

# INSTRUCTOR'S SOLUTIONS MANUAL

JUDITH A. PENNA

## ALGEBRA & TRIGONOMETRY GRAPHS AND MODELS

SIXTH EDITION

## PRECALCULUS GRAPHS AND MODELS, A RIGHT TRIANGLE APPROACH

SIXTH EDITION

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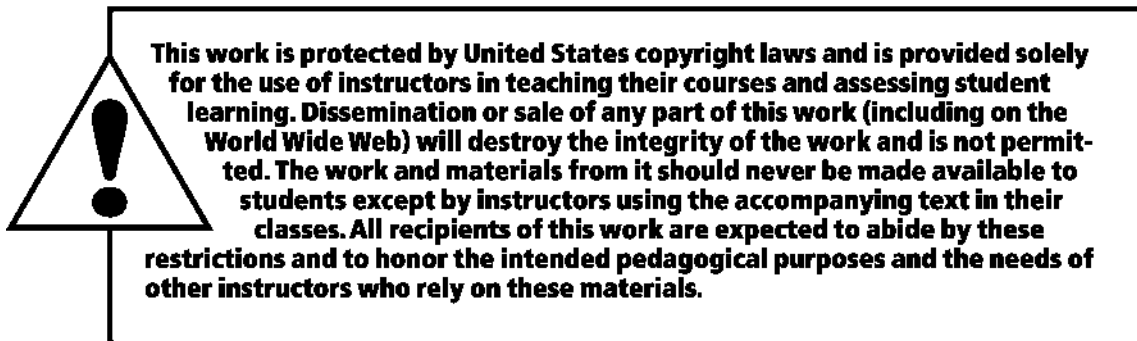
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# Just-in-Time Review

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## 1. Real Numbers

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1. Rational numbers:  $\frac{2}{3}$ , 6,  $-2.45$ ,  $18.\bar{4}$ ,  $-11$ ,  $\sqrt[3]{27}$ ,  $5\frac{1}{6}$ ,  $-\frac{8}{7}$ , 0,  $\sqrt{16}$
2. Rational numbers but not integers:  $\frac{2}{3}$ ,  $-2.45$ ,  $18.\bar{4}$ ,  $5\frac{1}{6}$ ,  $-\frac{8}{7}$
3. Irrational numbers:  $\sqrt{3}$ ,  $\sqrt[4]{26}$ ,  $7.151551555\dots$ ,  $-\sqrt{35}$ ,  $\sqrt[5]{3}$   
(Although there is a pattern in  $7.151551555\dots$ , there is no repeating block of digits.)
4. Integers: 6,  $-11$ ,  $\sqrt[3]{27}$ , 0,  $\sqrt{16}$
5. Whole numbers: 6,  $\sqrt[3]{27}$ , 0,  $\sqrt{16}$
6. Real numbers: All of them

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## 2. Properties of Real Numbers

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1.  $-24 + 24 = 0$  illustrates the additive inverse property.
2.  $7(xy) = (7x)y$  illustrates the associative property of multiplication.
3.  $9(r - s) = 9r - 9s$  illustrates a distributive property.
4.  $11 + z = z + 11$  illustrates the commutative property of addition.
5.  $-20 \cdot 1 = -20$  illustrates the multiplicative identity property.
6.  $5(x + y) = (x + y)5$  illustrates the commutative property of multiplication.
7.  $q + 0 = q$  illustrates the additive identity property.
8.  $75 \cdot \frac{1}{75} = 1$  illustrates the multiplicative inverse property.
9.  $(x + y) + w = x + (y + w)$  illustrates the associative property of addition.
10.  $8(a + b) = 8a + 8b$  illustrates a distributive property.

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## 3. Order on the Number Line

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1. 9 is to the right of  $-9$  on the number line, so it is false that  $9 < -9$ .
2.  $-10$  is to the left of  $-1$  on the number line, so it is true that  $-10 \leq -1$ .

3.  $-5 = -\sqrt{25}$ , and  $-\sqrt{26}$  is to the left of  $-\sqrt{25}$ , or  $-5$ , on the number line. Thus it is true that  $-\sqrt{26} < -5$ .
4.  $\sqrt{6} = \sqrt{6}$ , so it is true that  $\sqrt{6} \leq \sqrt{6}$ .
5.  $-30$  is to the left of  $-25$  on the number line, so it is false that  $-30 > -25$ .
6.  $-\frac{4}{5} = -\frac{16}{20}$  and  $-\frac{5}{4} = -\frac{25}{20}$ ;  $-\frac{16}{20}$  is to the right of  $-\frac{25}{20}$ , so it is true that  $-\frac{4}{5} > -\frac{5}{4}$ .

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## 4. Absolute Value

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1.  $|-98| = 98$  ( $|a| = -a$ , if  $a < 0$ .)
2.  $|0| = 0$  ( $|a| = a$ , if  $a \geq 0$ .)
3.  $|4.7| = 4.7$  ( $|a| = a$ , if  $a \geq 0$ .)
4.  $\left| -\frac{2}{3} \right| = \frac{2}{3}$  ( $|a| = -a$ , if  $a < 0$ .)
5.  $|-7 - 13| = |-20| = 20$ , or  
 $|13 - (-7)| = |13 + 7| = |20| = 20$
6.  $|2 - 14.6| = |-12.6| = 12.6$ , or  
 $|14.6 - 2| = |12.6| = 12.6$
7.  $|-39 - (-28)| = |-39 + 28| = |-11| = 11$ , or  
 $|-28 - (-39)| = |-28 + 39| = |11| = 11$
8.  $\left| -\frac{3}{4} - \frac{15}{8} \right| = \left| -\frac{6}{8} - \frac{15}{8} \right| = \left| -\frac{21}{8} \right| = \frac{21}{8}$ , or  
 $\left| \frac{15}{8} - \left( -\frac{3}{4} \right) \right| = \left| \frac{15}{8} + \frac{6}{8} \right| = \left| \frac{21}{8} \right| = \frac{21}{8}$

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## 5. Operations with Real Numbers

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1.  $8 - (-11) = 8 + 11 = 19$
2.  $-\frac{3}{10} \cdot \left( -\frac{1}{3} \right) = \frac{3 \cdot 1}{10 \cdot 3} = \frac{3}{3} \cdot \frac{1}{10} = 1 \cdot \frac{1}{10} = \frac{1}{10}$
3.  $15 \div (-3) = -5$
4.  $-4 - (-1) = -4 + 1 = -3$
5.  $7 \cdot (-50) = -350$
6.  $-0.5 - 5 = -0.5 + (-5) = -5.5$
7.  $-3 + 27 = 24$
8.  $-400 \div -40 = 10$
9.  $4.2 \cdot (-3) = -12.6$

10.  $-13 - (-33) = -13 + 33 = 20$
11.  $-60 + 45 = -15$
12.  $\frac{1}{2} - \frac{2}{3} = \frac{1}{2} + \left(-\frac{2}{3}\right) = \frac{3}{6} + \left(-\frac{4}{6}\right) = -\frac{1}{6}$
13.  $-24 \div 3 = -8$
14.  $-6 + (-16) = -22$
15.  $-\frac{1}{2} \div \left(-\frac{5}{8}\right) = -\frac{1}{2} \cdot \left(-\frac{8}{5}\right) = \frac{1 \cdot 8}{2 \cdot 5} = \frac{1 \cdot \cancel{2} \cdot 4}{\cancel{2} \cdot 5} = \frac{4}{5}$

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## 6. Interval Notation

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- This is a closed interval, so we use brackets. Interval notation is  $[-5, 5]$ .
- This is a half-open interval. We use a parenthesis on the left and a bracket on the right. Interval notation is  $(-3, -1]$ .
- This interval is of unlimited extent in the negative direction, and the endpoint  $-2$  is included. Interval notation is  $(-\infty, -2]$ .
- This interval is of unlimited extent in the positive direction, and the endpoint  $3.8$  is not included. Interval notation is  $(3.8, \infty)$ .
- $\{x|7 < x\}$ , or  $\{x|x > 7\}$ .  
This interval is of unlimited extent in the positive direction and the endpoint  $7$  is not included. Interval notation is  $(7, \infty)$ .
- The endpoints  $-2$  and  $2$  are not included in the interval, so we use parentheses. Interval notation is  $(-2, 2)$ .
- The endpoints  $-4$  and  $5$  are not included in the interval, so we use parentheses. Interval notation is  $(-4, 5)$ .
- The interval is of unlimited extent in the positive direction, and the endpoint  $1.7$  is included. Interval notation is  $[1.7, \infty)$ .
- The endpoint  $-5$  is not included in the interval, so we use a parenthesis before  $-5$ . The endpoint  $-2$  is included in the interval, so we use a bracket after  $-2$ . Interval notation is  $(-5, -2]$ .
- This interval is of unlimited extent in the negative direction, and the endpoint  $\sqrt{5}$  is not included. Interval notation is  $(-\infty, \sqrt{5})$ .

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## 7. Integers as Exponents

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- $3^{-6} = \frac{1}{3^6}$  Using  $a^{-m} = \frac{1}{a^m}$
- $\frac{1}{(0.2)^{-5}} = (0.2)^5$  Using  $a^{-m} = \frac{1}{a^m}$

- $\frac{w^{-4}}{z^{-9}} = \frac{z^9}{w^4}$  Using  $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$
- $\left(\frac{z}{y}\right)^2 = \frac{z^2}{y^2}$  Raising a quotient to a power
- $100^0 = 1$  Using  $a^0 = 1, a \neq 0$
- $\frac{a^5}{a^{-3}} = a^{5-(-3)} = a^{5+3} = a^8$  Using the quotient rule
- $(2xy^3)(-3x^{-5}y) = 2(-3)x \cdot x^{-5} \cdot y^3 \cdot y$   
 $= -6x^{1+(-5)}y^{3+1}$   
 $= -6x^{-4}y^4$ , or  $-\frac{6y^4}{x^4}$
- $x^{-4} \cdot x^{-7} = x^{-4+(-7)} = x^{-11}$ , or  $\frac{1}{x^{11}}$
- $(mn)^{-6} = m^{-6}n^{-6}$ , or  $\frac{1}{m^6n^6}$
- $(t^{-5})^4 = t^{-5 \cdot 4} = t^{-20}$ , or  $\frac{1}{t^{20}}$

---

## 8. Scientific Notation

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- Convert 18,500,000 to scientific notation.  
We want the decimal point to be positioned between the 1 and the 8, so we move it 7 places to the left. Since 18,500,000 is greater than 10, the exponent must be positive.  
 $18,500,000 = 1.85 \times 10^7$
- Convert 0.000786 to scientific notation.  
We want the decimal point to be positioned between the 7 and the 8, so we move it 4 places to the right. Since 0.000786 is between 0 and 1, the exponent must be negative.  
 $0.000786 = 7.86 \times 10^{-4}$
- Convert 0.000000023 to scientific notation.  
We want the decimal point to be positioned between the 2 and the 3, so we move it 9 places to the right. Since 0.000000023 is between 0 and 1, the exponent must be negative.  
 $0.000000023 = 2.3 \times 10^{-9}$
- Convert 8,927,000,000 to scientific notation.  
We want the decimal point to be positioned between the 8 and the 9, so we move it 9 places to the left. Since 8,927,000,000 is greater than 10, the exponent must be positive.  
 $8,927,000,000 = 8.927 \times 10^9$
- Convert  $4.3 \times 10^{-8}$  to decimal notation.  
The exponent is negative, so the number is between 0 and 1. We move the decimal point 8 places to the left.  
 $4.3 \times 10^{-8} = 0.000000043$

6. Convert  $5.17 \times 10^6$  to decimal notation.

The exponent is positive, so the number is greater than 10. We move the decimal point 6 places to the right.

$$5.17 \times 10^6 = 5,170,000$$

7. Convert  $6.203 \times 10^{11}$  to decimal notation.

The exponent is positive, so the number is greater than 10. We move the decimal point 11 places to the right.

$$6.203 \times 10^{11} = 620,300,000,000$$

8. Convert  $2.94 \times 10^{-5}$  to scientific notation.

The exponent is negative, so the number is between 0 and 1. We move the decimal point 5 places to the left.

$$2.94 \times 10^{-5} = 0.0000294$$

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## 9. Order of Operations

---

1.  $3 + 18 \div 6 - 3 = 3 + 3 - 3$  Dividing  
 $= 6 - 3 = 3$  Adding and subtracting
2.  $= 5 \cdot 3 + 8 \cdot 3^2 + 4(6 - 2)$   
 $= 5 \cdot 3 + 8 \cdot 3^2 + 4 \cdot 4$  Working inside parentheses  
 $= 5 \cdot 3 + 8 \cdot 9 + 4 \cdot 4$  Evaluating  $3^2$   
 $= 15 + 72 + 16$  Multiplying  
 $= 87 + 16$  Adding in order  
 $= 103$  from left to right
3.  $5[3 - 8 \cdot 3^2 + 4 \cdot 6 - 2]$   
 $= 5[3 - 8 \cdot 9 + 4 \cdot 6 - 2]$   
 $= 5[3 - 72 + 24 - 2]$   
 $= 5[-69 + 24 - 2]$   
 $= 5[-45 - 2]$   
 $= 5[-47]$   
 $= -235$
4.  $16 \div 4 \cdot 4 \div 2 \cdot 256$   
 $= 4 \cdot 4 \div 2 \cdot 256$  Multiplying and dividing  
in order from left to right  
 $= 16 \div 2 \cdot 256$   
 $= 8 \cdot 256$   
 $= 2048$
5.  $2^6 \cdot 2^{-3} \div 2^{10} \div 2^{-8}$   
 $= 2^3 \div 2^{10} \div 2^{-8}$   
 $= 2^{-7} \div 2^{-8}$   
 $= 2$

6. 
$$\frac{4(8-6)^2 - 4 \cdot 3 + 2 \cdot 8}{3^1 + 19^0}$$

$$= \frac{4 \cdot 2^2 - 4 \cdot 3 + 2 \cdot 8}{3 + 1}$$
 Calculating in the numerator and in the denominator  

$$= \frac{4 \cdot 4 - 4 \cdot 3 + 2 \cdot 8}{4}$$

$$= \frac{16 - 12 + 16}{4}$$

$$= \frac{4 + 16}{4}$$

$$= \frac{20}{4}$$

$$= 5$$

7.  $64 \div [(-4) \div (-2)] = 64 \div 2 = 32$

8.  $6[9 - (3 - 2)] + 4(2 - 3)$   
 $= 6[9 - 1] + 4(2 - 3)$   
 $= 6 \cdot 8 + 4(-1)$   
 $= 48 - 4$   
 $= 44$

---

## 10. Introduction to Polynomials

---

1.  $5 - x^6$   
The term of highest degree is  $-x^6$ , so the degree of the polynomial is 6.
2.  $x^2y^5 - x^7y + 4$   
The degree of  $x^2y^5$  is 2 + 5, or 7; the degree of  $-x^7y$  is 7 + 1, or 8; the degree of 4 is 0 ( $4 = 4x^0$ ). Thus the degree of the polynomial is 8.
3.  $2a^4 - 3 + a^2$   
The term of highest degree is  $2a^4$ , so the degree of the polynomial is 4.
4.  $-41 = -41x^0$ , so the degree of the polynomial is 0.
5.  $4x - x^3 + 0.1x^8 - 2x^5$   
The term of highest degree is  $0.1x^8$ , so the degree of the polynomial is 8.
6.  $x - 3$  has two terms. It is a binomial.
7.  $14y^5$  has one term. It is a monomial.
8.  $2y - \frac{1}{4}y^2 + 8$  has three terms. It is a trinomial.

---

## 11. Add and Subtract Polynomials

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1.  $(8y - 1) - (3 - y)$   
 $= (8y - 1) + (-3 + y)$   
 $= (8 + 1)y + (-1 - 3)$   
 $= 9y - 4$

2.  $(3x^2 - 2x - x^3 + 2) - (5x^2 - 8x - x^3 + 4)$   
 $= (3x^2 - 2x - x^3 + 2) + (-5x^2 + 8x + x^3 - 4)$   
 $= (3 - 5)x^2 + (-2 + 8)x + (-1 + 1)x^3 + (2 - 4)$   
 $= -2x^2 + 6x - 2$
3.  $(2x + 3y + z - 7) + (4x - 2y - z + 8) +$   
 $(-3x + y - 2z - 4)$   
 $= (2 + 4 - 3)x + (3 - 2 + 1)y + (1 - 1 - 2)z +$   
 $(-7 + 8 - 4)$   
 $= 3x + 2y - 2z - 3$
4.  $(3ab^2 - 4a^2b - 2ab + 6) +$   
 $(-ab^2 - 5a^2b + 8ab + 4)$   
 $= (3 - 1)ab^2 + (-4 - 5)a^2b + (-2 + 8)ab + (6 + 4)$   
 $= 2ab^2 - 9a^2b + 6ab + 10$
5.  $(5x^2 + 4xy - 3y^2 + 2) - (9x^2 - 4xy + 2y^2 - 1)$   
 $= (5x^2 + 4xy - 3y^2 + 2) + (-9x^2 + 4xy - 2y^2 + 1)$   
 $= (5 - 9)x^2 + (4 + 4)xy + (-3 - 2)y^2 + (2 + 1)$   
 $= -4x^2 + 8xy - 5y^2 + 3$

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## 12. Multiply Polynomials

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1.  $(3a^2)(-7a^4) = [3(-7)](a^2 \cdot a^4)$   
 $= -21a^6$
2.  $(y - 3)(y + 5)$   
 $= y^2 + 5y - 3y - 15$  Using FOIL  
 $= y^2 + 2y - 15$  Collecting like terms
3.  $(x + 6)(x + 3)$   
 $= x^2 + 3x + 6x + 18$  Using FOIL  
 $= x^2 + 9x + 18$  Collecting like terms
4.  $(2a + 3)(a + 5)$   
 $= 2a^2 + 10a + 3a + 15$  Using FOIL  
 $= 2a^2 + 13a + 15$  Collecting like terms
5.  $(2x + 3y)(2x + y)$   
 $= 4x^2 + 2xy + 6xy + 3y^2$  Using FOIL  
 $= 4x^2 + 8xy + 3y^2$
6.  $(11t - 1)(3t + 4)$   
 $= 33t^2 + 44t - 3t - 4$  Using FOIL  
 $= 33t^2 + 41t - 4$

---

## 13. Special Products of Binomials

---

1.  $(x + 3)^2$   
 $= x^2 + 2 \cdot x \cdot 3 + 3^2$   
 $[(A + B)^2 = A^2 + 2AB + B^2]$   
 $= x^2 + 6x + 9$

2.  $(5x - 3)^2$   
 $= (5x)^2 - 2 \cdot 5x \cdot 3 + 3^2$   
 $[(A - B)^2 = A^2 - 2AB + B^2]$   
 $= 25x^2 - 30x + 9$
3.  $(2x + 3y)^2$   
 $= (2x)^2 + 2(2x)(3y) + (3y)^2$   
 $[(A + B)^2 = A^2 + 2AB + B^2]$   
 $= 4x^2 + 12xy + 9y^2$
4.  $(a - 5b)^2$   
 $= a^2 - 2 \cdot a \cdot 5b + (5b)^2$   
 $[(A - B)^2 = A^2 - 2AB + B^2]$   
 $= a^2 - 10ab + 25b^2$
5.  $(n + 6)(n - 6)$   
 $= n^2 - 6^2$   $[(A + B)(A - B) = A^2 - B^2]$   
 $= n^2 - 36$
6.  $(3y + 4)(3y - 4)$   
 $= (3y)^2 - 4^2$   $[(A + B)(A - B) = A^2 - B^2]$   
 $= 9y^2 - 16$

---

## 14. Factor Polynomials; The FOIL Method

---

1.  $3x + 18 = 3 \cdot x + 3 \cdot 6 = 3(x + 6)$
2.  $2z^3 - 8z^2 = 2z^2 \cdot z - 2z^2 \cdot 4 = 2z^2(z - 4)$
3.  $3x^3 - x^2 + 18x - 6$   
 $= x^2(3x - 1) + 6(3x - 1)$   
 $= (3x - 1)(x^2 + 6)$
4.  $t^3 + 6t^2 - 2t - 12$   
 $= t^2(t + 6) - 2(t + 6)$   
 $= (t + 6)(t^2 - 2)$
5.  $w^2 - 7w + 10$

We look for two numbers with a product of 10 and a sum of  $-7$ . By trial, we determine that they are  $-5$  and  $-2$ .

$$w^2 - 7w + 10 = (w - 5)(w - 2)$$

6.  $t^2 + 8t + 15$

We look for two numbers with a product of 15 and a sum of 8. By trial, we determine that they are 3 and 5.

$$t^2 + 8t + 15 = (t + 3)(t + 5)$$

7.  $2n^2 - 20n - 48 = 2(n^2 - 10n - 24)$

Now factor  $n^2 - 10n - 24$ . We look for two numbers with a product of  $-24$  and a sum of  $-10$ . By trial, we determine that they are 2 and  $-12$ . Then  $n^2 - 10n - 24 = (n + 2)(n - 12)$ . We must include the common factor, 2, to have a factorization of the original trinomial.

$$2n^2 - 20n - 48 = 2(n + 2)(n - 12)$$



8.  $y^4 - 9y^3 + 14y^2 = y^2(y^2 - 9y + 14)$

Now factor  $y^2 - 9y + 14$ . Look for two numbers with a product of 14 and a sum of  $-9$ . The numbers are  $-2$  and  $-7$ . Then  $y^2 - 9y + 14 = (y - 2)(y - 7)$ . We must include the common factor,  $y^2$ , in order to have a factorization of the original trinomial.

$$y^4 - 9y^3 + 14y^2 = y^2(y - 2)(y - 7)$$

9.  $2n^2 + 9n - 56$

1. There is no common factor other than 1 or  $-1$ .
2. The factorization must be of the form  $(2n + \quad)(n + \quad)$ .
3. Factor the constant term,  $-56$ . The possibilities are  $-1 \cdot 56$ ,  $1(-56)$ ,  $-2 \cdot 28$ ,  $2(-28)$ ,  $-4 \cdot 16$ ,  $4(-16)$ ,  $-7 \cdot 8$ , and  $7(-8)$ . The factors can be written in the opposite order as well:  $56(-1)$ ,  $-56 \cdot 1$ ,  $28(-2)$ ,  $-28 \cdot 2$ ,  $16(-4)$ ,  $-16 \cdot 4$ ,  $8(-7)$ , and  $-8 \cdot 7$ .
4. Find a pair of factors for which the sum of the outer and the inner products is the middle term,  $9n$ . By trial, we determine that the factorization is  $(2n - 7)(n + 8)$ .

10.  $2y^2 + y - 6$

1. There is no common factor other than 1 or  $-1$ .
2. The factorization must be of the form  $(2y + \quad)(y + \quad)$ .
3. Factor the constant term,  $-6$ . The possibilities are  $-1 \cdot 6$ ,  $1(-6)$ ,  $-2 \cdot 3$ , and  $2(-3)$ . The factors can be written in the opposite order as well:  $6(-1)$ ,  $-6 \cdot 1$ ,  $3(-2)$  and  $-3 \cdot 2$ .
4. Find a pair of factors for which the sum of the outer and the inner products is the middle term,  $y$ . By trial, we determine that the factorization is  $(2y - 3)(y + 2)$ .

11.  $b^2 - 6bt + 5t^2$

We look for two numbers with a product of 5 and a sum of  $-6$ . By trial, we determine that they are  $-1$  and  $-5$ .

$$b^2 - 6bt + 5t^2 = (b - t)(b - 5t)$$

12.  $x^4 - 7x^2 - 30 = (x^2)^2 - 7x^2 - 30$

We look for two numbers with a product of  $-30$  and a sum of  $-7$ . By trial, we determine that they are 3 and  $-10$ .

$$x^4 - 7x^2 - 30 = (x^2 + 3)(x^2 - 10)$$

---

### 15. Factoring Polynomials; The $ac$ -Method

---

1.  $8x^2 - 6x - 9$

1. There is no common factor other than 1 or  $-1$ .
2. Multiply the leading coefficient and the constant:  $8(-9) = -72$ .
3. Try to factor  $-72$  so that the sum of the factors is the coefficient of the middle term,  $-6$ . The factors we want are  $-12$  and 6.

4. Split the middle term using the numbers found in step (3):

$$-6x = -12x + 6x$$

5. Factor by grouping.

$$\begin{aligned} 8x^2 - 6x - 9 &= 8x^2 - 12x + 6x - 9 \\ &= 4x(2x - 3) + 3(2x - 3) \\ &= (2x - 3)(4x + 3) \end{aligned}$$

2.  $10t^2 + 4t - 6$

1. Factor out the largest common factor, 2.

$$10t^2 + 4t - 6 = 2(5t^2 + 2t - 3)$$

Now factor  $5t^2 + 2t - 3$ .

2. Multiply the leading coefficient and the constant:  $5(-3) = -15$ .
3. Try to factor  $-15$  so that the sum of the factors is the coefficient of the middle term, 2. The factors we want are 5 and  $-3$ .
4. Split the middle term using the numbers found in step (3):

$$2t = 5t - 3t.$$

5. Factor by grouping.

$$\begin{aligned} 5t^2 + 2t - 3 &= 5t^2 + 5t - 3t - 3 \\ &= 5t(t + 1) - 3(t + 1) \\ &= (t + 1)(5t - 3) \end{aligned}$$

Include the largest common factor in the final factorization.

$$10t^2 + 4t - 6 = 2(t + 1)(5t - 3)$$

3.  $18a^2 - 51a + 15$

1. Factor out the largest common factor, 3.

$$18a^2 - 51a + 15 = 3(6a^2 - 17a + 5)$$

Now factor  $6a^2 - 17a + 5$ .

2. Multiply the leading coefficient and the constant:  $6(5) = 30$ .
3. Try to factor 30 so that the sum of the factors is the coefficient of the middle term,  $-17$ . The factors we want are  $-2$  and  $-15$ .
4. Split the middle term using the numbers found in step (3):

$$-17a = -2a - 15a.$$

5. Factor by grouping.

$$\begin{aligned} 6a^2 - 17a + 5 &= 6a^2 - 2a - 15a + 5 \\ &= 2a(3a - 1) - 5(3a - 1) \\ &= (3a - 1)(2a - 5) \end{aligned}$$

Include the largest common factor in the final factorization.

$$18a^2 - 51a + 15 = 3(3a - 1)(2a - 5)$$

### 16. Special Factorizations

- $z^2 - 81 = z^2 - 9^2 = (z + 9)(z - 9)$
- $16x^2 - 9 = (4x)^2 - 3^2 = (4x + 3)(4x - 3)$
- $$\begin{aligned} 7pq^4 - 7py^4 &= 7p(q^4 - y^4) \\ &= 7p[(q^2)^2 - (y^2)^2] \\ &= 7p(q^2 + y^2)(q^2 - y^2) \\ &= 7p(q^2 + y^2)(q + y)(q - y) \end{aligned}$$
- $x^2 + 12x + 36 = x^2 + 2 \cdot x \cdot 6 + 6^2 = (x + 6)^2$
- $9z^2 - 12z + 4 = (3z)^2 - 2 \cdot 3z \cdot 2 + 2^2 = (3z - 2)^2$
- $$\begin{aligned} a^3 + 24a^2 + 144a &= a(a^2 + 24a + 144) \\ &= a(a^2 + 2 \cdot a \cdot 12 + 12^2) \\ &= a(a + 12)^2 \end{aligned}$$
- $x^3 + 64 = x^3 + 4^3 = (x + 4)(x^2 - 4x + 16)$
- $m^3 - 216 = m^3 - 6^3 = (m - 6)(m^2 + 6m + 36)$
- $3a^5 - 24a^2 = 3a^2(a^3 - 8) = 3a^2(a^3 - 2^3) = 3a^2(a - 2)(a^2 + 2a + 4)$
- $t^6 + 1 = (t^2)^3 + 1^3 = (t^2 + 1)(t^4 - t^2 + 1)$

### 17. Equation-Solving Principles

- $7t = 70$   
 $t = 10$  Dividing by 7  
The solution is 10.
- $x - 5 = 7$   
 $x = 12$  Adding 5  
The solution is 12.
- $3x + 4 = -8$   
 $3x = -12$  Subtracting 4  
 $x = -4$  Dividing by 3  
The solution is  $-4$ .
- $6x - 15 = 45$   
 $6x = 60$  Adding 15  
 $x = 10$  Dividing by 6  
The solution is 10.

- $7y - 1 = 23 - 5y$   
 $12y - 1 = 23$  Adding  $5y$   
 $12y = 24$  Adding 1  
 $y = 2$  Dividing by 12

The solution is 2.

- $3m - 7 = -13 + m$   
 $2m - 7 = -13$  Subtracting  $m$   
 $2m = -6$  Adding 7  
 $m = -3$  Dividing by 2

The solution is  $-3$ .

- $2(x + 7) = 5x + 14$   
 $2x + 14 = 5x + 14$   
 $-3x + 14 = 14$  Subtracting  $5x$   
 $-3x = 0$  Subtracting 14  
 $x = 0$

The solution is 0.

- $5y - (2y - 10) = 25$   
 $5y - 2y + 10 = 25$   
 $3y + 10 = 25$  Collecting like terms  
 $3y = 15$  Subtracting 10  
 $y = 5$  Dividing by 3

The solution is 5.

### 18. Inequality-Solving Principles

- $p + 25 \geq -100$   
 $p \geq -125$  Subtracting 25  
The solution set is  $[-125, \infty)$ .
- $-\frac{2}{3}x > 6$   
 $x < -\frac{3}{2} \cdot 6$  Multiplying by  $-\frac{3}{2}$  and reversing the inequality symbol  
 $x < -9$   
The solution set is  $(-\infty, -9)$ .
- $9x - 1 < 17$   
 $9x < 18$  Adding 1  
 $x < 2$  Dividing by 9  
The solution set is  $(-\infty, 2)$ .
- $-x - 16 \geq 40$   
 $-x \geq 56$  Adding 6  
 $x \leq -56$  Multiplying by  $-1$  and reversing the inequality symbol  
The solution set is  $(-\infty, -56]$ .

5.  $\frac{1}{3}y - 6 < 3$

$$\frac{1}{3}y < 9 \quad \text{Adding 6}$$

$$y < 27 \quad \text{Multiplying by 3}$$

The solution set is  $(-\infty, 27)$ .

6.  $8 - 2w \leq -14$

$$-2w \leq -22 \quad \text{Subtracting 8}$$

$$w \geq 11 \quad \text{Dividing by } -2 \text{ and} \\ \text{reversing the inequality symbol}$$

The solution set is  $[11, \infty)$ .

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### 19. The Principle of Zero Products

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1.  $2y^2 + 42y = 0$

$$2y(y + 21) = 0$$

$$2y = 0 \quad \text{or} \quad y + 21 = 0$$

$$y = 0 \quad \text{or} \quad y = -21$$

The solutions are 0 and  $-21$ .

2.  $(a + 7)(a - 1) = 0$

$$a + 7 = 0 \quad \text{or} \quad a - 1 = 0$$

$$a = -7 \quad \text{or} \quad a = 1$$

The solutions are  $-7$  and  $1$ .

3.  $(5y + 3)(y - 4) = 0$

$$5y + 3 = 0 \quad \text{or} \quad y - 4 = 0$$

$$5y = -3 \quad \text{or} \quad y = 4$$

$$y = -\frac{3}{5} \quad \text{or} \quad y = 4$$

The solutions are  $-\frac{3}{5}$  and  $4$ .

4.  $6x^2 + 7x - 5 = 0$

$$(3x + 5)(2x - 1) = 0$$

$$3x + 5 = 0 \quad \text{or} \quad 2x - 1 = 0$$

$$3x = -5 \quad \text{or} \quad 2x = 1$$

$$x = -\frac{5}{3} \quad \text{or} \quad x = \frac{1}{2}$$

The solutions are  $-\frac{5}{3}$  and  $\frac{1}{2}$ .

5.  $t(t - 8) = 0$

$$t = 0 \quad \text{or} \quad t - 8 = 0$$

$$t = 0 \quad \text{or} \quad t = 8$$

The solutions are 0 and 8.

6.  $x^2 - 8x - 33 = 0$

$$(x + 3)(x - 11) = 0$$

$$x + 3 = 0 \quad \text{or} \quad x - 11 = 0$$

$$x = -3 \quad \text{or} \quad x = 11$$

The solutions are  $-3$  and  $11$ .

7.  $x^2 + 13x = 30$

$$x^2 + 13x - 30 = 0$$

$$(x + 15)(x - 2) = 0$$

$$x + 15 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = -15 \quad \text{or} \quad x = 2$$

The solutions are  $-15$  and  $2$ .

8.  $12x^2 - 7x - 12 = 0$

$$(4x + 3)(3x - 4) = 0$$

$$4x + 3 = 0 \quad \text{or} \quad 3x - 4 = 0$$

$$4x = -3 \quad \text{or} \quad 3x = 4$$

$$x = -\frac{3}{4} \quad \text{or} \quad x = \frac{4}{3}$$

The solutions are  $-\frac{3}{4}$  and  $\frac{4}{3}$ .

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### 20. The Principle of Square Roots

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1.  $x^2 - 36 = 0$

$$x^2 = 36$$

$$x = \sqrt{36} \quad \text{or} \quad x = -\sqrt{36}$$

$$x = 6 \quad \text{or} \quad x = -6$$

The solutions are 6 and  $-6$ , or  $\pm 6$ .

2.  $2y^2 - 20 = 0$

$$2y^2 = 20$$

$$y^2 = 10$$

$$y = \sqrt{10} \quad \text{or} \quad y = -\sqrt{10}$$

The solutions are  $\sqrt{10}$  and  $-\sqrt{10}$ , or  $\pm\sqrt{10}$ .

3.  $6z^2 = 18$

$$z^2 = 3$$

$$z = \sqrt{3} \quad \text{or} \quad z = -\sqrt{3}$$

The solutions are  $\sqrt{3}$  and  $-\sqrt{3}$ , or  $\pm\sqrt{3}$ .

4.  $3t^2 - 15 = 0$

$$3t^2 = 15$$

$$t^2 = 5$$

$$t = \sqrt{5} \quad \text{or} \quad t = -\sqrt{5}$$

The solutions are  $\sqrt{5}$  and  $-\sqrt{5}$ , or  $\pm\sqrt{5}$ .

5.  $z^2 - 1 = 24$

$$z^2 = 25$$

$$z = \sqrt{25} \quad \text{or} \quad z = -\sqrt{25}$$

The solutions are 5 and  $-5$ , or  $\pm 5$ .

6.  $5x^2 - 75 = 0$

$$5x^2 = 75$$

$$x^2 = 15$$

$$x = \sqrt{15} \quad \text{or} \quad x = -\sqrt{15}$$

The solutions are  $\sqrt{15}$  and  $-\sqrt{15}$ , or  $\pm\sqrt{15}$ .

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**21. Simplify Rational Expressions**


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1.  $\frac{3x-3}{x(x-1)}$

The denominator is 0 when the factor  $x = 0$  and also when  $x - 1 = 0$ , or  $x = 1$ . The domain is the set of all real numbers except 0 and 1.

2.  $\frac{y+6}{y^2+4y-21} = \frac{y+6}{(y+7)(y-3)}$

The denominator is 0 when  $y = -7$  or  $y = 3$ . The domain is the set of all real numbers except  $-7$  and  $3$ .

3.  $\frac{x^2-4}{x^2-4x+4} = \frac{(x+2)(\cancel{x-2})}{(x-2)(\cancel{x-2})} = \frac{x+2}{x-2}$

4.  $\frac{x^2+2x-3}{x^2-9} = \frac{(x-1)(\cancel{x+3})}{(\cancel{x+3})(x-3)} = \frac{x-1}{x-3}$

5.  $\frac{x^3-6x^2+9x}{x^3-3x^2} = \frac{x(x^2-6x+9)}{x^2(x-3)}$   
 $= \frac{\cancel{x}(x-3)(x-3)}{\cancel{x} \cdot x(\cancel{x-3})}$   
 $= \frac{x-3}{x}$

6.  $\frac{6y^2+12y-48}{3y^2-9y+6} = \frac{6(y^2+2y-8)}{3(y^2-3y+2)}$   
 $= \frac{2 \cdot \cancel{3} \cdot (y+4)(\cancel{y-2})}{\cancel{3}(y-1)(\cancel{y-2})}$   
 $= \frac{2(y+4)}{y-1}$

---

**22. Multiply and Divide Rational Expressions**


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1.  $\frac{r-s}{r+s} \cdot \frac{r^2-s^2}{(r-s)^2} = \frac{(r-s)(r^2-s^2)}{(r+s)(r-s)^2}$   
 $= \frac{(\cancel{r-s})(\cancel{r-s})(r+s) \cdot 1}{(\cancel{r+s})(\cancel{r-s})(\cancel{r-s})}$   
 $= 1$

2.  $\frac{m^2-n^2}{r+s} \div \frac{m-n}{r+s}$   
 $= \frac{m^2-n^2}{r+s} \cdot \frac{r+s}{m-n}$   
 $= \frac{(m+n)(\cancel{m-n})(\cancel{r+s})}{(\cancel{r+s})(\cancel{m-n})}$   
 $= m+n$

3.  $\frac{4x^2+9x+2}{x^2+x-2} \cdot \frac{x^2-1}{3x^2+x-2}$   
 $= \frac{(4x+1)(\cancel{x+2})(\cancel{x+1})(\cancel{x-1})}{(\cancel{x+2})(\cancel{x-1})(3x-2)(\cancel{x+1})}$   
 $= \frac{4x+1}{3x-2}$

4.  $\frac{3x+12}{2x-8} \div \frac{(x+4)^2}{(x-4)^2}$   
 $= \frac{3x+12}{2x-8} \cdot \frac{(x-4)^2}{(x+4)^2}$   
 $= \frac{3(\cancel{x+4})(\cancel{x-4})(x-4)}{2(\cancel{x-4})(\cancel{x+4})(x+4)}$   
 $= \frac{3(x-4)}{2(x+4)}$

5.  $\frac{a^2-a-2}{a^2-a-6} \div \frac{a^2-2a}{2a+a^2}$   
 $= \frac{a^2-a-2}{a^2-a-6} \cdot \frac{2a+a^2}{a^2-2a}$   
 $= \frac{(\cancel{a-2})(a+1)(\cancel{a})(2+\cancel{a})}{(a-3)(\cancel{a+2})(\cancel{a})(\cancel{a-2})}$   
 $= \frac{a+1}{a-3}$

6.  $\frac{x^2-y^2}{x^3-y^3} \cdot \frac{x^2+xy+y^2}{x^2+2xy+y^2}$   
 $= \frac{(x+y)(x-y)(x^2+xy+y^2)}{(x-y)(x^2+xy+y^2)(x+y)(x+y)}$   
 $= \frac{1}{x+y} \cdot \frac{(x+y)(x-y)(x^2+xy+y^2)}{(x+y)(x-y)(x^2+xy+y^2)}$   
 $= \frac{1}{x+y} \cdot 1$  Removing a factor of 1  
 $= \frac{1}{x+y}$

---

**23. Add and Subtract Rational Expressions**


---

1.  $\frac{a-3b}{a+b} + \frac{a+5b}{a+b} = \frac{2a+2b}{a+b}$   
 $= \frac{2(\cancel{a+b})}{1 \cdot (\cancel{a+b})}$   
 $= 2$

2.  $\frac{x^2-5}{3x^2-5x-2} + \frac{x+1}{3x-6}$   
 $= \frac{x^2-5}{(3x+1)(x-2)} + \frac{x+1}{3(x-2)}$   
 $= \frac{x^2-5}{(3x+1)(x-2)} \cdot \frac{3}{3} + \frac{x+1}{3(x-2)} \cdot \frac{3x+1}{3x+1}$   
 $= \frac{3(x^2-5) + (x+1)(3x+1)}{3(3x+1)(x-2)}$   
 $= \frac{3x^2-15+3x^2+4x+1}{3(3x+1)(x-2)}$   
 $= \frac{6x^2+4x-14}{3(3x+1)(x-2)}$

$$\begin{aligned}
 3. \quad & \frac{a^2 + 1}{a^2 - 1} - \frac{a - 1}{a + 1} \\
 &= \frac{a^2 + 1}{(a + 1)(a - 1)} - \frac{a - 1}{a + 1}, \text{ LCD is } (a + 1)(a - 1) \\
 &= \frac{a^2 + 1 - (a - 1)(a - 1)}{(a + 1)(a - 1)} \\
 &= \frac{a^2 + 1 - a^2 + 2a - 1}{(a + 1)(a - 1)} \\
 &= \frac{2a}{(a + 1)(a - 1)}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \frac{9x + 2}{3x^2 - 2x - 8} + \frac{7}{3x^2 + x - 4} \\
 &= \frac{9x + 2}{(3x + 4)(x - 2)} + \frac{7}{(3x + 4)(x - 1)}, \\
 & \quad \text{LCD is } (3x + 4)(x - 2)(x - 1) \\
 &= \frac{9x + 2}{(3x + 4)(x - 2)} \cdot \frac{x - 1}{x - 1} + \frac{7}{(3x + 4)(x - 1)} \cdot \frac{x - 2}{x - 2} \\
 &= \frac{9x^2 - 7x - 2}{(3x + 4)(x - 2)(x - 1)} + \frac{7x - 14}{(3x + 4)(x - 1)(x - 2)} \\
 &= \frac{9x^2 - 16}{(3x + 4)(x - 2)(x - 1)} \\
 &= \frac{\cancel{(3x + 4)}(3x - 4)}{\cancel{(3x + 4)}(x - 2)(x - 1)} \\
 &= \frac{3x - 4}{(x - 2)(x - 1)}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \frac{y}{y^2 - y - 20} - \frac{2}{y + 4} \\
 &= \frac{y}{(y + 4)(y - 5)} - \frac{2}{y + 4}, \text{ LCD is } (y + 4)(y - 5) \\
 &= \frac{y}{(y + 4)(y - 5)} - \frac{2}{y + 4} \cdot \frac{y - 5}{y - 5} \\
 &= \frac{y}{(y + 4)(y - 5)} - \frac{2y - 10}{(y + 4)(y - 5)} \\
 &= \frac{y - (2y - 10)}{(y + 4)(y - 5)} \\
 &= \frac{y - 2y + 10}{(y + 4)(y - 5)} \\
 &= \frac{-y + 10}{(y + 4)(y - 5)}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \frac{3y}{y^2 - 7y + 10} - \frac{2y}{y^2 - 8y + 15} \\
 &= \frac{3y}{(y - 2)(y - 5)} - \frac{2y}{(y - 5)(y - 3)}, \\
 & \quad \text{LCD is } (y - 2)(y - 5)(y - 3) \\
 &= \frac{3y(y - 3) - 2y(y - 2)}{(y - 2)(y - 5)(y - 3)} \\
 &= \frac{3y^2 - 9y - 2y^2 + 4y}{(y - 2)(y - 5)(y - 3)} \\
 &= \frac{y^2 - 5y}{(y - 2)(y - 5)(y - 3)} \\
 &= \frac{y\cancel{(y - 5)}}{(y - 2)\cancel{(y - 5)}(y - 3)} \\
 &= \frac{y}{(y - 2)(y - 3)}
 \end{aligned}$$

---

## 24. Simplify Complex Rational Expressions

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$$\begin{aligned}
 1. \quad & \frac{\frac{x}{y} - \frac{y}{x}}{\frac{1}{y} + \frac{1}{x}} = \frac{\frac{x}{y} - \frac{y}{x}}{\frac{1}{y} + \frac{1}{x}} \cdot \frac{xy}{xy}, \text{ LCM is } xy \\
 &= \frac{\left(\frac{x}{y} - \frac{y}{x}\right)(xy)}{\left(\frac{1}{y} + \frac{1}{x}\right)(xy)} \\
 &= \frac{x^2 - y^2}{x + y} \\
 &= \frac{(x + y)(x - y)}{\cancel{(x + y)} \cdot 1} \\
 &= x - y
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \frac{\frac{a - b}{b}}{a^2 - b^2} = \frac{a - b}{b} \cdot \frac{ab}{a^2 - b^2} \\
 &= \frac{a - b}{b} \cdot \frac{ab}{(a + b)(a - b)} \\
 &= \frac{\cancel{a} \cancel{b} (a - b)}{\cancel{b} (a + b) \cancel{(a - b)}} \\
 &= \frac{a}{a + b}
 \end{aligned}$$

$$3. \frac{w + \frac{8}{w^2}}{1 + \frac{2}{w}} = \frac{w \cdot \frac{w^2}{w^2} + \frac{8}{w^2}}{1 \cdot \frac{w}{w} + \frac{2}{w}}$$

$$= \frac{\frac{w^3 + 8}{w^2}}{\frac{w + 2}{w}} = \frac{w^3 + 8}{w^2} \cdot \frac{w}{w + 2} = \frac{(w+2)(w^2 - 2w + 4)w}{w^2 \cdot w(w+2)} = \frac{w^2 - 2w + 4}{w}$$

$$4. \frac{\frac{x^2 - y^2}{xy}}{\frac{x - y}{y}} = \frac{x^2 - y^2}{xy} \cdot \frac{y}{x - y} = \frac{(x + y)(x - y)y}{xy(x - y)} = \frac{x + y}{x}$$

$$5. \frac{\frac{a}{1} - \frac{b}{1}}{\frac{1}{a} - \frac{1}{b}} = \frac{a^2 - b^2}{b - a} \quad \text{Multiplying by } \frac{ab}{ab} = \frac{(a + b)(a - b)}{b - a} = \frac{(a + b)(a - b)}{-1 \cdot (a - b)} = -a - b$$

$$11. \sqrt{x^2 - 4x + 4} = \sqrt{(x - 2)^2} = x - 2$$

$$12. \sqrt{2x^3y}\sqrt{12xy} = \sqrt{24x^4y^2} = \sqrt{4x^4y^2 \cdot 6} = 2x^2y\sqrt{6}$$

$$13. \sqrt[3]{3x^2y}\sqrt[3]{36x} = \sqrt[3]{108x^3y} = \sqrt[3]{27x^3 \cdot 4y} = 3x\sqrt[3]{4y}$$

$$14. \begin{aligned} 5\sqrt{2} + 3\sqrt{32} &= 5\sqrt{2} + 3\sqrt{16 \cdot 2} \\ &= 5\sqrt{2} + 3 \cdot 4\sqrt{2} \\ &= 5\sqrt{2} + 12\sqrt{2} \\ &= (5 + 12)\sqrt{2} \\ &= 17\sqrt{2} \end{aligned}$$

$$15. 7\sqrt{12} - 2\sqrt{3} = 7 \cdot 2\sqrt{3} - 2\sqrt{3} = 14\sqrt{3} - 2\sqrt{3} = 12\sqrt{3}$$

$$16. 2\sqrt{32} + 3\sqrt{8} - 4\sqrt{18} = 2 \cdot 4\sqrt{2} + 3 \cdot 2\sqrt{2} - 4 \cdot 3\sqrt{2} = 8\sqrt{2} + 6\sqrt{2} - 12\sqrt{2} = 2\sqrt{2}$$

$$17. \begin{aligned} 6\sqrt{20} - 4\sqrt{45} + \sqrt{80} &= 6\sqrt{4 \cdot 5} - 4\sqrt{9 \cdot 5} + \sqrt{16 \cdot 5} \\ &= 6 \cdot 2\sqrt{5} - 4 \cdot 3\sqrt{5} + 4\sqrt{5} \\ &= 12\sqrt{5} - 12\sqrt{5} + 4\sqrt{5} \\ &= (12 - 12 + 4)\sqrt{5} \\ &= 4\sqrt{5} \end{aligned}$$

$$18. \begin{aligned} (2 + \sqrt{3})(5 + 2\sqrt{3}) &= 2 \cdot 5 + 2 \cdot 2\sqrt{3} + \sqrt{3} \cdot 5 + \sqrt{3} \cdot 2\sqrt{3} \\ &= 10 + 4\sqrt{3} + 5\sqrt{3} + 3 \cdot 2 \\ &= 10 + 9\sqrt{3} + 6 \\ &= 16 + 9\sqrt{3} \end{aligned}$$

$$19. \begin{aligned} (\sqrt{8} + 2\sqrt{5})(\sqrt{8} - 2\sqrt{5}) &= (\sqrt{8})^2 - (2\sqrt{5})^2 \\ &= 8 - 4 \cdot 5 \\ &= 8 - 20 \\ &= -12 \end{aligned}$$

$$20. \begin{aligned} (1 + \sqrt{3})^2 &= 1^2 + 2 \cdot 1 \cdot \sqrt{3} + (\sqrt{3})^2 \\ &= 1 + 2\sqrt{3} + 3 \\ &= 4 + 2\sqrt{3} \end{aligned}$$

## 25. Simplify Radical Expressions

$$1. \sqrt{(-21)^2} = |-21| = 21$$

$$2. \sqrt{9y^2} = \sqrt{(3y)^2} = |3y| = 3y$$

$$3. \sqrt{(a - 2)^2} = a - 2$$

$$4. \sqrt[3]{-27x^3} = \sqrt[3]{(-3x)^3} = -3x$$

$$5. \sqrt[4]{81x^8} = \sqrt[4]{(3x^2)^4} = 3x^2$$

$$6. \sqrt[5]{32} = \sqrt[5]{2^5} = 2$$

$$7. \sqrt[4]{48x^6y^4} = \sqrt[4]{16x^4y^4 \cdot 3x^2} = 2xy\sqrt[4]{3x^2} = 2xy\sqrt[4]{3x^2}$$

$$8. \sqrt{15}\sqrt{35} = \sqrt{15 \cdot 35} = \sqrt{3 \cdot 5 \cdot 5 \cdot 7} = \sqrt{5^2 \cdot 3 \cdot 7} = \sqrt{5^2} \cdot \sqrt{3 \cdot 7} = 5\sqrt{21}$$

$$9. \frac{\sqrt{40xy}}{\sqrt{8x}} = \sqrt{\frac{40xy}{8x}} = \sqrt{5y}$$

$$10. \frac{\sqrt[3]{3x^2}}{\sqrt[3]{24x^5}} = \sqrt[3]{\frac{3x^2}{24x^5}} = \sqrt[3]{\frac{1}{8x^3}} = \frac{1}{2x}$$

## 26. Rationalizing Denominators

$$1. \frac{4}{\sqrt{11}} = \frac{4}{\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}} = \frac{4\sqrt{11}}{11}$$

$$2. \sqrt{\frac{3}{7}} = \sqrt{\frac{3}{7} \cdot \frac{7}{7}} = \sqrt{\frac{21}{49}} = \frac{\sqrt{21}}{\sqrt{49}} = \frac{\sqrt{21}}{7}$$

$$3. \frac{\sqrt[3]{7}}{\sqrt[3]{2}} = \frac{\sqrt[3]{7}}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{\sqrt[3]{28}}{\sqrt[3]{8}} = \frac{\sqrt[3]{28}}{2}$$

$$4. \sqrt[3]{\frac{16}{9}} = \sqrt[3]{\frac{16}{9} \cdot \frac{3}{3}} = \sqrt[3]{\frac{48}{27}} = \frac{\sqrt[3]{48}}{\sqrt[3]{27}} = \frac{\sqrt[3]{8 \cdot 6}}{3} = \frac{2\sqrt[3]{6}}{3}$$

$$\begin{aligned}
 5. \quad \frac{3}{\sqrt{30}-4} &= \frac{3}{\sqrt{30}-4} \cdot \frac{\sqrt{30}+4}{\sqrt{30}+4} \\
 &= \frac{3\sqrt{30}+12}{(\sqrt{30})^2-4^2} \\
 &= \frac{3\sqrt{30}+12}{30-16} \\
 &= \frac{3\sqrt{30}+12}{14}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \frac{4}{\sqrt{7}-\sqrt{3}} &= \frac{4}{\sqrt{7}-\sqrt{3}} \cdot \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}} \\
 &= \frac{4\sqrt{7}+4\sqrt{3}}{(\sqrt{7})^2-(\sqrt{3})^2} \\
 &= \frac{4\sqrt{7}+4\sqrt{3}}{7-3} \\
 &= \frac{4\sqrt{7}+4\sqrt{3}}{4} = \frac{4(\sqrt{7}+\sqrt{3})}{4} \\
 &= \sqrt{7}+\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \frac{6}{\sqrt{m}-\sqrt{n}} &= \frac{6}{\sqrt{m}-\sqrt{n}} \cdot \frac{\sqrt{m}+\sqrt{n}}{\sqrt{m}+\sqrt{n}} \\
 &= \frac{6(\sqrt{m}+\sqrt{n})}{(\sqrt{m})^2-(\sqrt{n})^2} \\
 &= \frac{6\sqrt{m}+6\sqrt{n}}{m-n}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad \frac{1-\sqrt{2}}{\sqrt{3}-\sqrt{6}} &= \frac{1-\sqrt{2}}{\sqrt{3}-\sqrt{6}} \cdot \frac{\sqrt{3}+\sqrt{6}}{\sqrt{3}+\sqrt{6}} \\
 &= \frac{\sqrt{3}+\sqrt{6}-\sqrt{6}-\sqrt{12}}{3-6} \\
 &= \frac{\sqrt{3}+\sqrt{6}-\sqrt{6}-2\sqrt{3}}{3-6} \\
 &= \frac{-\sqrt{3}}{-3} = \frac{\sqrt{3}}{3}
 \end{aligned}$$

---

## 27. Rational Exponents

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$$1. \quad y^{5/6} = \sqrt[6]{y^5}$$

$$2. \quad x^{2/3} = \sqrt[3]{x^2}$$

$$3. \quad 16^{3/4} = (16^{1/4})^3 = (\sqrt[4]{16})^3 = 2^3 = 8$$

$$4. \quad 4^{7/2} = (\sqrt{4})^7 = 2^7 = 128$$

$$5. \quad 125^{-1/3} = \frac{1}{125^{1/3}} = \frac{1}{\sqrt[3]{125}} = \frac{1}{5}$$

$$6. \quad 32^{-4/5} = (\sqrt[5]{32})^{-4} = 2^{-4} = \frac{1}{16}$$

$$7. \quad \sqrt[12]{y^4} = y^{4/12} = y^{1/3}$$

$$8. \quad \sqrt{x^5} = x^{5/2}$$

$$9. \quad x^{1/2} \cdot x^{2/3} = x^{1/2+2/3} = x^{3/6+4/6} = x^{7/6} = \sqrt[6]{x^7} = x^{\sqrt[6]{x}}$$

$$\begin{aligned}
 10. \quad (a-2)^{9/4}(a-2)^{-1/4} &= (a-2)^{9/4+(-1/4)} = \\
 (a-2)^{8/4} &= (a-2)^2
 \end{aligned}$$

$$\begin{aligned}
 11. \quad (m^{1/2}n^{5/2})^{2/3} &= m^{\frac{1}{2} \cdot \frac{2}{3}} n^{\frac{5}{2} \cdot \frac{2}{3}} = m^{1/3} n^{5/3} = \\
 \sqrt[3]{m} \sqrt[3]{n^5} &= \sqrt[3]{mn^5} = n \sqrt[3]{mn^2}
 \end{aligned}$$

---

## 28. The Pythagorean Theorem

---

$$1. \quad a^2 + b^2 = c^2$$

$$8^2 + 15^2 = c^2$$

$$64 + 225 = c^2$$

$$289 = c^2$$

$$17 = c$$

$$2. \quad a^2 + b^2 = c^2$$

$$4^2 + 4^2 = c^2$$

$$16 + 16 = c^2$$

$$32 = c^2$$

$$\sqrt{32} = c$$

$$5.657 \approx c$$

$$3. \quad a^2 + b^2 = c^2$$

$$5^2 + b^2 = 13^2$$

$$25 + b^2 = 169$$

$$b^2 = 144$$

$$b = 12$$

$$4. \quad a^2 + b^2 = c^2$$

$$a^2 + 12^2 = 13^2$$

$$a^2 + 144 = 169$$

$$a^2 = 25$$

$$a = 5$$

$$5. \quad a^2 + b^2 = c^2$$

$$(\sqrt{5})^2 + b^2 = 6^2$$

$$5 + b^2 = 36$$

$$b^2 = 31$$

$$b = \sqrt{31} \approx 5.568$$





# Chapter 1

## Graphs, Functions, and Models

### Exercise Set 1.1

1. Point A is located 5 units to the left of the  $y$ -axis and 4 units up from the  $x$ -axis, so its coordinates are  $(-5, 4)$ .

Point B is located 2 units to the right of the  $y$ -axis and 2 units down from the  $x$ -axis, so its coordinates are  $(2, -2)$ .

Point C is located 0 units to the right or left of the  $y$ -axis and 5 units down from the  $x$ -axis, so its coordinates are  $(0, -5)$ .

Point D is located 3 units to the right of the  $y$ -axis and 5 units up from the  $x$ -axis, so its coordinates are  $(3, 5)$ .

Point E is located 5 units to the left of the  $y$ -axis and 4 units down from the  $x$ -axis, so its coordinates are  $(-5, -4)$ .

Point F is located 3 units to the right of the  $y$ -axis and 0 units up or down from the  $x$ -axis, so its coordinates are  $(3, 0)$ .

2. G:  $(2, 1)$ ; H:  $(0, 0)$ ; I:  $(4, -3)$ ; J:  $(-4, 0)$ ; K:  $(-2, 3)$ ; L:  $(0, 5)$

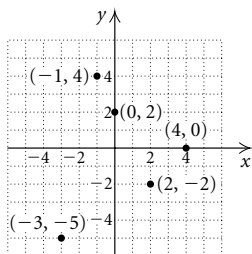
3. To graph  $(4, 0)$  we move from the origin 4 units to the right of the  $y$ -axis. Since the second coordinate is 0, we do not move up or down from the  $x$ -axis.

To graph  $(-3, -5)$  we move from the origin 3 units to the left of the  $y$ -axis. Then we move 5 units down from the  $x$ -axis.

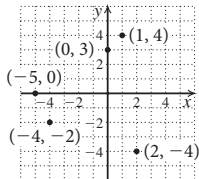
To graph  $(-1, 4)$  we move from the origin 1 unit to the left of the  $y$ -axis. Then we move 4 units up from the  $x$ -axis.

To graph  $(0, 2)$  we do not move to the right or the left of the  $y$ -axis since the first coordinate is 0. From the origin we move 2 units up.

To graph  $(2, -2)$  we move from the origin 2 units to the right of the  $y$ -axis. Then we move 2 units down from the  $x$ -axis.



4.



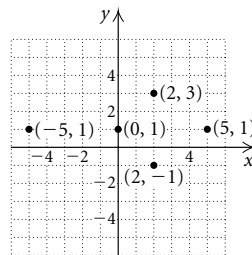
5. To graph  $(-5, 1)$  we move from the origin 5 units to the left of the  $y$ -axis. Then we move 1 unit up from the  $x$ -axis.

To graph  $(5, 1)$  we move from the origin 5 units to the right of the  $y$ -axis. Then we move 1 unit up from the  $x$ -axis.

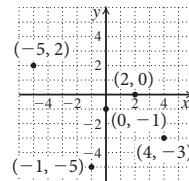
To graph  $(2, 3)$  we move from the origin 2 units to the right of the  $y$ -axis. Then we move 3 units up from the  $x$ -axis.

To graph  $(2, -1)$  we move from the origin 2 units to the right of the  $y$ -axis. Then we move 1 unit down from the  $x$ -axis.

To graph  $(0, 1)$  we do not move to the right or the left of the  $y$ -axis since the first coordinate is 0. From the origin we move 1 unit up.



6.



7. The first coordinate represents the year and the corresponding second coordinate represents the number of cities served by Southwest Airlines. The ordered pairs are  $(1971, 3)$ ,  $(1981, 15)$ ,  $(1991, 32)$ ,  $(2001, 59)$ ,  $(2011, 72)$ , and  $(2014, 96)$ .

8. The first coordinate represents the year and the second coordinate represents the percent of Marines who are women. The ordered pairs are  $(1960, 1\%)$ ,  $(1970, 0.9\%)$ ,  $(1980, 3.6\%)$ ,  $(1990, 4.9\%)$ ,  $(2000, 6.1\%)$ ,  $(2011, 6.8\%)$ , and  $(2014, 7.6\%)$ .

9. To determine whether  $(-1, -9)$  is a solution, substitute  $-1$  for  $x$  and  $-9$  for  $y$ .

$$\begin{array}{r|l}
 y = 7x - 2 & \\
 -9 \stackrel{?}{=} 7(-1) - 2 & \\
 -9 & -7 - 2 \\
 -9 & -9 \qquad \text{TRUE}
 \end{array}$$

The equation  $-9 = -9$  is true, so  $(-1, -9)$  is a solution.

To determine whether  $(0, 2)$  is a solution, substitute 0 for  $x$  and 2 for  $y$ .

$$\begin{array}{r|l} y = 7x - 2 & \\ \hline 2 \text{ ? } 7 \cdot 0 - 2 & \\ 0 - 2 & \\ 2 \text{ | } -2 & \text{FALSE} \end{array}$$

The equation  $2 = -2$  is false, so  $(0, 2)$  is not a solution.

10. For  $(\frac{1}{2}, 8)$ :

$$\begin{array}{r|l} y = -4x + 10 & \\ \hline 8 \text{ ? } -4 \cdot \frac{1}{2} + 10 & \\ -2 + 10 & \\ 8 \text{ | } 8 & \text{TRUE} \end{array}$$

$(\frac{1}{2}, 8)$  is a solution.

For  $(-1, 6)$ :

$$\begin{array}{r|l} y = -4x + 10 & \\ \hline 6 \text{ ? } -4(-1) + 10 & \\ 4 + 10 & \\ 6 \text{ | } 14 & \text{FALSE} \end{array}$$

$(-1, 6)$  is not a solution.

11. To determine whether  $(\frac{2}{3}, \frac{3}{4})$  is a solution, substitute  $\frac{2}{3}$  for  $x$  and  $\frac{3}{4}$  for  $y$ .

$$\begin{array}{r|l} 6x - 4y = 1 & \\ \hline 6 \cdot \frac{2}{3} - 4 \cdot \frac{3}{4} \text{ ? } 1 & \\ 4 - 3 & \\ 1 \text{ | } 1 & \text{TRUE} \end{array}$$

The equation  $1 = 1$  is true, so  $(\frac{2}{3}, \frac{3}{4})$  is a solution.

To determine whether  $(1, \frac{3}{2})$  is a solution, substitute 1 for  $x$  and  $\frac{3}{2}$  for  $y$ .

$$\begin{array}{r|l} 6x - 4y = 1 & \\ \hline 6 \cdot 1 - 4 \cdot \frac{3}{2} \text{ ? } 1 & \\ 6 - 6 & \\ 0 \text{ | } 1 & \text{FALSE} \end{array}$$

The equation  $0 = 1$  is false, so  $(1, \frac{3}{2})$  is not a solution.

12. For  $(1.5, 2.6)$ :

$$\begin{array}{r|l} x^2 + y^2 = 9 & \\ \hline (1.5)^2 + (2.6)^2 \text{ ? } 9 & \\ 2.25 + 6.76 & \\ 9.01 \text{ | } 9 & \text{FALSE} \end{array}$$

$(1.5, 2.6)$  is not a solution.

For  $(-3, 0)$ :

$$\begin{array}{r|l} x^2 + y^2 = 9 & \\ \hline (-3)^2 + 0^2 \text{ ? } 9 & \\ 9 + 0 & \\ 9 \text{ | } 9 & \text{TRUE} \end{array}$$

$(-3, 0)$  is a solution.

13. To determine whether  $(-\frac{1}{2}, -\frac{4}{5})$  is a solution, substitute  $-\frac{1}{2}$  for  $a$  and  $-\frac{4}{5}$  for  $b$ .

$$\begin{array}{r|l} 2a + 5b = 3 & \\ \hline 2(-\frac{1}{2}) + 5(-\frac{4}{5}) \text{ ? } 3 & \\ -1 - 4 & \\ -5 \text{ | } 3 & \text{FALSE} \end{array}$$

The equation  $-5 = 3$  is false, so  $(-\frac{1}{2}, -\frac{4}{5})$  is not a solution.

To determine whether  $(0, \frac{3}{5})$  is a solution, substitute 0 for  $a$  and  $\frac{3}{5}$  for  $b$ .

$$\begin{array}{r|l} 2a + 5b = 3 & \\ \hline 2 \cdot 0 + 5 \cdot \frac{3}{5} \text{ ? } 3 & \\ 0 + 3 & \\ 3 \text{ | } 3 & \text{TRUE} \end{array}$$

The equation  $3 = 3$  is true, so  $(0, \frac{3}{5})$  is a solution.

14. For  $(0, \frac{3}{2})$ :

$$\begin{array}{r|l} 3m + 4n = 6 & \\ \hline 3 \cdot 0 + 4 \cdot \frac{3}{2} \text{ ? } 6 & \\ 0 + 6 & \\ 6 \text{ | } 6 & \text{TRUE} \end{array}$$

$(0, \frac{3}{2})$  is a solution.

For  $(\frac{2}{3}, 1)$ :

$$\begin{array}{r|l} 3m + 4n = 6 & \\ \hline 3 \cdot \frac{2}{3} + 4 \cdot 1 \text{ ? } 6 & \\ 2 + 4 & \\ 6 \text{ | } 6 & \text{TRUE} \end{array}$$

The equation  $6 = 6$  is true, so  $(\frac{2}{3}, 1)$  is a solution.

15. To determine whether  $(-0.75, 2.75)$  is a solution, substitute  $-0.75$  for  $x$  and  $2.75$  for  $y$ .

$$\begin{array}{r|l} x^2 - y^2 = 3 & \\ \hline (-0.75)^2 - (2.75)^2 \text{ ? } 3 & \\ 0.5625 - 7.5625 & \\ -7 \text{ | } 3 & \text{FALSE} \end{array}$$

The equation  $-7 = 3$  is false, so  $(-0.75, 2.75)$  is not a solution.

To determine whether  $(2, -1)$  is a solution, substitute 2 for  $x$  and  $-1$  for  $y$ .

$$\begin{array}{r|l} x^2 - y^2 = 3 & \\ \hline 2^2 - (-1)^2 \stackrel{?}{=} 3 & \\ 4 - 1 & \\ \hline 3 & 3 \text{ TRUE} \end{array}$$

The equation  $3 = 3$  is true, so  $(2, -1)$  is a solution.

16. For  $(2, -4)$ :

$$\begin{array}{r|l} 5x + 2y^2 = 70 & \\ \hline 5 \cdot 2 + 2(-4)^2 \stackrel{?}{=} 70 & \\ 10 + 2 \cdot 16 & \\ \hline 10 + 32 & \\ \hline 42 & 70 \text{ FALSE} \end{array}$$

$(2, -4)$  is not a solution.

For  $(4, -5)$ :

$$\begin{array}{r|l} 5x + 2y^2 = 70 & \\ \hline 5 \cdot 4 + 2(-5)^2 \stackrel{?}{=} 70 & \\ 20 + 2 \cdot 25 & \\ \hline 20 + 50 & \\ \hline 70 & 70 \text{ TRUE} \end{array}$$

$(4, -5)$  is a solution.

17. Graph  $5x - 3y = -15$ .

To find the  $x$ -intercept we replace  $y$  with 0 and solve for  $x$ .

$$\begin{aligned} 5x - 3 \cdot 0 &= -15 \\ 5x &= -15 \\ x &= -3 \end{aligned}$$

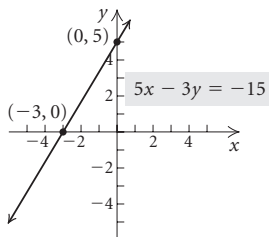
The  $x$ -intercept is  $(-3, 0)$ .

To find the  $y$ -intercept we replace  $x$  with 0 and solve for  $y$ .

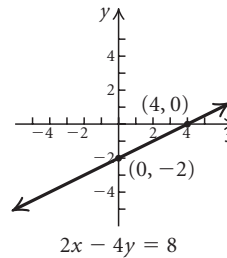
$$\begin{aligned} 5 \cdot 0 - 3y &= -15 \\ -3y &= -15 \\ y &= 5 \end{aligned}$$

The  $y$ -intercept is  $(0, 5)$ .

We plot the intercepts and draw the line that contains them. We could find a third point as a check that the intercepts were found correctly.



- 18.



19. Graph  $2x + y = 4$ .

To find the  $x$ -intercept we replace  $y$  with 0 and solve for  $x$ .

$$\begin{aligned} 2x + 0 &= 4 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

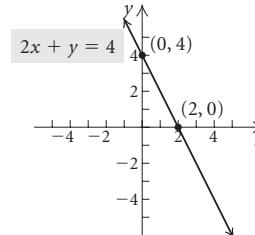
The  $x$ -intercept is  $(2, 0)$ .

To find the  $y$ -intercept we replace  $x$  with 0 and solve for  $y$ .

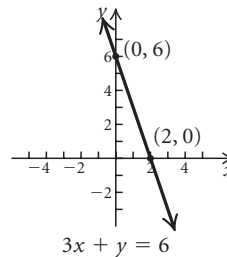
$$\begin{aligned} 2 \cdot 0 + y &= 4 \\ y &= 4 \end{aligned}$$

The  $y$ -intercept is  $(0, 4)$ .

We plot the intercepts and draw the line that contains them. We could find a third point as a check that the intercepts were found correctly.



- 20.



21. Graph  $4y - 3x = 12$ .

To find the  $x$ -intercept we replace  $y$  with 0 and solve for  $x$ .

$$\begin{aligned} 4 \cdot 0 - 3x &= 12 \\ -3x &= 12 \\ x &= -4 \end{aligned}$$

The  $x$ -intercept is  $(-4, 0)$ .

To find the  $y$ -intercept we replace  $x$  with 0 and solve for  $y$ .

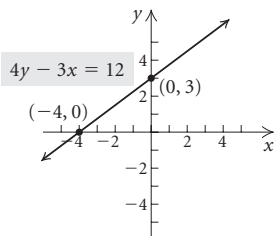
$$4y - 3 \cdot 0 = 12$$

$$4y = 12$$

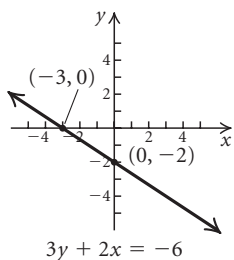
$$y = 3$$

The  $y$ -intercept is  $(0, 3)$ .

We plot the intercepts and draw the line that contains them. We could find a third point as a check that the intercepts were found correctly.



22.



23. Graph  $y = 3x + 5$ .

We choose some values for  $x$  and find the corresponding  $y$ -values.

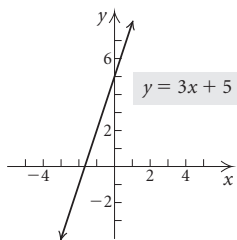
When  $x = -3$ ,  $y = 3x + 5 = 3(-3) + 5 = -9 + 5 = -4$ .

When  $x = -1$ ,  $y = 3x + 5 = 3(-1) + 5 = -3 + 5 = 2$ .

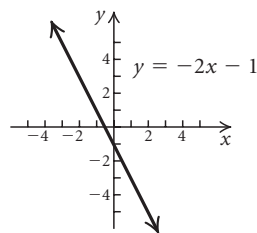
When  $x = 0$ ,  $y = 3x + 5 = 3 \cdot 0 + 5 = 0 + 5 = 5$

We list these points in a table, plot them, and draw the graph.

$x$	$y$	$(x, y)$
-3	-4	$(-3, -4)$
-1	2	$(-1, 2)$
0	5	$(0, 5)$



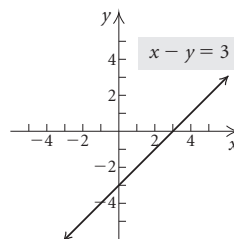
24.



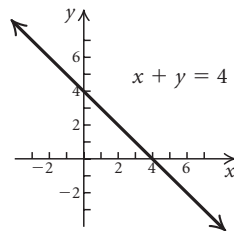
25. Graph  $x - y = 3$ .

Make a table of values, plot the points in the table, and draw the graph.

$x$	$y$	$(x, y)$
-2	-5	$(-2, -5)$
0	-3	$(0, -3)$
3	0	$(3, 0)$



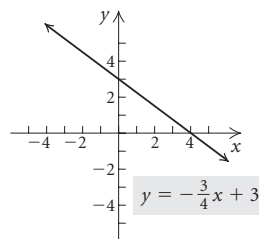
26.



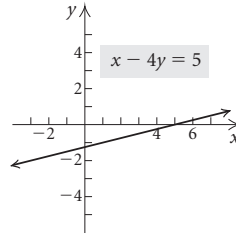
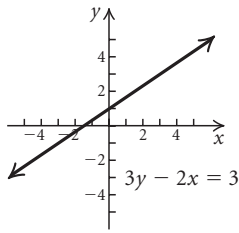
27. Graph  $y = -\frac{3}{4}x + 3$ .

By choosing multiples of 4 for  $x$ , we can avoid fraction values for  $y$ . Make a table of values, plot the points in the table, and draw the graph.

$x$	$y$	$(x, y)$
-4	6	$(-4, 6)$
0	3	$(0, 3)$
4	0	$(4, 0)$



28.



29. Graph  $5x - 2y = 8$ .

We could solve for  $y$  first.

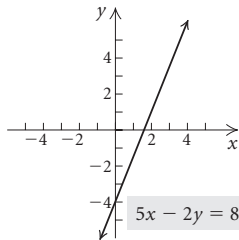
$$5x - 2y = 8$$

$$-2y = -5x + 8 \quad \text{Subtracting } 5x \text{ on both sides}$$

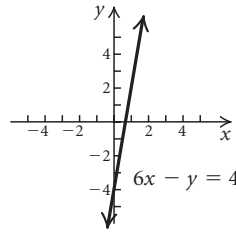
$$y = \frac{5}{2}x - 4 \quad \text{Multiplying by } -\frac{1}{2} \text{ on both sides}$$

By choosing multiples of 2 for  $x$  we can avoid fraction values for  $y$ . Make a table of values, plot the points in the table, and draw the graph.

$x$	$y$	$(x, y)$
0	-4	(0, -4)
2	1	(2, 1)
4	6	(4, 6)



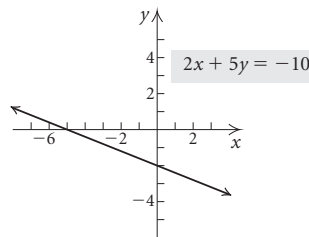
32.



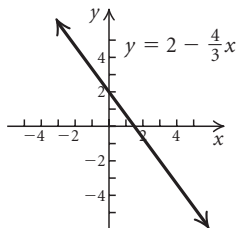
33. Graph  $2x + 5y = -10$ .

In this case, it is convenient to find the intercepts along with a third point on the graph. Make a table of values, plot the points in the table, and draw the graph.

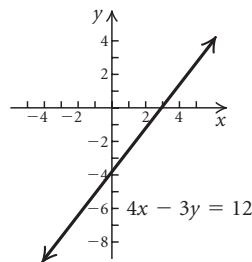
$x$	$y$	$(x, y)$
-5	0	(-5, 0)
0	-2	(0, -2)
5	-4	(5, -4)



30.



34.



31. Graph  $x - 4y = 5$ .

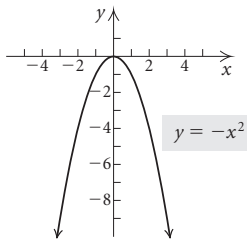
Make a table of values, plot the points in the table, and draw the graph.

$x$	$y$	$(x, y)$
-3	-2	(-3, -2)
1	-1	(1, -1)
5	0	(5, 0)

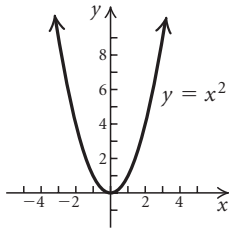
35. Graph  $y = -x^2$ .

Make a table of values, plot the points in the table, and draw the graph.

$x$	$y$	$(x, y)$
-2	-4	$(-2, -4)$
-1	-1	$(-1, -1)$
0	0	$(0, 0)$
1	-1	$(1, -1)$
2	-4	$(2, -4)$



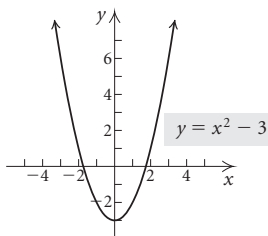
36.



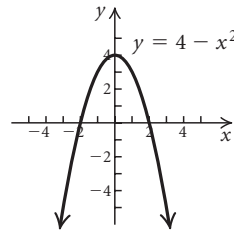
37. Graph  $y = x^2 - 3$ .

Make a table of values, plot the points in the table, and draw the graph.

$x$	$y$	$(x, y)$
-3	6	$(-3, 6)$
-1	-2	$(-1, -2)$
0	-3	$(0, -3)$
1	-2	$(1, -2)$
3	6	$(3, 6)$



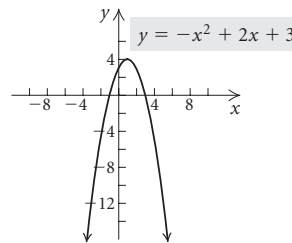
38.



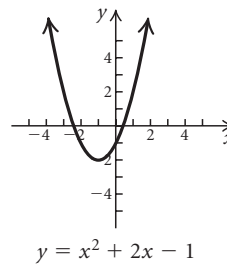
39. Graph  $y = -x^2 + 2x + 3$ .

Make a table of values, plot the points in the table, and draw the graph.

$x$	$y$	$(x, y)$
-2	-5	$(-2, -5)$
-1	0	$(-1, 0)$
0	3	$(0, 3)$
1	4	$(1, 4)$
2	3	$(2, 3)$
3	0	$(3, 0)$
4	-5	$(4, -5)$



40.



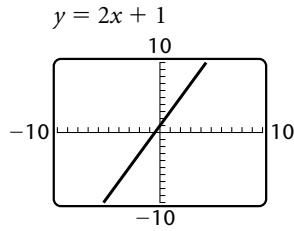
41. Graph (b) is the graph of  $y = 3 - x$ .

42. Graph (d) is the graph of  $2x - y = 6$ .

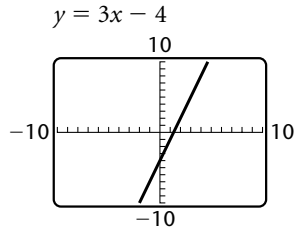
43. Graph (a) is the graph of  $y = x^2 + 2x + 1$ .

44. Graph (c) is the graph of  $y = 8 - x^2$ .

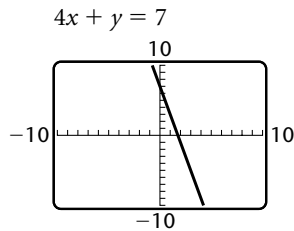
45. Enter the equation, select the standard window, and graph the equation.



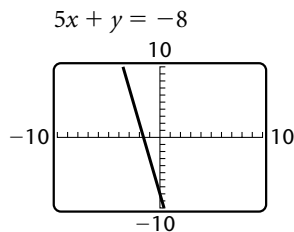
- 46.



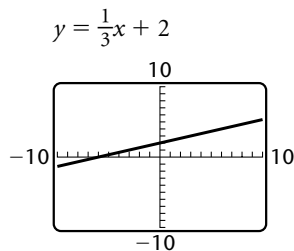
47. First solve the equation for  $y$ :  $y = -4x + 7$ . Enter the equation in this form, select the standard window, and graph the equation.



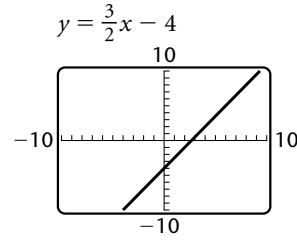
48.  $5x + y = -8$ , so  $y = -5x - 8$ .



49. Enter the equation, select the standard window, and graph the equation.



- 50.



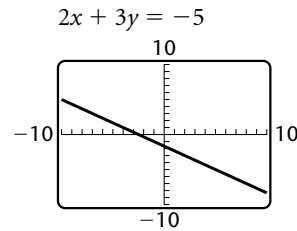
51. First solve the equation for  $y$ .

$$2x + 3y = -5$$

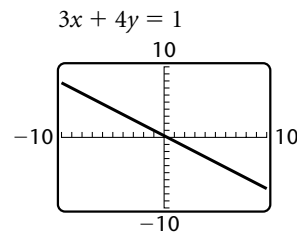
$$3y = -2x - 5$$

$$y = \frac{-2x - 5}{3}, \text{ or } \frac{1}{3}(-2x - 5)$$

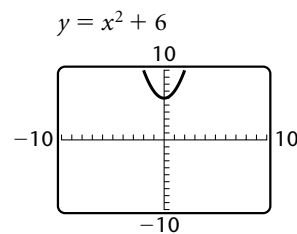
Enter the equation in “ $y =$ ” form, select the standard window, and graph the equation.



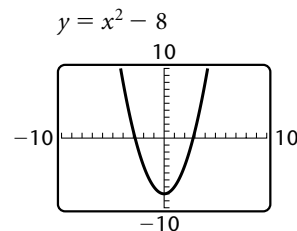
52.  $3x + 4y = 1$ , so  $y = \frac{-3x + 1}{4}$ , or  $y = -\frac{3}{4}x + \frac{1}{4}$



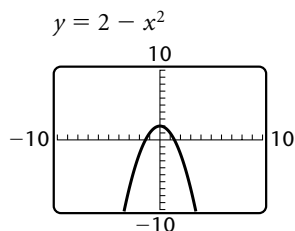
53. Enter the equation, select the standard window, and graph the equation.



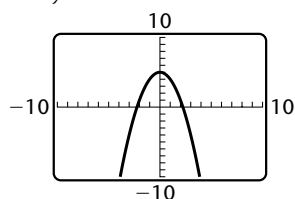
- 54.



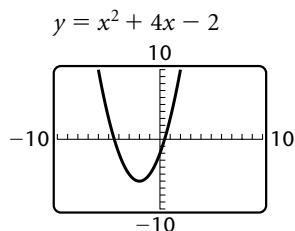
55. Enter the equation, select the standard window, and graph the equation.



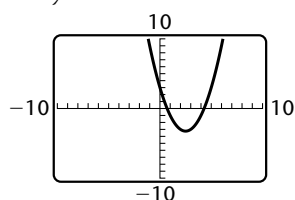
56.  $y = 5 - x^2$



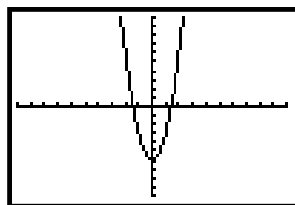
57. Enter the equation, select the standard window, and graph the equation.



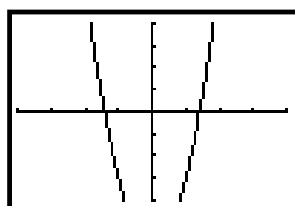
58.  $y = x^2 - 5x + 3$



59. Standard window:

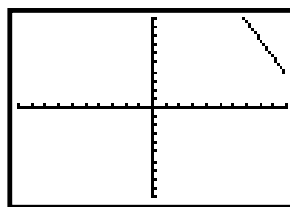


$[-4, 4, -4, 4]$

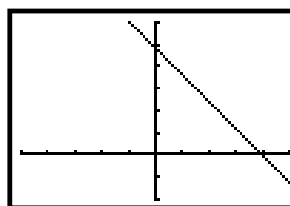


We see that the standard window is a better choice for this graph.

60. Standard window:

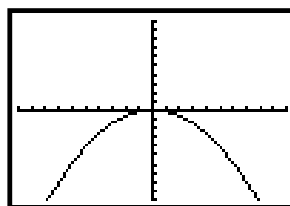


$[-15, 15, -10, 30]$ , Xscl = 3, Yscl = 5

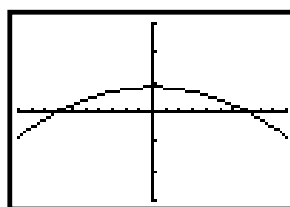


We see that  $[-15, 15, -10, 30]$  is a better choice for this graph.

61. Standard window:

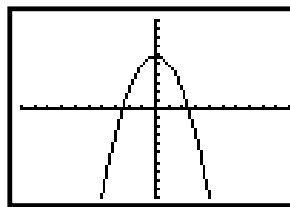


$[-1, 1, -0.3, 0.3]$ , Xscl = 0.1, Yscl = 0.1



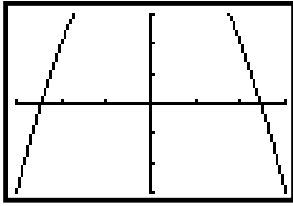
We see that  $[-1, 1, -0.3, 0.3]$  is a better choice for this graph.

62. Standard window:





$[-3, 3, -3, 3]$



We see that the standard window is a better choice for this graph.

63. Either point can be considered as  $(x_1, y_1)$ .

$$d = \sqrt{(4-5)^2 + (6-9)^2} \\ = \sqrt{(-1)^2 + (-3)^2} = \sqrt{10} \approx 3.162$$

64.  $d = \sqrt{(-3-2)^2 + (7-11)^2} = \sqrt{41} \approx 6.403$

65. Either point can be considered as  $(x_1, y_1)$ .

$$d = \sqrt{(-13 - (-8))^2 + (1 - (-11))^2} \\ = \sqrt{(-5)^2 + 12^2} = \sqrt{169} = 13$$

66.  $d = \sqrt{(-20 - (-60))^2 + (35 - 5)^2} = \sqrt{2500} = 50$

67. Either point can be considered as  $(x_1, y_1)$ .

$$d = \sqrt{(6-9)^2 + (-1-5)^2} \\ = \sqrt{(-3)^2 + (-6)^2} = \sqrt{45} \approx 6.708$$

68.  $d = \sqrt{(-4 - (-1))^2 + (-7 - 3)^2} = \sqrt{109} \approx 10.440$

69. Either point can be considered as  $(x_1, y_1)$ .

$$d = \sqrt{(-8-8)^2 + \left(\frac{7}{11} - \frac{7}{11}\right)^2} \\ = \sqrt{(-16)^2 + 0^2} = 16$$

70.  $d = \sqrt{\left(\frac{1}{2} - \frac{1}{2}\right)^2 + \left(-\frac{4}{25} - \left(-\frac{13}{25}\right)\right)^2} = \sqrt{\left(\frac{9}{25}\right)^2} = \frac{9}{25}$

71.  $d = \sqrt{\left[-\frac{3}{5} - \left(-\frac{3}{5}\right)\right]^2 + \left(-4 - \frac{2}{3}\right)^2} \\ = \sqrt{0^2 + \left(-\frac{14}{3}\right)^2} = \frac{14}{3}$

72.  $d = \sqrt{\left(-\frac{11}{3} - \frac{1}{3}\right)^2 + \left(-\frac{1}{2} - \frac{5}{2}\right)^2} = \sqrt{16+9} = \sqrt{25} = 5$

73. Either point can be considered as  $(x_1, y_1)$ .

$$d = \sqrt{(-4.2 - 2.1)^2 + [3 - (-6.4)]^2} \\ = \sqrt{(-6.3)^2 + (9.4)^2} = \sqrt{128.05} \approx 11.316$$

74.  $d = \sqrt{[0.6 - (-8.1)]^2 + [-1.5 - (-1.5)]^2} = \sqrt{(8.7)^2} = 8.7$

75. Either point can be considered as  $(x_1, y_1)$ .

$$d = \sqrt{(0-a)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$$

76.  $d = \sqrt{[r - (-r)]^2 + [s - (-s)]^2} = \sqrt{4r^2 + 4s^2} = 2\sqrt{r^2 + s^2}$

77. First we find the length of the diameter:

$$d = \sqrt{(-3-9)^2 + (-1-4)^2} \\ = \sqrt{(-12)^2 + (-5)^2} = \sqrt{169} = 13$$

The length of the radius is one-half the length of the diameter, or  $\frac{1}{2}(13)$ , or 6.5.

78. Radius =  $\sqrt{(-3-0)^2 + (5-1)^2} = \sqrt{25} = 5$

$$\text{Diameter} = 2 \cdot 5 = 10$$

79. First we find the distance between each pair of points.

For  $(-4, 5)$  and  $(6, 1)$ :

$$d = \sqrt{(-4-6)^2 + (5-1)^2} \\ = \sqrt{(-10)^2 + 4^2} = \sqrt{116}$$

For  $(-4, 5)$  and  $(-8, -5)$ :

$$d = \sqrt{(-4 - (-8))^2 + (5 - (-5))^2} \\ = \sqrt{4^2 + 10^2} = \sqrt{116}$$

For  $(6, 1)$  and  $(-8, -5)$ :

$$d = \sqrt{(6 - (-8))^2 + (1 - (-5))^2} \\ = \sqrt{14^2 + 6^2} = \sqrt{232}$$

Since  $(\sqrt{116})^2 + (\sqrt{116})^2 = (\sqrt{232})^2$ , the points could be the vertices of a right triangle.

80. For  $(-3, 1)$  and  $(2, -1)$ :

$$d = \sqrt{(-3-2)^2 + (1-(-1))^2} = \sqrt{29}$$

For  $(-3, 1)$  and  $(6, 9)$ :

$$d = \sqrt{(-3-6)^2 + (1-9)^2} = \sqrt{145}$$

For  $(2, -1)$  and  $(6, 9)$ :

$$d = \sqrt{(2-6)^2 + (-1-9)^2} = \sqrt{116}$$

Since  $(\sqrt{29})^2 + (\sqrt{116})^2 = (\sqrt{145})^2$ , the points could be the vertices of a right triangle.

81. First we find the distance between each pair of points.

For  $(-4, 3)$  and  $(0, 5)$ :

$$d = \sqrt{(-4-0)^2 + (3-5)^2} \\ = \sqrt{(-4)^2 + (-2)^2} = \sqrt{20}$$

For  $(-4, 3)$  and  $(3, -4)$ :

$$d = \sqrt{(-4-3)^2 + [3 - (-4)]^2} \\ = \sqrt{(-7)^2 + 7^2} = \sqrt{98}$$

For  $(0, 5)$  and  $(3, -4)$ :

$$d = \sqrt{(0-3)^2 + [5 - (-4)]^2} \\ = \sqrt{(-3)^2 + 9^2} = \sqrt{90}$$

The greatest distance is  $\sqrt{98}$ , so if the points are the vertices of a right triangle, then it is the hypotenuse. But  $(\sqrt{20})^2 + (\sqrt{90})^2 \neq (\sqrt{98})^2$ , so the points are not the vertices of a right triangle.

82. See the graph of this rectangle in Exercise 93.

The segments with endpoints  $(-3, 4)$ ,  $(2, -1)$  and  $(5, 2)$ ,  $(0, 7)$  are one pair of opposite sides. We find the length of each of these sides.

For  $(-3, 4)$ ,  $(2, -1)$ :

$$d = \sqrt{(-3 - 2)^2 + (4 - (-1))^2} = \sqrt{50}$$

For  $(5, 2)$ ,  $(0, 7)$ :

$$d = \sqrt{(5 - 0)^2 + (2 - 7)^2} = \sqrt{50}$$

The segments with endpoints  $(2, -1)$ ,  $(5, 2)$  and  $(0, 7)$ ,  $(-3, 4)$  are the second pair of opposite sides. We find their lengths.

For  $(2, -1)$ ,  $(5, 2)$ :

$$d = \sqrt{(2 - 5)^2 + (-1 - 2)^2} = \sqrt{18}$$

For  $(0, 7)$ ,  $(-3, 4)$ :

$$d = \sqrt{(0 - (-3))^2 + (7 - 4)^2} = \sqrt{18}$$

The endpoints of the diagonals are  $(-3, 4)$ ,  $(5, 2)$  and  $(2, -1)$ ,  $(0, 7)$ . We find the length of each.

For  $(-3, 4)$ ,  $(5, 2)$ :

$$d = \sqrt{(-3 - 5)^2 + (4 - 2)^2} = \sqrt{68}$$

For  $(2, -1)$ ,  $(0, 7)$ :

$$d = \sqrt{(2 - 0)^2 + (-1 - 7)^2} = \sqrt{68}$$

The opposite sides of the quadrilateral are the same length and the diagonals are the same length, so the quadrilateral is a rectangle.

83. We use the midpoint formula.

$$\left(\frac{4 + (-12)}{2}, \frac{-9 + (-3)}{2}\right) = \left(-\frac{8}{2}, -\frac{12}{2}\right) = (-4, -6)$$

84.  $\left(\frac{7 + 9}{2}, \frac{-2 + 5}{2}\right) = \left(8, \frac{3}{2}\right)$

85. We use the midpoint formula.

$$\left(\frac{0 + \left(-\frac{2}{5}\right)}{2}, \frac{\frac{1}{2} - 0}{2}\right) = \left(\frac{-\frac{2}{5}}{2}, \frac{\frac{1}{2}}{2}\right) = \left(-\frac{1}{5}, \frac{1}{4}\right)$$

86.  $\left(\frac{0 + \left(-\frac{7}{13}\right)}{2}, \frac{0 + \frac{2}{7}}{2}\right) = \left(-\frac{7}{26}, \frac{1}{7}\right)$

87. We use the midpoint formula.

$$\left(\frac{6.1 + 3.8}{2}, \frac{-3.8 + (-6.1)}{2}\right) = \left(\frac{9.9}{2}, -\frac{9.9}{2}\right) = (4.95, -4.95)$$

88.  $\left(\frac{-0.5 + 4.8}{2}, \frac{-2.7 + (-0.3)}{2}\right) = (2.15, -1.5)$

89. We use the midpoint formula.

$$\left(\frac{-6 + (-6)}{2}, \frac{5 + 8}{2}\right) = \left(-\frac{12}{2}, \frac{13}{2}\right) = \left(-6, \frac{13}{2}\right)$$

90.  $\left(\frac{1 + (-1)}{2}, \frac{-2 + 2}{2}\right) = (0, 0)$

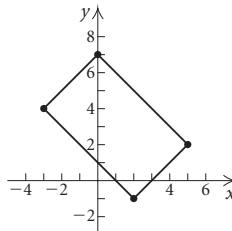
91. We use the midpoint formula.

$$\left(\frac{-\frac{1}{6} + \left(-\frac{2}{3}\right)}{2}, \frac{-\frac{3}{5} + \frac{5}{4}}{2}\right) = \left(\frac{-\frac{5}{6}, \frac{13}{20}}{2}\right) =$$

$$\left(-\frac{5}{12}, \frac{13}{40}\right)$$

92.  $\left(\frac{\frac{2}{9} + \left(-\frac{2}{5}\right)}{2}, \frac{\frac{1}{3} + \frac{4}{5}}{2}\right) = \left(-\frac{4}{45}, \frac{17}{30}\right)$

93.



For the side with vertices  $(-3, 4)$  and  $(2, -1)$ :

$$\left(\frac{-3 + 2}{2}, \frac{4 + (-1)}{2}\right) = \left(-\frac{1}{2}, \frac{3}{2}\right)$$

For the side with vertices  $(2, -1)$  and  $(5, 2)$ :

$$\left(\frac{2 + 5}{2}, \frac{-1 + 2}{2}\right) = \left(\frac{7}{2}, \frac{1}{2}\right)$$

For the side with vertices  $(5, 2)$  and  $(0, 7)$ :

$$\left(\frac{5 + 0}{2}, \frac{2 + 7}{2}\right) = \left(\frac{5}{2}, \frac{9}{2}\right)$$

For the side with vertices  $(0, 7)$  and  $(-3, 4)$ :

$$\left(\frac{0 + (-3)}{2}, \frac{7 + 4}{2}\right) = \left(-\frac{3}{2}, \frac{11}{2}\right)$$

For the quadrilateral whose vertices are the points found above, the diagonals have endpoints

$$\left(-\frac{1}{2}, \frac{3}{2}\right), \left(\frac{5}{2}, \frac{9}{2}\right) \text{ and } \left(\frac{7}{2}, \frac{1}{2}\right), \left(-\frac{3}{2}, \frac{11}{2}\right).$$

We find the length of each of these diagonals.

For  $\left(-\frac{1}{2}, \frac{3}{2}\right), \left(\frac{5}{2}, \frac{9}{2}\right)$ :

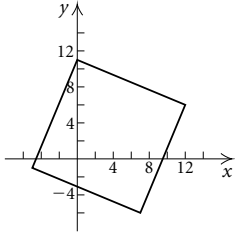
$$d = \sqrt{\left(-\frac{1}{2} - \frac{5}{2}\right)^2 + \left(\frac{3}{2} - \frac{9}{2}\right)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18}$$

For  $\left(\frac{7}{2}, \frac{1}{2}\right), \left(-\frac{3}{2}, \frac{11}{2}\right)$ :

$$d = \sqrt{\left(\frac{7}{2} - \left(-\frac{3}{2}\right)\right)^2 + \left(\frac{1}{2} - \frac{11}{2}\right)^2} = \sqrt{5^2 + (-5)^2} = \sqrt{50}$$

Since the diagonals do not have the same lengths, the midpoints are not vertices of a rectangle.

94.



For the side with vertices  $(-5, -1)$  and  $(7, -6)$ :

$$\left(\frac{-5+7}{2}, \frac{-1+(-6)}{2}\right) = \left(1, -\frac{7}{2}\right)$$

For the side with vertices  $(7, -6)$  and  $(12, 6)$ :

$$\left(\frac{7+12}{2}, \frac{-6+6}{2}\right) = \left(\frac{19}{2}, 0\right)$$

For the side with vertices  $(12, 6)$  and  $(0, 11)$ :

$$\left(\frac{12+0}{2}, \frac{6+11}{2}\right) = \left(6, \frac{17}{2}\right)$$

For the side with vertices  $(0, 11)$  and  $(-5, -1)$ :

$$\left(\frac{0+(-5)}{2}, \frac{11+(-1)}{2}\right) = \left(-\frac{5}{2}, 5\right)$$

For the quadrilateral whose vertices are the points found above, one pair of opposite sides has endpoints  $\left(1, -\frac{7}{2}\right)$ ,  $\left(\frac{19}{2}, 0\right)$  and  $\left(6, \frac{17}{2}\right)$ ,  $\left(-\frac{5}{2}, 5\right)$ . The length of each of these sides is  $\frac{\sqrt{338}}{2}$ . The other pair of opposite sides has endpoints  $\left(\frac{19}{2}, 0\right)$ ,  $\left(6, \frac{17}{2}\right)$  and  $\left(-\frac{5}{2}, 5\right)$ ,  $\left(1, -\frac{7}{2}\right)$ .

The length of each of these sides is also  $\frac{\sqrt{338}}{2}$ . The endpoints of the diagonals of the quadrilateral are  $\left(1, -\frac{7}{2}\right)$ ,  $\left(6, \frac{17}{2}\right)$  and  $\left(\frac{19}{2}, 0\right)$ ,  $\left(-\frac{5}{2}, 5\right)$ . The length of each diagonal is 13. Since the four sides of the quadrilateral are the same length and the diagonals are the same length, the midpoints are vertices of a square.

95. We use the midpoint formula.

$$\left(\frac{\sqrt{7} + \sqrt{2}}{2}, \frac{-4 + 3}{2}\right) = \left(\frac{\sqrt{7} + \sqrt{2}}{2}, -\frac{1}{2}\right)$$

96. 
$$\left(\frac{-3 + 1}{2}, \frac{\sqrt{5} + \sqrt{2}}{2}\right) = \left(-1, \frac{\sqrt{5} + \sqrt{2}}{2}\right)$$

97. Square the viewing window. For the graph shown, one possibility is  $[-12, 9, -4, 10]$ .

98. Square the viewing window. For the graph shown, one possibility is  $[-10, 20, -15, 5]$ .

99. 
$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 2)^2 + (y - 3)^2 = \left(\frac{5}{3}\right)^2 \quad \text{Substituting}$$

$$(x - 2)^2 + (y - 3)^2 = \frac{25}{9}$$

100. 
$$(x - 4)^2 + (y - 5)^2 = (4.1)^2$$
  

$$(x - 4)^2 + (y - 5)^2 = 16.81$$

101. The length of a radius is the distance between  $(-1, 4)$  and  $(3, 7)$ :

$$r = \sqrt{(-1 - 3)^2 + (4 - 7)^2}$$

$$= \sqrt{(-4)^2 + (-3)^2} = \sqrt{25} = 5$$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$[x - (-1)]^2 + (y - 4)^2 = 5^2$$

$$(x + 1)^2 + (y - 4)^2 = 25$$

102. Find the length of a radius:

$$r = \sqrt{(6 - 1)^2 + (-5 - 7)^2} = \sqrt{169} = 13$$

$$(x - 6)^2 + [y - (-5)]^2 = 13^2$$

$$(x - 6)^2 + (y + 5)^2 = 169$$

103. The center is the midpoint of the diameter:

$$\left(\frac{7 + (-3)}{2}, \frac{13 + (-11)}{2}\right) = (2, 1)$$

Use the center and either endpoint of the diameter to find the length of a radius. We use the point  $(7, 13)$ :

$$r = \sqrt{(7 - 2)^2 + (13 - 1)^2}$$

$$= \sqrt{5^2 + 12^2} = \sqrt{169} = 13$$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 2)^2 + (y - 1)^2 = 13^2$$

$$(x - 2)^2 + (y - 1)^2 = 169$$

104. The points  $(-9, 4)$  and  $(-1, -2)$  are opposite vertices of the square and hence endpoints of a diameter of the circle. We use these points to find the center and radius.

Center: 
$$\left(\frac{-9 + (-1)}{2}, \frac{4 + (-2)}{2}\right) = (-5, 1)$$

Radius: 
$$\frac{1}{2}\sqrt{(-9 - (-1))^2 + (4 - (-2))^2} = \frac{1}{2} \cdot 10 = 5$$

$$[x - (-5)]^2 + (y - 1)^2 = 5^2$$

$$(x + 5)^2 + (y - 1)^2 = 25$$

105. Since the center is 2 units to the left of the  $y$ -axis and the circle is tangent to the  $y$ -axis, the length of a radius is 2.

$$(x - h)^2 + (y - k)^2 = r^2$$

$$[x - (-2)]^2 + (y - 3)^2 = 2^2$$

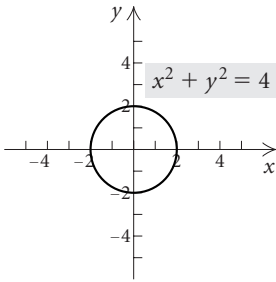
$$(x + 2)^2 + (y - 3)^2 = 4$$

106. Since the center is 5 units below the  $x$ -axis and the circle is tangent to the  $x$ -axis, the length of a radius is 5.

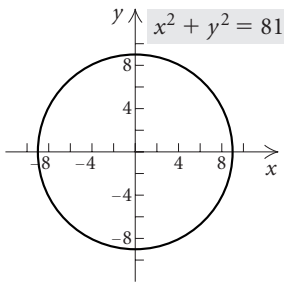
$$(x - 4)^2 + [y - (-5)]^2 = 5^2$$

$$(x - 4)^2 + (y + 5)^2 = 25$$

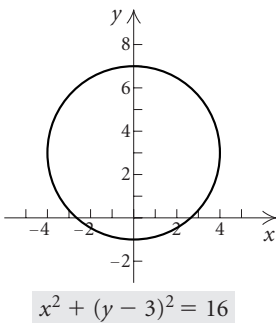
107.  $x^2 + y^2 = 4$   
 $(x - 0)^2 + (y - 0)^2 = 2^2$   
 Center:  $(0, 0)$ ; radius: 2



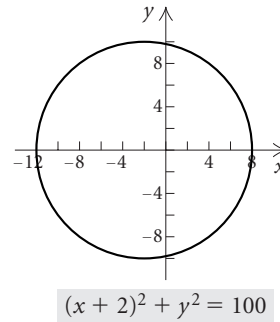
108.  $x^2 + y^2 = 81$   
 $(x - 0)^2 + (y - 0)^2 = 9^2$   
 Center:  $(0, 0)$ ; radius: 9



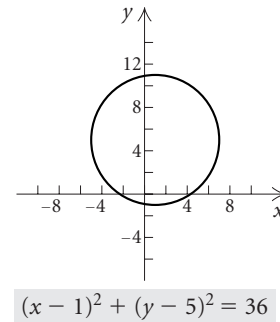
109.  $x^2 + (y - 3)^2 = 16$   
 $(x - 0)^2 + (y - 3)^2 = 4^2$   
 Center:  $(0, 3)$ ; radius: 4



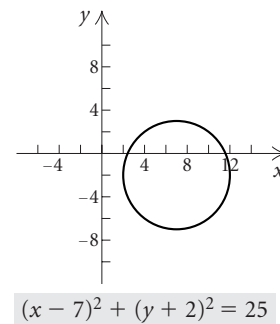
110.  $(x + 2)^2 + y^2 = 100$   
 $[x - (-2)]^2 + (y - 0)^2 = 10^2$   
 Center:  $(-2, 0)$ ; radius: 10



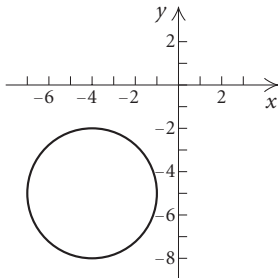
111.  $(x - 1)^2 + (y - 5)^2 = 36$   
 $(x - 1)^2 + (y - 5)^2 = 6^2$   
 Center:  $(1, 5)$ ; radius: 6



112.  $(x - 7)^2 + (y + 2)^2 = 25$   
 $(x - 7)^2 + [y - (-2)]^2 = 5^2$   
 Center:  $(7, -2)$ ; radius: 5

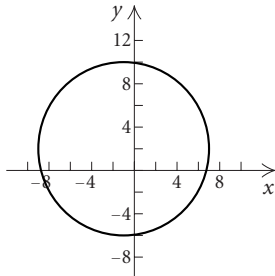


113.  $(x + 4)^2 + (y + 5)^2 = 9$   
 $[x - (-4)]^2 + [y - (-5)]^2 = 3^2$   
 Center:  $(-4, -5)$ ; radius: 3



$$(x + 4)^2 + (y + 5)^2 = 9$$

114.  $(x + 1)^2 + (y - 2)^2 = 64$   
 $[x - (-1)]^2 + (y - 2)^2 = 8^2$   
 Center:  $(-1, 2)$ ; radius: 8



$$(x + 1)^2 + (y - 2)^2 = 64$$

115. From the graph we see that the center of the circle is  $(-2, 1)$  and the radius is 3. The equation of the circle is  $[x - (-2)]^2 + (y - 1)^2 = 3^2$ , or  $(x + 2)^2 + (y - 1)^2 = 3^2$ .
116. Center:  $(3, -5)$ , radius: 4  
 Equation:  $(x - 3)^2 + [y - (-5)]^2 = 4^2$ , or  
 $(x - 3)^2 + (y + 5)^2 = 4^2$
117. From the graph we see that the center of the circle is  $(5, -5)$  and the radius is 15. The equation of the circle is  $(x - 5)^2 + [y - (-5)]^2 = 15^2$ , or  $(x - 5)^2 + (y + 5)^2 = 15^2$ .
118. Center:  $(-8, 2)$ , radius: 4  
 Equation:  $[x - (-8)]^2 + (y - 2)^2 = 4^2$ , or  
 $(x + 8)^2 + (y - 2)^2 = 4^2$
119. If the point  $(p, q)$  is in the fourth quadrant, then  $p > 0$  and  $q < 0$ . If  $p > 0$ , then  $-p < 0$  so both coordinates of the point  $(q, -p)$  are negative and  $(q, -p)$  is in the third quadrant.

120. Use the distance formula:

$$d = \sqrt{(a + h - a)^2 + \left(\frac{1}{a + h} - \frac{1}{a}\right)^2} =$$

$$\sqrt{h^2 + \left(\frac{-h}{a(a + h)}\right)^2} = \sqrt{h^2 + \frac{h^2}{a^2(a + h)^2}} =$$

$$\sqrt{\frac{h^2 a^2 (a + h)^2 + h^2}{a^2 (a + h)^2}} = \sqrt{\frac{h^2 (a^2 (a + h)^2 + 1)}{a^2 (a + h)^2}} =$$

$$\left| \frac{h}{a(a + h)} \right| \sqrt{a^2 (a + h)^2 + 1}$$

Find the midpoint:

$$\left(\frac{a + a + h}{2}, \frac{\frac{1}{a} + \frac{1}{a + h}}{2}\right) = \left(\frac{2a + h}{2}, \frac{2a + h}{2a(a + h)}\right)$$

121. Use the distance formula. Either point can be considered as  $(x_1, y_1)$ .

$$d = \sqrt{(a + h - a)^2 + (\sqrt{a + h} - \sqrt{a})^2}$$

$$= \sqrt{h^2 + a + h - 2\sqrt{a^2 + ah} + a}$$

$$= \sqrt{h^2 + 2a + h - 2\sqrt{a^2 + ah}}$$

Next we use the midpoint formula.

$$\left(\frac{a + a + h}{2}, \frac{\sqrt{a} + \sqrt{a + h}}{2}\right) = \left(\frac{2a + h}{2}, \frac{\sqrt{a} + \sqrt{a + h}}{2}\right)$$

122.  $C = 2\pi r$   
 $10\pi = 2\pi r$   
 $5 = r$

Then  $[x - (-5)]^2 + (y - 8)^2 = 5^2$ , or  $(x + 5)^2 + (y - 8)^2 = 25$ .

123. First use the formula for the area of a circle to find  $r^2$ :

$$A = \pi r^2$$

$$36\pi = \pi r^2$$

$$36 = r^2$$

Then we have:

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 2)^2 + [y - (-7)]^2 = 36$$

$$(x - 2)^2 + (y + 7)^2 = 36$$

124. Let the point be  $(x, 0)$ . We set the distance from  $(-4, -3)$  to  $(x, 0)$  equal to the distance from  $(-1, 5)$  to  $(x, 0)$  and solve for  $x$ .

$$\sqrt{(-4 - x)^2 + (-3 - 0)^2} = \sqrt{(-1 - x)^2 + (5 - 0)^2}$$

$$\sqrt{16 + 8x + x^2 + 9} = \sqrt{1 + 2x + x^2 + 25}$$

$$\sqrt{x^2 + 8x + 25} = \sqrt{x^2 + 2x + 26}$$

$$x^2 + 8x + 25 = x^2 + 2x + 26$$

Squaring both sides

$$8x + 25 = 2x + 26$$

$$6x = 1$$

$$x = \frac{1}{6}$$

The point is  $\left(\frac{1}{6}, 0\right)$ .

**125.** Let  $(0, y)$  be the required point. We set the distance from  $(-2, 0)$  to  $(0, y)$  equal to the distance from  $(4, 6)$  to  $(0, y)$  and solve for  $y$ .

$$\begin{aligned} \sqrt{[0 - (-2)]^2 + (y - 0)^2} &= \sqrt{(0 - 4)^2 + (y - 6)^2} \\ \sqrt{4 + y^2} &= \sqrt{16 + y^2 - 12y + 36} \\ 4 + y^2 &= 16 + y^2 - 12y + 36 \\ &\text{Squaring both sides} \\ -48 &= -12y \\ 4 &= y \end{aligned}$$

The point is  $(0, 4)$ .

**126.** We first find the distance between each pair of points.

For  $(-1, -3)$  and  $(-4, -9)$ :

$$\begin{aligned} d_1 &= \sqrt{[-1 - (-4)]^2 + [-3 - (-9)]^2} \\ &= \sqrt{3^2 + 6^2} = \sqrt{9 + 36} \\ &= \sqrt{45} = 3\sqrt{5} \end{aligned}$$

For  $(-1, -3)$  and  $(2, 3)$ :

$$\begin{aligned} d_2 &= \sqrt{(-1 - 2)^2 + (-3 - 3)^2} \\ &= \sqrt{(-3)^2 + (-6)^2} = \sqrt{9 + 36} \\ &= \sqrt{45} = 3\sqrt{5} \end{aligned}$$

For  $(-4, -9)$  and  $(2, 3)$ :

$$\begin{aligned} d_3 &= \sqrt{(-4 - 2)^2 + (-9 - 3)^2} \\ &= \sqrt{(-6)^2 + (-12)^2} = \sqrt{36 + 144} \\ &= \sqrt{180} = 6\sqrt{5} \end{aligned}$$

Since  $d_1 + d_2 = d_3$ , the points are collinear.

**127.** a) When the circle is positioned on a coordinate system as shown in the text, the center lies on the  $y$ -axis and is equidistant from  $(-4, 0)$  and  $(0, 2)$ .

Let  $(0, y)$  be the coordinates of the center.

$$\begin{aligned} \sqrt{(-4 - 0)^2 + (0 - y)^2} &= \sqrt{(0 - 0)^2 + (2 - y)^2} \\ 4^2 + y^2 &= (2 - y)^2 \\ 16 + y^2 &= 4 - 4y + y^2 \\ 12 &= -4y \\ -3 &= y \end{aligned}$$

The center of the circle is  $(0, -3)$ .

b) Use the point  $(-4, 0)$  and the center  $(0, -3)$  to find the radius.

$$\begin{aligned} (-4 - 0)^2 + [0 - (-3)]^2 &= r^2 \\ 25 &= r^2 \\ 5 &= r \end{aligned}$$

The radius is 5 ft.

**128.** The coordinates of  $P$  are  $\left(\frac{b}{2}, \frac{h}{2}\right)$  by the midpoint formula. By the distance formula, each of the distances from  $P$  to  $(0, h)$ , from  $P$  to  $(0, 0)$ , and from  $P$  to  $(b, 0)$  is  $\frac{\sqrt{b^2 + h^2}}{2}$ .

**129.**

$$\begin{array}{r|l} x^2 + y^2 = 1 & \\ \hline \left(\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 & ? 1 \\ \frac{3}{4} + \frac{1}{4} & \\ \hline 1 & | 1 \text{ TRUE} \end{array}$$

$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$  lies on the unit circle.

**130.**

$$\begin{array}{r|l} x^2 + y^2 = 1 & \\ \hline 0^2 + (-1)^2 & ? 1 \\ 1 & | 1 \text{ TRUE} \end{array}$$

$(0, -1)$  lies on the unit circle.

**131.**

$$\begin{array}{r|l} x^2 + y^2 = 1 & \\ \hline \left(-\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 & ? 1 \\ \frac{2}{4} + \frac{2}{4} & \\ \hline 1 & | 1 \text{ TRUE} \end{array}$$

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$  lies on the unit circle.

**132.**

$$\begin{array}{r|l} x^2 + y^2 = 1 & \\ \hline \left(\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2 & ? 1 \\ \frac{1}{4} + \frac{3}{4} & \\ \hline 1 & | 1 \text{ TRUE} \end{array}$$

$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$  lies on the unit circle.

**133.** See the answer section in the text.

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**Exercise Set 1.2**

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1. This correspondence is a function, because each member of the domain corresponds to exactly one member of the range.
2. This correspondence is a function, because each member of the domain corresponds to exactly one member of the range.
3. This correspondence is a function, because each member of the domain corresponds to exactly one member of the range.
4. This correspondence is not a function, because there is a member of the domain (1) that corresponds to more than one member of the range (4 and 6).
5. This correspondence is not a function, because there is a member of the domain (m) that corresponds to more than one member of the range (A and B).

