

## Chapter 2

1. How many neutrons and protons are there in the nuclei of the following atoms:  
 (a)  ${}^7\text{Li}$ ,  
 (b)  ${}^{24}\text{Mg}$ ,  
 (c)  ${}^{135}\text{Xe}$ ,  
 (d)  ${}^{209}\text{Bi}$ ,  
 (e)  ${}^{222}\text{Rn}$ ?

(a)  $Z=3, N=4$ ; (b)  $Z=12, N=12$ ; (c)  $Z=54, N=81$ ; (d)  $Z=83, N=126$ ; (e)  $Z=86, N=136$ .

3. How many atoms are there in 10 g of  ${}^{12}\text{C}$ ?

$10\text{ g} = 10/12$  mole and contains  $(10/12) \times 0.6022 \times 10^{24} = 0.502 \times 10^{24}$  atoms.

5. When  $\text{H}_2$  gas is formed from naturally occurring hydrogen, what percentages of the molecules have molecular weights of approximately 2, 3, and 4?

The molecules in question are  ${}^1\text{H}{}^1\text{H}$ ,  ${}^1\text{H}{}^2\text{H}$  or  ${}^2\text{H}{}^1\text{H}$ , and  ${}^2\text{H}{}^2\text{H}$ . Let  $x(1)$  and  $x(2)$  be the fractions of naturally-occurring hydrogen that are  ${}^1\text{H}$  and  ${}^2\text{H}$ . Combining atoms pulled successively from a large volume of hydrogen, the fraction that has weight 2 is  $x(1) \times x(1) = (0.99985)^2 = 0.9997 = 99.97\%$ . The fraction with weight 4 is  $x(2) \times x(2) = 2.25 \times 10^{-8} = 2.25 \times 10^{-6}\%$ . Weight 3 can be formed in two ways. The fraction of 3 is  $x(1)x(2) + x(2)x(1) = 2x(1)x(2) = 2 \times 0.99985 \times 0.00015 = 3.00 \times 10^{-4} = 0.03\%$ . Note that the fraction that have molecular weights of either 2, 3, or 4 is  $x^2(1) + 2x(1)x(2) + x^2(2) = (x(1) + x(2))^2 = 1$ .

7. A beaker contains 50 g of ordinary (i.e., naturally occurring) water.
- How many moles of water are present?
  - How many hydrogen atoms?
  - How many deuterium atoms?

(a) Number of moles =  $50 / 18.015 = 2.775$ .

(b) No. of molecules =  $2.775 N_A = 1.671 \times 10^{24}$ .

There are 2 atoms of H per molecule, so no. of atoms of H =  $2 \times 1.671 \times 10^{24} = 3.342 \times 10^{24}$ .

(c) No. of  $^2\text{H}$  atoms =  $1.5 \times 10^{-4} \times 3.342 \times 10^{24} = 5.013 \times 10^{20}$ .

9. Compute the mass of a proton in amu.

Mass of proton =  $1.67265 \times 10^{-27} \text{ kg}$

$1 \text{ amu} = 1.66057 \times 10^{-27} \text{ kg}$ . Mass of proton in amu is

$$m_p(\text{amu}) = \frac{1.67265 \times 10^{-27} \text{ kg}}{1.66057 \times 10^{-27} \text{ kg/amu}}$$

$$= 1.0073 \text{ amu}$$

11. Show that 1 amu is numerically equal to the reciprocal of Avogadro's number.

Avogadro's number is  $6.022 \times 10^{23}$

$$\frac{1}{6.022 \times 10^{23} \text{ atoms/mole}}$$

$$= 1.66057 \times 10^{-27} \text{ kg}$$

13. Using Eq. (2.3), estimate the density of nuclear matter in  $\text{g/cm}^3$ ; in  $\text{Kg/m}^3$ . Take the mass of each nucleon to be approximately  $1.5 \times 10^{-24}$  g.

The volume of the nucleus is

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (1.25 \times 10^{-13})^3 A.$$

The nucleon density is then

$$\frac{A}{V} = \frac{3 \times 10^{39}}{4\pi (1.25)^3} \text{ nucleons/cm}^3,$$

and the mass density is

$$\rho = \frac{3 \times 10^{39}}{4\pi (1.25)^3} \times 1.5 \times 10^{-24} = 1.96 \times 10^{14} \text{ g/cm}^3 \\ = 1.96 \times 10^{11} \text{ kg/m}^3$$

15. The complete combustion of 1 kg of bituminous coal releases about  $3 \times 10^7$  J in heat energy. The conversion of 1 g of mass into energy is equivalent to the burning of how much coal?

1 gram is equivalent to  $1 \times (3 \times 10^{10})^2 = 9 \times 10^{20}$  ergs =  $9 \times 10^{13}$  joules. From Table I.8,  $1 \text{ J} = 9.478 \times 10^{-4} \text{ Btu}$ . Thus  $1 \text{ g} \sim 9 \times 10^{13} \times 9.478 \times 10^{-4} = 8.530 \times 10^{10} \text{ Btu}$ . This is the heat released from  $8.530 \times 10^{10} / (13000 \times 2000) = 3280$  tons of coal.

17. Compute the neutron-proton mass difference in MeV.

Using values from Table II.1,  $\Delta M = M_n - M_p = 1.008665 - 1.007277 = 0.001388 \text{ amu}$ . Since  $1 \text{ amu} = 931.481 \text{ MeV}$ ,  $\Delta M = 0.001388 \times 931.481 = 1.293 \text{ MeV}$ .

19. Derive Eq. (2.18). [Hint: Square both sides of Eq. (2.5) and solve for  $mv$ .]

From Eq. (2.5),

$$\gamma = \frac{\gamma_0}{\sqrt{1 - v^2/c^2}}$$

$$\gamma^2 = \frac{\gamma_0^2}{1 - v^2/c^2}$$

$$\gamma^2 c^4 - \gamma^2 v^2 c^2 = \gamma_0^2 c^4.$$

But  $\gamma mc^2 = E_{\text{tot}}$ ,  $\gamma_0 mc^2 = E_{\text{rest}}$ , and  $\gamma mv = p$ .  
Thus,

$$p = \frac{1}{c} \sqrt{E_{\text{tot}}^2 - E_{\text{rest}}^2}.$$

21. Using the result derived in Problem 2.20, calculate the speed of a 1-MeV electron, one with a kinetic energy of 1 MeV.

For an electron,  $E_{\text{rest}} = 0.511 \text{ MeV}$ , and if its kinetic energy is 1 MeV, then  $E_{\text{tot}} = 1.511 \text{ MeV}$ .

From prob. 2.16,

$$v = c \sqrt{1 - \left(\frac{0.511}{1.511}\right)^2} = 0.941c = 2.82 \times 10^{10} \text{ cm/sec.}$$

$$= 2.82 \times 10^8 \text{ m/sec.}$$

23. Show that the wavelength of a relativistic particle is given by

$$\lambda = \lambda_C \frac{m_e c^2}{\sqrt{E_{\text{total}}^2 - E_{\text{rest}}^2}},$$

where  $\lambda_C = h/m_e c = 2.426 \times 10^{-10} \text{ cm}$  is called the Compton wavelength.

From Eq. (2.19),

$$\lambda = \frac{hc}{\sqrt{E_{\text{tot}}^2 - E_{\text{rest}}^2}} = \frac{h}{m_e c} \frac{m_e c^2}{\sqrt{E_{\text{tot}}^2 - E_{\text{rest}}^2}}$$

$$= \lambda_C \frac{m_e c^2}{\sqrt{E_{\text{tot}}^2 - E_{\text{rest}}^2}}$$



25. An electron moves with a kinetic energy equal to its rest-mass energy. Calculate the electron's
- total energy in units of  $m_e c^2$ ;
  - mass in units of  $m_e$ ;
  - speed in units of  $c$ ;
  - wavelength in units of the Compton wavelength.

(a)  $E_{tot} = 2m_e c^2$ . (b)  $m_e c^2 = E_{tot} = 2m_e c^2$ , and  
 $m = 2m_e$ . (c) From prob. 2.16,

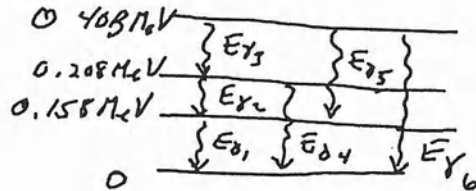
$$v = c \sqrt{1 - \left( \frac{m_e c^2}{2m_e c^2} \right)^2} = 0.866c.$$

(d) From prob. 2.19,

$$\lambda = \lambda_c \frac{m_e c^2}{\sqrt{(2m_e c^2)^2 - (m_e c^2)^2}} = 0.577 \lambda_c$$

27. The first three excited states of the nucleus of  $^{199}\text{Hg}$  are at 0.158 MeV, 0.208 MeV, and 0.403 MeV above the ground state. If all transitions between these states and ground occurred, what energy  $\gamma$ -rays would be observed?

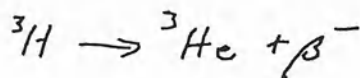
$^{199}\text{Hg}$



$$\begin{aligned} \gamma_1 &= 0.158 \text{ MeV} \\ \gamma_2 &= 0.05 \text{ MeV} \\ \gamma_3 &= 0.195 \text{ MeV} \\ \gamma_4 &= 0.208 \text{ MeV} \\ \gamma_5 &= 0.245 \text{ MeV} \\ \gamma_6 &= 0.403 \text{ MeV} \end{aligned}$$

29. Tritium ( ${}^3\text{H}$ ) decays by negative beta decay with a half-life of 12.26 years. The atomic weight of  ${}^3\text{H}$  is 3.016.
- (a) To what nucleus does  ${}^3\text{H}$  decay?  
 (b) What is the mass in grams of 1 mCi of tritium?

(a) Tritium  ${}^3\text{H}$  consists of a proton and two neutrons. Beta decay results in transformation of a n into a proton plus an electron. The product has 2 protons and 1 neutron and is thus an isotope of Helium



(b) Activity,  $\alpha$ , is  $\lambda n$ . For Tritium, the decay constant  $\lambda$  is

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

$$\lambda = \frac{\ln 2}{12.26 \text{ yr} \times \frac{365 \text{ days}}{\text{yr}} \times \frac{24 \text{ hr}}{\text{day}} \times \frac{3600 \text{ sec}}{\text{hr}}}$$

$$= 1.793 \times 10^{-9} \text{ sec}^{-1}$$

$$\alpha = 10^{-3} \text{ Ci} \times \frac{3.7 \times 10^{10} \text{ dis/sec}}{\text{Ci}} = 3.7 \times 10^7 \text{ dis/sec}$$

$$n_{{}^3\text{H}} = \frac{\alpha}{\lambda}$$

$$= \frac{3.7 \times 10^7 \text{ dis/sec}}{1.793 \times 10^{-9} \text{ sec}^{-1}}$$

$$= 2.064 \times 10^{16} \text{ atoms}$$

$$\text{Mass} = n_{{}^3\text{H}} \times \text{mass Tritium atom}$$

$$= 2.064 \times 10^{14} \text{ atoms} \times \frac{3 \text{ g/g atom}}{.6022 \times 10^{24} \text{ atoms/g-atom}}$$

$$= 1.03 \times 10^{-7} \text{ g}$$

31. Carbon tetrachloride labeled with  $^{14}\text{C}$  is sold commercially with an activity of 10 millicuries per millimole (10 mCi/mM). What fraction of the carbon atoms is  $^{14}\text{C}$ ?

Carbon tetrachloride is  $\text{CCl}_4$ . There is one atom of carbon per molecule. In 1 mM there are 1 mM of carbon atoms or  $.6022 \times 10^{24} \text{ atoms/mole} \times 10^{-3} \text{ mole/mM} = .6022 \times 10^{21} \text{ atoms C/mM}$ . If the sample contains 10 mCi then there are  $3.7 \times 10^{10} \frac{\text{dis/sec}}{\text{Ci}} \times 10^{-3} \text{ Ci/mCi}$  or  $3.7 \times 10^7 \text{ dis/sec}$  due to  $^{14}\text{C}$ . The activity  $A = \lambda N$ .

$\lambda = 0.693/T_{1/2}$ .  $T_{1/2} = 5730 \text{ y} \times \frac{365 \text{ d}}{\text{y}} \times \frac{24 \text{ hr}}{\text{d}} \times \frac{3600 \text{ sec}}{\text{hr}} = 1.808 \times 10^{11} \text{ s}$

$\lambda = 3.83 \times 10^{-12} \text{ sec}^{-1}$ ,  $N = \frac{A}{\lambda} = \frac{3.7 \times 10^7}{3.83 \times 10^{-12}} = 9.66 \times 10^{18} \text{ atoms}$

The fraction of carbon that is carbon  $^{14}$  is 1 mM is then  $9.66 \times 10^{18} / .6022 \times 10^{21} = 1.6 \times 10^{-2}$  or 1.6%.



33. After the initial cleanup effort at Three Mile Island, approximately 400,000 gallons of radioactive water remained in the basement of the containment building of the Three Mile Island Unit 2 nuclear plant. The principal sources of this radioactivity were  $^{137}\text{Cs}$  at  $156 \mu\text{Ci}/\text{cm}^3$  and  $^{134}\text{Cs}$  at  $26 \mu\text{Ci}/\text{cm}^3$ . How many atoms per  $\text{cm}^3$  of these radionuclides were in the water at that time?

$$\alpha(^{137}\text{Cs}) = 156 \mu\text{Ci}/\text{cm}^3 \text{ and } t_{1/2}^{137} = 30.2 \text{ yr}$$

$$\lambda_{137} = \frac{\ln 2}{t_{1/2}^{137}} = \frac{\ln 2}{30.2 \text{ yr}} = 0.0230 \text{ yr}^{-1}$$

$$\alpha(^{134}\text{Cs}) = 26 \mu\text{Ci}/\text{cm}^3 \text{ and } t_{1/2}^{134} = 2.06 \text{ yr}$$

$$\lambda_{134} = \frac{\ln 2}{t_{1/2}^{134}} = \frac{\ln 2}{2.06 \text{ yr}} = 0.3365 \text{ yr}^{-1}$$

$$\alpha_{137} = \lambda_{137} = \lambda_{137}$$

$$\lambda_{137} = \frac{\alpha_{137}}{\lambda_{137}} = \frac{(156 \times 10^{-6} \text{ Ci}/\text{cm}^3)(3.7 \times 10^{10} \text{ dis/sec-Ci})}{(0.0230 \text{ yr}^{-1})} \times$$

$$\left( \frac{3.15 \times 10^7 \text{ sec}}{\text{yr}} \right)$$

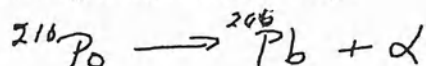
$$= 7.91 \times 10^{15} \text{ atoms}/\text{cm}^3$$

$$\lambda_{134} = \frac{\alpha_{134}}{\lambda_{134}} = \frac{(26 \times 10^{-6} \text{ Ci}/\text{cm}^3)(3.7 \times 10^{10} \text{ dis/sec-Ci})}{0.3365 \text{ yr}^{-1}} \times$$

$$\left( \frac{3.15 \times 10^7 \text{ s}}{\text{yr}} \right)$$

$$= 9.01 \times 10^{13} \text{ atoms}/\text{cm}^3$$

35. Polonium-210 decays to the ground state of  $^{206}\text{Pb}$  by the emission of a 5.305-MeV  $\alpha$ -particle with a half-life of 138 days. What mass of  $^{210}\text{Po}$  is required to produce 1 MW of thermal energy from its radioactive decay?



$$\alpha(^{210}\text{Po}) = \# \text{ of disint / sec}$$

$$\Sigma = \text{energy released / disint} = 5.305 \text{ MeV}$$

$$\text{Power} = \alpha \Sigma = P$$

$$\alpha = \lambda_{210} N_{210}$$

$$N_{210} = \frac{P}{\lambda_{210} \Sigma}$$

$$\lambda_{210} = \frac{\ln 2}{t_{1/2}^{210}} = \frac{\ln 2}{138 \text{ da.}} \left( \frac{1 \text{ da.}}{24 \text{ hr}} \right) \left( \frac{\text{hr}}{3600 \text{ sec}} \right)$$

$$= 5.81 \times 10^{-8} \text{ sec}^{-1}$$

$$N_{210} = \frac{1 \text{ MW}}{(5.81 \times 10^{-8} \text{ sec}^{-1}) (5.305 \text{ MeV})} \times$$

$$\left( \frac{10^6 \text{ W}}{1 \text{ MW}} \right) \left( \frac{1 \text{ J/s}}{1 \text{ W}} \right) \left( \frac{\text{MeV}}{10^6 \text{ eV}} \right) \frac{\text{eV}}{1.6 \times 10^{-19} \text{ J}}$$

$$\text{mass } ^{210}\text{Po} = \frac{N_{210} M_{210}}{M_A}$$

$$= \frac{(2.03 \times 10^{25} \text{ atoms}) (210 \text{ g/mole})}{(6.02 \times 10^{23} \text{ atoms/mole})}$$

$$= 7.08 \times 10^3 \text{ g} = 7.08 \text{ Kg.}$$

37. Since the half-life of  $^{235}\text{U}$  ( $7.13 \times 10^8$  years) is less than that of  $^{238}\text{U}$  ( $4.51 \times 10^9$  years), the isotopic abundance of  $^{235}\text{U}$  has been steadily decreasing since the earth was formed about 4.5 billion years ago. How long ago was the isotopic abundance of  $^{235}\text{U}$  equal to 3.0 a/o, the enrichment of the uranium used in many nuclear power plants?

$$t_{1/2}^{235} = 7.13 \times 10^8 \text{ yrs}$$

$$\lambda^{235} = \ln 2 / t_{1/2}^{235} = 9.72 \times 10^{-10} \text{ yr}^{-1}$$

$$t_{1/2}^{238} = 4.51 \times 10^9 \text{ yrs}$$

$$\lambda^{238} = \ln 2 / t_{1/2}^{238} = 1.54 \times 10^{-10} \text{ yr}^{-1}$$

$$n^{235}(t) = n_0^{235} e^{-\lambda^{235} t}$$

$$n^{238}(t) = n_0^{238} e^{-\lambda^{238} t}$$

At  $t=0$  isotopic abundance  $^{235}\text{U} = 0.03 = \delta_{235}$   
 Since  $n^{235}$  and  $n^{238} \gg n^{234}$

$$\delta_{235} = \frac{n_0^{235}}{n_0^{234} + n_0^{235} + n_0^{238}} = \frac{n_0^{235}}{n_0^{235} + n_0^{238}} = 0.03$$

$$n_0^{235} = 0.03 n_0^{235} + 0.03 n_0^{238}$$

$$\Rightarrow \frac{n_0^{235}}{n_0^{238}} = \frac{0.03}{0.97} = 0.03093 \text{ at } t=0$$

$$\text{Today } \delta_{235} = 0.0072 = \frac{n^{235}}{n^{235} + n^{238}}$$

$$\Rightarrow 0.9928 n^{235} = 0.0072 n^{238}$$

$$\frac{n^{238}}{n^{235}} = \frac{0.9928}{0.0072} = \frac{1}{0.00725}$$

Dividing the 2 decay equations yields

$$\frac{n^{235}(t)}{n^{238}(t)} = \frac{n_0^{235} (\lambda^{238} - \lambda^{235}) t}{n_0^{238} e}$$

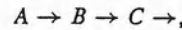
Solving for  $t$

$$t = \frac{1}{\lambda^{238} - \lambda^{235}} \ln \left( \frac{n^{235}(t) / n^{238}(t)}{n_0^{235} / n_0^{238}} \right)$$

$$t = \frac{1}{1.54 \times 10^{-10} \text{ yr}^{-1} - 9.72 \times 10^{-10} \text{ yr}^{-1}} \ln \left( \frac{0.00725}{0.03093} \right)$$

$$t = 1.77 \times 10^9 \text{ yr}$$

39. Consider the chain decay



with no atoms of B present at  $t = 0$ .

(a) Show that the activity of B rises to a maximum value at the time  $t_m$  given by

$$t_m = \frac{1}{\lambda_B - \lambda_A} \ln \left( \frac{\lambda_B}{\lambda_A} \right),$$

at which time the activities of A and B are equal.

(b) Show that, for  $t < t_m$ , the activity of B is less than that of A, whereas the reverse is the case for  $t > t_m$ .

Q. From Eq. 2.33, the activity of B is given by:

$$\alpha_B = \alpha_{B0} e^{-\lambda_B t} + \frac{\alpha_{A0} \lambda_B}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t})$$

Assuming no initial atoms of B drops the first term since  $\alpha_{B0} = 0$  then. The activity of B is a maximum

(or minimum) when  $\frac{d\alpha_B}{dt} = 0$ . Taking the derivative w.r.t. time of  $\alpha_B$  gives:

$$\frac{d\alpha_B}{dt} = \frac{\alpha_{A0} \lambda_B}{\lambda_B - \lambda_A} (-\lambda_A e^{-\lambda_A t} + \lambda_B e^{-\lambda_B t}) = 0$$

rearranging

$$e^{-(\lambda_A - \lambda_B)t} = \frac{\lambda_B}{\lambda_A}$$

taking ln of both sides and solve for  $t_m$  given

$$t_m = \frac{1}{\lambda_B - \lambda_A} \left( \ln \frac{\lambda_B}{\lambda_A} \right)$$

b.) For  $t < t_m$ ,  $\frac{d\alpha_B}{dt}$  has to be  $> 0$  since at  $t = t_m$   $\frac{d\alpha_B}{dt} = 0$  and  $\alpha_B$  is a maximum. For

$$\frac{d\alpha_B}{dt} > 0 \text{ then for } \frac{d\alpha_B}{dt} > 0 \text{ need } \frac{dn_B}{dt} > 0$$

$$\text{Since } \alpha_B = \lambda_B n_B \quad \text{But } \frac{dn_B}{dt} = -\lambda_B n_B + \lambda_A n_A \\ = -\alpha_B + \alpha_A$$

Then  $\alpha_A > \alpha_B$ . For  $\frac{d\alpha_B}{dt} < 0$  reverse is true



41.. Show that the abundance of  $^{234}\text{U}$  can be explained by assuming that this isotope originates solely from the decay of  $^{238}\text{U}$ .

$^{238}\text{U}$  and  $^{234}\text{U}$  are nuclides in the Uranium Series. The isotopes in the first portion of the series are  $^{238}\text{U}$ ,  $^{234}\text{Th}$ ,  $^{234\text{m}}\text{Pa}$ ,  $^{234}\text{Pa}$ , and  $^{234}\text{U}$ . The half-life of  $^{238}\text{U}$  is  $4.51 \times 10^9$  yrs. The half-lives of the other isotopes are much shorter than  $^{238}\text{U}$ . Thus, the decay constant of  $^{238}\text{U}$  is much, much less than the decay constants of the other members of the series. The daughter products of  $^{238}\text{U}$  are in secular equilibrium and all decay at the rate set by  $^{238}\text{U}$ . One can show that

$$\lambda_{238} N_{238} = \lambda_{234} N_{234}$$

or

$$\frac{\lambda_{238}}{\lambda_{234}} = \frac{N_{234}}{N_{238}}$$

and

$$\frac{T_{1/2, 234}}{T_{1/2, 238}} = \frac{N_{234}}{N_{238}}$$

$$\text{Since } T_{1/2, 234} = 2.48 \times 10^5 \text{ yrs}$$

$$\frac{2.48 \times 10^5 \text{ yrs}}{4.51 \times 10^9 \text{ yrs}} = 5.5 \times 10^{-5}$$

From Appendix II table II.2 the abundance of  $^{234}\text{U}$  is 0.0055 at a total of  $^{238}\text{U}$  99.27% the ratio is then  $5.5 \times 10^{-5}$ . Thus  $^{234}\text{U}$  can be assumed to originate from the decay of  $^{238}\text{U}$ .

43. According to U.S. Nuclear Regulatory Commission regulations, the maximum permissible concentration of radon-222 in air in equilibrium with its short-lived daughters is 3 pCi/liter for nonoccupational exposure. This corresponds to how many atoms of radon-222 per cm<sup>3</sup>?

An activity of 3 pCi/liter =  $3 \times 10^{-12}$  Ci/liter.

From the definition of Ci =  $3.7 \times 10^{10}$  dis/sec then the concentration of 3 pCi/liter yields  $11.1 \times 10^{-1}$  dis/sec-liter. This is the  $^{222}\text{Rn}$  activity in a liter. But the activity is  $\lambda_{\text{Rn}} N_{\text{Rn}}$  then the number of atoms in a liter that yield 1.11 dis/sec is

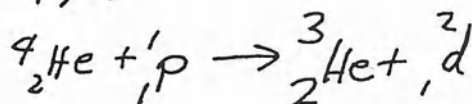
$$N_{\text{Rn}} = \frac{1.11 \text{ dis/sec}}{\lambda_{\text{Rn}}}$$

From the chart of the nucleus,  $T_{1/2} = 3.8$  days then  $\lambda = 2.11 \times 10^{-6} \text{ sec}^{-1}$  the number of atoms is  $\frac{5.26 \times 10^5}{\text{liter}}$

45. Complete the following reactions and calculate their  $Q$  values. [Note: The atomic weight of  $^{14}\text{C}$  is 14.003242.]

- (a)  $^4\text{He}(p, d)$   
 (b)  $^9\text{Be}(\alpha, n)$   
 (c)  $^{14}\text{N}(n, p)$   
 (d)  $^{115}\text{In}(d, p)$   
 (e)  $^{207}\text{Pb}(\gamma, n)$

(a)  $^4\text{He}(p, d)$

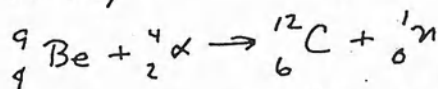
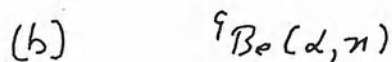


$$Q = [M_{\text{He}} + m_p] - (M_{^3\text{He}} + m_d) \times 931 \text{ MeV/a.m.u.}$$

Use mass hydrogen for  $m_p$  to account for  $e^-$  on deuterium

$$= [(4.002604 + 1.007825) - (3.01603 + 2.014102)] \times 931$$

$$= -18.3 \text{ MeV}$$

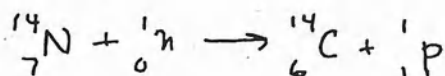
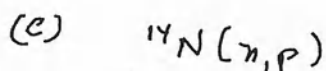


$$Q = [(M_{\text{Be}} + m_{\alpha}) - (M_{\text{C}} + m_n)] 931 \text{ MeV}$$

Use mass of helium for  $\alpha$  particle

$$= [(9.01219 + 4.002603) - (12.000 + 1.008665)] 931$$

$$= 5.705 \text{ MeV}$$

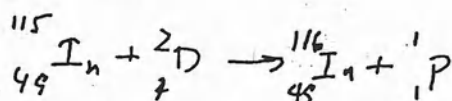
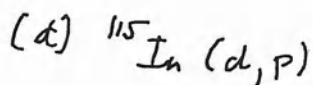


$$Q = [(M_{\text{N}} + m_n) - (M_{\text{C}} + m_p)] 931 \text{ MeV/amu}$$

Use mass of hydrogen for proton

$$= [(14.003074 + 1.008665) - (14.003242 + 1.007825)] 931$$

$$= 6.256 \text{ MeV}$$

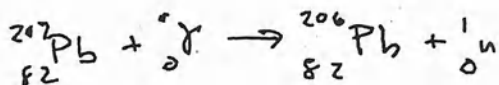
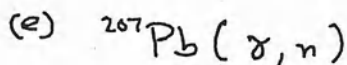


Use mass of hydrogen for proton

$$Q = [(M_{\text{In}} + m_{\text{D}}) - (M_{\text{In}} + m_{\text{H}})] 931 \text{ MeV/amu}$$

$$= [(114.90387 + 2.014102) - (115.905 + 1.007825)] 931$$

$$= 4.547 \text{ MeV}$$



$$Q = [(M_{\text{Pb}} + 0) - (M_{\text{Pb}} + m_n)] 931 \text{ MeV/amu}$$

$$= (206.975897 + 0) - (206.975897 + 1.008665)] 931$$

$$= -1.466 \text{ MeV}$$

49. The atomic weight of  $^{206}\text{Pb}$  is 205.9745. Using the data in Problem 2.35, calculate the atomic weight of  $^{210}\text{Po}$ . [Caution: See Problem 2.46]

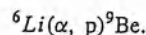
$$Q = [M_{\text{Po}} - (M_{\text{Pb}} + m_{\alpha})] \times 931 \text{ MeV/amu}$$

The energy is divided between the  $\alpha$  particle which is 5.305 MeV and the recoil energy of the Pb atom. To determine the weight of Po need total Q value. Can show the  $KE_{\text{Pb}}$  is given by  $\frac{m_{\alpha}}{M_{\text{Pb}}} KE_{\alpha}$

$$Q = KE_{\alpha} \left(1 + \frac{m_{\alpha}}{M_{\text{Pb}}}\right) \approx 5.305 \left(1 + \frac{4}{206}\right) = 5.408 \text{ MeV}$$

$$M_{\text{Po}} = M_{\text{Pb}} + m_{\alpha} + \frac{Q}{931 \text{ MeV}} = 205.9745 + 4.002604 + \frac{5.408}{931} \\ = 209.983 \text{ amu}$$

51. Consider the reaction



Using atomic mass data, compute:

- (a) the total binding energy of  ${}^6\text{Li}$ ,  ${}^9\text{Be}$ , and  ${}^4\text{He}$ ;  
 (b) the Q value of the reaction using the results of part (a).

$$BE = [Z M({}^1\text{H}) + N m_n - M(\text{atom})] 931 \text{ MeV/amu}$$

$$M({}^1\text{H}) = 1.007825$$

$$m_{\text{Li}} = 5.012537$$

$$m_{\text{He}} = 4.002604$$

$$m_n = 1.00866$$

$$m_{\text{Be}} = 9.012186$$

Evaluating for each

$$BE_{\text{Li}} = 31.993$$

$$BE_{\text{Be}} = 58.163 \quad BE_{\text{He}} = 28.296$$

$$Q = BE_{\text{Be}} - (BE_{\text{Li}} + BE_{\alpha})$$

$$= -2.126 \text{ MeV}$$

53. Using the mass formula, compute the binding energy per nucleon for the nuclei in Problem 2.52. Compare the results with those obtained in that problem.

Example.

${}^{12}\text{C}$

$$M = Nm_n + Zm_p - \alpha A + \beta A^{2/3} + \gamma Z^2/A^{1/3} + \delta (A - 2Z)/A + \delta$$

for  ${}^{12}\text{C}$   $Z=6$  and  $N=6$



$$= 6 \times 939,573 + 6 \times 938,280 - 15,56 \times 12 + 17,23(12)^{2/3} \\ + 0,697(6)^2 / (6)^{1/3} + 23,285(12-12)/12 \\ - 12$$

The last term is negative since both Z and N are even

$$= 1,1594 \times 10^4$$

Dividing by 931 MeV/amu

$$= 11,986 \text{ which compares to } 12,000$$

57. Calculate the atom density of graphite having density of 1.60 g/cm<sup>3</sup>.

$$N = \frac{\rho N_A}{M} \text{ For graphite, } M = 12,01115$$

$$N = \frac{1,60 \text{ g/cm}^3}{12,01115 \text{ grams/mole}} \times 6,022 \times 10^{23} \text{ atoms/mole} = 8,0219 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3}$$

59. What is the atom density of <sup>235</sup>U in uranium enriched to 2.5 a/o in this isotope if the physical density of the uranium is 19.0 g/cm<sup>3</sup>?

$$N_i = \frac{\delta_i \rho N_A}{100 M}$$

Use for M, the mass of uranium from Table 11.3

$$= \frac{(2,5)(19,0 \text{ g/cm}^3)(6,022 \times 10^{23} \text{ atoms/mole})}{100 \times 238,03 \text{ g/mole}}$$

$$= 1,201 \times 10^{21} \text{ atoms/cm}^3$$



61. It has been proposed to use uranium carbide (UC) for the initial fuel in certain types of breeder reactors, with the uranium enriched to 25 w/o. The density of UC is 13.6 g/cm<sup>3</sup>.

- (a) What is the atomic weight of the uranium?  
 (b) What is the atom density of the <sup>235</sup>U?

(a) From Eq. 2.25

$$\frac{1}{M} = \frac{1}{100} \left( \frac{25}{235.0439} + \frac{75}{238.0508} \right)$$

$$M = 237.292$$

(b) The # of UC atoms is given by

$\omega \frac{\rho N_A}{M(UC)}$

where  $\omega$  is w/o of uranium in the compound,  $\rho$  is the density of UC, and  $N_A$  is Avogadro's number.

$$\omega = \frac{M_U \times 100}{M_U + M_C} = \frac{237.292 \times 100}{237.292 + 12.000} = 95.19\%$$

Weight of uranium in each cc is then

$$\rho_u = \omega \text{ Uranium} \times \rho_{uc} \\ = 12.95 \text{ g/cc}$$

$$\# \text{ of } ^{235}\text{U atoms/cc} = \frac{\omega \rho_u N_A}{100 M_{u235}} = \frac{25 \times 12.95 \times 6.022 \times 10^{23}}{100 \times 235.0439} \\ = 1.84 \times 10^{23} \text{ atoms/cc}$$

63. The fuel for a certain breeder reactor consists of pellets composed of mixed oxides, UO<sub>2</sub> and PuO<sub>2</sub>, with the PuO<sub>2</sub> comprising approximately 30 w/o of the mixture. The uranium is essentially all <sup>238</sup>U, whereas the plutonium contains the following isotopes: <sup>239</sup>Pu (70.5 w/o), <sup>240</sup>Pu (21.3 w/o), <sup>241</sup>Pu (5.5 w/o), and <sup>242</sup>Pu (2.7 w/o). Calculate the number of atoms of each isotope per gram of the fuel.

PuO<sub>2</sub> comprises 30 w/o of the fuel or each gram of fuel contains 0.3 gm of PuO<sub>2</sub>. The w/o of the isotopes are <sup>239</sup>Pu = 70.5 w/o, <sup>240</sup>Pu = 21.3 w/o, <sup>241</sup>Pu = 5.5 w/o and <sup>242</sup>Pu = 2.7 w/o. The number density per gram of fuel is then the respective w/o times the w/o that is Pu times 13

w/o of  $\text{PuO}_2$  that  $\hookrightarrow \text{Pu} = \frac{M_{\text{Pu}}}{M_{\text{Pu}} + 2M_{\text{O}}}$

$$\frac{1}{M_{\text{Pu}}} = \frac{1}{100} \left[ \frac{70.5}{239.052775} + \frac{21.3}{240.053836} + \frac{5.5}{241.056973} + \frac{2.7}{242.058765} \right]$$

$M_{\text{Pu}} = 239.455$

w/o  $\text{PuO}_2$   $100 \times \frac{239.455}{31.9984 + 9.455} = 882 \times 100$

# atoms / g is then

$$\frac{\text{w/o of isotope} \times (.3) \times (882) \times (6.022 \times 10^{23})}{100 \quad M_i}$$

where  $M_i$  = mass of the  $\text{Pu}$  isotope and  $w_i$  is the w/o of the  $i^{\text{th}}$  isotope