

INSTRUCTOR'S
SOLUTIONS MANUAL

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THOMAS' CALCULUS
EARLY TRANSCENDENTALS
FOURTEENTH EDITION

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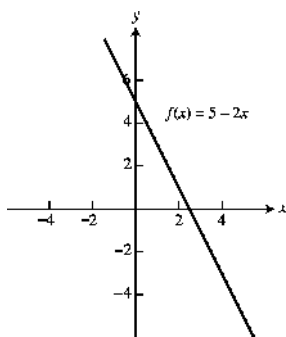
CHAPTER 1 FUNCTIONS

1.1 FUNCTIONS AND THEIR GRAPHS

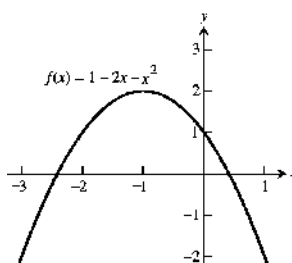
- domain = $(-\infty, \infty)$; range = $[1, \infty)$
- domain = $[0, \infty)$; range = $(-\infty, 1]$
- domain = $[-2, \infty)$; y in range and $y = \sqrt{5x+10} \geq 0 \Rightarrow y$ can be any positive real number \Rightarrow range = $[0, \infty)$.
- domain = $(-\infty, 0] \cup [3, \infty)$; y in range and $y = \sqrt{x^2 - 3x} \geq 0 \Rightarrow y$ can be any positive real number \Rightarrow range = $[0, \infty)$.
- domain = $(-\infty, 3) \cup (3, \infty)$; y in range and $y = \frac{4}{3-t}$, now if $t < 3 \Rightarrow 3-t > 0 \Rightarrow \frac{4}{3-t} > 0$, or if $t > 3 \Rightarrow 3-t < 0 \Rightarrow \frac{4}{3-t} < 0 \Rightarrow y$ can be any nonzero real number \Rightarrow range = $(-\infty, 0) \cup (0, \infty)$.
- domain = $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$; y in range and $y = \frac{2}{t^2-16}$, now if $t < -4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0$, or if $-4 < t < 4 \Rightarrow -16 \leq t^2 - 16 < 0 \Rightarrow -\frac{2}{16} \geq \frac{2}{t^2-16}$, or if $t > 4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0 \Rightarrow y$ can be any nonzero real number \Rightarrow range = $(-\infty, -\frac{1}{8}] \cup (0, \infty)$.
- (a) Not the graph of a function of x since it fails the vertical line test.
(b) Is the graph of a function of x since any vertical line intersects the graph at most once.
- (a) Not the graph of a function of x since it fails the vertical line test.
(b) Not the graph of a function of x since it fails the vertical line test.
- base = x ; (height) $^2 + (\frac{x}{2})^2 = x^2 \Rightarrow$ height = $\frac{\sqrt{3}}{2}x$; area is $a(x) = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$;
perimeter is $p(x) = x + x + x = 3x$.
- $s =$ side length $\Rightarrow s^2 + s^2 = d^2 \Rightarrow s = \frac{d}{\sqrt{2}}$; and area is $a = s^2 \Rightarrow a = \frac{1}{2}d^2$
- Let $D =$ diagonal length of a face of the cube and $\ell =$ the length of an edge. Then $\ell^2 + D^2 = d^2$ and $D^2 = 2\ell^2 \Rightarrow 3\ell^2 = d^2 \Rightarrow \ell = \frac{d}{\sqrt{3}}$. The surface area is $6\ell^2 = \frac{6d^2}{3} = 2d^2$ and the volume is $\ell^3 = \left(\frac{d}{\sqrt{3}}\right)^3 = \frac{d^3}{3\sqrt{3}}$.
- The coordinates of P are (x, \sqrt{x}) so the slope of the line joining P to the origin is $m = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}} (x > 0)$.
Thus, $(x, \sqrt{x}) = \left(\frac{1}{m^2}, \frac{1}{m}\right)$.
- $2x + 4y = 5 \Rightarrow y = -\frac{1}{2}x + \frac{5}{4}$; $L = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + \left(-\frac{1}{2}x + \frac{5}{4}\right)^2} = \sqrt{x^2 + \frac{1}{4}x^2 - \frac{5}{4}x + \frac{25}{16}}$
 $= \sqrt{\frac{5}{4}x^2 - \frac{5}{4}x + \frac{25}{16}} = \sqrt{\frac{20x^2 - 20x + 25}{16}} = \frac{\sqrt{20x^2 - 20x + 25}}{4}$
- $y = \sqrt{x-3} \Rightarrow y^2 + 3 = x$; $L = \sqrt{(x-4)^2 + (y-0)^2} = \sqrt{(y^2 + 3 - 4)^2 + y^2} = \sqrt{(y^2 - 1)^2 + y^2}$
 $= \sqrt{y^4 - 2y^2 + 1 + y^2} = \sqrt{y^4 - y^2 + 1}$

2 Chapter 1 Functions

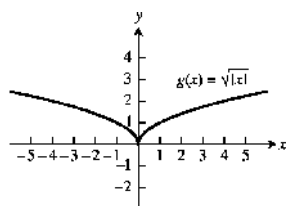
15. The domain is $(-\infty, \infty)$.



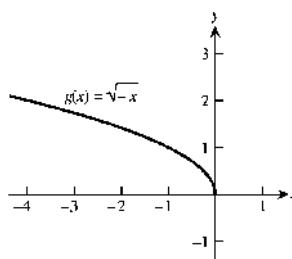
16. The domain is $(-\infty, \infty)$.



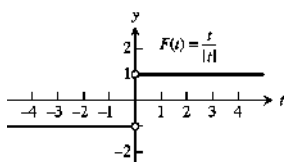
17. The domain is $(-\infty, \infty)$.



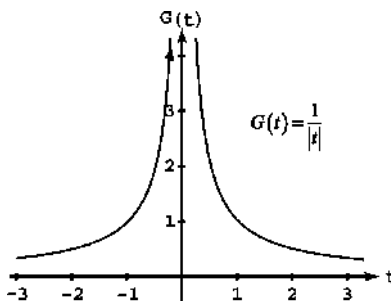
18. The domain is $(-\infty, 0]$.



19. The domain is $(-\infty, 0) \cup (0, \infty)$.



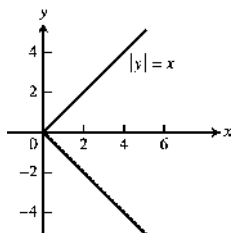
20. The domain is $(-\infty, 0) \cup (0, \infty)$.



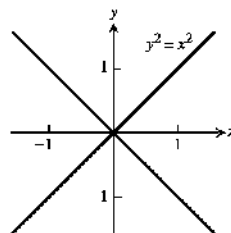
21. The domain is $(-\infty, -5) \cup (-5, -3] \cup [3, 5) \cup (5, \infty)$ 22. The range is $[5, \infty)$.

23. Neither graph passes the vertical line test

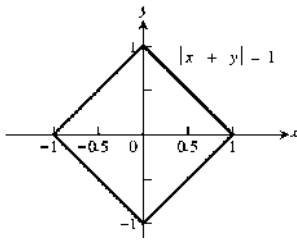
(a)



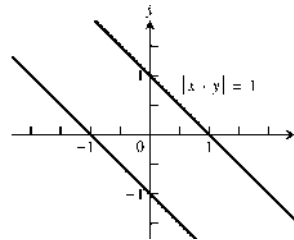
(b)



24. Neither graph passes the vertical line test
(a)



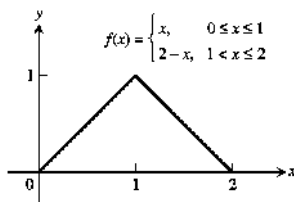
- (b)



$$|x + y| = 1 \Leftrightarrow \begin{cases} x + y = 1 \\ \text{or} \\ x + y = -1 \end{cases} \Leftrightarrow \begin{cases} y = 1 - x \\ \text{or} \\ y = -1 - x \end{cases}$$

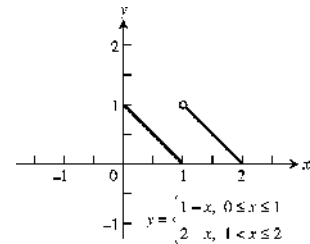
25.

x	0	1	2
y	0	1	0

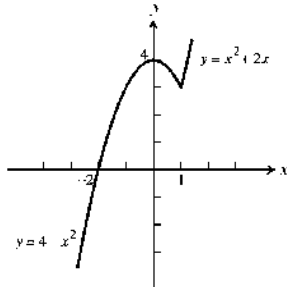


26.

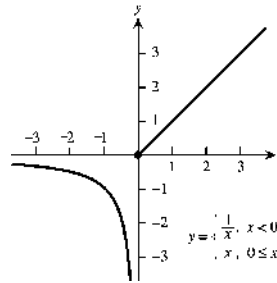
x	0	1	2
y	1	0	0



27. $F(x) = \begin{cases} 4 - x^2, & x \leq 1 \\ x^2 + 2x, & x > 1 \end{cases}$



28. $G(x) = \begin{cases} \frac{1}{x}, & x < 0 \\ x, & 0 \leq x \end{cases}$



29. (a) Line through (0, 0) and (1, 1): $y = x$; Line through (1, 1) and (2, 0): $y = -x + 2$

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ -x + 2, & 1 < x \leq 2 \end{cases}$$

(b) $f(x) = \begin{cases} 2, & 0 \leq x < 1 \\ 0, & 1 \leq x < 2 \\ 2, & 2 \leq x < 3 \\ 0, & 3 \leq x \leq 4 \end{cases}$

30. (a) Line through (0, 2) and (2, 0): $y = -x + 2$

Line through (2, 1) and (5, 0): $m = \frac{0-1}{5-2} = -\frac{1}{3}$, so $y = -\frac{1}{3}(x-2) + 1 = -\frac{1}{3}x + \frac{5}{3}$

$$f(x) = \begin{cases} -x + 2, & 0 < x \leq 2 \\ -\frac{1}{3}x + \frac{5}{3}, & 2 < x \leq 5 \end{cases}$$

4 Chapter 1 Functions

(b) Line through $(-1, 0)$ and $(0, -3)$: $m = \frac{-3 - 0}{0 - (-1)} = -3$, so $y = -3x - 3$

Line through $(0, 3)$ and $(2, -1)$: $m = \frac{-1 - 3}{2 - 0} = \frac{-4}{2} = -2$, so $y = -2x + 3$

$$f(x) = \begin{cases} -3x - 3, & -1 < x \leq 0 \\ -2x + 3, & 0 < x \leq 2 \end{cases}$$

31. (a) Line through $(-1, 1)$ and $(0, 0)$: $y = -x$

Line through $(0, 1)$ and $(1, 1)$: $y = 1$

Line through $(1, 1)$ and $(3, 0)$: $m = \frac{0 - 1}{3 - 1} = \frac{-1}{2} = -\frac{1}{2}$, so $y = -\frac{1}{2}(x - 1) + 1 = -\frac{1}{2}x + \frac{3}{2}$

$$f(x) = \begin{cases} -x & -1 \leq x < 0 \\ 1 & 0 < x \leq 1 \\ -\frac{1}{2}x + \frac{3}{2} & 1 < x < 3 \end{cases}$$

(b) Line through $(-2, -1)$ and $(0, 0)$: $y = \frac{1}{2}x$

Line through $(0, 2)$ and $(1, 0)$: $y = -2x + 2$

Line through $(1, -1)$ and $(3, -1)$: $y = -1$

$$f(x) = \begin{cases} \frac{1}{2}x & -2 \leq x \leq 0 \\ -2x + 2 & 0 < x \leq 1 \\ -1 & 1 < x \leq 3 \end{cases}$$

32. (a) Line through $(\frac{T}{2}, 0)$ and $(T, 1)$: $m = \frac{1 - 0}{T - (T/2)} = \frac{2}{T}$, so $y = \frac{2}{T}(x - \frac{T}{2}) + 0 = \frac{2}{T}x - 1$

$$f(x) = \begin{cases} 0, & 0 \leq x \leq \frac{T}{2} \\ \frac{2}{T}x - 1, & \frac{T}{2} < x \leq T \end{cases}$$

(b) $f(x) = \begin{cases} A, & 0 \leq x < \frac{T}{2} \\ -A, & \frac{T}{2} \leq x < T \\ A, & T \leq x < \frac{3T}{2} \\ -A, & \frac{3T}{2} \leq x \leq 2T \end{cases}$

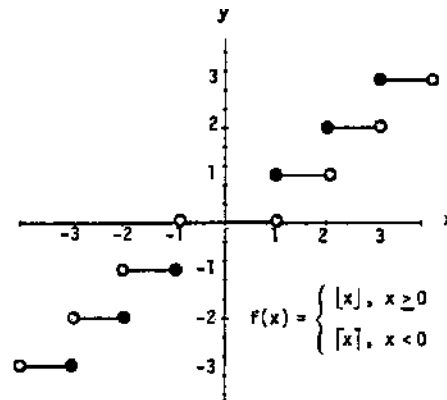
33. (a) $\lfloor x \rfloor = 0$ for $x \in [0, 1)$

(b) $\lceil x \rceil = 0$ for $x \in (-1, 0]$

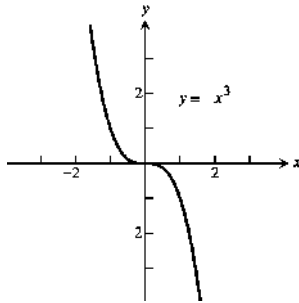
34. $\lfloor x \rfloor = \lceil x \rceil$ only when x is an integer.

35. For any real number x , $n \leq x \leq n + 1$, where n is an integer. Now: $n \leq x \leq n + 1 \Rightarrow -(n + 1) \leq -x \leq -n$.
By definition: $\lceil -x \rceil = -n$ and $\lfloor x \rfloor = n \Rightarrow -\lfloor x \rfloor = -n$. So $\lceil -x \rceil = -\lfloor x \rfloor$ for all real x .

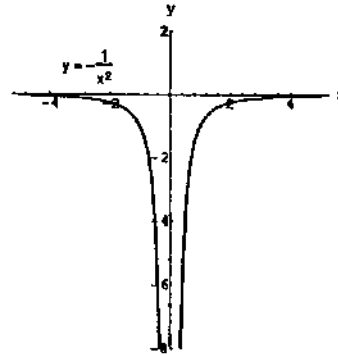
36. To find $f(x)$ you delete the decimal or fractional portion of x , leaving only the integer part.



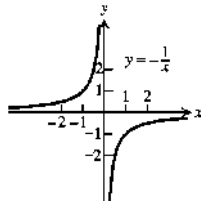
37. Symmetric about the origin
 Dec: $-\infty < x < \infty$
 Inc: nowhere



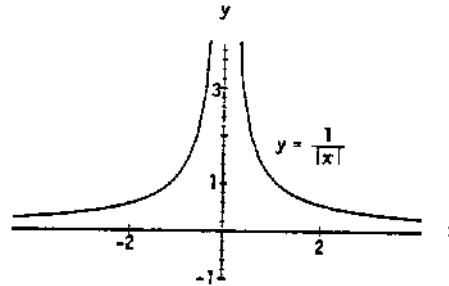
38. Symmetric about the y-axis
 Dec: $-\infty < x < 0$
 Inc: $0 < x < \infty$



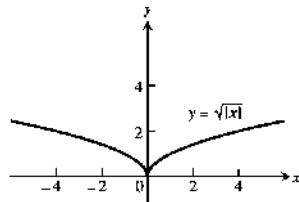
39. Symmetric about the origin
 Dec: nowhere
 Inc: $-\infty < x < 0$
 $0 < x < \infty$



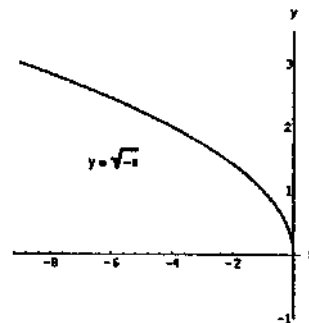
40. Symmetric about the y-axis
 Dec: $0 < x < \infty$
 Inc: $-\infty < x < 0$



41. Symmetric about the y-axis
 Dec: $-\infty < x \leq 0$
 Inc: $0 \leq x < \infty$

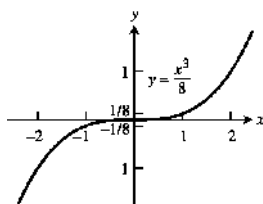


42. No symmetry
 Dec: $-\infty < x \leq 0$
 Inc: nowhere

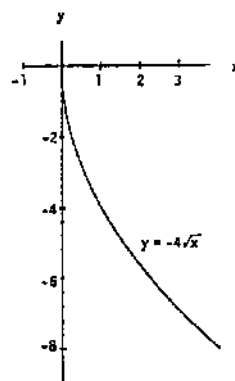


6 Chapter 1 Functions

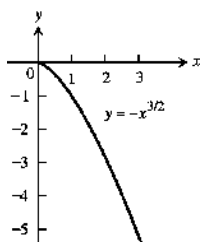
43. Symmetric about the origin
Dec: nowhere
Inc: $-\infty < x < \infty$



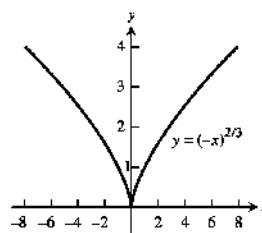
44. No symmetry
Dec: $0 \leq x < \infty$
Inc: nowhere



45. No symmetry
Dec: $0 \leq x < \infty$
Inc: nowhere



46. Symmetric about the y-axis
Dec: $-\infty < x \leq 0$
Inc: $0 \leq x < \infty$



47. Since a horizontal line not through the origin is symmetric with respect to the y-axis, but not with respect to the origin, the function is even.

48. $f(x) = x^{-5} = \frac{1}{x^5}$ and $f(-x) = (-x)^{-5} = \frac{1}{(-x)^5} = -\left(\frac{1}{x^5}\right) = -f(x)$. Thus the function is odd.

49. Since $f(x) = x^2 + 1 = (-x)^2 + 1 = f(-x)$. The function is even.

50. Since $[f(x) = x^2 + x] \neq [f(-x) = (-x)^2 - x]$ and $[f(x) = x^2 + x] \neq [-f(x) = -(x)^2 - x]$ the function is neither even nor odd.

51. Since $g(x) = x^3 + x$, $g(-x) = -x^3 - x = -(x^3 + x) = -g(x)$. So the function is odd.

52. $g(x) = x^4 + 3x^2 - 1 = (-x)^4 + 3(-x)^2 - 1 = g(-x)$, thus the function is even.

53. $g(x) = \frac{1}{x^2 - 1} = \frac{1}{(-x)^2 - 1} = g(-x)$. Thus the function is even.

54. $g(x) = \frac{x}{x^2 - 1}$; $g(-x) = -\frac{x}{x^2 - 1} = -g(x)$. So the function is odd.

55. $h(t) = \frac{1}{t - 1}$; $h(-t) = \frac{1}{-t - 1}$; $-h(t) = \frac{1}{1 - t}$. Since $h(t) \neq -h(t)$ and $h(t) \neq h(-t)$, the function is neither even nor odd.

56. Since $|t^3| = |(-t)^3|$, $h(t) = h(-t)$ and the function is even.
57. $h(t) = 2t + 1$, $h(-t) = -2t + 1$. So $h(t) \neq h(-t)$. $-h(t) = -2t - 1$, so $h(t) \neq -h(t)$. The function is neither even nor odd.
58. $h(t) = 2|t| + 1$ and $h(-t) = 2|-t| + 1 = 2|t| + 1$. So $h(t) = h(-t)$ and the function is even.
59. $g(x) = \sin 2x$; $g(-x) = -\sin 2x = -g(x)$. So the function is odd.
60. $g(x) = \sin x^2$; $g(-x) = \sin x^2 = g(x)$. So the function is even.
61. $g(x) = \cos 3x$; $g(-x) = \cos 3x = g(x)$. So the function is even.
62. $g(x) = 1 + \cos x$; $g(-x) = 1 + \cos x = g(x)$. So the function is even.
63. $s = kt \Rightarrow 25 = k(75) \Rightarrow k = \frac{1}{3} \Rightarrow s = \frac{1}{3}t$; $60 = \frac{1}{3}t \Rightarrow t = 180$
64. $K = cv^2 \Rightarrow 12960 = c(18)^2 \Rightarrow c = 40 \Rightarrow K = 40v^2$; $K = 40(10)^2 = 4000$ joules
65. $r = \frac{k}{s} \Rightarrow 6 = \frac{k}{4} \Rightarrow k = 24 \Rightarrow r = \frac{24}{s}$; $10 = \frac{24}{s} \Rightarrow s = \frac{12}{5}$
66. $P = \frac{k}{V} \Rightarrow 14.7 = \frac{k}{1000} \Rightarrow k = 14700 \Rightarrow P = \frac{14700}{V}$; $23.4 = \frac{14700}{V} \Rightarrow V = \frac{24500}{39} \approx 628.2 \text{ in}^3$
67. $V = f(x) = x(14 - 2x)(22 - 2x) = 4x^3 - 72x^2 + 308x$; $0 < x < 7$.
68. (a) Let h = height of the triangle. Since the triangle is isosceles, $(\overline{AB})^2 + (\overline{AB})^2 = 2^2 \Rightarrow \overline{AB} = \sqrt{2}$. So,
 $h^2 + 1^2 = (\sqrt{2})^2 \Rightarrow h = 1 \Rightarrow B$ is at $(0, 1) \Rightarrow$ slope of $AB = -1 \Rightarrow$ The equation of AB is
 $y = f(x) = -x + 1$; $x \in [0, 1]$.
- (b) $A(x) = 2xy = 2x(-x + 1) = -2x^2 + 2x$; $x \in [0, 1]$.
69. (a) Graph h because it is an even function and rises less rapidly than does Graph g .
 (b) Graph f because it is an odd function.
 (c) Graph g because it is an even function and rises more rapidly than does Graph h .
70. (a) Graph f because it is linear.
 (b) Graph g because it contains $(0, 1)$.
 (c) Graph h because it is a nonlinear odd function.

71. (a) From the graph, $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow x \in (-2, 0) \cup (4, \infty)$

(b) $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow \frac{x}{2} - 1 - \frac{4}{x} > 0$

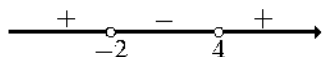
$$x > 0: \frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2 - 2x - 8}{2x} > 0 \Rightarrow \frac{(x-4)(x+2)}{2x} > 0$$

$$\Rightarrow x > 4 \text{ since } x \text{ is positive;}$$

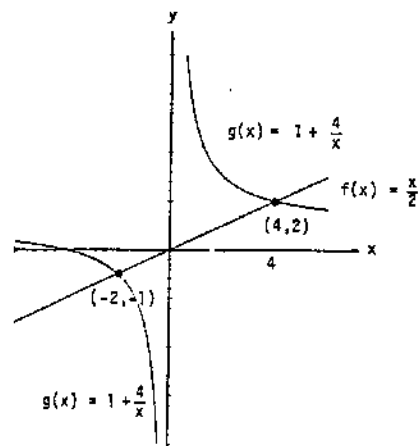
$$x < 0: \frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2 - 2x - 8}{2x} < 0 \Rightarrow \frac{(x-4)(x+2)}{2x} < 0$$

$$\Rightarrow x < -2 \text{ since } x \text{ is negative;}$$

$$\text{sign of } (x-4)(x+2)$$



Solution interval: $(-2, 0) \cup (4, \infty)$



72. (a) From the graph, $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow x \in (-\infty, -5) \cup (-1, 1)$

(b) Case $x < -1$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} > 2$

$$\Rightarrow 3x + 3 < 2x - 2 \Rightarrow x < -5.$$

Thus, $x \in (-\infty, -5)$ solves the inequality.

Case $-1 < x < 1$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} < 2$

$$\Rightarrow 3x + 3 > 2x - 2 \Rightarrow x > -5 \text{ which}$$

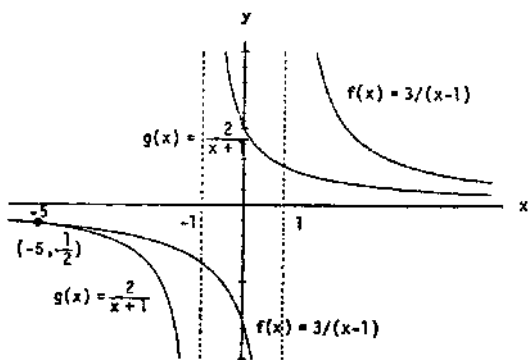
$$\text{is true if } x > -1. \text{ Thus, } x \in (-1, 1)$$

$$\text{solves the inequality.}$$

Case $1 < x$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow 3x + 3 < 2x - 2 \Rightarrow x < -5$

which is never true if $1 < x$,
so no solution here.

In conclusion, $x \in (-\infty, -5) \cup (-1, 1)$.



73. A curve symmetric about the x -axis will not pass the vertical line test because the points (x, y) and $(x, -y)$ lie on the same vertical line. The graph of the function $y = f(x) = 0$ is the x -axis, a horizontal line for which there is a single y -value, 0, for any x .

74. price = $40 + 5x$, quantity = $300 - 25x \Rightarrow R(x) = (40 + 5x)(300 - 25x)$

75. $x^2 + x^2 = h^2 \Rightarrow x = \frac{h}{\sqrt{2}} = \frac{\sqrt{2}h}{2}$; cost = $5(2x) + 10h \Rightarrow C(h) = 10\left(\frac{\sqrt{2}h}{2}\right) + 10h = 5h(\sqrt{2} + 2)$

76. (a) Note that 2 mi = 10,560 ft, so there are $\sqrt{800^2 + x^2}$ feet of river cable at \$180 per foot and $(10,560 - x)$ feet of land cable at \$100 per foot. The cost is $C(x) = 180\sqrt{800^2 + x^2} + 100(10,560 - x)$.

(b) $C(0) = \$1,200,000$

$$C(500) \approx \$1,175,812$$

$$C(1000) \approx \$1,186,512$$

$$C(1500) \approx \$1,212,000$$

$$C(2000) \approx \$1,243,732$$

$$C(2500) \approx \$1,278,479$$

$$C(3000) \approx \$1,314,870$$

Values beyond this are all larger. It would appear that the least expensive location is less than 2000 feet from the point P .

1.2 COMBINING FUNCTIONS; SHIFTING AND SCALING GRAPHS

1. $D_f: -\infty < x < \infty, D_g: x \geq 1 \Rightarrow D_{f+g} = D_{fg}: x \geq 1. R_f: -\infty < y < \infty, R_g: y \geq 0, R_{f+g}: y \geq 1, R_{fg}: y \geq 0$
2. $D_f: x+1 \geq 0 \Rightarrow x \geq -1, D_g: x-1 \geq 0 \Rightarrow x \geq 1. \text{ Therefore } D_{f+g} = D_{fg}: x \geq 1.$
 $R_f = R_g: y \geq 0, R_{f+g}: y \geq \sqrt{2}, R_{fg}: y \geq 0$
3. $D_f: -\infty < x < \infty, D_g: -\infty < x < \infty, D_{f/g}: -\infty < x < \infty, D_{g/f}: -\infty < x < \infty, R_f: y = 2, R_g: y \geq 1, R_{f/g}: 0 < y \leq 2,$
 $R_{g/f}: \frac{1}{2} \leq y < \infty$
4. $D_f: -\infty < x < \infty, D_g: x \geq 0, D_{f/g}: x \geq 0, D_{g/f}: x \geq 0; R_f: y = 1, R_g: y \geq 1, R_{f/g}: 0 < y \leq 1, R_{g/f}: 1 \leq y < \infty$
5. (a) 2 (b) 22 (c) $x^2 + 2$
 (d) $(x+5)^2 - 3 = x^2 + 10x + 22$ (e) 5 (f) -2
 (g) $x+10$ (h) $(x^2 - 3)^2 - 3 = x^4 - 6x^2 + 6$
6. (a) $-\frac{1}{3}$ (b) 2 (c) $\frac{1}{x+1} - 1 = \frac{-x}{x+1}$
 (d) $\frac{1}{x}$ (e) 0 (f) $\frac{3}{4}$
 (g) $x-2$ (h) $\frac{1}{\frac{1}{x+1} + 1} = \frac{1}{\frac{x+2}{x+1}} = \frac{x+1}{x+2}$
7. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(4-x)) = f(3(4-x)) = f(12-3x) = (12-3x) + 1 = 13-3x$
8. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x^2)) = f(2(x^2)-1) = f(2x^2-1) = 3(2x^2-1) + 4 = 6x^2 + 1$
9. $(f \circ g \circ h)(x) = f(g(h(x))) = f\left(g\left(\frac{1}{x}\right)\right) = f\left(\frac{1}{\frac{1}{x} + 4}\right) = f\left(\frac{x}{1+4x}\right) = \sqrt{\frac{x}{1+4x} + 1} = \sqrt{\frac{5x+1}{1+4x}}$
10. $(f \circ g \circ h)(x) = f(g(h(x))) = f\left(g\left(\sqrt{2-x}\right)\right) = f\left(\frac{(\sqrt{2-x})^2}{(\sqrt{2-x})^2 + 1}\right) = f\left(\frac{2-x}{3-x}\right) = \frac{\frac{2-x}{3-x} + 2}{3 - \frac{2-x}{3-x}} = \frac{8-3x}{7-2x}$
11. (a) $(f \circ g)(x)$ (b) $(j \circ g)(x)$ (c) $(g \circ g)(x)$
 (d) $(j \circ j)(x)$ (e) $(g \circ h \circ f)(x)$ (f) $(h \circ j \circ f)(x)$
12. (a) $(f \circ j)(x)$ (b) $(g \circ h)(x)$ (c) $(h \circ h)(x)$
 (d) $(f \circ f)(x)$ (e) $(j \circ g \circ f)(x)$ (f) $(g \circ f \circ h)(x)$
13.

$g(x)$	$f(x)$	$(f \circ g)(x)$
(a) $x-7$	\sqrt{x}	$\sqrt{x-7}$
(b) $x+2$	$3x$	$3(x+2) = 3x+6$
(c) x^2	$\sqrt{x-5}$	$\sqrt{x^2-5}$
(d) $\frac{x}{x-1}$	$\frac{x}{x-1}$	$\frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1} = \frac{x}{x - (x-1)} = x$
(e) $\frac{1}{x-1}$	$1 + \frac{1}{x}$	x

(f) $\frac{1}{x} \quad \frac{1}{x} \quad x$

14. (a) $(f \circ g)(x) = |g(x)| = \frac{1}{|x-1|}$.

(b) $(f \circ g)(x) = \frac{g(x)-1}{g(x)} = \frac{x}{x+1} \Rightarrow 1 - \frac{1}{g(x)} = \frac{x}{x+1} \Rightarrow 1 - \frac{x}{x+1} = \frac{1}{g(x)} \Rightarrow \frac{1}{x+1} = \frac{1}{g(x)}$, so $g(x) = x+1$.

(c) Since $(f \circ g)(x) = \sqrt{g(x)} = |x|$, $g(x) = x^2$.

(d) Since $(f \circ g)(x) = f(\sqrt{x}) = |x|$, $f(x) = x^2$. (Note that the domain of the composition is $[0, \infty)$.)

The completed table is shown. Note that the absolute value sign in part (d) is optional.

$g(x)$	$f(x)$	$(f \circ g)(x)$
$\frac{1}{x-1}$	$ x $	$\frac{1}{ x-1 }$
$x+1$	$\frac{x-1}{x}$	$\frac{x}{x+1}$
x^2	\sqrt{x}	$ x $
\sqrt{x}	x^2	$ x $

15. (a) $f(g(-1)) = f(1) = 1$ (b) $g(f(0)) = g(-2) = 2$ (c) $f(f(-1)) = f(0) = -2$
 (d) $g(g(2)) = g(0) = 0$ (e) $g(f(-2)) = g(1) = -1$ (f) $f(g(1)) = f(-1) = 0$

16. (a) $f(g(0)) = f(-1) = 2 - (-1) = 3$, where $g(0) = 0 - 1 = -1$
 (b) $g(f(3)) = g(-1) = -(-1) = 1$, where $f(3) = 2 - 3 = -1$
 (c) $g(g(-1)) = g(1) = 1 - 1 = 0$, where $g(-1) = -(-1) = 1$
 (d) $f(f(2)) = f(0) = 2 - 0 = 2$, where $f(2) = 2 - 2 = 0$
 (e) $g(f(0)) = g(2) = 2 - 1 = 1$, where $f(0) = 2 - 0 = 2$
 (f) $f\left(g\left(\frac{1}{2}\right)\right) = f\left(-\frac{1}{2}\right) = 2 - \left(-\frac{1}{2}\right) = \frac{5}{2}$, where $g\left(\frac{1}{2}\right) = \frac{1}{2} - 1 = -\frac{1}{2}$

17. (a) $(f \circ g)(x) = f(g(x)) = \sqrt{\frac{1}{x} + 1} = \sqrt{\frac{1+x}{x}}$
 $(g \circ f)(x) = g(f(x)) = \frac{1}{\sqrt{x+1}}$
 (b) Domain $(f \circ g)$: $(-\infty, -1] \cup (0, \infty)$, domain $(g \circ f)$: $(-1, \infty)$
 (c) Range $(f \circ g)$: $(1, \infty)$, range $(g \circ f)$: $(0, \infty)$

18. (a) $(f \circ g)(x) = f(g(x)) = 1 - 2\sqrt{x} + x$
 $(g \circ f)(x) = g(f(x)) = 1 - |x|$
 (b) Domain $(f \circ g)$: $[0, \infty)$, domain $(g \circ f)$: $(-\infty, \infty)$
 (c) Range $(f \circ g)$: $(0, \infty)$, range $(g \circ f)$: $(-\infty, 1]$

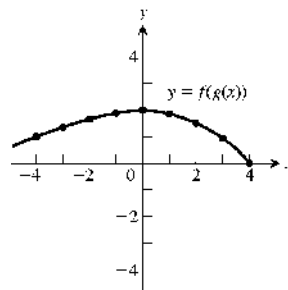
19. $(f \circ g)(x) = x \Rightarrow f(g(x)) = x \Rightarrow \frac{g(x)}{g(x)-2} = x \Rightarrow g(x) = (g(x)-2)x = x \cdot g(x) - 2x$
 $\Rightarrow g(x) - x \cdot g(x) = -2x \Rightarrow g(x) = -\frac{2x}{1-x} = \frac{2x}{x-1}$

20. $(f \circ g)(x) = x+2 \Rightarrow f(g(x)) = x+2 \Rightarrow 2(g(x))^3 - 4 = x+2 \Rightarrow (g(x))^3 = \frac{x+6}{2} \Rightarrow g(x) = \sqrt[3]{\frac{x+6}{2}}$

21. $V = V(s) = V(s(t)) = V(2t-3)$
 $= (2t-3)^2 + 2(2t-3) + 3$
 $= 4t^2 - 8t + 6$

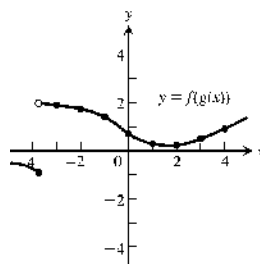
22. (a)

x	-4	-3	-2	-1	0	1	2	3	4
$g(x)$	-2	-1	-0.5	-0.2	0	0.2	0.5	1	2
$f(g(x))$	1	1.3	1.6	1.8	2	1.8	1.5	1	0



(b)

x	-4	-3	-2	-1	0	1	2	3	4
$g(x)$	1.5	0.3	-0.7	-1.5	-2.4	-2.8	-3	-2.7	-2
$f(g(x))$	-0.8	1.9	1.7	1.5	0.7	0.3	0.2	0.5	0.9



23. (a) $y = -(x+7)^2$

(b) $y = -(x-4)^2$

24. (a) $y = x^2 + 3$

(b) $y = x^2 - 5$

25. (a) Position 4

(b) Position 1

(c) Position 2

(d) Position 3

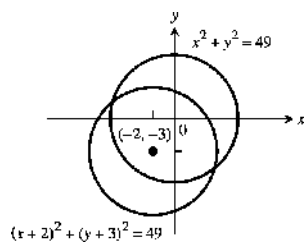
26. (a) $y = -(x-1)^2 + 4$

(b) $y = -(x+2)^2 + 3$

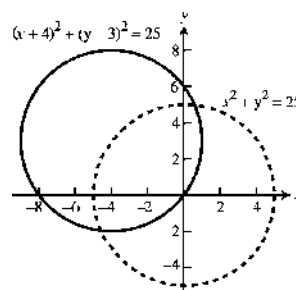
(c) $y = -(x+4)^2 - 1$

(d) $y = -(x-2)^2$

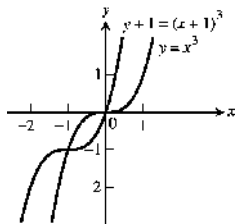
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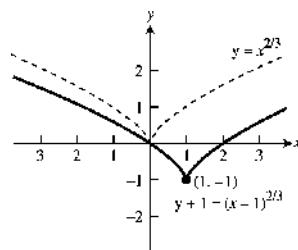
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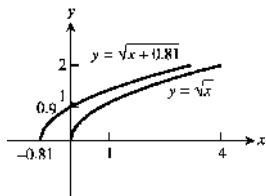
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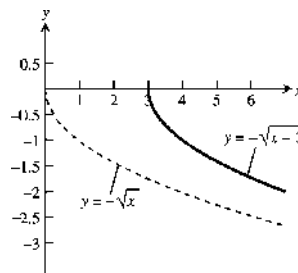
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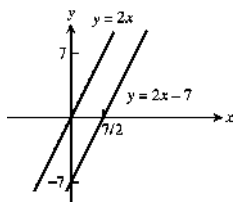
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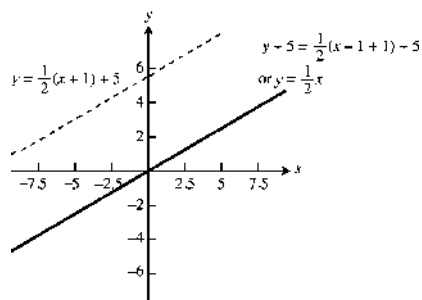
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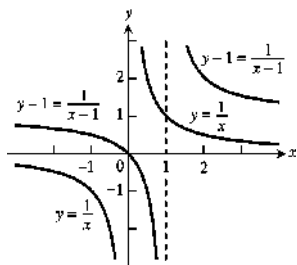
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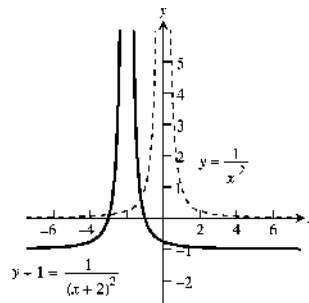
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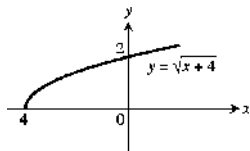
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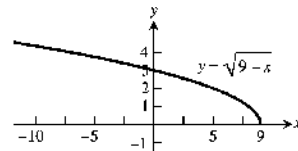
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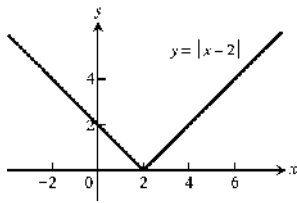
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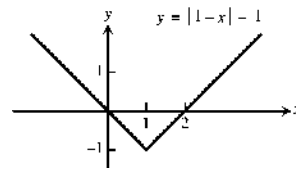
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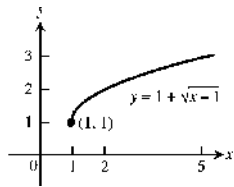
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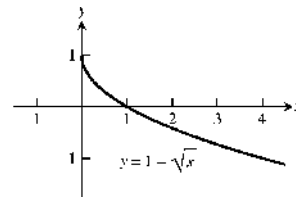
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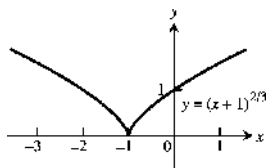
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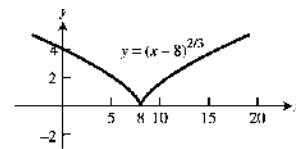
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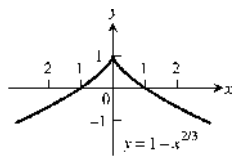
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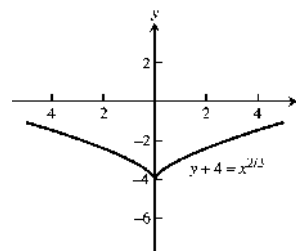
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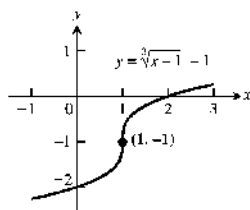
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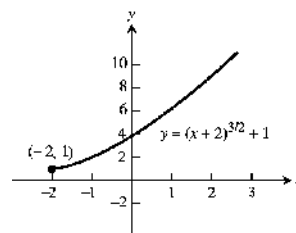
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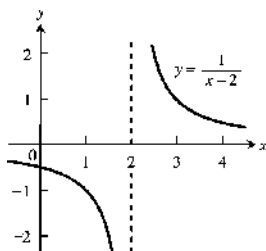
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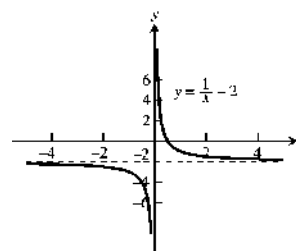
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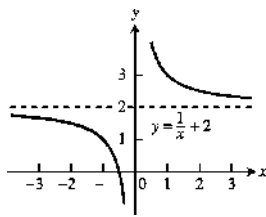
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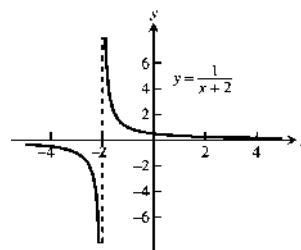
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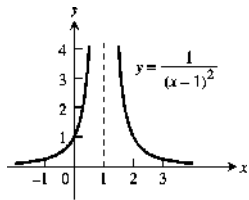
51.



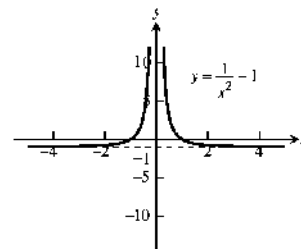
52.



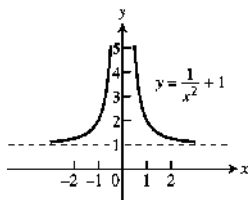
53.



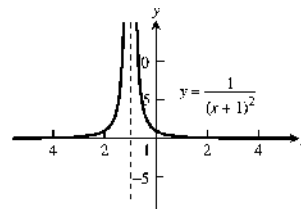
54.



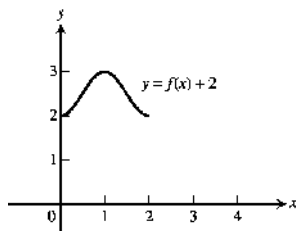
55.



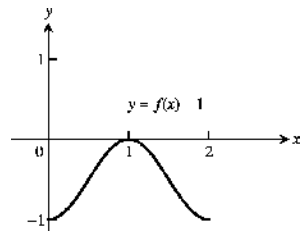
56.



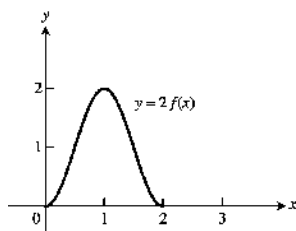
57. (a) domain: $[0, 2]$; range: $[2, 3]$



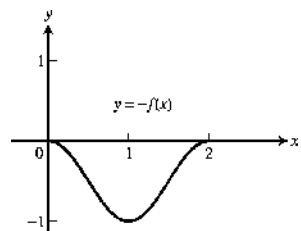
(b) domain: $[0, 2]$; range: $[-1, 0]$



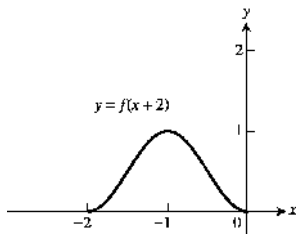
(c) domain: $[0, 2]$; range: $[0, 2]$



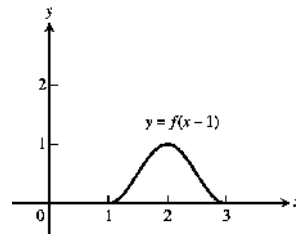
(d) domain: $[0, 2]$; range: $[-1, 0]$



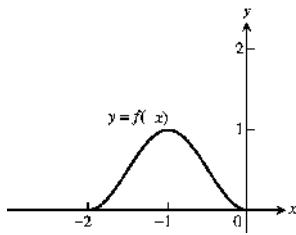
(e) domain: $[-2, 0]$; range: $[0, 1]$



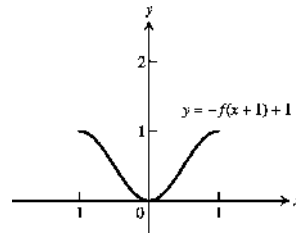
(f) domain: $[1, 3]$; range: $[0, 1]$



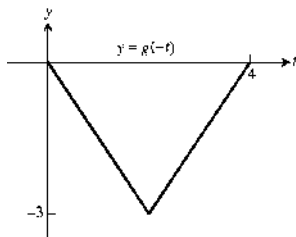
(g) domain: $[-2, 0]$; range: $[0, 1]$



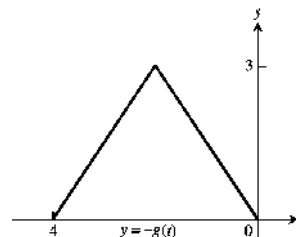
(h) domain: $[-1, 1]$; range: $[0, 1]$



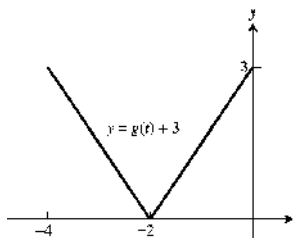
58. (a) domain: $[0, 4]$; range: $[-3, 0]$



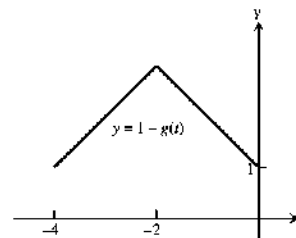
(b) domain: $[-4, 0]$; range: $[0, 3]$



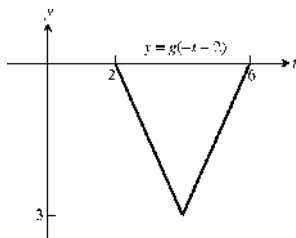
(c) domain: $[-4, 0]$; range: $[0, 3]$



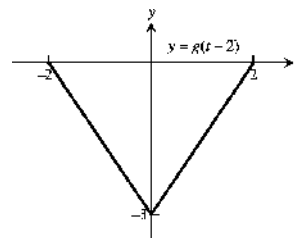
(d) domain: $[-4, 0]$; range: $[1, 4]$



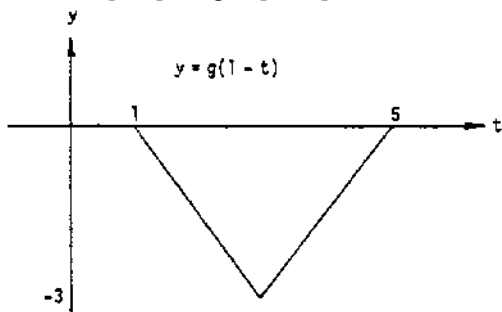
(e) domain: $[2, 4]$; range: $[-3, 0]$



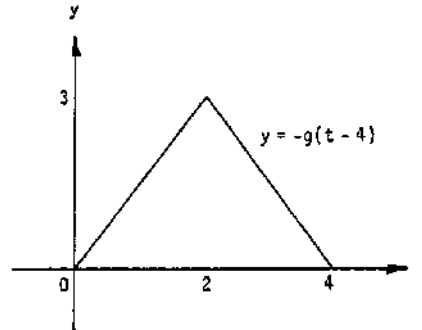
(f) domain: $[-2, 2]$; range: $[-3, 0]$



(g) domain: $[1, 5]$; range: $[-3, 0]$



(h) domain: $[0, 4]$; range: $[0, 3]$



59. $y = 3x^2 - 3$

60. $y = (2x)^2 - 1 = 4x^2 - 1$

61. $y = \frac{1}{2}\left(1 + \frac{1}{x^2}\right) = \frac{1}{2} + \frac{1}{2x^2}$

62. $y = 1 + \frac{1}{(x/3)^2} = 1 + \frac{9}{x^2}$

63. $y = \sqrt{4x+1}$

64. $y = 3\sqrt{x+1}$

65. $y = \sqrt{4 - \left(\frac{x}{2}\right)^2} = \frac{1}{2}\sqrt{16 - x^2}$

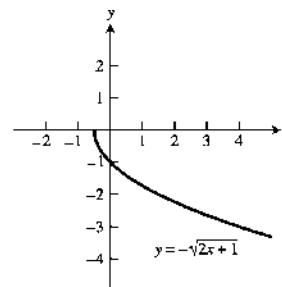
66. $y = \frac{1}{3}\sqrt{4 - x^2}$

67. $y = 1 - (3x)^3 = 1 - 27x^3$

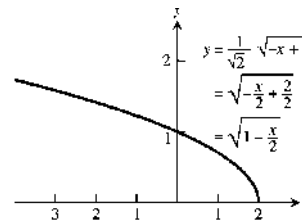
68. $y = 1 - \left(\frac{x}{2}\right)^3 = 1 - \frac{x^3}{8}$

69. Let $y = -\sqrt{2x+1} = f(x)$ and let $g(x) = x^{1/2}$,
 $h(x) = \left(x + \frac{1}{2}\right)^{1/2}$, $i(x) = \sqrt{2}\left(x + \frac{1}{2}\right)^{1/2}$, and
 $j(x) = -\left[\sqrt{2}\left(x + \frac{1}{2}\right)^{1/2}\right] = f(x)$. The graph of $h(x)$

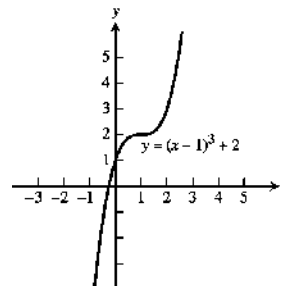
is the graph of $g(x)$ shifted left $\frac{1}{2}$ unit; the graph of $i(x)$ is the graph of $h(x)$ stretched vertically by a factor of $\sqrt{2}$; and the graph of $j(x) = f(x)$ is the graph of $i(x)$ reflected across the x -axis.



70. Let $y = \sqrt{1 - \frac{x}{2}} = f(x)$. Let $g(x) = (-x)^{1/2}$,
 $h(x) = (-x+2)^{1/2}$, and $i(x) = \frac{1}{\sqrt{2}}(-x+2)^{1/2} =$
 $\sqrt{1 - \frac{x}{2}} = f(x)$. The graph of $g(x)$ is the graph of $y = \sqrt{x}$ reflected across the x -axis. The graph of $h(x)$ is the graph of $g(x)$ shifted right two units. And the graph of $i(x)$ is the graph of $h(x)$ compressed vertically by a factor of $\sqrt{2}$.

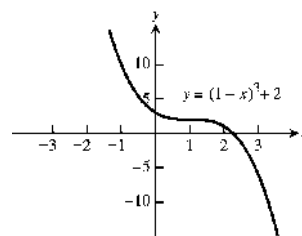


71. $y = f(x) = x^3$. Shift $f(x)$ one unit right followed by a shift two units up to get $g(x) = (x-1)^3 + 2$.

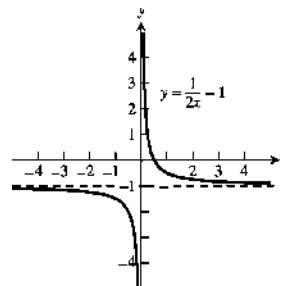


72. $y = (1-x)^3 + 2 = -[(x-1)^3 + (-2)] = f(x)$.

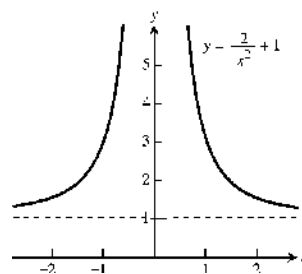
Let $g(x) = x^3$, $h(x) = (x-1)^3$, $i(x) = (x-1)^3 + (-2)$, and $j(x) = -[(x-1)^3 + (-2)]$. The graph of $h(x)$ is the graph of $g(x)$ shifted right one unit; the graph of $i(x)$ is the graph of $h(x)$ shifted down two units; and the graph of $f(x)$ is the graph of $i(x)$ reflected across the x -axis.



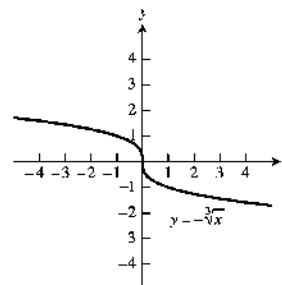
73. Compress the graph of $f(x) = \frac{1}{x}$ horizontally by a factor of 2 to get $g(x) = \frac{1}{2x}$. Then shift $g(x)$ vertically down 1 unit to get $h(x) = \frac{1}{2x} - 1$.



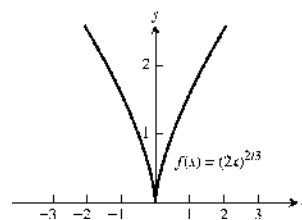
74. Let $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{2}{x^2} + 1 = \frac{1}{\left(\frac{x}{\sqrt{2}}\right)^2} + 1$
 $= \frac{1}{(x/\sqrt{2})^2} + 1 = \frac{1}{[(1/\sqrt{2})x]^2} + 1$. Since $\sqrt{2} \approx 1.4$, we see that the graph of $f(x)$ stretched horizontally by a factor of 1.4 and shifted up 1 unit is the graph of $g(x)$.



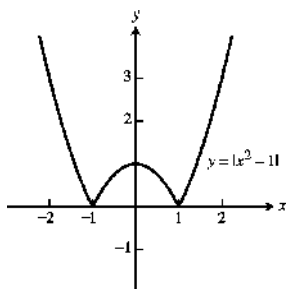
75. Reflect the graph of $y = f(x) = \sqrt[3]{x}$ across the x -axis to get $g(x) = -\sqrt[3]{x}$.



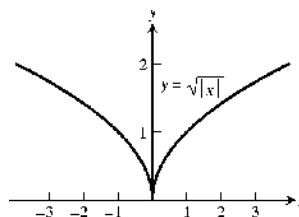
76. $y = f(x) = (-2x)^{2/3} = [(-1)(2x)]^{2/3} = (-1)^{2/3} (2x)^{2/3} = (2x)^{2/3}$. So the graph of $f(x)$ is the graph of $g(x) = x^{2/3}$ compressed horizontally by a factor of 2.



77.



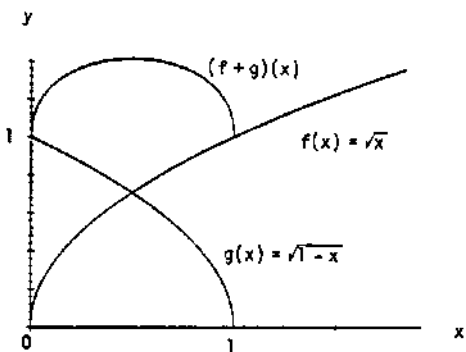
78.



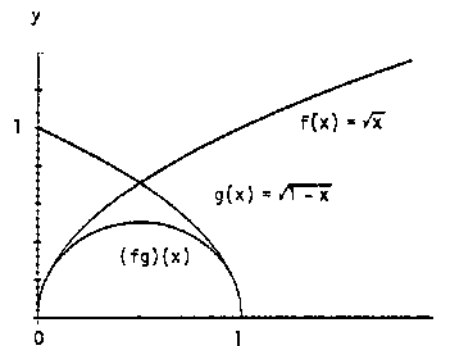
79. (a) $(fg)(-x) = f(-x)g(-x) = f(x)(-g(x)) = -(fg)(x)$, odd
 (b) $\left(\frac{f}{g}\right)(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\left(\frac{f}{g}\right)(x)$, odd
 (c) $\left(\frac{g}{f}\right)(-x) = \frac{g(-x)}{f(-x)} = \frac{-g(x)}{f(x)} = -\left(\frac{g}{f}\right)(x)$, odd
 (d) $f^2(-x) = f(-x)f(-x) = f(x)f(x) = f^2(x)$, even
 (e) $g^2(-x) = (g(-x))^2 = (-g(x))^2 = g^2(x)$, even
 (f) $(f \circ g)(-x) = f(g(-x)) = f(-g(x)) = f(g(x)) = (f \circ g)(x)$, even
 (g) $(g \circ f)(-x) = g(f(-x)) = g(f(x)) = (g \circ f)(x)$, even
 (h) $(f \circ f)(-x) = f(f(-x)) = f(f(x)) = (f \circ f)(x)$, even
 (i) $(g \circ g)(-x) = g(g(-x)) = g(-g(x)) = -g(g(x)) = -(g \circ g)(x)$, odd

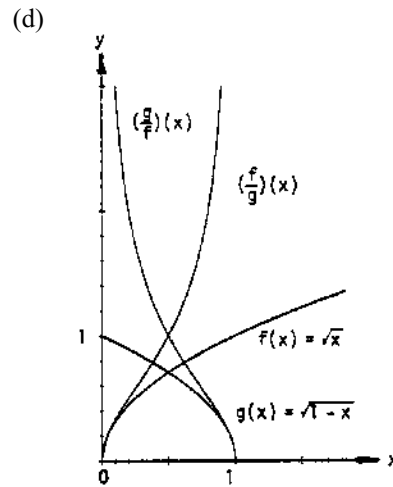
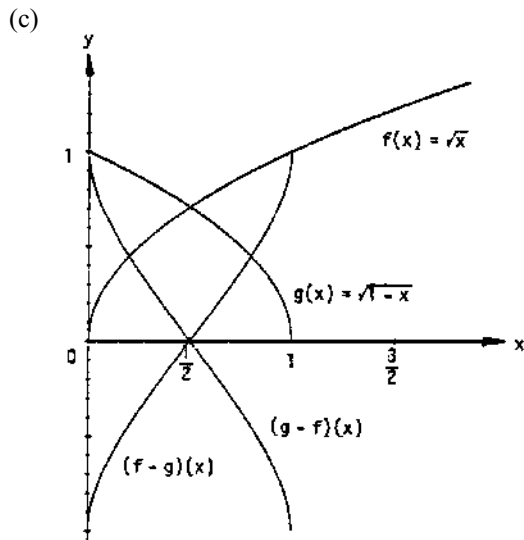
80. Yes, $f(x) = 0$ is both even and odd since $f(-x) = 0 = f(x)$ and $f(-x) = 0 = -f(x)$.

81. (a)

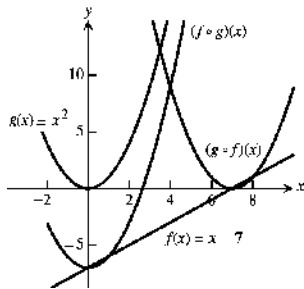


(b)





82.



1.3 TRIGONOMETRIC FUNCTIONS

- (a) $s = r\theta = (10)\left(\frac{4\pi}{5}\right) = 8\pi$ m

(b) $s = r\theta = (10)(110^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{110\pi}{18} = \frac{55\pi}{9}$ m
- $\theta = \frac{s}{r} = \frac{10\pi}{8} = \frac{5\pi}{4}$ radians and $\frac{5\pi}{4}\left(\frac{180^\circ}{\pi}\right) = 225^\circ$
- $\theta = 80^\circ \Rightarrow \theta = 80^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{4\pi}{9} \Rightarrow s = (6)\left(\frac{4\pi}{9}\right) = 8.4$ in. (since the diameter = 12 in. \Rightarrow radius = 6 in.)
- $d = 1$ meter $\Rightarrow r = 50$ cm $\Rightarrow \theta = \frac{s}{r} = \frac{30}{50} = 0.6$ rad or $0.6\left(\frac{180^\circ}{\pi}\right) \approx 34^\circ$

5.

θ	$-\pi$	$-\frac{2\pi}{3}$	0	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$\sin \theta$	0	$-\frac{\sqrt{3}}{2}$	0	1	$\frac{1}{\sqrt{2}}$
$\cos \theta$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{\sqrt{2}}$
$\tan \theta$	0	$\sqrt{3}$	0	und.	-1
$\cot \theta$	und.	$\frac{1}{\sqrt{3}}$	und.	0	-1
$\sec \theta$	-1	-2	1	und.	$-\sqrt{2}$
$\csc \theta$	und.	$-\frac{2}{\sqrt{3}}$	und.	1	$\sqrt{2}$

6.

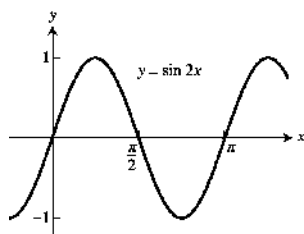
θ	$-\frac{3\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{5\pi}{6}$
$\sin \theta$	1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\cos \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$
$\tan \theta$	und.	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	1	$-\frac{1}{\sqrt{3}}$
$\cot \theta$	0	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	1	$-\sqrt{3}$
$\sec \theta$	und.	2	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	$-\frac{2}{\sqrt{3}}$
$\csc \theta$	1	$-\frac{2}{\sqrt{3}}$	-2	$\sqrt{2}$	2

7. $\cos x = -\frac{4}{5}, \tan x = -\frac{3}{4}$

9. $\sin x = -\frac{\sqrt{8}}{3}, \tan x = -\sqrt{8}$

11. $\sin x = -\frac{1}{\sqrt{5}}, \cos x = -\frac{2}{\sqrt{5}}$

13.



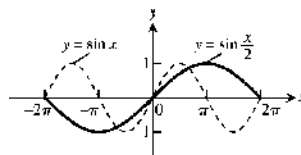
period = π

8. $\sin x = \frac{2}{\sqrt{5}}, \cos x = \frac{1}{\sqrt{5}}$

10. $\sin x = \frac{12}{13}, \tan x = -\frac{12}{5}$

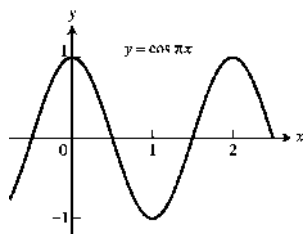
12. $\cos x = -\frac{\sqrt{3}}{2}, \tan x = \frac{1}{\sqrt{3}}$

14.



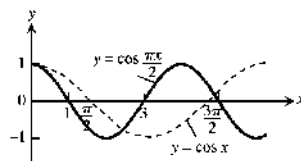
period = 4π

15.



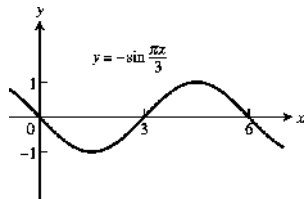
period = 2

16.



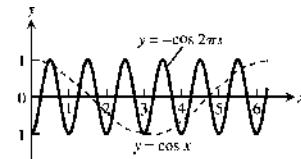
period = 4

17.



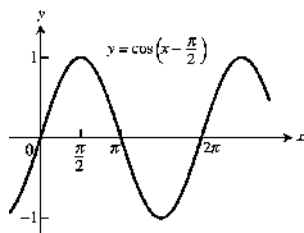
period = 6

18.



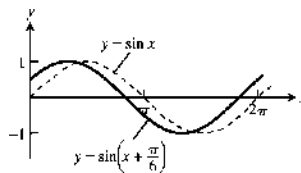
period = 1

19.



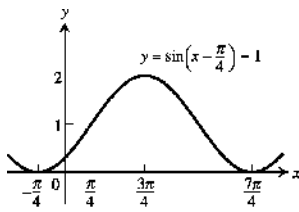
period = 2π

20.



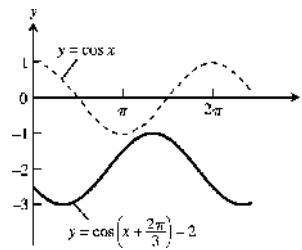
period = 2π

21.



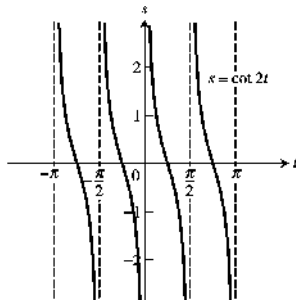
period = 2π

22.

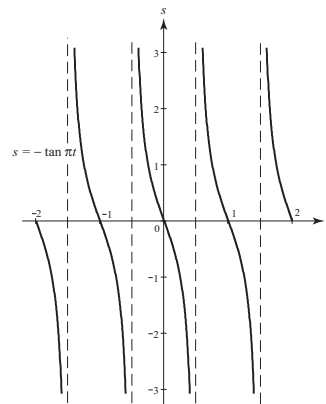


period = 2π

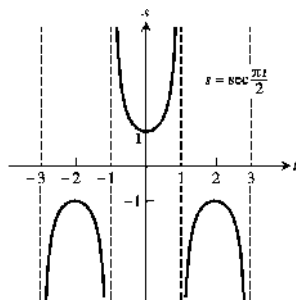
23. period = $\frac{\pi}{2}$, symmetric about the origin



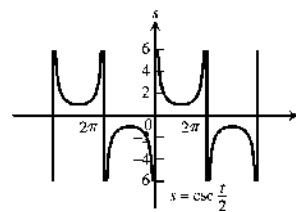
24. period = 1, symmetric about the origin



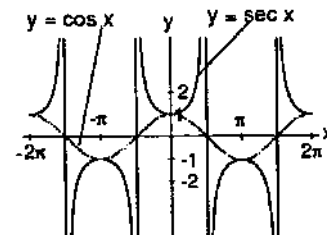
25. period = 4, symmetric about the s -axis



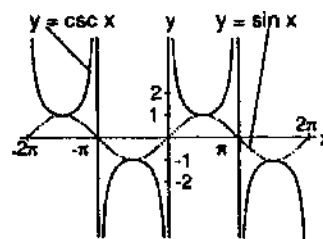
26. period = 4π , symmetric about the origin



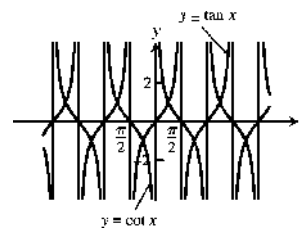
27. (a) $\cos x$ and $\sec x$ are positive for x in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$; and $\cos x$ and $\sec x$ are negative for x in the intervals $(-\frac{3\pi}{2}, -\frac{\pi}{2})$ and $(\frac{\pi}{2}, \frac{3\pi}{2})$. $\sec x$ is undefined when $\cos x$ is 0. The range of $\sec x$ is $(-\infty, -1] \cup [1, \infty)$; the range of $\cos x$ is $[-1, 1]$.



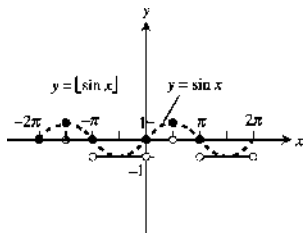
- (b) $\sin x$ and $\csc x$ are positive for x in the intervals $(-\frac{3\pi}{2}, -\pi)$ and $(0, \pi)$; and $\sin x$ and $\csc x$ are negative for x in the intervals $(-\pi, 0)$ and $(\pi, \frac{3\pi}{2})$. $\csc x$ is undefined when $\sin x$ is 0. The range of $\csc x$ is $(-\infty, -1] \cup [1, \infty)$; the range of $\sin x$ is $[-1, 1]$.



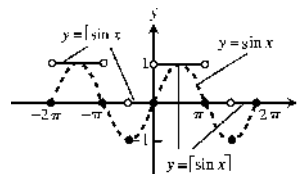
28. Since $\cot x = \frac{1}{\tan x}$, $\cot x$ is undefined when $\tan x = 0$ and is zero when $\tan x$ is undefined. As $\tan x$ approaches zero through positive values, $\cot x$ approaches infinity. Also, $\cot x$ approaches negative infinity as $\tan x$ approaches zero through negative values.



29. $D: -\infty < x < \infty; R: y = -1, 0, 1$



30. $D: -\infty < x < \infty; R: y = -1, 0, 1$



31. $\cos\left(x - \frac{\pi}{2}\right) = \cos x \cos\left(-\frac{\pi}{2}\right) - \sin x \sin\left(-\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(-1) = \sin x$
32. $\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos\left(\frac{\pi}{2}\right) - \sin x \sin\left(\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(1) = -\sin x$
33. $\sin\left(x + \frac{\pi}{2}\right) = \sin x \cos\left(\frac{\pi}{2}\right) + \cos x \sin\left(\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(1) = \cos x$
34. $\sin\left(x - \frac{\pi}{2}\right) = \sin x \cos\left(-\frac{\pi}{2}\right) + \cos x \sin\left(-\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(-1) = -\cos x$
35. $\cos(A - B) = \cos(A + (-B)) = \cos A \cos(-B) - \sin A \sin(-B) = \cos A \cos B - \sin A(-\sin B)$
 $= \cos A \cos B + \sin A \sin B$
36. $\sin(A - B) = \sin(A + (-B)) = \sin A \cos(-B) + \cos A \sin(-B) = \sin A \cos B + \cos A(-\sin B)$
 $= \sin A \cos B - \cos A \sin B$
37. If $B = A$, $A - B = 0 \Rightarrow \cos(A - B) = \cos 0 = 1$. Also $\cos(A - B) = \cos(A - A) = \cos A \cos A + \sin A \sin A$
 $= \cos^2 A + \sin^2 A$. Therefore, $\cos^2 A + \sin^2 A = 1$.
38. If $B = 2\pi$, then $\cos(A + 2\pi) = \cos A \cos 2\pi - \sin A \sin 2\pi = (\cos A)(1) - (\sin A)(0) = \cos A$ and
 $\sin(A + 2\pi) = \sin A \cos 2\pi + \cos A \sin 2\pi = (\sin A)(1) + (\cos A)(0) = \sin A$. The result agrees with the fact that the cosine and sine functions have period 2π .
39. $\cos(\pi + x) = \cos \pi \cos x - \sin \pi \sin x = (-1)(\cos x) - (0)(\sin x) = -\cos x$

