INSTRUCTOR'S SOLUTIONS MANUAL

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THOMAS' CALCULUS EARLY TRANSCENDENTALS

FOURTEENTH EDITION

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Pearson

Reproduced by Pearson from electronic files supplied by the author.

Publishing as Pearson, 330 Hudson Street, NY NY 10013

Pearson

ISBN-13: 978-0-13-443932-7
ISBN-10: 0-13-443932-5

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CHAPTER 1 FUNCTIONS

1.1 FUNCTIONS AND THEIR GRAPHS

1. domain $= (-\infty, \infty)$; range $= [1, \infty)$

2. domain $= [0, \infty)$; range $= (-\infty, 1]$

3. domain $= [-2, \infty)$; y in range and $y = \sqrt{5x+10} \geq 0 \Rightarrow$ y can be any positive real number $\Rightarrow$ range $= [0, \infty)$.

4. domain $= (-\infty, 0] \cup [3, \infty)$; y in range and $y = \sqrt{x^2 - 3x} \geq 0 \Rightarrow y$ can be any positive real number $\Rightarrow$ range $= [0, \infty)$.

5. domain $= (-\infty, 3) \cup (3, \infty)$; y in range and $y = \frac{4}{3-t}$, now if $t < 3 \Rightarrow 3 - t > 0 \Rightarrow \frac{4}{3-t} > 0$, or if $t > 3 \Rightarrow$ $3 - t < 0 \Rightarrow \frac{4}{3-t} < 0 \Rightarrow y$ can be any nonzero real number $\Rightarrow$ range $= (-\infty, 0) \cup (0, \infty)$.

6. domain $= (-\infty, -4) \cup (-4, 4) \cup (4, \infty)$; y in range and $y = \frac{2}{t^2-16}$, now if $t < -4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0$, or if $-4 < t < 4 \Rightarrow -16 \leq t^2 - 16 < 0 \Rightarrow -\frac{2}{16} \geq \frac{2}{t^2-16}$, or if $t > 4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0 \Rightarrow y$ can be any nonzero real number $\Rightarrow$ range $= (-\infty, -\frac{1}{8}] \cup (0, \infty)$.

7. (a) Not the graph of a function of x since it fails the vertical line test.
   (b) Is the graph of a function of x since any vertical line intersects the graph at most once.

8. (a) Not the graph of a function of x since it fails the vertical line test.
   (b) Not the graph of a function of x since it fails the vertical line test.

9. base $= x$; $(\text{height})^2 + \left(\frac{x}{2}\right)^2 = x^2 \Rightarrow$ height $= \frac{\sqrt{3}}{2}x$; area is $a(x) = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$; perimeter is $p(x) = x + x + x = 3x$.

10. s = side length $\Rightarrow s^2 + s^2 = d^2 \Rightarrow s = \frac{d}{\sqrt{2}}$; and area is $a = s^2 \Rightarrow a = \frac{1}{2}d^2$

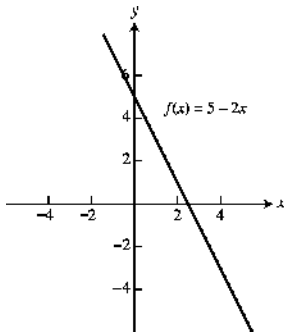
11. Let D = diagonal length of a face of the cube and ℓ = the length of an edge. Then $\ell^2 + D^2 = d^2$ and $D^2 = 2\ell^2 \Rightarrow 3\ell^2 = d^2 \Rightarrow \ell = \frac{d}{\sqrt{3}}$. The surface area is $6\ell^2 = \frac{6d^2}{3} = 2d^2$ and the volume is $\ell^3 = \left(\frac{d^2}{3}\right)^{3/2} = \frac{d^3}{3\sqrt{3}}$.

12. The coordinates of P are $\left(x, \sqrt{x}\right)$ so the slope of the line joining P to the origin is $m = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}\ (x > 0)$. Thus, $\left(x, \sqrt{x}\right) = \left(\frac{1}{m^2}, \frac{1}{m}\right)$.

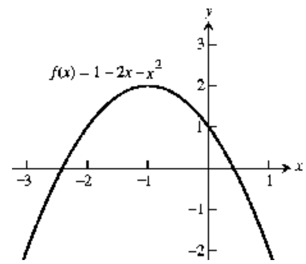
13. $2x + 4y = 5 \Rightarrow y = -\frac{1}{2}x + \frac{5}{4}; L = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + (-\frac{1}{2}x + \frac{5}{4})^2} = \sqrt{x^2 + \frac{1}{4}x^2 - \frac{5}{4}x + \frac{25}{16}}$
$= \sqrt{\frac{5}{4}x^2 - \frac{5}{4}x + \frac{25}{16}} = \sqrt{\frac{20x^2 - 20x + 25}{16}} = \frac{\sqrt{20x^2 - 20x + 25}}{4}$

14. $y = \sqrt{x-3} \Rightarrow y^2 + 3 = x; L = \sqrt{(x-4)^2 + (y-0)^2} = \sqrt{(y^2 + 3 - 4)^2 + y^2} = \sqrt{(y^2 - 1)^2 + y^2}$
$= \sqrt{y^4 - 2y^2 + 1 + y^2} = \sqrt{y^4 - y^2 + 1}$

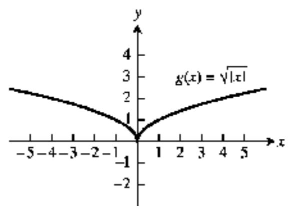
15. The domain is $(-\infty, \infty)$.



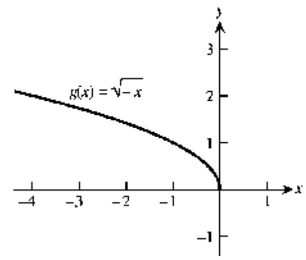
16. The domain is $(-\infty, \infty)$.



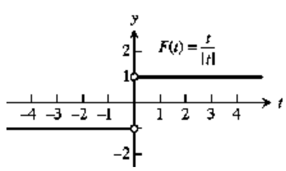
17. The domain is $(-\infty, \infty)$.



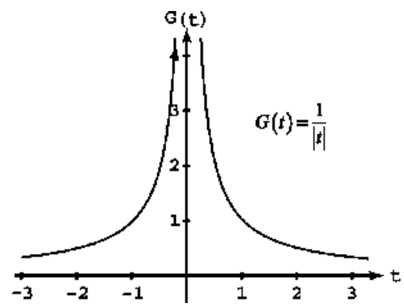
18. The domain is $(-\infty, 0]$.



19. The domain is $(-\infty, 0) \cup (0, \infty)$.



20. The domain is $(-\infty, 0) \cup (0, \infty)$.

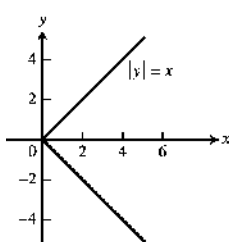


21. The domain is $(-\infty, -5) \cup (-5, -3] \cup [3, 5) \cup (5, \infty)$

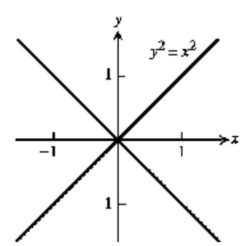
22. The range is $[5, \infty)$.

23. Neither graph passes the vertical line test

(a)

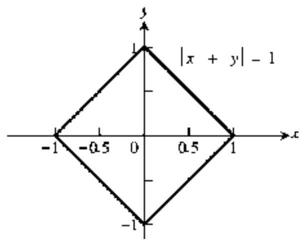


(b)

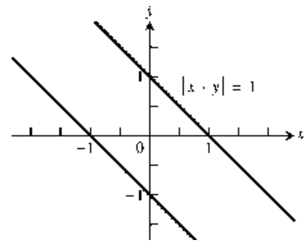


24. Neither graph passes the vertical line test

(a)



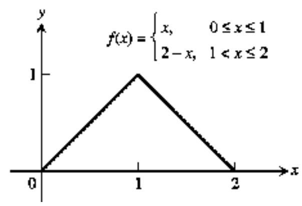
(b)



$$|x+y| = 1 \Leftrightarrow \left\{\begin{matrix} x+y=1 \\ \text{or} \\ x+y=-1 \end{matrix}\right\} \Leftrightarrow \left\{\begin{matrix} y=1-x \\ \text{or} \\ y=-1-x \end{matrix}\right\}$$

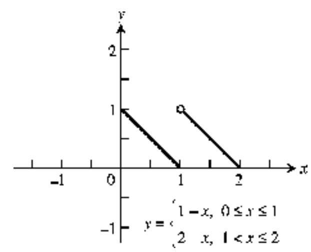
25.

x	0	1	2
y	0	1	0

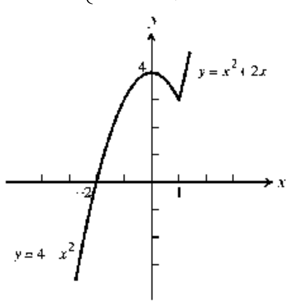


26.

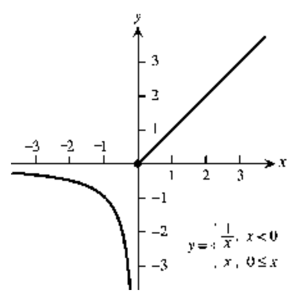
x	0	1	2
y	1	0	0



27. $F(x) = \begin{cases} 4-x^2, & x \le 1 \\ x^2+2x, & x > 1 \end{cases}$



28. $G(x) = \begin{cases} \frac{1}{x}, & x < 0 \\ x, & 0 \le x \end{cases}$



29. (a) Line through (0, 0) and (1, 1): $y = x$; Line through (1, 1) and (2, 0): $y = -x + 2$

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ -x+2, & 1 < x \le 2 \end{cases}$$

(b) $f(x) = \begin{cases} 2, & 0 \le x < 1 \\ 0, & 1 \le x < 2 \\ 2, & 2 \le x < 3 \\ 0, & 3 \le x \le 4 \end{cases}$

30. (a) Line through (0, 2) and (2, 0): $y = -x + 2$

Line through (2, 1) and (5, 0): $m = \frac{0-1}{5-2} = \frac{-1}{3} = -\frac{1}{3}$, so $y = -\frac{1}{3}(x-2) + 1 = -\frac{1}{3}x + \frac{5}{3}$

$$f(x) = \begin{cases} -x+2, & 0 < x \le 2 \\ -\frac{1}{3}x + \frac{5}{3}, & 2 < x \le 5 \end{cases}$$

(b) Line through $(-1, 0)$ and $(0, -3)$: $m = \frac{-3-0}{0-(-1)} = -3$, so $y = -3x - 3$

Line through $(0, 3)$ and $(2, -1)$: $m = \frac{-1-3}{2-0} = \frac{-4}{2} = -2$, so $y = -2x + 3$

$$f(x) = \begin{cases} -3x-3, & -1 < x \le 0 \\ -2x+3, & 0 < x \le 2 \end{cases}$$

31. (a) Line through $(-1, 1)$ and $(0, 0)$: $y = -x$

Line through $(0, 1)$ and $(1, 1)$: $y = 1$

Line through $(1, 1)$ and $(3, 0)$: $m = \frac{0-1}{3-1} = \frac{-1}{2} = -\frac{1}{2}$, so $y = -\frac{1}{2}(x-1) + 1 = -\frac{1}{2}x + \frac{3}{2}$

$$f(x) = \begin{cases} -x & -1 \le x < 0 \\ 1 & 0 < x \le 1 \\ -\frac{1}{2}x + \frac{3}{2} & 1 < x < 3 \end{cases}$$

(b) Line through $(-2, -1)$ and $(0, 0)$: $y = \frac{1}{2}x$

Line through $(0, 2)$ and $(1, 0)$: $y = -2x + 2$

Line through $(1, -1)$ and $(3, -1)$: $y = -1$

$$f(x) = \begin{cases} \frac{1}{2}x & -2 \le x \le 0 \\ -2x+2 & 0 < x \le 1 \\ -1 & 1 < x \le 3 \end{cases}$$

32. (a) Line through $\left(\frac{T}{2}, 0\right)$ and $(T, 1)$: $m = \frac{1-0}{T-(T/2)} = \frac{2}{T}$, so $y = \frac{2}{T}\left(x - \frac{T}{2}\right) + 0 = \frac{2}{T}x - 1$

$$f(x) = \begin{cases} 0, & 0 \le x \le \frac{T}{2} \\ \frac{2}{T}x - 1, & \frac{T}{2} < x \le T \end{cases}$$

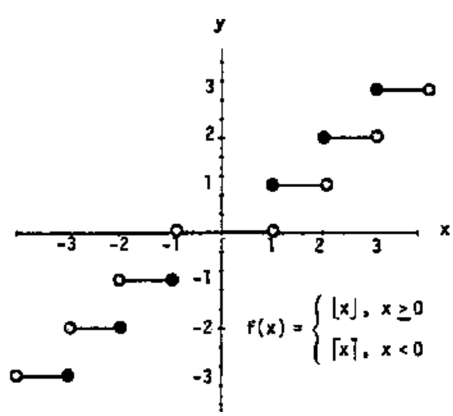
(b) $$f(x) = \begin{cases} A, & 0 \le x < \frac{T}{2} \\ -A, & \frac{T}{2} \le x < T \\ A, & T \le x < \frac{3T}{2} \\ -A, & \frac{3T}{2} \le x \le 2T \end{cases}$$

33. (a) $\lfloor x \rfloor = 0$ for $x \in [0, 1)$ (b) $\lceil x \rceil = 0$ for $x \in (-1, 0]$

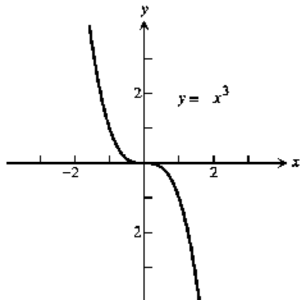
34. $\lfloor x \rfloor = \lceil x \rceil$ only when x is an integer.

35. For any real number x, $n \le x \le n+1$, where n is an integer. Now: $n \le x \le n+1 \Rightarrow -(n+1) \le -x \le -n$. By definition: $\lceil -x \rceil = -n$ and $\lfloor x \rfloor = n \Rightarrow -\lfloor x \rfloor = -n$. So $\lceil -x \rceil = -\lfloor x \rfloor$ for all real x.

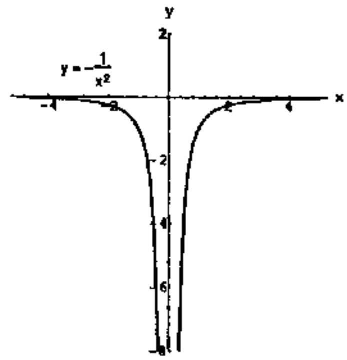
36. To find $f(x)$ you delete the decimal or fractional portion of x, leaving only the integer part.



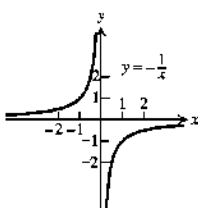
37. Symmetric about the origin
Dec: $-\infty < x < \infty$
Inc: nowhere



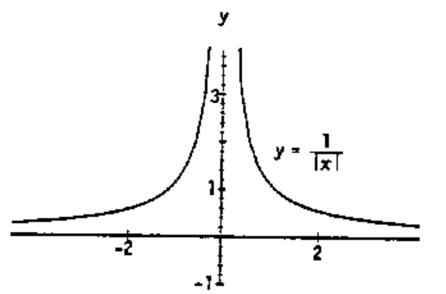
38. Symmetric about the y-axis
Dec: $-\infty < x < 0$
Inc: $0 < x < \infty$



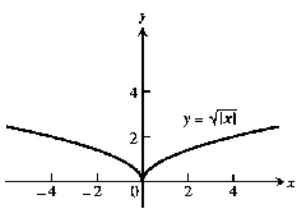
39. Symmetric about the origin
Dec: nowhere
Inc: $-\infty < x < 0$
$0 < x < \infty$



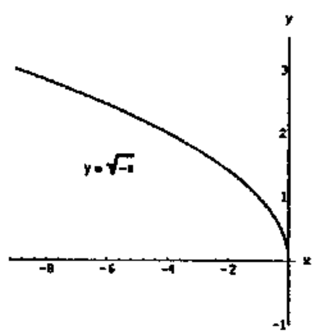
40. Symmetric about the y-axis
Dec: $0 < x < \infty$
Inc: $-\infty < x < 0$



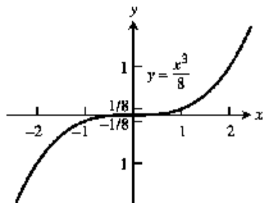
41. Symmetric about the y-axis
Dec: $-\infty < x \le 0$
Inc: $0 \le x < \infty$



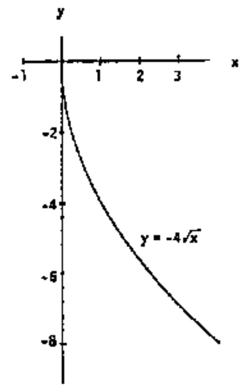
42. No symmetry
Dec: $-\infty < x \le 0$
Inc: nowhere



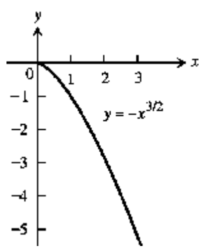
43. Symmetric about the origin
Dec: nowhere
Inc: $-\infty < x < \infty$



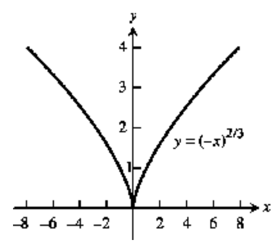
44. No symmetry
Dec: $0 \le x < \infty$
Inc: nowhere



45. No symmetry
Dec: $0 \le x < \infty$
Inc: nowhere



46. Symmetric about the y-axis
Dec: $-\infty < x \le 0$
Inc: $0 \le x < \infty$



47. Since a horizontal line not through the origin is symmetric with respect to the y-axis, but not with respect to the origin, the function is even.

48. $f(x) = x^{-5} = \frac{1}{x^5}$ and $f(-x) = (-x)^{-5} = \frac{1}{(-x)^5} = -\left(\frac{1}{x^5}\right) = -f(x)$. Thus the function is odd.

49. Since $f(x) = x^2 + 1 = (-x)^2 + 1 = f(-x)$. The function is even.

50. Since $[f(x) = x^2 + x] \neq [f(-x) = (-x)^2 - x]$ and $[f(x) = x^2 + x] \neq [-f(x) = -(x)^2 - x]$ the function is neither even nor odd.

51. Since $g(x) = x^3 + x$, $g(-x) = -x^3 - x = -(x^3 + x) = -g(x)$. So the function is odd.

52. $g(x) = x^4 + 3x^2 - 1 = (-x)^4 + 3(-x)^2 - 1 = g(-x)$, thus the function is even.

53. $g(x) = \frac{1}{x^2 - 1} = \frac{1}{(-x)^2 - 1} = g(-x)$. Thus the function is even.

54. $g(x) = \frac{x}{x^2 - 1}$; $g(-x) = -\frac{x}{x^2 - 1} = -g(x)$. So the function is odd.

55. $h(t) = \frac{1}{t - 1}$; $h(-t) = \frac{1}{-t - 1}$; $-h(t) = \frac{1}{1 - t}$. Since $h(t) \neq -h(t)$ and $h(t) \neq h(-t)$, the function is neither even nor odd.

56. Since $|t^3| = |(-t)^3|, h(t) = h(-t)$ and the function is even.

57. $h(t) = 2t + 1$, $h(-t) = -2t + 1$. So $h(t) \neq h(-t)$. $-h(t) = -2t - 1$, so $h(t) \neq -h(t)$. The function is neither even nor odd.

58. $h(t) = 2|t| + 1$ and $h(-t) = 2|-t| + 1 = 2|t| + 1$. So $h(t) = h(-t)$ and the function is even.

59. $g(x) = \sin 2x$; $g(-x) = -\sin 2x = -g(x)$. So the function is odd.

60. $g(x) = \sin x^2$; $g(-x) = \sin x^2 = g(x)$. So the function is even.

61. $g(x) = \cos 3x$; $g(-x) = \cos 3x = g(x)$. So the function is even.

62. $g(x) = 1 + \cos x$; $g(-x) = 1 + \cos x = g(x)$. So the function is even.

63. $s = kt \Rightarrow 25 = k(75) \Rightarrow k = \frac{1}{3} \Rightarrow s = \frac{1}{3}t$; $60 = \frac{1}{3}t \Rightarrow t = 180$

64. $K = c\,v^2 \Rightarrow 12960 = c(18)^2 \Rightarrow c = 40 \Rightarrow K = 40v^2$; $K = 40(10)^2 = 4000$ joules

65. $r = \frac{k}{s} \Rightarrow 6 = \frac{k}{4} \Rightarrow k = 24 \Rightarrow r = \frac{24}{s}$; $10 = \frac{24}{s} \Rightarrow s = \frac{12}{5}$

66. $P = \frac{k}{V} \Rightarrow 14.7 = \frac{k}{1000} \Rightarrow k = 14700 \Rightarrow P = \frac{14700}{V}$; $23.4 = \frac{14700}{V} \Rightarrow V = \frac{24500}{39} \approx 628.2 \text{ in}^3$

67. $V = f(x) = x(14 - 2x)(22 - 2x) = 4x^3 - 72x^2 + 308x$; $0 < x < 7$.

68. (a) Let h = height of the triangle. Since the triangle is isosceles, $\left(\overline{AB}\right)^2 + \left(\overline{AB}\right)^2 = 2^2 \Rightarrow \overline{AB} = \sqrt{2}$. So, $h^2 + 1^2 = \left(\sqrt{2}\right)^2 \Rightarrow h = 1 \Rightarrow B$ is at $(0, 1) \Rightarrow$ slope of $AB = -1 \Rightarrow$ The equation of AB is $y = f(x) = -x + 1$; $x \in [0, 1]$.
(b) $A(x) = 2xy = 2x(-x + 1) = -2x^2 + 2x$; $x \in [0, 1]$.

69. (a) Graph h because it is an even function and rises less rapidly than does Graph g.
(b) Graph f because it is an odd function.
(c) Graph g because it is an even function and rises more rapidly than does Graph h.

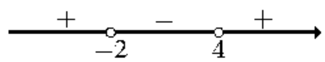
70. (a) Graph f because it is linear.
(b) Graph g because it contains (0, 1).
(c) Graph h because it is a nonlinear odd function.

71. (a) From the graph, $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow x \in (-2, 0) \cup (4, \infty)$

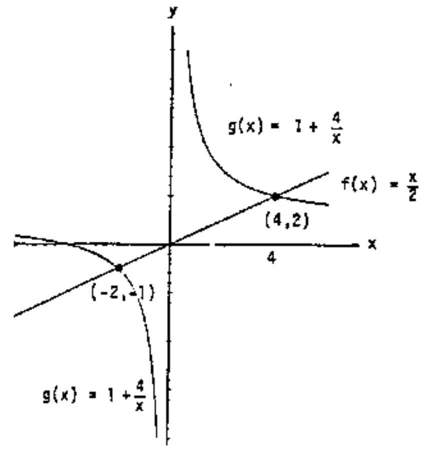
(b) $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow \frac{x}{2} - 1 - \frac{4}{x} > 0$

$x > 0$: $\frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2 - 2x - 8}{2x} > 0 \Rightarrow \frac{(x-4)(x+2)}{2x} > 0$
$\Rightarrow x > 4$ since x is positive;

$x < 0$: $\frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2 - 2x - 8}{2x} < 0 \Rightarrow \frac{(x-4)(x+2)}{2x} < 0$
$\Rightarrow x < -2$ since x is negative;
sign of $(x-4)(x+2)$



Solution interval: $(-2, 0) \cup (4, \infty)$



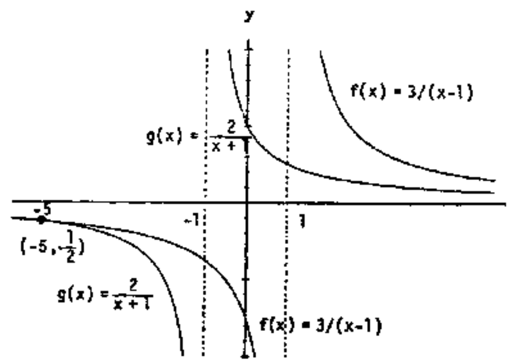
72. (a) From the graph, $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow x \in (-\infty, -5) \cup (-1, 1)$

(b) Case $x < -1$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} > 2$
$\Rightarrow 3x + 3 < 2x - 2 \Rightarrow x < -5.$
Thus, $x \in (-\infty, -5)$ solves the inequality.

Case $-1 < x < 1$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} < 2$
$\Rightarrow 3x + 3 > 2x - 2 \Rightarrow x > -5$ which is true if $x > -1$. Thus, $x \in (-1, 1)$ solves the inequality.

Case $1 < x$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow 3x + 3 < 2x - 2 \Rightarrow x < -5$
which is never true if $1 < x$, so no solution here.

In conclusion, $x \in (-\infty, -5) \cup (-1, 1)$.



73. A curve symmetric about the x-axis will not pass the vertical line test because the points (x, y) and $(x, -y)$ lie on the same vertical line. The graph of the function $y = f(x) = 0$ is the x-axis, a horizontal line for which there is a single y-value, 0, for any x.

74. price $= 40 + 5x$, quantity $= 300 - 25x \Rightarrow R(x) = (40 + 5x)(300 - 25x)$

75. $x^2 + x^2 = h^2 \Rightarrow x = \frac{h}{\sqrt{2}} = \frac{\sqrt{2}h}{2}$; cost $= 5(2x) + 10h \Rightarrow C(h) = 10\left(\frac{\sqrt{2}h}{2}\right) + 10h = 5h\left(\sqrt{2} + 2\right)$

76. (a) Note that 2 mi $= 10{,}560$ ft, so there are $\sqrt{800^2 + x^2}$ feet of river cable at \$180 per foot and $(10{,}560 - x)$ feet of land cable at \$100 per foot. The cost is $C(x) = 180\sqrt{800^2 + x^2} + 100(10{,}560 - x)$.

(b) $C(0) = \$1{,}200{,}000$
$C(500) \approx \$1{,}175{,}812$
$C(1000) \approx \$1{,}186{,}512$
$C(1500) \approx \$1{,}212{,}000$
$C(2000) \approx \$1{,}243{,}732$
$C(2500) \approx \$1{,}278{,}479$
$C(3000) \approx \$1{,}314{,}870$
Values beyond this are all larger. It would appear that the least expensive location is less than 2000 feet from the point P.

1.2 COMBINING FUNCTIONS; SHIFTING AND SCALING GRAPHS

1. $D_f: -\infty < x < \infty, D_g: x \ge 1 \Rightarrow D_{f+g} = D_{fg}: x \ge 1.$ $R_f: -\infty < y < \infty,$ $R_g: y \ge 0, R_{f+g}: y \ge 1, R_{fg}: y \ge 0$

2. $D_f: x+1 \ge 0 \Rightarrow x \ge -1, D_g: x-1 \ge 0 \Rightarrow x \ge 1.$ Therefore $D_{f+g} = D_{fg}: x \ge 1.$
$R_f = R_g: y \ge 0, R_{f+g}: y \ge \sqrt{2}, R_{fg}: y \ge 0$

3. $D_f: -\infty < x < \infty, D_g: -\infty < x < \infty, D_{f/g}: -\infty < x < \infty, D_{g/f}: -\infty < x < \infty, R_f: y = 2, R_g: y \ge 1, R_{f/g}: 0 < y \le 2,$
$R_{g/f}: \frac{1}{2} \le y < \infty$

4. $D_f: -\infty < x < \infty, D_g: x \ge 0, D_{f/g}: x \ge 0, D_{g/f}: x \ge 0; R_f: y = 1, R_g: y \ge 1, R_{f/g}: 0 < y \le 1, R_{g/f}: 1 \le y < \infty$

5. (a) 2 (b) 22 (c) x^2+2
(d) $(x+5)^2-3 = x^2+10x+22$ (e) 5 (f) -2
(g) $x+10$ (h) $(x^2-3)^2-3 = x^4-6x^2+6$

6. (a) $-\frac{1}{3}$ (b) 2 (c) $\frac{1}{x+1}-1 = \frac{-x}{x+1}$
(d) $\frac{1}{x}$ (e) 0 (f) $\frac{3}{4}$
(g) $x-2$ (h) $\frac{1}{\frac{1}{x+1}+1} = \frac{1}{\frac{x+2}{x+1}} = \frac{x+1}{x+2}$

7. $(f\circ g\circ h)(x) = f(g(h(x))) = f(g(4-x)) = f(3(4-x)) = f(12-3x) = (12-3x)+1 = 13-3x$

8. $(f\circ g\circ h)(x) = f(g(h(x))) = f(g(x^2)) = f(2(x^2)-1) = f(2x^2-1) = 3(2x^2-1)+4 = 6x^2+1$

9. $(f\circ g\circ h)(x) = f(g(h(x))) = f\left(g\left(\frac{1}{x}\right)\right) = f\left(\frac{1}{\frac{1}{x}+4}\right) = f\left(\frac{x}{1+4x}\right) = \sqrt{\frac{x}{1+4x}+1} = \sqrt{\frac{5x+1}{1+4x}}$

10. $(f\circ g\circ h)(x) = f(g(h(x))) = f\left(g\left(\sqrt{2-x}\right)\right) = f\left(\frac{\left(\sqrt{2-x}\right)^2}{\left(\sqrt{2-x}\right)^2+1}\right) = f\left(\frac{2-x}{3-x}\right) = \frac{\frac{2-x}{3-x}+2}{3-\frac{2-x}{3-x}} = \frac{8-3x}{7-2x}$

11. (a) $(f\circ g)(x)$ (b) $(j\circ g)(x)$ (c) $(g\circ g)(x)$
(d) $(j\circ j)(x)$ (e) $(g\circ h\circ f)(x)$ (f) $(h\circ j\circ f)(x)$

12. (a) $(f\circ j)(x)$ (b) $(g\circ h)(x)$ (c) $(h\circ h)(x)$
(d) $(f\circ f)(x)$ (e) $(j\circ g\circ f)(x)$ (f) $(g\circ f\circ h)(x)$

13.

	$g(x)$	$f(x)$	$(f\circ g)(x)$
(a)	$x-7$	$\sqrt{x}$	$\sqrt{x-7}$
(b)	$x+2$	$3x$	$3(x+2) = 3x+6$
(c)	x^2	$\sqrt{x-5}$	$\sqrt{x^2-5}$
(d)	$\frac{x}{x-1}$	$\frac{x}{x-1}$	$\frac{\frac{x}{x-1}}{\frac{x}{x-1}-1} = \frac{x}{x-(x-1)} = x$
(e)	$\frac{1}{x-1}$	$1+\frac{1}{x}$	x

(f) $\frac{1}{x}$ $\frac{1}{x}$ x

14. (a) $(f\circ g)(x)=|g(x)|=\frac{1}{|x-1|}$.
(b) $(f\circ g)(x)=\frac{g(x)-1}{g(x)}=\frac{x}{x+1}\Rightarrow 1-\frac{1}{g(x)}=\frac{x}{x+1}\Rightarrow 1-\frac{x}{x+1}=\frac{1}{g(x)}\Rightarrow \frac{1}{x+1}=\frac{1}{g(x)}$, so $g(x)=x+1$.
(c) Since $(f\circ g)(x)=\sqrt{g(x)}=|x|$, $g(x)=x^2$.
(d) Since $(f\circ g)(x)=f\left(\sqrt{x}\right)=|x|$, $f(x)=x^2$. (Note that the domain of the composition is $[0,\infty)$.)

The completed table is shown. Note that the absolute value sign in part (d) is optional.

$g(x)$	$f(x)$	$(f\circ g)(x)$
$\frac{1}{x-1}$	$\lvert x\rvert$	$\frac{1}{\lvert x-1\rvert}$
$x+1$	$\frac{x-1}{x}$	$\frac{x}{x+1}$
x^2	$\sqrt{x}$	$\lvert x\rvert$
$\sqrt{x}$	x^2	$\lvert x\rvert$

15. (a) $f(g(-1))=f(1)=1$ (b) $g(f(0))=g(-2)=2$ (c) $f(f(-1))=f(0)=-2$
(d) $g(g(2))=g(0)=0$ (e) $g(f(-2))=g(1)=-1$ (f) $f(g(1))=f(-1)=0$

16. (a) $f(g(0))=f(-1)=2-(-1)=3$, where $g(0)=0-1=-1$
(b) $g(f(3))=g(-1)=-(-1)=1$, where $f(3)=2-3=-1$
(c) $g(g(-1))=g(1)=1-1=0$, where $g(-1)=-(-1)=1$
(d) $f(f(2))=f(0)=2-0=2$, where $f(2)=2-2=0$
(e) $g(f(0))=g(2)=2-1=1$, where $f(0)=2-0=2$
(f) $f\left(g\left(\frac{1}{2}\right)\right)=f\left(-\frac{1}{2}\right)=2-\left(-\frac{1}{2}\right)=\frac{5}{2}$, where $g\left(\frac{1}{2}\right)=\frac{1}{2}-1=-\frac{1}{2}$

17. (a) $(f\circ g)(x)=f(g(x))=\sqrt{\frac{1}{x}+1}=\sqrt{\frac{1+x}{x}}$
$(g\circ f)(x)=g(f(x))=\frac{1}{\sqrt{x+1}}$
(b) Domain $(f\circ g)$: $(-\infty,-1]\cup(0,\infty)$, domain $(g\circ f)$: $(-1,\infty)$
(c) Range $(f\circ g)$: $(1,\infty)$, range $(g\circ f)$: $(0,\infty)$

18. (a) $(f\circ g)(x)=f(g(x))=1-2\sqrt{x}+x$
$(g\circ f)(x)=g(f(x))=1-|x|$
(b) Domain $(f\circ g)$: $[0,\infty)$, domain $(g\circ f)$: $(-\infty,\infty)$
(c) Range $(f\circ g)$: $(0,\infty)$, range $(g\circ f)$: $(-\infty,1]$

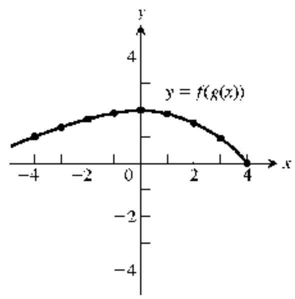
19. $(f\circ g)(x)=x\Rightarrow f(g(x))=x\Rightarrow \frac{g(x)}{g(x)-2}=x\Rightarrow g(x)=(g(x)-2)x=x\cdot g(x)-2x$
$\Rightarrow g(x)-x\cdot g(x)=-2x\Rightarrow g(x)=-\frac{2x}{1-x}=\frac{2x}{x-1}$

20. $(f\circ g)(x)=x+2\Rightarrow f(g(x))=x+2\Rightarrow 2(g(x))^3-4=x+2\Rightarrow (g(x))^3=\frac{x+6}{2}\Rightarrow g(x)=\sqrt[3]{\frac{x+6}{2}}$

21. $V=V(s)=V(s(t))=V(2t-3)$
$=(2t-3)^2+2(2t-3)+3$
$=4t^2-8t+6$

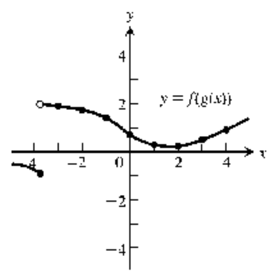
22. (a)

x	–4	–3	–2	–1	0	1	2	3	4
$g(x)$	–2	–1	–0.5	–0.2	0	0.2	0.5	1	2
$f(g(x))$	1	1.3	1.6	1.8	2	1.8	1.5	1	0



(b)

x	–4	–3	–2	–1	0	1	2	3	4
$g(x)$	1.5	0.3	–0.7	–1.5	–2.4	–2.8	–3	–2.7	–2
$f(g(x))$	–0.8	1.9	1.7	1.5	0.7	0.3	0.2	0.5	0.9



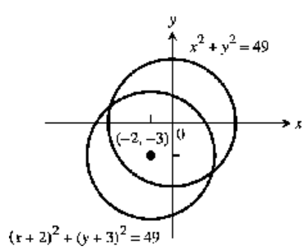
23. (a) $y=-(x+7)^2$ (b) $y=-(x-4)^2$

24. (a) $y=x^2+3$ (b) $y=x^2-5$

25. (a) Position 4 (b) Position 1 (c) Position 2 (d) Position 3

26. (a) $y=-(x-1)^2+4$ (b) $y=-(x+2)^2+3$ (c) $y=-(x+4)^2-1$ (d) $y=-(x-2)^2$

27.



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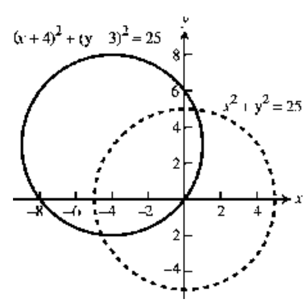

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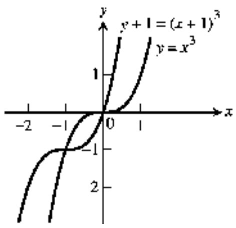

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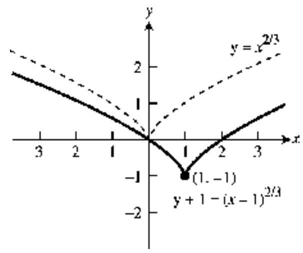

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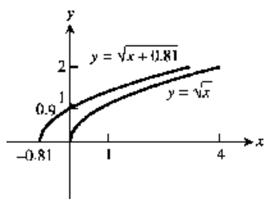

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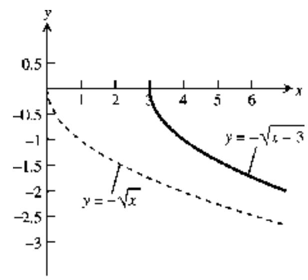

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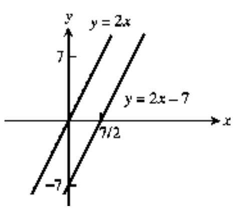

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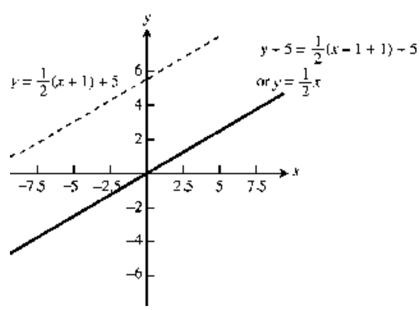

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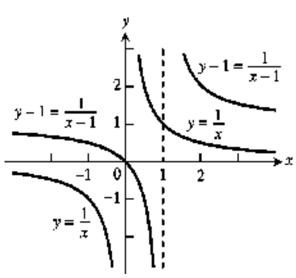

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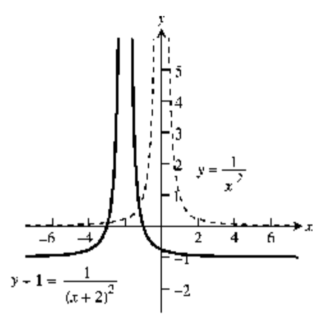

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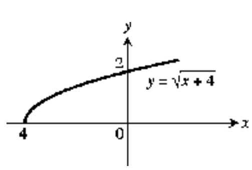

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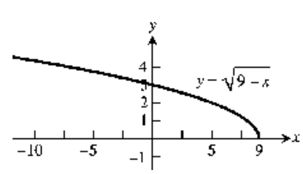

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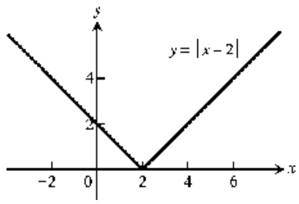

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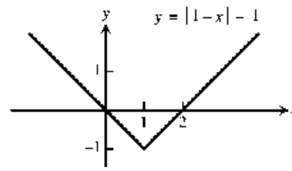

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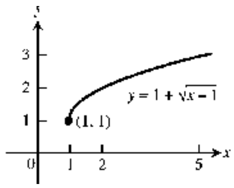

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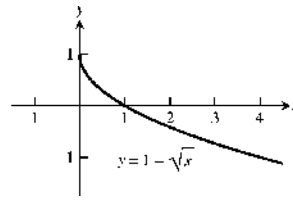

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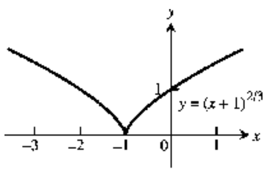

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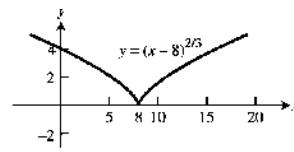

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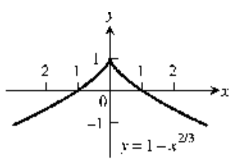

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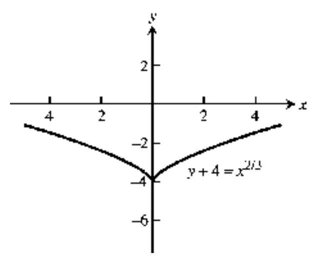

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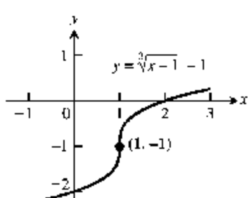

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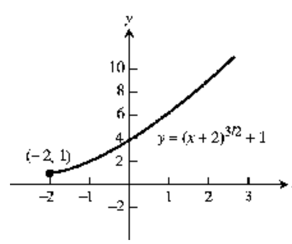

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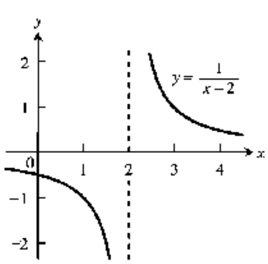

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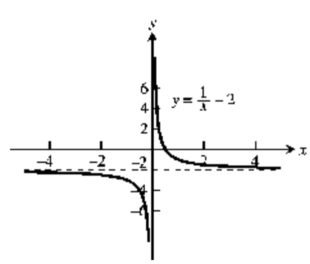

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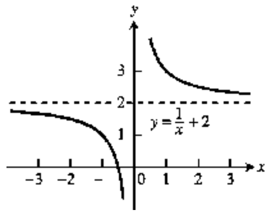

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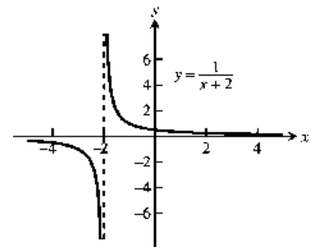

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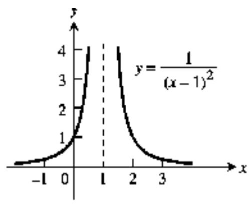

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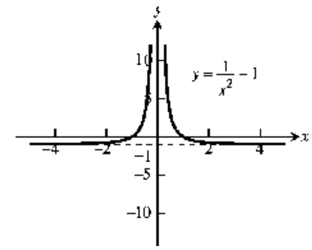

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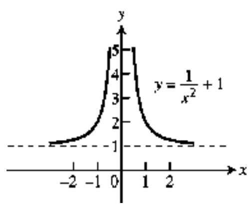

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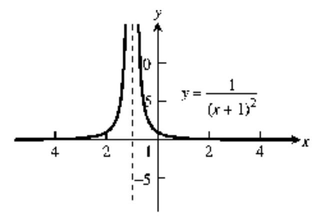
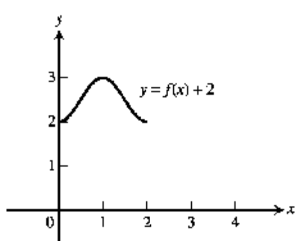
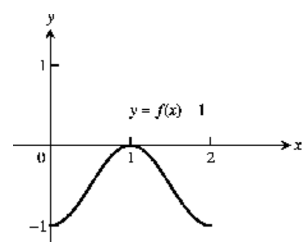


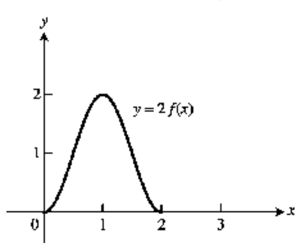
57. (a) domain: [0, 2]; range: [2, 3]



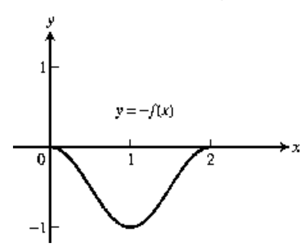
(b) domain: [0, 2]; range: [–1, 0]



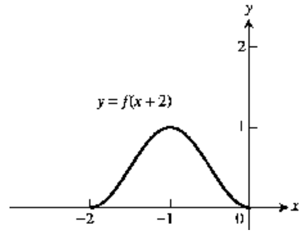
(c) domain: [0, 2]; range: [0, 2]



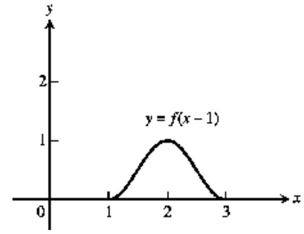
(d) domain: [0, 2]; range: [–1, 0]



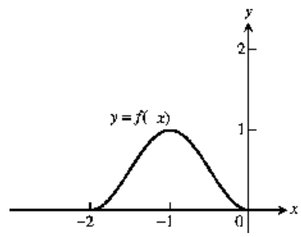
(e) domain: [–2, 0]; range: [0, 1]



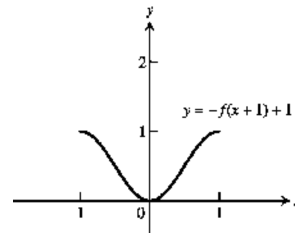
(f) domain: [1, 3]; range: [0,1]



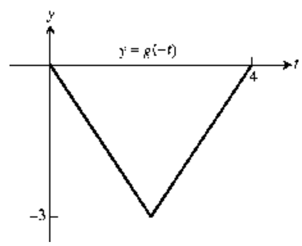
(g) domain: [–2, 0]; range: [0, 1]



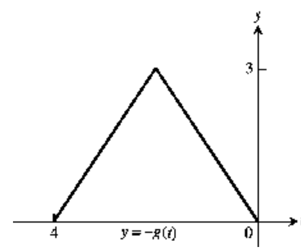
(h) domain: [–1, 1]; range: [0, 1]



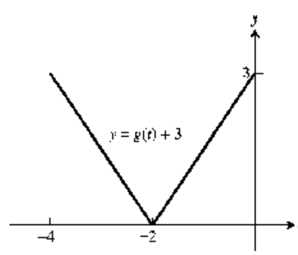
58. (a) domain: [0, 4]; range: [–3, 0]



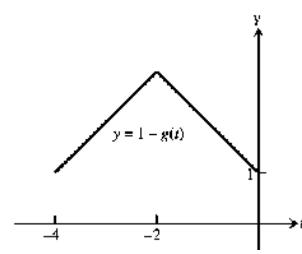
(b) domain: [–4, 0]; range: [0, 3]



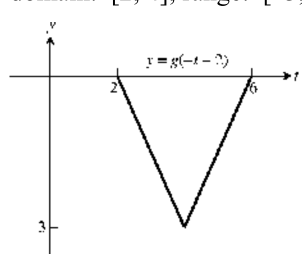
(c) domain: [–4, 0]; range: [0, 3]



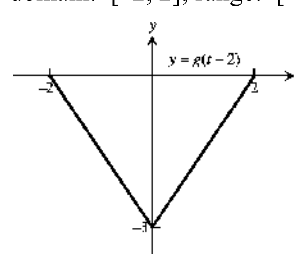
(d) domain: [–4, 0]; range: [1, 4]



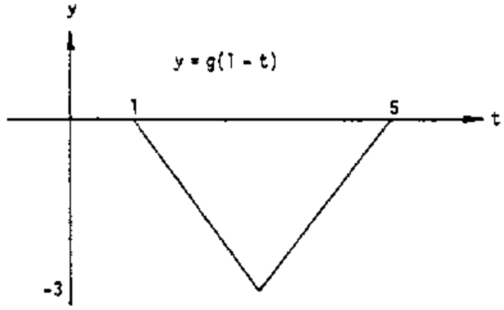
(e) domain: [2, 4]; range: [–3, 0]



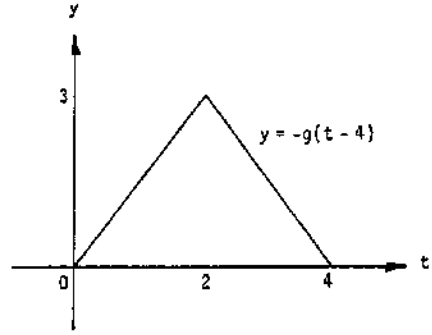
(f) domain: [–2, 2]; range: [–3, 0]



(g) domain: [1, 5]; range: [–3, 0]



(h) domain: [0, 4]; range: [0, 3]



59. $y = 3x^2 - 3$

60. $y = (2x)^2 - 1 = 4x^2 - 1$

61. $y = \frac{1}{2}\left(1 + \frac{1}{x^2}\right) = \frac{1}{2} + \frac{1}{2x^2}$

62. $y = 1 + \frac{1}{(x/3)^2} = 1 + \frac{9}{x^2}$

63. $y = \sqrt{4x+1}$

64. $y = 3\sqrt{x+1}$

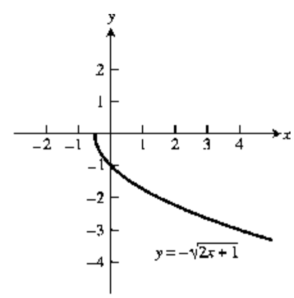
65. $y = \sqrt{4 - \left(\frac{x}{2}\right)^2} = \frac{1}{2}\sqrt{16 - x^2}$

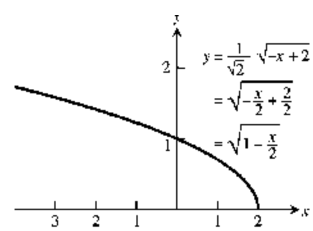
66. $y = \frac{1}{3}\sqrt{4 - x^2}$

67. $y = 1 - (3x)^3 = 1 - 27x^3$

68. $y = 1 - \left(\frac{x}{2}\right)^3 = 1 - \frac{x^3}{8}$

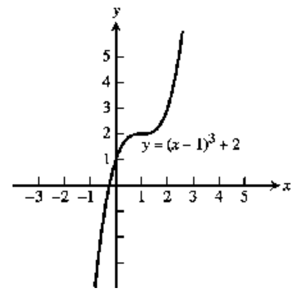
69. Let $y = -\sqrt{2x+1} = f(x)$ and let $g(x) = x^{1/2}$, $h(x) = \left(x + \frac{1}{2}\right)^{1/2}$, $i(x) = \sqrt{2}\left(x + \frac{1}{2}\right)^{1/2}$, and $j(x) = -\left[\sqrt{2}\left(x + \frac{1}{2}\right)^{1/2}\right] = f(x)$. The graph of $h(x)$ is the graph of $g(x)$ shifted left $\frac{1}{2}$ unit; the graph of $i(x)$ is the graph of $h(x)$ stretched vertically by a factor of $\sqrt{2}$; and the graph of $j(x) = f(x)$ is the graph of $i(x)$ reflected across the x-axis.



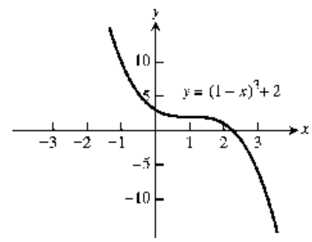


70. Let $y = \sqrt{1 - \frac{x}{2}} = f(x)$. Let $g(x) = (-x)^{1/2}$, $h(x) = (-x+2)^{1/2}$, and $i(x) = \frac{1}{\sqrt{2}}(-x+2)^{1/2} = \sqrt{1 - \frac{x}{2}} = f(x)$. The graph of $g(x)$ is the graph of $y = \sqrt{x}$ reflected across the x-axis. The graph of $h(x)$ is the graph of $g(x)$ shifted right two units. And the graph of $i(x)$ is the graph of $h(x)$ compressed vertically by a factor of $\sqrt{2}$.

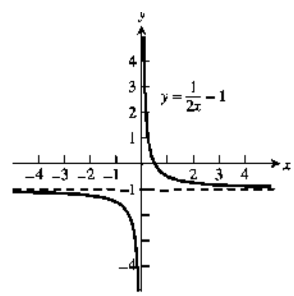
71. $y = f(x) = x^3$. Shift $f(x)$ one unit right followed by a shift two units up to get $g(x) = (x-1)^3 + 2$.



72. $y = (1-x)^3 + 2 = -[(x-1)^3 + (-2)] = f(x)$.
Let $g(x) = x^3$, $h(x) = (x-1)^3$, $i(x) = (x-1)^3 + (-2)$, and $j(x) = -[(x-1)^3 + (-2)]$. The graph of $h(x)$ is the graph of $g(x)$ shifted right one unit; the graph of $i(x)$ is the graph of $h(x)$ shifted down two units; and the graph of $f(x)$ is the graph of $i(x)$ reflected across the x-axis.

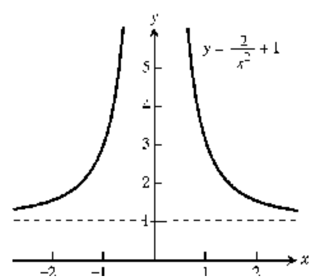


73. Compress the graph of $f(x) = \frac{1}{x}$ horizontally by a factor of 2 to get $g(x) = \frac{1}{2x}$. Then shift $g(x)$ vertically down 1 unit to get $h(x) = \frac{1}{2x} - 1$.

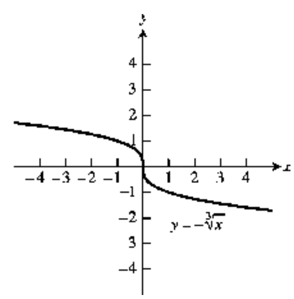


74. Let $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{2}{x^2} + 1 = \frac{1}{\left(\frac{x^2}{2}\right)} + 1$

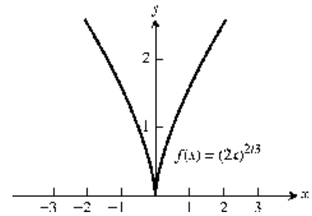
$= \frac{1}{\left(x/\sqrt{2}\right)^2} + 1 = \frac{1}{\left[\left(1/\sqrt{2}\right)x\right]^2} + 1.$ Since $\sqrt{2} \approx 1.4$, we see that the graph of $f(x)$ stretched horizontally by a factor of 1.4 and shifted up 1 unit is the graph of $g(x)$.



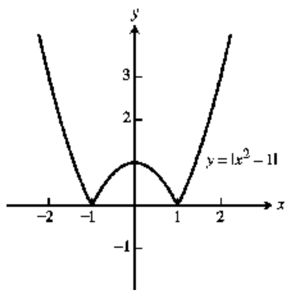
75. Reflect the graph of $y = f(x) = \sqrt[3]{x}$ across the x-axis to get $g(x) = -\sqrt[3]{x}$.



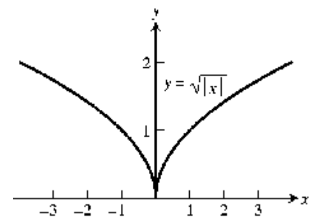
76. $y = f(x) = (-2x)^{2/3} = [(-1)(2)x]^{2/3} = (-1)^{2/3}(2x)^{2/3}$ $= (2x)^{2/3}$. So the graph of $f(x)$ is the graph of $g(x) = x^{2/3}$ compressed horizontally by a factor of 2.



77.



78.



79. (a) $(fg)(-x) = f(-x)g(-x) = f(x)(-g(x)) = -(fg)(x)$, odd

(b) $\left(\frac{f}{g}\right)(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\left(\frac{f}{g}\right)(x)$, odd

(c) $\left(\frac{g}{f}\right)(-x) = \frac{g(-x)}{f(-x)} = \frac{-g(x)}{f(x)} = -\left(\frac{g}{f}\right)(x)$, odd

(d) $f^2(-x) = f(-x)f(-x) = f(x)f(x) = f^2(x)$, even

(e) $g^2(-x) = (g(-x))^2 = (-g(x))^2 = g^2(x)$, even

(f) $(f \circ g)(-x) = f(g(-x)) = f(-g(x)) = f(g(x)) = (f \circ g)(x)$, even

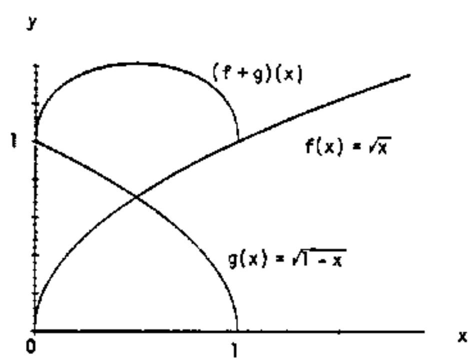
(g) $(g \circ f)(-x) = g(f(-x)) = g(f(x)) = (g \circ f)(x)$, even

(h) $(f \circ f)(-x) = f(f(-x)) = f(f(x)) = (f \circ f)(x)$, even

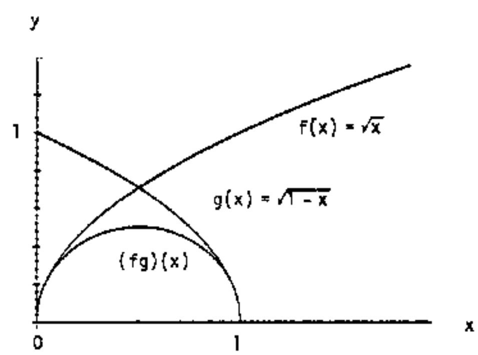
(i) $(g \circ g)(-x) = g(g(-x)) = g(-g(x)) = -g(g(x)) = -(g \circ g)(x)$, odd

80. Yes, $f(x) = 0$ is both even and odd since $f(-x) = 0 = f(x)$ and $f(-x) = 0 = -f(x)$.

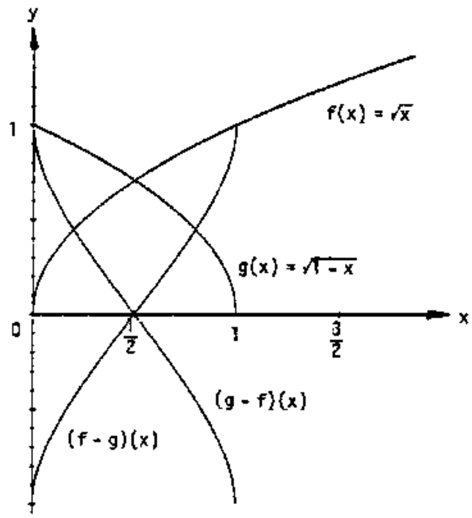
81. (a)



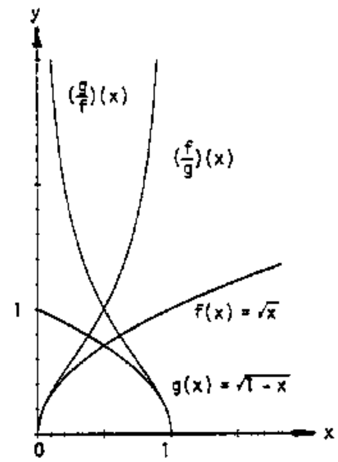
(b)



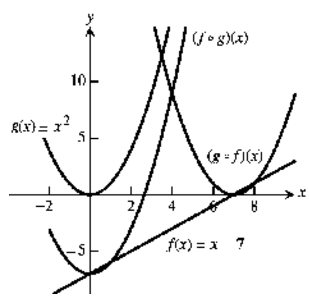
(c)



(d)



82.



1.3 TRIGONOMETRIC FUNCTIONS

1. (a) $s = r\theta = (10)\left(\frac{4\pi}{5}\right) = 8\pi$ m (b) $s = r\theta = (10)(110°)\left(\frac{\pi}{180°}\right) = \frac{110\pi}{18} = \frac{55\pi}{9}$ m

2. $\theta = \frac{s}{r} = \frac{10\pi}{8} = \frac{5\pi}{4}$ radians and $\frac{5\pi}{4}\left(\frac{180°}{\pi}\right) = 225°$

3. $\theta = 80° \Rightarrow \theta = 80°\left(\frac{\pi}{180°}\right) = \frac{4\pi}{9} \Rightarrow s = (6)\left(\frac{4\pi}{9}\right) = 8.4$ in. (since the diameter = 12 in. $\Rightarrow$ radius = 6 in.)

4. $d = 1$ meter $\Rightarrow r = 50$ cm $\Rightarrow \theta = \frac{s}{r} = \frac{30}{50} = 0.6$ rad or $0.6\left(\frac{180°}{\pi}\right) \approx 34°$

5.

θ	$-\pi$	$-\frac{2\pi}{3}$	0	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$\sin\theta$	0	$-\frac{\sqrt{3}}{2}$	0	1	$\frac{1}{\sqrt{2}}$
$\cos\theta$	−1	$-\frac{1}{2}$	1	0	$-\frac{1}{\sqrt{2}}$
$\tan\theta$	0	$\sqrt{3}$	0	und.	−1
$\cot\theta$	und.	$\frac{1}{\sqrt{3}}$	und.	0	−1
$\sec\theta$	−1	−2	1	und.	$-\sqrt{2}$
$\csc\theta$	und.	$-\frac{2}{\sqrt{3}}$	und.	1	$\sqrt{2}$

6.

θ	$-\frac{3\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{5\pi}{6}$
$\sin\theta$	1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\cos\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$
$\tan\theta$	und.	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	1	$-\frac{1}{\sqrt{3}}$
$\cot\theta$	0	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	1	$-\sqrt{3}$
$\sec\theta$	und.	2	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	$-\frac{2}{\sqrt{3}}$
$\csc\theta$	1	$-\frac{2}{\sqrt{3}}$	−2	$\sqrt{2}$	2

7. $\cos x = -\frac{4}{5}$, $\tan x = -\frac{3}{4}$

8. $\sin x = \frac{2}{\sqrt{5}}$, $\cos x = \frac{1}{\sqrt{5}}$

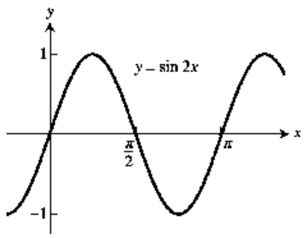
9. $\sin x = -\frac{\sqrt{8}}{3}$, $\tan x = -\sqrt{8}$

10. $\sin x = \frac{12}{13}$, $\tan x = -\frac{12}{5}$

11. $\sin x = -\frac{1}{\sqrt{5}}$, $\cos x = -\frac{2}{\sqrt{5}}$

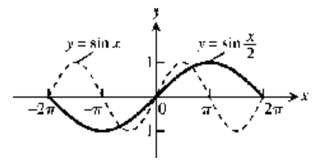
12. $\cos x = -\frac{\sqrt{3}}{2}$, $\tan x = \frac{1}{\sqrt{3}}$

13.



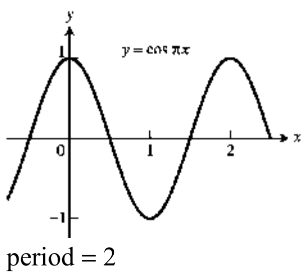
period = π

14.



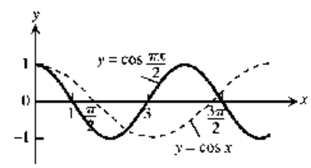
period = 4π

15.



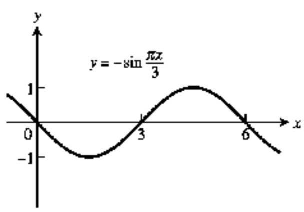
period = 2

16.



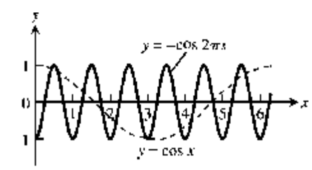
period = 4

17.



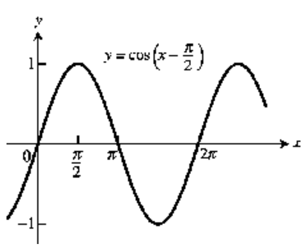
period = 6

18.



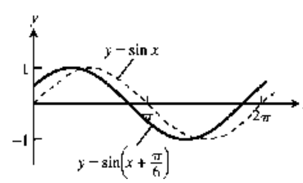
period = 1

19.



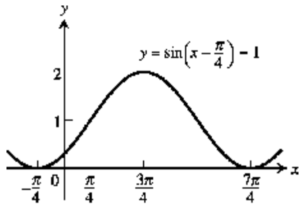
period = 2π

20.



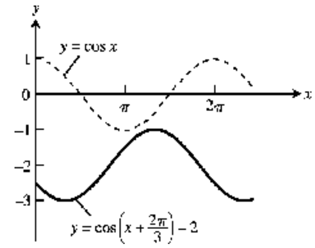
period = 2π

21.



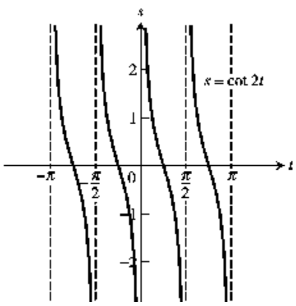
period $= 2\pi$

22.

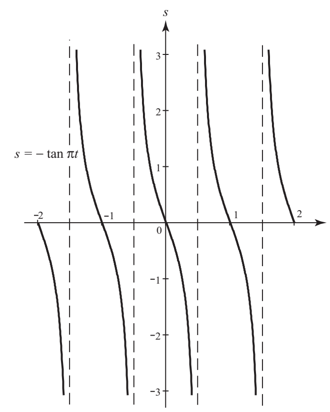


period $= 2\pi$

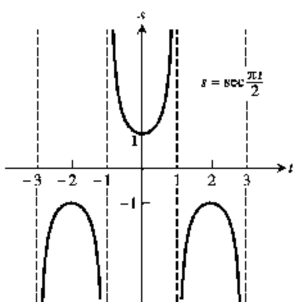
23. period $= \frac{\pi}{2}$, symmetric about the origin



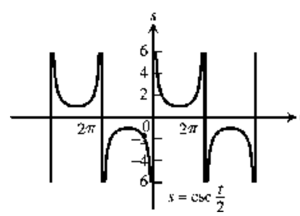
24. period $= 1$, symmetric about the origin



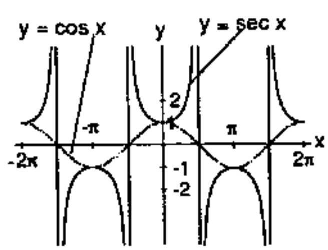
25. period $= 4$, symmetric about the s-axis



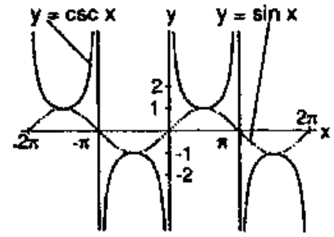
26. period $= 4\pi$, symmetric about the origin



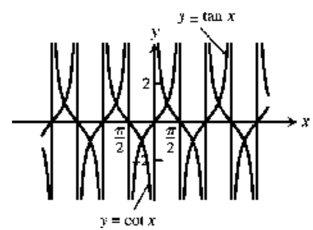
27. (a) Cos x and sec x are positive for x in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$; and cos x and sec x are negative for x in the intervals $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right)$ and $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$. Sec x is undefined when cos x is 0. The range of sec x is $(-\infty, -1] \cup [1, \infty)$; the range of cos x is $[-1, 1]$.



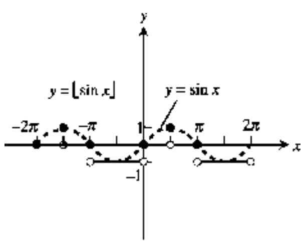
(b) Sin x and csc x are positive for x in the intervals $\left(-\frac{3\pi}{2}, -\pi\right)$ and $(0, \pi)$; and sin x and csc x are negative for x in the intervals $(-\pi, 0)$ and $\left(\pi, \frac{3\pi}{2}\right)$. Csc x is undefined when sin x is 0. The range of csc x is $(-\infty, -1] \cup [1, \infty)$; the range of sin x is $[-1, 1]$.



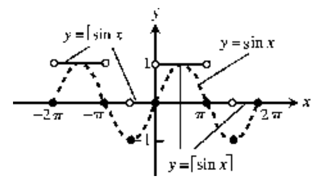
28. Since $\cot x = \frac{1}{\tan x}$, cot x is undefined when tan $x = 0$ and is zero when tan x is undefined. As tan x approaches zero through positive values, cot x approaches infinity. Also, cot x approaches negative infinity as tan x approaches zero through negative values.



29. $D: -\infty < x < \infty;\ R:\ y = -1, 0, 1$



30. $D: -\infty < x < \infty;\ R:\ y = -1, 0, 1$



31. $\cos\left(x - \frac{\pi}{2}\right) = \cos x \cos\left(-\frac{\pi}{2}\right) - \sin x \sin\left(-\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(-1) = \sin x$

32. $\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos\left(\frac{\pi}{2}\right) - \sin x \sin\left(\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(1) = -\sin x$

33. $\sin\left(x + \frac{\pi}{2}\right) = \sin x \cos\left(\frac{\pi}{2}\right) + \cos x \sin\left(\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(1) = \cos x$

34. $\sin\left(x - \frac{\pi}{2}\right) = \sin x \cos\left(-\frac{\pi}{2}\right) + \cos x \sin\left(-\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(-1) = -\cos x$

35. $\cos(A - B) = \cos(A + (-B)) = \cos A \cos(-B) - \sin A \sin(-B) = \cos A \cos B - \sin A(-\sin B)$
$= \cos A \cos B + \sin A \sin B$

36. $\sin(A - B) = \sin(A + (-B)) = \sin A \cos(-B) + \cos A \sin(-B) = \sin A \cos B + \cos A(-\sin B)$
$= \sin A \cos B - \cos A \sin B$

37. If $B = A$, $A - B = 0 \Rightarrow \cos(A - B) = \cos 0 = 1$. Also $\cos(A - B) = \cos(A - A) = \cos A \cos A + \sin A \sin A$ $= \cos^2 A + \sin^2 A$. Therefore, $\cos^2 A + \sin^2 A = 1$.

38. If $B = 2\pi$, then $\cos(A + 2\pi) = \cos A \cos 2\pi - \sin A \sin 2\pi = (\cos A)(1) - (\sin A)(0) = \cos A$ and $\sin(A + 2\pi) = \sin A \cos 2\pi + \cos A \sin 2\pi = (\sin A)(1) + (\cos A)(0) = \sin A$. The result agrees with the fact that the cosine and sine functions have period 2π.

39. $\cos(\pi + x) = \cos\pi \cos x - \sin\pi \sin x = (-1)(\cos x) - (0)(\sin x) = -\cos x$