

INSTRUCTOR'S SOLUTIONS MANUAL

DISCRETE MATHEMATICS

FIFTH EDITION

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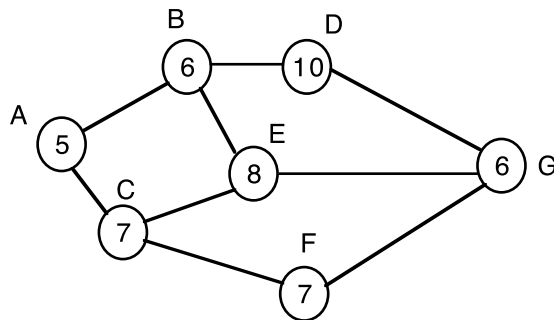
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Chapter 1

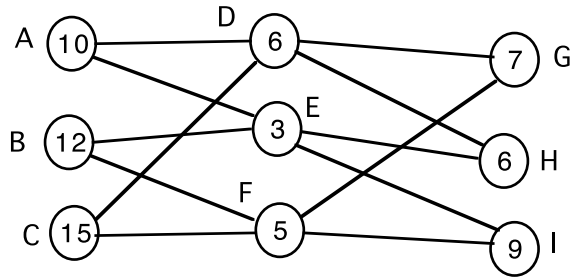
An Introduction to Combinatorial Problems and Techniques

1.1 THE TIME TO COMPLETE A PROJECT

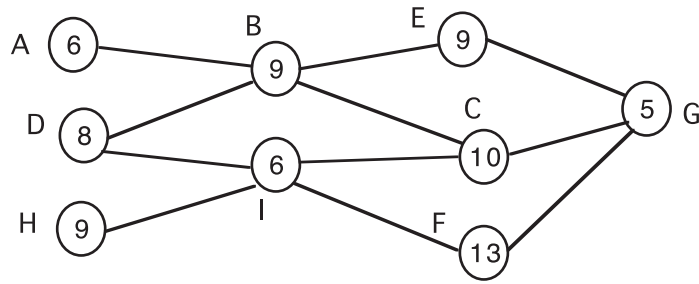
- 2. 31; A-B-E-G
- 4. 39; A-C-G-H
- 6. 16; B-D-F-H
- 8. 27; A-D-E-H
- 10. 27; A-B-D-G



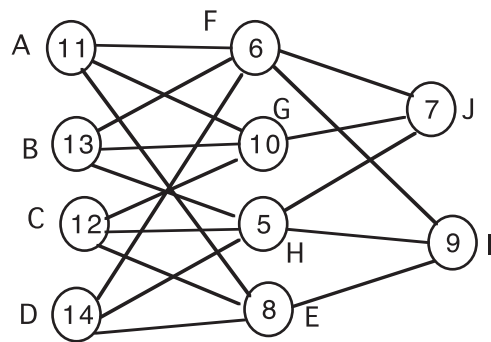
12. 29; C-F-I



14. 33; H-I-F-G



16. 31; D-E-I



18. 20 minutes

1.2 A MATCHING PROBLEM

2. 720 4. 210 6. 84 8. 1680
10. 19,958,400 12. $\frac{1}{8}$ 14. 5040 16. 126
18. 210 20. 119 22. 1320 24. 5040
26. 240 28. 1200

1.3 A KNAPSACK PROBLEM

2. T 4. F 6. F 8. F
10. T 12. T 14. T 16. no
18. yes; 32
20. \emptyset , {1}, {2}, {3}, {4}, {1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, {1, 2, 3, 4}; 16
22. 128 24. 1024 26. 256 28. 26
30. {1, 4, 6, 7, 8, 9, 10, 11, 12}

1.4 ALGORITHMS AND THEIR EFFICIENCY

2. yes; 0 4. no
6. no 8. -1, 9, 84; 3, 17, 84
10. -4, -4, 41, 95; 2, 11, 33, 95 12. 111000
14. 001010

16.

k	j	a_1	a_2	a_3
3		1	1	1
2		1	1	1
1		1	1	1
0		1	1	1

18.

k	j	a_1	a_2	a_3	a_4
4		1	1	1	0
		1	1	1	1

20. The circled numbers in the table below indicated the items being compared.

a_1	a_2	a_3	a_4	j	k
23	5	17	12	1	3
23	5	12	17		2
23	5	12	17		1
5	23	12	17	2	3
5	23	12	17		2
5	12	23	17	3	3
5	12	17	23		

22. The circled numbers in the table below indicated the items being compared.

a_1	a_2	a_3	a_4	a_5	j	k
88	2	75	10	48	1	4
88	2	75	10	48		3
88	2	10	75	48		2
88	2	10	75	48		1
2	88	10	75	48	2	4
2	88	10	48	75		3
2	88	10	48	75		2
2	10	88	48	75	3	4
2	10	88	48	75		3
2	10	48	88	75	4	4
2	10	48	75	88		

24. 6.5 years, 2.7 seconds

26. 2.3×10^{10} years, 12.5 seconds

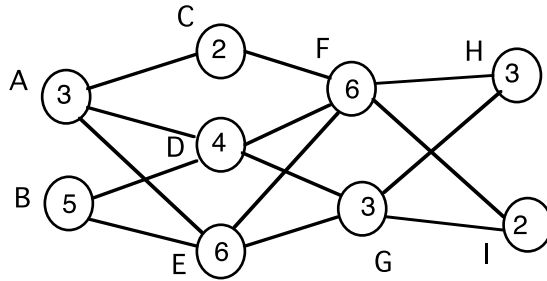
28. $4n - 3$

30. $3n - 2$

32. $-4, -4, 41, 95$

SUPPLEMENTARY EXERCISES

2. 20; B-E-F-H



4. 336

6. 40

8. 14040

10. T

12. F

14. T

16. T

18. 16

20. no

22. yes; 0

24. $-5, 7, 7, 88$

26. $\emptyset, \{4\}, \{3\}, \{3,4\}, \{2\}, \{2,4\}, \{2,3\}, \{2,3,4\}, \{1\}, \{1,4\}, \{1,3\}, \{1,3,4\}, \{1,2\}, \{1,2,4\}, \{1,2,3\}, \{1,2,3,4\}$

28. 4.92×10^8 years

30. 4

32. $4r - 3$

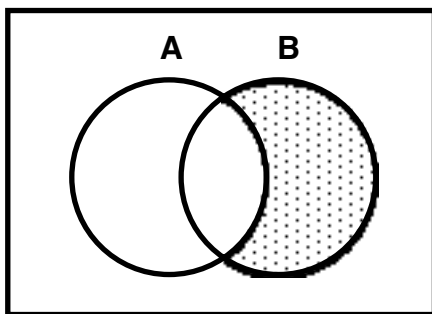
Chapter 2

Sets, Relations, and Functions

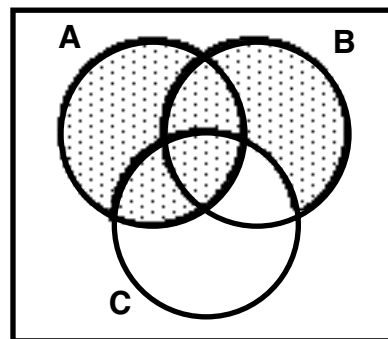
2.1 SET OPERATIONS

2. $A \cup B = \{1, 2, 4, 5, 6, 7, 9\}$, $A \cap B = \{1, 4, 6, 9\}$, $A - B = \emptyset$, $\bar{A} = \{2, 3, 5, 7, 8\}$, and $\bar{B} = \{3, 8\}$
4. $A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$, $A \cap B = \{7, 9\}$, $A - B = \{3, 4, 6, 8\}$, $\bar{A} = \{1, 2, 5\}$, and $\bar{B} = \{1, 3, 4, 6, 8\}$
6. $\{(3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3)\}$
8. $\{(p, a), (p, c), (p, e), (q, a), (q, c), (q, e), (r, a), (r, c), (r, e), (s, a), (s, c), (s, e)\}$

10.



12.

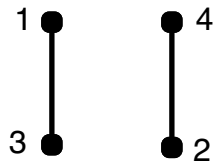


14. $A = \{1, 2\}$, $B = \{1, 3\}$, and $C = \{1\}$
16. $A = \{1, 2\}$, $B = \{1, 3\}$, and $C = \{1\}$

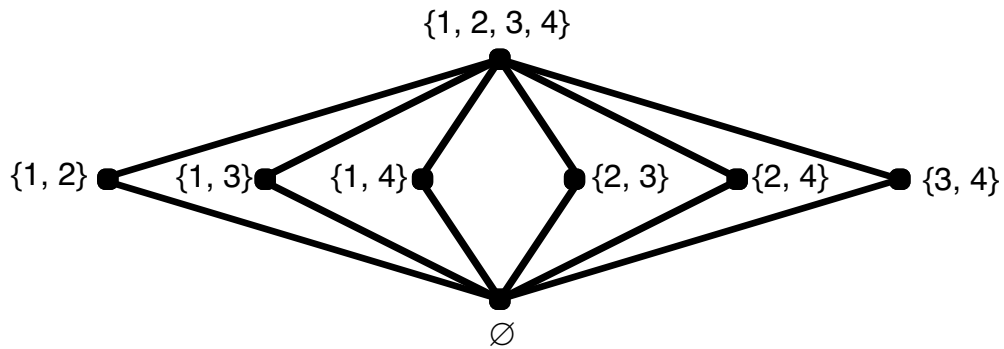
2.3 PARTIAL ORDERING RELATIONS

- 2. not antisymmetric
- 4. partial ordering
- 6. not antisymmetric
- 8. not antisymmetric

10.

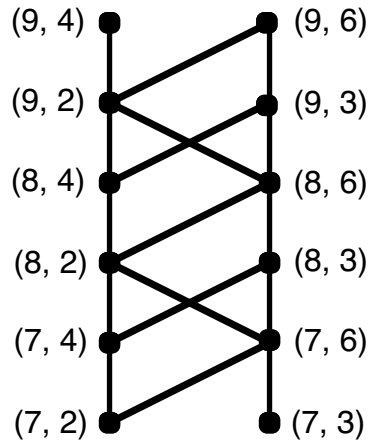


12.



- 14. $R = \{(a, a), (b, b), (b, a), (c, c), (c, b), (c, a), (d, d), (d, a)\}$
- 16. $R = \{(1, 1), (2, 2), (2, 1), (2, 4), (4, 4), (8, 8), (8, 4)\}$
- 18. The maximal elements are $\{1\}$, $\{2\}$, and $\{3\}$; the only minimal element is $\{1, 2, 3\}$.
- 20. The only minimal element is 0; there are no maximal elements.
- 22. One possible sequence is 1, 3, 2, 4.
- 24. One possible sequence is 1, 3, 2, 6, 4, 12.
- 26. Let S denote the set of residents of Illinois and R be defined so that $x R y$ means that x is a sister of y .
- 28. Every prime integer is a minimal element; there are no maximal elements.
- 30. $(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4)$

32.



36. (a) Suppose that y is element of S such that $x R y$ is false. If there is no element y_1 in S such that $y_1 R y$, then y is a minimal element of S , contradicting that x is the unique minimal element of S . Thus there must be such an element y_1 . If there is no element y_2 in S such that $y_2 R y_1$, then y_1 is a minimal element of S , another contradiction. So there must be such an element y_2 . Because S is finite, continuing in this manner must produce a minimal element y_k of S different from x . Because x is the unique minimal element of S , the assumption that there is an element y of S such that $x R y$ is false must be incorrect. Thus $x R s$ is true for every $s \in S$.
- (b) Let Z denote the set of integers and $S = Z \cup \{\emptyset\}$. Let R be the relation defined on S by $x R y$ if and only if one of the following holds: (i) $x, y \in Z$ and $x \leq y$, or (ii) $x = y = \emptyset$. Then \emptyset is the unique minimal element in S , but $\emptyset R z$ is false for every $z \in Z$.

40. 2^n 42. $2^n \cdot 3^{n(n-1)/2}$