PART I:
SOLUTIONS TO PROBLEMS

1 INTRODUCTION

NOTE: Answers for some of these problems, and some in later chapters, can be obtained by consulting the bibliographies, later chapters, websites, or professional surveyors.

1.1 List 10 uses for surveying in areas other than boundary surveying.

Answers may vary many are included in Section 1.6, which lists control, topographic hydrographic, alignment, construction, as-built, mine, solar, optical tooling, ground, aerial, and satellite surveys. This list is not complete and could also include other types of surveys such as hydrographic surveys, for example.

1.2 Explain the difference between geodetic and plane surveys.

From Section 1.4:
In geodetic surveys the curved surface of the earth is considered by performing the computations on an ellipsoid (curve surface approximating the size and shape of the earth). In plane surveys, except for leveling, the reference base for fieldwork and computations is assumed to be a flat horizontal surface. The direction of a plumb line (and thus gravity) is considered parallel throughout the survey region, and all measured angles are presumed to be plane angles.

1.3 Describe some surveying applications in:

(a) Archeology
There are many different uses of surveying in archeology. Some include using sonar to identify possible underground or underwater archeology sites, LiDAR to help identify possible ancient human settlements in unexplored forest and jungles, and traditional surveying and laser scanning to help locate artifacts in site excavations.

(b) Gas exploration
There are several stages of surveying in gas exploration, which include but are not limited to determining anomalies in the gravity field, which identify possible gas deposits, boundary surveys identifying properties that have mineral rights to the gas deposits, alignment surveys for placement of pipelines to transport extracted gas.

(c) Agriculture
In agriculture, surveying is used to determine the acreage of fields, to locate lines of constant elevation for strip farming, to track harvesting machinery to enable the size of the harvest, and to track the position of the planting equipment to allow for precise applications of seeds and fertilizers. The field is known as high-precision agriculture.

1.4 List some application of surveying in geology, forestry, and archeology.

Applications in each are multiple. For some in geology and archeology see the answer to Problem 1.3 (a) and (b). Some uses of surveying in forestry identifying forest boundaries, locating spread of diseases and insects through remote sensing, using GIS to help inventory and keep records on resources in forested regions.

1.5 Why is it important to make accurate surveys of underground utilities?

To provide an accurate record of the locations of these utilities so they can be found if repairs or servicing is needed, and to prevent their accidental destruction during excavation for other projects.

1.6 Discuss the uses for topographic surveys.

Topographic surveys are used whenever elevation data is required in the end product. Some examples include (1) creating maps for highway design; (2) creating maps for construction surveys; (3) creating maps for flood plain delineation; (4) creating maps for site location of buildings; and so on.

1.7 What are hydrographic surveys, and why are they important?

From Section 1.6, hydrographic surveys define shorelines and depths of lakes, streams, oceans, reservoirs, and other bodies of water. Sea surveying is associated with port and offshore industries and the marine environment, including measurements and marine investigations made by ship borne personnel.

1.8 Print a view of your location using Google Earth.®

Answers will vary but should be an image in your region.

1.9 Briefly explain the procedure used by Eratosthenes in determining the Earth's circumference.

From Section 1.3, paragraph 8 of text: His procedure, which occurred about 200 B.C., is illustrated in Figure 1.3. Eratosthenes had concluded that the Egyptian cities of Alexandria and Syene were located approximately on the same meridian, and he had also observed that at noon on the summer solstice, the sun was directly overhead at Syene. (This was apparent because at that time of that day, the image of the sun could be seen reflecting from the bottom of a deep vertical well there.) He reasoned that at that moment, the sun, Syene, and Alexandria were in a common meridian plane, and if he could measure the arc length between the two cities, and the angle it subtended at the earth's center, he could compute the earth's circumference. He determined the angle by measuring the length of the shadow cast at Alexandria from a tall vertical staff of known length. The arc length was found from multiplying the number of caravan days between Syene and Alexandria by the average daily distance traveled. From these measurements

Eratosthenes calculated the earth's circumference to be about 25,000 mi. Subsequent precise geodetic measurements using better instruments, but techniques similar geometrically to Eratosthenes', have shown his value, though slightly too large, to be amazingly close to the currently accepted one.

1.10 Describe the steps a land surveyor would need to do when performing a boundary survey.

Briefly, the steps should include (1) preliminary walking of property with owner; (2) courthouse research to locate deed of property and adjoiners to determine ownership, possible easements, right-of-ways, conflicts of interest, and so on; (3) location survey of property noting any encroachments; conflicting elements; and so on; (4) resolution of conflicting elements between deed and survey; (5) delivery of surveying report to owner.

1.11 What is the name of the state-level professional surveying organization in your state or region?

Answer will vary by location.

1.12 What organizations in your state furnish maps and reference data to surveyors and engineers?

Responses will vary but some common organizations are the (1) county surveyor, (2) register of deeds, (3) county engineer or county highway department (4) Department of Transportation, (5) Department of Natural Resources of its equivalent, and so on.

1.13 List the legal requirements for registration as a land surveyor in your state.

Responses will vary. Contact with your licensing board can be found on the NCEES website at http://www.ncees.org/licensure/licensing_boards/.

1.14 Briefly describe an Earth-Centered, Earth-Fixed coordinate system.

From Section 1.4 and 13.4.3, a ECEF coordinate system is an Earth-based three-dimensional coordinate system with its origin a the mass-center of the Earth, it Z axis aligned with the semi-minor (spin) axis of the Earth defined at some epoch, it X axis in the plane of the equator passing through mean Greenwich meridian, and it Y axis in the plane of the equator and creating a right-handed coordinate system. At this stage of their introduction to surveying it should be sufficient for students to simply know that it is an Earth-based three-dimensional coordinate system.

1.15 List the professional societies representing the geospatial industry in the

(a) United States.

There are several including AAGS, ASCE, ASPRS, GLIS, NSPS, and SaGES.

(b) Canada.

Canadian Institute of Geomatics (CIG)

(c) International.

International Federation of Surveyors (FIG)

1.15 Explain how aerial photographs and satellite images can be valuable in surveying.

Photogrammetry presently has many applications in surveying. It is used, for example, in land surveying to compute coordinates of section corners, boundary corners, or point of evidence that help locate these corners. Large–scale maps are made by photogrammetric procedures for many uses, one being subdivision design. Photogrammetry is used to map shorelines, in hydrographic surveying, to determine precise ground coordinates of points in control surveying, and to develop maps and cross sections for route and engineering surveys. Photogrammetry is playing an important role in developing the necessary data for modern Land and Geographic Information Systems.

1.16 Search the Internet and define a Very Long Baseline Interferometry (VLBI) station. Discuss why these stations are important to the geospatial industry.

VLBI stands for Very Long Baseline Interferometry. Responses will vary. These stations provide extremely accurate locations on the surface of the Earth. The stations are used to develop world-wide reference frameworks such as ITRF08 and thus provide a worldwide coordinate system that links continents. They also may provide tracking information for satellites.

1.17 Describe how a GIS can be used in flood emergency planning.

Responses will vary but may mention the capabilities of a GIS to overlay soil type and their permeability with slopes, soil saturation, and watershed regions. A GIS can also be used to provide a list of business and residences that will be affected by possible flooding for evacuation purposes. It can provide "best" routes out of a flooded region.

1.19 Visit one of the surveying websites listed in Table 1.1, and write a brief summary of its contents. Briefly explain the value of the available information to surveyors.

Responses will vary with time, but below are brief responses to the question

- NGS – control data sheets, CORS data, surveying software
- USGS – maps, software
- BLM – cadastral maps, software, ephemerides
- U.S. Coast Guard Navigation Center - GPS information
- U.S. Naval Observatory –Notice Advisory for NAVSTAR Users (NANU) and other GPS related links
- National Society of Professional Surveyors – professional organization for boundary and construction
- American Association for Geodetic Surveying – professional organization for control surveying
- Geographic and Land Information Society – professional organization for developers and users of geographic and land information systems

- American Society for Photogrammetry and Remote Sensing – professional organization for photogrammetry and remote sensing
- The Pearson Prentice Hall publishers access to software and support materials that accompany this book.
- SaGES – An organization to advance surveying/geomatics education

1.20 Read one of the articles cited in the bibliography for this chapter, or another of your choosing, that describes an application where satellite surveying methods were used. Write a brief summary of the article.

Answer will vary.

1.21 Same as Problem 1.20, except the article should be on safety as related to surveying.

Answers will vary but should be related to safety issues in surveying.

1.20 Read one of the articles cited in the bibliography for this chapter, or another of your choosing, that describes an application where satellite surveying methods were used. Write a brief summary of the article.

Answers will vary. Students should be told to look in trade journals for articles.

1.21 Same as Problem 1.20, except the article should be on safety as related to surveying.

Answers will vary. Students should be told to look in trade journals for articles.

2 UNITS, SIGNIFICANT FIGURES, AND FIELD NOTES

2.1 List the five types of measurements that form the basis of traditional plane surveying.

From Section 2.1, they are (1) horizontal angles, (2) horizontal distances, (3) vertical (altitude or zenith) angles, (4) vertical distances, and (5) slope (or slant) distances.

2.2 Give the basic units that are used in surveying for length, area, volume, and angles in

(a) The English system of units.

From Section 2.2:

length (U.S. survey ft or in some states international foot), area (sq. ft. or acres), volume (cu. ft. or cu. yd.), angle (sexagesimal)

(b) The SI system of units.

From Section 2.3:

length (m), area (sq. m. or hectare), volume (cu. m.), angle (sexagesimal, grad, or radian)

2.3 The easting coordinate for a point is 725,316.911 m. What is the coordinate using the

(a) Survey foot definition?

(b) International foot definition?

(c) Why was the survey foot definition maintained in the United States?

(a) **2,379,643.90 sft**; $725{,}316.911\left(\frac{39.37}{12}\right) = 2{,}379{,}643.899$ sft

(b) **2,379,648,66 ft**; $725{,}316.911/0.3048 = 2{,}379{,}648.658$ ft

(c) From Section 2.2: "Because of the vast number of surveys performed prior to 1959, it would have been extremely difficult and confusing to change all related documents and maps that already existed. Thus the old standard, now called the *U.S. survey foot*, is still used."

2.4 Convert the following distances given in meters to U.S. survey feet:

***(a)** 4129.574 m **13,548.44 sft**

(b) 588.234 m **1929.90 sft**

(c) 102,302.103 m **335,636.15 sft**

2.5 Convert the following distances given in survey feet to meters:

*__(a)__ 537.52 sft **163.836 m**

(b) 2,405,687.82 sft **733,255.114 m**

(c) 5783.12 sft **1762.699 m or 1762.70 m**

2.6 Compute the lengths in survey feet corresponding to the following distances measured with a Gunter's chain:

*__(a)__ 10 ch 13 lk **668.6 sft**

(b) 56 ch 83 lk **3750.8 sft**

(c) 124 ch 35 lk **8207.1 sft**

2.7 Express 5,377,700 sft^2 in:

*__(a)__ acres **123.46 ac**

(b) hectares **49.961 ha**

(c) square Gunter's chains **1234.6 sq. ch.**

2.8 Convert 23.4587 ha to:

(a) square survey feet **2,525,070 sft^2**

(b) acres **57.9676 ac**

(b) square Gunter's chains **579.676 sq. ch**

2.9 What are the lengths in feet and decimals for the following distances shown on a building blueprint:

(a) 12 ft 6-1/4 in. **12.5 ft** 601/4/12

(b) 10 ft 6-1/2 in. **10.5 ft** 253/2/12

2.10 What is the area in acres of a rectangular parcel of land measured with a Gunter's chain if the recorded sides are as follows:

*__(a)__ 9.17 ch and 10.64 ch **9.76 ac**

(b) 16 ch 78 lk and 52 ch 49 lk **88.08 ac**

2.11 Compute the area in acres of triangular lots shown on a plat having the following recorded right-angle sides:

(a) 335.36 ft and 804.02 ft **3.0945 ac**

(b) 93.064 m and 30.346 m **0.69785 ac**

2.12 A distance is expressed as 9756.12 sft. What is the length in

*__(a)__ international feet? **9756.14 ft**

(b) meters? **2973.67 m**

2.13 What are the radian and degree-minute-second equivalents for the following angles given in grads:

(a)* 136.000 grads **122°24′00"; 2.13628 rad**

(b) 115.089 grads **103°34′50″; 1.80781 rad**

(c) 363.809 grads **327°25′40″; 5.71469 rad**

2.14 Give answers to the following problems in the correct number of significant figures:

***(a)** sum of 23.15, 0.984, 124, and 12.5 **161**

(b) sum of 2.115, 23.04, 13.8, and 199.66 **238.6**

(c) product of 127.08 and 13.1 **1660**

(d) quotient of 4466.83 divided by 35.61 **125.4**

2.15 Express the value or answer in powers of 10 to the correct number of significant figures:

(a) 4586.49 **4.58679×10^3**

(b) 2450 **2.45×10^3**

(c) square of 199.99 **3.9996×10^4**

(d) sum of (32.087 + 1.56 + 206.44) divided by 2.3 **1.95×10^1**

2.16 Convert the angles of a triangle to radians and show a computational check:

***(a)** 39°41′54, 91°30'16", 48°47′50" **0.692867, 1.59705,** and **0.851672**

0.6928666 + 1.597054 + 0.8516721 = 3.14159 check

(b) 96°23'18, 44°56′53", 38°39′49" **1.68229, 0.784492**, and **0.674807**

1.682294 + 0.784492 + 0.674807 = 3.14159 check

2.17 Why should a ball point pen not be used in field notekeeping?

From Section 2.7: "Books so prepared (with 3h or higher pencil) will withstand damp weather in the field (or even a soaking) and still be legible, whereas graphite from a soft pencil, or ink from a pen or ballpoint, leaves an undecipherable smudge under such circumstances."

2.18 Explain why one number should not be superimposed over another or the lines of sketches.

From Section 2.7: This can be explained with the need for integrity since it would raise the issue of what are you hiding, legibility since the numbers are often hard to interpret when so written, or by clarity since the notes are being crowded.

2.19 Explain why data should always be entered directly into the field book at the time measurements are made, rather than on scrap paper for neat transfer to the field book later.

From Section 2.7: Data should always be entered into the field book directly at the time of the measurements to avoid loss of data.

2.20 Why should the field notes show the precision of the measurements?

Field notes should show the precision of the measurements made to indicate the accuracy of the measurements.

2.21 Explain the reason for item 7 in Section 2.11 when recording field notes.

In general a sketch will show more than a table of numbers. As the saying goes, "A picture is worth a thousand words."

2.22 Explain the reason for item 13 in Section 2.11 when recording field notes.

A standard set of symbols and signs improve the clarity of drawings.

2.23 Explain the reason for item 18 in Section 2.11 when recording field notes.

A zero should be placed before a decimal point for the sake of clarity.

2.24 When should sketches be made instead of just recording data?

Sketches should be made instead of recording data anytime observations need to be clarified so that the personnel interpreting the notes can have a clear understanding of the field conditions. This also serves as a reminder of the work performed and any unusual conditions in later references to the project.

2.25 Justify the requirement to list in a field book the makes and serial numbers of all instruments used on a survey.

Listing the makes and serial numbers of the instruments used in the survey may help isolate instrumental errors later when reviewing the project.

2.26 Discuss the advantages of survey controllers that can communicate with several different types of instruments.

The ability of survey controllers to communicate with several different types of instruments allows the surveyor to match the specific conditions of the project with the instrument that this is ideally suited for the job. Thus total station, digital levels, and GNSS receivers can all be used in a single project.

2.27 Discuss why data should always be backed up at regular intervals.

From Section 2.13, paragraph 1: "At regular intervals, usually at lunchtime and at the end of a day's work, or when a survey has been completed, the information stored in files within a data collector is transferred to another device. This is a safety precaution to avoid accidentally losing substantial amounts of data."

2.28 Search the Internet and find at least two sites related to
(a) Manufacturers of survey controllers.
(b) Manufacturers of total stations.
(c) Manufacturers of global navigation satellite system (GNSS) receivers.

Answers should vary with students.

2.29 Why do many survey controllers contain digital cameras?

From Section 2.15: "Many modern survey controllers also contain digital cameras that allow field personnel to capture a digital image of the survey."

2.30 What are the dangers involved in using a survey controller?

From Section 2.15: "Although survey controllers have many advantages, they also present some dangers and problems. There is the slight chance, for example, the files could be accidentally erased through carelessness or lost because of malfunction or damage to the unit."

2.31 Describe what is meant by the phrase "field-to-finish."

From Section 2.15, "The field codes can instruct the drafting software to draw a map of the data complete with lines, curves and mapping symbols. The process of collecting field data with field codes that can be interpreted later by software is known as a *field-to-finish* survey. This greatly reduces the time needed to complete a project."

2.32 Why are sketches in field books not usually drawn to scale?

This is true since this would require an overwhelming amount of time. The sketches are simply to provide readers of the notes an approximate visual reference to the measurements.

3 THEORY OF ERRORS IN OBSERVATIONS

3.1 Discuss the difference between an error and a residual.

From Section 3.3, an error is the difference between the observation and its true value, or $E = X - \bar{X}$ whereas a residual, which is defined in Section 3.11 is the difference between the mean of a set of observations and the observation or $v = \bar{M} - M$

3.2 Give two examples of **(a)** direct and **(b)** indirect measurements.

From Section 3.2: A direct observation is made by applying a measurement instrument directly to a quantity to be measured and an indirect observation is made by computing a quantity from direct observations.

Examples should vary by student response.

3.3 Define the term *systematic error*, and give two surveying examples of a systematic error.

See Section 3.6

3.4 Define the term *random error*, and give two surveying examples of a random error.

See Section 3.6

3.5 Discuss the difference between accuracy and precision.

From Section 3.7, accuracy is the nearness of the observed quantities to the true value, which is never known. Precision is the degree of refinement or consistency of a group of observations and is evaluated on the basis of discrepancy size.

3.6 The observations of 124.53, 124.55, 142.51, and 124.52 are obtained when taping the length of a line. What should the observer consider doing before a mean length is determined from the set of observations?

It appears that the observation 142.51 is an outlier and a possible mistake in the data set. The observer should collect another tape observation of the line and discard the offending observation(s).

A distance *AB* is observed repeatedly using the same equipment and procedures, and the results, in meters, are listed in Problems 3.7 through 3.10. Calculate **(a)** the line's most probable length, **(b)** the standard deviation and **(c)** the standard deviation of the mean for each set of results.

***3.7** 65.401, 65.400, 65.402, 65.396, 65.406, 65.401, 65.396, 65.401, 65.405, and 65.404

(a) **65.401** $\sum 654.012$

(b) **<u>±0.003</u>** $\sum v^2 = 0.000091$

(c) **<u>±0.001</u>**

3.8 Same as Problem 3.7 but discard only one 65.396 observation.

(a) **<u>65.402</u>** $\sum 588.616$

(b) **<u>±0.003</u>** $\sum v^2 = 0.000064$

(c) **<u>±0.0009</u>**

3.9 Same as Problem 3.7, but discard both 65.396 observations.

(a) **<u>65.402</u>** $\sum 523.220$

(b) **<u>±0.002</u>** $\sum v^2 = 0.000030$

(c) **<u>±0.0007</u>**

3.10 Same as Problem 3.7, but include two additional observations, 65.402 and 65.405.

(a) **<u>65.402</u>** $\sum 784.819$

(b) **<u>±0.003</u>** $\sum v^2 = 0.000115$

(c) **<u>±0.0009</u>**

In Problems 3.11 through 3.14, determine the range within which observations should fall (a) 90% of the time and (b) 95% of the time. List the percentage of values that actually fall within these ranges.

3.11 For the data of Problem 3.7.

***(a)** **<u>65.4012±0.0052 (65.3960, 65.4064), 100%</u>**

(b) **<u>65.4012±0.0062 (65.3950, 65.4074), 100%</u>**

3.12 For the data of Problem 3.8.

(a) **<u>65.4018±0.0046 (65.3971, 65.4064), 90%, 65.396 outside of range</u>**

(b) **<u>65.4018±0.0055 (65.3963, 65.4073), 90%, 65.396 outside of range</u>**

3.13 For the data of Problem 3.9.

(a) **<u>65.4025±0.0034 (65.3991, 65.4059), 90%, 65.406 outside of range</u>**

(b) **<u>65.4025±0.0040 (65.3985, 65.4065), 100%</u>**

3.14 For the data of Problem 3.10.

(a) **<u>65.4016±0.0053 (65.3963, 65.4069), 83.3%, both 65.396 outside of range</u>**

(b) **<u>65.4016±0.0063 (65.3952, 65.4079), 100%</u>**

In Problems 3.15 through 3.17, an angle is observed repeatedly using the same equipment and procedures. Calculate **(a)** the angle's most probable value, **(b)** the standard deviation, and **(c)** the standard deviation of the mean.

***3.15** $23°30'00''$, $23°30'10''$, $23°30'10''$, and $23°29'55''$.

(a) **<u>23°30′04″</u>**

(b) **<u>±7.5″</u>**

(c) **<u>±3.8″</u>**

3.16 Same as Problem 3.15, but with three additional observations, $23°29'55''$, $23°29'50''$ and $23°30'05''$.

(a) **<u>23°30′01″</u>**

(b) **<u>±7.9″</u>**

(c) **<u>±3.0″</u>**

3.17 Same as Problem 3.15, but with two additional observations, $23°30'05''$ and $23°29'55''$.

(a) **<u>23°30′02″</u>**

(b) **<u>±6.9″</u>**

(c) **<u>±2.8″</u>**

3.18* A field party is capable of making taping observations with a standard deviation of ±0.02 ft per 100 ft tape length. What standard deviation would be expected in a distance of 400 ft taped by this party?

By Equation (3.12): **<u>±0.04 ft</u>** $= 0.020\sqrt{400/100}$

3.19 Repeat Problem 3.18, except that the standard deviation per 30-m tape length is ±3mm and a distance of 60 m is taped. What is the expected 95% error in 60 m?

S = by Equation (3.12): **<u>±0.004 m</u>** $= 0.003\sqrt{60/30}$

S_{95} = by Equation (3.8): **<u>±0.008 m</u>** = 0.0042(1.9599)

3.20 A distance of 200 ft must be taped in a manner to ensure a standard deviation smaller than ±0.04 ft. What must be the standard deviation per 100 ft tape length to achieve the desired precision?

<u>±0.028 ft</u> = $\pm 0.04/\sqrt{200/100}$ by Equation (3.12) rearranged.

3.21 Lines of levels were run requiring n instrument setups. If the rod reading for each backsight and foresight has a standard deviation σ, what is the standard deviation in each of the following level lines?

(a) $n = 12$, $\sigma = \pm 0.005$ ft; By Equation (3.12): **<u>±0.017 ft</u>** $= 0.005\sqrt{12}$.

(b) n = 32, σ = ±3 mm; By Equation (3.12): **±17.0 mm** = $3\sqrt{32}$.

3.22 A line *AC* was observed in 2 sections *AB* and *BC*, with lengths and standard deviations listed below. What is the total length *AC*, and its standard deviation?

*__(a)__ $AB = 60.00 \pm 0.015$ ft; $BC = 86.13 \pm 0.018$ ft; **146.13±0.023 ft** by Equation (3.11)

(b) $AB = 30.000 \pm 0.004$ m; $BC = 23.150 \pm 0.003$ m; **53.150±0.005 m** by Equation (3.11)

3.23 Line *AD* is observed in three sections, *AB, BC*, and *CD*, with lengths and standard deviations as listed below. What is the total length *AD* and its standard deviation?

(a) $AB = 456.78 \pm 0.03$ ft; $BC = 524.56 \pm 0.04$ ft; $CD = 692.35 \pm 0.05$ ft
1673.69 ± 0.071 ft by Equation (3.11)

(b) $AB = 229.090 \pm 0.005\ m$; $BC = 336.447 \pm 0.006$ m; $CD = 465.837 \pm 0.008$ m

1031.374 ± 0.011 m by Equation (3.11)

3.24 The difference in elevation between *A* and *B* was observed four times as 32.05, 32.03, 32.08, and 32.01 ft. The observations were given weights of 2, 1, 3 and 2, respectively, by the observer. ***(a)** Calculate the weighted mean for distance *AB*. (**b**) What difference results if later judgment revises the weights to 2, 3, 1, and 1, respectively?

By Equation (3.17):

*__(a)__ **32.036 ft**; $m_w = \frac{32.05(2)+32.03(1)+32.08(3)+32.01(2)}{2+1+3+2}$

(b) **32.040 ft**; $m_w = \frac{32.05(2)+32.03(3)+32.08(1)+32.01(1)}{2+3+1+1}$

3.25 Determine the weighted mean for the following angles:

By Equation (3.17):

(a) $222°12'36''$, wt 2; $222°12'42''$, wt 1; $222°12'34''$, wt 3; **222°12′40.2″**; $m_w = \frac{36(2)+42(1)+34(3)}{2+1+3}$

(b) $96°14'20'' \pm 3''$; $96°14'24'' \pm 2''$; $96°14'18'' \pm 1''$; **96°14′19.3″**; $m_w = \frac{20\left(\frac{1}{3}\right)^2+42\left(\frac{1}{2}\right)^2+34\left(\frac{1}{1}\right)\text{^}2}{\frac{1}{3^2}+\frac{1}{2^2}+\frac{1}{1^2}}$

3.26 Specifications for observing angles of an *n*-sided polygon limit the total angular misclosure to *E*. How accurately must each angle be observed for the following values of *n* and *E*?

By rearranged Equation (3.12):

(a) n = 6, E = ±10″; **±4.1″**; $10/\sqrt{6}$

(b) n = 10, E = ±10″; **±3.2″**; $10/\sqrt{10}$

3.27 What is the area of a rectangular field and its estimated error for the following recorded values:

By Equation (3.13):

***(a)** 243.89 ± 0.05 ft by 208.65 ± 0.04 ft; **50,888 ± 14 ft²** or **1.1682 ± 0.0003 ac**;

$\sqrt{[243.89(0.04)]^2 + [208.65(0.05)]^2}$

(b) 1203.45 ± 0.08 ft by 906.78 ± 0.06 ft; **1,091,300 ± 100 ft²** or **25.052 ± 0.002 ac**; $\sqrt{[1203.45(0.06)]^2 + [906.78(0.08)]^2}$

(c) 344.092 ± 0.006 m by 180.403 ± 0.005 m; **62,075.0 ± 2.0 m²** or **6.2075 ± 0.00020 ha**; $\sqrt{[344.092(0.005)]^2 + [180.403(0.006)]^2}$

3.28 Adjust the angles of triangle *ABC* for the following angular values and weights:

By Equation (3.17):

***(a)** $A = 49°24'22''$, wt 2; $B = 39°02'16''$, wt 1; $C = 91°33'00''$, wt 3

Misclosure = −22″

	Obs. Ang.	Wt	Corr.	Num. Cor.	Rnd. Cor.	Adj. Ang.
A	49°24′22″	2	3x	6″	6″	**49°24′28″**
B	39°02′16″	1	6x	12″	12″	**39°02′28″**
C	91°33′00″	3	2x	4″	4″	**91°33′04″**
	179°59′38″	6	11x			
			11x = 22″	x = 2″		

(b) $A = 81°06'44''$, wt 2; $B = 53°33'56''$, wt 2; $C = 45°19'20''$, wt 3

Misclosure = –10″

	Obs. Ang.	Wt	Corr.	Num. Cor.	Rnd Cor.	Adj. Ang.
A	81°06′44″	2	21x	3.8″	4″	**81°06′48″**
B	53°33′56″	2	21x	3.8″	4″	**53°34′00″**
C	45°19′10″	3	14x	2.5″	2″	**45°19′12″**
	179°59′53″	7	56x		10″	
			56x = 10″	x = 0.178″		

3.29 Determine relative weights and perform a weighted adjustment (to the nearest second) for angles *A, B*, and *C* of a plane triangle, given the following four observations for each angle:

Angle *A*	Angle *B*	Angle *C*
44°28′16″	65°56′13″	69°35′20″
44°28′12″	65°56′10″	69°35′24″
44°28′17″	65°56′06″	69°35′18″
44°28′11″	65°56′08″	69°35′24″

A = 44°28′14.0″ ± 2.9″; ***B*** = 65°56′09.3″ ± 3.0″; ***C*** = 69°35′21.5″ ± 3″

Misclosure = −15.3″

	Obs. Ang.	Wt	Corr. Multiplier	Num. Cor.	Rnd. Cor.	Adj. Ang.
A	44°28′14.0″	0.115385	0.338645/wt = 2.93	4.97″	5.0″	44°28′19″
B	65°56′09.3″	0.11215	0.338645/wt = 3.02	5.12″	5.1″	65°56′14″
C	69°35′21.5″	0.111111	0.338645/wt= 3.05	5.16″	5.2″	69°35′27″
	179°59′44.7″	0.338645	9.00x			
			9.0 = 15.3″	x = 1.69″		

3.30 A line of levels was run from benchmarks *A* to *B, B* to *C*, and *C* to *D*. The elevation differences obtained between benchmarks, with their standard deviations, are listed below. What is the difference in elevation from benchmark *A* to *D* and the standard deviation of that elevation difference?

(a) BM *A* to BM *B* = +37.78 ± 0.12 ft; BM *B* to BM *C* = –73.50 ± 0.16 ft; and BM *C* to BM *D* = –84.09 ± 0.08 ft

By Equation (3.11): **−119.81 ± 0.22 ft**

(b) BM *A* to BM *B* = –60.821 ± 0.015 m; BM *B* to BM *C* = +94.378 241 ± 0.020 m; and BM *C* to BM *D* +56.805 ± 0.015 m

By Equation (3.11): **90.362 ± 0.029 m**

4 LEVELING THEORY, METHODS, AND EQUIPMENT

4.1 Define the following leveling terms: (**a**) vertical line, (**b**) level surface, and (**c**) benchmark.

From Section 4.2:

(a) Vertical line: "A line that follows the local direction of gravity as indicated by a plumb line"

(b) Level surface: ". A curved surface that at every point is perpendicular to the local plumb line (the direction in which gravity acts)."

(c) Benchmark: "A relatively permanent object, natural or artificial, having a marked point whose elevation above or below a reference datum is known or assumed."

***4.2** How far will a horizontal line depart from the Earth's surface in 1 km? 5 km? 10 km? (Apply both curvature and refraction)

1 km? $C_m = 0.0675(1)^2 = 0.068$ m

5 km? $C_m = 0.0675(5)^2 = 1.688$ m

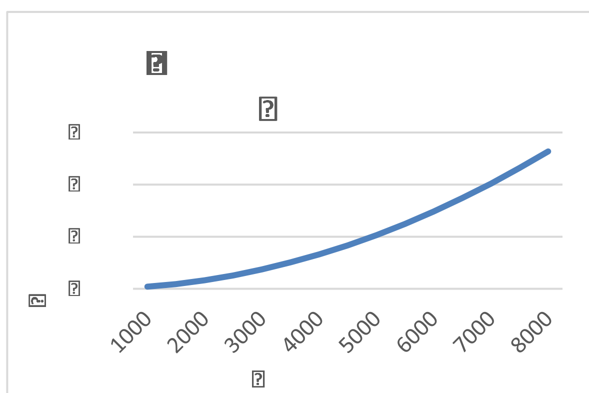
10 km? $C_m = 0.0675(10)^2 = 6.750$ m

4.3 Visit the website of the National Geodetic Survey, and obtain a data sheet description of a benchmark in your local area.

Solutions should vary.

4.4 Create plot of the curvature and refraction correction for sight lines going from 0 ft to 10,000 ft in 500 ft increments.

Dist. (ft)	CR (ft)
1000	0.02
1500	0.05
2000	0.08
2500	0.13
3000	0.19
3500	0.25
4000	0.33
4500	0.42
5000	0.52
5500	0.62



6000	0.74
6500	0.87
7000	1.01
7500	1.16
8000	1.32

4.5 Create a plot of curvature and refraction corrections for sight lines going from 0 m to 10,000 m in 500 m increments.

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
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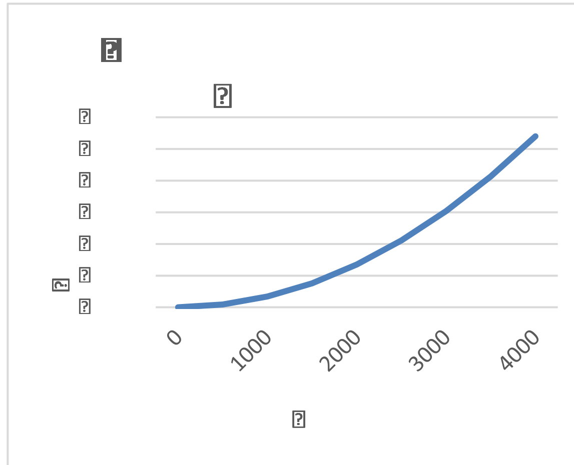

4.6 Why are elevations today not referred to as mean sea-level heights?

From Section 4.3: "This adjustment (NAVD88) shifted the position of the reference surface from the mean of the 26 tidal gage stations to a single tidal gage benchmark called *Father Point*, which is in Rimouski, Quebec, Canada, near the mouth of the St. Lawrence Seaway. Thus, elevations in NAVD88 are no longer referenced to mean sea level. Benchmark elevations that were defined by the NGVD29 datum have changed by relatively small, but nevertheless significant amounts in the eastern half of the continental United States (see Figure 19.7). However, the changes are much greater in the western part of the country and reach 1.5 m in the Rocky Mountain region. It is therefore imperative that surveyors positively identify the datum to which their elevations are referred. After the adjustment, adjusted elevations are properly known as orthometric heights."

***4.7** On a large lake without waves, how far from shore is a sailboat when the top of its 30-ft mast disappears from the view of a person lying at the water's edge?

38,160 ft or 7.3 mi; $F = 1000\sqrt{\frac{30}{0.0206}} = 38{,}161\ ft = \mathbf{7.228\ mi}$

4.8 Similar to Problem 4.7, except for a 5-m mast and a person whose eye height is 1.5 m above the water's edge.

13.3 km; $K = \sqrt{1.5/0.0675} + \sqrt{5/0.0675} = \mathbf{13.32\ km}$

4.9 Readings on a line of differential levels are taken to the nearest 0.2 mm. For what maximum distance can the Earth's curvature and refraction be neglected?

$$M = 1000\sqrt{0.0002/0.0675} = \mathbf{54.4\ m}$$

4.10 Similar to Problem 4.9 except readings are to the 0.001 ft.

$$F = 1000\sqrt{0.001/0.0206} = \mathbf{220.3\ ft}$$

4.11 Describe how readings are determined in a digital level when using a bar coded rod.

From Section 4.11: "At the press of a button, the image of bar codes in the telescope's field of view is captured and processed. This processing consists of an on-board computer comparing the captured image to the rod's entire pattern, which is stored in memory. When a match is found, which takes about 4 sec, the rod reading is displayed digitally."

Successive plus and minus sights taken on a downhill line of levels are listed in Problems 4.12 and 4.13. The values represent the horizontal distances between the instrument and either the plus or minus sights. What error results from curvature and refraction?

***4.12** 20, 225; 50, 195; 40, 135; 30, 250 ft.

Plus	CR (ft)	Minus	CR (ft)
20	0.00000824	225	0.001043
50	0.0000515	195	0.000783
40	0.00003296	135	0.000375
30	0.00001854	250	0.001288
Sum	0.00011124		0.003489

Combined **−0.003 ft**

4.13 20, 70; 25, 60; 20, 55; 15, 60 m.

Plus	CR (mm)	Minus	CR (mm)
20	0.027	70	0.33075
25	0.042188	60	0.243
20	0.027	55	0.204188
15	0.015188	60	0.243
	0.111375		1.020938

Combined: –0.91 mm

4.14 What error results if the curvature and refraction correction is neglected in trigonometric leveling for sights: (**a**) 3000 ft long (**b**) 1200 m long (**c**) 4500 ft long?

(**a**) $h_f = 0.0206\left(\frac{3000}{1000}\right)^2 = \mathbf{0.18\ ft}$

(**b**) $h_m = 0.0675\left(\frac{1200}{1000}\right)^2 = \mathbf{0.097}\ \boldsymbol{m}$

(**c**) $h_f = 0.0206\left(\frac{4500}{1000}\right)^2 = \mathbf{0.42\ ft}$

***4.15** The slope distance and zenith angle observed from point P to point Q were 2406.787 m and 84°13′07″ respectively. The instrument and rod target heights were equal. If the elevation of point P is 30.245 m, above datum, what is the elevation of point Q?

$$Elev_Q = 30.245 + 2406.787\cos(84°13'07'') + 0.0675\left(\frac{2406.787\sin(84°13'07'')}{1000}\right)^2$$

$$Elev_Q = 30.245 + 242.443 + 0.387 = \mathbf{273.075\ m}$$

4.16 The slope distance and zenith angle observed from point X to point Y were 2907.45 ft and 97°25'36″. The instrument and rod target heights were equal. If the elevation of point X is 6547.89 ft above datum, what is the elevation of point Y?

$$Elev_y = 6547.89 + 2907.45\cos(97°25'36'') + 0.0206\left(\frac{2907.45\sin(97°25'36'')}{1000}\right)^2$$

$$= 6547.89 - 375.809 + 0.171 = \mathbf{6172.25\ ft}$$

4.17 Similar to Problem 4.15, except the slope distance was 1543.853 m, the zenith angle was 83°44'08″ and the elevation of point P was 1850.567 m above datum.

$$Elev_y = 1850.567 + 1543.853\cos(83°44'08'') + 0.0675\left(\frac{1543.853\sin(83°44'08'')}{1000}\right)^2$$

$$= 1850.567 + 168.462 + 0.159 = \mathbf{2016.187\ m}$$

4.18 In trigonometric leveling from point A to point B, the slope distance and zenith angle measured at A were 5462.46 ft and 94°08'36″. At B these measurements were 5462.58 ft and 85°□ '□ ″, respectively. If the instrument and rod target heights were equal, calculate the difference in elevation from A to B.

$$z_{avg} = \frac{94°08'36'' + 180° - 85°51'47''}{2} = 94°08'24''$$

$$S_{avg} = \frac{5462.58 + 5462.46}{2} = 5462.52$$

$$\Delta Elev = 5462.52\cos(94°08'24'') = \mathbf{-394.36\ ft}$$

4.19 Describe how parallax in the viewing system of a level can be detected and removed.

From Section 4.7:

"After focusing, if the cross hairs appear to travel over the object sighted when the eye is shifted slightly in any direction, *parallax* exists. The objective lens, the eyepiece, or both must be refocused to eliminate this effect if accurate work is to be done."

4.20 What is the sensitivity of a level vial with 2-mm divisions for: (a) a radius of 13.75 m (b) a radius of 10.31 m?

(a) $\theta = \left[\frac{2}{13.75(1000)}\right]206264.8 = \mathbf{30''}$

(b) $\theta = \left[\frac{2}{10.31(1000)}\right]206264.8 = \mathbf{40''}$

***4.21** An observer fails to check the bubble, and it is off two divisions on a 500-ft sight. What error in elevation difference results with a 10-sec bubble?

angular error = 2(10) = 20 sec

Error = 250 tan(20) = **0.048 ft**

4.22 An observer fails to check the bubble, and it is off two divisions on a 300-m sight. What error results for a 20-sec bubble?

angular error = 2(20) = 40 sec

Error = 200 tan(40) = **0.058 m**

4.23 Similar to Problem 4.22, except a 30-sec bubble is off three divisions on a 300-ft sight.

angular error = 3(30) = 90 sec

Error = 300 tan(90) = **0.13 ft**

4.24 With the bubble centered, a 100-m sight gives a reading of 1.352 m. After moving the bubble three divisions off center, the reading is 1.396 m. For 2-mm vial divisions, what is: (**a**) the vial radius of curvature in meters (**b**) the angle in seconds subtended by one division?

Δrdg = 1.410 − 1.352 = 0.058 m

$$4\theta = \text{atan}\left(\frac{0.058}{100}\right) = 120''$$

(**a**) $R = 0.002/\tan(120'') = \mathbf{3.438\ m}$

(**b**) 120″/4 = **30″**

4.25 Similar to Problem 4.24, except the sight length was 300 ft, the initial reading was 5.132 ft, and the final reading was 5.176 ft.

Δrdg = 5.176 – 5.132 = 0.0.044 ft

$$3\theta = \text{atan}\left(\frac{0.044}{300}\right) = 3\mathbf{0}''$$

(**a**) $R = 0.002/\tan(30'') = \mathbf{13.75\ ft}$

(**b**) 30″/3 = **10″**

4.26 Sunshine on the forward end of a 20″/2 mm level vial bubble draws it off 2 divisions, giving a plus sight reading of 4.63 ft on a 250-ft sight. Compute the correct reading.

Correction = 200 tan(2·20″) = 0.048 ft

Correct reading = 4.63 − 0.048 = **4.58 ft**

Note: the correction is subtracted since the bubble was drawn off on the forward end of the level, thus raising the line of sight.

4.27 List in tabular form, for comparison, the advantages and disadvantages of an automatic level versus a digital level.

See Section 4.10 and 4.11.

***4.28** If a plus sight of 3.54 ft is taken on BM *A*, elevation 850.48 ft, and a minus sight of 7.84

ft is read on point X, calculate the HI and the elevation of point X.

HI = 850.48 + 3.54 = **854.02 ft**

Elev = 854.02 − 7.84 = **846.18 ft**

4.29 If a plus sight of 0687 m is taken on BM A, elevation 85.476 m, and a minus sight of 1.564 m is read on point X, calculate the HI and the elevation of point X.

HI = 85.476 + 0.687 = **86.163 m**

Elev = 86.163 – 1.564 = **84.599 m**

4.30 Similar to Problem 4.28, except a plus sight of 8.98 ft is taken on BM A, elevation 606.33 ft, and a minus sight of 4.32 ft read on point X.

HI = 606.33 + 8.98 = **615.31 ft**

Elev = 615.31 – 4.32 = **610.99 ft**

4.31 Describe the procedure used to test if the level vial is perpendicular to the vertical axis of the instrument.

See Section 4.15.5

4.32 A horizontal collimation test is performed on an automatic level following the procedures described in Section 4.15.5. With the instrument setup at point 1, the rod reading at A was 5.548 ft, and to B it was 5.126 ft. After moving and leveling the instrument at point 2, the rod reading to A was 5.540 ft and to B was 5.126 ft. What is the collimation error of the instrument and the corrected reading to A from point 2?

$$\varepsilon = \frac{5.126 - 5.548 - 5.126 + 5.540}{2} = -0.004$$

Correct reading at A = 5.540 – 2(–0.004) = **5.548 ft**

4.33 The instrument tested in Problem 4.32 was used in a survey immediately before the test where the observed elevation difference between two benchmarks was +44.65 ft. The sum of the plus sight distances between the benchmarks was 250 ft and the sum of the minus sight distances was 490 ft. What is the corrected elevation difference between the two benchmarks?

+44.64 ft; = 44.65– 0.004/100(250 − 490) = 44.64 ft

4.34 Similar to Problem 4.32 except that the rod readings are 1.894 m and 1.923 m to A and B, respectively, from point 1, and 1.083 m and 1.100 m to A and B, respectively, from point 2. The distance between the points in the test was 100 m.

$$\varepsilon = \frac{1.923 - 1.894 - 1.100 + 1.083}{2} = 0.006 \text{ m}$$

Correct reading at A = 1.083 – 2(0.006) = **1.071 m**

4.35 The instrument tested in Problem 4.34 was used in a survey immediately before the test where the observed elevation difference between two benchmarks was −13.068 m. The sum of the plus sight distances between the benchmarks was 1540 m and the sum of the minus sight distances was 545 m. What is the corrected elevation difference between the two benchmarks?

$-$13.128 m; $= -13.068 - 0.006/100(1540 - 545)$

5 LEVELING — FIELD PROCEDURES AND COMPUTATIONS

Asterisks (*) indicate problems that have answers given in Appendix G.

5.1 Explain the left-thumb rule when centering a level.

From Section 5.2, paragraph 3: A simple but useful rule in centering a bubble, illustrated in Figure 5.1, is: *A bubble follows the left thumb when turning the screws*.

5.2 What is the difference between a benchmark and a turning point in differential leveling?

A benchmark is a relatively permanent object, natural or artificial, having a marked point whose elevation above or below a reference datum is known, assumed, or will be established during the leveling process whereas a turning point is an intermediate, temporary point between benchmarks which are created to perform the differential leveling process. They are usually temporary points whose elevations are lost after the differential leveling process is complete.

5.3 Discuss how stadia can be used to determine the plus and minus sight distances in differential leveling.

From Section 5.4: "The stadia method determines the horizontal distance to points through the use of readings on the upper and lower (stadia) wires on the reticle. The method is based on the principle that in similar triangles, corresponding sides are proportional. … Thus the equation for a distance on a horizontal stadia sight reduces to $D = KI$ (5.2) … It should be realized by the reader that in differential leveling, the actual sight distances to the rod are not important. All one needs to balance is the rod intervals on the plus and minus sights between benchmarks to ensure that the sight distances are balanced."

5.4 What is the collimation error, and how can it be removed from the differential leveling process.

From Section 5.12.1: It is caused by the line of sight not being parallel with the axis of the level vial. When this condition exists, the line of sight will not be horizontal and thus result in incorrect readings. This is a systematic error and can be removed by balancing the backsight and foresight distances between benchmarks.

5.5 Discuss how errors due to Earth curvature and refraction can be eliminated from the differential leveling process.

From Section 5.4: “Balancing plus and minus sight distances will eliminate errors due to instrument maladjustment (most important) and the combined effects of the Earth’s curvature and refraction, as shown in Figure 5.6. Here e_1 and e_2 are the combined curvature and refraction errors for the plus and minus sights, respectively. If D_1 and D_2 are made equal, e_1 and e_2 are also equal. In calculations, e_1 is added and e_2 subtracted; thus they cancel each other."

5.6 When is it appropriate to use the reciprocal leveling procedure?

From Section 5.7: "Sometimes in leveling across topographic features such as rivers, lakes, and canyons, it is difficult or impossible to keep plus and minus sights short and equal. Reciprocal leveling may be utilized at such locations."

5.7 List four considerations that govern a rodperson's selection of TPs and BMs.

1. From Chapter 4: BMs must be permanent.
2. From Section 5.4: "Turning points should be solid objects with a definite high point."
3. From Section 5.6: "...it is recommended that some turning points or benchmarks used in the first part of the circuit be included again on the return run. This creates a multi-loop circuit, and if a blunder or large error exists, its location can be isolated to one of the smaller loops."
4. From Section 5.12.2: "It (settlement) can be avoided by selecting firm, solid turning points or, if none are available, using a steel turning pin set firmly in the ground."
5. Find turning points that aid in the balancing of plus and minus sight distances.

***5.8** What error is created by a rod leaning 10 min from plumb at a 12.51-ft reading on the leaning rod?

<u>Error = 0.000 ft</u>

Correct reading = 12.51 cos(10') = 12.50995; So error is 0.00005 ft, or 0.000 ft

Problem is designed to show that even for a high reading and a mislevelment outside of a typical circular bubble, the resulting error is negligble.

5.9 Similar to Problem 5.6, except for a 5-m reading.

<u>Error is 0.000 m</u>

Correct reading = 5 cos(10') = 4.9999785, so error is 0.000021. The error is negligible.

Problem is designed to show that even for a high reading and a mislevelment outside of a typical circular bubble, the resulting error is negligble.

5.10 What error results on a 30-m sight with a level if the rod reading is 2.865 m but the top of the 3 m rod is 0.3 m out of plumb?

Correct reading = $\frac{0.3}{3} 2.865 = 2.8506$ m

<u>Error = 0.014 m</u>

5.11 What error results on a 200-ft sight with a level if the rod reading is 6.307 ft but the top of the 7-ft rod is 0.2 ft out of plumb?

Correct reading = $\frac{0.2}{7} 6.307 = 6.3044$

<u>Error = 0.0026 ft</u>

5.12 Prepare a set of level notes for the data listed. Perform a check and adjust the misclosure. Elevation of BM 7 is 2303.45 ft. If the total loop length is 2400 ft, what order of leveling is represented? (Assume all readings are in feet)

POINT	+S (BS)	–S (FS)
BM 7	5.68	
TP 1	9.42	7.58
TP 2	9.26	5.81
BM 8	6.45	4.59
TP 3	9.59	8.50
BM 7		13.95

STA	Plus	HI	Minus	ELEV
BM 7	5.68			2303.45
		2309.13	(0.006)	(2301.556)
TP 1	9.42		7.58	2301.55
		2310.97	(0.012)	(2305.172)
TP 2	9.26		5.81	2305.16
		2314.42	(0.018)	(2309.848)
BM 8	6.45		4.590	2309.83
		2316.28	(0.024)	(2307.804)
TP 3	9.59		8.500	2307.78
		2317.37	(0.03)	(2303.45)
BM 7			13.950	2303.42
	40.40		40.43	

Page check 2303.45+40.4-40.43 = 2303.42
Misclosure = 2303.42 – 2303.45 = –0.03
Correction = – –0.03/5 = 0.006

2400 ft ≈ 0.7315 km and 0.03 ft ≈ 9.1 mm; From Equation 5.3: $m = 12/\sqrt{0.7315} =$ 10.7 mm, **Third Order**

***5.13** Similar to Problem 5.12, except the elevation of BM 7 is 132.05 ft and the loop length 2400 ft. (Assume all readings are in feet)

STA	Plus	HI	Minus	ELEV
BM7	5.68			132.05
		137.73	(0.006)	(130.156)
TP1	9.420		7.58	130.15
		139.57	(0.012)	(133.772)
BM 8	9.26		5.81	133.76
		143.02	(0.018)	(138.448)
TP2	6.45		4.590	138.43