# INSTRUCTOR'S Solutions Manual DANIEL S. MILLER <br> Niagara County Community College 

# Thinking Mathematically Seventh Edition 

Robert Blitzer<br>Miami Dade College



The author and publisher of this book have used their best efforts in preparing this book. These efforts include the development, research, and testing of the theories and programs to determine their effectiveness. The author and publisher make no warranty of any kind, expressed or implied, with regard to these programs or the documentation contained in this book. The author and publisher shall not be liable in any event for incidental or consequential damages in connection with, or arising out of, the furnishing, performance, or use of these programs.

Reproduced by Pearson from electronic files supplied by the author.

Copyright © 2019, 2015, 2011 Pearson Education, Inc.
Publishing as Pearson, 330 Hudson Street, NY NY 10013

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the publisher. Printed in the United States of America.

TABLE OF CONTENTS for INSTRUCTOR'S SOLUTION MANUAL Thinking Mathematically, 7E
Chapter 1: Problem Solving and Critical Thinking ..... 1
Chapter 2: Set Theory ..... 25
Chapter 3: Logic ..... 51
Chapter 4: Number Representation and Calculation ..... 125
Chapter 5: Number Theory and the Real Number System ..... 159
Chapter 6: Algebra: Equations and Inequalities ..... 227
Chapter 7: Algebra: Graphs, Functions, and Linear Systems ..... 285
Chapter 8: Personal Finance ..... 353
Chapter 9: Measurement ..... 411
Chapter 10: Geometry ..... 429
Chapter 11: Counting Methods and Probability Theory ..... 469
Chapter 12: Statistics ..... 527
Chapter 13: Voting and Apportionment ..... 587
Chapter 14: Graph Theory ..... 643

# Chapter 1 <br> Problem Solving and Critical Thinking 

## Check Points 1.1

1. Counterexamples will vary. Example: $40 \times 40=1600$
2. a. Add 6 each time.
$3+6=9$
$9+6=15$
$15+6=21$
$21+6=27$
$27+6=33$
$3,9,15,21,27, \underline{33}$
c. Cycle multiplying by $2,3,4$.
$3 \times 2=6$
$6 \times 3=18$
$18 \times 4=72$
$72 \times 2=144$
$144 \times 3=432$
$432 \times 4=1728$
$1728 \times 2=3456$
$6,18,72,144,432,1728, \underline{3456}$
$6 \times 3=18$
$18 \times 4=72$
$72 \times 2=144$
$432 \times 4=1728$
$1728 \times 2=3456$
$6,18,72,144,432,1728, \underline{3456}$
b. Multiply by 5 each time.
$2 \times 5=10$
$10 \times 5=50$
$50 \times 5=250$
$250 \times 5=1250$
$2,10,50,250, \underline{1250}$
d. Cycle adding 8 , adding 8 , subtracting 14.
$1+8=9$
$9+8=17$
$17-14=3$
$3+8=11$
$11+8=19$
$19-14=5$
$5+8=13$
$13+8=21$
$21-14=7$
9
$9,17,3,11,19,5,13,21,7$
3. a. Starting with the third number, each number is the sum of the previous two numbers, $29+47=76$
b. Starting with the second number, each number one less than twice the previous number, $2(129)-1=257$
4. The shapes alternate between rectangle and triangle.

The number of little legs cycles from 1 to 2 to 3 and then back to 1 .
Therefore the next figure will be a rectangle with 2 little legs.

5. a. Conjecture based on results: The original number is doubled.

| Select a number. | 4 | 10 | 0 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| Multiply the number by 4. | $4 \times 4=16$ | $10 \times 4=40$ | $0 \times 4=0$ | $3 \times 4=12$ |
| Add 6 to the product. | $16+6=22$ | $40+6=46$ | $0+6=6$ | $12+6=18$ |
| Divide this sum by 2. | $22 \div 2=11$ | $46 \div 2=23$ | $6 \div 2=3$ | $18 \div 2=9$ |
| Subtract 3 from the quotient. | $11-3=8$ | $23-3=20$ | $3-3=0$ | $9-3=6$ |
| Summary of results: | $4 \rightarrow 8$ | $10 \rightarrow 20$ | $0 \rightarrow 0$ | $3 \rightarrow 6$ |

b. Select a number: $n$

Multiply the number by 4: $4 n$
Add 6 to the product: $\quad 4 n+6$
Divide this sum by 2 : $\quad \frac{4 n+6}{2}=\frac{4 n}{2}+\frac{6}{2}=2 n+3$
Subtract 3 from the quotient: $2 n+3-3=2 n$

## Concept and Vocabulary Check 1.1

1. counterexample
2. deductive
3. inductive
4. true

## Exercise Set 1.1

1. Counterexamples will vary. Example: President Obama was younger than 65 at the time of his inauguration.
2. Counterexamples will vary. Example: Beyoncé Knowles is a singer who appears in movies.
3. Counterexamples will vary. Example: 3 multiplied by itself is 9 , which is not even.
4. Counterexamples will vary. Example: 100 is a three-digit number and $100+100=200$, which is not a four-digit number.
5. Counterexamples will vary. Example: Adding 1 to the numerator and denominator of $\frac{1}{2}$ results in $\frac{2}{3}$ which is not equal to $\frac{1}{2}$.
6. Counterexamples will vary. Example: $8-3=5$, which is odd, but 8 and 3 are not both odd.
7. Counterexamples will vary. Example: When -1 is added to itself, the result is -2 , which is less than -1 .
8. Counterexamples will vary. Example: When 1 is divided by -2 , the result is $-\frac{1}{2}$, which is greater than -2 .
9. Pattern: Add 4
$24+4=28$
$8,12,16,20,24, \underline{28}$
10. Pattern: Add 5
$39+5=44$
$19,24,29,34,39,44$
11. Pattern: Subtract 5
$17-5=12$
$37,32,27,22,17, \underline{12}$
12. Pattern: Subtract 4
$17-4=13$
$33,29,25,21,17,13$
13. Pattern: Multiply by 3
$243 \times 3=729$
$3,9,27,81,243,729$
14. Pattern: Multiply by 4
$512 \times 4=2048$
$2,8,32,128,512, \underline{2048}$
15. Pattern: Multiply by 2
$16 \times 2=32$
$1,2,4,8,16, \underline{32}$
16. Pattern: Multiply by 5
$125 \times 5=625$
$1,5,25,125, \underline{625}$
17. Pattern: 1 alternates with numbers that are multiplied by 2
$16 \times 2=32$
$1,4,1,8,1,16,1, \underline{32}$
18. Pattern: 1 alternates with numbers that are increased
by 3
$10+3=13$
$1,4,1,7,1,10,1, \underline{13}$
19. Pattern: Subtract 2
$-4-2=-6$
$4,2,0,-2,-4, \underline{6}$
20. Pattern: Subtract 3
$-6-3=-9$
$6,3,0,-3,-6, \underline{-9}$
21. Pattern: Add 4 to the denominator
$\frac{1}{18+4}=\frac{1}{22}$
$\frac{1}{2}, \frac{1}{6}, \frac{1}{10}, \frac{1}{14}, \frac{1}{18}, \frac{1}{22}$
22. Pattern: Add 1 to the denominator
$\frac{1}{5+1}=\frac{1}{6}$
$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{\underline{6}}$
23. Pattern: Multiply the denominator by 3
$\frac{1}{27 \times 3}=\frac{1}{81}$
$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}$
24. Pattern: Multiply the denominator by 2
$\frac{1}{8 \times 2}=\frac{1}{16}$
$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{\underline{16}}$
25. Pattern: The second number is obtained by adding 4 to the first number. The third number is obtained by adding 5 to the second number. The number being added to the previous number increases by 1 each time. $33+9=\underline{42}$
26. Pattern: The second number is obtained by adding 3 to the first number. The third number is obtained by adding 4 to the second number. The number being added to the previous number increases by 1 each time. $27+8=\underline{35}$
27. Pattern: The second number is obtained by adding 3 to the first number. The third number is obtained by adding 5 to the second number. The number being added to the previous number increases by 2 each time. $38+13=\underline{51}$
28. Pattern: The second number is obtained by adding 3 to the first number. The third number is obtained by adding 5 to the second number. The number being added to the previous number increases by 2 each time. $37+13=\underline{50}$
29. Pattern: Starting with the third number, each number is the sum of the previous two numbers. $27+44=\underline{71}$
30. Pattern: Starting with the third number, each number is the sum of the previous two numbers. $19+31=50$
31. Pattern: Cycle by adding 5, adding 5, then subtracting 7. $13+5=\underline{18}$
32. Pattern: Cycle by adding 6 , adding 6 , then subtracting 10. $13+6=\underline{19}$
33. Pattern: The second number is obtained by multiplying the first number by 2 . The third number is obtained by subtracting 1 from the second number. Then multiply by 2 and then subtract 1 , repeatedly. $34-1=\underline{33}$
34. Pattern: The second number is obtained by multiplying the first number by 3 . The third number is obtained by subtracting 1 from the second number. Then multiply by 3 and then subtract 1 , repeatedly. $123-1=\underline{122}$
35. Pattern: Divide by -4
$-1 \div(-4)=\frac{1}{4}$
$64,-16,4,-1, \frac{1}{4}$
36. Pattern: Divide by -5
$-1 \div(-5)=\frac{1}{5}$
$125,-25,5,-1, \frac{1}{5}$
37. Pattern: The second value of each pair is 4 less than the first.
$3-4=-1$
$(6,2),(0,-4),\left(7 \frac{1}{2}, 3 \frac{1}{2}\right),(2,-2),(3,-1)$
38. Pattern: The second value of each pair is the square of the first.
$\left(-\frac{4}{7}\right)^{2}=\frac{16}{49}$
$\left(\frac{2}{3}, \frac{4}{9}\right),\left(\frac{1}{5}, \frac{1}{25}\right),(7,49),\left(-\frac{5}{6}, \frac{25}{36}\right),\left(-\frac{4}{7}, \underline{16}\right)$
39. The figure cycles from square to triangle to circle and then repeats. So the next figure is

40. The figure rotates $90^{\circ}$ counterclockwise. So the next figure is
41. The pattern is to add one more letter to the previous figure and use the next consecutive letter in the alphabet. The next figure is shown at right.

42. The figure alternates from triangle to square and gains one line on the bottom. The next figure is ${ }^{4111}$.
43. a. Conjecture based on results: The original number is doubled.

| Select a number. | 4 | 10 | 0 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| Multiply the number by 4. | $4 \times 4=16$ | $10 \times 4=40$ | $0 \times 4=0$ | $3 \times 4=12$ |
| Add 8 to the product. | $16+8=24$ | $40+8=48$ | $0+8=8$ | $12+8=20$ |
| Divide this sum by 2. | $24 \div 2=12$ | $48 \div 2=24$ | $8 \div 2=4$ | $20 \div 2=10$ |
| Subtract 4 from the quotient. | $12-4=8$ | $24-4=20$ | $4-4=0$ | $10-4=6$ |
| Summary of results: | $4 \rightarrow 8$ | $10 \rightarrow 20$ | $0 \rightarrow 0$ | $3 \rightarrow 6$ |

b. $4 n$
$4 n+8$
$\frac{4 n+8}{2}=\frac{4 n}{2}+\frac{8}{2}=2 n+4$
$2 n+4-4=2 n$
44. a. Conjecture based on results: The result is always 2 .

| Select a number. | 4 | 10 | 0 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| Multiply the number by 3. | $4 \times 3=12$ | $10 \times 3=30$ | $0 \times 3=0$ | $3 \times 3=9$ |
| Add 6 to the product. | $12+6=18$ | $30+6=36$ | $0+6=6$ | $9+6=15$ |
| Divide this sum by 3. | $18 \div 3=6$ | $36 \div 3=12$ | $6 \div 3=2$ | $15 \div 3=5$ |
| Subtract the original from the quotient. | $6-4=2$ | $12-10=2$ | $2-0=2$ | $5-3=2$ |
| Summary of results: | $4 \rightarrow 2$ | $10 \rightarrow 2$ | $0 \rightarrow 2$ | $3 \rightarrow 2$ |

b. $3 n$
$3 n+6$
$\frac{3 n+6}{3}=\frac{3 n}{3}+\frac{6}{3}=n+2$
$n+2-n=2$
45. a. Conjecture based on results: The result is always 3 .

| Select a number. | 4 | 10 | 0 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| Add 5 to the number. | $4+5=9$ | $10+5=15$ | $0+5=5$ | $3+5=8$ |
| Double the result. | $9 \times 2=18$ | $15 \times 2=30$ | $5 \times 2=10$ | $8 \times 2=16$ |
| Subtract 4. | $18-4=14$ | $30-4=26$ | $10-4=6$ | $16-4=12$ |
| Divide the result by 2. | $14 \div 2=7$ | $26 \div 2=13$ | $6 \div 2=3$ | $12 \div 2=6$ |
| Subtract the original number. | $7-4=3$ | $13-10=3$ | $3-0=3$ | $6-3=3$ |
| Summary of results: | $4 \rightarrow 3$ | $10 \rightarrow 3$ | $0 \rightarrow 3$ | $3 \rightarrow 3$ |

b. $n+5$
$2(n+5)=2 n+10$
$2 n+10-4=2 n+6$
$\frac{2 n+6}{2}=\frac{2 n}{2}+\frac{6}{2}=n+3$
$n+3-n=3$
46. a. Conjecture based on results: The result is always 5.

| Select a number. | 4 | 10 | 0 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| Add 3 to the number. | $4+3=7$ | $10+3=13$ | $0+3=3$ | $3+3=6$ |
| Double the result. | $7 \times 2=14$ | $13 \times 2=26$ | $3 \times 2=6$ | $6 \times 2=12$ |
| Add 4. | $14+4=18$ | $26+4=30$ | $6+4=10$ | $12+4=16$ |
| Divide the result by 2. | $18 \div 2=9$ | $30 \div 2=15$ | $10 \div 2=5$ | $16 \div 2=8$ |
| Subtract the original number. | $9-4=5$ | $15-10=5$ | $5-0=5$ | $8-3=5$ |
| Summary of results: | $4 \rightarrow 5$ | $10 \rightarrow 5$ | $0 \rightarrow 5$ | $3 \rightarrow 5$ |

b. $n+3$

$$
\begin{aligned}
& 2(n+3)=2 n+6 \\
& 2 n+6+4=2 n+10 \\
& \frac{2 n+10}{2}=\frac{2 n}{2}+\frac{10}{2}=n+5 \\
& n+5-n=5
\end{aligned}
$$

47. Using inductive reasoning we predict $1+2+3+4+5+6=\frac{6 \times 7}{2}$.

Arithmetic verifies this result: $21=21$
48. Using inductive reasoning we predict $3+6+9+12+15+18=\frac{18 \times 7}{2}$.

Arithmetic verifies this result: $63=63$
49. Using inductive reasoning we predict $1+3+5+7+9+11=6 \times 6$.

Arithmetic verifies this result: $36=36$
50. Using inductive reasoning we predict $5 \cdot 9+(5+9)=59$.

Arithmetic verifies this result:

$$
\begin{aligned}
5 \cdot 9+(5+9) & =59 \\
5 \cdot 9+(14) & =59 \\
45+(14) & =59 \\
59 & =59
\end{aligned}
$$

51. Using inductive reasoning we predict $98765 \times 9+3=888,888$.

Arithmetic verifies this result:
$98765 \times 9+3=888,888$

$$
\begin{aligned}
888,885+3 & =888,888 \\
888,888 & =888,888
\end{aligned}
$$

52. Using inductive reasoning we predict $54321 \times 9-1=488,888$.

Arithmetic verifies this result:
$54321 \times 9-1=488,888$
$488,889-1=488,888$
$488,888=488,888$
53. The first multiplier increases by 33 .
$132+33=165$
The second multiplier is 3367 .
The product increases by 111,111 .
$165 \times 3367=555,555$ is correct.
54. The pattern implies we should attach a 6 to the right of the first multiplier. The second multiplier is always 8 . The pattern implies we should add 6 to that product to obtain 987,654.
$123,456 \times 8+6=987,654$ is correct.
55. $b$; The resulting exponent is always the first exponent added to twice the second exponent.
56. $c$; The resulting exponent is always half the sum of the three exponents.
57. deductive; The specific value was based on a general formula.
58. inductive; The general conclusion for all HMO patients was based on specific observations.
59. inductive; The general conclusion for all full-time four-year colleges was based on specific observations.
60. deductive; The specific grade was based on a general course policy.
61. a. $1,3,6,10,15$, and 21 are followed by
$21+7=28$
$28+8=36$
$36+9=45$
$45+10=55$
$55+11=66$
$1,3,6,10,15,21,28,36,45,55$, and 66.
b. $4-1=3$
$9-4=5$
$16-9=7$
$25-16=9$
The successive differences increase by 2 .
$25+11=36$
$36+13=49$
$49+15=64$
$64+17=81$
$81+19=100$
c. The successive differences are 4,7 , and 10 . Since these differences are increasing by 3 each time. The next five numbers will be found by using differences of $13,16,19,22$, and 25.
$22+13=35$
$35+16=51$
$51+19=70$
$70+22=92$
$92+25=117$
d. If a triangular number is multiplied by 8 and then 1 is added to the product, a square number is obtained.
62. Each row begins and ends with 1 . Other numbers are the sum of the two values that are diagonally above.

66. does not make sense; Explanations will vary. Sample explanation: Such conclusions would be certain.
67. makes sense
68. does not make sense; Explanations will vary. Sample explanation: Though this sample was $51 \%$, it is not certain that this exact percentage will hold for the entire population.
69. makes sense
70. The pattern suggests that the compatible expression is the square of the first number minus twice the product of the two numbers, plus the square of the second number.
$(11-7)^{2}=121-154+49$
71. a. The sums are all 30 :

| 16 | 3 | 11 |
| :---: | :---: | :---: |
| 5 | 10 | 15 |
| 9 | 17 | 4 |

b. The sums are all 36 :

| 17 | 5 | 14 |
| :---: | :---: | :---: |
| 9 | 12 | 15 |
| 10 | 19 | 7 |

c. For any values of $a, b$, and $c$, the sums of all rows, all columns, and both diagonals are the same.
d. The sums of the expressions in each row, each column, and each diagonal is $3 a$.
e. Finding each sum verifies the conjecture that they are all $3 a$.

First row: $(a+b)+(a-b-c)+(a+c)=3 a$
Second row: $(a-b+c)+(a)+(a+b-c)=3 a$
Third row: $(a-c)+(a+b+c)+(a-b)=3 a$
First Column: $(a+b)+(a-b+c)+(a-c)=3 a$
Second Column: $(a-b-c)+(a)+(a+b+c)=3 a$
Third Column: $(a+c)+(a+b-c)+(a-b)=3 a$
First Diagonal: $(a+b)+(a)+(a-b)=3 a$
Second Diagonal: $(a-c)+(a)+(a+c)=3 a$
72. Answers will vary. Possible answer: $5,10,15$ or $5,10,20$.
$5 \times 1=5$
$5 \times 2^{0}=5$
$5 \times 2=10$
$5 \times 2^{1}=10$
$5 \times 3=15$
$5 \times 2^{2}=20$
73. a. The result is a three- or four- digit number in which the thousands and hundreds places represent the month of the birthday and the tens and ones places represent the day of the birthday.
b. $\quad 5[4(5 M+6)+9]+D-165$
$=5[20 M+24+9]+D-165$
$=5[20 M+33]+D-165$
$=100 M+165+D-165$
$=100 M+D$
74. a. $6 \times 6=36$
$66 \times 66=4356$
$666 \times 666=443,556$
$6666 \times 6666=44,435,556$
b. An additional digit of 6 is attached to the numbers being multiplied. An additional digit of 4 is attached to the left of the result and an additional digit of 5 is placed between the 3 and the 6.
c. $66666 \times 66666=4,444,355,556$
$666,666 \times 666,666=444,443,555,556$
d. Inductive reasoning; it uses an observed pattern and draws conclusions from that pattern.
75. a. $3367 \times 3=10101$
$3367 \times 6=20202$
$3367 \times 9=30303$
$3367 \times 12=40404$
b. The first multiplier is always 3367 . The second multipliers are successive multiples of 3 . The product increases by 10101.
c. $3367 \times 15=50505$
$3367 \times 18=60606$
d. Inductive reasoning; it uses an observed pattern and draws conclusions from that pattern.

## Check Points 1.2

1. a. The digit to the right of the billions digit is less than 5. Thus, replace all the digits to the right with zeroes. $7,476,242,056$ rounded to the nearest billion is $7,000,000,000$.
b. The digit to the right of the ten millions digit is 5 or greater. Thus, add 1 to the ten millions digit and replace all the digits to the right with zeroes. $7,48,242,056$ rounded to the nearest hundred millions is $7,480,000,000$.
2. a. The digit to the right of the tenths digit is less than 5. Thus, 3.141593 rounded to the nearest tenth is 3.1.
b. The digit to the right of the ten-thousandths digit is greater than 5 . Thus, 3.141593 rounded to the nearest ten-thousandth is 3.1416 .
3. a. $\$ 3.40+\$ 2.25+\$ 5.60+\$ 5.40+\$ 3.40$
$+\$ 2.85+3.95$
$\approx \$ 3+\$ 2+\$ 6+\$ 5+\$ 3+\$ 3+4$
$\approx \$ 26$
b. The given bill is not reasonable. It is too high.
4. a. Round $\$ 52$ per hour to $\$ 50$ per hour and assume 40 hours per week.
$\frac{40 \text { hours }}{\text { week }} \times \frac{\$ 50}{\text { hour }}=\frac{\$ 2000}{\text { week }}$
The architect's salary is $\approx \$ 2000$ per week.
b. Round 52 weeks per year to 50 weeks per year.
$\frac{\$ 2000}{\text { week }} \times \frac{50 \text { weeks }}{\text { year }}=\frac{\$ 100,000}{\text { year }}$
The architect's salary is $\approx \$ 100,000$ per year.
5. a. $0.48 \times 2148.72$
b. $\quad 0.5 \times 2100=1050$

Your family spent approximately $\$ 1050$ on heating and cooling last year.
6. a. The yearly increase in life expectancy can be approximated by dividing the change in life expectancy by the change in time from 1950 to 2020. $\frac{81.9-71.1}{2020-1950}=\frac{10.8}{70} \approx 0.15 \mathrm{yr}$ for each subsequent birth year.

$$
\begin{aligned}
& \text { b. } \quad \overbrace{71.1}^{\begin{array}{c}
\text { life expectancy } \\
\text { in } 1950
\end{array}}+\overbrace{0.15}^{\begin{array}{c}
\text { yearly } \\
\text { increase }
\end{array}}(\overbrace{2050-1950}^{\begin{array}{c}
\text { number of years } \\
\text { from } 1950 \\
\text { to } 2050
\end{array}}) \\
& =71.1+0.15(100) \\
& =71.1+15 \\
& =86.1 \mathrm{yr}
\end{aligned}
$$

7. a. about $22 \%$
b. The greatest rate of increase in the percentage of college students who smoked cigarettes can be found by identifying the portion of the graph with the largest upward slope. This occurs between 1994 and 1998.
c. Approximately $24 \%$ of college students smoked cigarettes in 1982 and 1994.
d. 2014 was the year in which the least percentage of college students smoked cigarettes. The percentage that year was $13 \%$.
8. a. The yearly increase in tuition and fees can be approximated by dividing the change in tuition and fees by the change in time from 2000 to
9. $\frac{\$ 33,480-\$ 15,518}{2016-2000}=\frac{\$ 17,962}{16} \approx \$ 1123$
b. $T=\overbrace{15,518}^{\begin{array}{c}\text { Cost in } \\ 2000\end{array}}+\overbrace{1123} x$
c. 2020 is 20 years after 2000 . Thus, $T=15,518+1123 x$
$=15,518+1123(20)$
$=\$ 37,978$

## Concept and Vocabulary Check 1.2

1. estimation
2. circle graph
3. mathematical model
4. true
5. true
6. false

## Exercise Set 1.2

1. a. $39,144,200$
b. $39,145,000$
c. $39,140,000$
d. $39,100,000$
e. $39,000,000$
f. $40,000,000$
2. a-f. Answers will vary depending upon state chosen.
3. 2.718
4. 2.7183
5. 2.71828
6. 2.718282
7. 2.718281828
8. 2.7182818285
9. $350+600=950$

Actual answer of 955 compares reasonably well
10. $250+800=1050$

Actual answer of 1045 compares reasonably well
11. $9+1+19=29$

Actual answer of 29.23 compares quite well
12. $8+3+24=35$

Actual answer of 35.34 compares quite well
13. $32-11=21$

Actual answer of 20.911 compares quite well
14. $46-15=31$

Actual answer of 30.893 compares quite well
15. $40 \times 6=240$

Actual answer of 218.185 compares not so well
16. $80 \times 7=560$

Actual answer of 512.98 compares not so well
17. $0.8 \times 400=320$

Actual answer of 327.06 compares reasonably well
18. $0.7 \times 200=140$

Actual answer of 141.37 compares quite well
19. $48 \div 3=16$

Actual answer of 16.49 compares quite well
20. $55 \div 5=11$

Actual answer of 11.62 compares quite well
21. $30 \%$ of 200,000 is 60,000

Actual answer of $59,920.96$ compares quite well
22. $40 \%$ of 300,000 is 120,000

Actual answer of $122,432.52$ compares reasonably well
23. $\$ 3.47+\$ 5.89+\$ 19.98+\$ 2.03+\$ 11.85+\$ 0.23$
$\approx \$ 3+\$ 6+\$ 20+\$ 2+\$ 12+\$ 0$
$\approx \$ 43$
24. $\$ 4.23+\$ 7.79+\$ 28.97+\$ 4.06+\$ 13.03+\$ 0.74$
$\approx \$ 4+\$ 8+\$ 29+\$ 4+\$ 13+\$ 1$
$\approx \$ 59$
25. Round $\$ 19.50$ to $\$ 20$ per hour.

40 hours per week
$(40 \times \$ 20)$ per week $=\$ 800 /$ week
Round 52 weeks to 50 weeks per year.
50 weeks per year
$(50 \times \$ 800)$ per year $=\$ 40,000$
$\$ 19.50$ per hour $\approx \$ 40,000$ per year
26. Round $\$ 29.85$ to $\$ 30$ per hour.

40 hours per week
$(40 \times \$ 30)$ per week $=\$ 1200 /$ week
Round 52 weeks to 50 weeks per year.
50 weeks per year
$(50 \times \$ 1200)$ per year $=\$ 60,000$
$\$ 29.85$ per hour $\approx \$ 60,000$ per year
27. Round the $\$ 605$ monthly payment to $\$ 600$. 3 years is 36 months.
Round the 36 months to 40 months.
$\$ 600 \times 40$ months $=\$ 24,000$ total cost.
$\$ 605$ monthly payment for 3 years $\approx \$ 24,000$ total cost.
28. Round the $\$ 415$ monthly payment to $\$ 400$.

4 years is 48 months.
Round the 48 months to 50 months.
$\$ 400 \times 50$ months $=\$ 20,000$ total cost.
$\$ 415$ monthly payment for 3 years $\approx \$ 20,000$ total cost.
29. Round the raise of $\$ 310,000$ to $\$ 300,000$.

Round the 294 professors to 300.
$\$ 300,000 \div 300$ professors $=\$ 1000$ per professor.
$\$ 310,000$ raise $\approx \$ 1000$ per professor.
30. Round the raise of $\$ 310,000$ to $\$ 300,000$.

Round the 196 professors to 200.
$\$ 300,000 \div 200$ professors $=\$ 1500$ per professor.
$\$ 310,000$ raise $\approx \$ 1500$ per professor.
31. Round $\$ 61,500$ to $\$ 60,000$ per year.

Round 52 weeks per year to 50 weeks per year.
50 weeks $\times 40$ hours per week $=2000$ hours
$\$ 60,000 \div 2000$ hours $=\$ 30$ per hour
$\$ 61,500$ per year $\approx \$ 30$ per hour
32. Round $\$ 38,950$ to $\$ 40,000$ per year.

Round 52 weeks per year to 50 weeks per year.
50 weeks $\times 40$ hours per week $=2000$ hours
$\$ 40,000 \div 2000$ hours $=\$ 20$ per hour
$\$ 38,950$ per year $\approx \$ 20$ per hour
33. $80 \times 365 \times 24=700,800 \mathrm{hr}$
34. $40 \times 365 \times 24=350,400 \mathrm{hr}$
35. $\frac{0.2 \times 100}{0.5}=\frac{20}{0.5}=40$

Actual answer of 42.03 compares quite reasonable.
36. $\frac{0.5 \times 90}{0.25}=\frac{45}{0.25}=180$

Actual answer of 169.62 compares somewhat reasonable.
37. The given information suggests $\$ 30$ would be a good estimate per calculator.
$\$ 30 \times 10=\$ 300$ which is closest to choice b.
38. The given information suggests $\$ 7$ would be a good estimate per calculator.
$\$ 7 \times 10=\$ 70$ which is closest to choice c .
39. The given information suggests 65 mph would be a good rate estimate and 3.5 would be a good time estimate.
$65 \times 3.5=227.5$ which is closest to choice $c$.
40. The given information suggests 45 mph would be a good rate estimate and 3.5 would be a good time estimate.
$45 \times 3.5=157.5$ which is closest to choice $b$.
41. The given information suggests you can count 1 number per second.
$\frac{10000}{60 \times 60} \approx 2.77$ or 3 hours
42. The given information suggests you can count 1 number per second.
$\frac{1,000,000}{60 \times 60 \times 24} \approx 11.57$ or 12 days
43. $\approx 0.10 \times 16,000,000=1,600,000$
$10 \%$ of $16,000,000$ is $1,600,000$ high school teenagers.
44. $\approx 0.20 \times 16,000,000=3,200,000$
$20 \%$ of $16,000,000$ is $3,200,000$ high school teenagers.
45. a. about 85 people per 100
b. $(85-23) \times 87 \approx 5400$
46. a. about 66 people per 100
b. $(66-27) \times 72 \approx 2800$
47. a. $\frac{26.3-9.7}{2012-1980}=\frac{16.6}{32} \approx 0.5$

The annual increase is about $0.5 \%$.
b. $\quad 9.7+0.5(2020-1980)=9.7+0.5(40)$

$$
=29.7
$$

In 2020 the percentage will be approximately 29.7\%
48. a. $\frac{21.7-6.7}{2012-1980}=\frac{15.9}{32} \approx 0.5$

The annual increase is about $0.5 \%$.
b. $\quad 6.7+0.5(2020-1980)=6.7+0.5(40)$

$$
=26.7
$$

In 2020 the percentage will be approximately 26.7\%.
49. a. The percent body fat in 45 -year-old women is about $37 \%$.
b. The percent body fat in women reaches a maximum at age 55 of about $38 \%$
c. Women have $34 \%$ body fat at age 25 .
50. a. The percent body fat in 25 -year-old men is about $23 \%$.
b. The percent body fat in men reaches a maximum at age 65 of about $26 \%$
c. Men have $24 \%$ body fat at age 35 .
51. a. $\frac{401-310}{2015-1950}=\frac{91}{65} \approx 1.4 \mathrm{ppm}$ per year
b. $\quad C=310+1.4 x$
c. 2050 is 100 years after 1950 .

$$
C=310+1.4(100)=450 \mathrm{ppm}
$$

52. a. $\frac{58.44-56.98}{2015-1950}=\frac{1.46}{65} \approx 0.02^{\circ} \mathrm{F}$
b. $\quad T=56.98+0.02 x$
c. 2050 is 100 years after 1950 .

$$
T=56.98+0.02(100)=58.98^{\circ} \mathrm{F}
$$

66. makes sense
67. does not make sense; Explanations will vary. Sample explanation: Very large numbers and very small numbers often must be estimated when using a calculator.
68. makes sense
69. does not make sense; Explanations will vary. Sample explanation: Some mathematical models can break down over time.
70. Since there are infinitely many digits, the digits can not be reversed.
71. a
72. d
73. b
74. c
75. $20 \times 16 \times 50=16,000$ hours.

$$
\frac{16,000}{24} \approx 667 \text { days }
$$

$$
\frac{667}{365} \approx 1.8 \mathrm{yr}
$$

76. Round days in a year to 400 .

$$
\begin{aligned}
\frac{\$ 1,000,000,000}{\$ 1000 / \text { day }} & =1,000,000 \text { days } \\
& \approx \frac{1,000,000 \text { days }}{400 \text { days } / \text { year }} \\
& \approx 2500 \text { years }
\end{aligned}
$$

## Check Points 1.3

1. The amount of money given to the cashier is unknown.
2. Step 1: Understand the problem.

Bottles: 128 ounces costs $\$ 5.39$
Boxes: a 9-pack of 6.75 ounce boxes costs $\$ 3.15$
We must determine whether bottles or boxes are the better value.
Step 2: Devise a plan.
Dividing the cost by the number of ounces will give us the cost per ounce. We will need to multiply 9 by 6.75 to determine the total number of ounces the boxes contain. The lower cost per ounce is the best value.
Step 3: Carry out the plan and solve the problem. Unit price for the bottles:
$\frac{\$ 5.39}{128 \text { ounces }} \approx \$ 0.042$ per ounce
Unit price for the boxes:
$\frac{\$ 3.15}{9 \times 6.75 \text { ounces }}=\frac{\$ 3.15}{60.75 \text { ounces }} \approx \$ 0.052$ per ounce
Bottles have a lower price per ounce and are the better value.
Step 4: Look back and check the answer.
This answer satisfies the conditions of the problem.
3. Step 1: Understand the problem.

We are given the cost of the computer, the amount of cash paid up front, and the amount paid each month. We must determine the number of months it will take to finish paying for the computer.
Step 2: Devise a plan.
Subtract the amount paid in cash from the cost of the computer. This results in the amount still to be paid. Because the monthly payments are $\$ 45$, divide the amount still to be paid by 45 . This will give the number of months required to pay for the computer. Step 3: Carry out the plan and solve the problem. The balance is $\$ 980-\$ 350=\$ 630$. Now divide the $\$ 630$ balance by $\$ 45$, the monthly payment.

$$
\$ 630 \div \frac{\$ 45}{\text { month }}=\$ 630 \times \frac{\text { month }}{\$ 45}=\frac{630 \text { months }}{45}=14 \text { months } .
$$

Step 4: Look back and check the answer. This answer satisfies the conditions of the problem. 14 monthly payments at $\$ 45$ each gives $14 \times \$ 45=\$ 630$. Adding in the up front cash payment of $\$ 350$ gives us $\$ 630+\$ 350=\$ 980$. $\$ 980$ is the cost of the computer.
4. Step 1: Understand the problem.

The total change must always be 30 cents. One possible coin combination is six nickels. Another is three dimes. We need to count all such combinations.
Step 2: Devise a plan.
Make a list of all possible coin combinations. Begin with the coins of larger value and work toward the coins of smaller value.
Step 3: Carry out the plan and solve the problem.

| Quarters | Dimes | Nickels |
| :---: | :---: | :---: |
| 1 | 0 | 1 |
| 0 | 3 | 0 |
| 0 | 2 | 2 |
| 0 | 1 | 4 |
| 0 | 0 | 6 |

There are 5 combinations.
Step 4: Look back and check the answer.
Check to see that no combinations are omitted, and that those given total 30 cents. Also double-check the count.
5. Step 1: Understand the problem.

We must determine the number of jeans/T-shirt combinations that we can make.
For example, one such combination would be to wear the blue jeans with the beige shirt.
Step 2: Devise a plan.

Each pair of jeans could be matched with any of the three shirts. We will make a tree diagram to show all combinations.
Step 3: Carry out the plan and solve the problem.


There are 6 different outfits possible.
Step 4: Look back and check the answer. Check to see that no combinations are omitted, and double-check the count.
6. Step 1: Understand the problem.

There are many possible ways to visit each city once and then return home. We must find a route that costs less than $\$ 1460$.
Step 2: Devise a plan.
From city $A$ fly to the city with the cheapest
available flight. Repeat this until all cities have been visited and then fly home. If this cost is above $\$ 1460$ then use trial and error to find other alternative routes.
Step 3: Carry out the plan and solve the problem.
$A$ to $D$ costs $\$ 185, D$ to $E$ costs $\$ 302, E$ to $C$ costs
$\$ 165, C$ to $B$ costs $\$ 305, B$ back to $A$ costs $\$ 500$
$\$ 185+\$ 302+\$ 165+\$ 305+\$ 500=\$ 1457$
The route $A, D, E, C, B, A$ costs less than $\$ 1460$
Step 4: Look back and check the answer.
This answer satisfies the conditions of the problem.

## Trick Questions 1.3

1. The farmer has 12 sheep left since all but 12 sheep died.
2. All 12 months have [at least] 28 days.
3. The doctor and brother are brother and sister.
4. You should light the match first.

## Concept and Vocabulary Check 1.3

1. understand
2. devise a plan
3. false
4. false

## Exercise Set 1.3

1. The price of the computer is needed.
2. The weight of the steak is needed.
3. The number of words per page is needed.
4. The amount of the payments is needed.
5. Weekly salary is unnecessary information.
$212-200=12$ items sold in excess of 200
$12 \times \$ 15=\$ 180$ extra is received.
6. Tire weight is unnecessary information.
$4 \times(\$ 42+\$ 2.50)=4 \times \$ 44.50=\$ 178.00$
$\$ 250-\$ 178=\$ 72$ remaining after purchase.
7. How much the attendant was given is not necessary.

There were 5 hours of parking.
1st hour is $\$ 2.50$
4 hours at $\$ 0.50 / \mathrm{hr}$

$$
\begin{aligned}
\$ 2.50+(4 \times \$ 0.50) & =\$ 2.50+\$ 2.00 \\
& =\$ 4.50
\end{aligned}
$$

$\$ 4.50$ was charged.
8. The width of the house is not necessary.

90 feet $=15 \times 6$ feet
The line representing the length is 15 inches.
9. a. Step 1: Understand the problem.

Box \#1: 15.3 ounces costs $\$ 3.37$
Box \#2: 24 ounces costs $\$ 4.59$
We must determine whether Box \#1 or Box \#2 is the better value.
Step 2: Devise a plan.
Dividing the cost by the number of ounces will give us the cost per ounce. The lower cost per ounce is the best value.
Step 3: Carry out the plan and solve the problem.
Unit price for Box \#1:
$\frac{\$ 3.37}{15.3 \text { ounces }} \approx \$ 0.22$ per ounce
Unit price for Box \#2:
$\frac{\$ 4.59}{24 \text { ounces }} \approx \$ 0.19$ per ounce
The cereal that is 24 ounces for $\$ 4.59$ is the better value.
Step 4: Look back and check the answer.
This answer satisfies the conditions of the problem.
b. Unit price for Box \#1: $\$ 0.22$ per ounce

Unit price for Box \#2:
$\frac{\$ 4.59}{24 \text { ounces }} \times \frac{16 \text { ounces }}{\text { pound }} \approx \$ 3.06$ per pound
c. No, explanations will vary.
10. a. Step 1: Understand the problem.

Jar \#1: 12 ounces costs $\$ 2.25$
Jar \#2: 18 ounces costs \$3.24
We must determine whether Jar \#1 or Jar \#2 is the better value.
Step 2: Devise a plan.
Dividing the cost by the number of ounces will give us the cost per ounce. The lower cost per ounce is the best value.
Step 3: Carry out the plan and solve the problem.
Unit price for Jar \#1:
$\frac{\$ 2.25}{12 \text { ounces }} \approx \$ 0.19$ per ounce
Unit price for Jar \#2:
$\frac{\$ 3.24}{18 \text { ounces }} \approx \$ 0.18$ per ounce
The honey that is 18 ounces for $\$ 3.24$ is the better value.
Step 4: Look back and check the answer.
This answer satisfies the conditions of the problem.
b. Unit price for Jar \#1: $\$ 0.19$ per ounce

Unit price for Jar \#2:
$\frac{\$ 3.24}{18 \text { ounces }} \times \frac{32 \text { ounces }}{\text { quart }} \approx \$ 5.76$ per quart
c. No, explanations will vary.
11. Step 1: Comparing two yearly salaries

Step 2:
Convert the second person's wages to yearly salary. Step 3:
The person that earns $\$ 3750 /$ month earns
$12 \times \$ 3750=\$ 45,000 /$ year. The person that earns $\$ 48,000 /$ year gets $\$ 3000$ more per year.
Step 4:
It appears to satisfy the conditions of the problem.
12. Step 1:

Find the car's mileage for one year, and gas usage.
Step 2:
Subtract beginning odometer reading from ending reading.
Step 3:
37,364 miles $-25,124$ miles $=12,240$ miles
$\frac{12,240 \mathrm{miles}}{24 \mathrm{mpg}}=510$ gallons
Step 4:
It satisfies the conditions of the problem.
13. Step 1:

Find the difference between two options.
Step 2:
Compute total costs and compare two figures.
Step 3:
Redbox: 7 nights plus 3 additional nights is 10
nights, or $\$ 10$.
Netflix: $\$ 7.99$ per month.
$\$ 10-7.99=\$ 2.01$
Netflix would be $\$ 2.01$ less.
Step 4:
It satisfies the conditions of problem.
14. Step 1:

Find the sum of costs of each service.
Step 2:
Compute total costs.
Step 3:
Netflix: $\$ 7.99$ per month.
iTunes: $\$ 3.99 \times 2=\$ 7.98$
Total: $\$ 7.99+(\$ 3.99 \times 2)=\$ 15.97$
It satisfies the conditions of problem.
15. a. $\frac{3250 \mathrm{mg} / \text { day }}{500 \mathrm{mg} / \text { pill }}=6.5$ pills per day $=6$ pills per day limit
b. First find mg from 3 Tylenol pills:
$3 \times 500 \mathrm{mg}=1500 \mathrm{mg}$
Next, subtract that result from the daily maximum:
$3250 \mathrm{mg}-1500 \mathrm{mg}=1750 \mathrm{mg}$
Finally, calculate the number of NyQuil pills that could be taken:

$$
\begin{aligned}
\frac{1750 \mathrm{mg} \text { remaining }}{325 \mathrm{mg} / \mathrm{pill}} & \approx 5.38 \text { pills remaining } \\
& =5 \text { pill limit }
\end{aligned}
$$

16. a. $\frac{3250 \mathrm{mg} / \text { day }}{325 \mathrm{mg} / \mathrm{pill}}=10$ pills per day
b. First find mg from 4 NyQuil pills:
$4 \times 325 \mathrm{mg}=1500 \mathrm{mg}$
Next, subtract that result from the daily maximum:
$3250 \mathrm{mg}-1500 \mathrm{mg}=1750 \mathrm{mg}$
Finally, calculate the number of Tylenol pills that could be taken:

$$
\begin{aligned}
\frac{1750 \mathrm{mg} \text { remaining }}{500 \mathrm{mg} / \mathrm{pill}} & =3.5 \text { pills remaining } \\
& =3 \text { pill limit }
\end{aligned}
$$

17. Step 1:

Find the difference between two methods of payment.
Step 2:
Compute total costs and compare two figures.
Step 3:
By spreading purchase out, the total comes to:
$\$ 100+14(\$ 50)=\$ 100+\$ 700=\$ 800$
$\$ 800-\$ 750=\$ 50$ saved by paying all at once
Step 4:
It satisfies the conditions of problem.
18. Step 1: Determine which team won.

Step 2: Compile point totals and compare.
Step 3:
Bulldogs: $34 \times 2+13=68+13=81$ points
Panthers: $38 \times 2+8=76+8=84$ points
Panthers won by 3 points
Step 4:
It satisfies the conditions of the problem.
19. Step 1 :

Determine profit on goods sold.
Step 2:
Find total cost of buying product and comparing with gross sales.
Step 3:
Purchased: (\$65 per dozen)(6 dozen) = \$390
Sold: 6 dozen $=72$ calculators
$\frac{72}{3}=24$ groups of 3 at $\$ 20$ per group.
$24 \times \$ 20=\$ 480$
$\$ 480-\$ 390=\$ 90$ profit
Step 4:
It satisfies the conditions of the problem.
20. Step 1: Determine profit.

Step 2:
Determine cost of purchase and compare with gross sales.
Step 3:
Bought:
$(\$ 0.95 /$ dozen $)(15$ dozen $)=\$ 14.25$
Sold:
15 dozen $=180$ pens
$\frac{180}{4}=45$ packs
45 packs $\times \$ 2.25=\$ 101.25$
Profit $=\$ 101.25-\$ 14.25=\$ 87$
Step 4:
It satisfies the conditions of the problem.
21. Step 1: Determine profit for ten-day period.

Step 2: Compare totals.
Step 3:
$(200$ slices $)(\$ 1.50)=\$ 300$ for pizza
$(85$ sandwiches $)(\$ 2.50)=\$ 212.50$ for sandwiches
For 10 day period:
Gross: $10(\$ 300)+10(\$ 212.50)=\$ 3000+\$ 2125.00$

$$
=\$ 5125.00
$$

Expenses: $10(\$ 60)=\$ 600$
Profit: $\$ 5125.00-\$ 600=\$ 4525$
Step 4:
It satisfies the conditions of the problem.
22. Step 1: Determine how much was earned over twoweek period.
Step 2: Compute each week's earnings and total.
Step 3:
1st week:
(40 hours)(\$8.15 per hour) + (2 hours) $(\$ 8.15+\$ 2.20)$
$=\$ 326+(2$ hours $)(\$ 10.35)$
$=\$ 326+\$ 20.70$
= \$346.70
2nd week:
(40 hours)(\$8.15 per hour) $+(5$ hours)(\$10.35)
$=\$ 326+\$ 51.75$
$=\$ 377.75$
Total is $\$ 346.70+\$ 377.75=\$ 724.45$
Step 4:
It satisfies the conditions of problem.
23. Step 1:

Compute total rental cost.
Step 2:
Add rental cost and mileage cost to get total cost.
Step 3:
Rental costs:
$(2$ weeks $)(\$ 220$ per week $)=\$ 440$
Mileage: $(500$ miles $)(\$ 0.25)=\$ 125$
Total: $\$ 440+\$ 125=\$ 565$
Step 4:
It satisfies the conditions of problem.
24. Step 1:

We are trying to figure out an annual budget and see how much is left to buy stock and how many shares of stock.
Step 2:
Determine annual earnings and compare with
budget.
Step 3:
Salary:
$(12$ months/year)(\$2750/month $)=\$ 33,000$
Expenses:
$\$ 4800+\$ 8200+\$ 3750+\$ 4250+\$ 3000$
$=\$ 24,000$
Available to buy stocks:
$\$ 33,000-\$ 24,000=\$ 9000$
Number of stocks: $\frac{\$ 9000}{\$ 375 / \text { share }}=24$ shares
Step 4:
It satisfies the conditions of problem.
25. Step 1:

A round trip was made; we need to determine how much was walked or ridden.
Step 2:
Add up the totals walked and ridden and compare.
Step 3:
It is 5 miles between the homes or a 10 mile round trip. The first 3 were covered with the bicycle, leaving 7 miles covered by walking.
7 miles -3 miles $=4$ miles more that was walked.
Step 4:
It satisfies the conditions of the problem.
26. Step 1:

Determine the profit on goods sold.
Step 2:
Determine the cost of obtaining product and compare with gross sales.
Step 3:
Cost:
200 containers at $\$ 0.75$ apiece $=\$ 150$
Cost $=\$ 150$
Gross sales: $(150$ containers $)(\$ 1.25)=\$ 187.50$
50 containers returned for $\$ 0.50$ refund:
$50(\$ 0.50)=\$ 25.00$
Total received: $\$ 187.50+\$ 25.00=\$ 212.50$
Total Profit: $\$ 212.50-\$ 150.00=\$ 62.50$
Step 4:
It satisfies the conditions of the problem.
27. Step 1 :

Determine profit by comparing expenses with gross sales.
Step 2:
Calculate expenses and gross sales and compare.
Step 3:
Expense:
$(25$ calculators $)(\$ 30)=\$ 750$
Gross Sales:
$(22$ calculators $)(\$ 35.00)=\$ 770$
The storeowner receives $\$ 30-\$ 2=\$ 28$ for each returned calculator.
(3 calculators) $(\$ 28)=\$ 84$
Total Income:
$\$ 770+\$ 84=\$ 854$
Profit=Income-Expenses

$$
\begin{aligned}
& =\$ 854-\$ 750 \\
& =\$ 104
\end{aligned}
$$

Step 4:
It satisfies the conditions of the problem.
28. When the drivers meet for lunch they have been traveling for 2 hours and 24 minutes or 2.4 hours.
The car from New York City has traveled ( 2.4 hours) $(55$ miles per hour) $=132$ miles.
The car from D.C. has traveled
$(2.4$ hours $)(45$ miles per hour $)=108$ miles.
29. The car depreciates at
$\frac{23,000-2700}{7}=\$ 2900$ per year .
$23,000-3(2900)=\$ 14,300$
30. The car depreciates at
$\frac{34,800-8550}{7}=\$ 3750$ per year .
$34,800-3(3750)=\$ 23,550$
31. Use a list.

| 2 Quarters | 3 Dimes | 5 Nickels |
| :---: | :---: | :---: |
| 1 | 2 | 0 |
| 1 | 1 | 2 |
| 1 | 0 | 4 |
| 0 | 3 | 3 |
| 0 | 2 | 5 |

There are 5 ways.
32. Use a list.

| Pennies | Nickels | Dimes |
| :---: | :---: | :---: |
| 25 | 0 | 0 |
| 20 | 1 | 0 |
| 15 | 2 | 0 |
| 15 | 0 | 1 |
| 10 | 3 | 0 |
| 10 | 1 | 1 |
| 5 | 4 | 0 |
| 5 | 2 | 1 |
| 5 | 0 | 2 |
| 0 | 5 | 0 |
| 0 | 3 | 1 |
| 0 | 1 | 2 |

There are 12 ways.
33. Make a list of all possible selections: Watson/Clooney, Watson/DiCaprio, Watson/Lawrence,
Clooney/DiCaprio, Clooney/Lawrence, DiCaprio/Lawrence
There are 6 ways.
34. Make a list of all possible selections: Gates/Winfrey, Gates/Zuckerberg, Gates/Clinton, Gates/Curry,
Winfrey/Zuckerberg, Winfrey/Clinton, Winfrey/Curry,
Zuckerberg/Clinton, Zuckerberg/Curry, Clinton/Curry, There are 10 ways.
35. Use a list.

| Pennies | Nickels | Dimes |
| :---: | :---: | :---: |
| 21 | 0 | 0 |
| 16 | 1 | 0 |
| 11 | 2 | 0 |
| 11 | 0 | 1 |
| 6 | 3 | 0 |
| 6 | 1 | 1 |
| 1 | 4 | 0 |
| 1 | 2 | 1 |
| 1 | 0 | 2 |

There are 9 ways.
36. Use a list

| Pennies | Nickels | Dimes | Quarters |
| :---: | :---: | :---: | :---: |
| 26 | 0 | 0 | 0 |
| 21 | 1 | 0 | 0 |
| 16 | 2 | 0 | 0 |
| 16 | 0 | 1 | 0 |
| 11 | 3 | 0 | 0 |
| 11 | 1 | 1 | 0 |
| 6 | 4 | 0 | 0 |
| 6 | 2 | 1 | 0 |
| 6 | 0 | 2 | 0 |
| 1 | 5 | 0 | 0 |
| 1 | 3 | 1 | 0 |
| 1 | 1 | 2 | 0 |
| 1 | 0 | 0 | 1 |

There are 13 ways.
37. Use a list.

| 1 pt | 5 pt | $10-\mathrm{pt}$ | Total |
| :---: | :---: | :---: | :---: |
| 3 | 0 | 0 | 3 |
| 2 | 1 | 0 | 7 |
| 1 | 2 | 0 | 11 |
| 2 | 0 | 1 | 12 |
| 0 | 3 | 0 | 15 |
| 1 | 1 | 1 | 16 |
| 0 | 2 | 1 | 20 |
| 1 | 0 | 2 | 21 |
| 0 | 1 | 2 | 25 |
| 0 | 0 | 3 | 30 |

There are 10 different totals.
38. Answers will vary. An example is 61, 9, 7, 23.
39. The average expense is
$\frac{42+10+26+32+30}{5}=\$ 28$
Thus, B owes $\$ 18$ and C owes $\$ 2$, A is owed $\$ 14$, D is owed $\$ 4$, and E is owed $\$ 2$.
To resolve these discrepancies, B should give A \$14 and give $\mathrm{D} \$ 4$, while C should give $\mathrm{E} \$ 2$.
40. At 5 minutes between houses, it will take 30 minutes to walk all the way around.
41. Make a list of all possible orders:

TFFF, FTFF, FFTF, FFFT
The "True" could be written $1^{\text {st }}, 2^{\text {nd }}, 3^{\text {rd }}$, or $4^{\text {th }}$.
There are 4 ways.
42. Represent the 5 people as A, B, C, D, and E and make a list of all possible handshakes:
$\mathrm{AB}, \mathrm{AC}, \mathrm{AD}, \mathrm{AE}, \mathrm{BC}, \mathrm{BD}, \mathrm{BE}, \mathrm{CD}, \mathrm{CE}$ and DE . There are 10 handshakes exchanged.
43. The order the racers finished was; Andy, Darnell, Caleb, Beth, Ella.

44. To determine a winner, seven teams must be eliminated. Since each game eliminates one team, this tournament will take seven games.
45. Home $\rightarrow$ Bank $\rightarrow$ Post Office $\rightarrow$ Dry Cleaners $\rightarrow$ Home will take 11.5 miles.
46. Home $\rightarrow$ Post Office $\rightarrow$ Dry Cleaners $\rightarrow$ Bank $\rightarrow$ Home will take 12.5 miles.
47. $\mathrm{CO} \rightarrow \mathrm{WY} \rightarrow \mathrm{UT} \rightarrow \mathrm{AZ} \rightarrow \mathrm{NM} \rightarrow \mathrm{CO} \rightarrow \mathrm{UT}$

48. One possible route is indicated:

49. The problem states that the psychology major knocks on Jose's wall, and Jose's dorm is adjacent to Bob's dorm but not Tony's. Therefore Bob is the psychology major.
50. A minimum of 4 colors are needed.
51. a.

| 5 | 22 | 18 |
| :---: | :---: | :---: |
| 28 | 15 | 2 |
| 12 | 8 | 25 |

b.

| 4 | 9 | 8 |
| :---: | :---: | :---: |
| 11 | 7 | 3 |
| 6 | 5 | 10 |

52. a.

| 96 | 64 | 37 | 45 |
| :--- | :--- | :--- | :--- |
| 39 | 43 | 98 | 62 |
| 84 | 76 | 25 | 57 |
| 23 | 59 | 82 | 78 |

b.

| 69 | 46 | 73 | 54 |
| :--- | :--- | :--- | :--- |
| 93 | 34 | 89 | 26 |
| 48 | 67 | 52 | 75 |
| 32 | 95 | 28 | 87 |

53. 

| 9 | 6 | 7 |
| :--- | :--- | :--- |
| 8 | 1 | 4 |
| 3 | 2 | 5 |

54. 

| 2 | 3 | 3 | 6 |
| :--- | :--- | :--- | :--- |
| 4 | 1 | 4 | 6 |
| 1 | 1 | 4 | 8 |
| 9 | 8 | 2 | 5 |

55. 

$2 8 \longdiv { 4 3 6 8 }$
$\underline{28}$

140
168
$\underline{168}$
0
60. does not make sense; Explanations will vary. Sample explanation: Polya's four steps are a guide.

They will not necessarily make the solution quick and/or easy.
61. makes sense
62. makes sense
63. does not make sense; Explanations will vary. Sample explanation: When you are bogged down with a problem, it can often be helpful to stop working on it and return to it later.
64. A total of twenty 7 s are needed.

Possible solution: Notice that the digit 7 will occur 10 times in the tens digit and 10 times in the ones digit.
An alternative solution would be to make a list of every instance of a seven; $7,17,27,37,47,57,67$, $70,71,72,73,74,75,76,77,78,79,87,97$; and then count the occurrences of the digit 7 .
65. You should choose the dentist whose teeth show the effects of poor dental work because he took good care of the other dentist's teeth.
66. Use one plank diagonally across one corner of the square. Use the other plank to go from the first plank to the stone.
67. It is Friday. The first person is lying (as expected) because he told the truth on Thursday. The second is truthfully admitting that he lied the previous day.
68. The farmer can use the following strategy: Take the goat to the other side of the stream and return to get either the wolf or cabbage. The farmer should take that across the stream and bring the goat back to the original side. He then takes across the cabbage or wolf, whichever remains, and leaves it on the other side while he returns to get the goat.
69. Answers will vary.

70. Sudoku Puzzle solution:

| 2 | 3 | 1 | 4 | 9 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 4 | 5 | 7 | 6 | 3 | 8 | 9 | 2 | 1 |
| 8 | 9 | 6 | 7 | 1 | 2 | 3 | 5 | 4 |
| 5 | 7 | 2 | 3 | 8 | 6 | 1 | 4 | 9 |
| 3 | 6 | 4 | 1 | 7 | 9 | 2 | 8 | 5 |
| 9 | 1 | 8 | 5 | 2 | 4 | 7 | 3 | 6 |
| 1 | 8 | 3 | 9 | 4 | 7 | 5 | 6 | 2 |
| 6 | 2 | 9 | 8 | 5 | 3 | 4 | 1 | 7 |
| 7 | 4 | 5 | 2 | 6 | 1 | 8 | 9 | 3 |

71. There is no missing dollar; in the end the customers paid a total of $\$ 27$ of which $\$ 25$ went to the restaurant and $\$ 2$ was stolen by the waiter.
72. From the middle rung the firefighter went up 4, down 6 , up 7 , and up 4 to reach the top rung. This is a net climb of up 9 . Since there are 9 rungs above the middle rung, there must also be 9 rungs below. 9 above +9 below +1 middle $=19$ rungs.
73. Answers will vary. One method is to start by multiplying 30 by each state's fraction of the population.
State A: $30 \times \frac{275}{1890} \approx 4.365$ or 4
State B: $30 \times \frac{383}{1890} \approx 6.079$ or 6
State C: $30 \times \frac{465}{1890} \approx 7.381$ or 7
State D: $30 \times \frac{767}{1890} \approx 12.175$ or 12
Notice that $4,6,7$, and 12 add to 29 , so there is 1 more representative to be allocated. We could give this extra representative to state C because it had the largest decimal part (0.381). This leads to an allocation of state A: 4, state B: 6, state C: 8 , and state D: 12.

## Chapter 1 Review Exercises

1. Deductive; the specific conclusion about Carrie was based on a general statement about all Stephen King books.
2. Inductive; the general conclusion for this next book was based on past specific observations.
3. Pattern: Add 5
$19+5=24$
$4,9,14,19, \underline{24}$
4. Pattern: Multiply by 2
$56 \times 2=112$
$7,14,28,56,112$
5. Pattern: Numbers added increase by 1
$1+2=3$
$3+3=6$
$6+4=10$
$10+5=15$
$15+6=21$
$1,3,6,10,15, \underline{21}$
6. Notice that $\frac{1}{2}=\frac{3}{6}$

Pattern: Add 1 to the denominator
$\frac{3}{7+1}=\frac{3}{8}$
$\frac{3}{4}, \frac{3}{5}, \frac{3}{6}, \frac{3}{7}, \frac{3}{8}$ or
$\frac{3}{4}, \frac{3}{5}, \frac{1}{2}, \frac{3}{7}, \frac{3}{8}$
7. Pattern: Divide by -2
$-5 \div(-2)=\frac{-5}{-2}=\frac{5}{2}$ or $2 \frac{1}{2}$
$40,-20,10,-5, \frac{5}{\underline{2}}$
8. Pattern: Subtract 60
$-140-60=-200$
$40,-20,-80,-140, \underline{-200}$
9. Each number beginning with the third number is the sum of the previous two numbers. $16+26=\underline{42}$
10. To get the second number, multiply the first number by 3 . Then multiply the second number by 2 to get the third number. Then multiply by 3 and then by 2 , repeatedly. $216 \times 2=\underline{432}$
11. The pattern is alternating between square and circle while the line rotates $90^{\circ}$ clockwise. The next figure is
 shown at right.
12. Using inductive reasoning we predict
$2+4+8+16+32=64-2$.
Arithmetic verifies this result:
$2+4+8+16+32=64-2$
$62=62$
13. Using inductive reasoning we predict $444 \div 12=37$.

Arithmetic verifies this result: $444 \div 12=37$

$$
37=37
$$

14. a. Conjecture based on results: The result is the original number.

| Select a number. | 4 | 10 | 0 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| Double the number. | $4 \times 2=8$ | $10 \times 2=20$ |  | $0 \times 2=0$ |
|  | $8+4=12$ |  | $3 \times 2=6$ |  |
| Add 4 to the product. | $12 \div 2=6$ | $20+4=24$ | $0+4=4$ | $6+4=10$ |
| Divide this sum by 2. |  | $24 \div 2=12$ | $4 \div 2=2$ | $10 \div 2=5$ |
| Subtract 2 from the quotient. | $6-2=4$ | $12-2=10$ | $2-2=0$ | $5-2=3$ |
| Summary of results: | $4 \rightarrow 4$ | $10 \rightarrow 10$ | $0 \rightarrow 0$ | $3 \rightarrow 3$ |

b. $2 n$
$2 n+4$

$$
\begin{aligned}
& \frac{2 n+4}{2}=\frac{2 n}{2}+\frac{4}{2}=n+2 \\
& n+2-2=n
\end{aligned}
$$

15. a. $923,187,500$
b. $923,187,000$
c. $923,200,000$
d. $923,000,000$
e. $900,000,000$
16. a. 1.5
b. 1.51
c. $\quad 1.507$
d. 1.5065917
17. $2+4+10=16$

Actual answer: 15.71
quite reasonable
18. $9 \times 50=450$

Actual answer: 432.67
somewhat reasonable
19. $20 \div 4=5$

Actual answer: 4.79
quite reasonable
20. $0.60 \times 4000=2400$

Actual answer: 2397.0548
quite reasonable
21. $\$ 8.47+\$ 0.89+\$ 2.79+\$ 0.14+\$ 1.19+\$ 4.76$

$$
\begin{aligned}
& \approx \$ 8+\$ 1+\$ 3+\$ 0+\$ 1+\$ 5 \\
& \approx \$ 18
\end{aligned}
$$

22. Round 78 hours to 80 , round $\$ 9.95$ to $\$ 10.00$.

$$
\begin{aligned}
78 \times \$ 9.95 & \approx 80 \times \$ 10 \\
& \approx \$ 800
\end{aligned}
$$

23. Round book price to $\$ 1.00$ each.

Round chair price to $\$ 12.00$ each.
Round plate price to $\$ 15.00$.
$(21 \times \$ 0.85)+(2 \times \$ 11.95)+\$ 14.65$
$\approx(21 \times \$ 1)+(2 \times \$ 12)+\$ 15$
$\approx \$ 21+\$ 24+\$ 15$
$\approx \$ 60$
24. $31 \%$ of $20,207,375$ can be estimated as $30 \%$ of 20,000,000.
$0.3 \times 20,000,000 \approx 6,000,000$ students
25. The given information suggests $\$ 900$ would be a good estimate for weekly salary.
$\$ 900 \times 10 \times 4=\$ 36,000$ which is choice b.
26. $60 \times 60 \times 24=86,400$ which is closest to choice $c$.
27. a. The Asian group exceeds 100 . They have a population of about 122.
b. $30 \times 33$ million $=990$ million
28. a. $\frac{32.0-6.0}{2014-1950}=\frac{24.0}{64} \approx 0.4$
average increase: $0.4 \%$ per year
b. $\quad 6.0+0.4(2020-1950)=6.0+0.4(70)=34$

About $34 \%$ of people 25 years of age and older will be college graduates in 2020.
29. a. The woman's maximum heart rate was about 115 beats per minute. This occurred after about 10 minutes.
b. The woman's minimum heart rate was about 64 beats per minute. This occurred after about 8 minutes.
c. between 9 and 10 minutes
d. 9 minutes
30. a. $\frac{309.3-203.3}{2010-1970}=\frac{106}{40} \approx 2.65$ million per year
b. $\quad p=203.3+2.65 x$
c. $\quad p=203.3+2.65(2020-1970)=335.8$ million
31. The weight of the child is needed.
32. The unnecessary information is the customer giving the driver a $\$ 20$ bill.
For a 6 mile trip, the first mile is $\$ 3.00$, and the next 5 miles are $\$ 0.50 /$ half-mile or $\$ 1.00 /$ mile. The cost is

$$
\begin{aligned}
\$ 3.00+(5 \times \$ 1.00) & =\$ 3.00+\$ 5.00 \\
& =\$ 8.00
\end{aligned}
$$

33. Pay-per-Text Plan :
$\$ 0.20(40)+\$ 0.30(35)=\$ 18.50$
Since you are sending a total of 75 messages
$(40+35=75)$, the 200 message Package at $\$ 5.00$
will be sufficient.
Cost Difference:
$\$ 18.50-\$ 5.00=\$ 13.50$
You will save $\$ 13.50$ with the 200-message package.
34. Total of $28 \times 2=56$ frankfurters would be needed. $\frac{56}{7}=8$. Therefore, 8 pounds would be needed.
35. Rental for 3 weeks at $\$ 175$ per week is $3 \times \$ 175=\$ 525$. Mileage for 1200 miles at $\$ 0.30$ per mile is $1200 \times \$ 0.30=\$ 360$. Total cost is $\$ 525$ $+\$ 360=\$ 885$.
36. 12-year Solar Cost :
$\$ 29,700+\$ 200(12)=\$ 32,100$
12-year Electric Cost :
$\$ 5500+\$ 1800(12)=\$ 27,100$
Cost Difference:
$\$ 32,100-\$ 27,100=\$ 5000$
Solar costs would be $\$ 5000$ higher than electric.
37. The flight leaves Miami at 7:00 A.M. Pacific Standard Time. With a lay-over of 45 minutes, it arrives in San Francisco at 1:30 P.M. Pacific Standard Time, 6 hrs 30 min . $-45 \mathrm{~min}=5$ hours 45 minutes.
38. At steady decrease in value:
$\frac{\$ 37,000-\$ 2600}{8 \text { years }}=\frac{\$ 34,400}{8 \text { years }}=\$ 4300 /$ year
After 5 years: $\$ 4300 \times 5=\$ 21,500$ decrease in value
Value of car: $\$ 37,000-\$ 21,500=\$ 15,500$
39. The machine will accept nickels, dimes, quarters.

| nickels | dimes | quarters |
| :---: | :---: | :---: |
| 7 | 0 | 0 |
| 5 | 1 | 0 |
| 3 | 2 | 0 |
| 2 | 0 | 1 |
| 1 | 3 | 0 |
| 0 | 1 | 1 |

There are 6 combinations.

## Chapter 1 Test

1. deductive
2. inductive
3. $0+5=5$
$5+5=10$
$10+5=15$
$15+5=20$
$0,5,10,15, \underline{20}$
4. $\frac{1}{6 \times 2}=\frac{1}{12}$
$\frac{1}{12 \times 2}=\frac{1}{24}$
$\frac{1}{24 \times 2}=\frac{1}{48}$
$\frac{1}{48 \times 2}=\frac{1}{96}$
$\frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \frac{1}{48}, \frac{1}{\underline{96}}$
5. $3367 \times 15=50,505$
6. The outer figure is always a square. The inner figure appears to cycle from triangle to circle to square. The line segments at the bottom alternate from two to one. The next shape is shown at right.
7. a. Conjecture based on results: The original number is doubled.

| Select a number. | 4 | 10 | 3 |
| :--- | :---: | :---: | :---: |
| Multiply the number by 4. | $4 \times 4=16$ | $10 \times 4=40$ | $3 \times 4=12$ |
| Add 8 to the product. | $16+8=24$ | $40+8=48$ | $12+8=20$ |
| Divide this sum by 2. | $24 \div 2=12$ | $48 \div 2=24$ | $20 \div 2=10$ |
| Subtract 4 from the quotient. | $12-4=8$ | $24-4=20$ | $10-4=6$ |
| Summary of results: | $4 \rightarrow 8$ | $10 \rightarrow 20$ | $3 \rightarrow 6$ |

b. $4 n$
$4 n+8$
$\frac{4 n+8}{2}=\frac{4 n}{2}+\frac{8}{2}=2 n+4$
$2 n+4-4=2 n$
8. $3,300,000$
9. 706.38
10. Round $\$ 47.00$ to $\$ 50.00$.

Round \$311.00 to \$310.00.
Round $\$ 405.00$ to $\$ 410.00$.
Round $\$ 681.79$ to $\$ 680.00$.
Total needed for expenses:
$\$ 47.00+\$ 311.00+\$ 405.00$
$\approx \$ 50.00+\$ 310.00+\$ 410.00$
$\approx \$ 770.00$
Additional money needed:
$\$ 770.00-\$ 681.79 \approx \$ 770.00-\$ 680.00$

$$
\approx \$ 90
$$

11. Round $\$ 485,000$ to $\$ 500,000$.

Round number of people to 20.

$$
\begin{aligned}
\frac{\$ 485,000}{19 \text { people }} & \approx \frac{\$ 500,000}{20 \text { people }} \\
& \approx \$ 25,000 \text { per person }
\end{aligned}
$$

12. $0.48992 \times 120 \approx 0.5 \times 120 \approx 60$
13. $11 \%$ of 512 billion can be estimated by $10 \%$ of 500 billion.
$0.10 \times 500$ billion $=50$ billion
14. $72,000 \div 30=2400$ which is choice a.
15. a. 2001; about 1275 discharges
b. 2010; about 275 discharges
c. They decreased at the greatest rate where the graph has the steepest downward slope. This occurred between 2001 and 2002
d. There were about 1000 discharges in 1997 under this policy.
16. a. $\frac{53 \%-27 \%}{2013-1980}=\frac{26 \%}{33} \approx 0.8 \%$ per year
b. $\quad p=27+0.8 x$
c. $\quad p=27+0.8(2020-1980)=59 \%$
17. For 3 hours:

Estes: $\$ 9$ per $\frac{1}{4}$ hour
$3 \times 4=12$ quarter-hours $\rightarrow 12 \times \$ 9=\$ 108$
Ship and Shore: $\$ 20$ per $\frac{1}{2}$ hour
$3 \times 2=6$ half-hours $\rightarrow 6 \times \$ 20=\$ 120$
Estes is a better deal by
$\$ 120-\$ 108=\$ 12.00$.
18. 20 round trips mean 40 one-way trips at $\$ 11 /$ trip. (40 trips)(32 passengers)(\$11)
$=\$ 14,080$ in one day
19. $\$ 960-\$ 50=\$ 910$ remaining to pay
$\frac{\$ 910}{\$ 35 \text { per week }}=26$ weeks
20. Belgium will have 160,000 more people.

Greece: 10,600,000-28,000(35) $=9,620,000$
Belgium: 10,200,000-12,000(35) $=9,780,000$
$9,780,000-9,620,000=160,000$

## Chapter 2 <br> Set Theory

## Check Points 2.1

1. Set $L$ is the set of the first six lowercase letters in the English alphabet.
2. $M=\{$ April, August $\}$
3. $O=\{1,3,5,7,9\}$
4. a. not the empty set; Many numbers meet the criteria to belong to this set.
b. the empty set; No numbers meet the criteria, thus this set is empty
c. not the empty set; "nothing" is not a set.
d. not the empty set; This is a set that contains one element, that element is a set.
5. a. true; 8 is an element of the given set.
b. true; $r$ is not an element of the given set.
c. false; $\{$ Monday $\}$ is a set and the set $\{$ Monday $\}$ is not an element of the given set.
6. a. $A=\{1,2,3\}$
b. $B=\{15,16,17, \ldots\}$
c. $O=\{1,3,5, \ldots\}$
7. a. $\{1,2,3,4, \ldots, 199\}$
b. $\{51,52,53,54, \ldots, 200\}$
8. a. $n(A)=5$; the set has 5 elements
b. $\quad n(B)=1$; the set has only 1 element
c. $\quad n(C)=8$; Though this set lists only five elements, the three dots indicate 12,13 , and 14 are also elements.
d. $n(D)=0$ because the set has no elements.
9. No, the sets are not equivalent. Set A has 4 distinct elements yet set B has only 3 distinct elements.
10. a. true; $\{\mathrm{O}, \mathrm{L}, \mathrm{D}\}=\{\mathrm{D}, \mathrm{O}, \mathrm{L}\}$ because the sets contain exactly the same elements.
b. false; The two sets do not contain exactly the same elements.

## Concept and Vocabulary Check 2.1

1. roster; set builder
2. empty; $\varnothing$
3. is an element
4. natural numbers
5. cardinal; $n(A)$
6. equivalent
7. equal

## Exercise Set 2.1

1. This is well defined and therefore it is a set.
2. This is well defined and therefore it is a set.
3. This is a matter of opinion and not well defined, thus it is not a set.
4. This is a matter of opinion and not well defined, thus it is not a set.
5. This is well defined and therefore it is a set.
6. This is well defined and therefore it is a set.
7. The set of known planets in our Solar System. Note to student: This exercise did not forget Pluto. In 2006, based on the requirement that a planet must dominate its own orbit, the International Astronomical Union removed Pluto from the list of planets.
8. The set of weekend days.
9. The set of months that begin with J.
10. The set of months that begin with $A$.
11. The set of natural numbers greater than 5 .
12. The set of natural numbers greater than 8 .
13. The set of natural numbers between 6 and 20 , inclusive.
14. The set of natural numbers between 9 and 25 , inclusive.
15. $\{$ winter, spring, summer, fall $\}$
16. $\{$ April, June, September, November \}
17. $\{$ September, October, November, December\}
18. $\{e, f, g, h, i\}$
19. $\{1,2,3\}$
20. $\{1,2,3,4,5,6\}$
21. $\{1,3,5,7,9,11\}$
22. $\{2,4,6,8\}$
23. $\{1,2,3,4,5\}$
24. $\{1,2,3,4\}$
25. $\{6,7,8,9, \ldots\}$
26. $\{5,6,7,8, \ldots\}$
27. $\{7,8,9,10\}$
28. $\{8,9,10,11\}$
29. $\{10,11,12,13, \ldots, 79\}$
30. $\{15,16,17,18, \ldots, 59\}\}$
31. $\{2\}$
32. $\{6\}$
33. not the empty set
34. not the empty set
35. empty set
36. empty set
37. not the empty set

Note that the number of women who served as U.S. president before 2016 is 0 . Thus the number 0 is an element of the set.
38. not the empty set

Note that the number of living U.S. presidents born before 1200 is 0 . Thus the number 0 is an element of the set.
39. empty set
40. empty set
41. empty set
42. empty set
43. not the empty set
44. not the empty set
45. not the empty set
46. not the empty set
47. true

3 is a member of the set.
48. true

6 is a member of the set.
49. true

12 is a member of the set.
50. true

10 is a member of the set.
51. false

5 is not a member of the set.
52. false

8 is not a member of the set.
53. true

11 is not a member of the set.
54. true.

17 is not a member of the set.
55. false

37 is a member of the set.
56. false

26 is a member of the set.
57. false

4 is a member of the set.
58. false

2 is not a member of the set.
59. true

13 is not a member of the set.

