Chapter 2

Graphs and Functions

Section 2.1 Rectangular Coordinates and Graphs

Classroom Example 1 (page 184)

(a) (transportation, \$12,153)

(b) (health care, \$4917)

Classroom Example 2 (page 186)

$$d(P,Q) = \sqrt{(-2-3)^2 + [8-(-5)]^2} = \sqrt{25+169} = \sqrt{194}$$

Classroom Example 3 (page 186)

$$d(R,S) = \sqrt{(5-0)^2 + [1-(-2)]^2} = \sqrt{25+9} = \sqrt{34}$$

$$d(R,T) = \sqrt{(-4-0)^2 + [3-(-2)]^2} = \sqrt{16+25} = \sqrt{41}$$

$$d(S,T) = \sqrt{(-4-5)^2 + (3-1)^2} = \sqrt{81+4} = \sqrt{85}$$

The longest side has length $\sqrt{85}$

$$\left(\sqrt{34}\right)^2 + \left(\sqrt{41}\right)^2 \stackrel{?}{=} \left(\sqrt{85}\right)^2$$

$$34 + 41 \neq 85$$

The triangle formed by the three points is not a right triangle.

Classroom Example 4 (page 187)

The distance between $P(-2,5)$ and $Q(0,3)$ is

$$\sqrt{(-2-0)^2 + (5-3)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

The distance between $Q(0,3)$ and $R(8,-5)$ is

$$\sqrt{(8-0)^2 + (-5-3)^2} = \sqrt{64+64} = \sqrt{128} = 8\sqrt{2}$$

The distance between $P(-2,5)$ and $R(8,-5)$ is

$$\sqrt{(-2-8)^2 + [5-(-5)]^2} = \sqrt{100+100} = \sqrt{200} = 10\sqrt{2}$$

Because $2\sqrt{2} + 8\sqrt{2} = 10\sqrt{2}$, the points are collinear.

Classroom Example 5 (page 188)

(a) The coordinates of M are

$$\left(\frac{-7+(-2)}{2}, \frac{-5+13}{2}\right) = \left(-\frac{9}{2}, 4\right)$$

(b) Let (x, y) be the coordinates of Q. Use the midpoint formula to find the coordinates:

$$\left(\frac{8+x}{2}, \frac{-20+y}{2}\right) = (4,-4)$$

$$\frac{8+x}{2} = 4 \Rightarrow 8+x = 8 \Rightarrow x = 0$$

$$\frac{-20+y}{2} = -4 \Rightarrow -20+y = -8 \Rightarrow y = 12$$

The coordinates of Q are (0, 12).

Classroom Example 6 (page 188)

The year 2011 lies halfway between 2009 and 2013, so we must find the coordinates of the midpoint of the segment that has endpoints (2009, 124.0) and (2013, 137.4)

$$M = \left(\frac{2009+2013}{2}, \frac{124.0+137.4}{2}\right) = (2011, 130.7)$$

The estimate of \$130.7 billion is \$0.1 billion more than the actual amount.

Classroom Example 7 (page 189)

Choose any real number for x, substitute the value in the equation and then solve for y. Note that additional answers are possible.

(a)

x	$y = -2x + 5$
−1	$y = -2(-1) + 5 = 7$
0	$y = -2(0) + 5 = 5$
3	$y = -2(3) + 5 = -1$

Three ordered pairs that are solutions are (−1, 7), (0, 5), and (3, −1). Other answers are possible.

(b)

y	$x = \sqrt[3]{y+1}$
-9	$x = \sqrt[3]{-9+1} = \sqrt[3]{-8} = -2$
-2	$x = \sqrt[3]{-2+1} = \sqrt[3]{-1} = -1$
-1	$x = \sqrt[3]{-1+1} = \sqrt[3]{0} = 0$
0	$x = \sqrt[3]{0+1} = \sqrt[3]{1} = 1$
7	$x = \sqrt[3]{7+1} = \sqrt[3]{8} = 2$

Ordered pairs that are solutions are (−2, −9), (−1, −2), (0, −1), (1, 0) and (2, 7).
Other answers are possible.

(c)

x	$y = -x^2 + 1$
-2	$y = -(-2)^2 + 1 = -3$
-1	$y = -(-1)^2 + 1 = 0$
0	$y = -0^2 + 1 = 1$
1	$y = -1^2 + 1 = 0$
2	$y = -2^2 + 1 = -3$

Ordered pairs that are solutions are (−2, −3), (−1, 0), (0, 1), (1, 0) and (2, −3).
Other answers are possible.

Classroom Example 8 (page 190)

(a) Let $y = 0$ to find the x-intercept, and then let $x = 0$ to find the y-intercept:

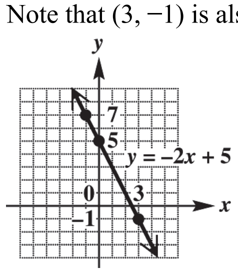
$0 = -2x + 5 \Rightarrow x = \frac{5}{2}$

$y = -2(0) + 5 \Rightarrow y = 5$

Find a third point on the graph by letting $x = -1$ and solving for y: $y = -2(-1) + 5 = 7$.

The three points are $\left(\frac{5}{2}, 0\right)$, (0, 5), and (−1, 7).

Note that (3, −1) is also on the graph.



(b) Let $y = 0$ to find the x-intercept, and then let $x = 0$ to find the y-intercept:

$x = \sqrt[3]{0+1} = \sqrt[3]{1} = 1$

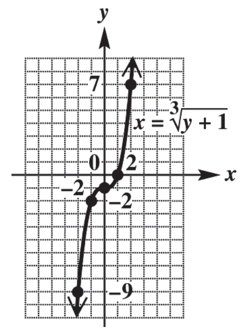
$0 = \sqrt[3]{y+1} \Rightarrow 0 = y + 1 \Rightarrow y = -1$

Find a third point by letting $x = 2$ and solving for y: $2 = \sqrt[3]{y+1} \Rightarrow 2^3 = y + 1 \Rightarrow 7 = y$.

Find a fourth point by letting $x = -2$ and solving for y:

$-2 = \sqrt[3]{y+1} \Rightarrow (-2)^3 = y + 1 \Rightarrow -9 = y$

The points to be plotted are (0, −1), (1, 0), (2, 7), and (−2, −9). Note that (−1, −2) is also on the graph.



(c) Let $y = 0$ to find the x-intercept, and then let $x = 0$ to find the y-intercept:

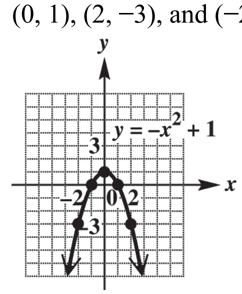
$0 = -x^2 + 1 \Rightarrow -1 = -x^2 \Rightarrow 1 = x^2 \Rightarrow \pm 1 = x$

$y = -\left(0^2\right) + 1 = 1$

Find a third point by letting $x = 2$ and solving for y: $y = -\left(2^2\right) + 1 = -3$. Find a fourth point by letting $x = -2$ and solving for y:

$y = -(-2)^2 + 1 = -3$

The points to be plotted are (−1, 0), (1, 0), (0, 1), (2, −3), and (−2, −3)



Section 2.2 Circles

Classroom Example 1 (page 195)

(a) $(h, k) = (1, -2)$ and $r = 3$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-1)^2 + [y-(-2)]^2 = 3^2$$

$$(x-1)^2 + (y+2)^2 = 9$$

(b) $(h, k) = (0, 0)$ and $r = 2$

$$(x-h)^2 + (y-k)^2 = r^2$$

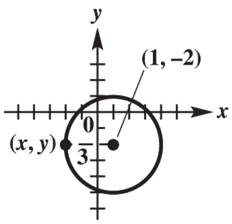
$$(x-0)^2 + (y-0)^2 = 2^2$$

$$x^2 + y^2 = 4$$

Classroom Example 2 (page 196)

(a) $(x-1)^2+(y+2)^2=9$

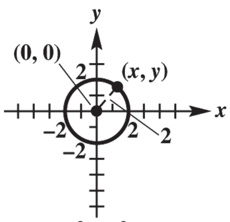
This is a circle with center (1, −2) and radius 3.



$(x-1)^2+(y+2)^2=9$

(b) $x^2+y^2=4$

This is a circle with center (0, 0) and radius 2.



$x^2+y^2=4$

Classroom Example 3 (page 197)

Complete the square twice, once for x and once for y:

$$x^2+4x+y^2-8y-44=0$$
$$\left(x^2+4x+4\right)+\left(y^2-8y+16\right)=44+4+16$$
$$(x+2)^2+(y-4)^2=64$$

Because $c = 64$ and $64 > 0$, the graph is a circle. The center is (−2, 4) and the radius is 8.

Classroom Example 4 (page 198)

$$2x^2+2y^2+2x-6y=45$$

Group the terms, factor out 2, and then complete the square:

$$2\left(x^2+x+\frac{1}{4}\right)+2\left(y^2-3y+\frac{9}{4}\right)=45+2\left(\frac{1}{4}\right)+2\left(\frac{9}{4}\right)$$

Factor and then divide both sides by 2:

$$2\left(x+\frac{1}{2}\right)^2+2\left(y-\frac{3}{2}\right)^2=50$$
$$\left(x+\frac{1}{2}\right)^2+\left(y-\frac{3}{2}\right)^2=25$$

Because $c = 25$ and $25 > 0$, the graph is a circle. The center is $\left(-\frac{1}{2},\frac{3}{2}\right)$ and the radius is 5.

Classroom Example 5 (page 198)

Complete the square twice, once for x and once for y:

$$x^2-6x+y^2+2y+12=0$$
$$\left(x^2-6x+9\right)+\left(y^2+2y+1\right)=-12+9+1$$
$$(x-3)^2+(y+1)^2=-2$$

Because $c = -2$ and $-2 < 0$, the graph is nonexistent.

Classroom Example 6 (page 199)

Determine the equation for each circle and then substitute $x = -3$ and $y = 4$.

Station A:

$$(x-1)^2+(y-4)^2=4^2$$
$$(-3-1)^2+(4-4)^2=4^2$$
$$(-4)^2=4^2$$
$$16=16$$

Station B:

$$\left[x-(-6)\right]^2+(y-0)^2=5^2$$
$$(x+6)^2+y^2=25$$
$$(-3+6)^2+4^2=25$$
$$3^2+4^2=25$$
$$9+16=25$$
$$25=25$$

Station C:

$$(x-5)^2+\left[y-(-2)\right]^2=10^2$$
$$(x-5)^2+(y+2)^2=100$$
$$(-3-5)^2+(4+2)^2=100$$
$$(-8)^2+6^2=100$$
$$64+36=100$$
$$100=100$$

Because (−3, 4) satisfies all three equations, we can conclude that the epicenter is (−3, 4).

Section 2.3 Functions

Classroom Example 1 (page 204)

$M = \{(-4,0), (-3,1), (3,1)\}$

M is a function because each distinct *x* value has exactly one *y* value.

$N = \{(2,3),(3,2),(4,5),(5,4)\}$

N is a function because each distinct *x* value has exactly one *y* value.

$P = \{(-4,3),(0,6),(2,8),(-4,-3)\}$

P is not a function because there are two *y*-values for $x = -4$.

Classroom Example 2 (page 205)

(a) Domain: {−4, −1, 1, 3}
Range: {−2, 0, 2, 5}
The relation is a function.

(b) Domain: {1, 2, 3}
Range: {4, 5, 6, 7}
The relation is not a function because 2 maps to 5 and 6.

(c) Domain: {−3, 0, 3, 5}
Range: {5}
The relation is a function.

Classroom Example 3 (page 206)

(a) Domain: {−2, 4}; range: {0, 3}

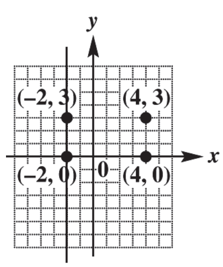
(b) Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

(c) Domain: [−5, 5]; range: [−3, 3]

(d) Domain: $(-\infty, \infty)$; range: $(-\infty, 4]$

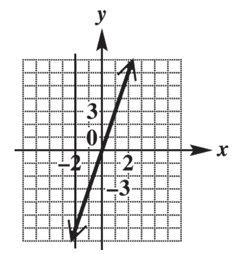
Classroom Example 4 (page 207)

(a)



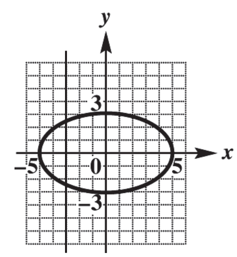
Not a function

(b)



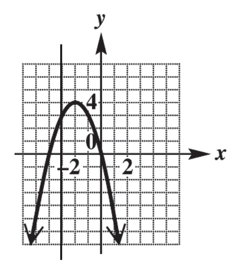
Function

(c)



Not a function

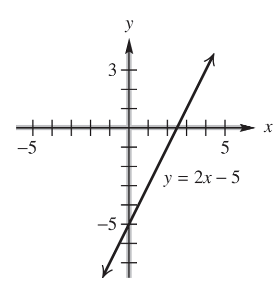
(d)



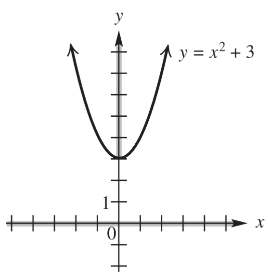
Function

Classroom Example 5 (page 208)

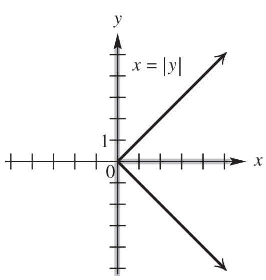
(a) $y = 2x - 5$ represents a function because *y* is always found by multiplying *x* by 2 and subtracting 5. Each value of *x* corresponds to just one value of *y*. *x* can be any real number, so the domain is all real numbers or $(-\infty, \infty)$. Because *y* is twice *x*, less 5, *y* also may be any real number, and so the range is also all real numbers, $(-\infty, \infty)$.



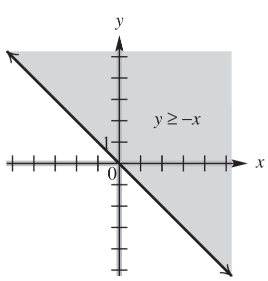
(b) For any choice of x in the domain of $y = x^2 + 3$, there is exactly one corresponding value for y, so the equation defines a function. The function is defined for all values of x, so the domain is $(-\infty, \infty)$. The square of any number is always positive, so the range is $[3, \infty)$.



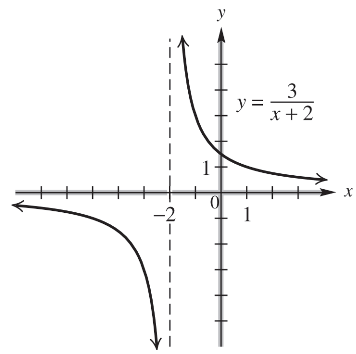
(c) For any choice of x in the domain of $x = |y|$, there are two possible values for y. Thus, the equation does not define a function. The domain is $[0, \infty)$ while the range is $(-\infty, \infty)$.



(d) By definition, y is a function of x if every value of x leads to exactly one value of y. Substituting a particular value of x, say 1, into $y \geq -x$ corresponds to many values of y. The ordered pairs (0, 2) (1, 1) (1, 0) (3, −1) and so on, all satisfy the inequality. This does not represent a function. Any number can be used for x or for y, so the domain and range of this relation are both all real numbers, $(-\infty, \infty)$.



(e) For $y = \dfrac{3}{x+2}$, we see that y can be found by dividing $x + 2$ into 3. This process produces one value of y for each value of x in the domain. The domain includes all real numbers except those that make the denominator equal to zero, namely $x = -2$. Therefore, the domain is $(-\infty, -2) \cup (-2, \infty)$. Values of y can be negative or positive, but never zero. Therefore the range is $(-\infty, 0) \cup (0, \infty)$.



Classroom Example 6 (page 210)

(a) $f(-3) = -(-3)^2 - 6(-3) + 4 = 13$

(b) $f(r) = -r^2 - 6r + 4$

(c) $g(r+2) = 3(r+2) + 1 = 3r + 7$

Classroom Example 7 (page 210)

(a) $f(-1) = 2(-1)^2 - 9 = -7$

(b) $f(-1) = 6$

(c) $f(-1) = 5$

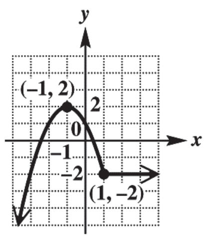
(d) $f(-1) = 0$

Classroom Example 8 (page 211)

(a) $f(x) = x^2 + 2x - 3$
$f(-5) = (-5)^2 + 2(-5) - 3 = 12$
$f(t) = t^2 + 2t - 3$

(b) $2x - 3y = 6 \Rightarrow y = \dfrac{2}{3}x - 2$
$f(x) = \dfrac{2}{3}x - 2$
$f(-5) = \dfrac{2}{3}(-5) - 2 = -\dfrac{16}{3}$
$f(t) = \dfrac{2}{3}t - 2$

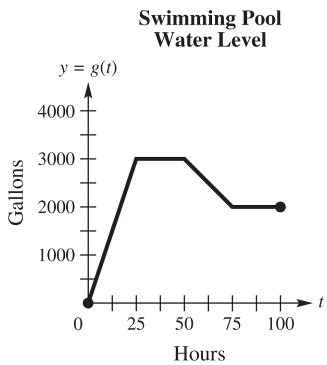
Classroom Example 9 (page 213)



The function is increasing on $(-\infty, -1)$, decreasing on $(-1, 1)$ and constant on $(1, \infty)$.

Classroom Example 10 (page 213)

The example refers to the following figure.



(a) The water level is changing most rapidly from 0 to 25 hours.

(b) The water level starts to decrease after 50 hours.

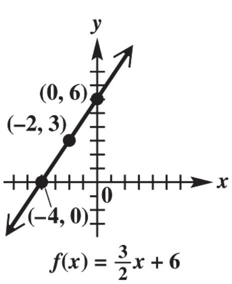
(c) After 75 hours, there are 2000 gallons of water in the pool.

Section 2.4 Linear Functions

Classroom Example 1 (page 220)

$f(x) = \frac{3}{2}x + 6$; Use the intercepts to graph the function. $f(0) = \frac{3}{2}(0) + 6 = 6$: y-intercept

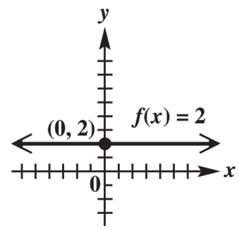
$0 = \frac{3}{2}x + 6 \Rightarrow -6 = \frac{3}{2}x \Rightarrow x = -4$: x-intercept



Domain: $(-\infty, \infty)$, range: $(-\infty, \infty)$

Classroom Example 2 (page 220)

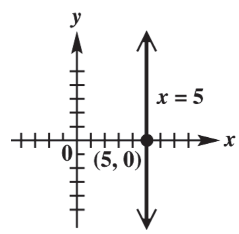
$f(x) = 2$ is a constant function. Its graph is a horizontal line with a y-intercept of 2.



Domain: $(-\infty, \infty)$, range: $\{2\}$

Classroom Example 3 (page 221)

$x = 5$ is a vertical line intersecting the x-axis at (5, 0).



Domain: $\{5\}$, range: $(-\infty, \infty)$

Classroom Example 4 (page 221)

$3x + 4y = 0$; Use the intercepts.

$3(0) + 4y = 0 \Rightarrow 4y = 0 \Rightarrow y = 0$: y-intercept

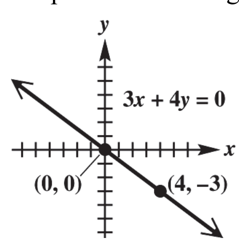
$3x + 4(0) = 0 \Rightarrow 3x = 0 \Rightarrow x = 0$: x-intercept

The graph has just one intercept. Choose an additional value, say 4, for x.

$3(4) + 4y = 0 \Rightarrow 12 + 4y = 0$

$4y = -12 \Rightarrow y = -3$

Graph the line through (0, 0) and (4, −3).



Domain: $(-\infty, \infty)$, range: $(-\infty, \infty)$

Classroom Example 5 (page 223)

(a) $m = \frac{4-(-6)}{-2-2} = \frac{10}{-4} = -\frac{5}{2}$

(b) $m = \frac{8-8}{5-(-3)} = \frac{0}{8} = 0$

(c) $m = \frac{-10-10}{-4-(-4)} = \frac{-20}{0} \Rightarrow$ the slope is undefined.

Classroom Example 6 (page 224)

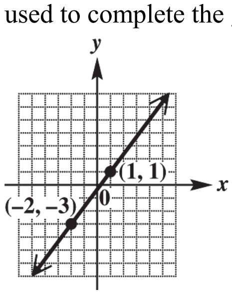
$2x - 5y = 10$
Solve the equation for y.

$$2x - 5y = 10$$
$$-5y = -2x + 10$$
$$y = \frac{2}{5}x - 2$$

The slope is $\frac{2}{5}$, the coefficient of x.

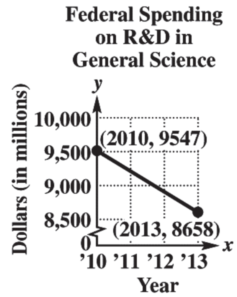
Classroom Example 7 (page 224)

First locate the point (−2, −3). Because the slope is $\frac{4}{3}$, a change of 3 units horizontally (3 units to the right) produces a change of 4 units vertically (4 units up). This gives a second point, (1, 1), which can be used to complete the graph.



Classroom Example 8 (page 225)

The average rate of change per year is
$\frac{8658-9547}{2013-2010} = \frac{-889}{3} = -296.33$ million
The graph confirms that the line through the ordered pairs fall from left to right, and therefore has negative slope. Thus, the amount spent by the federal government on R&D for general science decreased by an average of \$296.33 million (or \$296,330,000) each year from 2010 to 2013.



Classroom Example 9 (page 226)

(a) $C(x) = 120x + 2400$

(b) $R(x) = 150x$

(c)
$$P(x) = R(x) - C(x)$$
$$= 150x - (120x + 2400)$$
$$= 30x - 2400$$

(d) $P(x) > 0 \Rightarrow 30x - 2400 > 0 \Rightarrow x > 80$
At least 81 items must be sold to make a profit.

Section 2.5 Equations of Lines and Linear Models

Classroom Example 1 (page 234)

$$y - (-5) = -2(x - 3)$$
$$y + 5 = -2x + 6$$
$$y = -2x + 1$$

Classroom Example 2 (page 234)

First find the slope: $m = \frac{3-(-1)}{-4-5} = -\frac{4}{9}$

Now use either point for (x_1, y_1):

$$y - 3 = -\tfrac{4}{9}[x - (-4)]$$
$$9(y - 3) = -4(x + 4)$$
$$9y - 27 = -4x - 16$$
$$9y = -4x + 11$$
$$4x + 9y = 11$$

Classroom Example 3 (page 235)

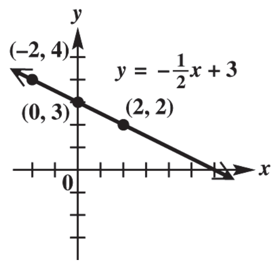
Write the equation in slope-intercept form:
$3x - 4y = 12 \Rightarrow -4y = -3x + 12 \Rightarrow y = \frac{3}{4}x - 3$

The slope is $\frac{3}{4}$, and the y-intercept is (0, −3).

Classroom Example 4 (page 236)

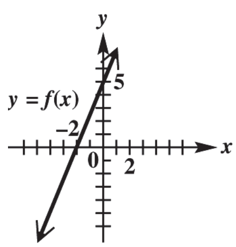
First find the slope: $m = \dfrac{4-2}{-2-2} = -\dfrac{1}{2}$

Now, substitute $-\frac{1}{2}$ for m and the coordinates of one of the points (say, (2, 2)) for x and y into the slope-intercept form $y = mx + b$, then solve for b:
$2 = -\frac{1}{2} \cdot 2 + b \Rightarrow 3 = b$. The equation is
$y = -\frac{1}{2}x + 3$.



Classroom Example 5 (page 236)

The example refers to the following figure:



(a) The line rises 5 units each time the x-value increases by 2 units. So the slope is $\frac{5}{2}$. The y-intercept is (0, 5), and the x-intercept is (−2, 0).

(b) An equation defining f is $f(x) = \frac{5}{2}x + 5$.

Classroom Example 6 (page 238)

(a) Rewrite the equation $3x - 2y = 5$ in slope-intercept form to find the slope:
$3x - 2y = 5 \Rightarrow y = \frac{3}{2}x + \frac{5}{2}$ The slope is $\frac{3}{2}$.
The line parallel to the equation also has slope $\frac{3}{2}$. An equation of the line through (2, −4) that is parallel to $3x - 2y = 5$ is
$y - (-4) = \frac{3}{2}(x-2) \Rightarrow y + 4 = \frac{3}{2}x - 3 \Rightarrow$
$y = \frac{3}{2}x - 7$ or $3x - 2y = 14$.

(b) The line perpendicular to the equation has slope $-\frac{2}{3}$. An equation of the line through (2, −4) that is perpendicular to $3x - 2y = 5$ is
$y - (-4) = -\frac{2}{3}(x-2) \Rightarrow y + 4 = -\frac{2}{3}x + \frac{4}{3} \Rightarrow$
$y = -\frac{2}{3}x - \frac{8}{3}$ or $2x + 3y = -8$.

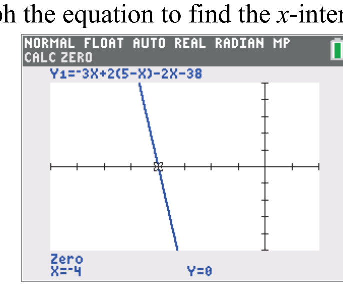
Classroom Example 7 (page 240)

(a) First find the slope: $m = \dfrac{7703 - 6695}{3-1} = 504$
Now use either point for (x_1, y_1):
$y - 6695 = 504(x-1)$
$y - 6695 = 504x - 504$
$y = 504x + 6191$

(b) The year 2015 is represented by $x = 6$.
$y = 504(6) + 6191 = 9215$
According to the model, average tuition and fees for 4-year colleges in 2015 will be about \$9215.

Classroom Example 8 (page 242)

Write the equation as an equivalent equation with 0 on one side.
$-3x + 2(5-x) = 2x + 38 \Rightarrow$
$-3x + 2(5-x) - 2x - 38 = 0$
Now graph the equation to find the x-intercept.



The solution set is $\{-4\}$.

Section 2.6 Graphs of Basic Functions

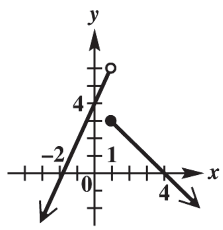
Classroom Example 1 (page 249)

(a) The function is continuous over $(-\infty, 0) \cup (0, \infty)$

(b) The function is continuous over its entire domain $(-\infty, \infty)$.

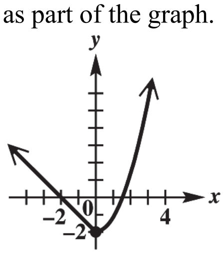
Classroom Example 2 (page 252)

(a) Graph each interval of the domain separately. If $x < 1$, the graph of $f(x) = 2x + 4$ has an endpoint at (1, 6), which is not included as part of the graph. To find another point on this part of the graph, choose $x = 0$, so $y = 4$. Draw the ray starting at (1, 6) and extending through (0, 4). Graph the function for $x \geq 1$, $f(x) = 4 - x$ similarly. This part of the graph has an endpoint at (1, 3), which is included as part of the graph. Find another point, say (4, 0), and draw the ray starting at (1, 3) which extends through (4, 0).



$$f(x) = \begin{cases} 2x + 4 & \text{if } x < 1 \\ 4 - x & \text{if } x \geq 1 \end{cases}$$

(b) Graph each interval of the domain separately. If $x \leq 0$, the graph of $f(x) = -x - 2$ has an endpoint at (0, −2), which is included as part of the graph. To find another point on this part of the graph, choose $x = -2$, so $y = 0$. Draw the ray starting at (0, −2) and extending through (−2, 0). Graph the function for $x > 0$, $f(x) = x^2 - 2$ similarly. This part of the graph has an endpoint at (0, −2), which is not included as part of the graph. Find another point, say (2, 2), and draw the curve starting at (0, −2) which extends through (2, 2). Note that the two endpoints coincide, so (0, −2) is included as part of the graph.

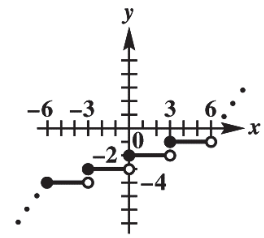


$$f(x) = \begin{cases} -x - 2 & \text{if } x \leq 0 \\ x^2 - 2 & \text{if } x > 0 \end{cases}$$

Classroom Example 3 (page 254)

Create a table of sample ordered pairs:

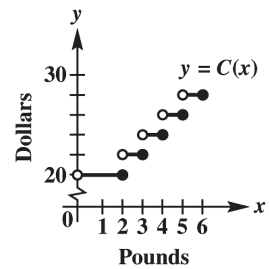
x	−6	−3	$-\frac{3}{2}$	0	$\frac{3}{2}$	3	6
$y = \left[\!\left[\frac{1}{3}x - 2\right]\!\right]$	−4	−3	−3	−2	−2	−1	0



$$f(x) = \left[\!\left[\tfrac{1}{3}x - 2\right]\!\right]$$

Classroom Example 4 (page 254)

For x in the interval (0, 2], $y = 20$. For x in (2, 3], $y = 20 + 2 = 22$. For x in (3, 4], $y = 22 + 2 = 24$. For x in (4, 5], $y = 24 + 2 = 26$. For x in (5, 6], $y = 26 + 2 = 28$.



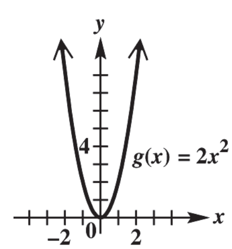
Section 2.7 Graphing Techniques

Classroom Example 1 (page 260)

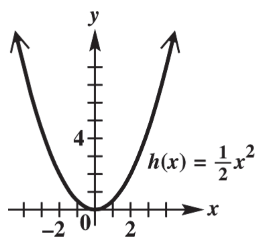
Use this table of values for parts (a)–(c)

x	$g(x) = 2x^2$	$h(x) = \frac{1}{2}x^2$	$k(x) = \left(\frac{1}{2}x\right)^2$
−2	8	2	1
−1	2	$\frac{1}{2}$	$\frac{1}{4}$
0	0	0	0
1	2	$\frac{1}{2}$	$\frac{1}{4}$
2	8	2	1

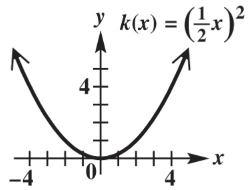
(a) $g(x) = 2x^2$



(b) $h(x) = \frac{1}{2}x^2$



(c) $k(x) = \left(\frac{1}{2}x\right)^2$

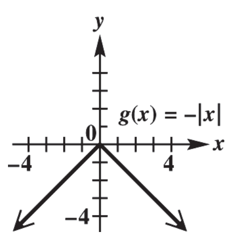


Classroom Example 2 (page 262)

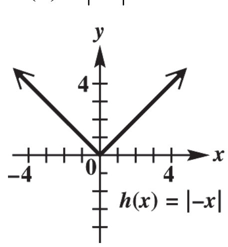
Use this table of values for parts (a) and (b)

x	$g(x) = -\lvert x\rvert$	$h(x) = \lvert -x\rvert$
−2	−2	2
−1	−1	1
0	0	0
1	−1	1
2	−2	2

(a) $g(x) = -|x|$



(b) $h(x) = |-x|$



Classroom Example 3 (page 263)

(a) $x = |y|$

Replace x with $-x$ to obtain $-x = |y|$. The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. Replace y with $-y$ to obtain $x = |-y| \Rightarrow x = |y|$ The result is the same as the original equation, so the graph is symmetric with respect to the x-axis. The graph is symmetric with respect to the x-axis only.

(b) $y = |x| - 3$

Replace x with $-x$ to obtain $y = |-x| - 3 \Rightarrow y = |x| - 3$. The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Replace y with $-y$ to obtain $-y = |x| - 3$. The result is not the same as the original equation, so the graph is not symmetric with respect to the x-axis. Therefore, the graph is symmetric with respect to the y-axis only.

(c) $2x - y = 6$

Replace x with $-x$ to obtain $2(-x) - y = 6 \Rightarrow -2x - y = 6$. The result is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. Replace y with $-y$ to obtain $2x - (-y) = 6 \Rightarrow 2x + y = 6$. The result is not the same as the original equation, so the graph is not symmetric with respect to the x-axis. Therefore, the graph is not symmetric with respect to either axis.

(d) $x^2 + y^2 = 25$

Replace x with $-x$ to obtain $(-x)^2 + y^2 = 25 \Rightarrow x^2 + y^2 = 25$. The result is the same as the original equation, so the graph is symmetric with respect to the y-axis. Replace y with $-y$ to obtain $x^2 + (-y)^2 = 25 \Rightarrow x^2 + y^2 = 25$. The result is the same as the original equation, so the graph is symmetric with respect to the x-axis. Therefore, the graph is symmetric with respect to both axes. Note that the graph is a circle of radius 5, centered at the origin.

Classroom Example 4 (page 265)

(a) $y = -2x^3$

Replace x with $-x$ and y with $-y$ to obtain $(-y) = -2(-x)^3 \Rightarrow -y = 2x^3 \Rightarrow y = -2x^3$. The result is the same as the original equation, so the graph is symmetric with respect to the origin.

(b) $y = -2x^2$

Replace x with $-x$ and y with $-y$ to obtain $(-y) = -2(-x)^2 \Rightarrow -y = -2x^2 \Rightarrow y = 2x^2$. The result is not the same as the original equation, so the graph is not symmetric with respect to the origin.

Classroom Example 5 (page 266)

(a) $g(x) = x^5 + 2x^3 - 3x$

Replace x with $-x$ to obtain

$$g(-x) = (-x)^5 + 2(-x)^3 - 3(-x)$$
$$= -x^5 - 2x^3 + 3x$$
$$= -(x^5 + 2x^3 - 3x) = -g(x) \Rightarrow$$

$g(x)$ is an odd function.

(b) $h(x) = 2x^2 - 3$

Replace x with $-x$ to obtain $h(-x) = 2(-x)^2 - 3 = 2x^2 - 3 = h(x) \Rightarrow h(x)$ is an even function.

(c) $k(x) = x^2 + 6x + 9$

Replace x with $-x$ to obtain

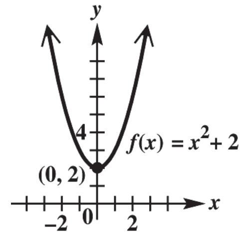
$$k(-x) = (-x)^2 + 6(-x) + 9$$
$$= x^2 - 6x + 9 \neq k(x) \text{ and } \neq -k(x) \Rightarrow$$

$k(x)$ is neither even nor odd.

Classroom Example 6 (page 267)

Compare a table of values for $g(x) = x^2$ with $f(x) = x^2 + 2$. The graph of $f(x)$ is the same as the graph of $g(x)$ translated 2 units up.

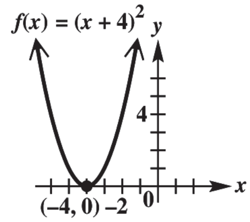
x	$g(x) = x^2$	$f(x) = x^2 + 2$
−2	4	6
−1	1	3
0	0	2
1	1	3
2	4	6



Classroom Example 7 (page 268)

Compare a table of values for $g(x) = x^2$ with $f(x) = (x+4)^2$. The graph of $f(x)$ is the same as the graph of $g(x)$ translated 4 units left.

x	$g(x) = x^2$	$f(x) = (x+4)^2$
−7	49	9
−6	36	4
−5	25	1
−4	16	0
−3	9	1
−2	4	4
−1	1	9



Classroom Example 8 (page 269)

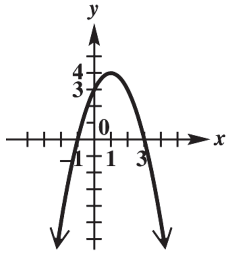
(a) $f(x) = -(x-1)^2 + 4$

This is the graph of $g(x) = x^2$, translated one unit to the right, reflected across the x-axis, and then translated four units up.

x	$g(x) = x^2$	$f(x) = -(x-1)^2 + 4$
−2	4	−5
−1	1	0
0	0	3
1	1	4
2	4	3
3	9	0
4	16	−5

(*continued on next page*)

(*continued*)

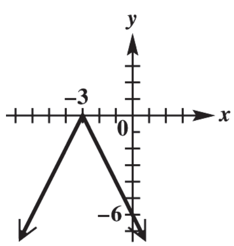


$f(x) = -(x-1)^2 + 4$

(b) $f(x) = -2|x+3|$

This is the graph of $g(x) = |x|$, translated three units to the left, reflected across the *x*-axis, and then stretched vertically by a factor of two.

x	$g(x) = \|x\|$	$f(x) = -2\|x+3\|$
−6	6	−6
−5	5	−4
−4	4	−2
−3	3	0
−2	2	−2
−1	1	−4
0	0	−6

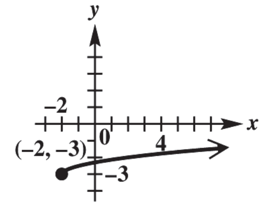


$g(x) = -2|x+3|$

(c) $h(x) = \frac{1}{2}\sqrt{x+2} - 3$

This is the graph of $g(x) = \sqrt{x}$, translated two units to the left, shrunk vertically by a factor of 2, and then translated 3 units down.

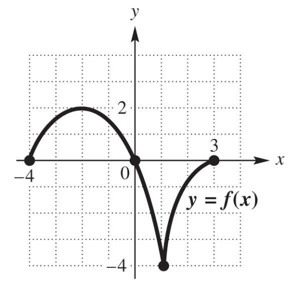
x	$g(x) = \sqrt{x}$	$h(x) = \frac{1}{2}\sqrt{x+2} - 3$
−2	undefined	−3
−1	undefined	−2.5
0	0	$\frac{1}{2}\sqrt{2} - 3 \approx -2.3$
2	$\sqrt{2} \approx 1.4$	−2
6	$\sqrt{6} \approx 2.4$	$\frac{1}{2}\sqrt{8} - 3 \approx -1.6$
7	$\sqrt{7} \approx 2.6$	−1.5



$h(x) = \frac{1}{2}\sqrt{x+2} - 3$

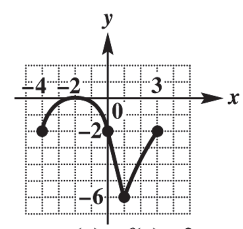
Classroom Example 9 (page 270)

The graphs in the exercises are based on the following graph.



(a) $g(x) = f(x) - 2$

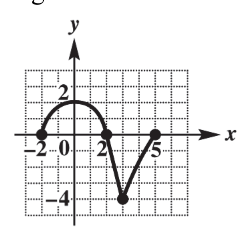
This is the graph of $f(x)$ translated two units down.



$g(x) = f(x) - 2$

(b) $h(x) = f(x-2)$

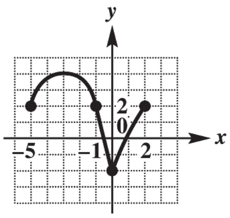
This is the graph of $f(x)$ translated two units right.



$h(x) = f(x-2)$

(c) $k(x) = f(x+1) + 2$

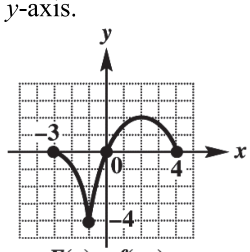
This is the graph of $f(x)$ translated one unit left, and then translated two units up.



$k(x) = f(x + 1) + 2$

(d) $F(x) = f(-x)$

This is the graph of $f(x)$ reflected across the y-axis.



$F(x) = f(-x)$

Section 2.8 Function Operations and Composition

Classroom Example 1 (page 278)

For parts (a)–(d), $f(x) = 3x - 4$ and $g(x) = 2x^2 - 1$

(a) $f(0) = 3(0) - 4 = -4$ and

$g(0) = 2(0)^2 - 1 = -1$, so

$(f+g)(0) = -4 - 1 = -5$

(b) $f(4) = 3(4) - 4 = 8$ and $g(4) = 2(4)^2 - 1 = 31$, so $(f-g)(4) = 8 - 31 = -23$

(c) $f(-2) = 3(-2) - 4 = -10$ and

$g(-2) = 2(-2)^2 - 1 = 7$, so

$(fg)(-2) = (-10)(7) = -70$

(d) $f(3) = 3(3) - 4 = 5$ and $g(3) = 2(3)^2 - 1 = 17$,

so $\left(\dfrac{f}{g}\right)(3) = \dfrac{5}{17}$

Classroom Example 2 (page 279)

For parts (a)–(e), $f(x) = x^2 - 3x$ and $g(x) = 4x + 5$

(a) $(f+g)(x) = (x^2 - 3x) + (4x + 5) = x^2 + x + 5$

(b) $(f-g)(x) = (x^2 - 3x) - (4x + 5) = x^2 - 7x - 5$

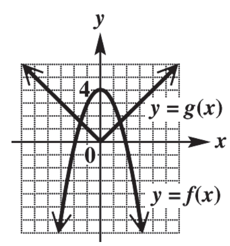
(c) $$\begin{aligned}(fg)(x) &= (x^2 - 3x)(4x + 5)\\ &= 4x^3 + 5x^2 - 12x^2 - 15x\\ &= 4x^3 - 7x^2 - 15x\end{aligned}$$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2 - 3x}{4x + 5}$

(e) The domains of f and g are both $(-\infty, \infty)$. So, the domains of $f+g$, $f-g$, and fg are the intersection of the domains of f and g, $(-\infty, \infty)$. The domain of $\frac{f}{g}$ includes those real numbers in the intersection of the domains of f and g for which $g(x) = 4x + 5 \neq 0 \Rightarrow x \neq -\frac{5}{4}$. So the domain of $\frac{f}{g}$ is $\left(-\infty, -\frac{5}{4}\right) \cup \left(-\frac{5}{4}, \infty\right)$.

Classroom Example 3 (page 280)

(a)



From the figure, we have $f(1) = 3$ and $g(1) = 1$, so $(f+g)(1) = 3 + 1 = 4$.
$f(0) = 4$ and $g(0) = 0$, so $(f-g)(0) = 4 - 0 = 4$.
$f(-1) = 3$ and $g(-1) = 1$, so
$(fg)(-1) = (3)(1) = 3$
$f(-2) = 0$ and $g(-2) = 2$, so $\left(\frac{f}{g}\right)(-2) = \frac{0}{2} = 0$.

(b)

x	$f(x)$	$g(x)$
−2	−5	0
−1	−3	2
0	−1	4
1	1	6

From the table, we have $f(1) = 1$ and $g(1) = 6$, so $(f+g)(1) = 1 + 6 = 7$.
$f(0) = -1$ and $g(0) = 4$, so
$(f-g)(1) = -1 - 4 = -5$.
$f(-1) = -3$ and $g(-1) = 2$,
so $(fg)(-1) = (-3)(2) = -6$
$f(-2) = -5$ and $g(-2) = 0$, so
$\left(\frac{f}{g}\right)(-2) = \frac{-5}{0} \Rightarrow \frac{f}{g}$ is undefined.

(c) $f(x) = 3x + 4, g(x) = -|x|$

From the formulas, we have

$f(1) = 3(1) + 4 = 7$ and $g(1) = -|1| = -1$, so

$(f + g)(1) = 7 + (-1) = 6.$

$f(0) = 3(0) + 4 = 4$ and $g(0) = -|0| = 0$, so

$(f - g)(1) = 4 - 0 = 4.$

$f(-1) = 3(-1) + 4 = 1$ and $g(-1) = -|-1| = -1$,

so $(fg)(-1) = (1)(-1) = -1.$

$f(-2) = 3(-2) + 4 = -2$ and

$g(-2) = -|-2| = -2$, so $\left(\frac{f}{g}\right)(-2) = \frac{-2}{-2} = 1.$

Classroom Example 4 (page 281)

Step 1: Find $f(x+h)$:

$$f(x+h) = 3(x+h)^2 - 2(x+h) + 4$$
$$= 3(x^2 + 2xh + h^2) - 2x - 2h + 4$$
$$= 3x^2 + 6xh + 3h^2 - 2x - 2h + 4$$

Step 2: Find $f(x+h) - f(x)$:

$$f(x+h) - f(x)$$
$$= (3x^2 + 6xh + 3h^2 - 2x - 2h + 4) - (3x^2 - 2x + 4)$$
$$= 6xh + 3h^2 - 2h$$

Step 3: Find the difference quotient:

$$\frac{f(x+h) - f(x)}{h} = \frac{6xh + 3h^2 - 2h}{h} = 6x + 3h - 2$$

Classroom Example 5 (page 283)

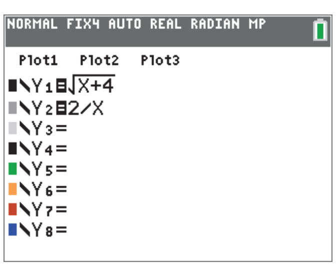
For parts (a) and (b), $f(x) = \sqrt{x+4}$ and $g(x) = \frac{2}{x}$

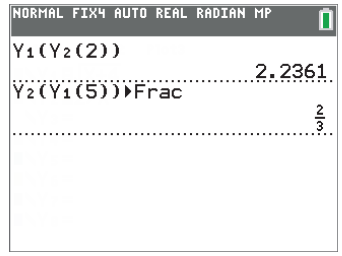
(a) First find g(2): $g(2) = \frac{2}{2} = 1$. Now find

$(f \circ g)(2) = f(g(2)) = f(1) = \sqrt{1+4} = \sqrt{5}$

(b) First find *f*(5): $f(5) = \sqrt{5+4} = \sqrt{9} = 3$. Now

find $(g \circ f)(5) = g(f(5)) = g(3) = \frac{2}{3}$





The screens show how a graphing calculator evaluates the expressions in this classroom example.

Classroom Example 6 (page 283)

For parts (a) and (b), $f(x) = \sqrt{x-1}$ and $g(x) = 2x + 5$

(a) $(f \circ g)(x) = f(g(x)) = \sqrt{(2x+5) - 1} = \sqrt{2x+4}$

The domain and range of *g* are both $(-\infty, \infty)$. However, the domain of *f* is $[1, \infty)$. Therefore, $g(x)$ must be greater than or equal to 1: $2x + 5 \geq 1 \Rightarrow x \geq -2$. So, the domain of $f \circ g$ is $[-2, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = 2\sqrt{x-1} + 5$

The domain of *f* is $[1, \infty)$, while the range of *f* is $[0, \infty)$. The domain of *g* is $(-\infty, \infty)$. Therefore, the domain of $(g \circ f)$ is restricted to that portion of the domain of *g* that intersects with the domain of *f*, that is $[1, \infty)$.

Classroom Example 7 (page 284)

For parts (a) and (b), $f(x) = \frac{5}{x+4}$ and $g(x) = \frac{2}{x}$

(a) $(f \circ g)(x) = f(g(x)) = \frac{5}{(2/x) + 4} = \frac{5x}{2 + 4x}$

The domain and range of *g* are both all real numbers except 0. The domain of *f* is all real numbers except −4. Therefore, the expression for $g(x)$ cannot equal −4. So,

$\frac{2}{x} \neq -4 \Rightarrow x \neq -\frac{1}{2}$. So, the domain of $f \circ g$ is the set of all real numbers except for $-\frac{1}{2}$, and 0. This is written

$\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, 0\right) \cup (0, \infty)$

(b) $(g \circ f)(x) = g(f(x)) = \dfrac{2}{5/(x+4)} = \dfrac{2x+8}{5}$

The domain of *f* is all real numbers except −4, while the range of *f* is all real numbers except 0. The domain and range of *g* are both all real numbers except 0, which is not in the range of *f*. So, the domain of $g \circ f$ is the set of all real numbers except for −4. This is written $(-\infty, -4) \cup (-4, \infty)$

Classroom Example 8 (page 285)

$f(x) = 2x - 5$ and $g(x) = 3x^2 + x$

$$\begin{aligned}(g \circ f)(x) &= g(2x-5) = 3(2x-5)^2 + (2x-5)\\ &= 3(4x^2 - 20x + 25) + 2x - 5\\ &= 12x^2 - 58x + 70\end{aligned}$$

$$\begin{aligned}(f \circ g)(x) &= f(3x^2 + x) = 2(3x^2 + x) - 5\\ &= 6x^2 + 2x - 5\end{aligned}$$

In general, $12x^2 - 58x + 70 \neq 6x^2 + 2x - 5$, so $(g \circ f)(x) \neq (f \circ g)(x)$.

Classroom Example 9 (page 286)

$(f \circ g)(x) = 4(3x+2)^2 - 5(3x+2) - 8$

Answers may vary. Sample answer:

$f(x) = 4x^2 - 5x - 8$ and $g(x) = 3x + 2$.