Chapter 2
Problem Solutions

Concept Problems

1. When does $x[n] = x(nT)$? When does it not?
Solution: See Equation (2.1).

2. If a continuous-time signal is sampled and the sample values applied in a digital computer, why is the sample value used in the computer generally not equal to the value of the continuous-time signal at the sampling instant.
Solution: Refer to Shannon's Sampling Theorem.

3. Discuss the relationship between the sampling period and the numerical accuracy of the calculation of the integral of a continuous-time signal using the sampled values.
Solution: Consider Figure 2.2. The numerical accuracy of the integration algorithm also may place a limit on the sampling frequency.

4. Discuss the differences between the continuous-time function $x(t) = u(t) - u(t - t_0)$ and the discrete-time function $x[n] = u[n] - u[n - n_0]$.
Solution: $x(t) = 1$ for $0 \le t \le t_0$, $x[n] = 1$ for $0 \le n < n_0$.

5. Discuss the difference between the continuous-time impulse $\delta(t - t_0)$ and the discrete-time impulse $\delta[n - n_0]$.
Solution: See Equations (2.11)-(2.13).

6. Explain why a discrete-time signal formed by sampling a periodic continuous-time signal may not be periodic. What are the conditions required for the discrete-time signal to be periodic?
Solution: See Equations (2.34)-(2.35).

7. Why is the time-average value of a signal always contained in the even part of the signal?
Solution: By definition (2.27), the odd part of a signal has a time-average value of zero.

8. Give an example of a periodic continuous-time signal that sampled with $T = 1$ ms resulting in a non-periodic discrete-time signal.
Solution: Consider any periodic, continuous-time signal for which $T_0/0.001$ is irrational.

9. Describe, in English, the process of convolution with respect to a signal being processed through a system.
Solution: Consider the sum of a continuing series of impulse responses.

10. Give an example of a finite impulse response system that is different from the system discussed in Example 2.7.

Solution: Revise Figure 2.24 and Equation (2.88).

Analysis and Design Problems

Section 2.1 Problems: Discrete-Time Signals and Systems

2.1. Determine which of the following discrete-time functions is different from all the others.

$(a)\ x_a[n] = u[n+1] - u[n-2]$

$(b)\ x_b[n] = \sum_{k=-1}^{1} \delta[n-k]$

$(c)\ x_c[n] = \begin{cases} 1, & n \in [-1, 1] \\ 0, & \text{otherwise} \end{cases}$

$(d)\ x_d[n] = \delta[n+1] + \delta[n] + \delta[n-1] + \delta[n-2]$

Solution

Only function (d) has a non-zero value at $n = 2$.

2.2. Write a MATLAB® program to generate a plot of $u[n-4] - u[n-10]$ over the discrete sequence $0 \le n \le 15$.

Solution

```
for m=1:16
  y(m)=0;
  n=m-1;
  k(m)=n;
  if n-4>=0, y(m)=1;
  end
  w(m)=0;
  if n-10>=0, w(m)=1;
  end
  x(m)=y(m)-w(m);
end
stem(k,x)
```

Section 2.1 Problems

2.3. The trapezoidal rule for numerical integration is shown in Figure P2.3. The value of the integral at time nT is equal to the value at (n-1)T plus the area of the trapezoid shown.

(a)

Write a difference-equation model for the integration operation that relates y[n] to x[n].

(b)

Write a MATLAB program that integrates $e^{-t}, 0 \le t \le 5$ seconds, with a sampling period of $T = 0.1$ second.

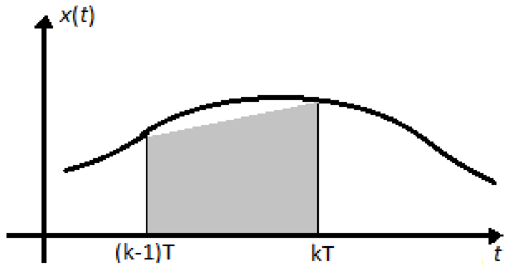


Figure P2.3

Solution P2.3

(a) $y[n] = y[n-1] + \frac{T}{2}\left(x[n] + x[n-1]\right)$

(b)

```
y(1)=0;
T=0.1;
for n=1:51
  y(n+1)=y(n)+T/2*(exp(-n*T)+exp(-(n-1)*T));
end
y
```

Final result: $y(5) \approx y[51] = 0.9941$.

Section 2.1 Problems

2.4. The sequence shown in Figure P2.4 was acquired by sampling a continuous-time signal $x(t) = e^{6.9315t}[u(t) - u(t-0.55)]$ using a sampling period of 0.1 second.

(a) Approximate the integral of the signal using rectangular (Euler) integration.
(b) Approximate the integral of the signal using trapezoidal integration.
(c) Calculate the integral of the continuous-time signal $x(t)$.
(d) Compare the results of (a), (b), and (c).

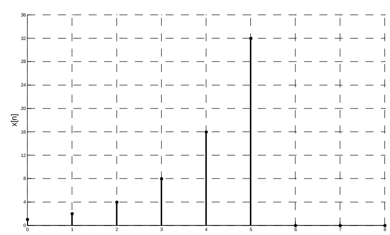
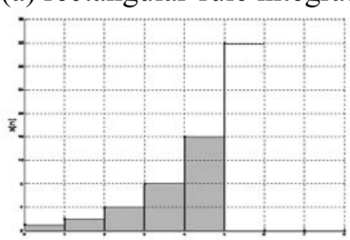


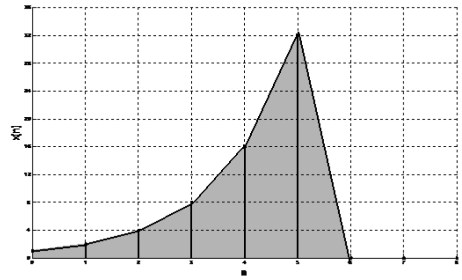
Figure P2.4

Solution

(a) rectangular-rule integration:



(b) trapezoidal itegration:



$x[-1]=0; y[-1]=0; T=0.1$

$y[0]=y[-1]+\frac{T}{2}(x[0]+x[-1])=0+0.05\times(1+0)=0.05$

$y[1]=y[0]+\frac{T}{2}(x[1]+x[0])=0.05+0.05\times(2+1)=0.2$

$y[2]+\frac{T}{2}(x[2]+x[1])=0.2+0.05\times(4+2)=0.5$

M

$y[6]=y[5]+\frac{T}{2}(x[6]+x[5])=4.7+0.05\times(0+32)=6.3$

(c) $y=\int_0^{0.55} e^{6.915t}dt=\frac{1}{6.9315}e^{6.9315t}\Big|_0^{0.55}=6.385$

Section 2.2 Problems: Transformations of Discrete-Time Signals

2.5. The discrete-time signals shown in Figure P2.5 are zero except as shown. Plot the following:

$(a)\ x[2n]$	$(b)\ x[n/2]$
$(c)\ x[-n]$	$(d)\ x[-2n]$
$(e)\ x[n-2]$	$(f)\ x[2-n]$
$(g)\ 2-3x[n]$	$(h)\ 1+2x[n-2]$

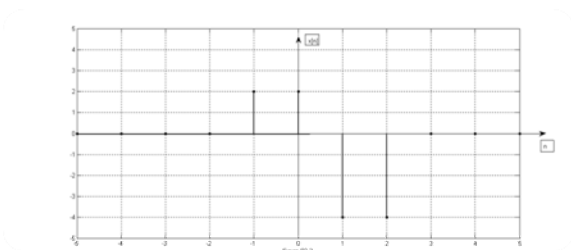
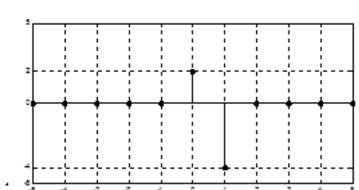


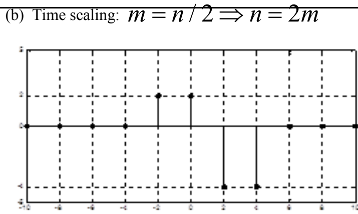
Figure P2.5

Solution P2.5

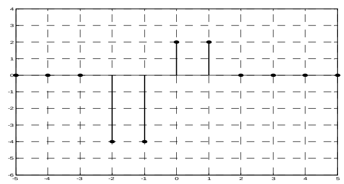
(a) Time scaling: $m=2n \Rightarrow n=m/2$



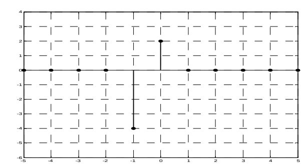
(b) Time scaling: $m=n/2 \Rightarrow n=2m$



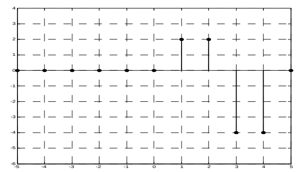
(c) Time reversal: $n = -m$



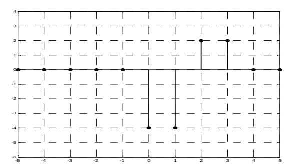
(d) Time reversal and scaling: $m = -2n \Rightarrow n = -m/2$



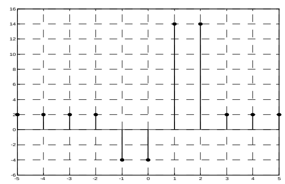
(e) Time shift: $m = n - 2 \Rightarrow n = m + 2$



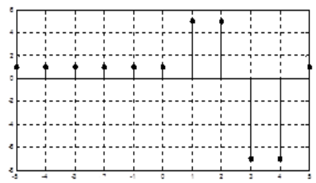
(f) Time shift and time reversal: $m = 2 - n \Rightarrow n = -m + 2$



(g) Amplitude shift and amplitude scaling: $y(n) = 2 - 3x(n)$



(h) Amplitude shift, amplitude scaling, and time shift: $y(n) = 1 + 2x(n-2)$



Section 2.2 Problems

2.6 The signal in Figure P2.5 is zero except as shown. Plot:

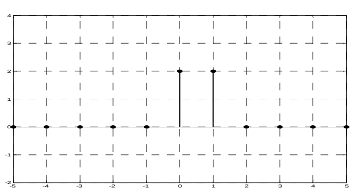
$(a) x[-n]u[n]$ $(b) x[n]u[-n]$

$(c) x[n]u[n-2]$ $(d) x[n]u[2-n]$

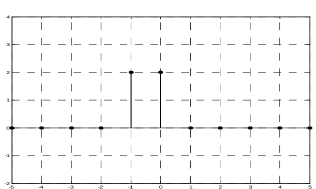
$(e) x[n]\delta[n-1]$ $(f) x[n](\delta[n]+\delta[n-2])$

Solution

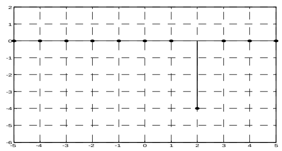
(a) Time reversal on $x[n]$, followed by multiplication by a unit step function:



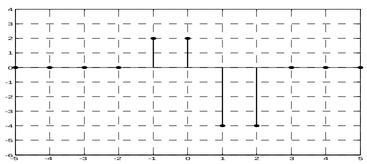
(b) Multiply $x[n]$ by a time reversed unit step function:



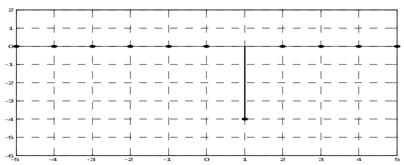
(c) Multiply $x[n]$ by a time-shifted unit step function:



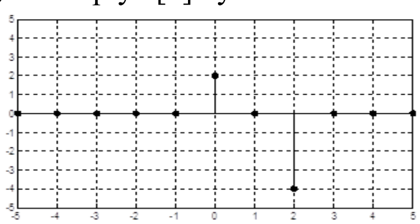
(d) Multiply $x[n]$ by a time reversed and time-shifted unit step function: $u[2-n]=\begin{cases}1, n\le 2\\ 0, n>2\end{cases}$



(e) Multiply $x[n]$ by a time-shifted impulse $\delta[n-n_0]=\begin{cases}1, & n=n_0\\ 0, otherwise\end{cases}$.



(f) Multiply $x[n]$ by the sum of two impulses, one occurring at $n = 0$ and one at $n = 2$.



Section 2.2 Problems

2.7. The sequence $x[n]$ is shown in Figure P2.4. For each of the following, write an equation describing the sequence as a sum of discrete-time impulses and sketch the sequence.

(a) $y_a[n]=x[n-2]$ vs. n.

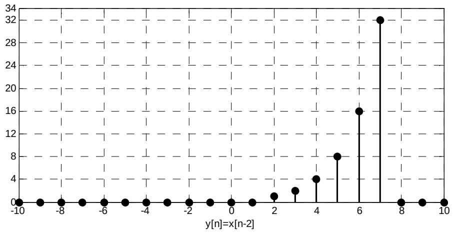
(b) $y_b[n]=x[n+3]$ vs. n.

(c) $y_c[n]=x[-n]$ vs. n.

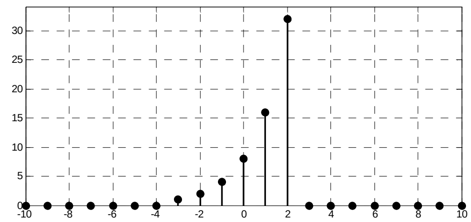
(d). $y_d[n]=x[-n+3]$ vs. n.

Solution P2.7

(a)

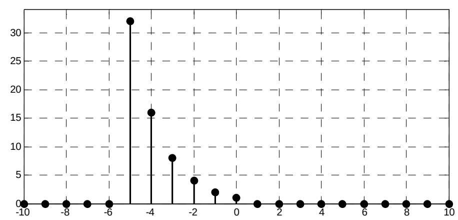


(b)



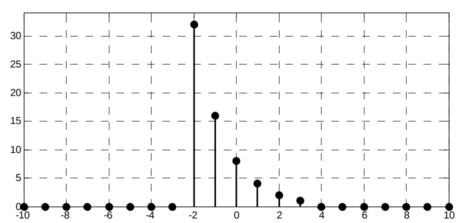
y[n]=x[n+3]

(c)



y[n]=x[-n]

(d)



y[n]=x[-n+3]

Section 2.2 Problems

2.8. Suppose that the sequence of Figure P2.5,

$$x[n] = 3v[0.5n - 2] - 2$$

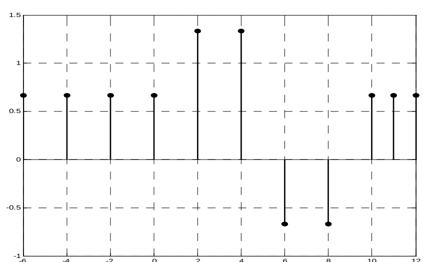
find and sketch $v[n]$.

Solution P2.8

$$v[0.5n-2] = \frac{x[n]+2}{3}$$

$$\textit{time transformation}: m = 0.5n - 2 \Rightarrow n = 2m + 4$$

m	**n**	**Av**	**Ax**
-4	-4	2	4/3
-3	-2	0	2/3
-2	0	-4	-2/3
-1	2		
0	4		
1	6		
2	8		
3	10		



Section 2.2 Problems

2.9. The general case of a signal transformed in both time and amplitude can be expressed as

$$y[n] = Ax[an+b] + B$$

where a is rational and b is an integer. Solve this expression for $x[n]$ in terms of $y[n]$.

Solution P2.9

$$y[n] = Ax[an+b] + B; \text{ let } m = an + b \Rightarrow n = \frac{m-b}{a},$$

$$x[n] = \frac{1}{A} y\left[\frac{m-b}{a}\right] - \frac{B}{A}$$

Section 2.3 Problems: Characteristics of Discrete-Time Signals

2.10 Give proofs of the following statements:
(a) The sum of two even functions is even.
(b) The sum of two odd functions is odd.
(c) The sum of an even function and an odd function is neither even nor odd.
(d) The product of two odd functions is even.
(e) The product of two even functions is even.
(f) The product of an even function and an odd function is odd.

Solution P2.10

(a)
$$x_t[n] = x_{e1}[n] + x_{e2}[n];$$
$$x_t[-n] = x_{e1}[-n] + x_{e2}[-n] = x_{e1}[n] + x_{e2}[n] = x_t[n], \therefore \textit{even}$$

(b)
$$x_t[n] = x_{o1}[n] + x_{o2}[n];$$
$$x_t[-n] = x_{o1}[-n] + x_{o2}[-n] = -x_{o1}[n] - x_{o2}[n] = -x_t[n], \therefore \textit{odd}$$

(c)
$$x_t[n] = x_e[n] + x_o[n]$$
$$x_t[-n] = x_e[-n] + x_o[-n] = x_e[n] - x_o[n] \neq x_t[n], \therefore \textit{neither}$$

(d)
$$x_t[n] = x_{e1}[n] x_{e2}[n]$$
$$x_t[-n] = x_{e1}[-n] x_{e2}[-n] = x_{e1}[n] x_{e2}[n] = x_t[n], \therefore \textit{even}$$

(e)
$$x_t[n] = x_{o1}[n] x_{o2}[n]$$
$$x_t[-n] = x_{o1}[-n] x_{o2}[-n] = \left[-x_{o1}[n]\right]\left[-x_{o2}[n]\right] = x_t[n], \therefore \textit{even}$$

(f)
$$x_t[n] = x_e[n] x_o[n]$$
$$x_t[-n] = x_e[-n] x_o[-n] = x_e[n]\left[-x_o[n]\right] = -x_t[n], \therefore \textit{odd}$$

Section 2.3 Problems

2.11. (a) Determine if each of the signals listed is even, odd, or neither.

$(i)\ x_1[n] = 3u[n-2]$ $\qquad (ii)\ x_2[n] = n$

$(iii)\ x_3[n] = (0.7)^{|n|}$ $\qquad (iv)\ x_4[n] = 3 + (0.7)^n + (0.7)^{-n}$

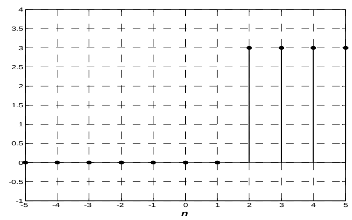
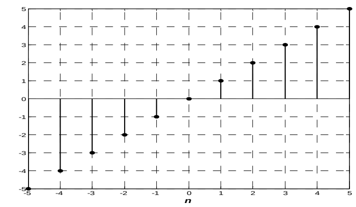
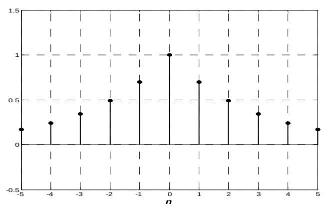
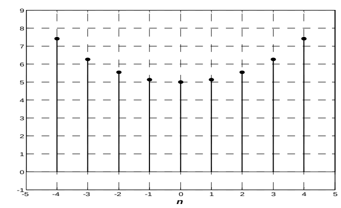
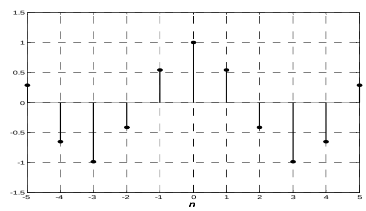
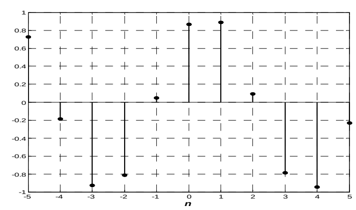
$(v)\ x_5[n] = \cos(n)$ $\qquad (vi)\ x_6[n] = \cos(n - \pi/6)$

(b) Sketch the signals and verify the results of (a).
(c) Find the even part and the odd part of each signal.

Solution P2.11

(a) (i) neither, (ii) odd, (iii) even, (iv) even,(v) even,(vi) neither

(b)

(i) Neither 	(ii) Odd 
(iii) Even 	(iv) Even 
(v) Even 	(vi) Neither 

(c)

$(i)\ x_e[n] = 1.5u[n-2] + 1.5u[-n-2], \quad x_o[n] = 1.5u[n-2] - 1.5u[-n-2]$

$(ii)\ x_e[n] = 0, \quad x_o[n] = n$

$(iii)\ x_e[n] = x[n], \quad x_o[n] = 0$

$(iv)\ x_e[n] = x[n], \quad x_o[n] = 0$

$(v)\ x_e[n] = x[n], \quad x_o[n] = 0$

$(vi)\ x_e[n] = 0.5\cos(n-\pi/6) + 0.5\cos(-n-\pi/6)$
$x_o[n] = 0.5\cos(n-\pi/6) - 0.5\cos(-n-\pi/6)$

Section 2.3 Problems

2.12. (a) Determine and plot, separately, the even and the odd parts of the discrete-time signal $x[n]$ shown in Figure P2.12.

(b) Write a MATLAB® program to calculate and plot the even and odd parts of the signal shown in Figure P2.12.

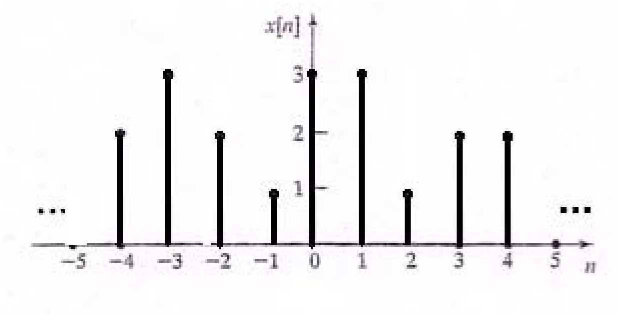
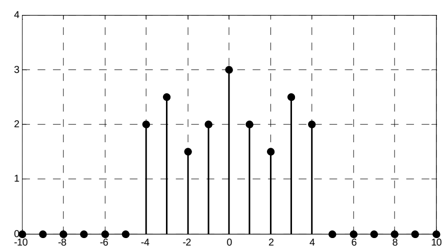
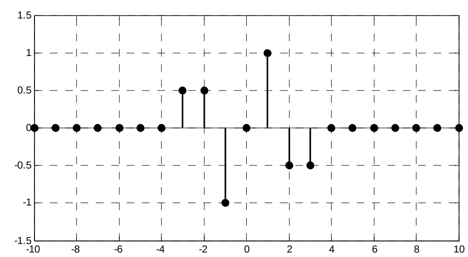


Figure P2.12

Solution P2.12

(a) $x_e[n] = (x[n] + x[-n])/2; \quad x_o[n] = (x[n] - x[-n])/2$





(b)

```
>> x=[0 0 0 0 0 0 2 3 2 1 3 3 1 2 2 0 0 0 0 0 0];
>> n=[-10:10];
>> xm=fliplr(x);
>> xe=(x+xm)/2;
>> xo=(x-xm)/2;
>> stem(n,xe);
>> stem(n,xo);
>> stem(xe+x0);
>> stem(n,xe+xo);
```

<u>Section 2.3 Problems</u>

2.13. The signals $x_1[n]$ and $x_2[n]$ are given: $x_1 = \cos(0.2\pi n)$ and $x_2 = \cos(0.125\pi n)$

(a) Determine if each of the signals is periodic, and if so, determine the number of samples per fundamental period.

(b) Determine if the sum of the two signals is periodic. If so, determine the number of samples per fundamental period.

Solution P2.13

(a)

$$x_1[n] = \cos(0.2\pi n + 0.2\pi N_{01}) = \cos(0.2\pi n + 2\pi k)$$
$$\therefore 0.2N_{01} = 2k \Rightarrow N_{01} = 10, k = 1,\ x_1[n]\ is\ periodic.$$

$$x_2[n] = \cos(0.125\pi n + 0.125\pi N_{02}) = \cos(0.125\pi n + 2\pi k)$$
$$\therefore 0.125N_{02} = 2k \Rightarrow N_{02} = \frac{2}{0.125} = 16, k = 1,\ x_2[n]\ is\ periodic.$$

$$x_3[n] = \cos(0.4\pi n + 0.4\pi N_{03}) = \cos(0.4\pi n + 2\pi k)$$
$$\therefore 0.4N_{03} = 2k \Rightarrow N_{03} = 5, k = 1,\ x_3[n]\ is\ periodic.$$

(b)

$$k_2/k_1 = 0.125/0.2 = 5/8,\ k_3/k_1 = 0.4/0.2 = 2/1 = 16/8$$
$$0.2N_{01} = 2k_1 \Rightarrow N_{01} = \left(2/0.2\right)8 = 80$$
$$0.125N_{02} = 2k_2 \Rightarrow N_{02} = \left(2/0.125\right)5 = 80$$
$$0.4N_{03} = 2k_3 \Rightarrow N_{03} = \left(2/0.4\right)16 = 80$$
$$\therefore the\ sum\ is\ periodic.$$

Section 2.3 Problems

2.14. (a) A continuous-time signal, $x(t) = \cos(2\pi t)$, is sampled every T seconds. Determine if the resulting discrete-time signal, $x[n] = x(nT)$, is periodic if:

$(i) T = 5s.$ $(ii) T = 0.125s$

$(iii) T = 0.1s$ $(iv) T = 0.130s$

$(v) T = 4/3\ s$

(b) For the sampled signals in part (a) that are periodic, find the number of periods of $x(t)$ in one period of $x[n]$.
(c) For the sampled signals in part (a) that are periodic, find the number of samples in one period of $x[n]$.

Solution P2.14

(a)(i)

$x[n] = \cos(10\pi n) = \cos(10\pi n + 10\pi N_0) = \cos(10\pi n + 2\pi k)$

$\therefore 2\pi k = 10\pi N_0,\ N_0 = k/5 \Rightarrow N_0 = 1, k = 5 \therefore$ *periodic* (constant)

(ii)

$x[n] = \cos(0.25\pi n) = \cos(0.25\pi n + 0.25\pi N_0) = \cos(0.25\pi n + 2\pi k)$

$\therefore 2\pi k = 0.25\pi N_0,\ N_0 = 2k/0.25 \Rightarrow N_0 = 8, k = 1 \therefore$ *periodic*

(iii)

$x[n] = \cos(0.2\pi n) = \cos(0.2\pi n + 0.2\pi N_0) = \cos(0.2\pi n + 2\pi k)$

$\therefore 2\pi k = 0.2\pi N_0,\ N_0 = 2k/0.2 \Rightarrow N_0 = 10, k = 1 \therefore$ *periodic*

(iv)

$x[n] = \cos(0.26\pi n) = \cos(0.26\pi n + 0.26\pi N_0) = \cos(0.26\pi n + 2\pi k)$

$\therefore 2\pi k = 0.26\pi N_0,\ N_0 = 2k/0.26 \Rightarrow N_0 = 100, k = 13, \therefore$ *periodic*

(v)

$x[n] = \cos\left((8/3)\pi n\right) = \cos\left((8/3)\pi n + (8/3)\pi N_0\right) = \cos\left((8/3)\pi n + 2\pi k\right)$

$\therefore 2\pi k = (8/3)\pi N_0,\ N_0 = 2k/(8/3) \Rightarrow N_0 = 3, k = 4, \therefore$ *periodic*

(b) (i) $k = 5$, (ii) $k = 1$, (iii) $k = 1$, (iv) $k = 13$, (v) $k = 4$.

(c) (i) $N_0 = 1$, (ii) $N_0 = 8$, (iii) $N_0 = 10$, (iv) $N_0 = 100$, (v) $N_0 = 3$.

Section 2.3 Problems

2.15. (a) Determine which of the discrete-time signals listed below are periodic in Ω.

$(i)\cos(\pi n)$	$(ii)\cos(3\pi n/2 + \pi/4)$
$(iii)\sin(0.01\pi n)$	$(iv)\sin(3.15n)$
$(v)\sin(3.15\pi n)$	$(vi)\cos(\pi n/2) + 1$

(b) For those signals in Part (a) that are periodic, determine the number of samples per period.

Solution P2.15

(a) All are periodic in Ω, the following is a test for periodicity in n.

(i)

$\cos(\pi n + \pi N_0) = \cos(\pi n + 2k\pi) \Rightarrow N_0 = 2k, \therefore k = 1, N_0 = 2,$ *periodic*

(ii)

$\cos(3\pi n/2 + \pi/4 + 3\pi N_0/2) = \cos(3\pi n/2 + \pi/4 + 2\pi k) \Rightarrow 3N_0/2 = 2k,$

$\therefore k = 3, N_0 = 4,$ *periodic*

(iii)

$\sin(0.01\pi + 0.01\pi N_0) = \sin(0.01\pi + 2\pi k) \Rightarrow 2k = 0.01N_0, \therefore k = 1, N_0 = 200,$ *periodic*

(iv)

$$\sin(3.15n + 3.15N_0) = \sin(3.15n + 2\pi k) \Rightarrow {}^{2\pi}\!/_{3.15} = {}^{N_0}\!/_{k},$$

$$N_0/k \textit{ is irrational}, \ \therefore \ \textit{nonperiodic.}$$

(v)

$$\sin(3.15\pi n + 3.15\pi N_0) = \sin(3.15\pi n + 2\pi k) \Rightarrow 2k = 3.15N_0,$$

$$N_0/k = 200/315 = 40/63, k = 63, N_0 = 40, \ \textit{periodic}$$

(vi)

$$\cos(0.5\pi n + 0.5\pi N_0) + 1 = \cos(0.5\pi n + 2\pi k) + 1 \Rightarrow 0.5N_0 = 2k, k = 1, N_0 = 4, \ \textit{periodic}$$

(b)

(i) $N_0 = 2$ (ii) $N_0 = 4$ (iii) $N_0 = 200$ (iv) not periodic in n (v) $N_0 = 40$ (vi) $N_0 = 4$

Section 2.4 Problems: Common Discrete-Time Signals

2.16. Given the discrete-time sequence $x[n] = \begin{cases} 0.5^n, -6 < n < 6 \\ 0, \quad \text{otherwise} \end{cases}$

(a) sketch the sequence as a function of *n*,

(b) sketch $x[n]u[n]$ vs. *n*.

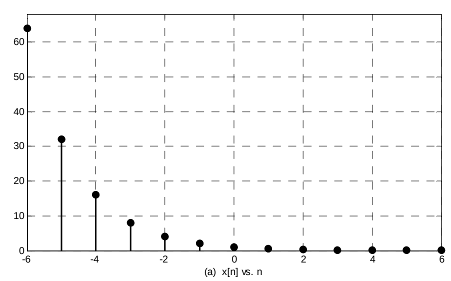
(c) sketch $x[n]u[-n]$ vs. *n*.

(d) sketch $x[-n]u[-n]$ vs. *n*.

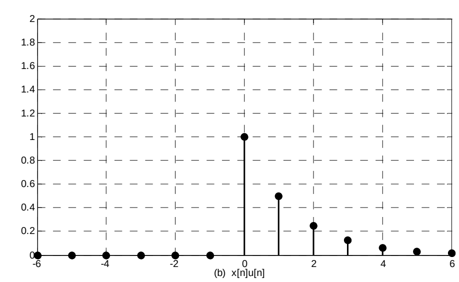
(e) sketch $x[2-n]u[2-n]$ vs. *n*.

Solution P2.16

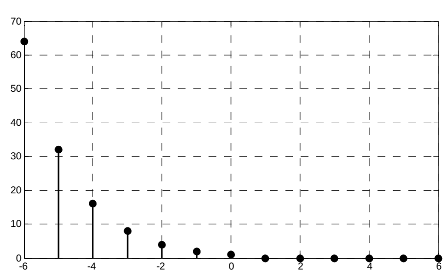
(a)



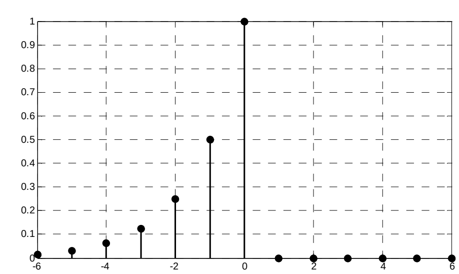
(b)



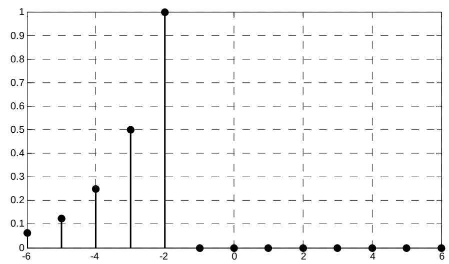
(c)



(d)



(e)



(e) x[2-n]u[2-n]

Section 2.4 Problems

2.17. The discrete-time signals listed below are the result of sampling a continuous-time signal with a sampling period $T = 0.1$ s. Find the time constant τ for each signal and the frequency ω for each of the sinusoidal signals.

$$(a)\ (0.3)^n \qquad (b)\ (0.3)^n\cos(n)$$
$$(c)\ (-0.3)^n \qquad (d)\ (0.3)^n\sin(n+1)$$
$$(e)\ (0.5)^n\cos(3n) \qquad (f)\ (0.5)^n\sin(120\pi n+\pi/4)$$

Solution P2.17

$T = 0.1$ s.

$e^{-at}\big|_{t=nT} = e^{-anT} = e^{(-aT)n}$, where $\tau = 1/a$. $\cos(\omega t)\big|_{t=nT} = \cos\left((\omega T)n\right) = \cos(bn)$, where $\omega = b/T$.

(a) $(0.3)^n = \left(e^{-T/\tau}\right)^n \Rightarrow \ln(0.3) = -T/\tau, \therefore \tau = -0.1/\ln(0.3) = 0.0106$ s.

(b) from (a), $\tau = 0.0106$ s. $\omega = b/T = 1/0.1 = 10$ rad/s

(c) $(-0.3)^n = (0.3)^n(-1)^n = (0.3)^n\cos(\pi n)$. From (a), $\tau = 0.0106$ s. $\omega = \pi/T = 10\pi$ rad/s.

(d) $(0.3)^n\sin(n+1)$. From (a), $\tau = 0.0106$. $\omega = b/T = 1/0.1 = 10$ rad/s.

(e) $(0.5)^n\cos(3n)$. $\tau = -0.1/\ln(0.5) = 0.144$ s. $\omega = 3/0.1 = 30$ rad/s.

(f) From (e), $(0.5)^n\sin(120\pi n + \pi/4)$. $\tau = 0.144$ s. $\omega = 120\pi/0.1$ rad/s.

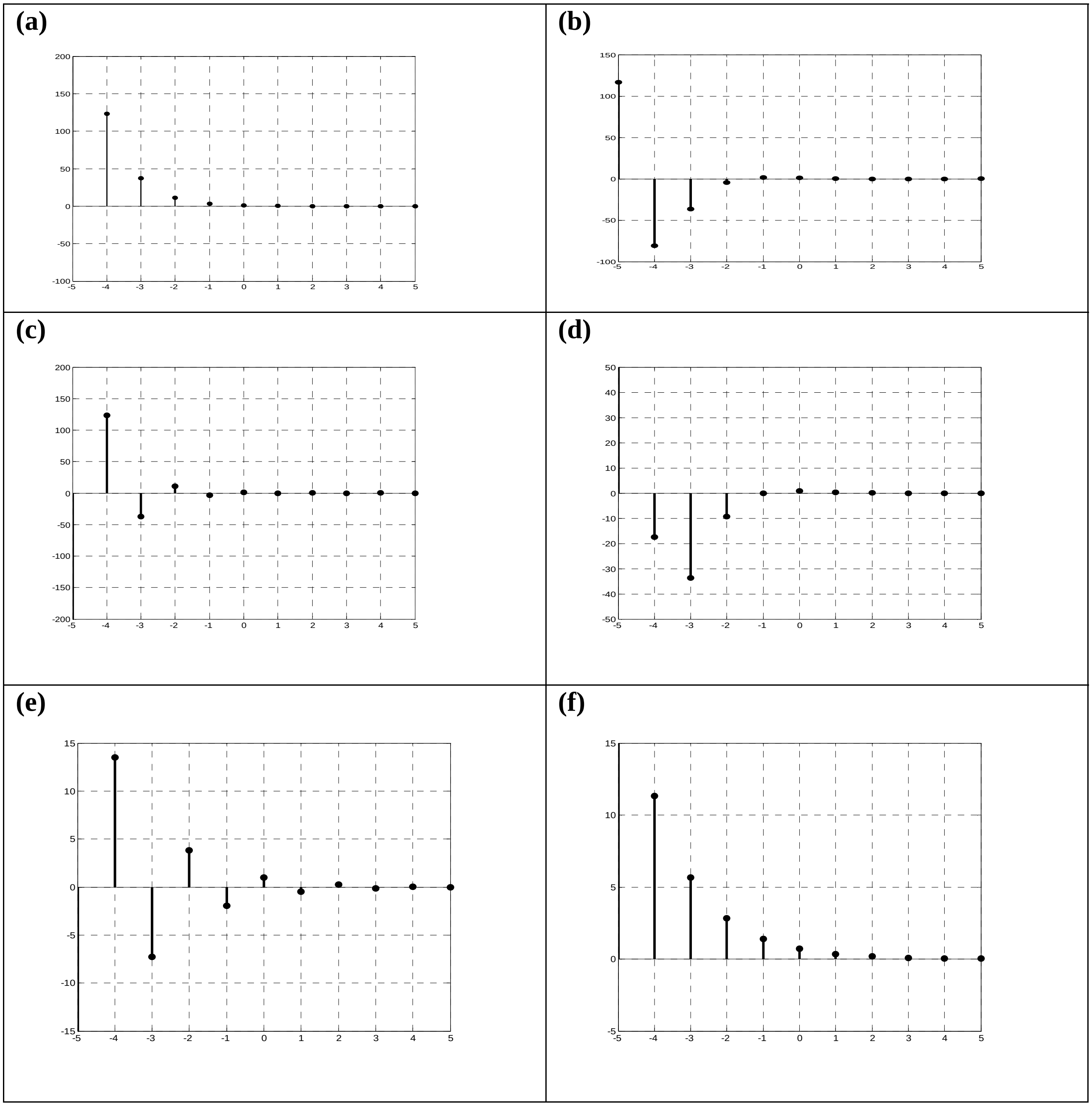
Section 2.4 Problems

2.18. Use MATLAB® to generate and plot the signals of Problem 2.16(a)-(f).

Solution P2.18

(MATLAB® commands for Part (e)

```
>> for m=-5:5
     k=m+6;
     x(k)=(0.5)^m*cos(3*m);
   end
>> stem(n,x)
```



Section 2.4 Problems

2.19. The discrete-time signal $x[n] = 4(0.6)^n \cos(4n)$ is formed by sampling a continuous-time signal, $x(t)$, with a sampling frequency of 10 samples-per-second.

(a) What is the time constant associated with the signal $x(t)$?

(b) What is the frequency of oscillations in $x(t)$?

(c) Plot $x[n]$ using MATLAB® and use the plot to confirm your answers to (a) and (b).

Solution P2.19

$x[n] = 4(0.6)^n \cos(4n) = 4a^n \cos(bT), \quad T = 0.1$ s.

(a) $\tau = {-T}/{\ln(a)} = {-0.1}/{\ln(0.6)} = 0.196$ s.

(b) $\omega = {b}/{T} = {4}/{0.1} = 40$ rad/s.

Section 2.4 Problems

2.20.(a) Write a mathematical model of each of the sequences shown in Figure P2.20, defining the sequence as a function of n.

(b) Use MATLAB® to generate and plot the function of n that you wrote in Part (a). Compare your plots with Figure P2.20 to check your answers for Part (a).

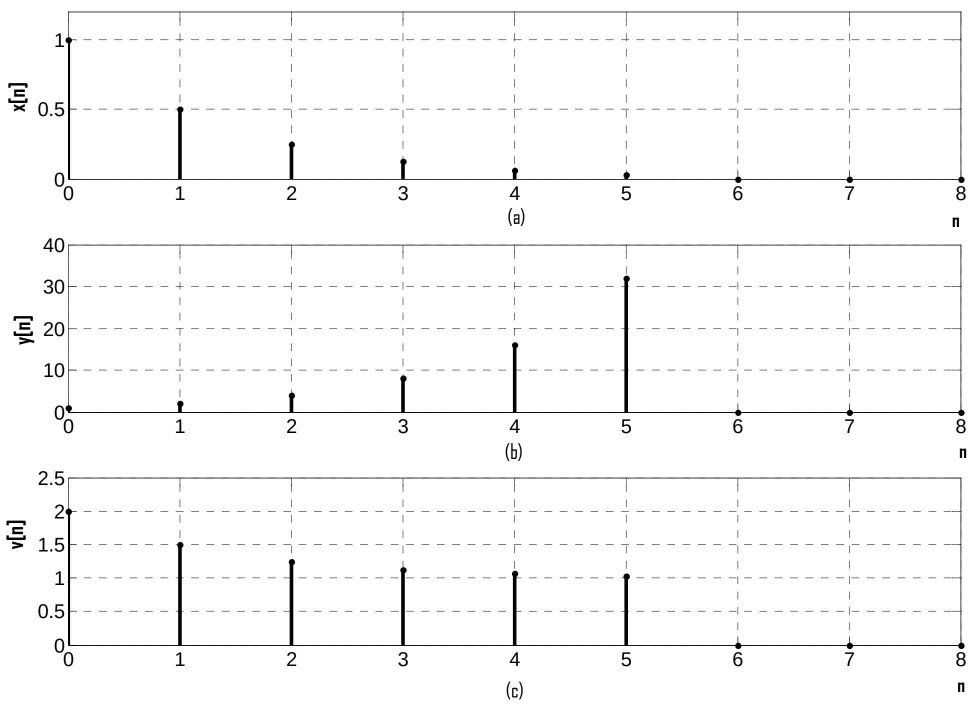


Figure P2.20.

Solution P2.20

(a) $x[n] = (0.5)^n u[n]$

(b) $y[n] = 30(0.5)^{-(5-n)}\left(u[6-n] - u[-n]\right) = 30(2)^{n-5}\left(u[n] - u[n-6]\right)$

(c) $v[n] = \left(1 + (0.5)^n\right)\left(u[n] - u[n-6]\right)$

Section 2.5 Problems: Discrete-Time Systems

2.21. A system's output can be calculated using the difference equation

$$y[n] = 5y[n-1] + y[n-2] + 3x[n-1] - 4x[n-2]$$

(a) Is the system linear? Justify your answer.
(b) Is the system time invariant? Justify your answer.
(b) Is the system causal? Justify your answer.
(c) Is the system stable? Justify your answer.

Solution P2.21

$$y[n] = 5y[n-1] + y[n-2] + 3x[n-1] - 4x[n-2]$$

(a) The system is linear

$$ay_1[n] + by_2[n] = 5\left(ay_1[n-1] + by_2[n-1]\right) + ay_1[n-2] + by_2[n-2] + 3\left(ax_1[n-1] + bx_2[n-1]\right)$$
$$-4\left(ax_1[n-1] + bx_2[n-1]\right)$$
$$\Rightarrow a\left(y_1[n] - 5y_1[n-1] - 3x_1[n-1] + 4x_1[n-2]\right) + b\left(y_2[n] - 5y_2[n-1] - 3x_2[n-1] + 4x_2[n-2]\right) = 0$$
$$a(0) + b(0) = 0$$

(b) $y[n-n_0] = 5y[n-1-n_0] + y[n-2-n_0] + 3x[n-1-n_0] - 4x[n-2-n_0], \therefore$ time invariant

(c) The system is causal since $y[n]$ is not a function of future values of $x[n]$.

(d) y is bounded if x is bounded. The system is BIBO stable.

Section 2.5 Problems

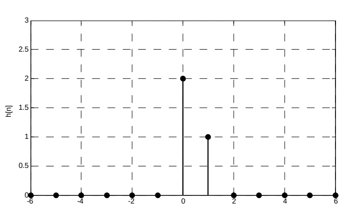
2.22. The impulse response of a LTID system is $h[n] = (0.5)^{n-1}(u[n] - u[n-2])$. The input signal is $x[n] = \delta[n] + \delta[n-1]$.

(a) Is the system causal? Justify your answer.

(b) Is the system stable? Justify your answer.

Solution P2.22

$$h[n] = (0.5)^{n-1}\left(u[n] - u[n-2]\right)$$



(a) The system is causal. The impulse response is zero for all $n < 0$.

(b) The system is stable. The output will be finite for finite inputs.

Section 2.5 Problems

2.23. Determine if each of the systems described by the equations given below is causal.
(a) $y[n] = 0.3x[n-2] + 0.1x[n-1]$
(b) $y[n] = x[n+2]$
(c) $y[n] = -0.2x[n+1] + 2x[n] + 0.5x[n-1] + 0.25x[n-2]$
(d) $y[n] = 0.5x[n-1] + 0.25x[n-2] + 0.125x[n-3] + 0.0625x[n+1]$

Solution P2.23
(a) Causal. The output, $y[n]$, is determined by past or current values of the input $x[n]$.

(b) Not Causal. The output, $y[n]$, is determined by future values of the input, $x[n]$.

(c) Not Causal. The output, $y[n]$, is partially determined by future values of the input, $x[n]$.

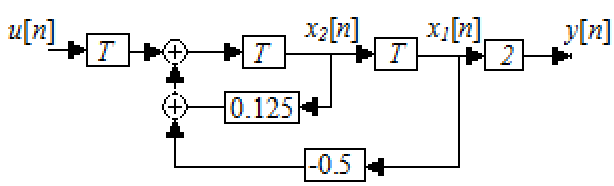
(d) Not Causal. The output, $y[n]$, is partially determined by future values of the input, $x[n]$.

Section 2.5 Problems

2.24. Sketch a block diagram for a system that will solve the difference equations:

$$x_1[n+1] = x_2[n]$$
$$x_2[n+1] = -0.5x_1[n] + 0.125x_2[n] + u[n]$$
$$y[n] = 2x_1[n]$$

Solution P2.24



Section 2.5 Problems

2.25. Write a difference equation model for the system shown in Figure P2.25.

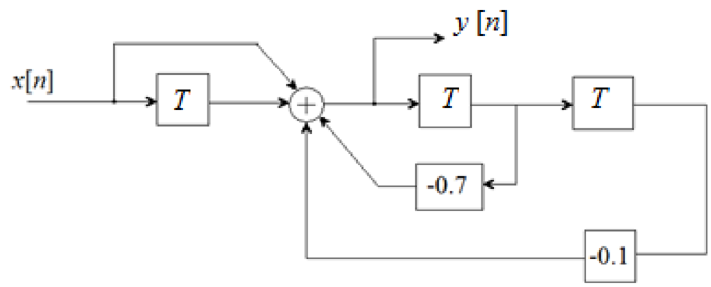


Figure P2.25

Solution P2.25

1. Recognize that the output of each block is the input to that block delayed by one sample period.

2. Write the equation for the output of the summing junction.

$$y[n] = -0.7y[n-1] - 0.1y[n-2] + x[n] + x[n-1]$$

Section 2.6 Problems: Convolution for Discrete-Time Systems

2.26. A linear, time-invariant system has the input signal $x[n]$, the output signal $y[n]$, and the impulse response $h[n]$. For each of the cases described below, find the output $y[n]$. The signals are plotted in Figure P2.26.

(a) $x[n]$ in (a), $h[n]$ in (b)
(b) $x[n]$ in (b), $h[n]$ in (d)
(c) $x[n]$ in (a), $h[n]$ in (c)
(d) $x[n]$ in (d), $h[n]$ in (e)
(e) $x[n]$ in (b), $h[n]$ in (f)

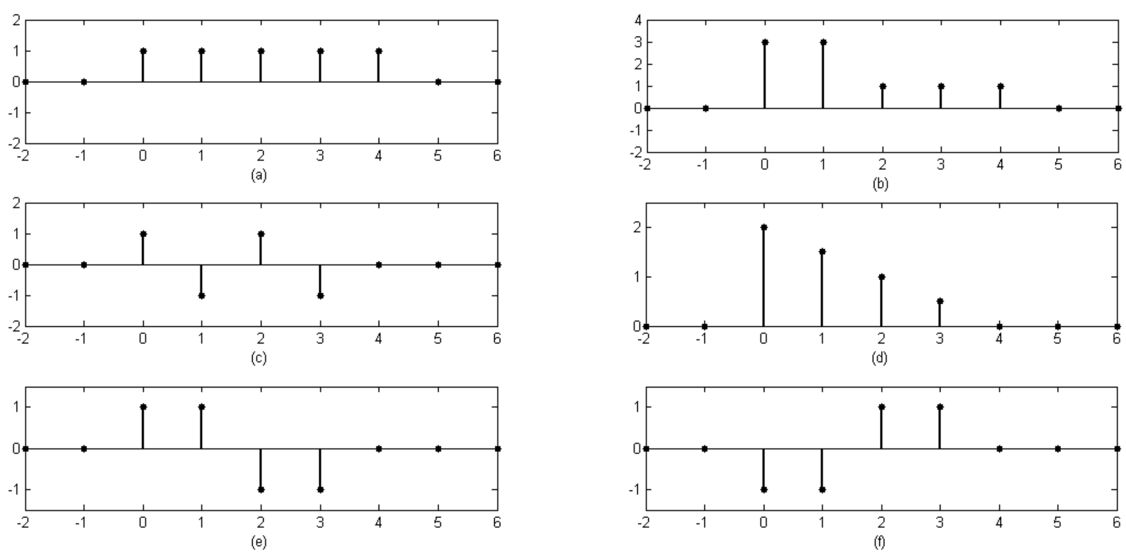


Figure P2.26

Solution P2.26

(a) $y[n]=0, n<0,$ $y[0]=1\times 3=3$ $y[1]=1\times 3+1\times 3=6$ $y[2]=1\times 3+1\times 3+1\times 1=7$ $y[3]=1\times 3+1\times 3+1\times 1+1\times 1=8$ $y[4]=1\times 3+1\times 3+1\times 1+1\times 1+1\times 1=9$ $y[5]=0\times 3+1\times 3+1\times 1+1\times 1+1\times 1=6$ $y[6]=0\times 3+0\times 3+1\times 1+1\times 1+1\times 1=3$ $y[7]=0\times 3+0\times 3+0\times 1+1\times 1+1\times 1=2$ $y[8]=\times 3+0\times 3+0\times 1+0\times 1+1\times 1=1$ $y[n]=0, n\geq 9$	(b) $y[n]=0, n<0,$ $y[0]=2\times 3=6$ $y[1]=2\times 3+1.5\times 3=10.5$ $y[2]=2\times 1+1.5\times 3+1\times 3=9.5$ $y[3]=2\times 1+1.5\times 1+1\times 3+0.5\times 3=8$ $y[4]=2\times 1+1.5\times 1+1\times 3+0.5\times 3+0\times 3=8$ $y[5]=2\times 0+1.5\times 1+1\times 1+0.5\times 3+0\times 3=5$ $y[6]=2\times 0+1.5\times 0+1\times 1+0.5\times 1+0\times 3=1.5$ $y[7]=2\times 0+1.5\times 0+1\times 0+0.5\times 1=0.5$ $y[n]=0, n\geq 8$
(c) $y[n]=0, n<0$ $n=$ 0 1 2 3 4 5 6 7 8 9 y[n]= 1 0 1 0 0 -1 0 -1 0 0 $y[n]=0, n\geq 8$	(d) $y[n]=0, n<0$ $n=$ 0 1 2 3 4 5 6 7 8 9 y[n]= 2 3.5 0.5 -2 -2 -1.5 -0.5 0 0 0 $y[n]=0, n\geq 7$

(e)
$y[n]=0, n<0$
$n=$ 0 1 2 3 4 5 6 7 8 9
y[n]= -3 -6 -1 4 2 1 2 1 0 0
$y[n]=0, n\geq 8$
	MATLAB Solution:
	(a)
	`x= [1 1 1 1 1 0 0 0 0 0];`
	`h=[3 3 1 1 1 0 0 0 0 0];`
	`y=conv(x,h)`

Section 2.6 Problems

2.27. (a) Consider two LTI systems connected in cascade. The impulse responses of the two systems are identical, with $h_1[n] = h_2[n] = (0.8)^n$. Find the impulse response of the total system.

(b) Repeat part (a) for $h_1[n] = h_2[n] = \delta[n\text{-}3]$.

(c) Repeat part (a) for $h_1[n] = h_2[n] = u[n] - u[n\text{-}2]$.

Solution P2.27

(a) $h[n]=h_1[n]*h_2[n]=\sum_{k=-\infty}^{\infty}0.8^k u[k]\times 0.8^{n-k}u[n-k]=\sum_{k=0}^{n}0.8^k\times 0.8^{n-k}=n0.8^n u[n]$

(b) $h[n]=\delta[n-3]*\delta[n-3]=\delta[n-6]$

(c) $h[n]=u[n-2]*u[n-2]=\sum_{k=-\infty}^{\infty}u[k-2]u[n-2-k]=\sum_{k=2}^{n-2}1=(n-4)u(n-4)$

Section 2.6 Problems

2.28. Calculate and write a mathematical expression for the output, $y[n]$, of a linear, time-invariant, discrete- time system with the impulse response, $h[n]=3\delta[n]+\delta[n-1]+\delta[n-2]$ if the input signal is $x[n]=2\delta[n]+3\delta[n-1]+4\delta[n-2]+\delta[n-3]$.

Solution P2.28

$h[n] = [3 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \ldots]$

$x[n] = [2 \quad 3 \quad 4 \quad 1 \quad 0 \quad 0 \quad 0 \ldots]$

$y[n] = x[n] * h[n]$

$y[n] = 0, n < 0,$

$y[0] = 3 \times 2 = 6$

$y[1] = 3 \times 3 + 1 \times 2 = 11$

$y[2] = 3 \times 4 + 1 \times 3 + 1 \times 2 = 17$

$y[3] = 3 \times 1 + 1 \times 4 + 1 \times 3 + 0 \times 2 = 10$

$y[4] = 3 \times 0 + 1 \times 1 + 1 \times 4 + 0 \times 2 = 5$

$y[5] = 3 \times 0 + 1 \times 0 + 1 \times 1 = 1$

$y[n] = 0, n > 5$

n = [0 1 2 3 4 5 6 7 8 …]
y[n]=[6 11 17 10 5 1 0 0 0 . . .]

Section 2.6 Problems

2.29. The impulse response of a LTID system is $h[n] = (0.5)^{n-1}(u[n] - u[n-2])$. The input signal is $x[n] = \delta[n] + \delta[n-1]$. Calculate and plot the output signal, $y[n]$.

Solution P2.29

$h[n] = (0.5)^{n-1}(u[n] - u[n-2]) = 2\delta[n] + 1\delta[n-1]$

$x[n] = 1\delta[n] + 1\delta[n-1]$

$y[n] = x[n] * h[n]$

$y[n] = 0, n < 0$

$y[0] = 2 \times 1 = 2$

$y[1] = 2 \times 1 + 1 \times 1 = 3$

$y[2] = 2 \times 0 + 1 \times 1 = 1$

$y[n] = 0, n > 2$

Section 2.6 Problems

2.30. The impulse response of a LTID system is $h[n] = (0.5)^{n-1}(u[n] - u[n-2])$. The input signal is $x[n] = \delta[n] + \delta[n-1]$.

(a) Is the system causal?
(b) Is the system stable?

Solution P2.30

(a) The system is causal. The response is determined by present and past inputs.

(b) The system is stable. The impulse response decays to zero with increasing time and it is time limited.