Chapter 2 Polynomial and Rational Functions

Section 2.1 Quadratic Functions and Applications

1. quadratic

2. axis

3. (h, k)

4. downward; maximum; *k*

5. upward; minimum; *k*

6. $x = h$

7. $f(x) = -(x-4)^2 + 1$

$a = -1, h = 4, k = 1$

a. Since $a < 0$, the parabola opens downward.

b. The vertex is $(h, k) = (4, 1)$.

c.

$$\begin{aligned} f(x) &= -(x-4)^2 + 1 \\ 0 &= -(x-4)^2 + 1 \\ -1 &= -(x-4)^2 \\ 1 &= (x-4)^2 \\ \pm\sqrt{1} &= x - 4 \\ 4 \pm 1 &= x \\ x = 3 \quad &\text{or} \quad x = 5 \end{aligned}$$

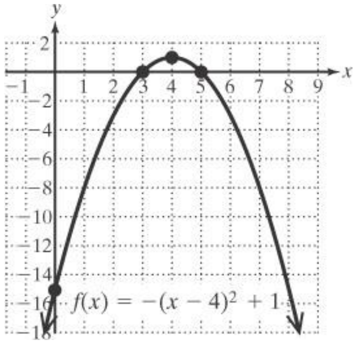
The *x*-intercepts are $(3, 0)$ and $(5, 0)$.

d. $f(0) = -(0-4)^2 + 1 = -(-4)^2 + 1$

$= -16 + 1 = -15$

The *y*-intercept is $(0, -15)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = 4$.

g. The maximum value is 1.

h. The domain is $(-\infty, \infty)$.

The range is $(-\infty, 1]$.

8. $g(x) = -(x+2)^2 + 4 = -[x - (-2)]^2 + 4$

$a = -1, h = -2, k = 4$

a. Since $a < 0$, the parabola opens downward.

b. The vertex is $(h, k) = (-2, 4)$.

c.

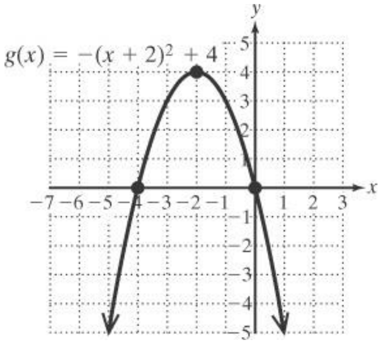
$$\begin{aligned} g(x) &= -(x+2)^2 + 4 \\ 0 &= -(x+2)^2 + 4 \\ -4 &= -(x+2)^2 \\ 4 &= (x+2)^2 \\ \pm\sqrt{2} &= x + 2 \\ -2 \pm 2 &= x \\ x = -4 \quad &\text{or} \quad x = 0 \end{aligned}$$

The *x*-intercepts are $(-4, 0)$ and $(0, 0)$.

d. $g(0) = -(0+2)^2 + 4 = -(2)^2 + 4$
$= -4 + 4 = 0$

The y-intercept is $(0, 0)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = -2$.

g. The maximum value is 4.

h. The domain is $(-\infty, \infty)$.

The range is $(-\infty, 4]$.

9. $h(x) = 2(x+1)^2 - 8$
$= 2[x - (-1)]^2 + (-8)$
$a = 2, h = -1, k = -8$

a. Since $a > 0$, the parabola opens upward.

b. The vertex is $(h, k) = (-1, -8)$.

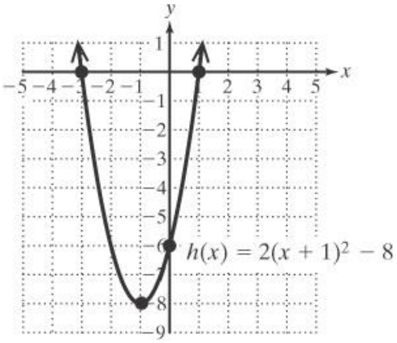
c.
$$\begin{aligned} h(x) &= 2(x+1)^2 - 8 \\ 0 &= 2(x+1)^2 - 8 \\ 8 &= 2(x+1)^2 \\ 4 &= (x+1)^2 \\ \pm\sqrt{4} &= x + 1 \\ -1 \pm 2 &= x \\ x = -3 \quad &\text{or} \quad x = 1 \end{aligned}$$

The x-intercepts are $(-3, 0)$ and $(1, 0)$.

d. $h(0) = 2(0+1)^2 - 8 = 2(1)^2 - 8$
$= 2 - 8 = -6$

The y-intercept is $(0, -6)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = -1$.

g. The minimum value is -8.

h. The domain is $(-\infty, \infty)$.

The range is $[-8, \infty)$.

10. $k(x) = 2(x-3)^2 - 2 = 2(x-3)^2 + (-2)$
$a = 2, h = 3, k = -2$

a. Since $a > 0$, the parabola opens upward.

b. The vertex is $(h, k) = (3, -2)$.

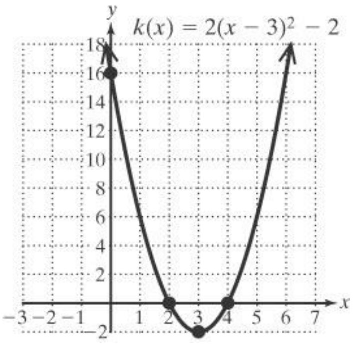
c.
$$\begin{aligned} k(x) &= 2(x-3)^2 - 2 \\ 0 &= 2(x-3)^2 - 2 \\ 2 &= 2(x-3)^2 \\ 1 &= (x-3)^2 \\ \pm\sqrt{1} &= x - 3 \\ 3 \pm 1 &= x \\ x = 2 \quad &\text{or} \quad x = 4 \end{aligned}$$

The x-intercepts are $(2, 0)$ and $(4, 0)$.

d. $k(0) = 2(0-3)^2 - 2 = 2(-3)^2 - 2$
$= 18 - 2 = 16$

The y-intercept is $(0, 16)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = 3$.

g. The minimum value is -2 .

h. The domain is $(-\infty, \infty)$.

The range is $[-2, \infty)$.

11. $m(x) = 3(x-1)^2 = 3(x-1)^2 + 0$

$a = 3,\ h = 1,\ k = 0$

a. Since $a > 0$, the parabola opens upward.

b. The vertex is $(h, k) = (1, 0)$.

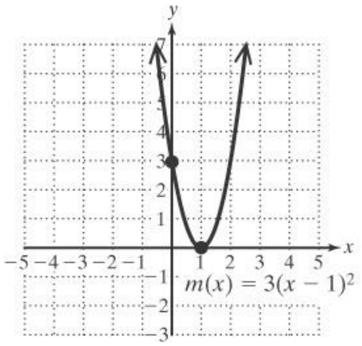
c.
$$\begin{aligned} m(x) &= 3(x-1)^2 \\ 0 &= 3(x-1)^2 \\ 0 &= (x-1)^2 \\ 0 &= (x-1)^2 \\ \pm\sqrt{0} &= x-1 \\ 1 &= x \end{aligned}$$

The x-intercept is $(1, 0)$.

d. $m(0) = 3(0-1)^2 = 3(-1)^2 = 3$

The y-intercept is $(0, 3)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = 1$.

g. The minimum value is 0.

h. The domain is $(-\infty, \infty)$.

The range is $[0, \infty)$.

12. $n(x) = \frac{1}{2}(x+2)^2 = \frac{1}{2}\left[(x-(-2))^2\right] + 0$

$a = \frac{1}{2},\ h = -2,\ k = 0$

a. Since $a > 0$, the parabola opens upward.

b. The vertex is $(h, k) = (-2, 0)$.

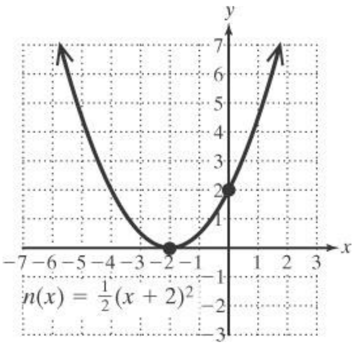
c.
$$\begin{aligned} n(x) &= \frac{1}{2}(x+2)^2 \\ 0 &= \frac{1}{2}(x+2)^2 \\ 2 &= (x+2)^2 \\ \pm\sqrt{0} &= x+2 \\ -2 &= x \end{aligned}$$

The x-intercept is $(-2, 0)$.

d. $n(0) = \frac{1}{2}(0+2)^2 = \frac{1}{2}(2)^2 = 2$

The y-intercept is $(0, 2)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = -2$.

g. The minimum value is 0.

h. The domain is $(-\infty, \infty)$.

The range is $[0, \infty)$.

13. $p(x) = -\frac{1}{5}(x+4)^2 + 1$

$$= -\frac{1}{5}\left[\left(x-(-4)\right)^2\right] + 1$$

$$a = -\frac{1}{5}, h = -4, k = 1$$

a. Since $a < 0$, the parabola opens downward.

b. The vertex is $(h, k) = (-4, 1)$.

c.
$$\begin{aligned} p(x) &= -\frac{1}{5}(x+4)^2 + 1 \\ 0 &= -\frac{1}{5}(x+4)^2 + 1 \\ -1 &= -\frac{1}{5}(x+4)^2 \\ 5 &= (x+4)^2 \\ \pm\sqrt{5} &= x+4 \end{aligned}$$

$$-4+\sqrt{5} = x \quad \text{or} \quad x = -4-\sqrt{5}$$

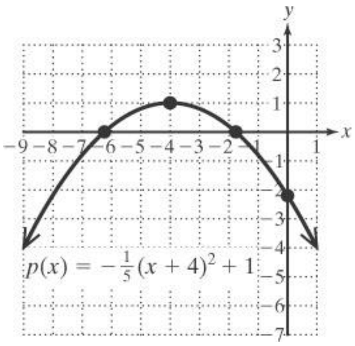
The x-intercepts are $\left(-4-\sqrt{5}, 0\right)$ and $\left(-4+\sqrt{5}, 0\right)$.

d. $p(0) = -\frac{1}{5}(0+4)^2 + 1 = -\frac{1}{5}(4)^2 + 1$

$$= -\frac{16}{5} + \frac{5}{5} = -\frac{11}{5}$$

The y-intercept is $\left(0, -\frac{11}{5}\right)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = -4$.

g. The maximum value is 1.

h. The domain is $(-\infty, \infty)$.

The range is $(-\infty, 1]$.

14. $q(x) = -\frac{1}{3}(x-1)^2 + 1$

$$a = -\frac{1}{3}, h = 1, k = 1$$

a. Since $a < 0$, the parabola opens downward.

b. The vertex is $(h, k) = (1, 1)$.

c.
$$\begin{aligned} q(x) &= -\frac{1}{3}(x-1)^2 + 1 \\ 0 &= -\frac{1}{3}(x-1)^2 + 1 \\ -1 &= -\frac{1}{3}(x-1)^2 \end{aligned}$$

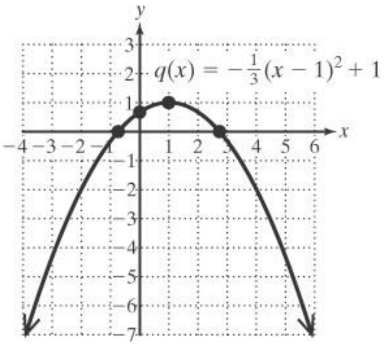
$$3 = (x-1)^2$$
$$\pm\sqrt{3} = x - 1$$
$$1 \pm \sqrt{3} = x$$
$$1 + \sqrt{3} = x \quad \text{or} \quad x = 1 - \sqrt{3}$$

The x-intercepts are $\left(1-\sqrt{3}, 0\right)$ and $\left(1+\sqrt{3}, 0\right)$.

d. $q(0) = -\frac{1}{3}(0-1)^2 + 1 = -\frac{1}{3}(-1)^2 + 1$
$= -\frac{1}{3} + 1 = \frac{2}{3}$

The y-intercept is $\left(0, \frac{2}{3}\right)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = 1$.

g. The maximum value is 1.

h. The domain is $(-\infty, \infty)$.

The range is $(-\infty, 1]$.

15. a. $f(x) = x^2 + 6x + 5$

$$f(x) = \left(x^2 + 6x\right) + 5 \quad \boxed{\left[\frac{1}{2}(6)\right]^2 = 9}$$

$$f(x) = \left(x^2 + 6x + 9\right) - 9 + 5$$
$$f(x) = (x+3)^2 - 4$$

b. The vertex is $(h, k) = (-3, -4)$.

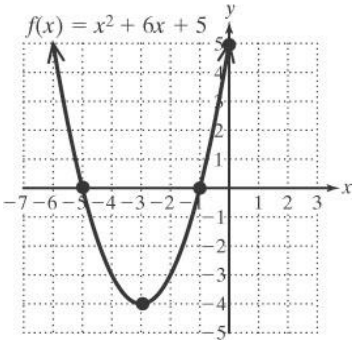
c. $f(x) = x^2 + 6x + 5$
$$0 = x^2 + 6x + 5$$
$$0 = (x+5)(x+1)$$
$$x = -5 \quad \text{or} \quad x = -1$$

The x-intercepts are $(-5, 0)$ and $(-1, 0)$.

d. $f(0) = (0)^2 + 6(0) + 5 = 5$

The y-intercept is $(0, 5)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = -3$.

g. The minimum value is -4.

h. The domain is $(-\infty, \infty)$.

The range is $[-4, \infty)$.

16. a. $g(x) = x^2 + 8x + 7$

$$g(x) = \left(x^2 + 8x\right) + 7 \quad \boxed{\left[\frac{1}{2}(8)\right]^2 = 16}$$

$$g(x) = \left(x^2 + 8x + 16\right) - 16 + 7$$
$$g(x) = (x+4)^2 - 9$$

b. The vertex is $(h, k) = (-4, -9)$.

c. $g(x) = x^2 + 8x + 7$

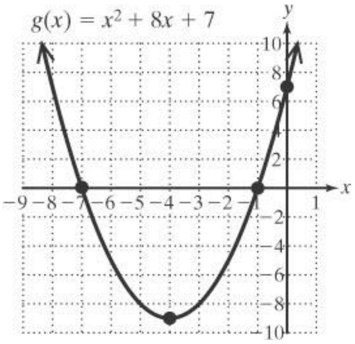
$$0 = x^2 + 8x + 7$$
$$0 = (x+7)(x+1)$$
$$x = -7 \quad \text{or} \quad x = -1$$

The x-intercepts are $(-7, 0)$ and $(-1, 0)$.

d. $g(0) = (0)^2 + 8(0) + 7 = 7$

The y-intercept is $(0, 7)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = -4$.

g. The minimum value is -9.

h. The domain is $(-\infty, \infty)$.

The range is $[-9, \infty)$.

17. a. $p(x) = 3x^2 - 12x - 7$

$$p(x) = 3(x^2 - 4x) - 7 \quad \boxed{\left[\frac{1}{2}(-4)\right]^2 = 4}$$
$$p(x) = 3(x^2 - 4x + 4 - 4) - 7$$
$$p(x) = 3(x^2 - 4x + 4) + 3(-4) - 7$$
$$p(x) = 3(x-2)^2 - 19$$

b. The vertex is $(h, k) = (2, -19)$.

c. $p(x) = 3x^2 - 12x - 7$

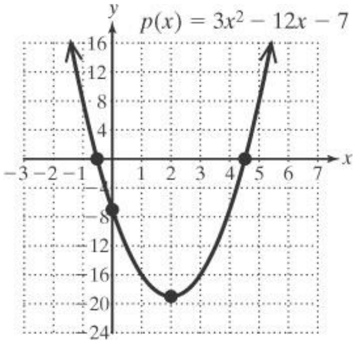
$$0 = 3x^2 - 12x - 7$$
$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(-7)}}{2(3)}$$
$$= \frac{12 \pm \sqrt{228}}{6}$$
$$= \frac{12 \pm 2\sqrt{57}}{6}$$
$$= \frac{6 \pm \sqrt{57}}{3}$$
$$x = \frac{6 - \sqrt{57}}{3} \quad \text{or} \quad x = \frac{6 + \sqrt{57}}{3}$$

The x-intercepts are $\left(\frac{6 - \sqrt{57}}{3}, 0\right)$ and $\left(\frac{6 + \sqrt{57}}{3}, 0\right)$.

d. $p(0) = 3(0)^2 - 12(0) - 7 = -7$

The y-intercept is $(0, -7)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = 2$.

g. The minimum value is -19.

h. The domain is $(-\infty, \infty)$.

The range is $[-19, \infty)$.

18. a. $q(x) = 2x^2 - 4x - 3$

$$q(x) = 2(x^2 - 2x) - 3 \quad \boxed{\left[\frac{1}{2}(-2)\right]^2 = 1}$$

$$q(x) = 2(x^2 - 2x + 1 - 1) - 3$$

$$q(x) = 2(x^2 - 2x + 1) + 2(-1) - 3$$

$$q(x) = 2(x - 1)^2 - 5$$

b. The vertex is $(h, k) = (1, -5)$.

c. $q(x) = 2x^2 - 4x - 3$

$$0 = 2x^2 - 4x - 3$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2(2)}$$

$$= \frac{4 \pm \sqrt{40}}{4}$$

$$= \frac{4 \pm 2\sqrt{10}}{4}$$

$$= \frac{2 \pm \sqrt{10}}{2}$$

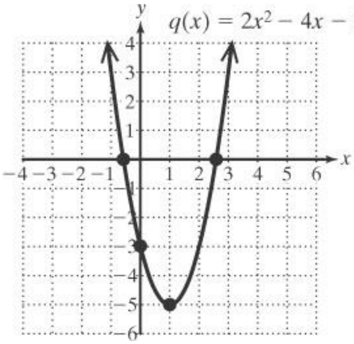
$$x = \frac{2 - \sqrt{10}}{2} \quad \text{or} \quad x = \frac{2 + \sqrt{10}}{2}$$

The x-intercepts are $\left(\frac{2 - \sqrt{10}}{2}, 0\right)$ and $\left(\frac{2 + \sqrt{10}}{2}, 0\right)$.

d. $q(0) = 2(0)^2 - 4(0) - 3 = -3$

The y-intercept is $(0, -3)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = 1$.

g. The minimum value is -5.

h. The domain is $(-\infty, \infty)$.

The range is $[-5, \infty)$.

19. a. $c(x) = -2x^2 - 10x + 4$

$$c(x) = -2(x^2 + 5x) + 4 \quad \boxed{\left[\frac{1}{2}(5)\right]^2 = \frac{25}{4}}$$

$$c(x) = -2\left(x^2 + 5x + \frac{25}{4} - \frac{25}{4}\right) + 4$$

$$c(x) = -2\left(x^2 + 5x + \frac{25}{4}\right) - 2\left(-\frac{25}{4}\right) + 4$$

$$c(x) = -2\left(x + \frac{5}{2}\right)^2 + \frac{33}{2}$$

b. The vertex is $(h, k) = \left(-\frac{5}{2}, \frac{33}{2}\right)$.

c. $c(x) = -2x^2 - 10x + 4$

$$0 = -2x^2 - 10x + 4$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(-2)(4)}}{2(-2)}$$

$$= \frac{10 \pm \sqrt{132}}{-4}$$

$$= \frac{-10 \pm 2\sqrt{33}}{4} = \frac{-5 \pm \sqrt{33}}{2}$$

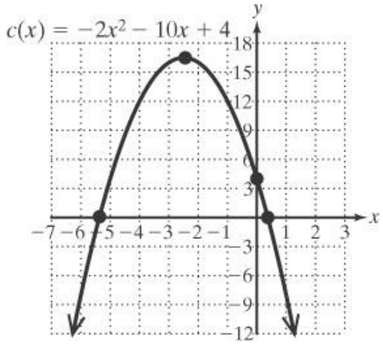
$$x = \frac{-5 - \sqrt{33}}{2} \quad \text{or} \quad x = \frac{2 + \sqrt{33}}{2}$$

The x-intercepts are $\left(\frac{-5 - \sqrt{33}}{2}, 0\right)$ and $\left(\frac{-5 + \sqrt{33}}{2}, 0\right)$.

d. $c\left(0\right)=-2\left(0\right)^2-10\left(0\right)+4=4$

The *y*-intercept is $\left(0, 4\right)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x=-\dfrac{5}{2}$.

g. The maximum value is $\dfrac{33}{2}$.

h. The domain is $\left(-\infty, \infty\right)$.

The range is $\left[-\infty, \dfrac{33}{2}\right)$.

20. a. $d\left(x\right)=-3x^2-9x+8$

$$d\left(x\right)=-3\left(x^2+3x\right)+8 \qquad \boxed{\left[\frac{1}{2}(3)\right]^2=\frac{9}{4}}$$

$$d\left(x\right)=-3\left(x^2+3x+\frac{9}{4}-\frac{9}{4}\right)+8$$

$$d\left(x\right)=-3\left(x^2+3x+\frac{9}{4}\right)-3\left(-\frac{9}{4}\right)+8$$

$$d\left(x\right)=-3\left(x+\frac{3}{2}\right)^2+\frac{59}{4}$$

b. The vertex is $\left(h, k\right)=\left(-\dfrac{3}{2}, \dfrac{59}{4}\right)$.

c. $d\left(x\right)=-3x^2-9x+8$

$$0=-3x^2-9x+8$$

$$x=\frac{-\left(-3\right)\pm\sqrt{\left(-3\right)^2-4\left(-3\right)\left(8\right)}}{2\left(-3\right)}$$

$$=\frac{9\pm\sqrt{177}}{-6}=\frac{-9\pm\sqrt{177}}{6}$$

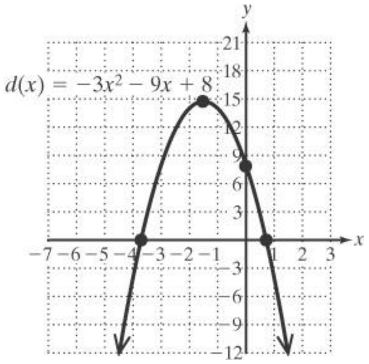
$$x=\frac{-9-\sqrt{177}}{6} \quad \text{or} \quad x=\frac{-9+\sqrt{177}}{6}$$

The *x*-intercepts are $\left(\dfrac{-9-\sqrt{177}}{6}, 0\right)$ and $\left(\dfrac{-9+\sqrt{177}}{6}, 0\right)$.

d. $d\left(0\right)=-3\left(0\right)^2-9\left(0\right)+8=8$

The *y*-intercept is $\left(0, 8\right)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x=-\dfrac{3}{2}$.

g. The maximum value is $\dfrac{59}{4}$

h. The domain is $\left(-\infty, \infty\right)$.

The range is $\left(-\infty, \dfrac{59}{4}\right]$.

21. a. $h(x) = -2x^2 + 7x$

$$h(x) = -2\left(x^2 - \frac{7}{2}x\right) \quad \boxed{\left[\frac{1}{2}\left(\frac{7}{2}\right)\right]^2 = \frac{49}{16}}$$

$$h(x) = -2\left(x^2 - \frac{7}{2}x + \frac{49}{16} - \frac{49}{16}\right)$$

$$h(x) = -2\left(x^2 - \frac{7}{2}x + \frac{49}{16}\right) - 2\left(-\frac{49}{16}\right)$$

$$h(x) = -2\left(x - \frac{7}{4}\right)^2 + \frac{49}{8}$$

b. The vertex is $(h, k) = \left(\frac{7}{4}, \frac{49}{8}\right)$.

c. $h(x) = -2x^2 + 7x$

$$0 = -2x^2 + 7x + 0$$

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(-2)(0)}}{2(-2)}$$

$$= \frac{-7 \pm 7}{-4} = \frac{7 \pm 7}{4}$$

$$x = \frac{7-7}{4} \quad \text{or} \quad x = \frac{7+7}{4}$$

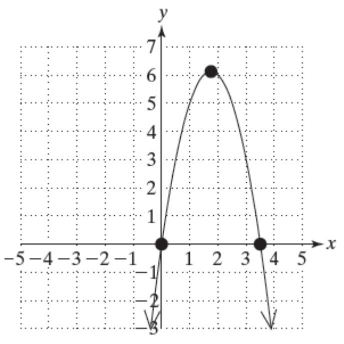
$$x = 0 \text{ or} \qquad x = \frac{7}{2}$$

The x-intercepts are $(0, 0)$ and $\left(\frac{7}{2}, 0\right)$.

d. $h(0) = -2(0)^2 + 7(0) = 0$

The y-intercept is $(0, 0)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = \frac{7}{4}$.

g. The maximum value is $\frac{49}{8}$.

h. The domain is $(-\infty, \infty)$.

The range is $\left(-\infty, \frac{49}{8}\right]$.

22. a. $k(x) = 3x^2 - 8x$

$$k(x) = 3\left(x^2 - \frac{8}{3}x\right) \quad \boxed{\left[\frac{1}{2}\left(\frac{8}{3}\right)\right]^2 = \frac{64}{36}}$$

$$k(x) = 3\left(x^2 - \frac{8}{3}x + \frac{64}{36} - \frac{64}{36}\right)$$

$$k(x) = 3\left(x^2 - \frac{8}{3}x + \frac{64}{36}\right) + 3\left(-\frac{64}{36}\right)$$

$$k(x) = 3\left(x - \frac{4}{3}\right)^2 - \frac{16}{3}$$

b. The vertex is $(h, k) = \left(\frac{4}{3}, -\frac{16}{3}\right)$.

c. $k(x) = 3x^2 - 8x$

$$0 = 3x^2 - 8x + 0$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(0)}}{2(3)}$$

$$= \frac{8 \pm 8}{6}$$

$$x = \frac{8-8}{6} \quad \text{or} \quad x = \frac{8+8}{6}$$

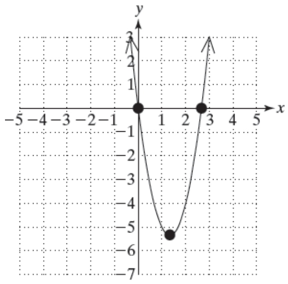
$$x = 0 \text{ or} \qquad x = \frac{8}{3}$$

The x-intercepts are $(0, 0)$ and $\left(\frac{8}{3}, 0\right)$.

d. $k(0) = 3(0)^2 - 8(0) = 0$

The y-intercept is $(0, 0)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = \frac{4}{3}$.

g. The minimum value is $-\frac{16}{3}$.

h. The domain is $\left(-\infty, \infty\right)$.

The range is $\left[-\frac{16}{3}, \infty\right)$.

23. a. $p\left(x\right) = x^2 + 9x + 17$

$$p\left(x\right) = \left(x^2 + 9x\right) + 17 \quad \boxed{\left[\frac{1}{2}(9)\right]^2 = \frac{81}{4}}$$

$$p\left(x\right) = \left(x^2 + 9x + \frac{81}{4} - \frac{81}{4}\right) + 17$$

$$p\left(x\right) = \left(x^2 + 9x + \frac{81}{4}\right) + 1\left(-\frac{81}{4}\right) + 17$$

$$p\left(x\right) = \left(x + \frac{9}{2}\right)^2 - \frac{13}{4}$$

b. The vertex is $\left(h, k\right) = \left(-\frac{9}{2}, -\frac{13}{4}\right)$.

c. $p\left(x\right) = x^2 + 9x + 17$

$$0 = x^2 + 9x + 17$$

$$x = \frac{-(9) \pm \sqrt{(9)^2 - 4(1)(17)}}{2(1)}$$

$$= \frac{-9 \pm \sqrt{13}}{2}$$

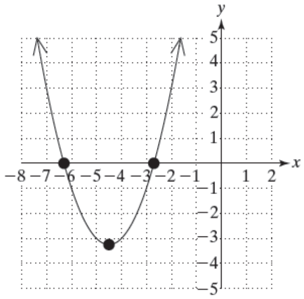
$$x = \frac{-9 - \sqrt{13}}{2} \quad \text{or} \quad x = \frac{-9 + \sqrt{13}}{2}$$

The x-intercepts are $\left(\frac{-9 - \sqrt{13}}{2}, 0\right)$ and $\left(\frac{-9 + \sqrt{13}}{2}, 0\right)$.

d. $p\left(0\right) = \left(0\right)^2 + 9\left(0\right) + 17 = 17$

The y-intercept is $\left(0, 17\right)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = -\frac{9}{2}$.

g. The minimum value is $-\frac{13}{4}$.

h. The domain is $\left(-\infty, \infty\right)$.

The range is $\left[-\frac{13}{4}, \infty\right)$.

24. a. $q\left(x\right) = x^2 + 11x + 26$

$$q\left(x\right) = \left(x^2 + 11x\right) + 26 \quad \boxed{\left[\frac{1}{2}(11)\right]^2 = \frac{121}{4}}$$

$$q\left(x\right) = \left(x^2 + 11x + \frac{121}{4} - \frac{121}{4}\right) + 26$$

$$q\left(x\right) = \left(x^2 + 11x + \frac{121}{4}\right) + 1\left(-\frac{121}{4}\right) + 26$$

$$q\left(x\right) = \left(x + \frac{11}{2}\right)^2 - \frac{17}{4}$$

b. The vertex is $\left(h, k\right) = \left(-\frac{11}{2}, -\frac{17}{4}\right)$.

c. $q(x) = x^2 + 11x + 26$

$$0 = x^2 + 11x + 26$$

$$x = \frac{-(11) \pm \sqrt{(11)^2 - 4(1)(26)}}{2(1)}$$

$$= \frac{-11 \pm \sqrt{17}}{2}$$

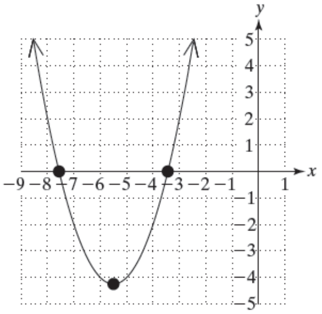
$$x = \frac{-11 - \sqrt{17}}{2} \text{ or } x = \frac{-11 + \sqrt{17}}{2}$$

The x-intercepts are $\left(\frac{-11 - \sqrt{17}}{2}, 0\right)$ and $\left(\frac{-11 + \sqrt{17}}{2}, 0\right)$.

d. $q(0) = (0)^2 + 11(0) + 26 = 26$

The y-intercept is $(0, 26)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = -\frac{11}{2}$.

g. The minimum value is $-\frac{17}{4}$.

h. The domain is $(-\infty, \infty)$.

The range is $\left[-\frac{17}{4}, \infty\right)$.

25. $f(x) = 3x^2 - 42x - 91$

x-coordinate: $\frac{-b}{2a} = \frac{-(-42)}{2(3)} = \frac{42}{6} = 7$

y-coordinate:

$f(7) = 3(7)^2 - 42(7) - 91 = -238$

The vertex is $(7, -238)$.

26. $g(x) = 4x^2 - 64x + 107$

x-coordinate: $\frac{-b}{2a} = \frac{-(-64)}{2(4)} = \frac{64}{8} = 8$

y-coordinate:

$g(8) = 4(8)^2 - 64(8) + 107 = -149$

The vertex is $(8, -149)$.

27. $k(a) = -\frac{1}{3}a^2 + 6a + 1$

x-coordinate: $\frac{-b}{2a} = \frac{-(6)}{2\left(-\frac{1}{3}\right)} = \frac{-6}{-\frac{2}{3}} = 9$

y-coordinate:

$k(9) = -\frac{1}{3}(9)^2 + 6(9) + 1 = 28$

The vertex is $(9, 28)$.

28. $j(t) = -\frac{1}{4}t^2 + 10t - 5$

x-coordinate: $\frac{-b}{2a} = \frac{-(10)}{2\left(-\frac{1}{4}\right)} = \frac{-10}{-\frac{1}{2}} = 20$

y-coordinate:

$j(20) = -\frac{1}{4}(20)^2 + 10(20) - 5 = 95$

The vertex is $(20, 95)$.

29. $f(c) = 4c^2 - 5$

x-coordinate: $\frac{-b}{2a} = \frac{-(0)}{2(4)} = 0$

y-coordinate: $f(0) = 4(0)^2 - 5 = -5$

The vertex is $(0, -5)$.

30. $h(a)=2a^2+14$

x-coordinate: $\frac{-b}{2a}=\frac{-(0)}{2(2)}=0$

y-coordinate: $h(0)=2(0)^2+14=14$

The vertex is $(0, 14)$.

31. $P(x)=1.2x^2+1.8x-3.6$

x-coordinate: $\frac{-b}{2a}=\frac{-(1.8)}{2(1.2)}=-0.75$

y-coordinate:

$$P(-0.75)=\begin{bmatrix}1.2(-0.75)^2 \\ \quad +1.8(-0.75)-3.6\end{bmatrix}$$
$$=-4.275$$

The vertex is $(-0.75, -4.275)$.

32. $Q(x)=7.5x^2-2.25x+4.75$

x-coordinate: $\frac{-b}{2a}=\frac{-(-2.25)}{2(7.5)}=0.15$

y-coordinate:

$$Q(0.15)=\begin{bmatrix}7.5(0.15)^2 \\ \quad -2.25(0.15)+4.75\end{bmatrix}$$
$$=4.58125$$

The vertex is $(0.15, 4.58125)$.

33. $g(x)=-x^2+2x-4$

a. Since $a<0$, the parabola opens downward.

b. x-coordinate: $\frac{-b}{2a}=\frac{-(2)}{2(-1)}=\frac{-2}{-2}=1$

y-coordinate:

$g(1)=-(1)^2+2(1)-4=-3$

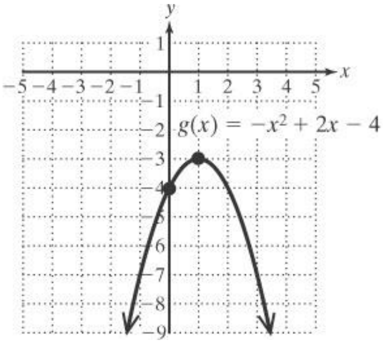
The vertex is $(1, -3)$.

c. Since the vertex of the parabola is below the x-axis and the parabola opens downward, the parabola cannot cross or touch the x-axis. Therefore, there are no x-intercepts.

d. $g(0)=-(0)^2+2(0)-4=-4$

The y-intercept is $(0, -4)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x=1$.

g. The maximum value is -3.

h. The domain is $(-\infty, \infty)$.

The range is $(-\infty, -3]$.

34. $h(x)=-x^2-6x-10$

a. Since $a<0$, the parabola opens downward.

b. x-coordinate: $\frac{-b}{2a}=\frac{-(-6)}{2(-1)}=\frac{6}{-2}=-3$

y-coordinate:

$h(-3)=-(-3)^2-6(-3)-10=-1$

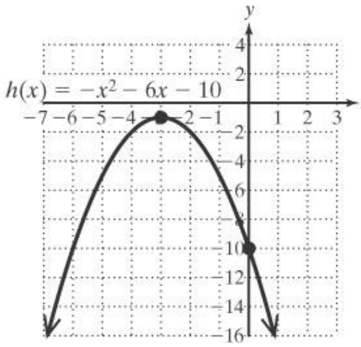
The vertex is $(1, -3)$.

c. Since the vertex of the parabola is below the x-axis and the parabola opens downward, the parabola cannot cross or touch the x-axis. Therefore, there are no x-intercepts.

d. $h(0) = -(0)^2 - 6(0) - 10 = -10$

The *y*-intercept is $(0, -10)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = -3$.

g. The maximum value is -1.

h. The domain is $(-\infty, \infty)$.

The range is $(-\infty, -1]$.

35. $f(x) = 5x^2 - 15x + 3$

a. Since $a > 0$, the parabola opens upward.

b. *x*-coordinate: $\frac{-b}{2a} = \frac{-(-15)}{2(5)} = \frac{15}{10} = \frac{3}{2}$

y-coordinate:

$$f\left(\frac{3}{2}\right) = 5\left(\frac{3}{2}\right)^2 - 15\left(\frac{3}{2}\right) + 3 = -\frac{33}{4}$$

The vertex is $\left(\frac{3}{2}, -\frac{33}{4}\right)$.

c. $f(x) = 5x^2 - 15x + 3$

$$0 = 5x^2 - 15x + 3$$

$$x = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(5)(3)}}{2(5)}$$

$$= \frac{15 \pm \sqrt{165}}{10}$$

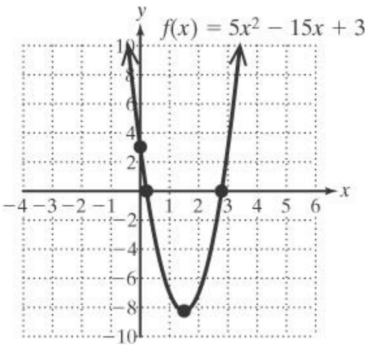
$$x = \frac{15 - \sqrt{165}}{10} \quad \text{or} \quad x = \frac{15 + \sqrt{165}}{10}$$

The *x*-intercepts are $\left(\frac{15 - \sqrt{165}}{10}, 0\right)$ and $\left(\frac{15 + \sqrt{165}}{10}, 0\right)$.

d. $f(0) = 5(0)^2 - 15(0) + 3 = 3$

The *y*-intercept is $(0, 3)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = \frac{3}{2}$.

g. The maximum value is $-\frac{33}{4}$.

h. The domain is $(-\infty, \infty)$.

The range is $\left[-\frac{33}{4}, \infty\right)$.

36. $k(x) = 2x^2 - 10x - 5$

a. Since $a > 0$, the parabola opens upward.

b. x-coordinate:

$$\frac{-b}{2a}=\frac{-(-10)}{2(2)}=\frac{10}{4}=\frac{5}{2}$$

y-coordinate:

$$k\left(\frac{5}{2}\right)=2\left(\frac{5}{2}\right)^2-10\left(\frac{5}{2}\right)-5=-\frac{35}{2}$$

The vertex is $\left(\frac{5}{2},-\frac{35}{2}\right)$.

c. $k(x)=2x^2-10x-5$

$$0=2x^2-10x-5$$

$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(2)(-5)}}{2(2)}$$

$$=\frac{10\pm\sqrt{140}}{4}$$

$$=\frac{10\pm2\sqrt{35}}{4}=\frac{5\pm\sqrt{35}}{2}$$

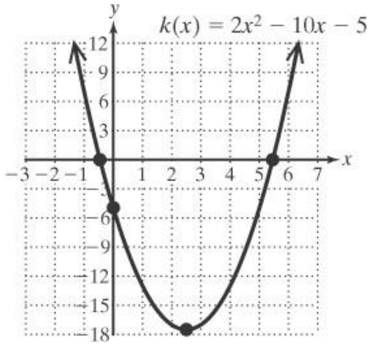
$$x=\frac{5-\sqrt{35}}{2}\quad\text{or}\quad x=\frac{5+\sqrt{35}}{2}$$

The x-intercepts are $\left(\frac{5-\sqrt{35}}{2},0\right)$ and $\left(\frac{5+\sqrt{35}}{2},0\right)$.

d. $k(0)=2(0)^2-10(0)-5=-5$

The y-intercept is $(0,-5)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x=\frac{5}{2}$.

g. The maximum value is $-\frac{35}{2}$.

h. The domain is $(-\infty,\infty)$.

The range is $\left[-\frac{35}{2},\infty\right)$.

37. $f(x)=2x^2+3$

a. Since $a>0$, the parabola opens upward.

b. x-coordinate: $\frac{-b}{2a}=\frac{-(0)}{2(2)}=0$

y-coordinate: $f(0)=2(0)^2+3=3$

The vertex is $(0,3)$.

c. $f(x)=2x^2+3$

$$0=2x^2+3$$

$$-3=2x^2$$

$$x^2=-\frac{3}{2}$$

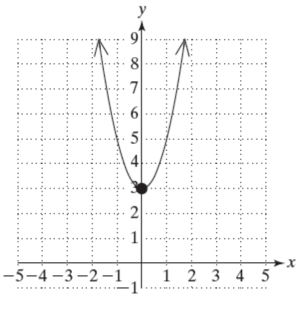
$$x=\sqrt{-\frac{3}{2}}$$

The x-intercepts does not exist.

d. $f(0)=2(0)^2+3=3$

The y-intercept is $(0,3)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = 0$.

g. The maximum value is 3.

h. The domain is $\left(-\infty, \infty\right)$.

The range is $\left[3, \infty\right)$.

38. $g\left(x\right) = -x^2 - 1$

a. Since $a < 0$, the parabola opens downward.

b. x-coordinate: $\frac{-b}{2a} = \frac{-\left(0\right)}{2\left(-1\right)} = 0$

y-coordinate: $g\left(0\right) = -\left(0\right)^2 - 1 = -1$

The vertex is $\left(0, -1\right)$.

c. $g\left(x\right) = -x^2 - 1$

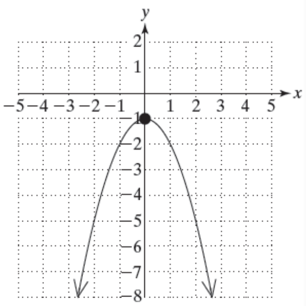
$$0 = -x^2 - 3$$
$$3 = -x^2$$
$$x^2 = -3$$
$$x = \pm\sqrt{-3}$$

The x-intercept does not exist.

d. $g\left(0\right) = -\left(0\right)^2 - 1 = -1$

The y-intercept is $\left(0, -1\right)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = 0$.

g. The maximum value is -1.

h. The domain is $\left(-\infty, \infty\right)$.

The range is $\left(-\infty, -1\right]$.

39. $f\left(x\right) = -2x^2 - 20x - 50$

a. Since $a < 0$, the parabola opens downward.

b. x-coordinate:

$$\frac{-b}{2a} = \frac{-\left(-20\right)}{2\left(-2\right)} = \frac{20}{-4} = -5$$

y-coordinate:

$$f\left(-5\right) = -2\left(-5\right)^2 - 20\left(-5\right) - 50 = 0$$

The vertex is $\left(-5, 0\right)$.

c. $f\left(x\right) = -2x^2 - 20x - 50$

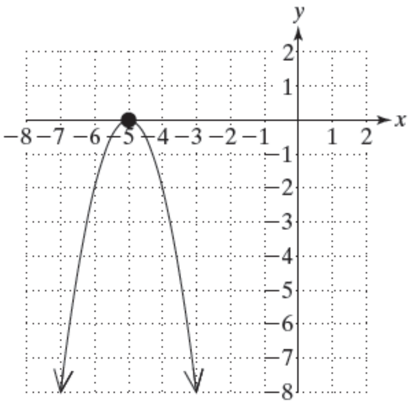
$$0 = -2x^2 - 20x - 50$$
$$x = \frac{-\left(-20\right) \pm \sqrt{\left(-20\right)^2 - 4\left(-2\right)\left(-50\right)}}{2\left(-2\right)}$$
$$= \frac{20 \pm 0}{-4}$$
$$x = -5$$

The x-intercept is $\left(-5, 0\right)$.

d. $f\left(0\right) = -2\left(0\right)^2 - 20\left(0\right) - 50 = -50$

The y-intercept is $\left(0, -50\right)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = -5$.

g. The maximum value is 0.

h. The domain is $(-\infty, \infty)$.The range is $(-\infty, 0]$.

40. $m(x) = 2x^2 - 8x + 8$

a. Since $a > 0$, the parabola opens upward.

b. x-coordinate: $\frac{-b}{2a} = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2$

y-coordinate:

$m(2) = 2(2)^2 - 8(2) + 8 = 0$

The vertex is $(2, 0)$.

c. $m(x) = 2x^2 - 8x + 8$

$$0 = 2x^2 - 8x + 8$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(8)}}{2(2)}$$

$$= \frac{8 \pm 0}{4}$$

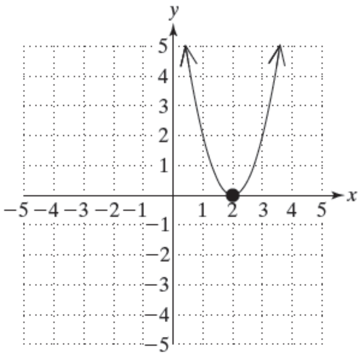
$$x = 2$$

The x-intercept is $(2, 0)$.

d. $m(0) = 2(0)^2 - 8(0) + 8 = 8$

The y-intercept is $(0, 8)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = 2$.

g. The maximum value is 0.

h. The domain is $(-\infty, \infty)$.

The range is $[0, \infty)$.

41. $n(x) = x^2 - x + 3$

a. Since $a > 0$, the parabola opens upward.

b. x-coordinate: $\frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}$

y-coordinate:

$$n\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) + 3 = \frac{11}{4}$$

The vertex is $\left(\frac{1}{2}, \frac{11}{4}\right)$.

c. $n(x) = x^2 - x + 3$

$$0 = x^2 - x + 3$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(3)}}{2(1)}$$

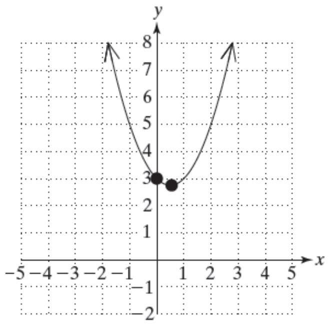
$$= \frac{1 \pm \sqrt{-11}}{2}$$

The x-intercept does not exist.

d. $n(0) = (0)^2 - (0) + 3 = 3$

The y-intercept is $(0, 3)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = \frac{1}{2}$.

g. The minimum value is $\frac{11}{4}$.

h. The domain is $(-\infty, \infty)$.

The range is $\left[\frac{11}{4}, \infty\right)$.

42. $r(x) = x^2 - 5x + 7$

a. Since $a > 0$, the parabola opens upward.

b. x-coordinate: $\frac{-b}{2a} = \frac{-(-5)}{2(1)} = \frac{5}{2}$

y-coordinate:

$$r\left(\frac{5}{2}\right) = \left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) + 7 = \frac{3}{4}$$

The vertex is $\left(\frac{5}{2}, \frac{3}{4}\right)$.

c. $r(x) = x^2 - 5x + 7$

$$0 = x^2 - 5x + 7$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(7)}}{2(2)}$$

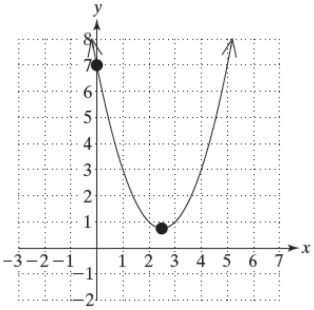
$$= \frac{5 \pm \sqrt{-3}}{4}$$

The x-intercept does not exist.

d. $r(0) = (0)^2 - 5(0) + 7 = 7$

The y-intercept is $(0, 7)$.

e.



f. The axis of symmetry is the vertical line through the vertex: $x = \frac{5}{2}$.

g. The minimum value is $\frac{3}{4}$.

h. The domain is $(-\infty, \infty)$.

The range is $\left[\frac{3}{4}, \infty\right)$.

43. a. The price that generates the maximum profit is the p-coordinate of the vertex.

$$p = \frac{-b}{2a} = \frac{-(1700)}{2(-50)}$$

$$= \frac{1700}{100} = 17$$

\$17

b. The maximum profit is the value of $f(p)$ at the vertex.

$$f(17) = -50(17)^2 + 1700(17) - 12{,}000$$

$$= 2450$$

\$2450

c. The prices that would enable the company to break even is the value of p when $f(p) = 0$.

$$0 = -50p^2 + 1700p - 12000$$
$$0 = -50\left(p^2 - 34p + 240\right)$$
$$0 = p^2 - 34p + 240$$
$$0 = \left(p - 10\right)\left(p - 24\right)$$
$$p = 10 \text{ or } p = 24$$
$$\$10 \text{ and } \$24$$

44. a. The price that generates the maximum profit is the p-coordinate of the vertex.

$$p = \frac{-b}{2a} = \frac{-(3440)}{2(-80)}$$
$$= \frac{3440}{160} = 21.50$$

\$21.50

b. The maximum profit is the value of $f(p)$ at the vertex.

$$f(21.50) = \begin{bmatrix} -80(21.50)^2 \\ +3440(21.50) - 36,000 \end{bmatrix}$$
$$= 980$$

\$980

c. The prices that would enable the company to break even is the value of p when $f(p) = 0$.

$$0 = -80p^2 + 3440p - 36000$$
$$0 = -80\left(p^2 - 43p + 450\right)$$
$$0 = p^2 - 43p + 450$$
$$0 = \left(p - 18\right)\left(p - 25\right)$$
$$p = 18 \text{ or } p = 25$$
$$\$18 \text{ and } \$25$$

45. a. The horizontal distance at which the jumper will be at a maximum height is the x-coordinate of the vertex.

$$x = \frac{-b}{2a} = \frac{-(0.364)}{2(-0.046)}$$
$$= \frac{-0.364}{-0.092} \approx 3.96 \text{ m}$$

b. The maximum height is the value of $h(x)$ at the vertex.

$$h(3.96)$$
$$= -0.046(3.96)^2 + 0.364(3.96)$$
$$\approx 0.72 \text{ m}$$

c. The length of the jump is the value of x when $h(x) = 0$.

$$0 = -0.046x^2 + 0.364x$$
$$0 = x\left(-0.046x + 0.364\right)$$
$$x = 0 \quad \text{or} -0.046x + 0.364 = 0$$
$$-0.046x = -0.364$$
$$x \approx 7.9$$

Since we want the length of the jump, we use the second value of x. The jump is 7.9 m.

46. a. The horizontal distance at which the water will be at its maximum height is the x- coordinate of the vertex.

$$x = \frac{-b}{2a} = \frac{-(0.576)}{2(-0.026)}$$
$$= \frac{-0.576}{-0.052} \approx 11.1 \text{ m}$$

b. The maximum height is the value of $h(x)$ at the vertex.

$$h(11.1) = \begin{bmatrix} -0.026(11.1)^2 \\ +0.576(11.1) + 3 \end{bmatrix}$$
$$\approx 6.2 \text{ m}$$

c. The distance of the firefighter from the house is the value of x when $h(x) = 6$.

$$6 = -0.026x^2 + 0.576x + 3$$
$$0 = -0.026x^2 + 0.576x - 3$$
$$x = \frac{-0.576 \pm \sqrt{(0.576)^2 - 4(-0.026)(-3)}}{2(-0.026)}$$
$$= \frac{0.576 \pm \sqrt{0.019776}}{0.052}$$
$$x \approx 8 \quad \text{or} \quad x \approx 14$$

Since we want the downward branch, we use the second value of x. The firefighter is 14 m from the house.

47. a. The time at which the population will be at a maximum is the t-coordinate of the vertex.

$$t = \frac{-b}{2a} = \frac{-(82{,}000)}{2(-1718)}$$
$$= \frac{82{,}000}{3436} \approx 24 \text{ hr}$$

b. The maximum population is the value of $P(t)$ at the vertex.

$$P(24) = \begin{bmatrix} -1718(24)^2 \\ \quad +82{,}000(24)+10{,}000 \end{bmatrix}$$
$$\approx 988{,}000$$

48. a. The speed at which the gas mileage will be at a maximum is the x-coordinate of the vertex.

$$x = \frac{-b}{2a} = \frac{-(2.688)}{2(-0.028)}$$
$$= \frac{-2.688}{-0.056} = 48 \text{ mph}$$

b. The maximum gas mileage is the value of $m(x)$ at the vertex.

$$m(x) = \begin{bmatrix} -0.028(48)^2 \\ \quad +2.688(48)-35.012 \end{bmatrix}$$
$$= 29.5 \text{ mpg}$$

49. Let x represent the first number. Let y represent the second number. We know that

$$x + y = 24$$
$$y = 24 - x$$

Let P represent the product.

$$P = xy$$
$$P(x) = x(24 - x)$$
$$= 24x - x^2$$
$$= -x^2 + 24x$$

Function P is a quadratic function with a negative leading coefficient. The graph of the parabola opens downward, so the vertex is the maximum point on the function. The x-coordinate of the vertex is the value of x that will maximize the product.

$$x = \frac{-b}{2a} = \frac{-(24)}{2(-1)}$$
$$= \frac{-24}{-2} = 12$$
$$y = 24 - x = 24 - 12 = 12$$

The numbers are 12 and 12.

50. Let x represent the first number. Let y represent the second number. We know that

$$x + y = 1$$
$$y = 1 - x$$

Let P represent the product.

$$P = xy$$
$$P(x) = x(1 - x)$$
$$= x - x^2$$
$$= -x^2 + x$$

Function P is a quadratic function with a negative leading coefficient. The graph of parabola opens downward, so the vertex is the maximum point on the function. The x-coordinate of the vertex is the value of x that will maximize the product.

$$x=\frac{-b}{2a}=\frac{-(1)}{2(-1)}$$
$$=\frac{-1}{-2}=0.5$$
$$y=1-x=1-0.5=0.5$$

The numbers are 0.5 and 0.5.

51. Let x represent the first number. Let y represent the second number. We know that

$$x-y=10$$
$$-y=10-x$$
$$y=x-10$$

Let P represent the product.

$$P=xy$$
$$P(x)=x(x-10)$$
$$=x^2-10x$$

Function P is a quadratic function with a positive leading coefficient. The graph of the parabola opens upward, so the vertex is the minimum point on the function. The x-coordinate of the vertex is the value of x that will minimize the product.

$$x=\frac{-b}{2a}=\frac{-(-10)}{2(1)}=\frac{10}{2}=5$$
$$y=x-10=5-10=-5$$

The numbers are 5 and -5.

52. Let x represent the first number. Let y represent the second number. We know that

$$x-y=30$$
$$-y=30-x$$
$$y=x-30$$

Let P represent the product.

$$P=xy$$
$$P(x)=x(x-30)=x^2-30x$$

Function P is a quadratic function with a positive leading coefficient. The graph of the parabola opens upward, so the vertex is the minimum point on the function. The x-coordinate of the vertex is the value of x that will minimize the product.

$$x=\frac{-b}{2a}=\frac{-(-30)}{2(1)}=\frac{30}{2}=15$$
$$y=x-30=15-30=-15$$

The numbers are 15 and -15.

53. **a.** We know that

$$2x+y=160$$
$$y=160-2x$$

Let A represent the area.

$$A=xy$$
$$A(x)=x(160-2x)$$
$$=160x-2x^2$$
$$=-x^2+160x$$

Function A is a quadratic function with a negative leading coefficient. The graph of the parabola opens downward, so the vertex is the maximum point on the function. The x-coordinate of the vertex is the value of x that will maximize the area.

$$x=\frac{-b}{2a}=\frac{-(160)}{2(-2)}$$
$$=\frac{-160}{-4}=40$$

$$\begin{aligned} y &= 160 - 2x \\ &= 160 - 2(40) \\ &= 160 - 80 = 80 \end{aligned}$$

The dimensions are 40 ft by 80 ft.

b. The maximum area is 3200 ft^2.

54. a. We know that

$$\begin{aligned} 3x + 4y &= 120 \\ 4y &= 120 - 3x \\ y &= 30 - \frac{3}{4}x \end{aligned}$$

Let *A* represent the area.

$$\begin{aligned} A &= xy \\ A(x) &= x\left(30 - \frac{3}{4}x\right) \\ &= 30x - \frac{3}{4}x^2 \\ &= -\frac{3}{4}x^2 + 30x \end{aligned}$$

Function *A* is a quadratic function with a negative leading coefficient. The graph of the parabola opens downward, so the vertex is the maximum point on the function. The *x*-coordinate of the vertex is the value of *x* that will maximize the area.

$$\begin{aligned} x &= \frac{-b}{2a} = \frac{-(30)}{2\left(-\frac{3}{4}\right)} \\ &= \frac{-30}{-\frac{3}{2}} = 20 \end{aligned}$$

$$\begin{aligned} y &= 30 - \frac{3}{4}x \\ &= 30 - \frac{3}{4}(20) \\ &= 30 - 15 = 15 \end{aligned}$$

The dimensions are 20 ft by 15 ft.

b. The maximum area is 300 ft^2.

55. a. Let *V* represent the volume.

$$\begin{aligned} V &= lwh \\ V(x) &= 20(12 - 2x)(x) \\ &= 20(12x - 2x^2) \\ &= 240x - 40x^2 \\ &= -40x^2 + 240x \end{aligned}$$

b. Function *V* is a quadratic function with a negative leading coefficient. The graph of the parabola opens downward, so the vertex is the maximum point on the function. The *x*-coordinate of the vertex is the value of *x* that will maximize the volume.

$$\begin{aligned} x &= \frac{-b}{2a} = \frac{-(240)}{2(-40)} \\ &= \frac{-240}{-80} = 3 \end{aligned}$$

The sheet of aluminum should be folded 3 in. from each end.

c. $V(3) = -40(3)^2 + 240(3)$

$= 360 \text{ in.}^3$

56. a. The perimeter of the box is 36 in.

$$\begin{aligned} 2x + 2y &= 36 \\ 2y &= 36 - 2x \\ y &= \frac{36 - 2x}{2} \end{aligned}$$

Let *A* represent the area.

$$\begin{aligned} A &= lw \\ A(x) &= xy \\ &= x\left(\frac{36 - 2x}{2}\right) = x(18 - x) \\ &= 18x - x^2 = -x^2 + 18x \end{aligned}$$

b. Function *A* is a quadratic function with a negative leading coefficient.

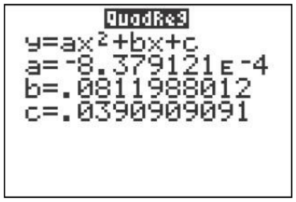
The graph of the parabola opens downward, so the vertex is the maximum point on the function. The x-coordinate of the vertex is the value of x that will maximize the area.

$$x = \frac{-b}{2a} = \frac{-(18)}{2(-1)}$$
$$= \frac{-18}{-2} = 9$$
$$y = \frac{36-2x}{2} = \frac{36-2(9)}{2}$$
$$= \frac{18}{2} = 9$$

The dimensions 9 in. by 9 in. should be used. That is, the shadow box should be square.

c. $A(9) = -(9)^2 + 18(9)$
$$= -81 + 162 = 81 \text{ in.}^2$$

57. a.



```
QuadReg
y=ax²+bx+c
a=-8.379121E-4
b=.0811988012
c=.0390909091
```

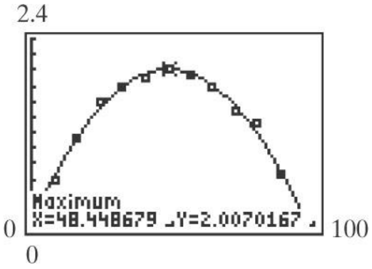
$$y(t) = \begin{bmatrix} -0.000838t^2 \\ +0.0812t + 0.040 \end{bmatrix}$$

b. From the graph, the time when the population is the greatest is the t-coordinate of the vertex.

$$t = \frac{-b}{2a} = \frac{-(0.0812)}{2(-0.000838)}$$
$$= \frac{-0.0812}{-0.001676} \approx 48 \text{ hr}$$

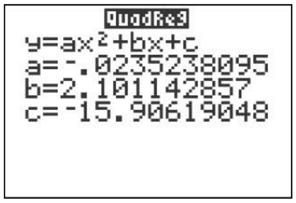
c. The maximum population of the bacteria is the $y(t)$ value at the vertex.

$$y(48) = \begin{bmatrix} -0.000838(48)^2 \\ +0.0812(48) + 0.040 \end{bmatrix}$$
$$\approx 2 \text{ g}$$



```
Maximum
X=48.448679  Y=2.0070167
```

58. a.



```
QuadReg
y=ax²+bx+c
a=-.0235238095
b=2.101142857
c=-15.90619048
```

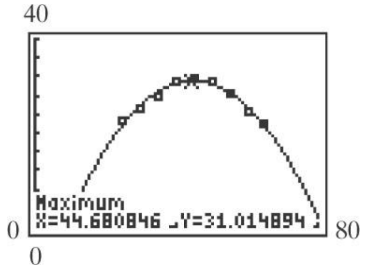
$$m(x) = -0.0235x^2 + 2.10x - 15.9$$

b. From the graph, the speed when the gas mileage is the greatest is the x-coordinate of the vertex.

$$x = \frac{-b}{2a} = \frac{-(2.10)}{2(-0.0235)}$$
$$= \frac{-2.10}{-0.047} \approx 45 \text{ mph}$$

c. The maximum gas mileage is the $m(x)$ value at the vertex.

$$m(45) = \begin{bmatrix} -0.0235(45)^2 \\ +2.10(45) - 15.9 \end{bmatrix}$$
$$\approx 31 \text{ mpg}$$



59. a. $x = 0$ cm. From the table,

$v(0) = 195.6$ cm/sec.

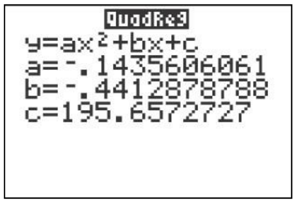
$$t = \frac{d}{r} = \frac{3000\text{ cm}}{195.6\text{ cm/sec}} \approx 15.3\text{ sec}$$

b. $x = 9$ cm. From the table,

$v(9) = 180.0$ cm/sec.

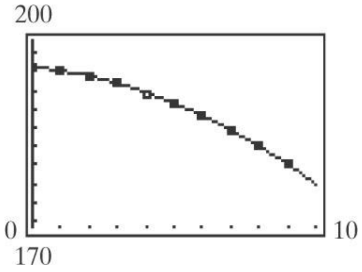
$$t = \frac{d}{r} = \frac{3000\text{ cm}}{180.0\text{ cm/sec}} \approx 16.7\text{ sec}$$

c.



```
QuadReg
y=ax²+bx+c
a=-.1435606061
b=-.4412878788
c=195.6572727
```

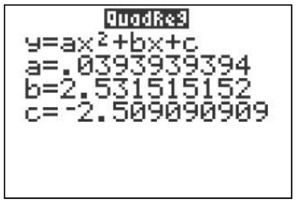
$$v(x) = \begin{bmatrix} -0.1436x^2 \\ -0.4413x + 195.7 \end{bmatrix}$$



d. $$v(5.5) = \begin{bmatrix} -0.1436(5.5)^2 \\ -0.4413(5.5) + 195.7 \end{bmatrix}$$

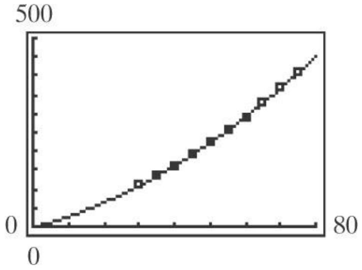
≈ 188.9 cm/sec

60. a.



```
QuadReg
y=ax²+bx+c
a=.0393939394
b=2.531515152
c=-2.509090909
```

$$d(s) = 0.039x^2 + 2.53s - 2.51$$



b. $$d(62) = \begin{bmatrix} 0.039(62)^2 \\ +2.53(62) - 2.51 \end{bmatrix}$$

≈ 304 ft

c. $$d(53) = \begin{bmatrix} 0.039(53)^2 \\ +2.53(53) - 2.51 \end{bmatrix}$$

≈ 241 ft

No, the stopping distance required is 242 ft (rounding up), so the car would miss the deer by approximately 3 ft.

61. False

If a relation has two *y*-intercepts, then it would fail the vertical line test and the relation would not define *y* as a function of *x*.

62. True

63. True

64. False

The axis of symmetry is a vertical line, whereas the graph of $y = c$ is a horizontal line. The axis of symmetry should be $x = -\frac{b}{2a}$.

65. $b^2 - 4ac = (12)^2 - 4(4)(9)$
$= 144 - 144 = 0$
Discriminant is 0; one *x*-intercept

66. $b^2 - 4ac = (-20)^2 - 4(25)(4)$
$= 400 - 400 = 0$
Discriminant is 0; one *x*-intercept

67. $b^2 - 4ac = (-5)^2 - 4(-1)(8)$
$= 25 + 32 = 57$
Discriminant is 57; two *x*-intercepts

68. $b^2 - 4ac = (4)^2 - 4(-3)(9)$
$= 16 + 108 = 124$
Discriminant is 124; two *x*-intercepts

69. $b^2 - 4ac = (6)^2 - 4(-3)(-11)$
$= 36 - 132 = -96$
Discriminant is -96; no *x*-intercepts

70. $b^2 - 4ac = (5)^2 - 4(-2)(-10)$
$= 25 - 80$
$= -55$
Discriminant is -55; no *x*-intercepts

71. Graph g

72. Graph d

73. Graph h

74. Graph b

75. Graph f

76. Graph e

77. Graph c

78. Graph a

79. For a parabola opening upward, such as the graph of $f(x) = x^2$, the minimum value is the *y*-coordinate of the vertex. There is no maximum value because the *y* values of the function become arbitrarily large for large values of $|x|$.

80. The vertex $(4, -3)$ is below the *x*-axis. If the parabola opens downward, then the vertex is the maximum point. If the maximum point is below the *x*-axis, then no points on the curve will intersect the *x*-axis.

81. No function defined by $y = f(x)$ can have two *y*-intercepts because the graph would fail the vertical line test.

82. The *x*-intercepts are the solutions of the equation $ax^2 + bx + c = 0$. The discriminant $b^2 - 4ac$ determines the number and type of solutions to the equation and thus the number of *x*-intercepts. If $b^2 - 4ac < 0$, then the solutions to the equation are imaginary and the function will have no *x*-intercepts. If $b^2 - 4ac = 0$, then the equation has one real solution and the function has one *x*-intercept. If $b^2 - 4ac > 0$, then the equation has two real solutions and the function has two *x*-intercepts.

83. Because a parabola is symmetric with respect to the vertical line through the vertex, the *x*-coordinate of the vertex must be equidistant from the *x*-intercepts. Therefore, given $y = f(x)$, the *x*-coordinate of the vertex is 4 because 4 is midway between 2 and 6. The *y*-coordinate of the vertex is $f(4)$.

84. Given an equation of a parabola in the form $y = a \cdot f\left(x-h\right)^2 + k$, the orientation is determined by a and the vertex is (h, k). If the vertex is above the x-axis and the parabola opens upward ($a > 0$), then the graph has no x-intercepts. Likewise, if the vertex is below the x-axis and the parabola opens downward ($a < 0$), then the graph has no x-intercepts.

85. $\left(h, k\right) = \left(2, -3\right)$

$$f\left(x\right) = a\left(x-2\right)^2 + \left(-3\right) = a\left(x-2\right)^2 - 3$$
$$f\left(0\right) = 5$$
$$a\left(0-2\right)^2 - 3 = 5$$
$$4a = 8$$
$$a = 2$$
$$f\left(x\right) = 2\left(x-2\right)^2 - 3$$

86. $\left(h, k\right) = \left(-3, 1\right)$

$$f\left(x\right) = a\left[x - \left(-3\right)\right]^2 + 1$$
$$= a\left(x+3\right)^2 + 1$$
$$f\left(0\right) = -17$$
$$a\left(0+3\right)^2 + 1 = -17$$
$$9a = -18$$
$$a = -2$$
$$f\left(x\right) = -2\left(x+3\right)^2 + 1$$

87. $\left(h, k\right) = \left(4, 6\right)$

$$f\left(x\right) = a\left(x-4\right)^2 + 6$$
$$f\left(1\right) = 3$$
$$a\left(1-4\right)^2 + 6 = 3$$
$$9a = -3$$
$$a = -\frac{1}{3}$$
$$f\left(x\right) = -\frac{1}{3}\left(x-4\right)^2 + 6$$

88. $\left(h, k\right) = \left(-2, 5\right)$

$$f\left(x\right) = a\left[x - \left(-2\right)\right]^2 + 5$$
$$= a\left(x+2\right)^2 + 5$$
$$f\left(2\right) = 13$$
$$a\left(2+2\right)^2 + 5 = 13$$
$$16a = 8$$
$$a = \frac{1}{2}$$
$$f\left(x\right) = \frac{1}{2}\left(x+2\right)^2 + 5$$

89. Find the x-coordinate of the vertex:

$$\frac{-b}{2a} = \frac{-12}{2\left(2\right)}$$
$$= \frac{-12}{4} = -3$$
$$f\left(-3\right) = -9$$
$$2\left(-3\right)^2 + 12\left(-3\right) + c = -9$$
$$18 - 36 + c = -9$$
$$-18 + c = -9$$
$$c = 9$$

90. Find the x-coordinate of the vertex:

$$\frac{-b}{2a} = \frac{-12}{2\left(3\right)} = \frac{-12}{6} - 2$$
$$f\left(-2\right) = -4$$
$$3\left(-2\right)^2 + 12\left(-2\right) + c = -4$$
$$12 - 24 + c = -4$$
$$-12 + c = -4$$
$$c = 8$$

91. Find the x-coordinate of the vertex:

$$\frac{-b}{2a}=\frac{-b}{2(-1)}=\frac{-b}{-2}=\frac{b}{2}$$

$$f\left(\frac{b}{2}\right)=8$$

$$-\left(\frac{b}{2}\right)^2+b\left(\frac{b}{2}\right)+4=8$$

$$-\frac{b^2}{4}+\frac{b^2}{2}=4$$

$$-b^2+2b^2=16$$

$$b^2=16$$

$$b=\pm 4$$

$$b=-4 \text{ or } b=4$$

92. Find the x-coordinate of the vertex:

$$\frac{-b}{2a}=\frac{-b}{2(-1)}=\frac{-b}{-2}=\frac{b}{2}$$

$$f\left(\frac{b}{2}\right)=7$$

$$-\left(\frac{b}{2}\right)^2+b\left(\frac{b}{2}\right)-2=7$$

$$-\frac{b^2}{4}+\frac{b^2}{2}=9$$

$$-b^2+2b^2=36$$

$$b^2=36$$

$$b=\pm 6$$

$$b=-6 \text{ or } b=6$$

Section 2.2 Introduction to Polynomial Functions

1. polynomial

2. is

3. is not

4. 2

5. 1

6. zeros

7. 5

8. 4

9. cross

10. touch (without crossing)

11. f has at least one zero on the interval $[a,b]$.

12. Multiply the leading terms from each factor.

$$-\frac{1}{3}(x)^4(3x)^2=-\frac{1}{3}(x^4)(9x^2)$$
$$=-3x^6$$

13. The leading coefficient is negative and the degree is even. The end behavior is down to the left and down to the right.

14. The leading coefficient is negative and the degree is even. The end behavior is down to the left and down to the right.

15. The leading coefficient is positive and the degree is odd. The end behavior is down to the left and up to the right.

16. The leading coefficient is positive and the degree is odd. The end behavior is down to the left and up to the right.

17. Multiply the leading terms from each factor.

$$-4(x)(2x)^2(x)^4=-4(x)(4x^2)(x^4)$$
$$=-16x^7$$

The leading coefficient is negative and

the degree is odd. The end behavior is up to the left and down to the right.

18. Multiply the leading terms from each factor.

$$-2(x)(3x)^3(x) = -2(x)(27x^3)(x)$$
$$= -54x^5$$

The leading coefficient is negative and the degree is odd. The end behavior is up to the left and down to the right.

19. Multiply the leading terms from each factor.

$$-2x^2(-x)(2x)^3 = -2x^2(-x)(8x^3)$$
$$= 16x^6$$

The leading coefficient is positive and the degree is even. The end behavior is up to the left and up to the right.

20. Multiply the leading terms from each factor.

$$-5x^4(-x)^3(2x) = -5x^4(-x^3)(2x)$$
$$= 10x^8$$

The leading coefficient is positive and the degree is even. The end behavior is up to the left and up to the right.

21. 3; 4

22. 5; 3

23. $f(x) = x^3 + 2x^2 - 25x - 50$

$$0 = x^3 + 2x^2 - 25x - 50$$
$$0 = x^2(x+2) - 25(x+2)$$
$$0 = (x+2)(x^2 - 25)$$
$$0 = (x+2)(x-5)(x+5)$$
$$x = -2, x = 5, x = -5$$

Each zero is of multiplicity 1.

24. $g(x) = x^3 + 5x^2 - x - 5$

$$0 = x^3 + 5x^2 - x - 5$$
$$0 = x^2(x+5) - 1(x+5)$$
$$0 = (x+5)(x^2 - 1)$$
$$0 = (x+5)(x-1)(x+1)$$
$$x = -5, x = 1, x = -1$$

Each zero is of multiplicity 1.

25. $h(x) = -6x^3 - 9x^2 + 60x$

$$0 = -6x^3 - 9x^2 + 60x$$
$$0 = -3x(2x^2 + 3x - 20)$$
$$0 = -3x(2x-5)(x+4)$$
$$x = 0, x = \frac{5}{2}, x = -4$$

Each zero is of multiplicity 1.

26. $k(x) = -6x^3 + 26x^2 - 28x$

$$0 = -6x^3 + 26x^2 - 28x$$
$$0 = -2x(3x^2 - 13x + 14)$$
$$0 = -2x(3x-7)(x-2)$$
$$x = 0, x = \frac{7}{3}, x = 2$$

Each zero is of multiplicity 1.

27. $m(x) = x^5 - 10x^4 + 25x^3$

$$0 = x^5 - 10x^4 + 25x^3$$
$$0 = x^3(x^2 - 10x + 25)$$
$$0 = x^3(x-5)^2$$
$$x = 0, x = 5$$

The zero 0 has multiplicity 3. The zero 5 has multiplicity 2.

28. $n(x) = x^6 + 4x^5 + 4x^4$

$$0 = x^6 + 4x^5 + 4x^4$$

$$0 = x^4\left(x^2 + 4x + 4\right)$$
$$0 = x^4\left(x + 2\right)^2$$
$$x = 0, x = -2$$

The zero 0 has multiplicity 4. The zero -2 has multiplicity 2.

29. $p\left(x\right) = -3x\left(x+2\right)^3\left(x+4\right)$
$$0 = -3x\left(x+2\right)^3\left(x+4\right)$$
$$x = 0, x = -2, x = -4$$

The zero 0 has multiplicity 1. The zero -2 has multiplicity 3. The zero -4 has multiplicity 1.

30. $q\left(x\right) = -2x^4\left(x+1\right)^3\left(x-2\right)^2$
$$0 = -2x^4\left(x+1\right)^3\left(x-2\right)^2$$
$$x = 0, x = -1, x = 2$$

The zero 0 has multiplicity 4. The zero -1 has multiplicity 3. The zero 2 has multiplicity 2.

31. $t\left(x\right) = \left[\begin{array}{c} 5x\left(3x-5\right)\left(2x+9\right) \\ \left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right) \end{array}\right]$
$$0 = \left[\begin{array}{c} 5x\left(3x-5\right)\left(2x+9\right) \\ \left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right) \end{array}\right]$$
$$x = 0, x = \frac{5}{3} x = -\frac{9}{2}, x = \sqrt{3}, x = -\sqrt{3}$$

Each zero is of multiplicity 1.

32. $z\left(x\right) = \left[\begin{array}{c} 4x\left(5x-1\right)\left(3x+8\right) \\ \left(x-\sqrt{5}\right)\left(x+\sqrt{5}\right) \end{array}\right]$
$$0 = \left[\begin{array}{c} 4x\left(5x-1\right)\left(3x+8\right) \\ \left(x-\sqrt{5}\right)\left(x+\sqrt{5}\right) \end{array}\right]$$
$$x = 0, x = \frac{1}{5} x = -\frac{8}{3}, x = \sqrt{5}, x = -\sqrt{5}$$

Each zero is of multiplicity 1.

33. $c\left(x\right) = \left[x - \left(3 - \sqrt{5}\right)\right]\left[x - \left(3 + \sqrt{5}\right)\right]$
$$0 = \left[x - \left(3 - \sqrt{5}\right)\right]\left[x - \left(3 + \sqrt{5}\right)\right]$$
$$x = 3 - \sqrt{5}, x = 3 + \sqrt{5}$$

Each zero is of multiplicity 1.

34. $d\left(x\right) = \left[x - \left(2 - \sqrt{11}\right)\right]\left[x - \left(2 + \sqrt{11}\right)\right]$
$$0 = \left[x - \left(2 - \sqrt{11}\right)\right]\left[x - \left(2 + \sqrt{11}\right)\right]$$
$$x = 2 - \sqrt{11}, x = 2 + \sqrt{11}$$

Each zero is of multiplicity 1.

35. $f\left(x\right) = 4x^4 - 37x^2 + 9$
$$0 = 4x^4 - 37x^2 + 9$$
$$0 = 4\left(x^2\right)^2 - 37x^2 + 9$$

Let $x^2 = y$.

$$0 = 4y^2 - 37y + 9$$
$$0 = 4y^2 - 36y - y + 9$$
$$0 = \left(4y - 1\right)\left(y - 9\right)$$
$$y = \frac{1}{4} \quad \text{or} \quad y = 9$$
$$x^2 = \frac{1}{4} \quad \text{or} \quad x^2 = 9$$
$$x = \pm\frac{1}{2} \qquad x = \pm 3$$
$$x = -3, -\frac{1}{2}, \frac{1}{2}, 3$$

Each zero is of multiplicity 1.

36. $k\left(x\right) = 4x^4 - 65x^2 + 16$
$$0 = 4x^4 - 65x^2 + 16$$
$$0 = 4\left(x^2\right)^2 - 37x^2 + 9$$

Let $x^2 = y$.

$$0 = 4y^2 - 65y + 16$$
$$0 = 4y^2 - 64y - y + 16$$
$$0 = \left(4y - 1\right)\left(y - 16\right)$$

$$y = \frac{1}{4} \quad \text{or} \quad y = 16$$
$$x^2 = \frac{1}{4} \quad \text{or} \quad x^2 = 16$$
$$x = \pm\frac{1}{2} \qquad x = \pm 4$$
$$x = -4, -\frac{1}{2}, \frac{1}{2}, 4$$

Each zero is of multiplicity 1.

37. $n(x) = x^6 - 7x^4$
$$0 = x^4(x^2 - 7)$$
$$0 = x^4(x - \sqrt{7})(x + \sqrt{7})$$
$$x = 0, -\sqrt{7}, \sqrt{7}$$

The zero 0 has multiplicity 4. The zero $-\sqrt{7}$ has multiplicity 1. The zero $\sqrt{7}$ has multiplicity 1.

38. $m(x) = x^5 - 5x^3$
$$0 = x^3(x^2 - 5)$$
$$0 = x^3(x - \sqrt{5})(x + \sqrt{5})$$
$$x = 0, -\sqrt{5}, \sqrt{5}$$

The zero 0 has multiplicity 3. The zero $-\sqrt{5}$ has multiplicity 1. The zero $\sqrt{5}$ has multiplicity 1.

39. $f(x) = 2x^3 - 7x^2 - 14x + 30$
$$f(1) = 2(1)^3 - 7(1)^2 - 14(1) + 30$$
$$= 2 - 7 - 14 + 30 = 11$$
$$f(2) = 2(2)^3 - 7(2)^2 - 14(2) + 30$$
$$= 16 - 28 - 28 + 30 = -10$$
$$f(3) = 2(3)^3 - 7(3)^2 - 14(3) + 30$$
$$= 54 - 63 - 42 + 30 = -21$$
$$f(4) = 2(4)^3 - 7(4)^2 - 14(4) + 30$$
$$= 128 - 112 - 56 + 30 = -10$$
$$f(5) = 2(5)^3 - 7(5)^2 - 14(5) + 30$$
$$= 250 - 175 - 70 + 30 = 35$$

a. Yes. Since $f(1)$ and $f(2)$ have opposite signs, the intermediate value theorem guarantees that the function has at least one zero on the interval $[1, 2]$.

b. No. Since $f(2)$ and $f(3)$ have the same sign, the intermediate value theorem does not guarantee that the function has at least one zero on the interval $[2, 3]$.

c. No. Since $f(3)$ and $f(4)$ have the same sign, the intermediate value theorem does not guarantee that the function has at least one zero on the interval $[3, 4]$.

d. Yes. Since $f(4)$ and $f(5)$ have opposite signs, the intermediate value theorem guarantees that the function has at least one zero on the interval $[4, 5]$.

40. $g(x) = 2x^3 - 13x^2 + 18x + 5$
$$g(1) = 2(1)^3 - 13(1)^2 + 18(1) + 5$$
$$= 2 - 13 + 18 + 5 = 12$$
$$g(2) = 2(2)^3 - 13(2)^2 + 18(2) + 5$$
$$= 16 - 52 + 36 + 5 = 5$$
$$g(3) = 2(3)^3 - 13(3)^2 + 18(3) + 5$$
$$= 54 - 117 + 54 + 5 = -4$$
$$g(4) = 2(4)^3 - 13(4)^2 + 18(4) + 5$$
$$= 128 - 208 + 72 + 5 = -3$$
$$g(5) = 2(5)^3 - 13(5)^2 + 18(5) + 5$$
$$= 250 - 325 + 90 + 5 = 20$$

a. No. Since $g(1)$ and $g(2)$ have same sign, the intermediate value theorem does not guarantee that the function has at least one zero on the interval $\left[1, 2\right]$.

b. Yes. Since $g\left(2\right)$ and $g\left(3\right)$ have the opposite signs, the intermediate value theorem guarantees that the function has at least one zero on the interval $\left[2, 3\right]$.

c. No. Since $g\left(3\right)$ and $g\left(4\right)$ have the same sign, the intermediate value theorem does not guarantee that the function has at least one zero on the interval $\left[3, 4\right]$.

d. Yes. Since $g\left(4\right)$ and $g\left(5\right)$ have opposite signs, the intermediate value theorem guarantees that the function has at least one zero on the interval $\left[4, 5\right]$.

41. a. Yes. Since $\text{Y}_1\left(-4\right)$ and $\text{Y}_1\left(-3\right)$ have opposite signs, the intermediate value theorem guarantees that the function has at least one zero on the interval $\left[-4, -3\right]$.

b. Yes. Since $\text{Y}_1\left(-3\right)$ and $\text{Y}_1\left(-2\right)$ have opposite signs, the intermediate value theorem guarantees that the function has at least one zero on the interval $\left[-3, -2\right]$.

c. No. Since $\text{Y}_1\left(-2\right)$ and $\text{Y}_1\left(-1\right)$ have same sign, the intermediate value theorem does not guarantee that the function has at least one zero on the interval $\left[-2, -1\right]$.

d. No. Since $\text{Y}_1\left(-1\right)$ and $\text{Y}_1\left(0\right)$ have same sign, the intermediate value theorem does not guarantee that the function has at least one zero on the interval $\left[-1, 0\right]$.

42. a. Yes. Since $\text{Y}_1\left(-4\right)$ and $\text{Y}_1\left(-3\right)$ have opposite signs, the intermediate value theorem guarantees that the function has at least one zero on the interval $\left[-4, -3\right]$.

b. Yes. Since $\text{Y}_1\left(-3\right)$ and $\text{Y}_1\left(-2\right)$ have opposite signs, the intermediate value theorem guarantees that the function has at least one zero on the interval $\left[-3, -2\right]$.

c. No. Since $\text{Y}_1\left(-2\right)$ and $\text{Y}_1\left(-1\right)$ have same sign, the intermediate value theorem does not guarantee that the function has at least one zero on the interval $\left[-2, -1\right]$.

d. No. Since $\text{Y}_1\left(-1\right)$ and $\text{Y}_1\left(0\right)$ have same sign, the intermediate value theorem does not guarantee that the function has at least one zero on the interval $\left[-1, 0\right]$.

43. $f\left(x\right) = 4x^3 - 8x^2 - 25x + 50$

$$\begin{aligned} f\left(-3\right) &= 4\left(-3\right)^3 - 8\left(-3\right)^2 - 25\left(-3\right) + 50 \\ &= -108 - 72 + 75 + 50 = -55 \\ f\left(-2\right) &= 4\left(-2\right)^3 - 8\left(-2\right)^2 - 25\left(-2\right) + 50 \\ &= -32 - 32 + 50 + 50 = 36 \end{aligned}$$

a. Yes. Since $f\left(-3\right)$ and $f\left(-2\right)$ have opposite signs, the intermediate value theorem guarantees that the function has at least one zero on the interval $\left[-3,-2\right]$.

b. $f\left(x\right)=4x^3-8x^2-25x+50$

$$0=4x^3-8x^2-25x+50$$
$$0=4x^2\left(x-2\right)-25\left(x-2\right)$$
$$0=\left(x-2\right)\left(4x^2-25\right)$$
$$0=\left(x-2\right)\left(2x-5\right)\left(2x+5\right)$$
$$x=2, x=\frac{5}{2}, x=-\frac{5}{2}$$

The zero on the interval $\left[-3,-2\right]$ is $-\frac{5}{2}$.

44. $f\left(x\right)=9x^3-18x^2-100x+200$

$$f\left(-4\right)=\begin{bmatrix}9\left(-4\right)^3-18\left(-4\right)^2\\-100\left(-4\right)+200\end{bmatrix}$$
$$=-576-288+400+200$$
$$=-264$$
$$f\left(-3\right)=\begin{bmatrix}9\left(-3\right)^3-18\left(-3\right)^2\\-100\left(-3\right)+200\end{bmatrix}$$
$$=-243-162+300+200$$
$$=95$$

a. Yes. Since $f\left(-4\right)$ and $f\left(-3\right)$ have opposite signs, the intermediate value theorem guarantees that the function has at least one zero on the interval $\left[-4,-3\right]$.

b. $f\left(x\right)=9x^3-18x^2-100x+200$

$$0=9x^3-18x^2-100x+200$$
$$0=9x^2\left(x-2\right)-100\left(x-2\right)$$
$$0=\left(x-2\right)\left(9x^2-100\right)$$
$$0=\left(x-2\right)\left(3x-10\right)\left(3x+10\right)$$
$$x=2, x=\frac{10}{3}, x=-\frac{10}{3}$$

The zero on the interval $\left[-4,-3\right]$ is $-\frac{10}{3}$.

45. The graph is not smooth. It cannot represent a polynomial function.

46. The graph is not smooth. It cannot represent a polynomial function.

47. The graph is smooth and continuous. It can represent a polynomial function.

a. The function has 2 turning points, so the minimum degree is 3.

b. The end behavior is down on the left and up on the right. The leading coefficient is positive and the degree is odd.

c. –4 (odd multiplicity); -1 (odd multiplicity); 3 (odd multiplicity)

48. The graph is smooth and continuous. It can represent a polynomial function.

a. The function has 3 turning points, so the minimum degree is 4.

b. The end behavior is up on the left and up on the right. The leading coefficient is positive and the degree is even.

c. –4 (odd multiplicity); -1 (odd multiplicity); 3 (even multiplicity)

49. The graph is smooth and continuous. It can represent a polynomial function.

a. The function has 5 turning points, so the minimum degree is 6.

b. The end behavior is down on the left and down on the right. The leading coefficient is negative and the degree is even.

c. -4 (odd multiplicity); -3 (odd multiplicity); -1 (even multiplicity); 2 (odd multiplicity); $\frac{7}{2}$ (odd multiplicity).

50. The graph is smooth and continuous. It can represent a polynomial function.

a. The function has 2 turning points, so the minimum degree is 3.

b. The end behavior is up on the left and down on the right. The leading coefficient is negative and the degree is odd.

c. -4 (odd multiplicity); -1 (odd multiplicity); 2 (odd multiplicity)

51. The graph is not smooth. It cannot represent a polynomial function.

52. The graph is not smooth. It cannot represent a polynomial function.

53. a. $y = x^6$

b. Shrink $y = x^6$ vertically by a factor of $\frac{1}{3}$. Reflect across the x-axis. Shift downward 2 units.

c. Graph iii

54. a. $y = x^4$

b. Shift $y = x^4$ to the right 3 units. Shrink vertically by a factor of $\frac{1}{2}$. Reflect across the x-axis.

c. Graph v

55. a. $y = x^3$

b. Shift $y = x^3$ to the left 2 units. Reflect across the x-axis. Shift upward 3 units.

c. Graph i

56. a. $y = x^3$

b. Shift $y = x^3$ to the left 4 units. Stretch vertically by a factor of 2. Shift downward 3 units.

c. Graph vi

57. a. $y = x^5$

b. Shift $y = x^5$ to the right 3 units. Reflect across the y-axis. Shift upward 1 unit.

c. Graph iv

58. a. $y = x^4$

b. Shift $y = x^4$ to the left 3 units. Reflect across the y-axis. Shift downward 1 unit.

c. Graph ii

59. $f(x) = x^3 - 5x^2$

The leading term is x^3. The end behavior is down to the left and up to the right.

$f(0) = (0)^3 - 5(0)^2 = 0$

The y-intercept is $(0, 0)$.

$0 = x^3 - 5x^2$

$0 = x^2(x-5)$

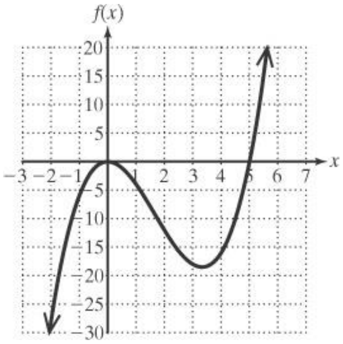
$x = 0, x = 5$

The zeros of the function are 0 (multiplicity 2) and 5 (multiplicity 1).

Test for symmetry:

$f(-x) = (-x)^3 - 5(-x)^2 = -x^3 - 5x^2$

$f(x)$ is neither even nor odd.



60. $g(x) = x^5 - 2x^4$

The leading term is x^5. The end behavior is down to the left and up to the right.

$g(0) = (0)^5 - 2(0)^4 = 0$

The *y*-intercept is $(0, 0)$.

$0 = x^5 - 2x^4$

$0 = x^4(x-2)$

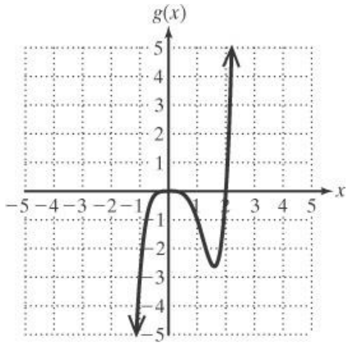
$x = 0, x = 2$

The zeros of the function are 0 (multiplicity 4) and 2 (multiplicity 1).

Test for symmetry:

$g(-x) = (-x)^5 - 2(-x)^4 = -x^5 - 2x^4$

$g(x)$ is neither even nor odd.



61. $f(x) = \frac{1}{2}(x-2)(x+1)(x+3)$

The leading term is $\frac{1}{2}(x)(x)(x) = \frac{1}{2}x^3$.

The end behavior is down to the left and up to

the right.

$$f(0) = \frac{1}{2}(0-2)(0+1)(0+3)$$
$$= \frac{1}{2}(-2)(1)(3)$$
$$= -3$$

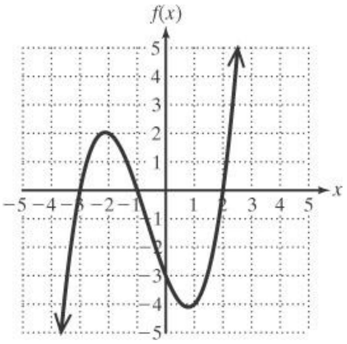
The *y*-intercept is $(0, -3)$.

The zeros of the function are 2 (multiplicity 1), -1 (multiplicity 1), and -3 (multiplicity 1).

Test for symmetry:

$f(-x) = \frac{1}{2}(-x-2)(-x+1)(-x+3)$

$f(x)$ is neither even nor odd.



62. $h(x)=\frac{1}{4}(x-1)(x-4)(x+2)$

The leading term is $\frac{1}{4}(x)(x)(x)=\frac{1}{4}x^3.$

The end behavior is down to the left and up to

the right.

$$h(0)=\frac{1}{4}(0-1)(0-4)(0+2)$$
$$=\frac{1}{4}(-1)(-4)2$$
$$=2$$

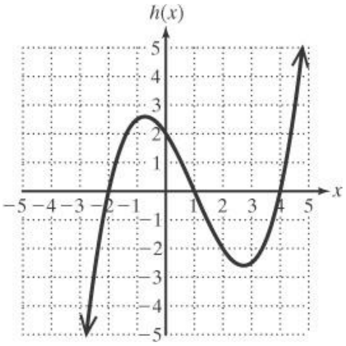
The y-intercept is $(0, 2)$.

The zeros of the function are 1 (multiplicity 1), 4 (multiplicity 1), and -2 (multiplicity 1).

Test for symmetry:

$$h(-x)=\frac{1}{4}(-x-1)(-x-4)(-x+2)$$

$h(x)$ is neither even nor odd.



63. $k(x)=x^4+2x^3-8x^2$

The leading term is x^4. The end behavior is up to the left and up to the right.

$$k(0)=(0)^4+2(0)^3-8(0)^2=0$$

The y-intercept is $(0, 0)$.

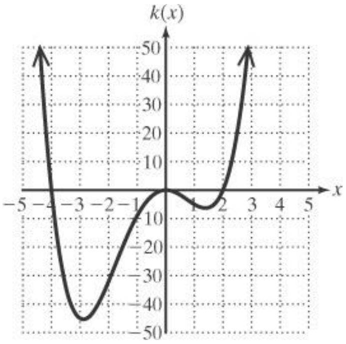
$$0=x^4+2x^3-8x^2$$
$$0=x^2(x^2+2x-8)$$
$$0=x^2(x+4)(x-2)$$
$$x=0, x=-4, x=2$$

The zeros of the function are 0 (multiplicity 2), -4 (multiplicity 1), and 2 (multiplicity 1).

Test for symmetry:

$$k(-x)=(-x)^4+2(-x)^3-8(-x)^2$$
$$=x^4-2x^3-8x^2$$

$k(x)$ is neither even nor odd.



64. $h(x)=x^4-x^3-6x^2$

The leading term is x^4. The end behavior is up to the left and up to the right.

$$h(0)=(0)^4-(0)^3-6(0)^2=0$$

The y-intercept is $(0, 0)$.

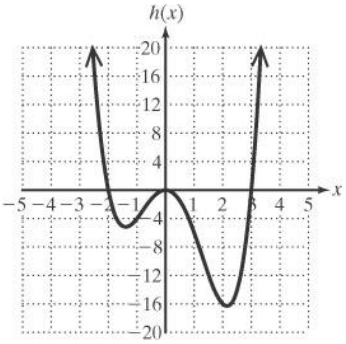
$$0=x^4-x^3-6x^2$$
$$0=x^2(x^2-x-6)$$
$$0=x^2(x+2)(x-3)$$
$$x=0, x=-2, x=3$$

The zeros of the function are 0 (multiplicity 2), -2 (multiplicity 1), and 3 (multiplicity 1).

Test for symmetry:

$$h(-x) = (-x)^4 - (-x)^3 - 6(-x)^2$$
$$= x^4 + x^3 - 6x^2$$

$h(x)$ is neither even nor odd.



65. $k(x) = 0.2(x+2)^2(x-4)^3$

The leading term is $0.2(x)^2(x)^3 = 0.2x^5$. The end behavior is down to the left and up to the right.

$$k(0) = 0.2(0+2)^2(0-4)^3$$
$$= 0.2(2)^2(-4)^3 = -51.2$$

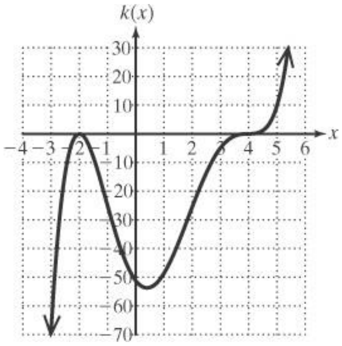
The y-intercept is $(0, -51.2)$.

The zeros of the function are -2 (multiplicity 2) and 4 (multiplicity 3).

Test for symmetry:

$$k(-x) = 0.2(-x+2)^2(-x-4)^3$$

$k(x)$ is neither even nor odd.



66. $m(x) = 0.1(x-3)^2(x+1)^3$

The leading term is $0.1(x)^2(x)^3 = 0.1x^5$. The end behavior is down to the left and up to the right.

$$m(0) = 0.1(0-3)^2(0+1)^3$$
$$= 0.1(-3)^2(1)^3$$
$$= 0.9$$

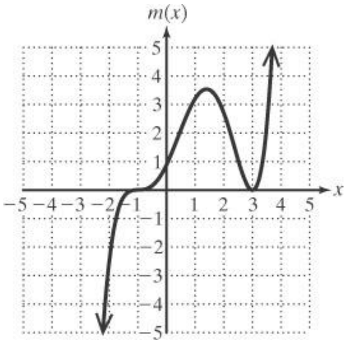
The y-intercept is $(0, 0.9)$.

The zeros of the function are 3 (multiplicity 2) and -1 (multiplicity 3).

Test for symmetry:

$$m(-x) = 0.1(-x-3)^2(-x+1)^3$$

$m(x)$ is neither even nor odd.



67. $p(x) = 9x^5 + 9x^4 - 25x^3 - 25x^2$

The leading term is x^5. The end behavior is down to the left and up to the right.

$$p(0)=\begin{bmatrix}9(0)^5+9(0)^4\\ -25(0)^3-25(0)^2\end{bmatrix}$$
$$=0$$

The y-intercept is $(0,0)$.

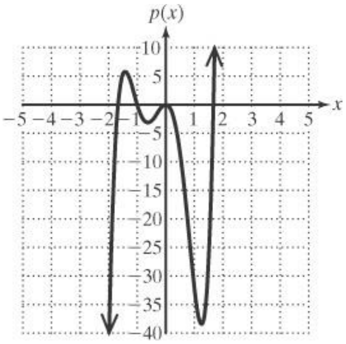
$$0=9x^5+9x^4-25x^3-25x^2$$
$$0=x^2(9x^3+9x^2-25x-25)$$
$$0=x^2\left[9x^2(x+1)-25(x+1)\right]$$
$$0=x^2(x+1)(9x^2-25)$$
$$0=x^2(x+1)(3x-5)(3x+5)$$
$$x=0,x=-1,x=\frac{5}{3},x=-\frac{5}{3}$$

The zeros of the function are 0 (multiplicity 2), -1 (multiplicity 1), $\frac{5}{3}$ (multiplicity 1), and $-\frac{5}{3}$ (multiplicity).

Test for symmetry:

$$p(-x)=\begin{bmatrix}9(-x)^5+9(-x)^4\\ -25(-x)^3-25(-x)^2\end{bmatrix}$$
$$=-9x^5+9x^4+25x^3-25x^2$$

$p(x)$ is neither even nor odd.



68. $q(x)=9x^5+18x^4-4x^3-8x^2$

The leading term is x^5. The end behavior is down to the left and up to the right.

$$q(0)=\begin{bmatrix}9(0)^5+18(0)^4\\ -4(0)^3-8(0)^2\end{bmatrix}$$
$$=0$$

The y-intercept is $(0,0)$.

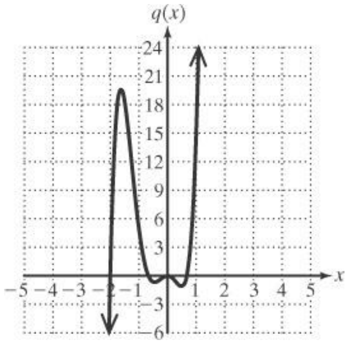
$$0=9x^5+18x^4-4x^3-8x^2$$
$$0=x^2(9x^3+18x^2-4x-8)$$
$$0=x^2\left[9x^2(x+2)-4(x+2)\right]$$
$$0=x^2(x+2)(9x^2-4)$$
$$0=x^2(x+2)(3x-2)(3x+2)$$
$$x=0,x=-2,x=\frac{2}{3},x=-\frac{2}{3}$$

The zeros of the function are 0 (multiplicity 2), -2 (multiplicity 1), $\frac{2}{3}$ (multiplicity 1), and $-\frac{2}{3}$ (multiplicity).

Test for symmetry:

$$q(-x)=\begin{bmatrix}9(-x)^5+18(-x)^4\\ -4(-x)^3-8(-x)^2\end{bmatrix}$$
$$=-9x^5+18x^4+4x^3-8x^2$$

$q(x)$ is neither even nor odd.



69. $t(x)=-x^4+11x^2-28$

The leading term is $-x^4$. The end behavior is down to the left and down to the right.

$$t(0) = -(0)^4 + 11(0)^2 - 28$$
$$= -28$$

The y-intercept is $(0, -28)$.

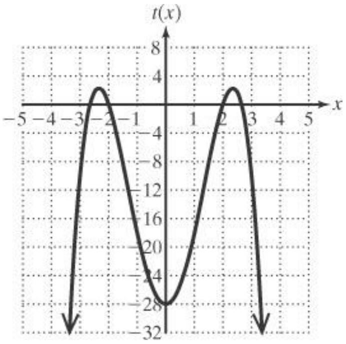
$$0 = -x^4 + 11x^2 - 28$$
$$0 = x^4 - 11x^2 + 28$$
$$0 = (x^2 - 4)(x^2 - 7)$$
$$0 = (x - 2)(x + 2)(x^2 - 7)$$
$$x = 2, x = -2, x = \sqrt{7}, x = -\sqrt{7}$$

The zeros of the function are 2 (multiplicity 1), -2 (multiplicity 1), $\sqrt{7}$ (multiplicity 1), and $-\sqrt{7}$ (multiplicity 1).

Test for symmetry:

$$t(-x) = -(-x)^4 + 11(-x)^2 - 28$$
$$= -x^4 + 11x^2 - 28$$

$t(x)$ is even, so it is symmetric with respect to the y-axis.



70. $v(x) = -x^4 + 15x^2 - 44$

The leading term is $-x^4$. The end behavior is down to the left and down to the right.

$$v(0) = -(0)^4 + 15(0)^2 - 44$$
$$= -44$$

The y-intercept is $(0, -44)$.

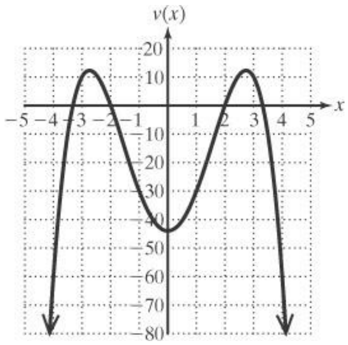
$$0 = -x^4 + 15x^2 - 44$$
$$0 = x^4 - 15x^2 + 44$$
$$0 = (x^2 - 4)(x^2 - 11)$$
$$0 = (x - 2)(x + 2)(x^2 - 11)$$
$$x = 2, x = -2, x = \sqrt{11}, x = -\sqrt{11}$$

The zeros of the function are 2 (multiplicity 1), -2 (multiplicity 1), $\sqrt{11}$ (multiplicity 1), and $-\sqrt{11}$ (multiplicity 1).

Test for symmetry:

$$v(-x) = -(-x)^4 + 15(-x)^2 - 44$$
$$= -x^4 + 15x^2 - 44$$

$v(x)$ is even, so it is symmetric with respect to the y-axis.



71. $g(x) = -x^4 + 5x^2 - 4$

The leading term is $-x^4$. The end behavior is down to the left and down to the right.

$$g(0) = -(0)^4 + 5(0)^2 - 4 = -4$$

The y-intercept is $(0, -4)$.

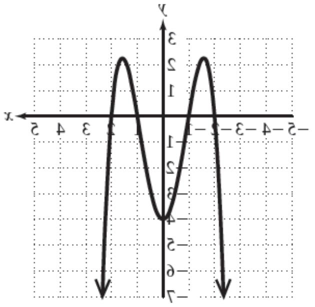
$$0 = -x^4 + 5x^2 - 4$$
$$0 = x^4 - 5x^2 + 4$$
$$0 = \left(x^2 - 4\right)\left(x^2 - 1\right)$$
$$0 = \left(x - 2\right)\left(x + 2\right)\left(x - 1\right)\left(x + 1\right)$$
$$x = 2, x = -2, x = 1, x = -1$$

The zeros of the function are 2 (multiplicity 1), -2 (multiplicity 1), 1 (multiplicity 1), and -1 (multiplicity).

Test for symmetry:

$$g\left(-x\right) = -\left(-x\right)^4 + 5\left(-x\right)^2 - 4$$
$$= -x^4 + 5x^2 - 4$$

$g\left(x\right)$ is even, so it is symmetric with respect to the *y*-axis.



72. $h\left(x\right) = -x^4 + 10x^2 - 9$

The leading term is $-x^4$. The end behavior is down to the left and down to the right.

$$g\left(0\right) = -\left(0\right)^4 + 10\left(0\right)^2 - 9 = -9$$

The *y*-intercept is $\left(0, -9\right)$.

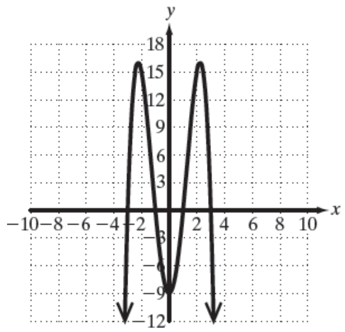
$$0 = -x^4 + 10x^2 - 9$$
$$0 = x^4 - 10x^2 + 9$$
$$0 = \left(x^2 - 9\right)\left(x^2 - 1\right)$$
$$0 = \left(x - 3\right)\left(x + 3\right)\left(x - 1\right)\left(x + 1\right)$$
$$x = 3, x = -3, x = 1, x = -1$$

The zeros of the function are 3 (multiplicity 1), -3 (multiplicity 1), 1 (multiplicity 1), and -1 (multiplicity).

Test for symmetry:

$$h\left(-x\right) = -\left(-x\right)^4 + 10\left(-x\right)^2 - 9$$
$$= -x^4 + 10x^2 - 9$$

$h\left(x\right)$ is even, so it is symmetric with respect to the *y*-axis.



73. $c\left(x\right) = 0.1x\left(x - 2\right)^4\left(x + 2\right)^3$

The leading term is $0.1x\left(x\right)^4\left(x\right)^3 = 0.1x^8$. The end behavior is up to the left and up to the right.

$$c\left(0\right) = 0.1\left(0\right)\left(0 - 2\right)^4\left(0 + 2\right)^3 = 0$$

The *y*-intercept is $\left(0, 0\right)$.

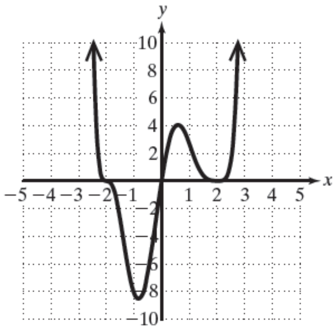
$$0 = 0.1x\left(x - 2\right)^4\left(x + 2\right)^3$$
$$x = 0,\ \left(x - 2\right)^4 = 0,\ \left(x + 2\right)^3 = 0$$
$$x = 0,\ x = 2,\ x = -2$$

The zeros of the function are 0 (multiplicity 1), 2 (multiplicity 4) and -2 (multiplicity 3).

Test for symmetry:

$$c\left(-x\right) = 0.1\left(-x\right)\left(-x - 2\right)^4\left(-x + 2\right)^3$$

$c\left(x\right)$ is neither even nor odd.



74. $d(x) = 0.05x(x-2)^4(x+3)^2$

The leading term is $0.05x(x)^4(x)^2 = 0.05x^7$. The end behavior is down to the left and up to the right.

$d(0) = 0.05(0)(0-2)^4(0+3)^2 = 0$

The y-intercept is $(0, 0)$.

$0 = 0.05x(x-2)^4(x+3)^2$

$x = 0,\ (x-2)^4 = 0,\ (x+3)^2 = 0$

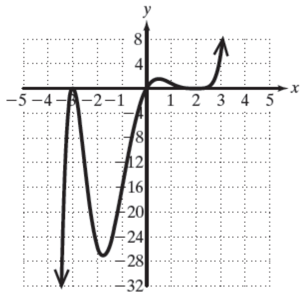
$x = 0,\ x = 2,\ x = -3$

The zeros of the function are 0 (multiplicity 1), 2 (multiplicity 4) and -3 (multiplicity 2).

Test for symmetry:

$d(-x) = 0.05(-x)(-x-2)^4(-x+3)^2$

$d(x)$ is neither even nor odd.



75. $m(x) = -\frac{1}{10}(x+3)(x-3)(x+1)^3$

The leading term is $-\frac{1}{10}(x)(x)(x)^3 = -\frac{1}{10}x^5$. The end behavior is up to the left and down to the right.

$$\begin{aligned} m(0) &= -\frac{1}{10}(0+3)(0-3)(0+1)^3 \\ &= -\frac{1}{10}(3)(-3)(1)^3 \\ &= \frac{9}{10} = 0.9 \end{aligned}$$

The y-intercept is $(0, 0.9)$.

$0 = -\frac{1}{10}(x+3)(x-3)(x+1)^3$

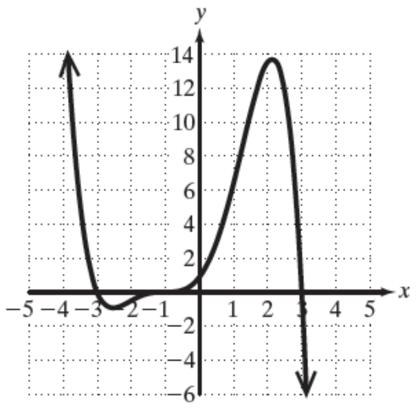
$x = -3,\ x = 3,\ x = -1$

The zeros of the function are 3 (multiplicity 1), -3 (multiplicity 1), and -1 (multiplicity 3).

Test for symmetry:

$m(-x) = -\frac{1}{10}(-x+3)(-x-3)(-x+1)^3$

$m(x)$ is neither even nor odd.



76. $f(x) = -\frac{1}{10}(x-1)(x+3)(x-4)^2$

The leading term is

$-\frac{1}{10}(x)(x)(x)^2=-\frac{1}{10}x^4$. The end behavior is down to the left and down to the right.

$$\begin{aligned} f(0)&=-\frac{1}{10}(0-1)(0+3)(0-4)^2\\ &=-\frac{1}{10}(-1)(3)(-4)^2\\ &=-\frac{48}{10}=-4.8 \end{aligned}$$

The y-intercept is $(0,-4.8)$.

$$0=-\frac{1}{10}(x-1)(x+3)(x-4)^2$$

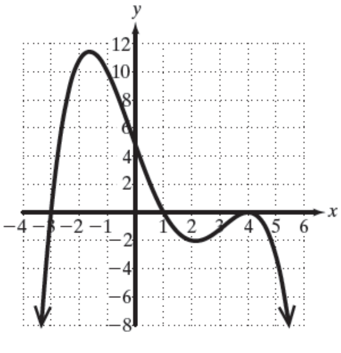
$x=1,\ x=-3,\ x=4$

The zeros of the function are 1 (multiplicity 1), –3 (multiplicity 1), and 4 (multiplicity 2).

Test for symmetry:

$$f(-x)=-\frac{1}{10}(-x-1)(-x+3)(-x-4)^2$$

$f(x)$ is neither even nor odd.



77. False. The value 5 is a zero with even multiplicity. Therefore, the graph touches but does not cross the x-axis at 5.

78. True

79. False. An nth-degree polynomial has at most $n-1$ turning points. Therefore, a third-degree polynomial has at most 2 turning points.

80. False. A third-degree polynomial may have at most 2 turning points, but may have fewer. For example the graph of $y=x^3$ has no turning points.

81. True

82. False. There are infinitely many polynomials. For example, $f(x)=(x-2)(x-4)(x-6)$ and $g(x)=2(x-2)(x-4)(x-6)$ are two polynomials with the required zeros.

83. False. If the leading coefficient is negative, the graph will be down to the far left and down to the far right.

84. True

85. False. The only real solution to the equation $x^3-27=0$ is $x=3$. Therefore, the graph of $f(x)=x^3-27$ has only one x-intercept.

86. True

87. True

88. False. A fourth-degree polynomial may have at most 3 turning points and consequently 3 relative maxima or minima.

89. **a.** The acceleration is increasing over the interval $(0,12)\cup(68,184)$.

b. The acceleration is decreasing over the interval $(12,68)\cup(184,200)$.

c. The graph shows 3 turning points.

d. The minimum degree of a polynomial function with 3 turning points is 4. The leading coefficient would be negative since the end behavior of the graph is down on the left and down on the right.

e. The acceleration is greatest approximately 184 sec after launch.

f. The maximum acceleration is approximately 2.85 G-forces.

90. a. The interval over which the number of new AIDS cases among 20- to 24-yr-olds increased is $\left(0, 8\right) \cup \left(14, 20\right)$.

b. The interval over which the number of new AIDS cases among 20- to 24-yr-olds decreased is $\left(8, 14\right)$.

c. The graph shows 2 turning points.

d. The minimum degree of a polynomial function with 2 turning points is 3. The leading coefficient would be positive since the end behavior of the graph is down on the left and up on the right.

e. The the number of new AIDS cases among 20- to 24- yr-olds will be greatest 8 years after the study began.

f. The maximum number of new case diagnosed in a single year is approximately 2648.

91. The *x*-intercepts are the real solutions to the equation $f\left(x\right) = 0$.

92. An *x*-intercept is a cross point if the corresponding zero of the polynomial has an odd multiplicity. An *x*-intercept is a touch point if the corresponding zero of the polynomial has an even multiplicity.

93. A function is continuous if its graph can be drawn without lifting the pencil from the paper.

94. All polynomial functions $y = f\left(x\right)$ are continuous. Therefore, for $a < b$, if $f\left(a\right)$ and $f\left(b\right)$ have different signs, then at some point on the interval $\left[a, b\right]$, $f\left(x\right)$ must be 0. That is, for the function to change sign from positive to negative or from negative to positive, the function must have a *y* value of 0 somewhere in between.

95. a. $f\left(3\right) = \left(3\right)^2 - 3\left(3\right) + 2 = 2$

$f\left(4\right) = \left(4\right)^2 - 3\left(4\right) + 2 = 6$

b. By the intermediate value theorem, because $f\left(3\right) = 2$ and $f\left(4\right) = 6$, then *f* must take on every value between 2 and 6 on the interval $\left[3, 4\right]$.

c. $f\left(x\right) = x^2 - 3x + 2$

$$4 = x^2 - 3x + 2$$
$$0 = x^2 - 3x - 2$$
$$x = \frac{-\left(-3\right) \pm \sqrt{\left(-3\right)^2 - 4\left(1\right)\left(-2\right)}}{2\left(1\right)}$$
$$= \frac{3 \pm \sqrt{17}}{2} \approx 3.56 \text{ or } -0.56$$

On the interval $\left[3, 4\right]$,

$$x = \frac{3 + \sqrt{17}}{2} \approx 3.56.$$

96. a. $f\left(-4\right)=-\left(-4\right)^2-4\left(-4\right)+3=3$

$f\left(-3\right)=-\left(-3\right)^2-4\left(-3\right)+3=6$

b. By the intermediate value theorem, because $f\left(-4\right)=3$ and $f\left(-3\right)=6$, then *f* must take on every value between 3 and 6 on the interval $\left[-4,-3\right]$.

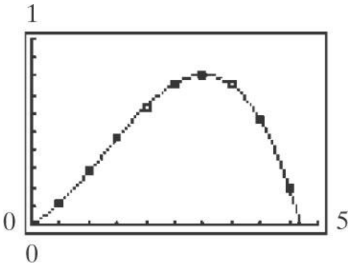
c. $f\left(x\right)=-x^2-4x+3$

$$5=-x^2-4x+3$$
$$0=-x^2-4x-2$$
$$0=x^2+4x+2$$
$$x=\frac{-4\pm\sqrt{\left(4\right)^2-4\left(1\right)\left(2\right)}}{2\left(1\right)}$$
$$=\frac{-4\pm\sqrt{8}}{2}=\frac{-4\pm2\sqrt{2}}{2}$$
$$=-2\pm\sqrt{2}\approx-0.59\text{ or }-3.41$$

On the interval $\left[-4,-3\right]$,

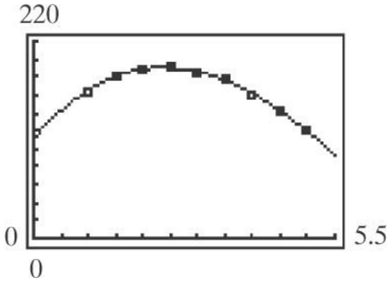
$x=-2-\sqrt{2}\approx-3.41$.

97.



$$V\left(t\right)=\left[\begin{array}{l}-0.0406t^3+0.154t^2\\ \quad+0.173t-0.0024\end{array}\right]$$

98.



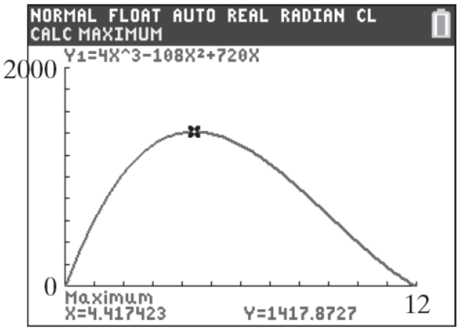
$$T\left(x\right)=\left[\begin{array}{l}0.76x^3-17.7x^2\\ \quad+71.2x+111\end{array}\right]$$

99. a. $V=lwh$

$$=\left(30-2x\right)\left(24-2x\right)\left(x\right)$$
$$=4x^3-108x^2+720x$$

The domain is restricted to $0<x<12$ because the width of the rectangular sheet is 24 in. The maximum amount that can be removed from each end would be half of 24 in.

b.



NORMAL FLOAT AUTO REAL RADIAN CL
CALC MAXIMUM
Y1=4X^3-108X²+720X
Maximum
X=4.417423 Y=1417.8727

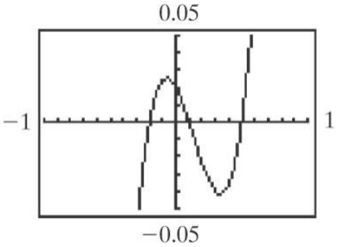
4.4 in.

c. $V\left(4.4\right)=\left[\begin{array}{l}4\left(4.4\right)^3-108\left(4.4\right)^2\\ \quad+720\left(4.4\right)\end{array}\right]$

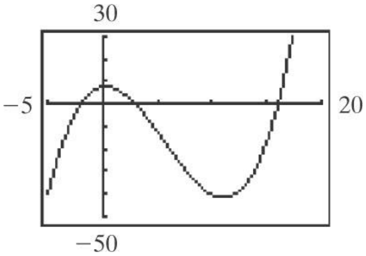
$$=\left[\begin{array}{l}340.736-2090.88\\ \quad+3168\end{array}\right]$$
$$=1417.856$$
$$=1418$$

1418 in.^{3}

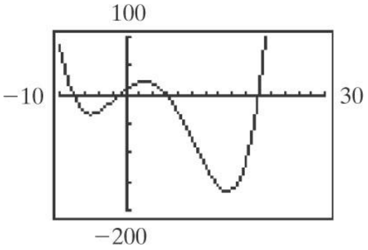
100. Window b is better.



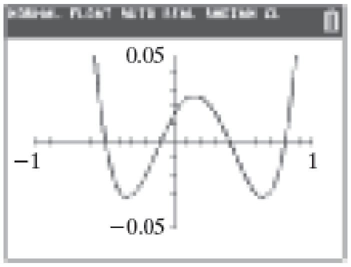
101. Window b is better.



102.



103.



Section 2.3 Division of Polynomials and the Remainder and Factor Theorems

1. Dividend: $f(x)$; Divisor: $d(x)$;
Quotient: $q(x)$; Remainder: $r(x)$

2. $(x-3)(2x^2+x-3)+(-8)$

$$=\left[\begin{array}{l}2x^3+x^2-3x\\ \quad -6x^2-3x+9-8\end{array}\right]$$

$$=2x^3-5x^2-6x+1$$

3. $f(c)$

4. factor; 0

5. True

6. False

7. a.
$$\begin{array}{r} 3x+12 \\ 2x-5\overline{)6x^2+9x+5} \\ -(6x^2-15x) \\ \hline 24x+5 \\ -(24x-60) \\ \hline 65 \end{array}$$

b. Dividend: $6x^2+9x+5$; Divisor: $2x-5$; Quotient: $3x+12$; Remainder: 65

c. $(2x-5)(3x+12)+65$

$$=6x^2+24x-15x-60+65$$

$$=6x^2+9x+5$$

8. a.

$$\begin{array}{r} 4x-2 \\ 3x+4\overline{)12x^2+10x+3} \\ -\left(12x^2+16x\right) \\ \hline -6x+3 \\ -\left(-6x-8\right) \\ \hline 11 \end{array}$$

b. Dividend: $12x^2+10x+3$; Divisor: $3x+4$; Quotient: $4x-2$; Remainder: 11

c. $\left(3x+4\right)\left(4x-2\right)+11$

$$=12x^2-6x+16x-8+11$$

$$=12x^2+10x+3$$

9.

$$\begin{array}{r} 3x^2+x+4 \\ x-4\overline{)3x^3-11x^2+0x-10} \\ -\left(3x^3-12x^2\right) \\ \hline x^2+0x \\ -\left(x^2-4x\right) \\ \hline 4x-10 \\ -\left(4x-16\right) \\ \hline 6 \end{array}$$

$$3x^2+x+4+\frac{6}{x-4}$$

10.

$$\begin{array}{r} 2x^2+3x+15 \\ x-5\overline{)2x^3-7x^2+0x-65} \\ -\left(2x^3-10x^2\right) \\ \hline 3x^2+0x \\ -\left(3x^2-15x\right) \\ \hline 15x-65 \\ -\left(15x-75\right) \\ \hline 10 \end{array}$$

$$2x^2+3x+15+\frac{10}{x-5}$$

11.

$$\begin{array}{r} 4x^3-20x^2+13x+4 \\ x+2\overline{)4x^4-12x^3-27x^2+30x+8} \\ -\left(4x^4+8x^3\right) \\ \hline -20x^3-27x^2 \\ -\left(-20x^3-40x^2\right) \\ \hline 13x^2+30x \\ -\left(13x^2+26x\right) \\ \hline 4x+8 \\ -\left(4x+8\right) \\ \hline 0 \end{array}$$

$$4x^3-20x^2+13x+4$$

12.

$$\begin{array}{r} 3x^3+8x^2-4x-16 \\ x+3\overline{)3x^4+17x^3+20x^2-28x-48} \\ -\left(3x^4+9x^3\right) \\ \hline 8x^3+20x^2 \\ -\left(8x^3+24x^2\right) \\ \hline -4x^2-28x \\ -\left(-4x^2-12x\right) \\ \hline -16x-48 \\ -\left(-16x-48\right) \\ \hline 0 \end{array}$$

$$3x^3+8x^2-4x-16$$

13.

$$\begin{array}{r}
3x^3-6x^2+2x-4\\
2x+4\overline{\big)6x^4+\ \ 0x^3-20x^2+0x-16}\\
-\underline{\left(6x^4+12x^3\right)}\qquad\qquad\qquad\qquad\\
-12x^3-20x^2\qquad\qquad\quad\\
-\underline{\left(-12x^3-24x^2\right)}\qquad\qquad\quad\\
4x^2+0x\qquad\quad\\
-\underline{\left(4x^2+8x\right)}\qquad\quad\\
-8x-16\\
-\underline{\left(-8x-16\right)}\\
0
\end{array}$$

$3x^3-6x^2+2x-4$

14.

$$\begin{array}{r}
4x^3+12x^2+6x+18\\
2x-6\overline{\big)8x^4+\ \ 0x^3-60x^2+\ \ 0x-108}\\
-\underline{\left(8x^4-24x^3\right)}\qquad\qquad\qquad\qquad\\
24x^3-60x^2\qquad\qquad\quad\\
-\underline{\left(24x^3-72x^2\right)}\qquad\qquad\quad\\
12x^2+\ \ 0x\qquad\quad\\
-\underline{\left(12x^2-36x\right)}\qquad\quad\\
36x-108\\
-\underline{\left(36x-108\right)}\\
0
\end{array}$$

$4x^3+12x^2+6x+18$

15.

$$\begin{array}{r}
x^3+4x^2-5x-2\\
x^2+5\overline{\big)x^5+4x^4+0x^3+18x^2-20x-10}\\
-\underline{\left(x^5+0x^4+5x^3\right)}\qquad\qquad\qquad\qquad\qquad\\
4x^4-5x^3+18x^2\qquad\qquad\qquad\\
-\underline{\left(4x^4+0x^3+20x^2\right)}\qquad\qquad\qquad\\
-5x^3-\ \ 2x^2-20x\qquad\quad\\
-\underline{\left(-5x^3\ +\ 0x^2-25x\right)}\qquad\quad\\
-2x^2\ +\ 5x-10\\
-\underline{\left(-2x^2\ +\ 0x-10\right)}\\
5x
\end{array}$$

$$x^3+4x^2-5x-2+\frac{5x}{x^2+5}$$

16.

$$\begin{array}{r}
x^3-2x^2+4x-6\\
x^2-3\overline{\big)x^5-2x^4+\ \ x^3+0x^2-\ 8x+18}\\
-\underline{\left(x^5+0x^4-3x^3\right)}\qquad\qquad\qquad\qquad\quad\\
-2x^4+4x^3+0x^2\qquad\qquad\qquad\\
-\underline{\left(-2x^4+0x^3+6x^2\right)}\qquad\qquad\qquad\\
4x^3-6x^2-\ \ 8x\qquad\quad\\
-\underline{\left(4x^3+\ 0x^2-12x\right)}\qquad\quad\\
-6x^2\ +\ 4x+18\\
-\underline{\left(-6x^2\ +\ 0x+18\right)}\\
4x
\end{array}$$

$$x^3-2x^2+4x-6+\frac{4x}{x^2-3}$$

17.

$$\begin{array}{r}
3x^2+1\\
2x^2+x-3\overline{\big)6x^4+3x^3-7x^2+6x-5}\\
-\underline{\left(6x^4+3x^3-9x^2\right)}\qquad\qquad\\
2x^2+6x-5\\
-\underline{\left(2x^2+\ \ x-3\right)}\\
5x-2
\end{array}$$

$$3x^2+1+\frac{5x-2}{2x^2+x-3}$$

18.
$$\begin{array}{r}
4x^2-1 \\
3x^2-x+4\overline{)12x^4-4x^3+13x^2+2x+1} \\
-\left(12x^4-4x^3+16x^2\right) \qquad\qquad \\
\hline
-3x^2+2x+1 \\
-\left(-3x^2+\ \ x-4\right) \\
\hline
x+5
\end{array}$$

$$4x^2-1+\frac{x+5}{3x^2-x+4}$$

19.
$$\begin{array}{r}
x^2+3x+9 \\
x-3\overline{)x^3+0x^2+0x-27} \\
-\left(x^3-3x^2\right) \qquad\qquad \\
\hline
3x^2+0x \qquad \\
-\left(3x^2-9x\right) \qquad \\
\hline
9x-27 \\
-\left(9x-27\right) \\
\hline
0
\end{array}$$

$$x^2+3x+9$$

20.
$$\begin{array}{r}
x^2-4x+16 \\
x+4\overline{)x^3+0x^2+0x+64} \\
-\left(x^3+4x^2\right) \qquad\qquad \\
\hline
-4x^2+\ 0x \qquad \\
-\left(-4x^2-16x\right) \qquad \\
\hline
16x+64 \\
-\left(16x+64\right) \\
\hline
0
\end{array}$$

$$x^2-4x+16$$

21.
$$\begin{array}{r}
\frac{5}{2}x^2+\frac{1}{4}x+\frac{1}{8} \\
2x-1\overline{)5x^3-2x^2+0x+3} \\
-\left(5x^3-\frac{5}{2}x^2\right) \qquad\qquad \\
\hline
\frac{1}{2}x^2+0x \qquad \\
-\left(\frac{1}{2}x^2-\frac{1}{4}x\right) \qquad \\
\hline
\frac{1}{4}x+3 \\
-\left(\frac{1}{4}x-\frac{1}{8}\right) \\
\hline
\frac{25}{8}
\end{array}$$

$$\frac{5}{2}x^2+\frac{1}{4}x+\frac{1}{8}+\frac{\frac{25}{8}}{2x-1}$$

22.
$$\begin{array}{r}
\frac{2}{3}x^2+\frac{1}{9}x-\frac{1}{27} \\
3x+1\overline{)2x^3+x^2+0x+1} \\
-\left(2x^3+\frac{2}{3}x^2\right) \qquad\qquad \\
\hline
\frac{1}{3}x^2+0x \qquad \\
-\left(\frac{1}{3}x^2+\frac{1}{9}x\right) \qquad \\
\hline
-\frac{1}{9}x+1 \\
-\left(-\frac{1}{9}x-\frac{1}{27}\right) \\
\hline
\frac{28}{27}
\end{array}$$

$$\frac{2}{3}x^2+\frac{1}{9}x-\frac{1}{27}+\frac{\frac{28}{27}}{3x+1}$$

23. a. Dividend: $2x^4-5x^3-5x^2-4x+29$

b. Divisor: $x-3$

c. Quotient: $2x^3+x^2-2x-10$

d. Remainder: -1

24. a. Dividend: $x^4-5x^3+2x^2-x+2$

b. Divisor: $x-2$

c. Quotient: x^3-3x^2-4x-9

d. Remainder: 2

25. a. Dividend: $x^3-2x^2-25x-4$

b. Divisor: $x+4$

c. Quotient: x^2-6x-1

d. Remainder: 0

26. a. Dividend: $3x^3+13x^2-14x-20$

b. Divisor: $x+5$

c. Quotient: $3x^2-2x-4$

d. Remainder: 0

27.
$$\begin{array}{r|rrr} -6 & 4 & 15 & 1 \\ & & -24 & 54 \\ \hline & 4 & -9 & \boxed{55} \end{array}$$

$$4x-9+\frac{55}{x+6}$$

28.
$$\begin{array}{r|rrr} -5 & 6 & 25 & -19 \\ & & -30 & 25 \\ \hline & 6 & -5 & \boxed{6} \end{array}$$

$$6x-5+\frac{6}{x+5}$$

29.
$$\begin{array}{r|rrr} 4 & 5 & -17 & -12 \\ & & 20 & 12 \\ \hline & 5 & 3 & \boxed{0} \end{array}$$

$$5x+3$$

30.
$$\begin{array}{r|rrr} 3 & 2 & 1 & -21 \\ & & 6 & 21 \\ \hline & 2 & 7 & \boxed{0} \end{array}$$

$$2x+7$$

31.
$$\begin{array}{r|rrrrr} -2 & -5 & 0 & -3 & -8 & 4 \\ & & 10 & -20 & 46 & -76 \\ \hline & -5 & 10 & -23 & 38 & \boxed{-72} \end{array}$$

$$-5x^3+10x^2-23x+38+\frac{-72}{x+2}$$

32.
$$\begin{array}{r|rrrrr} -1 & -2 & 5 & 0 & 2 & -5 \\ & & 2 & -7 & 7 & -9 \\ \hline & -2 & 7 & -7 & 9 & \boxed{-14} \end{array}$$

$$-2x^3+7x^2-7x+9+\frac{-14}{x+1}$$

33.
$$\begin{array}{r|rrrrrr} 3 & 4 & -25 & -58 & 232 & 198 & -63 \\ & & 12 & -39 & -291 & -177 & 63 \\ \hline & 4 & -13 & -97 & -59 & 21 & \boxed{0} \end{array}$$

$$4x^4-13x^3-97x^2-59x+21$$

34.
$$\begin{array}{r|rrrrrr} 2 & 2 & 13 & -3 & -58 & -20 & 24 \\ & & 4 & 34 & 62 & 8 & -24 \\ \hline & 2 & 17 & 31 & 4 & -12 & \boxed{0} \end{array}$$

$$2x^4+17x^3+31x^2+4x-12$$

35.
$$\begin{array}{r|rrrrrr} -2 & 1 & 0 & 0 & 0 & 0 & 32 \\ & & -2 & 4 & -8 & 16 & -32 \\ \hline & 1 & -2 & 4 & -8 & 16 & \boxed{0} \end{array}$$

$$x^4-2x^3+4x^2-8x+16$$

36.
$$\begin{array}{r|rrrrr} -3 & 1 & 0 & 0 & 0 & -81 \\ & & -3 & 9 & -27 & 81 \\ \hline & 1 & -3 & 9 & -27 & \boxed{0} \end{array}$$

$$x^3-3x^2+9x-27$$

37.
$$\begin{array}{r|rrrrr} \frac{3}{2} & 2 & -7 & -56 & 37 & 84 \\ & & 3 & -6 & -93 & 84 \\ \hline & 2 & -4 & -62 & -56 & \boxed{0} \end{array}$$

$$2x^3-4x^2-62x-56$$

38.
$$\begin{array}{r|rrrrr} \frac{2}{5} & -5 & -18 & 63 & 128 & -60 \\ & & -2 & -8 & 22 & 60 \\ \hline & -5 & -20 & 55 & 150 & \boxed{0} \end{array}$$

$-5x^3-20x^2+55x+150$

39. The remainder is 39.

40. 12

41. a. $f(x)=2x^4-5x^3+x^2-7$

$$\begin{aligned} f(4)&=2(4)^4-5(4)^3+(4)^2-7 \\ &=2(256)-5(64)+16-7 \\ &=512-320+16-7=201 \end{aligned}$$

b.
$$\begin{array}{r|rrrrr} 4 & 2 & -5 & 1 & 0 & -7 \\ & & 8 & 12 & 52 & 208 \\ \hline & 2 & 3 & 13 & 52 & \boxed{201} \end{array}$$

The remainder is 201.

42. a. $g(x)=-3x^5+2x^4+6x^2-x+4$

$$\begin{aligned} g(2)&=\left[\begin{array}{l} -3(2)^5+2(2)^4 \\ \quad +6(2)^2-(2)+4 \end{array}\right] \\ &=\left[\begin{array}{l} -3(32)+2(16) \\ \quad +6(4)-2+4 \end{array}\right] \\ &=-96+32+24-2+4 \\ &=-38 \end{aligned}$$

b.
$$\begin{array}{r|rrrrrr} 2 & -3 & 2 & 0 & 6 & -1 & 4 \\ & & -6 & -8 & -16 & -20 & -42 \\ \hline & -3 & -4 & -8 & -10 & -21 & \boxed{-38} \end{array}$$

The remainder is -38.

43. a.
$$\begin{array}{r|rrrrr} -1 & 2 & 1 & -49 & 79 & 15 \\ & & -2 & 1 & 48 & -127 \\ \hline & 2 & -1 & -48 & 127 & \boxed{-112} \end{array}$$

By the remainder theorem, $f(-1)=-112$.

b.
$$\begin{array}{r|rrrrr} 3 & 2 & 1 & -49 & 79 & 15 \\ & & 6 & 21 & -84 & -15 \\ \hline & 2 & 7 & -28 & -5 & \boxed{0} \end{array}$$

By the remainder theorem, $f(3)=0$.

c.
$$\begin{array}{r|rrrrr} 4 & 2 & 1 & -49 & 79 & 15 \\ & & 8 & 36 & -52 & 108 \\ \hline & 2 & 9 & -13 & 27 & \boxed{123} \end{array}$$

By the remainder theorem, $f(4)=123$.

d.
$$\begin{array}{r|rrrrr} \frac{5}{2} & 2 & 1 & -49 & 79 & 15 \\ & & 5 & 15 & -85 & -15 \\ \hline & 2 & 6 & -34 & -6 & \boxed{0} \end{array}$$

By the remainder theorem, $f\left(\frac{5}{2}\right)=0$.

44. a.
$$\begin{array}{r|rrrrr} -1 & 3 & -22 & 51 & -42 & 8 \\ & & -3 & 25 & -76 & 118 \\ \hline & 3 & -25 & 76 & -118 & \boxed{126} \end{array}$$

By the remainder theorem, $g(-1)=126$.

b.
$$\begin{array}{r|rrrrr} 2 & 3 & -22 & 51 & -42 & 8 \\ & & 6 & -32 & 38 & -8 \\ \hline & 3 & -16 & 19 & -4 & \boxed{0} \end{array}$$

By the remainder theorem, $g(2)=0$.

c.
$$\begin{array}{r|rrrrr} 1 & 3 & -22 & 51 & -42 & 8 \\ & & 3 & -19 & 32 & -10 \\ \hline & 3 & -19 & 32 & -10 & \boxed{-2} \end{array}$$

By the remainder theorem, $g(1)=-2$.

d. $$\begin{array}{r|rrrrr} \frac{4}{3} & 3 & -22 & 51 & -42 & 8 \\ & & 4 & -24 & 36 & -8 \\ \hline & 3 & -18 & 27 & -6 & \lfloor 0 \end{array}$$

By the remainder theorem,

$g\left(\frac{4}{3}\right)=0$.

45. a. $$\begin{array}{r|rrrr} 1 & 5 & -4 & -15 & 12 \\ & & 5 & 1 & -14 \\ \hline & 5 & 1 & -14 & \lfloor -2 \end{array}$$

By the remainder theorem, $h(1)=-2$.

b. $$\begin{array}{r|rrrr} \frac{4}{5} & 5 & -4 & -15 & 12 \\ & & 4 & 0 & -12 \\ \hline & 5 & 0 & -15 & \lfloor 0 \end{array}$$

By the remainder theorem,

$h\left(\frac{4}{5}\right)=0$.

c. $$\begin{array}{r|rrrr} \sqrt{3} & 5 & -4 & -15 & 12 \\ & & 5\sqrt{3} & 15-4\sqrt{3} & -12 \\ \hline & 5 & -4+5\sqrt{3} & -4\sqrt{3} & \lfloor 0 \end{array}$$

By the remainder theorem,

$h\left(\sqrt{3}\right)=0$.

d. $$\begin{array}{r|rrrr} -1 & 5 & -4 & -15 & 12 \\ & & -5 & 9 & 6 \\ \hline & 5 & -9 & -6 & \lfloor 18 \end{array}$$

By the remainder theorem,

$h(-1)=18$.

46. a. $$\begin{array}{r|rrrr} 2 & 2 & -1 & -14 & 7 \\ & & 4 & 6 & -16 \\ \hline & 2 & 3 & -8 & \lfloor -9 \end{array}$$

By the remainder theorem,

$k(2)=-9$.

b. $$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & -14 & 7 \\ & & 1 & 0 & -7 \\ \hline & 2 & 0 & -14 & \lfloor 0 \end{array}$$

By the remainder theorem,

$k\left(\frac{1}{2}\right)=0$.

c. $$\begin{array}{r|rrrr} \sqrt{7} & 2 & -1 & -14 & 7 \\ & & 2\sqrt{7} & 14-\sqrt{7} & -7 \\ \hline & 2 & -1+2\sqrt{7} & -\sqrt{7} & \lfloor 0 \end{array}$$

By the remainder theorem,

$k\left(\sqrt{7}\right)=0$.

d. $$\begin{array}{r|rrrr} -2 & 2 & -1 & -14 & 7 \\ & & -4 & 10 & 8 \\ \hline & 2 & -5 & -4 & \lfloor 15 \end{array}$$

By the remainder theorem,

$k(-2)=15$.

47. a. $$\begin{array}{r|rrrrr} 2 & 1 & 3 & -7 & 13 & -10 \\ & & 2 & 10 & 6 & 38 \\ \hline & 1 & 5 & 3 & 19 & \lfloor 28 \end{array}$$

By the remainder theorem,

$f(2)=28$. Since $f(2)\neq 0$, 2 is not a zero of $f(x)$.

b. $$\begin{array}{r|rrrrr} -5 & 1 & 3 & -7 & 13 & -10 \\ & & -5 & 10 & -15 & 10 \\ \hline & 1 & -2 & 3 & -2 & \lfloor 0 \end{array}$$

By the remainder theorem,

$f(-5)=0$. Since $f(-5)=0$, -5 is a zero of $f(x)$.

48. a. $$\begin{array}{r|rrrrr} -2 & 2 & 13 & -10 & -19 & 14 \\ & & -4 & -18 & 56 & -74 \\ \hline & 2 & 9 & -28 & 37 & \lfloor -60 \end{array}$$

By the remainder theorem,

$g(-2)=-60$. Since $g(-2)\neq 0$, -2 is not a zero of $g(x)$.

b.
$$\begin{array}{r|rrrrr} -7 & 2 & 13 & -10 & -19 & 14 \\ & & -14 & 7 & 21 & -14 \\ \hline & 2 & -1 & -3 & 2 & \boxed{0} \end{array}$$

By the remainder theorem, $g(-7)=0$. Since $g(-7)=0$, -7 is a zero of $g(x)$.

49. a.
$$\begin{array}{r|rrrr} -2 & 2 & 3 & -22 & -33 \\ & & -4 & 2 & 40 \\ \hline & 2 & -1 & -20 & \boxed{7} \end{array}$$

By the remainder theorem, $p(-2)=7$. Since $p(-2)\neq 0$, -2 is not a zero of $p(x)$.

b.
$$\begin{array}{r|rrrr} -\sqrt{11} & 2 & 3 & -22 & -33 \\ & & -2\sqrt{11} & 22-3\sqrt{11} & 33 \\ \hline & 2 & 3-2\sqrt{11} & -3\sqrt{11} & \boxed{0} \end{array}$$

By the remainder theorem, $p\left(-\sqrt{11}\right)=0$. Since $p\left(-\sqrt{11}\right)=0$, $-\sqrt{11}$ is a zero of $p(x)$.

50. a.
$$\begin{array}{r|rrrr} -3 & 3 & 1 & -30 & -10 \\ & & -9 & 24 & 18 \\ \hline & 3 & -8 & -6 & \boxed{8} \end{array}$$

By the remainder theorem, $q(-3)=8$. Since $q(-3)\neq 0$, -3 is not a zero of $q(x)$.

b.
$$\begin{array}{r|rrrr} -\sqrt{10} & 3 & 1 & -30 & -10 \\ & & -3\sqrt{10} & 30-\sqrt{10} & 10 \\ \hline & 3 & 1-3\sqrt{10} & -\sqrt{10} & \boxed{0} \end{array}$$

By the remainder theorem, $q\left(-\sqrt{10}\right)=0$. Since $q\left(-\sqrt{10}\right)=0$, $-\sqrt{10}$ is a zero of $q(x)$.

51. a.
$$\begin{array}{r|rrrr} 5i & 1 & -2 & 25 & -50 \\ & & 5i & -25-10i & 50 \\ \hline & 1 & -2+5i & -10i & \boxed{0} \end{array}$$

By the remainder theorem, $m(5i)=0$. Since $m(5i)=0$, $5i$ is a zero of $m(x)$.

b.
$$\begin{array}{r|rrrr} -5i & 1 & -2 & 25 & -50 \\ & & -5i & -25+10i & 50 \\ \hline & 1 & -2-5i & 10i & \boxed{0} \end{array}$$

By the remainder theorem, $m(-5i)=0$. Since $m(-5i)=0$, $-5i$ is a zero of $m(x)$.

52. a.
$$\begin{array}{r|rrrr} 3i & 1 & 4 & 9 & 36 \\ & & 3i & -9+12i & -36 \\ \hline & 1 & 4+3i & 12i & \boxed{0} \end{array}$$

By the remainder theorem, $n(3i)=0$. Since $n(3i)=0$, $3i$ is a zero of $n(x)$.

b.
$$\begin{array}{r|rrrr} -3i & 1 & 4 & 9 & 36 \\ & & -3i & -9-12i & -36 \\ \hline & 1 & 4-3i & -12i & \boxed{0} \end{array}$$

By the remainder theorem, $n(-3i)=0$. Since $n(-3i)=0$, $-3i$ is a zero of $n(x)$.

53. a.
$$\begin{array}{r|rrrr} 6+i & 1 & -11 & 25 & 37 \\ & & 6+i & -31+i & -37 \\ \hline & 1 & -5+i & -6+i & \boxed{0} \end{array}$$

By the remainder theorem, $g(6+i)=0$. Since $g(6+i)=0$, $6+i$ is a zero of $g(x)$.

b. $$\begin{array}{r|rrrr} 6-i & 1 & -11 & 25 & 37 \\ & & 6-i & -31-i & -37 \\ \hline & 1 & -5-i & -6-i & \boxed{0} \end{array}$$

By the remainder theorem, $g\left(6-i\right)=0$. Since $g\left(6-i\right)=0$, $6-i$ is a zero of $g\left(x\right)$.

54. a. $$\begin{array}{r|rrrr} 1+5i & 2 & -5 & 54 & -26 \\ & & 2+10i & 53-5i & 26 \\ \hline & 2 & -3+10i & 1-5i & \boxed{0} \end{array}$$

By the remainder theorem, $f\left(1+5i\right)=0$. Since $f\left(1+5i\right)=0$, $1+5i$ is a zero of $f\left(x\right)$.

b. $$\begin{array}{r|rrrr} 1-5i & 2 & -5 & 54 & -26 \\ & & 2-10i & 53+5i & 26 \\ \hline & 2 & -3-10i & 1+5i & \boxed{0} \end{array}$$

By the remainder theorem, $f\left(1-5i\right)=0$. Since $f\left(1-5i\right)=0$, $1-5i$ is a zero of $f\left(x\right)$.

55. a. $$\begin{array}{r|rrrrr} -5 & 1 & 11 & 41 & 61 & 30 \\ & & -5 & -30 & -55 & -30 \\ \hline & 1 & 6 & 11 & 6 & \boxed{0} \end{array}$$

By the factor theorem, since $f\left(-5\right)=0$, $x+5$ is a factor of $f(x)$.

b. $$\begin{array}{r|rrrrr} 2 & 1 & 11 & 41 & 61 & 30 \\ & & 2 & 26 & 134 & 390 \\ \hline & 1 & 13 & 67 & 195 & \boxed{420} \end{array}$$

By the factor theorem, since $f\left(2\right)\neq 0$, $x-2$ is not a factor of $f(x)$.

56. a. $$\begin{array}{r|rrrrr} 4 & 1 & -10 & 35 & -50 & 24 \\ & & 4 & -24 & 44 & -24 \\ \hline & 1 & -6 & 11 & -6 & \boxed{0} \end{array}$$

By the factor theorem, since $g\left(4\right)=0$, $x-4$ is a factor of $g(x)$.

b. $$\begin{array}{r|rrrrr} -1 & 1 & -10 & 35 & -50 & 24 \\ & & -1 & 11 & -46 & 96 \\ \hline & 1 & -11 & 46 & -96 & \boxed{120} \end{array}$$

By the factor theorem, since $g\left(-1\right)\neq 0$, $x+1$ is not a factor of $g(x)$.

57. a. $$\begin{array}{r|rrrr} 4 & 1 & 0 & 0 & 64 \\ & & 4 & 16 & 64 \\ \hline & 1 & 4 & 16 & \boxed{128} \end{array}$$

By the factor theorem, since $f\left(4\right)\neq 0$, $x-4$ is not a factor of $f(x)$.

b. $$\begin{array}{r|rrrr} -4 & 1 & 0 & 0 & 64 \\ & & -4 & 16 & -64 \\ \hline & 1 & -4 & 16 & \boxed{0} \end{array}$$

By the factor theorem, since $f\left(-4\right)=0$, $x+4$ is a factor of $f(x)$.

58. a. $$\begin{array}{r|rrrrr} 3 & 1 & 0 & 0 & 0 & -81 \\ & & 3 & 9 & 27 & 81 \\ \hline & 1 & 3 & 9 & 27 & \boxed{0} \end{array}$$

By the factor theorem, since $f\left(3\right)=0$, $x-3$ is a factor of $f(x)$.

b. $$\begin{array}{r|rrrrr} -3 & 1 & 0 & 0 & 0 & -81 \\ & & -3 & 9 & -27 & 81 \\ \hline & 1 & -3 & 9 & -27 & \boxed{0} \end{array}$$

By the factor theorem, since $f\left(-3\right)=0$, $x+3$ is a factor of $f(x)$.

59. a. $$\begin{array}{r|rrrr} 1 & 2 & 1 & -16 & -8 \\ & & 2 & 3 & -13 \\ \hline & 2 & 3 & -13 & \boxed{-21} \end{array}$$

By the factor theorem, since

$f(1) \neq 0$, $x-1$ is not a factor of $f(x)$.

b. $$\begin{array}{r|rrrr} 2\sqrt{2} & 2 & 1 & -16 & -8 \\ & & 4\sqrt{2} & 16+2\sqrt{2} & 8 \\ \hline & 2 & 1+4\sqrt{2} & 2\sqrt{2} & \underline{|0} \end{array}$$

By the factor theorem, since $f\left(2\sqrt{2}\right)=0$, $x-2\sqrt{2}$ is a factor of $f(x)$.

60. a. $$\begin{array}{r|rrrr} 2 & 3 & -1 & -54 & 18 \\ & & 6 & 10 & -88 \\ \hline & 3 & 5 & -44 & \underline{|-70} \end{array}$$

By the factor theorem, since $f(2) \neq 0$, $x-2$ is not a factor of $f(x)$.

b. $$\begin{array}{r|rrrr} 3\sqrt{2} & 3 & -1 & -54 & 18 \\ & & 9\sqrt{2} & 54-3\sqrt{2} & -18 \\ \hline & 3 & -1+9\sqrt{2} & -3\sqrt{2} & \underline{|0} \end{array}$$

By the factor theorem, since $f\left(3\sqrt{2}\right)=0$, $x-3\sqrt{2}$ is a factor of $f(x)$.

61. $g(x)=x^4-14x^2+45$

a. $g\left(\sqrt{5}\right)=\left(\sqrt{5}\right)^4-14\left(\sqrt{5}\right)^2+45$
$=25-70+45=0$

b. $g\left(-\sqrt{5}\right)=\left(-\sqrt{5}\right)^4-14\left(-\sqrt{5}\right)^2+45$
$=25-70+45=0$

c. $g(x)=x^4-14x^2+45$
$0=x^4-14x^2+45$

Let $x^2=y$.
$0=y^2-14y+45$
$0=(y-9)(y-5)$
$y=9$ or $y=5$
$x^2=9$ $\quad$ $x^2=5$
$x=\pm 3$ $\quad$ $x=\pm\sqrt{5}$
$\left\{-3,-\sqrt{5},\sqrt{5},3\right\}$

62. $h(x)=x^4-15x^2+44$

a. $h\left(\sqrt{11}\right)=\left(\sqrt{11}\right)^4-15\left(\sqrt{11}\right)^2+44$
$=121-165+44=0$

b. $h\left(-\sqrt{11}\right)=\begin{bmatrix}\left(-\sqrt{11}\right)^4 \\ -15\left(-\sqrt{11}\right)^2+44\end{bmatrix}$
$=121-165+44=0$

c. $h(x)=x^4-15x^2+44$
$0=x^4-15x^2+44$
Let $x^2=y$.
$0=y^2-15y+44$
$0=(y-11)(y-4)$
$y=11$ or $y=4$
$x^2=11$ $\quad$ $x^2=4$
$x=\pm\sqrt{11}$ $\quad$ $x=\pm 2$
$\left\{-\sqrt{11},-2,2,\sqrt{11}\right\}$

63. a. $$\begin{array}{r|rrr} 2+5i & 1 & -4 & 29 \\ & & 2+5i & -29 \\ \hline & 1 & -2+5i & \underline{|0} \end{array}$$

Yes, $2+5i$ is a factor of $f(x)$.

b. $$\begin{array}{r|rrr} 2-5i & 1 & -4 & 29 \\ & & 2-5i & -29 \\ \hline & 1 & -2-5i & \underline{|0} \end{array}$$

Yes, $2-5i$ is a factor of $f(x)$.

c. $x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(29)}}{2(1)}$

$= \dfrac{4 \pm \sqrt{-100}}{2}$

$= \dfrac{4 \pm 10i}{2}$

$= 2 \pm 5i$

The solution set is $\{2 \pm 5i\}$.

d. The zeroes are $2 \pm 5i$.

64. a.
$$\begin{array}{r|rrr} 3+4i & 1 & -6 & 25 \\ & & 3+4i & -25 \\ \hline & 1 & -3+4i & \lfloor 0 \end{array}$$

Yes, $3+4i$ is a factor of $f(x)$.

b.
$$\begin{array}{r|rrr} 3-4i & 1 & -6 & 25 \\ & & 3-4i & -25 \\ \hline & 1 & -3-4i & \lfloor 0 \end{array}$$

Yes, $3-4i$ is a factor of $f(x)$.

c. $x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(25)}}{2(1)}$

$= \dfrac{6 \pm \sqrt{-64}}{2} = \dfrac{6 \pm 8i}{2} = 3 \pm 4i$

The solution set is $\{3 \pm 4i\}$.

d. The zeroes are $3 \pm 4i$.

65. a.
$$\begin{array}{r|rrrr} -1 & 2 & 1 & -37 & -36 \\ & & -2 & 1 & 36 \\ \hline & 2 & -1 & -36 & \lfloor 0 \end{array}$$

$f(x) = (x+1)(2x^2 - x - 36)$
$f(x) = (x+1)(2x-9)(x+4)$

b. $(x+1)(2x-9)(x+4) = 0$

$x = -1, x = \dfrac{9}{2}, x = -4$

The solution set is $\left\{-1, \dfrac{9}{2}, -4\right\}$.

66. a.
$$\begin{array}{r|rrrr} -2 & 3 & 16 & -5 & -50 \\ & & -6 & -20 & 50 \\ \hline & 3 & 10 & -25 & \lfloor 0 \end{array}$$

$f(x) = (x+2)(3x^2 + 10x - 25)$
$f(x) = (x+2)(3x-5)(x+5)$

b. $(x+2)(3x-5)(x+5) = 0$

$x = -2, x = \dfrac{5}{3}, x = -5$

The solution set is $\left\{-2, \dfrac{5}{3}, -5\right\}$.

67. a.
$$\begin{array}{r|rrrr} \frac{1}{4} & 20 & 39 & -3 & -2 \\ & & 5 & 11 & 2 \\ \hline & 20 & 44 & 8 & \lfloor 0 \end{array}$$

$f(x) = \left(x - \dfrac{1}{4}\right)(20x^2 + 44x + 8)$

$f(x) = 4\left(x - \dfrac{1}{4}\right)(5x^2 + 11x + 2)$

$f(x) = (4x-1)(5x+1)(x+2)$

b. $(4x-1)(5x+1)(x+2) = 0$

$x = \dfrac{1}{4}, x = -\dfrac{1}{5}, x = -2$

The solution set is $\left\{\dfrac{1}{4}, -\dfrac{1}{5}, -2\right\}$.

68. a.
$$\begin{array}{r|rrrr} \frac{3}{4} & 8 & -18 & -11 & 15 \\ & & 6 & -9 & -15 \\ \hline & 8 & -12 & -20 & \lfloor 0 \end{array}$$

$f(x) = \left(x - \dfrac{3}{4}\right)(8x^2 - 12x - 20)$

$f(x) = 4\left(x - \dfrac{3}{4}\right)(2x^2 - 3x - 5)$

$f(x) = (4x-3)(2x-5)(x+1)$

b. $(4x-3)(2x-5)(x+1)=0$

$$x=\frac{3}{4}, x=\frac{5}{2}, x=-1$$

The solution set is $\left\{\frac{3}{4}, \frac{5}{2}, -1\right\}$.

69. a.

$$\begin{array}{r|rrrr} 3 & 9 & -33 & 19 & -3 \\ & & 27 & -18 & 3 \\ \hline & 9 & -6 & 1 & \boxed{0} \end{array}$$

$$f(x)=(x-3)(9x^2-6x+1)$$
$$f(x)=(x-3)(3x-1)^2$$

b. $(x-3)(3x-1)^2=0$

$$x=3, x=\frac{1}{3}$$

The solution set is $\left\{3, \frac{1}{3}\right\}$.

70. a.

$$\begin{array}{r|rrrr} 2 & 4 & -20 & 33 & -18 \\ & & 8 & -24 & 18 \\ \hline & 4 & -12 & 9 & \boxed{0} \end{array}$$

$$f(x)=(x-2)(4x^2-12x+9)$$
$$f(x)=(x-2)(2x-3)^2$$

b. $(x-2)(2x-3)^2=0$

$$x=2, x=\frac{3}{2}$$

The solution set is $\left\{2, \frac{3}{2}\right\}$.

71. $f(x)=(x-2)(x-3)(x+4)$
$$=(x-2)(x^2+x-12)$$
$$=x^3+x^2-12x-2x^2-2x+24$$
$$=x^3-x^2-14x+24$$

72. $f(x)=(x-1)(x+6)(x-3)$
$$=(x-1)(x^2+3x-18)$$
$$=x^3+3x^2-18x-x^2-3x+18$$
$$=x^3+2x^2-21x+18$$

73. $f(x)=(x-1)\left(x-\frac{3}{2}\right)(x)^2$
$$=x^2\left(x^2-\frac{5}{2}x+\frac{3}{2}\right)$$
$$=x^4-\frac{5}{2}x^3+\frac{3}{2}x^2$$
or $f(x)=a\left(x^4-\frac{5}{2}x^3+\frac{3}{2}x^2\right)$
$$=2\left(x^4-\frac{5}{2}x^3+\frac{3}{2}x^2\right)$$
$$=2x^4-5x^3+3x^2$$

74. $f(x)=(x-2)\left(x-\frac{5}{2}\right)(x)^3$
$$=x^3\left(x^2-\frac{9}{2}x+5\right)$$
$$=x^5-\frac{9}{2}x^3+5x^3$$
or $f(x)=a\left(x^5-\frac{9}{2}x^3+5x^3\right)$
$$=2\left(x^5-\frac{9}{2}x^3+5x^3\right)$$
$$=2x^5-9x^3+10x^3$$

75. $f(x)=(x-2\sqrt{11})(x+2\sqrt{11})$
$$=x^2-(2\sqrt{11})^2$$
$$=x^2-44$$

76. $f(x)=(x-5\sqrt{2})(x+5\sqrt{2})$
$$=x^2-(5\sqrt{2})^2$$
$$=x^2-50$$

77. $f(x)=(x+2)(x-3i)(x+3i)$
$$=(x+2)(x^2+9)$$
$$=x^3+2x^2+9x+18$$

78. $f(x)=(x-4)(x-2i)(x+2i)$
$$=(x-4)(x^2+4)$$
$$=x^3-4x^2+4x-16$$

79. $$\begin{aligned} f(x) &= a\left(x+\frac{2}{3}\right)\left(x-\frac{1}{2}\right)(x-4) \\ &= 6\left(x+\frac{2}{3}\right)\left(x-\frac{1}{2}\right)(x-4) \\ &= 3\left(x+\frac{2}{3}\right)\cdot 2\left(x-\frac{1}{2}\right)(x-4) \\ &= (3x+2)(2x-1)(x-4) \\ &= (3x+2)(2x^2-9x+4) \\ &= 6x^3-27x^2+12x+4x^2-18x+8 \\ &= 6x^3-23x^2-6x+8 \end{aligned}$$

80. $$\begin{aligned} f(x) &= a\left(x+\frac{2}{5}\right)\left(x-\frac{3}{2}\right)(x-6) \\ &= 10\left(x+\frac{2}{5}\right)\left(x-\frac{3}{2}\right)(x-6) \\ &= 5\left(x+\frac{2}{5}\right)\cdot 2\left(x-\frac{3}{2}\right)(x-6) \\ &= (5x+2)(2x-3)(x-6) \\ &= (5x+2)(2x^2-15x+18) \\ &= 10x^3-75x^2+90x+4x^2-30x+36 \\ &= 10x^3-71x^2+60x+36 \end{aligned}$$

81. $$\begin{aligned} f(x) &= [x-(7+8i)][x-(7-8i)] \\ &= [(x-7)-8i][(x-7)+8i] \\ &= (x-7)^2-(8i)^2 \\ &= x^2-14x+49+64 \\ &= x^2-14x+113 \end{aligned}$$

82. $$\begin{aligned} f(x) &= [x-(5+6i)][x-(5-6i)] \\ &= [(x-5)-6i][(x-5)+6i] \\ &= (x-5)^2-(6i)^2 \\ &= x^2-10x+25+36 \\ &= x^2-10x+61 \end{aligned}$$

83. Direct substitution is easier.

$$\begin{aligned} p(x) &= 2x^{452}-4x^{92} \\ p(1) &= 2(1)^{452}-4(1)^{92} \\ &= 2-4 \\ &= -2 \end{aligned}$$

84. Direct substitution is easier.

$$\begin{aligned} q(x) &= 5x^{721}-2x^{450} \\ q(-1) &= 5(-1)^{721}-2(-1)^{450} \\ &= 5(-1)-2(1) \\ &= -7 \end{aligned}$$

85. a. $f(x)=x^{100}-1$

$f(1)=(1)^{100}-1=1-1=0$

Yes, since $f(1)=0$, $(x-1)$ is a factor of $f(x)$.

b. $f(-1)=(-1)^{100}-1=1-1=0$

Yes, since $f(-1)=0$, $(x+1)$ is a factor of $f(x)$.

c. $g(x)=x^{99}-1$

$g(1)=(1)^{99}-1=1-1=0$

Yes, since $g(1)=0$, $(x-1)$ is a factor of $g(x)$.

d. $g(-1)=(-1)^{99}-1=-1-1=-2$

Yes, since $g(-1)\neq 0$, $(x+1)$ is a factor of $g(x)$.

e. Yes

f. No

86. third

87. Since x is not a factor, zero is not a zero of the polynomial. False.

88. Since x is a factor, zero is a zero of the polynomial. True.

89. $$\begin{array}{r|rrrr} \underline{-4} & 4 & 13 & -5 & m \\ & & -16 & 12 & -28 \\ \hline & 4 & -3 & 7 & \boxed{0} \end{array}$$

$$\begin{aligned} m+(-28) &= 0 \\ m &= 28 \end{aligned}$$

90. $$\begin{array}{r|rrrrr} -5 & -3 & -10 & 20 & -22 & m \\ & & 15 & -25 & 25 & -15 \\ \hline & -3 & 5 & -5 & 3 & \boxed{0} \end{array}$$

$$\begin{aligned} m+(-15)&=0 \\ m&=15 \end{aligned}$$

91. $$\begin{array}{r|rrrr} -2 & 4 & 5 & m & 2 \\ & & -8 & 6 & -2m-12 \\ \hline & 4 & -3 & m+6 & \boxed{0} \end{array}$$

$$\begin{aligned} 2+(-2m-12)&=0 \\ -2m-10&=0 \\ -2m&=10 \\ m&=-5 \end{aligned}$$

92. $$\begin{array}{r|rrrr} 3 & 2 & -7 & m & 6 \\ & & 6 & -3 & 3m-9 \\ \hline & 2 & -1 & m-3 & \boxed{0} \end{array}$$

$$\begin{aligned} 6+(3m-9)&=0 \\ 3m-3&=0 \\ 3m&=3 \\ m&=1 \end{aligned}$$

93. $$\begin{aligned} \frac{x^2-x-12}{x-4}&=x+3+\frac{r}{x-4} \\ \frac{(x+3)(x-4)}{x-4}&=x+3+\frac{r}{x-4} \\ x+3&=x+3+\frac{r}{x-4} \\ 0&=\frac{r}{x-4} \\ 0&=r \end{aligned}$$

94. $$\begin{array}{r|rrr} 2 & 1 & -5 & -8 \\ & & 2 & -6 \\ \hline & 1 & -3 & \boxed{-14} \end{array}$$

$$\begin{aligned} \frac{x^2-5x-8}{x-2}&=x-3+\frac{r}{x-2} \\ x-3+\frac{-14}{x-4}&=x-3+\frac{r}{x-4} \\ -14&=r \end{aligned}$$

95. a. $$\begin{aligned} V(x)&=x(x+1)(2x+1)-2(1)(x) \\ &=x(2x^2+3x+1)-2x \\ &=2x^3+3x^2+x-2x \\ &=2x^3+3x^2-x \end{aligned}$$

b. $$\begin{array}{r|rrrr} 6 & 2 & 3 & -1 & 0 \\ & & 12 & 90 & 534 \\ \hline & 2 & 15 & 89 & \boxed{534} \end{array}$$

The volume is 534 cm^3.

96. a. $$\begin{aligned} V(x)&=x(x-1)(x-4)-2(2)(x-5) \\ &=x(x^2-5x+4)-4(x-5) \\ &=x^3-5x^2+4x-4x+20 \\ &=x^3-5x^2+20 \end{aligned}$$

b. $$\begin{array}{r|rrrr} 10 & 1 & -5 & 0 & 20 \\ & & 10 & 50 & 500 \\ \hline & 1 & 5 & 50 & \boxed{520} \end{array}$$

The volume is 520 ft^3.

97. The divisor must be of the form $(x-c)$, where c is a constant.

98. Multiply the quotient times the divisor and add the remainder. The result should be equal to the dividend.

99. Compute $f(c)$ either by direct substitution or by using the remainder theorem. The remainder theorem states that $f(c)$ is equal to the remainder obtained after dividing $f(x)$ by $(x-c)$.

100. Given a polynomial $f(x)$, if c is a zero of $f(x)$, then $(x-c)$ is a factor of $f(x)$. The converse is also true. If $(x-c)$ is a factor of $f(x)$, then c is a zero of $f(x)$.

101. a.
$$\begin{array}{r|rrrr} 5 & 1 & -5 & 1 & -5 \\ & & 5 & 0 & 5 \\ \hline & 1 & 0 & 1 & \underline{|0} \end{array}$$

$$f(x) = (x-5)(x^2+1)$$
$$x^2+1=0$$
$$x^2=-1$$
$$x=\pm\sqrt{-1}=\pm i$$
$$f(x)=(x-5)(x-i)(x+i)$$

b. $(x-5)(x-i)(x+i)=0$

$x=5, x=i, x=-i$

The solution set is $\{5, i, -i\}$.

102. a.
$$\begin{array}{r|rrrr} 3 & 1 & -3 & 100 & -300 \\ & & 3 & 0 & 300 \\ \hline & 1 & 0 & 100 & \underline{|0} \end{array}$$

$$f(x)=(x-3)(x^2+100)$$
$$x^2+100=0$$
$$x^2=-100$$
$$x=\pm\sqrt{-100}=\pm 10i$$
$$f(x)=(x-3)(x-10i)(x+10i)$$

b. $(x-3)(x-10i)(x+10i)=0$

$x=3, x=10i, x=-10i$

The solution set is $\{3, 10i, -10i\}$.

103. a.
$$\begin{array}{r|rrrrr} -1 & 1 & 2 & -2 & -6 & -3 \\ & & -1 & -1 & 3 & 3 \\ \hline & 1 & 1 & -3 & -3 & \underline{|0} \end{array}$$

$$\begin{aligned} f(x) &= (x+1)(x^3+x^2-3x-3) \\ &= (x+1)\left[x^2(x+1)-3(x+1)\right] \\ &= (x+1)(x+1)(x^2-3) \\ &= (x+1)^2\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right) \end{aligned}$$

b. $(x+1)^2\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)=0$

$x=-1, x=\sqrt{3}, x=-\sqrt{3}$

The solution set is $\left\{-1, \sqrt{3}, -\sqrt{3}\right\}$.

104. a.
$$\begin{array}{r|rrrrr} -2 & 1 & 4 & -1 & -20 & -20 \\ & & -2 & -4 & 10 & 20 \\ \hline & 1 & 2 & -5 & -10 & \underline{|0} \end{array}$$

$$\begin{aligned} f(x) &= (x+2)(x^3+2x^2-5x-10) \\ &= (x+2)\left[x^2(x+2)-5(x+2)\right] \\ &= (x+2)(x+2)(x^2-5) \\ &= (x+2)^2\left(x-\sqrt{5}\right)\left(x+\sqrt{5}\right) \end{aligned}$$

b. $(x+2)^2\left(x-\sqrt{5}\right)\left(x+\sqrt{5}\right)=0$

$x=-2, x=\sqrt{5}, x=-\sqrt{5}$

The solution set is $\left\{-2, \sqrt{5}, -\sqrt{5}\right\}$.

105. a. The solution appears to be 3.

b.
$$\begin{array}{r|rrrr} 3 & 5 & 7 & -58 & -24 \\ & & 15 & 66 & 24 \\ \hline & 5 & 22 & 8 & \underline{|0} \end{array}$$

Since the remainder is zero, 3 is a solution.

c. $f(x)=(x-3)(5x^2+22x+8)$
$$=(x-3)(x+4)(5x+2)$$
$$(x-3)(x+4)(5x+2)=0$$
$$x=3, x=-4, x=-\frac{2}{5}$$

The solution set is $\left\{3, -4, -\frac{2}{5}\right\}$.

106. a. The solution appears to be -5.

b.
$$\begin{array}{r|rrrr} -5 & 2 & -1 & -41 & 70 \\ & & -10 & 55 & -70 \\ \hline & 2 & -11 & 14 & \underline{|0} \end{array}$$

Since the remainder is zero, -5 is a solution.

c. $$\begin{aligned} f(x) &= (x+5)(2x^2-11x+14) \\ &= (x+5)(2x-7)(x-2) \end{aligned}$$

$$(x+5)(2x-7)(x-2)=0$$

$$x=-5, x=\frac{7}{2}, x=2$$

The solution set is $\left\{-5, \frac{7}{2}, 2\right\}$.

Section 2.4 Zeros of Polynomials

1. zeros

2. n

3. $a-bi$

4. upper

5. greater than

6. 7 is not the ration of any of the factors over the factors of 2.

7. $\frac{\text{Factors of 4}}{\text{Factors of 1}} = \frac{\pm 1, \pm 2, \pm 4}{\pm 1} = \pm 1, \pm 2, \pm 4$

8. $\frac{\text{Factors of } -9}{\text{Factors of 1}} = \frac{\pm 1, \pm 3, \pm 9}{\pm 1} = \pm 1, \pm 3, \pm 9$

9. $\frac{\text{Factors of } -6}{\text{Factors of 4}}$

$$= \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4}$$

$$= \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$$

10. $\frac{\text{Factors of 10}}{\text{Factors of 25}} = \frac{\pm 1, \pm 2, \pm 5, \pm 10}{\pm 1, \pm 5, \pm 25}$

$$= \pm 1, \pm 2, \pm 5, \pm 10, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{1}{25}, \pm\frac{2}{25}$$

11. $\frac{\text{Factors of 8}}{\text{Factors of } -12}$

$$= \frac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}$$

$$= \pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}, \pm\frac{1}{6}, \pm\frac{1}{12}$$

12. $\frac{\text{Factors of 6}}{\text{Factors of } -16}$

$$= \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4, \pm 8, \pm 16}$$

$$= \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{1}{8}, \pm\frac{3}{8}, \pm\frac{1}{16}, \pm\frac{3}{16}$$

13. 7 and $\frac{5}{3}$ are not ratios of any of the factors of 12 over the factors of 2.

14. 3 and $\frac{3}{2}$ are not ratios of any of the factors of 10 over the factors of 4.

15. $\frac{\text{Factors of 2}}{\text{Factors of 2}} = \frac{\pm 1, \pm 2}{\pm 1, \pm 2} = \pm 1, \pm 2, \pm\frac{1}{2}$

Use synthetic division and the remainder theorem to determine if any of the numbers in the list is a zero of $p(x)$. (Successful tests of zeros are shown.)

$$\begin{array}{r|rrrrr} 1 & 2 & -1 & -5 & 2 & 2 \\ & & 2 & 1 & -4 & -2 \\ \hline & 2 & 1 & -4 & -2 & \boxed{0} \end{array}$$

The remainder is zero. Therefore, 1 is a zero of $p(x)$.

$$\begin{array}{r|rrrrr} -\frac{1}{2} & 2 & -1 & -5 & 2 & 2 \\ & & -1 & 1 & 2 & -2 \\ \hline & 2 & -2 & -4 & 4 & \boxed{0} \end{array}$$

The remainder is zero. Therefore, $-\frac{1}{2}$ is a zero of $p(x)$.

16. $$\frac{\text{Factors of } 10}{\text{Factors of } 1} = \frac{\pm 1, \pm 2, \pm 5, \pm 10}{\pm 1}$$
$$= \pm 1, \pm 2, \pm 5, \pm 10$$

Use synthetic division and the remainder theorem to determine if any of the numbers in the list is a zero of $q(x)$. (Successful tests of zeros are shown.)

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & -7 & -5 & 10 \\ & & 1 & 2 & -5 & -10 \\ \hline & 1 & 2 & -5 & -10 & \boxed{0} \end{array}$$

The remainder is zero. Therefore, 1 is a zero of $q(x)$.

$$\begin{array}{r|rrrrr} -2 & 1 & 1 & -7 & -5 & 10 \\ & & -2 & 2 & 10 & -10 \\ \hline & 1 & -1 & -5 & 5 & \boxed{0} \end{array}$$

The remainder is zero. Therefore, -2 is a zero of $q(x)$.

17. $$\frac{\text{Factors of } -24}{\text{Factors of } 2}$$
$$= \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24}{\pm 1, \pm 2}$$
$$= \pm 1, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$$
$$= \pm \frac{1}{2}, \pm \frac{3}{2}, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$$

$$\begin{array}{r|rrrrr} 2 & 2 & -7 & -17 & 58 & -24 \\ & & 4 & -6 & -46 & 24 \\ \hline & 2 & -3 & -23 & 12 & \boxed{0} \end{array}$$

$$c(x) = (x-2)(2x^3 - 3x^2 - 23x + 12)$$

Find the zeros of the quotient.

$$\begin{array}{r|rrrr} 4 & 2 & -3 & -23 & 12 \\ & & 8 & 20 & -12 \\ \hline & 2 & 5 & -3 & \boxed{0} \end{array}$$

$$c(x) = (x-2)(x-4)(2x^2 + 5x - 3)$$
$$c(x) = (x-2)(x-4)(2x-1)(x+3)$$
$$0 = (x-2)(x-4)(2x-1)(x+3)$$
$$x = 2, x = 4, x = \frac{1}{2}, x = -3$$

The zeros are $\left\{-3, \frac{1}{2}, 2, 4,\right\}$.

18. $$\frac{\text{Factors of } 12}{\text{Factors of } 3}$$
$$= \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1, \pm 3}$$
$$= \pm 1, \pm 2, \pm 4, \pm 6, \pm 12$$
$$\pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm 2, \pm 4$$

$$\begin{array}{r|rrrrr} -1 & 3 & -2 & -21 & -4 & 12 \\ & & -3 & 5 & 16 & -12 \\ \hline & 3 & -5 & -16 & 12 & \boxed{0} \end{array}$$

$$d(x) = (x+1)(3x^3 - 5x^2 - 16x + 12)$$

Find the zeros of the quotient.

$$\begin{array}{r|rrrr} -2 & 3 & -5 & -16 & 12 \\ & & -6 & 22 & -12 \\ \hline & 3 & -11 & 6 & \boxed{0} \end{array}$$

$$d(x)=(x+1)(x+2)(3x^2-11x+6)$$
$$d(x)=(x+1)(x+2)(3x-2)(x-3)$$
$$0=(x+1)(x+2)(3x-2)(x-3)$$
$$x=-1, x=-2, x=\frac{2}{3}, x=-3$$

The zeros are $\left\{-1,-2,\frac{2}{3},3,\right\}$.

19. $\dfrac{\text{Factors of }20}{\text{Factors of }1}$

$$=\frac{\pm1,\pm2,\pm4,\pm5,\pm10,\pm20}{\pm1}$$
$$=\pm1,\pm2,\pm4,\pm5,\pm10,\pm20$$

$$\begin{array}{r|rrrr} \underline{5} & 1 & -7 & 6 & 20 \\ & & 5 & -10 & -20 \\ \hline & 1 & -2 & -4 & \underline{|0} \end{array}$$

$$f(x)=(x-5)(x^2-2x-4)$$
$$0=(x-5)(x^2-2x-4)$$

$$x=5 \quad\text{or}\quad x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-4)}}{2(1)}$$
$$x=\frac{2\pm\sqrt{20}}{2}=\frac{2\pm2\sqrt{5}}{2}=1\pm\sqrt{5}$$

The zeros are 5, $1\pm\sqrt{5}$.

20. $\dfrac{\text{Factors of }6}{\text{Factors of }1}=\dfrac{\pm1,\pm2,\pm3,\pm6}{\pm1}$

$$=\pm1,\pm2,\pm3,\pm6$$

$$\begin{array}{r|rrrr} \underline{3} & 1 & -7 & 14 & -6 \\ & & 3 & -12 & 6 \\ \hline & 1 & -4 & 2 & \underline{|0} \end{array}$$

$$g(x)=(x-3)(x^2-4x+2)$$
$$0=(x-3)(x^2-4x+2)$$

$$x=3 \quad\text{or}\quad x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(2)}}{2(1)}$$
$$x=\frac{4\pm\sqrt{8}}{2}=\frac{4\pm2\sqrt{2}}{2}=2\pm\sqrt{2}$$

The zeros are 3, $2\pm\sqrt{2}$.

21. $\dfrac{\text{Factors of }7}{\text{Factors of }5}=\dfrac{\pm1,\pm7}{\pm1,\pm5}$

$$=\pm1,\pm7,\pm\frac{1}{5},\pm\frac{7}{5}$$

$$\begin{array}{r|rrrr} \underline{\frac{1}{5}} & 5 & -1 & -35 & 7 \\ & & 1 & 0 & -7 \\ \hline & 5 & 0 & -35 & \underline{|0} \end{array}$$

$$h(x)=\left(x-\frac{1}{5}\right)(5x^2-35)$$
$$0=\left(x-\frac{1}{5}\right)(5x^2-35)$$
$$x=\frac{1}{5} \quad\text{or}\quad 5x^2=35$$
$$x^2=7$$
$$x=\pm\sqrt{7}$$

The zeros are $\frac{1}{5},\pm\sqrt{7}$.

22. $\dfrac{\text{Factors of }3}{\text{Factors of }7}=\dfrac{\pm1,\pm3}{\pm1,\pm7}$

$$=\pm1,\pm3,\pm\frac{1}{7},\pm\frac{3}{7}$$

$$\begin{array}{r|rrrr} \underline{\frac{1}{7}} & 7 & -1 & -21 & 3 \\ & & 1 & 0 & -3 \\ \hline & 7 & 0 & -21 & \underline{|0} \end{array}$$

$$k(x)=\left(x-\frac{1}{7}\right)(7x^2-21)$$
$$0=\left(x-\frac{1}{7}\right)(7x^2-21)$$

$$x = \frac{1}{7} \text{ or } 7x^2 = 21$$
$$x^2 = 3$$
$$x = \pm\sqrt{3}$$

The zeros are $\frac{1}{7}, \pm\sqrt{3}$.

23. $\dfrac{\text{Factors of } -16}{\text{Factors of } 3}$

$$= \frac{\pm1, \pm2, \pm4, \pm8, \pm16}{\pm1, \pm3}$$

$$= \pm1, \pm2, \pm4, \pm8, \pm16,$$
$$\pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}, \pm\frac{16}{3}$$

$$\begin{array}{r|rrrrr} \underline{2} & 3 & -1 & -36 & 60 & -16 \\ & & 6 & 10 & -52 & 16 \\ \hline & 3 & 5 & -26 & 8 & \boxed{0} \end{array}$$

$$m(x) = (x-2)(3x^3 + 5x^2 - 26x + 8)$$

Find the zeros of the quotient.

$$\begin{array}{r|rrrr} \underline{2} & 3 & 5 & -26 & 8 \\ & & 6 & 22 & -8 \\ \hline & 3 & 11 & -4 & \boxed{0} \end{array}$$

$$m(x) = (x-2)^2(3x^2 + 11x - 4)$$
$$m(x) = (x-2)^2(3x-1)(x+4)$$
$$0 = (x-2)^2(3x-1)(x+4)$$
$$x = 2, x = \frac{1}{3}, x = -4$$

The zeros are 2 (multiplicity 2), $\frac{1}{3}$, -4.

24. $\dfrac{\text{Factors of } -45}{\text{Factors of } 2}$

$$= \frac{\pm1, \pm3, \pm5, \pm9, \pm15, \pm45}{\pm1, \pm2}$$

$$= \pm1, \pm3, \pm5, \pm9, \pm15,$$
$$\pm45, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{9}{2},$$
$$\pm\frac{15}{2}, \pm\frac{45}{2}$$

$$\begin{array}{r|rrrrr} \underline{-3} & 2 & 9 & -5 & -57 & -45 \\ & & -6 & -9 & 42 & 45 \\ \hline & 2 & 3 & -14 & -15 & \boxed{0} \end{array}$$

$$n(x) = (x+3)(2x^3 + 3x^2 - 14x - 15)$$

Find the zeros of the quotient.

$$\begin{array}{r|rrrr} \underline{-3} & 2 & 3 & -14 & -15 \\ & & -6 & 9 & 15 \\ \hline & 2 & -3 & -5 & \boxed{0} \end{array}$$

$$n(x) = (x+3)^2(2x^2 - 3x - 5)$$
$$n(x) = (x+3)^2(2x-5)(x+1)$$
$$0 = (x+3)^2(2x-5)(x+1)$$
$$x = -3, x = \frac{5}{2}, x = -1$$

The zeros are -3 (multiplicity 2), $\frac{5}{2}$, -1.

25. $\dfrac{\text{Factors of } 20}{\text{Factors of } 1}$

$$= \frac{\pm1, \pm2, \pm4, \pm5, \pm10, \pm20}{\pm1}$$
$$= \pm1, \pm2, \pm4, \pm5, \pm10, \pm20$$

$$\begin{array}{r|rrrr} \underline{-2} & 1 & -4 & -2 & 20 \\ & & -2 & 12 & -20 \\ \hline & 1 & -6 & 10 & \boxed{0} \end{array}$$

$$q(x) = (x+2)(x^2 - 6x + 10)$$
$$0 = (x+2)(x^2 - 6x + 10)$$

$$x=-2 \text{ or } x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(10)}}{2(1)}$$

$$x=\frac{6\pm\sqrt{-4}}{2}=\frac{6\pm 2i}{2}=3\pm i$$

The zeros are -2, $3\pm i$.

26. $\frac{\text{Factors of } -52}{\text{Factors of } 1}$

$$=\frac{\pm1,\pm2,\pm4,\pm13,\pm26,\pm52}{\pm1}$$

$$=\pm1,\pm2,\pm4,\pm13,\pm26,\pm52$$

$$\begin{array}{r|rrrr} 4 & 1 & -8 & 29 & -52 \\ & & 4 & -16 & 52 \\ \hline & 1 & -4 & 13 & \boxed{0} \end{array}$$

$$p(x)=(x-4)(x^2-4x+13)$$

$$0=(x-4)(x^2-4x+13)$$

$$x=4 \quad \text{or} \quad x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(13)}}{2(1)}$$

$$x=\frac{4\pm\sqrt{-36}}{2}=\frac{4\pm 6i}{2}=2\pm 3i$$

The zeros are 4, $2\pm3i$.

27. $t(x)=x^4-x^2-90$

$$t(x)=(x^2-10)(x^2+9)$$

$$0=(x^2-10)(x^2+9)$$

$x^2=10$ or $x^2=-9$

$x=\pm\sqrt{10}$ or $x=\pm3i$

The zeros are $\pm\sqrt{10}, \pm3i$.

28. $v(x)=x^4-12x^2-13$

$$v(x)=(x^2-13)(x^2+1)$$

$$0=(x^2-13)(x^2+1)$$

$x^2=13$ or $x^2=-1$

$x=\pm\sqrt{13}$ or $x=\pm i$

The zeros are $\pm\sqrt{13}, \pm i$.

29. one

30. 3

31. 7

32. 9

33. a.

$$\begin{array}{r|rrrrr} 2-5i & 1 & -4 & 22 & 28 & -203 \\ & & 2-5i & -29 & -14+35i & 203 \\ \hline & 1 & -2-5i & -7 & 14+35i & \boxed{0} \end{array}$$

Since $2-5i$ is a zero, $2+5i$ is also a zero.

$$\begin{array}{r|rrrr} 2+5i & 1 & -2-5i & -7 & 14+35i \\ & & 2+5i & 0 & -14-35i \\ \hline & 1 & 0 & -7 & \boxed{0} \end{array}$$

$$f(x)=[x-(2-5i)][x-(2+5i)](x^2-7)$$

Find the remaining two zeros.

$$x^2-7=0$$

$$x^2=7$$

$$x=\pm\sqrt{7}$$

The zeros are $2\pm5i$, $\pm\sqrt{7}$.

b. $f(x)=\left\{\begin{array}{c}\left[x-(2-5i)\right]\left[x-(2+5i)\right]\\ \left(x-\sqrt{7}\right)\left(x+\sqrt{7}\right)\end{array}\right\}$

c. The solution set is $\left\{2\pm 5i, \pm\sqrt{7}\right\}$.

34. a.

$$\begin{array}{r|rrrrr} 3-i & 1 & -6 & 5 & 30 & -50 \\ & & 3-i & -10 & -15+5i & 50 \\ \hline & 1 & -3-i & -5 & 15+5i & \lfloor 0 \end{array}$$

Since $3-i$ is a zero, $3+i$ is also a zero.

$$\begin{array}{r|rrrr} 3+i & 1 & -3-i & -5 & 15+5i \\ & & 3+i & 0 & -15-5i \\ \hline & 1 & 0 & -5 & \lfloor 0 \end{array}$$

$f(x)=\left[x-(3-i)\right]\left[x-(3+i)\right]\left(x^2-5\right)$

Find the remaining two zeros.

$x^2-5=0$

$x^2=5$

$x=\pm\sqrt{5}$

The zeros are $3\pm i$, $\pm\sqrt{5}$.

b. $f(x)=\left\{\begin{array}{c}\left[x-(3-i)\right]\left[x-(3+i)\right]\\ \left(x-\sqrt{5}\right)\left(x+\sqrt{5}\right)\end{array}\right\}$

c. The solution set is $\left\{3\pm i, \pm\sqrt{5}\right\}$.

35. a.

$$\begin{array}{r|rrrr} 4+i & 3 & -28 & 83 & -68 \\ & & 12+3i & -67-4i & 68 \\ \hline & 3 & -16+3i & 16-4i & \lfloor 0 \end{array}$$

Since $4+i$ is a zero, $4-i$ is also a zero.

$$\begin{array}{r|rrr} 4-i & 3 & -16+3i & 16-4i \\ & & 12-3i & -16+4i \\ \hline & 3 & -4 & \lfloor 0 \end{array}$$

$f(x)=\left[x-(4+i)\right]\left[x-(4-i)\right](3x-4)$

Find the remaining two zeros.

$3x-4=0$

$3x=4$

$x=\dfrac{4}{3}$

The zeros are $4\pm i$, $\dfrac{4}{3}$.

b. $f(x)=\left\{\begin{array}{c}\left[x-(4+i)\right]\left[x-(4-i)\right]\\ \left(x-\dfrac{4}{3}\right)\end{array}\right\}$

c. The solution set is $\left\{4\pm i, \dfrac{4}{3}\right\}$.

36. a.

$$\begin{array}{r|rrrr} 5+i & 5 & -54 & 170 & -104 \\ & & 25+5i & -150-4i & 104 \\ \hline & 5 & -29+5i & 20-4i & \lfloor 0 \end{array}$$

Since $5+i$ is a zero, $5-i$ is also a zero.

$$\begin{array}{r|rrr} 5-i & 5 & -29+5i & 20-4i \\ & & 25-5i & -20+4i \\ \hline & 5 & -4 & \lfloor 0 \end{array}$$

$f(x)=\left[x-(5+i)\right]\left[x-(5-i)\right](5x-4)$

Find the remaining two zeros.

$5x-4=0$

$5x=4$

$x=\dfrac{4}{5}$

The zeros are $5\pm i$, $\dfrac{4}{5}$.

b. $f(x)=\left\{\begin{array}{c}\left[x-(5+i)\right]\left[x-(5-i)\right]\\ \left(x-\dfrac{4}{5}\right)\end{array}\right\}$

c. The solution set is $\left\{5 \pm i, \frac{4}{5}\right\}$.

37. a. $$\begin{array}{r|rrrrrr} -\frac{1}{4} & 4 & 37 & 117 & 87 & -193 & -52 \\ & & -1 & -9 & -27 & -15 & 52 \\ \hline & 4 & 36 & 108 & 60 & -208 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$\begin{array}{r|rrrrr} -3+2i & 4 & 36 & 108 & 60 & -208 \\ & & -12+8i & -88+24i & -108-32i & 208 \\ \hline & 4 & 24+8i & 20+24i & -48-32i & \boxed{0} \end{array}$$

Since $-3+2i$ is a zero, $-3-2i$ is also a zero.

$$\begin{array}{r|rrrr} -3-2i & 4 & 24+8i & 20+24i & -48-32i \\ & & -12-8i & -36-24i & 48+32i \\ \hline & 4 & 12 & -16 & \boxed{0} \end{array}$$

$$f(x) = \left[x-(-3+2i)\right]\left[x-(-3-2i)\right]\left(x+\frac{1}{4}\right)\left(4x^2+12x-16\right)$$
$$f(x) = \left[x-(-3+2i)\right]\left[x-(-3-2i)\right](4x+1)\left(x^2+3x-4\right)$$
$$f(x) = \left[x-(-3+2i)\right]\left[x-(-3-2i)\right](4x+1)(x-1)(x+4)$$
$$0 = \left[x-(-3+2i)\right]\left[x-(-3-2i)\right](4x+1)(x-1)(x+4)$$
$$x = -3+2i, x = -3-2i, x = -\frac{1}{4}, x = 1, x = -4$$

The zeros are $-3 \pm 2i, -\frac{1}{4}, 1, -4$

b. $f(x) = \left[x-(-3+2i)\right]\left[x-(-3-2i)\right](4x+1)(x-1)(x+4)$

c. The solution set is $\left\{-3 \pm 2i, -\frac{1}{4}, 1, -4\right\}$.

38. a. $$\begin{array}{r|rrrrrr} \frac{5}{2} & 2 & -5 & -4 & -22 & 50 & 75 \\ & & 5 & 0 & -10 & -80 & -75 \\ \hline & 2 & 0 & -4 & -32 & -30 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$\begin{array}{r|rrrrr} -1-2i & 2 & 0 & -4 & -32 & -30 \\ & & -2-4i & -6+8i & 26+12i & 30 \\ \hline & 2 & -2-4i & -10+8i & -6+12i & \boxed{0} \end{array}$$

Since $-1-2i$ is a zero, $-1+2i$ is also a zero.

$$\begin{array}{r|rrrr} -1+2i & 2 & -2-4i & -10+8i & -6+12i \\ & & -2+4i & 4-8i & 6-12i \\ \hline & 2 & -4 & -6 & \boxed{0} \end{array}$$

$$f(x)=\left[x-(-1-2i)\right]\left[x-(-1+22i)\right]\left(x-\frac{5}{2}\right)\left(2x^2-4x-6\right)$$
$$f(x)=\left[x-(-1-2i)\right]\left[x-(-1+22i)\right](2x-5)\left(x^2-2x-3\right)$$
$$f(x)=\left[x-(-1-2i)\right]\left[x-(-1+22i)\right](2x-5)(x+1)(x-3)$$
$$0=\left[x-(-1-2i)\right]\left[x-(-1+22i)\right](2x-5)(x+1)(x-3)$$
$$x=-1-2i, x=-1+2i, x=\frac{5}{2}, x=-1, x=3$$

The zeros are $-1\pm 2i, \frac{5}{2}, -1, 3$

b. $f(x)=\left[x-(-1-2i)\right]\left[x-(-1+22i)\right](2x-5)(x+1)(x-3)$

c. The solution set is $\left\{-1\pm 2i, \frac{5}{2}, -1, 3\right\}$.

39. $f(x)=a(x-6i)(x+6i)\left(x-\frac{4}{5}\right)$

$$=a\left(x^2+36\right)\left(x-\frac{4}{5}\right)$$
$$=a\left(x^3-\frac{4}{5}x^2+36x-\frac{144}{5}\right) \text{ Let } a=5.$$
$$=5\left(x^3-\frac{4}{5}x^2+36x-\frac{144}{5}\right)$$
$$=5x^3-4x^2+180x-144$$

40. $f(x)=a(x+4i)(x-4i)\left(x-\frac{3}{2}\right)$

$$=a\left(x^2+16\right)\left(x-\frac{3}{2}\right)$$
$$=a\left(x^3-\frac{3}{2}x^2+16x-24\right) \text{ Let } a=2.$$
$$=2\left(x^3-\frac{3}{2}x^2+16x-24\right)$$
$$=2x^3-3x^2+32x-48$$

41. $f(x)=a(x+4)(x-2)^3$

$$
\begin{aligned}
&= a(x+4)(x-2)(x-2)^2 \\
&= a(x+4)(x-2)(x^2-4x+4) \\
&= a(x+4)\begin{pmatrix} x^3-4x^2+4x \\ -2x^2+8x-8 \end{pmatrix} \\
&= a(x+4)(x^3-6x^2+12x-8) \\
&= a\begin{pmatrix} x^4-6x^3+12x^2-8x+4x^3 \\ -24x^2+48x-32 \end{pmatrix} \\
&= a(x^4-2x^3-12x^2+40x-32)
\end{aligned}
$$

$$f(0) = a\begin{bmatrix} (0)^4-2(0)^3-12(0)^2 \\ +40(0)-32 \end{bmatrix}$$

$$= 160$$

$$-32a = 160$$

$$a = -5$$

$$f(x) = -5(x^4-2x^3-12x^2+40x-32)$$

$$f(x) = -5x^4+10x^3+60x^2-200x+160$$

42. $f(x) = a(x-5)^2(x+3)^2$

$$
\begin{aligned}
&= a(x^2-10x+25)(x^2+6x+9) \\
&= a\begin{pmatrix} x^4+6x^3+9x^2-10x^3-60x^2 \\ -90x+25x^2+150x+225 \end{pmatrix} \\
&= a(x^4-4x^3-26x^2+60x+225)
\end{aligned}
$$

$$f(0) = a\begin{bmatrix} (0)^4-4(0)^3-26(0)^2 \\ +60(0)+225 \end{bmatrix}$$

$$= -450$$

$$225a = -450$$

$$a = -2$$

$$f(x) = -2(x^4-4x^3-26x^2+60x+225)$$

$$f(x) = -2x^4+8x^3+52x^2-120x-450$$

43. $$\begin{aligned} f(x) &= a\left(x+\frac{4}{3}\right)^2\left(x-\frac{1}{2}\right) \\ &= a\left(x^2+\frac{8}{3}x+\frac{16}{9}\right)\left(x-\frac{1}{2}\right) \\ &= a\left(x^3-\frac{1}{2}x^2+\frac{8}{3}x^2-\frac{4}{3}x+\frac{16}{9}x-\frac{8}{9}\right) \\ &= a\left(x^3+\frac{13}{6}x^2+\frac{4}{9}x-\frac{8}{9}\right) \end{aligned}$$

$$\begin{aligned} f(0) &= a\left[(0)^3+\frac{13}{6}(0)^2+\frac{4}{9}(0)-\frac{8}{9}\right] \\ &= -16 \end{aligned}$$

$$-\frac{8}{9}a = -16$$

$$a = 18$$

$$f(x) = 18\left(x^3+\frac{13}{6}x^2+\frac{4}{9}x-\frac{8}{9}\right)$$

$$f(x) = 18x^3+39x^2+8x-16$$

44. $$\begin{aligned} f(x) = a\left(x+\frac{5}{6}\right)^2\left(x-\frac{1}{3}\right) &= a\left(x^2+\frac{5}{3}x+\frac{25}{36}\right)\left(x-\frac{1}{3}\right) \\ &= a\left(x^3-\frac{1}{3}x^2+\frac{5}{3}x^2-\frac{5}{9}x+\frac{25}{36}x-\frac{25}{108}\right) \\ &= a\left(x^3+\frac{4}{3}x^2+\frac{5}{36}x-\frac{25}{108}\right) \end{aligned}$$

$$\begin{aligned} f(0) &= a\left[(0)^3+\frac{4}{3}(0)^2+\frac{5}{36}(0)-\frac{25}{108}\right] \\ &= -25 \end{aligned}$$

$$-\frac{25}{108}a = -25$$

$$a = 108$$

$$f(x) = 108\left(x^3+\frac{4}{3}x^2+\frac{5}{36}x-\frac{25}{108}\right)$$

$$f(x) = 108x^3+144x^2+15x-25$$

45. $$\begin{aligned} f(x) &= x^4\left[x-(7-4i)\right]\left[x-(7+4i)\right] \\ &= x^4(x-7)^2-(4i)^2 \\ &= x^4\left(x^2-14x+49+16\right) \\ &= x^4\left(x^2-14x+65\right) \\ &= x^6-14x^5+65x^4 \end{aligned}$$

46. $$\begin{aligned} f(x) &= x^3\left[x-(5-10i)\right]\left[x-(5+10i)\right] \\ &= x^3(x-5)^2-(10i)^2 \\ &= x^3\left(x^2-10x+25+100\right) \\ &= x^3\left(x^2-10x+125\right) \\ &= x^5-10x^4+125x^3 \end{aligned}$$

47. $$\begin{aligned} f(x) &= \begin{bmatrix} (x-5i)(x+5i) & \\ & (x-(6-i))(x-(6+i)) \end{bmatrix} \\ &= \left(x^2+25\right)\left[(x-6)^2-(i)^2\right] \\ &= \left(x^2+25\right)\left(x^2-12x+36+1\right) \\ &= \left(x^2+25\right)\left(x^2-12x+37\right) \\ &= x^4-12x^3+62x^2-300x+925 \end{aligned}$$

48. $$\begin{aligned} f(x) &= \begin{bmatrix} (x+3i)(x-3i) & \\ & (x-(5+2i))(x-(5-2i)) \end{bmatrix} \\ &= \left(x^2+9\right)\left[(x-5)^2-(2i)^2\right] \\ &= \left(x^2+9\right)\left(x^2-10x+25+4\right) \\ &= \left(x^2+9\right)\left(x^2-10x+29\right) \\ &= x^4-10x^3+38x^2-90x+261 \end{aligned}$$

49. $$f(x) = x^6 \underset{\smile}{-} 2x^4 \underset{\smile}{+} 4x^3 \underset{\smile}{-} 2x^2 - 5x - 6$$

3 sign changes in $f(x)$. The number of possible positive real zeros is either 3 or 1.

$$\begin{aligned} f(-x) &= (-x)^6-2(-x)^4+4(-x)^3 \\ &\qquad -2(-x)^2-5(-x)-6 \end{aligned}$$

$$f(-x) = x^6 \underset{\smile}{-} 2x^4 - 4x^3 - 2x^2 \underset{\smile}{+} 5x \underset{\smile}{-} 6$$

3 sign changes in $f(-x)$. The number of possible negative real zeros is either 3 or 1.

50. $$g(x) = 3x^7 + 4x^4 \underset{\smile}{-} 6x^3 \underset{\smile}{+} 5x^2 \underset{\smile}{-} 6x \underset{\smile}{+} 1$$

4 sign changes in $g(x)$. The number of possible positive real zeros is either 4, 2, or 0.

$$\begin{aligned} g(-x) &= 3(-x)^7+4(-x)^4-6(-x)^3 \\ &\qquad +5(-x)^2-6(-x)+1 \end{aligned}$$

$$g(-x) = -3x^7 \underset{\smile}{+} 4x^4 + 6x^3 + 5x^2 + 6x + 1$$

1 sign change in $g(-x)$. The number of possible negative real zeros is 1.

51. $k(x) = -8x^7 + 5x^6 - 3x^4 + 2x^3 - 11x^2$
$+ 4x - 3$

$\smile \quad \smile \quad \smile \quad \smile \quad \smile \quad \smile$

6 sign changes in $k(x)$. The number of possible positive real zeros is either 6, 4, 2, or 0.

$k(-x) = -8(-x)^7 + 5(-x)^6 - 3(-x)^4$
$+ 2(-x)^3 - 11(-x)^2 + 4(-x) - 3$

$k(-x) = 8x^7 + 5x^6 - 3x^4 - 2x^3 - 11x^2$
$- 4x - 3$

$\smile$

1 sign change in $k(-x)$. The number of possible negative real zeros is 1.

52. $h(x) = -4x^9 + 6x^8 - 5x^5 - 2x^4 + 3x^2 - x + 8$

$\smile \quad \smile \qquad \smile \quad \smile \smile$

5 sign changes in $h(x)$. The number of possible positive real zeros is either 5, 3, or 1.

$h(-x) = -4(-x)^9 + 6(-x)^8 - 5(-x)^5$
$- 2(-x)^4 + 3(-x)^2 - (-x) + 8$

$h(-x) = 4x^9 + 6x^8 + 5x^5 - 2x^4 + 3x^2 + x + 8$

$\smile \quad \smile$

2 sign changes in $h(-x)$. The number of possible negative real zeros is 2 or 0.

53. $p(x) = 0.11x^4 + 0.04x^3 + 0.31x^2 + 0.27x + 1.1$

0 sign changes in $p(x)$. The number of possible positive real zeros is 0.

$p(-x) = 0.11(-x)^4 + 0.04(-x)^3$
$+ 0.31(-x)^2 + 0.27(-x) + 1.1$

$p(-x) = 0.11x^4 - 0.04x^3 + 0.31x^2 - 0.27x + 1.1$

$\smile \quad \smile \quad \smile \quad \smile$

4 sign changes in $p(-x)$. The number of possible negative real zeros is either 4, 2, or 0.

54. $q(x) = -0.6x^4 + 0.8x^3 - 0.6x^2 + 0.1x - 0.4$

$\smile \quad \smile \quad \smile \quad \smile$

4 sign changes in $q(x)$. The number of possible positive real zeros is either 4, 2, or 0.

$q(-x) = -0.6(-x)^4 + 0.8(-x)^3 - 0.6(-x)^2$
$+ 0.1(-x) - 0.4$

$q(-x) = -0.6x^4 - 0.8x^3 - 0.6x^2 - 0.1x - 0.4$

0 sign changes in $q(-x)$. The number of possible negative real zeros is 0.

55. $v(x) = \frac{1}{8}x^6 + \frac{1}{6}x^4 + \frac{1}{3}x^2 + \frac{1}{10}$

0 sign changes in $v(x)$. The number of possible positive real zeros is 0.

$$v(-x) = \frac{1}{8}(-x)^6 + \frac{1}{6}(-x)^4 + \frac{1}{3}(-x)^2 + \frac{1}{10}$$
$$v(-x) = \frac{1}{8}x^6 + \frac{1}{6}x^4 + \frac{1}{3}x^2 + \frac{1}{10}$$

0 sign changes in $v(-x)$. The number of possible negative real zeros is 0.

56. $t(x) = \frac{1}{1000}x^6 + \frac{1}{100}x^4 + \frac{1}{10}x^2 + 1$

0 sign changes in $t(x)$. The number of possible positive real zeros is 0.

$$t(-x) = \begin{bmatrix} \frac{1}{1000}(-x)^6 + \frac{1}{100}(-x)^4 \\ + \frac{1}{10}(-x)^2 + 1 \end{bmatrix}$$
$$t(-x) = \frac{1}{1000}x^6 + \frac{1}{100}x^4 + \frac{1}{10}x^2 + 1$$

0 sign changes in $t(-x)$. The number of possible negative real zeros is 0.

57. $f(x) = x^8 + 5x^6 + 6x^4 - x^3$

$f(x) = x^3\left(x^5 + 5x^3 \underbrace{+ 6x - 1}\right)$

1 sign change. The number of possible positive real zeros is 1.

$$(-x)^5 + 5(-x)^3 + 6(-x) - 1$$
$$= -x^5 - 5x^3 - 6x - 1$$

0 sign changes. The number of possible negative real zeros is 0.

$f(x)$ has 4 real zeros; 1 positive real zero, no negative real zeros, and the number 0 is a zero of multiplicity 3.

58. $f(x) = -5x^8 - 3x^6 - 4x^2 + x$

$f(x) = x\left(-5x^7 - 3x^5 \underbrace{- 4x + 1}\right)$

1 sign change. The number of possible positive real zeros is 1.

$$-5(-x)^7 - 3(-x)^5 - 4(-x) + 1$$
$$= 5x^7 + 3x^5 + 4x + 1$$

0 sign changes. The number of possible negative real zeros is 0.

$f(x)$ has 2 real zeros; 1 positive real zero, no negative real zeros, and the number 0 is a zero of multiplicity 1.

59. a.
$$\begin{array}{r|rrrrrr} 2 & 1 & 6 & 0 & 5 & 1 & -3 \\ & & 2 & 16 & 32 & 74 & 150 \\ \hline & 1 & 8 & 16 & 37 & 75 & \boxed{147} \end{array}$$

The remainder and all coefficients of the quotient are nonnegative. Therefore, 2 is an upper bound for the real zeros of $f(x)$.

b. $$\begin{array}{r|rrrrrr} -2 & 1 & 6 & 0 & 5 & 1 & -3 \\ & & -2 & -8 & 16 & -42 & 82 \\ \hline & 1 & 4 & -8 & 21 & -41 & \boxed{79} \end{array}$$

The signs of the quotient do not alternate. Therefore, we cannot conclude that -2 is a lower bound for the real zeros of $f(x)$.

60. a. $$\begin{array}{r|rrrrr} 3 & 1 & 8 & -4 & 7 & -3 \\ & & 3 & 33 & 87 & 282 \\ \hline & 1 & 11 & 29 & 94 & \boxed{279} \end{array}$$

The remainder and all coefficients of the quotient are nonnegative. Therefore, 3 is an upper bound for the real zeros of $f(x)$.

b. $$\begin{array}{r|rrrrr} -3 & 1 & 8 & -4 & 7 & -3 \\ & & -3 & -15 & 57 & -192 \\ \hline & 1 & 5 & -19 & 64 & \boxed{-195} \end{array}$$

The signs of the quotient do not alternate. Therefore, we cannot conclude that -3 is a lower bound for the real zeros of $f(x)$.

61. a. $$\begin{array}{r|rrrr} 6 & 8 & -42 & 33 & 28 \\ & & 48 & 36 & 414 \\ \hline & 8 & 6 & 69 & \boxed{442} \end{array}$$

The remainder and all coefficients of the quotient are nonnegative. Therefore, 6 is an upper bound for the real zeros of $f(x)$.

b. $$\begin{array}{r|rrrr} -1 & 8 & -42 & 33 & 28 \\ & & -8 & 50 & -83 \\ \hline & 8 & -50 & 83 & \boxed{-55} \end{array}$$

The signs of the quotient alternate. Therefore, -1 is a lower bound for the real zeros of $f(x)$.

62. a. $$\begin{array}{r|rrrr} 4 & 6 & -1 & -57 & 70 \\ & & 24 & 92 & 140 \\ \hline & 6 & 23 & 35 & \boxed{210} \end{array}$$

The remainder and all coefficients of the quotient are nonnegative. Therefore, 4 is an upper bound for the real zeros of $f(x)$.

b. $$\begin{array}{r|rrrr} -4 & 6 & -1 & -57 & 70 \\ & & -24 & 100 & -172 \\ \hline & 6 & -25 & 43 & \boxed{-102} \end{array}$$

The signs of the quotient alternate. Therefore, -4 is a lower bound for the real zeros of $f(x)$.

63. a. $$\begin{array}{r|rrrrrr} 3 & 2 & 11 & 0 & -63 & -50 & 40 \\ & & 6 & 51 & 153 & 270 & 660 \\ \hline & 2 & 17 & 51 & 90 & 220 & \boxed{700} \end{array}$$

The remainder and all coefficients of the quotient are nonnegative. Therefore, 3 is an upper bound for the real zeros of $f(x)$.

b. $$\begin{array}{r|rrrrrr} -6 & 2 & 11 & 0 & -63 & -50 & 40 \\ & & -12 & 6 & -36 & 594 & -3264 \\ \hline & 2 & -1 & 6 & -99 & 544 & \boxed{-3224} \end{array}$$

The signs of the quotient alternate. Therefore, -6 is a lower bound for the real zeros of $f(x)$.

64. a. $$\begin{array}{r|rrrrrr} 6 & 3 & -16 & 5 & 90 & -138 & 36 \\ & & 18 & 12 & 102 & 1152 & 6084 \\ \hline & 3 & 2 & 17 & 192 & 1014 & \boxed{6120} \end{array}$$

The remainder and all coefficients of the quotient are nonnegative. Therefore, 6 is an upper bound for the real zeros of $f(x)$.

b.

$$\begin{array}{r|rrrrrr} -3 & 3 & -16 & 5 & 90 & -138 & 36 \\ & & -9 & 75 & -240 & 450 & -936 \\ \hline & 3 & -25 & 80 & -150 & 312 & \boxed{-900} \end{array}$$

The signs of the quotient alternate. Therefore, -3 is a lower bound for the real zeros of $f(x)$.

65. True

66. False. Only numbers greater than 5 are also guaranteed to be upper bounds.

67. False. Only numbers less than -3 are also guaranteed to be lower bounds.

68. True

69. $\dfrac{\text{Factors of } 28}{\text{Factors of } 8}$

$$= \frac{\pm 1, \pm 2, \pm 4, \pm 7, \pm 14, \pm 28}{\pm 1, \pm 2, \pm 4, \pm 8}$$

$$= \pm 1, \pm 2, \pm 4, \pm 7, \pm 14, \pm 28, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{1}{8}, \pm\frac{7}{2}, \pm\frac{7}{4}$$

From Exercise 61, we know that 6 and -1 are upper and lower bounds of $f(x)$, respectively. We can restrict the list of possible rational zeros to those on the interval $(-1, 6)$:

$$1, 2, 4, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{1}{8}, \frac{7}{2}, \frac{7}{4}$$

$$\begin{array}{r|rrrr} 4 & 8 & -42 & 33 & 28 \\ & & 32 & -40 & -28 \\ \hline & 8 & -10 & -7 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$0 = 8x^2 - 10x - 7$$
$$0 = (4x - 7)(2x + 1)$$
$$x = \frac{7}{4}, x = -\frac{1}{2}$$

The zeros are $\frac{7}{4}, -\frac{1}{2}$, and 4 (each with multiplicity 1).

70. $\dfrac{\text{Factors of } 70}{\text{Factors of } 6}$

$$= \frac{\pm 1, \pm 2, \pm 5, \pm 7, \pm 10, \pm 14, \pm 35, \pm 70}{\pm 1, \pm 2, \pm 3, \pm 6}$$

$$= \pm 1, \pm 2, \pm 5, \pm 7, \pm 10, \pm 14, \pm 35, \pm 70, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{2}, \pm\frac{5}{3}, \pm\frac{5}{6}, \pm\frac{7}{2}, \pm\frac{7}{3}, \pm\frac{7}{6}, \pm\frac{10}{3}, \pm\frac{14}{3}, \pm\frac{35}{2}, \pm\frac{35}{3}, \pm\frac{35}{6}, \pm\frac{70}{3}, \pm\frac{70}{6}$$

From Exercise 62, we know that 4 and -4 are upper and lower bounds of $f(x)$, respectively. We can restrict the list of possible rational zeros to those on the interval $(-4, 4)$:

$$\pm 1, \pm 2, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{2}, \pm\frac{5}{3}, \pm\frac{5}{6}, \pm\frac{7}{2}, \pm\frac{7}{3}, \pm\frac{7}{6}, \pm\frac{10}{3}$$

$$\begin{array}{r|rrrr} 2 & 6 & -1 & -57 & 70 \\ & & 12 & 22 & -70 \\ \hline & 6 & 11 & -35 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$0 = 6x^2 + 11x - 35$$
$$0 = (2x + 7)(3x - 5)$$
$$x = -\frac{7}{2}, x = \frac{5}{3}$$

The zeros are $-\frac{7}{2}, \frac{5}{3}$, and 2 (each with multiplicity 1).

71. $\dfrac{\text{Factors of 40}}{\text{Factors of 2}}$

$$= \frac{\pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40}{\pm 1, \pm 2}$$

$$= \pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40,$$
$$\pm\frac{1}{2}, \pm\frac{5}{2}$$

From Exercise 63, we know that 3 and -6 are upper and lower bounds of $f(x)$, respectively. We can restrict the list of possible rational zeros to those on the interval $(-6, 3)$:

$$\pm 1, \pm 2, -4, -5, \pm\frac{1}{2}, \pm\frac{5}{2}$$

$$\begin{array}{r|rrrrrr} -2 & 2 & 11 & 0 & -63 & -50 & 40 \\ & & -4 & -14 & 28 & 70 & -40 \\ \hline & 2 & 7 & -14 & -35 & 20 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$\begin{array}{r|rrrrr} -4 & 2 & 7 & -14 & -35 & 20 \\ & & -8 & 4 & 40 & -20 \\ \hline & 2 & -1 & -10 & 5 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & -10 & 5 \\ & & 1 & 0 & -5 \\ \hline & 2 & 0 & -10 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$\begin{aligned} 2x^2 - 10 &= 0 \\ 2x^2 &= 10 \\ x^2 &= 5 \\ x &= \pm\sqrt{5} \end{aligned}$$

The zeros are $\pm\sqrt{5}, \frac{1}{2}, -2,$ and -4 (each with multiplicity 1).

72. $\dfrac{\text{Factors of 36}}{\text{Factors of 3}}$

$$= \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36}{\pm 1, \pm 3}$$

$$= \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36,$$
$$\pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$$

From Exercise 64, we know that 6 and -3 are upper and lower bounds of $f(x)$, respectively. We can restrict the list of possible rational zeros to those on the interval $(-3, 6)$:

$$\pm 1, \pm 2, 3, 4, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$$

$$\begin{array}{r|rrrrrr} 2 & 3 & -16 & 5 & 90 & -138 & 36 \\ & & 6 & -20 & -30 & 120 & -36 \\ \hline & 3 & -10 & -15 & 60 & -18 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$\begin{array}{r|rrrrr} 3 & 3 & -10 & -15 & 60 & -18 \\ & & 9 & -3 & -54 & 18 \\ \hline & 3 & -1 & -18 & 6 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -1 & -18 & 6 \\ & & 1 & 0 & -6 \\ \hline & 3 & 0 & -18 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$\begin{aligned} 3x^2 - 18 &= 0 \\ 3x^2 &= 18 \\ x^2 &= 6 \\ x &= \pm\sqrt{6} \end{aligned}$$

The zeros are $\pm\sqrt{6}, \frac{1}{3}, 2,$ and 3 (each with multiplicity 1).

73. $$\frac{\text{Factors of 9}}{\text{Factors of 4}}=\frac{\pm1,\pm3,\pm9}{\pm1,\pm2,\pm4}$$

$$=\pm1,\pm3,\pm9,\pm\frac{1}{2},\pm\frac{1}{4},$$
$$\pm\frac{3}{2},\pm\frac{3}{4},\pm\frac{9}{2},\pm\frac{9}{4}$$

$$\begin{array}{r|rrrrr} -3 & 4 & 20 & 13 & -30 & 9 \\ & & -12 & -24 & 33 & -9 \\ \hline & 4 & 8 & -11 & 3 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$\begin{array}{r|rrrr} -3 & 4 & 8 & -11 & 3 \\ & & -12 & 12 & -3 \\ \hline & 4 & -4 & 1 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$4x^2-4x+1=0$$
$$(2x-1)^2=0$$
$$x=\frac{1}{2}$$

The zeros are -3 and $\frac{1}{2}$ (each with multiplicity 2).

74. $$\frac{\text{Factors of 4}}{\text{Factors of 9}}=\frac{\pm1,\pm2,\pm4}{\pm1,\pm3,\pm9}$$

$$=\pm1,\pm2,\pm4,\pm\frac{1}{3},\pm\frac{1}{9},$$
$$\pm\frac{2}{3},\pm\frac{2}{9},\pm\frac{4}{3},\pm\frac{4}{9}$$

$$\begin{array}{r|rrrrr} -2 & 9 & 30 & 13 & -20 & 4 \\ & & -18 & -24 & 22 & -4 \\ \hline & 9 & 12 & -11 & 2 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$\begin{array}{r|rrrr} -2 & 9 & 12 & -11 & 2 \\ & & -18 & 12 & -2 \\ \hline & 9 & -6 & 1 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$9x^2-6x+1=0$$
$$(3x-1)^2=0$$
$$x=\frac{1}{3}$$

The zeros are -2 and $\frac{1}{3}$ (each with multiplicity 2).

75. $$\frac{\text{Factors of 15}}{\text{Factors of 2}}=\frac{\pm1,\pm3,\pm5,\pm15}{\pm1,\pm2}$$

$$=\pm1,\pm3,\pm5,\pm15$$
$$\pm\frac{1}{2},\pm\frac{3}{2},\pm\frac{5}{2},\pm\frac{15}{2}$$

$$\begin{array}{r|rrrrr} 3 & 2 & -11 & 27 & -41 & 15 \\ & & 6 & -15 & 36 & -15 \\ \hline & 2 & -5 & 12 & -5 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -5 & 12 & -5 \\ & & 1 & -2 & 5 \\ \hline & 2 & -4 & 10 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$2x^2-4x+10=0$$
$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(2)(10)}}{2(2)}$$
$$x=\frac{4\pm8i}{4}$$
$$x=1\pm2i$$

The zeros are $3,\frac{1}{2}$, and $1\pm2i$ (each with multiplicity 1).

76. $\dfrac{\text{Factors of 20}}{\text{Factors of 3}}$
$= \dfrac{\pm1, \pm2, \pm5, \pm10, \pm20}{\pm1, \pm3}$
$= \pm1, \pm2, \pm5, \pm10, \pm20,$
$\pm\dfrac{1}{3},\ \pm\dfrac{2}{3}, \pm\dfrac{5}{3}, \pm\dfrac{10}{3}, \pm\dfrac{20}{3}$

$$\begin{array}{r|rrrrr} 2 & 3 & -20 & 51 & -56 & 20 \\ & & 6 & -28 & 46 & -20 \\ \hline & 3 & -14 & 23 & -10 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$\begin{array}{r|rrrr} \frac{2}{3} & 3 & -14 & 23 & -10 \\ & & 2 & -8 & 10 \\ \hline & 3 & -12 & 15 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$3x^2 - 12x + 15 = 0$

$x = \dfrac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(15)}}{2(3)}$

$x = \dfrac{12 \pm 6i}{6}$

$x = 2 \pm i$

The zeros are $2, \dfrac{2}{3}$, and $2 \pm i$ (each with multiplicity 1).

77. $\dfrac{\text{Factors of 50}}{\text{Factors of 4}}$
$= \dfrac{\pm1, \pm2, \pm5, \pm10, \pm25, \pm50}{\pm1, \pm2, \pm4}$
$= \pm1, \pm2, \pm5, \pm10, \pm25, \pm50, \pm\dfrac{1}{2},$
$\pm\dfrac{5}{2}, \pm\dfrac{5}{4}, \pm\dfrac{10}{4}, \pm\dfrac{25}{2},\ \pm\dfrac{25}{4}, \pm\dfrac{50}{4}$

$$\begin{array}{r|rrrrr} \frac{5}{2} & 4 & -28 & 73 & -90 & 50 \\ & & 10 & -45 & 70 & -50 \\ \hline & 4 & -18 & 28 & -20 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$\begin{array}{r|rrrr} \frac{5}{2} & 4 & -18 & 28 & -20 \\ & & 10 & -20 & 20 \\ \hline & 4 & -8 & 8 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$4x^2 - 8x + 8 = 0$

$x = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(4)8}}{2(4)}$

$x = \dfrac{8 \pm 8i}{8}$

$x = 1 \pm i$

The zeros are $\dfrac{5}{2}$ (multiplicity 2), and $1 \pm i$ (each with multiplicity 1).

78. $\dfrac{\text{Factors of 5}}{\text{Factors of 9}}$
$= \dfrac{\pm1, \pm5}{\pm1, \pm3, \pm9}$
$= \pm1, \pm5, \pm\dfrac{1}{3}, \pm\dfrac{5}{3}, \pm\dfrac{1}{9}, \pm\dfrac{5}{9}$

$$\begin{array}{r|rrrrr} \frac{1}{3} & 9 & -42 & 70 & -34 & 5 \\ & & 3 & -13 & 19 & -5 \\ \hline & 9 & -39 & 57 & -15 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$\begin{array}{r|rrrr} \frac{1}{3} & 9 & -39 & 57 & -15 \\ & & 3 & -12 & 15 \\ \hline & 9 & -36 & 45 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$9x^2 - 36x + 45 = 0$

$$x=\frac{-(-36)\pm\sqrt{(-36)^2-4(9)(45)}}{2(9)}$$

$$x=\frac{36\pm18i}{18}$$

$$x=2\pm i$$

The zeros are $\frac{1}{3}$ (multiplicity 2), and $2\pm i$ (each with multiplicity 1).

79. $f(x)=x^2\left(x^4+2x^3+11x^2+20x+10\right)$

$$\frac{\text{Factors of }10}{\text{Factors of }1}=\frac{\pm1,\pm2,\pm5,\pm10}{\pm1}$$
$$=\pm1,\pm2,\pm5,\pm10$$

$$\begin{array}{r|rrrrr} \underline{-1} & 1 & 2 & 11 & 20 & 10 \\ & & -1 & -1 & -10 & -10 \\ \hline & 1 & 1 & 10 & 10 & \lfloor 0 \end{array}$$

Find the zeros of the quotient.

$$\begin{array}{r|rrrr} \underline{-1} & 1 & 1 & 10 & 10 \\ & & -1 & 0 & -10 \\ \hline & 1 & 0 & 10 & \lfloor 0 \end{array}$$

Find the zeros of the quotient.

$$x^2+10=0$$
$$x^2=-10$$
$$x=\pm i\sqrt{10}$$

The zeros are 0 (with multiplicity 2), -1 (with multiplicity 2), and $\pm i\sqrt{10}$ (each with multiplicity 1).

80. $f(x)=x^2\left(x^4+6x^3+12x^2+18x+27\right)$

$$\frac{\text{Factors of }27}{\text{Factors of }1}=\frac{\pm1,\pm3,\pm9,\pm27}{\pm1}$$
$$=\pm1,\pm3,\pm9,\pm27$$

$$\begin{array}{r|rrrrr} \underline{-3} & 1 & 6 & 12 & 18 & 27 \\ & & -3 & -9 & -9 & -27 \\ \hline & 1 & 3 & 3 & 9 & \lfloor 0 \end{array}$$

Find the zeros of the quotient.

$$\begin{array}{r|rrrr} \underline{-3} & 1 & 3 & 3 & 9 \\ & & -3 & 0 & -9 \\ \hline & 1 & 0 & 3 & \lfloor 0 \end{array}$$

Find the zeros of the quotient.

$$x^2+3=0$$
$$x^2=-3$$
$$x=\pm i\sqrt{3}$$

The zeros are 0 (with multiplicity 2), -3 (with multiplicity 2), and $\pm i\sqrt{3}$ (each with multiplicity 1).

81. $f(x)=x^3\left(x^2-10x+34\right)$

Find the zeros of $x^2-10x+34$.

$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(1)(34)}}{2(1)}$$
$$=\frac{10\pm\sqrt{-36}}{2}$$
$$=\frac{10\pm6i}{2}$$
$$=5\pm3i$$

The zeros are 0 (with multiplicity 3), and $5\pm3i$ (each with multiplicity 1).

82. $f(x)=x^4\left(x^2-12x+40\right)$

Find the zeros of $x^2-12x+40$.

$$x=\frac{-(-12)\pm\sqrt{(-12)^2-4(1)(40)}}{2(1)}$$
$$=\frac{12\pm\sqrt{-16}}{2}$$
$$=\frac{12\pm4i}{2}$$
$$=6\pm2i$$

The zeros are 0 (with multiplicity 4), and $6\pm2i$ (each with multiplicity 1).

83. $f(x)=-\left(x^3-3x^2+9x+13\right)$

$$\frac{\text{Factors of }13}{\text{Factors of }1}=\frac{\pm1,\pm13}{\pm1}=\pm1,\pm13$$

$$\begin{array}{r|rrrr} \underline{-1} & 1 & -3 & 9 & 13 \\ & & -1 & 4 & -13 \\ \hline & 1 & -4 & 13 & \boxed{0} \end{array}$$

Find the zeros of $x^2-4x+13$.

$$\begin{aligned} x &= \frac{-(-4)\pm\sqrt{(-4)^2-4(1)(13)}}{2(1)} \\ &= \frac{4\pm\sqrt{-36}}{2} \\ &= \frac{4\pm 6i}{2} \\ &= 2\pm 3i \end{aligned}$$

The zeros are -1 and $2\pm 3i$ (each with multiplicity 1).

84. $f(x)=-(x^3-5x^2+11x-15)$

$$\frac{\text{Factors of } 15}{\text{Factors of } 1}=\frac{\pm1,\pm3,\pm5,\pm15}{\pm1}=\pm1,\pm3,\pm5,\pm15$$

$$\begin{array}{r|rrrr} \underline{3} & 1 & -5 & 11 & -15 \\ & & 3 & -6 & 15 \\ \hline & 1 & -2 & 5 & \boxed{0} \end{array}$$

Find the zeros of x^2-2x+5.

$$\begin{aligned} x &= \frac{-(-2)\pm\sqrt{(-2)^2-4(1)(5)}}{2(1)} \\ &= \frac{2\pm\sqrt{-16}}{2}=\frac{2\pm 4i}{2}=1\pm 2i \end{aligned}$$

The zeros are 3 and $1\pm 2i$ (each with multiplicity 1).

85. False. For example, the graph of $f(x)=x^4+1$ has no x-intercepts. Thus, x^4+1 has no real zeros.

86. False. This is true only if the polynomial has real coefficients.

87. False. For example, the graph of $f(x)=x^{10}+1$ ha no x-intercepts.

88. False. The graph of $y=f(x)$ may touch or cross the *x*-axis between *a* and *b*. Thus, $f(x)$ may have one or more zeros between *a* and *b*.

89. True

90. False. This might be true in special cases (for example, if the graph of the polynomial were symmetric with respect to the *y*-axis), but the statement is false in general.

91. a. True

b. True

c. True

d. True

92. a.
$$\begin{aligned} x &= \frac{-(-7)\pm\sqrt{(-7)^2-4(1)(5)}}{2(1)} \\ &= \frac{7\pm\sqrt{29}}{2} \end{aligned}$$

b.
$$x^2-7x+5=\left[\left(x-\frac{7+\sqrt{29}}{2}\right)\left(x-\frac{7-\sqrt{29}}{2}\right)\right]$$

93. a.
$$\begin{aligned} f(2) &= 2(2)^2-7(2)+4 \\ &= 8-14+4 \\ &= -2 \end{aligned}$$

$$\begin{aligned} f(3) &= 2(3)^2-7(3)+4 \\ &= 18-21+4 \\ &= 1 \end{aligned}$$

Since $f(2)$ and $f(3)$ have opposite signs, the intermediate value theorem guarantees that *f* has at least one real zero between 2 and 3.

b. $x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(4)}}{2(2)}$

$= \dfrac{7 \pm \sqrt{17}}{4}$

Furthermore, $\dfrac{7+\sqrt{17}}{4} \approx 2.78$ is on the interval [2, 3].

94. Let $f(x) = x^n - a^n$. Then $f(a) = a^n - a^n = 0$. By the factor theorem, since $f(a) = 0$, then $x - a$ is a factor of $x^n - a^n$.

95. If a polynomial has real coefficients, then all imaginary zeros must come in conjugate pairs. This means that if the polynomial has imaginary zeros, there would be an even number of them. A third-degree polynomial has 3 zeros (including multiplicities). Therefore, it would have either 2 or 0 imaginary zeros, leaving room for either 1 or 3 real zeros.

96. The zeros of a second-degree polynomial can be found by applying the quadratic formula.

97. $f(x)$ has no variation in sign, nor does $f(-x)$. By Descartes' rule of signs, there are no positive or negative real zeros. Furthermore, 0 itself is not a zero of $f(x)$ because x is not a factor of $f(x)$. Therefore, there are no real zeros of $f(x)$.

98. The expression $\sqrt{x} + 3$ is not a polynomial. The term $\sqrt{x} = x^{1/2}$ has an exponent that is not a positive integer.

99. $f(x) = x^n - 1$ has one sign change. The number of possible positive real zeros is 1. Since n is a positive even integer, $f(-x) = (-x)^n - 1 = x^n - 1$ has one sign change. The number of possible negative real zeros is 1. Therefore, there are 2 possible real roots, and $n - 2$ possible imaginary roots.

100. $f(x) = x^n - 1$ has one sign change. The number of possible positive real zeros is 1. Since n is a positive odd integer, $f(-x) = (-x)^n - 1 = -x^n - 1$ has no sign changes. The number of possible negative real zeros is 0. Therefore, there is 1 possible real root, and $n - 1$ possible imaginary roots.

101. $108 = l\left(\frac{1}{2}bh\right)$

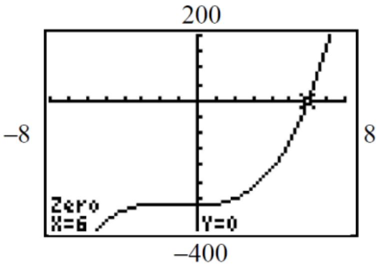
$108 = (x+3)\left(\frac{1}{2}\right)(x)\left(\frac{2}{3}x\right)$

$108 = \frac{1}{3}\left(x^3 + 3x^2\right)$

$324 = x^3 + 3x^2$

$0 = x^3 + 3x^2 - 324$

Graph the function $f(x) = x^3 + 3x^2 - 324$ on a graphing utility. Use the Zero feature to find positive real roots of the equation.



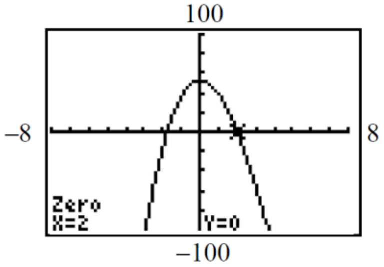
$$\frac{2}{3}x=\frac{2}{3}(6)=4$$
$$x+3=6+3=9$$

The triangular front has a base of 6 ft and a height of 4 ft. The length is 9 ft.

102. $\frac{104\pi}{3}=\frac{4}{3}\pi r^3+\pi r^2 h$

$$\frac{104\pi}{3}=\frac{4}{3}\pi r^3+\pi r^2(10-2r)$$
$$\frac{104\pi}{3}=\frac{4}{3}\pi r^3+10\pi r^2-2\pi r^3$$
$$\frac{104}{3}=\frac{4}{3}r^3+10r^2-2r^3$$
$$52=2r^3+15r^2-3r^3$$
$$0=r^3-15r^2+52$$

Graph the function $f(r)=r^3-15r^2+52$ on a graphing utility. Use the Zero feature to find positive real roots of the equation.

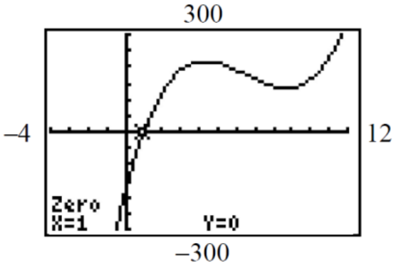


The radius is 2 ft.

103. $81=(10-x)(7-x)\left(\frac{5}{2}-x\right)$

$$-162=(x-10)(x-7)(2x-5)$$
$$-162=(x-10)(2x^2-19x+35)$$
$$-162=\begin{bmatrix}2x^3-19x^2+35x-20x^2\\ +190x-350\end{bmatrix}$$
$$0=2x^3-39x^2+225x-188$$

Graph the function $f(x)=2x^3-39x^2+225x-188$ on a graphing utility. Use the Zero feature to find positive real roots of the equation.

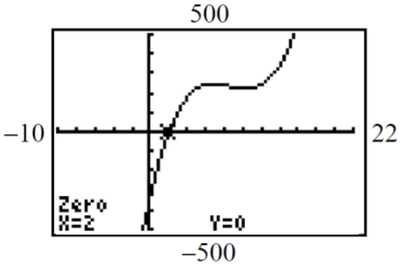


Each dimension was decreased by 1 in.

104. $240=(12-x)(8-x)(6-x)$

$$-240=(x-12)(x-8)(x-6)$$
$$-240=(x-12)(x^2-14x+48)$$
$$-240=\begin{bmatrix}x^3-14x^2+48x-12x^2\\ +168x-576\end{bmatrix}$$
$$0=x^3-26x^2+216x-336$$

Graph the function $f(x)=x^3-26x^2+216x-336$ on a graphing utility. Use the Zero feature to find positive real roots of the equation.



Each dimension is decreased by 2 ft. The smaller truck is 10 ft by 6 ft by 4 ft.

105. $6 = 2x\left(4 - x^2\right)$

$3 = x\left(4 - x^2\right)$

$3 = 4x - x^3$

$0 = x^3 - 4x + 3$

$$\frac{\text{Factors of } 3}{\text{Factors of } 1} = \frac{\pm 1, \pm 3}{\pm 1} = \pm 1, \pm 3$$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -4 & 3 \\ & & 1 & 1 & -3 \\ \hline & 1 & 1 & -3 & \underline{|0} \end{array}$$

Find the zeros of $x^2 + x - 3$.

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-3)}}{2(1)} = \frac{-1 \pm \sqrt{13}}{2}$$

The width must be positive, so use the positive zero.

The width of the rectangle is 2*x*, which is 2 or $2\left(\dfrac{-1+\sqrt{13}}{2}\right) = -1 + \sqrt{13}$.

When $x = 1$, $y = 4 - x^2 = 4 - (1)^2 = 3$.

When $x = \dfrac{-1+\sqrt{13}}{2}$,

$$y = 4 - x^2 = 4 - \left(\frac{-1+\sqrt{13}}{2}\right)^2$$

$$= 4 - \frac{1 - 2\sqrt{13} + 13}{4}$$

$$= \frac{16}{4} - \frac{14 - 2\sqrt{13}}{4}$$

$$= \frac{2 - 2\sqrt{13}}{4} = \frac{1 - 1\sqrt{13}}{2}.$$

The dimensions are either 2 cm by 3 cm or $-1 + \sqrt{13}$ cm by $\dfrac{1 - 1\sqrt{13}}{2}$ cm.

106. $12 = (5 - x)\left(x^2\right)$

$12 = 5x^2 - x^3$

$0 = x^3 - 5x^2 + 12$

$$\frac{\text{Factors of } 12}{\text{Factors of } 1} = \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1}$$

$$= \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$$

$$\begin{array}{r|rrrr} 2 & 1 & -5 & 0 & 12 \\ & & 2 & -6 & -12 \\ \hline & 1 & -3 & -6 & \underline{|0} \end{array}$$

Find the zeros of $x^2 - 3x - 6$.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{33}}{2}$$

The width must be positive, so use the positive zero.

The width of the rectangle is $5 - x$, which is $5 - 2 = 3$ or

$$5 - \frac{3+\sqrt{33}}{2} = \frac{10}{2} - \frac{3+\sqrt{33}}{2}$$

$$= \frac{7 - \sqrt{33}}{2}$$

When $x = 2$, $y = x^2 = (2)^2 = 4$.

When $x = \dfrac{3+\sqrt{33}}{2}$,

$$y = x^2 = \left(\frac{3+\sqrt{33}}{2}\right)^2$$

$$= \frac{9 + 6\sqrt{33} + 33}{4}$$

$$= \frac{42 + 6\sqrt{33}}{4} = \frac{21 + 3\sqrt{33}}{2}.$$

The dimensions are either 3 in. by 4 in. or $\dfrac{7 - \sqrt{33}}{2}$ in. by $\dfrac{21 + 3\sqrt{33}}{2}$ in.

107. a. $\dfrac{\text{Factors of } -12}{\text{Factors of } 1}$

$$= \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1}$$

$$= \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$$

$$\begin{array}{r|rrrrr} 1 & 1 & 2 & 1 & 8 & -12 \\ & & 1 & 3 & 4 & 12 \\ \hline & 1 & 3 & 4 & 12 & \underline{|0} \end{array}$$

Factor the quotient.

$$\begin{array}{r|rrrr} -3 & 1 & 3 & 4 & 12 \\ & & -3 & 0 & -12 \\ \hline & 1 & 0 & 4 & \underline{|0} \end{array}$$

$$f(x) = (x+3)(x-1)(x^2+4)$$

b. Factor $x^2 + 4$.

$$x^2 + 4 = 0$$

$$x^2 = -4$$

$$x = \pm 2i$$

$$f(x) = \begin{bmatrix} (x+3)(x-1) \\ (x+2i)(x-2i) \end{bmatrix}$$

108. a. $\dfrac{\text{Factors of } 8}{\text{Factors of } 1} = \dfrac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1}$

$$= \pm 1, \pm 2, \pm 4, \pm 8$$

$$\begin{array}{r|rrrrr} 2 & 1 & -6 & 9 & -6 & 8 \\ & & 2 & -8 & 2 & -8 \\ \hline & 1 & -4 & 1 & -4 & \underline{|0} \end{array}$$

Factor the quotient.

$$\begin{array}{r|rrrr} 4 & 1 & -4 & 1 & -4 \\ & & 4 & 0 & 4 \\ \hline & 1 & 0 & 1 & \underline{|0} \end{array}$$

$$f(x) = (x-4)(x-2)(x^2+1)$$

b. Factor $x^2 + 1$.

$$x^2 + 1 = 0$$

$$x^2 = -1$$

$$x = \pm i$$

$$f(x) = (x-4)(x-2)(x+i)(x-i)$$

109. a. $f(x) = x^4 + 2x^2 - 35$

$$f(x) = (x^2-5)(x^2+7)$$

$$0 = (x^2-5)(x^2+7)$$

$$x^2 - 5 = 0 \quad \text{or} \quad x^2 + 7 = 0$$

$$x^2 = 5 \qquad x^2 = -7$$

$$x = \pm\sqrt{5} \qquad x = \pm\sqrt{7}i$$

$$f(x) = (x-\sqrt{5})(x+\sqrt{5})(x^2+7)$$

b. $f(x) =$

$$\begin{bmatrix} (x-\sqrt{5})(x+\sqrt{5}) \\ (x+\sqrt{7}i)(x-\sqrt{7}i) \end{bmatrix}$$

110. a. $f(x) = x^4 + 8x^2 - 33$

$$f(x) = (x^2-3)(x^2+11)$$

$$0 = (x^2-3)(x^2+11)$$

$$x^2 - 3 = 0 \quad \text{or} \quad x^2 + 11 = 0$$

$$x^2 = 3 \qquad x^2 = -11$$

$$x = \pm\sqrt{3} \qquad x = \pm\sqrt{11}i$$

$$f(x) = (x-\sqrt{3})(x+\sqrt{3})(x^2+11)$$

b. $f(x) = \begin{bmatrix} (x-\sqrt{3})(x+\sqrt{3}) \\ (x+\sqrt{11}i)(x-\sqrt{11}i) \end{bmatrix}$

111. $\dfrac{\text{Factors of } -1}{\text{Factors of } 1} = \dfrac{\pm 1}{\pm 1} = \pm 1$

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & 0 & 0 & -1 \\ & & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & \underline{|0} \end{array}$$

Factor the quotient.

$$\begin{array}{r|rrrr} -1 & 1 & 1 & 1 & 1 \\ & & -1 & 0 & -1 \\ \hline & 1 & 0 & 1 & \boxed{0} \end{array}$$

Factor the quotient.

$$x^2+1=0$$
$$x^2=-1$$
$$x=\pm i$$

The fourth roots of 1 are 1, -1, i, and $-i$.

112. $f(x)=x^6-1$

$$=(x^3-1)(x^3+1)$$
$$=\big((x-1)(x^2+x+1)$$
$$(x+1)(x^2-x+1)\big)$$

Factor x^2+x+1.

$$x=\frac{-(1)\pm\sqrt{(1)-4(1)(1)}}{2(1)}$$
$$=\frac{-1\pm\sqrt{-3}}{2}=\frac{-1\pm i\sqrt{3}}{2}$$

Factor x^2-x+1.

$$x=\frac{-(-1)\pm\sqrt{(-1)-4(1)(1)}}{2(1)}$$
$$=\frac{1\pm\sqrt{-3}}{2}=\frac{1\pm i\sqrt{3}}{2}$$

The sixth roots of 1 are 1, -1, $\frac{-1\pm i\sqrt{3}}{2}$, and $\frac{1\pm i\sqrt{3}}{2}$.

113. The number $\sqrt{5}$ is a real solution to the equation $x^2-5=0$ and a zero of the polynomial $f(x)=x^2-5$. However, by the rational zeros theorem, the only possible rational zeros of $f(x)$ are ±1 and ±5. This means that $\sqrt{5}$ is irrational.

114. a. $ax+b=0$

$$ax=-b$$
$$x=-\frac{b}{a}$$

b. $x=\dfrac{-b\pm\sqrt{b-4ac}}{2a}$

115. $x^3-3x=-2$

$$x^3+(-3)x=(-2)$$
$$m=-3,\ n=-2$$

$$x=\left[\begin{array}{l}\sqrt[3]{\sqrt{\left(\frac{n}{2}\right)^2+\left(\frac{m}{3}\right)^3}+\frac{n}{2}}\\ -\sqrt[3]{\sqrt{\left(\frac{n}{2}\right)^2+\left(\frac{m}{3}\right)^3}-\frac{n}{2}}\end{array}\right]$$

$$=\left[\begin{array}{l}\sqrt[3]{\sqrt{\left(\frac{-2}{2}\right)^2+\left(\frac{-3}{3}\right)^3}+\frac{-2}{2}}\\ -\sqrt[3]{\sqrt{\left(\frac{-2}{2}\right)^2+\left(\frac{-3}{3}\right)^3}-\frac{-2}{2}}\end{array}\right]$$

$$=\sqrt[3]{\sqrt{1+(-1)}-1}-\sqrt[3]{\sqrt{1+(-1)}+1}$$
$$=\sqrt[3]{-1}-\sqrt[3]{1}=-1+-1=-2$$

116. $x^3+9x=26$

$$m=9,\ n=26$$

$$x=\left[\begin{array}{l}\sqrt[3]{\sqrt{\left(\frac{n}{2}\right)^2+\left(\frac{m}{3}\right)^3}+\frac{n}{2}}\\ -\sqrt[3]{\sqrt{\left(\frac{n}{2}\right)^2+\left(\frac{m}{3}\right)^3}-\frac{n}{2}}\end{array}\right]$$

$$=\left[\begin{array}{l}\sqrt[3]{\sqrt{\left(\frac{26}{2}\right)^2+\left(\frac{9}{3}\right)^3}+\frac{26}{2}}\\ \quad -\sqrt[3]{\sqrt{\left(\frac{26}{2}\right)^2+\left(\frac{9}{3}\right)^3}-\frac{26}{2}}\end{array}\right]$$

$$=\left[\begin{array}{l}\sqrt[3]{\sqrt{169+27}+13}\\ \quad -\sqrt[3]{\sqrt{169+27}-13}\end{array}\right]$$

$$=\sqrt[3]{\sqrt{196}+13}-\sqrt[3]{\sqrt{196}-13}$$

$$=\sqrt[3]{14+13}-\sqrt[3]{14-13}$$

$$=\sqrt[3]{27}-\sqrt[3]{1}$$

$$=3-1=2$$

Section 2.5 Rational Functions

1. $q(x)$

2. x approaches infinity

3. x approaches 5 from the left

4. vertical; c

5. nonzero

6. <; >

7. $x-5\neq 0$

$x\neq 5$

$(-\infty, 5)\cup(5, \infty)$

8. $x-3\neq 0$

$x\neq 3$

$(-\infty, 3)\cup(3, \infty)$

9. $4x^2+3x-1\neq 0$

$(x+1)(4x-1)\neq 0$

$x\neq -1, x\neq \frac{1}{4}$

$\left(-\infty, -1\right)\cup\left(-1, \frac{1}{4}\right)\cup\left(\frac{1}{4}, \infty\right)$

10. $2x^2+5x-7\neq 0$

$(2x+7)(x-1)\neq 0$

$x\neq -\frac{7}{2}, x\neq 1$

$\left(-\infty, -\frac{7}{2}\right)\cup\left(-\frac{7}{2}, 1\right)\cup\left(1, \infty\right)$

11. $x^2+100\neq 0$

True for all real x.

$(-\infty, \infty)$

12. $x^2+49\neq 0$

True for all real x.

$(-\infty, \infty)$

13. a. 2

b. $-\infty$

c. ∞

d. 2

e. Never increasing

f. $(-\infty, 4)\cup(4, \infty)$

g. $\left(-\infty, 4\right) \cup \left(4, \infty\right)$

h. $\left(-\infty, 2\right) \cup \left(2, \infty\right)$

i. $x = 4$

j. $y = 2$

14. a. -2

b. ∞

c. $-\infty$

d. -2

e. $\left(-\infty, -3\right) \cup \left(-3, \infty\right)$

f. Never decreasing

g. $\left(-\infty, -3\right) \cup \left(-3, \infty\right)$

h. $\left(-\infty, -2\right) \cup \left(-2, \infty\right)$

i. $x = -3$

j. $y = -2$

15. a. -1

b. ∞

c. ∞

d. -1

e. $\left(-\infty, -3\right)$

f. $\left(-3, \infty\right)$

g. $\left(-\infty, -3\right) \cup \left(-3, \infty\right)$

h. $\left(-1, \infty\right)$

i. $x = -3$

j. $y = -1$

16. a. 4

b. $-\infty$

c. $-\infty$

d. 4

e. $\left(1, \infty\right)$

f. $\left(-\infty, 1\right)$

g. $\left(-\infty, 1\right) \cup \left(1, \infty\right)$

h. $\left(-\infty, 4\right)$

i. $x = 1$

j. $y = 4$

17. $x - 4 = 0$

$x = 4$

18. $x + 7 = 0$

$x = -7$

19.
$$\begin{aligned} 2x^2 - 9x - 5 &= 0 \\ \left(x-5\right)\left(2x+1\right) &= 0 \\ x = 5, x &= -\frac{1}{2} \end{aligned}$$

20.
$$\begin{aligned} 3x^2 + 8x - 3 &= 0 \\ \left(x+3\right)\left(3x-1\right) &= 0 \\ x = -3, x &= \frac{1}{3} \end{aligned}$$

21. $x^2 + 5$ has no real zeros. The function has no vertical asymptotes.

22. $x^4 + 1$ has no real zeros. The function has no vertical asymptotes.

23. $2t^2 + 4t - 3 = 0$

$$\begin{aligned} t &= \frac{-\left(4\right) \pm \sqrt{\left(4\right)^2 - 4\left(2\right)\left(-3\right)}}{2\left(2\right)} \\ &= \frac{-4 \pm \sqrt{40}}{4} \\ &= \frac{-4 \pm 2\sqrt{10}}{4} = \frac{-2 \pm \sqrt{10}}{2} \\ t &= \frac{-2 + \sqrt{10}}{2}, t = \frac{-2 - \sqrt{10}}{2} \end{aligned}$$

24. $3a^2 + 4a - 1 = 0$

$$\begin{aligned} a &= \frac{-\left(4\right) \pm \sqrt{\left(4\right)^2 - 4\left(3\right)\left(-1\right)}}{2\left(3\right)} \\ &= \frac{-4 \pm \sqrt{28}}{6} = \frac{-4 \pm 2\sqrt{7}}{6} \end{aligned}$$

$$=\frac{-2\pm\sqrt{7}}{3}$$

$$a=\frac{-2+\sqrt{7}}{3}, a=\frac{-2-\sqrt{7}}{3}$$

25. a

26. c

27. d

28. b

29. a. The degree of the numerator is 0.
The degree of the denominator is 2.
Since $n < m$, the line $y = 0$ is a horizontal asymptote of *p*.

b. $\frac{5}{x^2+2x+1}=0$

$5 = 0$ No solution

The graph does not cross $y = 0$.

30. a. The degree of the numerator is 0.
The degree of the denominator is 2.
Since $n < m$, the line $y = 0$ is a horizontal asymptote of *q*.

b. $\frac{8}{x^2+4x+4}=0$

$8 = 0$ No solution

The graph does not cross $y = 0$.

31. a. The degree of the numerator is 2.
The degree of the denominator is 2.
Since $n = m$, the line $y = \frac{3}{1}$ or equivalently $y = 3$ is a horizontal asymptote of *h*.

b. $\frac{3x^2+8x-5}{x^2+3}=3$

$$3x^2+8x-5=3x^2+9$$
$$8x=14$$
$$x=\frac{7}{4}$$

The graph crosses $y = 3$ at $\left(\frac{7}{4},3\right)$.

32. a. The degree of the numerator is 2.
The degree of the denominator is 2.
Since $n = m$, the line $y = \frac{-4}{1}$ or equivalently $y = -4$ is a horizontal asymptote of *r*.

b. $\frac{-4x^2+5x-1}{x^2+2}=-4$

$$-4x^2+5x-1=-4x^2-8$$
$$5x=-7$$
$$x=-\frac{7}{5}$$

The graph crosses $y = -4$ at $\left(-\frac{7}{5},-4\right)$.

33. a. The degree of the numerator is 4.
The degree of the denominator is 1.
Since $n > m$, the function has no horizontal asymptotes.

b. Not applicable

34. a. The degree of the numerator is 3.
The degree of the denominator is 1.
Since $n > m$, the function has no horizontal asymptotes.

b. Not applicable

35. a. The degree of the numerator is 1.
The degree of the denominator is 2.

Since $n < m$, the line $y = 0$ is a horizontal asymptote of *t*.

b. $\dfrac{2x+4}{x^2+7x-4} = 0$

$$2x+4=0$$
$$2x=-4$$
$$x=-2$$

The graph crosses $y = 0$ at $(-2,0)$.

35. a. The degree of the numerator is 1. The degree of the denominator is 2. Since $n < m$, the line $y = 0$ is a horizontal asymptote of *s*.

b. $\dfrac{x+3}{2x^2-3x-5} = 0$

$$x+3=0$$
$$x=-3$$

The graph crosses $y = 0$ at $(-3,0)$.

37. a. $\dfrac{x^2+3x+1}{2x^2+5} = \dfrac{\frac{x^2}{x^2}+\frac{3x}{x^2}+\frac{1}{x^2}}{\frac{2x^2}{x^2}+\frac{5}{x^2}}$

$$= \frac{1+\frac{3}{x}+\frac{1}{x^2}}{2+\frac{5}{x^2}}$$

b. All of the terms approach 0 as $|x| \to \infty$.

c. Since the terms from part (b) approach 0 as $|x| \to \infty$, the horizontal asymptote is

$y = \dfrac{1+0+0}{2+0}$ or $y = \dfrac{1}{2}$.

38. a. $\dfrac{3x^3-2x^2+7x}{5x^3+1} = \dfrac{\frac{3x^3}{x^3}-\frac{2x^2}{x^3}+\frac{7x}{x^3}}{\frac{5x^3}{x^3}+\frac{1}{x^3}}$

$$= \frac{3-\frac{2}{x}+\frac{7}{x^2}}{5+\frac{1}{x^3}}$$

b. All of the terms approach 0 as $|x| \to \infty$.

c. Since the terms from part (b) approach 0 as $|x| \to \infty$, the horizontal asymptote is

$y = \dfrac{3-0+0}{5+0}$ or $y = \dfrac{3}{5}$.

39. The expression $\dfrac{2x^2+3}{x}$ is in lowest terms and the denominator is 0 at $x = 0$. *f* has a vertical asymptote at $x = 0$. The degree of the numerator is exactly one greater than the degree of the denominator. Therefore, *f* has no horizontal asymptote, but does have a slant asymptote.

$$\frac{2x^2+3}{x} = \frac{2x^2}{x}+\frac{3}{x} = 2x+\frac{3}{x}$$

The slant asymptote is $y = 2x$.

40. The expression $\dfrac{3x^2+2}{x}$ is in lowest terms and the denominator is 0 at $x = 0$. *g* has a vertical asymptote at $x = 0$. The degree of the numerator is exactly one greater than the degree of the denominator. Therefore, *g* has no horizontal asymptote, but does have a slant asymptote.

$$\frac{3x^2+2}{x} = \frac{3x^2}{x}+\frac{2}{x} = 3x+\frac{2}{x}$$

The slant asymptote is $y = 3x$.

41. The expression $\frac{-3x^2+4x-5}{x+6}$ is in lowest terms and the denominator is 0 at $x=-6$. h has a vertical asymptote at $x=-6$. The degree of the numerator is exactly one greater than the degree of the denominator. Therefore, h has no horizontal asymptote, but does have a slant asymptote.

$$\begin{array}{r|rrr} -6 & -3 & 4 & -5 \\ & & 18 & -132 \\ \hline & -3 & 22 & \boxed{-137} \end{array}$$

The quotient is $-3x+22$.

The slant asymptote is $y=-3x+22$.

42. The expression $\frac{-2x^2-3x+7}{x+3}$ is in lowest terms and the denominator is 0 at $x=-3$. k has a vertical asymptote at $x=-3$. The degree of the numerator is exactly one greater than the degree of the denominator. Therefore, k has no horizontal asymptote, but does have a slant asymptote.

$$\begin{array}{r|rrr} -3 & -2 & -3 & 7 \\ & & 6 & -9 \\ \hline & -2 & 3 & \boxed{-2} \end{array}$$

The quotient is $-2x+3$.

The slant asymptote is $y=-2x+3$.

43. The expression $\frac{x^3+5x^2-4x+1}{x^2-5}$ is in lowest terms.

$x^2-5=0$

$x^2=5$

$x=\pm\sqrt{5}$

The denominator is 0 at $x=\pm\sqrt{5}$. p has vertical asymptotes at $x=\sqrt{5}$ and $x=-\sqrt{5}$.

The degree of the numerator is exactly one greater than the degree of the denominator.

Therefore, p has no horizontal asymptote, but does have a slant asymptote.

$$\begin{array}{r} x+5 \\ x^2+0x-5\overline{\smash{)}x^3+5x^2-4x+1} \\ -\left(x^3+0x^2-5x\right) \\ \hline 5x^2+x+1 \\ -\left(5x^2+0x-25\right) \\ \hline x+26 \end{array}$$

The quotient is $x+5$.

The slant asymptote is $y=x+5$.

44. The expression $\frac{x^3+3x^2-2x-4}{x^2-7}$ is in lowest terms.

$x^2-7=0$

$x^2=7$

$x=\pm\sqrt{7}$

The denominator is 0 at $x=\pm\sqrt{7}$. q has vertical asymptotes at $x=\sqrt{7}$ and $x=-\sqrt{7}$.

The degree of the numerator is exactly one greater than the degree of the denominator.

Therefore, q has no horizontal asymptote, but does have a slant asymptote.

$$\begin{array}{r@{}l} & x+3 \\ x^2+0x-7 & \overline{)x^3+3x^2-2x-4} \\ & \underline{-\left(x^3+0x^2-7x\right)} \\ & 3x^2+5x-4 \\ & \underline{-\left(3x^2+0x-21\right)} \\ & 5x+17 \end{array}$$

The quotient is $x+3$.

The slant asymptote is $y=x+3$.

45. The expression $\dfrac{2x+1}{x^3+x^2-4x-4}$ is in lowest terms.

$$\begin{aligned}\frac{2x+1}{x^3+x^2-4x-4}&=\frac{2x+1}{x^2\left(x+1\right)-4\left(x+1\right)}\\&=\frac{2x+1}{\left(x+1\right)\left(x^2-4\right)}\end{aligned}$$

The denominator is 0 at $x=-1$, $x=-2$, and $x=2$. *r* has vertical asymptotes at $x=-1$, $x=-2$, and $x=2$.

The degree of the numerator is 1.

The degree of the denominator is 3.

Since $n<m$, the line $y=0$ is a horizontal asymptote of *r*. *r* has no slant asymptote.

46. The expression $\dfrac{3x-4}{x^3+2x^2-9x-18}$ is in lowest terms.

$$\begin{aligned}\frac{3x-4}{x^3+2x^2-9x-18}&=\frac{3x-4}{x^2\left(x+2\right)-9\left(x+2\right)}\\&=\frac{3x-4}{\left(x+2\right)\left(x^2-9\right)}\end{aligned}$$

The denominator is 0 at $x=-2$, $x=-3$, and $x=3$. *t* has vertical asymptotes at $x=-2$, $x=-3$, and $x=3$.

The degree of the numerator is 1.

The degree of the denominator is 3.

Since $n<m$, the line $y=0$ is a horizontal asymptote of *t*. *t* has no slant asymptote.

47. The expression $\dfrac{4x^3-2x^2+7x-3}{2x^2+4x+3}$ is in lowest terms.

$$2x^2+4x+3=0$$

$$x=\frac{-\left(4\right)\pm\sqrt{\left(4\right)^2-4\left(2\right)\left(3\right)}}{2\left(2\right)}=\frac{-4\pm\sqrt{-8}}{4}$$

The denominator is never 0, so *f* has no vertical asymptotes. The degree of the numerator is exactly one greater than the degree of the denominator. Therefore, *f* has no horizontal asymptote, but does have a slant asymptote.

$$\begin{array}{r@{}l} & 2x-5 \\ 2x^2+4x+3 & \overline{)4x^3-2x^2+7x-3} \\ & \underline{-\left(4x^3+8x^2+6x\right)} \\ & -10x^2+x-3 \\ & \underline{-\left(-10x^2-20x-15\right)} \\ & 21x+12 \end{array}$$

The quotient is $2x-5$.

The slant asymptote is $y=2x-5$.

48. The expression $\dfrac{9x^3-5x+4}{3x^2+2x+1}$ is in lowest terms.

$$3x^2+2x+1=0$$

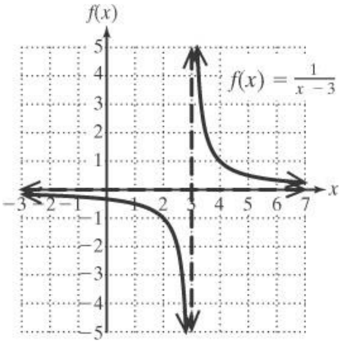
$$x=\frac{-\left(2\right)\pm\sqrt{\left(2\right)^2-4\left(3\right)\left(1\right)}}{2\left(3\right)}=\frac{-2\pm\sqrt{-8}}{6}$$

The denominator is never 0, so *a* has no vertical asymptotes. The degree of the numerator is exactly one greater than the degree of the denominator. Therefore, *a* has no horizontal asymptote, but does have a slant asymptote.

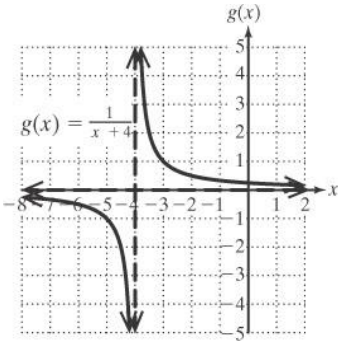
$$\begin{array}{r} 3x-2 \\ 3x^2+2x+1\overline{)9x^3+0x^2-5x+4} \\ -\left(9x^3+6x^2+3x\right) \\ \hline -6x^2-8x+4 \\ -\left(-6x^2-4x-2\right) \\ \hline -4x+6 \end{array}$$

The quotient is $3x-2$.
The slant asymptote is $y=3x-2$.

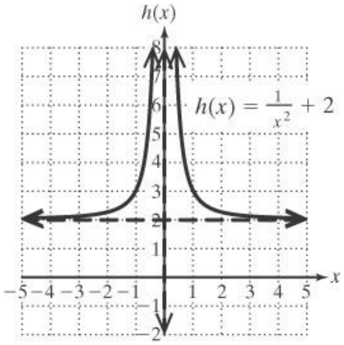
49. The graph of *f* is the graph of $y=\frac{1}{x}$ with a shift right 3 units.



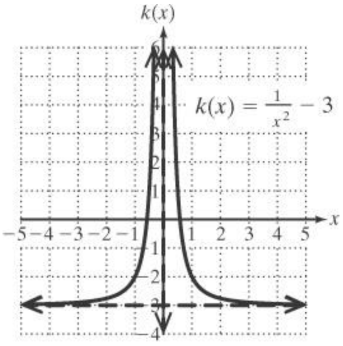
50. The graph of *g* is the graph of $y=\frac{1}{x}$ with a shift left 4 units.



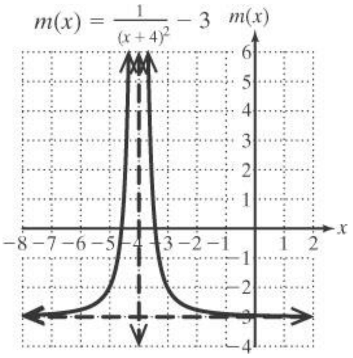
51. The graph of *h* is the graph of $y=\frac{1}{x^2}$ with a shift upward 2 units.



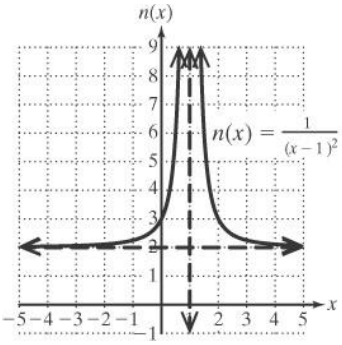
52. The graph of *k* is the graph of $y=\frac{1}{x^2}$ with a shift downward 3 units.



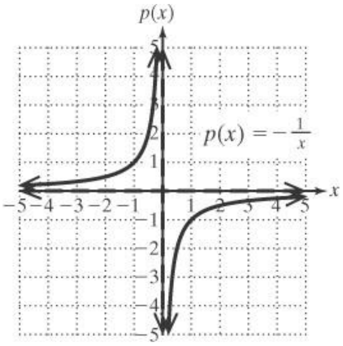
53. The graph of *m* is the graph of $y=\frac{1}{x^2}$ with a shift to the left 4 units and a shift downward 3 units.



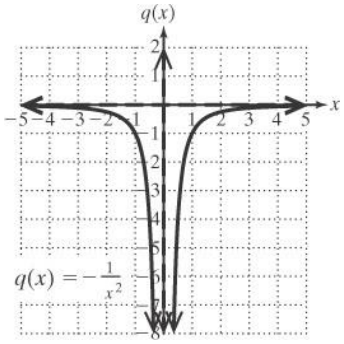
54. The graph of n is the graph of $y = \frac{1}{x^2}$ with a shift to the right 1 unit and a shift upward 2 units.



55. The graph of p is the graph of $y = \frac{1}{x}$ reflected across the x-axis.



56. The graph of q is the graph of $y = \frac{1}{x^2}$ reflected across the x-axis.



57. a. The numerator is 0 at $x = -3$ and $x = \frac{7}{2}$.

The x-intercepts are $(-3, 0)$ and $\left(\frac{7}{2}, 0\right)$.

b. The denominator is 0 at $x = -2$ and $x = -\frac{1}{4}$. f has vertical asymptotes at $x = -2$ and $x = -\frac{1}{4}$.

c. The degree of the numerator is 2. The degree of the denominator is 2. Since $n = m$, the line $y = \frac{2}{4}$ or equivalently $y = \frac{1}{2}$ is a horizontal asymptote of f.

d. The constant term in the numerator is -21. The constant term in the denominator is 2. All other terms are 0 when evaluated at $x = 0$. The y-intercept is $\left(0, -\frac{21}{2}\right)$.

58. a. The numerator is 0 at $x = \frac{4}{3}$ and $x = 6$.

The x-intercepts are $\left(\frac{4}{3}, 0\right)$ and $\left(6, 0\right)$.

b. The denominator is 0 at $x = \frac{3}{2}$ and $x = -5$. f has vertical asymptotes at $x = \frac{3}{2}$ and $x = -5$.

c. The degree of the numerator is 2. The degree of the denominator is 2. Since $n = m$, the line $y = \frac{3}{2}$ is a horizontal asymptote of f.

d. The constant term in the numerator is 24. The constant term in the denominator is 15. All other terms are 0 when evaluated at $x = 0$. The y-intercept is $\left(0, \frac{24}{-15}\right) = \left(0, -\frac{8}{5}\right)$.

59. a. The numerator is 0 at $x = \frac{9}{4}$.

The x-intercept is $\left(\frac{9}{4}, 0\right)$.

b. The denominator is 0 at $x = 3$ and $x = -3$. f has vertical asymptotes at $x = 3$ and $x = -3$.

c. The degree of the numerator is 1. The degree of the denominator is 2. Since $n < m$, the line $y = 0$ is a horizontal asymptote of f.

d. The constant term in the numerator is -9. The constant term in the denominator is -9. All other terms are 0 when evaluated at $x = 0$. The y-intercept is $\left(0, \frac{-9}{-9}\right) = \left(0, 1\right)$.

60. a. The numerator is 0 at $x = \frac{8}{5}$.

The x-intercept is $\left(\frac{8}{5}, 0\right)$.

b. The denominator is 0 at $x = 2$ and $x = -2$. f has vertical asymptotes at $x = 2$ and $x = -2$.

c. The degree of the numerator is 1. The degree of the denominator is 2. Since $n < m$, the line $y = 0$ is a horizontal asymptote of f.

d. The constant term in the numerator is -8. The constant term in the denominator is -4. All other terms are 0 when evaluated at $x = 0$.

The y-intercept is $\left(0, \frac{-8}{-4}\right) = \left(0, 2\right)$.

61. a. The numerator is 0 at $x = \frac{1}{5}$ and $x = -3$.

The x-intercepts are $\left(\frac{1}{5}, 0\right)$ and $\left(-3, 0\right)$.

b. The denominator is 0 at $x = -2$. f has a vertical asymptote at $x = -2$.

c.
$$f\left(x\right) = \frac{\left(5x-1\right)\left(x+3\right)}{x+2} = \frac{5x^2 + 14x - 3}{x+2}$$

The degree of the numerator is exactly one greater than the degree of the denominator. Therefore, f has no horizontal asymptote, but does have a slant asymptote.

$$\begin{array}{r|rrr} -2 & 5 & 14 & -3 \\ & & -10 & -8 \\ \hline & 5 & 4 & \boxed{-11} \end{array}$$

The quotient is $5x+4$. The slant asymptote is $y=5x+4$.

d. The constant term in the numerator is -3. The constant term in the denominator is 2. All other terms are 0 when evaluated at $x=0$.

The y-intercept is $\left(0, \frac{-3}{-2}\right)=\left(0, -\frac{3}{2}\right)$.

62. a. The numerator is 0 at $x=-\frac{3}{4}$ and $x=-2$.

The x-intercepts are $\left(-\frac{3}{4},0\right)$ and $(-2,0)$.

b. The denominator is 0 at $x=-3$. f has a vertical asymptote at $x=-3$.

c. $$f(x)=\frac{(4x+3)(x+2)}{x+3}$$
$$=\frac{4x^2+11x+6}{x+3}$$

The degree of the numerator is exactly one greater than the degree of the denominator. Therefore, f has no horizontal asymptote, but does have a slant asymptote.

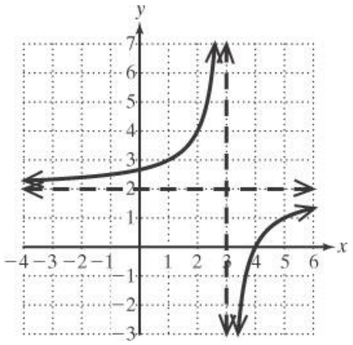
$$\begin{array}{r|rrr} -3 & 4 & 11 & 6 \\ & & -12 & 3 \\ \hline & 4 & -1 & \boxed{9} \end{array}$$

The quotient is $4x-1$. The slant asymptote is $y=4x-1$.

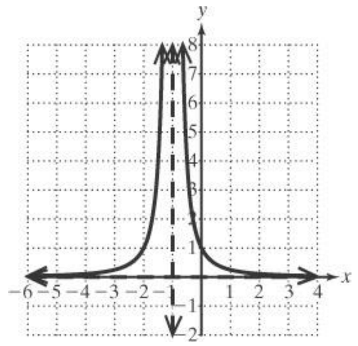
d. The constant term in the numerator is 6. The constant term in the denominator is 3. All other terms are 0 when evaluated at $x=0$.

The y-intercept is $\left(0, \frac{6}{3}\right)=(0,2)$.

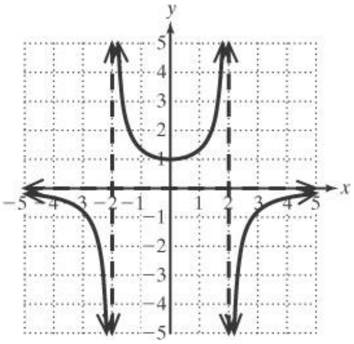
63.



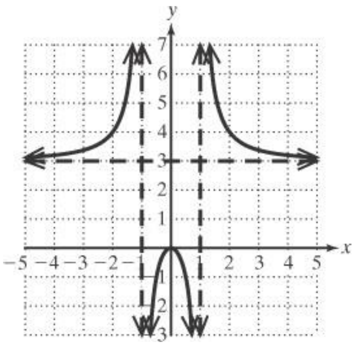
64.



65.



66.



67. $n(0) = \frac{-3}{2(0)+7} = \frac{-3}{7} = -\frac{3}{7}$

The y-intercept is $\left(0, -\frac{3}{7}\right)$.

n is never 0. It has no x-intercepts.

n is in lowest terms, and $2x+7$ is 0 for $x = -\frac{7}{2}$, which is the vertical asymptote.

The degree of the numerator is 0. The degree of the denominator is 1.
Since $n < m$, the line $y = 0$ is a horizontal asymptote of n.

$$0 = \frac{-3}{2x+7}$$

$0 = -3$ No solution

n does not cross its horizontal asymptote.

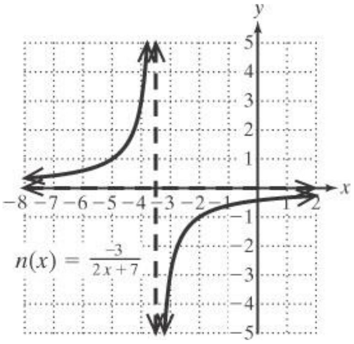
$$n(-x) = \frac{-3}{2(-x)+7} = \frac{-3}{-2x+7}$$

$n(-x) \neq n(x)$, $n(-x) \neq -n(x)$

n is neither even nor odd.

Interval	**Test Point**	**Comments**
$\left(-\infty, -\frac{7}{2}\right)$	$(-4, 3)$	$n(x)$ is positive. $n(x)$ must approach the horizontal asymptote $y = 0$ from above as $x \to -\infty$. As x approaches the vertical asymptote $x = -\frac{7}{2}$ from the left, $n(x) \to \infty$.
$\left(-\frac{7}{2}, \infty\right)$	$(-2, -1)$	$n(x)$ is negative. $n(x)$ must approach the horizontal asymptote $y = 0$ from below as $x \to \infty$.

		As x approaches the vertical asymptote $x=-\frac{7}{2}$ from the right, $n(x)\to-\infty$.



68. $m(0)=\frac{-4}{2(0)-5}=\frac{-4}{-5}=\frac{4}{5}$

The *y*-intercept is $\left(0,\frac{4}{5}\right)$.

m is never 0. It has no *x*-intercepts.

m is in lowest terms, and $2x-5$ is 0 for $x=\frac{5}{2}$, which is the vertical asymptote.

The degree of the numerator is 0. The degree of the denominator is 1.
Since $n<m$, the line $y=0$ is a horizontal asymptote of *m*.

$$0=\frac{-4}{2x-5}$$

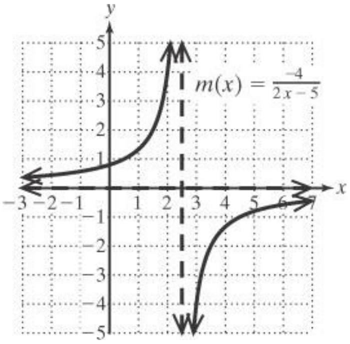
$0=-4$ No solution
m does not cross its horizontal asymptote.

$$m(-x)=\frac{-4}{2(-x)-5}=\frac{-4}{-2x-5}$$

$m(-x)\neq m(x)$, $m(-x)\neq -m(x)$
n is neither even nor odd.

Interval	**Test Point**	**Comments**
$\left(-\infty,\frac{5}{2}\right)$	$(2,4)$	*m*(*x*) is positive. *m*(*x*) must approach the horizontal asymptote $y=0$ from above as $x\to-\infty$.

		As x approaches the vertical asymptote $x = \frac{5}{2}$ from the left, $m(x) \to \infty$.
$\left(\frac{5}{2}, \infty\right)$	$(3, -4)$	$m(x)$ is negative. $m(x)$ must approach the horizontal asymptote $y = 0$ from below as $x \to \infty$. As x approaches the vertical asymptote $x = \frac{5}{2}$ from the right, $m(x) \to -\infty$.



69. $f(0) = \frac{0-4}{0-2} = \frac{-4}{-2} = 2$

The y-intercept is $(0, 2)$.

$$x - 4 = 0$$
$$x = 4$$

The x-intercept is $(4, 0)$.

f is in lowest terms, and $x - 2$ is 0 for $x = 2$, which is the vertical asymptote.

The degree of the numerator is 1.

The degree of the denominator is 1.

Since $n = m$, the line $y = 1$ is a horizontal asymptote of f.

$$\frac{1}{1} = \frac{x-4}{x-2}$$
$$x - 2 = x - 4$$
$$-2 = -4 \text{ No solution}$$

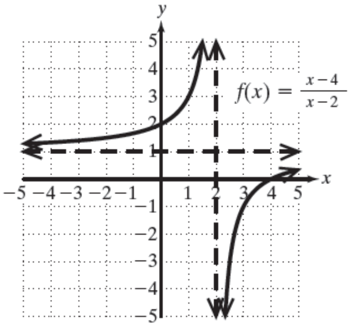
f does not cross its horizontal asymptote.

$$f(-x)=\frac{(-x)-4}{(-x)-2}=\frac{-x-4}{-x-2}$$

$f(-x)\neq f(x)$, $f(-x)\neq -f(x)$

f is neither even nor odd.

Interval	**Test Point**	**Comments**
$(-\infty, 2)$	$\left(-1, \frac{5}{3}\right)$	*f*(*x*) is positive. *f*(*x*) must approach the horizontal asymptote $y=1$ from down as $x\to-\infty$.
$(2, 4)$	$(3, -1)$	*f*(*x*) is negative. As *x* approaches the vertical asymptote $x=2$ from the left, $f(x)\to-\infty$.
$(4, \infty)$	$\left(6, \frac{1}{2}\right)$	*f*(*x*) is positive. *f*(*x*) must approach the horizontal asymptote $y=1$ from down as $x\to\infty$. As *x* approaches the vertical asymptote $x=2$ from the right, $f(x)\to\infty$.



70. $g(0)=\frac{0-3}{0-1}=\frac{-3}{-1}=3$

The *y*-intercept is $(0, 3)$.

$x-3=0$

$x=3$

The *x*-intercept is $(3, 0)$.

g is in lowest terms, and $x-1$ is 0 for $x=1$, which is the vertical asymptote.

The degree of the numerator is 1.

The degree of the denominator is 1.

Since $n = m$, the line $y = 1$ is a horizontal asymptote of *g*.

$$\frac{1}{1} = \frac{x-3}{x-1}$$

$$x - 1 = x - 3$$

$$-1 = -3 \text{ No solution}$$

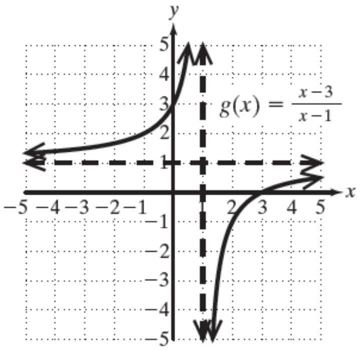
g does not cross its horizontal asymptote.

$$g(-x) = \frac{(-x)-3}{(-x)-1} = \frac{-x-3}{-x-1}$$

$$g(-x) \neq g(x),\ g(-x) \neq -g(x)$$

g is neither even nor odd.

Interval	**Test Point**	**Comments**
$(-\infty, 1)$	$(-1, 2)$	*g*(*x*) is positive. *g*(*x*) must approach the horizontal asymptote $y = 1$ from down as $x \to -\infty$.
$(1, 3)$	$(2, -1)$	*g*(*x*) is negative. As *x* approaches the vertical asymptote $x = 1$ from the left, $g(x) \to -\infty$.
$(3, \infty)$	$\left(4, \frac{1}{3}\right)$	*g*(*x*) is positive. *g*(*x*) must approach the horizontal asymptote $y = 1$ from above as $x \to \infty$. As *x* approaches the vertical asymptote $x = 1$ from the right, $g(x) \to \infty$.



71. $h(0)=\dfrac{2(0)-4}{0+3}=\dfrac{-4}{3}=-\dfrac{4}{3}$

The y-intercept is $\left(0,-\dfrac{4}{3}\right)$.

$$2x-4=0$$
$$x=2$$

The x-intercept is $(2,0)$.

h is in lowest terms, and $x+3$ is 0 for $x=-3$, which is the vertical asymptote.

The degree of the numerator is 1.

The degree of the denominator is 1.

Since $n=m$, the line $y=2$ is a horizontal asymptote of h.

$$\frac{2}{1}=\frac{2x-4}{x+3}$$
$$2x+6=2x-4$$
$$6=-4 \text{ No solution}$$

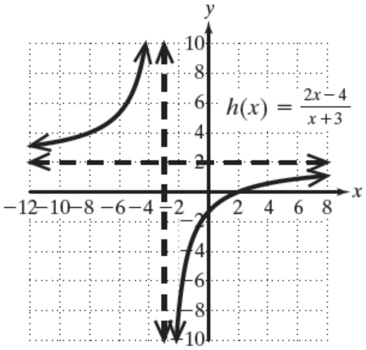
h does not cross its horizontal asymptote.

$$h(-x)=\frac{2(-x)-4}{(-x)+3}=\frac{-2x-4}{-x+3}$$

$h(-x)\neq h(x)$, $h(-x)\neq -h(x)$

f is neither even nor odd.

Interval	**Test Point**	**Comments**
$(-\infty,-3)$	$(-4,12)$	$h(x)$ is negative. $h(x)$ must approach the horizontal asymptote $y=2$ from down as $x\to-\infty$.
$(-3,2)$	$\left(1,-\frac{1}{2}\right)$	$h(x)$ is negative. As x approaches the vertical asymptote $x=-3$ from the left, $h(x)\to-\infty$.
$(2,\infty)$	$\left(4,\frac{4}{7}\right)$	$h(x)$ is positive. $h(x)$ must approach the horizontal asymptote $y=2$ from above as $x\to\infty$. As x approaches the vertical asymptote $x=-3$ from the left, $h(x)\to\infty$.



72. $k(0)=\dfrac{3(0)-9}{0+2}=\dfrac{-9}{2}=-\dfrac{9}{2}$

The y-intercept is $\left(0,-\dfrac{9}{2}\right)$.

$$\begin{aligned}3x-9&=0\\3x&=9\\x&=3\end{aligned}$$

The x-intercept is $(3,0)$.

k is in lowest terms, and $x+2$ is 0 for $x=-2$, which is the vertical asymptote.

The degree of the numerator is 1.

The degree of the denominator is 1.

Since $n=m$, the line $y=3$ is a horizontal asymptote of k.

$$\begin{aligned}\frac{3}{1}&=\frac{3x-9}{x+2}\\3x+6&=3x-9\\6&=-9\ \text{ No solution}\end{aligned}$$

k does not cross its horizontal asymptote.

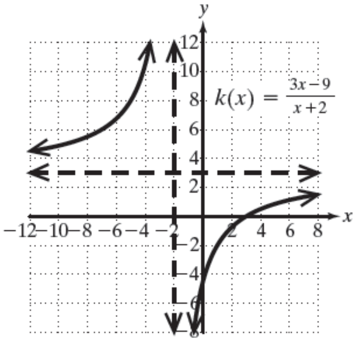
$k(-x)=\dfrac{3(-x)-9}{(-x)+2}=\dfrac{-3x-9}{-x+2}$

$k(-x)\neq k(x),\ k(-x)\neq -k(x)$

k is neither even nor odd.

Interval	**Test Point**	**Comments**
$(-\infty,-2)$	$(-3,18)$	$k(x)$ is positive. $k(x)$ must approach the horizontal asymptote $y=3$ from down as $x\to-\infty$.

$\left(-2, 3\right)$	$\left(2, -\frac{3}{4}\right)$	$k(x)$ is negative. As x approaches the vertical asymptote $x = -2$ from the left, $k\left(x\right) \to -\infty$.
$\left(3, \infty\right)$	$\left(4, \frac{1}{2}\right)$	$k(x)$ is positive. $k(x)$ must approach the horizontal asymptote $y = 3$ from above as $x \to \infty$. As x approaches the vertical asymptote $x = -2$ from the left, $k\left(x\right) \to \infty$.



73. $p\left(0\right) = \dfrac{6}{\left(0\right)^2 - 9} = \dfrac{6}{-9} = -\dfrac{2}{3}$

The y-intercept is $\left(0, -\frac{2}{3}\right)$.

p is never 0. It has no x-intercepts.

p is in lowest terms, and $x^2 - 9$ is 0 for $x = 3$ and $x = -3$, which are the vertical asymptotes.

The degree of the numerator is 0.
The degree of the denominator is 2.
Since $n < m$, the line $y = 0$ is a horizontal asymptote of p.

$$0 = \frac{6}{x^2 - 9}$$

$0 = 6$ No solution

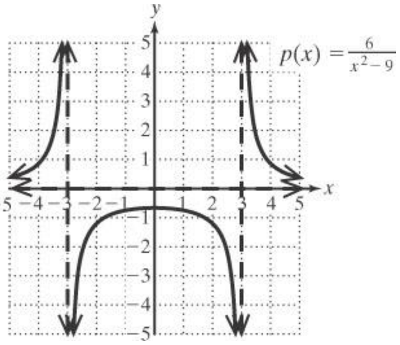
p does not cross its horizontal asymptote.

$$p\left(-x\right) = \frac{6}{\left(-x\right)^2 - 9} = \frac{6}{x^2 - 9}$$

$$p\left(-x\right) = p\left(x\right)$$

p is even.

Interval	**Test Point**	**Comments**
$(-\infty, -3)$	$\left(-5, \frac{3}{8}\right)$	$p(x)$ is positive. $p(x)$ must approach the horizontal asymptote $y = 0$ from above as $x \to -\infty$. As x approaches the vertical asymptote $x = -3$ from the left, $p(x) \to \infty$.
$(-3, 0)$	$\left(-2, -\frac{6}{5}\right)$	$p(x)$ is negative. As x approaches the vertical asymptote $x = -3$ from the right, $p(x) \to -\infty$.
$(0, 3)$	$\left(2, -\frac{6}{5}\right)$	$p(x)$ is negative. As x approaches the vertical asymptote $x = 3$ from the left, $p(x) \to -\infty$.
$(3, \infty)$	$\left(5, \frac{3}{8}\right)$	$p(x)$ is positive. $p(x)$ must approach the horizontal asymptote $y = 0$ from above as $x \to \infty$. As x approaches the vertical asymptote $x = 3$ from the right, $p(x) \to \infty$.



74. $q(0) = \dfrac{4}{(0)^2 - 16} = \dfrac{4}{-16} = -\dfrac{1}{4}$

The y-intercept is $\left(0, -\frac{1}{4}\right)$.

q is never 0. It has no x-intercepts.

q is in lowest terms, and $x^2 - 16$ is 0 for $x = 4$ and $x = -4$, which are the vertical asymptotes.

The degree of the numerator is 0.

The degree of the denominator is 2.

Since $n < m$, the line $y = 0$ is a horizontal asymptote of q.

$0 = \dfrac{4}{x^2 - 16}$

$0 = 4$ No solution

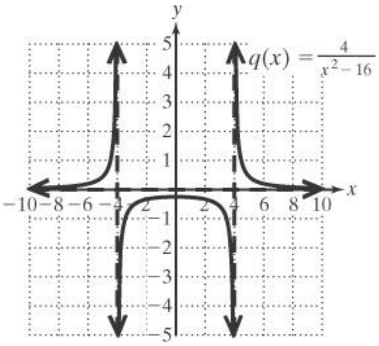
q does not cross its horizontal asymptote.

$q(-x) = \dfrac{4}{(-x)^2 - 16} = \dfrac{4}{x^2 - 16}$

$q(-x) = q(x)$

q is even.

Interval	**Test Point**	**Comments**
$(-\infty, -4)$	$\left(-6, \frac{1}{5}\right)$	$q(x)$ is positive. $q(x)$ must approach the horizontal asymptote $y = 0$ from above as $x \to -\infty$. As x approaches the vertical asymptote $x = -4$ from the left, $q(x) \to \infty$.
$(-4, 0)$	$\left(-2, -\frac{6}{5}\right)$	$q(x)$ is negative. As x approaches the vertical asymptote $x = -4$ from the right, $q(x) \to -\infty$.
$(0, 4)$	$\left(2, -\frac{6}{5}\right)$	$q(x)$ is negative. As x approaches the vertical asymptote $x = 4$ from the left, $q(x) \to -\infty$.
$(4, \infty)$	$\left(6, \frac{1}{5}\right)$	$q(x)$ is positive. $q(x)$ must approach the horizontal asymptote $y = 0$ from above as $x \to \infty$. As x approaches the vertical asymptote $x = 4$ from the right, $q(x) \to \infty$.



75. $r(0)=\dfrac{5(0)}{(0)^2-(0)-6}=0$

The *y*-intercept is $(0,0)$.

The *x*-intercept is $(0,0)$.

r is in lowest terms, and $x^2-x-6=(x-3)(x+2)$ is 0 for $x=3$ and $x=-2$, which are the vertical asymptotes.

The degree of the numerator is 0.

The degree of the denominator is 2.

Since $n<m$, the line $y=0$ is a horizontal asymptote of *r*.

$$0=\frac{5x}{x^2-x-6}$$
$$5x=0$$
$$x=0$$

r crosses its horizontal asymptote at $(0,0)$.

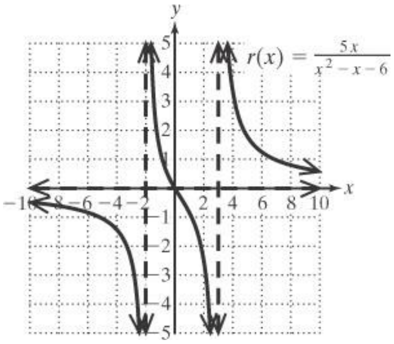
$$r(-x)=\frac{5(-x)}{(-x)^2-(-x)-6}=\frac{-5x}{x^2+x-6}$$

$$r(-x)\neq r(x),\ r(-x)\neq -r(x)$$

r is neither even nor odd.

Interval	**Test Point**	**Comments**
$(-\infty,-2)$	$\left(-3,-\frac{5}{2}\right)$	*r*(*x*) is negative. *r*(*x*) must approach the horizontal asymptote $y=0$ from below as $x\to-\infty$. As *x* approaches the vertical asymptote $x=-2$ from the left, $r(x)\to-\infty$.

$(-2, 0)$	$\left(-1, \frac{5}{4}\right)$	$r(x)$ is positive. As x approaches the vertical asymptote $x = -2$ from the right, $r(x) \to \infty$.
$(0, 3)$	$\left(1, -\frac{5}{6}\right)$	$r(x)$ is negative. As x approaches the vertical asymptote $x = 3$ from the left, $r(x) \to -\infty$.
$(3, \infty)$	$\left(6, \frac{5}{4}\right)$	$r(x)$ is positive. $r(x)$ must approach the horizontal asymptote $y = 0$ from above as $x \to \infty$. As x approaches the vertical asymptote $x = 3$ from the right, $r(x) \to \infty$.



76. $t(0) = \dfrac{4(0)}{(0)^2 - 2(0) - 3} = 0$

The y-intercept is $(0, 0)$.

The x-intercept is $(0, 0)$.

t is in lowest terms, and $x^2 - 2x - 3 = (x - 3)(x + 1)$ is 0 for $x = 3$ and $x = -1$, which are the vertical asymptotes.

The degree of the numerator is 0.

The degree of the denominator is 2.

Since $n < m$, the line $y = 0$ is a horizontal asymptote of t.

$$0 = \frac{4x}{x^2 - 2x - 3}$$

$$4x = 0$$

$$x = 0$$

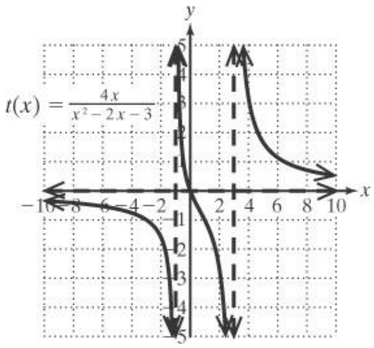
t crosses its horizontal asymptote at $(0, 0)$.

$$t(-x) = \frac{4(-x)}{(-x)^2 - 2(-x) - 3} = \frac{-4x}{x^2 - 2x - 3}$$

$t(-x) \neq t(x)$, $n(-x) \neq -t(x)$

t is neither even nor odd.

Interval	**Test Point**	**Comments**
$(-\infty, -1)$	$(-3, -1)$	$t(x)$ is negative. $t(x)$ must approach the horizontal asymptote $y = 0$ from below as $x \to -\infty$. As x approaches the vertical asymptote $x = -1$ from the left, $t(x) \to -\infty$.
$(-1, 0)$	$\left(-\frac{3}{4}, \frac{16}{5}\right)$	$t(x)$ is positive. As x approaches the vertical asymptote $x = -1$ from the right, $t(x) \to \infty$.
$(0, 3)$	$(1, -1)$	$t(x)$ is negative. As x approaches the vertical asymptote $x = 3$ from the left, $t(x) \to -\infty$.
$(3, \infty)$	$\left(5, \frac{5}{3}\right)$	$t(x)$ is positive. $t(x)$ must approach the horizontal asymptote $y = 0$ from above as $x \to \infty$. As x approaches the vertical asymptote $x = 3$ from the right, $t(x) \to \infty$.



77. $k(0) = \frac{5(0) - 3}{2(0) - 7} = \frac{3}{7}$

The y-intercept is $\left(0, \frac{3}{7}\right)$.

$$\begin{aligned} 5x - 3 &= 0 \\ 5x &= 3 \\ x &= \frac{3}{5} \end{aligned}$$

The x-intercept is $\left(\frac{3}{5}, 0\right)$.

k is in lowest terms, and $2x - 7$ is 0 for $x = \frac{7}{2}$, which is the vertical asymptote.

The degree of the numerator is 1.

The degree of the denominator is 1.

Since $n = m$, the line $y = \frac{5}{2}$ is a horizontal asymptote of k.

$$\begin{aligned} \frac{5}{2} &= \frac{5x - 3}{2x - 7} \\ 10x - 35 &= 10x - 6 \\ -35 &= -6 \quad \text{No solution} \end{aligned}$$

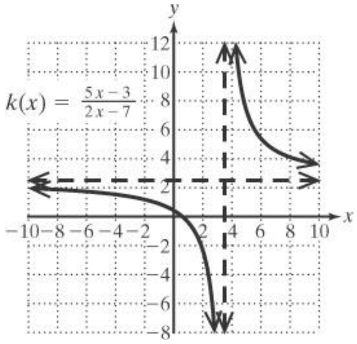
k does not cross its horizontal asymptote.

$$k(-x) = \frac{5(-x) - 3}{2(-x) - 7} = \frac{-5x - 3}{-2x - 7}$$

$$k(-x) \neq k(x),\ k(-x) \neq -k(x)$$

k is neither even nor odd.

Interval	**Test Point**	**Comments**
$\left(-\infty, \frac{3}{5}\right)$	$\left(-1, \frac{8}{9}\right)$	$k(x)$ is positive. $k(x)$ must approach the horizontal asymptote $y = \frac{5}{2}$ from above as $x \to -\infty$.
$\left(\frac{3}{5}, \frac{7}{2}\right)$	$(3, -12)$	$k(x)$ is negative. As x approaches the vertical asymptote $x = \frac{7}{2}$ from the left, $k(x) \to -\infty$.
$\left(\frac{7}{2}, \infty\right)$	$(4, 17)$	$k(x)$ is positive. $k(x)$ must approach the horizontal asymptote $y = \frac{5}{2}$ from above as $x \to \infty$. As x approaches the vertical asymptote $x = \frac{7}{2}$ from the right, $k(x) \to \infty$.



78. $h(0) = \dfrac{4(0)+3}{3(0)-5} = \dfrac{3}{-5} = -\dfrac{3}{5}$

The *y*-intercept is $\left(0, -\dfrac{3}{5}\right)$.

$$4x + 3 = 0$$
$$4x = -3$$
$$x = -\frac{3}{4}$$

The *x*-intercept is $\left(-\dfrac{3}{4}, 0\right)$.

h is in lowest terms, and $3x - 5$ is 0 for $x = \dfrac{5}{3}$, which is the vertical asymptote.

The degree of the numerator is 1.
The degree of the denominator is 1.

Since $n = m$, the line $y = \dfrac{4}{3}$ is a horizontal asymptote of *h*.

$$\frac{4}{3} = \frac{4x+3}{3x-5}$$
$$12x - 20 = 12x + 9$$
$$-20 = 9 \text{ No solution}$$

h does not cross its horizontal asymptote.

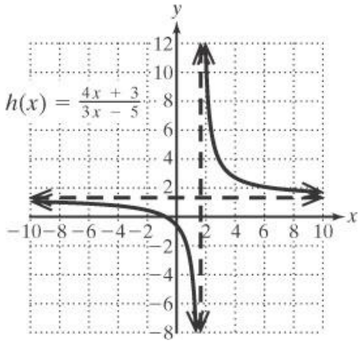
$$h(-x) = \frac{4(-x)+3}{3(-x)-5} = \frac{-4x+3}{-3x-5}$$

$h(-x) \neq h(x),\ h(-x) \neq -h(x)$

h is neither even nor odd.

Interval	**Test Point**	**Comments**

$\left(-\infty, -\frac{3}{4}\right)$	$\left(-1, \frac{8}{9}\right)$	$h(x)$ is positive. $h(x)$ must approach the horizontal asymptote $y = \frac{4}{3}$ from above as $x \to -\infty$.
$\left(-\frac{3}{4}, \frac{5}{3}\right)$	$(3, -12)$	$h(x)$ is negative. As x approaches the vertical asymptote $x = \frac{5}{3}$ from the left, $h(x) \to -\infty$.
$\left(\frac{5}{3}, \infty\right)$	$(4, 17)$	$h(x)$ is positive. $h(x)$ must approach the horizontal asymptote $y = \frac{4}{3}$ from above as $x \to \infty$. As x approaches the vertical asymptote $x = \frac{5}{3}$ from the right, $h(x) \to \infty$.



79. $g(0) = \dfrac{3(0)^2 - 5(0) - 2}{(0)^2 + 1} = \dfrac{-2}{1} = -2$

The y-intercept is $(0, -2)$.

$$3x^2 - 5x - 2 = 0$$
$$(3x+1)(x-2) = 0$$
$$x = -\frac{1}{3}, x = 2$$

The x-intercepts are $\left(-\frac{1}{3}, 0\right)$ and $(2, 0)$.

g is in lowest terms, and $x^2 + 1$ is never 0, so there are no vertical asymptotes.

The degree of the numerator is 2.

The degree of the denominator is 2.

Since $n = m$, the line $y = \frac{3}{1}$, or equivalently $y = 3$, is a horizontal asymptote of g.

$$3 = \frac{3x^2 - 5x - 2}{x^2 + 1}$$
$$3x^2 + 3 = 3x^2 - 5x - 2$$
$$5x = -5$$
$$x = -1$$

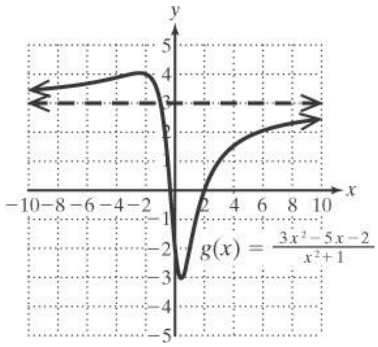
g crosses its horizontal asymptote at $\left(-1, 3\right)$.

$$g\left(-x\right) = \frac{3\left(-x\right)^2 - 5\left(-x\right) - 2}{\left(-x\right)^2 + 1} = \frac{3x^2 + 5x - 2}{x^2 + 1}$$

$g\left(-x\right) \neq g\left(x\right),\ g\left(-x\right) \neq -g\left(x\right)$

g is neither even nor odd.

Interval	**Test Point**	**Comments**
$\left(-\infty, -1\right)$	$\left(-2, 4\right)$	Since $g\left(-2\right) = 4$ is above the horizontal asymptote $y = 3$, $g(x)$ must approach the horizontal asymptote from above as $x \to -\infty$.
$\left(-1, -\frac{1}{3}\right)$	$\left(-\frac{1}{2}, 1\right)$	Plot the point $\left(-\frac{1}{2}, 1\right)$ between the horizontal asymptote and the x-intercept of $\left(-\frac{1}{3}, 0\right)$.
$\left(-\frac{1}{3}, 2\right)$	$\left(0, -2\right)$	The point $\left(0, -2\right)$ is the y-intercept.
$\left(2, \infty\right)$	$\left(3, 1\right)$	Since $g\left(3\right) = 1$ is below the horizontal asymptote $y = 3$, $g(x)$ must approach the horizontal asymptote from below as $x \to \infty$.



80. $c(0)=\dfrac{2(0)^2-5(0)-3}{(0)^2+1}=\dfrac{-3}{1}=-3$

The y-intercept is $(0,-3)$.

$$2x^2-5x-3=0$$
$$(2x+1)(x-3)=0$$
$$x=-\frac{1}{2},\ x=3$$

The x-intercepts are $\left(-\frac{1}{2},0\right)$ and $(3,0)$.

c is in lowest terms, and x^2+1 is never 0, so there are no vertical asymptotes.

The degree of the numerator is 2.

The degree of the denominator is 2.

Since $n=m$, the line $y=\frac{2}{1}$, or equivalently $y=2$, is a horizontal asymptote of c.

$$2=\frac{2x^2-5x-3}{x^2+1}$$
$$2x^2+2=2x^2-5x-3$$
$$5x=-5$$
$$x=-1$$

c crosses its horizontal asymptote at $(-1,2)$.

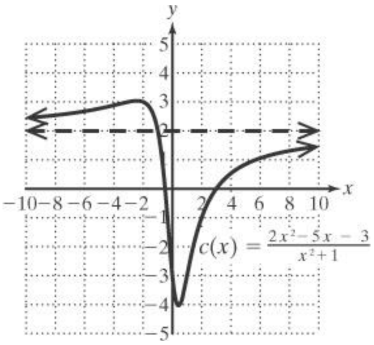
$$c(-x)=\frac{2(-x)^2-5(-x)-3}{(-x)^2+1}=\frac{2x^2+5x-3}{x^2+1}$$

$c(-x)\neq c(x),\ c(-x)\neq -c(x)$

c is neither even nor odd.

Interval	**Test Point**	**Comments**
$(-\infty,-1)$	$(-2,3)$	Since $c(-2)=3$ is above the horizontal asymptote $y=2$, $c(x)$ must approach the horizontal asymptote from above as $x\to-\infty$.
$\left(-1,-\frac{1}{2}\right)$	$\left(-\frac{3}{4},\frac{6}{5}\right)$	Plot the point $\left(-\frac{3}{4},\frac{6}{5}\right)$ between the horizontal asymptote and the x-intercept of $\left(-\frac{1}{2},0\right)$.

$\left(-\frac{1}{2}, 3\right)$	$\left(0, -2\right)$	The point $\left(0, -3\right)$ is the *y*-intercept.
$\left(3, \infty\right)$	$\left(7, \frac{6}{5}\right)$	Since $c\left(7\right) = \frac{6}{5}$ is below the horizontal asymptote $y = 2$, *c*(*x*) must approach the horizontal asymptote from below as $x \to \infty$.



81. $n\left(0\right) = \frac{\left(0\right)^2 + 2\left(0\right) + 1}{\left(0\right)}$ undefined

There is no *y*-intercept.

$$x^2 + 2x + 1 = 0$$
$$\left(x + 1\right)\left(x + 1\right) = 0$$
$$x = -1$$

The *x*-intercept is $\left(-1, 0\right)$. *n* is in lowest terms, and the denominator is 0 when $x = 0$, which is the vertical asymptote. The degree of the numerator is exactly one greater than the degree of the denominator. Therefore, *n* has no horizontal asymptote, but does have a slant asymptote.

$$n\left(x\right) = \frac{x^2 + 2x + 1}{x}$$
$$= \frac{x^2}{x} + \frac{2x}{x} + \frac{1}{x}$$
$$= x + 2 + \frac{1}{x}$$

The quotient is $x + 2$.

The slant asymptote is $y = x + 2$.

$$x + 2 = \frac{x^2 + 2x + 1}{x}$$
$$x^2 + 2x = x^2 + 2x + 1$$
$$0 = 1 \text{ No solution}$$

n does not cross its slant asymptote.

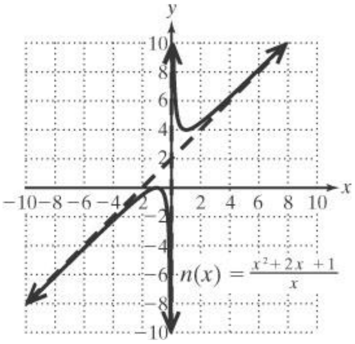
$$n\left(-x\right) = \frac{\left(-x\right)^2 + 2\left(-x\right) + 1}{\left(-x\right)} = \frac{x^2 - 2x + 1}{-x}$$

$n\left(-x\right) \neq n\left(x\right)$, $n\left(-x\right) \neq -n\left(x\right)$

n is neither even nor odd.

Select test points from each interval.

Interval	**Test Point**	**Test Point**
$\left(-\infty, -1\right)$	$\left(-4, -\frac{9}{4}\right)$	$\left(-2, -\frac{1}{2}\right)$
$\left(-1, 0\right)$	$\left(-\frac{1}{2}, -\frac{1}{2}\right)$	$\left(-\frac{1}{4}, -\frac{9}{4}\right)$
$\left(0, \infty\right)$	$\left(\frac{1}{4}, \frac{25}{4}\right)$	$\left(2, \frac{9}{2}\right)$



82. $m(0)=\frac{(0)^2-4(0)+4}{(0)}$ undefined

There is no *y*-intercept.

$$x^2-4x+4=0$$
$$(x-2)(x-2)=0$$
$$x=2$$

The *x*-intercept is $(2, 0)$.

m is in lowest terms, and the denominator is 0 when $x=0$, which is the vertical asymptote. The degree of the numerator is exactly one greater than the degree of the denominator. Therefore, *m* has no horizontal asymptote, but does have a slant asymptote.

$$m(x)=\frac{x^2-4x+4}{x}$$
$$=\frac{x^2}{x}-\frac{4x}{x}+\frac{4}{x}$$
$$=x-4+\frac{4}{x}$$

The quotient is $x-4$.

The slant asymptote is $y=x-4$.

$$x-4=\frac{x^2-4x+4}{x}$$
$$x^2-4x=x^2-4x+4$$
$$0=4 \text{ No solution}$$

m does not cross its slant asymptote.

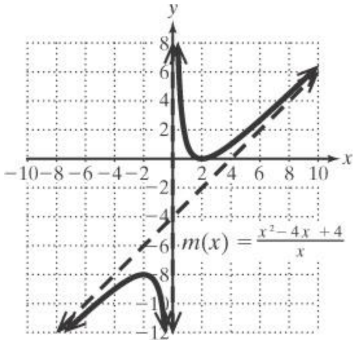
$$m(-x)=\frac{(-x)^2-4(-x)+4}{(-x)}$$
$$=\frac{x^2+4x+4}{-x}$$

$m(-x)\neq m(x)$, $m(-x)\neq -m(x)$

m is neither even nor odd.

Select test points from each interval.

Interval	Test Point	Test Point
$(-\infty, 0)$	$(-4, -9)$	$(-2, -8)$
$(0, 2)$	$\left(\frac{1}{2}, \frac{9}{2}\right)$	$(1, 1)$
$(2, \infty)$	$\left(3, \frac{1}{3}\right)$	$\left(8, \frac{9}{2}\right)$



83. $f(0)=\frac{(0)^2+7(0)+10}{(0)+3}=\frac{10}{3}$

The *y*-intercept is $\left(0, \frac{10}{3}\right)$.

$$x^2+7x+10=0$$
$$(x+5)(x+2)=0$$
$$x=-5, x=-2$$

The *x*-intercepts are $(-5, 0)$ and $(-2, 0)$. *f* is in lowest terms, and $x+3$ is 0 when $x=-3$, which is the vertical asymptote.

The degree of the numerator is exactly one greater than the degree of the denominator. Therefore, f has no horizontal asymptote, but does have a slant asymptote.

$$\begin{array}{r|rrr} \underline{-3} & 1 & 7 & 10 \\ & & -3 & -12 \\ \hline & 1 & 4 & \boxed{-2} \end{array}$$

The quotient is $x+4$.

The slant asymptote is $y=x+4$.

$$\begin{aligned} x+4 &= \frac{x^2+7x+10}{x+3} \\ x^2+7x+12 &= x^2+7x+10 \\ 12 &= 10 \quad \text{No solution} \end{aligned}$$

f does not cross its slant asymptote.

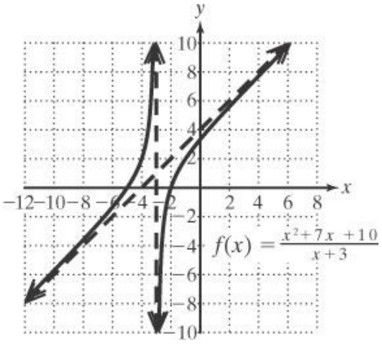
$$\begin{aligned} f(-x) &= \frac{(-x)^2+7(-x)+10}{(-x)+3} \\ &= \frac{x^2-7x+10}{-x+3} \end{aligned}$$

$f(-x) \neq f(x)$, $f(-x) \neq -f(x)$

f is neither even nor odd.

Select test points from each interval.

Interval	**Test Point**	**Test Point**
$(-\infty, -5)$	$\left(-7, -\frac{5}{2}\right)$	$\left(-6, -\frac{4}{3}\right)$
$(-5, -3)$	$(-4, 2)$	$\left(-\frac{7}{2}, \frac{9}{2}\right)$
$(-3, -2)$	$\left(-\frac{5}{2}, -\frac{5}{2}\right)$	$\left(-\frac{7}{3}, -\frac{4}{3}\right)$
$(-2, \infty)$	$(-1, 2)$	$\left(1, \frac{9}{2}\right)$



84. $$\begin{aligned} d(0) &= \frac{(0)^2-(0)-12}{(0)-2} \\ &= \frac{-12}{-2} = 6 \end{aligned}$$

The y-intercept is $(0, 6)$.

$$\begin{aligned} x^2-x-12 &= 0 \\ (x+3)(x-4) &= 0 \\ x &= -3, x = 4 \end{aligned}$$

The x-intercepts are $(-3, 0)$ and $(4, 0)$.

d is in lowest terms, and $x-2$ is 0 when $x=2$, which is the vertical asymptote. The degree of the numerator is exactly one greater than the degree of the denominator. Therefore, d has no horizontal asymptote, but does have a slant asymptote.

$$\begin{array}{r|rrr} \underline{2} & 1 & -1 & -12 \\ & & 2 & 2 \\ \hline & 1 & 1 & \boxed{-10} \end{array}$$

The quotient is $x+1$.

The slant asymptote is $y=x+1$.

$$\begin{aligned} x+1 &= \frac{x^2-x-12}{x-2} \\ x^2-x-2 &= x^2-x-12 \\ -2 &= -12 \quad \text{No solution} \end{aligned}$$

d does not cross its slant asymptote.

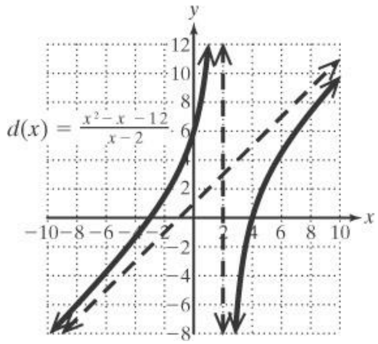
$$d\left(-x\right)=\frac{\left(-x\right)^{2}-\left(-x\right)-12}{\left(-x\right)-2}=\frac{x^{2}+x-12}{-x-2}$$

$d\left(-x\right)\neq d\left(x\right)$, $d\left(-x\right)\neq -d\left(x\right)$

d is neither even nor odd.

Select test points from each interval.

Interval	**Test Point**	**Test Point**
$\left(-\infty,-3\right)$	$\left(-8,-6\right)$	$\left(-4,-\frac{4}{3}\right)$
$\left(-3,2\right)$	$\left(-2,\frac{3}{2}\right)$	$\left(1,12\right)$
$\left(2,4\right)$	$\left(3,-6\right)$	$\left(\frac{7}{2},-\frac{13}{6}\right)$
$\left(4,\infty\right)$	$\left(6,\frac{9}{2}\right)$	$\left(7,6\right)$



85. $w\left(0\right)=\frac{-4\left(0\right)^{2}}{\left(0\right)^{2}+4}=0$

The *y*-intercept is $\left(0,0\right)$.

The *x*-intercept is $\left(0,0\right)$.

w is in lowest terms, and $x^{2}+4$ is never 0, so there are no vertical asymptotes.

The degree of the numerator is 2.

The degree of the denominator is 2.

Since $n=m$, the line $y=\frac{-4}{1}$, or equivalently $y=-4$, is a horizontal asymptote of *w*.

$$-4=\frac{-4x^{2}}{x^{2}+4}$$
$$-4x^{2}-16=-4x^{2}$$
$$-16=0 \text{ No solution}$$

w does not cross its horizontal asymptote.

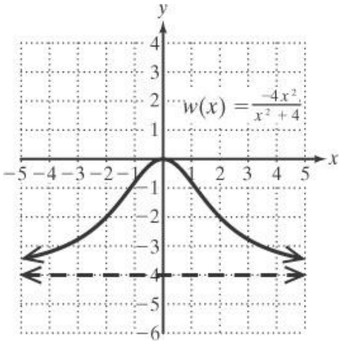
$$w\left(-x\right)=\frac{-4\left(-x\right)^{2}}{\left(-x\right)^{2}+4}=\frac{-4x^{2}}{x^{2}+4}$$

$w\left(-x\right)=w\left(x\right)$

w is even.

Interval	**Test Point**	**Comments**

$\left(-\infty, 0\right)$	$\left(-2, -2\right)$	Since $w\left(-2\right) = -2$ is above the horizontal asymptote $y = -4$, $w(x)$ must approach the horizontal asymptote from above as $x \to -\infty$.
$\left(0, \infty\right)$	$\left(2, -2\right)$	Since $w\left(2\right) = -2$ is above the horizontal asymptote $y = -4$, $w(x)$ must approach the horizontal asymptote from above as $x \to \infty$.



86. $u\left(0\right) = \dfrac{-3\left(0\right)^2}{\left(0\right)^2 + 1} = 0$

The y-intercept is $\left(0, 0\right)$.

The x-intercept is $\left(0, 0\right)$.

u is in lowest terms, and $x^2 + 1$ is never 0, so there are no vertical asymptotes.

The degree of the numerator is 2.

The degree of the denominator is 2.

Since $n = m$, the line $y = \dfrac{-3}{1}$, or equivalently $y = -3$, is a horizontal asymptote of u.

$$-3 = \frac{-3x^2}{x^2 + 1}$$
$$-3x^2 - 3 = -3x^2$$
$$-3 = 0 \quad \text{No solution}$$

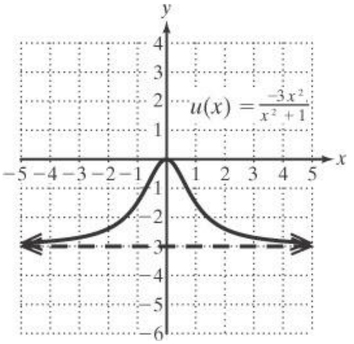
u does not cross its horizontal asymptote.

$$u\left(-x\right) = \frac{-3\left(-x\right)^2}{\left(-x\right)^2 + 1} = \frac{-3x^2}{x^2 + 1}$$

$u\left(-x\right) = u\left(x\right)$

u is even.

Interval	**Test Point**	**Comments**
$(-\infty, 0)$	$\left(-1, -\frac{3}{2}\right)$	Since $u(-1) = -\frac{3}{2}$ is above the horizontal asymptote $y = -3$, $u(x)$ must approach the horizontal asymptote from above as $x \to -\infty$.
$(0, \infty)$	$\left(1, -\frac{3}{2}\right)$	Since $u(1) = -\frac{3}{2}$ is above the horizontal asymptote $y = -3$, $u(x)$ must approach the horizontal asymptote from above as $x \to \infty$.



87. $f(0) = \dfrac{(0)^3 + (0)^2 - 4(0) - 4}{(0)^2 + 3(0)}$

$= \dfrac{-4}{0}$ undefined

There is no y-intercept.

$$\begin{aligned} x^3 + x^2 - 4x - 4 &= 0 \\ x^2(x+1) - 4(x+1) &= 0 \\ (x+1)(x^2-4) &= 0 \\ (x+1)(x+2)(x-2) &= 0 \\ x = -1, x = -2, x &= 2 \end{aligned}$$

The x-intercepts are $(-1, 0)$, $(-2, 0)$ and $(2, 0)$. f is in lowest terms, and $x^2 + 3x = x(x+3)$ is 0 when $x = 0$ and $x = -3$, which are the vertical asymptotes.

The degree of the numerator is exactly one greater than the degree of the denominator. Therefore, f has no horizontal asymptote, but does have a slant asymptote.

$$\begin{array}{r} x - 2 \\ x^2 + 3x\overline{\smash{)}x^3 + x^2 - 4x - 4} \\ -(x^3 + 3x^2) \\ \hline -2x^2 - 4x \\ -(-2x^2 - 6x) \\ \hline 2x - 4 \end{array}$$

The quotient is $x - 2$.

The slant asymptote is $y = x - 2$.

$x - 2 = \dfrac{x^3 + x^2 - 4x - 4}{x^2 + 3x}$

$$x^3+x^2-6x=x^3+x^2-4x-4$$
$$-2x=-4$$
$$x=2$$
$$y=x-2=2-2=0$$

f crosses its slant asymptote at $(2, 0)$.

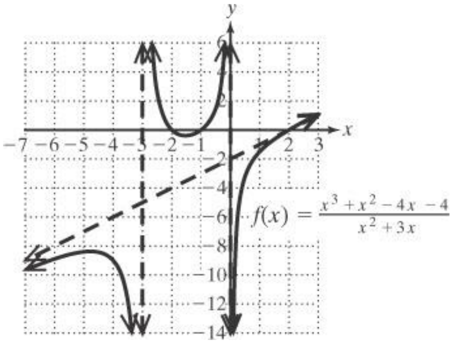
$$f(-x)=\frac{(-x)^3+(-x)^2-4(-x)-4}{(-x)^2+3(-x)}$$
$$=\frac{-x^3+x^2+4x-4}{x^2-3x}$$

$f(-x)\neq f(x)$, $f(-x)\neq -f(x)$

f is neither even nor odd.

Select test points from each interval.

Interval	**Test Point**	**Test Point**
$(-\infty, -3)$	$\left(-8, -\frac{21}{2}\right)$	$(-4, -9)$
$(-3, -2)$	$\left(-\frac{5}{2}, \frac{27}{10}\right)$	
$(-2, -1)$	$\left(-\frac{4}{3}, -\frac{1}{3}\right)$	
$(-1, 0)$	$\left(-\frac{1}{2}, \frac{3}{2}\right)$	
$(0, 2)$	$\left(\frac{1}{3}, -\frac{14}{3}\right)$	$\left(1, -\frac{3}{2}\right)$
$(2, \infty)$	$\left(3, \frac{10}{9}\right)$	$\left(4, \frac{15}{7}\right)$



88. $g(0)=\frac{(0)^3+3(0)^2-(0)-3}{(0)^2-2(0)}$

$$=\frac{-3}{0}\ \text{undefined}$$

There is no *y*-intercept.

$$x^3+3x^2-x-3=0$$
$$x^2(x+3)-1(x+3)=0$$
$$(x+3)(x^2-1)=0$$
$$(x+3)(x+1)(x-1)=0$$
$$x=-3, x=-1, x=1$$

The *x*-intercepts are $(-3, 0)$, $(-1, 0)$ and $(1, 0)$. *g* is in lowest terms, and $x^2-2x=x(x-2)$ is 0 when $x=0$ and $x=2$, which are the vertical asymptotes. The degree of the numerator is exactly one greater than the degree of the denominator. Therefore, *g* has no horizontal asymptote, but does have a slant asymptote.

$$\begin{array}{r}
x+5 \\
x^2-2x\overline{\smash{)}\,x^3+3x^2-x-3} \\
-\underline{(x^3-2x^2)} \\
5x^2-x \\
-\underline{(5x^2-10x)} \\
9x-3
\end{array}$$

The quotient is $x+5$.

The slant asymptote is $y=x+5$.

$$x+5=\frac{x^3+3x^2-x-3}{x^2-2x}$$
$$x^3+3x^2-10x=x^3+3x^2-x-3$$
$$-9x=-3$$
$$x=\frac{1}{3}$$

$$y = x + 5 = \frac{1}{3} + 5 = \frac{16}{3}$$

g crosses its slant asymptote at

$\left(\frac{1}{3}, \frac{16}{3}\right)$.

$$g\left(-x\right) = \frac{\left(-x\right)^3 + 3\left(-x\right)^2 - \left(-x\right) - 3}{\left(-x\right)^2 - 2\left(-x\right)}$$

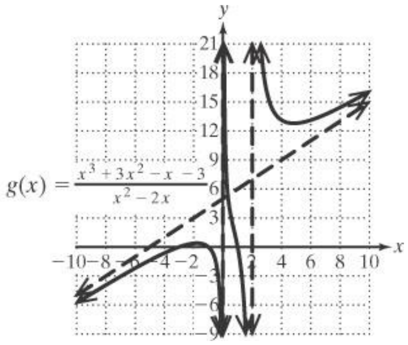
$$= \frac{-x^3 + 3x^2 + x - 3}{x^2 + 2x}$$

$g\left(-x\right) \neq g\left(x\right),\ g\left(-x\right) \neq -g\left(x\right)$

g is neither even nor odd.

Select test points from each interval.

Interval	**Test Point**	**Test Point**
$\left(-\infty, -3\right)$	$\left(-5, -\frac{48}{35}\right)$	$\left(-4, -\frac{5}{8}\right)$
$\left(-3, -1\right)$	$\left(-2, \frac{3}{8}\right)$	
$\left(-1, 0\right)$	$\left(-\frac{1}{2}, -\frac{3}{2}\right)$	
$\left(0, \frac{1}{3}\right)$	$\left(\frac{1}{5}, \frac{128}{15}\right)$	
$\left(\frac{1}{3}, 1\right)$	$\left(\frac{1}{2}, \frac{7}{2}\right)$	
$\left(1, 2\right)$	$\left(\frac{3}{2}, -\frac{15}{2}\right)$	
$\left(2, \infty\right)$	$\left(3, 16\right)$	$\left(5, 12.8\right)$



89. $v\left(0\right) = \frac{2\left(0\right)^4}{\left(0\right)^2 + 9} = 0$

The y-intercept is $\left(0, 0\right)$.

The x-intercept is $\left(0, 0\right)$.

v is in lowest terms, and $x^2 + 9$ is never 0, so there are no vertical asymptotes.

The degree of the numerator is 4.

The degree of the denominator is 2.

Since $n > m$, the function has no horizontal asymptotes. Since $n - m > 1$, the function has no slant asymptote.

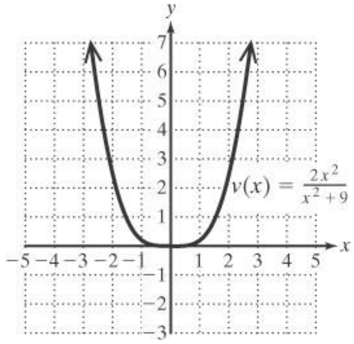
$$v\left(-x\right) = \frac{2\left(-x\right)^4}{\left(-x\right)^2 + 9} = \frac{2x^4}{x^2 + 9}$$

$v\left(-x\right) = v\left(x\right)$

v is even.

Select test points.

The graph passes through $\left(-3, 9\right)$, $\left(-1, \frac{1}{5}\right)$, $\left(1, \frac{1}{5}\right)$, $\left(3, 9\right)$.



90. $g(0) = \dfrac{4(0)^4}{(0)^2 + 8} = 0$

The *y*-intercept is $(0, 0)$.

The *x*-intercept is $(0, 0)$.

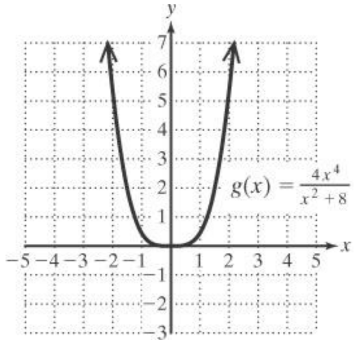
g is in lowest terms, and $x^2 + 8$ is never 0, so there are no vertical asymptotes.
The degree of the numerator is 4.
The degree of the denominator is 2.
Since $n > m$, the function has no horizontal asymptotes. Since $n - m > 1$, the function has no slant asymptote.

$$g(-x) = \frac{4(-x)^4}{(-x)^2 + 8} = \frac{4x^4}{x^2 + 8}$$

$g(-x) = g(x)$

g is even.
Select test points.

The graph passes through $\left(-2, \dfrac{16}{3}\right)$, $\left(-1, \dfrac{4}{9}\right)$, $\left(1, \dfrac{4}{9}\right)$, $\left(2, \dfrac{16}{3}\right)$.



91. a. $C(x) = 20x + 69.95 + 39.99$

$C(x) = 20x + 109.94$

b. $\overline{C}(x) = \dfrac{20x + 109.94}{x}$

c. $\overline{C}(5) = \dfrac{20(5) + 109.94}{5} \approx 41.99$

$\overline{C}(30) = \dfrac{20(30) + 109.94}{30} \approx 23.66$

$\overline{C}(120) = \dfrac{20(120) + 109.94}{120} \approx 20.92$

d. The average cost would approach \$20 per session. This is the same as the fee paid to the gym in the absence of fixed costs.

92. a. $C(x) = 40x + 1200 + 420 + 100 + 200$

$C(x) = 40x + 1920$

b. $\overline{C}(x) = \dfrac{40x + 1920}{x}$

c. $\overline{C}(20) = \dfrac{40(20) + 1920}{20} = 136$

$\overline{C}(50) = \dfrac{40(50) + 1920}{50} = 78.4$

$\overline{C}(100) = \dfrac{40(100) + 1920}{100} = 59.2$

$\overline{C}(200) = \dfrac{40(200) + 1920}{200} = 49.6$

d. $\overline{C}(200) = 49.6$ means that if 200,000 pages are printed, then the average cost per thousand pages is \$49.60 (or equivalently \$0.0496 per page).

e. The average cost would approach \$40 per thousand pages or equivalently \$0.04 per page. This is the cost per page in the absence of fixed costs.

93. a. $R(x) = \dfrac{6x}{x+6}$

b.

x	6	12	18	30
$R(x)$	3	4	4.5	5

c. $R(x) = \dfrac{6x}{x+6} = \dfrac{x(6)}{x\left(1+\dfrac{6}{x}\right)}$

$$R(x) = \frac{6}{\left(1+\dfrac{6}{x}\right)}$$

$$R(\infty) = \frac{6}{\left(1+\dfrac{6}{\infty}\right)}$$

$$R(\infty) = 6\ \Omega$$

Even for large values of *x*, the total resistance will always be less than $6\ \Omega$.
This is consistent with the statement that the total resistance is always less than the resistance in any individual branch of the circuit.

94. a. $R(x) = \dfrac{96x}{20x+96}$

b. $R(x) = \dfrac{96x}{20x+96} = \dfrac{x(96)}{x\left(20+\dfrac{96}{x}\right)}$

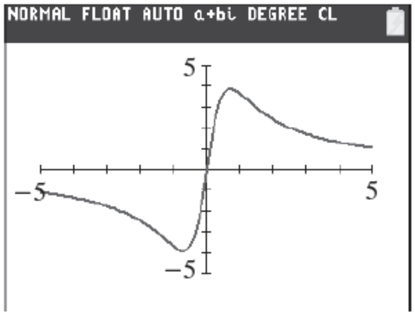
$$R(x) = \frac{96}{\left(20+\dfrac{96}{x}\right)}$$

$$R(\infty) = \frac{96}{\left(20+\dfrac{96}{\infty}\right)}$$

$$R(\infty) = \frac{96}{20} = 4.8\ \Omega$$

Even for large values of *x*, the total resistance will always be less than $4.8\ \Omega$.

95. a.

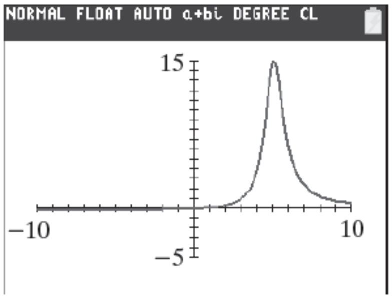


b. $t \geq 0$

c. 4 mg/L

d. 0 mg/L

96. a.



b. $t \geq 0$

c. 15 mg/L

d. 0 mg/L

97. a. $C(25) = \dfrac{600(25)}{100-25} = 200$

b. $C(50) = \dfrac{600(50)}{100-50} = 600$; \$600,000

$C(75) = \dfrac{600(75)}{100-75} = 1800$; \$1,800,000

$C(90) = \dfrac{600(90)}{100-90} = 5400$; \$5,400,000

c.

$$\begin{aligned} C(x) &= \frac{600x}{100-x} \\ 1400 &= \frac{600x}{100-x} \\ 140,000 - 1400x &= 600x \\ 140,000 &= 2000x \\ 70 &= x \end{aligned}$$

70% of the air pollutants can be removed.

98. a. $C(20) = \dfrac{80(20)}{100-20} = 20$; \$20,000

$C(40) = \dfrac{80(40)}{100-40} \approx 53$; \$53,000

$C(90) = \dfrac{80(90)}{100-90} = 720$; \$720,000

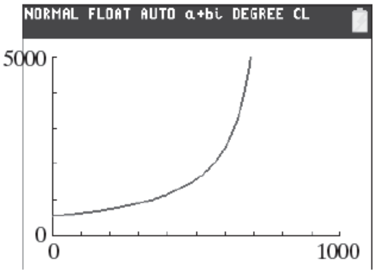
b.

$$\begin{aligned} C(x) &= \frac{80x}{100-x} \\ 320 &= \frac{80x}{100-x} \\ 32,000 - 320x &= 80x \\ 32,000 &= 400x \\ 80 &= x \end{aligned}$$

80% of the waste can be removed.

99. a. $F(v) = 560\left(\dfrac{772.4}{772.4-v}\right)$

b.



c. The frequency increases, making the pitch of the siren higher to the observer.

100. a. $F(v) = 600\left(\dfrac{772.4}{772.4-v}\right)$

b.
$$\begin{aligned}664 &= 600\left(\frac{772.4}{772.4-v}\right)\\ \frac{664}{600} &= \left(\frac{772.4}{772.4-v}\right)\\ 772.4-v &= \frac{(772.4)(600)}{664}\\ 772.4-v &= 697.95\\ v &= 74.4 \text{ mph}\end{aligned}$$

c. If the speed of the police car were to approach the speed of sound in the air, the frequency would increase infinitely and a sonic boom would occur.

101. a. $f(x)=\frac{1}{\left[x-(-1)\right]^2}+3$

$f(x)=\frac{1}{(x+1)^2}+3$

b. Domain: $(-\infty,-1)\cup(-1,\infty)$;

Range: $(3,\infty)$

102. a. $f(x)=\frac{1}{x-(4)}-3$

$f(x)=\frac{1}{x-4}-3$

b. Domain: $(-\infty,4)\cup(4,\infty)$;

Range: $(-\infty,-3)\cup(-3,\infty)$

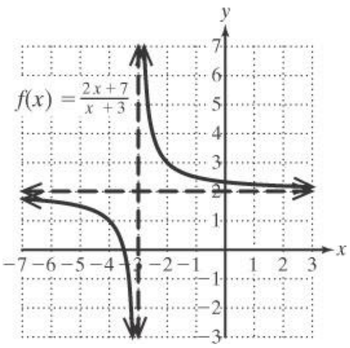
103. $f(x)=\frac{2x+7}{x+3}$

Find the quotient.

$$\begin{array}{r|rr} -3 & 2 & 7\\ & & -6\\ \hline & 2 & \boxed{1}\end{array}$$

$f(x)=2+\frac{1}{x+3}$

The graph of *f* is the graph of $y=\frac{1}{x}$ with a shift to the left 3 units and a shift upward 2 units



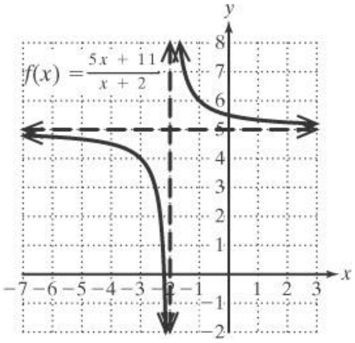
104. $f(x)=\frac{5x+11}{x+2}$

Find the quotient.

$$\begin{array}{r|rr} -2 & 5 & 11\\ & & -10\\ \hline & 5 & \boxed{1}\end{array}$$

$f(x)=5+\frac{1}{x+2}$

The graph of *f* is the graph of $y=\frac{1}{x}$ with a shift to the left 2 units and a shift upward 5 units.



105. The numerator and denominator share a common factor of $x+2$. The value -2 is not in the domain of *f*. The graph will have a “hole” at $x = -2$ rather than a vertical asymptote.

106. A horizontal asymptote is a horizontal line that the function approaches as $|x|$ becomes very large—that is, as x approaches infinity or as x approaches negative infinity.

107. Factors of the numerator:

$(x+3)(x+1)$

Factor of the denominator: $(x-2)$

The degree of the numerator equals the degree of the denominator.

$$f(x) = \frac{(x+3)(x+1)}{(x-2)(x-a)}$$
$$f(0) = \frac{3}{4}$$
$$\frac{(0+3)(0+1)}{(0-2)(0-a)} = \frac{3}{4}$$
$$\frac{3}{2a} = \frac{3}{4}$$
$$2a = 4$$
$$a = 2$$
$$f(x) = \frac{(x+3)(x+1)}{(x-2)(x-2)} = \frac{x^2+4x+3}{x^2-4x+4}$$

108. Factors of the numerator:

$(x-4)(x-2)$

Factor of the denominator: $(x-1)$

The degree of the numerator equals the degree of the denominator.

$$f(x) = \frac{(x-4)(x-2)}{(x-1)(x-a)}$$
$$f(0) = 8$$
$$\frac{(0-4)(0-2)}{(0-1)(0-a)} = 8$$
$$\frac{8}{a} = 8$$
$$a = 1$$
$$f(x) = \frac{(x-4)(x-2)}{(x-1)(x-1)} = \frac{x^2-6x+8}{x^2-2x+1}$$

109. Factor of the numerator:

$\left(x-\frac{3}{2}\right)$ or $(2x-3)$

Factors of the denominator:

$(x+2)(x-5)$

The degree of the numerator is less than the degree of the denominator.

$$f(x) = a\left[\frac{2x-3}{(x+2)(x-5)}\right]$$
$$f(0) = 3$$
$$a\left[\frac{2(0)-3}{(0+2)(0-5)}\right] = 3$$
$$a\left(\frac{-3}{-10}\right) = 3$$
$$a = 10$$
$$f(x) = 10\left[\frac{2x-3}{(x+2)(x-5)}\right]$$
$$= \frac{20x-30}{x^2-3x-10}$$

110. Factor of the numerator:

$\left(x-\frac{4}{3}\right)$ or $(3x-4)$

Factors of the denominator:

$(x+3)(x+4)$

The degree of the numerator is less

than the degree of the denominator.

$$f(x)=a\left[\frac{3x-4}{(x+3)(x+4)}\right]$$

$$f(0)=-1$$

$$a\left[\frac{3(0)-4}{(0+3)(0+4)}\right]=-1$$

$$a\left(\frac{-4}{12}\right)=-1$$

$$a=3$$

$$f(x)=3\left[\frac{3x-4}{(x+3)(x+4)}\right]$$

$$=\frac{9x-12}{x^2+7x+12}$$

111. a. $(-\infty,2)\cup(2,\infty)$

b. $f(x)=x+3$ where $x\neq 2$

c. None

d. $x=2$

e. Graph iii

112. a. $(-\infty,-1)\cup(-1,\infty)$

b. $f(x)=-x+3$ where $x\neq -1$

c. None

d. $x=-1$

e. Graph ii

113. a. $(-\infty,-5)\cup(-5,-4)\cup(-4,\infty)$

b. $f(x)=\frac{2}{x+4}$ where $x\neq -5$

c. $x=-4$

d. $x=-5$

e. Graph iv

114. a. $(-\infty,-3)\cup(-3,1)\cup(1,\infty)$

b. $f(x)=\frac{2}{x+3}$ where $x\neq 1$

c. $x=-3$

d. $x=1$

e. Graph i

Problem Recognition Exercises: Polynomial and Rational Functions

1. $$\frac{\text{Factors of }-8}{\text{Factors of }1}=\frac{\pm1,\pm2,\pm4,\pm8}{\pm1}$$
$$=\pm1,\pm2,\pm4,\pm8$$

$$\begin{array}{r|rrrr} \underline{2} & 1 & 3 & -6 & -8 \\ & & 2 & 10 & 8 \\ \hline & 1 & 5 & 4 & \underline{|0} \end{array}$$

Factor the quotient.

$$x^2+5x+4=0$$
$$(x+1)(x+4)=0$$
$$x=-1,x=-4$$

The zeros are 2, -1, and -4.

2. $$\frac{\text{Factors of }6}{\text{Factors of }1}=\frac{\pm1,\pm2,\pm3,\pm6}{\pm1}$$
$$=\pm1,\pm2,\pm3,\pm6$$

$$\begin{array}{r|rrrr} \underline{-2} & 1 & -2 & -5 & 6 \\ & & -2 & 8 & -6 \\ \hline & 1 & -4 & 3 & \underline{|0} \end{array}$$

Factor the quotient.

$$x^2-4x+3=0$$
$$(x-1)(x-3)=0$$
$$x=1,x=3$$

The zeros are -2, 1, and 3.

3. The x-intercepts of q are $(-2,0)$, $(1,0)$, and $(3,0)$.

4. The x-intercepts of p are $(2, 0)$, $(-1, 0)$, and $(-4, 0)$.

5. The x-intercepts of f are $(2, 0)$, $(-1, 0)$, and $(-4, 0)$.

6. The vertical asymptote of f occur where $q(x) = 0$, at $x = -2$, $x = 1$, and $x = 3$.

7. Since the degree of the numerator is the same as the degree of the denominator, the line $y = \frac{1}{1}$, or equivalently $y = 1$, is a horizontal asymptote of f.

8. $$\frac{x^3 + 3x^2 - 6x - 8}{x^3 - 2x^2 - 5x + 6} = 1$$
$$x^3 + 3x^2 - 6x - 8 = x^3 - 2x^2 - 5x + 6$$
$$5x^2 - x - 14 = 0$$
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(5)(-14)}}{2(5)}$$
$$= \frac{1 \pm \sqrt{281}}{10}$$
$$\approx 1.78 \text{ or } -1.58$$

The graph crosses the horizontal asymptote at $\left(\frac{1+\sqrt{281}}{10}, 1\right)$ and $\left(\frac{1-\sqrt{281}}{10}, 1\right)$, or $(1.78, 1)$ and $(-1.58, 1)$.

9. $$\frac{\text{Factors of } 8}{\text{Factors of } 1} = \frac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1}$$
$$= \pm 1, \pm 2, \pm 4, \pm 8$$

$$\begin{array}{r|rrrr} 4 & 1 & -4 & -2 & 8 \\ & & 4 & 0 & -8 \\ \hline & 1 & 0 & -2 & \boxed{0} \end{array}$$

Factor the quotient.

$$x^2 - 2 = 0$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

The zeros are 4, $\sqrt{2}$, and $-\sqrt{2}$.

10. $$\frac{\text{Factors of } -4}{\text{Factors of } 1} = \frac{\pm 1, \pm 2, \pm 4}{\pm 1}$$
$$= \pm 1, \pm 2, \pm 4$$

$$\begin{array}{r|rrrr} 1 & 1 & 3 & 0 & -4 \\ & & 1 & 4 & 4 \\ \hline & 1 & 4 & 4 & \boxed{0} \end{array}$$

Factor the quotient.

$$x^2 + 4x + 4 = 0$$
$$(x+2)(x+2) = 0$$
$$x = -2$$

The zeros are 1 and -2 (multiplicity 2).

11. The x-intercepts of d are $(1, 0)$ and $(-2, 0)$.

12. The x-intercepts of c are $(4, 0)$, $(\sqrt{2}, 0)$, and $(-\sqrt{2}, 0)$.

13. The x-intercepts of g are $(4, 0)$, $(\sqrt{2}, 0)$, and $(-\sqrt{2}, 0)$.

14. The vertical asymptotes of g occur where $d(x) = 0$, at $x = 1$ and $x = -2$.

15. Since the degree of the numerator is the same as the degree of the denominator, the line $y = \frac{1}{1}$, or equivalently $y = 1$, is a horizontal asymptote of g.

16. $\dfrac{x^3-4x^2-2x+8}{x^3+3x^2-4}=1$

$$\begin{aligned}
x^3-4x^2-2x+8&=x^3+3x^2-4\\
7x^2+2x-12&=0\\
x&=\frac{-(2)\pm\sqrt{(2)^2-4(7)(-12)}}{2(7)}\\
&=\frac{-2\pm\sqrt{340}}{14}\\
&=\frac{-2\pm2\sqrt{85}}{14}\\
&=\frac{-1\pm\sqrt{85}}{7}\\
&\approx 1.17 \text{ or } -1.46
\end{aligned}$$

The graph crosses the horizontal asymptote at $\left(\dfrac{-1+\sqrt{85}}{7},1\right)$ and $\left(\dfrac{-1-\sqrt{85}}{7},1\right)$, or $(1.17,1)$ and $(-1.46,1)$.

17. Graph b has the intercepts and asymptotes of function *f*.

18. Graph a has the intercepts and asymptotes of function *g*.

19.

$$\begin{array}{r}
2x-4\\
x^2+0x-11\overline{\smash{)}2x^3-4x^2-10x+12}\\
-\left(2x^3+0x^2-22x\right)\\
\hline
-4x^2+12x+12\\
-\left(-4x^2+\ 0x+44\right)\\
\hline
12x-32
\end{array}$$

a. The quotient is $q(x)=2x-4$.

b. The remainder is $r(x)=12x-32$.

20. The slant asymptote is $y=2x-4$.

21. $\dfrac{2x^3-4x^2-10x+12}{x^2-11}=2x-4$

$$\begin{aligned}
\begin{bmatrix}2x^3-4x^2\\ \quad -10x+12\end{bmatrix}&=\begin{bmatrix}2x^3-4x^2\\ \quad -22x+44\end{bmatrix}\\
-10x+12&=-22x+44\\
12x&=32\\
x&=\frac{32}{12}=\frac{8}{3}
\end{aligned}$$

$$\begin{aligned}
y&=2x-4\\
y&=2\left(\frac{8}{3}\right)-4=\frac{16}{3}-\frac{12}{3}=\frac{4}{3}
\end{aligned}$$

The graph crosses the slant asymptote at $\left(\dfrac{8}{3},\dfrac{4}{3}\right)$.

22.

$$\begin{aligned}
r(x)&=12x-32\\
0&=12x-32\\
12x&=32\\
x&=\frac{32}{12}=\frac{8}{3}
\end{aligned}$$

The solution to $r(x)=0$ gives the *x*-coordinate of the point where the graph of *f* crosses its slant asymptote.

Section 2.6 Polynomial and Rational Inequalities

1. polynomial; 2

2. rational

3. $(-\infty,\infty)$; $\{\ \}$

4. negative

5. a. $(-4,-1)$

b. $[-4,-1]$

c. $(-\infty,-4)\cup(-1,\infty)$

d. $(-\infty,-4]\cup[-1,\infty)$

6. a. $(-\infty,-1)\cup(3,\infty)$

b. $(-\infty,-1]\cup[3,\infty)$

c. $(-1,3)$

d. $[-1,3]$

7. a. $(-\infty,-3)\cup(-3,\infty)$

b. $(-\infty,\infty)$

c. $\{\ \}$

d. $\{-3\}$

8. a. $\{\ \}$

b. $\{4\}$

c. $(-\infty,4)\cup(4,\infty)$

d. $(-\infty,\infty)$

9. a. $(-\infty,-2)\cup(0,3)$

b. $(-\infty,-2]\cup[0,3]$

c. $(-2,0)\cup(3,\infty)$

d. $[-2,0]\cup[3,\infty)$

10. a. $(-\infty,-4)\cup(0,1)$

b. $(-\infty,-4]\cup[0,1]$

c. $(-4,0)\cup(1,\infty)$

d. $[-4,0]\cup[1,\infty)$

11. a. $(0,3)\cup(3,\infty)$

b. $[0,\infty)$

c. $(-\infty,0)$

d. $(-\infty,0]\cup\{3\}$

12. a. $(0,\infty)$

b. $\{-2\}\cup[0,\infty)$

c. $(-\infty,-2)\cup(-2,0)$

d. $(-\infty,0]$

13. a. $(-\infty,\infty)$

b. $(-\infty,\infty)$

c. $\{\ \}$

d. $\{\ \}$

14. a. $\{\ \}$

b. $\{\ \}$

c. $(-\infty,\infty)$

d. $(-\infty,\infty)$

15. $(5x-3)(x-5)=0$

$$x=\frac{3}{5} \quad \text{or} \quad x=5$$

The boundary points are $\frac{3}{5}$ and 5.

Sign of $(5x-3)$:	$-$	$+$	$+$
Sign of $(x-5)$:	$-$	$-$	$+$
Sign of $(5x-3)(x-5)$:	$+$	$-$	$+$

$\frac{3}{5}$ 5

a. $(5x-3)(x-5)=0$ $\left\{\frac{3}{5},5\right\}$

b. $(5x-3)(x-5)<0$ $\left(\frac{3}{5},5\right)$

c. $(5x-3)(x-5)\le 0$ $\left[\frac{3}{5},5\right]$

d. $(5x-3)(x-5)>0$

$$\left(-\infty,\frac{3}{5}\right)\cup(5,\infty)$$

e. $(5x-3)(x-5) \geq 0$

$\left(-\infty, \frac{3}{5}\right] \cup \left[5, \infty\right)$

16. $(3x+7)(x-2) = 0$

$$x = -\frac{7}{3} \text{ or } x = 2$$

The boundary points are $-\frac{7}{3}$ and 2.

Sign of $(3x+7)$:	$-$	$+$	$+$
Sign of $(x-2)$:	$-$	$-$	$+$
Sign of $(3x+7)(x-2)$:	$+$	$-$	$+$

$-\frac{7}{3}$ 2

a. $(3x+7)(x-2) = 0$ $\left\{-\frac{7}{3}, 2\right\}$

b. $(3x+7)(x-2) < 0$ $\left(-\frac{7}{3}, 2\right)$

c. $(3x+7)(x-2) \leq 0$ $\left[-\frac{7}{3}, 2\right]$

d. $(3x+7)(x-2) > 0$

$\left(-\infty, -\frac{7}{3}\right) \cup \left(2, \infty\right)$

e. $(3x+7)(x-2) \geq 0$

$\left(-\infty, -\frac{7}{3}\right] \cup \left[2, \infty\right)$

17. $-x^2 + x + 12 = 0$

$-\left(x^2 - x - 12\right) = 0$

$-(x+3)(x-4) = 0$

$x = -3$ or $x = 4$

The boundary points are -3 and 4.

Sign of $(x+3)$:	$-$	$+$	$+$
Sign of $(x-4)$:	$-$	$-$	$+$
Sign of $(x+3)(x-4)$:	$+$	$-$	$+$
Sign of $-(x+3)(x-4)$:	$-$	$+$	$-$

-3 4

a. $-x^2 + x + 12 = 0$ $\{-3, 4\}$

b. $-x^2 + x + 12 < 0$ $(-\infty, -3) \cup (4, \infty)$

c. $-x^2 + x + 12 \leq 0$ $(-\infty, -3] \cup [4, \infty)$

d. $-x^2 + x + 12 > 0$ $(-3, 4)$

e. $-x^2 + x + 12 \geq 0$ $[-3, 4]$

18. $-x^2 - 10x - 9 = 0$

$-\left(x^2 + 10x + 9\right) = 0$

$-(x+9)(x+1) = 0$

$x = -9$ or $x = -1$

The boundary points are -9 and -1.

Sign of $(x+9)$:	$-$	$+$	$+$
Sign of $(x+1)$:	$-$	$-$	$+$
Sign of $(x+9)(x+1)$:	$+$	$-$	$+$
Sign of $-(x+9)(x+1)$:	$-$	$+$	$-$

-9 -1

a. $-x^2 - 10x - 9 = 0$ $\{-9, -1\}$

b. $-x^2 - 10x - 9 < 0$

$(-\infty, -9) \cup (-1, \infty)$

c. $-x^2 - 10x - 9 \leq 0$

$(-\infty, -9] \cup [-1, \infty)$

d. $-x^2 - 10x - 9 > 0$ $(-9, -1)$

e. $-x^2 - 10x - 9 \geq 0$ $[-9, -1]$

19. $a^2 + 12a + 36 = 0$

$(a+6)^2 = 0$

$a = -6$

The boundary point is -6.

a. $a^2 + 12a + 36 = 0$

$\{-6\}$

b. $a^2 + 12a + 36 < 0$

The square of any real number is nonnegative. Therefore, this inequality has no solution.

$\{\ \}$

c. $a^2+12a+36\leq 0$

The inequality in part (b) is the same as the inequality in part (a) except that equality is included. The expression $(a+6)^2=0$ for $a=-6$.

$\{-6\}$

d. $a^2+12a+36>0$

The expression $(a+6)^2>0$ for all real numbers except where $(a+6)^2=0$. Therefore, the solution set is all real numbers except -6.

$(-\infty, -6)\cup(-6, \infty)$

e. $a^2+12a+36\geq 0$

The square of any real number is greater than or equal to zero. Therefore, the solution set is all real numbers.

$(-\infty, \infty)$

20. $t^2-14t+49=0$

$$(t-7)^2=0$$

$t=7$

The boundary point is -6.

a. $t^2-14t+49=0$

$\{7\}$

b. $t^2-14t+49<0$

The square of any real number is nonnegative. Therefore, this inequality has no solution.

$\{\ \}$

c. $t^2-14t+49\leq 0$

The inequality in part (b) is the same as the inequality in part (a) except that equality is included. The expression $(t-7)^2=0$ for $t=7$.

$\{7\}$

d. $t^2-14t+49>0$

The expression $(t-7)^2>0$ for all real numbers except where $(t-7)^2=0$.

Therefore, the solution set is all real numbers except 7.

$(-\infty, 7)\cup(7, \infty)$

e. $t^2-14t+49\geq 0$

The square of any real number is greater than or equal to zero. Therefore, the solution set is all real numbers.

$(-\infty, \infty)$

21. $4w^2-9\geq 0$

Find the real zeros of the related equation.

$$4w^2-9=0$$

$$(2w-3)(2w+3)=0$$

$$w=\frac{3}{2} \text{ or } w=-\frac{3}{2}$$

The boundary points are $-\frac{3}{2}$ and $\frac{3}{2}$.

Sign of $(2w-3)$:	$-$	$+$	$+$
Sign of $(2w+3)$:	$-$	$-$	$+$
Sign of $(2w-3)(2w+3)$:	$+$	$-$	$+$

$-\frac{3}{2}$ $\frac{3}{2}$

The solution set is $\left(-\infty, -\frac{3}{2}\right] \cup \left[\frac{3}{2}, \infty\right)$.

22. $16z^2 - 25 < 0$

Find the real zeros of the related equation.

$$16z^2 - 25 = 0$$

$$(4z-5)(4z+5) = 0$$

$$z = \frac{5}{4} \text{ or } w = -\frac{5}{4}$$

The boundary points are $-\frac{5}{4}$ and $\frac{5}{4}$.

Sign of $(4z-5)$:	$-$	$+$	$+$
Sign of $(4z+5)$:	$-$	$-$	$+$
Sign of $(4z-5)(4z+5)$:	$+$	$-$	$+$

$-\frac{5}{4}$ $\frac{5}{4}$

The solution set is $\left(-\frac{5}{4}, \frac{5}{4}\right)$.

23.

$$3w^2 + w < 2(w+2)$$

$$3w^2 + w < 2w + 4$$

$$3w^2 - w - 4 < 0$$

Find the real zeros of the related equation.

$$3w^2 - w - 4 = 0$$

$$(w+1)(3w-4) = 0$$

$$w = -1 \text{ or } w = \frac{4}{3}$$

The boundary points are -1 and $\frac{4}{3}$.

Sign of $(w+1)$:	$-$	$+$	$+$
Sign of $(3w-4)$:	$-$	$-$	$+$
Sign of $(w+1)(3w-4)$:	$+$	$-$	$+$

-1 $\frac{4}{3}$

The solution set is $\left(-1, \frac{4}{3}\right)$.

24.

$$5y^2 + 7y < 3(y+4)$$

$$5y^2 + 7y < 3y + 12$$

$$5y^2 + 4y - 12 < 0$$

Find the real zeros of the related equation.

$$5y^2 + 4y - 12 = 0$$

$$(y+2)(5y-6) = 0$$

$$y = -2 \text{ or } y = \frac{6}{5}$$

The boundary points are -2 and $\frac{6}{5}$.

Sign of $(y+2)$:	$-$	$+$	$+$
Sign of $(5y-6)$:	$-$	$-$	$+$
Sign of $(y+2)(5y-6)$:	$+$	$-$	$+$

-2 $\frac{6}{5}$

The solution set is $\left(-2, \frac{6}{5}\right)$.

25.

$$a^2 \geq 3a$$

$$a^2 - 3a \geq 0$$

Find the real zeros of the related equation.

$$a^2 - 3a = 0$$

$$a(a-3) = 0$$

$$a = 0 \text{ or } a = 3$$

The boundary points are 0 and 3.

Sign of (a):	$-$	$+$	$+$
Sign of $(a-3)$:	$-$	$-$	$+$
Sign of $(a)(a-3)$:	$+$	$-$	$+$

0 3

The solution set is $(-\infty, 0] \cup [3, \infty)$.

26.
$$d^2 \geq 6d$$
$$d^2 - 6d \geq 0$$

Find the real zeros of the related equation.

$$d^2 - 6d = 0$$
$$d(d-6) = 0$$
$$d = 0 \text{ or } d = 6$$

The boundary points are 0 and 6.

Sign of (d):	$-$	$+$	$+$
Sign of $(d-6)$:	$-$	$-$	$+$
Sign of $(d)(d-6)$:	$+$	$-$	$+$

0 6

The solution set is $(-\infty, 0] \cup [6, \infty)$.

27.
$$10 - 6x > 5x^2$$
$$-5x^2 - 6x + 10 > 0$$
$$5x^2 + 6x - 10 < 0$$

Find the real zeros of the related equation.

$$5x^2 + 6x - 10 = 0$$
$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(5)(-10)}}{2(5)}$$
$$= \frac{-6 \pm \sqrt{236}}{10}$$
$$= \frac{-6 \pm 2\sqrt{59}}{10}$$
$$= \frac{-3 \pm \sqrt{59}}{5}$$
$$x = \frac{-3 - \sqrt{59}}{5} \text{ or } x = \frac{-3 + \sqrt{59}}{5}$$

The boundary points are $\frac{-3 - \sqrt{59}}{5}$ and $\frac{-3 + \sqrt{59}}{5}$.

Sign of $\left[x-\left(\frac{-3-\sqrt{59}}{5}\right)\right]$:	$-$	$+$	$+$
Sign of $\left[x-\left(\frac{-3+\sqrt{59}}{5}\right)\right]$:	$-$	$-$	$+$
Sign of $\left[x-\left(\frac{-3-\sqrt{59}}{5}\right)\right]\left[x-\left(\frac{-3+\sqrt{59}}{5}\right)\right]$:	$+$	$-$	$+$

$\frac{-3-\sqrt{59}}{5}$ $\frac{-3+\sqrt{59}}{5}$

The solution set is $\left(\frac{-3-\sqrt{59}}{5}, \frac{-3+\sqrt{59}}{5}\right)$.

28. $6-4x>3x^2$

$-3x^2-4x+6>0$

$3x^2+4x-6<0$

Find the real zeros of the related equation.

$3x^2+4x-6=0$

$$\begin{aligned}x&=\frac{-(4)\pm\sqrt{(4)^2-4(3)(-6)}}{2(3)}\\&=\frac{-4\pm\sqrt{88}}{6}\\&=\frac{-4\pm2\sqrt{22}}{6}\\&=\frac{-2\pm\sqrt{22}}{3}\end{aligned}$$

$x=\frac{-2-\sqrt{22}}{3}$ or $x=\frac{-2+\sqrt{22}}{3}$

The boundary points are $\frac{-2-\sqrt{22}}{3}$ and $\frac{-2+\sqrt{22}}{3}$.

Sign of $\left[x-\left(\frac{-2-\sqrt{22}}{3}\right)\right]$:	−	+	+
Sign of $\left[x-\left(\frac{-2+\sqrt{22}}{3}\right)\right]$:	−	−	+
Sign of $\left[x-\left(\frac{-2-\sqrt{22}}{3}\right)\right]\left[x-\left(\frac{-2+\sqrt{22}}{3}\right)\right]$:	+	−	+

$\frac{-2-\sqrt{22}}{3}$ $\frac{-2+\sqrt{22}}{3}$

The solution set is $\left(\frac{-2-\sqrt{22}}{3}, \frac{-2+\sqrt{22}}{3}\right)$.

29. $m^2 < 49$

Find the real zeros of the related equation.

$$m^2 - 49 = 0$$
$$(m-7)(m+7) = 0$$
$$m = -7 \text{, or } m = 7$$

The boundary points are -7, and 7.

Sign of $(m-7)$:	−	+	+
Sign of $(m+7)$:	−	−	+
Sign of $(m-7)(m+7)$:	+	−	+

7 −7

The solution set is $(-7, 7)$.

30. $y^2 \geq 9$

Find the real zeros of the related equation.

$$y^2 - 9 = 0$$
$$(y-3)(y+3) = 0$$
$$y = -3 \text{, or } y = 3$$

The boundary points are -3, and 3.

Sign of $(y-3)$:	−	+	+
Sign of $(y+3)$:	−	−	+
Sign of $(y-3)(y+3)$:	+	−	+

3 −3

The solution set is $(-\infty, -3] \cup [3, \infty)$.

31. $16p^2 \geq 2$

Find the real zeros of the related equation.

$$16p^2 - 2 = 0$$
$$\left(4p-\sqrt{2}\right)\left(4p+\sqrt{2}\right) = 0$$
$$p = -\frac{\sqrt{2}}{4} \text{, or } p = \frac{\sqrt{2}}{4}$$

The boundary points are $-\frac{\sqrt{2}}{4}$, and $\frac{\sqrt{2}}{4}$.

Sign of $\left(4p-\sqrt{2}\right)$:	−	+	+
Sign of $\left(4p+\sqrt{2}\right)$:	−	−	+
Sign of $\left(4p-\sqrt{2}\right)\left(4p+\sqrt{2}\right)$:	+	−	+

$\frac{\sqrt{2}}{4}$ $-\frac{\sqrt{2}}{4}$

The solution set is

$$\left(-\infty,-\frac{\sqrt{2}}{4}\right]\cup\left[\frac{\sqrt{2}}{4},\infty\right).$$

32. $54q^2 \le 50$

Find the real zeros of the related equation.

$$54q^2 - 50 = 0$$
$$2\left(27q^2 - 25\right) = 0$$
$$2\left(\sqrt{27}q - 5\right)\left(\sqrt{27}q + 5\right) = 0$$
$$2\left(3\sqrt{3}q - 5\right)\left(3\sqrt{3}q + 5\right) = 0$$

$q = \frac{5}{3\sqrt{3}} = \frac{5\sqrt{3}}{9}$, or $q = -\frac{5}{3\sqrt{3}} = -\frac{5\sqrt{3}}{9}$

The boundary points are $-\frac{5\sqrt{3}}{9}$, and $\frac{5\sqrt{3}}{9}$.

Sign of $\left(9q-5\sqrt{3}\right)$:	$-$	$+$	$+$
Sign of $\left(9q+5\sqrt{3}\right)$:	$-$	$-$	$+$
Sign of $\left(9q-5\sqrt{3}\right)\left(9q+5\sqrt{3}\right)$:	$+$	$-$	$+$

$\frac{5\sqrt{3}}{9} \quad -\frac{5\sqrt{3}}{9}$

The solution set is $\left(-\frac{5\sqrt{3}}{9},\frac{5\sqrt{3}}{9}\right)$.

33. $(x+4)(x-1)(x-3) \ge 0$

Find the real zeros of the related equation.

$(x+4)(x-1)(x-3) = 0$

$x = -4$, $x = 1$, or $x = 3$

The boundary points are -4, 1, and 3.

Sign of $(x+4)$:	$-$	$+$	$+$	$+$
Sign of $(x-1)$:	$-$	$-$	$+$	$+$
Sign of $(x-3)$:	$-$	$-$	$-$	$+$
Sign of $(x+4)(x-1)(x-3)$:	$-$	$+$	$-$	$+$

$-4 \quad 1 \quad 3$

The solution set is $\left[-4, 1\right]\cup\left[3, \infty\right)$.

34. $(x+2)(x+5)(x-4) \ge 0$

Find the real zeros of the related equation.

$(x+2)(x+5)(x-4) = 0$

$x = -2$, $x = -5$, or $x = 4$

The boundary points are -5, -2, and 4.

Sign of $(x+5)$:	$-$	$+$	$+$	$+$
Sign of $(x+2)$:	$-$	$-$	$+$	$+$
Sign of $(x-4)$:	$-$	$-$	$-$	$+$
Sign of $(x+5)(x+2)(x-4)$:	$-$	$+$	$-$	$+$

$-5 \quad -2 \quad 4$

The solution set is $\left[-5, -2\right]\cup\left[4, \infty\right)$.

35. $-5c(c+2)^2(4-c) > 0$

$5c(c+2)^2(c-4) > 0$

Find the real zeros of the related equation.

$5c(c+2)^2(c-4) = 0$

$c(c+2)^2(c-4) = 0$

$c = 0$, $c = -2$, or $c = 4$

The boundary points are -2, 0, and 4.

Sign of $(c+2)^2$:	$+$	$+$	$+$	$+$
Sign of (c):	$-$	$-$	$+$	$+$
Sign of $(c-4)$:	$-$	$-$	$-$	$+$
Sign of $(c)(c+2)^2(c-4)$:	$+$	$+$	$-$	$+$

$-2 \quad 0 \quad 4$

The solution set is

$(-\infty, -2) \cup (-2, 0) \cup (4, \infty)$.

36. $-6u(u+1)^2(3-u) > 0$

$6u(u+1)^2(u-3) > 0$

Find the real zeros of the related equation.

$6u(u+1)^2(u-3) = 0$

$u(u+1)^2(u-3) = 0$

$u = 0$, $u = -1$, or $u = 3$

The boundary points are -1, 0, and 3.

Sign of $(u+1)^2$:	$+$	$+$	$+$	$+$
Sign of (u):	$-$	$-$	$+$	$+$
Sign of $(u-3)$:	$-$	$-$	$-$	$+$
Sign of $(u)(u+1)^2(u-3)$:	$+$	$+$	$-$	$+$

$-1 \quad 0 \quad 3$

The solution set is

$(-\infty, -1) \cup (-1, 0) \cup (3, \infty)$.

37. $t^4 - 10t^2 + 9 \le 0$

Find the real zeros of the related equation.

$$t^4 - 10t^2 + 9 = 0$$

$$(t^2 - 1)(t^2 - 9) = 0$$

$$(t+1)(t-1)(t+3)(t-3) = 0$$

$t = -1$, $t = 1$, $t = -3$, or $t = 3$

The boundary points are -3, -1, 1, and 3.

Sign of $(t+3)$:	$-$	$+$	$+$	$+$	$+$
Sign of $(t+1)$:	$-$	$-$	$+$	$+$	$+$
Sign of $(t-1)$:	$-$	$-$	$-$	$+$	$+$
Sign of $(t-3)$:	$-$	$-$	$-$	$-$	$+$
Sign of $((t+1)(t-1)$ $(t+3)(t-3))$:	$+$	$-$	$+$	$-$	$+$

$-3 \quad -1 \quad 1 \quad 3$

The solution set is $[-3, -1] \cup [1, 3]$.

38. $w^4 - 20w^2 + 64 \le 0$

Find the real zeros of the related equation.

$$w^4 - 20w^2 + 64 = 0$$

$$(w^2 - 4)(w^2 - 16) = 0$$

$$(w+2)(w-2)(w+4)(w-4) = 0$$

$w = -2$, $w = 2$, $w = -4$, or $w = 4$

The boundary points are -4, -2, 2, and 4.

Sign of $(w+4)$:	$-$	$+$	$+$	$+$	$+$
Sign of $(w+2)$:	$-$	$-$	$+$	$+$	$+$
Sign of $(w-2)$:	$-$	$-$	$-$	$+$	$+$
Sign of $(w-4)$:	$-$	$-$	$-$	$-$	$+$
Sign of $((w+2)(w-2)$ $(w+4)(w-4))$:	$+$	$-$	$+$	$-$	$+$

$-4 \quad -2 \quad 2 \quad 4$

The solution set is $[-4, -2] \cup [2, 4]$.

39. $2x^3 + 5x^2 < 8x + 20$

$2x^3 + 5x^2 - 8x - 20 < 0$

Find the real zeros of the related equation.

$$2x^3+5x^2-8x-20=0$$
$$x^2\left(2x+5\right)-4\left(2x+5\right)=0$$
$$\left(2x+5\right)\left(x^2-4\right)=0$$
$$\left(2x+5\right)\left(x+2\right)\left(x-2\right)=0$$
$$x=-\frac{5}{2},\ x=-2\text{, or }x=2$$

The boundary points are $-\frac{5}{2}$, -2, and 2.

Sign of $\left(2x+5\right)$:	−	+	+	+
Sign of $\left(x+2\right)$:	−	−	+	+
Sign of $\left(x-2\right)$:	−	−	−	+
Sign of $\left(2x+5\right)\left(x+2\right)\left(x-2\right)$:	−	+	−	+

$-\frac{5}{2}$ $\quad -2 \quad 2$

The solution set is $\left(-\infty,-\frac{5}{2}\right)\cup\left(-2,2\right)$.

40. $3x^3-3x<4x^2-4$

$$3x^3-4x^2-3x+4<0$$

Find the real zeros of the related equation.

$$3x^3-4x^2-3x+4=0$$
$$x^2\left(3x-4\right)-1\left(3x-4\right)=0$$
$$\left(x^2-1\right)\left(3x-4\right)=0$$
$$\left(x+1\right)\left(x-1\right)\left(3x-4\right)=0$$
$$x=-1,\ x=1\text{, or }x=\frac{4}{3}$$

The boundary points are -1, 1, and $\frac{4}{3}$.

Sign of $\left(x+1\right)$:	−	+	+	+
Sign of $\left(x-1\right)$:	−	−	+	+
Sign of $\left(3x-4\right)$:	−	−	−	+
Sign of $\left(x+1\right)\left(x-1\right)\left(3x-4\right)$:	−	+	−	+

$-1 \quad 1 \quad \frac{4}{3}$

The solution set is $\left(-\infty,-1\right)\cup\left(1,\frac{4}{3}\right)$.

41. $-2x^4+10x^3-6x^2-18x\geq 0$

$$-2x\left(x^3-5x^2+3x+9\right)\geq 0$$
$$x\left(x^3-5x^2+3x+9\right)\leq 0$$

Find the real zeros of the related equation.

$$x\left(x^3-5x^2+3x+9\right)=0$$

$$\frac{\text{Factors of 9}}{\text{Factors of 1}}=\frac{\pm1,\pm3,\pm9}{\pm1}$$
$$=\pm1,\pm3,\pm9$$

$$\begin{array}{r|rrrr} -1 & 1 & -5 & 3 & 9 \\ & & -1 & 6 & -9 \\ \hline & 1 & -6 & 9 & \boxed{0} \end{array}$$

$$x\left(x+1\right)\left(x^2-6x+9\right)=0$$
$$x\left(x+1\right)\left(x-3\right)^2=0$$
$$x=0,\ x=-1\text{, or }x=3$$

The boundary points are -1, 0, and 3.

Sign of $\left(x+1\right)$:	−	+	+	+
Sign of $\left(x\right)$:	−	−	+	+
Sign of $\left(x-3\right)^2$:	+	+	+	+
Sign of $x\left(x+1\right)\left(x-3\right)^2$:	+	−	+	+

$-1 \quad 0 \quad 3$

The solution set is $\left[-1,0\right]\cup\left\{3\right\}$.

42. $-4x^4+4x^3+64x^2+80x\geq 0$

$$-4x\left(x^3-x^2-16x-20\right)\geq 0$$
$$x\left(x^3-x^2-16x-20\right)\leq 0$$

Find the real zeros of the related equation.

$$x\left(x^3-x^2-16x-20\right)=0$$

$$\frac{\text{Factors of }-20}{\text{Factors of 1}}$$
$$=\frac{\pm1,\pm2,\pm4,\pm5,\pm10,\pm20}{\pm1}$$
$$=\pm1,\pm2,\pm4,\pm5,\pm10,\pm20$$

$$\begin{array}{r|rrrr} -2 & 1 & -1 & -16 & -20 \\ & & -2 & 6 & 20 \\ \hline & 1 & -3 & -10 & \boxed{0} \end{array}$$

$$x\left(x+2\right)\left(x^2-3x-10\right)=0$$
$$x\left(x+2\right)\left(x+2\right)\left(x-5\right)=0$$
$$x\left(x+2\right)^2\left(x-5\right)=0$$

$x=0$, $x=-2$, or $x=5$

The boundary points are -2, 0, and 5.

Sign of $\left(x+2\right)^2$:	$+$	$+$	$+$	$+$
Sign of $\left(x\right)$:	$-$	$-$	$+$	$+$
Sign of $\left(x-5\right)$:	$-$	$-$	$-$	$+$
Sign of $x\left(x+2\right)^2\left(x-5\right)$:	$+$	$+$	$-$	$+$

$-2 \quad 0 \quad 5$

The solution set is $\left\{-2\right\}\cup\left[0, 5\right]$.

43. $-5u^6+28u^5-15u^4\le 0$

$$-u^4\left(5u^2-28u+15\right)\le 0$$
$$u^4\left(5u^2-28u+15\right)\ge 0$$

Find the real zeros of the related equation.

$$u^4\left(5u^2-28u+15\right)=0$$
$$u^4\left(5u-3\right)\left(u-5\right)=0$$

$u=0$, $u=\frac{3}{5}$, or $u=5$

The boundary points are 0, $\frac{3}{5}$, and 5.

Sign of $\left(u^4\right)$:	$+$	$+$	$+$	$+$
Sign of $\left(5u-3\right)$:	$-$	$-$	$+$	$+$
Sign of $\left(u-5\right)$:	$-$	$-$	$-$	$+$
Sign of $\left(u^4\right)\left(5u-3\right)\left(u-5\right)$:	$+$	$+$	$-$	$+$

$0 \quad \frac{3}{5} \quad 5$

The solution set is $\left(-\infty, \frac{3}{5}\right]\cup\left[5, \infty\right)$.

44. $-3w^6+8w^5-4w^4\le 0$

$$-w^4\left(3w^2-8w+4\right)\le 0$$
$$w^4\left(3w^2-8w+4\right)\ge 0$$

Find the real zeros of the related equation.

$$w^4\left(3w^2-8w+4\right)=0$$
$$w^4\left(3w-2\right)\left(w-2\right)=0$$

$w=0$, $w=\frac{2}{3}$, or $w=2$

The boundary points are 0, $\frac{2}{3}$, and 2.

Sign of $\left(w^4\right)$:	$+$	$+$	$+$	$+$
Sign of $\left(3w-2\right)$:	$-$	$-$	$+$	$+$
Sign of $\left(w-2\right)$:	$-$	$-$	$-$	$+$
Sign of $\left(w^4\right)\left(5u-3\right)\left(u-5\right)$:	$+$	$+$	$-$	$+$

$0 \quad \frac{2}{3} \quad 2$

The solution set is $\left(-\infty, \frac{2}{3}\right]\cup\left[2, \infty\right)$.

45. $6x\left(2x-5\right)^4\left(3x+1\right)^5\left(x-4\right)<0$

$$x\left(2x-5\right)^4\left(3x+1\right)^5\left(x-4\right)<0$$

Find the real zeros of the related equation.

$$x\left(2x-5\right)^4\left(3x+1\right)^5\left(x-4\right)=0$$

$x=0$, $x=\frac{5}{2}$, $x=-\frac{1}{3}$, or $x=4$

The boundary points are $-\frac{1}{3}$, 0, $\frac{5}{2}$, and 4.

Sign of $(3x+1)^5$:	$-$	$+$	$+$	$+$	$+$
Sign of (x):	$-$	$-$	$+$	$+$	$+$
Sign of $(2x-5)^4$:	$+$	$+$	$+$	$+$	$+$
Sign of $(x-4)$:	$-$	$-$	$-$	$-$	$+$
Sign of $\left(x(2x-5)^4 (3x+1)^5(x-4)\right)$:	$-$	$+$	$-$	$-$	$+$

$\frac{1}{3}$ $\quad 0 \quad$ $\frac{5}{2}$ $\quad 4$

The solution set is $\left(-\infty, -\frac{1}{3}\right) \cup \left(0, \frac{5}{2}\right) \cup \left(\frac{5}{2}, 4\right)$.

46. $5x(3x-2)^2(4x+1)^3(x-3)^4 < 0$

Find the real zeros of the related equation.

$$5x(3x-2)^2(4x+1)^3(x-3)^4 = 0$$
$$x(3x-2)^2(4x+1)^3(x-3)^4 = 0$$
$$x = 0,\ x = \frac{2}{3},\ x = -\frac{1}{4}, \text{ or } x = 3$$

The boundary points are $-\frac{1}{4}$, 0, $\frac{2}{3}$, and 3.

Sign of $(4x+1)^3$:	$-$	$+$	$+$	$+$	$+$
Sign of $(5x)$:	$-$	$-$	$+$	$+$	$+$
Sign of $(3x-2)^2$:	$+$	$+$	$+$	$+$	$+$
Sign of $(x-3)^4$:	$+$	$+$	$+$	$+$	$+$
Sign of $\left(5x(3x-2)^2 (4x+1)^3(x-3)^4\right)$:	$+$	$-$	$+$	$+$	$+$

$-\frac{1}{4}$ $\quad 0 \quad$ $\frac{2}{3}$ $\quad 3$

The solution set is $\left(-\frac{1}{4}, 0\right)$.

47. $(5x-3)^2 > -2$

The square of any real number is nonnegative. Therefore, $(5x-3)^2$ is nonnegative, and always greater than 0. The solution set is $(-\infty, \infty)$.

48. $(4x+1)^2 > -6$

The square of any real number is nonnegative. Therefore, $(4x+1)^2$ is nonnegative, and always greater than 0. The solution set is $(-\infty, \infty)$.

49. $-4 \geq (x-7)^2$

The square of any real number is nonnegative. Therefore, $(x-7)^2$ is nonnegative, and always greater than 0. The solution set is $\{\ \}$.

50. $-1 \geq (x+2)^2$

The square of any real number is nonnegative. Therefore, $(x+2)^2$ is nonnegative, and always greater than 0. The solution set is $\{\ \}$.

51. $16y^2 > 24y - 9$

$16y^2 - 24y + 9 > 0$

$(4y - 3)^2 > 0$

The expression $(4y-3)^2 > 0$ for all real numbers except where $(4y-3)^2 = 0$. Therefore, the solution set is all real numbers except $\frac{3}{4}$.

The solution set is $\left(-\infty, \frac{3}{4}\right) \cup \left(\frac{3}{4}, \infty\right)$.

52. $4w^2 > 20w - 25$

$4w^2 - 20w + 25 > 0$

$(2w - 5)^2 > 0$

The expression $(2w-5)^2 > 0$ for all real numbers except where $(2w-5)^2 = 0$ .Therefore, the solution set is all real numbers except $\frac{5}{2}$.

The solution set is $\left(-\infty, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$.

53. $(x+3)(x+1) \le -1$

$x^2 + 4x + 3 + 1 \le 0$

$x^2 + 4x + 4 \le 0$

$(x+2)^2 \le 0$

The square of any real number is nonnegative. Therefore, $(x+2)^2$ is nonnegative, and always greater than 0. However, the equality is included. The solution set is $\{-2\}$.

54. $(x+2)(x+4) \le -1$

$x^2 + 6x + 8 + 1 \le 0$

$x^2 + 6x + 9 \le 0$

$(x+3)^2 \le 0$

The square of any real number is nonnegative. Therefore, $(x+3)^2$ is nonnegative, and always greater than 0. However, the equality is included. The solution set is $\{-3\}$.

55. a. $(2, 3)$

b. $[2, 3)$

c. $(-\infty, -2) \cup (3, \infty)$

d. $(-\infty, -2] \cup (3, \infty)$

56. a. $(-\infty, -2) \cup (2, \infty)$

b. $(-\infty, -2] \cup (2, \infty)$

c. $(-2, 2)$

d. $[-2, 2)$

57. a. $(-\infty, 2) \cup (2, \infty)$

b. $(-\infty, 2) \cup (2, \infty)$

c. $\{\ \}$

d. $\{\ \}$

58. a. $\{\ \}$

b. $\{\ \}$

c. $(-\infty, -3) \cup (-3, \infty)$

d. $(-\infty, -3) \cup (-3, \infty)$

59. $\frac{x+2}{x-3} = 0$

$x + 2 = 0$

$x = -2$

-2 is a boundary point.

The expression is undefined for $x = 3$.

This is also a boundary point.

Sign of $(x+2)$:	$-$	$+$	$+$
Sign of $(x-3)$:	$-$	$-$	$+$
Sign of $\frac{x+2}{x-3}$:	$+$	$-$	$+$

$-2 \quad 3$

a. $\frac{x+2}{x-3} \le 0$

$[-2, 3)$

b. $\frac{x+2}{x-3} < 0$

$(-2, 3)$

c. $\frac{x+2}{x-3} \ge 0$

$(-\infty, -2] \cup (3, \infty)$

d. $\frac{x+2}{x-3} > 0$

$(-\infty, -2) \cup (3, \infty)$

60. $\frac{x+4}{x-1} = 0$

$x + 4 = 0$

$x = -4$

-4 is a boundary point.

The expression is undefined for $x = 3$.

This is also a boundary point.

Sign of $(x+4)$:	$-$	$+$	$+$
Sign of $(x-1)$:	$-$	$-$	$+$
Sign of $\frac{x+4}{x-1}$:	$+$	$-$	$+$

$-4 \quad 1$

a. $\frac{x+4}{x-1} \le 0$

$[-4, 1)$

b. $\frac{x+4}{x-1} < 0$

$(-4, 1)$

c. $\frac{x+4}{x-1} \ge 0$

$(-\infty, -4] \cup (1, \infty)$

d. $\frac{x+4}{x-1} > 0$

$(-\infty, -4) \cup (1, \infty)$

61. $\frac{x^4}{x^2+9} = 0$

$x^4 = 0$

$x = 0$

0 is a boundary point.

The denominator is nonzero for all real numbers. Therefore, the only boundary point is 0.

Sign of x^4:	$+$	$+$
Sign of x^2+9:	$+$	$+$
Sign of $\frac{x^4}{x^2+9}$:	$+$	$+$

0

a. $\frac{x^4}{x^2+9} \le 0$

$\{0\}$

b. $\frac{x^4}{x^2+9} < 0$

$\{\ \}$

c. $\frac{x^4}{x^2+9} \ge 0$

$(-\infty, \infty)$

d. $\frac{x^4}{x^2+9} > 0$

$(-\infty, 0) \cup (0, \infty)$

62. $\frac{-x^2}{x^2+16}=0$

$x^2=0$

$x=0$

0 is a boundary point.

The denominator is nonzero for all real numbers. Therefore, the only boundary point is 0.

Sign of $-x^2$:	$-$	$-$
Sign of x^2+16:	$+$	$+$
Sign of $\frac{-x^2}{x^2+16}$:	$-$	$-$

0

a. $\frac{-x^2}{x^2+16}\le 0$

$(-\infty, \infty)$

b. $\frac{-x^2}{x^2+16}<0$

$(-\infty, 0)\cup(0, \infty)$

c. $\frac{-x^2}{x^2+16}\ge 0$

$\{0\}$

d. $\frac{-x^2}{x^2+16}>0$

$\{\ \}$

63. $\frac{5-x}{x+1}\ge 0$

$\frac{x-5}{x+1}\le 0$

The expression is undefined for $x=-1$.

This is a boundary point.

Find the real zeros of the related equation.

$\frac{x-5}{x+1}=0$

$x-5=0$

$x=5$

The boundary points are -1 and 5.

Sign of $(x+1)$:	$-$	$+$	$+$
Sign of $(x-5)$:	$-$	$-$	$+$
Sign of $\frac{x-5}{x+1}$:	$+$	$-$	$+$

-1 5

The solution set is $(-1, 5]$.

64. $\frac{2-x}{x+6}\ge 0$

$\frac{x-2}{x+6}\le 0$

The expression is undefined for $x=-6$.

This is a boundary point.

Find the real zeros of the related equation.

$\frac{x-2}{x+6}=0$

$x-2=0$

$x=2$

The boundary points are –6 and 2.

Sign of $(x+6)$:	$-$	$+$	$+$
Sign of $(x-2)$:	$-$	$-$	$+$
Sign of $\frac{x-2}{x+6}$:	$+$	$-$	$+$

-6 2

The solution set is $(-6, 2]$.

65. $\frac{4-2x}{x^2}\le 0$

$\frac{2x-4}{x^2}\ge 0$

$\frac{x-2}{x^2}\ge 0$

The expression is undefined for $x=0$.

This is a boundary point.

Find the real zeros of the related equation.

$$\frac{x-2}{x^2}=0$$

$$x-2=0$$

$$x=2$$

The boundary points are 0 and 2.

Sign of x^2:	$+$	$+$	$+$
Sign of $x-2$:	$-$	$-$	$+$
Sign of $\frac{x-2}{x^2}$:	$-$	$-$	$+$

0 2

The solution set is $\left[2,\infty\right)$.

66. $\dfrac{9-3x}{x^2}\le 0$

$$\frac{3x-9}{x^2}\ge 0$$

$$\frac{x-3}{x^2}\ge 0$$

The expression is undefined for $x=0$. This is a boundary point.

Find the real zeros of the related equation.

$$\frac{x-3}{x^2}=0$$

$$x-3=0$$

$$x=3$$

The boundary points are 0 and 3.

Sign of x^2:	$+$	$+$	$+$
Sign of $x-3$:	$-$	$-$	$+$
Sign of $\frac{x-3}{x^2}$:	$-$	$-$	$+$

0 3

The solution set is $\left[3,\infty\right)$.

67. $\dfrac{w^2-w-2}{w+3}\ge 0$

$$\frac{(w+1)(w-2)}{w+3}\ge 0$$

The expression is undefined for $w=-3$. This is a boundary point.

Find the real zeros of the related equation.

$$\frac{(w+1)(w-2)}{w+3}=0$$

$$(w+1)(w-2)=0$$

$$w=-1 \text{ or } w=2$$

The boundary points are -3, -1, and 2.

Sign of $(x+3)$:	$-$	$+$	$+$	$+$
Sign of $(w+1)$:	$-$	$-$	$+$	$+$
Sign of $(w-2)$:	$-$	$-$	$-$	$+$
Sign of $\frac{(w+1)(w-2)}{(w+3)}$:	$-$	$+$	$-$	$+$

-3 -1 2

The solution set is $\left(-3,-1\right]\cup\left[2,\infty\right)$.

68. $\dfrac{p^2-2p-8}{p-1}\ge 0$

$$\frac{(p+2)(p-4)}{p-1}\ge 0$$

The expression is undefined for $p=1$. This is a boundary point.

Find the real zeros of the related equation.

$$\frac{(p+2)(p-4)}{p-1}=0$$

$$(p+2)(p-4)=0$$

$$p=-2 \text{ or } p=4$$

The boundary points are -2, 1, and 4.

Sign of $(p+2)$:	$-$	$+$	$+$	$+$
Sign of $(p-1)$:	$-$	$-$	$+$	$+$
Sign of $(p-4)$:	$-$	$-$	$-$	$+$
Sign of $\dfrac{(p+2)(p-4)}{(p-1)}$:	$-$	$+$	$-$	$+$

$-2 \quad 1 \quad 4$

The solution set is $[-2, 1) \cup [4, \infty)$.

Sign of $(2t-7)$:	$-$	$+$	$+$
Sign of $(t-6)$:	$-$	$-$	$+$
Sign of $\dfrac{(t-6)}{(2t-7)}$:	$+$	$-$	$+$

$\frac{7}{2} \quad 6$

The solution set is $\left(\frac{7}{2}, 6\right)$.

69.

$$\frac{5}{2t-7} > 1$$
$$\frac{5}{2t-7} - 1 > 0$$
$$\frac{5}{2t-7} - \frac{2t-7}{2t-7} > 0$$
$$\frac{12-2t}{2t-7} > 0$$
$$\frac{-2(t-6)}{2t-7} > 0$$
$$\frac{t-6}{2t-7} < 0$$

The expression is undefined for $t = \frac{7}{2}$.

This is a boundary point.

Find the real zeros of the related equation.

$$\frac{t-6}{2t-7} = 0$$
$$t-6 = 0$$
$$t = 6$$

The boundary points are $\frac{7}{2}$ and 6.

70.

$$\frac{4}{3c-8} > 1$$
$$\frac{4}{3c-8} - 1 > 0$$
$$\frac{4}{3c-8} - \frac{3c-8}{3c-8} > 0$$
$$\frac{12-3c}{3c-8} > 0$$
$$\frac{-3(c-4)}{3c-8} > 0$$
$$\frac{c-4}{3c-8} < 0$$

The expression is undefined for $c = \frac{8}{3}$.

This is a boundary point.

Find the real zeros of the related equation.

$$\frac{c-4}{3c-8} = 0$$
$$c-4 = 0$$
$$c = 4$$

The boundary points are $\frac{8}{3}$ and 4.

Sign of $(3c-8)$:	$-$	$+$	$+$
Sign of $(c-4)$:	$-$	$-$	$+$
Sign of $\frac{(c-4)}{(3c-8)}$:	$+$	$-$	$+$

$\frac{8}{3}$ 4

The solution set is $\left(\frac{8}{3}, 4\right)$.

71.

$$\begin{aligned}\frac{2x}{x-2} &\le 2\\ \frac{2x}{x-2}-2 &\le 0\\ \frac{2x}{x-2}-2\cdot\frac{x-2}{x-2} &\le 0\\ \frac{4}{x-2} &\le 0\end{aligned}$$

The expression is undefined for $x=2$.

This is a boundary point.

The related equation has no real zeros.

Sign of 4:	$+$	$+$
Sign of $(x-2)$:	$-$	$+$
Sign of $\frac{4}{(x-2)}$:	$-$	$+$

2

The solution set is $(-\infty, 2)$.

72.

$$\begin{aligned}\frac{3x}{3x-7} &\le 1\\ \frac{3x}{3x-7}-1 &\le 0\\ \frac{3x}{3x-7}-\frac{3x-7}{3x-7} &\le 0\\ \frac{7}{3x-7} &\le 0\end{aligned}$$

The expression is undefined for $x=\frac{7}{3}$.

This is a boundary point.

The related equation has no real zeros.

Sign of 7:	$+$	$+$
Sign of $(3x-7)$:	$-$	$+$
Sign of $\frac{7}{(3x-7)}$:	$-$	$+$

$\frac{7}{3}$

The solution set is $\left(-\infty, \frac{7}{3}\right)$.

73.

$$\begin{aligned}\frac{4-x}{x+5} &\ge 2\\ \frac{4-x}{x+5}-2 &\ge 0\\ \frac{4-x}{x+5}-2\cdot\frac{x+5}{x+5} &\ge 0\\ \frac{-3x-6}{x+5} &\ge 0\\ \frac{x+2}{x+5} &\le 0\end{aligned}$$

The expression is undefined for $x=-5$. This is a boundary point.

Find the real zeros of the related equation.

$$\begin{aligned}\frac{x+2}{x+5} &= 0\\ x+2 &= 0\\ x &= -2\end{aligned}$$

The boundary points are -5 and -2.

Sign of $(x+5)$:	$-$	$+$	$+$
Sign of $(x+2)$:	$-$	$-$	$+$
Sign of $\frac{(x+2)}{(x+5)}$:	$+$	$-$	$+$

-5 -2

The solution set is $(-5, -2]$.

74. $$\frac{3-x}{x+2}\ge 4$$
$$\frac{3-x}{x+2}-4\ge 0$$
$$\frac{3-x}{x+2}-4\cdot\frac{x+2}{x+2}\ge 0$$
$$\frac{-5x-5}{x+2}\ge 0$$
$$\frac{x+1}{x+2}\le 0$$

The expression is undefined for $x=-2$.

This is a boundary point.

Find the real zeros of the related equation.

$$\frac{x+1}{x+2}=0$$
$$x+1=0$$
$$x=-1$$

The boundary points are -2 and -1.

Sign of $(x+2)$:	$-$	$+$	$+$
Sign of $(x+1)$:	$-$	$-$	$+$
Sign of $\frac{(x+1)}{(x+2)}$:	$+$	$-$	$+$

-2 -1

The solution set is $(-2,-1]$.

75. $$\frac{a-2}{a^2+4}\le 0$$

Find the real zeros of the related equation.

$$\frac{a-2}{a^2+4}=0$$
$$a-2=0$$
$$a=2$$

2 is a boundary point.

The denominator is nonzero for all real numbers. Therefore, the only boundary point is 2.

Sign of $a-2$:	$-$	$+$
Sign of a^2+4:	$+$	$+$
Sign of $\frac{a-2}{a^2+4}$:	$-$	$+$

2

The solution set is $(-\infty, 2]$.

76. $$\frac{d-3}{d^2+1}\le 0$$

Find the real zeros of the related equation.

$$\frac{d-3}{d^2+1}=0$$
$$d-3=0$$
$$d=3$$

3 is a boundary point.

The denominator is nonzero for all real numbers. Therefore, the only boundary point is 3.

Sign of $d-3$:	$-$	$+$
Sign of d^2+1:	$+$	$+$
Sign of $\frac{d-3}{d^2+1}$:	$-$	$+$

3

The solution set is $(-\infty, 3]$.

77. $$\frac{10}{x+2}\ge\frac{2}{x+2}$$
$$\frac{10}{x+2}-\frac{2}{x+2}\ge 0$$
$$\frac{8}{x+2}\ge 0$$

The expression is undefined for $x=-2$.

This is a boundary point.

Sign of 10:	$+$	$+$
Sign of $(x+2)$:	$-$	$+$
Sign of $\dfrac{10}{(x+2)}$:	$-$	$+$

-2

The solution set is $(-2, \infty)$.

78.
$$\frac{4}{x-3} \ge \frac{1}{x-3}$$
$$\frac{4}{x-3} - \frac{1}{x-3} \ge 0$$
$$\frac{3}{x-3} \ge 0$$

The expression is undefined for $x = 3$. This is a boundary point.

Sign of 3:	$+$	$+$
Sign of $(x-3)$:	$-$	$+$
Sign of $\dfrac{3}{(x-3)}$:	$-$	$+$

3

The solution set is $(3, \infty)$.

79.
$$\frac{4}{y+3} > -\frac{2}{y}$$
$$\frac{4}{y+3} + \frac{2}{y} > 0$$
$$\frac{4y}{y(y+3)} + \frac{2(y+3)}{y(y+3)} > 0$$
$$\frac{6(y+1)}{y(y+3)} > 0$$

The expression is undefined for $y = 0$ and $y = -3$. These are boundary points.

Find the real zeros of the related equation.

$$\frac{6(y+1)}{y(y+3)} = 0$$
$$y+1 = 0$$
$$y = -1$$

The boundary points are -3, -1, and 0.

Sign of $(y+3)$:	$-$	$+$	$+$	$+$
Sign of $6(y+1)$:	$-$	$-$	$+$	$+$
Sign of y:	$-$	$-$	$-$	$+$
Sign of $\dfrac{6(y+1)}{y(y+3)}$:	$-$	$+$	$-$	$+$

$-3 \quad -1 \quad 0$

The solution set is $(-3, -1) \cup (0, \infty)$.

80.
$$\frac{2}{z-1} > -\frac{4}{z}$$
$$\frac{2}{z-1} + \frac{4}{z} > 0$$
$$\frac{2z}{z(z-1)} + \frac{4(z-1)}{z(z-1)} > 0$$
$$\frac{2(3z-2)}{z(z-1)} > 0$$

The expression is undefined for $z = 0$ and $z = 1$. These are boundary points. Find the real zeros of the related equation.

$$\frac{2(3z-2)}{z(z-1)} = 0$$
$$3z - 2 = 0$$
$$z = \frac{2}{3}$$

The boundary points are 0, $\frac{2}{3}$, and 1.

Sign of z:	$-$	$+$	$+$	$+$
Sign of $(3z-2)$:	$-$	$-$	$+$	$+$
Sign of $(z-1)$:	$-$	$-$	$-$	$+$
Sign of $\frac{2(3z-2)}{z(z-1)}$:	$-$	$+$	$-$	$+$

$0 \quad \frac{2}{3} \quad 1$

The solution set is $\left(0, \frac{2}{3}\right) \cup (1, \infty)$.

81. $\frac{3}{4-x} \le \frac{6}{1-x}$

$$\begin{aligned} \frac{3}{x-4} &\ge \frac{6}{x-1} \\ \frac{3}{x-4} - \frac{6}{x-1} &\ge 0 \\ \frac{3(x-1)}{(x-4)(x-1)} - \frac{6(x-4)}{(x-4)(x-1)} &\ge 0 \\ \frac{-3x+21}{(x-4)(x-1)} &\ge 0 \\ \frac{-3(x-7)}{(x-4)(x-1)} &\ge 0 \\ \frac{3(x-7)}{(x-4)(x-1)} &\le 0 \end{aligned}$$

The expression is undefined for $x = 1$ and $x = 4$. These are boundary points. Find the real zeros of the related equation.

$$\begin{aligned} \frac{3(x-7)}{(x-4)(x-1)} &= 0 \\ 3(x-7) &= 0 \\ x &= 7 \end{aligned}$$

The boundary points are 1, 4, and 7.

Sign of $(x-1)$:	$-$	$+$	$+$	$+$
Sign of $(x-4)$:	$-$	$-$	$+$	$+$
Sign of $(x-7)$:	$-$	$-$	$-$	$+$
Sign of $\frac{3(x-7)}{(x-4)(x-1)}$:	$-$	$+$	$-$	$+$

$1 \quad 4 \quad 7$

The solution set is $(-\infty, 1) \cup (4, 7]$.

82. $\frac{5}{2-x} \le \frac{3}{3-x}$

$$\begin{aligned} \frac{5}{x-2} &\ge \frac{3}{x-3} \\ \frac{5}{x-2} - \frac{3}{x-3} &\ge 0 \\ \frac{5(x-3)}{(x-2)(x-3)} - \frac{3(x-2)}{(x-2)(x-3)} &\ge 0 \\ \frac{2x-9}{(x-2)(x-3)} &\ge 0 \end{aligned}$$

The expression is undefined for $x = 2$ and $x = 3$. These are boundary points. Find the real zeros of the related equation.

$$\begin{aligned} \frac{2x-9}{(x-2)(x-3)} &= 0 \\ 2x-9 &= 0 \\ x &= \frac{9}{2} \end{aligned}$$

The boundary points are 2, 3, and $\frac{9}{2}$.

Sign of $(x-2)$:	$-$	$+$	$+$	$+$
Sign of $(x-3)$:	$-$	$-$	$+$	$+$
Sign of $(2x-9)$:	$-$	$-$	$-$	$+$
Sign of $\frac{2x-9}{(x-2)(x-3)}$:	$-$	$+$	$-$	$+$

$2 \quad 3 \quad \frac{9}{2}$

The solution set is $(2, 3) \cup \left[\frac{9}{2}, \infty\right)$.

83. $\dfrac{(2-x)(2x+1)^2}{(x-4)^4} \le 0$

$\dfrac{(x-2)(2x+1)^2}{(x-4)^4} \ge 0$

The expression is undefined for $x = 4$.
This is a boundary point.
Find the real zeros of the related equation.

$\dfrac{(x-2)(2x+1)^2}{(x-4)^4} = 0$

$(x-2)(2x+1)^2 = 0$

$x = 2$ or $x = -\dfrac{1}{2}$

The boundary points are $-\dfrac{1}{2}$, 2, and 4.

Sign of $(2x+1)^2$:	+	+	+	+
Sign of $(x-2)$:	−	−	+	+
Sign of $(x-4)^4$:	+	+	+	+
Sign of $\dfrac{(x-2)(2x+1)^2}{(x-4)^4}$:	−	−	+	+

$-\dfrac{1}{2}$ 2 4

The solution set is $[2, 4) \cup (4, \infty)$.

84. $\dfrac{(3-x)(4x-1)^4}{(x+2)^2} \le 0$

$\dfrac{(x-3)(4x-1)^4}{(x+2)^2} \ge 0$

The expression is undefined for $x = -2$.
This is a boundary point.
Find the real zeros of the related equation.

$\dfrac{(x-3)(4x-1)^4}{(x+2)^2} = 0$

$(x-3)(4x-1)^4 = 0$

$x = 3$ or $x = \dfrac{1}{4}$

The boundary points are -2, 3, and $\dfrac{1}{4}$.

Sign of $(x+2)^2$:	+	+	+	+
Sign of $(4x-1)^4$:	+	+	+	+
Sign of $(x-3)$:	−	−	−	+
Sign of $\dfrac{(x-3)(4x-1)^4}{(x+2)^2}$:	−	−	−	+

-2 $\dfrac{1}{4}$ 3

The solution set is $[3, \infty) \cup \left\{\dfrac{1}{4}\right\}$.

85. a. $s(t) = -\dfrac{1}{2}(32)t^2 + (216)t + (0)$

$s(t) = -16t^2 + 216t$

b. $t = \dfrac{-b}{2a} = \dfrac{-(216)}{2(-16)} = \dfrac{216}{32} = 6.75$

The shell will explode 6.75 sec after launch.

c. $-16t^2 + 216t > 200$

$-16t^2 + 216t - 200 > 0$

$2t^2 - 27t + 25 < 0$

Find the real zeros of the related equation.

$2t^2 - 27t + 25 = 0$

$(t-1)(2t-25) = 0$

$t = 1$ or $t = \dfrac{25}{2}$

The boundary points are 1 and $\frac{25}{2}$.

Sign of $(t-1)$:	$-$	$+$	$+$
Sign of $(2t-25)$:	$-$	$-$	$+$
Sign of $(t-1)(2t-25)$:	$+$	$-$	$+$

$1 \quad \frac{25}{2}$

The solution set is $\left(1, \frac{25}{2}\right)$.

The spectators can see the shell between 1 sec and 6.75 sec after launch.

86. a. $s(t) = -\frac{1}{2}(32)t^2 + (16)t + (0)$

$s(t) = -16t^2 + 16t$

b. $-16t^2 + 16t > 3$

$-16t^2 + 16t - 3 > 0$

$16t^2 - 16t + 3 < 0$

Find the real zeros of the related equation.

$16t^2 - 16t + 3 = 0$

$(4t-1)(4t-3) = 0$

$t = \frac{1}{4}$ or $t = \frac{3}{4}$

The boundary points are $\frac{1}{4}$ and $\frac{3}{4}$.

Sign of $(4t-1)$:	$-$	$+$	$+$
Sign of $(4t-3)$:	$-$	$-$	$+$
Sign of $(4t-1)(4t-3)$:	$+$	$-$	$+$

$\frac{1}{4} \quad \frac{3}{4}$

The solution set is $\left(\frac{1}{4}, \frac{3}{4}\right)$.

The player will be more than 3 ft in the air between 0.25 sec and 0.75 sec after leaving the ground.

87. $d(v) = 0.06v^2 + 2v$

$0.06v^2 + 2v < 250$

$0.06v^2 + 2v - 250 < 0$

$6v^2 + 200v - 25{,}000 < 0$

$3v^2 + 100v - 12{,}500 < 0$

Find the real zeros of the related equation.

$3v^2 + 100v - 12{,}500 = 0$

$(3v + 250)(v - 50) = 0$

$v = -\frac{250}{3}$ or $v = 50$

Sign of $(3v+250)$:	$-$	$+$	$+$
Sign of $(v-50)$:	$-$	$-$	$+$
Sign of $(3v+250)(v-50)$:	$+$	$-$	$+$

$-\frac{250}{3} \quad 50$

The solution set is $\left(-\frac{250}{3}, 50\right)$.

The car will stop within 250 ft if the car is traveling less than 50 mph.

88. $P(t) = -1500t^2 + 60{,}000t + 10{,}000$

$-1500t^2 + 60{,}000t + 10{,}000 > 460{,}000$

$-1500t^2 + 60{,}000t - 450{,}000 > 0$

$t^2 - 40t + 300 < 0$

Find the real zeros of the related equation.

$t^2 - 40t + 300 = 0$

$(t-10)(t-30) = 0$

$t = 10$ or $t = 30$

Sign of $(t-10)$:	$-$	$+$	$+$
Sign of $(t-30)$:	$-$	$-$	$+$
Sign of $(t-10)(t-30)$:	$+$	$-$	$+$

$10 \quad 30$

The solution set is $(10, 30)$.

The population will be greater than 460,000 organisms between 10 hr and 30 hr after the culture is started.

89. a. The degree of the numerator is less than the degree of the denominator. The horizontal asymptote is $y = 0$ and means that the temperature will approach $0°\text{C}$ as time increases without bound.

b.
$$\frac{320}{x^2+3x+10} < 5$$
$$\frac{320}{x^2+3x+10} - 5 < 0$$

Find the real zeros of the related equation.

$$\frac{320}{x^2+3x+10} - 5 = 0$$
$$\frac{320}{x^2+3x+10} = 5$$
$$5x^2+15x+50 = 320$$
$$5x^2+15x-270 = 0$$
$$x^2+3x-54 = 0$$
$$(x+9)(x-6) = 0$$

$x = -9$ or $x = 6$

More than 6 hr is required for the temperature to fall below $5°\text{C}$.

90. a. $S = \dfrac{2d}{\frac{d}{r_1}+\frac{d}{r_2}}$

$$\frac{2(200)}{\frac{200}{50_1}+\frac{200}{r_2}} \ge 60$$
$$\frac{400}{4+\frac{200}{r_2}} \ge 60$$
$$\frac{400r_2}{4r_2+200} \ge 60$$
$$400r_2 \ge 240r_2 + 12{,}000$$
$$160r_2 \ge 12{,}000$$
$$r_2 \ge 75$$

The motorist must travel at least 75 mph on the return trip to average at least 60 mph for the round trip.

b. No.

91. Let *w* represent the width. Then $1.2w$ represents the length.

$$72 \le w(1.2w) \le 96$$
$$72 \le 1.2w^2 \le 96$$
$$60 \le w^2 \le 80$$
$$\sqrt{60} \le w \le \sqrt{80}$$
$$2\sqrt{15} \le w \le 4\sqrt{5}$$

The width should be between $2\sqrt{15}$ ft and $4\sqrt{5}$ ft. This is between approximately 7.7 ft and 8.9 ft.

92. Let *w* represent the width. Then $1.5w$ represents the length.

$$480 \le w(1.5w) \le 720$$
$$480 \le 1.5w^2 \le 720$$
$$320 \le w^2 \le 480$$
$$\sqrt{320} \le w \le \sqrt{480}$$
$$8\sqrt{5} \le w \le 4\sqrt{30}$$

The width should be between $8\sqrt{5}$ yd and $4\sqrt{30}$ yd. This is between approximately 17.9 yd and 21.9 yd.

93. $9 - x^2 \ge 0$

$x^2 - 9 \le 0$

Find the real zeros of the related equation.

$$x^2 - 9 = 0$$
$$(x+3)(x-3) = 0$$

$x = 3$ or $x = -3$

The boundary points are -3 and 3.

Sign of $(x+3)$:	$-$	$+$	$+$
Sign of $(x-3)$:	$-$	$-$	$+$
Sign of $(x+3)(x-3)$:	$+$	$-$	$+$

$-3 \quad 3$

$\left[-3, 3\right]$

94. $1-t^2 \ge 0$

$t^2-1 \le 0$

Find the real zeros of the related equation.

$$t^2-1=0$$

$$(t+1)(t-1)=0$$

$t=1$ or $t=-1$

The boundary points are -1 and 1.

Sign of $(x+1)$:	$-$	$+$	$+$
Sign of $(x-1)$:	$-$	$-$	$+$
Sign of $(x+1)(x-1)$:	$+$	$-$	$+$

$-1 \quad 1$

$\left[-1, 1\right]$

95. $a^2-5 \ge 0$

Find the real zeros of the related equation.

$$a^2-5=0$$

$$\left(a+\sqrt{5}\right)\left(a-\sqrt{5}\right)=0$$

$a=-\sqrt{5}$ or $a=\sqrt{5}$

The boundary points are $-\sqrt{5}$ and $\sqrt{5}$.

Sign of $\left(x+\sqrt{5}\right)$:	$-$	$+$	$+$
Sign of $\left(x-\sqrt{5}\right)$:	$-$	$-$	$+$
Sign of $\left(x+\sqrt{5}\right)\left(x-\sqrt{5}\right)$:	$+$	$-$	$+$

$-\sqrt{5} \quad \sqrt{5}$

$\left(-\infty, -\sqrt{5}\right] \cup \left[\sqrt{5}, \infty\right)$

96. $u^2-7 \ge 0$

Find the real zeros of the related equation.

$$u^2-7=0$$

$$\left(u+\sqrt{7}\right)\left(u-\sqrt{7}\right)=0$$

$a=-\sqrt{7}$ or $a=\sqrt{7}$

The boundary points are $-\sqrt{7}$ and $\sqrt{7}$.

Sign of $\left(x+\sqrt{7}\right)$:	$-$	$+$	$+$
Sign of $\left(x-\sqrt{7}\right)$:	$-$	$-$	$+$
Sign of $\left(x+\sqrt{7}\right)\left(x-\sqrt{7}\right)$:	$+$	$-$	$+$

$-\sqrt{7} \quad \sqrt{7}$

$\left(-\infty, -\sqrt{7}\right] \cup \left[\sqrt{7}, \infty\right)$

97. $2x^2+9x-18 \ge 0$

Find the real zeros of the related equation.

$$2x^2+9x-18=0$$

$$(x+6)(2x-3)=0$$

$x=-6$ or $x=\dfrac{3}{2}$

The boundary points are -6 and $\dfrac{3}{2}$.

Sign of $(x+6)$:	$-$	$+$	$+$
Sign of $(2x-3)$:	$-$	$-$	$+$
Sign of $(x+6)(2x-3)$:	$+$	$-$	$+$

$-6 \quad \dfrac{3}{2}$

The solution set is $\left(-\infty, -6\right] \cup \left[\dfrac{3}{2}, \infty\right)$.

98. $4x^2+7x-2 \ge 0$

Find the real zeros of the related equation.

$$4x^2 + 7x - 2 = 0$$
$$(x+2)(4x-1) = 0$$
$$x = -2 \text{ or } x = \frac{1}{4}$$

The boundary points are -2 and $\frac{1}{4}$.

Sign of $(x+2)$:	$-$	$+$	$+$
Sign of $(4x-1)$:	$-$	$-$	$+$
Sign of $(x+2)(4x-1)$:	$+$	$-$	$+$

$$-2 \quad \frac{1}{4}$$

The solution set is $\left(-\infty, -2\right] \cup \left[\frac{1}{4}, \infty\right)$.

99. $2x^2 + 9x - 18 > 0$

Find the real zeros of the related equation.

$$2x^2 + 9x - 18 = 0$$
$$(x+6)(2x-3) = 0$$
$$x = -6 \text{ or } x = \frac{3}{2}$$

The boundary points are -6 and $\frac{3}{2}$.

Sign of $(x+6)$:	$-$	$+$	$+$
Sign of $(2x-3)$:	$-$	$-$	$+$
Sign of $(x+6)(2x-3)$:	$+$	$-$	$+$

$$-6 \quad \frac{3}{2}$$

The solution set is $\left(-\infty, -6\right) \cup \left(\frac{3}{2}, \infty\right)$.

100. $4x^2 + 7x - 2 > 0$

Find the real zeros of the related equation.

$$4x^2 + 7x - 2 = 0$$
$$(x+2)(4x-1) = 0$$
$$x = -2 \text{ or } x = \frac{1}{4}$$

The boundary points are -2 and $\frac{1}{4}$.

Sign of $(x+2)$:	$-$	$+$	$+$
Sign of $(4x-1)$:	$-$	$-$	$+$
Sign of $(x+2)(4x-1)$:	$+$	$-$	$+$

$$-2 \quad \frac{1}{4}$$

The solution set is $\left(-\infty, -2\right) \cup \left(\frac{1}{4}, \infty\right)$.

101. $\frac{3x}{x+2} \geq 0$

The expression is undefined for $x = -2$. This is a boundary point. Find the real zeros of the related equation.

$$\frac{3x}{x+2} = 0$$
$$3x = 0$$
$$x = 0$$

The boundary points are -2 and 0.

Sign of $(x+2)$:	$-$	$+$	$+$
Sign of $(3x)$:	$-$	$-$	$+$
Sign of $\frac{3x}{x+2}$:	$+$	$-$	$+$

$$-2 \quad 0$$

The solution set is $\left(-\infty, -2\right) \cup \left[0, \infty\right)$.

102. $\frac{2x}{x+1} \geq 0$

The expression is undefined for $x = -1$. This is a boundary point. Find the real zeros of the related equation.

$$\frac{2x}{x+1} = 0$$
$$2x = 0$$
$$x = 0$$

The boundary points are -1 and 0.

Sign of $(x+1)$:	$-$	$+$	$+$
Sign of $(2x)$:	$-$	$-$	$+$
Sign of $\dfrac{2x}{x+1}$:	$+$	$-$	$+$

$-1 \quad 0$

The solution set is $(-\infty, -1) \cup [0, \infty)$.

103. a. $f(x) = (x-a)^2 (b-x)(x-c)^3$

Sign of $(x-a)^2$:	$+$	$+$	$+$	$+$
Sign of $(b-x)$:	$+$	$+$	$-$	$-$
Sign of $(x-c)^3$:	$-$	$-$	$-$	$+$
Sign of $(x-a)^2 (b-x)(x-c)^3$:	$-$	$-$	$+$	$-$

$a \quad b \quad c$

b. (b, c)

c. $(-\infty, a) \cup (a, b) \cup (c, \infty)$

104. a. $g(x) = \dfrac{(a-x)(x-b)^2}{(c-x)^5}$

Sign of $(a-x)$:	$+$	$-$	$-$	$-$
Sign of $(x-b)^2$:	$+$	$+$	$+$	$+$
Sign of $(c-x)^5$:	$+$	$+$	$+$	$-$
Sign of $\dfrac{(a-x)(x-b)^2}{(c-x)^5}$:	$+$	$-$	$-$	$+$

$a \quad b \quad c$

b. $(-\infty, a) \cup (c, \infty)$

c. $(a, b) \cup (b, c)$

105. The solution set to the inequality $f(x) < 0$ corresponds to the values of x for which the graph of $y = f(x)$ is below the x-axis.

106. The solution set to the inequality $f(x) \ge 0$ corresponds to the values of x for which the graph of $y = f(x)$ is on or above the x-axis.

107. Both the numerator and denominator of the rational expression are positive for all real numbers x. Therefore, the expression cannot be negative for any real number.

108. The rational expression is not defined for $x = 1$. Therefore, $x = 1$ is not included in the solution set. At $x = 3$, the expression equals zero, indicating that 3 is in the solution set. The solution set is (1, 3].

109. $\sqrt{2x-6} - 2 < 0$

The radical is real when

$$2x - 6 \ge 0$$
$$2x \ge 6$$
$$x \ge 3$$

Find the real zeros of the related equation.

$$\sqrt{2x-6} - 2 = 0$$
$$\sqrt{2x-6} = 2$$
$$2x - 6 = 4$$
$$2x = 10$$
$$x = 5$$

The boundary point is 5.

$f\left(\frac{7}{2}\right) = -1$	$f\left(\frac{15}{2}\right) = 1$
$f(x) < 0$	$f(x) > 0$

5

The solution set is $[3, 5)$.

110. $\sqrt{3x-5}-4<0$

The radical is real when

$$3x-5\ge 0$$
$$3x\ge 5$$
$$x\ge \frac{5}{3}$$

Find the real zeros of the related equation.

$$\sqrt{3x-5}-4=0$$
$$\sqrt{3x-5}=4$$
$$3x-5=16$$
$$3x=21$$
$$x=7$$

The boundary point is 7.

$f(2)=-3$	$f(10)=1$
$f(x)<0$	$f(x)>0$

7

The solution set is $\left[\frac{5}{3}, 7\right)$.

111. $\sqrt{4-x}-6\ge 0$

The radical is real when

$$4-x\ge 0$$
$$-x\ge -4$$
$$x\le 4$$

Find the real zeros of the related equation.

$$\sqrt{4-x}-6=0$$
$$\sqrt{4-x}=6$$
$$4-x=36$$
$$-x=32$$
$$x=-32$$

The boundary point is -32.

$f(-45)=1$	$f(-21)=-1$
$f(x)>0$	$f(x)<0$

-32

The solution set is $\left(-\infty, -32\right]$.

112. $\sqrt{5-x}-7\ge 0$

The radical is real when

$$5-x\ge 0$$
$$-x\ge -5$$
$$x\le 5$$

Find the real zeros of the related equation.

$$\sqrt{5-x}-7=0$$
$$\sqrt{5-x}=7$$
$$5-x=49$$
$$-x=44$$
$$x=-44$$

The boundary point is -44.

$f(-59)=1$	$f(-31)=-1$
$f(x)>0$	$f(x)<0$

-44

The solution set is $\left(-\infty, -44\right]$.

113. $\frac{1}{\sqrt{x-2}-4}\le 0$

The radical is real when

$$x-2\ge 0$$
$$x\ge 2$$

The denominator is zero when

$$\sqrt{x-2}-4=0$$
$$\sqrt{x-2}=4$$
$$x-2=16$$
$$x=18$$

The related equation has no zeros.

The boundary point is 18.

$f(11)=-1$		$f(27)=1$
$f(x)<0$		$f(x)>0$
	18	

The solution set is $\left[2,18\right)$.

114. $\dfrac{1}{\sqrt{x-3}-5}\le 0$

The radical is real when

$$x-3\ge 0$$
$$x\ge 3$$

The denominator is zero when

$$\sqrt{x-3}-5=0$$
$$\sqrt{x-3}=5$$
$$x-3=25$$
$$x=28$$

The related equation has no zeros.

The boundary point is 28.

$f(19)=-1$		$f(39)=1$
$f(x)<0$		$f(x)>0$
	28	

The solution set is $\left[3,28\right)$.

115. $-3<x^2-6x+5\le 5$

Rewrite the left inequality $f(x)$.

$$-3<x^2-6x+5$$
$$0<x^2-6x+8$$
$$x^2-6x+8>0$$
$$(x-2)(x-4)>0$$

Find the real zeros of the related equation.

$$(x-2)(x-4)=0$$
$$x=2 \text{ or } x=4$$

Rewrite the right inequality $g(x)$.

$$x^2-6x+5\le 5$$
$$x^2-6x\le 0$$
$$x(x-6)\le 0$$

Find the real zeros of the related equation.

$$x(x-6)=0$$
$$x=0 \text{ or } x=6$$

The boundary points are 0, 2, 4, and 6.

$f(-1)=15$		$f(1)=3$		$f(3)=-1$		$f(5)=3$		$f(1)=15$
$f(x)>0$		$f(x)>0$		$f(x)<0$		$f(x)>0$		$f(x)>0$
$g(-1)=7$		$g(1)=-5$		$g(3)=-9$		$g(5)=-5$		$g(1)=7$
$g(x)>0$		$g(x)<0$		$g(x)<0$		$g(x)<0$		$g(x)>0$
	0		2		4		6	

The solution set is $\left[0,2\right)\cup\left(4,6\right]$.

116. $8\le x^2+4x+3<15$

Rewrite the left inequality $f\left(x\right)$.

$$8 \le x^2 + 4x + 3$$
$$0 \le x^2 + 4x - 5$$

$x^2 + 4x - 5 \ge 0$

Find the real zeros of the related equation.

$x^2 + 4x - 5 = 0$

$\left(x+5\right)\left(x-1\right) = 0$

$x = -5$ or $x = 1$

Rewrite the right inequality $g\left(x\right)$.

$x^2 + 4x + 3 < 15$

$x^2 + 4x - 12 < 0$

Find the real zeros of the related equation.

$x^2 + 4x - 12 = 0$

$\left(x+6\right)\left(x-2\right) = 0$

$x = -6$ or $x = 2$

The boundary points are -6, -5, 1, and 2.

$f\left(-7\right) = 16$	$f\left(-5.5\right) = 3.25$	$f\left(0\right) = -5$	$f\left(1.5\right) = 3.25$	$f\left(3\right) = 16$
$f\left(x\right) > 0$	$f\left(x\right) > 0$	$f\left(x\right) < 0$	$f\left(x\right) > 0$	$f\left(x\right) > 0$
$g\left(-7\right) = 9$	$g\left(-5.5\right) = -3.75$	$g\left(0\right) = -12$	$g\left(1.5\right) = -3.75$	$g\left(3\right) = 9$
$g\left(x\right) > 0$	$g\left(x\right) < 0$	$g\left(x\right) < 0$	$g\left(x\right) < 0$	$g\left(x\right) > 0$

$-6 \qquad -5 \qquad 1 \qquad 2$

The solution set is $\left(-6, -5\right] \cup \left[1, 2\right)$.

117. $\left|x^2 - 4\right| < 5$

$\left|x^2 - 4\right| - 5 < 0$

Find the real zeros of the related equation.

$\left|x^2 - 4\right| - 5 = 0$

$\left|x^2 - 4\right| = 5$

$x^2 - 4 = 5$ or $x^2 - 4 = -5$

$x^2 = 9 \qquad x^2 = -1$

$x = \pm 3$

The boundary points are -3 and 3.

$f(-4)=7$	$f(0)=-1$	$f(4)=7$
$f(x)>0$	$f(x)<0$	$f(x)>0$

$-3 \qquad 3$

The solution set is $(-3, 3)$.

118. $|x^2+1|<17$

$|x^2+1|-17<0$

Find the real zeros of the related equation.

$$|x^2+1|-17=0$$
$$|x^2+1|=17$$
$$x^2+1=17 \quad \text{or} \quad x^2+1=-17$$
$$x^2=16 \qquad x^2=-18$$
$$x=\pm 4$$

The boundary points are -4 and 4.

$f(-5)=9$	$f(0)=-16$	$f(5)=9$
$f(x)>0$	$f(x)<0$	$f(x)>0$

$-4 \qquad 4$

The solution set is $(-4, 4)$.

119. $|x^2-18|>2$

$|x^2-18|-2>0$

Find the real zeros of the related equation.

$$|x^2-18|-2=0$$
$$|x^2-18|=2$$
$$x^2-18=2 \quad \text{or} \quad x^2-18=-2$$
$$x^2=20 \qquad x^2=16$$
$$x=\pm\sqrt{20} \qquad x=\pm 4$$
$$x=\pm 2\sqrt{5}$$

The boundary points are $-2\sqrt{5}$, -4, 4, and $2\sqrt{5}$.

$f(-5)=5$	$f(-4.1)=-0.81$	$f(0)=16$	$f(4.1)=-0.81$	$f(5)=5$
$f(x)>0$	$f(x)<0$	$f(x)>0$	$f(x)<0$	$f(x)>0$

$-2\sqrt{5} \qquad -4 \qquad 4 \qquad 2\sqrt{5}$

The solution set is $(-\infty, -2\sqrt{5})\cup(-4, 4)\cup(2\sqrt{5}, \infty)$.

120. $|x^2-6|>3$

$|x^2-6|-3>0$

Find the real zeros of the related equation.

$$|x^2-6|-3=0$$
$$|x^2-6|=3$$
$$x^2-6=3 \quad \text{or} \quad x^2-6=-3$$
$$x^2=9 \qquad\qquad x^2=3$$
$$x=\pm 3 \qquad\qquad x=\pm\sqrt{3}$$

The boundary points are -3, $-\sqrt{3}$, $\sqrt{3}$, and 3.

$f(-4)=7$	$f(-2)=-1$	$f(0)=3$	$f(2)=-1$	$f(4)=7$
$f(x)>0$	$f(x)<0$	$f(x)>0$	$f(x)<0$	$f(x)>0$

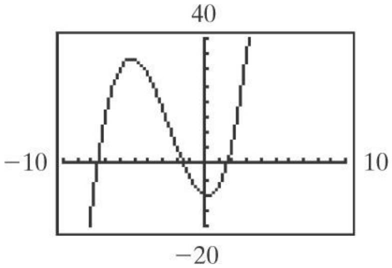
$-3 \qquad -\sqrt{3} \qquad \sqrt{3} \qquad 3$

The solution set is $(-\infty,-3)\cup(-\sqrt{3},\sqrt{3})\cup(3,\infty)$.

121. a. $\begin{bmatrix} 0.552x^3+4.13x^2 \\ -1.84x-3.5 \end{bmatrix}<6.7$

$\begin{bmatrix} 0.552x^3+4.13x^2 \\ -1.84x-10.2 \end{bmatrix}<0$

b.



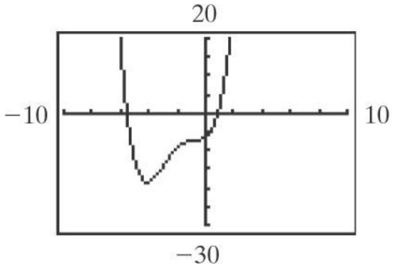
c. The real zeros are approximately -7.6, -1.5, and 1.6.

d. $(-\infty,-7.6)\cup(-1.5,1.6)$

122. a. $\begin{bmatrix} 0.24x^4+1.8x^3+3.3x^2 \\ +2.84x-1.8 \end{bmatrix}>4.5$

$\begin{bmatrix} 0.24x^4+1.8x^3+3.3x^2 \\ +2.84x-6.3 \end{bmatrix}>0$

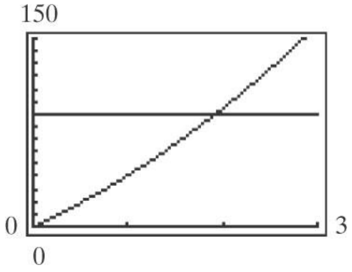
b.



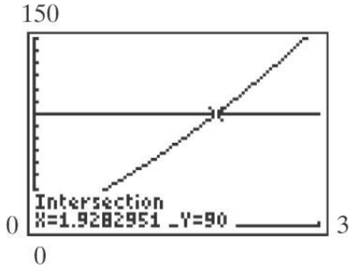
c. The real zeros are approximately -5.6, and 0.9.

d. $(-\infty,-5.6)\cup(0.9,\infty)$

123. a.

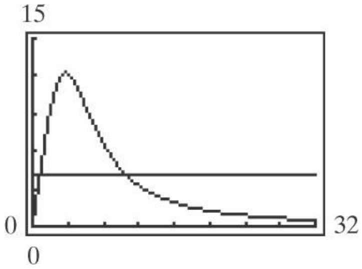


b.

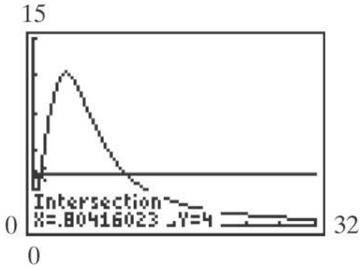


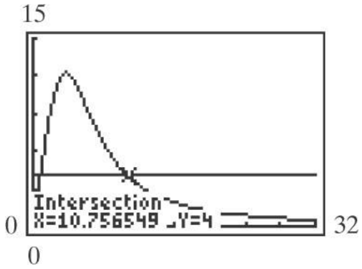
c. The radius should be no more than 1.9 in. to keep the amount of aluminum to at most 90 in.^{2}

124. a.



b.





c. It is safe to give a second dose 10.8 hr after the first dose.

Problem Recognition Exercises: Solving Equations and Inequalities

1.
$$-\frac{1}{2} \le -\frac{1}{4}x - 5 < 2$$
$$\frac{9}{2} \le -\frac{1}{4}x < 7$$
$$-18 \ge x > -28$$
$$-28 < x \le -18$$
$$\left(-28, -18\right]$$

2.
$$2x^2 - 6x = 5$$
$$2x^2 - 6x - 5 = 0$$
$$x = \frac{-\left(-6\right) \pm \sqrt{\left(-6\right)^2 - 4\left(2\right)\left(-5\right)}}{2\left(2\right)}$$
$$= \frac{6 \pm \sqrt{76}}{4}$$
$$= \frac{6 \pm 2\sqrt{19}}{4}$$
$$= \frac{3 \pm \sqrt{19}}{2}$$
$$\left\{\frac{3 \pm \sqrt{19}}{2}\right\}$$

3. $50x^3 - 25x^2 - 2x + 1 = 0$

$$25x^2(2x-1) - (2x-1) = 0$$

$$(2x-1)(25x^2-1) = 0$$

$$2x = 1 \quad \text{or} \quad 25x^2 = 1$$

$$x = \frac{1}{2} \qquad x^2 = \frac{1}{25}$$

$$x = \pm\frac{1}{5}$$

$$\left\{\pm\frac{1}{5}, \frac{1}{2}\right\}$$

4. $\dfrac{-5x(x-3)^2}{2+x} \le 0$

$$\frac{x(x-3)^2}{x+2} \ge 0$$

The expression is undefined for $x = -2$.
This is a boundary point.
Find the real zeros of the related equation.

$$\frac{x(x-3)^2}{x+2} = 0$$

$$x(x-3)^2 = 0$$

$$x = 0 \text{ or } x = 3$$

The boundary points are -2, 0, and 3.

Sign of $(x+2)$:	−	+	+	+
Sign of (x):	−	−	+	+
Sign of $(x-3)^2$:	+	+	+	+
Sign of $\dfrac{x(x-3)^2}{x+2}$:	+	−	+	+

$-2 \quad 0 \quad 3$

The solution set is $(-\infty, -2) \cup [0, \infty)$.

5. $\sqrt[4]{m+4} - 5 = -2$

$$\sqrt[4]{m+4} = 3$$

$$m + 4 = 3^4$$

$$m + 4 = 81$$

$$m = 77$$

$$\{77\}$$

6. $-5 < y \quad \text{and} \quad -3y + 4 \ge 7$

$$y > -5 \qquad -3y \ge 3$$

$$y \le -1$$

$$(-5, -1]$$

7. $|5t-4| + 2 = 7$

$$|5t-4| = 5$$

$$5t - 4 = 5 \quad \text{or} \quad 5t - 4 = -5$$

$$5t = 9 \qquad 5t = -1$$

$$t = \frac{9}{5} \qquad t = -\frac{1}{5}$$

$$\left\{-\frac{1}{5}, \frac{9}{5}\right\}$$

8. $3 - 4\{x - 5[x + 2(3-2x)]\}$
$= -2[4 - (x-1)]$

$$3 - 4\{x - 5[x + 6 - 4x]\} = -2[4 - x + 1]$$

$$3 - 4\{x - 5[-3x + 6]\} = -2[-x + 5]$$

$$3 - 4\{x + 15x - 30\} = 2x - 10$$

$$3 - 4\{16x - 30\} = 2x - 10$$

$$3 - 64x + 120 = 2x - 10$$

$$-66x = -133$$

$$x = \frac{133}{66}$$

$$\left\{\frac{133}{66}\right\}$$

9.
$$\begin{aligned} 10x\left(2x-14\right) &= -29x^2-100 \\ 20x^2-140x &= -29x^2-100 \\ 49x^2-140x+100 &= 0 \\ \left(7x-10\right)^2 &= 0 \\ 7x-10 &= 0 \\ x &= \frac{10}{7} \end{aligned}$$

$\left\{\frac{10}{7}\right\}$

10. $\dfrac{5}{y-4}=\dfrac{3y}{y+2}-\dfrac{2y^2-14y}{y^2-2y-8}$

$$\frac{5}{y-4}=\frac{3y}{y+2}-\frac{2y^2-14y}{\left(y-4\right)\left(y-2\right)}$$

$$\frac{5\left(y+2\right)}{\left(y-4\right)\left(y+2\right)} = \frac{3y\left(y-4\right)}{\left(y-4\right)\left(y+2\right)}-\frac{2y^2-14y}{\left(y-4\right)\left(y-2\right)}$$

$$\begin{aligned} 5y+10 &= 3y^2-12y-2y^2+14y \\ -y^2+3y+10 &= 0 \\ y^2-3y-10 &= 0 \\ \left(y-5\right)\left(y+2\right) &= 0 \\ y=5 &\text{ or } y=-2 \end{aligned}$$

$\left[5\right]$; The value -2 does not check.

11.
$$\begin{aligned} x\left(x-14\right) &\le -40 \\ x^2-14x+40 &\le 0 \end{aligned}$$

Find the real zeros of the related equation.

$$\begin{aligned} x^2-14x+40 &= 0 \\ \left(x-4\right)\left(x-10\right) &= 0 \\ x=4 &\text{ or } x=10 \end{aligned}$$

The boundary points are 4 and 10.

Sign of $\left(x-4\right)$:	$-$	$+$	$+$
Sign of $\left(x-10\right)$:	$-$	$-$	$+$
Sign of $\left(x-4\right)\left(x-10\right)$:	$+$	$-$	$+$

4 10

$\left[4, 10\right]$

12. $\dfrac{1}{x^2-14x+40}\le 0$

$$\frac{1}{\left(x-4\right)\left(x-10\right)}\le 0$$

The expression is undefined for $x=4$ and $x=10$. These are boundary points. The related equation has no real zeros.

Sign of $\left(x-4\right)$:	$-$	$+$	$+$
Sign of $\left(x-10\right)$:	$-$	$-$	$+$
Sign of $\dfrac{1}{\left(x-4\right)\left(x-10\right)}$:	$+$	$-$	$+$

4 10

$\left(4, 10\right)$

13. $\left|x-0.15\right|=\left|x+0.05\right|$

$$\begin{aligned} x-0.15=x+0.05 \quad &\text{or} \quad x-0.15=-\left(x+0.05\right) \\ -0.15=0.05 \quad & \quad x-0.15=-x-0.05 \\ & \quad 2x=0.1 \\ & \quad x=0.05 \end{aligned}$$

$\left\{0.05\right\}$

14. $\sqrt{t-1}-5\le 1$ and $t-1\ge 0$

$$\begin{aligned} \sqrt{t-1} &\le 6 \qquad t\ge 1 \\ t-1 &\le 36 \\ t &\le 37 \end{aligned}$$

$\left[1, 37\right]$

15. $n^{1/2}+7=10$

$$\begin{aligned} n^{1/2} &= 3 \\ n &= 9 \end{aligned}$$

$\left\{9\right\}$

16. $-4x(3-x)(x+2)^2(x-5)^3 \ge 0$

$x(x-3)(x+2)^2(x-5)^3 \ge 0$

Find the real zeros of the related equation.

$x(x-3)(x+2)^2(x-5)^3 = 0$

$x = 0$, $x = 3$, $x = -2$, or $x = 5$

The boundary points are -2, 0, 3, and 5.

Sign of $(x+2)^2$:	+	+	+	+	+
Sign of (x):	−	−	+	+	+
Sign of $(x-3)$:	−	−	−	+	+
Sign of $(x-5)^3$:	−	−	−	−	+
Sign of $x(x-3)(x+2)^2(x-5)^3$:	−	−	+	−	+

$-2 \quad 0 \quad 3 \quad 5$

$\{-2\} \cup [0, 3] \cup [5, \infty)$

17. $-2x - 5(x+3) = -4(x+2) - 3x$

$-2x - 5x - 15 = -4x - 8 - 3x$

$-7x - 15 = -7x - 8$

$-15 = 8$ No solution

$\{\ \}$

18. $\sqrt{7x+29} - 3 = x$

$\sqrt{7x+29} = x + 3$

$7x + 29 = x^2 + 6x + 9$

$-x^2 + x + 20 = 0$

$x^2 - x - 20 = 0$

$(x+4)(x-5) = 0$

$x = -4$ or $x = 5$

$\{5\}$; The value -4 does not check.

19. $(x^2-9)^2 - 5(x^2-9) - 14 = 0$

Let $u = (x^2 - 9)$.

$u^2 - 5u - 14 = 0$

$(u+2)(u-7) = 0$

$u = -2$ or $u = 7$

Back substitute (x^2-9) for u.

$x^2 - 9 = -2$ or $x^2 - 9 = 7$

$x^2 = 7 \qquad x^2 = 16$

$x = \pm\sqrt{7} \qquad x = \pm 4$

$\{\pm 4, \pm\sqrt{7}\}$

20. $2 + 7x^{-1} - 15x^{-2} = 0$

$15x^{-2} - 7x^{-1} - 2 = 0$

Let $u = x^{-1}$.

$15u^2 - 7u - 2 = 0$

$(5u+1)(3u-2) = 0$

$u = -\frac{1}{5}$ or $u = \frac{2}{3}$

Back substitute x^{-1} for u.

$x^{-1} = -\frac{1}{5}$ or $x^{-1} = \frac{2}{3}$

$x^{-1} = (-5)^{-1} \qquad x^{-1} = \left(\frac{3}{2}\right)^{-1}$

$x = -5 \qquad x = \frac{3}{2}$

$\left\{-5, \frac{3}{2}\right\}$

21. $|8x - 3| + 10 \le 7$

$|8x - 3| \le -3$

Since the absolute value of a number cannot be negative, this inequality has no solution.

$\{\ \}$

22. $2(x-1)^{3/4}=16$

$$(x-1)^{3/4}=8$$
$$x-1=8^{4/3}$$
$$x-1=16$$
$$x=17$$

$\{17\}$

23. $x^3-3x^2<6x-8$

$$x^3-3x^2-6x+8<0$$

Find the real zeros of the related equation.

$$x^3-3x^2-6x+8=0$$

$$\frac{\text{Factors of }-8}{\text{Factors of }1}=\frac{\pm1,\pm2,\pm4,\pm8}{\pm1}$$
$$=\pm1,\pm2,\pm4,\pm8$$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & -6 & 8 \\ & & 1 & -2 & -8 \\ \hline & 1 & -2 & -8 & \boxed{0} \end{array}$$

$$(x-1)(x^2-2x-8)=0$$
$$(x-1)(x+2)(x-4)=0$$

$x=1$, $x=-2$, or $x=4$

The boundary points are -2, 1, and 4.

Sign of $(x+2)$:	−	+	+	+
Sign of $(x-1)$:	−	−	+	+
Sign of $(x-4)$:	−	−	−	+
Sign of $(x-1)(x+2)(x-4)$:	−	+	−	+

-2 1 4

$(-\infty,-2)\cup(1,4)$

24. $\dfrac{3-x}{x+5}\ge1$

$$\frac{3-x}{x+5}-1\ge0$$
$$\frac{3-x}{x+5}-1\cdot\frac{x+5}{x+5}\ge0$$
$$\frac{-2x-2}{x+5}\ge0$$
$$\frac{x+1}{x+5}\le0$$

The expression is undefined for $x=-5$. This is a boundary point.

Find the real zeros of the related equation.

$$\frac{x+1}{x+5}=0$$
$$x+1=0$$
$$x=-1$$

The boundary points are -5 and -1.

Sign of $(x+5)$:	−	+	+
Sign of $(x+1)$:	−	−	+
Sign of $\dfrac{(x+1)}{(x+5)}$:	+	−	+

-5 -1

The solution set is $(-5,-1]$.

25. $15-3(x-1)=-2x-(x-18)$

$$15-3x+3=-2x-x+18$$
$$-3x+18=-3x+18$$
$$0=0$$

$(-\infty,\infty)$

26. $25x^2+70x>-49$

$$25x^2+70x+49>0$$
$$(5x+7)^2>0$$

The expression $(5x+7)^2>0$ for all real numbers except where $(5x+7)^2=0$. Therefore, the solution set is all real numbers except $-\frac{7}{5}$.

$\left(-\infty, -\frac{7}{5}\right) \cup \left(-\frac{7}{5}, \infty\right)$

27. $2 < |3 - x| - 9$

$11 < |3 - x|$

$|3 - x| > 11$

$3 - x < -11$ or $3 - x > 11$

$-x < -14$ $\quad$ $-x > 8$

$x > 14$ $\quad$ $x < -8$

$(-\infty, -8) \cup (14, \infty)$

28. $-4(x - 3) < 8$ or $-7 > x - 3$

$-4x + 12 < 8$ $\quad$ $-x > 4$

$-4x < -4$ $\quad$ $x < -4$

$x > 1$

$(-\infty, -4) \cup (1, \infty)$

29. $\frac{1}{3}x + \frac{2}{5} > \frac{5}{6}x - 1$

$10x + 12 > 25x - 30$

$-15x > -42$

$x < \frac{42}{15}$

$x < \frac{14}{5}$

$\left(-\infty, \frac{14}{5}\right)$

30. $2|2x + 1| - 2 \le 8$

$2|2x + 1| \le 10$

$|2x + 1| \le 5$

$-5 \le 2x + 1 \le 5$

$-6 \le 2x \le 4$

$-3 \le x \le 2$

$[-3, 2]$

Section 2.7 Variation

1. directly

2. inversely

3. constant; variation

4. jointly

5. a. $y = 2x$

$y = 2(1) = 2$

$y = 2(2) = 4$

$y = 2(3) = 6$

$y = 2(4) = 8$

$y = 2(5) = 10$

b. *y* is also doubled.

c. *y* is also tripled.

d. increases

e. decreases

6. a. $y = \frac{24}{x}$

$y = \frac{24}{(1)} = 24$

$y = \frac{24}{(2)} = 12$

$y = \frac{24}{(3)} = 8$

$y = \frac{24}{(4)} = 6$

$y = \frac{24}{(6)} = 4$

b. *y* is one-half its original value.

c. *y* is one-third its original value.

d. decreases

e. increases

7. inversely

8. directly

9. jointly

10. directly

11. $C = kr$

12. $I = kA$

13. $\overline{C} = \frac{k}{n}$

14. $t = \frac{k}{r}$

15. $V = khr^2$

16. $V = klw$

17. $E = \frac{ks}{\sqrt{n}}$

18. $n = \frac{k\sigma^2}{E^2}$

19. $c = \frac{kmn}{t^3}$

20. $d = \frac{kuv}{\sqrt[3]{T}}$

21.
$$\begin{aligned} y &= kx \\ 20 &= k(8) \\ \frac{20}{8} &= k \\ k &= \frac{5}{2} \end{aligned}$$

22.
$$\begin{aligned} y &= kx \\ 42 &= k(10) \\ \frac{42}{10} &= k \\ k &= \frac{21}{5} \end{aligned}$$

23.
$$\begin{aligned} p &= \frac{k}{q} \\ 54 &= \frac{k}{18} \\ k &= 972 \end{aligned}$$

24.
$$\begin{aligned} T &= \frac{k}{x} \\ 200 &= \frac{k}{50} \\ k &= 10{,}000 \end{aligned}$$

25.
$$\begin{aligned} y &= kwv \\ 40 &= k(40)(0.2) \\ 40 &= k(8) \\ k &= 5 \end{aligned}$$

26.
$$\begin{aligned} N &= ktp \\ 15 &= k(2)(2.5) \\ 15 &= k(5) \\ k &= 3 \end{aligned}$$

27. a.
$$\begin{aligned} y &= kx \\ 4 &= k(10) \\ \frac{4}{10} &= k \\ k &= \frac{2}{5} \\ y &= \frac{2}{5}x = \frac{2}{5}(5) = 2 \end{aligned}$$

b.
$$\begin{aligned} y &= \frac{k}{x} \\ 4 &= \frac{k}{10} \\ k &= 40 \\ y &= \frac{40}{x} = \frac{40}{5} = 8 \end{aligned}$$

28. a.
$$\begin{aligned} y &= kx \\ 24 &= k\left(\frac{1}{2}\right) \\ k &= 48 \\ y &= 48x = 48(3) = 144 \end{aligned}$$

b. $$\begin{aligned} y &= \frac{k}{x} \\ 24 &= \frac{k}{\frac{1}{2}} \\ k &= 12 \\ y &= \frac{12}{x} = \frac{12}{3} = 4 \end{aligned}$$

29. Let A represent the amount of the medicine. Let w represent the weight of the child.

$$\begin{aligned} A &= kw \\ 180 &= k(40) \\ \frac{180}{40} &= k \\ k &= 4.5 \\ A &= 4.5w \end{aligned}$$

a. $A = 4.5w = 4.5(50) = 225$ mg

b. $A = 4.5w = 4.5(60) = 270$ mg

c. $A = 4.5w = 4.5(70) = 315$ mg

d. $$\begin{aligned} A &= 4.5w \\ 135 &= 4.5w \\ \frac{135}{4.5} &= w \\ w &= 30 \text{ lb} \end{aligned}$$

30. Let S represent the number of people served. Let w represent the weight of the ham.

$$\begin{aligned} S &= kw \\ 20 &= k(8) \\ \frac{20}{8} &= k \\ k &= 2.5 \\ S &= 2.5w \end{aligned}$$

a. $S = 2.5w = 2.5(10) = 25$ people

b. $S = 2.5w = 2.5(15) = 37$ people (rounded down to have more food per person)

c. $S = 2.5w = 2.5(18) = 45$ people

d. $$\begin{aligned} S &= 2.5w \\ 30 &= 2.5w \\ \frac{30}{2.5} &= w \\ w &= 12 \text{ lb} \end{aligned}$$

31. Let C represent the cost per mile to rent a car. Let m represent the number of miles driven.

$$\begin{aligned} C &= \frac{k}{m} \\ 0.8 &= \frac{k}{100} \\ k &= 80 \\ C &= \frac{80}{m} \end{aligned}$$

a. $C = \frac{80}{m} = \frac{80}{200} = \$0.40$ per mile

b. $C = \frac{80}{m} = \frac{80}{300} \approx \$0.27$ per mile

c. $C = \frac{80}{m} = \frac{80}{400} = \$0.20$ per mile

d. $$\begin{aligned} C &= \frac{80}{m} \\ 0.16 &= \frac{80}{m} \\ 0.16m &= 80 \\ m &= \frac{80}{0.16} = 500 \text{ mi} \end{aligned}$$

32. Let C represent the number of books sold per month. Let p represent price of a book.

$$C = \frac{k}{p}$$
$$1500 = \frac{k}{8}$$
$$k = 12{,}000$$
$$C = \frac{12{,}000}{p}$$

a. $C = \frac{12{,}000}{p} = \frac{12{,}000}{12} = 1000 \text{ books}$

b. $C = \frac{12{,}000}{p} = \frac{12{,}000}{15} = 800 \text{ books}$

c. $C = \frac{12{,}000}{p} = \frac{12{,}000}{6} = 2000 \text{ books}$

d.
$$C = \frac{12{,}000}{p}$$
$$1200 = \frac{12{,}000}{p}$$
$$1200p = 12{,}000$$
$$p = \frac{12{,}000}{1200} = \$10$$

33. Let d represent the distance a bicycle travels in 1 min. Let r represent the rpm of the wheels.

$$d = kr$$
$$440 = k(60)$$
$$\frac{440}{60} = k$$

$$k = \frac{22}{3}$$
$$d = \frac{22}{3}r = \frac{22}{3}(87) = 638 \text{ ft}$$

34. Let p represent the amount of pollution entering the atmosphere. Let n represent the number of people.

$$p = kn$$
$$71{,}000 = k(100{,}000)$$
$$\frac{71{,}000}{100{,}000} = k$$
$$k = 0.71$$
$$d = 0.71n$$
$$= 0.71(750{,}000)$$
$$= 532{,}500 \text{ tons}$$

35. a. Let d represent the stopping distance of the car. Let s represent the speed of the car.

$$d = ks^2$$
$$170 = k(50)^2$$
$$\frac{170}{2500} = k$$
$$k = 0.068$$
$$d = 0.068s^2 = 0.068(70)^2$$
$$= 333.2 \text{ ft}$$

b.
$$d = 0.068s^2$$
$$244.8 = 0.068s^2$$
$$\frac{244.8}{0.068} = s^2$$
$$3600 = s^2$$
$$s = 60 \text{ mph}$$

36. a. Let A represent the area of a picture projected on a wall. Let d represent the distance from the projector to the wall.

$$\begin{aligned} A &= kd^2 \\ 36 &= k\left(15\right)^2 \\ \frac{36}{225} &= k \\ k &= 0.16 \\ A &= 0.16d^2 = 0.16\left(25\right)^2 \\ &= 100 \text{ ft}^2 \end{aligned}$$

b. $$\begin{aligned} A &= 0.16d^2 \\ 144 &= 0.16d^2 \\ \frac{144}{0.16} &= d^2 \\ 900 &= d^2 \\ d &= 30 \text{ ft} \end{aligned}$$

37. Let t represent the number of days to complete the job. Let n represent the number of people working on the job.

$$\begin{aligned} t &= \frac{k}{n} \\ 12 &= \frac{k}{8} \\ k &= 96 \\ t &= \frac{96}{n} \end{aligned}$$

a. $t = \frac{96}{n} = \frac{96}{15} = 6.4$ days

b. $$\begin{aligned} t &= \frac{96}{n} \\ 8 &= \frac{96}{n} \\ 8n &= 96 \\ n &= 12 \text{ people} \end{aligned}$$

38. Let y represent the yield of the bond. Let p represent the price of the bond.

$$\begin{aligned} y &= \frac{k}{p} \\ 0.05 &= \frac{k}{120} \\ k &= 6 \\ y &= \frac{6}{p} \end{aligned}$$

a. $y = \frac{6}{p} = \frac{6}{100} = 0.06 = 6\%$

b. $$\begin{aligned} y &= \frac{6}{p} \\ 0.075 &= \frac{6}{p} \\ 0.075p &= 6 \\ p &= \$80 \end{aligned}$$

39. Let I represent the current. Let V represent the voltage. Let R represent the resistance.

$$\begin{aligned} I &= \frac{kV}{R} \\ 9 &= \frac{k\left(90\right)}{10} \\ 90 &= 90k \\ k &= 1 \\ I &= \frac{V}{R} = \frac{160}{5} = 32 \text{ A} \end{aligned}$$

40. Let R represent the resistance of the wire. Let ℓ represent the length of the wire. Let d represent the diameter of the wire.

$$\begin{aligned}R &= \frac{k\ell}{d^2}\\ 0.0125 &= \frac{k(50)}{(0.2)^2}\\ 0.0005 &= 50k\\ k &= 0.00001\\ R &= \frac{0.00001\ell}{d^2}\\ &= \frac{0.00001(40)}{(0.1)^2}\\ &= 0.04\ \Omega\end{aligned}$$

41. Let *I* represent the amount of interest owed. Let *P* represent the amount of the principal borrowed. Let *t* represent the amount of time (in years) that the money is borrowed.

$$\begin{aligned}I &= kPt\\ 480 &= k(4000)(2)\\ 480 &= 8000k\\ \frac{480}{8000} &= k\\ k &= 0.06\\ I &= 0.06(6000)(4) = \$1440\end{aligned}$$

42. Let *I* represent the amount of interest earned. Let *P* represent the amount of the principal invested. Let *t* represent the amount of time (in years) that the money is invested.

$$\begin{aligned}I &= kPt\\ 750 &= k(5000)(6)\\ 750 &= 30{,}000k\\ \frac{750}{30{,}000} &= k\\ k &= 0.025\\ I &= 0.025(8000)(4) = \$800\end{aligned}$$

43. Let *B* represent the BMI of an individual. Let *w* represent the weight of the individual. Let *h* represent the weight of the individual.

$$\begin{aligned}B &= \frac{kw}{h^2}\\ 21.52 &= \frac{k(150)}{(70)^2}\\ 105{,}448 &= 150k\\ \frac{105{,}448}{150} &= k\\ k &= 703\\ B &= \frac{703(180)}{(68)^2} \approx 27.37\end{aligned}$$

44. Let *S* represent the strength of the beam. Let *w* represent the width of the beam. Let *t* represent the thickness of the beam. Let ℓ represent the length of the beam.

$$\begin{aligned}S &= \frac{kwt^2}{\ell}\\ 417 &= \frac{k(6)(2)^2}{48}\\ 20{,}016 &= 24k\\ \frac{20{,}016}{24} &= k\\ k &= 834\\ S &= \frac{834(12)(4)^2}{72} = 2224\text{ lb}\end{aligned}$$

45. Let s represent the speed of the canoe. Let ℓ represent the length of the canoe.

$$\begin{aligned} s &= k\sqrt{\ell} \\ 6.2 &= k\sqrt{16} \\ \frac{6.2}{\sqrt{16}} &= k \\ \frac{6.2}{4} &= k \\ k &= 1.55 \\ s &= 1.55\sqrt{\ell} = 1.55\sqrt{25} \\ &= 7.75 \text{ mph} \end{aligned}$$

46. Let P represent the period of the pendulum. Let ℓ represent the length of the pendulum.

$$\begin{aligned} P &= k\sqrt{\ell} \\ 1.8 &= k\sqrt{0.81} \\ \frac{1.8}{\sqrt{0.81}} &= k \\ \frac{1.8}{0.9} &= k \\ k &= 2 \\ P &= 2\sqrt{\ell} = 2\sqrt{1} = 2 \text{ sec} \end{aligned}$$

47. Let C represent the cost to carpet the room. Let ℓ represent the length of the room. Let w represent the width of the room.

$$\begin{aligned} C &= k\ell w \\ 3870 &= k(10)(15) \\ \frac{3870}{150} &= k \\ k &= 25.8 \\ C &= k\ell w \\ &= 25.8(18)(24) \\ &= \$11{,}145.60 \end{aligned}$$

48. Let C represent the cost to tile the kitchen. Let ℓ represent the length of the kitchen. Let w represent the width of the kitchen.

$$\begin{aligned} C &= k\ell w \\ 1104 &= k(10)(12) \\ \frac{1104}{120} &= k \\ k &= 9.2 \\ C &= k\ell w = 9.2(20)(14) = \$2576 \end{aligned}$$

49. The data appears to show direct variation. Use the first row to find the constant of variation.

$$\begin{aligned} y &= kx \\ 16 &= k(5) \\ \frac{16}{5} &= k \\ k &= 3.2 \\ y &= 3.2x \end{aligned}$$

Check the remaining values in the equation. They check.

50. The data appears to show direct variation. Use the first row to find the constant of variation.

$$\begin{aligned} y &= kx \\ 24 &= k(5) \\ \frac{24}{5} &= k \\ k &= 4.8 \\ y &= 4.8x \end{aligned}$$

Check the remaining values in the equation. They check.

51. The data appears to show indirect variation. Use the first row to find the constant of variation.

$$\begin{aligned} y &= \frac{k}{x} \\ 6 &= \frac{k}{2} \end{aligned}$$

$$k = 12$$

$$y = \frac{12}{x}$$

Check the remaining values in the equation. They check.

52. The data appears to show indirect variation. Use the first row to find the constant of variation.

$$y = \frac{k}{x}$$

$$2 = \frac{k}{4}$$

$$k = 8$$

$$y = \frac{8}{x}$$

Check the remaining values in the equation. They check.

53. formulas a and c

54. formulas b and c

55. The variable *P* varies directly as the square of *v* and inversely as *t*.

56. The variable *E* varies directly as the square of *c* and inversely as the square root of *b*.

57. a. Let *S* represent the surface area of the sphere. Let *r* represent the radius of the sphere.

$$S = kr^2$$

$$400\pi = k\left(10\right)^2$$

$$\frac{400\pi}{100} = k$$

$$k = 4\pi$$

$$S = 4\pi r^2 = 4\pi\left(20\right)^2$$

$$= 1600\pi \text{ m}^2$$

b. The surface area is 4 times as great. Doubling the radius results in $\left(2\right)^2$ times the surface area of the sphere.

c. The intensity at 20 m should be $\frac{1}{4}$ the intensity at 10 m. This is because the energy from the light is distributed across an area 4 times as great.

d. Let *I* represent the intensity of light from a source. Let *d* represent the distance of the light from the source.

$$I = \frac{k}{d^2}$$

$$200 = \frac{k}{\left(10\right)^2}$$

$$k = 20{,}000$$

$$I = \frac{20{,}000}{d^2} = \frac{20{,}000}{\left(20\right)^2} = 50 \text{ lux}$$

58. $T^2 = kd^3$

$$\left(365\right)^2 = k\left(9.3 \times 10^7\right)^3$$

$$k = \frac{\left(365\right)^2}{\left(9.3 \times 10^7\right)^3} \approx 1.66 \times 10^{-19}$$

$$T^2 = 1.66 \times 10^{-19} d^3$$

a. $T^2 = 1.66 \times 10^{-19} d^3$

$$T = \sqrt{1.66 \times 10^{-19}\left(1.5 \times 9.3 \times 10^7\right)^3}$$

$$T \approx 671 \text{ Earth days}$$

b. $T^2 = 1.66 \times 10^{-19} d^3$

$$d^3 = \frac{T^2}{1.66 \times 10^{-19}}$$

$$d = \sqrt[3]{\frac{T^2}{1.66 \times 10^{-19}}} = \sqrt[3]{\frac{223^2}{1.66 \times 10^{-19}}}$$

$$d \approx 67 \text{ million miles}$$

59. $I = \frac{k}{d^2}$

$$I = \frac{k}{(10d)^2} = \frac{k}{100d^2}$$
$$= \left(\frac{1}{100}\right)\left(\frac{k}{d^2}\right)$$

The intensity is $\frac{1}{100}$ as great.

60. $y = \frac{k}{x^3}$

$$y = \frac{k}{\left(\frac{x}{4}\right)^3} = \frac{k}{\frac{x^3}{4^3}} = 64\left(\frac{k}{x^3}\right)$$

y will be 64 times as great.

61. $y = \frac{kx^2}{w^4}$

$$y = \frac{k(2x)^2}{(2w)^4} = \frac{k(4x^2)}{(16w^4)}$$
$$= \frac{1}{4}\left(\frac{kx^2}{w^4}\right)$$

y will be $\frac{1}{4}$ its original value.

62. $y = \frac{kx^5}{w^2}$

$$y = \frac{k(2x)^5}{(2w)^2} = \frac{k(32x^5)}{(4w^2)}$$
$$= 8\left(\frac{kx^5}{w^2}\right)$$

y will be 8 times its original value.

63. $y = kxw^3$

$$y = k\left(\frac{x}{3}\right)(3w)^3$$
$$= k\left(\frac{x}{3}\right)(27w^3)$$
$$= 9(kxw^3)$$

y will be 9 times its original value.

64. $y = kx^4w$

$$y = k\left(\frac{x}{4}\right)^4(4w)$$
$$= k\left(\frac{x^4}{256}\right)(4w)$$
$$= \frac{1}{64}(kx^4w)$$

y will be $\frac{1}{64}$ times its original value.

Chapter 2 Review Exercises

1. $f(x) = -(x+5)^2 + 2 = -[x-(-5)]^2 + 2$

Vertex: $(h, k) = (-5, 2)$

2. a. $f(x) = x^2 - 8x + 15$

$$f(x) = (x^2 - 8x \quad) + 15$$
$$\boxed{\left[\frac{1}{2}(-8)\right]^2 = 16}$$
$$f(x) = (x^2 - 8x + 16) - 16 + 15$$
$$f(x) = (x-4)^2 - 1$$

b. Since $a > 0$, the parabola opens upward.

c. The vertex is $(h, k) = (4, -1)$.

d. $f(x) = x^2 - 8x + 15$

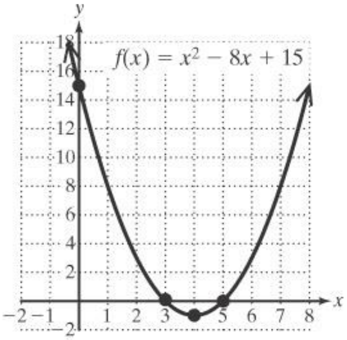
$$0 = x^2 - 8x + 15$$
$$0 = (x-3)(x-5)$$
$$x = 3 \quad \text{or} \quad x = 5$$

The x-intercepts are $(3, 0)$ and $(5, 0)$.

e. $f(0)=(0)^2-8(0)+15=15$

The y-intercept is $(0,15)$.

f.



g. The axis of symmetry is the vertical line through the vertex: $x=4$.

h. The minimum value is -1.

i. The domain is $(-\infty,\infty)$.

The range is $[-1,\infty)$.

3. a. $f(x)=-2x^2+4x+6$

$f(x)=-2\left(x^2-2x\quad\right)+6$

$$\boxed{\left[\frac{1}{2}(-2)\right]^2=1}$$

$f(x)=-2\left(x^2-2x+1-1\right)+6$

$f(x)=-2\left(x^2-2x+1\right)-2(-1)+6$

$f(x)=-2(x-1)^2+8$

b. Since $a<0$, the parabola opens downward.

c. The vertex is $(h,k)=(1,8)$.

d. $f(x)=-2x^2+4x+6$

$0=-2x^2+4x+6$

$0=x^2-2x-3$

$0=(x+1)(x-3)$

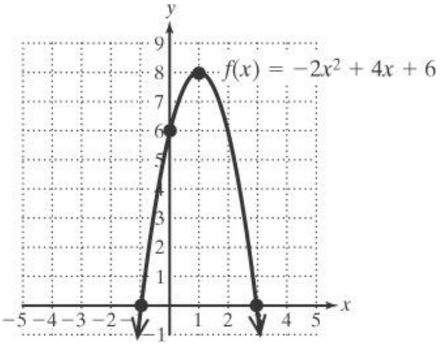
$x=-1$ or $x=3$

The x-intercepts are $(-1,0)$ and $(3,0)$.

e. $f(0)=-2(0)^2+4(0)+6=6$

The y-intercept is $(0,6)$.

f.



g. The axis of symmetry is the vertical line through the vertex: $x=1$.

h. The minimum value is 8.

i. The domain is $(-\infty,\infty)$.

The range is $[-\infty,8)$.

4. a. $f(x)=2x^2+12x+19$

$f(x)=2\left(x^2+6x\quad\right)+19$

$$\boxed{\left[\frac{1}{2}(6)\right]^2=9}$$

$f(x)=2\left(x^2+6x+9-9\right)+19$

$f(x)=2\left(x^2+6x+9\right)+2(-9)+19$

$f(x)=2(x+3)^2+1$

Vertex: $(h,k)=(-3,1)$

b. Since the vertex of the parabola is above the x-axis and the parabola opens upward, the parabola cannot cross or touch the x-axis. Therefore, there are no x-intercepts.

5. We know that

$$2x+y=180$$
$$y=180-2x$$

Let *A* represent the area.

$$A=xy$$
$$A(x)=x(180-2x)$$
$$=180x-2x^2$$
$$=-x^2+180x$$

Function *A* is a quadratic function with a negative leading coefficient. The graph of the parabola opens downward, so the vertex is the maximum point on the function. The *x*-coordinate of the vertex is the value of *x* that will maximize the area.

$$x=\frac{-b}{2a}=\frac{-(180)}{2(-2)}=\frac{-180}{-4}=45$$

$$y=180-2x$$
$$=180-2(45)$$
$$=180-90$$
$$=90$$

The dimensions are 45 yd by 90 yd.

6. a. $V(p)=100p(1-p)=100p-100p^2$

$$=-100p^2+100p$$

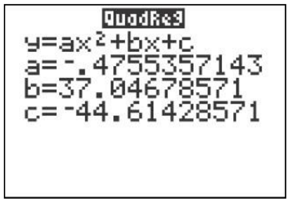
The value of *p* at which the variance will be at a maximum is the *p*-coordinate of the vertex.

$$p=\frac{-b}{2a}=\frac{-(100)}{2(-100)}=\frac{-100}{-200}=\frac{1}{2}$$

b. The maximum variance is the value of $V(p)$ at the vertex.

$$V\left(\frac{1}{2}\right)=-100\left(\frac{1}{2}\right)^2+100\left(\frac{1}{2}\right)$$
$$=-\frac{100}{4}+\frac{100}{2}$$
$$=-25+50$$
$$=25$$

7. a.



```
QuadReg
y=ax²+bx+c
a=-.4755357143
b=37.04678571
c=-44.61428571
```

$$E(a)=-0.476a^2+37.0a-44.6\backslash$$

b. The age when the yearly expenditure is the greatest is the *a*-coordinate of the vertex.

$$a=\frac{-b}{2a}=\frac{-(37.0)}{2(-0.476)}$$
$$=\frac{-37.0}{-0.952}\approx 39\text{ yr}$$

c. The maximum yearly expenditure is the $E(a)$ value at the vertex.

$$E(39)=\begin{bmatrix}-0.476(39)^2\\ \quad+37.0(39)-44.6\end{bmatrix}$$
$$\approx \$674$$

8. a. $f(x)=-4x^3+16x^2+25x-100$

The leading term is $-4x^3$. The end behavior is up to the left and down to the right.

b.
$$0=-4x^3+16x^2+25x-100$$
$$0=4x^3-16x^2-25x+100$$
$$0=4x^2(x-4)-25(x-4)$$
$$0=(x-4)(4x^2-25)$$
$$0=(x-4)(2x-5)(2x+5)$$
$$x=4,\ x=\frac{5}{2},\ x=-\frac{5}{2}$$

The zeros of the function are 4, $\frac{5}{2}$, and $-\frac{5}{2}$ (each with multiplicity 1).

c. The x-intercepts are $(4, 0)$, $\left(\frac{5}{2}, 0\right)$,

and $\left(-\frac{5}{2}, 0\right)$.

d. $f(0) = -4(0)^3 + 16(0)^2 + 25(0) - 100$

$= -100$

The y-intercept is $(0, -100)$.

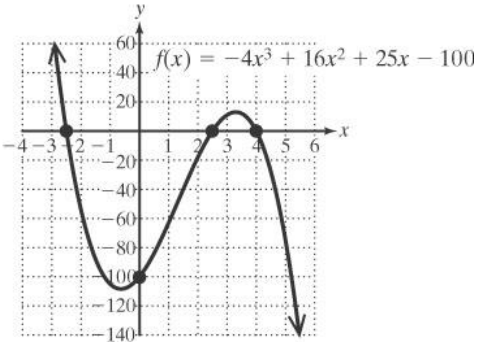
e. $f(-x)$

$$= \begin{bmatrix} -4(-x)^3 + 16(-x)^2 \\ + 25(-x) - 100 \end{bmatrix}$$

$= 4x^3 + 16x^2 - 25x - 100$

$f(x)$ is neither even nor odd.

f.



9. a. $f(x) = x^4 - 10x^2 + 9$

The leading term is x^4. The end behavior is up to the left and up to the right.

b. $0 = x^4 - 10x^2 + 9$

$0 = (x^2 - 9)(x^2 - 1)$

$0 = (x-3)(x+3)(x-1)(x+1)$

$x = 3, x = -3, x = 1, x = -1$

The zeros of the function are 3, -3, 1, and -1 (each with multiplicity 1).

c. The x-intercepts are $(3, 0)$, $(-3, 0)$, $(1, 0)$, and $(-1, 0)$.

d. $f(0) = (0)^4 - 10(0)^2 + 9 = 9$

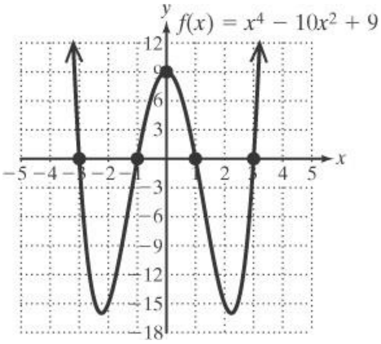
The y-intercept is $(0, 9)$.

e. $f(-x) = (-x)^4 - 10(-x)^2 + 9$

$= x^4 - 10x^2 + 9$

$f(x)$ is even.

f.



10. a. $f(x) = x^4 + 3x^3 - 3x^2 - 11x - 6$

The leading term is x^4. The end behavior is up to the left and up to the right.

b. Factor the polynomial to find the zeros.

Possible rational zeros:

$$\frac{\text{Factors of } -6}{\text{Factors of } 1} = \frac{\pm1, \pm2, \pm3}{\pm1} = \pm1, \pm2, \pm3$$

$$\begin{array}{r|rrrrr} -1 & 1 & 3 & -3 & -11 & -6 \\ & & -1 & -2 & 5 & 6 \\ \hline & 1 & 2 & -5 & -6 & \boxed{0} \end{array}$$

$f(x) = (x+1)(x^3 + 2x^2 - 5x - 6)$

Factor the quotient. Possible rational zeros:

$$\frac{\text{Factors of } -6}{\text{Factors of } 1} = \pm1, \pm2, \pm3$$

$$\begin{array}{r|rrrr} \underline{-1} & 1 & 2 & -5 & -6 \\ & & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & \underline{|0} \end{array}$$

$$f(x) = (x+1)^2(x^2+x-6)$$
$$f(x) = (x+1)^2(x-2)(x+3)$$
$$0 = (x+1)^2(x-2)(x+3)$$
$$x = -1, x = 2, x = -3$$

The zeros of the function are 2, –3 (each with multiplicity 1), and –1 (with multiplicity 2).

c. The x-intercepts are $(2, 0)$, $(-3, 0)$, and $(-1, 0)$.

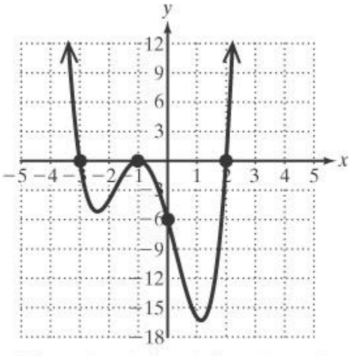
d. $f(0) = \begin{bmatrix} (0)^4 + 3(0)^3 \\ -3(0)^2 - 11(0) - 6 \end{bmatrix}$
$= -6$

The y-intercept is $(0, -6)$.

e. $f(-x)$
$= \begin{bmatrix} (-x)^4 + 3(-x)^3 - 3(-x)^2 \\ -11(-x) - 6 \end{bmatrix}$
$= x^4 - 3x^3 - 3x^2 + 11x - 6$

$f(x)$ is neither even nor odd.

f.



$f(x) = x^4 + 3x^3 - 3x^2 + 11x - 6$

11. a. $f(x) = x^5 - 8x^4 + 13x^3$

The leading term is x^5. The end behavior is down to the left and up to the right.

b. $0 = x^5 - 8x^4 + 13x^3$
$0 = x^3(x^2 - 8x + 13)$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(13)}}{2(1)}$$
$$= \frac{8 \pm \sqrt{12}}{2}$$
$$= \frac{8 \pm 2\sqrt{3}}{2}$$
$$= 4 \pm \sqrt{3}$$
$$x = 0, x = 4 + \sqrt{3}, x = 4 - \sqrt{3}$$

The zeros of the function are 0 (with multiplicity 3), $4 + \sqrt{3}$ and $4 - \sqrt{3}$ (each with multiplicity 1).

c. The x-intercepts are $(0, 0)$, $(4 + \sqrt{3}, 0)$, and $(4 - \sqrt{3}, 0)$.

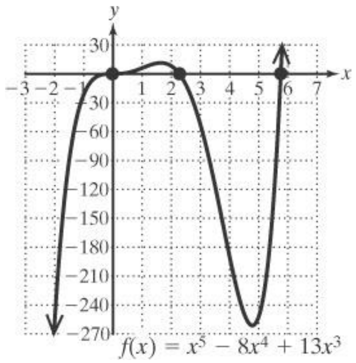
d. $f(0) = (0)^5 - 8(0)^4 + 13(0)^3 = 0$

The y-intercept is $(0, 0)$.

e. $f(-x) = (-x)^5 - 8(-x)^4 + 13(-x)^3$
$= -x^5 - 8x^4 - 13x^3$

$f(x)$ is neither even nor odd.

f.



$f(x) = x^5 - 8x^4 + 13x^3$

12. $f(x) = 2x^3 - 5x^2 - 6x + 2$

$$\begin{aligned} f(-2) &= \begin{bmatrix} 2(-2)^3 - 5(-2)^2 \\ -6(-2) + 2 \end{bmatrix} \\ &= -16 - 20 + 12 + 2 \\ &= -22 \end{aligned}$$

$$\begin{aligned} f(-1) &= \begin{bmatrix} 2(-1)^3 - 5(-1)^2 \\ -6(-1) + 2 \end{bmatrix} \\ &= -2 - 5 + 6 + 2 \\ &= 1 \end{aligned}$$

$$\begin{aligned} f(0) &= 2(0)^3 - 5(0)^2 - 6(0) + 2 \\ &= 2 \end{aligned}$$

$$\begin{aligned} f(1) &= 2(1)^3 - 5(1)^2 - 6(1) + 2 \\ &= 2 - 5 - 6 + 2 \\ &= -7 \end{aligned}$$

$$\begin{aligned} f(2) &= 2(2)^3 - 5(2)^2 - 6(2) + 2 \\ &= 16 - 20 - 12 + 2 \\ &= -14 \end{aligned}$$

a. Since $f(-2)$ and $f(-1)$ have opposite signs, the intermediate value theorem guarantees that the function has at least one zero on the interval $[-2, -1]$.

b. Since $f(-1)$ and $f(0)$ have the same sign, the intermediate value theorem does not guarantee that the function has at least one zero on the interval $[-1, 0]$.

c. Since $f(0)$ and $f(1)$ have opposite signs, the intermediate value theorem guarantees that the function has at least one zero on the interval $[0, 1]$.

d. Since $f(1)$ and $f(2)$ have the same sign, the intermediate value theorem does not guarantee that the function has at least one zero on the interval $[1, 2]$.

13. False. It may have three or fewer turning points.

14. True

15. False. There are infinitely many such polynomials. For example, any polynomial of the form $f(x) = a(x-2)(x-3)(x-4)$ has the required zeros.

16. True

17. a.

$$\begin{array}{r}
-2x^2 + 3x - 9 \\
x^2 + x - 3\overline{\smash{\big)}-2x^4 + x^3 + 0x^2 + 4x - 1} \\
-\left(-2x^4 - 2x^3 + 6x^2\right) \qquad\qquad \\
\hline
3x^3 - 6x^2 + 4x \qquad \\
-\left(3x^3 + 3x^2 - 9x\right) \qquad \\
\hline
-9x^2 + 13x - 1 \\
-\left(-9x^2 - 9x + 27\right) \\
\hline
22x - 28
\end{array}$$

$$-2x^2 + 3x - 9 + \frac{22x - 28}{x^2 + x - 3}$$

b. Dividend: $-2x^4 + x^3 + 4x - 1$;

Divisor: $x^2 + x - 3$;

Quotient: $-2x^2 + 3x - 9$;

Remainder: $22x - 28$

18. a.

$$\begin{array}{r} x^3-5x+4 \\ 3x-2\overline{)3x^4-2x^3-15x^2+22x-8} \\ -\left(3x^4-2x^3\right) \\ \hline 0\ -15x^2+22x \\ -\left(15x^2+10x\right) \\ \hline 12x-8 \\ -\left(12x-8\right) \\ \hline 0 \end{array}$$

x^3-5x+4

b. Dividend: $3x^4-2x^3-15x^2+22x-8$;

Divisor: $3x-2$;

Quotient: x^3-5x+4;

Remainder: 0

19.

$$\begin{array}{r|rrrrrr} -2 & 2 & 0 & 0 & 1 & -5 & 1 \\ & & -4 & 8 & -16 & 30 & -50 \\ \hline & 2 & -4 & 8 & -15 & 25 & \boxed{-49} \end{array}$$

$$2x^4-4x^3+8x^2-15x+25+\frac{-49}{x+2}$$

20.

$$\begin{array}{r|rrrrr} 3 & 1 & 3 & -1 & 7 & 2 \\ & & 3 & 18 & 51 & 174 \\ \hline & 1 & 6 & 17 & 58 & \boxed{176} \end{array}$$

$$x^3+6x^2+17x+58+\frac{176}{x-3}$$

21.

$$\begin{array}{r|rrrrr} -2 & 3 & 0 & 2 & -4 & 1 \\ & & -6 & 12 & -28 & 64 \\ \hline & 3 & -6 & 14 & -32 & \boxed{65} \end{array}$$

By the remainder theorem, $f\left(-2\right)=65$.

22.

$$\begin{array}{r|rrrrr} \sqrt{5} & 1 & 2 & -4 & -10 & -5 \\ & & \sqrt{5} & 5+2\sqrt{5} & 10+\sqrt{5} & 5 \\ \hline & 1 & 2+\sqrt{5} & 1+2\sqrt{5} & \sqrt{5} & \boxed{0} \end{array}$$

By the remainder theorem, $f\left(\sqrt{5}\right)=0$.

23. a.

$$\begin{array}{r|rrrrr} 2 & 3 & 13 & 2 & 52 & -40 \\ & & 6 & 38 & 80 & 264 \\ \hline & 3 & 19 & 40 & 132 & \boxed{224} \end{array}$$

By the factor theorem, since $f\left(2\right)\neq 0$, $x-2$ is not a factor of $f\left(x\right)$.

b.

$$\begin{array}{r|rrrrr} \frac{2}{3} & 3 & 13 & 2 & 52 & -40 \\ & & 2 & 10 & 8 & 40 \\ \hline & 3 & 15 & 12 & 60 & \boxed{0} \end{array}$$

By the remainder theorem, $f\left(\frac{2}{3}\right)=0$. Since $f\left(\frac{2}{3}\right)=0$, $\frac{2}{3}$ is a zero of $f\left(x\right)$.

24. a.

$$\begin{array}{r|rrrrr} -5 & 1 & 6 & 9 & 24 & 20 \\ & & -5 & -5 & -20 & -20 \\ \hline & 1 & 1 & 4 & 4 & \boxed{0} \end{array}$$

By the remainder theorem, $f\left(-5\right)=0$. Since $f\left(-5\right)=0$, -5 is a zero of $f\left(x\right)$.

b.

$$\begin{array}{r|rrrrr} 2i & 1 & 6 & 9 & 24 & 20 \\ & & 2i & -4+12i & -24+10i & -20 \\ \hline & 1 & 6+2i & 5+12i & 10i & \boxed{0} \end{array}$$

By the remainder theorem, $f\left(2i\right)=0$. Since $f\left(2i\right)=0$, $2i$ is a zero of $f\left(x\right)$.

25. a.
$$\begin{array}{r|rrrr} -4 & 1 & 4 & 9 & 36 \\ & & -4 & 0 & -36 \\ \hline & 1 & 0 & 9 & 0 \end{array}$$

By the factor theorem, since $f(-4)=0$, $x+4$ is a factor of $f(x)$.

b.
$$\begin{array}{r|rrrr} 3i & 1 & 4 & 9 & 36 \\ & & 3i & -9+12i & -36 \\ \hline & 1 & 4+3i & 12i & 0 \end{array}$$

By the factor theorem, since $f(3i)=0$, $x-3i$ is a factor of $f(x)$.

26. a.
$$\begin{array}{r|rrr} -2 & 1 & -4 & -46 \\ & & -2 & 12 \\ \hline & 1 & -6 & -34 \end{array}$$

By the factor theorem, since $f(-2)\neq 0$, $x+2$ is not a factor of $f(x)$.

b.
$$\begin{array}{r|rrr} 2-5\sqrt{2} & 1 & -4 & -46 \\ & & 2-5\sqrt{2} & 46 \\ \hline & 1 & -2-5\sqrt{2} & 0 \end{array}$$

By the factor theorem, since $f\left(2-5\sqrt{2}\right)=0$, $x-\left(2-5\sqrt{2}\right)$ is a factor of $f(x)$.

27.
$$\begin{array}{r|rrrr} \frac{2}{3} & 15 & -67 & 26 & 8 \\ & & 10 & -38 & -8 \\ \hline & 15 & -57 & -12 & 0 \end{array}$$

$$\begin{aligned} f(x) &= a\left(x-\frac{2}{3}\right)\left(15x^2-57x-12\right) \\ &= 3\left(x-\frac{2}{3}\right)\left(5x^2-19x-4\right) \\ &= (3x-2)(5x+1)(x-4) \end{aligned}$$

28.
$$\begin{aligned} f(x) &= (x+1)\left(x-3\sqrt{2}\right)\left(x+3\sqrt{2}\right) \\ &= (x+1)\left(x^2-18\right) \\ &= x^3+x^2-18x-18 \end{aligned}$$

29.
$$\begin{aligned} f(x) &= a\left(x-\frac{1}{4}\right)\left(x+\frac{1}{2}\right)(x-3) \\ &= 8\left(x-\frac{1}{4}\right)\left(x+\frac{1}{2}\right)(x-3) \\ &= 4\left(x-\frac{1}{4}\right)\cdot 2\left(x+\frac{1}{2}\right)(x-3) \\ &= (4x-1)(2x+1)(x-3) \\ &= (4x-1)\left(2x^2-5x-3\right) \\ &= 8x^3-20x^2-12x-2x^2+5x+3 \\ &= 8x^3-22x^2-7x+3 \end{aligned}$$

30. a. $f(x)$ is a fifth-degree polynomial. It has 5 zeros.

b.
$$\begin{aligned} &\frac{\text{Factors of } 40}{\text{Factors of } 2} \\ &= \frac{\pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40}{\pm 1, \pm 2} \\ &= \pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \\ &\quad \pm 20, \pm 40, \pm\frac{1}{2}. \pm\frac{5}{2} \end{aligned}$$

c.
$$\begin{array}{r|rrrrrr} -1 & 2 & -7 & 9 & -18 & 4 & 40 \\ & & -2 & 9 & -18 & 36 & -40 \\ \hline & 2 & -9 & 18 & -36 & 40 & 0 \end{array}$$

Factor the quotient.

$$\begin{array}{r|rrrrr} 2 & 2 & -9 & 18 & -36 & 40 \\ & & 4 & -10 & 16 & -40 \\ \hline & 2 & -5 & 8 & -20 & 0 \end{array}$$

Factor the quotient.

$$\begin{array}{r|rrrr} \frac{5}{2} & 2 & -5 & 8 & -20 \\ & & 5 & 0 & 20 \\ \hline & 2 & 0 & 8 & 0 \end{array}$$

The rational zeros are $\frac{5}{2}$, 2, -1.

d. Find the remaining two zeros.

$$\begin{aligned} 2x^2+8&=0 \\ 2x^2&=-8 \\ x^2&=-4 \\ x&=\pm 2i \end{aligned}$$

The zeros are $\frac{5}{2}$, 2, -1, $2i$, $-2i$.

31. a. $f(x)$ is a fourth-degree polynomial. It has 4 zeros.

b. $$\frac{\text{Factors of } -8}{\text{Factors of } 1}=\frac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1}$$
$$=\pm 1, \pm 2, \pm 4, \pm 8$$

c. $$\begin{array}{r|rrrrr} -2 & 1 & 4 & 2 & -8 & -8 \\ & & -2 & -4 & 4 & 8 \\ \hline & 1 & 2 & -2 & -4 & \boxed{0} \end{array}$$

Factor the quotient.

33. a. $$\begin{array}{r|rrrrr} 11-i & 1 & -22 & 119 & 66 & -366 \\ & & 11-i & -122 & -33+i & 366 \\ \hline & 1 & -11-i & -3 & 33+3i & \boxed{0} \end{array}$$

Since $3-i$ is a zero, $3+i$ is also a zero.

$$\begin{array}{r|rrrr} 11+i & 1 & -11-i & -3 & 33+3i \\ & & 11+i & 0 & -33-3i \\ \hline & 1 & 0 & -3 & \boxed{0} \end{array}$$

$$f(x)=\left[x-(11-i)\right]\left[x-(11+i)\right]\left(x^2-3\right)$$

Find the remaining two zeros.

$$\begin{aligned} x^2-3&=0 \\ x^2&=3 \\ x&=\pm\sqrt{3} \end{aligned}$$

The zeros are $11 \pm i$, $\pm\sqrt{3}$.

b. $f(x)=\left[x-(11-i)\right]\left[x-(11+i)\right]\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)$

c. The solution set is $\left\{11 \pm i, \pm\sqrt{3}\right\}$.

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -2 & -4 \\ & & -2 & 0 & 4 \\ \hline & 1 & 0 & -2 & \boxed{0} \end{array}$$

The rational zero is -2 (with multiplicity 2).

d. Find the remaining two zeros.

$$\begin{aligned} x^2-2&=0 \\ x^2&=2 \\ x&=\pm\sqrt{2} \end{aligned}$$

The zeros are -2 (with multiplicity 2), $\pm\sqrt{2}$.

32. Since $2+7i$ is a zero, then $2-7i$ is also a zero. The total number of zeros is 6, so the minimum degree of $f(x)$ is 6.

34. $f(x) = x^2\left[x-(2-3i)\right]\left[x-(2+3i)\right]$

$$\begin{aligned} f(x) &= x^2\left[x-(2-3i)\right]\left[x-(2+3i)\right] \\ &= x^2(x-2)^2-(3i)^2 \\ &= x^2\left(x^2-4x+4+9\right) \\ &= x^2\left(x^2-4x+13\right) \\ &= x^4-4x^3+13x^2 \end{aligned}$$

35. $f(x) = a(x-2i)(x+2i)\left(x-\frac{5}{3}\right)$

$$\begin{aligned} &= a\left(x^2+4\right)\left(x-\frac{5}{3}\right) \\ &= a\left(x^3-\frac{5}{3}x^2+4x-\frac{20}{3}\right) \quad \text{Let } a = 3. \\ &= 3\left(x^3-\frac{5}{3}x^2+4x-\frac{20}{3}\right) \\ &= 3x^3-5x^2+12x-20 \end{aligned}$$

36. $g(x) = -3x^7+4x^6-2x^2+5x-4$

4 sign changes in $g(x)$. The number of possible positive real zeros is either 4, 2, or 0.

$$g(-x) = \begin{bmatrix} -3(-x)^7+4(-x)^6 \\ -2(-x)^2+5(-x)-4 \end{bmatrix}$$

$$g(-x) = 3x^7+4x^6-2x^2-5x-4$$

1 sign change in $g(-x)$. The number of possible negative real zeros is 1.

37. $n(x) = x^6+\frac{1}{3}x^4+\frac{2}{7}x^3+4x^2+3$

0 sign changes in $n(x)$. The number of possible positive real zeros is 0.

$n(-x)$

$$\begin{aligned} &= \begin{bmatrix} (-x)^6+\frac{1}{3}(-x)^4+\frac{2}{7}(-x)^3 \\ +4(-x)^2+3 \end{bmatrix} \\ &= x^6+\frac{1}{3}x^4-\frac{2}{7}x^3+4x^2+3 \end{aligned}$$

2 sign changes in $n(-x)$. The number of possible negative real zeros is either 2 or 0.

38. a.

$$\begin{array}{r|rrrrr} 2 & 1 & -3 & 0 & 2 & -3 \\ & & 2 & -2 & -4 & -4 \\ \hline & 1 & -1 & -2 & -2 & \boxed{-7} \end{array}$$

The remainder and 3 of the coefficients of the quotient are negative. Therefore, 2 is not an upper bound for the real zeros of $f(x)$.

b.

$$\begin{array}{r|rrrrr} -2 & 1 & -3 & 0 & 2 & -3 \\ & & -2 & 10 & -20 & 36 \\ \hline & 1 & -5 & 10 & -18 & \boxed{33} \end{array}$$

The signs of the quotient alternate. Therefore, -2 is a lower bound for the real zeros of $f(x)$.

39. a.

$$\begin{array}{r|rrrr} 5 & 1 & -4 & 2 & 1 \\ & & 5 & 5 & 35 \\ \hline & 1 & 1 & 7 & \boxed{36} \end{array}$$

The remainder and 3 of the coefficients of the quotient are nonnegative. Therefore, 5 is an upper bound for the real zeros of $f(x)$.

b.

$$\begin{array}{r|rrrr} -2 & 1 & -4 & 2 & 1 \\ & & -2 & 12 & -28 \\ \hline & 1 & -6 & 14 & \boxed{-27} \end{array}$$

The signs of the quotient alternate.

Therefore, -2 is a lower bound for the real zeros of $f\left(x\right)$.

40. a. –3

b. ∞

c. –∞

d. –3

e. $\left(-\infty,-2\right)\cup\left(-2,\infty\right)$

f. Never decreasing

g. $\left(-\infty,-2\right)\cup\left(-2,\infty\right)$

h. $\left(-\infty,-3\right)\cup\left(-3,\infty\right)$

i. $x=-2$

j. $y=-3$

41.
$$2x^2+x-15=0$$
$$\left(2x-5\right)\left(x+3\right)=0$$
$$x=\frac{5}{2},\ x=-3$$

42. x^2+3 is never zero. There are no vertical asymptotes.

43. a. The degree of the numerator is 0. The degree of the denominator is 2. Since $n<m$, the line $y=0$ is a horizontal asymptote of *r*.

b.
$$\frac{3}{x^2+2x+1}=0$$
$$3=0 \quad \text{No solution}$$
The graph does not cross $y=0$.

44. a. The degree of the numerator is 2. The degree of the denominator is 2.

Since $n=m$, the line $y=\frac{-2}{1}$, or equivalently $y=-2$, is a horizontal asymptote of *q*.

b.
$$\frac{-2x^2-3x+4}{x^2+1}=-2$$
$$-2x^2-3x+4=-2x^2-2$$
$$-3x+4=-2$$
$$-3x=-6$$
$$x=2$$
The graph crosses $y=-2$ at $\left(2,-2\right)$.

45. a. The degree of the numerator is 3. The degree of the denominator is 1. Since $n>m$, *k* has no horizontal asymptotes.

b. Not applicable

46. The expression $\frac{2x^3-x^2-6x+7}{x^2-3}$ is in lowest terms.
$$x^2-3=0$$
$$x^2=3$$
$$x=\pm\sqrt{3}$$
The denominator is 0 at $x=\pm\sqrt{3}$. *m* has vertical asymptotes at $x=\sqrt{3}$ and $x=-\sqrt{3}$.

The degree of the numerator is exactly one greater than the degree of the denominator. Therefore, *m* has no horizontal asymptote, but does have a slant asymptote.

$$\begin{array}{r}
2x-1 \\
x^2+0x-3\overline{\big)\,2x^3-x^2-6x+7} \\
-\left(2x^3+0x^2-6x\right) \\
\hline
-x^2+0x+7 \\
-\left(-x^2+0x+3\right) \\
\hline
4
\end{array}$$

The quotient is $2x-1$.

The slant asymptote is $y=2x-1$.

47. The expression $\frac{-4x^2+5}{3x^2-14x-5}$ is in lowest terms.

$$3x^2-14x-5=0$$
$$(3x+1)(x-5)=0$$
$$x=-\frac{1}{3},\ x=5$$

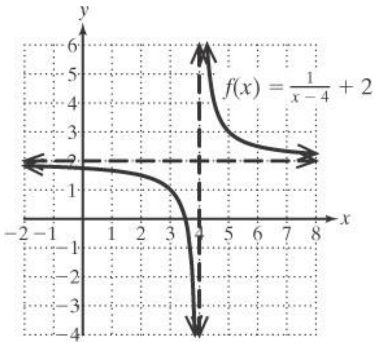
n has vertical asymptotes at $x=-\frac{1}{3}$ and $x=5$.

The degree of the numerator is 2.

The degree of the denominator is 2.

Since $n=m$, the line $y=\frac{-4}{3}$ or equivalently $y=-\frac{4}{3}$ is a horizontal asymptote of *n*.

48. The graph of *f* is the graph of $y=\frac{1}{x}$ with a shift to the right 4 units and a shift upward 2 units.



49. $k(0)=\frac{(0)^2}{(0)^2-(0)-12}=0$

The *y*-intercept is $(0,0)$.

The *x*-intercept is $(0,0)$.

k is in lowest terms, and $x^2-x-12=(x+3)(x-4)$ is 0 for $x=-3$ and $x=4$, which are the vertical asymptotes.

The degree of the numerator is 2.

The degree of the denominator is 2.

Since $n=m$, the line $y=\frac{1}{1}$, or equivalently $y=1$, is a horizontal asymptote of *k*.

$$1=\frac{x^2}{x^2-x-12}$$
$$x^2-x-12=x^2$$
$$x=-12$$

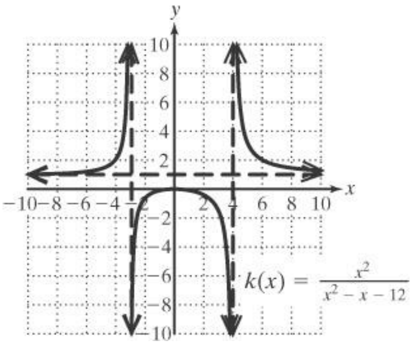
k crosses its horizontal asymptote at $(-12,1)$.

$$k(-x)=\frac{(-x)^2}{(-x)^2-(-x)-12}=\frac{x^2}{x^2+x-12}$$

$k(-x) \neq k(x)$, $k(-x) \neq -k(x)$

k is neither even nor odd.

Interval	**Test Point**	**Comments**
$(-\infty, -12)$	$\left(-14, \frac{98}{99}\right)$	$k(x)$ is less than 1. $k(x)$ must approach the horizontal asymptote $y=1$ from below as $x \to -\infty$.
$(-12, -3)$	$(-4, 2)$	$k(x)$ is positive. As x approaches the vertical asymptote $x = -3$ from the left, $k(x) \to \infty$.
$(-3, 0)$	$\left(-2, -\frac{2}{3}\right)$	$k(x)$ is negative. As x approaches the vertical asymptote $x = -3$ from the right, $k(x) \to -\infty$.
$(0, 4)$	$\left(3, -\frac{3}{2}\right)$	$k(x)$ is negative. As x approaches the vertical asymptote $x = 4$ from the left, $k(x) \to -\infty$.
$(4, \infty)$	$(6, 2)$	$k(x)$ is positive. As x approaches the vertical asymptote $x = 4$ from the right, $k(x) \to \infty$. $k(x)$ is greater than 1. $k(x)$ must approach the horizontal asymptote $y = 1$ from above as $x \to \infty$.



50. $m(0)=\dfrac{(0)^2+6(0)+9}{(0)}$ undefined

There is no *y*-intercept.

$$x^2+6x+9=0$$
$$(x+3)(x+3)=0$$
$$x=-3$$

The *x*-intercept is $(-3, 0)$.

m is in lowest terms, and the denominator is 0 when $x=0$, which is the vertical asymptote.

The degree of the numerator is exactly one greater than the degree of the denominator. Therefore, *m* has no horizontal asymptote, but does have a slant asymptote.

$$m(x)=\frac{x^2+6x+9}{x}$$
$$=\frac{x^2}{x}+\frac{6x}{x}+\frac{9}{x}$$
$$=x+6+\frac{9}{x}$$

The quotient is $x+6$.

The slant asymptote is $y=x+6$.

$$x+6=\frac{x^2+6x+9}{x}$$
$$x^2+6x=x^2+6x+9$$
$$0=9 \text{ No solution}$$

m does not cross its slant asymptote.

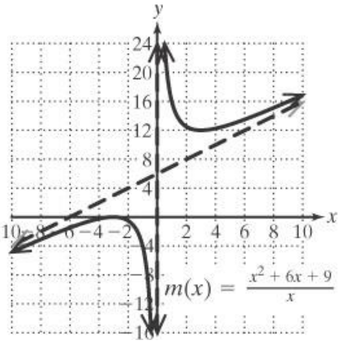
$$m(-x)=\frac{(-x)^2+6(-x)+9}{(-x)}=\frac{x^2-6x+9}{-x}$$

$m(-x)\neq m(x)$, $m(-x)\neq -m(x)$

m is neither even nor odd.

Select test points from each interval.

Interval	**Test Point**	**Test Point**
$(-\infty, -3)$	$\left(-6, -\frac{3}{2}\right)$	$\left(-4, -\frac{1}{4}\right)$
$(-3, 0)$	$\left(-2, -\frac{1}{2}\right)$	$(-1, -4)$
$(0, \infty)$	$(1, 16)$	$(3, 12)$



51. $q(0)=\dfrac{12}{(0)^2+6}=\dfrac{12}{6}=2$

The *y*-intercept is $(0, 2)$.

q is never 0. It has no *x*-intercepts nor vertical asymptotes.

The degree of the numerator is 0.

The degree of the denominator is 2.

Since $n<m$, the line $y=0$ is a horizontal asymptote of *q*.

$$0 = \frac{12}{x^2+6}$$

$0 = 12$ No solution

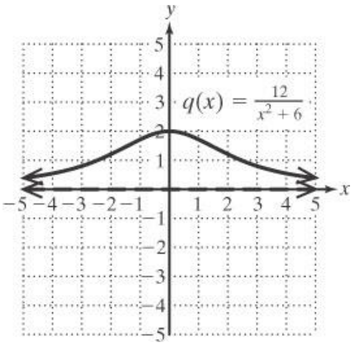
q does not cross its horizontal asymptote.

$$q(-x) = \frac{12}{(-x)^2+6} = \frac{12}{x^2+6}$$

$q(-x) = q(x)$

q is even.

Interval	**Test Point**	**Comments**
$(-\infty, 0)$	$\left(-3, \frac{4}{5}\right)$	$q(x)$ is positive. $q(x)$ must approach the horizontal asymptote $y = 0$ from above as $x \to -\infty$.
$(0, \infty)$	$\left(2, \frac{6}{5}\right)$	$q(x)$ is positive. $q(x)$ must approach the horizontal asymptote $y = 0$ from above as $x \to \infty$.



52. a. $P(1) = \frac{1+90}{0.16(1)+1} \approx 78\%$

$P(4) = \frac{4+90}{0.16(4)+1} \approx 57\%$

$P(6) = \frac{6+90}{0.16(6)+1} \approx 49\%$

b. Horizontal asymptote: $\frac{1}{0.16} = 6.25$

$P(t)$ will approach 6.25%.

53. a. $(-\infty, -4)$

b. $(-\infty, -4] \cup \{1\}$

c. $(-4, 1) \cup (1, \infty)$

d. $[-4, \infty)$

54. a. $(1, 2)$

b. $[1, 2)$

c. $(-\infty, 1) \cup (2, \infty)$

d. $(-\infty, 1] \cup (2, \infty)$

55. $x^2 + 7x + 10 = 0$

$(x+5)(x+2) = 0$

$x = -5$ or $x = -2$

The boundary points are -5 and -2.

Sign of $(x+5)$:	$-$	$+$	$+$
Sign of $(x+2)$:	$-$	$-$	$+$
Sign of $(x+5)(x+2)$:	$+$	$-$	$+$

$-5 \quad -2$

a. $x^2 + 7x + 10 = 0$

$\{-5, -2\}$

b. $x^2 + 7x + 10 < 0$

$(-5, -2)$

c. $x^2 + 7x + 10 \le 0$

$[-5, -2]$

d. $x^2 + 7x + 10 > 0$

$(-\infty, -5) \cup (-2, \infty)$

e. $x^2 + 7x + 10 \ge 0$

$(-\infty, -5] \cup [-2, \infty)$

56. $\dfrac{x+1}{x-5} = 0$

$x + 1 = 0$

$x = -1$

-1 is a boundary point.

The expression is undefined for $x = 5$.

This is also a boundary point.

Sign of $(x+1)$:	$-$	$+$	$+$
Sign of $(x-5)$:	$-$	$-$	$+$
Sign of $\dfrac{x+1}{x-5}$:	$+$	$-$	$+$

$-1 \quad 5$

a. $\dfrac{x+1}{x-5} \le 0$

$[-1, 5)$

b. $\dfrac{x+1}{x-5} < 0$

$(-1, 5)$

c. $\dfrac{x+1}{x-5} \ge 0$

$(-\infty, -1] \cup (5, \infty)$

d. $\dfrac{x+1}{x-5} > 0$

$(-\infty, -1) \cup (5, \infty)$

57. $t(t-3) \ge 18$

$t^2 - 3t - 18 \ge 0$

Find the real zeros of the related equation.

$t^2 - 3t - 18 = 0$

$(t+3)(t-6) = 0$

$t = -3$ or $t = 6$

The boundary points are -3 and 6.

Sign of $(t+3)$:	$-$	$+$	$+$
Sign of $(t-6)$:	$-$	$-$	$+$
Sign of $(t+3)(t-6)$:	$+$	$-$	$+$

$-3 \quad 6$

The solution set is $(-\infty, -3] \cup [6, \infty)$

58. $w^3 + w^2 - 9w - 9 > 0$

$w^3 + w^2 - 9w - 9 > 0$

$w^2(w+1) - 9(w+1) > 0$

$(w^2 - 9)(w+1) > 0$

$(w+3)(w-3)(w+1) > 0$

Find the real zeros of the related equation.

$w^3 + w^2 - 9w - 9 = 0$

$w^2(w+1) - 9(w+1) = 0$

$(w^2 - 9)(w+1) = 0$

$(w+3)(w-3)(w+1) = 0$

$w = -3$, $w = 3$, or $w = -1$

The boundary points are -3, -1, and 3.

Sign of $(w+3)$:	$-$	$+$	$+$	$+$
Sign of $(w+1)$:	$-$	$-$	$+$	$+$
Sign of $(w-3)$:	$-$	$-$	$-$	$+$
Sign of $(w+3)(w-3)(w+1)$:	$-$	$+$	$-$	$+$

$-3 \quad -1 \quad 3$

The solution set is $(-3,-1)\cup(3,\infty)$.

59. $x^2-2x+4\le 3$

$x^2-2x+1\le 0$

$(x-1)^2\le 0$

The square of any real number is nonnegative. Therefore, $(x-1)^2$ is nonnegative, and

always greater than 0. However, the equality is included.

The solution set is $\{1\}$.

60. $-6x^4(3x-4)^2(x+2)^3\le 0$

$6x^4(3x-4)^2(x+2)^3\ge 0$

Find the real zeros of the related equation.

$6x^4(3x-4)^2(x+2)^3=0$

$x^4(3x-4)^2(x+2)^3=0$

$x=0$, $x=\frac{4}{3}$, or $x=-2$

The boundary points are -2, 0, and $\frac{4}{3}$.

Sign of $(x+2)^3$:	$-$	$+$	$+$	$+$
Sign of $(6x^4)$:	$+$	$+$	$+$	$+$
Sign of $(3x-4)^2$:	$+$	$+$	$+$	$+$
Sign of $6x^4(3x-4)^2(x+2)^3$:	$-$	$+$	$+$	$+$

$-2 \quad 0 \quad \frac{4}{3}$

The solution set is $[-2,\infty)$.

61. $z^3-3z^2>10z-24$

$z^3-3z^2-10z+24>0$

Find the real zeros of the related equation.

$z^3-3z^2-10z+24=0$

$$\frac{\text{Factors of } 24}{\text{Factors of } 1}=\frac{\pm1,\pm2,\pm3,\pm4,\pm6,\pm8,\pm12,\pm24}{\pm1}$$

$=\pm1,\pm2,\pm3,\pm4,\pm6,\pm8,\pm12,\pm24$

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -10 & 24 \\ & & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & \boxed{0} \end{array}$$

$(x-2)(x^2-x-12)=0$

$(x-2)(x+3)(x-4)=0$

$x=2$, $x=-3$, or $x=4$

The boundary points are -3, 2, and 4.

Sign of $(x+3)$:	$-$	$+$	$+$	$+$
Sign of $(x-2)$:	$-$	$-$	$+$	$+$
Sign of $(x-4)$:	$-$	$-$	$-$	$+$
Sign of $(x-2)(x+3)(x-4)$:	$-$	$+$	$-$	$+$

$-3 \quad 2 \quad 4$

The solution set is $(-3,2)\cup(4,\infty)$.

62. $(4x-5)^4>0$

The expression $(4x-5)^4>0$ for all real numbers except where $(4x-5)^4=0$. Therefore, the solution set is all real numbers except $\frac{5}{4}$.

$\left(-\infty,\frac{5}{4}\right)\cup\left(\frac{5}{4},\infty\right)$

63. $$\frac{6-2x}{x^2}\ge 0$$
$$\frac{2x-6}{x^2}\le 0$$
$$\frac{x-3}{x^2}\le 0$$

The expression is undefined for $x=0$.

This is a boundary point.

Find the real zeros of the related equation.

$$\frac{x-3}{x^2}=0$$
$$x-3=0$$
$$x=3$$

The boundary points are 0 and 3.

Sign of x^2:	+	+	+
Sign of $x-3$:	−	−	+
Sign of $\frac{x-3}{x^2}$:	−	−	+

0 3

The solution set is $(-\infty,0)\cup(0,3]$.

64. $$\frac{8}{3x-4}\le 1$$
$$\frac{8}{3x-4}-1\le 0$$
$$\frac{8}{3x-4}-\frac{3x-4}{3x-4}\le 0$$
$$\frac{12-3x}{3x-4}\le 0$$
$$\frac{-3(x-4)}{3x-4}\le 0$$
$$\frac{x-4}{3x-4}\ge 0$$

The expression is undefined for $x=\frac{4}{3}$.

This is a boundary point.

Find the real zeros of the related equation.

$$\frac{x-4}{3x-4}=0$$
$$x-4=0$$
$$x=4$$

The boundary points are $\frac{4}{3}$ and 4.

Sign of $(3x-4)$:	−	+	+
Sign of $(x-4)$:	−	−	+
Sign of $\frac{(x-4)}{(3x-4)}$:	+	−	+

$\frac{4}{3}$ 4

The solution set is $\left(-\infty,\frac{4}{3}\right)\cup[4,\infty)$.

65. $$\frac{3}{x-2}<-\frac{2}{x}$$
$$\frac{3}{x-2}+\frac{2}{x}<0$$
$$\frac{3x}{x(x-2)}+\frac{2(x-2)}{x(x-2)}<0$$
$$\frac{5x-4}{x(x-2)}<0$$

The expression is undefined for $x=0$ and $x=2$. These are boundary points.

Find the real zeros of the related equation.

$$\frac{5x-4}{x(x-2)}=0$$
$$5x-4=0$$
$$x=\frac{4}{5}$$

The boundary points are 0, $\frac{4}{5}$, and 2.

Sign of (x):	$-$	$+$	$+$	$+$
Sign of $6(5x-4)$:	$-$	$-$	$+$	$+$
Sign of $(x-2)$:	$-$	$-$	$-$	$+$
Sign of $\frac{6(x+1)}{x(x+3)}$:	$-$	$+$	$-$	$+$

$0 \quad \frac{4}{5} \quad 2$

The solution set is $(-\infty, 0) \cup \left(\frac{4}{5}, 2\right)$.

66. $\frac{(1-x)(3x+5)^2}{(x-3)^4} < 0$

$\frac{(x-1)(3x+5)^2}{(x-3)^4} > 0$

The expression is undefined for $x = 3$. This is a boundary point.

Find the real zeros of the related equation.

$\frac{(x-1)(3x+5)^2}{(x-3)^4} = 0$

$(x-1)(3x+5)^2 = 0$

$x = 1$ or $x = -\frac{5}{3}$

The boundary points are $-\frac{5}{3}$, 1, and 3.

Sign of $(3x+5)^2$:	$+$	$+$	$+$	$+$
Sign of $(x-1)$:	$-$	$-$	$+$	$+$
Sign of $(x-3)^4$:	$+$	$+$	$+$	$+$
Sign of $\frac{(x-1)(3x+5)^2}{(x-3)^4}$:	$-$	$-$	$+$	$+$

$-\frac{5}{3} \quad 1 \quad 3$

The solution set is $(1, 3) \cup (3, \infty)$.

67. a. $\overline{C}(x) = \frac{80+40+15x}{x}$

$\overline{C}(x) = \frac{120+15x}{x}$

b. $\frac{120+15x}{x} < 16$

$\frac{120+15x}{x} - 16 < 0$

$\frac{120+15x-16x}{x} < 0$

$\frac{120-x}{x} < 0$

$\frac{x-120}{x} > 0$

The expression is undefined for $x = 0$. This is a boundary point.

Find the real zeros of the related equation

$\frac{x-120}{x} = 0$

$x - 120 = 0$

$x = 120$

The boundary points are 0 and 120.

Sign of $(x-120)$:	$-$	$+$	$+$
Sign of (x):	$-$	$-$	$+$
Sign of $\frac{x-120}{x}$:	$+$	$-$	$+$

$0 \quad 120$

The solution set is $(-\infty, 0) \cup (120, \infty)$.

The trainer must have more than 120 sessions with his clients for his average cost to drop below \$16 per session.

68. a. $s(t) = -\frac{1}{2}(32)t^2 + (40)t + (2)$

$s(t) = -16t^2 + 40t + 2$

b. $-16t^2+40t+2>18$

$$-16t^2+40t-16>0$$

$$2t^2-5t+2<0$$

Find the real zeros of the related equation.

$$2t^2-5t+2=0$$

$$(2t-1)(t-2)=0$$

$$t=\frac{1}{2} \text{ or } t=2$$

The boundary points are $\frac{1}{2}$ and 2.

Sign of $(2t-1)$:	$-$	$+$	$+$
Sign of $(t-2)$:	$-$	$-$	$+$
Sign of $(2t-1)(t-2)$:	$+$	$-$	$+$

$\frac{1}{2}$ 2

The solution set is $\left(\frac{1}{2},2\right)$ or $0.5<t<2$ sec.

The ball will be more than 18 ft high 0.5 sec and 2 sec after it is thrown.

69. $m=kw$

70. $x=\frac{k}{p^2}$

71. $y=\frac{kx\sqrt{z}}{t^3}$

72. $Q=kp\sqrt{t}$

$$132=k(11)\sqrt{9}$$

$$\frac{132}{33}=k$$

$$k=4$$

73. $d=\frac{kc}{x^2}$

$$1.8=\frac{k(3)}{(2)^2}$$

$$7.2=3k$$

$$k=2.4$$

74. Let w represent the weight of the ball. Let r represent the radius of the ball.

$$w=kr^3$$

$$3.24=k(3)^3$$

$$\frac{3.24}{27}=k$$

$$k=0.12$$

$$w=0.12r^3=0.12(5)^3=15 \text{ lb}$$

75. $F=\frac{k}{L}$

$$6.25=\frac{k}{1.6}$$

$$k=10$$

$$F=\frac{10}{L}=\frac{10}{2}=5 \text{ lb}$$

76. Let P represent power in a circuit. Let I represent the current in the circuit. Let R represent the resistance in the circuit.

$$P=kIR^2$$

$$144=k(4)(6)^2$$

$$\frac{144}{144}=k$$

$$k=1$$

$$P=IR^2=(3)(10)^2=300 \text{ W}$$

77. $F = \dfrac{kq_1q_2}{d^2}$

$$F = \frac{k(2q_1)(2q_2)}{\left(\frac{d}{2}\right)^2}$$

$$= \frac{4kq_1q_2}{\frac{d^2}{4}}$$

$$= 16\left(\frac{kq_1q_2}{d^2}\right)$$

The force will be 16 times as great.

Chapter 2 Test

1. a. $f(x) = 2x^2 - 12x + 16$

$$f(x) = 2(x^2 - 6x \qquad) + 16$$

$$\boxed{\left[\frac{1}{2}(-6)\right]^2 = 9}$$

$$f(x) = 2(x^2 - 6x + 9 - 9) + 16$$

$$f(x) = 2(x^2 - 6x + 9) + 2(-9) + 16$$

$$f(x) = 2(x - 3)^2 - 2$$

b. Since $a > 0$, the parabola opens upward.

c. The vertex is $(h, k) = (3, -2)$.

d. $f(x) = 2x^2 - 12x + 16$

$$0 = 2x^2 - 12x + 16$$

$$0 = x^2 - 6x + 8$$

$$0 = (x - 2)(x - 4)$$

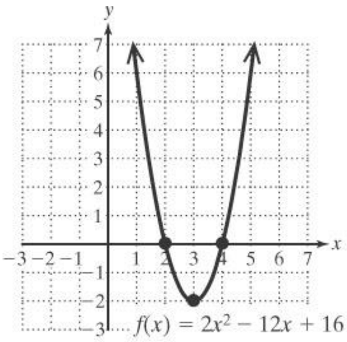
$$x = 2 \quad \text{or} \quad x = 4$$

The *x*-intercepts are $(2, 0)$ and $(4, 0)$.

e. $f(0) = 2(0)^2 - 12(0) + 16 = 16$

The *y*-intercept is $(0, 16)$.

f.



g. The axis of symmetry is the vertical line through the vertex: $x = 3$.

h. The minimum value is -2.

i. The domain is $(-\infty, \infty)$.

The range is $[-2, \infty)$.

2. a. $f(x) = 2x^4 - 5x^3 - 17x^2 + 41x - 21$

The leading coefficient is positive and the degree is even. The end behavior is up to the left and up to the right.

b. Possible rational zeros:

$$\frac{\text{Factors of } -21}{\text{Factors of } 2} == \frac{\pm 1, \pm 3, \pm 7, \pm 21}{\pm 1, \pm 2}$$

$$= \pm 1, \pm 3, \pm 7, \pm 21, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{7}{2}, \pm \frac{21}{2}$$

c.

$$\begin{array}{r|rrrrr} 1 & 2 & -5 & -17 & 41 & -21 \\ & & 2 & -3 & -20 & 21 \\ \hline & 2 & -3 & -20 & 21 & \boxed{0} \end{array}$$

$$\begin{array}{r|rrrr} 1 & 2 & -3 & -20 & 21 \\ & & 2 & -1 & -21 \\ \hline & 2 & -1 & -21 & \boxed{0} \end{array}$$

$$f(x) = (x-1)^2(2x^2 - x - 21)$$

$$f(x) = (x-1)^2(2x-7)(x+3)$$

$$(x-1)^2(2x-7)(x+3) = 0$$

$$x = 1, x = \frac{7}{2}, x = -3$$

The zero 1 has multiplicity 2. The zeros $\frac{7}{2}$ and -3 have multiplicity 1.

d. The x-intercepts are $\left(\frac{7}{2}, 0\right)$, $(-3, 0)$, and $(1, 0)$.

e. $$f(0) = \begin{bmatrix} 2(0)^4 - 5(0)^3 - 17(0)^2 \\ +41(0) - 21 \end{bmatrix}$$

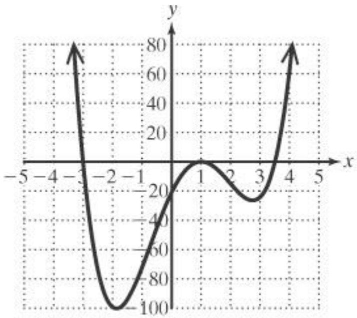
$$= -21$$

f. $$f(-x) = \begin{bmatrix} 2(-x)^4 - 5(-x)^3 - 17(-x)^2 \\ +41(-x) - 21 \end{bmatrix}$$

$$= 2x^4 + 5x^3 - 17x^2 - 41x - 21$$

$f(x)$ is neither even nor odd.

g.



$f(x) = 2x^4 - 5x^3 - 17x^2 + 41x - 21$

3. a. Multiply the leading terms from each factor.

$$-0.25x^3(x)^2(x)^4 = -0.25x^9$$

b. The leading coefficient is negative and the degree is odd. The end behavior is up to the left and down to the right.

c. $f(x) = -0.25x^3(x-2)^2(x+1)^4$

$$0 = -0.25x^3(x-2)^2(x+1)^4$$

$$x = 0, x = 2, x = -1$$

The zero 0 has multiplicity 3. The zero 2 has multiplicity 2. The zero -1 has multiplicity 4.

4. a. The leading term has degree 4. There are 4 zeros.

b. $0 = x^4 + 5x^2 - 36$

$$0 = (x^2 - 4)(x^2 + 9)$$

$$0 = (x-2)(x+2)(x-3i)(x+3i)$$

$$x = 2, x = -2, x = 3i, x = -3i$$

The zeros are 2, -2, $3i$, and $-3i$.

c. The x-intercepts are $(2,0)$ and $(-2,0)$.

d. $f(-x) = (-x)^4 + 5(-x)^2 - 36$

$$= x^4 + 5x^2 - 36$$

$f(x)$ is even.

5. $f(x)=x^3-5x^2+2x+5$

$$f(-2)=(-2)^3-5(-2)^2+2(-2)+5$$
$$=-8-20-4+5=-27$$
$$f(-1)=(-1)^3-5(-1)^2+2(-1)+5$$
$$=-1-5-2+5=-3$$
$$f(0)=(0)^3-5(0)^2+2(0)+5=5$$
$$f(1)=(1)^3-5(1)^2+2(1)+5$$
$$=1-5+2+5=3$$
$$f(2)=(2)^3-5(2)^2+2(2)+5$$
$$=8-20+4+5=-3$$

a. Since $f(-2)$ and $f(-1)$ have the same sign, the intermediate value theorem does not guarantee that the function has at least one zero on the interval $[-2,-1]$.

b. Since $f(-1)$ and $f(0)$ have opposite signs, the intermediate value theorem guarantees that the function has at least one zero on the interval $[-1,0]$.

c. Since $f(0)$ and $f(1)$ have the same sign, the intermediate value theorem does not guarantee that the function has at least one zero on the interval $[0,1]$.

d. Since $f(1)$ and $f(2)$ have opposite signs, the intermediate value theorem guarantees that the function has at least one zero on the interval $[1,2]$.

6. a.

$$\begin{array}{r}
2x^2+2x+4 \\
x^2-3x+1\overline{)\,2x^4-4x^3+0x^2+x-5} \\
\underline{-\left(-2x^4-6x^3+2x^2\right)}\phantom{+x-5} \\
2x^3-2x^2+x \\
\underline{-\left(2x^3-6x^2+2x\right)} \\
4x^2-x-5 \\
\underline{-\left(4x^2-12x+4\right)} \\
11x-9
\end{array}$$

$$2x^2+2x+4+\frac{11x-9}{x^2-3x+1}$$

b. Dividend: $2x^4-4x^3+x-5$;

Divisor: x^2-3x+1;

Quotient: $2x^2+2x+4$;

Remainder: $11x-9$

7. a.

$$\begin{array}{r|rrrrr}
\frac{3}{5} & 5 & 47 & 80 & -51 & -9 \\
 & & 3 & 30 & 66 & 9 \\
\hline
 & 5 & 50 & 110 & 15 & \boxed{0}
\end{array}$$

Since $f\left(\frac{3}{5}\right)=0$, $\frac{3}{5}$ is a zero of $f(x)$.

b.

$$\begin{array}{r|rrrrr}
-1 & 5 & 47 & 80 & -51 & -9 \\
 & & -5 & -42 & -38 & 89 \\
\hline
 & 5 & 42 & 38 & -89 & \boxed{80}
\end{array}$$

Since $f(-1)\neq 0$, -1 is a zero of $f(x)$.

c. Since $f(-1)\neq 0$, $x+1$ is not a factor of $f(x)$.

d.

$$\begin{array}{r|rrrrr}
-3 & 5 & 47 & 80 & -51 & -9 \\
 & & -15 & -96 & 48 & 9 \\
\hline
 & 5 & 32 & -16 & -3 & \boxed{0}
\end{array}$$

Since $f(-3)=0$, $x+3$ is a factor of $f(x)$.

e.
$$\begin{array}{r|rrrrr} -2 & 5 & 47 & 80 & -51 & -9 \\ & & -10 & -74 & -12 & 126 \\ \hline & 5 & 37 & 6 & -63 & \boxed{117} \end{array}$$

By the remainder theorem,
$f(-2)=117$.

8. a.
$$\begin{array}{r|rrrrr} 2i & 1 & -8 & 21 & -32 & 68 \\ & & 2i & -4-16i & 32+34i & -68 \\ \hline & 1 & -8+2i & 17-16i & 34i & \boxed{0} \end{array}$$

Since $2i$ is a zero, $-2i$ is also a zero.

$$\begin{array}{r|rrrr} -2i & 1 & -8+2i & 17-16i & 34i \\ & & -2i & 16i & -34i \\ \hline & 1 & -8 & 17 & \boxed{0} \end{array}$$

$f(x)=(x-2i)(x+2i)(x^2-8x+17)$

Find the remaining two zeros.

$$x^2-8x+17=0$$
$$x=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)(17)}}{2(1)}$$
$$=\frac{8\pm\sqrt{-4}}{2}=\frac{8\pm 2i}{2}=4\pm i$$

The zeros are $\pm 2i$, $4\pm i$.

b. $f(x)=\big((x-2i)(x+2i)$
$[x-(4+i)][x-(4-i)]\big)$

c. The solution set is $\{\pm 2i, 4\pm i\}$.

9. a. $f(x)$ is a fourth-degree polynomial. It has 4 zeros.

b.
$$\frac{\text{Factors of 12}}{\text{Factors of 3}}=\frac{\pm1,\pm2,\pm3,\pm4,\pm6,\pm12}{\pm1,\pm3}$$
$$=\pm1,\pm2,\pm3,\pm4,\pm6,\pm12,$$
$$\pm\frac{1}{3},\pm\frac{2}{3},\pm\frac{4}{3}$$

c.
$$\begin{array}{r|rrrrr} 2 & 3 & 7 & -12 & -14 & 12 \\ & & 6 & 26 & 28 & 28 \\ \hline & 3 & 13 & 14 & 14 & \boxed{40} \end{array}$$

The remainder and all coefficients of the quotient are nonnegative. Therefore, 2 is an upper bound for the real zeros of $f(x)$.

d. $$\begin{array}{r|rrrrr} -4 & 3 & 7 & -12 & -14 & 12 \\ & & -12 & 20 & -32 & 184 \\ \hline & 3 & -5 & 8 & -46 & \boxed{196} \end{array}$$

The signs of the quotient alternate. Therefore, -4 is a lower bound for the real zeros of $f(x)$.

e. We can restrict the list of possible rational zeros to those on the interval $(-4, 2)$:

$$\pm 1, -2, -3, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$$

From part (c), the value 2 itself is not a zero of $f(x)$. Likewise, from part (d), the value -4 itself is not a zero. Therefore, 2 and -4 are also eliminated from the list of possible rational zeros.

f. $$\begin{array}{r|rrrrr} -3 & 3 & 7 & -12 & -14 & 12 \\ & & -9 & 6 & 18 & -12 \\ \hline & 3 & -2 & -6 & 4 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$$\begin{array}{r|rrrr} \frac{2}{3} & 3 & -2 & -6 & 4 \\ & & 2 & 0 & -4 \\ \hline & 3 & 0 & -6 & \boxed{0} \end{array}$$

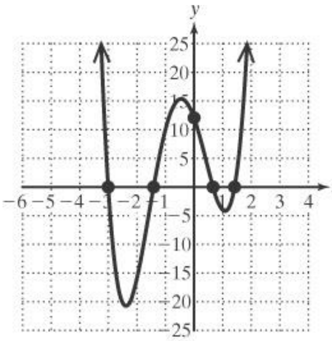
The rational zeros are $\frac{2}{3}$ and -3.

g. Factor $3x^2 - 6$.

$$3x^2 - 6 = 0$$
$$3x^2 = 6$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

The zeros are $\frac{2}{3}$, -3, $\pm\sqrt{2}$.

h.



$f(x) = 3x^4 + 7x^3 - 12x^2 - 14x + 12$

10. $f(x) = a\left(x - \frac{1}{5}\right)\left(x + \frac{2}{3}\right)(x - 4)$

Let $a = 15$.

$$= 15\left(x - \frac{1}{5}\right)\left(x + \frac{2}{3}\right)(x - 4)$$
$$= (5x - 1)(3x + 2)(x - 4)$$
$$= (5x - 1)(3x^2 - 10x - 8)$$
$$= 15x^3 - 50x^2 - 40x - 3x^2 + 10x + 8$$
$$= 15x^3 - 53x^2 - 30x + 8$$

11. $f(x) = -6x^7 - 4x^5 + 2x^4 - 3x^2 + 1$
$\smile \quad \smile \quad \smile$

3 sign changes in $f(x)$. The number of possible positive real zeros is either 3 or 1.

$$f(-x) = -6(-x)^7 - 4(-x)^5 + 2(-x)^4$$
$$-3(-x)^2 + 1$$
$$f(-x) = 6x^7 + 4x^5 + 2x^4 - 3x^2 + 1$$
$\smile \ \smile$

2 sign changes in $f(-x)$. The number of possible negative real zeros is either 2 or 0.

12. The expression $\frac{2x^2-3x+5}{x-7}$ is in lowest terms and the denominator is 0 at $x=7$. r has a vertical asymptote at $x=7$. The degree of the numerator is exactly one greater than the degree of the denominator. Therefore, r has no horizontal asymptote, but does have a slant asymptote.

$$\begin{array}{r|rrr} 7 & 2 & -3 & 5 \\ & & 14 & 77 \\ \hline & 2 & 11 & \boxed{82} \end{array}$$

The quotient is $2x+11$.

The slant asymptote is $y=2x+11$.

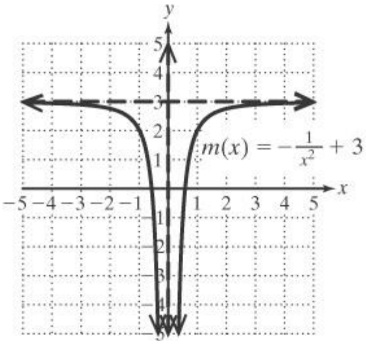
13. The expression $\frac{-3x+1}{4x^2-1}$ is in lowest terms.

$$\frac{-3x+1}{4x^2-1}=\frac{-3x+1}{(2x+1)(2x-1)}$$

The denominator is 0 at $x=-\frac{1}{2}$, and $x=\frac{1}{2}$. p has vertical asymptotes at $x=-\frac{1}{2}$, and $x=\frac{1}{2}$. The degree of the numerator is 1. The degree of the denominator is 2. Since $n<m$, the line $y=0$ is a horizontal asymptote of *p*. *p* has no slant asymptote.

14. The expression $\frac{5x^2-2x+1}{3x^2+4}$ is in lowest terms. The denominator $3x^2+4$ is never zero. *k* has no vertical asymptotes. The degree of the numerator is 2. The degree of the denominator is 2. Since $n=m$, the line $y=\frac{5}{3}$ is a horizontal asymptote of *k*.

15. The graph of *m* is the graph of $y=\frac{1}{x^2}$ with a shift upward 3 units and a reflection across the *x*-axis.



16. $h(0)=\frac{-4}{(0)^2-4}=\frac{-4}{-4}=1$

The *y*-intercept is $(0,1)$.

h is never 0. It has no *x*-intercepts.

h is in lowest terms, and x^2-4 is 0 for $x=2$ and $x=-2$, which are the vertical asymptotes.

The degree of the numerator is 0.

The degree of the denominator is 2.

Since $n<m$, the line $y=0$ is a horizontal asymptote of *q*.

$$0=\frac{-4}{x^2-4}$$

$0=-4$ No solution

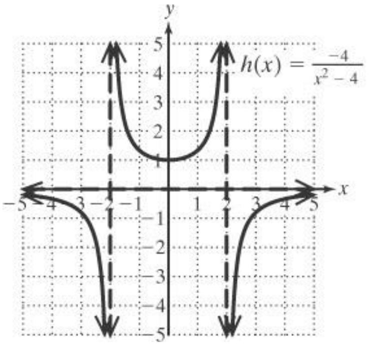
h does not cross its horizontal asymptote.

$$h(-x)=\frac{-4}{(-x)^2-4}=\frac{-4}{x^2-4}$$

$$h(-x)=h(x)$$

h is even.

Interval	**Test Point**	**Comments**
$(-\infty,-2)$	$\left(-4,-\frac{1}{3}\right)$	*h*(*x*) is negative. *h*(*x*) must approach the horizontal asymptote $y=0$ from below as $x\to-\infty$. As *x* approaches the vertical asymptote $x=-2$ from the left, $h(x)\to-\infty$.
$(-2,0)$	$\left(-1,\frac{4}{3}\right)$	*h*(*x*) is positive. As *x* approaches the vertical asymptote $x=-2$ from the right, $h(x)\to\infty$.
$(0,2)$	$\left(1,\frac{4}{3}\right)$	*h*(*x*) is positive. As *x* approaches the vertical asymptote $x=2$ from the left, $h(x)\to\infty$.
$(2,\infty)$	$\left(4,-\frac{1}{3}\right)$	*h*(*x*) is negative. *h*(*x*) must approach the horizontal asymptote $y=0$ from below as $x\to\infty$. As *x* approaches the vertical asymptote $x=2$ from the right, $h(x)\to-\infty$.



17. $k(0)=\dfrac{(0)^2+2(0)+1}{(0)}$ undefined

There is no *y*-intercept.

$$x^2+2x+1=0$$
$$(x+1)(x+1)=0$$
$$x=-1$$

The *x*-intercept is $(-1,0)$.

k is in lowest terms, and the denominator is 0 when $x = 0$, which is the vertical asymptote. The degree of the numerator is exactly one greater than the degree of the denominator. Therefore, *k* has no horizontal asymptote, but does have a slant asymptote.

$$k(x) = \frac{x^2 + 2x + 1}{x}$$
$$= \frac{x^2}{x} + \frac{2x}{x} + \frac{1}{x}$$
$$= x + 2 + \frac{1}{x}$$

The quotient is $x + 2$.

The slant asymptote is $y = x + 2$.

$$x + 2 = \frac{x^2 + 2x + 1}{x}$$
$$x^2 + 2x = x^2 + 2x + 1$$
$$0 = 1 \text{ No solution}$$

k does not cross its slant asymptote.

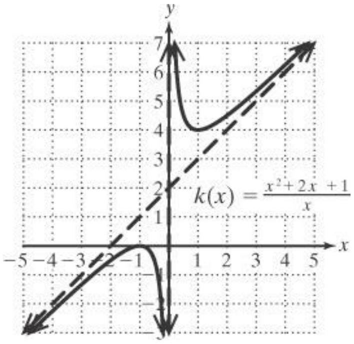
$$k(-x) = \frac{(-x)^2 + 2(-x) + 1}{(-x)} = \frac{x^2 - 2x + 1}{-x}$$

$$k(-x) \neq k(x),\ k(-x) \neq -k(x)$$

k is neither even nor odd.

Select test points from each interval.

Interval	**Test Point**	**Test Point**
$(-\infty, -1)$	$\left(-4, -\frac{9}{4}\right)$	$\left(-2, -\frac{1}{2}\right)$
$(-1, 0)$	$\left(-\frac{1}{2}, -\frac{1}{2}\right)$	$\left(-\frac{1}{4}, -\frac{9}{4}\right)$
$(0, \infty)$	$\left(\frac{1}{4}, \frac{25}{4}\right)$	$\left(2, \frac{9}{2}\right)$



18. $c^2 < c + 20$

$c^2 - c - 20 < 0$

Find the real zeros of the related equation.

$$c^2 - c - 20 = 0$$
$$(c + 4)(c - 5) = 0$$
$$c = -4 \text{ or } c = 5$$

The boundary points are -4 and 5.

Sign of $(c + 4)$:	$-$	$+$	$+$
Sign of $(c - 5)$:	$-$	$-$	$+$
Sign of $(c + 4)(c - 5)$:	$+$	$-$	$+$

$-4 \quad 5$

The solution set is $(-4, 5)$.

19. $y^3 > 13y - 12$

$y^3 - 13y + 12 > 0$

Find the real zeros of the related equation.

$y^3 - 13y + 12 = 0$

$$\frac{\text{Factors of } 12}{\text{Factors of } 1} = \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1}$$
$$= \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -13 & 12 \\ & & 1 & 1 & -12 \\ \hline & 1 & 1 & -12 & \boxed{0} \end{array}$$

$$(x - 1)(x^2 + x - 12) = 0$$
$$(x - 1)(x + 4)(x - 3) = 0$$

$x=1$, $x=-4$, or $x=3$

The boundary points are -4, 1, and 3.

Sign of $(x+4)$:	$-$	$+$	$+$	$+$
Sign of $(x-1)$:	$-$	$-$	$+$	$+$
Sign of $(x-3)$:	$-$	$-$	$-$	$+$
Sign of $(x-1)(x+4)(x-3)$:	$-$	$+$	$-$	$+$

$-4 \quad 1 \quad 3$

The solution set is $(-4, 1)\cup(3, \infty)$.

20. $-2x(x-4)^2(x+1)^3 \le 0$

$x(x-4)^2(x+1)^3 \ge 0$

Find the real zeros of the related equation.

$x(x-4)^2(x+1)^3 = 0$

$x=0$, $x=4$, or $x=-1$

The boundary points are -1, 0, and 4.

Sign of $(x+1)^3$:	$-$	$+$	$+$	$+$
Sign of (x):	$-$	$-$	$+$	$+$
Sign of $(x-4)^2$:	$+$	$+$	$+$	$+$
Sign of $x(x-4)^2(x+1)^3$:	$+$	$-$	$+$	$+$

$-1 \quad 0 \quad 4$

The solution set is $(-\infty, -1]\cup[0, \infty)$.

21. $9x^2+42x+49>0$

$(3x+7)^2>0$

The expression $(3x+7)^2>0$ for all real numbers except where $(3x+7)^2=0$. Therefore, the solution set is all real numbers except $-\frac{7}{3}$.

$\left(-\infty, -\frac{7}{3}\right)\cup\left(-\frac{7}{3}, \infty\right)$

22. $\frac{x+3}{2-x}\le 0$

$\frac{x+3}{x-2}\ge 0$

The expression is undefined for $x=2$. This is a boundary point.

Find the real zeros of the related equation.

$\frac{x+3}{x-2}=0$

$x+3=0$

$x=-3$

The boundary points are -3 and 2.

Sign of $(x+3)$:	$-$	$+$	$+$
Sign of $(x-2)$:	$-$	$-$	$+$
Sign of $\frac{x+3}{x-2}$:	$+$	$-$	$+$

$-3 \quad 2$

The solution set is $(-\infty, -3]\cup[2, \infty)$.

23. $\frac{-4}{x^2-9}\ge 0$

$\frac{4}{x^2-9}\le 0$

$\frac{4}{(x+3)(x-3)}\le 0$

The expression is undefined for $x=-3$ and $x=3$. These are boundary points.

The related equation has no real zeros.

Sign of $(x+3)$:	$-$	$+$	$+$
Sign of $(x-3)$:	$-$	$-$	$+$
Sign of $\frac{4}{(x+3)(x-3)}$:	$+$	$-$	$+$

$-3 \quad 3$

The solution set is $(-3, 3)$.

24.
$$\frac{4}{x-1} < -\frac{3}{x}$$
$$\frac{4}{x-1} + \frac{3}{x} < 0$$
$$\frac{4x}{x(x-1)} + \frac{3(x-1)}{x(x-1)} < 0$$
$$\frac{7x-3}{x(x-1)} < 0$$

The expression is undefined for $x = 0$ and $x = 1$. These are boundary points. Find the real zeros of the related equation.

$$\frac{7x-3}{x(x-1)} = 0$$
$$7x - 3 = 0$$
$$x = \frac{3}{7}$$

The boundary points are 0, $\frac{3}{7}$, and 1.

Sign of x:	$-$	$+$	$+$	$+$
Sign of $(7x-3)$:	$-$	$-$	$+$	$+$
Sign of $(x-1)$:	$-$	$-$	$-$	$+$
Sign of $\frac{7x-3}{x(x-1)}$:	$-$	$+$	$-$	$+$

$0 \quad \frac{3}{7} \quad 1$

The solution set is $(-\infty, 0) \cup \left(\frac{3}{7}, 1\right)$.

25. $E = kv^2$

26.
$$w = \frac{ky\sqrt{x}}{z}$$
$$7.2 = \frac{k(6)\sqrt{4}}{7}$$
$$50.4 = 12k$$
$$k = 4.2$$

27. Let A represent the surface area of a cube. Let s represent the length of an edge.

$$A = ks^2$$
$$24 = k(2)^2$$
$$\frac{24}{4} = k$$
$$k = 6$$
$$A = 6s^2 = 6(7)^2 = 294 \text{ ft}^2$$

28. Let w represent the weight of a body. Let d represent the distance from the center of the Earth.

$$w = \frac{k}{d^2}$$
$$180 = \frac{k}{(4000)^2}$$
$$k = 2{,}880{,}000{,}000$$
$$w = \frac{2{,}880{,}000{,}000}{d^2}$$
$$= \frac{2{,}880{,}000{,}000}{(4020)^2} \approx 178 \text{ lb}$$

29. Let P represent the pressure on the wall. Let A represent the area of the wall. Let v represent the velocity of the wind.

$$P = kAv^2$$
$$P = kA(3v)^2 = kA(9v^2) = 9(kAv^2)$$

The pressure is 9 times as great.

30. a.
$$P(1) = \frac{2000(1)}{1+1} = 1000$$
$$P(5) = \frac{2000(5)}{5+1} \approx 1667$$
$$P(10) = \frac{2000(10)}{10+1} \approx 1818$$

There will be 1000 rabbits after 1 yr, 1667 rabbits after 5 yr, and 1818 rabbits after 10 yr.

b. Horizontal asymptote: $\frac{2000}{1} = 2000$

The rabbit population will approach 2000 as t increases.

31. a. $y(20) = 140.3$ means that with 20,000 plants per acre, the yield will be 140.3 bushels per acre; $y(30) = 172$ means that with 30,000 plants per acre, the yield will be 172 bushels per acre; $y(60) = 143.5$ means that with 60,000 plants per acre, the yield will be 143.5 bushels per acre.

b. The number of plants per acre that will maximize the yield is the n-coordinate of the vertex.

$$n = \frac{-b}{2a} = \frac{-(8.32)}{2(-0.103)}$$
$$= \frac{-8.32}{-0.206}$$
$$\approx 40.4 \text{ thousand plants}$$
$$\approx 40{,}400 \text{ plants}$$

c. The maximum yield is the value of $y(n)$ at the vertex.

$$y(40.4) = -0.103(40.4)^2 + 8.32n + 15.1$$
$$\approx 183 \text{ bushels}$$

32. a. $s(t) = -\frac{1}{2}(9.8)t^2 + (98)t + (0)$

$s(t) = -4.9t^2 + 98t$

b. $t = \frac{-b}{2a} = \frac{-(98)}{2(-4.9)} = \frac{-98}{-9.8} = 10$

The rocket will reach maximum height 10 sec after launch.

c. $s(t) = -4.9t^2 + 98t$

$$s(10) = -4.9(10)^2 + 98(10)$$
$$= -490 + 980 = 490$$

The rocket's reach maximum height is 490 m.

d. $-4.9t^2 + 98t > 200$

$-4.9t^2 + 98t - 200 > 0$

$4.9t^2 - 98t + 200 < 0$

Find the real zeros of the related equation.

$4.9t^2 - 98t + 200 = 0$

$$t = \frac{-(-98) \pm \sqrt{(-98)^2 - 4(4.9)(200)}}{2(4.9)}$$
$$= \frac{98 \pm \sqrt{5684}}{9.8} \approx 2.3 \text{ or } 17.7$$

$4.9t^2 - 98t + 200 < 0$

$(t - 2.3)(t - 17.7) < 0$

The boundary points are 2.3 and 17.7.

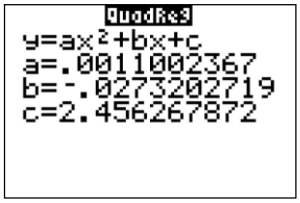
Sign of $(t-2.3)$:	$-$	$+$	$+$
Sign of $(t-17.7)$:	$-$	$-$	$+$
Sign of $(t-2.3)(t-17.7)$:	$+$	$-$	$+$

2.3 17.7

The solution set is $(2.3, 17.7)$.

The rocket will be more than 200 m high between 2.3 sec and 17.7 sec after launch.

33. a.



```
QuadReg
y=ax²+bx+c
a=.0011002367
b=-.0273202719
c=2.456267872
```

$$n(a) = 0.0011a^2 - 0.027a + 2.46$$

b. The age when the number of yearly visits is the least is the a-coordinate of the vertex.

$$a = \frac{-b}{2a} = \frac{-(-0.027)}{2(0.0011)}$$
$$= \frac{0.027}{0.0022} \approx 12 \text{ yr}$$

c. The minimum number of yearly visits is the $n(a)$ value at the vertex.

$$n(12) = \begin{bmatrix} 0.0011(12)^2 \\ \quad -0.027(12) + 2.46 \end{bmatrix}$$
$$\approx 2.3 \text{ visits per year}$$

Chapter 2 Cumulative Review Exercises

1. a. $x^2 - 16 = 0$

$x^2 = 16$

$x = 4,\ x = -4$

b. Since the degree of the numerator is greater than the degree of the denominator by more than 1, the horizontal asymptote is $y = \frac{2}{1} = 2$.

2. $$f(x) = \begin{Bmatrix} (x-2)\left[x-(3+2i)\right] \\ \left[x-(3-2i)\right] \end{Bmatrix}$$

$$f(x) = \begin{Bmatrix} (x-2)\left[(x-3)+2i\right] \\ \left[(x-3)-2i\right] \end{Bmatrix}$$

$$f(x) = (x-2)\left[(x-3)^2 + 4\right]$$
$$f(x) = (x-2)(x^2 - 6x + 9 + 4)$$
$$f(x) = (x-2)(x^2 - 6x + 13)$$
$$f(x) = x^3 - 6x^2 + 13x - 2x^2 + 12x - 26$$
$$f(x) = x^3 - 8x^2 + 25x - 26$$

3. $f(x) = 2x^3 - x^2 - 8x - 5$

a. The leading term is $2x^3$. The end behavior is down to the left and up to the right.

b. $$\frac{\text{Factors of } -5}{\text{Factors of } 2} = \frac{\pm 1, \pm 5}{\pm 1, \pm 2}$$
$$= \pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}$$

$$\begin{array}{r|rrrr} \underline{-1} & 2 & -1 & -8 & -5 \\ & & -2 & 3 & 5 \\ \hline & 2 & -3 & -5 & \boxed{0} \end{array}$$

Find the zeros of the quotient.

$2x^2 - 3x - 5 = 0$

$(2x - 5)(x + 1) = 0$

$x = \frac{5}{2},\ x = -1$

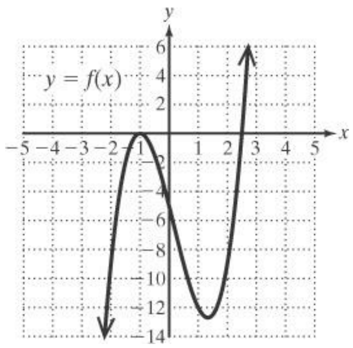
The zeros are $\frac{5}{2}$ (multiplicity 1) and -1 (multiplicity 2).

c. The x-intercepts are $\left(\frac{5}{2}, 0\right)$ and $(-1, 0)$.

d. $f\left(0\right)=2\left(0\right)^3-\left(0\right)^2-8\left(0\right)-5=-5$

The y-intercept is $\left(0,-5\right)$.

e.



4. $$\frac{3+2i}{4-i}=\frac{3+2i}{4-i}\cdot\frac{4+i}{4+i}$$
$$=\frac{12+3i+8i-2}{16+1}$$
$$=\frac{10+11i}{17}$$
$$=\frac{10}{17}+\frac{11}{17}i$$

5. $$x^2+y^2+8x-14y+56=0$$
$$\left(x^2+8x\qquad\right)+\left(y^2-14y\qquad\right)=-56$$
$$\boxed{\left[\frac{1}{2}\left(8\right)\right]^2=16\qquad\left[\frac{1}{2}\left(-14\right)\right]^2=49}$$
$$\begin{bmatrix}\left(x^2+8x+16\right)\\+\left(y^2-14y+49\right)\end{bmatrix}=-56+16+49$$
$$\left(x+4\right)^2+\left(y-7\right)^2=9$$

Center: $\left(-4,7\right)$; Radius: $\sqrt{9}=3$

6. $$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-3-\left(-8\right)}{2-4}$$
$$=\frac{5}{-2}=-\frac{5}{2}$$
$$y-y_1=m\left(x-x_1\right)$$
$$y-\left(-8\right)=-\frac{5}{2}\left(x-4\right)$$
$$y+8=-\frac{5}{2}x+10$$
$$y=-\frac{5}{2}x+2$$

7. $x=y^2-9$

$x=\left(0\right)^2-9$

$x=-9$

The x-intercept is $\left(-9,0\right)$.

$x=y^2-9$

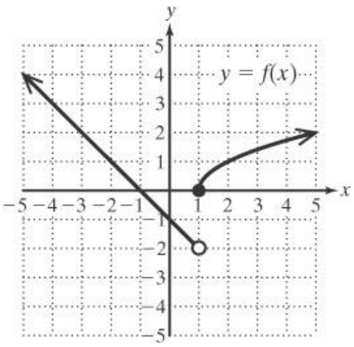
$0=y^2-9$

$y^2=9$

$y=\pm 3$

The y-intercepts are $\left(0,3\right)$ and $\left(0,-3\right)$.

8.



9. $$v\ t=\sqrt{m-t}$$
$$\left(v_0t\right)^2=m-t$$
$$m=\left(v_0t\right)^2+t$$

10. a. $f(x) = 2x^2 - 6x + 1$

$$2x^2 - 6x + 1 = 0$$

$$\begin{aligned} x &= \frac{-(-6) \pm \sqrt{(-6) - 4(2)(1)}}{2(2)} \\ &= \frac{6 \pm \sqrt{28}}{4} \\ &= \frac{6 \pm 2\sqrt{7}}{4} \\ &= \frac{3 \pm \sqrt{7}}{2} \end{aligned}$$

The x-intercepts are $\left(\frac{3+\sqrt{7}}{2}, 0\right)$ and $\left(\frac{3-\sqrt{7}}{2}, 0\right)$.

b. $f(0) = 2(0)^2 - 6(0) + 1 = 1$

The y-intercept is $(0, 1)$.

c. $x = \frac{-b}{2a} = \frac{-(-6)}{2(2)}$

$$= \frac{6}{4} = \frac{3}{2}$$

$$\begin{aligned} f\left(\frac{3}{2}\right) &= 2\left(\frac{3}{2}\right)^2 - 6\left(\frac{3}{2}\right) + 1 \\ &= \frac{9}{2} - 9 + 1 \\ &= -\frac{7}{2} \end{aligned}$$

The vertex is $\left(\frac{3}{2}, -\frac{7}{2}\right)$.

11. $125x^6 - y^9 = \left(5x^2\right)^3 - \left(y^3\right)^3$

$$= \left[\begin{array}{l} \left(5x^2 - y^3\right) \\ \quad \left(25x^4 + 5x^2y^3 + y^6\right) \end{array}\right]$$

12. $\left(\frac{4x^3y^{-5}}{z^{-2}}\right)^{-3}\left(\frac{4y^{-6}}{x^{-12}}\right)^{1/2}$

$$\begin{aligned} &= \left(\frac{4^{-3}x^{-9}y^{15}}{z^6}\right)\left(\frac{4^{1/2}y^{-3}}{x^{-6}}\right) \\ &= \left(\frac{y^{15}}{4^3x^9z^6}\right)\left(\frac{2x^6}{y^3}\right) \\ &= \frac{y^{12}}{32x^3z^6} \end{aligned}$$

13. $\sqrt[3]{250z^5xy^{21}} = \sqrt[3]{125z^3y^{21} \cdot 2z^2x}$

$$= 5zy^7\sqrt[3]{2z^2x}$$

14. $\dfrac{\frac{1}{3x} - \frac{1}{x^2}}{\frac{1}{3} - \frac{3}{x^2}} = \dfrac{\frac{1}{3x} - \frac{1}{x^2}}{\frac{1}{3} - \frac{3}{x^2}} \cdot \dfrac{3x^2}{3x^2} = \dfrac{x-3}{x^2-3}$

$$= \frac{\cancel{x-3}}{\cancel{(x-3)}(x+3)} = \frac{1}{x-3}$$

15. $-5 \le -\frac{1}{4}x + 3 < \frac{1}{2}$

$$20 \ge x - 12 > -2$$

$$32 \ge x > 10$$

$$10 < x \le 32$$

$$(10, 32]$$

16. $|x-3| + 4 \le 10$

$$|x-3| \le 6$$

$$-6 \le x - 3 \le 6$$

$$-3 \le x \le 9$$

$$[-3, 9]$$

17. $|2x+1|=|x-4|$

$2x+1=x-4$ or $2x+1=-(x-4)$

$x=-5$ $\qquad$ $2x+1=-x+4$

$3x=3$

$x=1$

$\{-5,1\}$

18. $c^2-5c+9<c(c+3)$

$c^2-5c+9<c^2+3c$

$-8c<-9$

$c>\frac{9}{8}$

$\left(\frac{9}{8},\infty\right)$

19. $\frac{49x^2+14x+1}{x}>0$

$\frac{(7x+1)^2}{x}>0$

The expression is undefined for $x=0$.

This is a boundary point.

Find the real zeros of the related equation.

$\frac{(7x+1)^2}{x}=0$

$(7x+1)^2=0$

$x=-\frac{1}{7}$

The boundary points are $-\frac{1}{7}$ and 0.

Sign of $(7x+1)^2$:	+	+	+
Sign of (x):	−	−	+
Sign of $\frac{(7x+1)^2}{x}$:	−	−	+

$-\frac{1}{7}$ $\quad$ 0

$(0,\infty)$

20. $\sqrt{4x-3}-\sqrt{x+12}=0$

$\sqrt{4x-3}=\sqrt{x+12}$

$4x-3=x+12$

$3x=15$

$x=5$

$\{5\}$