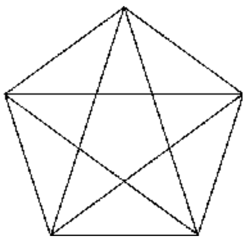
Chapter 1: Problem Solving: Strategies and Principles

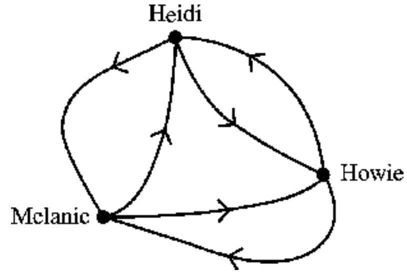
Section 1.1: Problem Solving

1. Drawings may vary.

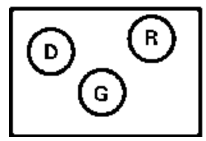
2. Drawings may vary.



3. Drawings may vary.



4. Drawings may vary.



5. Answers may vary. Let H be the hybrid automobiles, W be the windmill turbines, and S be solar energy.

6. Answers may vary. Let T be Tyrion, J be Jamie, C be Cersei, D be Daenerys, and S be Sansa.

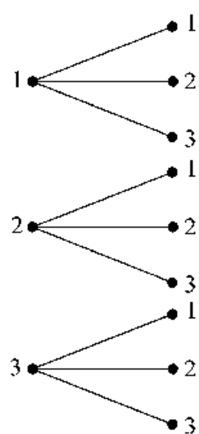
7. Answers may vary. Let s be the dollar amount invested in stocks and b be the dollar amount invested in bonds.

8. Answers may vary. Let c be the amount of calcium and p be the amount of protein.

9. Answers (order) may vary. Combinations would be HH, HT, TH, and TT.

Penny	Nickel
Heads	Heads
Heads	Tails
Tails	Heads
Tails	Tails

10. Answers (order) may vary. Pairs would be (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), and (3, 3).



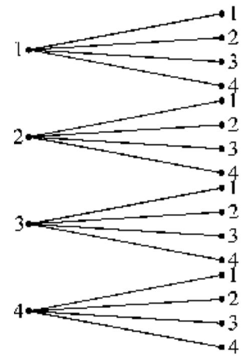
11. $2\times2\times2\times2\times2=32$; Graph not provided.

12. $6\times6=36$

13. Answers (order) may vary. Using the "Be Systematic" strategy, first list all pairs that begin with L, next all new pairs that begin with S, etc. Pairs would be LS, LB, LE, LD, SB, SE, SD, BE, BD, ED.

14. Answers (order) may vary. Routes you can take are: (Begin, A, D, H, End), (Begin, A, E, H, End), (Begin, A, E, I, End), (Begin, B, E, H, End), (Begin, B, E, I, End), (Begin, B, F, I, End), (Begin, B, F, J, End), (Begin, C, F, I, End), (Begin, C, F, J, End), and (Begin, C, G, J, End).

15. (1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)



16. (1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)

17. r_3 is the set of people who are good singers and appeared on "American Idol". r_4 is the set of people who have appeared on "American Idol" and are not good singers.

18. r_2 is the set of people who are working to reduce global warming but not striving to reduce world hunger. r_4 is the set of people who are striving to reduce world hunger but not working to reduce global warming.

19. 35, 42, 49, 56, 63; multiples of 7

20. 81, 243, 729, 2187, 6561; multiples of 3

21. bf, cd, ce, cf, cg

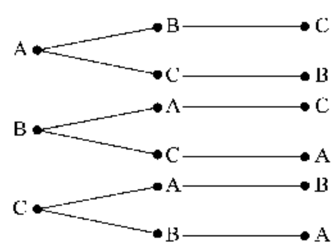
22. (2, 4), (2, 5), (2, 6), (3, 1), (3, 2)

23. 21, 34, 55, 89, 144; sum of previous two numbers

24. 19, 23, 29, 31, 37; prime numbers

25. Answers may vary.

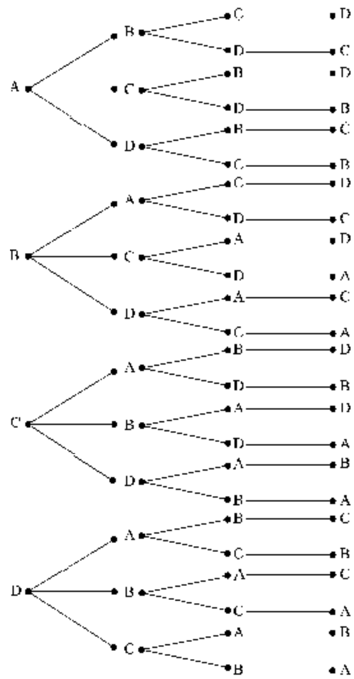
In how many ways can we line up three people for a picture? Let the people be labeled A, B, and C.



The possible orders are ABC, ACB, BAC, BCA, CAB, and CBA. There are 6 different ways.

In how many ways can we line up four people for a picture? Let the people be labeled A, B, C, and D.

25. (continued)



The possible orders are ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC, BCAD, BCDA, BDAC, BDCA, CABD, CADB, CBAD, CBDA, CDAB, CDBA, DABC, DACB, DBAC, DBCA, DCAB, and DCBA. There are 24 different ways.

26. Answers may vary. If you guess at 2 true-false questions, how many different ways can you fill in the 2 answers?

1	2
T	T
T	F
F	T
F	F

There are 4 different ways.

If you guess at 3 true-false questions, how many different ways can you fill in the 3 answers?

1	2	3
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

There are 8 different ways.

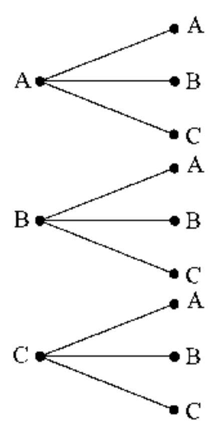
26. (continued)

If you guess at 4 true-false questions, how many different ways can you fill in the 4 answers?

1	2	3	4
T	T	T	T
T	T	T	F
T	T	F	T
T	T	F	F
T	F	T	T
T	F	T	F
T	F	F	T
T	F	F	F
F	T	T	T
F	T	T	F
F	T	F	T
F	T	F	F
F	F	T	T
F	F	T	F
F	F	F	T
F	F	F	F

There are 16 different ways.

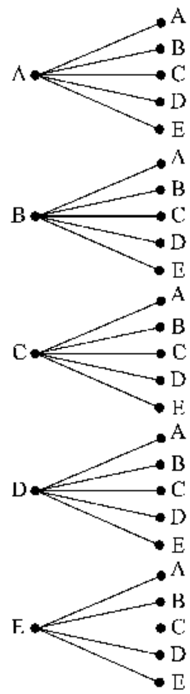
27. Answers may vary. Using the first three letters of the alphabet, how many two-letter codes can we form if we are allowed to use the same letter twice?



The possible codes are AA, AB, AC, BA, BB, BC, CA, CB, and CC. There are 9 different codes.

27. (continued)

Using the first five letters of the alphabet, how many two-letter codes can we form if we are allowed to use the same letter twice?



The possible codes are AA, AB, AC, AD, AE, BA, BB, BC, BD, BE, CA, CB, CC, CD, CE, DA, DB, DC, DD, DE, EA, EB, EC, ED, and EE. There are 25 different codes.

28. Answers may vary.

A family has three children. If we list the gender of the children, how many different lists are possible?

1	2	3
g	g	g
g	g	b
g	b	g
g	b	b
b	g	g
b	g	b
b	b	g
b	b	b

There are 8 different lists.

28. (continued)

A family has four children. If we list the gender of the children, how many different lists are possible?

1	2	3	4
g	g	g	g
g	g	g	b
g	g	b	g
g	g	b	b
g	b	g	g
g	b	g	b
g	b	b	g
g	b	b	b
b	g	g	g
b	g	g	b
b	g	b	g
b	g	b	b
b	b	g	g
b	b	g	b
b	b	b	g
b	b	b	b

There are 16 different lists.

29. Answers may vary.

An electric-blue Ferrari comes with two options: run flat tires and front heated seats. You may buy the car with any combination of the options (including none). How many different choices do you have? Let R be run flat tires and F be front heated seats. If a feature is not included, it is indicated by a "0". If it is included, it is indicated by a "1". There are 4 different choices.

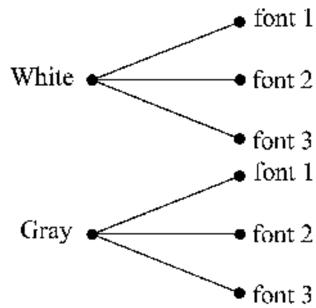
R	F
0	0
0	1
1	0
1	1

An electric-blue Ferrari comes with three options: run flat tires, front heated seats, and polished rims. You may buy the car with any combination of the options (including none). How many different choices do you have? Let R be run flat tires, F be front heated seats, and P be polished rims. If a feature is not included, it is indicated by a "0". If it is included, it is indicated by a "1". There are 8 different choices.

R	F	P
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

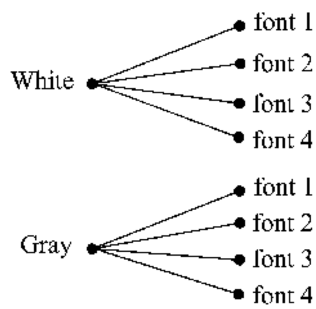
30. Answers may vary.

You have 2 different colors of paper to use and 3 different styles of font. How many different ways can you print your resume?



There are 6 different ways .

You have 2 different colors of paper to use and 4 different styles of font. How many different ways can you print your resume?



There are 8 different ways.

31. False, counterexamples may vary. June has 30 days.

32. False, counterexamples may vary. At the time this book was written, this was false because Bill Clinton is still alive.

33. False, counterexamples may vary. $\frac{1}{2}+\frac{3}{4}=\frac{2}{4}+\frac{3}{4}=\frac{5}{4}$, $\frac{1+3}{2+4}=\frac{4}{6}=\frac{2}{3}$, and $\frac{5}{4}\neq\frac{2}{3}$

34. False, counterexamples may vary. $-9<-5$ but $(-9)^2>(-5)^2$ since $81>25$.

35. False, A is the grandfather of C.

36. False, counterexamples may vary. You know your instructor and your instructor knows his/her mother, but (most likely) you don't know your instructor's mother.

37. False, counterexamples may vary. If the price of a $10.00 item is increased by 10%, its new price is $11.00. If the $11.00 item is then decreased by 10%, the new price would be $9.90, not $10.00.

38. False, counterexamples may vary. If your hourly rate of $10.00 is decreased by 20%, the new rate is $8.00. If the $8.00 rate is then increased by 20%, the new rate would be $9.60, not $10.00.

39. Explanations may vary. These two sequences do not give the same results. The question here is equivalent to asking if the algebraic statement, $x^2+5=(x+5)^2$, is true.

If we let $x=1$, we have a counterexample.

$$1^2+5\stackrel{?}{=}(1+5)^2$$
$$1+5\stackrel{?}{=}6^2$$
$$6\neq 36$$

Hence, the statement is false.

40. Explanations may vary. These two sequences do not give the same results. The question here is equivalent to asking if the algebraic statement, $x^2-y^2=(x-y)^2$, is true.
If we let $x=1$ and $y=3$, we have a counterexample.

$$1^2-3^2\stackrel{?}{=}(1-3)^2$$
$$1-9\stackrel{?}{=}(-2)^2$$
$$-8\neq 4$$

Hence, the statement is false.

41. Explanations may vary. These two sequences do give the same results. The question here is equivalent to asking if the algebraic statement, $\frac{x+y}{3}=\frac{x}{3}+\frac{y}{3}$, is true. If you think of dividing by 3 as equivalent to multiplying by $1/3$, then you can use the distributive property to prove this statement.

$$\frac{x+y}{3}=\frac{1}{3}(x+y)=\frac{1}{3}x+\frac{1}{3}y=\frac{x}{3}+\frac{y}{3}$$

42. Explanations may vary. These two sequences do give the same results. The question here is equivalent to asking if the algebraic statement, $x\cdot 5+y\cdot 5=(x+y)\cdot 5$, is true. The proof of this statement is equivalent to proving the distributive property.

43. Answers may vary. 5 is a number; $\{5\}$ is a number with braces around it. Moreover, 5 is a singly listed element, while $\{5\}$ is a set that contains the single element 5.

44. Answers may vary. The second A has a prime ($'$) on it. Moreover, A is a set (which is a subset of some universal set U), and A' is the complement of set A relative to some universal set U.

45. Answers may vary. One is uppercase and the other is lowercase. Moreover, U usually denotes the universal set, and the lower case letters are usually elements that appear in some set.

46. Answers may vary. $\{\ \}$ are different from $(\ )$. Moreover, $\{1,2\}$ is the set that contains only the elements 1 and 2. $(1,2)$ is the interval that contains all real numbers between 1 and 2 (not including 1 and 2 themselves). Note: Another interpretation of $(1,2)$ is being a point in a plane. However, with $\{1,2\}$ in this same problem, the interpretation should be relative to sets.

47. Answers may vary. The order of the numbers is different. Moreover, if you interpret $(2,3)$ and $(3,2)$ as intervals, they are both expressing the same set of numbers. The standard, however, is to express intervals (a,b) in such a way that $a<b$ (assuming a and b are real numbers). Note: Another interpretation of $(2,3)$ and $(3,2)$ is as points in the plane, and these two points would be different. They would be points that are reflected over the line $y=x$. The context of this question, however, is relative to intervals.

48. The symbol on the right is the number zero; the symbol on the left is not a number.

49. – 52. No solution provided.

53. Answers may vary. Let O be the age of the older building and Y be the age of the younger building.

Guesses for O and Y	Good Points	Weak Points
100, 221	Sum is 321.	The older is more than twice the younger.
110, 211	Sum is 321.	The older is less than twice the older.
107, 214	All conditions satisfied. We have the solution.	

The buildings are 107 and 214 years old.

54. Answers may vary. Let S be the shortest piece, M be the middle size piece, and L be the longest piece.

Guesses for S, M, and L	Good Points	Weak Points
3, 8, and 24	The longest is 3 times the middle size and the shortest is 5 less than the middle size.	Total length is less than 40.
5, 10, and 30	The longest is 3 times the middle size and the shortest is 5 less than the middle size.	Total length is more than 40.
4, 9, and 27	All conditions satisfied. We have the solution	

The three pieces should be 4, 9, and 27 inches long.

55. Answers may vary. Let T be the number of times Tom Brady threw a touchdown pass, P be the number of times Phillip Rivers threw a touchdown pass, and A be the number of times Aaron Rodgers threw a touchdown pass.

Guesses for T, P, and A	Good Points	Weak Points
30, 24, and 22	P is 2 more than A and 6 less than T.	Sum is less than 94.
40, 34, and 32	P is 2 more than A and 6 less than T.	Sum is more than 94.
36, 30, and 28	All conditions satisfied. We have the solution.	

Brady had 36 touchdowns.

56. Answers may vary. Let J be the number of home runs hit by Joc Pederson, T be the number of home runs hit by Todd Frazier and P be the number of home runs hit by Prince Fielder.

Guesses for J, T, and P	Good Points	Weak Points
30, 29 and 7	J is 1 more than T and J is 23 more than P.	Sum is less than 72.
40, 39 and 17	J is 1 more than J and J is 23 more than P.	Sum is more than 72.
32, 31 and 9	All conditions satisfied. We have the solution.	

Pederson had 32 hits.

57. Answers may vary. Let A be the amount invested at 8% and B be the amount invested at 6%.

Guesses for A and B	Good Points	Weak Points
\$3,000 and \$5,000	Sum is \$8,000.	Return is less than \$550.
\$4,000 and \$4,000	Sum is \$8,000.	Return is more than \$550.
\$3,500 and \$4,500	All conditions satisfied. We have the solution	

Heather invested \$3,500 at 8% and \$4,500 at 6%.

58. Answers may vary. Let A be the amount invested at 11% and B be the amount invested at 8%.

Guesses for A and B	Good Points	Weak Points
\$7,000 and \$2,000	Sum is \$9,000.	Return is less than \$936.
\$7,500 and \$1,500	Sum is \$9,000.	Return is more than \$936.
\$7,200 and \$1,800	All conditions satisfied. We have the solution.	

Carlos invested \$7,200 at 11% and \$1,800 at 8%.

59. Answers may vary. Let A be the number of administrators, S be the number of students, and F be the number of faculty members.

Guesses for A, S, and F	Good Points	Weak Points
10, 2, and 7	A is 5 times S and F is 5 more than S.	There are less than 26 people.
20, 4, and 9	A is 5 times S and F is 5 more than S.	There are more than 26 people.
15, 3, and 8	All conditions satisfied. We have the solution.	

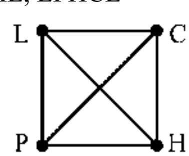
There are 3 students.

60. Answers may vary. Let S be the number of senior citizens, Y be the number of young adults, and M be the number of middle-aged adults.

Guesses for S, Y, and M	Good Points	Weak Points
18, 6, and 9	S is 3 times Y and M is one-half S.	There are less than 55 people.
36, 12, and 18	S is 3 times Y and M is one-half S.	There are more than 55 people.
30, 10, and 15	All conditions satisfied. We have the solution.	

There are 30 senior citizens.

61. LCHPL, LCPHL, LHCPL, LHPCL, LPCHL, LPHCL



62. $9\times8\times7\times6\times5\times4\times3\times2\times1=362,880$

63. PN, PD, PQ, ND, NQ, DQ

64. PND, PNQ, PDQ, NDQ, PNDQ

65. The possible schedules are given in the table below.

Math	English	Sociology	Art History
9	11	12	10
9	12	10	11
10	9	12	11
10	11	12	9
12	9	10	11
12	11	10	9

66. The possible schedules are given in the table below.

Math	English	Sociology	Art History
9	11	12	10
9	12	10	11
12	9	10	11

67. The possible schedules are given in the table below.

Math	English	Art History
9	11	10
9	12	10
9	12	11
10	9	11
10	11	9
10	12	9
10	12	11
12	9	10
12	9	11
12	11	9
12	11	10

68. The possible schedules are given in the table below.

Math	English	Sociology	Art History
9	11	12	10
9	12	11	10
10	9	12	11
10	11	12	9
10	12	11	9
12	9	11	10

69. 79; The top and bottom rows will both have 21 tiles, the middle row will have one tile, and the remaining 18 rows will have 2 tiles each.

$$2\times 21+1\times 1+18\times 2=79$$

70. 48; The top and bottom rows will both have 1 tile, and the remaining 23 rows will have 2 tiles each.

$$2\times 1+23\times 2=48$$

71. You will pay more, in total, over the two years if the 8% raise occurs first. As an example, if tuition is $100 and the 8% raise occurs first, you will pay $100+0.08(100)=\$108$ the first year and $108+0.05(108)$ $=\$113.40$ the second year, for a total of $221.40. If the 5% raise occurs first, you will pay $100+0.05(100)=\$105$ the first year and $105+0.08(105)=\$113.40$ the second year, for a total of $218.40. Note that in either case, you would pay the same tuition during the second year.

72. The board would not follow its mandate. As an example, if tuition is $100, you will pay $100+0.02(100)=\$102$ after the first increase, $102+0.03(102)=\$105.06$ after the second increase, and $105.06+0.05(105.06)=\$110.31$ after the third increase, which is an approximately 10.3% increase.

73. – 76. Answers will vary.

77. $6\times6\times6=216$

78. $12\times12\times12=1{,}728$

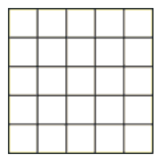
79. (41, 43), (59, 61); Pairs of sequential primes that differ by 2.

80. (23, 31), (29, 37); The first numbers are the primes in order, the second numbers come from taking the second prime number after the first.

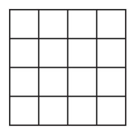
81. There are a total of 55 squares.

1×1	25
2×2	16
3×3	9

4×4	4
5×5	1



82. a) 12
b) 6
c) 100



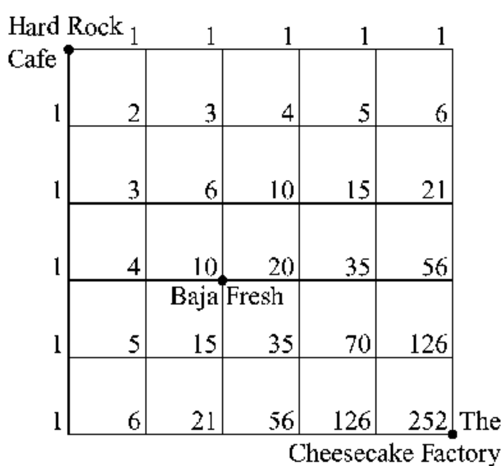
1×1	16
2×2	9
3×3	4
4×4	1

1×2	12
1×3	8
1×4	4
2×3	6

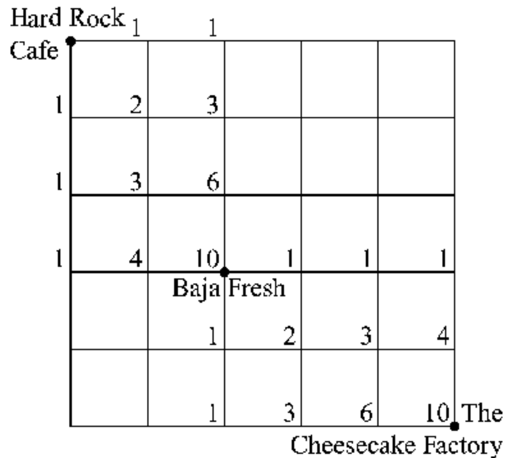
2×4	3
3×4	2
2×1	12
3×1	8

4×1	4
3×2	6
4×2	3
4×3	2

83. As you look at the intersections, you'll see that there is a pattern as to how many routes can be created as you leave the Hard Rock Cafe on the way to The Cheesecake Factory. To choose direct routes, you must always be traveling down and/or to the right. The numbers indicate how many ways there are to get to the intersection below and to the right of the number. For example, the "2" below and to the right of the Hard Rock Cafe indicates there are two ways to arrive at that intersection, one by going right, then down, and another by going down, and then right. There are a total of 252 possible routes.

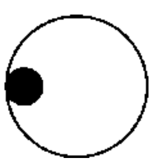


84. As you look at the intersections, you'll see that there is a pattern as to how many routes can be created as you leave the Hard Rock Cafe on the way to Baja Fresh and as you leave Baja Fresh on the way to The Cheesecake Factory. To choose direct routes, you must always be traveling down and/or to the right. The numbers indicate how many ways there are to get to the intersection below and to the right of the number. Since there are 10 ways to go from the Hard Rock Cafe to Baja Fresh and 10 ways to go from the Baja Fresh to The Cheesecake Factory , there are $10 \times 10 = 100$ possible routes.

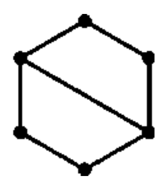


Section 1.2: Inductive and Deductive Reasoning

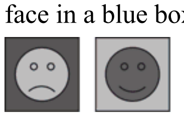
1. inductive
2. deductive
3. deductive
4. inductive
5. inductive
6. deductive
7. deductive
8. inductive
9. inductive
10. deductive
11. 16
12. 32
13. 96
14. 1,215
15. 21
16. 0.101010
17.



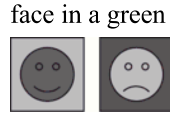
18.



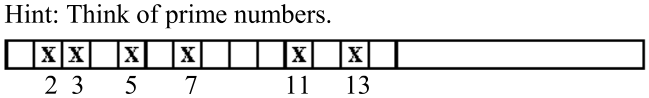
19. A blue face in a green box followed by a red face in a blue box.



20. A red face in a blue box followed by a blue face in a green box.



21. Hint: Think of prime numbers.



22. Hint: Think of the Fibonacci sequence.

X	X	X		X			X					X								X	
1	2	3		5			8					13								21	

23. $3+13$ or $5+11$

24. $5+13$ or $7+11$

25. $3+17$ or $7+13$

26. $3+23$, $7+19$, or $13+13$

27. $1+2+3+4+5=\dfrac{5\times 6}{2}$, $1+2+3+4+5+6=\dfrac{6\times 7}{2}$

28. $2+4+6+8+10=5\times 6$, $2+4+6+8+10+12=6\times 7$

29. $1+3+5+7+9=25$, $1+3+5+7+9+11=36$

30. $\dfrac{1}{1\times 2}+\dfrac{1}{2\times 3}+\dfrac{1}{3\times 4}+\dfrac{1}{4\times 5}+\dfrac{1}{5\times 6}=\dfrac{5}{6}$, $\dfrac{1}{1\times 2}+\dfrac{1}{2\times 3}+\dfrac{1}{3\times 4}+\dfrac{1}{4\times 5}+\dfrac{1}{5\times 6}+\dfrac{1}{6\times 7}=\dfrac{6}{7}$

31. $1+4+9+16=30$

32. $1+4+9+16+25+36=91$

33. a) The total of all the numbers in the square is $1+2+3+\ldots+16=136$.

b) The total of the numbers for each row, column, and diagonal would be $136/4=34$.

c) One can deduce the missing numbers to yield the following.

7	6	12	9
10	11	5	8
13	16	2	3
4	1	15	14

34. a) The total of all the numbers in the square is $1+2+3+\ldots+16=136$.

b) The total of the numbers for each row, column, and diagonal would be $136/4=34$.

c) One can deduce the missing numbers to yield the following.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

35.

	1	2	3	4
A	1	4	3	2
B	3	2	8+ 1	4
C	2	1- 1	4	1- 3
D	7+ 4	3	2	1

36.

	1	2	3	4
A	3- 1	9+ 4	2	3
B	4	2	3	1
C	2	3	1	9+ 4
D	2- 3	1	4	2

37. Yes; there are many ways to do this. Try to rewrite the puzzle in various ways with some numbers or clues missing to see if you can still solve it.

38. Yes; there are many ways to do this. Try to rewrite the puzzle in various ways with some numbers or clues missing to see if you can still solve it.

39. Adriana (political issues), Caleb (solar power), Ethan (water conservation), and Julia (recycling)

40. Jessica (third), Serena (second), Andre (fourth), and Emily (first)

41. 36644633; Reversing the number in the first pair yields the relationship $\begin{array}{cccccccc} G & G & A & G & L & L & G & A \\ 6 & 6 & 4 & 6 & 3 & 3 & 6 & 4 \end{array}$, so G, A, and L correspond to 6, 4, and 3, respectively. Assigning these values to the letters in the second word yields $\begin{array}{cccccccc} L & L & G & A & A & G & G & L \\ 3 & 3 & 6 & 4 & 4 & 6 & 6 & 3 \end{array}$. Reversing the order of the resulting number leads to the final value of 36644633.

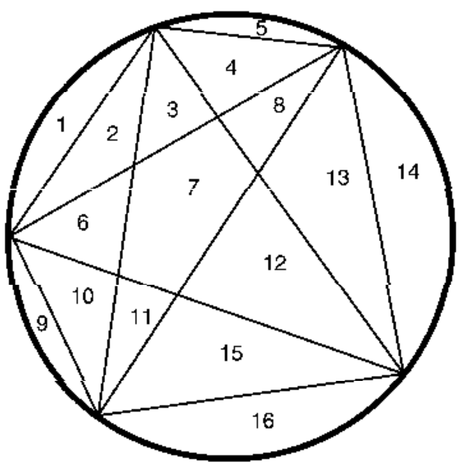
42. leader; Froofrug Merduc represents "Where is your"

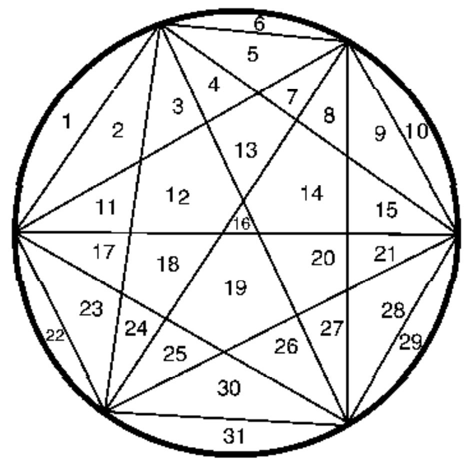
43. 986763; reverse the numbers and delete one number from any pair of numbers.

44. 20; the numbers are found by repeatedly adding 6 then subtracting 2.

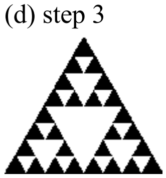
45. Answers may vary. Possible responses include that Sharifa would begin by visiting one of the six branches and then visit the five original cities in 120 ways. The total number of ways she could make her visits is $6 \times 120 = 720$ ways. The same reasoning would lead to $7 \times 720 = 5040$ ways for seven cities.

46. Reasons will vary.

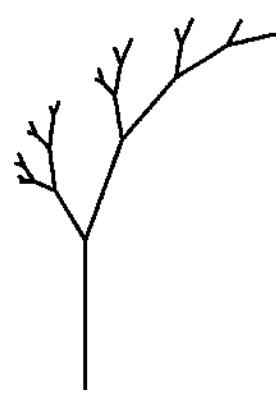




47. (d) step 3



48.



49. By looking at examples, inductive reasoning leads us to make conjectures which we then try to prove.

50. Consider examples; Look for patterns; Make a conjecture; Use deductive reasoning to prove the statement.

51. – 54. Answers will vary.

55. 53, 107, 213; Next term is the previous term plus twice the term before the previous term.

56. 5461, 21845, 87381; Next term is four times the previous term plus one.

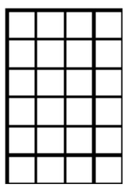
57. 47, 76, 123; Next term is sum of two previous terms.

58. 150, 298, 598; Next term is the previous term plus twice the term before the previous term.

59. There are a total of $2+6+12=20$ squares.

1×1	12
2×2	6
3×3	2

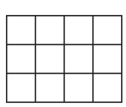
60. There are a total of $3+8+15+24=50$ squares.



1×1	24
2×2	15
3×3	8
4×4	3

61. a) There are a total of 60 rectangles.

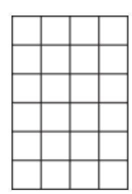
1×1	12
1×2	9
1×3	6
1×4	3
2×1	8
2×2	6
2×3	4
2×4	2
3×1	4
3×2	3
3×3	2
3×4	1



b) There are a total of 210 rectangles.

1×1	24
1×2	18
1×3	12
1×4	6
2×1	20
2×2	15
2×3	10
2×4	5
3×1	16
3×2	12
3×3	8
3×4	4

4×1	12
4×2	9
4×3	6
4×4	3
5×1	8
5×2	6
5×3	4
5×4	2
6×1	4
6×2	3
6×3	2
6×4	1

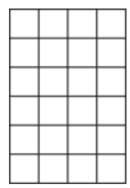


62. Answers may vary. One can count the rectangles in a systematic way like in Exercise 61. One can see that there is a pattern to the counting. Firstly, one needs to realize that the sum of the first n natural numbers is $1+2+3+...+n=\dfrac{n(n+1)}{2}$. In Exercise 61, the first four natural numbers occurred in the last four entries. Looking above these entries, you can see multiples of these numbers. One can view the sum of all the entries as

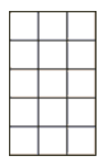
$$(1+2+3+4)+2\cdot(1+2+3+4)+3\cdot(1+2+3+4)+4\cdot(1+2+3+4)+5\cdot(1+2+3+4)+6\cdot(1+2+3+4)$$
$$=(1+2+3+4+5+6)\cdot(1+2+3+4)=\frac{6\cdot 7}{2}\cdot\frac{4\cdot 5}{2}=21\cdot 10=210.$$

In general, the number of rectangles of all types would be $\dfrac{m(m+1)}{2}\cdot\dfrac{n(n+1)}{2}$ for an $m\times n$ rectangle. For the case of a 10×6 rectangle, we would have $\dfrac{10\cdot(10+1)}{2}\cdot\dfrac{6\cdot(6+1)}{2}=55\cdot 21=1{,}155$ rectangles of all types.

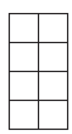
63. The base is a 6×4 rectangle. If you consider the diagram below as the base, we have $6\times 4=24$ baseballs. In order to build the next level, we are looking for the number of places in which four baseballs (squares) meet. There are $5\times 3=15$ such places.



For the next level we would have $4\times 2=8$ meeting places.



For the last level we would have $3\times 1=3$ meeting places.



This yields a total of $6\times 4+5\times 3+4\times 2+3\times 1=24+15+8+3=50$ baseballs.

64. Using the same reasoning as in Exercise 63, we have $7\times 5+6\times 4+5\times 3+4\times 2+3\times 1=$ $35+24+15+8+3=85$ baseballs.

65. – 66. No solution provided.

67. In this trick, you will always get the result three.

a) Call the number n.

b) $3n$

c) $3n+9$

d) $\dfrac{3n+9}{3}=\dfrac{3(n+3)}{3}=n+3$

e) $(n+3)-n=n+3-n=3$

68. In this trick, you will always get a result that is the number that you started with.

a) Call the number n.

b) $5n$

c) $5n+20$

d) $\dfrac{5n+20}{5}=\dfrac{5(n+4)}{5}=n+4$

e) $(n+4)-4=n+4-4=n$

Section 1.3: Estimation

1. $20+40+190+40=290$
2. $20+30+10+90=150$
3. $35-15=20$
4. $110-70=40$
5. $5\times 16=80$
6. $1000\times 15=15{,}000$
7. $18/3=6$
8. $2100/70=30$
9. $0.1\times 800=80$
10. $0.001\times 4000=4$
11. $9\%\times 1000=0.09\times 1000=90$
12. $20\%\times 700=0.20\times 700=140$
13. $4\times 5\times 6=120$ miles; The estimate is larger than the exact answer. The exact answer is 111 miles.
14. $18\times \$1.00=\$18.00$; The estimate is larger than the exact answer. The exact answer is \$16.92.
15. $325/50=6.5$ more hours, 7:30 PM; The estimate is earlier than the actual time. The actual answer is 7:50 PM.
16. $\$60.00/15=\$4.00$ per gallon; The estimate is less than the actual answer. The actual answer is \$3.89 per gallon.
17. $\$120.00\times 15\%=\$120.00\times 0.15=\$18.00$; The estimate is more than the exact answer. The exact answer is \$17.77.
18. $\$75.00/3=\$25.00$; The estimate is less than the exact answer. The exact answer is \$25.46.
19. $(3\times \$3.00)+(4\times \$1.50)+\$3.00=\$9.00+\$6.00+\$3.00=\$18.00$; The estimate is larger than the exact answer. The exact answer is \$17.20.
20. $\$1{,}400\times 5\%=\$1{,}400\times 0.05=\$70.00$; The estimated total is $\$1{,}400.00+\$70.00=\$1{,}470.00$. The estimate is less than the exact answer. The exact answer is $\$1{,}389.00+\$83.34=\$1{,}472.34$.
21. It seems safe. Alicia probably weighs less than 200 pounds, so that leaves $2{,}300-200=2{,}100$ pounds for the 21 students. They probably each weigh less than 100 pounds.
22. NFL linemen usually weigh at least 300 pounds. So the eight linemen would weigh at least $8\times 300=2{,}400$ pounds, which is above the elevator's capacity.
23. $\$40{,}000\times 4\%=\$40{,}000\times 0.04=\$1{,}600$; The estimate is larger than the exact answer. The exact answer is \$1324.40.
24. $(25\times 5)+15=125+15=140$; $42{,}000\div 140=300$ weeks; The estimate is less than the actual answer. The actual answer is 326 weeks.
25. $\frac{1{,}000}{1}=1{,}000$ times greater; The estimate is very close to the actual answer. The actual answer is 996.67 times greater.
26. $\frac{400}{100}=4$ times greater; The estimate is very close to the actual answer. The actual answer is 4.01 times greater.

27 Her total expenses are about \$100 per month; $100/7\approx 14$ $12\times \$14=\$168$; The estimate is larger than the actual answer. The actual answer is \$163 (you round to nearest dollar on deductions).

28. $\$20{,}000\times 0.025=\$500$, $\$500\times 20\%=\$500\times 0.20=\$100$; The estimate is less than the actual answer. The actual answer is \$104 (you round to nearest dollar on deductions).

29. Answers will vary. Actual answers are: Male high school graduates; \$44,300; Females with associates degrees; \$42,000
30. Answers will vary. Actual answer is \$17,200.
31. college and professional degrees, high school graduates
32. Answers will vary. Actual answer is \$47,300.
33. category 2
34. 500; $0.21 \times 2,309 \approx 485$
35. response 2
36. 139; $0.06 \times 2309 \approx 139$
37. Estimated answers may vary. Actual value is 69.3%.
38. Slightly decreases.
39. 2014; Estimated answers may vary. Actual value is 84.8%.
40. almost six times
41. Estimated answers may vary. The exact answer is $2165 \times 0.404 = 874.66$, or \$874.66 billion.
42. Estimated answers may vary. The exact answer is $2165 \times 0.432 = 935.28$, or \$935.28 billion.
43. Estimated answers may vary. The exact answer is $2165 \times 0.072 = 155.88$ or \$155.88 billion.
44. Estimated answers may vary. The exact answer is $2165 \times (0.404 + 0.432) = 2165 \times 0.836 = 1809.94$, or \$1,809.94 billion.
45. Estimated answers may vary. The actual number of immigrants was $705,361 \times 0.302 \approx 213,019$.
46. Estimated answers may vary. The actual number of immigrants was $705,361 \times 0.132 \approx 93,108$.
47. Estimated answers may vary. The actual percentage is $130,661/705,361 \approx 0.185$, or 18.5%.
48. Estimated answers may vary. $41,034/705,361 \approx 0.058$, or 5.8%.

49. – 54. Answers will vary.

55. Answers may vary. The amount of lawn that needs to be fertilized is represented by the size of the lot, less the non-grassy areas such as the garden, driveway and house. If you divide the grassy area into rectangles, you get $96 \cdot 169 - 96 \cdot 30 - 65 \cdot 28 - 18 \cdot 65 = 16,224 - 2,880 - 1,820 - 1,170 = 10,354$ square feet. They need $10,354/5000 \approx 2.07$, or slightly over two bags of fertilizer.

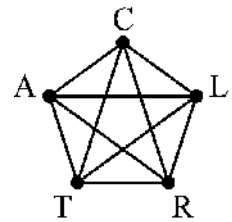
    $96 \cdot 169 - 96 \cdot 30 - 65 \cdot 28 - 18 \cdot 65 = 16,224 - 2,880 - 1,820 - 1,170 = 10,354$
56. Answers may vary. For our estimate we will not consider areas such as windows and doors that may not be painted. They need to cover exactly $7.75(2 \cdot 18.5 + 2 \cdot 11) = 7.75 \cdot 59 = 457.25$ square feet twice for a total of 914.5 square feet. They will need to purchase $914.5/200 \approx 4.57$, or 5 gallons of paint (no partial gallons).

57. – 60. No solution provided.

Chapter Review Exercises

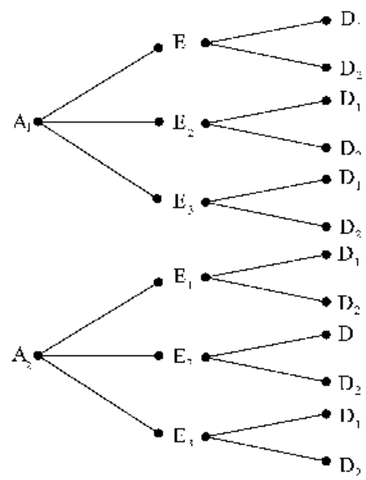
1. Understand the problem; devise a plan; carry out your plan; check your answer.
2. An example that shows a conjecture is false.

3. 10; Let A, C, L, R, and T represent Amber, Chris, Lawrence, Remy, and Travis.



The pairs are AC, AL, AR, AT, CL, CR, CT, LR, LT, and RT.

4. Answers may vary. At a restaurant, you have 2 appetizers, 3 entrees, and 2 desserts. How many different meals can you choose if you select one appetizer, one entrée, and one desert?



There are twelve different meals possible.

5. Answers may vary. Let P be the number of hours Picaboo worked as a stock person and I be the number of hours she worked as a ski instructor.

Guesses for P and I	Good Points	Weak Points
9 and 11	Sum is 20.	Amount earned is less than \$141.20.
7 and 13	Sum is 20.	Amount earned is more than \$141.20.
8 and 12	All conditions satisfied. We have the solution.	

6. false; Counterexamples may vary.

$$\frac{1}{2}+\frac{3}{4}=\frac{2}{4}+\frac{3}{4}=\frac{5}{4},\ \frac{1+3}{2+4}=\frac{4}{6}=\frac{2}{3},\ \text{and}\ \frac{5}{4}\neq\frac{2}{3}$$

7. No solution provided.

8. Answers may vary. Possible answers include: Inductive reasoning is the process of drawing a general conclusion by observing a pattern in specific instances. In deductive reasoning, we use accepted facts and general principles to arrive at a specific conclusion.

9. a) deductive
   b) inductive

10. a) 27; Add five to the previous number.
    b) 47; Add the previous two numbers.

11.

		X
		X

12. $5+43,\ 7+41,\ 11+37,\ 17+31,$ or $19+29$

13. In this trick, you will always get twice the number you started with.
  a) Call the number n.
  b) $8n$
  c) $8n+12$
  d) $\frac{8n+12}{4}=\frac{4(2n+3)}{4}=2n+3$
  e) $(2n+3)-3=2n+3-3=2n$

14. a) 46,000
  b) 28,000

15. a) $210-60=150$
  b) $6\times 15=90$

16. Estimated time left to travel would be $150/50=3$ hours, arriving at 7:00 PM.

17. a) Estimated answers may vary. The actual difference is 60 milligrams.
  b) a little more than twice as much.
  c) approximately four
  d) the gourmet coffee

Chapter Test

1. Answers will vary.

2. a) true; adding fractions with like denominators
  b) false; $\frac{3}{4+5}\neq\frac{3}{4}+\frac{3}{5}$

3. Answers may vary. Let W be the number of *Wii Sports* sold, S be the number of *Super Mario Brothers* sold, and P be the number of *Pokémons* sold.

Guesses for W, S, and P	Good Points	Weak Points
36, 29, and 20	*Super Mario Brothers* sold 9 more than *Pokémon* and 7 less than *Wii Sports*.	Total number is less than 118.
56, 49, and 40	*Super Mario Brothers* sold 9 more than *Pokémon* and 7 less than *Wii Sports*.	Total number is more than 118.
47, 40, and 31	All conditions satisfied. We have the solution.	

47 million *Wii Sports*, 40 million *Super Mario Brothers*, and 31 million *Pokémons* were sold.

4. a) 4,320
  b) 2,280

5. a) 36,000
  b) 36,500

6. If two terms are similar but sounds slightly different, they usually do not mean exactly the same thing.

7. Answers may vary. Possible answers include: Inductive reasoning is the process of drawing a general conclusion by observing a pattern in specific instances. In deductive reasoning, we use accepted facts and general principles to arrive at a specific conclusion.

8. a) inductive
   b) deductive

9. Mathematical ideas can be understood verbally, graphically, and through examples.

10. Answers will vary. One estimate is $\left(\frac{\$600+\$200}{3}\right)\cdot 12=(\$800)\cdot 4=\$3,200$. The true value is \$3,220.

11. $1+1^2=2,\ 2+2^2=6,\ 6+3^2=15,\ 15+4^2=31,\ 31+5^2=56,\ 56+6^2=92,$ and $92+7^2=141$

12. cde, cdf, cdg

13.

	X	X	
		X	X

14. $7+53=60,\ 13+47=60,\ 19+41=60,\ 23+37=60,$ or $29+31=60$

15. False; suppose the laptop costs \$1,000. After the 10% discount, the laptop would cost \$900. If that price is increased by 10%, the laptop would cost \$990, not \$1,000.

16. In this trick, you will always get twice the original number.
    a) Call the number n.
    b) $4n$
    c) $4n+40$
    d) $\frac{4n+40}{2}=\frac{2(2n+20)}{2}=2n+20$
    e) $(2n+20)-20=2n+20-20=2n$

Chapter 2: Set Theory: Using Mathematics to Classify Objects

Section 2.1: The Language of Sets

1. $\{10, 11, 12, 13, 14, 15\}$
2. $\{f, g, h, i, j\}$
3. $\{17, 18, 19, 20, 21, 22, 23, 24, 25\}$
4. $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$
5. $\{4, 8, 12, 16, 20, 24, 28\}$
6. $\{7, 9, 11, 13, 15, 17, 19\}$
7. {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}
8. {New Hampshire, New Jersey, New Mexico, New York}
9. $\varnothing$
10. $\{11, 13, 15, 17, 19, 21, 23, 25\}$
11. $\varnothing$
12. {Alaska, Hawaii}
13. Answers may vary. Possible answers include $\{x : x$ is a multiple of 3 between 3 and 12 inclusive$\}$.
14. $\{x : x$ is a color of the rainbow$\}$
15. $\{28, 29, 30, 31\}$
16. $\{-1, -2, -3, \ldots\}$
17. {January, February, March, April, May, June, July, August, September, October, November, December}
18. $\{x : x$ is a sign of the Zodiac$\}$
19. Answers may vary. Possible answers include $\{101, 102, 103, \ldots\}$.
20. Answers may vary. Possible answers include $\{x : x$ is an even natural number$\}$.
21. Answers may vary. Possible answers include $\{x : x$ is an even natural number between 1 and 101$\}$.
22. Answers may vary. Possible answers include $\{3, 6, 9, 12, 15, \ldots\}$.
23. well defined
24. well defined
25. not well defined
26. not well defined
27. not well defined
28. well defined
29. well defined
30. not well defined
31. $\notin$
32. $\in$
33. $\in$
34. $\notin$
35. $\notin$
36. $\notin$
37. $\in$
38. $\in$
39. $\notin$
40. $\notin$
41. $\in$
42. $\notin$; {Florida} is a subset, not an element.
43. 6
44. 11
45. 0
46. 48
47. 4
48. 5
49. 2 elements; $\{1, 2\}, \{1, 2, 3\}$
50. 4 elements; $\{1\}, \varnothing, 0, \{0\}$

51. 1 element; $\{\{\varnothing\}\}$

52. 4 elements; $\{1\}, \{2\}, \{3\}, \{1, 2, 3\}$

53. finite

54. finite

55. infinite

56. finite

57. Answers may vary. Possible answers include 4.5.

58. Answers may vary. Possible answers include the King(Queen) of England.

59. Answers may vary. Possible answers include Sony.

60. Answers may vary. Possible answers include a frog.

61. Answers may vary. Possible answers include Angela Merkel.

62. Answers may vary. Possible answers include Kia.

63. Answers may vary. Possible answers include Sunday.

64. Answers may vary. Possible answers include California.

65. $\{x : x \text{ is a humanities elective}\}$

66. $\{x : x \text{ does not satisfy a world culture requirement}\}$

67. $\{\text{History012, History223, Geography115, Anthropology111}\}$

68. $\{\text{History012, English010, English220, Psychology200}\}$

69. $L = \{\text{AZ, FL, GA, LA, NJ, NM, TX, VA}\}$

70. $G = \{\text{CA, MN, NY, PA}\}$

71. $\{x : x \text{ is a state with the price of gasoline above \$2.35}\}$

72. $\{x : x \text{ is a state with price of gasoline below \$2.10}\}$

73. $M = \{\text{jogging, jumping rope}\}$

74. $L = \{\text{calisthenics, leisure cycling, slow walking}\}$

75. $\{x : x \text{ burns less than 140 calories per one-half hour}\}$

76. $\{x : x \text{ burns at least 300 calories per one-half hour}\}$

77. Answers will vary.

78. When the set is too large or to complicated to list all the elements.

79. $\varnothing$ is the empty set, it contains no elements. $\{\varnothing\}$ is not empty, it contains 1 element, $\varnothing$.

80. a) n stands for the word *number*.
    b) A stands for the set A. Set names are always capitalized.
    c) $n(A)$ is the number of elements in set A.

81. – 84. Answers will vary.

85. If the barber shaves himself, then he (the barber) does not shave himself. If the barber does not shave himself, then he (the barber) does shave himself. Conclusion: This is a paradox.

86. If the sentence is true, then the sentence is false. If the sentence is false, then the sentence is true. Conclusion: This is a paradox.

87. If $S \in S$, then $S \notin S$. If $S \notin S$, then $S \in S$. Conclusion: This is a paradox.

Section 2.2: Comparing Sets

1. These two sets are equal. They have the same elements arranged in a different order.
2. These two sets are not equal. The first set is the set of vowels. The second set contains "b" along with other letters. Since "b" is not a vowel, these two sets cannot be equal.
3. These two sets are not equal. The second set contains (infinitely many) elements that don't appear in the first set.
4. These two sets are equal. They are both $\{3, 4, 5, 6, 7, 8, 9, 10\}$.
5. These two sets are equal. They are both $\{1, 3, 5, \ldots, 99\}$.
6. These two sets are not equal. The first set contains the first 5 multiples of 3 that are counting numbers. The second set contains all multiples of 3 that are counting numbers.
7. These two sets are equal. Common sense dictates that nobody born before 1800 should be living.
8. These two sets are not equal. The null set contains no elements. The set, $\{\varnothing\}$, contains one element, namely the null set.
9. true; All the elements of the first set are understood to be elements of the second set and moreover the first set is not equal to the second set.
10. true; All the elements of the first set are also elements of the second set.
11. false; The letter "y" is an element of the first set and not an element of the second set.
12. false; The set on the left and the set on the right are equal. They both represent the set $\{r, u, t, h\}$. The first set cannot be a <u>proper</u> subset of the second set.
13. true; The null set is a subset of all sets.
14. false; Although the null set is a subset of all sets, it is not a *proper* subset because it is equal to itself.
15. The first set is equivalent to the second set because they both have the same number of elements.
16. The first set is not equivalent to the second set. The first set has 6 elements while the second set is understood to have 11 elements.
17. The first set is equivalent to the second set because they both have that same number of elements, namely 4.
18. The first set is not equivalent to the second set. The first set has 7 elements while the second set has 6 elements.
19. The first set is not equivalent to the second set. The first set has 0 elements while the second set has 1 element.
20. The first set is equivalent to the second set. They both have 1 element.
21. The first set is equivalent to the second set. They both have 8 elements.
22. The first set is not equivalent to the second set. The first set has 26 elements while the second set has 24 elements.
23. The first set is not equivalent to the second set. The first set has 366 elements while the second set has 365 elements.
24. The first set is equivalent to the second set. The starting number of players for both teams is the same.

25. $\{1, 2\}, \{1, 3\}, \{2, 3\}$

26. $\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}$

27. $\{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$

28. $\{1, 2, 3\}$, $\{1, 2, 4\}$, $\{1, 2, 5\}$, $\{1, 3, 4\}$, $\{1, 3, 5\}$, $\{1, 4, 5\}$, $\{2, 3, 4\}$, $\{2, 3, 5\}$, $\{2, 4, 5\}$, $\{3, 4, 5\}$

29. There are $2^5 = 32$ subsets and $2^5 - 1 = 31$ proper subsets.

30. There are $2^7 = 128$ subsets and $2^7 - 1 = 127$ proper subsets.

31. T; $V = \{\text{Carmen, Frank, Ivana}\} = T$

32. Answers may vary. Note: the **boxed** values have the same cardinality as S. The set of students majoring in art $= A$. The set of students that are involved in drama $= D$.
$n(U) = 4,\ n(L) = 6,\ \boxed{n(S) = 2,}\ n(V) = 3,\ \boxed{n(A) = 2,}\ n(T) = 3,$ and $\boxed{n(D) = 2}$

33. The set of lowerclassmen $= L$; Note: the **boxed** value indicates the set with the largest cardinality.
$$n(U) = 4,\ \boxed{n(L) = 6,}\ n(S) = 2,\ n(V) = 3,\ n(A) = 2,\ n(T) = 3, \text{ and } n(D) = 2$$

34. Answers may vary. Note: the **boxed** values are the sets with the smallest cardinality. The set of students that are science majors $= S$. The set of students that are art majors $= A$. The set of students that are involved in drama $= D$.
$n(U) = 4,\ n(L) = 6,\ \boxed{n(S) = 2,}\ n(V) = 3,\ \boxed{n(A) = 2,}\ n(T) = 3,$ and $\boxed{n(D) = 2}$

35. $2^6 = 64$

36. $2^1 = 2$

37. $2^4 = 16$

38. $2^4 = 16$

39. $2^7 = 128$

40. 9; $2^9 = 512$

41. $2^8 = 256$

42. 10; $2^{10} = 1024$

43. 7

44. 6

45. $\{5\text{P}, 10\text{P}, 25\text{D}\}$

46. $\{1\text{S}, 25\text{S}, 50\text{D}\}$

47. $\{5\text{P}, 10\text{P}, 25\text{D}\}$ or $\{5\text{P}, 10\text{P}, 25\text{S}\}$

48. $\{5\text{P}, 10\text{S}, 50\text{D}\}$

49. 8

50. a) branch 2
b) branch 7

51. He didn't understand that the order of elements in a set does not matter.

52. He didn't understand that repetition of elements in a set does not matter.

53. a) 25 is not a power of 2.
b) He confused 5^2 with 2^5.
c) $2^5 = 32$

54. There are 2^n ways to flip n coins and also to answer an n question true-false test.

55. Answers will vary.

56. $2^{30} = 1{,}073{,}741{,}824$

57. Over 34 years; $\dfrac{1{,}073{,}741{,}824}{365 \times 24 \times 60 \times 60} = \dfrac{1{,}073{,}741{,}824}{31{,}536{,}000} \approx 34.05$

58. $\dfrac{2^{100}\text{ subsets}}{1{,}000{,}000{,}000\text{ subsets/sec}} \approx 1.26765\times10^{21}$ seconds; $\dfrac{1.26765\times10^{21}}{365\times24\times60\times60} \approx 4.0197\times10^{13}$ years

59. The fifth line counts the number of subsets of sizes 0, 1, 2, 3, 4, and 5 of a five-element set.

60. The sixth line, which is 1 6 15 20 15 6 1, counts the number of subsets of sizes 0, 1, 2, 3, 4, 5, and 6 of a six-element set.

61. The sixth, seventh, eighth, and ninth lines are:

$$\begin{array}{c}
1\ 6\ 15\ 20\ 15\ 6\ 1\\
1\ 7\ 21\ 35\ 35\ 21\ 7\ 1\\
1\ 8\ 28\ 56\ 70\ 56\ 28\ 8\ 1\\
1\ 9\ 36\ \mathbf{84}\ 126\ 126\ 84\ 36\ 9\ 1
\end{array}$$

So Tyra Banks can choose the three contestants in 84 different ways.

62. The sixth, seventh, eighth, ninth, and tenth lines are:

$$\begin{array}{c}
1\ 6\ 15\ 20\ 15\ 6\ 1\\
1\ 7\ 21\ 35\ 35\ 21\ 7\ 1\\
1\ 8\ 28\ 56\ 70\ 56\ 28\ 8\ 1\\
1\ 9\ 36\ 84\ 126\ 126\ 84\ 36\ 9\ 1\\
1\ 10\ 45\ 120\ \mathbf{210}\ 252\ 210\ 120\ 45\ 10\ 1
\end{array}$$

So the four dancers can be chosen in 210 different ways.

63.

$$\begin{array}{rcl}
1 & \longrightarrow & 1\\
1\ 1 & \longrightarrow & 2\\
1\ 2\ 1 & \longrightarrow & 4\\
1\ 3\ 3\ 1 & \longrightarrow & 8\\
1\ 4\ 6\ 4\ 1 & \longrightarrow & 16\\
1\ 5\ 10\ 10\ 5\ 1 & \longrightarrow & 32\\
1\ 6\ 15\ 20\ 15\ 6\ 1 & \longrightarrow & 64\\
1\ 7\ 21\ 35\ 35\ 21\ 7\ 1 & \longrightarrow & 128\\
1\ 8\ 28\ 56\ 70\ 56\ 28\ 8\ 1 & \longrightarrow & 256\\
1\ 9\ 36\ 84\ 126\ 126\ 84\ 36\ 9\ 1 & \longrightarrow & 512\\
1\ 10\ 45\ 120\ 210\ 252\ 210\ 120\ 45\ 10\ 1 & \longrightarrow & 1024
\end{array}$$

The sum across the rows is always a power of 2, specifically 2^n, where n is the number of the row that is being summed. Note: Recall we start counting these lines with 0, not 1.

64. Answers may vary. When we consider two numbers that are the same in a row of Pascal's triangle, one number is counting subsets of a certain size and the other number is counting the complements of those subsets.

65. 16; We are choosing 3, 4, or 5 senior partners from the five possible, so add the last three elements of the 5th row of Pascal's triangle. (Remember we begin numbering the rows with 0.)

66. 60; Using the 3rd element of the 5th row of Pascal's triangle, there are 10 ways to choose the senior partners. Using the 2nd element of the 4th row of Pascal's triangle, there are 6 ways to choose the associates. There will be $6\times10=60$ ways to form the committee.

67. Corresponding property: If $A\subseteq B$ and $B\subseteq C$, then $A\subseteq C$. If $x\in A$, then $x\in B$. Since $x\in B$, then $x\in C$. Therefore, if we have $x\in A$, we must also have $x\in C$. Note: We use capital letters for sets and lower case letters for elements.

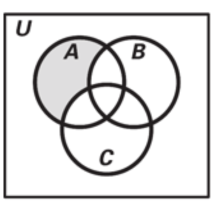
68. Corresponding property: If $A \subseteq B$ and $B \subseteq A$, then $A = B$. Since $A \subseteq B$, then each $x \in A$ is also an element of B. Also, since $B \subseteq A$ then each $x \in B$ is also an element of A. Therefore, each element of each set belongs to the other set, so the two sets must be equal. Note: We use capital letters for sets and lower case letters for elements.

69. Examples will vary. A three-element set has $3! = 6$ correspondences, a four element set has $4! = 24$, and so on. So, a set with n elements will have $n!$ one-to-one correspondences.

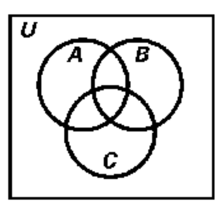
70. Answers will vary.

Section 2.3: Set Operations

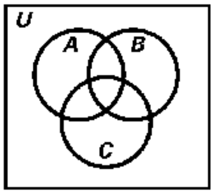
1. $A \cap B = \{1, 3, 5\}$
2. $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 9\}$
3. $B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$
4. $B \cap C = \{2, 4, 6\}$
5. $A \cup \varnothing = \{1, 3, 5, 7, 9\} = A$
6. $A \cap \varnothing = \varnothing$
7. $A \cup U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} = U$
8. $A \cap U = \{1, 3, 5, 7, 9\} = A$
9. $A \cap (B \cup C) = A \cap \{1, 2, 3, 4, 5, 6, 7, 8\} = \{1, 3, 5, 7\}$
10. $A' \cap (B \cup C') = A' \cap (B \cup \{1, 3, 5, 9, 10\}) = A' \cap \{1, 2, 3, 4, 5, 6, 9, 10\}$
$= \{2, 4, 6, 8, 10\} \cap \{1, 2, 3, 4, 5, 6, 9, 10\} = \{2, 4, 6, 10\}$
11. $(A - B) \cap (A - C) = \{7, 9\} \cap \{1, 3, 5, 9\} = \{9\}$
12. $A - (B \cup C) = A - \{1, 2, 3, 4, 5, 6, 7, 8\} = \{9\}$
13. $M \cap E = \{$potato chip, bread, pizza$\}$
14. $M - E = \{$flat-screen TV, hat, satellite radio, sofa, hybrid automobile, hammer$\}$
15. $E - M = \{$apple, fish, banana$\}$
16. $E' = \{$flat-screen TV, hat, satellite radio, sofa, hybrid automobile, hammer$\}$
17. $M' \cap G' = \{$fish$\}$
18. $G \cap (M' \cap E) = G \cap (\{$apple, fish, banana$\} \cap E) = G \cap \{$apple, fish, banana$\} = \{$apple, banana$\} = G$; Note: Since all things that grow on a plant from our universal set are edible, the outcome of G should be readily understood.
19. 7
20. 2
21. 5
22. 1
23. $A - (B \cup C)$



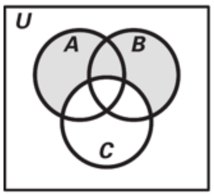
24. $A \cap (B - C)$



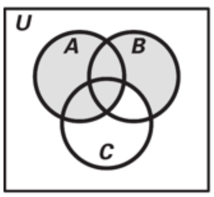
25. $(A\cap B)-C$



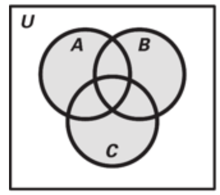
26. $(A\cup B)-C$



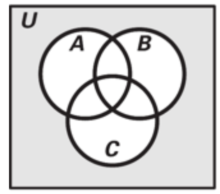
27. $A\cup(B-C)$



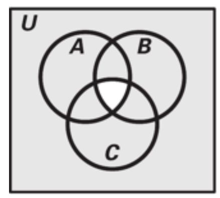
28. $A\cup(B\cup C)$



29. $\left(A\cup(B\cup C)\right)'$



30. $\left(A\cap(B\cap C)\right)'$



31. $B-A$

32. $(A\cup B)-(A\cap B)$

33. $(A\cup B)'$

34. $(A\cup B)'\cup(A\cap B)$

35. $A\cap B\cap C$

36. $(A\cap B)\cup(A\cap C)$

37. $(A\cup C)-B$

38. $(A\cup B\cup C)-(A\cap B\cap C)$

39. equal; Using the diagrams from Example 2, $(A\cup B')'$ consists of region r_4. $A'\cap B$ also consists of region r_4. so $(A\cup B')' = A'\cap B$.

40. unequal; Using the diagrams from Example 2, $(A'\cap B)'$ consists of regions r_1, r_2, and r_3, $A\cap B'$ consists of region r_2. so $(A'\cap B)' \neq A\cap B'$.

41. 30

42. 50

43. 28

44. 10

45. 20

46. 11

47. 27

48. 6

49. $P\cap C=$ the set of cars whose price is above $20,000 and is compact $=\{d, f, g\}$

50. $A\cup G=$ the set of cars that have an antitheft package or have a good safety rating $=\{a, b, c, g, h\}$; Note: This is an inclusive "or". The car can have both features.

51. $W\cap G'=$ the set of cars that have a warranty of at least three years and don't have a good safety rating $=\{c, d, f, g\}$

52. $G-A=$ the set of cars that don't have an antitheft package but do have a good safety rating $=\{b, h\}$

53. $P\cap(G\cup W)=$ the set of cars that have a price above \$20,000, and a good safety rating or a warranty of at least three years $=\{b, d, f, g, h\}=P$

54. $G'\cap C'=$ the set of cars that don't have a good safety rating and are not compact $=\{c, e\}$

55. $P-(G\cup A)=$ the set of cars that have a price above \$20,000 and don't have a good safety rating nor have an antitheft package $=\{d, f\}$

56. $P'-(G\cup C)=$ the set of cars that don't have a price above \$20,000 and don't have a good safety rating and aren't compact $=\{c, e\}$

57. $P\cap(B\cup A)=P\cap\{m, mc, hc\}=\{m, mc, hc\}$

58. $(P\cup C)\cap(B\cup A)=\{m, mc, bc, c, hc\}\cap\{m, mc, hc\}=\{m, mc, hc\}$

59. $P\cup C\cup B=\{m, mc, bc, c, hc\}$

60. $P\cap C\cap B=\{mc, hc\}$

61. $H\cap B=\{a, b, e\}$

62. $B'=\{c, d, f, h, j\}$

63. $H-B=\{c, d\}$

64. $L\cap(H\cup B)=\{g, i\}$

65. $M\cap L=$ the set of movies that earned more than \$1.5 billion and also earned less than \$2 billion $=\{c\}$

66. $M\cup B=$ the set of movies that earned more than \$1.5 billion or were made before 2015 $=\{a, b, c, d, g, h, i\}$

67. $M-L=$ the set of movies that earned more than \$1.5 billion and did not earn less than \$2 billion $=\{a, b\}$

68. $B'=$ the set of movies that were not made before 2015 $=\{c, e, f, j\}$

69. $L-(M\cup B)=$ set of movies that earned less than \$2 billion, but neither made more than \$1.5 billion nor were made before 2015 $=\{e, f, j\}$

70. $L\cap(M\cup B)=$ set of movies that earned less than \$2 billion, and either made more than \$1.5 billion or were made before 2015 $=\{c, d, g, h, i\}$

71. "Union" implies joining together . "Intersection" implies overlapping.

72. Set differences are found by removing the elements common to both sets.

73. Answers will vary. Possible answers include confusing DeMorgan's laws with the distributive property.

74. – 76. Answers will vary.

77. false; It is possible that $A=B$ and hence $n(A)=n(B)$. Counterexamples may vary.

78. true

79. true

80. false; Sample counterexample: Let $X=\{1,2,3,4\}$ and $Y=\{3,4,5,6\}$
$X-Y=\{1,2\}$, $n(X-Y)=2$, $n(X)=4$ and $n(Y)=4$, $2\neq 4-4$

81. A

82. $\varnothing$

83. B

84. B'