 - 2x^{-1/2} - 10x^{-3} - 0$$
$$\frac{dy}{dx} = 12x^3 - \frac{2}{\sqrt{x}} - \frac{10}{x^3}$$

(b) $y = (3x^3 - x + 1)(4 - x^2)$

$$\frac{dy}{dx} = (3x^3 - x + 1)(-2x) + (4 - x^2)(9x^2 - 1)$$
$$\frac{dy}{dx}$$
$$= -6x^4 + 2x^2 - 2x + 36x^2 - 9x^4 - 4 + x^2$$
$$\frac{dy}{dx} = -15x^4 + 39x^2 - 2x - 4$$

(c) $y = \dfrac{5x^2 - 3x + 2}{1 - 2x}$

$$\frac{dy}{dx} = \frac{(1-2x)(10x-3) - (5x^2 - 3x + 2)(-2)}{(1-2x)^2}$$
$$\frac{dy}{dx} = \frac{10x - 20x^2 - 3 + 6x + 10x^2 - 6x + 4}{(1-2x)^2}$$
$$\frac{dy}{dx} = \frac{-10x^2 + 10x + 1}{(1-2x)^2}$$

(d) $y=(3-4x+3x^2)^{3/2}$

$\frac{dy}{dx}=\frac{3}{2}(3-4x+3x^2)^{1/2}(-4+6x)$

$\frac{dy}{dx}=(9x-6)(3-4x+3x^2)^{1/2}$

2. $f(t)=t(2t+1)^2$

$f'(t)=t\cdot 2(2t+1)(2)+(2t+1)^2(1)$

$f'(t)=(2t+1)(4t+2t+1)$

$f'(t)=(2t+1)(6t+1)=12t^2+8t+1$

$f''(t)=24t+8$

3. $y=x^2-2x+1$

Slope $=\frac{dy}{dx}=2x-2$

When $x=-1$, $y=(-1)^2-2(-1)+1=4$ and $\frac{dy}{dx}=2(-1)-2=-4.$ The equation of the tangent line is

$y-4=-4(x+1)$, or

$y=-4x$.

4. $f(x)=\frac{x+1}{1-5x}$

$f'(x)=\frac{(1-5x)(1)-(x+1)(-5)}{(1-5x)^2}$

$f'(x)=\frac{1-5x+5x+5}{(1-5x)^2}=\frac{6}{(1-5x)^2}$

$f'(1)=\frac{6}{(1-5)^2}=\frac{3}{8}$

5. $T(x)=3x^2+40x+1800$

(a) $T'(x)=6x+40$

In 2013, $x=3$ and $T'(3)=6(3)+40=\$58$ per year.

(b) Need $100\frac{T'(3)}{T(3)}$

$T(3)=3(3)^2+40(3)+1800=1947$

$100\frac{T'(3)}{T(3)}=100\frac{58}{1947}\approx 2.98\%$ per year

6. $s(t)=2t^3-3t^2+2,\ t\geq 0$

(a) $v(t) = s'(t) = 6t^2 - 6t$
$a(t) = s''(t) = 12t - 6$

(b) When stationary, $v(t) = 0$
$6t^2 - 6t = 0$
$6t(t - 1) = 0$, or $t = 0, 1$
When retreating, $v(t) < 0$
$6t^2 - 6t < 0$
$6t(t - 1) < 0$, or $0 < t < 1$
When retreating, $v(t) > 0$
$6t^2 - 6t > 0$
$6t(t - 1) > 0$, or $t > 1$

(c) $|s(1) - s(0)| + |s(2) - s(1)| = 1 + 5 = 6$

7. $C(x) = 0.04x^2 + 5x + 73$
$C'(x) = 0.08x + 5$
$C'(4.09) = 0.08(4.09) + 5 = 5.3272$
From the marginal cost, $\Delta C \approx C'(x_0)\Delta x$
$C'(4.09)(0.01) = 0.053272$ thousand dollars The actual cost is
$C(4.10) - C(4.09)$
$= 94.1724 - 94.119124$
$= 0.053276$ thousand dollars

8. $Q = 500L^{3/4}$
$\Delta Q \approx Q'(L)\Delta L$
$Q'(L) = 375L^{-1/4} = \dfrac{375}{L^{1/4}}$
$Q'(2401) = \dfrac{375}{(2401)^{1/4}} = \dfrac{375}{7}$
Since $\Delta L = 200$, $\Delta Q \approx \dfrac{375}{7}(200) = \dfrac{75,000}{7}$, or an increase of approximately 10,714.29 units.

9. $S = 0.2029w^{0.425}$
$\dfrac{dS}{dt} = (0.2029)(0.425)w^{-0.575}\dfrac{dw}{dt}$
$= \dfrac{(0.2029)(0.425)}{(30)^{0.575}}(0.13)$
$\approx 0.001586,$
or an increasing at a rate of $0.001586\ \text{m}^2$ per week.

10. a. $V(r) = \frac{4}{3}\pi r^3$

$V'(r) = 4\pi r^2$

$$V'(0.75) = 4\pi(0.75)^2 = 2.25\pi \approx 7.069 \text{ cm}^3 \text{ per cm}$$

(b) $V = \frac{4}{3}\pi r^3$

Want $100\frac{\Delta V}{V} \le 8,$ where $\Delta V \approx V'(r)\Delta r,$ $V'(r) = 4\pi r^2$ and

$\Delta r = a \cdot r$, where a represents the % error in the measure of r (as a decimal).

$$100\frac{\Delta V}{V} \le 8$$

$$100\frac{4\pi r^2 \cdot ar}{\frac{4}{3}\pi r^3} \le 8$$

$$100a \le \frac{8}{3}$$

or $\frac{8}{3}\%$ error in the measurement of r.

Review Exercises

1. $f(x) = x^2 - 3x + 1$

$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{[(x+h)^2 - 3(x+h) + 1] - (x^2 - 3x + 1)}{h}$$
$$= \frac{x^2 + 2xh + h^2 - 3x - 3h + 1 - x^2 + 3x + 1}{h}$$
$$= \frac{2xh + h^2 - 3h}{h}$$
$$= 2x + h - 3$$
$$f'(x) = \lim_{h \to 0} 2x + h - 3 = 2x - 3$$

2. $f(x) = \frac{1}{x-2}$

As $h \to 0$ this difference quotient approaches $\frac{-1}{(x-2)^2}$, so $f'(x) = \frac{-1}{(x-2)^2}$

3. $f(x) = 6x^4 - 7x^3 + 2x + \sqrt{2}$

$f'(x) = 24x^3 - 21x^2 + 2$

4. $f(x)=x^3-\dfrac{1}{3x^5}+2\sqrt{x}-\dfrac{3}{x}+\dfrac{1-2x}{x^3}$

$=x^3-\dfrac{1}{3}x^{-5}+2x^{1/2}-3x^{-1}+x^{-3}-2x^{-2}$

$f'(x)=3x^2+\dfrac{5}{3}x^{-6}+x^{-1/2}+3x^{-2}-3x^{-4}+4x^{-3}$

$=3x^2+\dfrac{5}{3x^6}+\dfrac{1}{\sqrt{x}}+\dfrac{3}{x^2}-\dfrac{3}{x^4}+\dfrac{4}{x^3}$

5. $y=\dfrac{2-x^2}{3x^2+1}$

$\dfrac{dy}{dx}=\dfrac{(3x^2+1)(-2x)-(2-x^2)(6x)}{(3x^2+1)^2}$

$=\dfrac{-14x}{(3x^2+1)^2}$

6. $y=(x^3+2x-7)(3+x-x^2)$

$\dfrac{dy}{dx}=(x^3+2x-7)(1-2x)+(3+x-x^2)(3x^2+2)$

$=-5x^4+4x^3+3x^2+18x-1$

7. $f(x)=(5x^4-3x^2+2x+1)^{10}$

$f'(x)$

$=10(5x^4-3x^2+2x+1)^9(20x^3-6x+2)$

8. $f(x)=\sqrt{x^2+1}=\left(x^2+1\right)^{1/2}$

$f'(x)=\dfrac{1}{2}\left(x^2+1\right)^{-1/2}(2x)=\dfrac{x}{\sqrt{x^2+1}}$

9. $y=\left(x+\dfrac{1}{x}\right)^2-\dfrac{5}{\sqrt{3x}}$

$=(x+x^{-1})^2-\dfrac{5}{\sqrt{3}}x^{-1/2}$

$\dfrac{dy}{dx}=2(x+x^{-1})(1-x^{-2})+\dfrac{5}{2\sqrt{3}}x^{-3/2}$

$=2\left(x+\dfrac{1}{x}\right)\left(1-\dfrac{1}{x^2}\right)+\dfrac{5}{2\sqrt{3}x^{3/2}}$

10. $y=\left(\frac{x+1}{1-x}\right)^2$

$$\frac{dy}{dx}=2\left(\frac{x+1}{1-x}\right)\frac{d}{dx}\left(\frac{x+1}{1-x}\right)$$
$$=2\,\frac{x+1}{1-x}\,\frac{(1-x)-(x+1)(-1)}{(1-x)^2}$$
$$=2\,\frac{x+1}{1-x}\,\frac{2}{(1-x)^2}$$
$$=\frac{4(x+1)}{(1-x)^3}$$

11. $f(x)=(3x+1)\sqrt{6x+5}$

$$=(3x+1)(6x+5)^{1/2}$$
$$f'(x)=(3x+1)\left(\frac{1}{2}\right)(6x+5)^{1/2}(6)$$
$$+(6x+5)^{1/2}(3)$$
$$=\frac{3(3x+1)}{(6x+5)^{1/2}}+3(6x+5)^{1/2}$$
$$=\frac{3(3x+1)}{\sqrt{6x+5}}+3\sqrt{6x+5}$$

12. $f(x)=\frac{(3x+1)^3}{(1-3x)^4}$

$$f'(x)=\frac{1}{(1-3x)^8}$$
$$\left[(1-3x)^4\frac{d}{dx}(3x+1)^3-(3x+1)^3\frac{d}{dx}(1-3x)^4\right]$$
$$=\frac{1}{(1-3x)^8}\{(1-3x)^4[3(3x+1)^2(3)]-(3x+1)^3[4(1-3x)^3(-3)]\}$$
$$=\frac{3(3x+1)^2(3x+7)}{(1-3x)^5}$$

13. $y=\sqrt{\dfrac{1-2x}{3x+2}}=\left(\dfrac{1-2x}{3x+2}\right)^{1/2}$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{1-2x}{3x+2}\right)^{-1/2}\cdot\frac{(3x+2)(-2)-(1-2x)(3)}{(3x+2)^2}$$

$$=\frac{1}{2}\frac{(3x+2)^{1/2}}{(1-2x)^{1/2}}\cdot\frac{-7}{(3x+2)^2}$$

$$=\frac{-7}{2\sqrt{1-2x}\,(3x+2)^{3/2}}$$

14. $f(x)=x^2-3x+2$

$f'(x)=2x-3$

$f(1)=0.$ The slope of the tangent line at $(1,0)$ is $m=f'(1)=-1.$ The equation of the tangent line is $y-0=-(x-1)$ or $y=-x+1$

15. $f(x)=\dfrac{4}{x-3}$

$f'(x)=\dfrac{-4}{(x-3)^2}$

$f(1)=-2$

The slope of the tangent line at $(1,-2)$ is $f'(1)=-1.$ The equation of the tangent line is $y+2=-(x-1)$, or $y=-x-1$.

16. $f(x)=\dfrac{x}{x^2+1}$

$f'(x)=\dfrac{(x^2+1)(1)-x(2x)}{(x^2+1)^2}=\dfrac{1-x^2}{(x^2+1)^2}$

$f(0)=0.$ The slope of the tangent line at $(0,0)$ is $m=f'(0)=1.$ The equation of the tangent line is $y-0=x-0$ or $y=x$

17. $f(x)=\sqrt{x^2+5}=(x^2+5)^{1/2}$

$f'(x)=\dfrac{1}{2}(x^2+5)^{-1/2}(2x)=\dfrac{x}{\sqrt{x^2+5}}$

$f(-2)=3$

The slope of the tangent line at $(-2,3)$ is $f'(-2)=-\dfrac{2}{3}.$ The equation of the tangent line is

$y-3=-\dfrac{2}{3}(x+2),$ or $y=-\dfrac{2}{3}x+\dfrac{5}{3}.$

18. (a) The rate of change of

$$f(t) = t^3 - 4t^2 + 5t\sqrt{t} - 5$$
$$= t^3 - 4t^2 + 5t^{3/2} - 5$$

is $f'(t) = 3t^2 - 8t + \frac{15}{2}t^{1/2}$

at any value of $t \geq 0$ and when $t = 4$, $f'(4) = 48 - 32 + \frac{15}{2}(2) = 31.$

(b) The rate of change of $f(t) = \frac{2t^2 - 5}{1 - 3t}$

is $f'(t) = \frac{(1-3t)4t - (2t^2 - 5)(-3)}{(1-3t)^2}$

$$= \frac{-6t^2 + 4t - 15}{(1-3t)^2}$$

at any value of $t \neq \frac{1}{3}$. When $t = -1$,

$f'(-1) = \frac{-6 - 4 - 15}{4^2} = -\frac{25}{16}$.

19. (a) $f(t) = t^3(t^2 - 1)$, $t = 0$

The rate of change of *f* is

$f'(t) = (t^3)(2t) + (t^2 - 1)(3t^2)$.

When $t = 0$, the rate is

$f'(0) = (0^3)(2 \cdot 0) + (0^2 - 1)(3 \cdot 0^2) = 0.$

(b) $f(t) = (t^2 - 3t + 6)^{1/2}$, $t = 1$

The rate of change of *f* is

$$f'(t) = \frac{1}{2}(t^2 - 3t + 6)^{-1/2}(2t - 3)$$
$$= \frac{2t - 3}{2(t^2 - 3t + 6)^{1/2}}.$$

When $t = 1$, the rate is

$$f'(1) = \frac{2(1) - 3}{2\sqrt{1^2 - 3(1) + 6}} = -\frac{1}{4}.$$

20. (a) $f(t) = t^2 - 3t + \sqrt{t}$; $f(4) = 16 - 12 + 2 = 6$

$$f'(t) = 2t - 3 + \frac{1}{2\sqrt{t}};$$

$$f'(4) = 8 - 3 + \frac{1}{4} = \frac{21}{4}$$

$$100\frac{f'(t)}{f(t)} = 100\frac{\frac{21}{4}}{6} = 87.5$$

The percentage rate of change is 87.5%.

(b) $f(t) = \frac{t}{t-3}$; $f(4) = \frac{4}{4-3} = 4$

$$f'(t) = \frac{(t-3) - t}{(t-3)^2} = -\frac{3}{(t-3)^2};$$

$$f'(4) = -\frac{3}{(4-3)^2} = -3$$

$$100\frac{f'(t)}{f(t)} = 100\frac{-3}{4} = -75$$

The percentage rate of change is –75%.

21. (a) $f(t) = t^2(3 - 2t)^3$

$$f'(t) = t^2 \cdot 3(3 - 2t)^2(-2) + (3 - 2t)^3(2t)$$

$$f'(1) = 1 \cdot 3(3-2)^2(-2) + (3-2)^3(2)$$
$$= -4$$

$$f(1) = 1(3-2)^3 = 1$$

$$100\frac{f'(1)}{f(1)} = 100\frac{-4}{1} = -400\%$$

(b) $f(t) = \frac{1}{t+1} = (t+1)^{-1}$

$$f'(t) = -(t+1)^{-2} = \frac{-1}{(t+1)^2}$$

$$f'(0) = \frac{-1}{(0+1)^2} = -1$$

$$f(0) = \frac{1}{0+1} = 1$$

$$100\frac{f'(0)}{f(0)} = 100\frac{-1}{1} = -100\%$$

22. **(a)** $y = 5u^2 + u - 1,\ u = 3x+1$

$$\frac{dy}{du} = 10u+1,\ \frac{du}{dx} = 3,$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (10u+1)(3) = 3(30x+11)$$

(b) $y = \frac{1}{u^2},\ u = 2x+3,\ \frac{dy}{du} = \frac{-2}{u^3},\ \frac{du}{dx} = 2$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{-4}{(2x+3)^3}$$

23. **(a)** $y = (u+1)^2,\ u = 1-x$

$$\frac{dy}{du} = 2(u+1)(1),\ \frac{du}{dx} = -1$$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{dy}{dx} = 2(u+1)\cdot -1 = -2(u+1)$$

Since $u = 1 - x$,

$$\frac{dy}{dx} = -2[(1-x)+1] = -2(2-x).$$

(b) $y = \frac{1}{\sqrt{u}} = u^{-1/2},\ u = 2x+1$

$$\frac{dy}{du} = -\frac{1}{2}u^{-3/2},\ \frac{du}{dx} = 2$$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = -\frac{1}{2u^{3/2}}\cdot 2 = -\frac{1}{u^{3/2}}$$

$$\frac{dy}{dx} = -\frac{1}{(2x+1)^{3/2}}$$

24. **(a)** $y = u - u^2;\ u = x - 3$

When $x = 0$, $u = -3$.

$$\left.\frac{dy}{dx}\right|_{x=0} = \left.\frac{dy}{du}\right|_{u=-3}\left.\frac{du}{dx}\right|_{x=0} = (1-2u)|_{u=-3}(1)|_{x=0} = 1-2(-3) = 7$$

(b) $y = \left(\frac{u-1}{u+1}\right)^{1/2};\ u = \sqrt{x-1} = (x-1)^{1/2}$

$$\frac{dy}{du} = \frac{1}{2}\left(\frac{u-1}{u+1}\right)^{-1/2}\left(\frac{u+1-(u-1)}{(u+1)^2}\right) = \frac{1}{2}\left(\frac{u+1}{u-1}\right)^{1/2}\left(\frac{2}{(u+1)^2}\right) = \frac{1}{(u-1)^{1/2}(u+1)^{3/2}}$$

$$\frac{du}{dx} = \frac{1}{2}(x-1)^{-1/2}(1) = \frac{1}{2\sqrt{x-1}}$$

When $x = \frac{34}{9}$, $u = \sqrt{\frac{25}{9}} = \frac{5}{3}$.

$$\left.\frac{dy}{dx}\right|_{x=\frac{34}{9}} = \left.\frac{dy}{du}\right|_{u=\frac{5}{3}}\left.\frac{du}{dx}\right|_{x=\frac{34}{9}} = \frac{1}{\left(\frac{5}{3}-1\right)^{1/2}\left(\frac{5}{3}+1\right)^{3/2}}\cdot\frac{1}{2\sqrt{\frac{34}{9}-1}} = \frac{1}{\sqrt{\frac{2}{3}}\left(\frac{8}{3}\right)^{3/2}}\cdot\frac{1}{2\sqrt{\frac{25}{9}}} = \frac{(3^{1/2})(3)^{3/2}(3)}{(2^{1/2})(8^{3/2})(2)(5)} = \frac{27}{320}$$

25. **(a)** $y = u^3 - 4u^2 + 5u + 2, \quad u = x^2 + 1$

$$\frac{dy}{du} = 3u^2 - 8u + 5, \quad \frac{du}{dx} = 2x,$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

When $x = 1$, $u = 2$, and so

$$\frac{dy}{dx} = [3(2^2) - 8(2) + 5][2(1)] = 2.$$

(b) $y = \sqrt{u} = u^{1/2}, \quad u = x^2 + 2x - 4,$

$$\frac{dy}{du} = \frac{1}{2u^{1/2}}, \quad \frac{du}{dx} = 2x + 2,$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

When $x = 2$, $u = 4$, and so

$$\frac{dy}{dx} = \frac{1}{2(4)^{1/2}} \cdot [2(2) + 2] = \frac{3}{2}.$$

26. **(a)** $f(x) = 6x^5 - 4x^3 + 5x^2 - 2x + \dfrac{1}{x}$

$$f'(x) = 30x^4 - 12x^2 + 10x - 2 - \frac{1}{x^2}$$

$$f''(x) = 120x^3 - 24x + 10 + \frac{2}{x^3}$$

(b) $z = \dfrac{2}{1 + x^2} = 2(1 + x^2)^{-1}$

$$\frac{dz}{dx} = -2(1 + x^2)^{-2}(2x) = -\frac{4x}{(1 + x^2)^2}$$

$$\frac{d^2z}{dx^2} = -\frac{(1 + x^2)^2(4) - 4x[2(1 + x^2)(2x)]}{(1 + x^2)^4}$$

$$= -\frac{4(1 + x^2)[(1 + x^2) - 4x^2]}{(1 + x^2)^4}$$

$$= -\frac{4(1 - 3x^2)}{(1 + x^2)^3}$$

$$= \frac{4(3x^2 - 1)}{(1 + x^2)^3}$$

(c) $y = \left(3x^2 + 2\right)^4$

$$\frac{dy}{dx} = 4\left(3x^2 + 2\right)^3(6x) = 24x\left(3x^2 + 2\right)^3$$

$$\frac{d^2y}{dx^2} = 24x\left[\left(3x^2 + 2\right)^2(6x)(3)\right] + \left(3x^2 + 2\right)^3(24)$$

$$= 24\left(3x^2 + 2\right)^2\left(21x^2 + 2\right)$$

27. **(a)** $f(x) = 4x^3 - 3x$

$$f'(x) = 12x^2 - 3$$

$$f''(x) = 24x$$

(b) $f(x) = 2x(x+4)^3$

$$f'(x) = (2x)\cdot 3(x+4)^2(1) + (x+4)^3(2)$$

$$= 2(x+4)^2[3x + (x+4)]$$

$$= 2(x+4)^2(4x+4)$$

$$= 8(x+4)^2(x+1)$$

$f''(x)$

$$= 8[(x+4)^2(1) + (x+1)\cdot 2(x+4)(1)]$$

$$= 8(x+4)[(x+4) + 2(x+1)]$$

$$= 8(x+4)(3x+6)$$

$$= 24(x+4)(x+2)$$

(c) $f(x) = \dfrac{x-1}{(x+1)^2}$

$$f'(x) = \frac{(x+1)^2(1) - (x-1)\cdot 2(x+1)(1)}{[(x+1)^2]^2}$$

$$= \frac{(x+1)[(x+1) - 2(x-1)]}{(x+1)^4}$$

$$= \frac{3-x}{(x+1)^3}$$

$f''(x)$

$$= \frac{(x+1)^3(-1) - (3-x)\cdot 3(x+1)^2(1)}{[(x+1)^3]^2}$$

$$= \frac{(x+1)^2[-(x+1) - 3(3-x)]}{(x+1)^6}$$

$$= \frac{2x-10}{(x+1)^4}$$

$$= \frac{2(x-5)}{(x+1)^4}$$

28. **(a)** $5x + 3y = 12,\ 5+3\frac{dy}{dx}=0,$ or $\frac{dy}{dx}=-\frac{5}{3}.$

(b) $(2x+3y)^5 = x+1,$

$$592x+3y)^4\left(2+3\frac{dy}{dx}\right)=1$$

$$10(2x+3y)^4+15(2x+3y)^4\frac{dy}{dx}=1$$

$$\frac{dy}{dx}=\frac{1-10(2x+3y)^4}{15(2x+3y)^4}$$

29. **(a)** $x^2y=1,\ \ x^2\frac{dy}{dx}+y(2x)=0$

$$\frac{dy}{dx}=-\frac{2xy}{x^2}=-\frac{2y}{x}$$

(b) $(1-2xy^3)^5 = x+4y$

$$5(1-2xy^3)^4\left(-2x\cdot 3y^2\frac{dy}{dx}+y^3\cdot -2\right)$$

$$=1+4\frac{dy}{dx}$$

$$-30xy^2(1-2xy^3)^4\frac{dy}{dx}$$

$$-10y^3(1-2xy^3)^4$$

$$=1+4\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{1+10y^3(1-2xy^3)^4}{-30xy^2(1-2xy^3)^4-4}$$

30. **(a)**

$$xy^3=8$$

$$x\left(3y^2\frac{dy}{dx}\right)+y^3=0$$

$$\text{or } \frac{dy}{dx}=\frac{-y^3}{3xy^2}=-\frac{y}{3x}$$

To find the slope of the tangent line at the point (1, 2), substitute $x=1$ and $y=2$ into the equation for $\frac{dy}{dx}$ to get $m=\frac{dy}{dx}=-\frac{2}{3}$.

(b)

$$x^2y - 2xy^3 + 6 = 2x + 2y$$

$$x^2\frac{dy}{dx} + y(2x) - 2\left[x\left(3y^2\frac{dy}{dx}\right) + y^3(1)\right] = 2 + 2\frac{dy}{dx}$$

$$x^2\frac{dy}{dx} + 2xy - 6xy^2\frac{dy}{dx} - 2y^3 = 2 + 2\frac{dy}{dx}$$

To find the slope of the tangent line at $(0,3)$, substitute $x = 0$ and $y = 3$ into the derivative equation and solve for $\frac{dy}{dx}$ to get

$$-2(3)^3 = 2 + 2\frac{dy}{dx}$$

$$-54 = 2 + 2\frac{dy}{dx}$$

or the slope is $m = \frac{dy}{dx} = -28.$

31. (a) $x^2 + 2y^3 = \frac{3}{xy}$, (1, 1)

$$x^2 + 2y^3 = 3(xy)^{-1}$$

$$2x + 6y^2 \cdot \frac{dy}{dx} = -3(xy)^{-2}\left(x \cdot \frac{dy}{dx} + y \cdot 1\right)$$

$$2x + 6y^2 \cdot \frac{dy}{dx} = \frac{-3\left(x \cdot \frac{dy}{dx} + y\right)}{(xy)^2}$$

When $x = 1$ and $y = 1$

$$2(1) + 6(1)^2 \cdot \frac{dy}{dx} = \frac{-3\left(1 \cdot \frac{dy}{dx} + 1\right)}{(1 \cdot 1)^2}$$

$$2 + 6 \cdot \frac{dy}{dx} = -3 \cdot \frac{dy}{dx} - 3$$

$$9 \cdot \frac{dy}{dx} = -5$$

$$\frac{dy}{dx} = -\frac{5}{9}$$

The slope of the tangent to the curve at (1, 1) is $-\frac{5}{9}$.

(b) $y = \frac{x + y}{x - y}$, (6, 2)

$$\frac{dy}{dx} = \frac{(x-y)\left(1+\frac{dy}{dx}\right)-(x+y)\left(1-\frac{dy}{dx}\right)}{(x-y)^2}$$

$$\frac{dy}{dx} = \frac{x+x\frac{dy}{dx}-y-y\frac{dy}{dx}-\left(x-x\frac{dy}{dx}+y-y\frac{dy}{dx}\right)}{(x-y)^2}$$

$$\frac{dy}{dx} = \frac{2x\frac{dy}{dx}-2y}{(x-y)^2}$$

when $x = 6$ and $y = 2$.

$$\frac{dy}{dx} = \frac{2(6)\frac{dy}{dx}-2(2)}{(6-2)^2} = \frac{12\frac{dy}{dx}-4}{16}$$

$$16\frac{dy}{dx} = 12\frac{dy}{dx}-4$$

$$4\frac{dy}{dx} = -4$$

$$\frac{dy}{dx} = -1$$

The slope of the tangent to the curve at (6, 2) is –1.

32. $4x^2 + y^2 = 1$

$$8x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-8x}{2y} = \frac{-4x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{-4y+4x\frac{dy}{dx}}{y^2}$$

$$= \frac{-4y+4x\left(\frac{-4x}{y}\right)}{y^2}$$

$$= \frac{-4y^2-16x^2}{y^3}$$

33. $3x^2 - 2y^2 = 6,\ \ 6x - 4y\frac{dy}{dx} = 0,$ or $\frac{dy}{dx} = \frac{3x}{2y}.$

$$\frac{d^2y}{dx^2} = \frac{2y(3)-3x\left(2\frac{dy}{dx}\right)}{(2y)^2} = \frac{3y-3x\frac{dy}{dx}}{2y^2}$$

Since $\frac{dy}{dx} = \frac{3x}{2y},\ \ \frac{d^2y}{dx^2} = \frac{3y-3x\left(\frac{3x}{2y}\right)}{2y^2} = \frac{6y^2-9x^2}{4y^3}.$

From the original equation
$$\begin{aligned}6y^2 - 9x^2 &= 3(2y^2 - 3x^2)\\ &= -3(3x^2 - 2y^2)\\ &= -3(6)\\ &= -18\end{aligned}$$

and so $\dfrac{d^2y}{dx^2} = -\dfrac{18}{4y^3} = -\dfrac{9}{2y^3}$.

34. (a) $s(t) = -16t^2 + 160t = 0$ when $t = 0$ and $t = 10$. The projectile leaves the ground at $t = 0$ and returns 10 seconds later.

(b) $\dfrac{ds}{dt} = -32t + 160$, thus
$\dfrac{ds}{dt} = -160$ ft/sec at $t = 10$.

(c) $\dfrac{ds}{dt} = 0$ at $t = 5$ and the maximum height is $s(5) = -16(25) + 160(5) = 400$ ft .

35. $P(t) = -2t^3 + 9t^2 + 8t + 200$

(a) $P'(t) = -6t^2 + 18t + 8$
$P'(3) = -6(3)^2 + 18(3) + 8 = 8$, or increasing at a rate of 8,000 people per year.

(b) Letting R be rate of population growth, where
$$R(t) = P'(t) = -6t^2 + 18t + 8$$
$$R'(t) = -12t + 18$$
$$R'(3) = -12(3) + 18 = -18$$
or the rate of population growth is decreasing at a rate of 18,000 people per year.

36. (a) $s(t) = 2t^3 - 21t^2 + 60t - 25$ for $1 \le t \le 6$.
$v(t) = 6(t^2 - 7t + 10) = 6(t-2)(t-5)$
The positive roots are $t = 2, t = 5$. $v(t) > 0$ for $1 < t < 2$, $5 < t < 6$, so the object advances. For $2 < t < 5$, $v(t) < 0$ so the object retreats.
$a(t) = 6(2t - 7) = 0$ if $t = \dfrac{7}{2}$.
$a(t) > 0$ for $\dfrac{7}{2} < t < 6$ so the object accelerates. For $1 < t < \dfrac{7}{2}$ it decelerates.

(b) $s(1) = 2 - 21 + 60 - 25 = 16$,
$s(2) = 16 - 84 + 120 - 25 = 27$,
$s(5) = 250 - 21(25) + 300 - 25 = 0$,
$s(6) = 432 - 21(36) + 360 - 25 = 11$
$\Delta s = (27 - 16) + (27 - 0) + (11 - 0) = 49$

37. $s(t)=\dfrac{2t+1}{t^2+12}$ for $0\le t\le 4$

(a) $$v(t)=\frac{(t^2+12)(2)-(2t+1)(2t)}{(t^2+12)^2}$$
$$=\frac{-2t^2-2t+24}{(t^2+12)^2}$$
$$=\frac{-2(t+4)(t-3)}{(t^2+12)^2}$$
$$a(t)=\frac{(t^2+12)^2(-4t-2)}{(t^2+12)^4}-\frac{(-2t^2-2t+24)2(t^2+12)(2t)}{(t^2+12)^4}$$
$$=-2(t^2+12)\left[\frac{(t^2+12)(2t+1)}{(t^2+12)^4}+\frac{(-2t^2-2t+24)(2t)}{(t^2+12)^4}\right]$$
$$=\frac{2(2t^3+3t^2-72t-12)}{(t^2+12)^3}$$

Now, for $0\le t\le 4$, $v(t)=0$ when $t=3$ and $a(t)\ne 0$.
When $0\le t<3$, $v(t)>0$ and $a(t)<0$, so the object is advancing and decelerating.
When $3<t\le 4$, $v(t)<0$ and $a(t)<0$, so the object is retreating and decelerating.

(b) The distance for $0<t<3$ is $|s(3)-s(0)|=\left|\dfrac{1}{3}-\dfrac{1}{12}\right|=\dfrac{1}{4}$.

The distance for $3<t<4$ is $|s(4)-s(3)|=\left|\dfrac{9}{28}-\dfrac{1}{3}\right|=\dfrac{1}{84}$.

So, the total distance travelled is $\dfrac{1}{4}+\dfrac{1}{84}=\dfrac{22}{84}=\dfrac{11}{42}$.

38. **(a)** Since $N(x)=6x^3+500x+8{,}000$
is the number of people using the system after x weeks, the rate at which use of the system is changing after x weeks is
$N'(x)=18x^2+500$ people per week and the rate after 8 weeks is
$N'(8)=1{,}652$ people per week.

(b) The actual increase in the use of the system during the 8^{th} week is $N(8)-N(7)=1{,}514$ people.

39. **(a)** $Q(x) = 50x^2 + 9{,}000x$

$\Delta Q \approx Q'(x) = 100x + 9{,}000$

$Q'(30) = 12{,}000,$ or an increase of 12,000 units.

(b) The actual increase in output is
$Q(31) - Q(30) = 12{,}050$ units.

40. Since the population in t months will be $P(t) = 3t + 5t^{3/2} + 6{,}000,$ the rate of change of the population will be $P'(t) = 3 + \frac{15}{2}t^{1/2},$ and the percentage rate of change 4 months from now will be

$$100\frac{P'(4)}{P(4)} = 100\frac{18}{6{,}052}$$
$$\approx 0.30\% \text{ per month.}$$

41. $Q(L) = 20{,}000L^{1/2}$

$\Delta Q \approx Q'(L)\Delta L$

$Q'(L) = 10{,}000L^{-1/2} = \dfrac{10{,}000}{\sqrt{L}}$

$Q'(900) = \dfrac{10{,}000}{\sqrt{900}} = \dfrac{1{,}000}{3}$

Since L will decrease to 885,
$\Delta L = 885 - 900 = -15$

$\Delta Q \approx \left(\dfrac{1{,}000}{3}\right)(-15) = -5{,}000,$ or a decrease in output of 5,000 units.

42. The gross domestic product t years after 2004 is $N(t) = t^2 + 6t + 300$ billion dollars.
The derivative is $N'(t) = 2t + 6$
At the beginning of the second quarter of 2012, $t = 8.25.$
The change in t during this quarter is $h = 0.25.$ Hence the percentage change in N is

$$100\frac{N'(8.25)h}{N(8.25)} = 100\frac{[2(8.25)+6](0.25)}{8.25^2 + 6(8.25) + 300} \approx 1.347\%$$

43. Let A be the level of air pollution and p be the population.

$A = kp^2,$ where k is a constant of proportionality

$\Delta A \approx A'(p)\Delta p$

$A'(p) = 2kp$ and $\Delta p = .05p$, so $\Delta A \approx (2kp)(0.05p) = 0.1kp^2 = 0.1A,$ or a 10% increase in air pollution.

44. $C(t) = -170.36t^3 + 1{,}707.5t^2 + 1{,}998.4t + 4{,}404.8$

(a) $C'(t) = -511.08t^2 + 3{,}415t + 1{,}998.4$

$C'(t)$ represents the rate of change in the number of cases of AIDS at time t in units of reported cases per year.

(b) $C'(0)=1{,}998.4$. The epidemic was spreading at the rate of approximately 1,998 cases per year in 1984.

(c) The percentage rate of change in 1984 ($t=0$) was $100\dfrac{C'(0)}{C(0)}=100\left(\dfrac{1{,}998.4}{4{,}404.8}\right)\approx 45.4\%$.

The percentage rate of change in 1990 ($t=6$) was $100\dfrac{C'(6)}{C(6)}=100\left(\dfrac{4{,}089.52}{41{,}067.44}\right)\approx 9.96\%$.

45. $D=36m^{-1.14}$

(a) $D=36(70)^{-1.14}\approx 0.2837$ individuals per square kilometer.

(b) $(0.2837 \text{ individuals/km}^2)$
$(9.2\times 10^6)\text{km}^2$
≈ 2.61 million people

(c) The ideal population density would be $36(30)^{-1.14}\approx 0.7454$ animals/km^2.

Since the area of the island is $3{,}000\text{ km}^2$, the number of animals on the island for the ideal population density would be $(0.7454 \text{ animals/km}^2)(3{,}000\text{ km}^2)$
$\approx 2{,}235$ animals

Since the animal population is given by $P(t)=0.43t^2+13.37t+200$, this population is reached when

$$2236=0.43t^2+13.37t+200$$
$$0=0.43t^2+13.37t-2036$$

or, using the quadratic formula, when
$t\approx 55$ years. The rate the population is changing at this time is $P'(55)$, where
$P'(t)=0.86t+13.37$, or
0.86(55) + 13.37 = 60.67 animals per year.

46. $P(t)=1.035t^3+103.5t^2+6{,}900t+230{,}000$

(a) $P'(t)=3.105t^2+207t+6{,}900$. $P'(t)$ represents the rate of change of the population, in bacteria per day, after *t* days.

(b) After 1 day the population is changing at $P'(1)=7{,}110.105$ or about 7,110 bacteria per day. After 10 days the population is changing at $P'(10)=9{,}280.5$ or about 9,281 bacteria per day.

(c) The initial bacterial population is $P(0)=230{,}000$ bacteria. The population has doubled when $P(t)=2(230{,}000)=460{,}000$ or $1.035t^3+103.5t^2+6{,}900t-230{,}000=0$.
Using the solving features of a graphing calculator yields $t\approx 23.3$ days as the approximate time until the population doubles. At that time the rate of change is $P'(23.3)=13{,}409$ bacteria per day.

47. Need $100\frac{\Delta L}{L}$, given that $100\frac{\Delta Q}{Q}=1\%$, where $\Delta Q \approx Q'(L)\Delta L$. Since, $100\frac{Q'(L)\Delta L}{Q(L)}=1$, solving for ΔL yields

$\Delta L = \frac{Q(L)}{100Q'(L)}$ and $100\frac{\Delta L}{L}=100\frac{\frac{Q(L)}{100Q'(L)}}{L}=\frac{Q(L)}{Q'(L)\cdot L}$.

Since $Q(L)=600L^{2/3}$, $Q'(L)=400L^{-1/3}=\frac{400}{L^{1/3}}$

$100\frac{\Delta L}{L}=\frac{600L^{2/3}}{\left(\frac{400}{L^{1/3}}\right)(L)}=\frac{3}{2}$, or 1.5%

Increase labor by approximately 1.5%.

48. By the approximation formula, $\Delta y \approx \frac{dy}{dx}\Delta x$

To find $\frac{dy}{dx}$ differentiate the equation $Q = x^3+2xy^2+2y^3$ implicitly with respect to *x*. Since *Q* is to be held constant in this analysis, $\frac{dQ}{dx}=0$. Thus

$0=3x^2+4xy\frac{dy}{dx}+2y^2+6y^2\frac{dy}{dx}$

or $\frac{dy}{dx}=-\frac{3x^2+2y^2}{4xy+6y^2}$

At $x=10$ and $y=20$

$\frac{dy}{dx}=-\frac{3(10)^2+2(20)^2}{4(10)(20)+6(20)^2}\approx -0.344$

Use the approximation formula with

$\frac{dy}{dx}\approx -0.344$ and $\Delta x = 0.5$ to get $\Delta y = -0.344(0.5) = -0.172$ unit.

That is, to maintain the current level of output, input *y* should be decreased by approximately 0.172 unit to offset a 0.5 unit increase in input *x*.

49. Need $\Delta A \approx A'(r)\Delta r$

Since $A=\pi r^2$, $A'(r)=2\pi r$.

When r = 12, $A'(12)=2\pi(12)=24\pi$.

Since $\Delta r = \pm 0.03r$,

$\Delta r = \pm 0.03(12) = \pm 0.36$ and

$\Delta A \approx (24\pi)(\pm 0.36) \approx \pm 27.14 \text{ cm}^2$.

When r = 12, $A=\pi(12)^2=144\pi\approx 452.39$ square centimeters. The calculation of area is off by ± 27.14 at most, or $\frac{27.14}{452.39}\approx 0.06$ or 6% and $425.25 \le A \le 479.53$

50. $V = x^3$ and $dV = 3x^2 dx$, $\frac{dV}{V} = \frac{3x^2 dx}{x^3} = 3\frac{dx}{x} = 0.06$ or 6%

51. $Q = 600K^{1/2}L^{1/3}$

Need $100\frac{\Delta Q}{Q}$, where $\Delta Q \approx Q'(L)\Delta L$.

Treating K as a constant $Q'(L) = 200K^{1/2}L^{-2/3} = \frac{200K^{1/2}}{L^{2/3}}$ with

$\Delta L = 0.02L$.

$$100\frac{\Delta Q}{Q} = 100\frac{\left(\frac{200K^{1/2}}{L^{2/3}}\right)(0.02L)}{600K^{1/2}L^{1/3}} \approx 0.67\%$$

52. $S(R) = 1.8\left(10^5\right)R^2$

$R = 1.2\left(10^{-2}\right)$

$\Delta R = \pm 5\left(10^{-4}\right)$

$$\Delta S = S\left[1.2\left(10^{-2}\right) \pm 5\left(10^{-4}\right)\right] - S\left[1.2\left(10^{-2}\right)\right]$$
$$\approx S'\left[1.2\left(10^{-2}\right)\right]\left[\pm 5\left(10^{-4}\right)\right]$$

$S'(R) = 3.6\left(10^5\right)R$

$$S'\left[1.2\left(10^{-2}\right)\right] = \left[3.6\left(10^5\right)\right]\left[1.2\left(10^{-2}\right)\right]$$
$$= 4.32\left(10^3\right)$$

$$\Delta S \approx \left[4.3\left(10^3\right)\right]\left[\pm 5\left(10^{-4}\right)\right]$$
$$= \pm 2.15 \text{ cm/sec.}$$

53. The error in the calculation of the tumor's surface area, due to the error in measuring its radius is

$$\Delta S \approx S'(r)\Delta r$$
$$= 8\pi r(\Delta r)$$

Since $3\%r = 0.03r = 0.03(1.2) = 0.036$

$= 8\pi(1.2)(\pm 0.036)$

$= \pm 0.3456\pi$

The calculated surface area is

$S = 4\pi(1.2)^2 = 5.76\pi$

The true surface area is between

$S + \Delta S = 5.76\pi \pm 0.3456\pi$, or

$17.01 \le S \le 19.18$

The measurement is accurate within

$$\frac{0.3456\pi}{5.76\pi} = 0.06, \text{ or } 6\%$$

54. $V(t) = [C_1 + C_2P(t)]\left(\frac{3t^2}{T^2} - \frac{2t^3}{T^3}\right)$

$$\frac{dV}{dt} = [C_1 + C_2P(t)]\left(\frac{6t}{T^2} - \frac{6t^2}{T^3}\right) + C_2\left(\frac{3t^2}{T^2} - \frac{2t^3}{T^3}\right)\frac{dP}{dt}.$$

55. $75x^2 + 17p^2 = 5{,}300$

$$150x\frac{dx}{dt} + 34p\frac{dp}{dt} = 0$$

$$\frac{dx}{dt} = \frac{-34p\frac{dp}{dt}}{150x}$$

When $p = 7$, $75x^2 + 17(7)^2 = 5{,}300$ or $x \approx 7.717513$.

So, $\frac{dx}{dt} = \frac{-34(7)(-0.75)}{150(7.717513)}$

≈ 0.15419 hundred, or

≈ 15.1419 units/month.

56. At t hours past noon, the truck is $70t$ km north of the intersection while the car is $105(t-1)$ km east of the intersection. The distance between them is then

$$D(t) = \sqrt{(70t)^2 + [105(t-1)]^2} = \sqrt{4900t^2 + 11025(t-1)^2}.$$

The rate of change of the distance is

$$D'(t) = \frac{9800t + 22050(t-1)}{2\sqrt{4900t^2 + 11025(t-1)^2}}.$$

At 2 P.M., $t = 2$ and

$$D'(2) = \frac{9800(2) + 22050(1)}{2\sqrt{4900(4) + 11025(1)^2}} = 119 \text{ km/hr}$$

57. $P(t) = 20 - \frac{6}{t+1} = 20 - 6(t+1)^{-1}$

Need $100\frac{\Delta P}{P}$, where $\Delta P \approx P'(t)\Delta t$

$$P'(t) = 6(t+1)^{-2}(1) = \frac{6}{(t+1)^2}$$

The next quarter year is from $t = 0$ to $t = \frac{1}{4}$, so $P(0) = 14$, $P'(0) = 6$ and $\Delta t = \frac{1}{4}$.

$$100\frac{\Delta P}{P} = 100\frac{(6)\left(\frac{1}{4}\right)}{14} \approx 10.7\%$$

58. $Q(t) = -t^3 + 9t^2 + 12t$, where 8 A.M. corresponds to $t = 0$.

(a) $R(t) = Q'(t) = -3t^2 + 18t + 12$

(b) The rate at which the rate of production is changing is given by $R'(t) = Q''(t) = -6t + 18$

At 9 A.M., $t = 1$, and $R'(1) = 12$ units per hour per hour.

(c) From 9:00 A.M. to 9:06 A.M. the change in time is 6 minutes or

$\Delta t = \frac{1}{10}$ hour.

The change in the rate of production is approximated as

$\Delta R = R'(1)\Delta t = 12\left(\frac{1}{10}\right) = 1.2$ units per hour.

(d) The actual change, estimated in part (c), is

$$\begin{aligned} &R(1.1) - R(1) \\ &= Q'(1.1) - Q'(1) \\ &= (-3(1.1)^2 + 18(1.1) + 12) - (-3 + 18 + 12) \\ &= 1.17 \text{ units per hour} \end{aligned}$$

59. $s(t) = 88t - 8t^2$

$v(t) = s'(t) = 88 - 16t$

The car is stopped when $v(t) = 0$, so $0 = 88 - 16t$, or $t = 5.5$ seconds.

The distance travelled until it stops is

$s(5.5) = 88(5.5) - 8(5.5)^2 = 242$ feet.

60. (a) $S(t) = 50\left(1 - \frac{t^2}{15}\right)^3$

$S(0) = 50$ lbs.

(b) $S'(t) = 50(3)\left(1 - \frac{t^2}{15}\right)^2\left(-\frac{2}{15}\right)t$

$S'(1) = -150\left(1 - \frac{1}{15}\right)^2 \frac{2}{15}$

$= -17.42$ lbs/sec.

(c) The bag will be empty when $S(t) = 0$ at $t = \sqrt{15} = 3.873$ sec. The rate of leakage at that time is $S'(\sqrt{15}) = 0$.

61. $P(t) = -t^3 + 7t^2 + 200t + 300$

(a) $P'(t) = -3t^2 + 14t + 200$

$P'(5) = -3(5)^2 + 14(5) + 200 = 195,$

or increasing at a rate of \$195 per unit per month.

(b) $P''(t) = -6t + 14$

$P''(5) = -6(5) + 14 = -16,$ or

decreasing at a rate of \$16 per unit per month per month.

(c) Need $\Delta P' \approx P''(t)\Delta t$

Now, $P''(5) = -16$ and the first six months of the sixth year corresponds to $\Delta t = \frac{1}{2}.$

$\Delta P' \approx (-16)\left(\frac{1}{2}\right) = -8,$ or a decrease of \$8 per unit per month.

(d) Need $P'(5.5) - P'(5)$

$P'(5.5) = -3(5.5)^2 + 14(5.5) + 200$

$= 186.25$

The actual change in the rate of price increase is 186.25 – 195 = –8.75, or decreasing at a rate of \$8.75 per unit per month.

62. $C(q) = 0.1q^2 + 10q + 400,\ q(t) = t^2 + 50t$

By the chain rule

$\frac{dC}{dt} = \frac{dC}{dq}\frac{dq}{dt} = (0.2q + 10)(2t + 50)$

At $t = 2$, $q = q(t) = 2^2 + 50(2) = 104$ and

$\frac{dC}{dt} = [0.2(104) + 10][2(2) + 50]$

$= 1{,}663.2$ units per hour

63. $C(x) = 0.06x + 3x^{1/2} + 20$ hundred

$\frac{dx}{dt} = -11$ when $x = 2{,}500$

$\frac{dC}{dt} = \frac{dC}{dx} \cdot \frac{dx}{dt}$

$\frac{dC}{dx} = 0.06 + 1.5x^{-1/2} = 0.06 + \frac{1.5}{\sqrt{x}}$

$\frac{dC}{dt} = \left(0.06 + \frac{1.5}{\sqrt{2{,}500}}\right)(-11)$

$= -0.99$ hundred,

or decreasing at a rate of \$99 per month.

64. $S = 4\pi r^2$

$S' = 8\pi r$

$\Delta S \approx 8\pi r\, \Delta r$

The percentage error is

$\frac{100 \times 8\pi r \Delta r}{4\pi r^2} = 200\frac{\Delta r}{r}$

$8 = (2)\left(100\frac{\Delta r}{r}\right)$

$100\frac{\Delta r}{r} = \frac{8}{2} = 4$

The computations assumed a positive percentage error of 8% but –8% could also be used. The percentage error is then $\pm 4\%$.

65. Consider the volume of the shell as a change in volume, where $r = \frac{8.5}{2}$ and

$$\Delta r = \frac{1}{8} = 0.125.$$
$$\Delta V \approx V'(r)\Delta r$$
$$V(r) = \frac{4}{3}\pi r^3$$
$$V'(r) = 4\pi r^2$$
$$V'(4.25) = 4\pi(4.25)^2 = 72.25\pi$$
$$\Delta V = (72.25\pi)(0.125) \approx 28.37 \text{ in}^3$$

66. Let t be the time in hours and s the distance between the car and the truck. Then

$$s = \sqrt{(60t)^2 + (45t)^2}$$
$$= \sqrt{3,600t^2 + 2,025t^2} = 75t$$

and so $\frac{ds}{dt} = 75$ mph **.**

67. Let the length of string be the hypotenuse of the right triangle formed by the horizontal and vertical distance of the kite from the child's hand. Then,

$$s = x^2 + (80)^2$$
$$2s\frac{ds}{dt} = 2x\frac{dx}{dt}$$
$$\frac{ds}{dt} = \frac{2x\frac{dx}{dt}}{2s} = \frac{x\frac{dx}{dt}}{s}$$

When $s = 100$, $(100)^2 = x^2 + (80)^2$, or $x = 60$

$\frac{ds}{dt} = \frac{(60)(5)}{100} = 3$, or increasing at a rate of 3 feet per second.

68. We have a right triangle with legs 8 and x, the distance from the buoy to the pier, and hypotenuse y, the length of the rope. Thus $y^2 = x^2 + 8^2$ and through implicit differentiation,

$$2xx' = 2yy' \text{ or } x' = \frac{yy'}{x}$$

We know $y' = -2$ and at the moment in question, $x = 6$, so the rope is $y = 10$ ft long. Thus $x' = \frac{10(-2)}{6} = -\frac{10}{3}$. The buoy is approaching the pier at roughly 3.33 feet per minute.

69. Need $\frac{dx}{dt}$.

$$x^2 + y^2 = (10)^2$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$
$$\frac{dx}{dt} = \frac{-2y\frac{dy}{dt}}{2x} = \frac{-y\frac{dy}{dt}}{x}$$

When $y = 6$, $x^2 + 36 = 100$, or $x = 8$.

Since $\frac{dy}{dt} = -3$, $\frac{dx}{dt} = \frac{(-6)(-3)}{8} = 2.25$, or increasing at a rate of 2.25 feet per second.

70. Let x be the distance between the woman and the building, and s the length of the shadow. Since $h(t) = 150 - 16t^2$, the lantern will be 10 ft from the ground when $10 = 150 - 16t^2$ which leads to $t = \frac{1}{4}\sqrt{140}$ seconds.

When $h = 10$ and $x = 5t = \frac{5\sqrt{140}}{4}$ from similar right triangles we get

$$\frac{x}{h-5}=\frac{x+s}{h}$$

$$\frac{5\sqrt{140}}{4(10-5)}=\frac{\frac{5}{4}\sqrt{140}+s}{10}$$

$$\text{or } s=\frac{5\sqrt{140}}{4}$$

$$hx=hx+hs-5x-5s$$

$$hs'+h's=5\frac{dx}{dt}+5s'$$

$$(h-5)s'=5\frac{dx}{dt}-h's$$

$$5s'=5(5)+32\left(\frac{1}{4}\right)\sqrt{140}\left(\frac{5}{4}\sqrt{140}\right)$$

$$s'=5+2(140)=285 \text{ ft/sec}$$

71. Let x be the distance from the player to third base. Then,

$$s^2=x^2+(90)^2$$

$$2s\frac{ds}{dt}=2x\frac{dx}{dt}$$

$$\frac{ds}{dt}=\frac{2x\frac{dx}{dt}}{2s}=\frac{x\frac{dx}{dt}}{s}$$

When $x = 15$, $s^2=(15)^2+(90)^2$, or

$s=\sqrt{8325}.$

$\frac{ds}{dt}=\frac{(15)(-20)}{\sqrt{8325}}\approx -3.29$, or decreasing at a rate of 3.29 feet per second.

72. The total manufacturing cost C is a function of q (where q is the number of units produced) and q is a function of t (where t is the number of hours during which the factory operates). Hence,

(a) $\frac{dC}{dq}=$ the rate of change of cost with respect to the number of units produced in $\frac{\text{dollars}}{\text{unit}}$.

(b) $\frac{dq}{dt}=$ the rate of change of units produced with respect to time in $\frac{\text{units}}{\text{hour}}$.

(c) $\frac{dC}{dq}\frac{dq}{dt}$

$$=\frac{\text{dollars}}{\text{unit}}\,\frac{\text{units}}{\text{hour}}$$

$$=\frac{\text{dollars}}{\text{hour}}$$

= the rate of change of cost WRT time

73. Let x be the distance from point P to the object.

$V = ktx$

When $t = 5$ and $x = 20$, $V = 4$, so

$4 = k(5)(20)$, or $k=\frac{1}{25}$.

Since $a=V'$,

$$a=k\left(t\frac{dx}{dt}+x\cdot 1\right)$$

$$a=\frac{1}{25}(5\cdot 4+20)=\frac{8}{5}\text{ ft/sec}^2$$

74. $y=4x^2$ and $P(2, 0)$

Note that P is not on the graph of the curve (its coordinates do not satisfy the equation of the curve).

$y'=8x$

Let x_t be the abscissa of the point of tangency. The slope is $m=8x_t$

The point (x_t, y_t) lies on the curve through (2, 0) so its slope is

$$\frac{y_t-0}{x_t-2}=8x_t \text{ or } y_t=8x_t{}^2=16x_t$$

The point of contact (tangency) is both on the curve and on the tangent line. Thus
$4x_t^2 = 8x_t^2 - 16x_t$ or $4x_t(x_t - 4) = 0$.
This is satisfied for $x_t = x_1 = 0$ as well as $x_t = x_2 = 4$.
The two points of contact have coordinates (0, 0) and (4, 64).

75. Need $100\dfrac{y'}{y}$ as $x \to \infty$.

$$y = mx + b$$
$$y' = m$$
$$100\frac{y'}{y} = 100\frac{m}{mx+b}$$

As x approaches ∞, this value approaches zero.

76. $$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{2x}{a^2} - \frac{2y\frac{dy}{dx}}{b^2} = 0$$

$$\frac{dy}{dx} = \frac{b^2x}{a^2y}$$

Thus the slope at (x_0, y_0) is $m = \dfrac{b^2x_0}{a^2y_0}$ and the equation of the line becomes

$$y - y_0 = \frac{b^2x_0}{a^2y_0}(x - x_0)$$

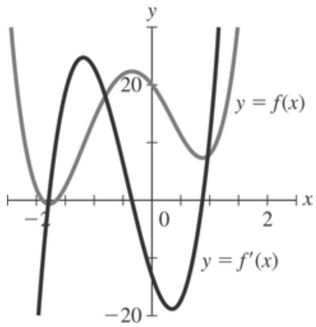
$$\frac{y_0y}{b^2} - \frac{y_0^2}{b^2} = \frac{x_0x}{a^2} - \frac{x_0^2}{a^2}$$

$$\frac{x_0x}{a^2} - \frac{y_0y}{b^2} = \frac{x_0^2}{a^2} - \frac{y_0^2}{b^2} = 1$$

because the point (x_0, y_0) is on the curve.

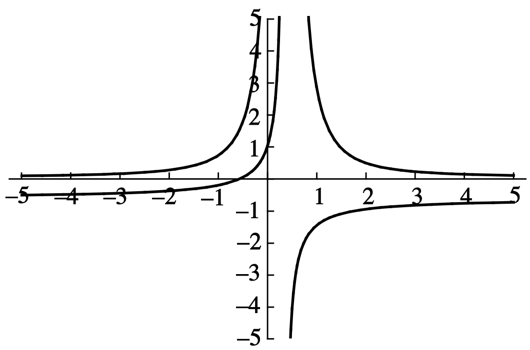
77. To use a graphing utility to graph *f* and f',
Press [y =] and input
(3*x* + 5)(2*x* ^ 3 – 5*x* + 4) for $y_1 =$
$f'(x)$
$= (3x+5)(6x^2 - 5) + (3)(2x^3 - 5x + 4)$
Input $f'(x)$ for $y_2 =$
Use window dimensions [–3, 2]1 by
[–20, 30]10
Use trace and zoom-in to find the *x*-intercepts of $f'(x)$ or use the zero function under the calc menu. In either case, make sure that y_2 is displayed in the upper left corner. The three zeros are

$x \approx -1.78$, $x \approx -0.35$, and $x \approx 0.88$.



78. $f(x) = \dfrac{2x+3}{1-3x}$

$$f'(x) = \frac{(1-3x)2-(2x+3)(-3)}{(1-3x)^2}$$
$$= \frac{11}{(1-3x)^2}$$



It is clear from the graph and the expression for $f'(x)$ that $f'(x)$ is never 0.

79. (a) To graph $y^2(2-x) = x^3$,

$$y^2 = \frac{x^3}{2-x}$$
$$y = \pm\sqrt{\frac{x^3}{2-x}}$$

Press [y =] and input $\sqrt{\ }((x) \div (2-x))$ for $y_1 =$ and input $-y_1$ for $y_2 =$ (you can find y_1 by pressing [vars] and selecting function under the y-vars menu). Use window dimensions