PROBLEM 2-1

Statement: Figure P2-1 shows stress-strain curves for three failed tensile-test specimens. All are plotted on the same scale.
(a) Characterize each material as brittle or ductile.
(b) Which is the stiffest?
(c) Which has the highest ultimate strength?
(d) Which has the largest modulus of resilience?
(e) Which has the largest modulus of toughness?

Solution: See Figure P2-1 and Mathcad file P0201.

1. The material in Figure P2-1(a) has a moderate amount of strain beyond the yield point, P2-1(b) has very little, and P2-1(c) has considerably more than either of the other two. Based on this observation, the material in Figure P2-1(a) is **mildly ductile**, that in P2-1(b)is **brittle**, and that in P2-1(c) is **ductile**.

2. The stiffest material is the one with the grearesr slope in the elastic range. Determine this by dividing the rise by the run of the straight-line portion of each curve. The material in Figure P2-1(c) has a slope of 5 stress units per strain unit, which is the greatest of the three. Therefore, **P2-1(c) is the stiffest**.

3. Ultimate strength corresponds to the highest stress that is achieved by a material under test. The material in Figure P2-1(b) has a maximum stress of 10 units, which is considerably more than either of the other two. Therefore, **P2-1(b) has the highest ultimate strength**.

4. The modulus of resilience is the area under the elastic portion of the stress-starin curve. From observation of the three graphs, the stress and strain values at the yield points are:

   P2-1(a) $\sigma_{ya} := 5$ $\varepsilon_{ya} := 5$

   P2-1(b) $\sigma_{yb} := 9$ $\varepsilon_{yb} := 2$

   P2-1(c) $\sigma_{yc} := 5$ $\varepsilon_{yc} := 1$

   Using equation (2.7), the modulus of resiliency for each material is, approximately,

   $$P21a := \frac{1}{2}\cdot\sigma_{ya}\cdot\varepsilon_{ya} \qquad P21a = 12.5$$

   $$P21b := \frac{1}{2}\cdot\sigma_{yb}\cdot\varepsilon_{yb} \qquad P21b = 9$$

   $$P21c := \frac{1}{2}\cdot\sigma_{yc}\cdot\varepsilon_{yc} \qquad P21c = 2.5$$

   P2-1 (a) has the largest modulus of resilience

5. The modulus of toughness is the area under the stress-starin curve up to the point of fracture. By inspection, P2-1 (c) has the largest area under the stress-strain curve therefore, it has the largest modulus of toughness.

PROBLEM 2-2

Statement: Determine an approximate ratio between the yield strength and ultimate strength for each material shown in Figure P2-1.

Solution: See Figure P2-1 and Mathcad file P0202.

1. The yield strength is the value of stress at which the stress-strain curve begins to be nonlinear. The ultimate strength is the maximum value of stress attained during the test. From the figure, we have the following values of yield strength and tensile strength:

Figure P2-1(a)	$S_{ya} := 5$	$S_{ua} := 6$
Figure P2-1(b)	$S_{yb} := 9$	$S_{ub} := 10$
Figure P2-1(c)	$S_{yc} := 5$	$S_{uc} := 8$

2. The ratio of yield strength to ultimate strength for each material is:

Figure P2-1(a)	$ratio_a := \dfrac{S_{ya}}{S_{ua}}$	$ratio_a = 0.83$
Figure P2-1(b)	$ratio_b := \dfrac{S_{yb}}{S_{ub}}$	$ratio_b = 0.90$
Figure P2-1(c)	$ratio_c := \dfrac{S_{yc}}{S_{uc}}$	$ratio_c = 0.63$

PROBLEM 2-3

Statement: Which of the steel alloys shown in Figure 2-19 would you choose to obtain
(a) Maximum strength
(b) Maximum modulus of resilience
(c) Maximum modulus of toughness
(d) Maximum stiffness

Given: Young's modulus for steel $E := 207 \cdot GPa$

Solution: See Figure 2-19 and Mathcad file P0203.

1. Determine from the graph: values for yield strength, ultimate strength and strain at fracture for each material.

Steel	Yield Strength	Ultimate Strength	Fracture Strain
AISI 1020:	$Sy_{1020} := 300 \cdot MPa$	$Sut_{1020} := 400 \cdot MPa$	$\varepsilon f_{1020} := 0.365$
AISI 1095:	$Sy_{1095} := 550 \cdot MPa$	$Sut_{1095} := 1050 \cdot MPa$	$\varepsilon f_{1095} := 0.11$
AISI 4142:	$Sy_{4142} := 1600 \cdot MPa$	$Sut_{4142} := 2430 \cdot MPa$	$\varepsilon f_{4142} := 0.06$

Note: The 0.2% offset method was used to define a yield strength for the AISI 1095 and the 4142 steels.

2. From the values of S_{ut} above it is clear that the **AISI 4142** has maximum strength.

3. Using equation (2-7) and the data above, determine the modulus of resilience.

$$U_{R1020} := \frac{1}{2} \cdot \frac{Sy_{1020}^{\,2}}{E} \qquad U_{R1020} = 0.22 \cdot \frac{MN \cdot m}{m^3}$$

$$U_{R1095} := \frac{1}{2} \cdot \frac{Sy_{1095}^{\,2}}{E} \qquad U_{R1095} = 0.73 \cdot \frac{MN \cdot m}{m^3}$$

$$U_{R4142} := \frac{1}{2} \cdot \frac{Sy_{4142}^{\,2}}{E} \qquad U_{R4142} = 6.18 \cdot \frac{MN \cdot m}{m^3}$$

Even though the data is approximate, the **AISI 4142** clearly has the largest modulus of resilience.

4. Using equation (2-8) and the data above, determine the modulus of toughness.

$$U_{T1020} := \frac{1}{2} \cdot \left(Sy_{1020} + Sut_{1020}\right) \cdot \varepsilon f_{1020} \qquad U_{T1020} = 128 \cdot \frac{MN \cdot m}{m^3}$$

$$U_{T1095} := \frac{1}{2} \cdot \left(Sy_{1095} + Sut_{1095}\right) \cdot \varepsilon f_{1095} \qquad U_{T1095} = 88 \cdot \frac{MN \cdot m}{m^3}$$

$$U_{T4142} := \frac{1}{2} \cdot \left(Sy_{4142} + Sut_{4142}\right) \cdot \varepsilon f_{4142} \qquad U_{T4142} = 121 \cdot \frac{MN \cdot m}{m^3}$$

Since the data is approximate, there is no significant difference between the 1020 and 4142 steels. Because of the wide difference in shape and character of the curves, one should also determine the area under the curves by graphical means. When this is done, the area under the curve is about 62 square units for 1020 and 66 for 4142. Thus, they seem to have about equal toughness, which is about 50% greater than that for the 1095 steel.

5. All three materials are steel therefore, the **stiffnesses are the same**.

PROBLEM 2-4

Statement: Which of the aluminum alloys shown in Figure 2-21 would you choose to obtain
(a) Maximum strength
(b) Maximum modulus of resilience
(c) Maximum modulus of toughness
(d) Maximum stiffness

Given: Young's modulus for aluminum $E := 71.7 \cdot GPa$

Solution: See Figure 2-21 and Mathcad file P0204.

1. Determine, from the graph, values for yield strength, ultimate strength and strain at fracture for each material.

Alum	Yield Strength	Ultimate Strength	Fracture Strain
1100:	$Sy_{1100} := 120 \cdot MPa$	$Sut_{1100} := 130 \cdot MPa$	$\varepsilon f_{1100} := 0.170$
2024-T351:	$Sy_{2024} := 330 \cdot MPa$	$Sut_{2024} := 480 \cdot MPa$	$\varepsilon f_{2024} := 0.195$
7075-T6:	$Sy_{7075} := 510 \cdot MPa$	$Sut_{7075} := 560 \cdot MPa$	$\varepsilon f_{7075} := 0.165$

Note: The 0.2% offset method was used to define a yield strength for all of the aluminums.

2. From the values of Sut above it is clear that the **7075-T6 has maximum strength**.

3. Using equation (2-7) and the data above, determine the modulus of resilience.

$$U_{R1100} := \frac{1}{2} \cdot \frac{Sy_{1100}^{\ 2}}{E} \qquad U_{R1100} = 0.10 \cdot \frac{MN \cdot m}{m^3}$$

$$U_{R2024} := \frac{1}{2} \cdot \frac{Sy_{2024}^{\ 2}}{E} \qquad U_{R2024} = 0.76 \cdot \frac{MN \cdot m}{m^3}$$

$$U_{R7075} := \frac{1}{2} \cdot \frac{Sy_{7075}^{\ 2}}{E} \qquad U_{R7075} = 1.81 \cdot \frac{MN \cdot m}{m^3}$$

Even though the data is approximate, the **7075-T6** clearly has the largest modulus of resilience.

4. Using equation (2-8) and the data above, determine the modulus of toughness.

$$U_{T1100} := \frac{1}{2} \cdot \left(Sy_{1100} + Sut_{1100}\right) \cdot \varepsilon f_{1100} \qquad U_{T1100} = 21 \cdot \frac{MN \cdot m}{m^3}$$

$$U_{T2024} := \frac{1}{2} \cdot \left(Sy_{2024} + Sut_{2024}\right) \cdot \varepsilon f_{2024} \qquad U_{T2024} = 79 \cdot \frac{MN \cdot m}{m^3}$$

$$U_{T7075} := \frac{1}{2} \cdot \left(Sy_{7075} + Sut_{7075}\right) \cdot \varepsilon f_{7075} \qquad U_{T7075} = 88 \cdot \frac{MN \cdot m}{m^3}$$

Even though the data is approximate, the **7075-T6** has the largest modulus of toughness.

5. All three materials are aluminum therefore, the **stiffnesses are the same**.

PROBLEM 2-5

Statement: Which of the thermoplastic polymers shown in Figure 2-22 would you choose to obtain
(a) Maximum strength
(b) Maximum modulus of resilience
(c) Maximum modulus of toughness
(d) Maximum stiffness

Solution: See Figure 2-22 and Mathcad file P0205.

1. Determine, from the graph, values for yield strength, ultimate strength, strain at fracture, and modulus of elasticity for each material.

Plastic	Yield Strength	Ultimate Strength	Fracture Strain	Mod of Elasticity
Nylon 101:	$Sy_{Nylon} := 63 \cdot MPa$	$Sut_{Nylon} := 80 \cdot MPa$	$\varepsilon f_{Nylon} := 0.52$	$E_{Nylon} := 1.1 \cdot GPa$
HDPE:	$Sy_{HDPE} := 15 \cdot MPa$	$Sut_{HDPE} := 23 \cdot MPa$	$\varepsilon f_{HDPE} := 3.0$	$E_{HDPE} := 0.7 \cdot GPa$
PTFE:	$Sy_{PTFE} := 8.3 \cdot MPa$	$Sut_{PTFE} := 13 \cdot MPa$	$\varepsilon f_{PTFE} := 0.51$	$E_{PTFE} := 0.8 \cdot GPa$

2. From the values of Sut above it is clear that the **Nylon 101 has maximum strength**.

3. Using equation (2-7) and the data above, determine the modulus of resilience.

$$U_{RNylon} := \frac{1}{2} \cdot \frac{Sy_{Nylon}^{\ 2}}{E_{Nylon}} \qquad U_{RNylon} = 1.8 \cdot \frac{MN \cdot m}{m^3}$$

$$U_{RHDPE} := \frac{1}{2} \cdot \frac{Sy_{HDPE}^{\ 2}}{E_{HDPE}} \qquad U_{RHDPE} = 0.16 \cdot \frac{MN \cdot m}{m^3}$$

$$U_{RPTFE} := \frac{1}{2} \cdot \frac{Sy_{PTFE}^{\ 2}}{E_{PTFE}} \qquad U_{RPTFE} = 0.04 \cdot \frac{MN \cdot m}{m^3}$$

Even though the data is approximate, the **Nylon 101** clearly has the largest modulus of resilience.

4. Using equation (2-8) and the data above, determine the modulus of toughness.

$$U_{TNylon} := \frac{1}{2} \cdot \left(Sy_{Nylon} + Sut_{Nylon} \right) \cdot \varepsilon f_{Nylon} \qquad U_{TNylon} = 37 \cdot \frac{MN \cdot m}{m^3}$$

$$U_{THDPE} := \frac{1}{2} \cdot \left(Sy_{HDPE} + Sut_{HDPE} \right) \cdot \varepsilon f_{HDPE} \qquad U_{THDPE} = 57 \cdot \frac{MN \cdot m}{m^3}$$

$$U_{TPTFE} := \frac{1}{2} \cdot \left(Sy_{PTFE} + Sut_{PTFE} \right) \cdot \varepsilon f_{PTFE} \qquad U_{TPTFE} = 5 \cdot \frac{MN \cdot m}{m^3}$$

Even though the data is approximate, the **HDPE** has the largest modulus of toughness.

5. The **Nylon 101** has the steepest slope in the (approximately) elastic range and is, therefore, the stiffest of the three materials..

PROBLEM 2-6

Statement: A metal has a strength of 414 MPa at its elastic limit and the strain at that point is 0.002. What is the modulus of elasticity? What is the strain energy at the elastic limit? Assume that the test speimen is 12.8-mm dia and has a 50-mm gage length. Can you define the type of metal based on the given data?

Given: Elastic limit: Strength $S_{el} := 414 \cdot MPa$ Strain $\varepsilon_{el} := 0.002$

Test specimen: Diameter $d_o := 12.8 \cdot mm$ Length $L_o := 50 \cdot mm$

Solution: See Mathcad file P0206.

1. The modulus of elasticity is the slope of the stress-strain curve, which is a straight line, in the elastic region. Since one end of this line is at the origin, the slope (modulus of elasticity) is

$$E := \frac{S_{el}}{\varepsilon_{el}} \qquad E = 207 \cdot GPa$$

2. The strain energy per unit volume at the elastic limit is the area under the stress-strain curve up to the elastic limit. Since the curve is a straight line up to this limit, the area is one-half the base times the height, or

$$U'_{el} := \frac{1}{2} \cdot S_{el} \cdot \varepsilon_{el} \qquad U'_{el} = 414 \cdot \frac{kN \cdot m}{m^3}$$

The total strain energy in the specimen is the strain energy per unit volume times the volume,

$$U_{el} := U'_{el} \cdot \frac{\pi \cdot {d_o}^2}{4} \cdot L_o \qquad U_{el} = 2.7 \cdot N \cdot m$$

3. Based on the modulus of elasticity and using Table C-1, the material is **steel**.

PROBLEM 2-7

Statement: A metal has a strength of 41.2 kpsi (284 MPa) at its elastic limit and the strain at that point is 0.004. What is the modulus of elasticity? What is the strain energy at the elastic limit? Assume that the test speimen is 0.505-in dia and has a 2-in gage length. Can you define the type of metal based on the given data?

Given: Elastic limit: Strength $S_{el} := 41.2 \cdot ksi$ Strain $\varepsilon_{el} := 0.004$ $S_{el} = 284 \cdot MPa$

Test specimen: Diameter $d_o := 0.505 \cdot in$ Length $L_o := 2.00 \cdot in$

Solution: See Mathcad file P0207.

1. The modulus of elasticity is the slope of the stress-strain curve, which is a straight line, in the elastic region. Since one end of this line is at the origin, the slope (modulus of elasticity) is

$$E := \frac{S_{el}}{\varepsilon_{el}} \qquad E = 10.3 \cdot 10^6 \cdot psi \qquad E = 71 \cdot GPa$$

2. The strain energy per unit volume at the elastic limit is the area under the stress-strain curve up to the elastic limit. Since the curve is a straight line up to this limit, the area is one-half the base times the height, or

$$U'_{el} := \frac{1}{2} \cdot S_{el} \cdot \varepsilon_{el} \qquad U'_{el} = 82.4 \cdot \frac{lbf \cdot in}{in^3} \qquad U'_{el} = 568 \cdot \frac{kN \cdot m}{m^3}$$

The total strain energy in the specimen is the strain energy per unit volume times the volume,

$$U_{el} := U'_{el} \cdot \frac{\pi \cdot d_o^2}{4} \cdot L_o \qquad U_{el} = 33.0 \cdot in \cdot lbf$$

3. Based on the modulus of elasticity and using Table C-1, the material is **aluminum**.

PROBLEM 2-8

Statement: A metal has a strength of 134 MPa at its elastic limit and the strain at that point is 0.006. What is the modulus of elasticity? What is the strain energy at the elastic limit? Assume that the test speimen is 12.8-mm dia and has a 50-mm gage length. Can you define the type of metal based on the given data?

Given: Elastic limit: Strength $S_{el} := 134 \cdot MPa$ Strain $\varepsilon_{el} := 0.003$

Test specimen: Diameter $d_o := 12.8 \cdot mm$ Length $L_o := 50 \cdot mm$

Solution: See Mathcad file P0208.

1. The modulus of elasticity is the slope of the stress-strain curve, which is a straight line, in the elastic region. Since one end of this line is at the origin, the slope (modulus of elasticity) is

$$E := \frac{S_{el}}{\varepsilon_{el}} \qquad E = 45 \cdot GPa$$

2. The strain energy per unit volume at the elastic limit is the area under the stress-strain curve up to the elastic limit. Since the curve is a straight line up to this limit, the area is one-half the base times the height, or

$$U'_{el} := \frac{1}{2} \cdot S_{el} \cdot \varepsilon_{el} \qquad U'_{el} = 201 \cdot \frac{kN \cdot m}{m^3}$$

The total strain energy in the specimen is the strain energy per unit volume times the volume,

$$U_{el} := U'_{el} \cdot \frac{\pi \cdot d_o^2}{4} \cdot L_o \qquad U_{el} = 1.3 \cdot N \cdot m$$

3. Based on the modulus of elasticity and using Table C-1, the material is **magnesium**.

PROBLEM 2-9

Statement: A metal has a strength of 100 kpsi (689 MPa) at its elastic limit and the strain at that point is 0.006. What is the modulus of elasticity? What is the strain energy at the elastic limit? Assume that the test speimen is 0.505-in dia and has a 2-in gage length. Can you define the type of metal based on the given data?

Given: Elastic limit: Strength $S_{el} := 100 \cdot ksi$ Strain $\varepsilon_{el} := 0.006$

$S_{el} = 689 \cdot MPa$

Test specimen: Diameter $d_o := 0.505 \cdot in$ Length $L_o := 2.00 \cdot in$

Solution: See Mathcad file P0209.

1. The modulus of elasticity is the slope of the stress-strain curve, which is a straight line, in the elastic region. Since one end of this line is at the origin, the slope (modulus of elasticity) is

$$E := \frac{S_{el}}{\varepsilon_{el}} \qquad E = 16.7 \cdot 10^{6} \cdot psi \qquad E = 115 \cdot GPa$$

2. The strain energy per unit volume at the elastic limit is the area under the stress-strain curve up to the elastic limit. Since the curve is a straight line up to this limit, the area is one-half the base times the height, or

$$U'_{el} := \frac{1}{2} \cdot S_{el} \cdot \varepsilon_{el} \qquad U'_{el} = 300 \cdot \frac{lbf \cdot in}{in^3} \qquad U'_{el} = 2 \times 10^3 \cdot \frac{kN \cdot m}{m^3}$$

The total strain energy in the specimen is the strain energy per unit volume times the volume,

$$U_{el} := U'_{el} \cdot \frac{\pi \cdot d_o^{\,2}}{4} \cdot L_o \qquad U_{el} = 120.18 \cdot in \cdot lbf$$

3. Based on the modulus of elasticity and using Table C-1, the material is **titanium**.

PROBLEM 2-10

Statement: A material has a yield strength of 689 MPa at an offset of 0.6% strain. What is its modulus of resilience?

Units: $MJ := 10^6 \cdot joule$

Given: Yield strength $S_y := 689 \cdot MPa$

Yield strain $\varepsilon_y := 0.006$

Solution: See Mathcad file P0210.

1. The modulus of resilience (strain energy per unit volume) is given by Equation (2.7) and is approximately

$$U_R := \frac{1}{2} \cdot S_y \cdot \varepsilon_y \qquad U_R = 2.067 \cdot \frac{MJ}{m^3} \qquad U_R = 2.1 \cdot MPa$$

PROBLEM 2-11

Statement: A material has a yield strength of 60 ksi (414 MPa) at an offset of 0.2% strain. What is its modulus of resilience?

Units: $MJ := 10^6 \cdot joule$

Given: Yield strength $S_y := 60 \cdot ksi$ $S_y = 414 \cdot MPa$

Yield strain $\varepsilon_y := 0.002$

Solution: See Mathcad file P0211.

1. The modulus of resilience (strain energy per unit volume) is given by Equation (2.7) and is approximately

$$U_R := \frac{1}{2} \cdot S_y \cdot \varepsilon_y \qquad U_R = 60 \cdot \frac{in \cdot lbf}{in^3} \qquad U_R = 0.414 \cdot \frac{MJ}{m^3} \qquad U_R = 0.414 \cdot MPa$$

PROBLEM 2-12

Statement: A steel has a yield strength of 414 MPa, an ultimate tensile strength of 689 MPa, and an elongation at fracture of 15%. What is its approximate modulus of toughness? What is the approximate modulus of resilience?

Given: $S_y := 414 \cdot MPa$ $\quad S_{ut} := 689 \cdot MPa$ $\quad \varepsilon_f := 0.15$

Solution: See Mathcad file P0212.

1. Determine the modulus of toughness using Equation (2.8).

$$U_T := \left(\frac{S_y + S_{ut}}{2}\right) \cdot \varepsilon_f \qquad U_T = 82.7 \cdot \frac{MN \cdot m}{m^3} \qquad U_T = 82.7 \cdot MPa$$

2. Determine the modulus of resilience using Equation (2.7) and Young's modulus for steel: $E := 207 \cdot GPa$

$$U_R := \frac{1}{2} \cdot \frac{S_y^{\,2}}{E} \qquad U_R = 414 \cdot \frac{kN \cdot m}{m^3} \qquad U_R = 0.41 \cdot MPa$$

PROBLEM 2-13

Statement: The Brinell hardness of a steel specimen was measured to be 250 HB. What is the material's approximate tensile strength? What is the hardness on the Vickers scale? The Rockwell scale?

Given: Brinell hardness of specimen $HB := 250$

Solution: See Mathcad file P0213.

1. Determine the approximate tensile strength of the material from equations (2.10), not Table 2-3.

$$S_{ut} := 0.5 \cdot HB \cdot ksi \qquad S_{ut} = 125 \cdot ksi \qquad S_{ut} = 862 \cdot MPa$$

2. From Table 2-3 (using linear interpolation) the hardness on the Vickers scale is

$$HV := \frac{HB - 241}{277 - 241} \cdot (292 - 253) + 253 \qquad HV = 263$$

3. From Table 2-3 (using linear interpolation) the hardness on the Rockwell C scale is

$$HRC := \frac{HB - 241}{277 - 241} \cdot (28.8 - 22.8) + 22.8 \qquad HRC = 24.3$$

PROBLEM 2-14

Statement: The Brinell hardness of a steel specimen was measured to be 340 HB. What is the material's approximate tensile strength? What is the hardness on the Vickers scale? The Rockwell scale?

Given: Brinell hardness of specimen $HB := 340$

Solution: See Mathcad file P0214.

1. Determine the approximate tensile strength of the material from equations (2.10), not Table 2-3.

$$S_{ut} := 0.5 \cdot HB \cdot ksi \qquad S_{ut} = 170 \cdot ksi \qquad S_{ut} = 1172 \cdot MPa$$

2. From Table 2-3 (using linear interpolation) the hardness on the Vickers scale is

$$HV := \frac{HB - 311}{341 - 311} \cdot (360 - 328) + 328 \qquad HV = 359$$

3. From Table 2-3 (using linear interpolation) the hardness on the Rockwell C scale is

$$HRC := \frac{HB - 311}{341 - 311} \cdot (36.6 - 33.1) + 33.1 \qquad HRC = 36.5$$

PROBLEM 2-15

Statement: What are the principal alloy elements of an AISI 4340 steel? How much carbon does it have? Is it hardenable? By what techniques?

Solution: See Mathcad file P0215.

1. Determine the principal alloying elements from Table 2-5 for 43xx steel..

   1.82% Nickel
   0.50 or 0.80% Chromium
   0.25% Molybdenum

2. From "Steel Numbering Systems" in Section 2.6, the carbon content is

   From the last two digits, the carbon content is 0.40%.

3. Is it hardenable? Yes, all of the alloying elements increase the hardenability. By what techniques? It can be through hardened by heating, quenching and tempering; and it can also be case hardened (See Section 2.4).

PROBLEM 2-16

Statement: What are the principal alloy elements of an AISI 1095 steel? How much carbon does it have? Is it hardenable? By what techniques?

Solution: See Mathcad file P0216.

1. Determine the principal alloying elements from Table 2-5 for 10xx steel.

   Carbon only, no alloying elements

2. From "Steel Numbering Systems" in Section 2.6, the carbon content is

   From the last two digits, the carbon content is 0.95%.

3. Is it hardenable? Yes, as a high-carbon steel, it has sufficient carbon content for hardening. By what techniques? It can be through hardened by heating, quenching and tempering; and it can also be case hardened (See Section 2.4).

PROBLEM 2-17

Statement: What are the principal alloy elements of an AISI 6180 steel? How much carbon does it have? Is it hardenable? By what techniques?

Solution: See Mathcad file P0217.

1. Determine the principal alloying elements from Table 2-5 for 61xx steel..

   0.15% Vanadium
   0.60 to 0.95% Chromium

2. From "Steel Numbering Systems" in Section 2.6, the carbon content is

   From the last two digits, the carbon content is 0.80%.

3. Is it hardenable? Yes, all of the alloying elements increase the hardenability. By what techniques? It can be through hardened by heating, quenching and tempering; and it can also be case hardened (See Section 2.4).

PROBLEM 2-18

Statement: Which of the steels in Problems 2-15, 2-16, and 2-17 is the stiffest?

Solution: See Mathcad file P0218.

1. None. All steel alloys have the same Young's modulus, which determines stiffness.

PROBLEM 2-19

Statement: Calculate the *specific strength* and *specific stiffness* of the following materials and pick one for use in an aircraft wing spar.

Given:

Material	Code	Ultimate Strength	Young's Modulus	Weight Density
Steel	$st := 0$	$Sut_{st} := 80 \cdot ksi$	$E_{st} := 30 \cdot 10^6 \cdot psi$	$\gamma_{st} := 0.28 \cdot \frac{lbf}{in^3}$
Aluminum	$al := 1$	$Sut_{al} := 60 \cdot ksi$	$E_{al} := 10.4 \cdot 10^6 \cdot psi$	$\gamma_{al} := 0.10 \cdot \frac{lbf}{in^3}$
Titanium	$ti := 2$	$Sut_{ti} := 90 \cdot ksi$	$E_{ti} := 16.5 \cdot 10^6 \cdot psi$	$\gamma_{ti} := 0.16 \cdot \frac{lbf}{in^3}$

Index $i := 0, 1 .. 2$

Solution: See Mathcad file P0219.

1. *Specific strength* is the ultimate tensile strength divided by the weight density and *specific stiffness* is the modulus of elasticity divided by the weight density. The text does not give a symbol to these quantities.

Specific strength $\frac{Sut_i}{\gamma_i} \cdot \frac{1}{in} =$ Specific stiffness $\frac{E_i}{\gamma_i} \cdot \frac{1}{in} =$

Material	Specific strength	Specific stiffness
Steel	$286 \cdot 10^3$	$107 \cdot 10^6$
Aluminum	$600 \cdot 10^3$	$104 \cdot 10^6$
Titanium	$563 \cdot 10^3$	$103 \cdot 10^6$

2. Based on the results above, all three materials have the same specific stiffness but the aluminum has the largest specific strength. **Aluminum for the aircraft wing spar** is recommended.

PROBLEM 2-20

Statement: If maximum *impact resistance* were desired in a part, which material properties would you look for?

Solution: See Mathcad file P0220.

1. Ductility and a large modulus of toughness (see "Impact Resistance" in Section 2.1).

PROBLEM 2-21

Statement: Refer to the tables of material data in Appendix A and determine the strength-to-weight ratios of the following material alloys based on their tensile yield strengths: heat-treated 2024 aluminum, SAE 1040 cold-rolled steel, Ti-75A titanium, type 302 cold-rolled stainless steel.

Given:

Material	Yield Strength	Specific Weight	
$Mat_1 :=$ "2024 Aluminum, HT"	$Sy_1 := 290 \cdot MPa$	$\gamma_1 := 0.10 \cdot lbf \cdot in^{-3}$	$\gamma_1 = 27.14 \cdot \frac{kN}{m^3}$
$Mat_2 :=$ "1040 CR Steel"	$Sy_2 := 490 \cdot MPa$	$\gamma_2 := 0.28 \cdot lbf \cdot in^{-3}$	$\gamma_2 = 76.01 \cdot \frac{kN}{m^3}$
$Mat_3 :=$ "Ti-75A Titanium"	$Sy_3 := 517 \cdot MPa$	$\gamma_3 := 0.16 \cdot lbf \cdot in^{-3}$	$\gamma_3 = 43.43 \cdot \frac{kN}{m^3}$
$Mat_4 :=$ "Type 302 CR SS"	$Sy_4 := 1138 \cdot MPa$	$\gamma_4 := 0.28 \cdot lbf \cdot in^{-3}$	$\gamma_4 = 76.01 \cdot \frac{kN}{m^3}$

$i := 1, 2 .. 4$

Solution: See Mathcad file P0221.

1. Calculate the strength-to-weight ratio for each material as described in Section 2.1.

$$SWR_i := \frac{Sy_i}{\gamma_i}$$

$Mat_i =$	$\frac{SWR_i}{10^4 \cdot m} =$
"2024 Aluminum, HT"	1.068
"1040 CR Steel"	0.645
"Ti-75A Titanium"	1.190
"Type 302 CR SS"	1.497

PROBLEM 2-22

Statement: Refer to the tables of material data in Appendix A and determine the strength-to-weight ratios of the following material alloys based on their ultimate tensile strengths: heat-treated 2024 aluminum, SAE 1040 cold-rolled steel, unfilled acetal plastic, Ti-75A titanium, type 302 cold-rolled stainless steel.

Given:

Material	Tensile Strength	Specific Weight	
$Mat_1 :=$ "2024 Aluminum, HT"	$Sut_1 := 441 \cdot MPa$	$\gamma_1 := 0.10 \cdot lbf \cdot in^{-3}$	$\gamma_1 = 27.14 \cdot kN \cdot m^{-3}$
$Mat_2 :=$ "1040 CR Steel"	$Sut_2 := 586 \cdot MPa$	$\gamma_2 := 0.28 \cdot lbf \cdot in^{-3}$	$\gamma_2 = 76.01 \cdot kN \cdot m^{-3}$
$Mat_3 :=$ "Acetal, unfilled"	$Sut_3 := 60.7 \cdot MPa$	$\gamma_3 := 0.051 \cdot lbf \cdot in^{-3}$	$\gamma_3 = 13.84 \cdot kN \cdot m^{-3}$
$Mat_4 :=$ "Ti-75A Titanium"	$Sut_4 := 586 \cdot MPa$	$\gamma_4 := 0.16 \cdot lbf \cdot in^{-3}$	$\gamma_4 = 43.43 \cdot kN \cdot m^{-3}$
$Mat_5 :=$ "Type 302 CR SS"	$Sut_5 := 1310 \cdot MPa$	$\gamma_5 := 0.28 \cdot lbf \cdot in^{-3}$	$\gamma_5 = 76.01 \cdot kN \cdot m^{-3}$

$i := 1, 2 .. 5$

Solution: See Mathcad file P0222.

1. Calculate the strength-to-weight ratio for each material as described in Section 2.1.

$$SWR_i := \frac{Sut_i}{\gamma_i}$$

$Mat_i =$	$\frac{SWR_i}{10^4 \cdot m} =$
"2024 Aluminum, HT"	1.625
"1040 CR Steel"	0.771
"Acetal, unfilled"	0.438
"Ti-75A Titanium"	1.349
"Type 302 CR SS"	1.724

PROBLEM 2-23

Statement: Refer to the tables of material data in Appendix A and calculate the specific stiffness of aluminum, titanium, gray cast iron, ductile iron, bronze, carbon steel, and stainless steel. Rank them in increasing order of this property and discuss the engineering significance of these data.

Units: $Mg := 10^3 \cdot kg$

Given:

Material	Modulus of Elasticity	Density
$Mat_1 :=$ "Aluminum"	$E_1 := 71.7 \cdot GPa$	$\rho_1 := 2.8 \cdot Mg \cdot m^{-3}$
$Mat_2 :=$ "Titanium"	$E_2 := 113.8 \cdot GPa$	$\rho_2 := 4.4 \cdot Mg \cdot m^{-3}$
$Mat_3 :=$ "Gray cast iron"	$E_3 := 103.4 \cdot GPa$	$\rho_3 := 7.2 \cdot Mg \cdot m^{-3}$
$Mat_4 :=$ "Ductile iron"	$E_4 := 168.9 \cdot GPa$	$\rho_4 := 6.9 \cdot Mg \cdot m^{-3}$
$Mat_5 :=$ "Bronze"	$E_5 := 110.3 \cdot GPa$	$\rho_5 := 8.6 \cdot Mg \cdot m^{-3}$
$Mat_6 :=$ "Carbon steel"	$E_6 := 206.8 \cdot GPa$	$\rho_6 := 7.8 \cdot Mg \cdot m^{-3}$
$Mat_7 :=$ "Stainless steel"	$E_7 := 189.6 \cdot GPa$	$\rho_7 := 7.8 \cdot Mg \cdot m^{-3}$

$i := 1, 2 .. 7$

Solution: See Mathcad file P0223.

1. Calculate the specific stiffness for each material as described in Section 2.1.

$$E'_i := \frac{E_i}{\rho_i}$$

$Mat_i =$	$\frac{E'_i}{10^6} \cdot \frac{s^2}{m^2} =$
"Aluminum"	25.6
"Titanium"	25.9
"Gray cast iron"	14.4
"Ductile iron"	24.5
"Bronze"	12.8
"Carbon steel"	26.5
"Stainless steel"	24.3

2. Rank them in increasing order of specific stiffness.

$Mat_5 =$ "Bronze" $\quad \frac{E'_5}{10^6} \cdot \frac{s^2}{m^2} = 12.8$

$Mat_3 =$ "Gray cast iron" $\quad \frac{E'_3}{10^6} \cdot \frac{s^2}{m^2} = 14.4$

$Mat_7 =$ "Stainless steel" $\quad \frac{E'_7}{10^6} \cdot \frac{s^2}{m^2} = 24.3$

Mat_4 = "Ductile iron" $\quad \frac{E'_4}{10^6} \cdot \frac{s^2}{m^2} = 24.5$

Mat_1 = "Aluminum" $\quad \frac{E'_1}{10^6} \cdot \frac{s^2}{m^2} = 25.6$

Mat_2 = "Titanium" $\quad \frac{E'_2}{10^6} \cdot \frac{s^2}{m^2} = 25.9$

Mat_6 = "Carbon steel" $\quad \frac{E'_6}{10^6} \cdot \frac{s^2}{m^2} = 26.5$

3. Bending and axial deflection are inversely proportional to the modulus of elasticity. For the same shape and dimensions, the material with the highest specific stiffness will give the smallest deflection. Or, put another way, for a given deflection, using the material with the highest specific stiffness will result in the least weight.

PROBLEM 2-24

Statement: Call your local steel and aluminum distributors (consult the Yellow Pages) and obtain current costs per pound for round stock of consistent size in low-carbon (SAE 1020) steel, SAE 4340 steel, 2024-T4 aluminum, and 6061-T6 aluminum. Calculate a strength/dollar ratio and a stiffness/dollar ratio for each alloy. Which would be your first choice on a cost-efficiency basis for an axial-tension-loaded round rod

(a) If maximum strength were needed?
(b) If maximum stiffness were needed?

Solution: Left to the student as data will vary with time and location.

PROBLEM 2-25

Statement: Call your local plastic stock-shapes distributors (consult the Yellow Pages) and obtain current costs per pound for round rod or tubing of consistent size in plexiglass, acetal, nylon 6/6, and PVC. Calculate a strength/dollar ratio and a stiffness/dollar ratio for each alloy. Which would be your first choice on a cost-efficiency basis for an axial-tension-loaded round rod or tube of particular diameters.

(a) If maximum strength were needed?
(b) If maximum stiffness were needed?

Solution: Left to the student as data will vary with time and location.

PROBLEM 2-26

Statement: A part has been designed and its dimensions cannot be changed. To minimize its deflections under the same loading in all directions irrespective of stress levels, which material woulod you choose among the following: aluminum, titanium, steel, or stainless steel?

Solution: See Mathcad file P0226.

1. Choose the material with the highest modulus of elasticity because deflection is inversely proportional to modulus of elasticity. Thus, choose steel unless there is a corrosive atmosphere, in which case, choose stainless steel.

PROBLEM 2-27

Statement: Assuming that the mechanical properties data given in Appendix Table A-9 for some carbon steels represents mean values, what is the value of the tensile yield strength for 1050 steel quenched and tempered at 400F if a reliability of 99.9% is required?

Given: Mean yield strength $S_y := 117 \cdot ksi$ $S_y = 807 \cdot MPa$

Solution: See Mathcad file P0227.

1. From Table 2-2 the reliability factor for 99.9% is $Re := 0.753$. Applying this to the mean tensile strength gives

$$S_{y99.9} := S_y \cdot Re \qquad S_{y99.9} = 88.1 \cdot ksi \qquad S_{y99.9} = 607 \cdot MPa$$

PROBLEM 2-28

Statement: Assuming that the mechanical properties data given in Appendix Table A-9 for some carbon steels represents mean values, what is the value of the ultimate tensile strength for 4340 steel quenched and tempered at 800F if a reliability of 99.99% is required?

Given: Mean ultimate tensile strength $S_{ut} := 213 \cdot ksi$ $S_{ut} = 1469 \cdot MPa$

Solution: See Mathcad file P0228.

1. From Table 2-2 the reliability factor for 99.99% is $Re := 0.702$. Applying this to the mean ultimate tensile strength gives

$$S_{ut99.99} := S_{ut} \cdot Re \qquad S_{ut99.99} = 150 \cdot ksi \qquad S_{ut99.99} = 1031 \cdot MPa$$

PROBLEM 2-29

Statement: Assuming that the mechanical properties data given in Appendix Table A-9 for some carbon steels represents mean values, what is the value of the ultimate tensile strength for 4130 steel quenched and tempered at 400F if a reliability of 90% is required?

Given: Mean ultimate tensile strength $S_{ut} := 236 \cdot ksi$ $S_{ut} = 1627 \cdot MPa$

Solution: See Mathcad file P0229.

1. From Table 2-2 the reliability factor for 90% is $Re := 0.897$. Applying this to the mean ultimate tensile strength gives

$$S_{ut99.99} := S_{ut} \cdot Re \qquad S_{ut99.99} = 212 \cdot ksi \qquad S_{ut99.99} = 1460 \cdot MPa$$

PROBLEM 2-30

Statement: Assuming that the mechanical properties data given in Appendix Table A-9 for some carbon steels represents mean values, what is the value of the tensile yield strength for 4140 steel quenched and tempered at 800F if a reliability of 99.999% is required?

Given: Mean yield strength $S_y := 165 \cdot ksi$ $S_y = 1138 \cdot MPa$

Solution: See Mathcad file P0230.

1. From Table 2-2 the reliability factor for 99.999% is $Re := 0.659$. Applying this to the mean tensile strength gives

$S_{y99.9} := S_y \cdot Re$ $S_{y99.9} = 109 \cdot ksi$ $S_{y99.9} = 750 \cdot MPa$

PROBLEM 2-31

Statement: A steel part is to be plated to give it better corrosion resistance. Two materials are being considered: cadmium and nickel. Considering only the problem of galvanic action, which would you chose? Why?

Solution: See Mathcad file P0231.

1. From Table 2-4 we see that cadmium is closer to steel than nickel. Therefore, from the standpoint of reduced galvanic action, cadmium is the better choice. Also, since cadmium is less noble than steel it will be the material that is consumed by the galvanic action. If nickel were used the steel would be consumed by galvanic action.

PROBLEM 2-32

Statement: A steel part with many holes and sharp corners is to be plated with nickel. Two processes are being considered: electroplating and electroless plating. Which process would you chose? Why?

Solution: See Mathcad file P0232.

1. Electroless plating is the better choice since it will give a uniform coating thickness in the sharp corners and in the holes. It also provides a relatively hard surface of about 43 HRC.

PROBLEM 2-33

Statement: What is the common treatment used on aluminum to prevent oxidation? What other metals can also be treated with this method? What options are available with this method?

Solution: See Mathcad file P0233.

1. Aluminum is commonly treated by anodizing, which creates a thin layer of aluminum oxide on the surface. Titanium, magnesium, and zinc can also be anodized. Common options include tinting to give various colors to the surface and the use of "hard anodizing" to create a thicker, harder surface.

PROBLEM 2-34

Statement: Steel is often plated with a less nobel metal that acts as a sacrificial anode that will corrode instead of the steel. What metal is commonly used for this purpose (when the finished product will not be exposed to saltwater), what is the coating process called, and what are the common processes used to obtain the finished product?

Solution: See Mathcad file P0234.

1. The most commonly used metal is zinc. The process is called "galvanizing" and it is accomplished by electroplating or hot dipping.

PROBLEM 2-35

Statement: A low-carbon steel part is to be heat-treated to increase its strength. If an ultimate tensile strength of approximately 550 MPa is required, what mean Brinell hardness should the part have after treatment? What is the equivalent hardness on the Rockwell scale?

Given: Approximate tensile strength $S_{ut} := 550 \cdot MPa$

Solution: See Mathcad file P0235.

1. Use equation (2.10), solving for the Brinell hardness, HB.

$$S_{ut} = 3.45 \cdot HB \qquad HB := \frac{S_{ut}}{3.45 \cdot MPa} \qquad HB = 159$$

2. From Table 2-3, the equivalent hardness on the Rocwell scale is 83.9HRB.

PROBLEM 2-36

Statement: A low-carbon steel part has been tested for hardness using the Brinell method and is found to have a hardness of 220 HB. What are the approximate lower and upper limits of the ultimate tensile strength of this part in MPa?

Given: Hardness $HB := 220$

Solution: See Mathcad file P0236.

1. Use equation (2.10), solving for ultimate tensile strength.

Minimum: $S_{utmin} := (3.45 \cdot HB - 0.2 \cdot HB) \cdot MPa$ $S_{utmin} = 715 \cdot MPa$

Maximum: $S_{utmax} := (3.45 \cdot HB + 0.2 \cdot HB) \cdot MPa$ $S_{utmax} = 803 \cdot MPa$

PROBLEM 2-37

Statement: Figure 2-24 shows "guide lines" for minimum weight design when failure is the criterion. The guide line, or index, for minimizing the weight of a beam in bending is $\sigma_f^{2/3}/\rho$, where σ_f is the yield strength of a material and ρ is its mass density. For a given cross-section shape the weight of a beam with given loading will be minimized when this index is maximized. The following materials are being considered for a beam application: 5052 aluminum, cold rolled; CA-170 beryllium copper, hard plus aged; and 4130 steel, Q & T @ 1200F. The use of which of these three materials will result in the least-weight beam?

Units: $Mg := kg^3$

Given:

5052 Aluminum	$S_{ya} := 255 \cdot MPa$	$\rho_a := 2.8 \cdot Mg \cdot m^{-3}$
CA-170 beryllium copper	$S_{yb} := 1172 \cdot MPa$	$\rho_b := 8.3 \cdot Mg \cdot m^{-3}$
4130 steel	$S_{ys} := 703 \cdot MPa$	$\rho_s := 7.8 \cdot Mg \cdot m^{-3}$

Solution: See Mathcad file P0237.

1. The values for the mass density are taken from Appendix Table A-1 and the values of yield strength come from from Tables A-2, A-4, and A-9 for aluminum, beryllium copper, and steel, respectively.

2. Calculate the index value for each material.

$$Index(S_y, \rho) := \frac{S_y^{0.667}}{\rho} \cdot \frac{Mg \cdot m^{-3}}{MPa^{0.667}}$$

Aluminum	$I_a := Index(S_{ya}, \rho_a)$	$I_a = 14.4$
Beryllium copper	$I_b := Index(S_{yb}, \rho_b)$	$I_b = 13.4$
Steel	$I_s := Index(S_{ys}, \rho_s)$	$I_s = 10.2$

The 5052 aluminum has the highest value of the index and would be the best choice to minimize weight.

PROBLEM 2-38

Statement: Figure 2-24 shows "guide lines" for minimum weight design when failure is the criterion. The guide line, or index, for minimizing the weight of a member in tension is σ_f/ρ, where σ_f is the yield strength of a material and ρ is its mass density. The weight of a member with given loading will be minimized when this index is maximized. For the three materials given in Problem 2-37, which will result in the least weight tension member?

Units: $Mg := kg^3$

Given:

5052 Aluminum	$S_{ya} := 255 \cdot MPa$	$\rho_a := 2.8 \cdot Mg \cdot m^{-3}$
CA-170 beryllium copper	$S_{yb} := 1172 \cdot MPa$	$\rho_b := 8.3 \cdot Mg \cdot m^{-3}$
4130 steel	$S_{ys} := 703 \cdot MPa$	$\rho_s := 7.8 \cdot Mg \cdot m^{-3}$

Solution: See Mathcad file P0238.

1. The values for the mass density are taken from Appendix Table A-1 and the values of yield strength come from from Tables A-2, A-4, and A-9 for aluminum, beryllium copper, and steel, respectively.

2. Calculate the index value for each material.

$$Index(S_y, \rho) := \frac{S_y}{\rho} \cdot \frac{Mg \cdot m^{-3}}{MPa}$$

Aluminum	$I_a := Index(S_{ya}, \rho_a)$	$I_a = 91.1$
Beryllium copper	$I_b := Index(S_{yb}, \rho_b)$	$I_b = 141.2$
Steel	$I_s := Index(S_{ys}, \rho_s)$	$I_s = 90.1$

The beryllium copper has the highest value of the index and would be the best choice to minimize weight.

PROBLEM 2-39

Statement: Figure 2-23 shows "guide lines" for minimum weight design when stiffness is the criterion. The guide line, or index, for minimizing the weight of a beam in bending is $E^{1/2}/\rho$, where E is the modulus of elasticity of a material and ρ is its mass density. For a given cross-section shape the weight of a beam with given stiffness will be minimized when this index is maximized. The following materials are being considered for a beam application: 5052 aluminum, cold rolled; CA-170 beryllium copper, hard plus aged; and 4130 steel, Q & T @ 1200F. The use of which of these three materials will result in the least-weight beam?

Units: $Mg := kg^3$

Given:

5052 Aluminum	$E_a := 71.7 \cdot GPa$	$\rho_a := 2.8 \cdot Mg \cdot m^{-3}$
CA-170 beryllium copper	$E_b := 127.6 \cdot GPa$	$\rho_b := 8.3 \cdot Mg \cdot m^{-3}$
4130 steel	$E_s := 206.8 \cdot GPa$	$\rho_s := 7.8 \cdot Mg \cdot m^{-3}$

Solution: See Mathcad file P0239.

1. The values for the mass density and modulus are taken from Appendix Table A-1.
2. Calculate the index value for each material.

$$Index(E, \rho) := \frac{E^{0.5}}{\rho} \cdot \frac{Mg \cdot m^{-3}}{GPa^{0.5}}$$

Aluminum	$I_a := Index(E_a, \rho_a)$	$I_a = 3.0$
Beryllium copper	$I_b := Index(E_b, \rho_b)$	$I_b = 1.4$
Steel	$I_s := Index(E_s, \rho_s)$	$I_s = 1.8$

The 5052 aluminum has the highest value of the index and would be the best choice to minimize weight.

PROBLEM 2-40

Statement: Figure 2-24 shows "guide lines" for minimum weight design when stiffness is the criterion. The guide line, or index, for minimizing the weight of a member in tension is E/ρ, where E is the modulus of elasticity of a material and ρ is its mass density. The weight of a member with given stiffness will be minimized when this index is maximized. For the three materials given in Problem 2-39, which will result in the least weight tension member?

Units: $Mg := kg^3$

Given:

5052 Aluminum	$E_a := 71.7 \cdot GPa$	$\rho_a := 2.8 \cdot Mg \cdot m^{-3}$
CA-170 beryllium copper	$E_b := 127.6 \cdot GPa$	$\rho_b := 8.3 \cdot Mg \cdot m^{-3}$
4130 steel	$E_s := 206.8 \cdot GPa$	$\rho_s := 7.8 \cdot Mg \cdot m^{-3}$

Solution: See Mathcad file P0240.

1. The values for the mass density and modulus are taken from Appendix Table A-1.
2. Calculate the index value for each material.

$$Index(E, \rho) := \frac{E}{\rho} \cdot \frac{Mg \cdot m^{-3}}{GPa}$$

Aluminum	$I_a := Index(E_a, \rho_a)$	$I_a = 25.6$
Beryllium copper	$I_b := Index(E_b, \rho_b)$	$I_b = 15.4$
Steel	$I_s := Index(E_s, \rho_s)$	$I_s = 26.5$

The steel has the highest value of the index and would be the best choice to minimize weight.