INSTRUCTOR'S SOLUTIONS MANUAL

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ALGEBRA & TRIGONOMETRY

GRAPHS AND MODELS

SIXTH EDITION

PRECALCULUS

GRAPHS AND MODELS, A RIGHT TRIANGLE APPROACH

SIXTH EDITION

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Contents

Just-in-Time Review

1. Real Numbers

1. Rational numbers: $\frac{2}{3}$, 6, -2.45, $18.\overline{4}$, -11, $\sqrt[3]{27}$, $5\frac{1}{6}$, $-\frac{8}{7}$, 0, $\sqrt{16}$

2. Rational numbers but not integers: $\frac{2}{3}$, -2.45, $18.\overline{4}$, $5\frac{1}{6}$, $-\frac{8}{7}$

3. Irrational numbers: $\sqrt{3}$, $\sqrt[6]{26}$, $7.151551555\ldots$, $-\sqrt{35}$, $\sqrt[5]{3}$ (Although there is a pattern in $7.151551555\ldots$, there is no repeating block of digits.)

4. Integers: 6, -11, $\sqrt[3]{27}$, 0, $\sqrt{16}$

5. Whole numbers: 6, $\sqrt[3]{27}$, 0, $\sqrt{16}$

6. Real numbers: All of them

2. Properties of Real Numbers

1. $-24+24=0$ illustrates the additive inverse property.

2. $7(xy)=(7x)y$ illustrates the associative property of multiplication.

3. $9(r-s)=9r-9s$ illustrates a distributive property.

4. $11+z=z+11$ illustrates the commutative property of addition.

5. $-20\cdot 1=-20$ illustrates the multiplicative identity property.

6. $5(x+y)=(x+y)5$ illustrates the commutative property of multiplication.

7. $q+0=q$ illustrates the additive identity property.

8. $75\cdot\frac{1}{75}=1$ illustrates the multiplicative inverse property.

9. $(x+y)+w=x+(y+w)$ illustrates the associative property of addition.

10. $8(a+b)=8a+8b$ illustrates a distributive property.

3. Order on the Number Line

1. 9 is to the right of -9 on the number line, so it is false that $9<-9$.

2. -10 is to the left of -1 on the number line, so it is true that $-10\le -1$.

3. $-5=-\sqrt{25}$, and $-\sqrt{26}$ is to the left of $-\sqrt{25}$, or -5, on the number line. Thus it is true that $-\sqrt{26}<-5$.

4. $\sqrt{6}=\sqrt{6}$, so it is true that $\sqrt{6}\le\sqrt{6}$.

5. -30 is to the left of -25 on the number line, so it is false that $-30>-25$.

6. $-\frac{4}{5}=-\frac{16}{20}$ and $-\frac{5}{4}=-\frac{25}{20}$; $-\frac{16}{20}$ is to the right of $-\frac{25}{20}$, so it is true that $-\frac{4}{5}>-\frac{5}{4}$.

4. Absolute Value

1. $|-98|=98 \quad (|a|=-a$, if $a<0.)$

2. $|0|=0 \quad (|a|=a$, if $a\ge 0.)$

3. $|4.7|=4.7 \quad (|a|=a$, if $a\ge 0.)$

4. $\left|-\frac{2}{3}\right|=\frac{2}{3} \quad (|a|=-a$, if $a<0.)$

5. $|-7-13|=|-20|=20$, or
$|13-(-7)|=|13+7|=|20|=20$

6. $|2-14.6|=|-12.6|=12.6$, or
$|14.6-2|=|12.6|=12.6$

7. $|-39-(-28)|=|-39+28|=|-11|=11$, or
$|-28-(-39)|=|-28+39|=|11|=11$

8. $\left|-\frac{3}{4}-\frac{15}{8}\right|=\left|-\frac{6}{8}-\frac{15}{8}\right|=\left|-\frac{21}{8}\right|=\frac{21}{8}$, or
$\left|\frac{15}{8}-\left(-\frac{3}{4}\right)\right|=\left|\frac{15}{8}+\frac{6}{8}\right|=\left|\frac{21}{8}\right|=\frac{21}{8}$

5. Operations with Real Numbers

1. $8-(-11)=8+11=19$

2. $-\frac{3}{10}\cdot\left(-\frac{1}{3}\right)=\frac{3\cdot 1}{10\cdot 3}=\frac{3}{3}\cdot\frac{1}{10}=1\cdot\frac{1}{10}=\frac{1}{10}$

3. $15\div(-3)=-5$

4. $-4-(-1)=-4+1=-3$

5. $7\cdot(-50)=-350$

6. $-0.5-5=-0.5+(-5)=-5.5$

7. $-3+27=24$

8. $-400\div -40=10$

9. $4.2\cdot(-3)=-12.6$

10. $-13-(-33)=-13+33=20$

11. $-60+45=-15$

12. $\frac{1}{2}-\frac{2}{3}=\frac{1}{2}+\left(-\frac{2}{3}\right)=\frac{3}{6}+\left(-\frac{4}{6}\right)=-\frac{1}{6}$

13. $-24\div 3=-8$

14. $-6+(-16)=-22$

15. $-\frac{1}{2}\div\left(-\frac{5}{8}\right)=-\frac{1}{2}\cdot\left(-\frac{8}{5}\right)=\frac{1\cdot 8}{2\cdot 5}=\frac{1\cdot \not{2}\cdot 4}{\not{2}\cdot 5}=\frac{4}{5}$

6. Interval Notation

1. This is a closed interval, so we use brackets. Interval notation is $[-5,5]$.
2. This is a half-open interval. We use a parenthesis on the left and a bracket on the right. Interval notation is $(-3,-1]$.
3. This interval is of unlimited extent in the negative direction, and the endpoint -2 is included. Interval notation is $(-\infty,-2]$.
4. This interval is of unlimited extent in the positive direction, and the endpoint 3.8 is not included. Interval notation is $(3.8,\infty)$.
5. $\{x|7<x\}$, or $\{x|x>7\}$.

   This interval is of unlimited extent in the positive direction and the endpoint 7 is not included. Interval notation is $(7,\infty)$.
6. The endpoints -2 and 2 are not included in the interval, so we use parentheses. Interval notation is $(-2,2)$.
7. The endpoints -4 and 5 are not included in the interval, so we use parentheses. Interval notation is $(-4,5)$.
8. The interval is of unlimited extent in the positive direction, and the endpoint 1.7 is included. Internal notation is $[1.7,\infty)$.
9. The endpoint -5 is not included in the interval, so we use a parenthesis before -5. The endpoint -2 is included in the interval, so we use a bracket after -2. Interval notation is $(-5,-2]$.
10. This interval is of unlimited extent in the negative direction, and the endpoint $\sqrt{5}$ is not included. Interval notation is $(-\infty,\sqrt{5})$.

7. Integers as Exponents

1. $3^{-6}=\frac{1}{3^6}$ Using $a^{-m}=\frac{1}{a^m}$

2. $\frac{1}{(0.2)^{-5}}=(0.2)^5$ Using $a^{-m}=\frac{1}{a^m}$

3. $\frac{w^{-4}}{z^{-9}}=\frac{z^9}{w^4}$ Using $\frac{a^{-m}}{b^{-n}}=\frac{b^n}{a^m}$

4. $\left(\frac{z}{y}\right)^2=\frac{z^2}{y^2}$ Raising a quotient to a power

5. $100^0=1$ Using $a^0=1,\ a\neq 0$

6. $\frac{a^5}{a^{-3}}=a^{5-(-3)}=a^{5+3}=a^8$ Using the quotient rule

7.
$$\begin{aligned}(2xy^3)(-3x^{-5}y)&=2(-3)x\cdot x^{-5}\cdot y^3\cdot y\\&=-6x^{1+(-5)}y^{3+1}\\&=-6x^{-4}y^4,\text{ or }-\frac{6y^4}{x^4}\end{aligned}$$

8. $x^{-4}\cdot x^{-7}=x^{-4+(-7)}=x^{-11}$, or $\frac{1}{x^{11}}$

9. $(mn)^{-6}=m^{-6}n^{-6}$, or $\frac{1}{m^6n^6}$

10. $(t^{-5})^4=t^{-5\cdot 4}=t^{-20}$, or $\frac{1}{t^{20}}$

8. Scientific Notation

1. Convert 18,500,000 to scientific notation.

   We want the decimal point to be positioned between the 1 and the 8, so we move it 7 places to the left. Since 18,500,000 is greater than 10, the exponent must be positive.

   $18,500,000=1.85\times 10^7$
2. Convert 0.000786 to scientific notation.

   We want the decimal point to be positioned between the 7 and the 8, so we move it 4 places to the right. Since 0.000786 is between 0 and 1, the exponent must be negative.

   $0.000786=7.86\times 10^{-4}$
3. Convert 0.0000000023 to scientific notation.

   We want the decimal point to be positioned between the 2 and the 3, so we move it 9 places to the right. Since 0.0000000023 is between 0 and 1, the exponent must be negative.

   $0.0000000023=2.3\times 10^{-9}$
4. Convert 8,927,000,000 to scientific notation.

   We want the decimal point to be positioned between the 8 and the 9, so we move it 9 places to the left. Since 8,927,000,000 is greater than 10, the exponent must be positive.

   $8,927,000,000=8.927\times 10^9$
5. Convert 4.3×10^{-8} to decimal notation.

   The exponent is negative, so the number is between 0 and 1. We move the decimal point 8 places to the left.

   $4.3\times 10^{-8}=0.000000043$

6. Convert 5.17×10^6 to decimal notation.

The exponent is positive, so the number is greater than 10. We move the decimal point 6 places to the right.

$5.17 \times 10^6 = 5,170,000$

7. Convert 6.203×10^{11} to decimal notation.

The exponent is positive, so the number is greater than 10. We move the decimal point 11 places to the right.

$6.203 \times 10^{11} = 620,300,000,000$

8. Convert 2.94×10^{-5} to scientific notation.

The exponent is negative, so the number is between 0 and 1. We move the decimal point 5 places to the left.

$2.94 \times 10^{-5} = 0.0000294$

9. Order of Operations

1. $3 + 18 \div 6 - 3 = 3 + 3 - 3$ Dividing

$= 6 - 3 = 3$ Adding and subtracting

2.
$= 5 \cdot 3 + 8 \cdot 3^2 + 4(6-2)$

$= 5 \cdot 3 + 8 \cdot 3^2 + 4 \cdot 4$ Working inside parentheses

$= 5 \cdot 3 + 8 \cdot 9 + 4 \cdot 4$ Evaluating 3^2

$= 15 + 72 + 16$ Multiplying

$= 87 + 16$ Adding in order

$= 103$ from left to right

3.
$5[3 - 8 \cdot 3^2 + 4 \cdot 6 - 2]$

$= 5[3 - 8 \cdot 9 + 4 \cdot 6 - 2]$

$= 5[3 - 72 + 24 - 2]$

$= 5[-69 + 24 - 2]$

$= 5[-45 - 2]$

$= 5[-47]$

$= -235$

4.
$16 \div 4 \cdot 4 \div 2 \cdot 256$

$= 4 \cdot 4 \div 2 \cdot 256$ Multiplying and dividing in order from left to right

$= 16 \div 2 \cdot 256$

$= 8 \cdot 256$

$= 2048$

5.
$2^6 \cdot 2^{-3} \div 2^{10} \div 2^{-8}$

$= 2^3 \div 2^{10} \div 2^{-8}$

$= 2^{-7} \div 2^{-8}$

$= 2$

6.
$\dfrac{4(8-6)^2 - 4 \cdot 3 + 2 \cdot 8}{3^1 + 19^0}$

$= \dfrac{4 \cdot 2^2 - 4 \cdot 3 + 2 \cdot 8}{3 + 1}$ Calculating in the numerator and in the denominator

$= \dfrac{4 \cdot 4 - 4 \cdot 3 + 2 \cdot 8}{4}$

$= \dfrac{16 - 12 + 16}{4}$

$= \dfrac{4 + 16}{4}$

$= \dfrac{20}{4}$

$= 5$

7. $64 \div [(-4) \div (-2)] = 64 \div 2 = 32$

8.
$6[9 - (3 - 2)] + 4(2 - 3)$

$= 6[9 - 1] + 4(2 - 3)$

$= 6 \cdot 8 + 4(-1)$

$= 48 - 4$

$= 44$

10. Introduction to Polynomials

1. $5 - x^6$

The term of highest degree is $-x^6$, so the degree of the polynomial is 6.

2. $x^2y^5 - x^7y + 4$

The degree of x^2y^5 is $2 + 5$, or 7; the degree of $-x^7y$ is $7 + 1$, or 8; the degree of 4 is 0 ($4 = 4x^0$). Thus the degree of the polynomial is 8.

3. $2a^4 - 3 + a^2$

The term of highest degree is $2a^4$, so the degree of the polynomial is 4.

4. $-41 = -41x^0$, so the degree of the polynomial is 0.

5. $4x - x^3 + 0.1x^8 - 2x^5$

The term of highest degree is $0.1x^8$, so the degree of the polynomial is 8.

6. $x - 3$ has two terms. It is a binomial.

7. $14y^5$ has one term. It is a monomial.

8. $2y - \dfrac{1}{4}y^2 + 8$ has three terms. It is a trinomial.

11. Add and Subtract Polynomials

1.
$(8y - 1) - (3 - y)$

$= (8y - 1) + (-3 + y)$

$= (8 + 1)y + (-1 - 3)$

$= 9y - 4$

2. $(3x^2 - 2x - x^3 + 2) - (5x^2 - 8x - x^3 + 4)$
$= (3x^2 - 2x - x^3 + 2) + (-5x^2 + 8x + x^3 - 4)$
$= (3 - 5)x^2 + (-2 + 8)x + (-1 + 1)x^3 + (2 - 4)$
$= -2x^2 + 6x - 2$

3. $(2x + 3y + z - 7) + (4x - 2y - z + 8) +$
$(-3x + y - 2z - 4)$
$= (2 + 4 - 3)x + (3 - 2 + 1)y + (1 - 1 - 2)z +$
$(-7 + 8 - 4)$
$= 3x + 2y - 2z - 3$

4. $(3ab^2 - 4a^2b - 2ab + 6) +$
$(-ab^2 - 5a^2b + 8ab + 4)$
$= (3 - 1)ab^2 + (-4 - 5)a^2b + (-2 + 8)ab + (6 + 4)$
$= 2ab^2 - 9a^2b + 6ab + 10$

5. $(5x^2 + 4xy - 3y^2 + 2) - (9x^2 - 4xy + 2y^2 - 1)$
$= (5x^2 + 4xy - 3y^2 + 2) + (-9x^2 + 4xy - 2y^2 + 1)$
$= (5 - 9)x^2 + (4 + 4)xy + (-3 - 2)y^2 + (2 + 1)$
$= -4x^2 + 8xy - 5y^2 + 3$

12. Multiply Polynomials

1. $(3a^2)(-7a^4) = [3(-7)](a^2 \cdot a^4)$
$= -21a^6$

2. $(y - 3)(y + 5)$
$= y^2 + 5y - 3y - 15$ Using FOIL
$= y^2 + 2y - 15$ Collecting like terms

3. $(x + 6)(x + 3)$
$= x^2 + 3x + 6x + 18$ Using FOIL
$= x^2 + 9x + 18$ Collecting like terms

4. $(2a + 3)(a + 5)$
$= 2a^2 + 10a + 3a + 15$ Using FOIL
$= 2a^2 + 13a + 15$ Collecting like terms

5. $(2x + 3y)(2x + y)$
$= 4x^2 + 2xy + 6xy + 3y^2$ Using FOIL
$= 4x^2 + 8xy + 3y^2$

6. $(11t - 1)(3t + 4)$
$= 33t^2 + 44t - 3t - 4$ Using FOIL
$= 33t^2 + 41t - 4$

13. Special Products of Binomials

1. $(x + 3)^2$
$= x^2 + 2 \cdot x \cdot 3 + 3^2$
$[(A + B)^2 = A^2 + 2AB + B^2]$
$= x^2 + 6x + 9$

2. $(5x - 3)^2$
$= (5x)^2 - 2 \cdot 5x \cdot 3 + 3^2$
$[(A - B)^2 = A^2 - 2AB + B^2]$
$= 25x^2 - 30x + 9$

3. $(2x + 3y)^2$
$= (2x)^2 + 2(2x)(3y) + (3y)^2$
$[(A+B)^2 = A^2+2AB+B^2]$
$= 4x^2 + 12xy + 9y^2$

4. $(a - 5b)^2$
$= a^2 - 2 \cdot a \cdot 5b + (5b)^2$
$[(A-B)^2 = A^2-2AB+B^2]$
$= a^2 - 10ab + 25b^2$

5. $(n + 6)(n - 6)$
$= n^2 - 6^2$ $[(A + B)(A - B) = A^2 - B^2]$
$= n^2 - 36$

6. $(3y + 4)(3y - 4)$
$= (3y)^2 - 4^2$ $[(A + B)(A - B) = A^2 - B^2]$
$= 9y^2 - 16$

14. Factor Polynomials; The FOIL Method

1. $3x + 18 = 3 \cdot x + 3 \cdot 6 = 3(x + 6)$

2. $2z^3 - 8z^2 = 2z^2 \cdot z - 2z^2 \cdot 4 = 2z^2(z - 4)$

3. $3x^3 - x^2 + 18x - 6$
$= x^2(3x - 1) + 6(3x - 1)$
$= (3x - 1)(x^2 + 6)$

4. $t^3 + 6t^2 - 2t - 12$
$= t^2(t + 6) - 2(t + 6)$
$= (t + 6)(t^2 - 2)$

5. $w^2 - 7w + 10$

We look for two numbers with a product of 10 and a sum of -7. By trial, we determine that they are -5 and -2.

$$w^2 - 7w + 10 = (w - 5)(w - 2)$$

6. $t^2 + 8t + 15$

We look for two numbers with a product of 15 and a sum of 8. By trial, we determine that they are 3 and 5.

$$t^2 + 8t + 15 = (t + 3)(t + 5)$$

7. $2n^2 - 20n - 48 = 2(n^2 - 10n - 24)$

Now factor $n^2 - 10n - 24$. We look for two numbers with a product of -24 and a sum of -10. By trial, we determine that they are 2 and -12. Then $n^2 - 10n - 24 = (n + 2)(n - 12)$. We must include the common factor, 2, to have a factorization of the original trinomial.

$$2n^2 - 20n - 48 = 2(n + 2)(n - 12)$$

8. $y^4 - 9y^3 + 14y^2 = y^2(y^2 - 9y + 14)$

Now factor $y^2 - 9y + 14$. Look for two numbers with a product of 14 and a sum of -9. The numbers are -2 and -7. Then $y^2 - 9y + 14 = (y-2)(y-7)$. We must include the common factor, y^2, in order to have a factorization of the original trinomial.

$$y^4 - 9y^3 + 14y^2 = y^2(y-2)(y-7)$$

9. $2n^2 + 9n - 56$

1. There is no common factor other than 1 or -1.
2. The factorization must be of the form $(2n+\quad)(n+\quad)$.
3. Factor the constant term, -56. The possibilities are $-1\cdot 56$, $1(-56)$, $-2\cdot 28$, $2(-28)$, $-4\cdot 16$, $4(-16)$, $-7\cdot 8$, and $7(-8)$. The factors can be written in the opposite order as well: $56(-1)$, $-56\cdot 1$, $28(-2)$, $-28\cdot 2$, $16(-4)$, $-16\cdot 4$, $8(-7)$, and $-8\cdot 7$.
4. Find a pair of factors for which the sum of the outer and the inner products is the middle term, $9n$. By trial, we determine that the factorization is $(2n-7)(n+8)$.

10. $2y^2 + y - 6$

1. There is no common factor other than 1 or -1.
2. The factorization must be of the form $(2y+\quad)(y+\quad)$.
3. Factor the constant term, -6. The possibilities are $-1\cdot 6$, $1(-6)$, $-2\cdot 3$, and $2(-3)$. The factors can be written in the opposite order as well: $6(-1)$, $-6\cdot 1$, $3(-2)$ and $-3\cdot 2$.
4. Find a pair of factors for which the sum of the outer and the inner products is the middle term, y. By trial, we determine that the factorization is $(2y-3)(y+2)$.

11. $b^2 - 6bt + 5t^2$

We look for two numbers with a product of 5 and a sum of -6. By trial, we determine that they are -1 and -5.

$$b^2 - 6bt + 5t^2 = (b-t)(b-5t)$$

12. $x^4 - 7x^2 - 30 = (x^2)^2 - 7x^2 - 30$

We look for two numbers with a product of -30 and a sum of -7. By trial, we determine that they are 3 and -10.

$$x^4 - 7x^2 - 30 = (x^2+3)(x^2-10)$$

15. Factoring Polynomials; The ac-Method

1. $8x^2 - 6x - 9$

1. There is no common factor other than 1 or -1.
2. Multiply the leading coefficient and the constant: $8(-9) = -72$.
3. Try to factor -72 so that the sum of the factors is the coefficient of the middle term, -6. The factors we want are -12 and 6.
4. Split the middle term using the numbers found in step (3):

   $-6x = -12x + 6x$
5. Factor by grouping.

$$\begin{aligned} 8x^2 - 6x - 9 &= 8x^2 - 12x + 6x - 9 \\ &= 4x(2x-3) + 3(2x-3) \\ &= (2x-3)(4x+3) \end{aligned}$$

2. $10t^2 + 4t - 6$

1. Factor out the largest common factor, 2.

   $10t^2 + 4t - 6 = 2(5t^2 + 2t - 3)$

   Now factor $5t^2 + 2t - 3$.
2. Multiply the leading coefficient and the constant: $5(-3) = -15$.
3. Try to factor -15 so that the sum of the factors is the coefficient of the middle term, 2. The factors we want are 5 and -3.
4. Split the middle term using the numbers found in step (3):

   $2t = 5t - 3t.$
5. Factor by grouping.

$$\begin{aligned} 5t^2 + 2t - 3 &= 5t^2 + 5t - 3t - 3 \\ &= 5t(t+1) - 3(t+1) \\ &= (t+1)(5t-3) \end{aligned}$$

Include the largest common factor in the final factorization.

$$10t^2 + 4t - 6 = 2(t+1)(5t-3)$$

3. $18a^2 - 51a + 15$

1. Factor out the largest common factor, 3.

   $18a^2 - 51a + 15 = 3(6a^2 - 17a + 5)$

   Now factor $6a^2 - 17a + 5$.
2. Multiply the leading coefficient and the constant: $6(5) = 30$.
3. Try to factor 30 so that the sum of the factors is the coefficient of the middle term, -17. The factors we want are -2 and -15.
4. Split the middle term using the numbers found in step (3):

   $-17a = -2a - 15a.$
5. Factor by grouping.

$$\begin{aligned} 6a^2 - 17a + 5 &= 6a^2 - 2a - 15a + 5 \\ &= 2a(3a-1) - 5(3a-1) \\ &= (3a-1)(2a-5) \end{aligned}$$

Include the largest common factor in the final factorization.

$$18a^2 - 51a + 15 = 3(3a-1)(2a-5)$$

16. Special Factorizations

1. $z^2-81=z^2-9^2=(z+9)(z-9)$

2. $16x^2-9=(4x)^2-3^2=(4x+3)(4x-3)$

3. $$\begin{aligned}7pq^4-7py^4&=7p(q^4-y^4)\\&=7p[(q^2)^2-(y^2)^2]\\&=7p(q^2+y^2)(q^2-y^2)\\&=7p(q^2+y^2)(q+y)(q-y)\end{aligned}$$

4. $$\begin{aligned}x^2+12x+36&=x^2+2\cdot x\cdot 6+6^2\\&=(x+6)^2\end{aligned}$$

5. $9z^2-12z+4=(3z)^2-2\cdot 3z\cdot 2+2^2=(3z-2)^2$

6. $$\begin{aligned}&a^3+24a^2+144a\\&=a(a^2+24a+144)\\&=a(a^2+2\cdot a\cdot 12+12^2)\\&=a(a+12)^2\end{aligned}$$

7. $$\begin{aligned}x^3+64&=x^3+4^3\\&=(x+4)(x^2-4x+16)\end{aligned}$$

8. $$\begin{aligned}m^3-216&=m^3-6^3\\&=(m-6)(m^2+6m+36)\end{aligned}$$

9. $$\begin{aligned}3a^5-24a^2&=3a^2(a^3-8)\\&=3a^2(a^3-2^3)\\&=3a^2(a-2)(a^2+2a+4)\end{aligned}$$

10. $$\begin{aligned}t^6+1&=(t^2)^3+1^3\\&=(t^2+1)(t^4-t^2+1)\end{aligned}$$

17. Equation-Solving Principles

1. $$\begin{aligned}7t&=70\\t&=10 &&\text{Dividing by 7}\end{aligned}$$
The solution is 10.

2. $$\begin{aligned}x-5&=7\\x&=12 &&\text{Adding 5}\end{aligned}$$
The solution is 12.

3. $$\begin{aligned}3x+4&=-8\\3x&=-12 &&\text{Subtracting 4}\\x&=-4 &&\text{Dividing by 3}\end{aligned}$$
The solution is -4.

4. $$\begin{aligned}6x-15&=45\\6x&=60 &&\text{Adding 15}\\x&=10 &&\text{Dividing by 6}\end{aligned}$$
The solution is 10.

5. $$\begin{aligned}7y-1&=23-5y\\12y-1&=23 &&\text{Adding } 5y\\12y&=24 &&\text{Adding 1}\\y&=2 &&\text{Dividing by 12}\end{aligned}$$
The solution is 2.

6. $$\begin{aligned}3m-7&=-13+m\\2m-7&=-13 &&\text{Subtracting } m\\2m&=-6 &&\text{Adding 7}\\m&=-3 &&\text{Dividing by 2}\end{aligned}$$
The solution is -3.

7. $$\begin{aligned}2(x+7)&=5x+14\\2x+14&=5x+14\\-3x+14&=14 &&\text{Subtracting } 5x\\-3x&=0 &&\text{Subtracting 14}\\x&=0\end{aligned}$$
The solution is 0.

8. $$\begin{aligned}5y-(2y-10)&=25\\5y-2y+10&=25\\3y+10&=25 &&\text{Collecting like terms}\\3y&=15 &&\text{Subtracting 10}\\y&=5 &&\text{Dividing by 3}\end{aligned}$$
The solution is 5.

18. Inequality-Solving Principles

1. $$\begin{aligned}p+25&\geq -100\\p&\geq -125 &&\text{Subtracting 25}\end{aligned}$$
The solution set is $[-125,\infty)$.

2. $$\begin{aligned}-\frac{2}{3}x&>6\\x&<-\frac{3}{2}\cdot 6 &&\text{Multiplying by } -\frac{3}{2} \text{ and reversing the inequality symbol}\\x&<-9\end{aligned}$$
The solution set is $(-\infty,-9)$.

3. $$\begin{aligned}9x-1&<17\\9x&<18 &&\text{Adding 1}\\x&<2 &&\text{Dividing by 9}\end{aligned}$$
The solution set is $(-\infty,2)$.

4. $$\begin{aligned}-x-16&\geq 40\\-x&\geq 56 &&\text{Adding 6}\\x&\leq -56 &&\text{Multiplying by } -1 \text{ and reversing the inequality symbol}\end{aligned}$$
The solution set is $(-\infty,-56]$.

5. $\frac{1}{3}y - 6 < 3$

$\frac{1}{3}y < 9$ Adding 6

$y < 27$ Multiplying by 3

The solution set is $(-\infty, 27)$.

6. $8 - 2w \leq -14$

$-2w \leq -22$ Subtracting 8

$w \geq 11$ Dividing by -2 and reversing the inequality symbol

The solution set is $[11, \infty)$.

19. The Principle of Zero Products

1. $2y^2 + 42y = 0$

$2y(y + 21) = 0$

$2y = 0$ *or* $y + 21 = 0$

$y = 0$ *or* $y = -21$

The solutions are 0 and -21.

2. $(a + 7)(a - 1) = 0$

$a + 7 = 0$ *or* $a - 1 = 0$

$a = -7$ *or* $a = 1$

The solutions are -7 and 1.

3. $(5y + 3)(y - 4) = 0$

$5y + 3 = 0$ *or* $y - 4 = 0$

$5y = -3$ *or* $y = 4$

$y = -\frac{3}{5}$ *or* $y = 4$

The solutions are $-\frac{3}{5}$ and 4.

4. $6x^2 + 7x - 5 = 0$

$(3x + 5)(2x - 1) = 0$

$3x + 5 = 0$ *or* $2x - 1 = 0$

$3x = -5$ *or* $2x = 1$

$x = -\frac{5}{3}$ *or* $x = \frac{1}{2}$

The solutions are $-\frac{5}{3}$ and $\frac{1}{2}$.

5. $t(t - 8) = 0$

$t = 0$ *or* $t - 8 = 0$

$t = 0$ *or* $t = 8$

The solutions are 0 and 8.

6. $x^2 - 8x - 33 = 0$

$(x + 3)(x - 11) = 0$

$x + 3 = 0$ *or* $x - 11 = 0$

$x = -3$ *or* $x = 11$

The solutions are -3 and 11.

7. $x^2 + 13x = 30$

$x^2 + 13x - 30 = 0$

$(x + 15)(x - 2) = 0$

$x + 15 = 0$ *or* $x - 2 = 0$

$x = -15$ *or* $x = 2$

The solutions are -15 and 2.

8. $12x^2 - 7x - 12 = 0$

$(4x + 3)(3x - 4) = 0$

$4x + 3 = 0$ *or* $3x - 4 = 0$

$4x = -3$ *or* $3x = 4$

$x = -\frac{3}{4}$ *or* $x = \frac{4}{3}$

The solutions are $-\frac{3}{4}$ and $\frac{4}{3}$.

20. The Principle of Square Roots

1. $x^2 - 36 = 0$

$x^2 = 36$

$x = \sqrt{36}$ *or* $x = -\sqrt{36}$

$x = 6$ *or* $x = -6$

The solutions are 6 and -6, or ± 6.

2. $2y^2 - 20 = 0$

$2y^2 = 20$

$y^2 = 10$

$y = \sqrt{10}$ *or* $y = -\sqrt{10}$

The solutions are $\sqrt{10}$ and $-\sqrt{10}$, or $\pm\sqrt{10}$.

3. $6z^2 = 18$

$z^2 = 3$

$z = \sqrt{3}$ *or* $z = -\sqrt{3}$

The solutions are $\sqrt{3}$ and $-\sqrt{3}$, or $\pm\sqrt{3}$.

4. $3t^2 - 15 = 0$

$3t^2 = 15$

$t^2 = 5$

$t = \sqrt{5}$ *or* $t = -\sqrt{5}$

The solutions are $\sqrt{5}$ and $-\sqrt{5}$, or $\pm\sqrt{5}$.

5. $z^2 - 1 = 24$

$z^2 = 25$

$z = \sqrt{25}$ *or* $z = -\sqrt{25}$

The solutions are 5 and -5, or ± 5.

6. $5x^2 - 75 = 0$

$5x^2 = 75$

$x^2 = 15$

$x = \sqrt{15}$ *or* $x = -\sqrt{15}$

The solutions are $\sqrt{15}$ and $-\sqrt{15}$, or $\pm\sqrt{15}$.

21. Simplify Rational Expressions

1. $\dfrac{3x-3}{x(x-1)}$

The denominator is 0 when the factor $x = 0$ and also when $x - 1 = 0$, or $x = 1$. The domain is the set of all real numbers except 0 and 1.

2. $\dfrac{y+6}{y^2+4y-21} = \dfrac{y+6}{(y+7)(y-3)}$

The denominator is 0 when $y = -7$ or $y = 3$. The domain is the set of all real numbers except -7 and 3.

3. $\dfrac{x^2-4}{x^2-4x+4} = \dfrac{(x+2)\cancel{(x-2)}}{(x-2)\cancel{(x-2)}} = \dfrac{x+2}{x-2}$

4. $\dfrac{x^2+2x-3}{x^2-9} = \dfrac{(x-1)\cancel{(x+3)}}{\cancel{(x+3)}(x-3)} = \dfrac{x-1}{x-3}$

5.
$$\begin{aligned}\frac{x^3-6x^2+9x}{x^3-3x^2} &= \frac{x(x^2-6x+9)}{x^2(x-3)}\\ &= \frac{\cancel{x}\cancel{(x-3)}(x-3)}{\cancel{x}\cdot x\cancel{(x-3)}}\\ &= \frac{x-3}{x}\end{aligned}$$

6.
$$\begin{aligned}\frac{6y^2+12y-48}{3y^2-9y+6} &= \frac{6(y^2+2y-8)}{3(y^2-3y+2)}\\ &= \frac{2\cdot\cancel{3}\cdot(y+4)\cancel{(y-2)}}{\cancel{3}(y-1)\cancel{(y-2)}}\\ &= \frac{2(y+4)}{y-1}\end{aligned}$$

22. Multiply and Divide Rational Expressions

1.
$$\begin{aligned}\frac{r-s}{r+s}\cdot\frac{r^2-s^2}{(r-s)^2} &= \frac{(r-s)(r^2-s^2)}{(r+s)(r-s)^2}\\ &= \frac{\cancel{(r-s)}\cancel{(r-s)}\cancel{(r+s)}\cdot 1}{\cancel{(r+s)}\cancel{(r-s)}\cancel{(r-s)}}\\ &= 1\end{aligned}$$

2.
$$\begin{aligned}&\frac{m^2-n^2}{r+s} \div \frac{m-n}{r+s}\\ &= \frac{m^2-n^2}{r+s}\cdot\frac{r+s}{m-n}\\ &= \frac{(m+n)\cancel{(m-n)}\cancel{(r+s)}}{\cancel{(r+s)}\cancel{(m-n)}}\\ &= m+n\end{aligned}$$

3.
$$\begin{aligned}&\frac{4x^2+9x+2}{x^2+x-2}\cdot\frac{x^2-1}{3x^2+x-2}\\ &= \frac{(4x+1)\cancel{(x+2)}\cancel{(x+1)}\cancel{(x-1)}}{\cancel{(x+2)}\cancel{(x-1)}(3x-2)\cancel{(x+1)}}\\ &= \frac{4x+1}{3x-2}\end{aligned}$$

4.
$$\begin{aligned}&\frac{3x+12}{2x-8} \div \frac{(x+4)^2}{(x-4)^2}\\ &= \frac{3x+12}{2x-8}\cdot\frac{(x-4)^2}{(x+4)^2}\\ &= \frac{3\cancel{(x+4)}\cancel{(x-4)}(x-4)}{2\cancel{(x-4)}\cancel{(x+4)}(x+4)}\\ &= \frac{3(x-4)}{2(x+4)}\end{aligned}$$

5.
$$\begin{aligned}&\frac{a^2-a-2}{a^2-a-6} \div \frac{a^2-2a}{2a+a^2}\\ &= \frac{a^2-a-2}{a^2-a-6}\cdot\frac{2a+a^2}{a^2-2a}\\ &= \frac{\cancel{(a-2)}(a+1)\cancel{(a)}\cancel{(2+a)}}{(a-3)\cancel{(a+2)}\cancel{(a)}\cancel{(a-2)}}\\ &= \frac{a+1}{a-3}\end{aligned}$$

6.
$$\begin{aligned}&\frac{x^2-y^2}{x^3-y^3}\cdot\frac{x^2+xy+y^2}{x^2+2xy+y^2}\\ &= \frac{(x+y)(x-y)(x^2+xy+y^2)}{(x-y)(x^2+xy+y^2)(x+y)(x+y)}\\ &= \frac{1}{x+y}\cdot\frac{(x+y)(x-y)(x^2+xy+y^2)}{(x+y)(x-y)(x^2+xy+y^2)}\\ &= \frac{1}{x+y}\cdot 1 \quad \text{Removing a factor of 1}\\ &= \frac{1}{x+y}\end{aligned}$$

23. Add and Subtract Rational Expressions

1.
$$\begin{aligned}\frac{a-3b}{a+b}+\frac{a+5b}{a+b} &= \frac{2a+2b}{a+b}\\ &= \frac{2\cancel{(a+b)}}{1\cdot\cancel{(a+b)}}\\ &= 2\end{aligned}$$

2.
$$\begin{aligned}&\frac{x^2-5}{3x^2-5x-2}+\frac{x+1}{3x-6}\\ &= \frac{x^2-5}{(3x+1)(x-2)}+\frac{x+1}{3(x-2)}\\ &= \frac{x^2-5}{(3x+1)(x-2)}\cdot\frac{3}{3}+\frac{x+1}{3(x-2)}\cdot\frac{3x+1}{3x+1}\\ &= \frac{3(x^2-5)+(x+1)(3x+1)}{3(3x+1)(x-2)}\\ &= \frac{3x^2-15+3x^2+4x+1}{3(3x+1)(x-2)}\\ &= \frac{6x^2+4x-14}{3(3x+1)(x-2)}\end{aligned}$$

3.
$$\frac{a^2+1}{a^2-1}-\frac{a-1}{a+1}$$
$$=\frac{a^2+1}{(a+1)(a-1)}-\frac{a-1}{a+1},\ \text{LCD is } (a+1)(a-1)$$
$$=\frac{a^2+1-(a-1)(a-1)}{(a+1)(a-1)}$$
$$=\frac{a^2+1-a^2+2a-1}{(a+1)(a-1)}$$
$$=\frac{2a}{(a+1)(a-1)}$$

4.
$$\frac{9x+2}{3x^2-2x-8}+\frac{7}{3x^2+x-4}$$
$$=\frac{9x+2}{(3x+4)(x-2)}+\frac{7}{(3x+4)(x-1)},$$
$$\text{LCD is } (3x+4)(x-2)(x-1)$$
$$=\frac{9x+2}{(3x+4)(x-2)}\cdot\frac{x-1}{x-1}+\frac{7}{(3x+4)(x-1)}\cdot\frac{x-2}{x-2}$$
$$=\frac{9x^2-7x-2}{(3x+4)(x-2)(x-1)}+\frac{7x-14}{(3x+4)(x-1)(x-2)}$$
$$=\frac{9x^2-16}{(3x+4)(x-2)(x-1)}$$
$$=\frac{(\cancel{3x+4})(3x-4)}{(\cancel{3x+4})(x-2)(x-1)}$$
$$=\frac{3x-4}{(x-2)(x-1)}$$

5.
$$\frac{y}{y^2-y-20}-\frac{2}{y+4}$$
$$=\frac{y}{(y+4)(y-5)}-\frac{2}{y+4},\ \text{LCD is } (y+4)(y-5)$$
$$=\frac{y}{(y+4)(y-5)}-\frac{2}{y+4}\cdot\frac{y-5}{y-5}$$
$$=\frac{y}{(y+4)(y-5)}-\frac{2y-10}{(y+4)(y-5)}$$
$$=\frac{y-(2y-10)}{(y+4)(y-5)}$$
$$=\frac{y-2y+10}{(y+4)(y-5)}$$
$$=\frac{-y+10}{(y+4)(y-5)}$$

6.
$$\frac{3y}{y^2-7y+10}-\frac{2y}{y^2-8y+15}$$
$$=\frac{3y}{(y-2)(y-5)}-\frac{2y}{(y-5)(y-3)},$$
$$\text{LCD is } (y-2)(y-5)(y-3)$$
$$=\frac{3y(y-3)-2y(y-2)}{(y-2)(y-5)(y-3)}$$
$$=\frac{3y^2-9y-2y^2+4y}{(y-2)(y-5)(y-3)}$$
$$=\frac{y^2-5y}{(y-2)(y-5)(y-3)}$$
$$=\frac{y(\cancel{y-5})}{(y-2)(\cancel{y-5})(y-3)}$$
$$=\frac{y}{(y-2)(y-3)}$$

24. Simplify Complex Rational Expressions

1.
$$\frac{\frac{x}{y}-\frac{y}{x}}{\frac{1}{y}+\frac{1}{x}}=\frac{\frac{x}{y}-\frac{y}{x}}{\frac{1}{y}+\frac{1}{x}}\cdot\frac{xy}{xy},\ \text{LCM is } xy$$
$$=\frac{\left(\frac{x}{y}-\frac{y}{x}\right)(xy)}{\left(\frac{1}{y}+\frac{1}{x}\right)(xy)}$$
$$=\frac{x^2-y^2}{x+y}$$
$$=\frac{(\cancel{x+y})(x-y)}{(\cancel{x+y})\cdot 1}$$
$$=x-y$$

2.
$$\frac{\frac{a-b}{b}}{\frac{a^2-b^2}{ab}}=\frac{a-b}{b}\cdot\frac{ab}{a^2-b^2}$$
$$=\frac{a-b}{b}\cdot\frac{ab}{(a+b)(a-b)}$$
$$=\frac{a\,\cancel{b}\,(\cancel{a-b})}{\cancel{b}\,(a+b)(\cancel{a-b})}$$
$$=\frac{a}{a+b}$$

3. $$\frac{w+\frac{8}{w^2}}{1+\frac{2}{w}} = \frac{w\cdot\frac{w^2}{w^2}+\frac{8}{w^2}}{1\cdot\frac{w}{w}+\frac{2}{w}} = \frac{\frac{w^3+8}{w^2}}{\frac{w+2}{w}} = \frac{w^3+8}{w^2}\cdot\frac{w}{w+2} = \frac{(\cancel{w+2})(w^2-2w+4)\cancel{w}}{\cancel{w}\cdot w(\cancel{w+2})} = \frac{w^2-2w+4}{w}$$

4. $$\frac{\frac{x^2-y^2}{xy}}{\frac{x-y}{y}} = \frac{x^2-y^2}{xy}\cdot\frac{y}{x-y} = \frac{(x+y)(\cancel{x-y})\cancel{y}}{x\cancel{y}(\cancel{x-y})} = \frac{x+y}{x}$$

5. $$\frac{\frac{a}{b}-\frac{b}{a}}{\frac{1}{a}-\frac{1}{b}} = \frac{a^2-b^2}{b-a} \quad \text{Multiplying by } \frac{ab}{ab}$$
$$= \frac{(a+b)(a-b)}{b-a} = \frac{(a+b)(\cancel{a-b})}{-1\cdot(\cancel{a-b})} = -a-b$$

25. Simplify Radical Expressions

1. $\sqrt{(-21)^2} = |-21| = 21$

2. $\sqrt{9y^2} = \sqrt{(3y)^2} = |3y| = 3y$

3. $\sqrt{(a-2)^2} = a-2$

4. $\sqrt[3]{-27x^3} = \sqrt[3]{(-3x)^3} = -3x$

5. $\sqrt[4]{81x^8} = \sqrt[4]{(3x^2)^4} = 3x^2$

6. $\sqrt[5]{32} = \sqrt[5]{2^5} = 2$

7. $\sqrt[4]{48x^6y^4} = \sqrt[4]{16x^4y^4\cdot 3x^2} = 2xy\sqrt[4]{3x^2} =$
$2xy\sqrt[4]{3x^2}$

8. $\sqrt{15}\sqrt{35} = \sqrt{15\cdot 35} = \sqrt{3\cdot 5\cdot 5\cdot 7} = \sqrt{5^2\cdot 3\cdot 7} =$
$\sqrt{5^2}\cdot\sqrt{3\cdot 7} = 5\sqrt{21}$

9. $\dfrac{\sqrt{40xy}}{\sqrt{8x}} = \sqrt{\dfrac{40xy}{8x}} = \sqrt{5y}$

10. $\dfrac{\sqrt[3]{3x^2}}{\sqrt[3]{24x^5}} = \sqrt[3]{\dfrac{3x^2}{24x^5}} = \sqrt[3]{\dfrac{1}{8x^3}} = \dfrac{1}{2x}$

11. $\sqrt{x^2-4x+4} = \sqrt{(x-2)^2} = x-2$

12. $\sqrt{2x^3y}\sqrt{12xy} = \sqrt{24x^4y^2} = \sqrt{4x^4y^2\cdot 6} = 2x^2y\sqrt{6}$

13. $\sqrt[3]{3x^2y}\sqrt[3]{36x} = \sqrt[3]{108x^3y} = \sqrt[3]{27x^3\cdot 4y} = 3x\sqrt[3]{4y}$

14. $$5\sqrt{2}+3\sqrt{32} = 5\sqrt{2}+3\sqrt{16\cdot 2} = 5\sqrt{2}+3\cdot 4\sqrt{2} = 5\sqrt{2}+12\sqrt{2} = (5+12)\sqrt{2} = 17\sqrt{2}$$

15. $7\sqrt{12}-2\sqrt{3} = 7\cdot 2\sqrt{3}-2\sqrt{3} = 14\sqrt{3}-2\sqrt{3} = 12\sqrt{3}$

16. $2\sqrt{32}+3\sqrt{8}-4\sqrt{18} = 2\cdot 4\sqrt{2}+3\cdot 2\sqrt{2}-4\cdot 3\sqrt{2} =$
$8\sqrt{2}+6\sqrt{2}-12\sqrt{2} = 2\sqrt{2}$

17. $$6\sqrt{20}-4\sqrt{45}+\sqrt{80} = 6\sqrt{4\cdot 5}-4\sqrt{9\cdot 5}+\sqrt{16\cdot 5} = 6\cdot 2\sqrt{5}-4\cdot 3\sqrt{5}+4\sqrt{5} = 12\sqrt{5}-12\sqrt{5}+4\sqrt{5} = (12-12+4)\sqrt{5} = 4\sqrt{5}$$

18. $$(2+\sqrt{3})(5+2\sqrt{3}) = 2\cdot 5+2\cdot 2\sqrt{3}+\sqrt{3}\cdot 5+\sqrt{3}\cdot 2\sqrt{3} = 10+4\sqrt{3}+5\sqrt{3}+3\cdot 2 = 10+9\sqrt{3}+6 = 16+9\sqrt{3}$$

19. $$\left(\sqrt{8}+2\sqrt{5}\right)\left(\sqrt{8}-2\sqrt{5}\right) = \left(\sqrt{8}\right)^2-\left(2\sqrt{5}\right)^2 = 8-4\cdot 5 = 8-20 = -12$$

20. $$(1+\sqrt{3})^2 = 1^2+2\cdot 1\cdot\sqrt{3}+(\sqrt{3})^2 = 1+2\sqrt{3}+3 = 4+2\sqrt{3}$$

26. Rationalizing Denominators

1. $\dfrac{4}{\sqrt{11}} = \dfrac{4}{\sqrt{11}}\cdot\dfrac{\sqrt{11}}{\sqrt{11}} = \dfrac{4\sqrt{11}}{11}$

2. $\sqrt{\dfrac{3}{7}} = \sqrt{\dfrac{3}{7}\cdot\dfrac{7}{7}} = \sqrt{\dfrac{21}{49}} = \dfrac{\sqrt{21}}{\sqrt{49}} = \dfrac{\sqrt{21}}{7}$

3. $\dfrac{\sqrt[3]{7}}{\sqrt[3]{2}} = \dfrac{\sqrt[3]{7}}{\sqrt[3]{2}}\cdot\dfrac{\sqrt[3]{4}}{\sqrt[3]{4}} = \dfrac{\sqrt[3]{28}}{\sqrt[3]{8}} = \dfrac{\sqrt[3]{28}}{2}$

4. $\sqrt[3]{\dfrac{16}{9}} = \sqrt[3]{\dfrac{16}{9}\cdot\dfrac{3}{3}} = \sqrt[3]{\dfrac{48}{27}} = \dfrac{\sqrt[3]{48}}{\sqrt[3]{27}} =$
$\dfrac{\sqrt[3]{8\cdot 6}}{3} = \dfrac{2\sqrt[3]{6}}{3}$

5. $$\frac{3}{\sqrt{30}-4} = \frac{3}{\sqrt{30}-4}\cdot\frac{\sqrt{30}+4}{\sqrt{30}+4} = \frac{3\sqrt{30}+12}{(\sqrt{30})^2-4^2} = \frac{3\sqrt{30}+12}{30-16} = \frac{3\sqrt{30}+12}{14}$$

6. $$\frac{4}{\sqrt{7}-\sqrt{3}} = \frac{4}{\sqrt{7}-\sqrt{3}}\cdot\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}} = \frac{4\sqrt{7}+4\sqrt{3}}{(\sqrt{7})^2-(\sqrt{3})^2} = \frac{4\sqrt{7}+4\sqrt{3}}{7-3} = \frac{4\sqrt{7}+4\sqrt{3}}{4} = \frac{4(\sqrt{7}+\sqrt{3})}{4} = \sqrt{7}+\sqrt{3}$$

7. $$\frac{6}{\sqrt{m}-\sqrt{n}} = \frac{6}{\sqrt{m}-\sqrt{n}}\cdot\frac{\sqrt{m}+\sqrt{n}}{\sqrt{m}+\sqrt{n}} = \frac{6(\sqrt{m}+\sqrt{n})}{(\sqrt{m})^2-(\sqrt{n})^2} = \frac{6\sqrt{m}+6\sqrt{n}}{m-n}$$

8. $$\frac{1-\sqrt{2}}{\sqrt{3}-\sqrt{6}} = \frac{1-\sqrt{2}}{\sqrt{3}-\sqrt{6}}\cdot\frac{\sqrt{3}+\sqrt{6}}{\sqrt{3}+\sqrt{6}} = \frac{\sqrt{3}+\sqrt{6}-\sqrt{6}-\sqrt{12}}{3-6} = \frac{\sqrt{3}+\sqrt{6}-\sqrt{6}-2\sqrt{3}}{3-6} = \frac{-\sqrt{3}}{-3} = \frac{\sqrt{3}}{3}$$

27. Rational Exponents

1. $y^{5/6} = \sqrt[6]{y^5}$

2. $x^{2/3} = \sqrt[3]{x^2}$

3. $16^{3/4} = (16^{1/4})^3 = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$

4. $4^{7/2} = (\sqrt{4})^7 = 2^7 = 128$

5. $125^{-1/3} = \dfrac{1}{125^{1/3}} = \dfrac{1}{\sqrt[3]{125}} = \dfrac{1}{5}$

6. $32^{-4/5} = \left(\sqrt[5]{32}\right)^{-4} = 2^{-4} = \dfrac{1}{16}$

7. $\sqrt[12]{y^4} = y^{4/12} = y^{1/3}$

8. $\sqrt{x^5} = x^{5/2}$

9. $x^{1/2}\cdot x^{2/3} = x^{1/2+2/3} = x^{3/6+4/6} = x^{7/6} = \sqrt[6]{x^7} = x\sqrt[6]{x}$

10. $(a-2)^{9/4}(a-2)^{-1/4} = (a-2)^{9/4+(-1/4)} = (a-2)^{8/4} = (a-2)^2$

11. $(m^{1/2}n^{5/2})^{2/3} = m^{\frac{1}{2}\cdot\frac{2}{3}}n^{\frac{5}{2}\cdot\frac{2}{3}} = m^{1/3}n^{5/3} = \sqrt[3]{m}\sqrt[3]{n^5} = \sqrt[3]{mn^5} = n\sqrt[3]{mn^2}$

28. The Pythagorean Theorem

1. $$\begin{aligned} a^2+b^2 &= c^2 \\ 8^2+15^2 &= c^2 \\ 64+225 &= c^2 \\ 289 &= c^2 \\ 17 &= c \end{aligned}$$

2. $$\begin{aligned} a^2+b^2 &= c^2 \\ 4^2+4^2 &= c^2 \\ 16+16 &= c^2 \\ 32 &= c^2 \\ \sqrt{32} &= c \\ 5.657 &\approx c \end{aligned}$$

3. $$\begin{aligned} a^2+b^2 &= c^2 \\ 5^2+b^2 &= 13^2 \\ 25+b^2 &= 169 \\ b^2 &= 144 \\ b &= 12 \end{aligned}$$

4. $$\begin{aligned} a^2+b^2 &= c^2 \\ a^2+12^2 &= 13^2 \\ a^2+144 &= 169 \\ a^2 &= 25 \\ a &= 5 \end{aligned}$$

5. $$\begin{aligned} a^2+b^2 &= c^2 \\ (\sqrt{5})^2+b^2 &= 6^2 \\ 5+b^2 &= 36 \\ b^2 &= 31 \\ b &= \sqrt{31} \approx 5.568 \end{aligned}$$

Chapter 1

Graphs, Functions, and Models

Exercise Set 1.1

1. Point A is located 5 units to the left of the y-axis and 4 units up from the x-axis, so its coordinates are $(-5, 4)$.

Point B is located 2 units to the right of the y-axis and 2 units down from the x-axis, so its coordinates are $(2, -2)$.

Point C is located 0 units to the right or left of the y-axis and 5 units down from the x-axis, so its coordinates are $(0, -5)$.

Point D is located 3 units to the right of the y-axis and 5 units up from the x-axis, so its coordinates are $(3, 5)$.

Point E is located 5 units to the left of the y-axis and 4 units down from the x-axis, so its coordinates are $(-5, -4)$.

Point F is located 3 units to the right of the y-axis and 0 units up or down from the x-axis, so its coordinates are $(3, 0)$.

2. G: $(2, 1)$; H: $(0, 0)$; I: $(4, -3)$; J: $(-4, 0)$; K: $(-2, 3)$; L: $(0, 5)$

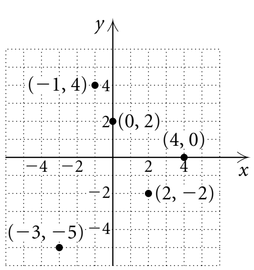
3. To graph $(4, 0)$ we move from the origin 4 units to the right of the y-axis. Since the second coordinate is 0, we do not move up or down from the x-axis.

To graph $(-3, -5)$ we move from the origin 3 units to the left of the y-axis. Then we move 5 units down from the x-axis.

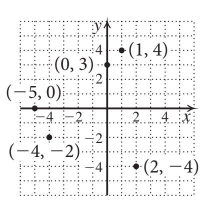
To graph $(-1, 4)$ we move from the origin 1 unit to the left of the y-axis. Then we move 4 units up from the x-axis.

To graph $(0, 2)$ we do not move to the right or the left of the y-axis since the first coordinate is 0. From the origin we move 2 units up.

To graph $(2, -2)$ we move from the origin 2 units to the right of the y-axis. Then we move 2 units down from the x-axis.



4.



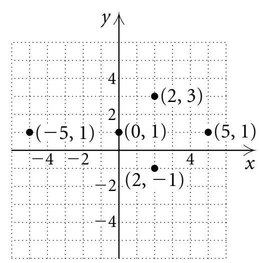
5. To graph $(-5, 1)$ we move from the origin 5 units to the left of the y-axis. Then we move 1 unit up from the x-axis.

To graph $(5, 1)$ we move from the origin 5 units to the right of the y-axis. Then we move 1 unit up from the x-axis.

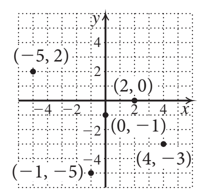
To graph $(2, 3)$ we move from the origin 2 units to the right of the y-axis. Then we move 3 units up from the x-axis.

To graph $(2, -1)$ we move from the origin 2 units to the right of the y-axis. Then we move 1 unit down from the x-axis.

To graph $(0, 1)$ we do not move to the right or the left of the y-axis since the first coordinate is 0. From the origin we move 1 unit up.



6.



7. The first coordinate represents the year and the corresponding second coordinate represents the number of cities served by Southwest Airlines. The ordered pairs are (1971, 3), (1981, 15), (1991, 32), (2001, 59), (2011, 72), and (2014, 96).

8. The first coordinate represents the year and the second coordinate represents the percent of Marines who are women. The ordered pairs are (1960, 1%), (1970, 0.9%), (1980, 3.6%), (1990, 4.9%), (2000, 6.1%), (2011, 6.8%), and (2014, 7.6%).

9. To determine whether $(-1, -9)$ is a solution, substitute -1 for x and -9 for y.

$$\begin{array}{c|ll} \multicolumn{2}{c}{y = 7x - 2} \\ \hline -9 \;?\; & 7(-1) - 2 & \\ & -7 - 2 & \\ -9 & -9 & \text{TRUE} \end{array}$$

The equation $-9 = -9$ is true, so $(-1, -9)$ is a solution.

To determine whether $(0, 2)$ is a solution, substitute 0 for x and 2 for y.

$$\begin{array}{r|l} \multicolumn{2}{c}{y = 7x - 2} \\ \hline 2 \ ? & 7 \cdot 0 - 2 \\ & 0 - 2 \\ 2 & -2 \quad \text{FALSE} \end{array}$$

The equation $2 = -2$ is false, so $(0, 2)$ is not a solution.

10. For $\left(\frac{1}{2}, 8\right)$:

$$\begin{array}{r|l} \multicolumn{2}{c}{y = -4x + 10} \\ \hline 8 \ ? & -4 \cdot \frac{1}{2} + 10 \\ & -2 + 10 \\ 8 & 8 \quad \text{TRUE} \end{array}$$

$\left(\frac{1}{2}, 8\right)$ is a solution.

For $(-1, 6)$:

$$\begin{array}{r|l} \multicolumn{2}{c}{y = -4x + 10} \\ \hline 6 \ ? & -4(-1) + 10 \\ & 4 + 10 \\ 6 & 14 \quad \text{FALSE} \end{array}$$

$(-1, 6)$ is not a solution.

11. To determine whether $\left(\frac{2}{3}, \frac{3}{4}\right)$ is a solution, substitute $\frac{2}{3}$ for x and $\frac{3}{4}$ for y.

$$\begin{array}{r|l} \multicolumn{2}{c}{6x - 4y = 1} \\ \hline 6 \cdot \frac{2}{3} - 4 \cdot \frac{3}{4} \ ? & 1 \\ 4 - 3 & \\ 1 & 1 \quad \text{TRUE} \end{array}$$

The equation $1 = 1$ is true, so $\left(\frac{2}{3}, \frac{3}{4}\right)$ is a solution.

To determine whether $\left(1, \frac{3}{2}\right)$ is a solution, substitute 1 for x and $\frac{3}{2}$ for y.

$$\begin{array}{r|l} \multicolumn{2}{c}{6x - 4y = 1} \\ \hline 6 \cdot 1 - 4 \cdot \frac{3}{2} \ ? & 1 \\ 6 - 6 & \\ 0 & 1 \quad \text{FALSE} \end{array}$$

The equation $0 = 1$ is false, so $\left(1, \frac{3}{2}\right)$ is not a solution.

12. For $(1.5, 2.6)$:

$$\begin{array}{r|l} \multicolumn{2}{c}{x^2 + y^2 = 9} \\ \hline (1.5)^2 + (2.6)^2 \ ? & 9 \\ 2.25 + 6.76 & \\ 9.01 & 9 \quad \text{FALSE} \end{array}$$

$(1.5, 2.6)$ is not a solution.

For $(-3, 0)$:

$$\begin{array}{r|l} \multicolumn{2}{c}{x^2 + y^2 = 9} \\ \hline (-3)^2 + 0^2 \ ? & 9 \\ 9 + 0 & \\ 9 & 9 \quad \text{TRUE} \end{array}$$

$(-3, 0)$ is a solution.

13. To determine whether $\left(-\frac{1}{2}, -\frac{4}{5}\right)$ is a solution, substitute $-\frac{1}{2}$ for a and $-\frac{4}{5}$ for b.

$$\begin{array}{r|l} \multicolumn{2}{c}{2a + 5b = 3} \\ \hline 2\left(-\frac{1}{2}\right) + 5\left(-\frac{4}{5}\right) \ ? & 3 \\ -1 - 4 & \\ -5 & 3 \quad \text{FALSE} \end{array}$$

The equation $-5 = 3$ is false, so $\left(-\frac{1}{2}, -\frac{4}{5}\right)$ is not a solution.

To determine whether $\left(0, \frac{3}{5}\right)$ is a solution, substitute 0 for a and $\frac{3}{5}$ for b.

$$\begin{array}{r|l} \multicolumn{2}{c}{2a + 5b = 3} \\ \hline 2 \cdot 0 + 5 \cdot \frac{3}{5} \ ? & 3 \\ 0 + 3 & \\ 3 & 3 \quad \text{TRUE} \end{array}$$

The equation $3 = 3$ is true, so $\left(0, \frac{3}{5}\right)$ is a solution.

14. For $\left(0, \frac{3}{2}\right)$:

$$\begin{array}{r|l} \multicolumn{2}{c}{3m + 4n = 6} \\ \hline 3 \cdot 0 + 4 \cdot \frac{3}{2} \ ? & 6 \\ 0 + 6 & \\ 6 & 6 \quad \text{TRUE} \end{array}$$

$\left(0, \frac{3}{2}\right)$ is a solution.

For $\left(\frac{2}{3}, 1\right)$:

$$\begin{array}{r|l} \multicolumn{2}{c}{3m + 4n = 6} \\ \hline 3 \cdot \frac{2}{3} + 4 \cdot 1 \ ? & 6 \\ 2 + 4 & \\ 6 & 6 \quad \text{TRUE} \end{array}$$

The equation $6 = 6$ is true, so $\left(\frac{2}{3}, 1\right)$ is a solution.

15. To determine whether $(-0.75, 2.75)$ is a solution, substitute -0.75 for x and 2.75 for y.

$$\begin{array}{r|l} \multicolumn{2}{c}{x^2 - y^2 = 3} \\ \hline (-0.75)^2 - (2.75)^2 \ ? & 3 \\ 0.5625 - 7.5625 & \\ -7 & 3 \quad \text{FALSE} \end{array}$$

The equation $-7 = 3$ is false, so $(-0.75, 2.75)$ is not a solution.

To determine whether $(2, -1)$ is a solution, substitute 2 for x and -1 for y.

$$\begin{array}{r|l} x^2 - y^2 & = 3 \\ \hline 2^2 - (-1)^2 & ?\ 3 \\ 4 - 1 & \\ 3 & 3 \quad \text{TRUE} \end{array}$$

The equation $3 = 3$ is true, so $(2, -1)$ is a solution.

16. For $(2, -4)$:

$$\begin{array}{r|l} 5x + 2y^2 & = 70 \\ \hline 5 \cdot 2 + 2(-4)^2 & ?\ 70 \\ 10 + 2 \cdot 16 & \\ 10 + 32 & \\ 42 & 70 \quad \text{FALSE} \end{array}$$

$(2, -4)$ is not a solution.

For $(4, -5)$:

$$\begin{array}{r|l} 5x + 2y^2 & = 70 \\ \hline 5 \cdot 4 + 2(-5)^2 & ?\ 70 \\ 20 + 2 \cdot 25 & \\ 20 + 50 & \\ 70 & 70 \quad \text{TRUE} \end{array}$$

$(4, -5)$ is a solution.

17. Graph $5x - 3y = -15$.

To find the x-intercept we replace y with 0 and solve for x.

$$\begin{aligned} 5x - 3 \cdot 0 &= -15 \\ 5x &= -15 \\ x &= -3 \end{aligned}$$

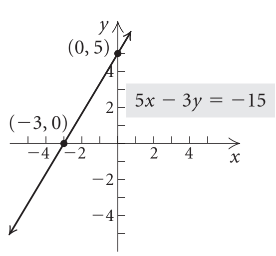
The x-intercept is $(-3, 0)$.

To find the y-intercept we replace x with 0 and solve for y.

$$\begin{aligned} 5 \cdot 0 - 3y &= -15 \\ -3y &= -15 \\ y &= 5 \end{aligned}$$

The y-intercept is $(0, 5)$.

We plot the intercepts and draw the line that contains them. We could find a third point as a check that the intercepts were found correctly.



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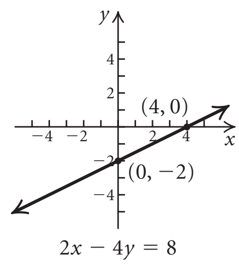


19. Graph $2x + y = 4$.

To find the x-intercept we replace y with 0 and solve for x.

$$\begin{aligned} 2x + 0 &= 4 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

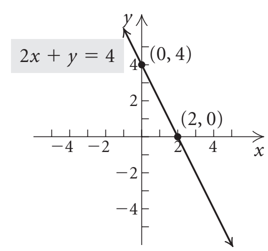
The x-intercept is $(2, 0)$.

To find the y-intercept we replace x with 0 and solve for y.

$$\begin{aligned} 2 \cdot 0 + y &= 4 \\ y &= 4 \end{aligned}$$

The y-intercept is $(0, 4)$.

We plot the intercepts and draw the line that contains them. We could find a third point as a check that the intercepts were found correctly.



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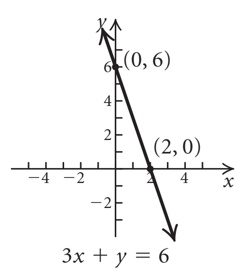


21. Graph $4y - 3x = 12$.

To find the x-intercept we replace y with 0 and solve for x.

$$\begin{aligned} 4 \cdot 0 - 3x &= 12 \\ -3x &= 12 \\ x &= -4 \end{aligned}$$

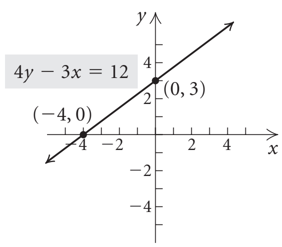
The x-intercept is $(-4, 0)$.

To find the y-intercept we replace x with 0 and solve for y.

$$4y - 3 \cdot 0 = 12$$
$$4y = 12$$
$$y = 3$$

The y-intercept is $(0, 3)$.

We plot the intercepts and draw the line that contains them. We could find a third point as a check that the intercepts were found correctly.



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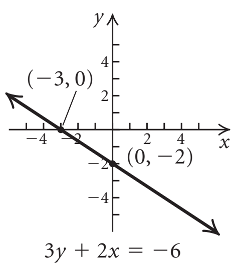


23. Graph $y = 3x + 5$.

We choose some values for x and find the corresponding y-values.

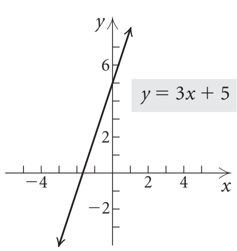
When $x = -3$, $y = 3x + 5 = 3(-3) + 5 = -9 + 5 = -4$.

When $x = -1$, $y = 3x + 5 = 3(-1) + 5 = -3 + 5 = 2$.

When $x = 0$, $y = 3x + 5 = 3 \cdot 0 + 5 = 0 + 5 = 5$

We list these points in a table, plot them, and draw the graph.

x	y	(x, y)
-3	-4	$(-3, -4)$
-1	2	$(-1, 2)$
0	5	$(0, 5)$



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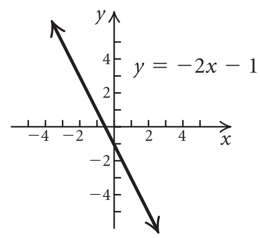
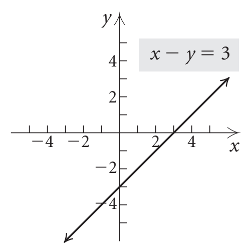


25. Graph $x - y = 3$.

Make a table of values, plot the points in the table, and draw the graph.

x	y	(x, y)
-2	-5	$(-2, -5)$
0	-3	$(0, -3)$
3	0	$(3, 0)$



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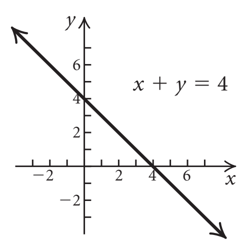
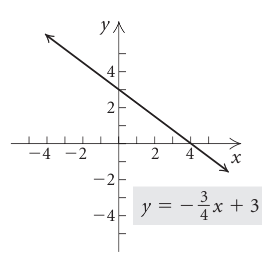


27. Graph $y = -\dfrac{3}{4}x + 3$.

By choosing multiples of 4 for x, we can avoid fraction values for y. Make a table of values, plot the points in the table, and draw the graph.

x	y	(x, y)
-4	6	$(-4, 6)$
0	3	$(0, 3)$
4	0	$(4, 0)$



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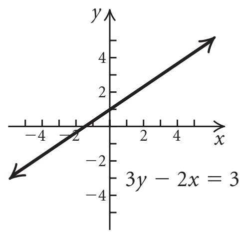


29. Graph $5x - 2y = 8$.

We could solve for y first.

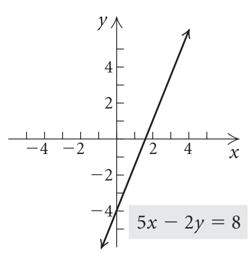
$5x - 2y = 8$

$-2y = -5x + 8$ Subtracting $5x$ on both sides

$y = \frac{5}{2}x - 4$ Multiplying by $-\frac{1}{2}$ on both sides

By choosing multiples of 2 for x we can avoid fraction values for y. Make a table of values, plot the points in the table, and draw the graph.

x	y	(x, y)
0	-4	$(0, -4)$
2	1	$(2, 1)$
4	6	$(4, 6)$



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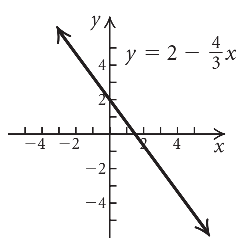
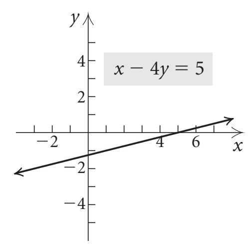


31. Graph $x - 4y = 5$.

Make a table of values, plot the points in the table, and draw the graph.

x	y	(x, y)
-3	-2	$(-3, -2)$
1	-1	$(1, -1)$
5	0	$(5, 0)$



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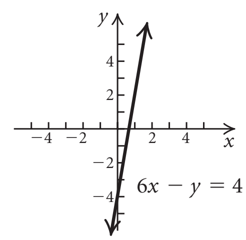
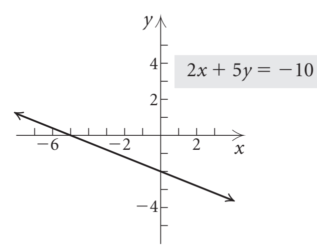


33. Graph $2x + 5y = -10$.

In this case, it is convenient to find the intercepts along with a third point on the graph. Make a table of values, plot the points in the table, and draw the graph.

x	y	(x, y)
-5	0	$(-5, 0)$
0	-2	$(0, -2)$
5	-4	$(5, -4)$



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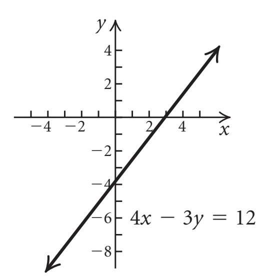
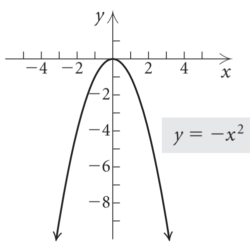


35. Graph $y = -x^2$.

Make a table of values, plot the points in the table, and draw the graph.

x	y	(x, y)
-2	-4	$(-2, -4)$
-1	-1	$(-1, -1)$
0	0	$(0, 0)$
1	-1	$(1, -1)$
2	-4	$(2, -4)$



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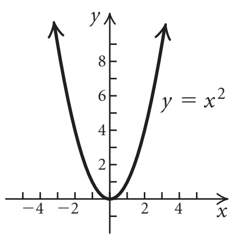
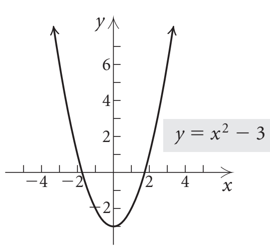


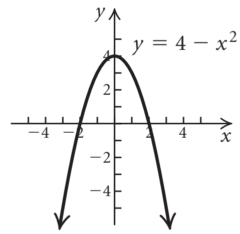
37. Graph $y = x^2 - 3$.

Make a table of values, plot the points in the table, and draw the graph.

x	y	(x, y)
-3	6	$(-3, 6)$
-1	-2	$(-1, -2)$
0	-3	$(0, -3)$
1	-2	$(1, -2)$
3	6	$(3, 6)$



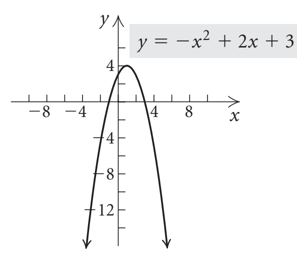
38.



39. Graph $y = -x^2 + 2x + 3$.

Make a table of values, plot the points in the table, and draw the graph.

x	y	(x, y)
-2	-5	$(-2, -5)$
-1	0	$(-1, 0)$
0	3	$(0, 3)$
1	4	$(1, 4)$
2	3	$(2, 3)$
3	0	$(3, 0)$
4	-5	$(4, -5)$



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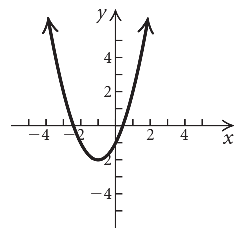


$y = x^2 + 2x - 1$

41. Graph (b) is the graph of $y = 3 - x$.

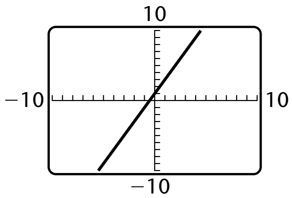
42. Graph (d) is the graph of $2x - y = 6$.

43. Graph (a) is the graph of $y = x^2 + 2x + 1$.

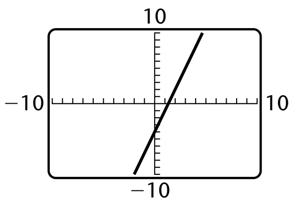
44. Graph (c) is the graph of $y = 8 - x^2$.

45. Enter the equation, select the standard window, and graph the equation.

$y = 2x + 1$

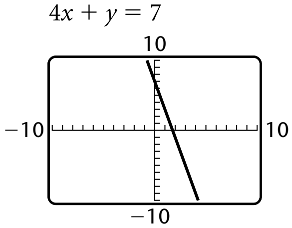


46. $y = 3x - 4$



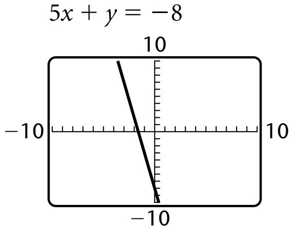
47. First solve the equation for y: $y = -4x + 7$. Enter the equation in this form, select the standard window, and graph the equation.

$4x + y = 7$



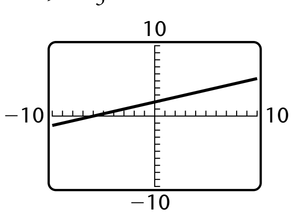
48. $5x + y = -8$, so $y = -5x - 8$.

$5x + y = -8$

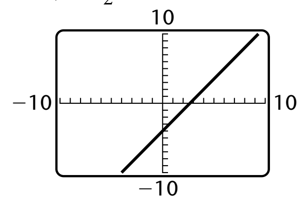


49. Enter the equation, select the standard window, and graph the equation.

$y = \frac{1}{3}x + 2$



50. $y = \frac{3}{2}x - 4$

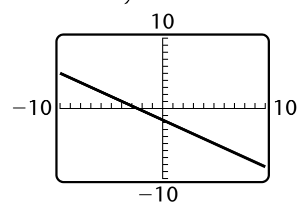


51. First solve the equation for y.

$$2x + 3y = -5$$
$$3y = -2x - 5$$
$$y = \frac{-2x - 5}{3}, \text{ or } \frac{1}{3}(-2x - 5)$$

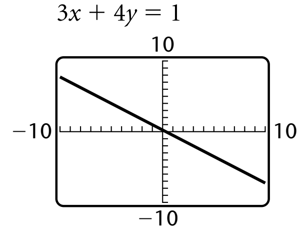
Enter the equation in "$y =$" form, select the standard window, and graph the equation.

$2x + 3y = -5$



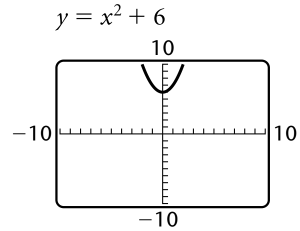
52. $3x + 4y = 1$, so $y = \dfrac{-3x + 1}{4}$, or $y = -\dfrac{3}{4}x + \dfrac{1}{4}$

$3x + 4y = 1$

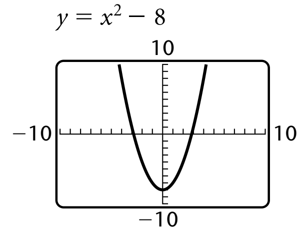


53. Enter the equation, select the standard window, and graph the equation.

$y = x^2 + 6$

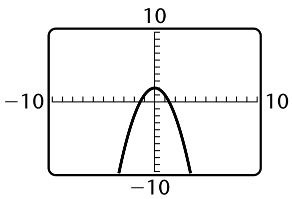


54. $y = x^2 - 8$

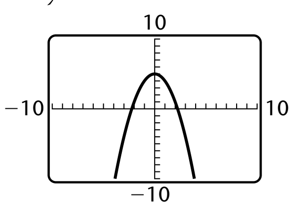


55. Enter the equation, select the standard window, and graph the equation.

$y = 2 - x^2$

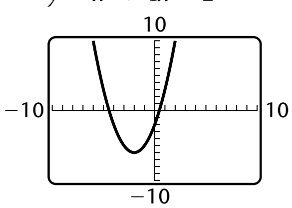


56. $y = 5 - x^2$

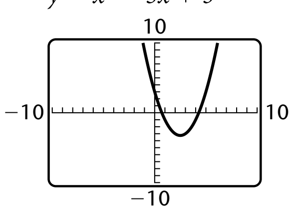


57. Enter the equation, select the standard window, and graph the equation.

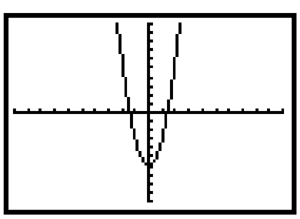
$y = x^2 + 4x - 2$



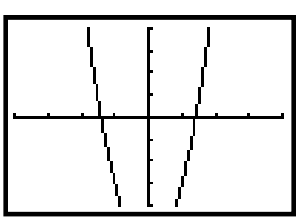
58. $y = x^2 - 5x + 3$



59. Standard window:

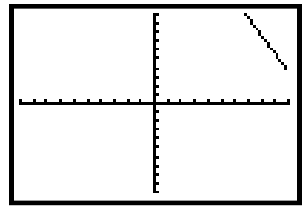


$[-4, 4, -4, 4]$

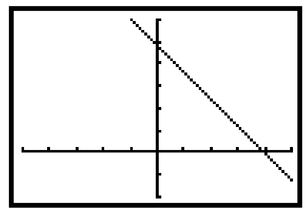


We see that the standard window is a better choice for this graph.

60. Standard window:

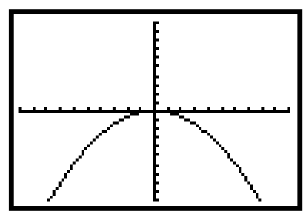


$[-15, 15, -10, 30]$, Xscl $= 3$, Yscl $= 5$

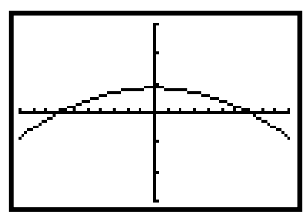


We see that $[-15, 15, -10, 30]$ is a better choice for this graph.

61. Standard window:

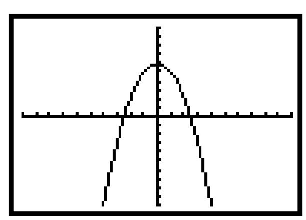


$[-1, 1, -0.3, 0.3]$, Xscl $= 0.1$, Yscl $= 0.1$

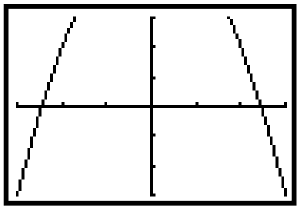


We see that $[-1, 1, -0.3, 0.3]$ is a better choice for this graph.

62. Standard window:



$[-3, 3, -3, 3]$



We see that the standard window is a better choice for this graph.

63. Either point can be considered as (x_1, y_1).

$$d = \sqrt{(4-5)^2 + (6-9)^2}$$
$$= \sqrt{(-1)^2 + (-3)^2} = \sqrt{10} \approx 3.162$$

64. $d = \sqrt{(-3-2)^2 + (7-11)^2} = \sqrt{41} \approx 6.403$

65. Either point can be considered as (x_1, y_1).

$$d = \sqrt{(-13-(-8))^2 + (1-(-11))^2}$$
$$= \sqrt{(-5)^2 + 12^2} = \sqrt{169} = 13$$

66. $d = \sqrt{(-20-(-60))^2 + (35-5)^2} = \sqrt{2500} = 50$

67. Either point can be considered as (x_1, y_1).

$$d = \sqrt{(6-9)^2 + (-1-5)^2}$$
$$= \sqrt{(-3)^2 + (-6)^2} = \sqrt{45} \approx 6.708$$

68. $d = \sqrt{(-4-(-1))^2 + (-7-3)^2} = \sqrt{109} \approx 10.440$

69. Either point can be considered as (x_1, y_1).

$$d = \sqrt{(-8-8)^2 + \left(\frac{7}{11} - \frac{7}{11}\right)^2}$$
$$= \sqrt{(-16)^2 + 0^2} = 16$$

70. $d = \sqrt{\left(\frac{1}{2} - \frac{1}{2}\right)^2 + \left(-\frac{4}{25} - \left(-\frac{13}{25}\right)\right)^2} = \sqrt{\left(\frac{9}{25}\right)^2} = \frac{9}{25}$

71. $d = \sqrt{\left[-\frac{3}{5} - \left(-\frac{3}{5}\right)\right]^2 + \left(-4 - \frac{2}{3}\right)^2}$

$$= \sqrt{0^2 + \left(-\frac{14}{3}\right)^2} = \frac{14}{3}$$

72. $d = \sqrt{\left(-\frac{11}{3} - \frac{1}{3}\right)^2 + \left(-\frac{1}{2} - \frac{5}{2}\right)^2} = \sqrt{16+9} = \sqrt{25} = 5$

73. Either point can be considered as (x_1, y_1).

$$d = \sqrt{(-4.2-2.1)^2 + [3-(-6.4)]^2}$$
$$= \sqrt{(-6.3)^2 + (9.4)^2} = \sqrt{128.05} \approx 11.316$$

74. $d = \sqrt{[0.6-(-8.1)]^2 + [-1.5-(-1.5)]^2} = \sqrt{(8.7)^2} = 8.7$

75. Either point can be considered as (x_1, y_1).

$d = \sqrt{(0-a)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$

76. $d = \sqrt{[r-(-r)]^2 + [s-(-s)]^2} = \sqrt{4r^2 + 4s^2} = 2\sqrt{r^2 + s^2}$

77. First we find the length of the diameter:

$$d = \sqrt{(-3-9)^2 + (-1-4)^2}$$
$$= \sqrt{(-12)^2 + (-5)^2} = \sqrt{169} = 13$$

The length of the radius is one-half the length of the diameter, or $\frac{1}{2}(13)$, or 6.5.

78. Radius $= \sqrt{(-3-0)^2 + (5-1)^2} = \sqrt{25} = 5$

Diameter $= 2 \cdot 5 = 10$

79. First we find the distance between each pair of points.

For $(-4, 5)$ and $(6, 1)$:

$$d = \sqrt{(-4-6)^2 + (5-1)^2}$$
$$= \sqrt{(-10)^2 + 4^2} = \sqrt{116}$$

For $(-4, 5)$ and $(-8, -5)$:

$$d = \sqrt{(-4-(-8))^2 + (5-(-5))^2}$$
$$= \sqrt{4^2 + 10^2} = \sqrt{116}$$

For $(6, 1)$ and $(-8, -5)$:

$$d = \sqrt{(6-(-8))^2 + (1-(-5))^2}$$
$$= \sqrt{14^2 + 6^2} = \sqrt{232}$$

Since $(\sqrt{116})^2 + (\sqrt{116})^2 = (\sqrt{232})^2$, the points could be the vertices of a right triangle.

80. For $(-3, 1)$ and $(2, -1)$:

$$d = \sqrt{(-3-2)^2 + (1-(-1))^2} = \sqrt{29}$$

For $(-3, 1)$ and $(6, 9)$:

$$d = \sqrt{(-3-6)^2 + (1-9)^2} = \sqrt{145}$$

For $(2, -1)$ and $(6, 9)$:

$$d = \sqrt{(2-6)^2 + (-1-9)^2} = \sqrt{116}$$

Since $(\sqrt{29})^2 + (\sqrt{116})^2 = (\sqrt{145})^2$, the points could be the vertices of a right triangle.

81. First we find the distance between each pair of points.

For $(-4, 3)$ and $(0, 5)$:

$$d = \sqrt{(-4-0)^2 + (3-5)^2}$$
$$= \sqrt{(-4)^2 + (-2)^2} = \sqrt{20}$$

For $(-4, 3)$ and $(3, -4)$:

$$d = \sqrt{(-4-3)^2 + [3-(-4)]^2}$$
$$= \sqrt{(-7)^2 + 7^2} = \sqrt{98}$$

For $(0, 5)$ and $(3, -4)$:

$$d = \sqrt{(0-3)^2 + [5-(-4)]^2}$$
$$= \sqrt{(-3)^2 + 9^2} = \sqrt{90}$$

The greatest distance is $\sqrt{98}$, so if the points are the vertices of a right triangle, then it is the hypotenuse. But $(\sqrt{20})^2 + (\sqrt{90})^2 \neq (\sqrt{98})^2$, so the points are not the vertices of a right triangle.

82. See the graph of this rectangle in Exercise 93.

The segments with endpoints $(-3,4)$, $(2,-1)$ and $(5,2)$, $(0,7)$ are one pair of opposite sides. We find the length of each of these sides.

For $(-3,4)$, $(2,-1)$:

$$d=\sqrt{(-3-2)^2+(4-(-1))^2}=\sqrt{50}$$

For $(5,2)$, $(0,7)$:

$$d=\sqrt{(5-0)^2+(2-7)^2}=\sqrt{50}$$

The segments with endpoints $(2,-1)$, $(5,2)$ and $(0,7)$, $(-3,4)$ are the second pair of opposite sides. We find their lengths.

For $(2,-1)$, $(5,2)$:

$$d=\sqrt{(2-5)^2+(-1-2)^2}=\sqrt{18}$$

For $(0,7)$, $(-3,4)$:

$$d=\sqrt{(0-(-3))^2+(7-4)^2}=\sqrt{18}$$

The endpoints of the diagonals are $(-3,4)$, $(5,2)$ and $(2,-1)$, $(0,7)$. We find the length of each.

For $(-3,4)$, $(5,2)$:

$$d=\sqrt{(-3-5)^2+(4-2)^2}=\sqrt{68}$$

For $(2,-1)$, $(0,7)$:

$$d=\sqrt{(2-0)^2+(-1-7)^2}=\sqrt{68}$$

The opposite sides of the quadrilateral are the same length and the diagonals are the same length, so the quadrilateral is a rectangle.

83. We use the midpoint formula.

$$\left(\frac{4+(-12)}{2},\frac{-9+(-3)}{2}\right)=\left(-\frac{8}{2},-\frac{12}{2}\right)=(-4,-6)$$

84. $\left(\frac{7+9}{2},\frac{-2+5}{2}\right)=\left(8,\frac{3}{2}\right)$

85. We use the midpoint formula.

$$\left(\frac{0+\left(-\frac{2}{5}\right)}{2},\frac{\frac{1}{2}-0}{2}\right)=\left(\frac{-\frac{2}{5}}{2},\frac{\frac{1}{2}}{2}\right)=\left(-\frac{1}{5},\frac{1}{4}\right)$$

86. $\left(\frac{0+\left(-\frac{7}{13}\right)}{2},\frac{0+\frac{2}{7}}{2}\right)=\left(-\frac{7}{26},\frac{1}{7}\right)$

87. We use the midpoint formula.

$$\left(\frac{6.1+3.8}{2},\frac{-3.8+(-6.1)}{2}\right)=\left(\frac{9.9}{2},-\frac{9.9}{2}\right)=$$
$$(4.95,-4.95)$$

88. $\left(\frac{-0.5+4.8}{2},\frac{-2.7+(-0.3)}{2}\right)=(2.15,-1.5)$

89. We use the midpoint formula.

$$\left(\frac{-6+(-6)}{2},\frac{5+8}{2}\right)=\left(-\frac{12}{2},\frac{13}{2}\right)=\left(-6,\frac{13}{2}\right)$$

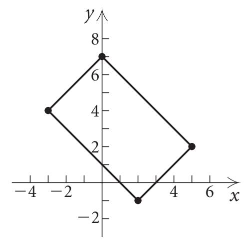
90. $\left(\frac{1+(-1)}{2},\frac{-2+2}{2}\right)=(0,0)$

91. We use the midpoint formula.

$$\left(\frac{-\frac{1}{6}+\left(-\frac{2}{3}\right)}{2},\frac{-\frac{3}{5}+\frac{5}{4}}{2}\right)=\left(\frac{-\frac{5}{6}}{2},\frac{\frac{13}{20}}{2}\right)=$$
$$\left(-\frac{5}{12},\frac{13}{40}\right)$$

92. $\left(\frac{\frac{2}{9}+\left(-\frac{2}{5}\right)}{2},\frac{\frac{1}{3}+\frac{4}{5}}{2}\right)=\left(-\frac{4}{45},\frac{17}{30}\right)$

93.



For the side with vertices $(-3,4)$ and $(2,-1)$:

$$\left(\frac{-3+2}{2},\frac{4+(-1)}{2}\right)=\left(-\frac{1}{2},\frac{3}{2}\right)$$

For the side with vertices $(2,-1)$ and $(5,2)$:

$$\left(\frac{2+5}{2},\frac{-1+2}{2}\right)=\left(\frac{7}{2},\frac{1}{2}\right)$$

For the side with vertices $(5,2)$ and $(0,7)$:

$$\left(\frac{5+0}{2},\frac{2+7}{2}\right)=\left(\frac{5}{2},\frac{9}{2}\right)$$

For the side with vertices $(0,7)$ and $(-3,4)$:

$$\left(\frac{0+(-3)}{2},\frac{7+4}{2}\right)=\left(-\frac{3}{2},\frac{11}{2}\right)$$

For the quadrilateral whose vertices are the points found above, the diagonals have endpoints

$\left(-\frac{1}{2},\frac{3}{2}\right)$, $\left(\frac{5}{2},\frac{9}{2}\right)$ and $\left(\frac{7}{2},\frac{1}{2}\right)$, $\left(-\frac{3}{2},\frac{11}{2}\right)$.

We find the length of each of these diagonals.

For $\left(-\frac{1}{2},\frac{3}{2}\right)$, $\left(\frac{5}{2},\frac{9}{2}\right)$:

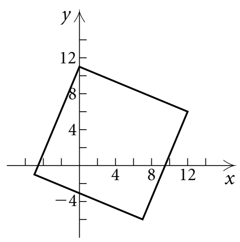
$$d=\sqrt{\left(-\frac{1}{2}-\frac{5}{2}\right)^2+\left(\frac{3}{2}-\frac{9}{2}\right)^2}$$
$$=\sqrt{(-3)^2+(-3)^2}=\sqrt{18}$$

For $\left(\frac{7}{2},\frac{1}{2}\right)$, $\left(-\frac{3}{2},\frac{11}{2}\right)$:

$$d=\sqrt{\left(\frac{7}{2}-\left(-\frac{3}{2}\right)\right)^2+\left(\frac{1}{2}-\frac{11}{2}\right)^2}$$
$$=\sqrt{5^2+(-5)^2}=\sqrt{50}$$

Since the diagonals do not have the same lengths, the midpoints are not vertices of a rectangle.

94.



For the side with vertices $(-5,-1)$ and $(7,-6)$:

$$\left(\frac{-5+7}{2},\frac{-1+(-6)}{2}\right)=\left(1,-\frac{7}{2}\right)$$

For the side with vertices $(7,-6)$ and $(12,6)$:

$$\left(\frac{7+12}{2},\frac{-6+6}{2}\right)=\left(\frac{19}{2},0\right)$$

For the side with vertices $(12,6)$ and $(0,11)$:

$$\left(\frac{12+0}{2},\frac{6+11}{2}\right)=\left(6,\frac{17}{2}\right)$$

For the side with vertices $(0,11)$ and $(-5,-1)$:

$$\left(\frac{0+(-5)}{2},\frac{11+(-1)}{2}\right)=\left(-\frac{5}{2},5\right)$$

For the quadrilateral whose vertices are the points found above, one pair of opposite sides has endpoints $\left(1,-\frac{7}{2}\right)$, $\left(\frac{19}{2},0\right)$ and $\left(6,\frac{17}{2}\right)$, $\left(-\frac{5}{2},5\right)$. The length of each of these sides is $\frac{\sqrt{338}}{2}$. The other pair of opposite sides has endpoints $\left(\frac{19}{2},0\right)$, $\left(6,\frac{17}{2}\right)$ and $\left(-\frac{5}{2},5\right)$, $\left(1,-\frac{7}{2}\right)$. The length of each of these sides is also $\frac{\sqrt{338}}{2}$. The endpoints of the diagonals of the quadrilateral are $\left(1,-\frac{7}{2}\right)$, $\left(6,\frac{17}{2}\right)$ and $\left(\frac{19}{2},0\right)$, $\left(-\frac{5}{2},5\right)$. The length of each diagonal is 13. Since the four sides of the quadrilateral are the same length and the diagonals are the same length, the midpoints are vertices of a square.

95. We use the midpoint formula.

$$\left(\frac{\sqrt{7}+\sqrt{2}}{2},\frac{-4+3}{2}\right)=\left(\frac{\sqrt{7}+\sqrt{2}}{2},-\frac{1}{2}\right)$$

96. $$\left(\frac{-3+1}{2},\frac{\sqrt{5}+\sqrt{2}}{2}\right)=\left(-1,\frac{\sqrt{5}+\sqrt{2}}{2}\right)$$

97. Square the viewing window. For the graph shown, one possibility is $[-12,9,-4,10]$.

98. Square the viewing window. For the graph shown, one possibility is $[-10,20,-15,5]$.

99.
$$\begin{aligned}(x-h)^2+(y-k)^2&=r^2\\(x-2)^2+(y-3)^2&=\left(\frac{5}{3}\right)^2 \quad \text{Substituting}\\(x-2)^2+(y-3)^2&=\frac{25}{9}\end{aligned}$$

100.
$$\begin{aligned}(x-4)^2+(y-5)^2&=(4.1)^2\\(x-4)^2+(y-5)^2&=16.81\end{aligned}$$

101. The length of a radius is the distance between $(-1,4)$ and $(3,7)$:

$$\begin{aligned}r&=\sqrt{(-1-3)^2+(4-7)^2}\\&=\sqrt{(-4)^2+(-3)^2}=\sqrt{25}=5\end{aligned}$$

$$\begin{aligned}(x-h)^2+(y-k)^2&=r^2\\ [x-(-1)]^2+(y-4)^2&=5^2\\(x+1)^2+(y-4)^2&=25\end{aligned}$$

102. Find the length of a radius:

$$r=\sqrt{(6-1)^2+(-5-7)^2}=\sqrt{169}=13$$

$$\begin{aligned}(x-6)^2+[y-(-5)]^2&=13^2\\(x-6)^2+(y+5)^2&=169\end{aligned}$$

103. The center is the midpoint of the diameter:

$$\left(\frac{7+(-3)}{2},\frac{13+(-11)}{2}\right)=(2,1)$$

Use the center and either endpoint of the diameter to find the length of a radius. We use the point $(7,13)$:

$$\begin{aligned}r&=\sqrt{(7-2)^2+(13-1)^2}\\&=\sqrt{5^2+12^2}=\sqrt{169}=13\end{aligned}$$

$$\begin{aligned}(x-h)^2+(y-k)^2&=r^2\\(x-2)^2+(y-1)^2&=13^2\\(x-2)^2+(y-1)^2&=169\end{aligned}$$

104. The points $(-9,4)$ and $(-1,-2)$ are opposite vertices of the square and hence endpoints of a diameter of the circle. We use these points to find the center and radius.

Center: $\left(\frac{-9+(-1)}{2},\frac{4+(-2)}{2}\right)=(-5,1)$

Radius: $\frac{1}{2}\sqrt{(-9-(-1))^2+(4-(-2))^2}=\frac{1}{2}\cdot 10=5$

$$\begin{aligned}[x-(-5)]^2+(y-1)^2&=5^2\\(x+5)^2+(y-1)^2&=25\end{aligned}$$

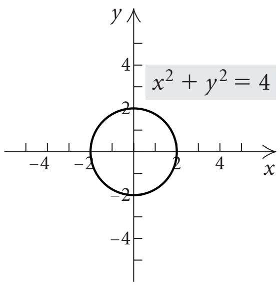
105. Since the center is 2 units to the left of the y-axis and the circle is tangent to the y-axis, the length of a radius is 2.

$$\begin{aligned}(x-h)^2+(y-k)^2&=r^2\\ [x-(-2)]^2+(y-3)^2&=2^2\\(x+2)^2+(y-3)^2&=4\end{aligned}$$

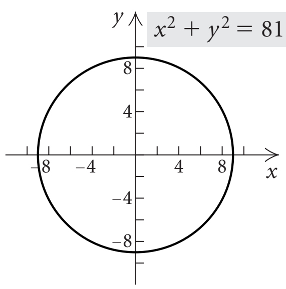
106. Since the center is 5 units below the x-axis and the circle is tangent to the x-axis, the length of a radius is 5.

$$\begin{aligned}(x-4)^2+[y-(-5)]^2&=5^2\\(x-4)^2+(y+5)^2&=25\end{aligned}$$

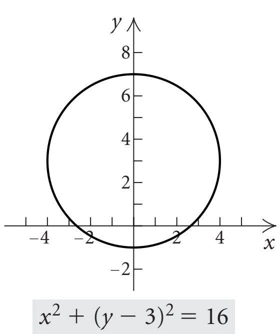
107.
$$x^2 + y^2 = 4$$
$$(x-0)^2 + (y-0)^2 = 2^2$$
Center: $(0,0)$; radius: 2



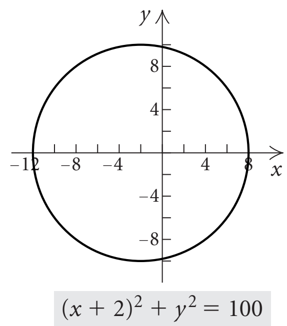
108.
$$x^2 + y^2 = 81$$
$$(x-0)^2 + (y-0)^2 = 9^2$$
Center: $(0,0)$; radius: 9



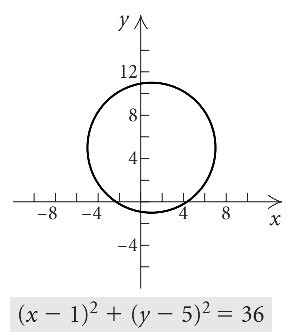
109.
$$x^2 + (y-3)^2 = 16$$
$$(x-0)^2 + (y-3)^2 = 4^2$$
Center: $(0,3)$; radius: 4



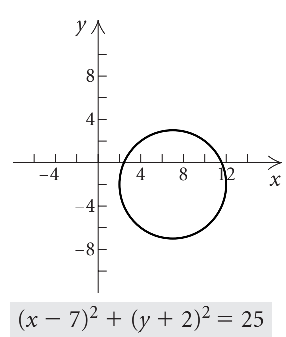
110.
$$(x+2)^2 + y^2 = 100$$
$$[x-(-2)]^2 + (y-0)^2 = 10^2$$
Center: $(-2,0)$; radius: 10



111.
$$(x-1)^2 + (y-5)^2 = 36$$
$$(x-1)^2 + (y-5)^2 = 6^2$$
Center: $(1,5)$; radius: 6



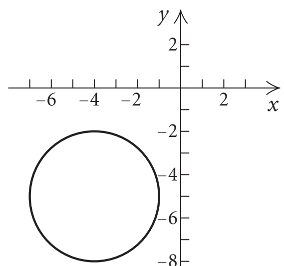
112.
$$(x-7)^2 + (y+2)^2 = 25$$
$$(x-7)^2 + [y-(-2)]^2 = 5^2$$
Center: $(7,-2)$; radius: 5



113. $(x+4)^2+(y+5)^2=9$

$[x-(-4)]^2+[y-(-5)]^2=3^2$

Center: $(-4,-5)$; radius: 3

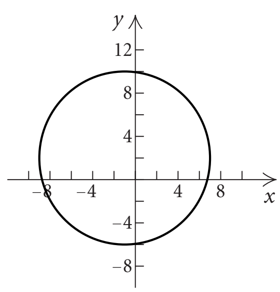


$(x+4)^2+(y+5)^2=9$

114. $(x+1)^2+(y-2)^2=64$

$[x-(-1)]^2+(y-2)^2=8^2$

Center: $(-1,2)$; radius: 8



$(x+1)^2+(y-2)^2=64$

115. From the graph we see that the center of the circle is $(-2,1)$ and the radius is 3. The equation of the circle is $[x-(-2)]^2+(y-1)^2=3^2$, or $(x+2)^2+(y-1)^2=3^2$.

116. Center: $(3,-5)$, radius: 4

Equation: $(x-3)^2+[y-(-5)]^2=4^2$, or

$(x-3)^2+(y+5)^2=4^2$

117. From the graph we see that the center of the circle is $(5,-5)$ and the radius is 15. The equation of the circle is $(x-5)^2+[y-(-5)]^2=15^2$, or $(x-5)^2+(y+5)^2=15^2$.

118. Center: $(-8,2)$, radius: 4

Equation: $[x-(-8)]^2+(y-2)^2=4^2$, or

$(x+8)^2+(y-2)^2=4^2$

119. If the point (p,q) is in the fourth quadrant, then $p>0$ and $q<0$. If $p>0$, then $-p<0$ so both coordinates of the point $(q,-p)$ are negative and $(q,-p)$ is in the third quadrant.

120. Use the distance formula:

$$d=\sqrt{(a+h-a)^2+\left(\frac{1}{a+h}-\frac{1}{a}\right)^2}=$$

$$\sqrt{h^2+\left(\frac{-h}{a(a+h)}\right)^2}=\sqrt{h^2+\frac{h^2}{a^2(a+h)^2}}=$$

$$\sqrt{\frac{h^2a^2(a+h)^2+h^2}{a^2(a+h)^2}}=\sqrt{\frac{h^2(a^2(a+h)^2+1)}{a^2(a+h)^2}}=$$

$$\left|\frac{h}{a(a+h)}\right|\sqrt{a^2(a+h)^2+1}$$

Find the midpoint:

$$\left(\frac{a+a+h}{2},\frac{\frac{1}{a}+\frac{1}{a+h}}{2}\right)=\left(\frac{2a+h}{2},\frac{2a+h}{2a(a+h)}\right)$$

121. Use the distance formula. Either point can be considered as (x_1,y_1).

$$\begin{aligned}d&=\sqrt{(a+h-a)^2+(\sqrt{a+h}-\sqrt{a})^2}\\&=\sqrt{h^2+a+h-2\sqrt{a^2+ah}+a}\\&=\sqrt{h^2+2a+h-2\sqrt{a^2+ah}}\end{aligned}$$

Next we use the midpoint formula.

$$\left(\frac{a+a+h}{2},\frac{\sqrt{a}+\sqrt{a+h}}{2}\right)=\left(\frac{2a+h}{2},\frac{\sqrt{a}+\sqrt{a+h}}{2}\right)$$

122.

$$\begin{aligned}C&=2\pi r\\10\pi&=2\pi r\\5&=r\end{aligned}$$

Then $[x-(-5)]^2+(y-8)^2=5^2$, or $(x+5)^2+(y-8)^2=25$.

123. First use the formula for the area of a circle to find r^2:

$$\begin{aligned}A&=\pi r^2\\36\pi&=\pi r^2\\36&=r^2\end{aligned}$$

Then we have:

$$\begin{aligned}(x-h)^2+(y-k)^2&=r^2\\(x-2)^2+[y-(-7)]^2&=36\\(x-2)^2+(y+7)^2&=36\end{aligned}$$

124. Let the point be $(x,0)$. We set the distance from $(-4,-3)$ to $(x,0)$ equal to the distance from $(-1,5)$ to $(x,0)$ and solve for x.

$$\begin{aligned}\sqrt{(-4-x)^2+(-3-0)^2}&=\sqrt{(-1-x)^2+(5-0)^2}\\\sqrt{16+8x+x^2+9}&=\sqrt{1+2x+x^2+25}\\\sqrt{x^2+8x+25}&=\sqrt{x^2+2x+26}\\x^2+8x+25&=x^2+2x+26\\&\quad\text{Squaring both sides}\\8x+25&=2x+26\\6x&=1\\x&=\frac{1}{6}\end{aligned}$$

The point is $\left(\frac{1}{6},0\right)$.

125. Let $(0, y)$ be the required point. We set the distance from $(-2, 0)$ to $(0, y)$ equal to the distance from $(4, 6)$ to $(0, y)$ and solve for y.

$$\begin{aligned}\sqrt{[0-(-2)]^2+(y-0)^2} &= \sqrt{(0-4)^2+(y-6)^2}\\ \sqrt{4+y^2} &= \sqrt{16+y^2-12y+36}\\ 4+y^2 &= 16+y^2-12y+36\\ &\quad \text{Squaring both sides}\\ -48 &= -12y\\ 4 &= y\end{aligned}$$

The point is $(0, 4)$.

126. We first find the distance between each pair of points.

For $(-1, -3)$ and $(-4, -9)$:

$$\begin{aligned}d_1 &= \sqrt{[-1-(-4)]^2+[-3-(-9)]^2}\\ &= \sqrt{3^2+6^2} = \sqrt{9+36}\\ &= \sqrt{45} = 3\sqrt{5}\end{aligned}$$

For $(-1, -3)$ and $(2, 3)$:

$$\begin{aligned}d_2 &= \sqrt{(-1-2)^2+(-3-3)^2}\\ &= \sqrt{(-3)^2+(-6)^2} = \sqrt{9+36}\\ &= \sqrt{45} = 3\sqrt{5}\end{aligned}$$

For $(-4, -9)$ and $(2, 3)$:

$$\begin{aligned}d_3 &= \sqrt{(-4-2)^2+(-9-3)^2}\\ &= \sqrt{(-6)^2+(-12)^2} = \sqrt{36+144}\\ &= \sqrt{180} = 6\sqrt{5}\end{aligned}$$

Since $d_1 + d_2 = d_3$, the points are collinear.

127. a) When the circle is positioned on a coordinate system as shown in the text, the center lies on the y-axis and is equidistant from $(-4, 0)$ and $(0, 2)$.

Let $(0, y)$ be the coordinates of the center.

$$\begin{aligned}\sqrt{(-4-0)^2+(0-y)^2} &= \sqrt{(0-0)^2+(2-y)^2}\\ 4^2+y^2 &= (2-y)^2\\ 16+y^2 &= 4-4y+y^2\\ 12 &= -4y\\ -3 &= y\end{aligned}$$

The center of the circle is $(0, -3)$.

b) Use the point $(-4, 0)$ and the center $(0, -3)$ to find the radius.

$$\begin{aligned}(-4-0)^2+[0-(-3)]^2 &= r^2\\ 25 &= r^2\\ 5 &= r\end{aligned}$$

The radius is 5 ft.

128. The coordinates of P are $\left(\frac{b}{2}, \frac{h}{2}\right)$ by the midpoint formula. By the distance formula, each of the distances from P to $(0, h)$, from P to $(0, 0)$, and from P to $(b, 0)$ is $\dfrac{\sqrt{b^2+h^2}}{2}$.

129.

$$\begin{array}{c|c} x^2+y^2 & 1 \\ \hline \left(\frac{\sqrt{3}}{2}\right)^2+\left(-\frac{1}{2}\right)^2 \;?& 1 \\ \frac{3}{4}+\frac{1}{4} & \\ 1 & 1 \quad \text{TRUE}\end{array}$$

$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ lies on the unit circle.

130.

$$\begin{array}{c|c} x^2+y^2 & 1 \\ \hline 0^2+(-1)^2 \;?& 1 \\ 1 & 1 \quad \text{TRUE}\end{array}$$

$(0, -1)$ lies on the unit circle.

131.

$$\begin{array}{c|c} x^2+y^2 & 1 \\ \hline \left(-\frac{\sqrt{2}}{2}\right)^2+\left(\frac{\sqrt{2}}{2}\right)^2 \;?& 1 \\ \frac{2}{4}+\frac{2}{4} & \\ 1 & 1 \quad \text{TRUE}\end{array}$$

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ lies on the unit circle.

132.

$$\begin{array}{c|c} x^2+y^2 & 1 \\ \hline \left(\frac{1}{2}\right)^2+\left(-\frac{\sqrt{3}}{2}\right)^2 \;?& 1 \\ \frac{1}{4}+\frac{3}{4} & \\ 1 & 1 \quad \text{TRUE}\end{array}$$

$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ lies on the unit circle.

133. See the answer section in the text.

Exercise Set 1.2

1. This correspondence is a function, because each member of the domain corresponds to exactly one member of the range.

2. This correspondence is a function, because each member of the domain corresponds to exactly one member of the range.

3. This correspondence is a function, because each member of the domain corresponds to exactly one member of the range.

4. This correspondence is not a function, because there is a member of the domain (1) that corresponds to more than one member of the range (4 and 6).

5. This correspondence is not a function, because there is a member of the domain (m) that corresponds to more than one member of the range (A and B).