CHAPTER 2

Exercises

E2.1 (a) R_2, R_3, and R_4 are in parallel. Furthermore R_1 is in series with the combination of the other resistors. Thus we have:

$$R_{eq} = R_1 + \frac{1}{1/R_2 + 1/R_3 + 1/R_4} = 3\,\Omega$$

(b) R_3 and R_4 are in parallel. Furthermore, R_2 is in series with the combination of R_3, and R_4. Finally R_1 is in parallel with the combination of the other resistors. Thus we have:

$$R_{eq} = \frac{1}{1/R_1 + 1/[R_2 + 1/(1/R_3 + 1/R_4)]} = 5\,\Omega$$

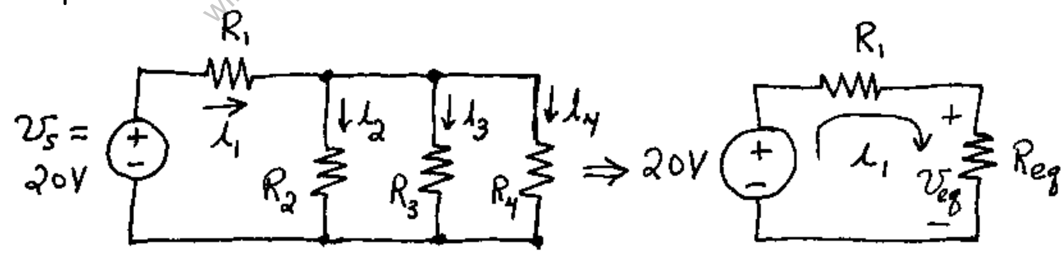
(c) R_1 and R_2 are in parallel. Furthermore, R_3, and R_4 are in parallel. Finally, the two parallel combinations are in series.

$$R_{eq} = \frac{1}{1/R_1 + 1/R_2} + \frac{1}{1/R_3 + 1/R_4} = 52.1\,\Omega$$

(d) R_1 and R_2 are in series. Furthermore, R_3 is in parallel with the series combination of R_1 and R_2.

$$R_{eq} = \frac{1}{1/R_3 + 1/(R_1 + R_2)} = 1.5\text{ k}\Omega$$

E2.2 (a) First we combine R_2, R_3, and R_4 in parallel. Then R_1 is in series with the parallel combination.

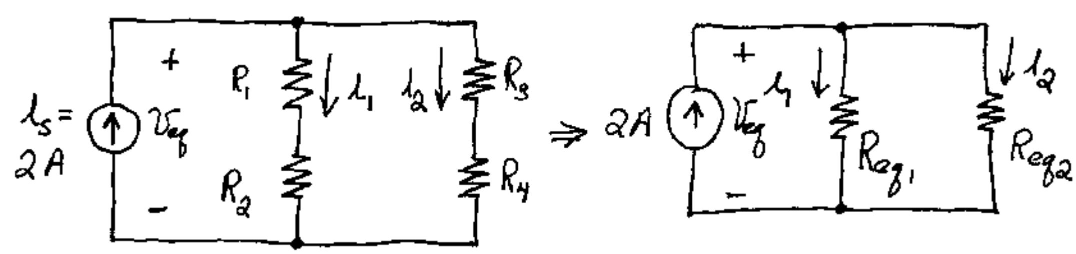


$$R_{eq} = \frac{1}{1/R_2 + 1/R_3 + 1/R_4} = 9.231\,\Omega \qquad i_1 = \frac{20\text{ V}}{R_1 + R_{eq}} = \frac{20}{10 + 9.231} = 1.04\text{ A}$$

$$v_{eq} = R_{eq} i_1 = 9.600\text{ V} \qquad i_2 = v_{eq}/R_2 = 0.480\text{ A} \qquad i_3 = v_{eq}/R_3 = 0.320\text{ A}$$

$$i_4 = v_{eq}/R_4 = 0.240\text{ A}$$

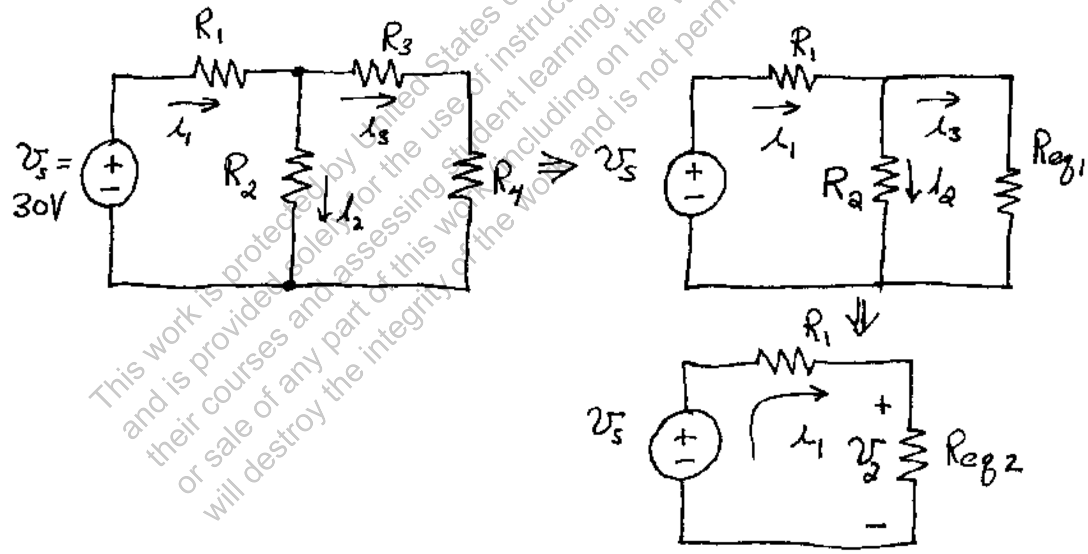
(b) R_1 and R_2 are in series. Furthermore, R_3, and R_4 are in series. Finally, the two series combinations are in parallel.



$$R_{eq1} = R_1 + R_2 = 20\ \Omega \qquad R_{eq2} = R_3 + R_4 = 20\ \Omega \quad R_{eq} = \frac{1}{1/R_{eq1} + 1/R_{eq2}} = 10\ \Omega$$

$$v_{eq} = 2 \times R_{eq} = 20\text{ V} \qquad i_1 = v_{eq}/R_{eq1} = 1\ A \qquad i_2 = v_{eq}/R_{eq2} = 1\ A$$

(c) R_3, and R_4 are in series. The combination of R_3 and R_4 is in parallel with R_2. Finally the combination of R_2, R_3, and R_4 is in series with R_1.



$$R_{eq1} = R_3 + R_4 = 40\ \Omega \quad R_{eq2} = \frac{1}{1/R_{eq1} + 1/R_2} = 20\ \Omega \quad i_1 = \frac{v_s}{R_1 + R_{eq2}} = 1\ A$$

$$v_2 = i_1 R_{eq2} = 20\text{ V} \quad i_2 = v_2/R_2 = 0.5\ A \quad i_3 = v_2/R_{eq1} = 0.5\ A$$

E2.3 (a) $v_1 = v_s \dfrac{R_1}{R_1 + R_2 + R_3 + R_4} = 10\text{ V}$. $v_2 = v_s \dfrac{R_2}{R_1 + R_2 + R_3 + R_4} = 20\text{ V}$.

Similarly, we find $v_3 = 30\text{ V}$ and $v_4 = 60\text{ V}$.

(b) First combine R_2 and R_3 in parallel: $R_{eq} = 1/(1/R_2 + 1/R_3) = 2.917\ \Omega$.

Then we have $v_1 = v_s \dfrac{R_1}{R_1 + R_{eq} + R_4} = 6.05\text{ V}$. Similarly, we find

$v_2 = v_s \dfrac{R_{eq}}{R_1 + R_{eq} + R_4} = 5.88\text{ V}$ and $v_4 = 8.07\text{ V}$.

E2.4 (a) First combine R_1 and R_2 in series: $R_{eq} = R_1 + R_2 = 30\ \Omega$. Then we have $i_1 = i_s \dfrac{R_3}{R_3 + R_{eq}} = \dfrac{15}{15 + 30} = 1\ A$ and $i_3 = i_s \dfrac{R_{eq}}{R_3 + R_{eq}} = \dfrac{30}{15 + 30} = 2\ A$.

(b) The current division principle applies to two resistances in parallel. Therefore, to determine i_1, first combine R_2 and R_3 in parallel: $R_{eq} = 1/(1/R_2 + 1/R_3) = 5\ \Omega$. Then we have $i_1 = i_s \dfrac{R_{eq}}{R_1 + R_{eq}} = \dfrac{5}{10 + 5} = 1\ A$.

Similarly, $i_2 = 1\ A$ and $i_3 = 1\ A$.

E2.5 Write KVL for the loop consisting of v_1, v_y, and v_2. The result is $-v_1 - v_y + v_2 = 0$ from which we obtain $v_y = v_2 - v_1$. Similarly we obtain $v_z = v_3 - v_1$.

E2.6 Node 1: $\dfrac{v_1 - v_3}{R_1} + \dfrac{v_1 - v_2}{R_2} = i_a$ Node 2: $\dfrac{v_2 - v_1}{R_2} + \dfrac{v_2}{R_3} + \dfrac{v_2 - v_3}{R_4} = 0$

Node 3: $\dfrac{v_3}{R_5} + \dfrac{v_3 - v_2}{R_4} + \dfrac{v_3 - v_1}{R_1} + i_b = 0$

E2.7 Following the step-by-step method in the book, we obtain

$$\begin{bmatrix} \dfrac{1}{R_1} + \dfrac{1}{R_2} & -\dfrac{1}{R_2} & 0 \\ -\dfrac{1}{R_2} & \dfrac{1}{R_2} + \dfrac{1}{R_3} + \dfrac{1}{R_4} & -\dfrac{1}{R_4} \\ 0 & -\dfrac{1}{R_4} & \dfrac{1}{R_4} + \dfrac{1}{R_5} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -i_s \\ 0 \\ i_s \end{bmatrix}$$

E2.8 Instructions for various calculators vary. The MATLAB solution is given in the book following this exercise.

E2.9 (a) Writing the node equations we obtain:

Node 1: $\frac{v_1 - v_3}{20} + \frac{v_1}{5} + \frac{v_1 - v_2}{10} = 0$

Node 2: $\frac{v_2 - v_1}{10} + 10 + \frac{v_2 - v_3}{5} = 0$

Node 3: $\frac{v_3 - v_1}{20} + \frac{v_3}{10} + \frac{v_3 - v_2}{5} = 0$

(b) Simplifying the equations we obtain:

$$0.35v_1 - 0.10v_2 - 0.05v_3 = 0$$
$$-0.10v_1 + 0.30v_2 - 0.20v_3 = -10$$
$$-0.05v_1 - 0.20v_2 + 0.35v_3 = 0$$

(c) and (d) Solving using Matlab:

```
>>clear
>>G = [0.35 -0.1 -0.05; -0.10 0.30 -0.20; -0.05 -0.20 0.35];
>>I = [0; -10; 0];
>>V = G\I
V =
  -27.2727
  -72.7273
  -45.4545
>>Ix = (V(1) - V(3))/20
Ix =
    0.9091
```


E2.10 Using determinants we can solve for the unknown voltages as follows:

$$v_1 = \frac{\begin{vmatrix} 6 & -0.2 \\ 1 & 0.5 \end{vmatrix}}{\begin{vmatrix} 0.7 & -0.2 \\ -0.2 & 0.5 \end{vmatrix}} = \frac{3 + 0.2}{0.35 - 0.04} = 10.32 \text{ V}$$

$$v_2 = \frac{\begin{vmatrix} 0.7 & 6 \\ -0.2 & 1 \end{vmatrix}}{\begin{vmatrix} 0.7 & -0.2 \\ -0.2 & 0.5 \end{vmatrix}} = \frac{0.7 + 1.2}{0.35 - 0.04} = 6.129 \text{ V}$$

Many other methods exist for solving linear equations.

E2.11 First write KCL equations at nodes 1 and 2:

Node 1: $$\frac{v_1 - 10}{2} + \frac{v_1}{5} + \frac{v_1 - v_2}{10} = 0$$

Node 2: $$\frac{v_2 - 10}{10} + \frac{v_2}{5} + \frac{v_2 - v_1}{10} = 0$$

Then, simplify the equations to obtain:

$$8v_1 - v_2 = 50 \quad \text{and} \quad -v_1 + 4v_2 = 10$$

Solving manually or with a calculator, we find v_1 = 6.77 V and v_2 = 4.19 V. The MATLAB session using the symbolic approach is:

```
>> clear all
syms V1 V2
[V1,V2] = solve((V1-10)/2+(V1)/5 +(V1 - V2)/10 == 0 , ...
                (V2-10)/10 +V2/5 +(V2-V1)/10 == 0)
V1 =
210/31
V2 =
130/31
```

Next, we solve using the numerical approach.

```
>> clear
G = [8 -1; -1 4];
I = [50; 10];
V = G\I
V =
   6.7742
   4.1935
```

E2.12 The equation for the supernode enclosing the 15-V source is:

$$\frac{v_3 - v_2}{R_3} + \frac{v_3 - v_1}{R_1} = \frac{v_1}{R_2} + \frac{v_2}{R_4}$$

This equation can be readily shown to be equivalent to Equation 2.37 in the book. (Keep in mind that v_3 = -15 V.)

E2.13 Write KVL from the reference to node 1 then through the 10-V source to node 2 then back to the reference node:

$$-v_1 + 10 + v_2 = 0$$

Then write KCL equations. First for a supernode enclosing the 10-V source, we have:

$$\frac{v_1}{R_1} + \frac{v_1 - v_3}{R_2} + \frac{v_2 - v_3}{R_3} = 1$$

Node 3:

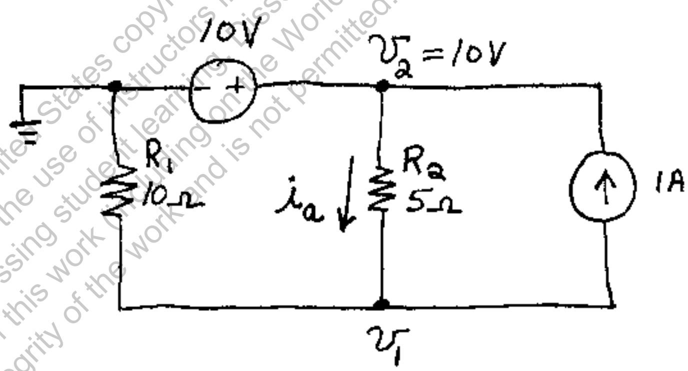
$$\frac{v_3}{R_4} + \frac{v_3 - v_1}{R_2} + \frac{v_3 - v_2}{R_3} = 0$$

Reference node:

$$\frac{v_1}{R_1} + \frac{v_3}{R_4} = 1$$

An independent set consists of the KVL equation and any two of the KCL equations.

E2.14 (a) Select the reference node at the left-hand end of the voltage source as shown at right.



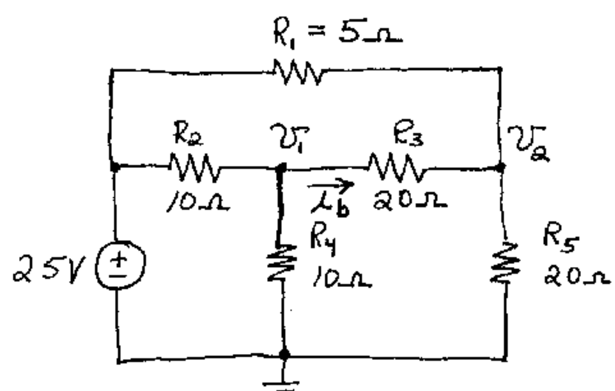
Then write a KCL equation at node 1.

$$\frac{v_1}{R_1} + \frac{v_1 - 10}{R_2} + 1 = 0$$

Substituting values for the resistances and solving, we find v_1 = 3.33 V.

Then we have $i_a = \dfrac{10 - v_1}{R_2} = 1.333$ A.

(b) Select the reference node and assign node voltages as shown.



Then write KCL equations at nodes 1 and 2.

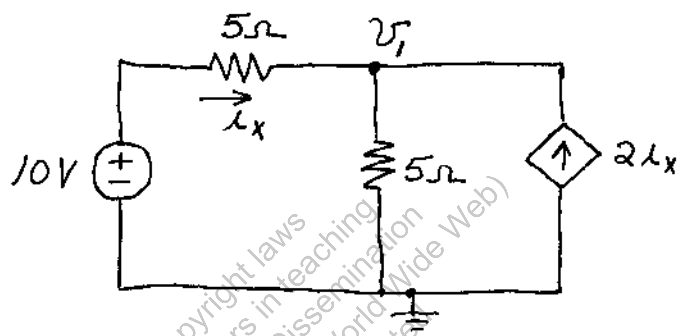
$$\frac{v_1 - 25}{R_2} + \frac{v_1}{R_4} + \frac{v_1 - v_2}{R_3} = 0$$

$$\frac{v_2 - 25}{R_1} + \frac{v_2 - v_1}{R_3} + \frac{v_2}{R_5} = 0$$

Substituting values for the resistances and solving, we find v_1 = 13.79 V and v_2 = 18.97 V. Then we have $i_b = \frac{v_1 - v_2}{R_3}$ = -0.259 A.

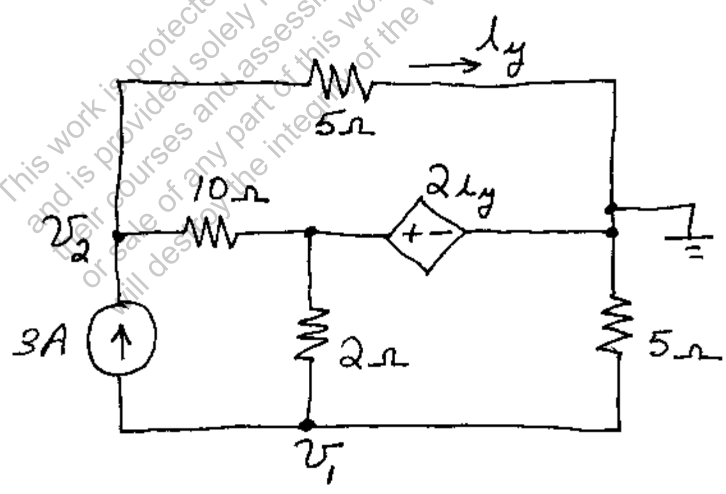
E2.15 (a) Select the reference node and node voltage as shown. Then write a KCL equation at node 1, resulting in

$$\frac{v_1}{5} + \frac{v_1 - 10}{5} - 2i_x = 0$$



Then use $i_x = (10 - v_1)/5$ to substitute and solve. We find v_1 = 7.5 V. Then we have $i_x = \frac{10 - v_1}{5} = 0.5$ A.

(b) Choose the reference node and node voltages shown:



Then write KCL equations at nodes 1 and 2:

$$\frac{v_1}{5} + \frac{v_1 - 2i_y}{2} + 3 = 0 \qquad \frac{v_2}{5} + \frac{v_2 - 2i_y}{10} = 3$$

Finally use $i_y = v_2 / 5$ to substitute and solve. This yields $v_2 = 11.54$ V and $i_y = 2.31$ A.

E2.16 These are the MATLAB commands:

```
clear all
syms V1 V2 V3 R1 R2 R3 R4 Is
[V1 V2 V3] = solve(V3/R4 + (V3 - V2)/R3 + (V3 - V1)/R1 == 0, ...
                   V1/R2 + V3/R4 == Is, ...
                   V1 == (1/2)*(V3 - V1) + V2 , V1, V2 , V3);
pretty(V1), pretty(V2), pretty(V3)
```

The results are;

```
2 Is R1 R2 R3 + 3 Is R1 R2 R4 + 2 Is R2 R3 R4
---------------------------------------------
3 R1 R2 + 2 R1 R3 + 3 R1 R4 + 2 R2 R3 + 2 R3 R4

    Is R2 (3 R1 R3 + 3 R1 R4 + 2 R3 R4)
---------------------------------------------
3 R1 R2 + 2 R1 R3 + 3 R1 R4 + 2 R2 R3 + 2 R3 R4

         Is R2 R4 (3 R1 + 2 R3)
---------------------------------------------
3 R1 R2 + 2 R1 R3 + 3 R1 R4 + 2 R2 R3 + 2 R3 R4
```


E2.17 Refer to Figure 2.34b in the book. (a) Two mesh currents flow through R_2: i_1 flows downward and i_4 flows upward. Thus the current flowing in R_2 referenced upward is $i_4 - i_1$. (b) Similarly, mesh current i_1 flows to the left through R_4 and mesh current i_2 flows to the right, so the total current referenced to the right is $i_2 - i_1$. (c) Mesh current i_3 flows downward through R_8 and mesh current i_4 flows upward, so the total current referenced downward is $i_3 - i_4$. (d) Finally, the total current referenced upward through R_8 is $i_4 - i_3$.

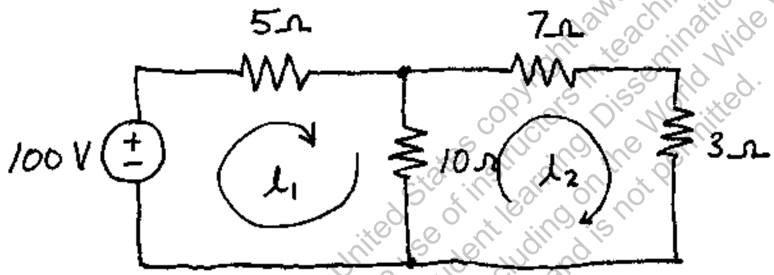
E2.18 Refer to Figure 2.34b in the book. Following each mesh current in turn, we have

$$R_1 i_1 + R_2(i_1 - i_4) + R_4(i_1 - i_2) - v_A = 0$$
$$R_5 i_2 + R_4(i_2 - i_1) + R_6(i_2 - i_3) = 0$$
$$R_7 i_3 + R_6(i_3 - i_2) + R_8(i_3 - i_4) = 0$$
$$R_3 i_4 + R_2(i_4 - i_1) + R_8(i_4 - i_3) = 0$$

In matrix form, these equations become

$$\begin{bmatrix} (R_1 + R_2 + R_4) & -R_4 & 0 & -R_2 \\ -R_4 & (R_4 + R_5 + R_6) & -R_6 & 0 \\ 0 & -R_6 & (R_6 + R_7 + R_8) & -R_8 \\ -R_2 & 0 & -R_8 & (R_2 + R_3 + R_8) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} v_A \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

E2.19 We choose the mesh currents as shown:

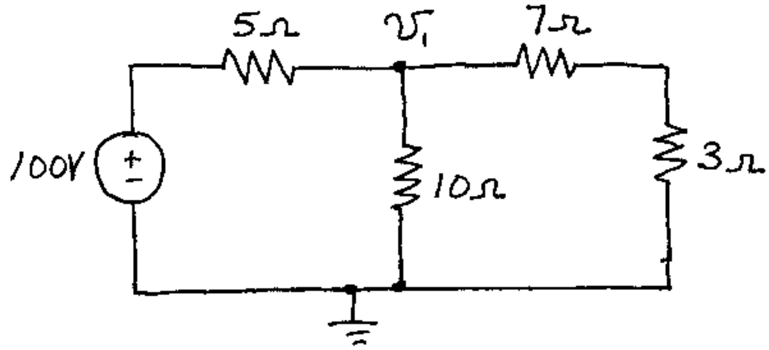


Then, the mesh equations are:

$$5i_1 + 10(i_1 - i_2) = 100 \quad \text{and} \quad 10(i_2 - i_1) + 7i_2 + 3i_2 = 0$$

Simplifying and solving these equations, we find that $i_1 = 10$ A and $i_2 = 5$ A. The net current flowing downward through the 10-Ω resistance is $i_1 - i_2 = 5$ A.

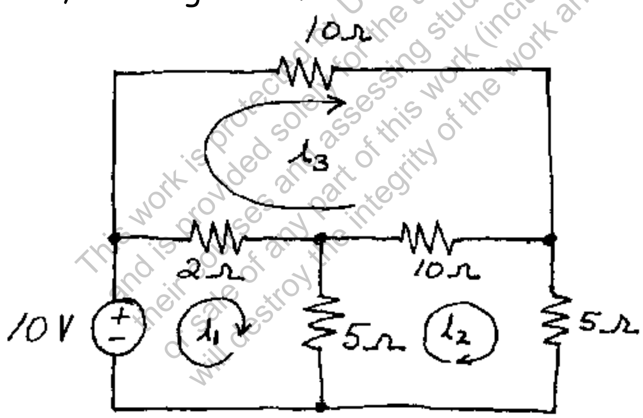
To solve by node voltages, we select the reference node and node voltage shown. (We do not need to assign a node voltage to the connection between the 7-Ω resistance and the 3-Ω resistance because we can treat the series combination as a single 10-Ω resistance.)



The node equation is $(v_1 - 10)/5 + v_1/10 + v_1/10 = 0$. Solving we find that v_1 = 50 V. Thus we again find that the current through the 10-Ω resistance is $i = v_1/10 = 5$ A.

Combining resistances in series and parallel, we find that the resistance "seen" by the voltage source is 10 Ω. Thus the current through the source and 5-Ω resistance is (100 V)/(10 Ω) = 10 A. This current splits equally between the 10-Ω resistance and the series combination of 7 Ω and 3 Ω.

E2.20 First, we assign the mesh currents as shown.



Then we write KVL equations following each mesh current:

$$2(i_1 - i_3) + 5(i_1 - i_2) = 10$$
$$5i_2 + 5(i_2 - i_1) + 10(i_2 - i_3) = 0$$
$$10i_3 + 10(i_3 - i_2) + 2(i_3 - i_1) = 0$$

Simplifying and solving, we find that i_1 = 2.194 A, i_2 = 0.839 A, and i_3 = 0.581 A. Thus the current in the 2-Ω resistance referenced to the right is i_1 - i_3 = 2.194 - 0.581 = 1.613 A.

E2.21 Following the step-by-step process, we obtain

$$\begin{bmatrix} (R_2 + R_3) & -R_3 & -R_2 \\ -R_3 & (R_3 + R_4) & 0 \\ -R_2 & 0 & (R_1 + R_2) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_A \\ -v_B \\ v_B \end{bmatrix}$$

E2.22 Refer to Figure 2.40 in the book. In terms of the mesh currents, the current directed to the right in the 5-A current source is i_1, however by the definition of the current source, the current is 5 A directed to the left. Thus, we conclude that i_1 = -5 A. Then we write a KVL equation following i_2, which results in $10(i_2 - i_1) + 5i_2 = 100$.

E2.23 Refer to Figure 2.41 in the book. First, for the current source, we have

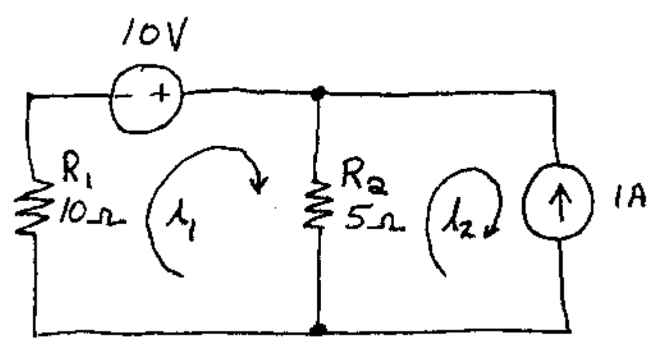
$$i_2 - i_1 = 1$$

Then, we write a KVL equation going around the perimeter of the entire circuit:

$$5i_1 + 10i_2 + 20 - 10 = 0$$

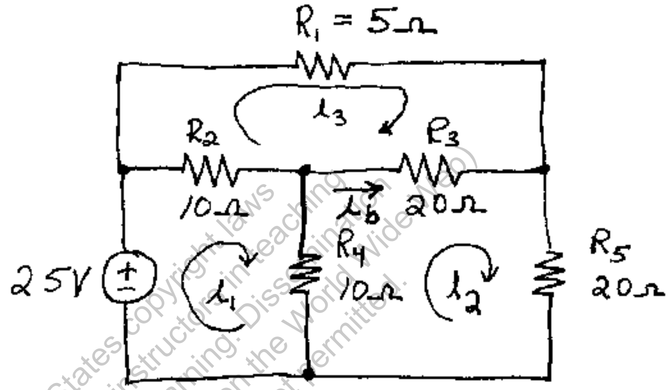
Simplifying and solving these equations we obtain i_1 = -4/3 A and i_2 = -1/3 A.

E2.24 (a) As usual, we select the mesh currents flowing clockwise around the meshes as shown. Then for the current source, we have i_2 = -1 A. This is because we defined the mesh



current i_2 as the current referenced downward through the current source. However, we know that the current through this source is 1 A flowing upward. Next we write a KVL equation around mesh 1: $10i_1 - 10 + 5(i_1 - i_2) = 0$. Solving, we find that i_1 = 1/3 A. Referring to Figure 2.31a in the book we see that the value of the current i_a referenced downward through the 5 Ω resistance is to be found. In terms of the mesh currents, we have $i_a = i_1 - i_2 = 4/3\,A$.

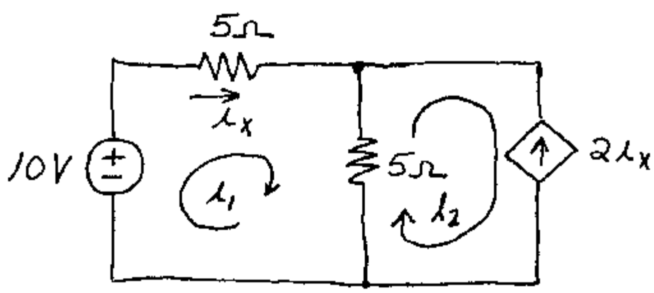
(b) As usual, we select the mesh currents flowing clockwise around the meshes as shown.
Then we write a KVL equation for each mesh.



$$-25 + 10(i_1 - i_3) + 10(i_1 - i_2) = 0$$
$$10(i_2 - i_1) + 20(i_2 - i_3) + 20i_2 = 0$$
$$10(i_3 - i_1) + 5i_3 + 20(i_3 - i_2) = 0$$

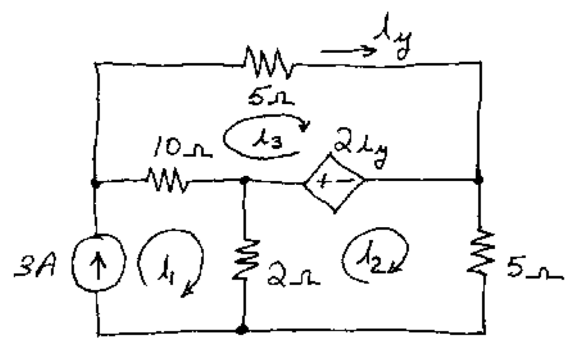
Simplifying and solving, we find i_1 = 2.3276 A, i_2 = 0.9483 A, and i_3 = 1.2069 A. Finally, we have $i_b = i_2 - i_3$ = -0.2586 A.

E2.25 (a) KVL mesh 1:
$-10 + 5i_1 + 5(i_1 - i_2) = 0$
For the current source:
$i_2 = -2i_x$
However, i_x and i_1 are the same current, so we also have $i_1 = i_x$.



Simplifying and solving, we find $i_x = i_1 = 0.5\,A$.

(b) First for the current source, we have: $i_1 = 3$ A
Writing KVL around meshes 2 and 3, we have:



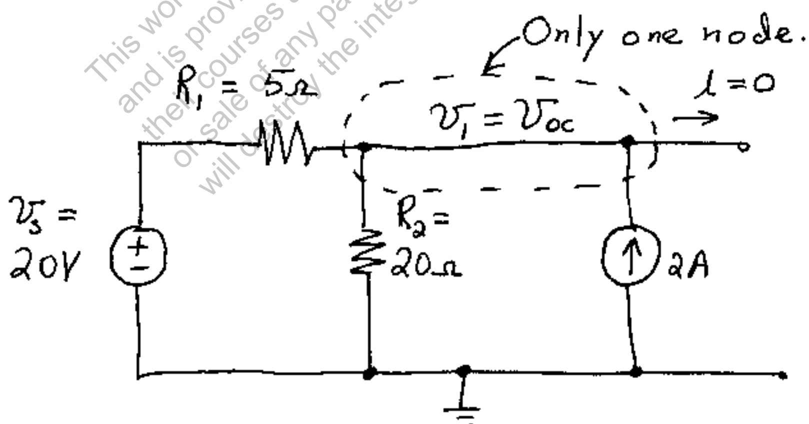
$$2(i_2 - i_1) + 2i_y + 5i_2 = 0$$
$$10(i_3 - i_1) + 5i_3 - 2i_y = 0$$

However i_3 and i_y are the same current: $i_y = i_3$. Simplifying and solving, we find that $i_3 = i_y = 2.31$ A.

E2.26 Under open-circuit conditions, 5 A circulates clockwise through the current source and the 10-Ω resistance. The voltage across the 10-Ω resistance is 50 V. No current flows through the 40-Ω resistance so the open circuit voltage is $V_t = 50$ V.

With the output shorted, the 5 A divides between the two resistances in parallel. The short-circuit current is the current through the 40-Ω resistance, which is $i_{sc} = 5\frac{10}{10+40} = 1$ A. Then, the Thévenin resistance is $R_t = v_{oc} / i_{sc} = 50\ \Omega$.

E2.27 Choose the reference node at the bottom of the circuit as shown:

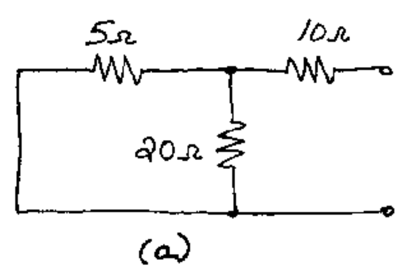


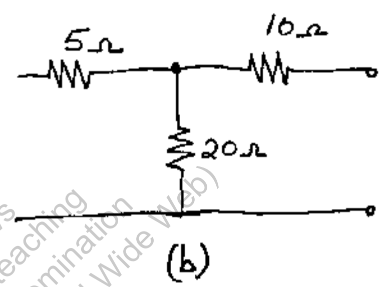
Notice that the node voltage is the open-circuit voltage. Then write a KCL equation:

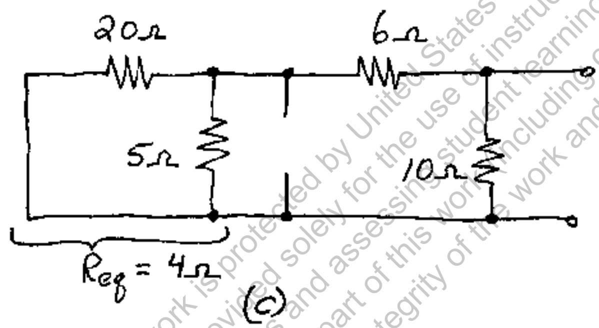
$$\frac{v_{oc} - 20}{5} + \frac{v_{oc}}{20} = 2$$

Solving we find that v_{oc} = 24 V which agrees with the value found in Example 2.19.

E2.28 To zero the sources, the voltage sources become short circuits and the current sources become open circuits. The resulting circuits are :





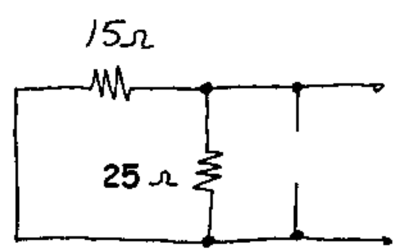


(a) $R_t = 10 + \frac{1}{1/5 + 1/20} = 14\ \Omega$ (b) $R_t = 10 + 20 = 30\ \Omega$

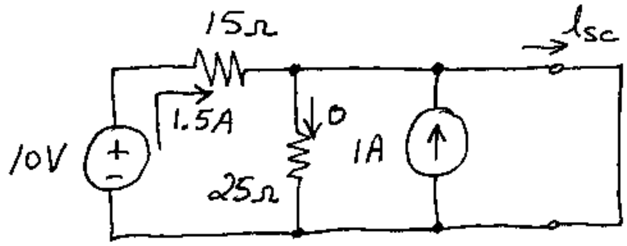
(c) $$R_t = \frac{1}{\frac{1}{10} + \frac{1}{6 + \frac{1}{(1/5 + 1/20)}}} = 5\ \Omega$$

E2.29 (a) Zero sources to determine Thévenin resistance. Thus

$$R_t = \frac{1}{1/15 + 1/25} = 9.375\ \Omega.$$

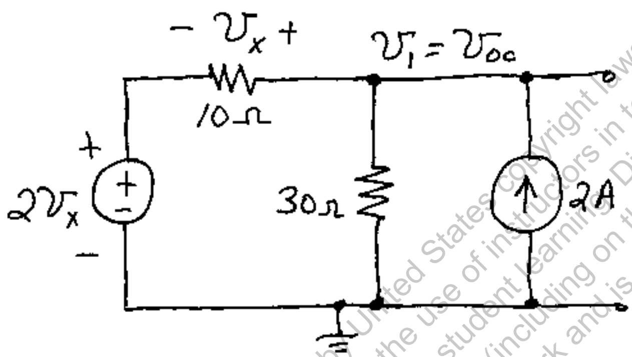


Then find short-circuit current:



$I_n = i_{sc} = 10/15 + 1 = 1.67$ A

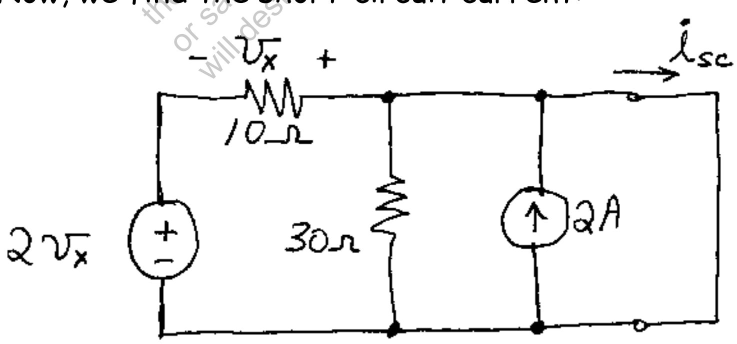
(b) We cannot find the Thévenin resistance by zeroing the sources, because we have a controlled source. Thus, we find the open-circuit voltage and the short-circuit current.



$$\frac{v_{oc} - 2v_x}{10} + \frac{v_{oc}}{30} = 2 \qquad v_{oc} = 3v_x$$

Solving, we find $V_t = v_{oc} = 30$ V.

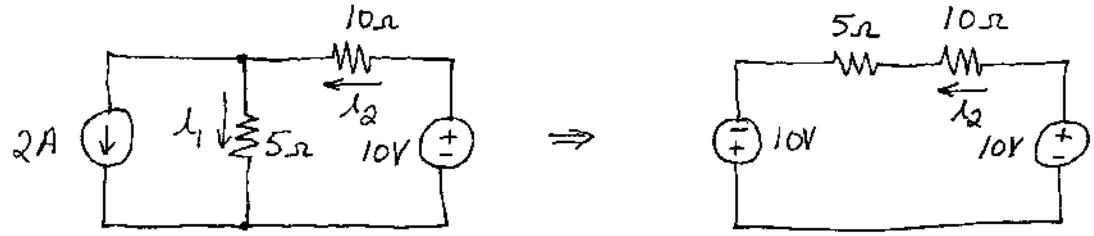
Now, we find the short-circuit current:



$2v_x + v_x = 0 \qquad \Rightarrow \qquad v_x = 0$

Therefore $i_{sc} = 2$ A. Then we have $R_t = v_{oc} / i_{sc} = 15\ \Omega$.

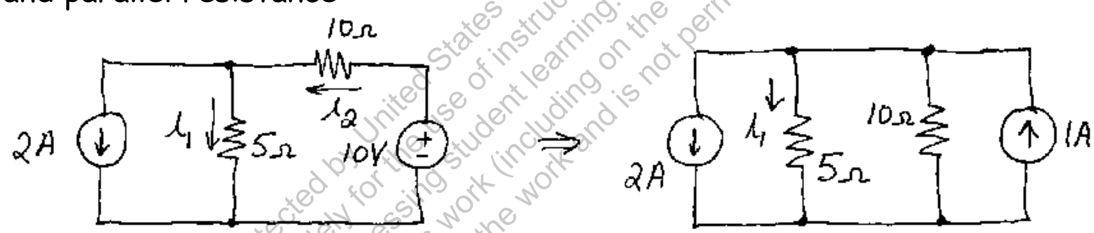
E2.30 First, we transform the 2-A source and the 5-Ω resistance into a voltage source and a series resistance:



Then we have $i_2 = \dfrac{10+10}{15} = 1.333$ A.

From the original circuit, we have $i_1 = i_2 - 2$, from which we find $i_1 = -0.667$ A.

The other approach is to start from the original circuit and transform the 10-Ω resistance and the 10-V voltage source into a current source and parallel resistance:



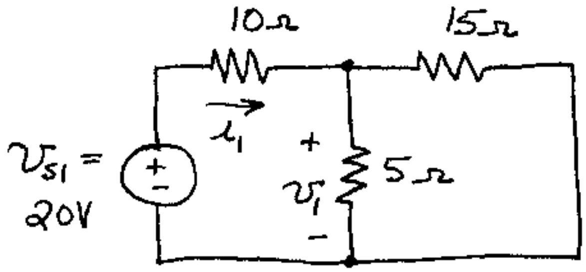
Then we combine the resistances in parallel. $R_{eq} = \dfrac{1}{1/5 + 1/10} = 3.333\ \Omega$.

The current flowing upward through this resistance is 1 A. Thus the voltage across R_{eq} referenced positive at the bottom is 3.333 V and $i_1 = -3.333/5 = -0.667$ A. Then from the original circuit we have $i_2 = 2 + i_1 = 1.333$ A, as before.

E2.31 Refer to Figure 2.64b. We have $i_1 = 15/15 = 1$ A.

Refer to Figure 2.64c. Using the current division principle, we have $i_2 = -2 \times \dfrac{5}{5+10} = -0.667$ A. (The minus sign is because of the reference direction of i_2.) Finally, by superposition we have $i_T = i_1 + i_2 = 0.333$ A.

E2.32 With only the first source active we have:

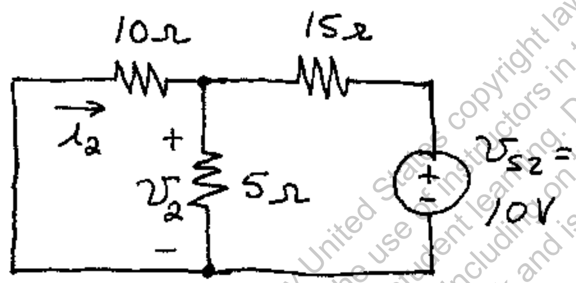


Then we combine resistances in series and parallel:

$$R_{eq} = 10 + \frac{1}{1/5 + 1/15} = 13.75\ \Omega$$

Thus, $i_1 = 20/13.75 = 1.455$ A, and $v_1 = 3.75 i_1 = 5.45$ V.

With only the second source active, we have:



Then we combine resistances in series and parallel:

$$R_{eq2} = 15 + \frac{1}{1/5 + 1/10} = 18.33\ \Omega$$

Thus, $i_s = 10/18.33 = 0.546$ A, and $v_2 = 3.33 i_s = 1.818$ V. Then, we have $i_2 = (-v_2)/10 = -0.1818$ A

Finally we have $v_T = v_1 + v_2 = 5.45 + 1.818 = 7.27$ V and $i_T = i_1 + i_2 = 1.455 - 0.1818 = 1.27$ A.

Problems

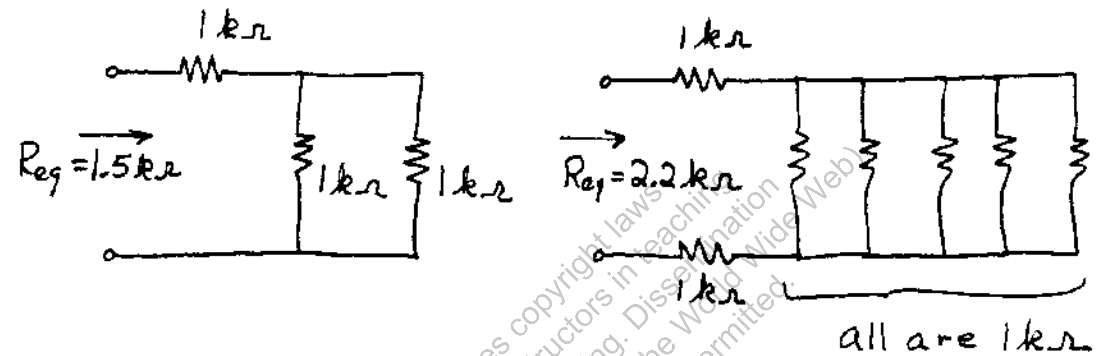
P2.1* (a) $R_{eq} = 20\ \Omega$ (b) $R_{eq} = 23\ \Omega$

P2.2* We have $4 + \frac{1}{1/20 + 1/R_x} = 8$ which yields $R_x = 5\ \Omega$.

P2.3* The 12-Ω and 6-Ω resistances are in parallel having an equivalent resistance of 4 Ω. Similarly, the 18-Ω and 9-Ω resistances are in parallel and have an equivalent resistance of 6 Ω. Finally, the two parallel combinations are in series, and we have

$$R_{ab} = 4 + 6 = 10\ \Omega$$

P2.4*

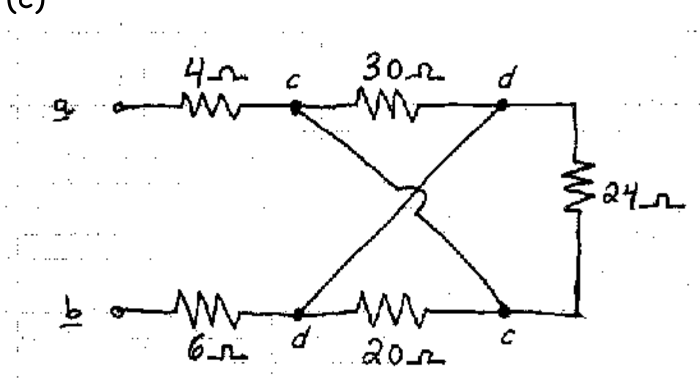


P2.5* The 20-Ω and 30-Ω resistances are in parallel and have an equivalent resistance of R_{eq1} = 12 Ω. Also the 40-Ω and 60-Ω resistances are in parallel with an equivalent resistance of R_{eq2} = 24 Ω. Next we see that R_{eq1} and the 4-Ω resistor are in series and have an equivalent resistance of $R_{eq3} = 4 + R_{eq1} = 16$ Ω. Finally R_{eq3} and R_{eq2} are in parallel and the overall equivalent resistance is

$$R_{ab} = \frac{1}{1/R_{eq1} + 1/R_{eq2}} = 9.6\ \Omega$$

P2.6 (a) $R_{eq} = 22\ \Omega$ (b) $R_{eq} = 20\ \Omega$

(c)



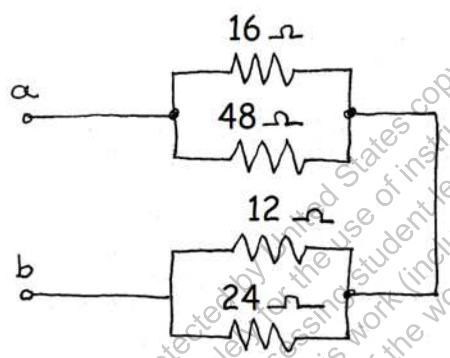
Notice that the points labeled c are the same node and that the points labeled d are another node. Thus, the 30-Ω, 24-Ω, and 20-Ω resistors are in parallel because they are each connected between nodes c and d. The equivalent resistance is 18 Ω.

P2.7 We have $\dfrac{1}{1/120 + 1/R_x} = 48$ which yields $R_x = 80\ \Omega$.

P2.8 (a) $R_{eq} = 18\ \Omega$ (b) $R_{eq} = 10\ \Omega$

P2.9 We have $R_{eq} = \dfrac{R(2R)}{R + 2R} = \dfrac{2R}{3}$. Clearly, for R_{eq} to be an integer, R must be an integer multiple of 3.

P2.10 $R_{ab} = 20\ \Omega$



P2.11 Because the resistances are in parallel, the same voltage v appears across both of them. The current through R_1 is $i_1 = v/90$. The current through R_2 is $i_2 = 3i_1 = 3v/90$. Finally, we have $R_2 = v/i_2 = v/(3v/90) = 30\ \Omega$.

P2.12 Combining the resistances shown in Figure P2.12b, we have

$$R_{eq} = 1 + \frac{1}{1/2 + 1/R_{eq}} + 1 = 2 + \frac{2R_{eq}}{2 + R_{eq}}$$

$$R_{eq}(2 + R_{eq}) = 2(2 + R_{eq}) + 2R_{eq}$$

$$(R_{eq})^2 - 2R_{eq} - 4 = 0$$

$$R_{eq} = 3.236\ \Omega$$

($R_{eq} = -1.236\ \Omega$ is another root, but is not physically reasonable.)

P2.13 $$R_{eq} = \frac{1}{\frac{1}{1000} + \frac{1}{1000} + \frac{1}{1000} + \ldots} = \frac{1}{\frac{n}{1000}} = \frac{1000}{n}$$

P2.14 In the lowest power mode, the power is $P_{lowest} = \dfrac{120^2}{R_1 + R_2} = 83.33$ W.

For the highest power mode, the two elements should be in parallel with an applied voltage of 240 V. The resulting power is

$$P_{highest} = \frac{240^2}{R_1} + \frac{240^2}{R_2} = 1000 + 500 = 1500 \text{ W}.$$

Some other modes and resulting powers are:

- R_1 operated separately from 240 V yielding 1000 W
- R_2 operated separately from 240 V yielding 500 W
- R_1 in series with R_2 operated from 240 V yielding 333.3 W
- R_1 operated separately from 120 V yielding 250 W

P2.15 For operation at the lowest power, we have

$$P = 240 = \frac{120^2}{R_1 + R_2}$$

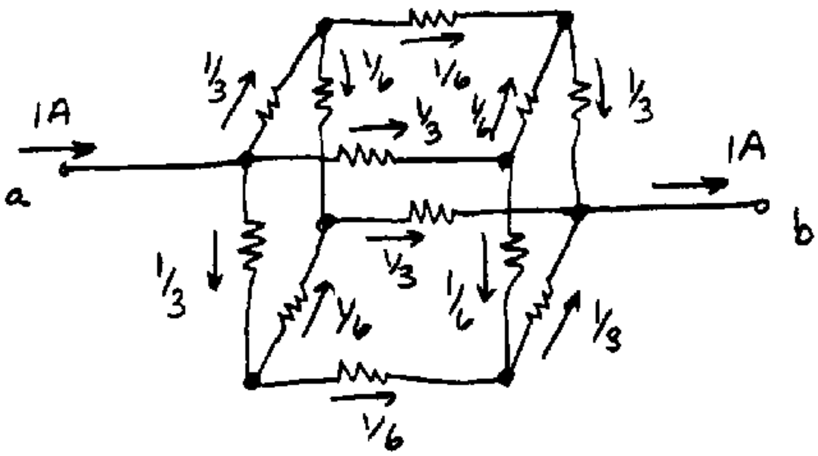
At the high power setting, we have

$$P = 1280 = \frac{120^2}{R_1} + \frac{120^2}{R_2}$$

Solving these equations, we find $R_1 = 15\,\Omega$ and $R_2 = 45\,\Omega$. (The second solution simply has the values of R_1 and R_2 interchanged.)

The intermediate power settings are obtained by operating one of the elements from 120 V resulting in powers of 320 W and 960 W.

P2.16 By symmetry, we find the currents in the resistors as shown below:



Then, the voltage between terminals a and b is

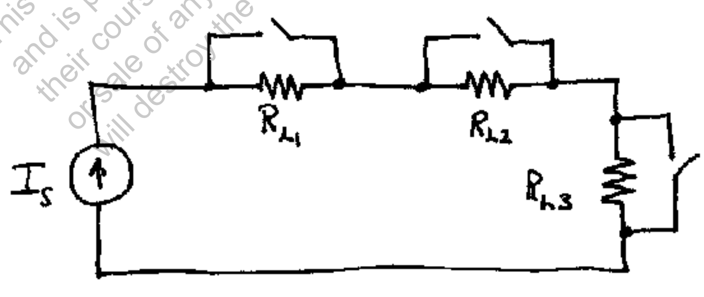
$$v_{ab} = R_{eq} = 1/3 + 1/6 + 1/3 = 5/6$$

P2.17 $R = 16\ \Omega$.

P2.18 (a) For a series combination $G_{eq} = \dfrac{1}{1/G_1 + 1/G_2 + 1/G_3}$

(b) For a parallel combination of conductances $G_{eq} = G_1 + G_2 + G_3$

P2.19 To supply the loads in such a way that turning one load on or off does not affect the other loads, we must connect the loads in series with a switch in parallel with each load:



To turn a load on, we open the corresponding switch, and to turn a load off, we close the switch.

P2.20 We have $R_a + R_b = R_{ab} = 50$, $R_b + R_c = R_{bc} = 100$ and $R_a + R_c = R_{ca} = 70$ These equations can be solved to find that $R_a = 10\ \Omega$, $R_b = 40\ \Omega$, and $R_c = 60\ \Omega$. After shorting terminals *b* and *c*, the equivalent resistance between terminal *a* and the shorted terminals is

$$R_{eq} = R_a + \frac{1}{1/R_b + 1/R_c} = 34\ \Omega$$

P2.21 The equations for the conductances are

$$G_b + G_c = \frac{1}{R_{as}} = \frac{1}{12} \qquad G_a + G_c = \frac{1}{R_{bs}} = \frac{1}{20} \qquad G_b + G_a = \frac{1}{R_{cs}} = \frac{1}{15}$$

Adding respective sides of the first two equations and subtracting the respective sides of the third equation yields

$2G_c = \frac{1}{12} + \frac{1}{20} - \frac{1}{15} = \frac{8}{120}$ from which we obtain $G_c = \frac{1}{30}$ S. Then we have $R_c = 30\,\Omega$. Similarly, we find $R_a = 60\,\Omega$ and $R_b = 20\,\Omega$.

P2.22 The steps in solving a circuit by network reduction are:

1. Find a series or parallel combination of resistances.
2. Combine them.
3. Repeat until the network is reduced to a single resistance and a single source (if possible).
4. Solve for the currents and voltages in the final circuit. Transfer results back along the chain of equivalent circuits, solving for more currents and voltages along the way.
5. Check to see that KVL and KCL are satisfied in the original network.

The method does not always work because some networks cannot be reduced sufficiently. Then, another method such as node voltages or mesh currents must be used.

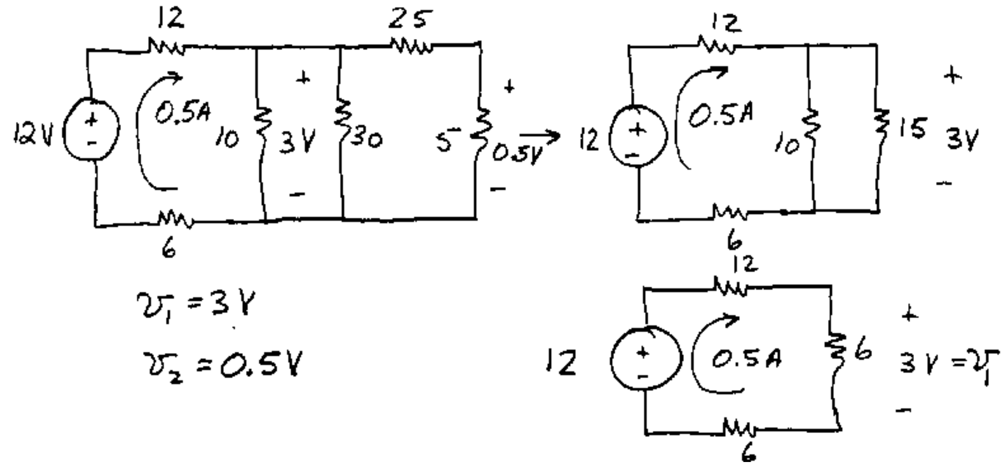
P2.23*

$$i_1 = \frac{10}{R_{eq}} = \frac{10}{10} = 1\text{ A}$$

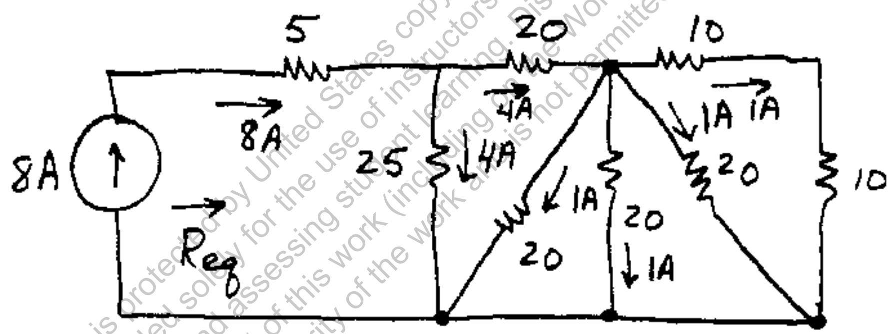
$$v_x = 4\text{ V}$$

$$i_2 = \frac{v_x}{8} = 0.5\ \text{A}$$

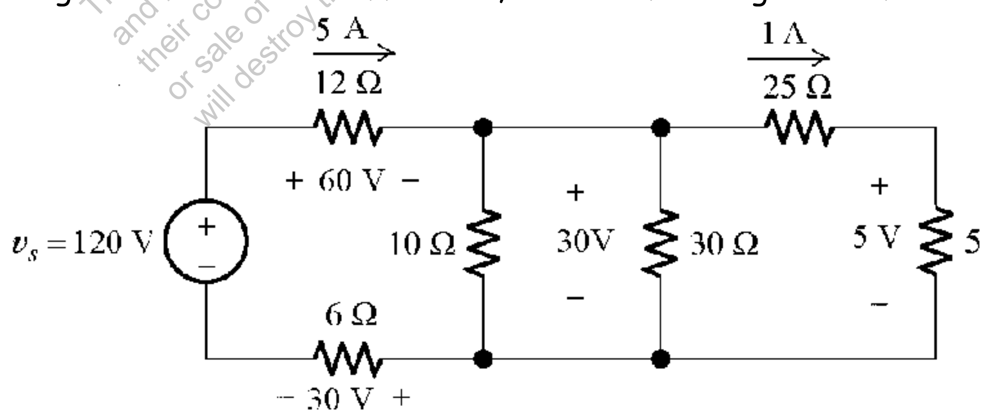
P2.24* We combine resistances in series and parallel until the circuit becomes an equivalent resistance across the voltage source. Then, we solve the simplified circuit and transfer information back along the chain of equivalents until we have found the desired results.



P2.25* Combining resistors in series and parallel, we find that the equivalent resistance seen by the current source is $R_{eq} = 17.5\ \Omega$. Thus, $v = 8 \times 17.5 = 140$ V. Also, $i = 1$ A.



P2.26 Using Ohm's and Kirchhoff's laws, we work from right to left resulting in



P2.27 The equivalent resistance seen by the current source is

$R_{eq} = 8 + \dfrac{1}{1/6 + 1/12} + \dfrac{1}{1/20 + 1/30} = 24\ \Omega$. Then, we have $v = 3R_{eq} = 72$ V, $i_2 = 1$ A, and $i_1 = 1.2$ A.

P2.28 The equivalent resistance seen by the current source is

$$R_{eq} = 6 + \frac{1}{1/10 + 1/(30+10)} = 14\ \Omega$$

Then, we have $v_s = 2R_{eq} = 28$ V $\quad v_2 = 2\dfrac{1}{1/10 + 1/(30+10)} = 16$ V

$i_2 = \dfrac{v_2}{10} = 1.6$ A $\quad i_1 = \dfrac{v_2}{10+30} = 0.4$ A $\quad v_1 = -10i_1 = -4$ V.

P2.29 The equivalent resistance seen by the voltage source is

$$R_{eq} = \frac{1}{1/18 + 1/(7+2)} + 4 = 10\ \Omega$$

Then, we have

$$i_1 = \frac{20\text{ V}}{R_{eq}} = 2\text{ A} \quad v_2 = i_1\frac{1}{1/18 + 1/(7+2)} = 12\text{ V} \quad i_2 = \frac{v_2}{18} = 0.667\text{ A}$$

P2.30 The currents through the 3-Ω resistance and the 4-Ω resistance are zero because they are series with an open circuit. Similiarly, the 6-Ω resistance is also in series with the open circuit, and its current is zero. Thus, we can consider the 8-Ω and the 7-Ω resistances to be in series. The current circulating clockwise in the left-hand loop is given by $i_1 = \dfrac{15}{7+8} = 1$ A, and we have $v_1 = 7i_1 = 7$ V. The current circulating counterclockwise in the right hand loop is 2 A. By Ohm's law, we have $v_2 = 4$ V. Then, using KVL we have $v_{ab} = v_1 - v_2 = 3$ V.

P2.31 $i_2 = \dfrac{20\text{ V}}{20\ \Omega} = 1$ A $\qquad i_1 = i_2 - 3 = -2$ A

Notice that current is referenced into the negative reference for the voltage for both sources, thus $P = -vi$ for both sources.

$P_{current-source} = -3\text{ A} \times 20\text{ V} = -60\text{ W} \qquad P_{voltage-source} = -20i_1 = 40\text{ W}$

Power is delivered by the current source and absorbed by the voltage source.

P2.32 $i = \frac{P}{v} = \frac{36\text{ mW}}{12\text{ V}} = 3\text{ mA}$ $\qquad R_{eq} = R + \frac{1}{1/R + 1/R + 1/R} = \frac{4}{3}R$

$i = 3 \times 10^{-3} = \frac{12}{R_{eq}} = \frac{12}{4R/3}$ $\qquad R = 3000\ \Omega$

P2.33 With the switch open, the current flowing clockwise in the circuit is given by $i = \frac{12}{6 + R_2}$, and we have $v_2 = R_2 i = \frac{12R_2}{6 + R_2} = 8$. Solving, we find $R_2 = 12$ Ω.

With the switch closed, R_2 and R_L are in parallel with an equivalent resistance given by $R_{eq} = \frac{1}{1/R_2 + 1/R_L} = \frac{1}{1/12 + 1/R_L}$. The current through R_{eq} is given by $i = \frac{12}{6 + R_{eq}}$ and we have $v_2 = R_{eq} i = \frac{12R_{eq}}{6 + R_{eq}} = 6$. Solving, we find $R_{eq} = 6\ \Omega$. Then, we can write $R_{eq} = \frac{1}{1/12 + 1/R_L} = 6$. Solving, we find $R_L = 12\ \Omega$.

P2.34* $R_{eq} = \frac{1}{1/5 + 1/15} = 3.75\,\Omega \qquad v_x = 2\,A \times R_{eq} = 7.5\text{ V}$

$i_1 = v_x/5 = 1.5\,A \qquad i_2 = v_x/15 = 0.5\,A$

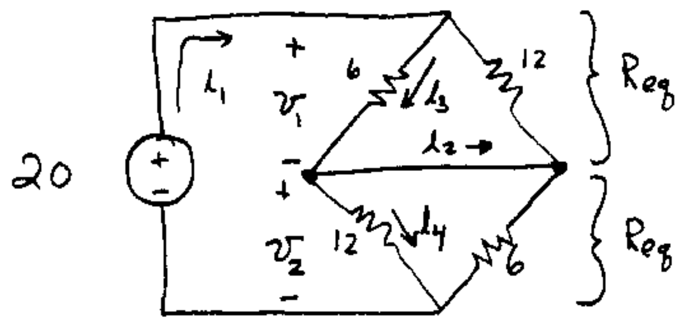
$P_{4A} = 4 \times 7.5 = 30$ W delivering

$P_{2A} = 2 \times 7.5 = 15$ W absorbing

$P_{5\Omega} = 7.5^2/5 = 11.25$ W absorbing

$P_{15\Omega} = (7.5)^2/15 = 3.75$ W absorbing

P2.35*



$$R_{eq} = \frac{1}{1/6 + 1/12} = 4\,\Omega \qquad i_1 = \frac{20\text{ V}}{2R_{eq}} = 2.5\text{ A}$$

$$v_1 = v_2 = R_{eq} i_1 = 10\text{ V} \qquad i_3 = 10/6 = 1.667\text{ A}$$

$$i_4 = 10/12 = 0.8333\text{ A} \qquad i_2 = i_3 - i_4 = 0.8333\text{ A}$$

P2.36* $v_1 = \dfrac{R_1}{R_1 + R_2 + R_3} \times v_s = 5\text{ V}$ $\qquad v_2 = \dfrac{R_2}{R_1 + R_2 + R_3} \times v_s = 7\text{ V}$

$$v_3 = \frac{R_3}{R_1 + R_2 + R_3} \times v_s = 13\text{ V}$$

P2.37* $i_1 = \dfrac{R_2}{R_1 + R_2} i_s = 1\text{ A}$ $\qquad i_2 = \dfrac{R_1}{R_1 + R_2} i_s = 2\text{ A}$

P2.38* Combining R_2 and R_3, we have an equivalent resistance

$R_{eq} = \dfrac{1}{1/R_2 + 1/R_3} = 10\,\Omega$. Then, using the voltage-division principle, we

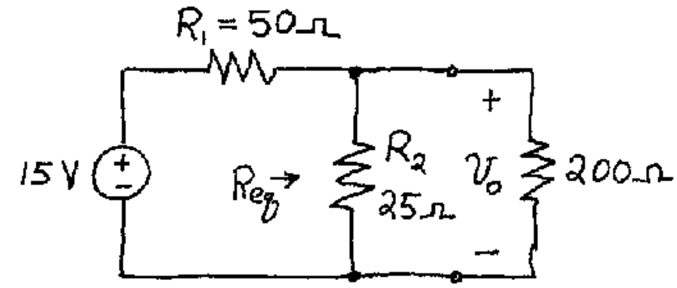
have $v = \dfrac{R_{eq}}{R_1 + R_{eq}} \times v_s = \dfrac{10}{20 + 10} \times 10 = 3.333\text{ V}$.

P2.39 $i_3 = \dfrac{R_2}{R_2 + R_3} \times i_s = \dfrac{25}{25 + 75} \times 20 = 5\text{ mA}$

P2.40 (a) $R_1 + R_2 = \dfrac{15\text{ V}}{0.2\text{ A}} = 75\,\Omega$ $\qquad \dfrac{R_2}{R_1 + R_2} \times 15 = 5$

Solving, we find $R_2 = 25\,\Omega$ and $R_1 = 50\,\Omega$.

(b)



The equivalent resistance for the parallel combination of R_2 and the load is

$$R_{eq} = \frac{1}{1/25 + 1/200} = 22.22\ \Omega$$

Then, using the voltage division principle, we have

$$v_o = \frac{R_{eq}}{R_1 + R_{eq}} \times 15\ \text{V} = 4.615\ \text{V}$$

P2.41 We have $120\frac{5}{10 + 5 + R_x} = 20$, which yields $R_x = 15\ \Omega$.

P2.42 First, we combine the 60 Ω and 20 Ω resistances in parallel yielding an equivalent resistance of 15 Ω, which is in parallel with R_x. Then, applying the current division principle, we have

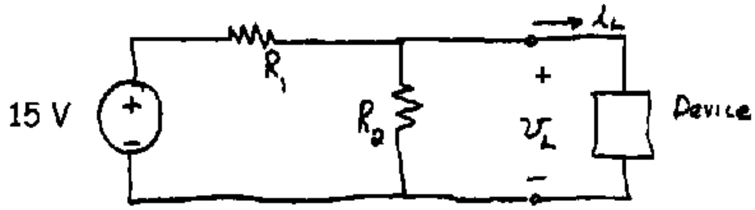
$$15\frac{15}{15 + R_x} = 10$$

which yields $R_x = 7.5\ \Omega$.

P2.43* $v = 0.1\ \text{mA} \times R_w = 50\ \text{mV}$

$$R_g = \frac{50\ \text{mV}}{2\ \text{A} - 0.1\ \text{mA}} = 25\ \text{m}\Omega$$

P2.44 The circuit diagram is:



With $i_L = 0$ and $v_L = 5\text{ V}$, we must have $\dfrac{R_2}{R_1 + R_2} \times 15 = 5\text{ V}$. Rearranging, this gives

$$\frac{R_1}{R_2} = 2 \qquad (1)$$

With $i_L = 50\text{ mA}$ and $v_L = 4.7\text{ V}$, we have $15 - R_1(4.7/R_2 + 50\text{ mA}) = 4.7$. Rearranging, this gives

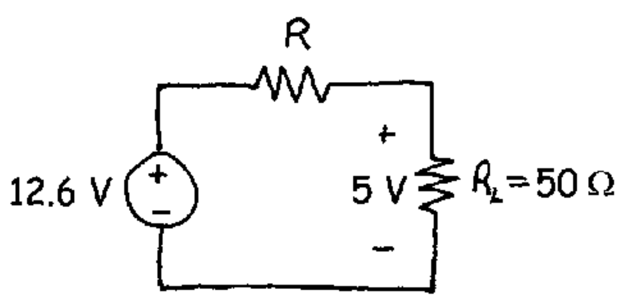
$$4.7\frac{R_1}{R_2} + R_1 \times 0.05 = 10.3. \qquad (2)$$

Using Equation (1) to substitute into Equation (2) and solving, we obtain $R_1 = 18\,\Omega$ and $R_2 = 9\,\Omega$.

Maximum power is dissipated in R_1 for $i_L = 50\text{ mA}$, for which the voltage across R_1 is 10.3 V. Thus, $P_{\max R1} = \dfrac{10.3^2}{18} = 5.89\text{ W}$. Thus, R_1 must be rated for at least 5.89 W of power dissipation.

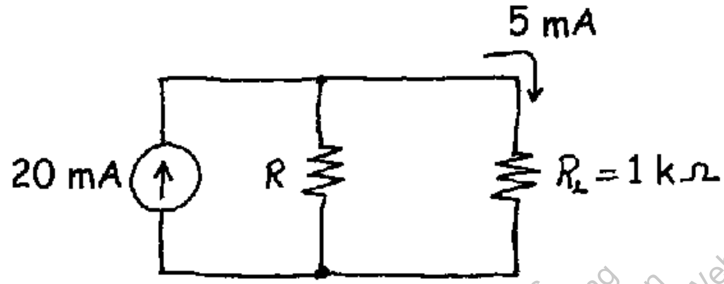
Maximum power is dissipated in R_2 for $i_L = 0$, in which case the voltage across R_2 is 5 V. Thus, $P_{\max R2} = \dfrac{5^2}{9} = 2.78\text{ W}$.

P2.45 We need to place a resistor in series with the load and the voltage source as shown:



Applying the voltage-division principle, we have $12.6\frac{50}{50+R}=5$. Solving, we find $R=76\ \Omega$.

P2.46 We have $P=25\times10^{-3}=I_L^2R_L=I_L^2 1000$. Solving, we find that the current through the load is $I_L=5$ mA. Thus, we must place a resistor in parallel with the current source and the load.



Then, we have $20\frac{R}{R+R_L}=5$ from which we find $R=333.3\ \Omega$.

P2.47 In a similar fashion to the solution for Problem P2.12, we can write the following expression for the resistance seen by the 16-V source.

$$R_{eq}=2+\frac{1}{1/R_{eq}+1/4}\ \text{k}\Omega$$

The solutions to this equation are $R_{eq}=4\,\text{k}\Omega$ and $R_{eq}=-2\,\text{k}\Omega$. However, we reason that the resistance must be positive and discard the negative root. Then, we have $i_1=\frac{16\text{ V}}{R_{eq}}=4$ mA, $i_2=i_1\frac{R_{eq}}{4+R_{eq}}=\frac{i_1}{2}=2$ mA, and $i_3=\frac{i_1}{2}=2$ mA. Similarly, $i_4=\frac{i_3}{2}=\frac{i_1}{2^2}=1$ mA. Clearly, $i_{n+2}=i_n/2$. Thus, $i_{18}=\frac{i_1}{2^9}=7.8125\ \mu\text{A}$.

P2.48* At node 1 we have: $\frac{v_1}{20}+\frac{v_1-v_2}{10}=1$

At node 2 we have: $\frac{v_2}{5}+\frac{v_2-v_1}{10}=2$

In standard form, the equations become

$$0.15v_1-0.1v_2=1$$
$$-0.1v_1+0.3v_2=2$$

Solving, we find $v_1=14.29$ V and $v_2=11.43$ V.

Then we have $i_1 = \frac{v_1 - v_2}{10} = 0.2857$ A.

P2.49* Writing a KVL equation, we have $v_1 - v_2 = 10$.

At the reference node, we write a KCL equation: $\frac{v_1}{5} + \frac{v_2}{10} = 1$.

Solving, we find $v_1 = 6.667$ and $v_2 = -3.333$.

Then, writing KCL at node 1, we have $i_s = \frac{v_2 - v_1}{5} - \frac{v_1}{5} = -3.333$ A.

P2.50 Writing KCL equations, we have

$$\frac{v_1}{21} + \frac{v_1 - v_2}{6} + \frac{v_1 - v_3}{9} = 0$$

$$\frac{v_2 - v_1}{6} + \frac{v_2}{28} = 3$$

$$\frac{v_3}{6} + \frac{v_3 - v_1}{9} + = -3$$

In standard form, we have:

$$0.3254v_1 - 0.1667v_2 - 0.1111v_3 = 0$$

$$-0.1667v_1 + 0.2024v_2 = 3$$

$$-0.1111v_1 + 0.2778v_3 = -3$$

G = [0.3254 -0.1667 -0.1111; -0.1667 0.2024 0; -0.1111 0 0.2778]

I = [0; 3; -3]

V = G\I

Solving, we find $v_1 = 8.847$ V, $v_2 = 22.11$ V, and $v_3 = -7.261$ V.

If the source is reversed, the algebraic signs are reversed in the **I** matrix and consequently, the node voltages are reversed in sign.

P2.51 Writing KCL equations at nodes 1, 2, and 3, we have

$$\frac{v_1}{R_4} + \frac{v_1 - v_2}{R_2} + \frac{v_1 - v_3}{R_1} = 0$$

$$\frac{v_2 - v_1}{R_2} + \frac{v_2 - v_3}{R_3} = I_s$$

$$\frac{v_3}{R_5} + \frac{v_3 - v_2}{R_3} + \frac{v_3 - v_1}{R_1} = 0$$

In standard form, we have:

$$0.55v_1 - 0.20v_2 - 0.25v_3 = 0$$

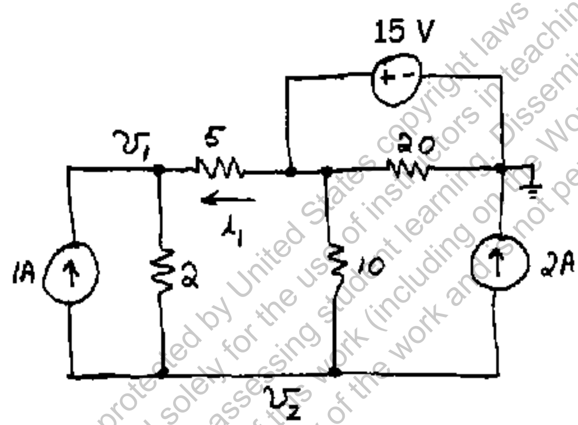
$$-0.20v_1 + 0.325v_2 - 0.125v_3 = 2$$

$$-0.25v_1 - 0.125v_2 + 0.875v_3 = 0$$

Using Matlab, we have

```
>> G = [0.55 -0.20 -0.25; -0.20 0.325 -0.125; -0.25 -0.125 0.875];
>> I = [0; 2; 0];
>> V = G\I
V =
   5.1563
  10.4688
   2.9688
```

P2.52 To minimize the number of unknowns, we select the reference node at one end of the voltage source. Then, we define the node voltages and write a KCL equation at each node.



$$\frac{v_1 - 15}{5} + \frac{v_1 - v_2}{2} = 1 \qquad \frac{v_2 - v_1}{2} + \frac{v_2 - 15}{10} = -3$$

In Matlab, we have

```
G = [0.7 -0.5; -0.5 0.6]
I = [4; -1.5]
V = G\I
I1 = (15 - V(1))/5
```

Then, we have $i_1 = 1.0588\ A$.

The 20-Ω resistance does not appear in the network equations and has no effect on the answer. The voltage at the top end of the 10-Ω resistance is 15 V regardless of the value of the 20-Ω resistance. Thus, any nonzero value could be substituted for the 20-Ω resistance without affecting the answer.

P2.53 Writing KCL equations at nodes 1, 2, and 3, we have

$$\frac{v_1}{R_3} + \frac{v_1 - v_2}{R_4} + I_s = 0$$

$$\frac{v_2 - v_1}{R_4} + \frac{v_2 - v_3}{R_6} + \frac{v_2}{R_5} = 0$$

$$\frac{v_3}{R_1 + R_2} + \frac{v_3 - v_2}{R_6} = I_s$$

In standard form, we have:

$$0.15v_1 - 0.10v_2 = -5$$

$$-0.10v_1 + 0.475v_2 - 0.25v_3 = 0$$

$$-0.25v_2 + 0.30v_3 = 5$$

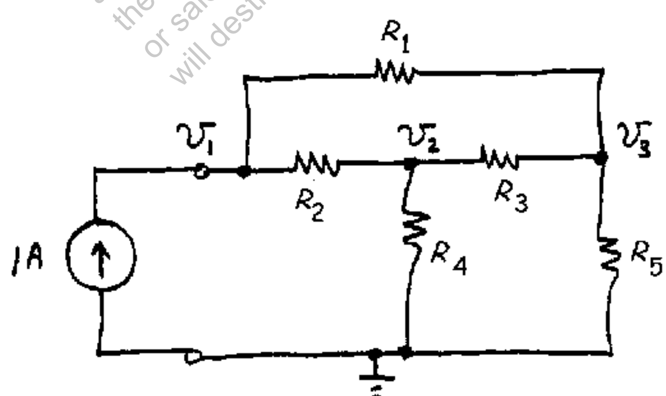
Solving using Matlab, we have

```
G = [ 0.15 -0.10 0; -0.10 0.475 -0.25; 0 -0.25 0.30]
I = [-5; 0; 5]
V = G\I
```

$v_1 = -30.56$ V $\quad v_2 = 4.167$ V $\quad v_3 = 20.14$ V

P2.54 We must not use all of the nodes (including those that are inside supernodes) in writing KCL equations. Otherwise, dependent equations result.

P2.55 The circuit with a 1-A source connected is:



$$\frac{v_1 - v_2}{R_2} + \frac{v_1 - v_3}{R_1} = 1 \qquad \frac{v_2}{R_4} + \frac{v_2 - v_1}{R_2} + \frac{v_2 - v_3}{R_3} = 0$$

$$\frac{v_3}{R_5} + \frac{v_3 - v_1}{R_1} + \frac{v_3 - v_2}{R_3} = 0$$

In Matlab, we use the commands

```
clear all
syms V1 V2 V3 R1 R2 R3 R4 R5
[V1,V2,V3] = solve((V1 - V2)/R2 + (V1 - V3)/R1 == 1 , ...
                V2/R4 + (V2 - V1)/R2 + (V2 - V3)/R3 == 0 , ...
                V3/R5 + (V3 - V1)/R1 + (V3 - V2)/R3 == 0, V1, V2, V3);
 pretty(V1)
```

This produces

```
R1 R2 R3 + R1 R2 R4 + R1 R2 R5 + R1 R3 R4 + R1 R4 R5 + R2 R3 R5 + R2 R4 R5 + R3 R4 R5
--------------------------------------------------------------------------------------
        R1 R3 + R1 R4 + R2 R3 + R1 R5 + R2 R4 + R2 R5 + R3 R4 + R3 R5
```

Then, the command

subs(V1, {'R1','R2','R3','R4','R5'}, {15,5,20,10,8})

yields $R_{eq} = 905/112 = 8.7946\ \Omega$.

P2.56* First, we can write: $i_x = \frac{v_1 - v_2}{5}$.

Then, writing KCL equations at nodes 1 and 2, we have:

$$\frac{v_1}{10} + i_x = 1 \text{ and } \frac{v_2}{20} + 0.5i_x - i_x = 0$$

Substituting for i_x and simplifying, we have

$$0.3v_1 - 0.2v_2 = 1$$

$$-0.1v_1 + 0.15v_2 = 0$$

Solving, we have $v_1 = 6$ and $v_2 = 4$.

Then, we have $i_x = \frac{v_1 - v_2}{5} = 0.4\ \text{A}$.

P2.57* $v_x = v_2 - v_1$

Writing KCL at nodes 1 and 2:

$$\frac{v_1}{5} + \frac{v_1 - 2v_x}{15} + \frac{v_1 - v_2}{10} = 1$$

$$\frac{v_2}{5} + \frac{v_2 - 2v_x}{10} + \frac{v_2 - v_1}{10} = 2$$

Substituting and simplifying, we have

$15v_1 - 7v_2 = 30$ and $v_1 + 2v_2 = 20$.

Solving, we find $v_1 = 5.405$ and $v_2 = 7.297$.

P2.58 First, we can write $i_x = -\frac{v_1}{10}$. Then writing KVL, we have $v_1 - 5i_x - v_2 = 0$. Writing KCL at the reference node, we have $\frac{v_2}{20} = i_x + 8$. Using the first equation to substitute for i_x and simplifying, we have

$$1.5v_1 - v_2 = 0$$
$$2v_1 + v_2 = 160$$

Solving, we find $v_1 = 45.71$ V, $v_2 = 68.57$ V, and $i_x = -\frac{v_1}{10} = 4.571$ A.

Finally, the power delivered to the 8-Ω resistance is

$$P = \frac{(v_1 - v_2)^2}{8} = 65.31 \text{ W}.$$

P2.59 First, we can write:

$$i_x = \frac{5i_x - v_2}{10}$$

Simplifying, we find $i_x = -0.2v_2$.

Then write KCL at nodes 1 and 2:

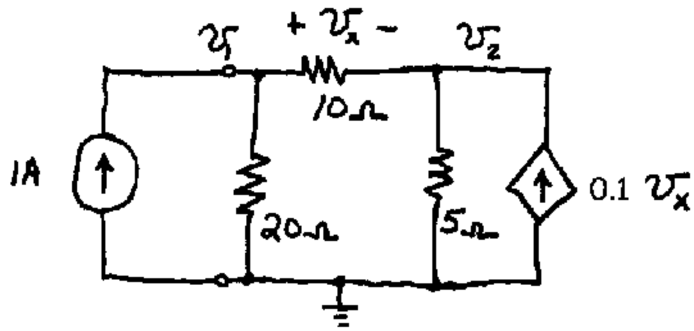
$$\frac{v_1 - 5i_x}{5} = 3 \qquad \frac{v_2}{10} - i_x = -1$$

Substituting for i_x and simplifying, we have

$$v_1 - v_2 = 15 \quad \text{and} \quad 0.3v_2 = -3$$

which yield $v_1 = 25$ V and $v_2 = -10$ V.

P2.60 The circuit with a 1-A current source connected is:



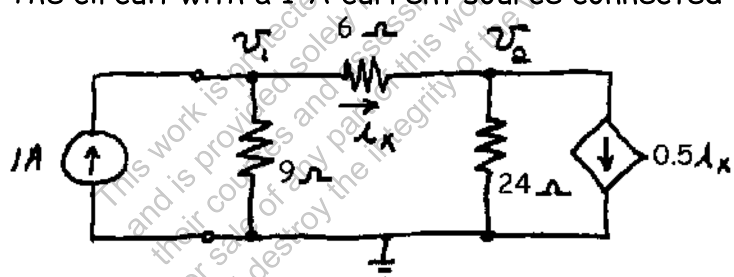
$$v_x = v_1 - v_2$$
$$\frac{v_1}{20} + \frac{v_1 - v_2}{10} = 1$$
$$\frac{v_2 - v_1}{10} + \frac{v_2}{5} - 0.1v_x = 0$$

Using the first equation to substitute for v_x and simplifying, we have

$$0.15v_1 - 0.1v_2 = 1$$
$$-0.2v_1 + 0.4v_2 = 0$$

Solving we find $v_1 = 10$. However, the equivalent resistance is equal in value to v_1 so we have $R_{eq} = 10\ \Omega$.

P2.61 The circuit with a 1-A current source connected is



$$i_x = \frac{v_1 - v_2}{6}$$
$$\frac{v_1}{9} + \frac{v_1 - v_2}{6} = 1$$
$$\frac{v_2 - v_1}{6} + \frac{v_2}{24} + 0.5i_x = 0$$

Using the first equation to substitute for i_x and simplifying, we have

$$5v_1 - 3v_2 = 18$$
$$-2v_1 + 3v_2 = 0$$

Solving we find $v_1 = 6$ V. However, the equivalent resistance is equal in value to v_1, so we have R_{eq} = 6 Ω.

P2.62 The Matlab commands:

```
clear all
syms V1 V2 Vin Vout R1 R2
SV = solve((V1 - Vin)/R1 + (V1 - Vout)/R1 + (V1 - V2)/R1 == 0, ...
        (V2 - V1)/R1 + V2/R1 + (V2 - Vout)/R1 == 0, ...
       (Vout - V1)/R1 + (Vout - V2)/R1 + Vout/R2 == 0 , V1, V2, Vout);
pretty(SV.Vout)
```

result in

```
        R2 Vin
1/2 -------
        R2 + R1
```

Thus, we have

$$\frac{V_{out}}{V_{in}} = \frac{R_2/2}{R_2 + R_1}$$

P2.63 We write equations in which voltages are in volts, resistances are in kΩ, and currents are in mA.

KCL node 2: $\frac{(v_2 - v_1)}{4} + \frac{(v_2 - v_3)}{3} + \frac{v_2}{2} = 0$

KCL node 3: $\frac{(v_3 - v_2)}{3} + \frac{(v_3 - v_1)}{1} + \frac{(v_3 - v_4)}{2} = 2$

KCL ref node: $\frac{v_2}{2} + \frac{v_4}{5} = 2$

KVL: $v_1 - v_4 = 10$

Using Matlab:

```
>> G = [-1/4 (1/2 + 1/3 + 1/4) -1/3 0; ...
-1 -1/3 (1 + 1/2 + 1/3) -1/2; 0 1/2 0 1/5; 1 0 0 -1]
>> I = [0; 2; 2; 10]
>> V = G\I
V =
   9.3659
   4.2537
   6.8000
  -0.6341
```

P2.64 Elements on the diagonal of ***G*** equal the sum of the conductances connected to any node, which is 3 S. Element g_{jk} off the diagonal is zero if no resistance is connected between nodes *j* and *k* and equal to −1 if there is a resistance connected between the nodes. ***G*** is the same for all three parts of the problem, only the node to which the current source is attached changes. We used the MATLAB Array Editor to enter the elements of *G*.

```
>> G
G =
     3    -1    -1     0     0     0     0
    -1     3     0    -1     0     0    -1
    -1     0     3    -1    -1     0     0
     0    -1    -1     3     0    -1     0
     0     0    -1     0     3    -1     0
     0     0     0    -1    -1     3    -1
     0    -1     0     0     0    -1     3
>> Ia = [1; 0; 0; 0; 0; 0; 0];
>> Ib = [0; 1; 0; 0; 0; 0; 0];
>> Ic = [0; 0; 0; 1; 0; 0; 0];
>> Va = G\Ia;
>> Vb = G\Ib;
>> Vc = G\Ic;
>> % Here are the answers:
>> Ra = Va(1)
Ra =
    0.5833
>> Rb = Vb(2)
Rb =
    0.7500
>> Rc = Vc(4)
Rc =
    0.8333
```

By symmetry, shorting nodes with equal node voltages, and series parallel combination, we can obtain $R_a = 7/12\ \Omega$, $R_b = 3/4\ \Omega$, and $R_c = 5/6\ \Omega$.

P2.65* Writing KVL equations around each mesh, we have

$$5i_1 + 15(i_1 - i_2) = 20 \quad \text{and} \quad 15(i_2 - i_1) + 10i_2 = 10$$

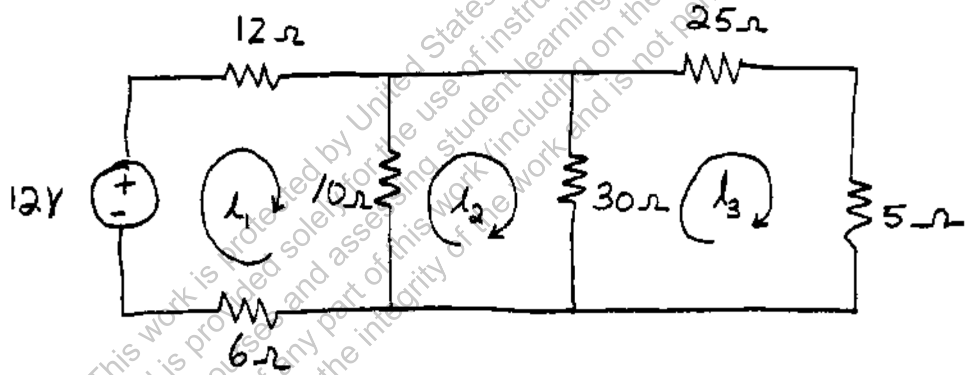
Putting the equations into standard form we have

$$20i_1 - 15i_2 = 20 \quad \text{and} \quad -15i_1 + 25i_2 = 10$$

Solving, we obtain $i_1 = 2.364$ A and $i_2 = 1.818$ A.
Then, the power delivered to the 15-Ω resistor is $P = (i_1 - i_2)^2 15 = 4.471$ W.

P2.66* Writing and simplifying the mesh-current equations, we have:

$$28i_1 - 10i_2 = 12$$
$$-10i_1 + 40i_2 - 30i_3 = 0$$
$$-30i_2 + 60i_3 = 0$$

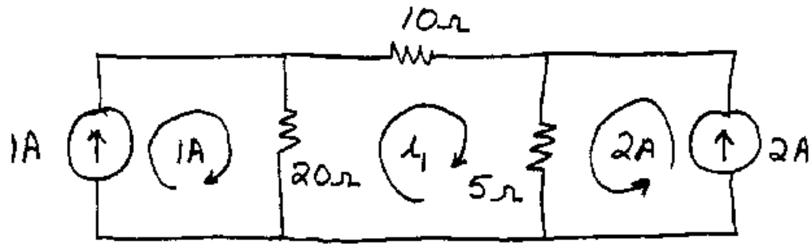


Solving, we obtain

$$i_1 = 0.500 \qquad i_2 = 0.200 \qquad i_3 = 0.100$$

Thus, $v_2 = 5i_3 = 0.500$ V and the power delivered by the source is $P = 12i_1 = 6$ W.

P2.67* Because of the current sources, two of the mesh currents are known.



Writing a KVL equation around the middle loop we have

$$20(i_1 - 1) + 10i_1 + 5(i_1 + 2) = 0$$

Solving, we find $i_1 = 0.2857$ A.

P2.68 Writing KVL equations around each mesh, we have

$$5i_1 + 7(i_1 - i_3) + 31 = 0$$
$$11(i_2 - i_3) + 3i_2 - 31 = 0$$
$$i_3 + 11(i_3 - i_2) + 7(i_3 - i_1) = 0$$

Putting the equations into standard form, we have

$$12i_1 - 7i_3 = -31$$
$$14i_2 - 11i_3 = 31$$
$$-7i_1 - 11i_2 + 19i_3 = 0$$

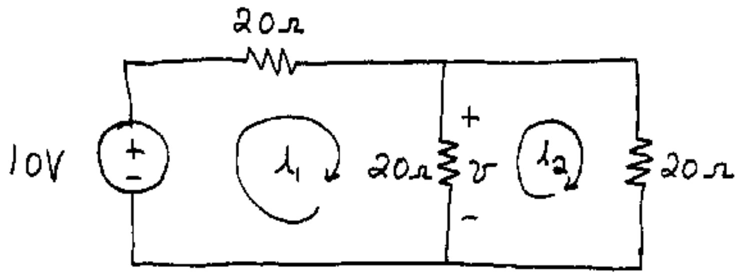
Using Matlab to solve, we have

```
>> R = [12 0 -7; 0 14 -11; -7 -11 19];
>> V = [-31; 31; 0];
>> I = R\V
I =
  -2.0000
   3.0000
   1.0000
```

Then, the power delivered by the source is $P = -31(i_1 - i_2) = 155$ W.

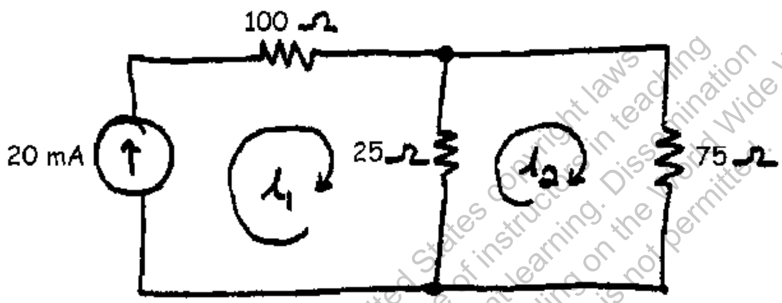
P2.69 Writing and simplifying the mesh equations, we obtain:

$$40i_1 - 20i_2 = 10 \qquad\qquad -20i_1 + 40i_2 = 0$$



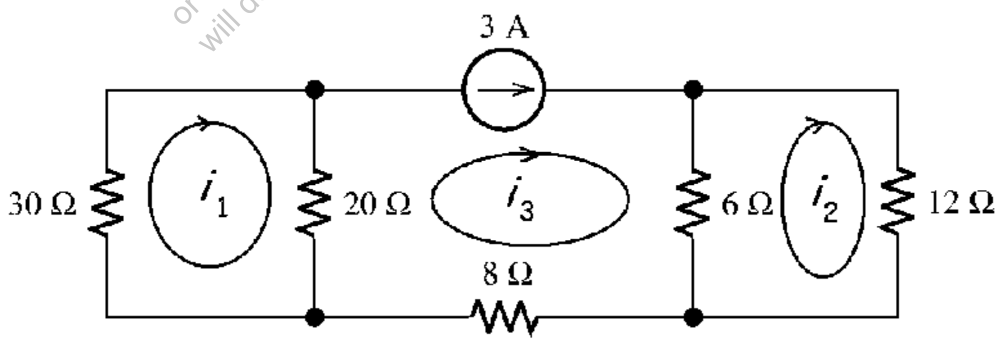
Solving, we find $i_1 = 0.3333$ and $i_2 = 0.1667$.
Thus, $v = 20(i_1 - i_2) = 3.333$ V.

P2.70 The mesh currents and corresponding equations are:



$i_1 = 20$ mA $\quad 25(i_2 - i_1) + 75i_2 = 0$
Solving, we find $i_2 = 5$ mA.
However, i_3 shown in Figure P2.39 is the same as i_2, so the answer is $i_3 = 5$ mA.

P2.71 First, we select the mesh currents and then write three equations.
Mesh 1: $30i_1 + 20(i_1 - i_3) = 0$



Mesh 2: $12i_2 + 6(i_2 - i_3) = 0$
However by inspection, we have $i_3 = 3$. Solving, we obtain
$i_1 = 1.2$ A and $i_2 = 1.0$ A.

P2.72 Writing and simplifying the mesh equations yields:

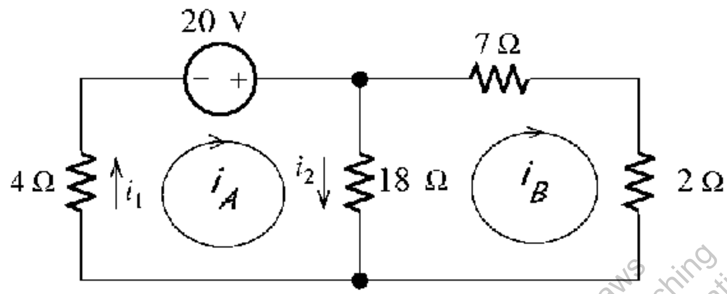
$14i_1 - 8i_2 = 10$

$-8i_1 + 16i_2 = 0$

Solving, we find $i_1 = 1.000$ and $i_2 = 0.500$.

Finally, the power delivered by the source is $P = 10i_1 = 10$ W.

P2.73



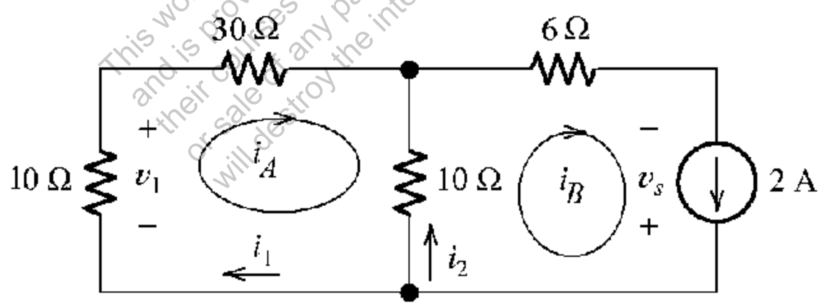
$4i_A + 18(i_A - i_B) = 20$

$18(i_B - i_A) + 7i_B + 2i_B = 0$

Solving we find $i_A = 2$ A and $i_B = 1.333$ A. Then we have $i_1 = i_A = 2$ A and $i_2 = i_A - i_B = 0.667$ A.

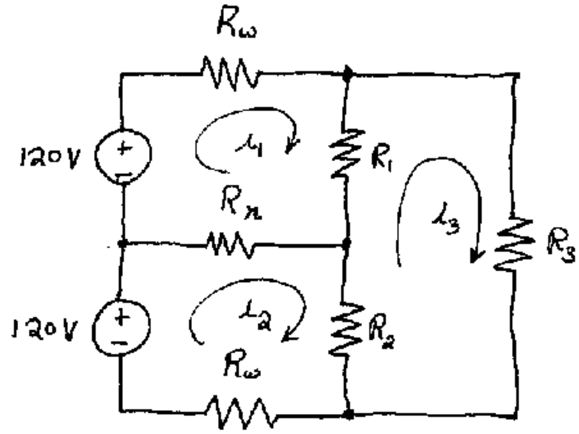
P2.74 Mesh A: $10i_A + 30i_A + 10(i_A - i_B) = 0$

By inspection: $i_B = 2$



Solving, we find $i_A = 0.4$ A. Then we have $i_1 = i_A = 0.4$ A and $i_2 = i_B - i_A = 1.6$ A.

P2.75 (a) First, we select mesh-current variables as shown.



Then, we can write

$$(R_w + R_n + R_1)i_1 - R_n i_2 - R_1 i_3 = 120$$
$$-R_n i_1 + (R_w + R_n + R_2)i_2 - R_2 i_3 = 120$$
$$-R_1 i_1 - R_2 i_2 + (R_1 + R_2 + R_3)i_3 = 0$$

Alternatively, because the network consists of independent voltage sources and resistances, and all of the mesh currents flow clockwise, we can enter the matrices directly into MATLAB.

```
>> Rw = 0.1; Rn=0.1; R1 = 15; R2 = 5; R3 = 8;
>> R = [Rw+Rn+R1 -Rn -R1; -Rn Rw+Rn+R2 -R2; -R1 -R2 R1+R2+R3];
>> V = [120; 120; 0];
>> I = R\V;
>> % Finally, we compute the voltages across the loads.
>> Vr1 = R1*(I(1) - I(3)), Vr2 = R2*(I(2) - I(3)), Vr3 = R3*I(3)
    Vr1 =
      117.8069
    Vr2 =
      113.3613
    Vr3 =
      231.1682
```

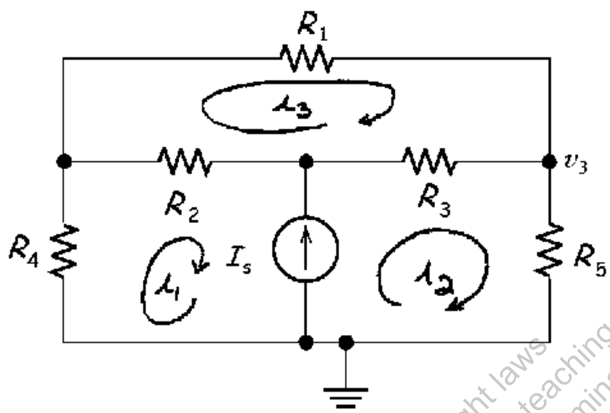
(b) Next, we change Rn to a very high value such as 10^9 which for practical calculations is equivalent to an open circuit, and again compute the voltages resulting in:

```
    Vr1 =
      173.9130
    Vr2 =
       57.9710
```

Vr3 =
  231.8841

The voltage across R_1 is certainly high enough to damage most loads designed to operate at 110 V.

P2.76



Current source in terms of mesh currents: $-i_1 + i_2 = I_s$

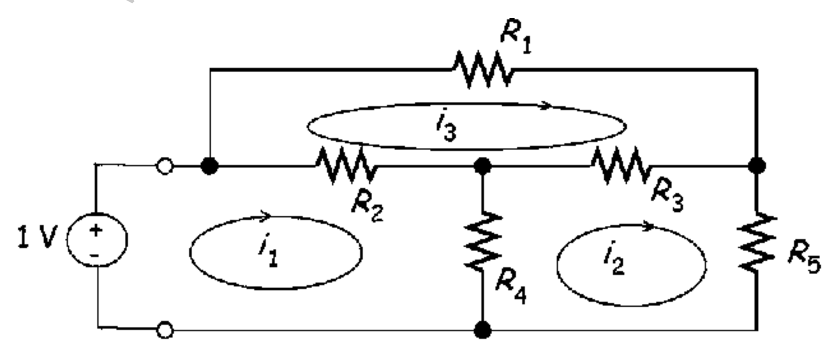
KVL for mesh 3: $-R_2 i_1 - R_3 i_2 + (R_1 + R_2 + R_3)i_3 = 0$

KVL around outside of network: $R_4 i_1 + R_5 i_2 + R_1 i_3 = 0$

Then using MATLAB:

```
>> R1 = 4; R2 = 5; R3 = 8; R4 = 10; R5 = 2; Is = 2;
>> R = [-1 1 0; -R2 -R3 (R1+R2+R3); R4 R5 R1];
>> V = [Is; 0; 0];
>> I = R\V;
>> V3 = R5*I(2)
V3 =
   2.9688
```


P2.77



$(R_2 + R_4)i_1 - R_4 i_2 - R_2 i_3 = 1$

$-R_4 i_1 + (R_3 + R_4 + R_5)i_2 - R_3 i_3 = 0$

$-R_2 i_1 - R_3 i_2 + (R_1 + R_2 + R_3) i_3 = 0$

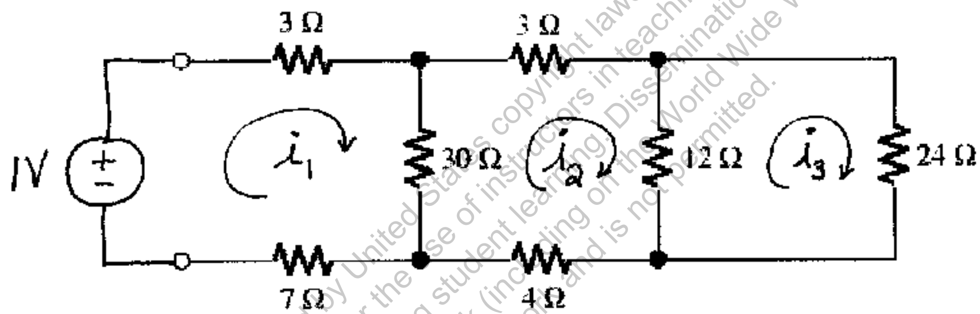
Now using MATLAB:

```
R1 = 6; R2 = 5; R3 = 4; R4 = 8; R5 = 2;
R = [(R2+R4) -R4 -R2; -R4 (R3+R4+R5) -R3; -R2 -R3 (R1+R2+R3)];
V = [1; 0; 0];
I =  R\V;
Req = 1/I(1) % Gives answer in ohms.
Req =
   4.5979
```

P2.78 Mesh 1: $3i_1 + 7i_1 + 30(i_1 - i_2) = 1$

Mesh 2: $3i_2 + 12(i_2 - i_3) + 4i_2 + 30(i_2 - i_1) = 0$

Mesh 3: $24i_3 + 12(i_3 - i_2) = 0$



Solving, we find $i_1 = 0.05$ A. Then $R_{eq} = 1/i_1 = 20\ \Omega$.

P2.79 We write equations in which voltages are in volts, resistances are in kΩ, and currents are in mA.

KVL mesh 1: $4i_1 + 1(i_1 - i_2) + 3(i_1 - i_3) = 0$

KVL mesh 2: $1(i_2 - i_1) + 2(i_2 - i_4) = -10$

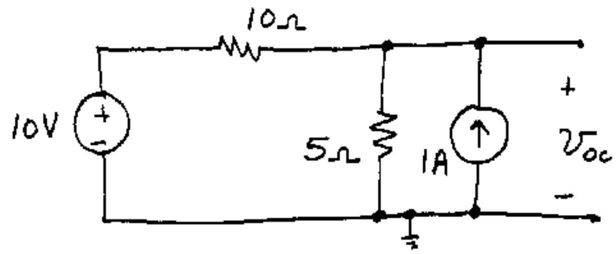
KVL supermesh: $2i_3 + 3(i_3 - i_1) + 2(i_4 - i_2) + 5i_4 = 0$

Current source: $i_4 - i_3 = 2$

Now, we proceed in Matlab.

```
>>  R = [8 -1 -3 0; -1 3 0 -2; -3 -2 5 7; 0 0 -1 1];
>>  V = [0; -10; 0; 2];
>>  I = R\V % This yields the mesh currents in mA.
I =
         0.4780
        4.6439
        -0.2732
        1.7268
```

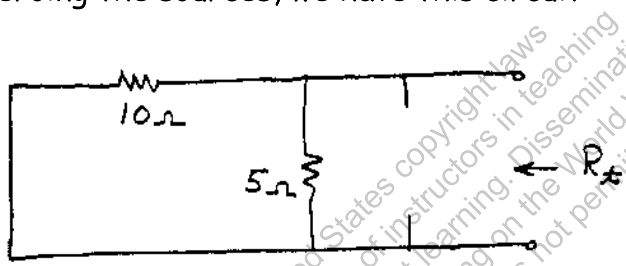
P2.80* First, we write a node voltage equation to solve for the open-circuit voltage:



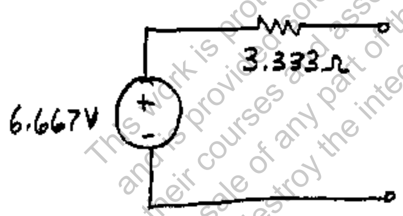
$$\frac{v_{oc}-10}{10}+\frac{v_{oc}}{5}=1$$

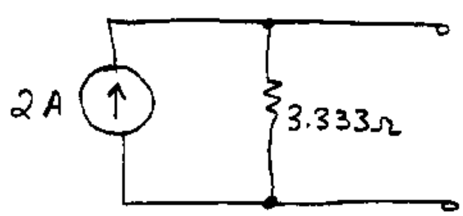
Solving, we find $v_{oc} = 6.667\text{ V}$.

Then zeroing the sources, we have this circuit:

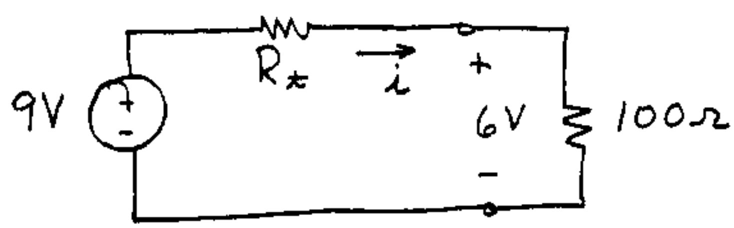


Thus, $R_t = \dfrac{1}{1/10+1/5} = 3.333\,\Omega$. The Thévenin and Norton equivalents are:





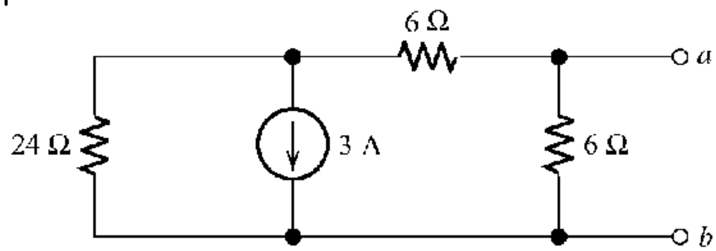
P2.81* The equivalent circuit of the battery with the resistance connected is



$i = 6/100 = 0.06\text{ A}$

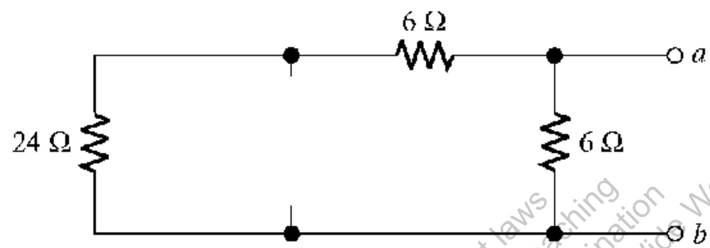
$$R_t = \frac{9-6}{0.06} = 50\,\Omega$$

P2.82 With open-circuit conditions:



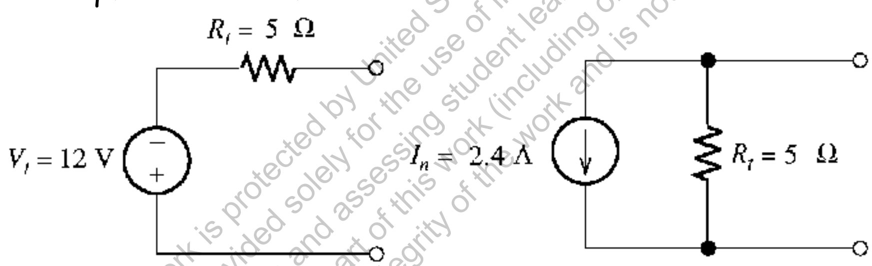
Solving, we find $v_{ab} = -12\ \text{V}$.

With the source zeroed:



$$R_t = \frac{1}{1/6 + 1/(6+24)} = 5\ \Omega$$

The equivalent circuits are:



Notice the source polarity relative to terminals *a* and *b*.

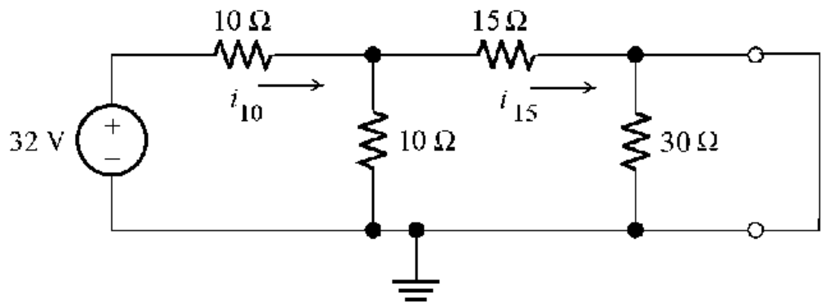
P2.83 First, we solve the network with a short circuit:

$$R_{eq} = 10 + \frac{1}{1/10 + 1/15} = 16\ \Omega$$

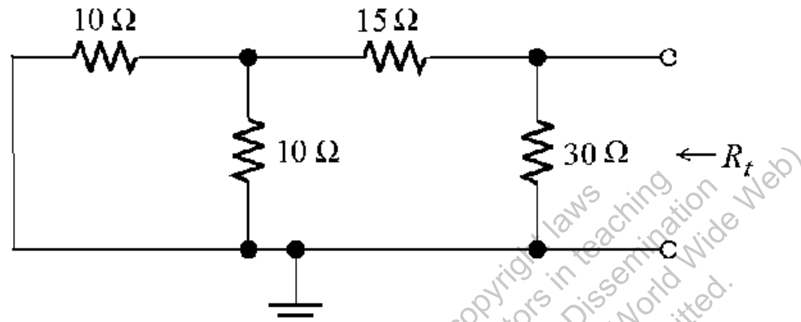
$$i_{10} = 32/R_{eq} = 2\ A$$

$$i_{15} = i_{10}\frac{10}{10+15} = 0.8\ \ A$$

$$i_{sc} = i_{15} = 0.8\ \ A$$

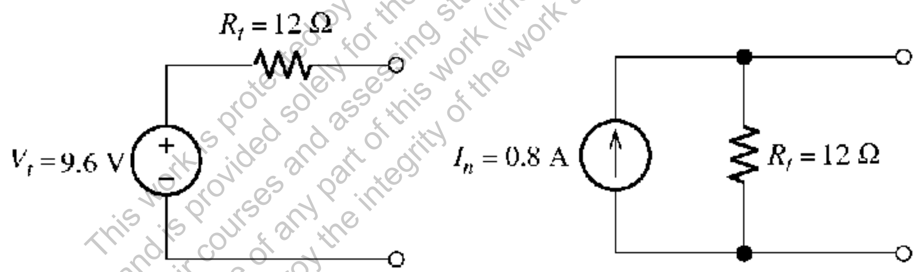


Zeroing the source, we have:

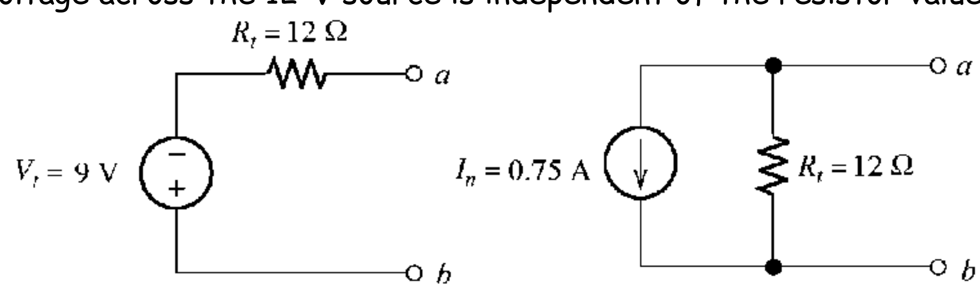


Combining resistances in series and parallel we find $R_t = 12\ \Omega$.

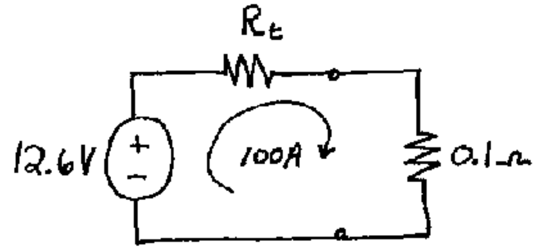
Then the Thévenin voltage is $v_t = i_{sc}R_t = 9.6\,\mathrm{V}$. The Thévenin and Norton equivalents are:



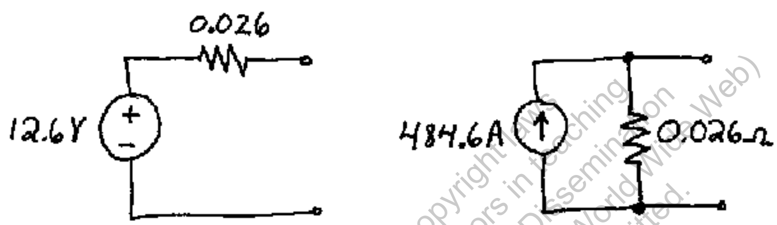
P2.84 The 7-Ω resistor has no effect on the equivalent circuits because the voltage across the 12-V source is independent of the resistor value.



P2.85 The Thévenin voltage is equal to the open-circuit voltage which is 12.6 V. The equivalent circuit with the 0.1-Ω load connected is:

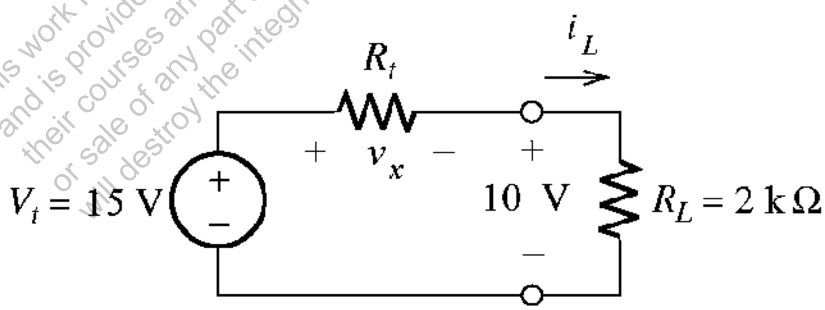


We have $12.6/(R_t + 0.1) = 100$ from which we find $R_t = 0.026\,\Omega$. The Thévenin and Norton equivalent circuits are:



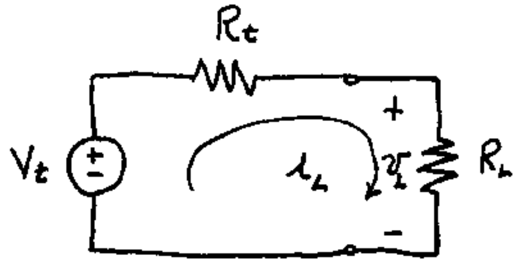
Because no energy is converted from chemical form to heat in a battery under open-circuit conditions, the Thévenin equivalent seems more realistic from an energy conversion standpoint.

P2.86 The Thévenin voltage is equal to the open-circuit voltage which is 15 V. The circuit with the load attached is:



We have $i_L = \dfrac{10}{2000} = 5\,\text{mA}$ and $v_x = V_t - 10 = 5\,\text{V}$. Thus, the Thévenin resistance is $R_t = \dfrac{5\,\text{V}}{5\,\text{mA}} = 1\,\text{k}\Omega$.

P2.87 The equivalent circuit with a load attached is:



For a load of 2.2 kΩ, we have $i_L = 4.4/2200 = 2\text{ mA}$, and we can write $v_L = V_t - R_t i_L$. Substituting values this becomes

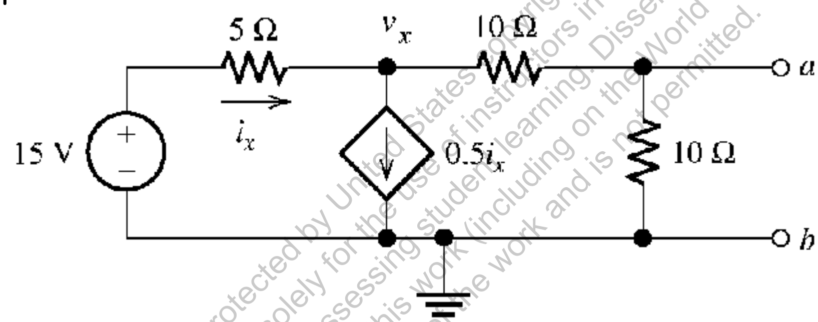
$$4.4 = V_t - 0.002R_t \quad (1)$$

Similarly, for the 10-kΩ load we obtain

$$5 = V_t - 0.0005R_t \quad (2)$$

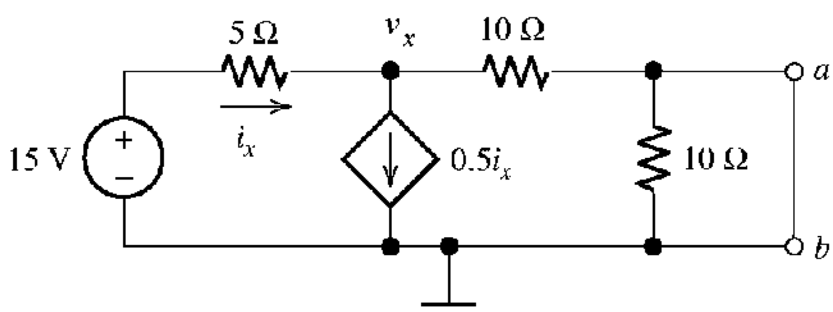
Solving Equations (1) and (2), we find $V_t = 5.2$ V and $R_t = 400\ \Omega$.

P2.88 Open-circuit conditions:

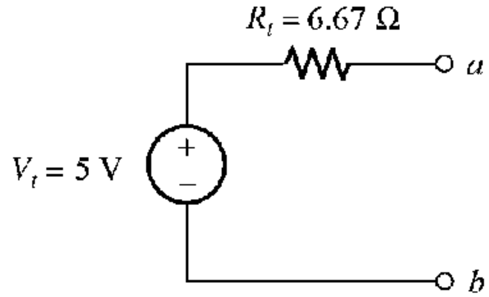


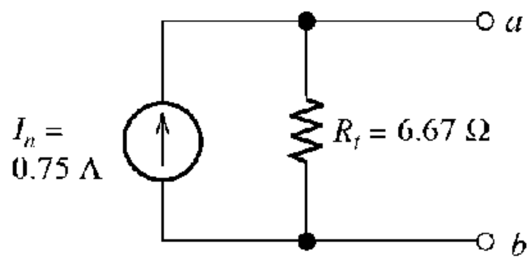
$$i_x = \frac{15 - v_x}{5} \qquad \frac{v_x}{10+10} - i_x + 0.5i_x = 0$$ Solving, we find $v_x = 10$ V and then we have $V_t = v_{oc} = v_x \dfrac{10}{10+10} = 5$ V.

Short-circuit conditions:

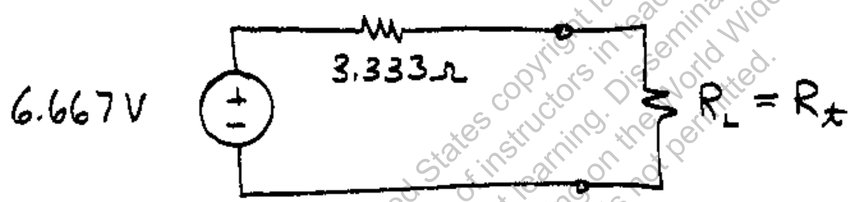


$i_x = \frac{15 - v_x}{5} \qquad \frac{v_x}{10} - i_x + 0.5 i_x = 0$ Solving, we find $v_x = 7.5$ V and then we have $i_{sc} = \frac{v_x}{10} = 0.75$ A. Then, we have $R_t = v_{oc}/i_{sc} = 6.67\ \Omega$. Thus the equivalents are:





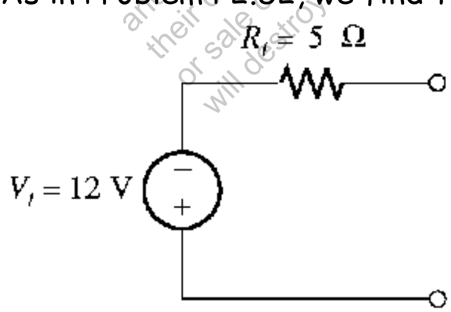
P2.89 As is Problem P2.80, we find the Thévenin equivalent:

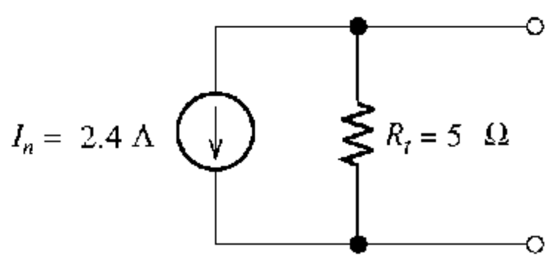


Then maximum power is obtained for a load resistance equal to the Thévenin resistance.

$$P_{max} = \frac{(v_t/2)^2}{R_t} = 3.333 \text{ W}$$

P2.90 As in Problem P2.82, we find the Thévenin equivalent:





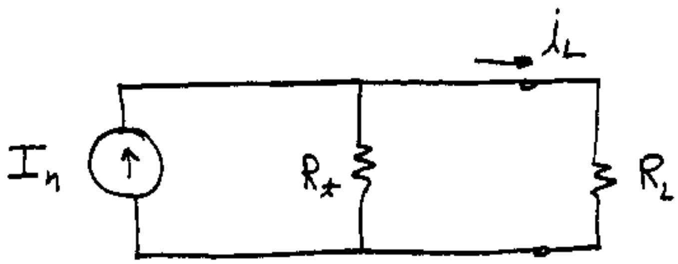
Then, maximum power is obtained for a load resistance equal to the Thévenin resistance.

$$P_{max} = \frac{(V_t/2)^2}{R_t} = 7.2 \text{ W}$$

P2.91* To maximize the power to R_L, we must maximize the voltage across it. Thus, we need to have $R_t = 0$. The maximum power is

$$P_{max} = \frac{20^2}{5} = 80 \text{ W}$$

P2.92 The circuit is



By the current division principle:

$$i_L = I_n \frac{R_t}{R_L + R_t}$$

The power delivered to the load is

$$P_L = (i_L)^2 R_L = (I_n)^2 \frac{(R_t)^2 R_L}{(R_L + R_t)^2}$$

Taking the derivative and setting it equal to zero, we have

$$\frac{dP_L}{dR_L} = 0 = (I_n)^2 \frac{(R_t)^2 (R_t + R_L)^2 - 2(R_t)^2 R_L (R_t + R_L)}{(R_t + R_L)^4}$$

which yields $R_L = R_t$.

The maximum power is $P_{L\max} = (I_n)^2 R_t / 4$.

P2.93 For maximum power conditions, we have $R_L = R_t$. The power taken from the voltage source is

$$P_s = \frac{(V_t)^2}{R_t + R_L} = \frac{(V_t)^2}{2R_t}$$

Then, half of V_t appears across the load and the power delivered to the load is

$$P_L = \frac{(0.5V_t)^2}{R_t}$$

Thus, the percentage of the power taken from the source that is delivered to the load is

$$\eta = \frac{P_L}{P_s} \times 100\% = 50\%$$

On the other hand, for $R_L = 9R_t$, we have

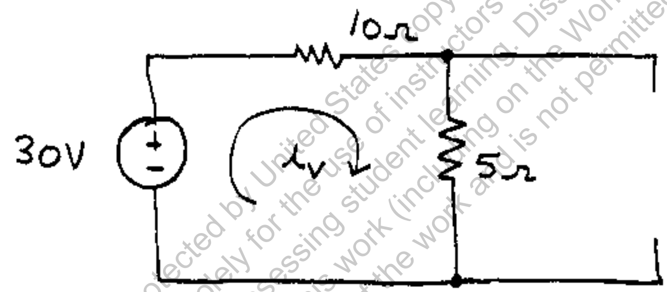
$$P_s = \frac{(V_t)^2}{R_t + R_L} = \frac{(V_t)^2}{10R_t}$$

$$P_L = \frac{(0.9V_t)^2}{9R_t}$$

$$\eta = \frac{P_L}{P_s} \times 100\% = 90\%$$

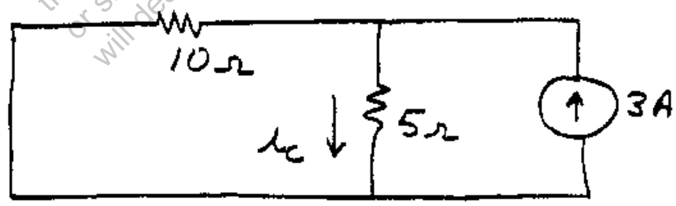
Design for maximum power transfer is relatively inefficient. Thus, systems in which power efficiency is important are almost never designed for maximum power transfer.

P2.94* First, we zero the current source and find the current due to the voltage source.



$$i_v = 30/15 = 2\ A$$

Then, we zero the voltage source and use the current-division principle to find the current due to the current source.

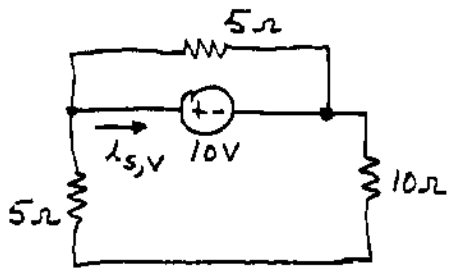


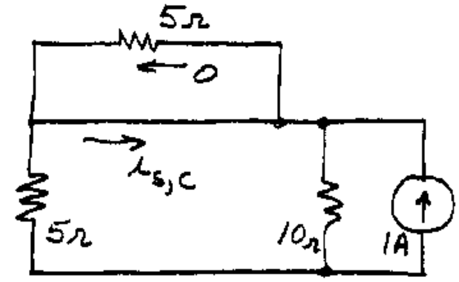
$$i_c = 3\frac{10}{5 + 10} = 2\ A$$

Finally, the total current is the sum of the contributions from each source.

$$i = i_v + i_c = 4\ A$$

P2.95* The circuits with only one source active at a time are:





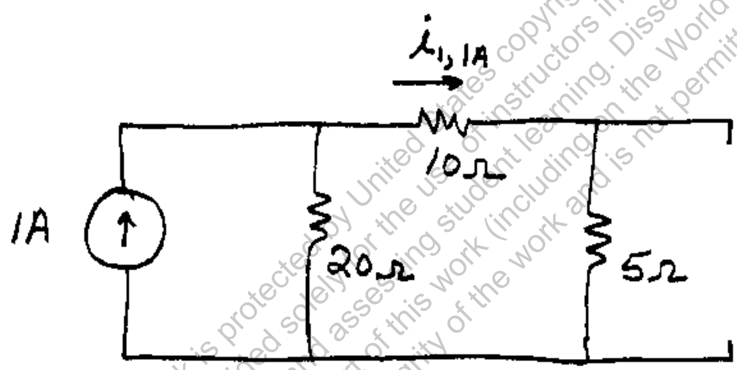
$$R_{eq} = \frac{1}{1/5 + 1/15} = 3.75\ \Omega$$

$$i_{s,v} = -\frac{10\text{ V}}{R_{eq}} = -2.667\text{ A}$$

$$i_{s,c} = -1\frac{10}{10+5} = -0.667\text{ A}$$

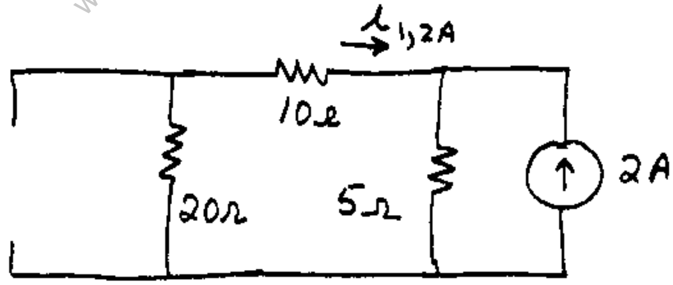
Then the total current due to both sources is $i_s = i_{s,v} + i_{s,c} = -3.333\text{ A}$.

P2.96 Zero the 2-A source and use the current-division principle:



$$i_{1,1A} = 1\frac{20}{15+20} = 0.5714\text{ A}$$

Then zero the 1 A source and use the current-division principle:

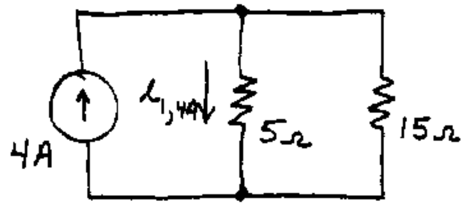


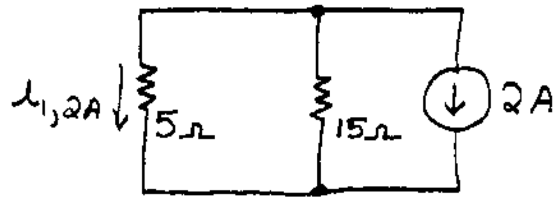
$$i_{1,2A} = -2\frac{5}{5+30} = -0.2857\text{ A}$$

Finally,

$$i_1 = i_{1,1A} + i_{1,2A} = 0.2857\text{ A}$$

P2.97 The circuits with only one source active at a time are:





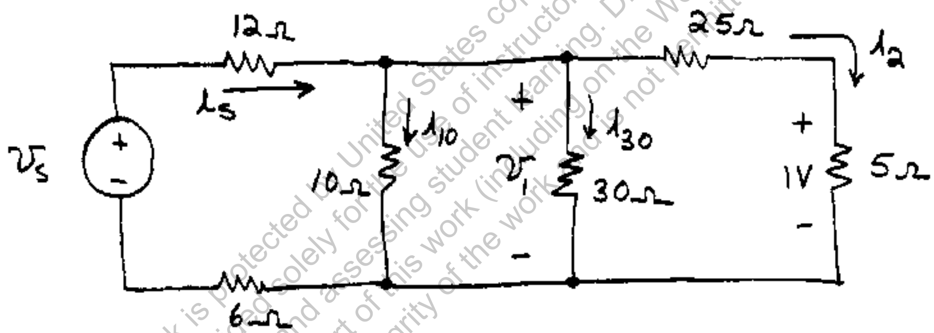
$$i_{1.4A} = 4 \times \frac{15}{15+5} = 3\ A$$

$$i_{1,2A} = -2 \times \frac{15}{15+5} = -1.5$$

Finally, we add the components to find the current with both sources active.

$$i_1 = i_{1,4A} + i_{1,2A} = 1.5\ A$$

P2.98 The circuit, assuming that $v_2 = 1\ \text{V}$ is:

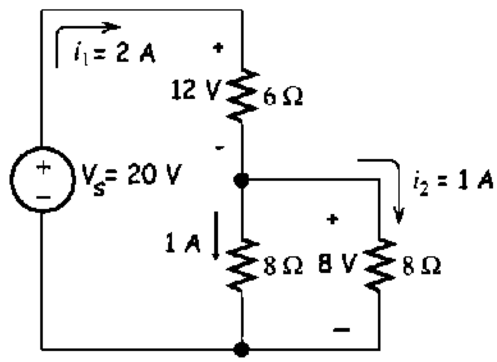


$$i_2 = (v_2/5) = 0.2\ A$$
$$v_1 = 30 i_2 = 6\ \text{V}$$
$$i_{10} = v_1/10 = 0.6\ A$$
$$i_{30} = v_1/30 = 0.2\ A$$
$$i_s = i_2 + i_{10} + i_{30} = 1\ A$$
$$v_s = 12 i_s + v_1 + 6 i_s = 24\ \text{V}$$

We have established that for $v_s = 24\ \text{V}$, we have $v_2 = 1\ \text{V}$. Thus, for $v_s = 12\ \text{V}$, we have:

$$v_2 = 1 \times \frac{12}{24} = 0.5\ \text{V}$$

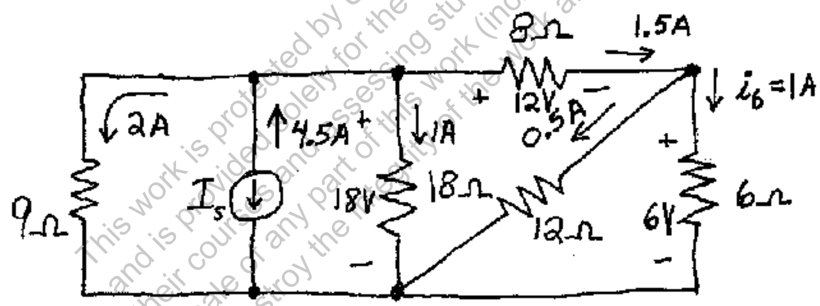
P2.99 We start by assuming $i_2 = 1$ A and work back through the circuit to determine the value of v_s. The results are shown on the circuit diagram.



However, the circuit actually has v_s = 10 V, so the actual value of i_2 is

$$\frac{10}{20} \times (1\ A) = 0.5\ A.$$

P2.100 We start by assuming $i_6 = 1$ A and work back through the circuit to determine the value of I_s. This results in $I_s = -4.5$ A.



However, the circuit actually has I_s = 10 A, so the actual value of i_6 is

$$\frac{10}{-4.5} \times (1\ A) = -2.222\ A.$$

P2.101 (a) With only the 2-A source activated, we have

$i_2 = 2$ and $v_2 = 3(i_2)^2 = 12$ V

(b) With only the 1-A source activated, we have

$i_1 = 1$ A and $v_1 = 3(i_1)^2 = 3$ V

(c) With both sources activated, we have

$i = 3$ A and $v = 3(i)^2 = 27$ V

Notice that $i \neq i_1 + i_2$. Superposition does not apply because device A has a nonlinear relationship between v and i.

P2.102 From Equation 2.90, we have

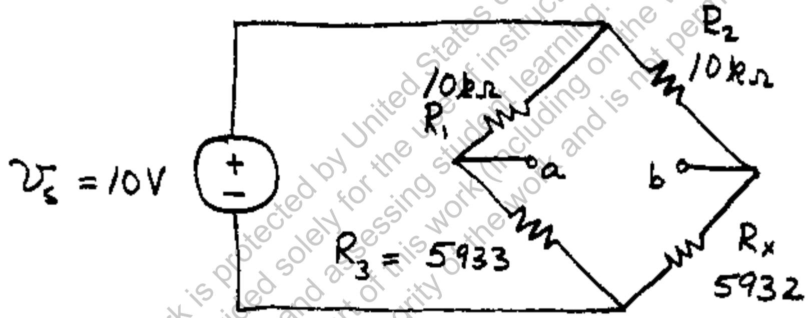
(a) $$R_x = \frac{R_2}{R_1} R_3 = \frac{1\ \text{k}\Omega}{10\ \text{k}\Omega} \times 3419 = 341.9\ \Omega$$

(b) $$R_x = \frac{R_2}{R_1} R_3 = \frac{100\ \text{k}\Omega}{10\ \text{k}\Omega} \times 3419 = 34.19\ \text{k}\Omega$$

P2.103* (a) Rearranging Equation 2.90, we have

$$R_3 = \frac{R_1}{R_2} R_x = \frac{10^4}{10^4} \times 5932 = 5932\ \Omega$$

(b) The circuit is:



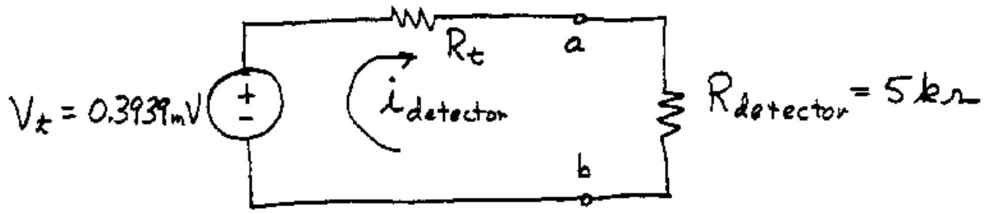
The Thévenin resistance is

$$R_t = \frac{1}{1/R_3 + 1/R_1} + \frac{1}{1/R_2 + 1/R_x} = 7447\ \Omega$$

The Thévenin voltage is

$$v_t = v_s \frac{R_3}{R_1 + R_3} - v_s \frac{R_x}{R_x + R_2}$$
$$= 0.3939\ \text{mV}$$

Thus, the equivalent circuit is:



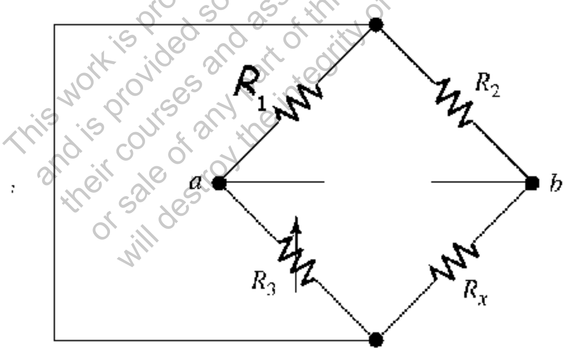
$$i_{\text{detector}} = \frac{V_t}{R_t + R_{\text{detector}}} = 31.65 \times 10^{-9}\ A$$

Thus, the detector must be sensitive to very small currents if the bridge is to be accurately balanced.

P2.104 If R_1 and R_3 are too small, large currents are drawn from the source. If the source were a battery, it would need to be replaced frequently. Large power dissipation could occur, leading to heating of the components and inaccuracy due to changes in resistance values with temperature.

If R_1 and R_3 are too large, we would have very small detector current when the bridge is not balanced, and it would be difficult to balance the bridge accurately.

P2.105 With the source replaced by a short circuit and the detector removed, the Wheatstone bridge circuit becomes



The Thévenin resistance seen looking back into the detector terminals is

$$R_t = \frac{1}{1/R_3 + 1/R_1} + \frac{1}{1/R_2 + 1/R_x}$$

The Thévenin voltage is zero when the bridge is balanced.

P2.106 Using the voltage-division principle, the voltage at node *a* is

$$v_a = V_s \frac{R_0 + \Delta R}{R_0 + \Delta R + R_0 - \Delta R} = V_s \frac{R_0 + \Delta R}{2R_0}$$

Similarly at node *b*, we have

$$v_b = V_s \frac{R_0 - \Delta R}{2R_0}$$

Then, the output voltage is

$$v_o = v_{ab} = v_a - v_b = V_s \frac{\Delta R}{R_0}$$

Finally using Equation 2.92 to substitute for ΔR, we have

$$v_o = V_s G \frac{\Delta L}{L}$$

P2.107 Before strain is applied, the resistance is

$$R_0 = \frac{\rho L}{A}$$

After strain is applied, the length becomes $L + \Delta L = L(1 + \Delta L / L)$, and the cross sectional area becomes $A/((1 + \Delta L / L)$ so the volume is constant. Thus, the resistance becomes

$$R_0 + \Delta R = R_0(1 + \frac{\Delta R}{R_0}) = \frac{\rho L(1 + \Delta L / L)}{A/(1 + \Delta L / L)} = R_0(1 + \Delta L / L)^2$$

$$R_0(1 + \frac{\Delta R}{R_0}) = R_0(1 + 2\Delta L / L + (\Delta L / L)^2)$$

However, we have $\Delta L / L << 1$ so we can neglect the $(\Delta L / L)^2$ term to a good approximation. This results in

$$R_0(1 + \frac{\Delta R}{R_0}) \cong R_0(1 + 2\Delta L / L)$$

$$G = \frac{\Delta R / R_0}{\Delta L / L} \cong 2$$

P2.108 In this case, the bridge would be balanced for any value of ΔR and the output voltage v_o would be zero regardless of the strain.

Practice Test

T2.1 (a) 6, (b) 10, (c) 2, (d) 7, (e) 10 or 13 (perhaps 13 is the better answer), (f) 1 or 4 (perhaps 4 is the better answer), (g) 11, (h) 3, (i) 8, (j) 15, (k) 17, (l) 14.

T2.2 The equivalent resistance seen by the voltage source is:

$$R_{eq} = R_1 + \frac{1}{1/R_2 + 1/R_3 + 1/R_4} = 16\ \Omega$$

$$i_s = \frac{v_s}{R_{eq}} = 6\ A$$

Then, using the current division principle, we have

$$i_4 = \frac{G_4}{G_2 + G_3 + G_4} i_s = \frac{1/60}{1/48 + 1/16 + 1/60} 6 = 1\ A$$

T2.3 Writing KCL equations at each node gives

$$\frac{v_1}{4} + \frac{v_1 - v_2}{5} + \frac{v_1 - v_3}{2} = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{10} = 2$$

$$\frac{v_3}{1} + \frac{v_3 - v_1}{2} = -2$$

In standard form, we have:

$$0.95v_1 - 0.20v_2 - 0.50v_3 = 0$$

$$-0.20v_1 + 0.30v_2 = 2$$

$$-0.50v_1 + 1.50v_3 = -2$$

In matrix form, we have

$$\mathbf{GV} = \mathbf{I}$$

$$\begin{bmatrix} 0.95 & -0.20 & -0.50 \\ -0.20 & 0.30 & 0 \\ -0.50 & 0 & 1.50 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}$$

The MATLAB commands needed to obtain the column vector of the node voltages are

```
G = [0.95 -0.20 -0.50; -0.20 0.30 0; -0.50 0 1.50]
I = [0; 2; -2]
V = G\I     % As an alternative we could use V = inv(G)*I
```

Actually, because the circuit contains only resistances and independent current sources, we could have used the short-cut method to obtain the **G** and **I** matrices.

T2.4 We can write the following equations:

KVL mesh 1: $R_1 i_1 - V_s + R_3(i_1 - i_3) + R_2(i_1 - i_2) = 0$

KVL for the supermesh obtained by combining meshes 2 and 3:

$$R_4 i_2 + R_2(i_2 - i_1) + R_3(i_3 - i_1) + R_5 i_3 = 0$$

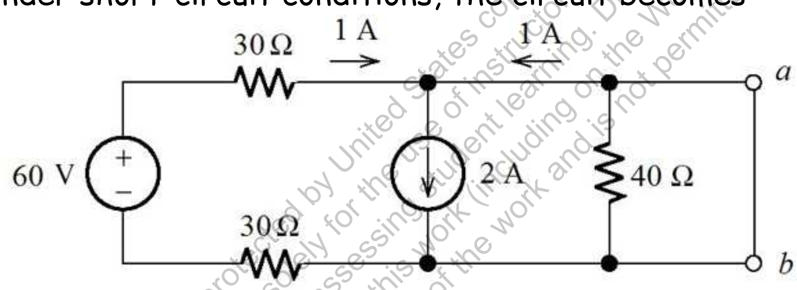
KVL around the periphery of the circuit:

$$R_1 i_1 - V_s + R_4 i_2 + R_5 i_3 = 0$$

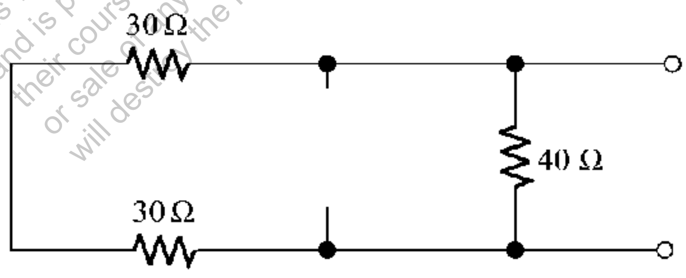
Current source: $i_2 - i_3 = I_s$

A set of equations for solving the network must include the current source equation plus two of the mesh equations. The three mesh equations are dependent and will not provide a solution by themselves.

T2.5 Under short-circuit conditions, the circuit becomes



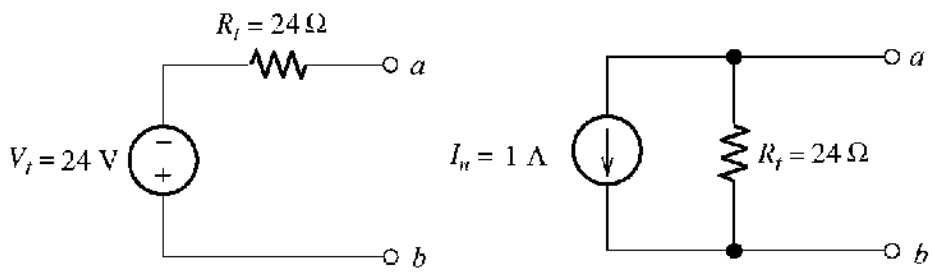
Thus, the short-circuit current is 1 A flowing out of *b* and into *a*. Zeroing the sources, we have



Thus, the Thévenin resistance is

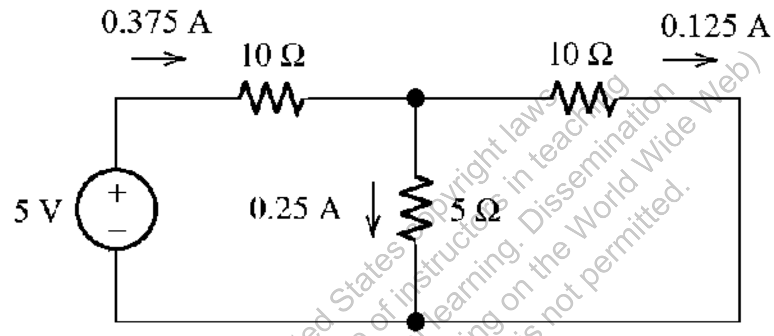
$$R_t = \frac{1}{1/40 + 1/(30+30)} = 24\ \Omega$$

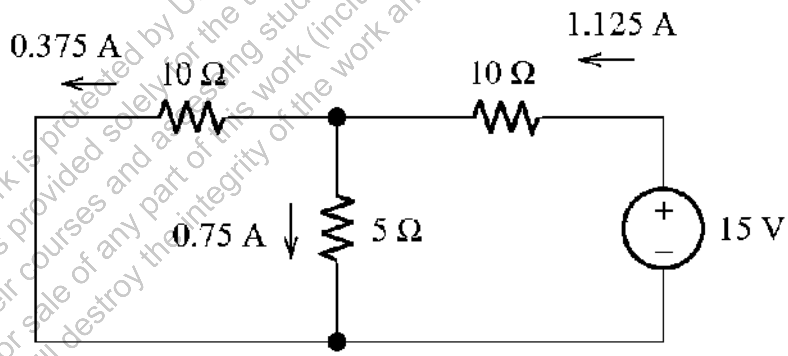
and the Thévenin voltage is $V_t = I_{sc}R_t = 24\ \text{V}$. The equivalent circuits are:



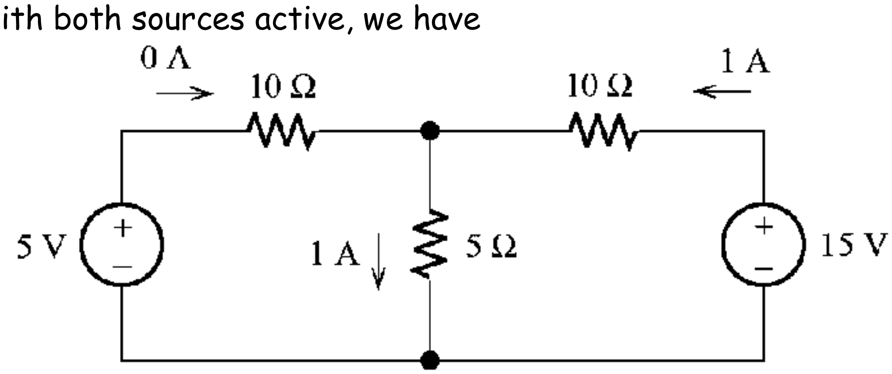
Because the short-circuit current flows out of terminal *b*, we have oriented the voltage polarity positive toward *b* and pointed the current source reference toward *b*.

T2.6 With one source active at a time, we have





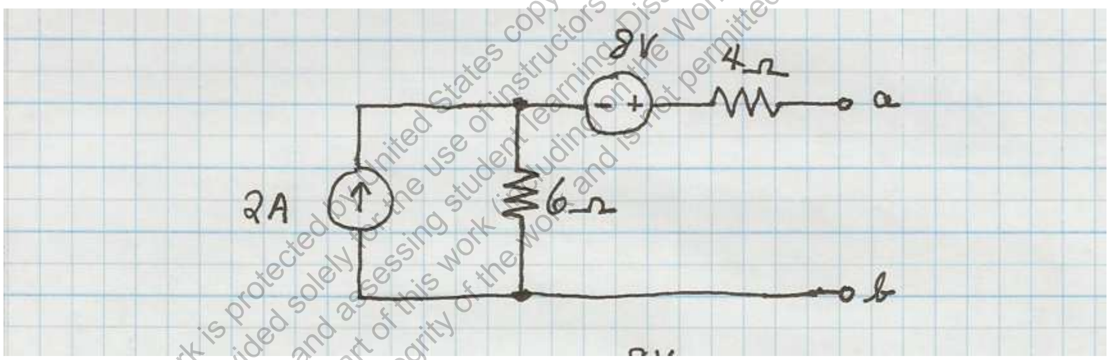
Then, with both sources active, we have



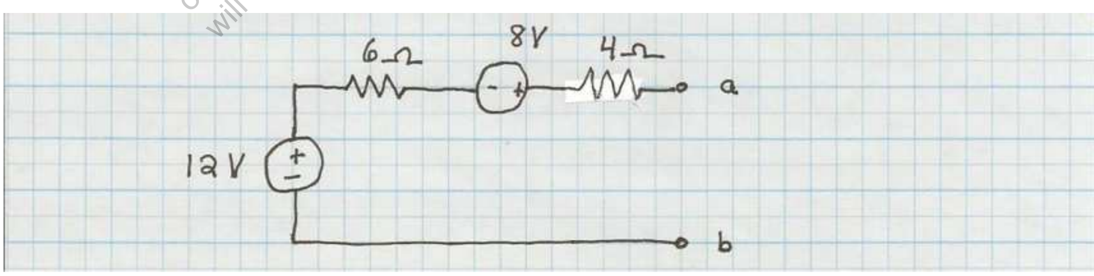
We see that the 5-V source produces 25% of the total current through the 5-Ω resistance. However, the power produced by the 5-V source with both sources active is zero. Thus, the 5-V source produces 0% of the power delivered to the 5-Ω resistance. Strange, but true! Because power is a nonlinear function of current (i.e., $P = Ri^2$), the superposition principle does not apply to power.

T2.7 First, the 10-Ω and the 15-Ω resistances are in parallel with an equivalent resistance of 10(15)/(10 +15) = 6 Ω. Next, the 60-Ω and the 30-Ω resistances are in parallel with an equivalent resistance of 60(30)/(60 +30) = 20 Ω. Finally, these equivalent resistances are in series, so the resistance between terminals *a* and *b* is R_{ab} = 6 + 20 = 26 Ω.

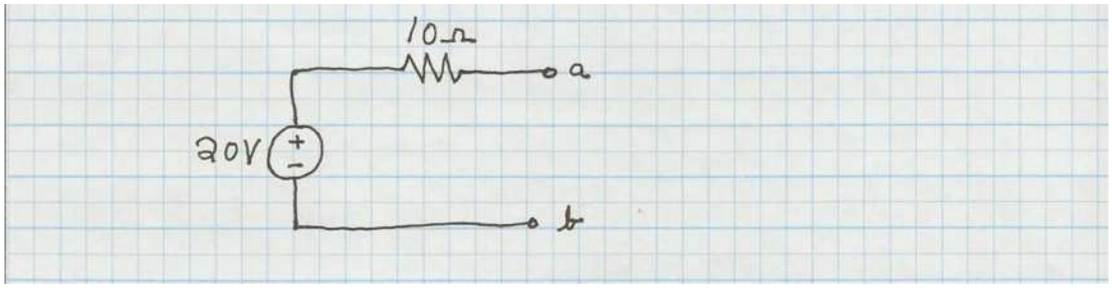
T2.8 The original circuit is:



Converting the 2-A current source and the 6-Ω resistance into an equivalent voltage source and series resistance, we have:



The voltage sources are in series and their voltages can be added. Similarly, the resistances are in series, so we have:


10Ω
a
20V
+
−
b