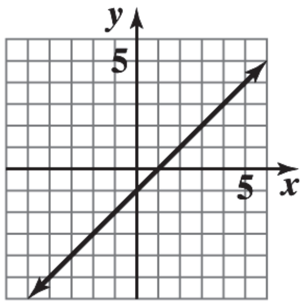
2 FUNCTIONS

EXERCISE 2-1

2.



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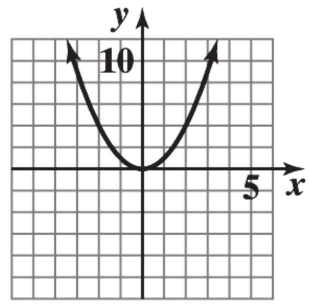

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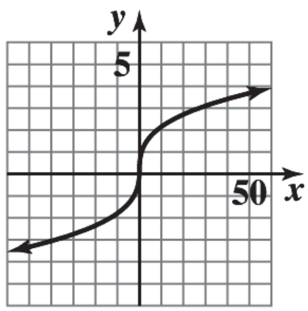
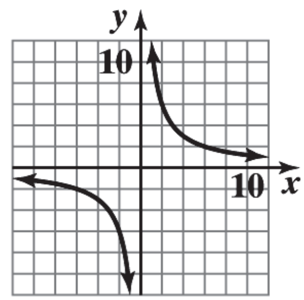


8.



10. The table specifies a function, since for each domain value there corresponds one and only one range value.

12. The table does not specify a function, since more than one range value corresponds to a given domain value.
(Range values 1, 2 correspond to domain value 9.)

14. This is a function.

16. The graph specifies a function; each vertical line in the plane intersects the graph in at most one point.

18. The graph does not specify a function. There are vertical lines which intersect the graph in more than one point. For example, the *y*-axis intersects the graph in two points.

20. The graph does not specify a function.

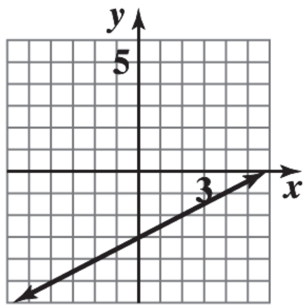
22. $y = 4x + \frac{1}{x}$ is neither linear nor constant.

24. $2x - 4y - 6 = 0$ is linear.

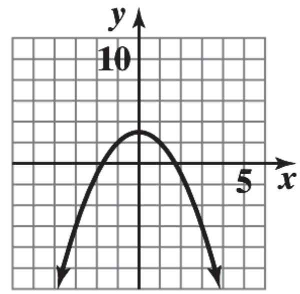
26. $x + xy + 1 = 0$ is neither linear nor constant.

28. $\frac{y-x}{2} + \frac{3+2x}{4} = 1$ simplifies to $y = \frac{1}{2}$ constant.

30.



32.



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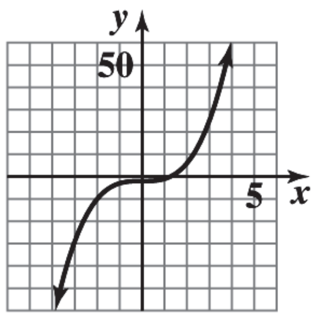
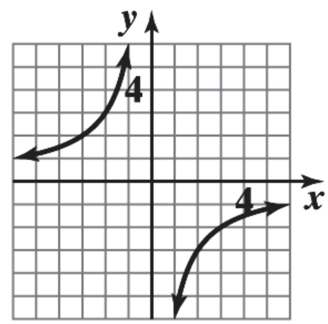


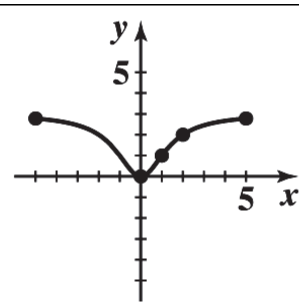
36.



38. $f(x) = \dfrac{3x^2}{x^2+2}$. Since the denominator is bigger than 1, we note that the values of f are between 0 and 3. Furthermore, the function f has the property that $f(-x) = f(x)$. So, adding points $x = 3$, $x = 4$, $x = 5$, we have:

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$F(x)$	2.78	2.67	2.45	2	1	0	1	2	2.45	2.67	2.78

The sketch is:



40. $y = f(4) = 0$

42. $y = f(-2) = 3$

44. $f(x) = 4$ at $x = 5$.

46. $f(x) = 0$ at $x = -5, 0, 4$.

48. Domain**:** all real numbers.

50. Domain: all real numbers except $x = 2$**.**

52. Domain: $x \geq -5$ or $[-5, \infty)$.

54. Given $6x - 7y = 21$. Solving for y we have: $-7y = 21 - 6x$ and $y = \dfrac{6}{7}x - 3$.

This equation specifies a function. The domain is R, the set of real numbers.

56. Given $x(x+y)=4$. Solving for y we have: $xy+x^2=4$ and $y=\dfrac{4-x^2}{x}$.

This equation specifies a function. The domain is all real numbers except 0

58. Given $x^2+y^2=9$. Solving for y we have: $y^2=9-x^2$ and $y=\pm\sqrt{9-x^2}$.

This equation does not define y as a function of x. For example, when $x=0$, $y=\pm 3$.

60. Given $\sqrt{x}-y^3=0.$. Solving for y we have: $y^3=\sqrt{x}$ and $y=x^{1/6}$.

This equation specifies a function. The domain is all nonnegative real numbers, i.e., $x\ge 0$.

62. $f(-3x)=(-3x)^2-4=9x^2-4$

64. $f(x-1)=(x-1)^2-4=x^2-2x+1-4=x^2-2x-3$

66. $f(x^3)=(x^3)^2-4=x^6-4$

68. $f(\sqrt[4]{x})=\left(x^{1/4}\right)^2-4=x^{1/2}-4=\sqrt{x}-4$

70. $f(-3)+f(h)=(-3)^2-4+h^2-4=5+h^2-4=h^2+1$

72. $f(-3+h)=(-3+h)^2-4=9-6h+h^2-4=5-6h+h^2$

74. $f(-3+h)-f(-3)=\left[(-3+h)^2-4\right]-\left[(-3)^2-4\right]=(9-6h+h^2-4)-(9-4)=-6h+h^2$

76. (A) $f(x+h)=-3(x+h)+9=-3x-3h+9$

(B) $f(x+h)-f(x)=(-3x-3h+9)-(-3x+9)=-3h$

(C) $\dfrac{f(x+h)-f(x)}{h}=\dfrac{-3h}{h}=-3$

78. (A) $f(x+h)=3(x+h)^2+5(x+h)-8$

$=3(x^2+2xh+h^2)+5x+5h-8$

$=3x^2+6xh+3h^2+5x+5h-8$

(B) $f(x+h)-f(x)=\left(3x^2+6xh+3h^2+5x+5h-8\right)-\left(3x^2+5x-8\right)$

$=6xh+3h^2+5h$

(C) $\dfrac{f(x+h)-f(x)}{h}=\dfrac{6xh+3h^2+5h}{h}=6x+3h+5$

80. (A) $f(x+h)=x^2+2xh+h^2+40x+40h$

(B) $f(x+h)-f(x)=2xh+h^2+40h$

(C) $\dfrac{f(x+h)-f(x)}{h}=2x+h+40$

82. Given $A = l\,w = 81$.

Thus, $w = \frac{81}{l}$. Now $P = 2l + 2w = 2l + 2\frac{81}{l} = 2l + \frac{162}{l}$.

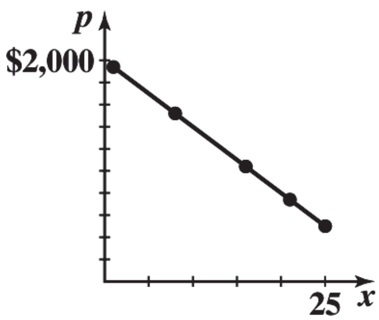
The domain is $l > 0$.

84. Given $P = 2\ell + 2w = 160$ or $\ell + w = 80$ and $\ell = 80 - w$.

Now $A = \ell w = (80 - w)w$ and $A = 80w - w^2$.

The domain is $0 \le w \le 80$. [Note: $w \le 80$ since $w > 80$ implies $\ell < 0$.]

86. (A)



(B) $p(11) = 1{,}340$ dollars per computer
$p(18) = 920$ dollars per computer

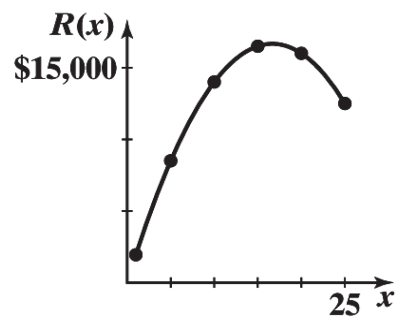
88. (A) $R(x) = xp(x)$
$= x(2{,}000 - 60x)$ thousands of dollars

Domain: $1 \le x \le 25$

(B) Table 11 Revenue

x(thousands)	$R(x)$(thousands)
1	$1,940
5	8,500
10	14,000
15	16,500
20	16,000
25	12,500

(C)



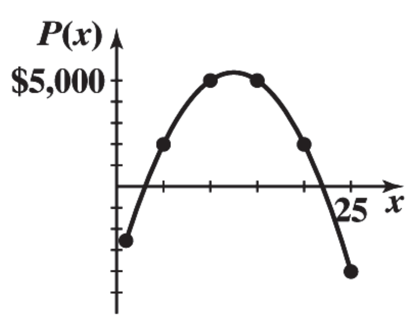
90. (A) $P(x) = R(x) - C(x)$
$= x(2{,}000 - 60x) - (4{,}000 + 500x)$ thousand dollars
$= 1{,}500x - 60x^2 - 4{,}000$

Domain: $1 \le x \le 25$

(B) Table 13 Profit

x (thousands)	$P(x)$ (thousands)
1	–$2,560
5	2,000
10	5,000
15	5,000
20	2,000
25	–4,000

(C)



92. (A) Given $5v - 2s = 1.4$. Solving for v, we have:
$v = 0.4s + 0.28$.
If $s = 0.51$, then $v = 0.4(0.51) + 0.28 = 0.484$ or 48.4%.

(B) Solving the equation for s, we have:
$s = 2.5v - 0.7$.
If $v = 0.51$, then $s = 2.5(0.51) - 0.7 = 0.575$ or 57.5%.

EXERCISE 2-2

2. $f(x) = 1 + \sqrt{x}$ Domain: $[0, \infty)$; range: $[1, \infty)$.

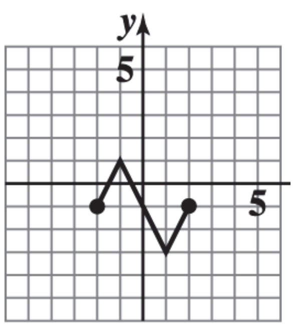
4. $f(x) = x^2 + 10$ Domain: all real numbers; range: $[10, \infty)$.

6. $f(x) = 5x + 3$ Domain: all real numbers; range: all real numbers.

8. $f(x) = 15 - 20|x|$ Domain: all real numbers; range: $(-\infty, 15]$.

10. $f(x) = -8 + \sqrt[3]{x}$ Domain: all real numbers; range: all real numbers.

12.



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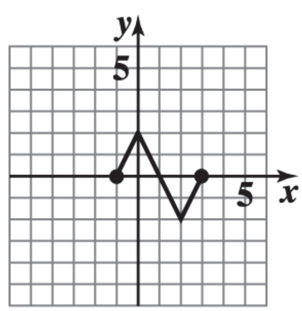

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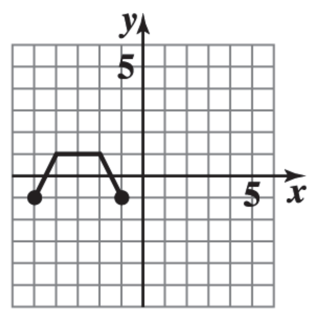

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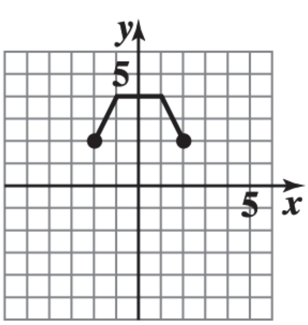

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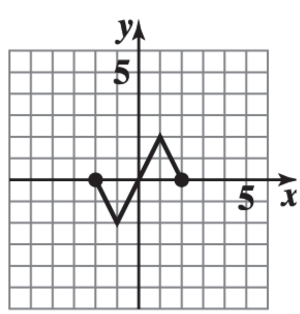

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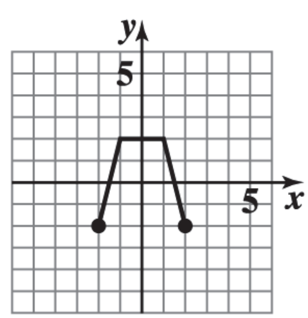

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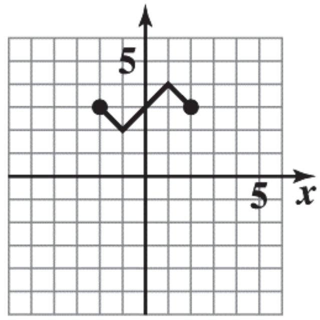

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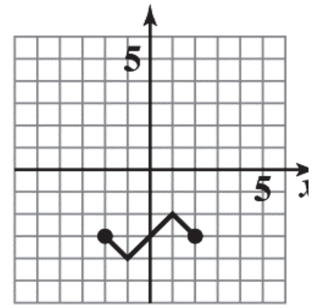
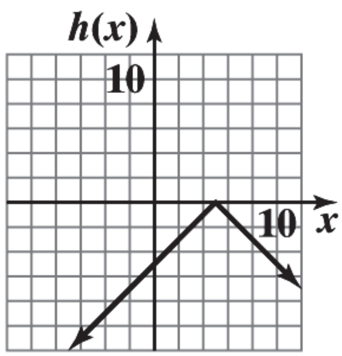
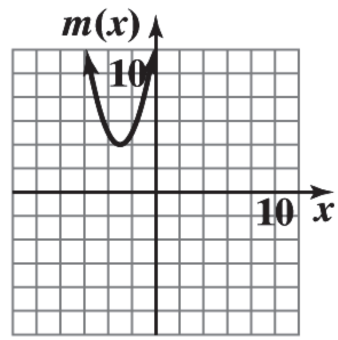


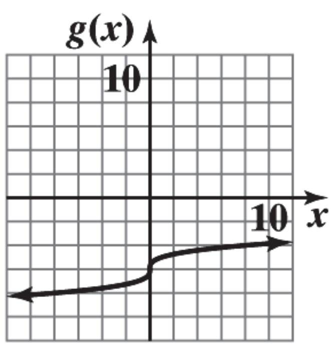
28. The graph of $h(x) = -|x - 5|$ is the graph of $y = |x|$ reflected in the x axis and shifted 5 units to the right.



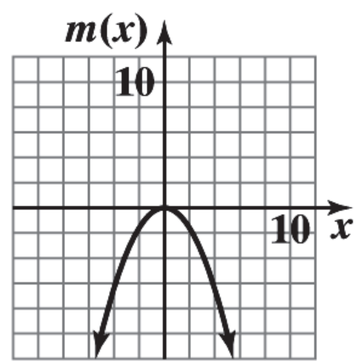
30. The graph of $m(x) = (x + 3)^2 + 4$ is the graph of $y = x^2$ shifted 3 units to the left and 4 units up.



32. The graph of $g(x) = -6 + \sqrt[3]{x}$ is the graph of $y = \sqrt[3]{x}$ shifted 6 units down.



34. The graph of $m(x) = -0.4x^2$ is the graph of $y = x^2$ reflected in the x axis and vertically contracted by a factor of 0.4.



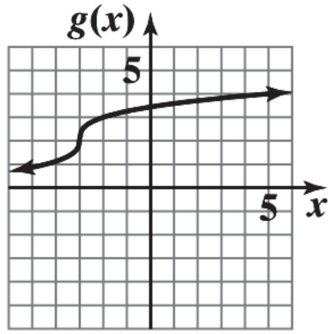
36. The graph of the basic function $y = |x|$ is shifted 3 units to the right and 2 units up. $y = |x - 3| + 2$

38. The graph of the basic function $y = |x|$ is reflected in the x axis, shifted 2 units to the left and 3 units up. Equation: $y = 3 - |x + 2|$

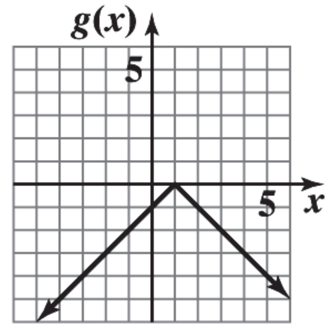
40. The graph of the basic function $\sqrt[3]{x}$ is reflected in the x axis and shifted up 2 units. Equation: $y = 2 - \sqrt[3]{x}$

42. The graph of the basic function $y = x^3$ is reflected in the x axis, shifted to the right 3 units and up 1 unit. Equation: $y = 1 - (x - 3)^3$

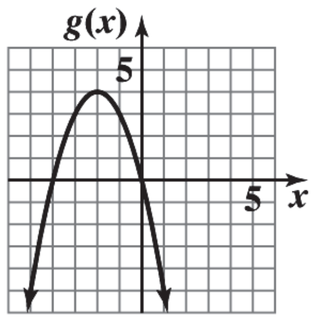
44. $g(x) = \sqrt[3]{x+3} + 2$



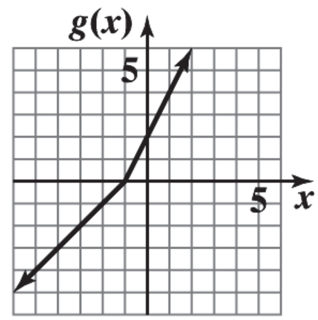
46. $g(x) = -|x - 1|$



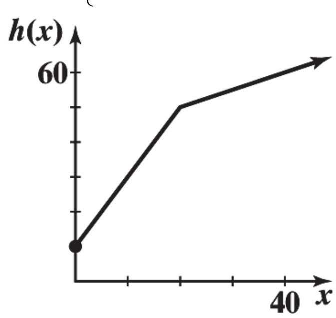
48. $g(x) = 4 - (x + 2)^2$



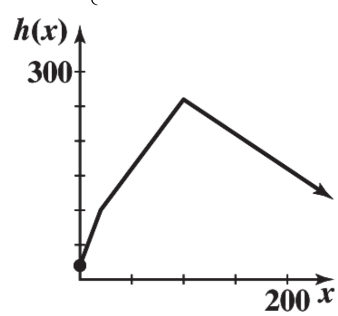
50. $g(x) = \begin{cases} x+1 & \text{if } x < -1 \\ 2+2x & \text{if } x \geq -1 \end{cases}$



52. $h(x) = \begin{cases} 10+2x & \text{if } \quad 0 \leq x \leq 20 \\ 40+0.5x & \text{if } \quad x > 20 \end{cases}$



54. $h(x) = \begin{cases} 4x+20 & \text{if } \quad 0 \leq x \leq 20 \\ 2x+60 & \text{if } \quad 20 < x \leq 100 \\ -x+360 & \text{if } \quad x > 100 \end{cases}$



56. The graph of the basic function $y = x$ is reflected in the x axis and vertically expanded by a factor of 2. Equation: $y = -2x$

58. The graph of the basic function $y = |x|$ is vertically expanded by a factor of 4. Equation: $y = 4|x|$

60. The graph of the basic function $y = x^3$ is vertically contracted by a factor of 0.25. Equation: $y = 0.25x^3$.

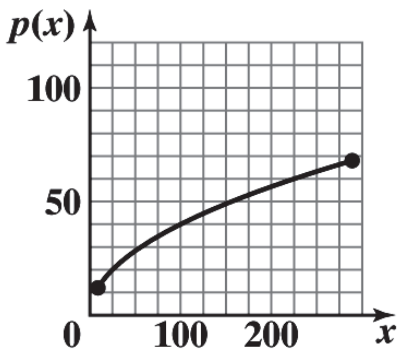
62. Vertical shift, reflection in y axis.
Reversing the order does not change the result. Consider a point (a, b) in the plane. A vertical shift of k units followed by a reflection in y axis moves (a, b) to $(a, b + k)$ and then to $(-a, b + k)$. In the reverse order, a reflection in y axis followed by a vertical shift of k units moves (a, b) to $(-a, b)$ and then to $(-a, b + k)$. The results are the same.

64. Vertical shift, vertical expansion.
Reversing the order can change the result. For example, let (a, b) be a point in the plane. A vertical shift of k units followed by a vertical expansion of h ($h > 1$) moves (a, b) to $(a, b + k)$ and then to $(a, bh + kh)$. In the reverse order, a vertical expansion of h followed by a vertical shift of k units moves (a, b) to (a, bh) and then to $(a, bh + k)$; $(a, bh + kh) \neq (a, bh + k)$.

66. Horizontal shift, vertical contraction.
Reversing the order does not change the result. Consider a point (a, b) in the plane. A horizontal shift of k units followed by a vertical contraction of h ($0 < h < 1$) moves (a, b) to $(a + k, b)$ and then to $(a + k, bh)$. In the reverse order, a vertical contraction of h followed by a horizontal shift of k units moves (a, b) to (a, bh) and then to $(a + k, bh)$. The results are the same.

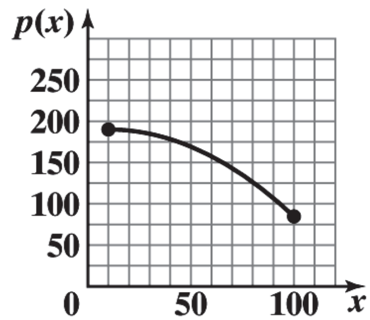
68. (A) The graph of the basic function $y = \sqrt{x}$ is vertically expanded by a factor of 4.

(B)



70. (A) The graph of the basic function $y = x^2$ is reflected in the x axis, vertically contracted by a factor of 0.013, and shifted 10 units to the right and 190 units up.

(B)

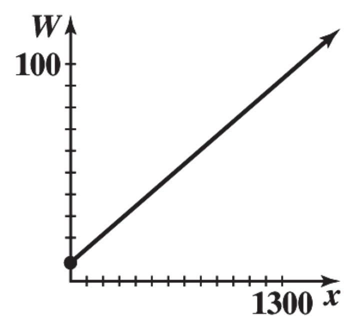


72. (A) Let x = number of kwh used in a winter month. For $0 \le x \le 700$, the charge is $8.5 + .065x$. At $x = 700$, the charge is \$54. For $x > 700$, the charge is $54 + .053(x - 700) = 16.9 + 0.053x$.

Thus,

$$W(x) = \begin{cases} 8.5 + .065x & \text{if } 0 \le x \le 700 \\ 16.9 + 0.053x & \text{if } x > 700 \end{cases}$$

(B)

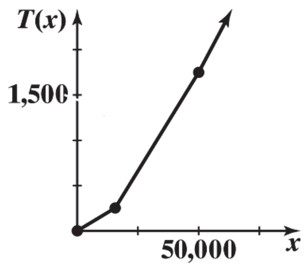


74. (A) Let x = taxable income.
If $0 \le x \le 12{,}500$, the tax due is \$$.02x$. At $x = 12{,}500$, the tax due is \$250. For $12{,}500 < x \le 50{,}000$, the tax due is $250 + .04(x - 12{,}500) = .04x - 250$. For $x > 50{,}000$, the tax due is $1{,}250 + .06(x - 50{,}000) = .06x - 1{,}250$.

Thus,

$$T(x) = \begin{cases} 0.02x & \text{if } 0 \le x \le 12{,}500 \\ 0.04x - 250 & \text{if } 12{,}500 < x \le 50{,}000 \\ 0.06x - 1{,}250 & \text{if } x > 50{,}000 \end{cases}$$

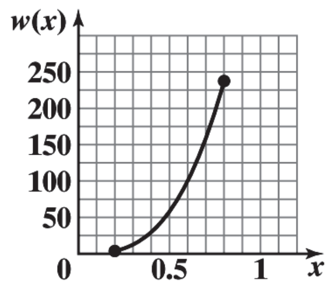
(B)



(C) $T(32{,}000) = \$1{,}030$
$T(64{,}000) = \$2{,}590$

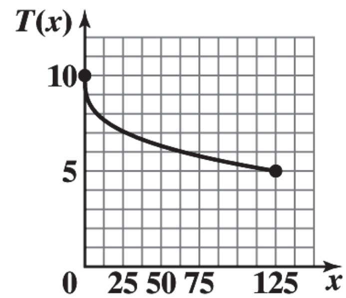
76. (A) The graph of the basic function $y = x^3$ is vertically expanded by a factor of 463.

(B)



78. (A) The graph of the basic function $y = \sqrt[3]{x}$ is reflected in the x axis and shifted up 10 units.

(B)



EXERCISE 2-3

2. $x^2 + 16x$ (standard form)
$x^2 + 16x + 64 - 64$ (completing the square)
$(x+8)^2 - 64$ (vertex form)

4. $x^2 - 12x - 8$ (standard form)
$(x^2 - 12x) - 8$
$(x^2 - 12x + 36) + 8 - 36$
(completing the square)
$(x-6)^2 - 44$ (vertex form)

6. $3x^2 + 18x + 21$ (standard form)

$3(x^2 + 6x) + 21$

$3(x^2 + 6x + 9 - 9) + 21$ (completing the square)

$3(x+3)^2 + 21 - 27$

$3(x+3)^2 - 6$ (vertex form)

8. $-5x^2 + 15x - 11$ (standard form)

$-5(x^2 - 3x) - 11$

$-5(x^2 - 3x + \frac{9}{4} - \frac{9}{4}) - 11$ (completing the square)

$-5(x - \frac{3}{2})^2 - 11 + \frac{45}{4}$

$-5(x - \frac{3}{2})^2 + \frac{1}{4}$ (vertex form)

10. The graph of $g(x)$ is the graph of $y = x^2$ shifted right 1 unit and down 6 units; $g(x) = (x-1)^2 - 6$.

12. The graph of $n(x)$ is the graph of $y = x^2$ reflected in the x axis, then shifted right 4 units and up 7 units;

$n(x) = -(x-4)^2 + 7$.

14. (A) g (B) m (C) n (D) f

16. (A) x intercepts: –5, –1; y intercept: –5 (B) Vertex: (–3, 4)
(C) Maximum: 4 (D) Range: $y \le 4$ or $(-\infty, 4]$

18. (A) x intercepts: 1, 5; y intercept: 5 (B) Vertex: (3, –4)
(C) Minimum: –4 (D) Range: $y \ge -4$ or $[-4, \infty)$

20. $g(x) = -(x+2)^2 + 3$

(A) x intercepts:
$$-(x+2)^2 + 3 = 0$$
$$(x+2)^2 = 3$$
$$x + 2 = \pm\sqrt{3}$$
$$x = -2 - \sqrt{3}, -2 + \sqrt{3}$$

y intercept: –1

(B) Vertex: (–2, 3) (C) Maximum: 3 (D) Range: $y \le 3$ or $(-\infty, 3]$

22. $n(x) = (x-4)^2 - 3$

(A) x intercepts:
$$(x-4)^2 - 3 = 0$$
$$(x-4)^2 = 3$$
$$x - 4 = \pm\sqrt{3}$$
$$x = 4 - \sqrt{3}, 4 + \sqrt{3}$$

y intercept: 13

(B) Vertex: (4, –3) (C) Minimum: –3 (D) Range: $y \ge -3$ or $[-3, \infty)$

24. $y = -(x-4)^2 + 2$

26. $y = [x - (-3)]^2 + 1$ or $y = (x+3)^2 + 1$

28. $g(x) = x^2 - 6x + 5 = x^2 - 6x + 9 - 4 = (x-3)^2 - 4$

(A) x intercepts:
$$(x-3)^2 - 4 = 0$$
$$(x-3)^2 = 4$$
$$x - 3 = \pm 2$$
$$x = 1, 5$$

y intercept: 5

(B) Vertex: (3, –4) (C) Minimum: –4 (D) Range: $y \ge -4$ or $[-4, \infty)$

30. $s(x) = -4x^2 - 8x - 3 = -4\left[x^2 + 2x + \frac{3}{4}\right] = -4\left[x^2 + 2x + 1 - \frac{1}{4}\right]$

$$= -4\left[(x+1)^2 - \frac{1}{4}\right] = -4(x+1)^2 + 1$$

(A) x intercepts:

$$-4(x+1)^2 + 1 = 0$$
$$4(x+1)^2 = 1$$
$$(x+1)^2 = \frac{1}{4}$$
$$x + 1 = \pm\frac{1}{2}$$
$$x = -\frac{3}{2}, -\frac{1}{2}$$

y intercept: –3

(B) Vertex: (–1, 1) (C) Maximum: 1 (D) Range: $y \le 1$ or $(-\infty, 1]$

32. $v(x) = 0.5x^2 + 4x + 10 = 0.5[x^2 + 8x + 20] = 0.5[x^2 + 8x + 16 + 4]$

$$= 0.5[(x+4)^2 + 4]$$
$$= 0.5(x+4)^2 + 2$$

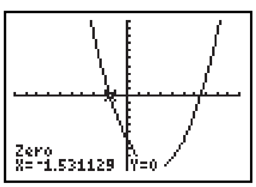
(A) x intercepts: none
y intercept: 10

(B) Vertex: (–4, 2) (C) Minimum: 2 (D) Range: $y \ge 2$ or $[2, \infty)$

34. $g(x) = -0.6x^2 + 3x + 4$

(A) $g(x) = -2$: $-0.6x^2 + 3x + 4 = -2$

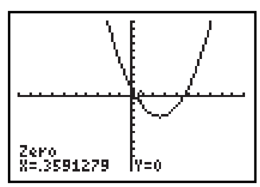
$$0.6x^2 - 3x - 6 = 0$$



$x = -1.53, 6.53$

(B) $g(x) = 5$: $-0.6x^2 + 3x + 4 = 5$

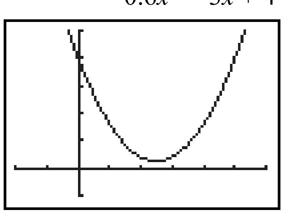
$$-0.6x^2 + 3x - 1 = 0$$
$$0.6x^2 - 3x + 1 = 0$$



$x = 0.36, 4.64$

(C) $g(x) = 8$: $-0.6x^2 + 3x + 4 = 8$

$$-0.6x^2 + 3x - 4 = 0$$
$$0.6x^2 - 3x + 4 = 0$$



No solution

36. Using a graphing utility with $y = 100x - 7x^2 - 10$ and the calculus option with maximum command, we obtain 347.1429 as the maximum value.

38. $m(x) = 0.20x^2 - 1.6x - 1 = 0.20(x^2 - 8x - 5)$
$= 0.20[(x-4)^2 - 21] = 0.20(x-4)^2 - 4.2$

(A) x intercepts: $0.20(x-4)^2 - 4.2 = 0$
$(x-4)^2 = 21$
$x - 4 = \pm\sqrt{21}$
$x = 4 - \sqrt{21} = -0.6,\ 4 + \sqrt{21} = 8.6;$

y intercept: –1

(B) Vertex: (4, –4.2) (C) Minimum: –4.2 (D) Range: $y \geq -4.2$ or $[-4.2, \infty)$

40. $n(x) = -0.15x^2 - 0.90x + 3.3 = -0.15(x^2 + 6x - 22) = -0.15[(x+3)^2 - 31] = -0.15(x+3)^2 + 4.65$

(A) x intercepts: $-0.15(x+3)^2 + 4.65 = 0$
$(x+3)^2 = 31$
$x + 3 = \pm\sqrt{31}$
$x = -3 - \sqrt{31} = -8.6, -3 + \sqrt{31} = 2.6;$

y intercept: 3.30

(B) Vertex: (–3, 4.65) (C) Maximum: 4.65 (D) Range: $x \leq 4.65$ or $(-\infty, 4.65]$

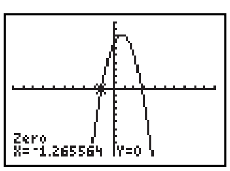
42. $(x+6)(x-3) < 0$

Therefore, either $(x+6) < 0$ and $(x-3) > 0$ or $(x+6) > 0$ and $(x-3) < 0$. The first case is impossible. The second case implies $-6 < x < 3$. Solution set: $(-6, 3)$.

44. $x^2 + 7x + 12 = (x+3)(x+4) \geq 0$

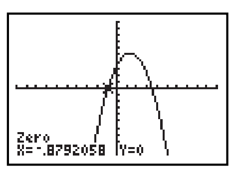
Therefore, either $(x+3) \geq 0$ and $(x+4) \geq 0$ or $(x+3) \leq 0$ and $(x+4) \leq 0$. The first case implies $x \geq -3$ and the second case implies $x \leq -4$. Solution set: $(-\infty, -4] \cup [-3, \infty)$.

46.



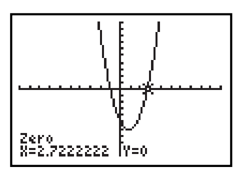
$x = -1.27, 2.77$

48.



$-0.88 \leq x \leq 3.52$

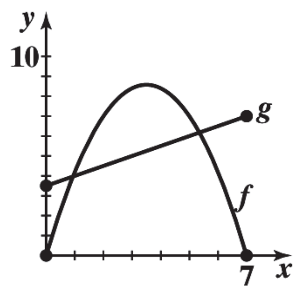
50.



$x < -1$ or $x > 2.72$

52. f is a quadratic function and max $f(x) = f(-3) = -5$
Axis: $x = -3$
Vertex: (–3, –5)
Range: $y \leq -5$ or $(-\infty, -5]$
x intercepts: None

54. (A)



(B) $f(x) = g(x)$: $-0.7x(x - 7) = 0.5x + 3.5$

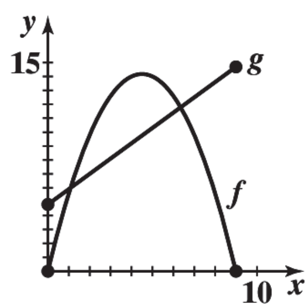
$$-0.7x^2 + 4.4x - 3.5 = 0$$

$$x = \frac{-4.4 \pm \sqrt{(4.4)^2 - 4(0.7)(3.5)}}{-1.4} = 0.93, 5.35$$

(C) $f(x) > g(x)$ for $0.93 < x < 5.35$

(D) $f(x) < g(x)$ for $0 \le x < 0.93$ or $5.35 < x \le 7$

56. (A)



(B) $f(x) = g(x)$: $-0.7x^2 + 6.3x = 1.1x + 4.8$

$$-0.7x^2 + 5.2x - 4.8 = 0$$

$$0.7x^2 - 5.2x + 4.8 = 0$$

$$x = \frac{-(-5.2) \pm \sqrt{(-5.2)^2 - 4(0.7)(4.8)}}{1.4} = 1.08, 6.35$$

(C) $f(x) > g(x)$ for $1.08 < x < 6.35$

(D) $f(x) < g(x)$ for $0 \le x < 1.08$ or $6.35 < x \le 9$

58. The graph of a quadratic with no real zeros will not intersect the x–axis.

60. Such an equation will have $b^2 - 4ac = 0$.

62. Such an equation will have $\frac{k}{a} < 0$.

64. $ax^2 + bx + c = a(x-h)^2 + k$

$= a(x^2 - 2hx + h^2) + k$

$= ax^2 - 2ahx + ah^2 + k$

Equating constant terms gives $k = c - ah^2$. Since h is the vertex, we have $h = -\dfrac{b}{2a}$. Substituting then gives

$$k = c - ah^2 = c - a\left(\frac{b^2}{4a^2}\right) = c - \frac{b^2}{4a}$$

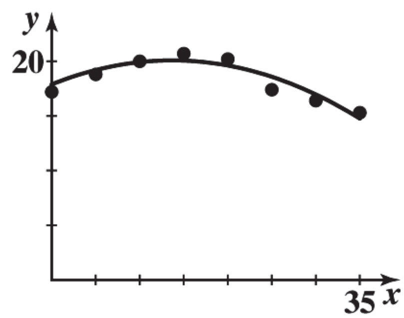
$$= \frac{4ac - b^2}{4a}$$

66. $f(x) = -0.0117x^2 + 0.32x + 17.9$

(A)

x	Mkt Share	$f(x)$
5	18.8	19.2
10	20.0	19.9
15	20.7	20.1
20	20.2	19.6
25	17.4	18.6
30	16.4	17
35	15.3	14.8

(B)

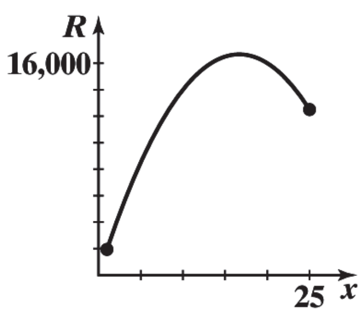


(C) For 2025, $x = 45$ and $f(45) = -0.0117(45)^2 + 0.32(45) + 17.9 = 8.6\%$
For 2028, $x = 48$ and $f(48) = -0.0117(48)^2 + 0.32(48) + 17.9 = 6.3\%$

(D) Market share rose from 18.8% in 1985 to a maximum of 20.7% in 1995 and then fell to 15.3% in 2010.

68. Verify

70. (A)



(B) $R(x) = 2{,}000x - 60x^2$

$= -60\left(x^2 - \frac{100}{3}x\right)$

$= -60\left[x^2 - \frac{100}{3}x + \frac{2500}{9} - \frac{2500}{9}\right]$

$= -60\left[\left(x - \frac{50}{3}\right)^2 - \frac{2500}{9}\right]$

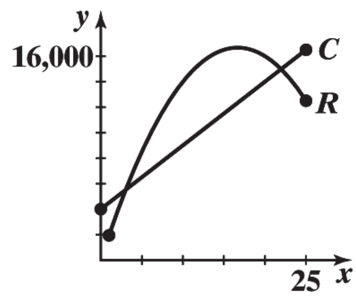
$= -60\left(x - \frac{50}{3}\right)^2 + \frac{50{,}000}{3}$

16.667 thousand computers (16,667 computers); 16,666.667 thousand dollars ($16,666,667)

(C) $2000 - 60(50/3) =$ $1,000

72. (A)

$$p\left(\frac{50}{3}\right) = 2{,}000 - 60\left(\frac{50}{3}\right) = \$1{,}000$$



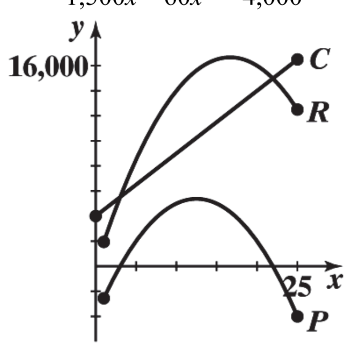
(B) $R(x) = C(x)$

$$x(2{,}000 - 60x) = 4{,}000 + 500x$$
$$2{,}000x - 60x^2 = 4{,}000 + 500x$$
$$60x^2 - 1{,}500x + 4{,}000 = 0$$
$$6x^2 - 150x + 400 = 0$$
$$x = 3.035,\ \ 21.965$$

Break-even at 3.035 thousand (3,035) and 21.965 thousand (21,965)

(C) Loss: $1 \le x < 3.035$ or $21.965 < x \le 25$;
Profit: $3.035 < x < 21.965$

74. (A) $P(x) = R(x) - C(x)$
$= 1{,}500x - 60x^2 - 4{,}000$

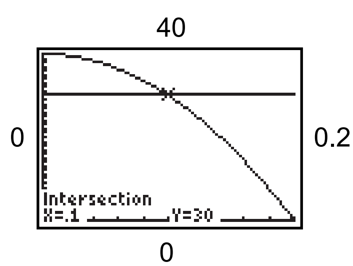


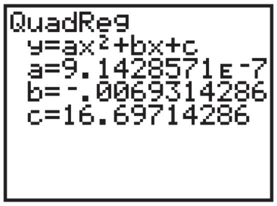
(B) and (C) Intercepts and break-even points: 3,035 computers and 21,965 computers

(D) Maximum profit is $5,375,000 when 12,500 computers are produced. This is much smaller than the maximum revenue of $16,666,667.

76. Solve: $f(x) = 1{,}000(0.04 - x^2) = 30$

$$40 - 1000x^2 = 30$$
$$1000x^2 = 10$$
$$x^2 = 0.01$$
$$x = 0.10 \text{ cm}$$



78. 

For $x = 2{,}300$, the estimated fuel consumption is
$y = a(2{,}300)^2 + b(2{,}300) + c = 5.6$ mpg.

EXERCISE 2-4

2. $f(x) = x^2 - 5x + 6$

(A) Degree: 2

(B) $x^2 - 5x + 6 = 0$
$(x-2)(x-3) = 30$
$x = 2, 3$

x-intercepts: $x = 2, 3$

(C) $f(0) = 0^2 - 5(0) + 6 = 6$
y-intercept: 6

4. $f(x) = 30 - 3x$

(A) Degree: 1

(B) $30 - 3x = 0$
$3x = 30$
$x = 10$

x-intercept: 10

(C) $f(0) = 30 - 3(0) = 30$
y-intercept: 30

6. $f(x) = 5x^6 + x^4 + x^8 + 10$

(A) Degree: 8

(B) $f(x) \geq 10$ for all x.
No x-intercepts.

(C) $f(0) = 5(0)^6 + (0)^4 + (0)^8 + 10 = 10$
y-intercept: 10

8. $f(x) = (x-5)^2(x+7)^2$

(A) Degree: 4

(B) $(x-5)^2(x+7)^2 = 0$
$x = 5, -7$
x-intercepts: $x = 5, -7$

(C) $f(0) = (0-5)^2(0+7)^2 = 1{,}225$
y-intercept: 1,225

10. $f(x) = (2x-5)^2(x^2-9)^4$

(A) Degree: 10

(B) $(2x-5)^2\left(x^2-9\right)^4 = 0$

$x = \dfrac{5}{2}, -3, 3 \quad x = -3, \tfrac{1}{2}$

x-intercepts: $-3, 5/2, 3$

(C) $f(0) = [2(0)-5]^2[(0)^2-9]^4 = 5^2 9^4 = 164{,}025$
y-intercept: 164,025

12. (A) Minimum degree: 2

(B) Negative – it must have even degree, and positive values in the domain are mapped to negative values in the range.

14. (A) Minimum degree: 3

(B) Negative – it must have odd degree, and positive values in the domain are mapped to negative values in the range.

16. (A) Minimum degree: 4

(B) Positive – it must have even degree, and positive values in the domain are mapped to positive values in the range.

18. (A) Minimum degree: 5

(B) Positive – it must have odd degree, and large positive values in the domain are mapped to positive values in the range.

20. A polynomial of degree 7 can have at most 7 x intercepts.

22. A polynomial of degree 6 may have no x intercepts. For example, the polynomial $f(x) = x^6 + 1$ has no x intercepts.

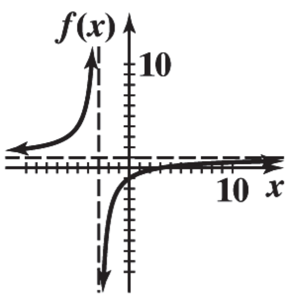
24. (A) Intercepts:

x-intercept(s):	y-intercept:
$x - 3 = 0$ $x = 3$ (3, 0)	$f(0) = \frac{0-3}{0+3} = -1$ (0, –1)

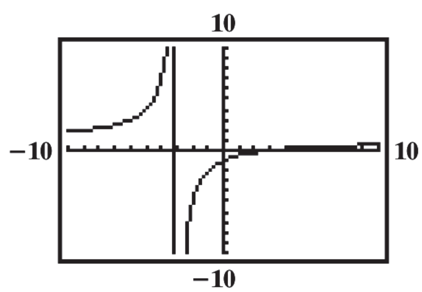
(B) Domain: all real numbers except $x = -3$

(C) Vertical asymptote at $x = -3$ by case 1 of the vertical asymptote procedure on page 57.
Horizontal asymptote at $y = 1$ by case 2 of the horizontal asymptote procedure on page 57.

(D)



(E)



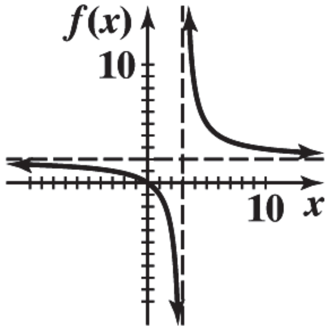
26. (A) Intercepts:

x-intercept(s):	y-intercept:
$2x = 0$ $x = 0$ (0, 0)	$f(0) = \frac{2(0)}{0-3} = 0$ (0, 0)

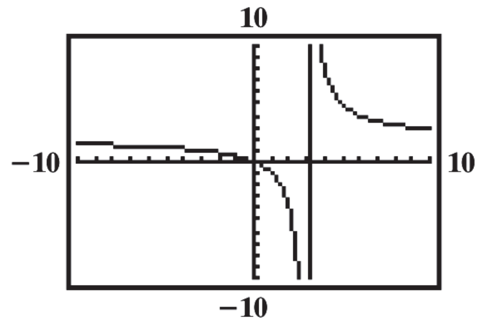
(B) Domain: all real numbers except $x = 3$.

(C) Vertical asymptote at $x = 3$ by case 1 of the vertical asymptote procedure on page 57.
Horizontal asymptote at $y = 2$ by case 2 of the horizontal asymptote procedure on page 57.

(D)



(E)



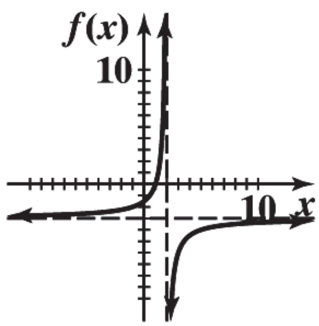
28. (A) Intercepts:

x-intercept:	y-intercept:
$3 - 3x = 0$ $x = 1$ (1, 0)	$f(0) = \dfrac{3 - 3(0)}{0 - 2} = -\dfrac{3}{2}$ $\left(0, -\dfrac{3}{2}\right)$

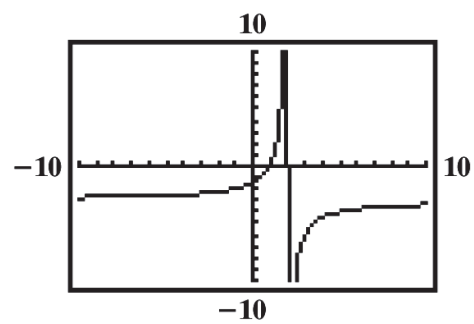
(B) Domain: all real numbers except $x = 2$

(C) Vertical asymptote at $x = 2$ by case 1 of the vertical asymptote procedure on page 57.
Horizontal asymptote at $y = -3$ by case 2 of the horizontal asymptote procedure on page 57.

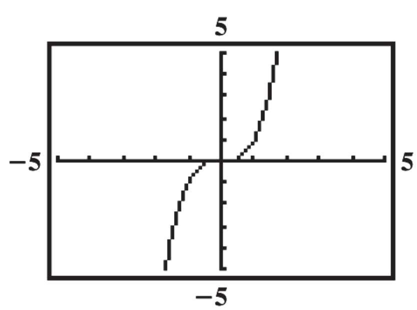
(D)

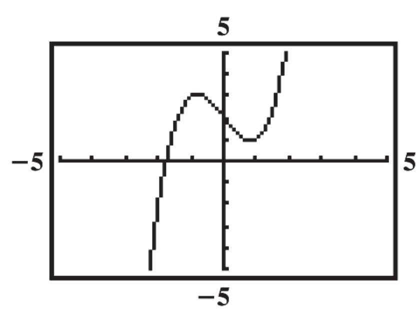


(E)

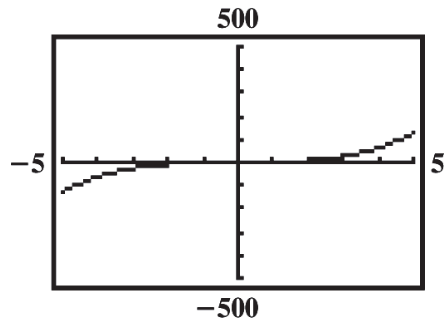


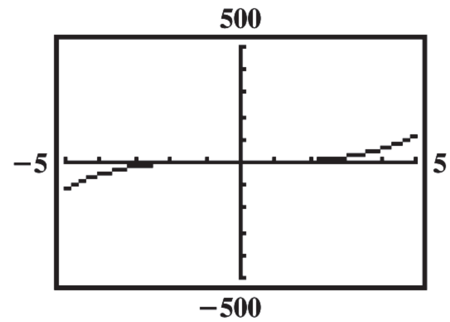
30. (A)



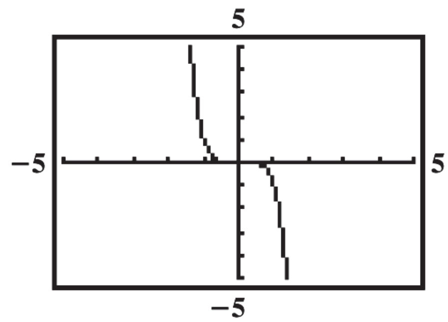


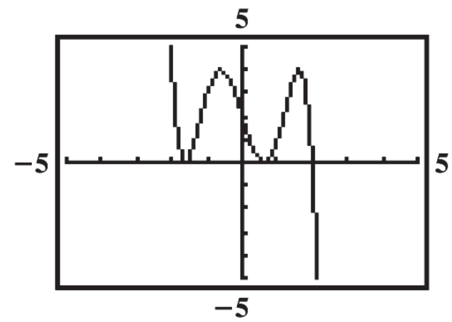
(B)



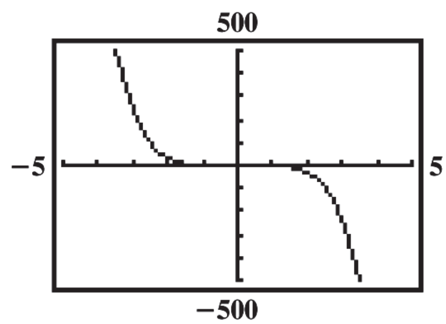


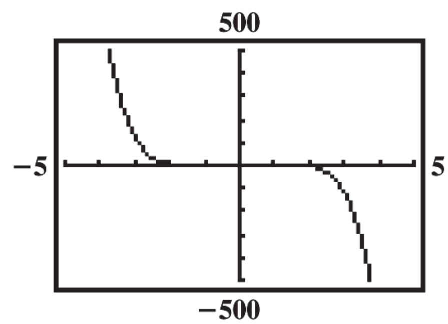
32. (A)





(B)





34. $y = \frac{6}{4}$, by case 2 for horizontal asymptotes on page 57.

36. $y = -\frac{1}{2}$, by case 2 for horizontal asymptotes on page 57.

38. $y = 0$, by case 1 for horizontal asymptotes on page 57.

40. No horizontal asymptote, by case 3 for horizontal asymptotes on page 57.

42. Here we have denominator $(x^2 - 4)(x^2 - 16) = (x-2)(x+2)(x-4)(x+4)$. Since none of these linear terms are factors of the numerator, the function has vertical asymptotes at $x = 2$, $x = -2$, $x = 4$, and $x = -4$.

44. Here we have denominator $x^2 + 7x - 8 = (x-1)(x+8)$. Also, we have numerator $x^2 - 8x + 7 = (x-1)(x-7)$. By case 2 of the vertical asymptote procedure on page 57, we conclude that the function has a vertical asymptote at $x = -8$.

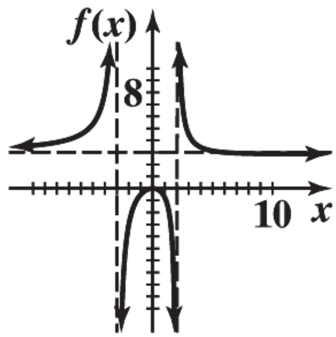
46. Here we have denominator $x^3 - 3x^2 + 2x = x(x^2 - 3x + 2) = x(x-2)(x-1)$. We also have numerator $x^2 + x - 2 = (x+2)(x-1)$. By case 2 of the vertical asymptote procedure on page 57, we conclude that the function has vertical asymptotes at $x = 0$ and $x = 2$.

48. (A) Intercepts:

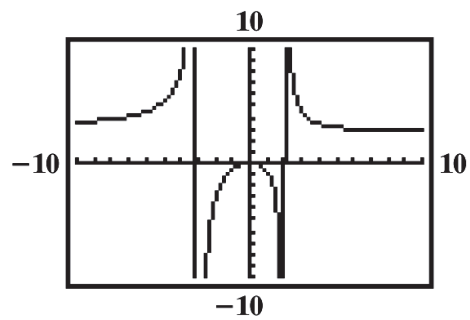
x-intercept(s):	y-intercept:
$3x^2 = 0$ $x = 0$ (0, 0)	$f(0) = 0$ (0, 0)

(B) Vertical asymptote when $x^2 + x - 6 = (x-2)(x+3) = 0$; so, vertical asymptotes at $x = 2$, $x = -3$.
Horizontal asymptote $y = 3$.

(C)



(D)

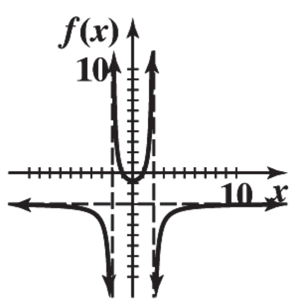


50. (A) Intercepts:

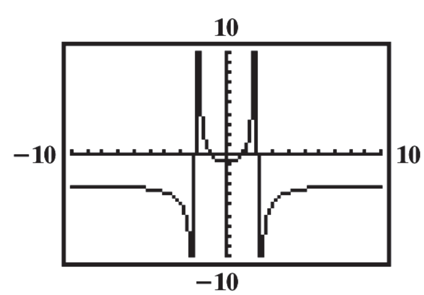
x-intercept(s):	y-intercept:
$3 - 3x^2 = 0$ $3x^2 = 3$ $x = \pm 1$ (1,0), (–1,0)	$f(0) = -\frac{3}{4}$ $\left(0, \frac{-3}{4}\right)$

(B) Vertical asymptotes when $x^2 - 4 = 0$; i.e. at $x = 2$ and $x = -2$.
Horizontal asymptote at $y = -3$

(C)



(D)

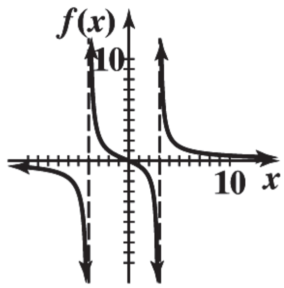


52. (A) Intercepts:

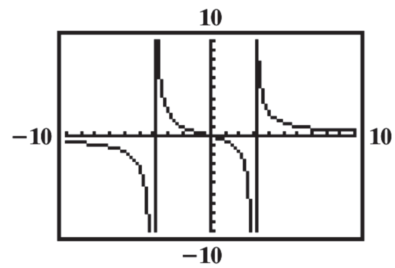
x-intercept(s):	y-intercept:
$5x - 10 = 0$ $x = 2$ (2,0)	$f(0) = \frac{-10}{-12} = \frac{5}{6}$ (0,5/6)

(B) Vertical asymptote when $x^2+x-12=(x+4)(x-3)=0$; i.e. when $x = -4$ and when $x = 3$. Horizontal asymptote at $y = 0$.

(C)



(D)



54. $f(x)=-(x+2)(x-1)=-x^2-x+2$

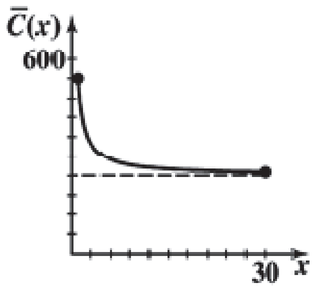
56. $f(x)=x(x+1)(x-1)=x\left(x^2-1\right)=x^3-x$

58. (A) We want $C(x)=mx+b$. Fix costs are $b=\$300$ per day. Given $C(20)=5{,}100$ we have

$$m(20)+300=5{,}100$$
$$20m=4800$$
$$m=240$$
$$C(x)=240x+300$$

(B) $\bar{C}(x)=\dfrac{C(x)}{x}=\dfrac{240x+300}{x}=240+\dfrac{300}{x}$

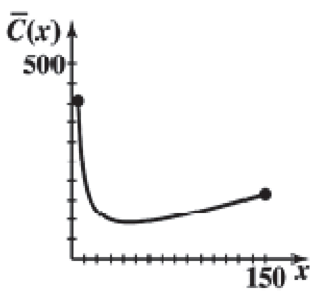
(C)



(D) Average cost tends towards \$240 as production increases.

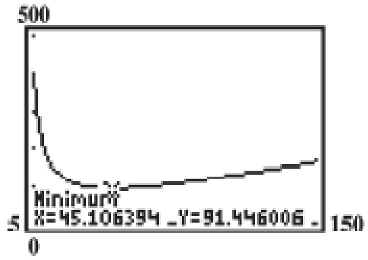
60. (A) $\bar{C}(x)=\dfrac{x^2+2x+2{,}000}{x}$

(B)



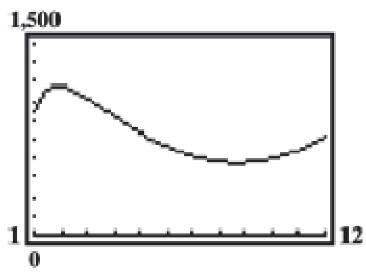
(C) A daily production level of $x = 45$ units per day, results in the lowest average cost of $\bar{C}(45)=\$91.44$ per unit

(D)



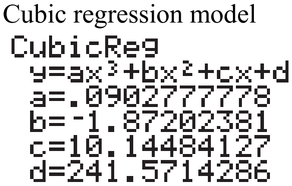
62. (A) $\bar{C}(x) = \dfrac{20x^3 - 360x^2 + 2{,}300x - 1{,}000}{x}$

(B)



(C) A minimum average cost of $566.84 is achieved at a production level of $x = 8.67$ thousand cases per month.

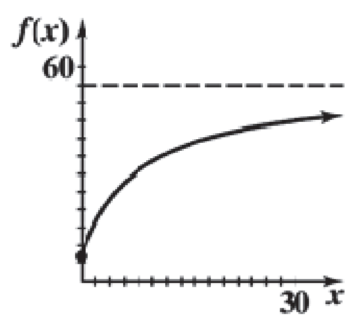
64. (A) Cubic regression model



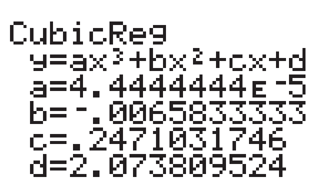
(B) $y(21) = 583$ eggs

66. (A) The horizontal asymptote is $y = 55$.

(B)



68. (A) Cubic regression model



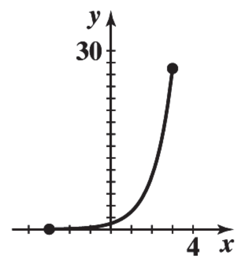
(B) This model gives an estimate of 2.5 divorces per 1,000 marriages.

EXERCISE 2-5

2. A. graph g B. graph f C. graph h D. graph k

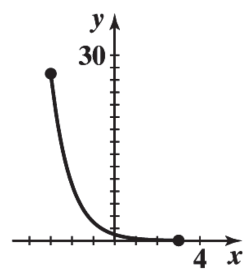
4. $y = 3^x; [-3, 3]$

x	y
–3	$\frac{1}{27}$
–1	$\frac{1}{3}$
0	1
1	3
3	27



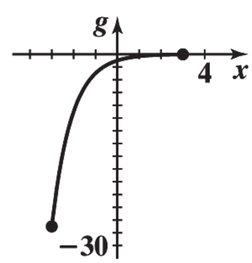
6. $y = 3^{-x}; [-3, 3]$

x	y
–3	27
–1	3
0	1
1	$\frac{1}{3}$
3	$\frac{1}{27}$



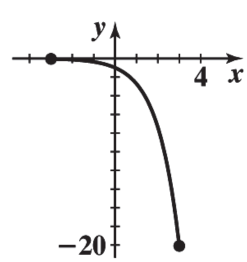
8. $g(x) = -3^{-x}; [-3, 3]$

x	$g(x)$
–3	–27
–1	–3
0	–1
1	$-\frac{1}{3}$
3	$-\frac{1}{27}$



10. $y = -e^x; [-3, 3]$

x	y
–3	≈ -0.05
–1	≈ -0.37
0	–1
1	≈ -2.72
3	≈ -20.09



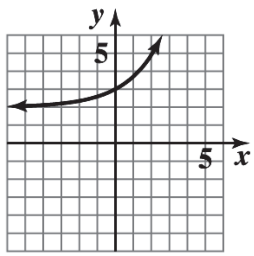
12. The graph of g is the graph of f shifted 2 units to the right.

14. The graph of g is the graph of f reflected in the x axis.

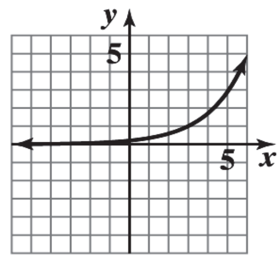
16. The graph of g is the graph of f shifted 2 units down.

18. The graph of g is the graph of f vertically contracted by a factor of 0.5 and shifted 1 unit to the right.

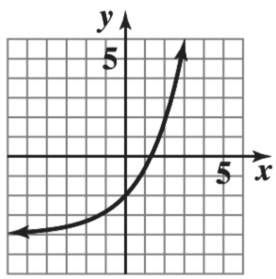
20. (A) $y = f(x)+2$



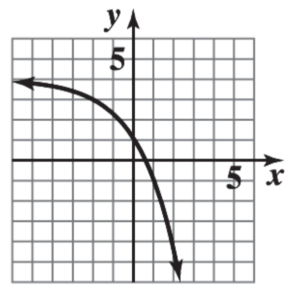
(B) $y = f(x-3)$



(C) $y = 2f(x)-4$

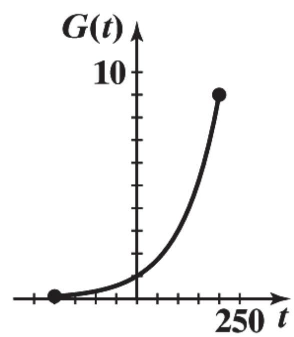


(D) $y = 4 - f(x+2)$



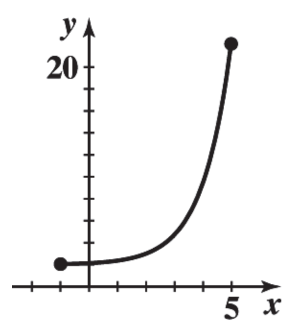
22. $G(t) = 3^{t/100}$; [−200, 200]

x	$G(t)$
−200	$\frac{1}{9}$
−100	$\frac{1}{3}$
0	1
100	3
200	9



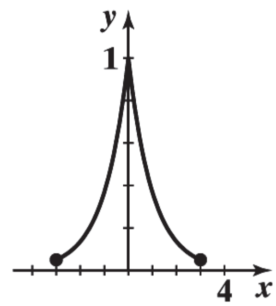
24. $y = 2 + e^{x-2}$; [−1, 5]

x	y
−1	≈ 2.05
0	≈ 2.14
1	≈ 2.37
3	≈ 4.72
5	≈ 22.09



26. $y = e^{-|x|}$; $[-3, 3]$

x	y
–3	≈ 0.05
–1	≈ 0.37
0	1
1	≈ 0.37
3	≈ 0.05



28. $a = 2,\ b = -2$ for example. The exponential function property: For $x \neq 0$, $a^x = b^x$ if and only if $a = b$ assumes $a > 0$ and $b > 0$.

30.
$$3^{x+4} = 3^{2x-5}$$
$$x + 4 = 2x - 5$$
$$-x = -9$$
$$x = 9$$

32.
$$5^{x^2-x} = 5^{42}$$
$$x^2 - x = 42$$
$$x^2 - x - 42 = 0$$
$$(x-7)(x+6) = 0$$
$$x = -6, 7$$

34.
$$(3x+4)^4 = (52)^4$$
$$3x + 4 = 52$$
$$3x = 48$$
$$x = 16$$

36.
$$(2x+1)^2 = (3x-1)^2$$
$$4x^2 + 4x + 1 = 9x^2 - 6x + 1$$
$$5x^2 - 10x = 0$$
$$x(x-2) = 0$$
$$x = 0, 2$$

38.
$$(4x+1)^4 = (5x-10)^4$$
$$(4x+1)^2 = (5x-10)^2$$
$$4x + 1 = \pm 5(x-2)$$
$$4x + 1 = 5(x-2),\ x = 11$$
$$4x + 1 = -5(x-2),\ x = 1$$

40.
$$10xe^x - 5e^x = 0$$
$$e^x(10x - 5) = 0$$
$$10x - 5 = 0 \quad (\text{since } e^x \neq 0)$$
$$x = \frac{1}{2}$$

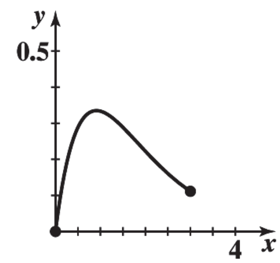
42.
$$x^2e^{-x} - 9e^{-x} = 0$$
$$e^{-x}(x^2 - 9) = 0$$
$$(x^2 - 9) = 0 \quad (\text{since } e^{-x} \neq 0)$$
$$x = -3, 3$$

44. $e^{4x} + e > 0$ for all x;
$e^{4x} + e = 0$ has no solutions.

46.
$$e^{3x-1} - e = 0$$
$$e^{3x-1} = e^1$$
$$3x - 1 = 1$$
$$x = 2/3$$

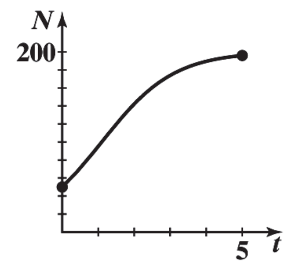
48. $m(x) = x(3^{-x}); [0, 3]$

x	$m(x)$
0	0
1	$\frac{1}{3}$
2	$\frac{2}{9}$
3	$\frac{1}{9}$



50. $N = \frac{200}{1+3e^{-t}};\ [0, 5]$

x	N
0	50
1	≈ 95.07
2	≈ 142.25
3	≈ 174.01
4	≈ 184.58
5	≈ 196.04



52. $A = Pe^{rt}$

$A = (24{,}000)e^{(0.0435)(7)}$

$A = (24{,}000)e^{0.3045}$

$A = (24{,}000)(1.35594686)$

$A = \$32{,}542.72$

54. (A) $A = P(1+\frac{r}{m})^{mt}$

$A = 4000(1+\frac{0.06}{52})^{(52)(0.5)}$

$A = 4000(1.0011538462)^{26}$

$A = 4000(1.030436713)$

$A = \$4121.75$

(B) $A = P(1+\frac{r}{m})^{mt}$

$A = 4000(1+\frac{0.06}{52})^{(52)(10)}$

$A = 4000(1.0011538462)^{520}$

$A = 4000(1.821488661)$

$A = \$7285.95$

56. $A = P(1+\frac{r}{m})^{mt}$

$40{,}000 = P(1+\frac{0.055}{365})^{(365)(17)}$

$40{,}000 = P(1.0001506849)^{6205}$

$40{,}000 = P(2.547034043)$

$P = \$15{,}705$

58. (A) $A = P(1+\frac{r}{m})^{mt}$

$A = 10{,}000(1+\frac{0.0135}{4})^{(4)(5)}$

$A = 10{,}000(1.003375)^{20}$

$A = 10{,}000(1.069709)$

$A = \$10{,}697.09$

(B) $A = P(1+\frac{r}{m})^{mt}$

$A = 10{,}000(1+\frac{0.0130}{12})^{(12)(5)}$

$A = 10{,}000(1.00108333)^{60}$

$A = 10{,}000(1.067121479)$

$A = \$10{,}671.21$

(C) $A = P(1+\frac{r}{m})^{mt}$

$A = 10,000(1+\frac{0.0125}{365})^{(365)(5)}$

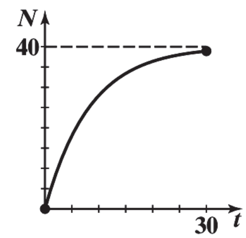
$A = 10,000(1.000034245)^{1825}$

$A = 10,000(1.06449332)$

$A = \$10,644.93$

60. $N = 40(1-e^{-0.12t})$; $[0, 30]$

x	N
0	0
10	≈ 27.95
20	≈ 36.37
30	≈ 38.91



The maximum number of boards an average employee can be expected to produce in 1 day is 40.

62. The exponential regression model

```
ExpReg
 y=a*b^x
 a=3.996184237
 b=1.523286295
```

(B) $y(10) = 268.8$ exabytes per month

64. (A) $I(50) = I_o e^{-0.00942(50)} \approx 62\ \%$

(B) $I(100) = I_o e^{-0.00942(100)} \approx 39\ \%$

66. (A) $P = 204e^{0.0077t}$.

(B) Population in 2030:
$P(15) = 204e^{0.0077(15)} \approx 229$ million.

68. (A) $P = 7.4e^{0.0113t}$

(B) Population in 2025: $P(10) = 7.4e^{0.0113(10)} \approx 8.29$ billion

Population in 2033: $P(18) = 7.4e^{0.0113(18)} \approx 9.07$ billion

EXERCISE 2-6

2. $\log_2 32 = 5 \Rightarrow 32 = 2^5$

4. $\log_e 1 = 0 \Rightarrow e^0 = 1$

6. $\log_9 27 = \frac{3}{2} \Rightarrow 27 = 9^{3/2}$

8. $36 = 6^2 \Rightarrow \log_6 36 = 2$

10. $9 = 27^{2/3} \Rightarrow \log_{27} 9 = \frac{2}{3}$

12. $M = b^x \Rightarrow \log_b M = x$

14. $\log_{10} \frac{1}{1000} = \log_{10} 10^{-3} = -3$

16. $\log_{10} 10,000 = \log_{10} 10^4 = 4$

18. $\log_2 \frac{1}{64} = \log_2 2^{-6} = -6$

20. $\ln(-1)$ is not defined.

22. $\ln(e^{-1}) = -1$

24. $\log_b FG = \log_b F + \log_b G$

26. $\log_b w^{15} = 15\log_b w$

28. $\dfrac{\log_3 P}{\log_3 R} = \log_R P$

30. $\log_{10} x = 1$

$x = 10^1 = 10$

32. $\log_b \dfrac{1}{25} = 2$

$b^2 = \dfrac{1}{25}$

$b = \dfrac{1}{5}$

34. $\log_{49} 7 = y$

$49^y = 7$

$y = 1/2$

36. $\log_b 10{,}000 = 2$

$b^2 = 10{,}000$

$b = 100$

38. $\log_8 x = \dfrac{5}{3}$

$x = 8^{5/3} = \left(8^{1/3}\right)^5 = 2^5 = 32$

40. False; an example of a polynomial function of odd degree that is not one–to–one is $f(x) = x^3 - x$. $f(-1) = f(0) = f(1) = 0$.

42. False; the graph of every function (not necessarily one–to–one) intersects each vertical line at most once.

For example, $f(x) = \dfrac{1}{x-1}$ *is a one-to-one function which does not intersect the vertical line* $x = 1$.

44. False; $x = -1$ is in the domain of *f*, but cannot be in the range of *g*.

46. True; since *g* is the inverse of *f*, then (*a*, *b*) is on the graph of *f* if and only if (*b*, *a*) is on the graph of *g*. Therefore, *f* is also the inverse of *g*.

48. $\log_b x = \dfrac{2}{3}\log_b 27 + 2\log_b 2 - \log_b 3$

$\log_b x = \log_b 27^{2/3} + \log_b 2^2 - \log_b 3$

$\log_b x = \log_b 9 + \log_b 4 - \log_b 3$

$\log_b x = \log_b \dfrac{(9)(4)}{3}$

$\log_b x = \log_b 12$

$x = 12$

50. $\log_b x = 3\log_b 2 + \dfrac{1}{2}\log_b 25 - \log_b 20$

$\log_b x = \log_b 2^3 + \log_b 25^{1/2} - \log_b 20$

$\log_b x = \log_b 8 + \log_b 5 - \log_b 20$

$\log_b x = \log_b \dfrac{(8)(5)}{20}$

$\log_b x = \log_b 2$

$x = 2$

52. $\log_b(x+2)+\log_b x=\log_b 24$

$$\log_b(x+2)x=\log_b 24$$
$$\log_b(x^2+2x)=\log_b 24$$
$$x^2+2x=24$$
$$x^2+2x-24=0$$
$$(x+6)(x-4)=0$$
$$x=-6,4$$

Since the domain of $\log_b$ is $(0,\infty)$, omit the negative solution. Therefore, the solution is $x=4$.

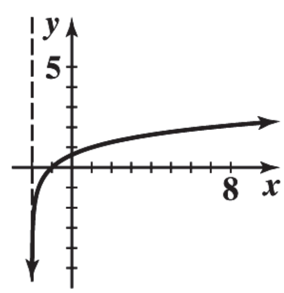
54. $\log_{10}(x+6)-\log_{10}(x-3)=1$

$$\log_{10}\frac{x+6}{x-3}=1$$
$$10^1=\frac{x+6}{x-3}$$
$$10(x-3)=x+6$$
$$10x-30=x+6$$
$$x=4$$

56. $y=\log_3(x+2)$

$$3^y=x+2$$
$$3^y-2=x$$

x	y
$-\frac{53}{27}$	-3
$-\frac{17}{9}$	-2
$-\frac{5}{3}$	-1
-1	0
1	1
7	2
25	3



58. The graph of $y=\log_3(x+2)$ is the graph of $y=\log_3 x$ shifted to the left 2 units.

60. The domain of logarithmic function is defined for positive values only. Therefore, the domain of the function is $x-1>0$ or $x>1$. The range of a logarithmic function is all real numbers. In interval notation the domain is $(1,\infty)$ and the range is $(-\infty,\infty)$.

62. (A) $\log 72.604=1.86096$ (B) $\log 0.033041=-1.48095$

(C) $\ln 40{,}257=10.60304$ (D) $\ln 0.0059263=-5.12836$

64. (A) $\log x=2.0832$

$$x=\log^{-1}(2.0832)$$
$$x=121.1156$$

(B) $\log x=-1.1577$

$$x=\log^{-1}(-1.1577)$$
$$x=0.0696$$

(C) $\ln x = 3.1336$

$x = \ln^{-1}(3.1336)$

$x = 22.9565$

(D) $\ln x = -4.3281$

$x = \ln^{-1}(-4.3281)$

$x = 0.0132$

66. $10^x = 153$

$\log 10^x = \log 153$

$x = 2.1847$

68. $e^x = 0.3059$

$\ln e^x = \ln 0.3059$

$x = -1.1845$

70. $1.02^{4t} = 2$

$\ln 1.02^{4t} = \ln 2$

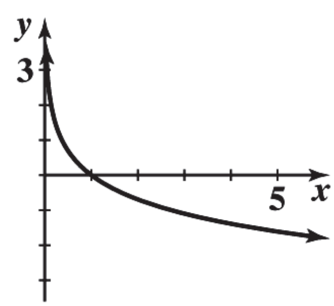
$4t \ln 1.02 = \ln 2$

$$t = \frac{\ln 2}{4 \ln 1.02}$$

$t = 8.7507$

72. $y = -\ln x;\ \ x > 0$

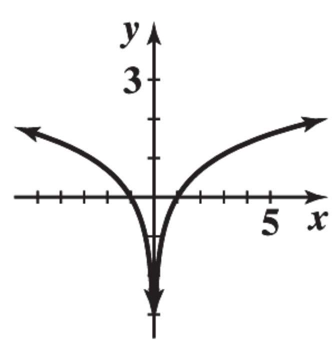
x	y
0.5	≈ 0.69
1	0
2	≈ -0.69
4	≈ -1.39
5	≈ -1.61



Based on the graph above, the function is decreasing on the interval $(0, \infty)$.

74. $y = \ln|x|$

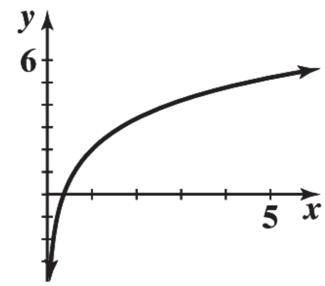
x	y
-5	≈ 1.61
-2	≈ 0.69
1	0
2	≈ 0.69
5	≈ 1.61



Based on the graph above, the function is decreasing on the interval $(-\infty, 0)$ and increasing on the interval $(0, \infty)$.

76. $y = 2\ln x + 2$

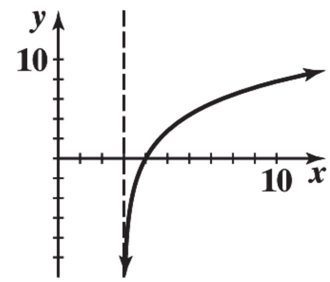
x	y
0.5	≈ 0.61
1	2
2	≈ 3.39
4	≈ 4.77
5	≈ 5.22



Based on the graph above, the function is increasing on the interval $(0, \infty)$.

78. $y = 4\ln(x-3)$

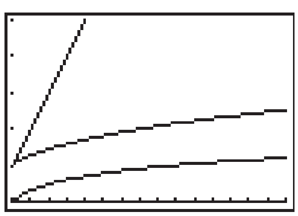
x	y
4	0
6	≈ 4.39
8	≈ 6.44
10	≈ 7.78
12	≈ 8.79



Based on the graph above, the function is increasing on the interval $(3, \infty)$.

80. It is not possible to find a power of 1 that is an arbitrarily selected real number, because 1 raised to any power is 1.

82.



A function *f* is "smaller than" a function *g* on an interval $[a, b]$ if $f(x) < g(x)$ for $a \le x \le b$. Based on the graph above, $\log x < \sqrt[3]{x} < x$ for $1 < x \le 16$.

84. Use the compound interest formula: $A = P(1+r)^t$. The problem is asking for the original amount to double, therefore $A = 2P$.

$$2P = P(1+0.0958)^t$$
$$2 = (1.0958)^t$$
$$\ln 2 = \ln(1.0958)^t$$
$$\ln 2 = t\ln(1.0958)$$
$$\frac{\ln 2}{\ln 1.0958} = t$$
$$7.58 \approx t$$

It will take approximately 8 years for the original amount to double.

86. Use the compound interest formula: $A = P(1+\frac{r}{m})^{mt}$.

(A)
$$\begin{aligned} 7500 &= 5000(1+\tfrac{0.08}{2})^{2t} \\ 1.5 &= (1.04)^{2t} \\ \ln 1.5 &= \ln(1.04)^{2t} \\ \ln 1.5 &= 2t\ln(1.04) \\ \frac{\ln 1.5}{2\ln 1.04} &= t \\ 5.17 &\approx t \end{aligned}$$

It will take approximately 5.17 years for \$5000 to grow to \$7500 if compounded semiannually.

(B)
$$\begin{aligned} 7500 &= 5000(1+\tfrac{0.08}{12})^{12t} \\ 1.5 &= (1.0066667)^{12t} \\ \ln 1.5 &= \ln(1.0066667)^{12t} \\ \ln 1.5 &= 12t\ln(1.0066667) \\ \frac{\ln 1.5}{12\ln 1.0066667} &= t \\ 5.09 &\approx t \end{aligned}$$

It will take approximately 5.09 years for \$5000 to grow to \$7500 if compounded monthly.

88. Use the compound interest formula: $A = Pe^{rt}$.

$$\begin{aligned} 41,000 &= 17,000e^{0.0295t} \\ \tfrac{41}{17} &= e^{0.0295t} \\ \ln\tfrac{41}{17} &= \ln e^{0.0295t} \\ \ln\tfrac{41}{17} &= 0.0295t \\ \frac{\ln\frac{41}{17}}{0.0295} &= t \\ 29.84 &\approx t \end{aligned}$$

It will take approximately 29.84 years for \$17,000 to grow to \$41,000 if compounded continuously.

90. Equilibrium occurs when supply and demand are equal. The models from Problem 85 have the demand and supply functions defined by $y = 256.4659159 - 24.03812068\ln x$ and $y = -127.8085281 + 20.01315349\ln x$, respectively. Set both equations equal to each other to yield:

$$\begin{aligned} 256.4659159 - 24.03812068\ln x &= -127.8085281 + 20.01315349\ln x \\ 384.274444 &= 44.05127417\ln x \\ \frac{384.274444}{44.05127417} &= \ln x \\ e^{384.274444/44.05127417} &= e^{\ln x} \\ 6145 &\approx x \end{aligned}$$

Substitute the value above into either equation.

$$y = 256.4659159 - 24.03812068 \ln x$$
$$y = 256.4659159 - 24.03812068 \ln(6145)$$
$$y = 256.4659159 - 24.03812068(8.723394022)$$
$$y = 46.77$$

Therefore, equilibrium occurs when 6145 units are produced and sold at a price of \$46.77.

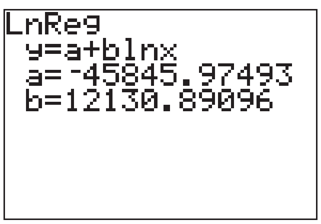
92. (A) $N = 10\log\frac{I}{I_0} = 10\log\frac{10^{-13}}{10^{-16}} = 10\log 10^3 = 30$

(B) $N = 10\log\frac{I}{I_0} = 10\log\frac{3.16\times10^{-10}}{10^{-16}} = 10\log 3.16\times10^6 \approx 65$

(C) $N = 10\log\frac{I}{I_0} = 10\log\frac{10^{-8}}{10^{-16}} = 10\log 10^8 = 80$

(D) $N = 10\log\frac{I}{I_0} = 10\log\frac{10^{-1}}{10^{-16}} = 10\log 10^{15} = 150$

94.



2024: $t = 124$; $y(124) \approx 12{,}628$. Therefore, according to the model, the total production in the year 2024 will be approximately 12,628 million bushels.

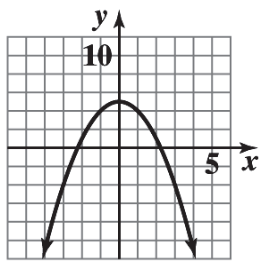
96.

$$A = A_0e^{-0.000124t}$$
$$0.1A_0 = A_0e^{-0.000124t}$$
$$0.1 = e^{-0.000124t}$$
$$\ln 0.1 = \ln e^{-0.000124t}$$
$$\ln 0.1 = -0.000124t$$
$$18{,}569 \approx t$$

If 10% of the original amount is still remaining, the skull would be approximately 18,569 years old.

CHAPTER 2 REVIEW

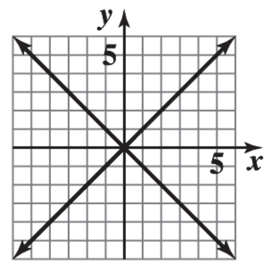
1.



(2-1)

2. $x^2 = y^2$:

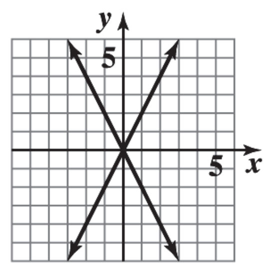
x	−3	−2	−1	0	1	2	3
y	±3	±2	±1	0	±1	±2	±3



(2-1)

3. $y^2 = 4x^2$:

x	−3	−2	−1	0	1	2	3
y	±6	±4	±2	0	±2	±4	±6



(2-1)

4. (A) Not a function; fails vertical line test

(B) A function

(C) A function

(D) Not a function; fails vertical line test (2-1)

5. $f(x) = 2x - 1$, $g(x) = x^2 - 2x$

(A) $f(-2) + g(-1) = 2(-2) - 1 + (-1)^2 - 2(-1) = -2$

(B) $f(0)\cdot g(4) = (2\cdot 0 - 1)(4^2 - 2\cdot 4) = -8$

(C) $\dfrac{g(2)}{f(3)} = \dfrac{2^2 - 2\cdot 2}{2\cdot 3 - 1} = 0$

(D) $\dfrac{f(3)}{g(2)}$ not defined because $g(2) = 0$ (2-1)

6. $u = e^v$
$v = \ln u$ (2-6)

7. $x = 10^y$
$y = \log x$ (2-6)

8. $\ln M = N$
$M = e^N$ (2-6)

9. $\log u = v$
$u = 10^v$ (2-6)

10. $\log_3 x = 2$
$x = 3^2 = 9$ (2-6)

11. $\log_x 36 = 2$
$x^2 = 36$
$x = 6$ (2-6)

12. $\log_2 16 = x$
$2^x = 16$
$x = 4$ (2-6)

13. $10^x = 143.7$
$x = \log 143.7$
$x \approx 2.157$ (2-6)

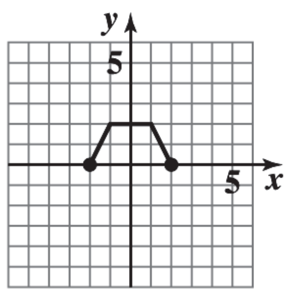
14. $e^x = 503{,}000$
$x = \ln 503{,}000 \approx 13.128$ (2-6)

15. $\log x = 3.105$
$x = 10^{3.105} \approx 1273.503$ (2-6)

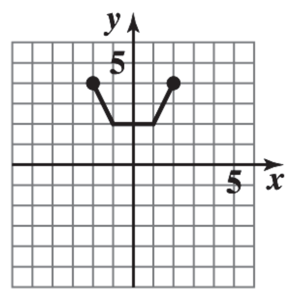
16. $\ln x = -1.147$
$x = e^{-1.147} \approx 0.318$ (2-6)

17. (A) $y = 4$ (B) $x = 0$ (C) $y = 1$ (D) $x = -1$ or 1
(E) $y = -2$ (F) $x = -5$ or 5 (2-1)

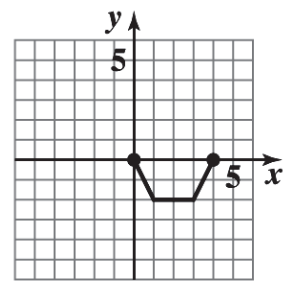
18. (A)



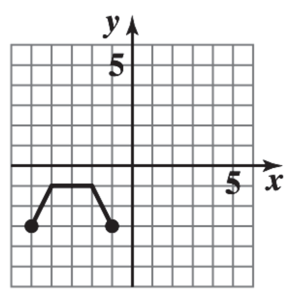
(B)



(C)



(D)



(2-2)

19. $f(x) = -x^2 + 4x = -(x^2 - 4x)$
$= -(x^2 - 4x + 4) + 4$
$= -(x - 2)^2 + 4$ (vertex form)
The graph of $f(x)$ is the graph of $y = x^2$ reflected in the x axis, then shifted right 2 units and up 4 units.
(2-3)

20. (A) g (B) m (C) n (D) f (2-2, 2-3)

21. $y = f(x) = (x + 2)^2 - 4$
(A) x intercepts: $(x + 2)^2 - 4 = 0$; y intercept: 0
$(x + 2)^2 = 4$
$x + 2 = -2$ or 2
$x = -4, 0$

(B) Vertex: (–2, –4) (C) Minimum: –4 (D) Range: $y \geq -4$ or $[-4, \infty)$ (2-3)

22. $y = 4 - x + 3x^2 = 3x^2 - x + 4$; quadratic function. (2-3)

23. $y = \frac{1+5x}{6} = \frac{5}{6}x + \frac{1}{6}$; linear function. (2-1, 2-3)

24. $y = \frac{7-4x}{2x} = \frac{7}{2x} - 2$; none of these. (2-1), (2-3)

25. $y = 8x + 2(10 - 4x) = 8x + 20 - 8x = 20$; constant function (2-1)

26. $\log(x + 5) = \log(2x - 3)$
$x + 5 = 2x - 3$
$-x = -8$
$x = 8$ (2-6)

27. $2\ln(x - 1) = \ln(x^2 - 5)$
$\ln(x - 1)^2 = \ln(x^2 - 5)$
$(x - 1)^2 = x^2 - 5$
$x^2 - 2x + 1 = x^2 - 5$
$-2x = -6$
$x = 3$ (2-6)

28. $9^{x-1} = 3^{1+x}$
$(3^2)^{x-1} = 3^{1+x}$
$3^{2x-2} = 3^{1+x}$
$2x - 2 = 1 + x$
$x = 3$ (2-5)

29. $e^{2x} = e^{x^2-3}$
$2x = x^2 - 3$
$x^2 - 2x - 3 = 0$
$(x - 3)(x + 1) = 0$
$x = 3, -1$ (2-5)

30. $2x^2e^x = 3xe^x$
$2x^2 = 3x$
$2x^2 - 3x = 0$
$x(2x - 3) = 0$
$x = 0, 3/2$ (2-5)

31. $\log_{1/3} 9 = x$
$\left(\frac{1}{3}\right)^x = 9$
$\frac{1}{3^x} = 9$
$3^x = \frac{1}{9}$
$x = -2$ (2-6)

32. $\log_x 8 = -3$
$x^{-3} = 8$
$\frac{1}{x^3} = 8$
$x^3 = \frac{1}{8}$
$x = \frac{1}{2}$ (2-6)

33. $\log_9 x = \frac{3}{2}$
$9^{3/2} = x$
$x = 27$ (2-6)

34. $x = 3(e^{1.49}) \approx 13.3113$ (2-5)

35. $x = 230(10^{-0.161}) \approx 158.7552$ (2-5)

36. $\log x = -2.0144$
$x \approx 10^{-2.0144} \approx 0.0097$ (2-6)

37. $\ln x = 0.3618$
$x = e^{0.3618} \approx 1.4359$ (2-6)

38.
$$35 = 7(3^x)$$
$$3^x = 5$$
$$\ln 3^x = \ln 5$$
$$x \ln 3 = \ln 5$$
$$x = \frac{\ln 5}{\ln 3} \approx 1.4650 \qquad (2\text{-}6)$$

39.
$$0.01 = e^{-0.05x}$$
$$\ln(0.01) = \ln(e^{-0.05x}) = -0.05x$$
$$\text{Thus, } x = \frac{\ln(0.01)}{-0.05} \approx 92.1034$$
(2-6)

40.
$$8{,}000 = 4{,}000(1.08)^x$$
$$(1.08)^x = 2$$
$$\ln(1.08)^x = \ln 2$$
$$x \ln 1.08 = \ln 2$$
$$x = \frac{\ln 2}{\ln 1.08} \approx 9.0065 \qquad (2\text{-}6)$$

41.
$$5^{2x-3} = 7.08$$
$$\ln(5^{2x-3}) = \ln 7.08$$
$$(2x - 3)\ln 5 = \ln 7.08$$
$$2x \ln 5 - 3 \ln 5 = \ln 7.08$$
$$x = \frac{\ln 7.08 + 3\ln 5}{2\ln 5}$$
$$x \approx 2.1081 \qquad (2\text{-}6)$$

42. (A) $x^2 - x - 6 = 0$ at $x = -2, 3$
Domain: all real numbers except $x = -2, 3$

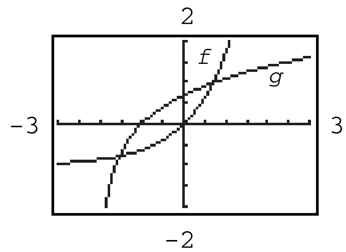
(B) $5 - x > 0$ for $x < 5$
Domain: $x < 5$ or $(-\infty, 5)$ (2-1)

43.
$$f(x) = 4x^2 + 4x - 3 = 4(x^2 + x) - 3$$
$$= 4\left(x^2 + x + \frac{1}{4}\right) - 3 - 1$$
$$= 4\left(x + \frac{1}{2}\right)^2 - 4 \text{ (vertex form)}$$

Intercepts:
y intercept: $f(0) = 4(0)^2 + 4(0) - 3 = -3$
x intercepts: $f(x) = 0$
$$4\left(x + \frac{1}{2}\right)^2 - 4 = 0$$
$$\left(x + \frac{1}{2}\right)^2 = 1$$
$$x + \frac{1}{2} = \pm 1$$
$$x = -\frac{1}{2} \pm 1 = -\frac{3}{2}, \frac{1}{2}$$

Vertex: $\left(-\frac{1}{2}, -4\right)$; minimum: -4; range: $y \geq -4$ or $[-4, \infty)$ (2-3)

44. $f(x) = e^x - 1$, $g(x) = \ln(x + 2)$

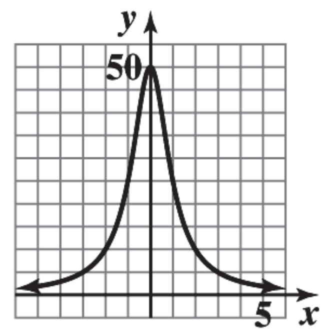


Points of intersection:
(–1.54, –0.79), (0.69, 0.99)

(2-5, 2-6)

45. $f(x) = \dfrac{50}{x^2 + 1}$:

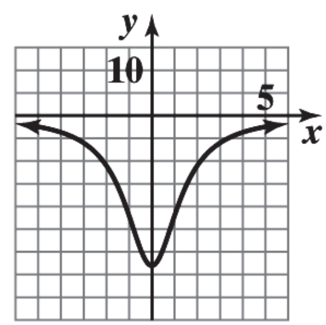
x	–3	–2	–1	0	1	2	3
$f(x)$	5	10	25	50	25	10	5



(2-1)

46. $f(x) = \dfrac{-66}{2 + x^2}$:

x	–3	–2	–1	0	1	2	3
$f(x)$	–6	–11	–22	–66	–22	–11	–6



(2-1)

For Problems 47–50, $f(x) = 5x + 1$.

47. $f(f(0)) = f(5(0) + 1) = f(1) = 5(1) + 1 = 6$ (2-1)

48. $f(f(-1)) = f(5(-1) + 1) = f(-4) = 5(-4) + 1 = -19$ (2-1)

49. $f(2x - 1) = 5(2x - 1) + 1 = 10x - 4$ (2-1)

50. $f(4 - x) = 5(4 - x) + 1 = 20 - 5x + 1 = 21 - 5x$ (2-1)

51. $f(x) = 3 - 2x$

(A) $f(2) = 3 - 2(2) = 3 - 4 = -1$

(B) $f(2 + h) = 3 - 2(2 + h) = 3 - 4 - 2h = -1 - 2h$

(C) $f(2 + h) - f(2) = -1 - 2h - (-1) = -2h$

(D) $\dfrac{f(2+h) - f(2)}{h} = -\dfrac{2h}{h} = -2$ (2-1)

52. $f(x) = x^2 - 3x + 1$

(A) $f(a) = a^2 - 3a + 1$

(B) $f(a+h) = (a+h)^2 - 3(a+h) + 1 = a^2 + 2ah + h^2 - 3a - 3h + 1$

(C) $f(a+h) - f(a) = a^2 + 2ah + h^2 - 3a - 3h + 1 - (a^2 - 3a + 1)$
$= 2ah + h^2 - 3h$

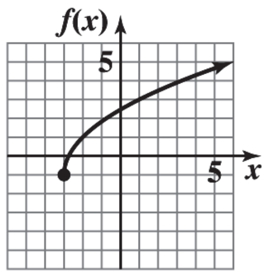
(D) $\dfrac{f(a+h)-f(a)}{h} = \dfrac{2ah+h^2-3h}{h} = \dfrac{h(2a+h-3)}{h} = 2a + h - 3$ (2-1)

53. The graph of m is the graph of $y = |x|$ reflected in the x axis and shifted 4 units to the right. (2-2)

54. The graph of g is the graph of $y = x^3$ vertically contracted by a factor of 0.3 and shifted up 3 units. (2-2)

55. The graph of $y = x^2$ is vertically expanded by a factor of 2, reflected in the x axis and shifted to the left 3 units.
Equation: $y = -2(x+3)^2$ (2-2)

56. Equation: $f(x) = 2\sqrt{x+3} - 1$



(2-2)

57. $f(x) = \dfrac{n(x)}{d(x)} = \dfrac{5x+4}{x^2-3x+1}$. Since degree $n(x) = 1 < 2 =$ degree $d(x)$, $y = 0$ is the horizontal asymptote. (2-4)

58. $f(x) = \dfrac{n(x)}{d(x)} = \dfrac{3x^2+2x-1}{4x^2-5x+3}$. Since degree $n(x) = 2 =$ degree $d(x)$, $y = \dfrac{3}{4}$ is the horizontal asymptote. (2-4)

59. $f(x) = \dfrac{n(x)}{d(x)} = \dfrac{x^2+4}{100x+1}$. Since degree $n(x) = 2 > 1 =$ degree $d(x)$, there is no horizontal asymptote. (2-4)

60. $f(x) = \dfrac{n(x)}{d(x)} = \dfrac{x^2+100}{x^2-100} = \dfrac{x^2+100}{(x-10)(x+10)}$. Since $n(x) = x^2 + 100$ has no real zeros and $d(10) = d(-10) = 0$, $x = 10$ and $x = -10$ are the vertical asymptotes of the graph of f. (2-4)

61. $f(x) = \dfrac{n(x)}{d(x)} = \dfrac{x^2+3x}{x^2+2x} = \dfrac{x(x+3)}{x(x+2)} = \dfrac{x+3}{x+2}$, $x \neq 0$. $x = -2$ is a vertical asymptote of the graph of f. (2-4)

62. True; $p(x) = \dfrac{p(x)}{1}$ is a rational function for every polynomial p. (2-4)

63. False; $f(x) = \frac{1}{x} = x^{-1}$ is not a polynomial function. (2-4)

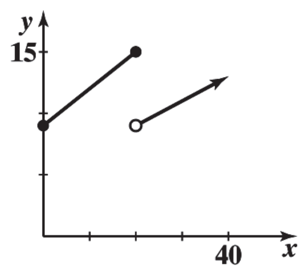
64. False; $f(x) = \frac{1}{x^2+1}$ has no vertical asymptotes. (2-4)

65. True: let $f(x) = b^x$, $(b > 0, b \neq 1)$, then the positive x-axis is a horizontal asymptote if $0 < b < 1$, and the negative x-axis is a horizontal asymptote if $b > 1$. (2-5)

66. True: let $f(x) = \log_b x$ $(b > 0, b \neq 1)$. If $0 < b < 1$, then the positive y-axis is a vertical asymptote; if $b > 1$, then the negative y-axis is a vertical asymptote. (2-6)

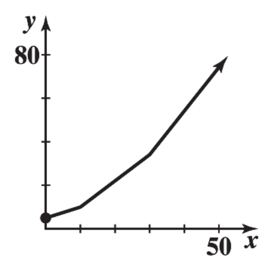
67. True; $f(x) = \frac{x}{x-1}$ has vertical asymptote $x = 1$ and horizontal asymptote $y = 1$. (2-4)

68.



(2-2)

69.



(2-2)

70. $y = -(x-4)^2 + 3$ (2-2, 2-3)

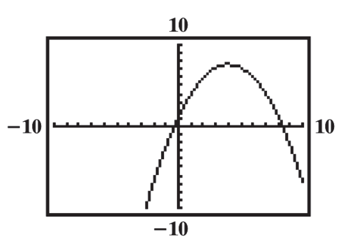
71. $f(x) = -0.4x^2 + 3.2x + 1.2 = -0.4(x^2 - 8x + 16) + 7.6$
$= -0.4(x-4)^2 + 7.6$

(A) y intercept: 1.2
x intercepts: $-0.4(x-4)^2 + 7.6 = 0$
$(x-4)^2 = 19$
$x = 4 + \sqrt{19} \approx 8.4,\ 4 - \sqrt{19} \approx -0.4$

(B) Vertex: (4.0, 7.6) (C) Maximum: 7.6 (D) Range: $y \leq 7.6$ or $(-\infty, 7.6]$

(2-3)

72.



(A) y intercept: 1.2
x intercepts: –0.4, 8.4

(B) Vertex: (4.0, 7.6)

(C) Maximum: 7.6

(D) Range: $y \leq 7.6$ or $(-\infty, 7.6]$ (2-3)

73. $\log 10^{\pi} = \pi \log 10 = \pi$
$10^{\log\sqrt{2}} = y$ is equivalent to $\log y = \log\sqrt{2}$
which implies $y = \sqrt{2}$
Similarly, $\ln e^{\pi} = \pi \ln e = \pi$ (Section 2-5, 4.b & g) and $e^{\ln\sqrt{2}} = y$ implies $\ln y = \ln\sqrt{2}$ and $y = \sqrt{2}$. (2-6)

74. $\log x - \log 3 = \log 4 - \log(x+4)$

$$\log\frac{x}{3} = \log\frac{4}{x+4}$$
$$\frac{x}{3} = \frac{4}{x+4}$$
$$x(x+4) = 12$$
$$x^2 + 4x - 12 = 0$$
$$(x+6)(x-2) = 0$$
$$x = -6,\ 2$$

Since log(−6) is not defined, −6 is not a solution. Therefore, the solution is $x = 2$. (2-6)

75. $\ln(2x-2) - \ln(x-1) = \ln x$

$$\ln\left(\frac{2x-2}{x-1}\right) = \ln x$$
$$\ln\left[\frac{2(x-1)}{x-1}\right] = \ln x$$
$$\ln 2 = \ln x$$
$$x = 2$$
(2-6)

76. $\ln(x+3) - \ln x = 2\ln 2$

$$\ln\left(\frac{x+3}{x}\right) = \ln(2^2)$$
$$\frac{x+3}{x} = 4$$
$$x + 3 = 4x$$
$$3x = 3$$
$$x = 1$$
(2-6)

77. $\log 3x^2 = 2 + \log 9x$

$$\log 3x^2 - \log 9x = 2$$
$$\log\left(\frac{3x^2}{9x}\right) = 2$$
$$\log\left(\frac{x}{3}\right) = 2$$
$$\frac{x}{3} = 10^2 = 100$$
$$x = 300$$
(2-6)

78. $\ln y = -5t + \ln c$

$$\ln y - \ln c = -5t$$
$$\ln\frac{y}{c} = -5t$$
$$\frac{y}{c} = e^{-5t}$$
$$y = ce^{-5t}$$
(2-6)

79. Let x be *any* positive real number and suppose $\log_1 x = y$. Then $1^y = x$.

But, $1^y = 1$, so $x = 1$, i.e., $x = 1$ for all positive real numbers x.
This is clearly impossible. (2-6)

80. The graph of $y = \sqrt[3]{x}$ is vertically expanded by a factor of 2, reflected in the x axis, shifted 1 unit to the left and 1 unit down.
Equation: $y = -2\sqrt[3]{x+1} - 1$ (2-2)

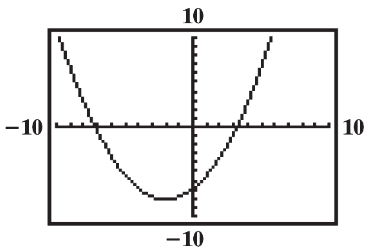
81. $G(x) = 0.3x^2 + 1.2x - 6.9 = 0.3(x^2 + 4x + 4) - 8.1$
$= 0.3(x + 2)^2 - 8.1$

(A) y intercept: -6.9
x intercepts: $0.3(x + 2)^2 - 8.1 = 0$
$(x + 2)^2 = 27$
$x = -2 + \sqrt{27} \approx 3.2, -2 - \sqrt{27} \approx -7.2$

(B) Vertex: $(-2, -8.1)$ (C) Minimum: -8.1 (D) Range: $y \geq -8.1$ or $[-8.1, \infty)$

(2-3)

82.



(A) y intercept: -6.9
x intercept: $-7.2, 3.2$

(B) Vertex: $(-2, -8.1)$

(C) Minimum: -8.1

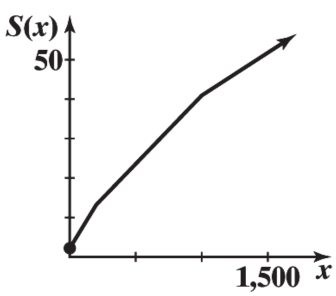
(D) Range: $y \geq -8.1$ or $[-8.1, \infty)$ (2-3)

83. (A) $S(x) = 3$ if $0 \leq x \leq 20$;
$S(x) = 3 + 0.057(x - 20)$
$= 0.057x + 1.86$ if $20 < x \leq 200$;
$S(200) = 13.26$
$S(x) = 13.26 + 0.0346(x - 200)$
$= 0.0346x + 6.34$ if $200 < x \leq 1000$;
$S(1000) = 40.94$

$S(x) = 40.94 + 0.0217(x - 1000)$
$= 0.0217x + 19.24$ if $x > 1000$

$$\text{Therefore, } S(x) = \begin{cases} 3 & \text{if} \quad 0 \leq x \leq 20 \\ 0.057x + 1.86 & \text{if} \quad 20 < x \leq 200 \\ 0.0346x + 6.34 & \text{if} \quad 200 < x \leq 1000 \\ 0.0217x + 19.24 & \text{if} \quad x > 1000 \end{cases}$$

(B)



(2-2)

84. $A = P\left(1 + \frac{r}{m}\right)^{mt}$; $P = 5{,}000,\ r = 0.0125,\ m = 4,\ t = 5.$

$$A = 5000\left(1 + \frac{0.0125}{4}\right)^{4(5)} = 5000\left(1 + \frac{0.0125}{4}\right)^{20} \approx 5321.95$$

After 5 years, the CD will be worth \$5,321.95 (2-5)

85. $A = P\left(1+\frac{r}{m}\right)^{mt}$; $P = 5{,}000$, $r = 0.0105$, $m = 365$, $t = 5$

$A = 5000\left(1+\frac{0.0105}{365}\right)^{365(5)} = 5000\left(1+\frac{0.0105}{365}\right)^{1825} \approx 5269.51$

After 5 years, the CD will be worth \$5,269.51. (2-5)

86. $A = P\left(1+\frac{r}{m}\right)^{mt}$, $r = 0.0659$, $m = 12$

Solve $P\left(1+\frac{0.0659}{12}\right)^{12t} = 3P$ or $(1.005492)^{12t} = 3$

for t:

$12t\ln(1.005492) = \ln 3$

$t = \frac{\ln 3}{12\ln(1.005492)} \approx 16.7$ year. (2-5)

87. $A = Pe^{rt}$, $r = 0.0739$. Solve $2P = Pe^{0.0739t}$ for t.

$2P = Pe^{0.0739t}$

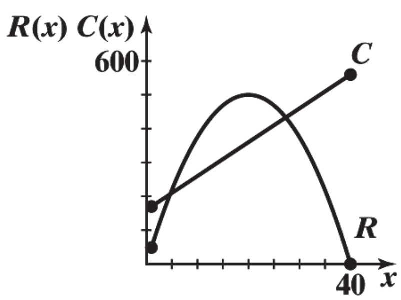
$e^{0.0739t} = 2$

$0.0739t = \ln 2$

$t = \frac{\ln 2}{0.0739} \approx 9.38$ years. (2-5)

88. $p(x) = 50 - 1.25x$ Price-demand function
$C(x) = 160 + 10x$ Cost function
$R(x) = xp(x)$
$= x(50 - 1.25x)$ Revenue function

(A)



(B) $R = C$

$x(50 - 1.25x) = 160 + 10x$

$-1.25x^2 + 50x = 160 + 10x$

$-1.25x^2 + 40x = 160$

$-1.25(x^2 - 32x + 256) = 160 - 320$

$-1.25(x - 16)^2 = -160$

$(x - 16)^2 = 128$

$x = 16 + \sqrt{128} \approx 27.314$,

$16 - \sqrt{128} \approx 4.686$

$R = C$ at $x = 4.686$ thousand units (4,686 units) and
$x = 27.314$ thousand units (27,314 units)
$R < C$ for $1 \le x < 4.686$ or $27.314 < x \le 40$
$R > C$ for $4.686 < x < 27.314$

(C) Max Rev: $50x - 1.25x^2 = R$

$-1.25(x^2 - 40x + 400) + 500 = R$

$-1.25(x - 20)^2 + 500 = R$

Vertex at (20, 500)

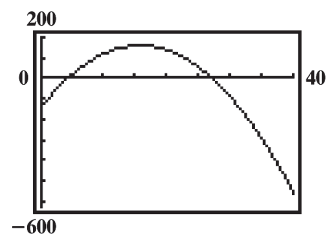
Max. Rev. = 500 thousand ($500,000) occurs when <u>output</u> is 20 thousand (20,000 units)

<u>Wholesale price</u> at this output: $p(x) = 50 - 1.25x$

$p(20) = 50 - 1.25(20) = \$25$ (2-3)

89. (A) $P(x) = R(x) - C(x) = x(50 - 1.25x) - (160 + 10x)$

$= -1.25x^2 + 40x - 160$



(B) $P = 0$ for $x = 4.686$ thousand units (4,686 units) and $x = 27.314$ thousand units (27,314 units)

$P < 0$ for $1 \le x < 4.686$ or $27.314 < x \le 40$

$P > 0$ for $4.686 < x < 27.314$

(C) Maximum profit is 160 thousand dollars ($160,000), and this occurs at $x = 16$ thousand units(16,000 units). The wholesale price at this output is $p(16) = 50 - 1.25(16) = \$30$, which is $5 greater than the $25 found in 88(C). (2-3)

90. (A) The area enclosed by the storage areas is given by

$A = (2y)x$

Now, $3x + 4y = 840$

so $y = 210 - \frac{3}{4}x$

Thus $A(x) = 2\left(210 - \frac{3}{4}x\right)x$

$= 420x - \frac{3}{2}x^2$

(B) Clearly x and y must be nonnegative; the fact that $y \ge 0$ implies

$210 - \frac{3}{4}x \ge 0$

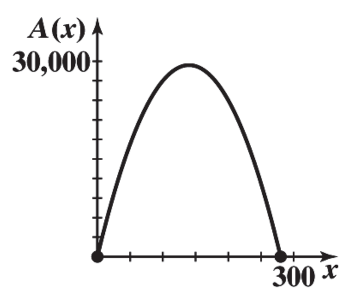
and $210 \ge \frac{3}{4}x$

$840 \ge 3x$

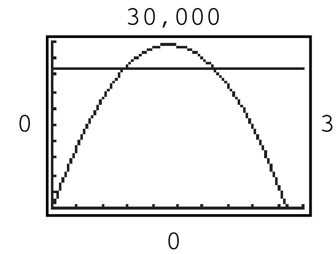
$280 \ge x$

Thus, domain A: $0 \le x \le 280$

(C)



(D) Graph $A(x) = 420x - \frac{3}{2}x^2$ and $y = 25{,}000$ together.



There are two values of x that will produce storage areas with a combined area of 25,000 square feet, one near $x = 90$ and the other near $x = 190$.

(E) $x = 86$, $x = 194$

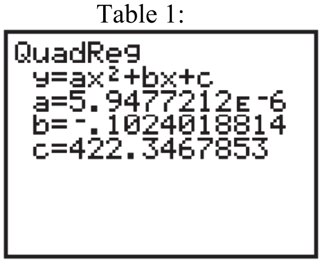
(F) $A(x) = 420x - \frac{3}{2}x^2 = -\frac{3}{2}(x^2 - 280x)$

Completing the square, we have

$$A(x) = -\frac{3}{2}(x^2 - 280x + 19{,}600 - 19{,}600)$$
$$= -\frac{3}{2}[(x-140)^2 - 19{,}600]$$
$$= -\frac{3}{2}(x-140)^2 + 29{,}400$$

The dimensions that will produce the maximum combined area are:
$x = 140$ ft, $y = 105$ ft. The maximum area is 29,400 sq. ft. (2-3)

91. (A) Quadratic regression model,
Table 1:

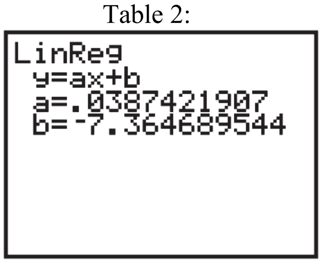


To estimate the demand at price level of \$180, we solve the equation

$$ax^2 + bx + c = 180$$

for x. The result is $x \approx 2{,}833$ sets.

(B) Linear regression model,
Table 2:



To estimate the supply at a price level of \$180, we solve the equation

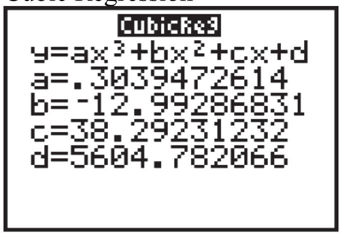
$$ax + b = 180$$

for x. The result is $x \approx 4{,}836$ sets.

(C) The condition is not stable; the price is likely to decrease since the supply at the price level of \$180 exceeds the demand at this level.

(D) Equilibrium price: \$131.59
Equilibrium quantity: 3,587 cookware set. (2-3)

92. (A) Cubic Regression



$y = 0.30395x^3 - 12.993x^2 + 38.292x + 5{,}604.8$

(B) $y = 0.30395(38)^3 - 12.993(38)^2 + 38.292(38) + 5{,}604.8 \approx 4{,}976$

The predicted crime index in 2025 is 4,976.

93. (A) $N(0) = 1$

$N\left(\frac{1}{2}\right) = 2$

$N(1) = 4 = 2^2$

$N\left(\frac{3}{2}\right) = 8 = 2^3$

$N(2) = 16 = 2^4$

$\vdots$

Thus, we conclude that

$N(t) = 2^{2t}$ or $N = 4^t$

(B) We need to solve:

$2^{2t} = 10^9$

$\log 2^{2t} = \log 10^9 = 9$

$2t \log 2 = 9$

$t = \frac{9}{2\log 2} \approx 14.95$

Thus, the mouse will die in 15 days.

(2-6)

94. Given $I = I_0 e^{-kd}$. When $d = 73.6$, $I = \frac{1}{2} I_0$. Thus, we have:

$\frac{1}{2} I_0 = I_0 e^{-k(73.6)}$

$e^{-k(73.6)} = \frac{1}{2}$

$-k(73.6) = \ln \frac{1}{2}$

$k = \frac{\ln(0.5)}{-73.6} \approx 0.00942$

Thus, $k \approx 0.00942$.

To find the depth at which 1% of the surface light remains, we set $I = 0.01I_0$ and solve

$0.01I_0 = I_0 e^{-0.00942d}$ for d:

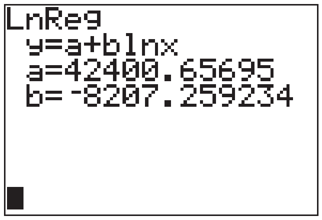
$0.01 = e^{-0.00942d}$

$-0.00942d = \ln 0.01$

$d = \frac{\ln 0.01}{-0.00942} \approx 488.87$

Thus, 1% of the surface light remains at approximately 489 feet. (2-6)

95. (A) Logarithmic regression model:



Year 2023 corresponds to $x = 83$; $y(83) \approx 6{,}134{,}000$ cows.

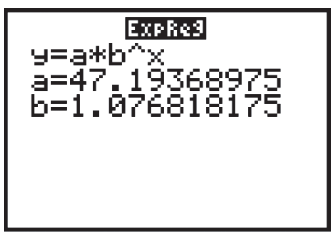
(B) ln (0) is not defined. (2-6)

96. Using the continuous compounding model, we have:

$$2P_0 = P_0 e^{0.03t}$$
$$2 = e^{0.03t}$$
$$0.03t = \ln 2$$
$$t = \frac{\ln 2}{0.03} \approx 23.1$$

Thus, the model predicts that the population will double in approximately 23.1 years. (2-5)

97. (A)



The exponential regression model is $y = 47.194(1.0768)^x$.

To estimate for the year 2025, let $x = 45 \Rightarrow y = 47.19368975(1.076818175)^{45} \approx 1{,}319.140047$. The estimated annual expenditure for Medicare by the U.S. government, rounded to the nearest billion, is approximately $1,319 billion. (This is $1.319 trillion.)

(B) To find the year, solve $47.194(1.0768)^x = 2{,}000$. Note: Use 2,000 because expenditures are in billions of dollars, and 2 trillion is 2,000 billion.

$$47.194(1.0768)^x = 2{,}000$$
$$1.0768^x = \frac{2{,}000}{47.194}$$
$$\ln(1.0768^x) = \ln\left(\frac{2{,}000}{47.194}\right)$$
$$x \ln 1.0768 = \ln\left(\frac{2{,}000}{47.194}\right)$$
$$x = \frac{\ln\left(\frac{2{,}000}{47.194}\right)}{\ln 1.0768} \approx 50.6 \text{ years}$$

1,980 + 50.63 = 2,030.63 Annual expenditures exceed two trillion dollars in the year 2031.
(2-5)