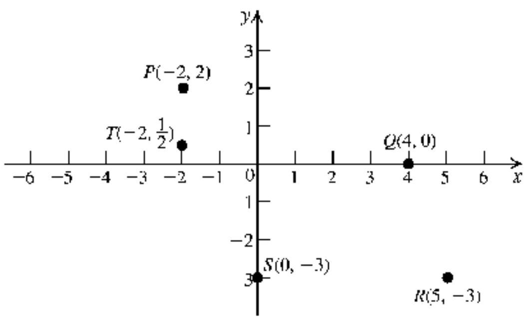
Chapter 2 Graphs and Functions

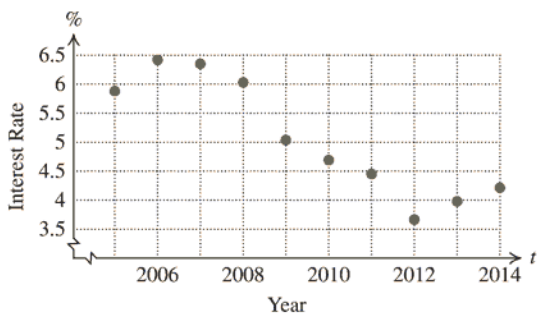
2.1 The Coordinate Plane

2.1 Practice Problems

1.



2.



3. $(x_1, y_1) = (-5, 2);\ (x_2, y_2) = (-4, 1)$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-4 - (-5))^2 + (1 - 2)^2} \\ &= \sqrt{1^2 + (-1)^2} = \sqrt{2} \approx 1.4 \end{aligned}$$

4. $(x_1, y_1) = (6, 2);\ (x_2, y_2) = (-2, 0)$
$(x_3, y_3) = (1, 5)$

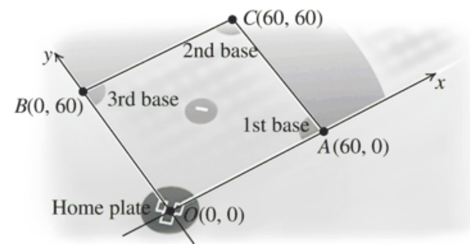
$$\begin{aligned} d_1 &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2 - 6)^2 + (0 - 2)^2} \\ &= \sqrt{(-8)^2 + (-2)^2} = \sqrt{68} \end{aligned}$$

$$\begin{aligned} d_2 &= \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2} \\ &= \sqrt{(1 - 6)^2 + (5 - 2)^2} \\ &= \sqrt{(-5)^2 + (3)^2} = \sqrt{34} \end{aligned}$$

$$\begin{aligned} d_3 &= \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2} \\ &= \sqrt{(1 - (-2))^2 + (5 - 0)^2} \\ &= \sqrt{(3)^2 + (5)^2} = \sqrt{34} \end{aligned}$$

Yes, the triangle is an isosceles triangle.

5.



We are asked to find the distance between the points $A(60, 0)$ and $B(0, 60)$.

$$\begin{aligned} d(A, B) &= \sqrt{(60 - 0)^2 + (0 - 60)^2} \\ &= \sqrt{(60)^2 + (-60)^2} = \sqrt{2(60)^2} \\ &= 60\sqrt{2} \approx 84.85 \text{ ft} \end{aligned}$$

6. $M = \left(\frac{5 + 6}{2}, \frac{-2 + (-1)}{2}\right) = \left(\frac{11}{2}, -\frac{3}{2}\right)$

2.1 Concepts and Vocabulary

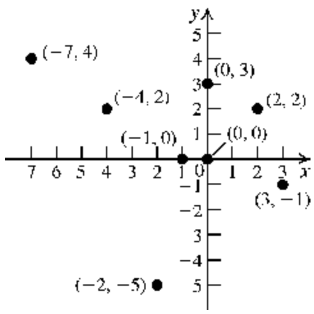
1. A point with a negative first coordinate and a positive second coordinate lies in the <u>second</u> quadrant.
2. Any point on the x-axis has second coordinate <u>0</u>.
3. The distance between the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by the formula $d(P, Q) = \underline{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$.
4. The coordinates of the midpoint $M(x, y)$ of the line segment joining $P(x_1, y_1)$ and $Q(x_2, y_2)$ are given by $(x, y) = \underline{\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$.
5. True
6. False. The point (7, −4) is 4 units to the right and 6 units below the point (3, 2).

7. False. Every point in quadrant II has a negative x-coordinate.

8. True.

2.1 Building Skills

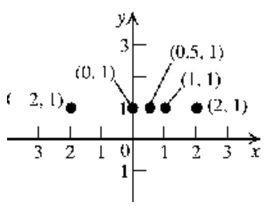
9.



(2, 2): Q1; (3, −1): Q4; (−1, 0): x-axis
(−2, −5): Q3; (0, 0): origin; (−7, 4): Q2
(0, 3): y-axis; (−4, 2): Q2

10. a. Answers will vary. Sample answer:
$(-2,0),\ (-1,0),(0,0),(1,0),(2,0)$
The y-coordinate is 0.

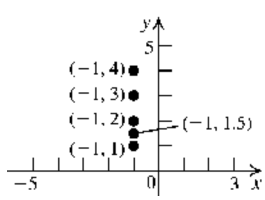
b.



The set of all points of the form (x, 1) is a horizontal line that intersects the y-axis at 1.

11. a. If the x-coordinate of a point is 0, the point lies on the y-axis.

b.



The set of all points of the form (–1, y) is a vertical line that intersects the x-axis at –1.

12. a. A vertical line that intersects the x-axis at –3.

b. A horizontal line that intersects the y-axis at 4.

13. a. $y > 0$ **b.** $y < 0$

c. $x < 0$ **d.** $x > 0$

14. a. Quadrant III **b.** Quadrant I

c. Quadrant IV **d.** Quadrant II

In Exercises 15−24, use the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ and the midpoint formula, $(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

15. a. $d = \sqrt{(2-2)^2 + (5-1)^2} = \sqrt{4^2} = 4$

b. $M = \left(\frac{2+2}{2}, \frac{1+5}{2}\right) = (2, 3)$

16. a. $d = \sqrt{(-2-3)^2 + (5-5)^2} = \sqrt{(-5)^2} = 5$

b. $M = \left(\frac{3+(-2)}{2}, \frac{5+5}{2}\right) = (0.5, 5)$

17. a. $d = \sqrt{(2-(-1))^2 + (-3-(-5))^2}$
$= \sqrt{3^2 + 2^2} = \sqrt{13}$

b. $M = \left(\frac{-1+2}{2}, \frac{-5+(-3)}{2}\right) = (0.5, -4)$

18. a. $d = \sqrt{(-7-(-4))^2 + (-9-1)^2}$
$= \sqrt{(-3)^2 + (-10)^2} = \sqrt{109}$

b. $M = \left(\frac{-4+(-7)}{2}, \frac{1+(-9)}{2}\right) = (-5.5, -4)$

19. a. $d = \sqrt{(3-(-1))^2 + (-6.5-1.5)^2}$
$= \sqrt{4^2 + (-8)^2} = \sqrt{80} = 4\sqrt{5}$

b. $M = \left(\frac{-1+3}{2}, \frac{1.5+(-6.5)}{2}\right) = (1, -2.5)$

20. a. $d = \sqrt{(1-0.5)^2 + (-1-0.5)^2}$
$= \sqrt{(0.5)^2 + (-1.5)^2} = \sqrt{2.5} = \sqrt{\frac{5}{2}} = \frac{\sqrt{10}}{2}$

b. $M = \left(\frac{0.5+1}{2}, \frac{0.5+(-1)}{2}\right) = (0.75, -0.25)$

21. a. $d = \sqrt{\left(\sqrt{2} - \sqrt{2}\right)^2 + (5-4)^2} = \sqrt{1^2} = 1$

b. $M = \left(\frac{\sqrt{2}+\sqrt{2}}{2}, \frac{4+5}{2}\right) = \left(\sqrt{2}, 4.5\right)$

22. a. $d = \sqrt{((v+w)-(v-w))^2 + (t-t)^2}$
$= \sqrt{(2w)^2} = 2|w|$

b. $M = \left(\frac{(v-w)+(v+w)}{2}, \frac{t+t}{2}\right) = (v, t)$

23. a. $d = \sqrt{(k-t)^2 + (t-k)^2}$
$= \sqrt{\left(k^2 - 2tk + t^2\right) + \left(t^2 - 2kt + k^2\right)}$
$= \sqrt{2t^2 - 4tk + 2k^2} = \sqrt{2\left(t^2 - 2tk + k^2\right)}$
$= \sqrt{2(t-k)^2} = |t-k|\sqrt{2}$

b. $M = \left(\frac{t+k}{2}, \frac{k+t}{2}\right)$

24. a. $d = \sqrt{(-n-m)^2 + (-m-n)^2}$
$= \sqrt{\left(n^2 + 2mn + m^2\right) + \left(m^2 + 2mn + n^2\right)}$
$= \sqrt{2m^2 + 4mn + 2n^2}$
$= \sqrt{2\left(m^2 + 2mn + n^2\right)}$
$= \sqrt{2(m+n)^2} = \sqrt{2}\,|m+n|$

b. $M = \left(\frac{m+(-n)}{2}, \frac{n+(-m)}{2}\right)$
$= \left(\frac{m-n}{2}, \frac{n-m}{2}\right)$

25. $P = (-1, -2), Q = (0, 0), R = (1, 2)$
$d(P,Q) = \sqrt{(0-(-1))^2 + (0-(-2))^2} = \sqrt{5}$
$d(Q,R) = \sqrt{(1-0)^2 + (2-0)^2} = \sqrt{5}$
$d(P,R) = \sqrt{(1-(-1))^2 + (2-(-2))^2}$
$= \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$
Because $d(P, Q) + d(Q, R) = d(P, R)$, the points are collinear.

26. $P = (-3, -4), Q = (0, 0), R = (3, 4)$
$d(P,Q) = \sqrt{(0-(-3))^2 + (0-(-4))^2} = \sqrt{25} = 5$
$d(Q,R) = \sqrt{(3-0)^2 + (4-0)^2} = \sqrt{25} = 5$
$d(P,R) = \sqrt{(3-(-3))^2 + (4-(-4))^2}$
$= \sqrt{6^2 + 8^2} = \sqrt{100} = 10$

Because $d(P, Q) + d(Q, R) = d(P, R)$, the points are collinear.

27. $P = (4, -2), Q = (1, 3), R = (-2, 8)$
$d(P,Q) = \sqrt{(1-4)^2 + (3-(-2))^2} = \sqrt{34}$
$d(Q,R) = \sqrt{(-2-1)^2 + (8-3)^2} = \sqrt{34}$

$d(P,R) = \sqrt{(-2-4)^2 + (8-(-2))^2}$
$= \sqrt{(-6)^2 + 10^2} = \sqrt{136} = 2\sqrt{34}$
Because $d(P, Q) + d(Q, R) = d(P, R)$, the points are collinear.

28. It is not possible to arrange the points in such a way so that $d(P, Q) + d(Q, R) = d(P, R)$, so the points are not collinear.

29. $P = (-1, 4), Q = (3, 0), R = (11, -8)$
$d(P,Q) = \sqrt{(3-(-1))^2 + (0-4)^2} = 4\sqrt{2}$
$d(Q,R) = \sqrt{(11-3)^2 + ((-8)-0)^2} = 8\sqrt{2}$
$d(P,R) = \sqrt{(11-(-1))^2 + (-8-4)^2}$
$= \sqrt{(12)^2 + (-12)^2} = \sqrt{288} = 12\sqrt{2}$
Because $d(P, Q) + d(Q, R) = d(P, R)$, the points are collinear.

30. It is not possible to arrange the points in such a way so that $d(P, Q) + d(Q, R) = d(P, R)$, so the points are not collinear.

31. It is not possible to arrange the points in such a way so that $d(P, Q) + d(Q, R) = d(P, R)$, so the points are not collinear.

32. $P = (1, 7), Q = (-3, 7.5), R = (-7, 8)$
$d(P,Q) = \sqrt{(-3-1)^2 + (7.5-7)^2} = \sqrt{16.25}$
$d(Q,R) = \sqrt{(-7-(-3))^2 + (8-7.5)^2}$
$= \sqrt{16.25}$
$d(P,R) = \sqrt{(-7-1)^2 + (8-7)^2}$
$= \sqrt{(-8)^2 + 1^2} = \sqrt{65} = 2\sqrt{16.25}$
Because $d(P, Q) + d(Q, R) = d(P, R)$, the points are collinear.

33. First, find the midpoint M of PQ.
$M = \left(\frac{-4+0}{2}, \frac{0+8}{2}\right) = (-2, 4)$
Now find the midpoint R of PM.
$R = \left(\frac{-4+(-2)}{2}, \frac{0+4}{2}\right) = (-3, 2)$
Finally, find the midpoint S of MQ.
$S = \left(\frac{-2+0}{2}, \frac{4+8}{2}\right) = (-1, 6)$
Thus, the three points are $(-3, 2)$, $(-2, 4)$, and $(-1, 6)$.

34. First, find the midpoint M of PQ.

$$M = \left(\frac{-8+16}{2}, \frac{4+(-12)}{2}\right) = (4, -4)$$

Now find the midpoint R of PM.

$$R = \left(\frac{-8+4}{2}, \frac{4+(-4)}{2}\right) = (-2, 0)$$

Finally, find the midpoint S of MQ.

$$S = \left(\frac{4+16}{2}, \frac{-4+(-12)}{2}\right) = (10, -8)$$

Thus, the three points are (−2, 0), (4, −4), and (10, −8).

35. $d(P,Q) = \sqrt{(-1-(-5))^2 + (4-5)^2} = \sqrt{17}$

$d(Q,R) = \sqrt{(-4-(-1))^2 + (1-4)^2} = 3\sqrt{2}$

$d(P,R) = \sqrt{(-4-(-5))^2 + (1-5)^2} = \sqrt{17}$

The triangle is isosceles.

36. $d(P,Q) = \sqrt{(6-3)^2 + (6-2)^2} = 5$

$d(Q,R) = \sqrt{(-1-6)^2 + (5-6)^2} = 5\sqrt{2}$

$d(P,R) = \sqrt{(-1-3)^2 + (5-2)^2} = 5$

The triangle is an isosceles triangle.

37. $d(P,Q) = \sqrt{(0-(-4))^2 + (7-8)^2} = \sqrt{17}$

$d(Q,R) = \sqrt{(-3-0)^2 + (5-7)^2} = \sqrt{13}$

$d(P,R) = \sqrt{(-3-(-4))^2 + (5-8)^2} = \sqrt{10}$

The triangle is scalene.

38. $d(P,Q) = \sqrt{(-1-6)^2 + (-1-6)^2} = 7\sqrt{2}$

$d(Q,R) = \sqrt{(-5-(-1))^2 + (3-(-1))^2} = 4\sqrt{2}$

$d(P,R) = \sqrt{(-5-6)^2 + (3-6)^2} = \sqrt{130}$

The triangle is scalene.

39. $d(P,Q) = \sqrt{(9-0)^2 + (-9-(-1))^2} = \sqrt{145}$

$d(Q,R) = \sqrt{(5-9)^2 + (1-(-9))^2} = 2\sqrt{29}$

$d(P,R) = \sqrt{(5-0)^2 + (1-(-1))^2} = \sqrt{29}$

The triangle is scalene.

40. $d(P,Q) = \sqrt{(4-(-4))^2 + (5-4)^2} = \sqrt{65}$

$d(Q,R) = \sqrt{(0-4)^2 + (-2-5)^2} = \sqrt{65}$

$d(P,R) = \sqrt{(0-(-4))^2 + (-2-4)^2} = 2\sqrt{13}$

The triangle is isosceles.

41. $d(P,Q) = \sqrt{(-1-1)^2 + (1-(-1))^2} = 2\sqrt{2}$

$$d(Q,R) = \sqrt{\left(-\sqrt{3}-(-1)\right)^2 + \left(-\sqrt{3}-1\right)^2} = \sqrt{\left(3-2\sqrt{3}+1\right)+\left(3+2\sqrt{3}+1\right)} = \sqrt{8} = 2\sqrt{2}$$

$$d(P,R) = \sqrt{\left(-\sqrt{3}-1\right)^2 + \left(-\sqrt{3}-(-1)\right)^2} = \sqrt{\left(3+2\sqrt{3}+1\right)+\left(3-2\sqrt{3}+1\right)} = \sqrt{8} = 2\sqrt{2}$$

The triangle is equilateral.

42. $d(P,Q) = \sqrt{(-1.5-(-0.5))^2 + (1-(-1))^2} = \sqrt{5}$

$$d(Q,R) = \sqrt{\left(\left(\sqrt{3}-1\right)-(-1.5)\right)^2 + \left(\frac{\sqrt{3}}{2}-1\right)^2}$$

$$= \sqrt{\left(\left(\sqrt{3}-1\right)^2 + 3\left(\sqrt{3}-1\right)+2.25\right) + \left(\frac{3}{4}-\sqrt{3}+1\right)}$$

$$= \sqrt{\left(3-2\sqrt{3}+1+3\sqrt{3}-3+2.25\right) + \left(1.75-\sqrt{3}\right)}$$

$$= \sqrt{5}$$

$$d(P,R) = \sqrt{\left(\left(\sqrt{3}-1\right)-(-0.5)\right)^2 + \left(\frac{\sqrt{3}}{2}-(-1)\right)^2}$$

$$= \sqrt{\left(\left(\sqrt{3}-1\right)^2 + \left(\sqrt{3}-1\right)+0.25\right) + \left(\frac{3}{4}+\sqrt{3}+1\right)}$$

$$= \sqrt{\left(3-2\sqrt{3}+1+\sqrt{3}-1+0.25\right) + \left(1.75+\sqrt{3}\right)}$$

$$= \sqrt{5}$$

The triangle is equilateral.

43. First find the lengths of the sides:

$d(P,Q) = \sqrt{(-1-7)^2 + (3-(-12))^2} = 17$

$d(Q,R) = \sqrt{(14-(-1))^2 + (11-3)^2} = 17$

$d(R,S) = \sqrt{(22-14)^2 + (-4-11)^2} = 17$

$d(S,P) = \sqrt{(22-7)^2 + (-4-(-12))^2} = 17$

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(*continued*)

All the sides are equal, so the quadrilateral is either a square or a rhombus. Now find the length of the diagonals:

$$d(P,R)=\sqrt{(14-7)^2+(11-(-12))^2}=17\sqrt{2}$$
$$d(Q,S)=\sqrt{(22-(-1))^2+(-4-3)^2}=17\sqrt{2}$$

The diagonals are equal, so the quadrilateral is a square.

44. First find the lengths of the sides:

$$d(P,Q)=\sqrt{(9-8)^2+(-11-(-10))^2}=\sqrt{2}$$
$$d(Q,R)=\sqrt{(8-9)^2+(-12-(-11))^2}=\sqrt{2}$$
$$d(R,S)=\sqrt{(7-8)^2+(-11-(-12))^2}=\sqrt{2}$$
$$d(S,P)=\sqrt{(8-7)^2+(-10-(-11))^2}=\sqrt{2}$$

All the sides are equal, so the quadrilateral is either a square or a rhombus. Now find the length of the diagonals.

$$d(P,R)=\sqrt{(8-8)^2+(-12-(-10))^2}=2$$
$$d(Q,S)=\sqrt{(7-9)^2+(-11-(-11))^2}=2$$

The diagonals are equal, so the quadrilateral is a square.

45. $5=\sqrt{(x-2)^2+(2-(-1))^2}$
$=\sqrt{x^2-4x+4+9}\Rightarrow$
$5=\sqrt{x^2-4x+13}\Rightarrow 25=x^2-4x+13\Rightarrow$
$0=x^2-4x-12\Rightarrow 0=(x-6)(x+2)\Rightarrow$
$x=-2$ or $x=6$

46. $13=\sqrt{(2-(-10))^2+(y-(-3))^2}$
$=\sqrt{144+y^2+6y+9}$
$=\sqrt{y^2+6y+153}\Rightarrow$
$169=y^2+6y+153$
$0=y^2+6y-16\Rightarrow 0=(y+8)(y-2)\Rightarrow$
$y=-8$ or $y=2$

47. $P=(-5,2)$, $Q=(2,3)$, $R=(x,0)$ (R is on the x-axis, so the y-coordinate is 0).

$$d(P,R)=\sqrt{(x-(-5))^2+(0-2)^2}$$
$$d(Q,R)=\sqrt{(x-2)^2+(0-3)^2}$$
$$\sqrt{(x-(-5))^2+(0-2)^2}=\sqrt{(x-2)^2+(0-3)^2}$$
$$(x+5)^2+(0-2)^2=(x-2)^2+(0-3)^2$$
$$x^2+10x+25+4=x^2-4x+4+9$$
$$10x+29=-4x+13$$
$$14x=-16$$
$$x=-\frac{8}{7}$$

The coordinates of R are $\left(-\frac{8}{7},0\right)$.

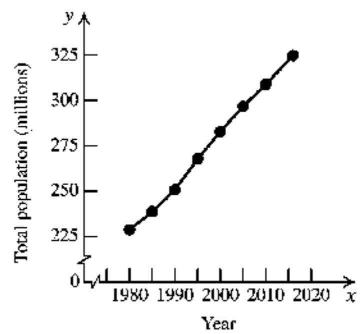
48. $P=(7,-4)$, $Q=(8,3)$, $R=(0,y)$ (R is on the y-axis, so the x-coordinate is 0).

$$d(P,R)=\sqrt{(0-7)^2+(y-(-4))^2}$$
$$d(Q,R)=\sqrt{(0-8)^2+(y-3)^2}$$
$$\sqrt{(0-7)^2+(y-(-4))^2}=\sqrt{(0-8)^2+(y-3)^2}$$
$$49+(y-(-4))^2=64+(y-3)^2$$
$$49+y^2+8y+16=64+y^2-6y+9$$
$$8y+65=-6y+73$$
$$14y=8$$
$$y=\frac{4}{7}$$

The coordinates of R are $\left(0,\frac{4}{7}\right)$.

2.1 Applying the Concepts

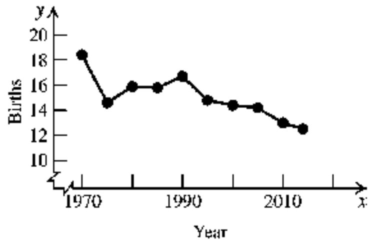
49.



50. $M=\left(\frac{2010+2016}{2},\frac{308+324}{2}\right)$
$=(2013, 316)$

The population is 2013 was about 316 million.

51.



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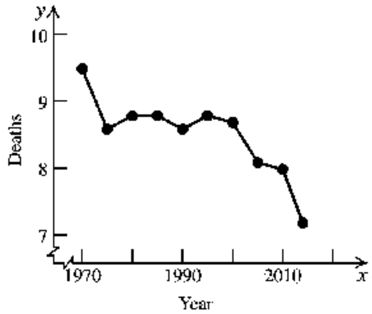
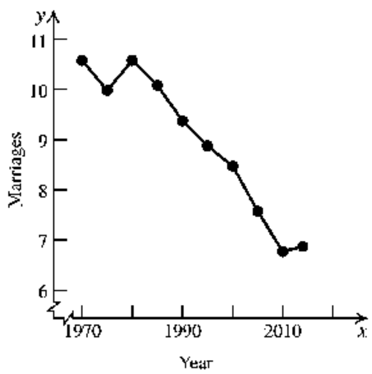
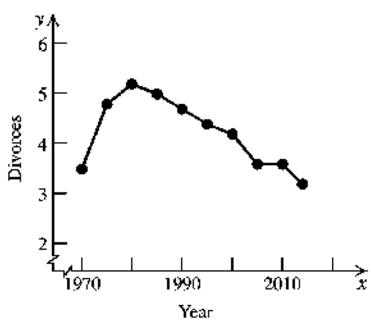


53.



54.



55. 2014 is the midpoint of the initial range, so

$$M = \left(\frac{2012+2016}{2}, \frac{326+425}{2}\right)$$
$$= (2014,\ 375.5)$$

Americans spent about \$376 billion on prescription drugs in 2014.

56. 2014 is the midpoint of the initial range, so

$$M = \left(\frac{2012+2016}{2}, \frac{2497+3696}{2}\right).$$
$$= (2014,\ 3096.5)$$

There were about 3097 million Internet users in 2014.

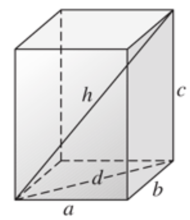
57. Percentage of Android sales in June 2013: 51.5%

58. Percentage of iPhone sales in December 2012: 49.7%

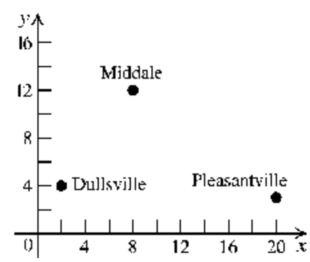
59. Android sales were at a maximum in June 2014.

60. iPhone sales were at a maximum in December 2012.

61. Denote the diagonal connecting the endpoints of the edges *a* and *b* by *d*. Then *a*, *b*, and *d* form a right triangle. By the Pythagorean theorem, $a^2 + b^2 = d^2$. The edge *c* and the diagonals *d* and *h* also form a right triangle, so $c^2 + d^2 = h^2$. Substituting d^2 from the first equation, we obtain $a^2 + b^2 + c^2 = h^2$.



62. a.



b. $d(D, M) = \sqrt{(800-200)^2 + (1200-400)^2}$
$= 1000$

$d(M, P) = \sqrt{(2000-800)^2 + (300-1200)^2}$
$= 1500$

The distance traveled by the pilot
= 1000 + 1500 = 2500 miles.

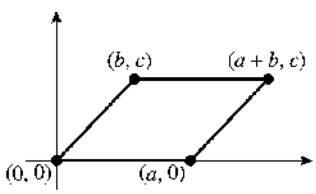
c. $d(D, P) = \sqrt{(2000-200)^2 + (300-400)^2}$
$= \sqrt{3,250,000} = \sqrt{325 \cdot 10000}$
$= 100\sqrt{325} = 100 \cdot 5\sqrt{13} = 500\sqrt{13}$
≈ 1802.78 miles

63. First, find the initial length of the rope using the Pythagorean theorem:
$c = \sqrt{24^2 + 10^2} = 26$.
After t seconds, the length of the rope is $26 - 3t$. Now find the distance from the boat to the dock, x, using the Pythagorean theorem again and solving for x:

$$(26 - 3t)^2 = x^2 + 10^2$$
$$676 - 156t + 9t^2 = x^2 + 100$$
$$576 - 156t + 9t^2 = x^2$$
$$\sqrt{576 - 156t + 9t^2} = x$$

2.1 Beyond the Basics

64. The midpoint of the diagonal connecting (0, 0) and $(a + b, c)$ is $\left(\frac{a+b}{2}, \frac{c}{2}\right)$. The midpoint of the diagonal connecting $(a, 0)$ and (b, c) is also $\left(\frac{a+b}{2}, \frac{c}{2}\right)$. Because the midpoints of the two diagonals are the same, the diagonals bisect each other.



65. a. If AB is one of the diagonals, then DC is the other diagonal, and both diagonals have the same midpoint. The midpoint of AB is $\left(\frac{2+5}{2}, \frac{3+4}{2}\right) = (3.5, 3.5)$. The midpoint of $DC = (3.5, 3.5) = \left(\frac{x+3}{2}, \frac{y+8}{2}\right)$.

So we have $3.5 = \frac{x+3}{2} \Rightarrow x = 4$ and $3.5 = \frac{y+8}{2} \Rightarrow y = -1$.

The coordinates of D are (4, –1).

b. If AC is one of the diagonals, then DB is the other diagonal, and both diagonals have the same midpoint. The midpoint of AC is $\left(\frac{2+3}{2}, \frac{3+8}{2}\right) = (2.5, 5.5)$. The midpoint of $DB = (2.5, 5.5) = \left(\frac{x+5}{2}, \frac{y+4}{2}\right)$.

So we have $2.5 = \frac{x+5}{2} \Rightarrow x = 0$ and $5.5 = \frac{y+4}{2} \Rightarrow y = 7$.

The coordinates of D are (0, 7).

c. If BC is one of the diagonals, then DA is the other diagonal, and both diagonals have the same midpoint. The midpoint of BC is $\left(\frac{5+3}{2}, \frac{4+8}{2}\right) = (4, 6)$. The midpoint of DA is $(4, 6) = \left(\frac{x+2}{2}, \frac{y+3}{2}\right)$. So we have $4 = \frac{x+2}{2} \Rightarrow x = 6$ and $6 = \frac{y+3}{2} \Rightarrow y = 9$.

The coordinates of D are (6, 9).

66. The midpoint of the diagonal connecting (0, 0) and (x, y) is $\left(\frac{x}{2}, \frac{y}{2}\right)$. The midpoint of the diagonal connecting $(a, 0)$ and (b, c) is $\left(\frac{a+b}{2}, \frac{c}{2}\right)$. Because the diagonals bisect each other, the midpoints coincide. So $\frac{x}{2} = \frac{a+b}{2} \Rightarrow x = a + b$, and $\frac{y}{2} = \frac{c}{2} \Rightarrow y = c$.
Therefore, the quadrilateral is a parallelogram.

67. a. The midpoint of the diagonal connecting (1, 2) and (5, 8) is $\left(\frac{1+5}{2}, \frac{2+8}{2}\right) = (3, 5)$.

The midpoint of the diagonal connecting (–2, 6) and (8, 4) is $\left(\frac{-2+8}{2}, \frac{6+4}{2}\right) = (3, 5)$. Because the midpoints are the same, the figure is a parallelogram.

b. The midpoint of the diagonal connecting (3, 2) and (x, y) is $\left(\frac{3+x}{2}, \frac{2+y}{2}\right)$. The midpoint of the diagonal connecting (6, 3) and (6, 5) is (6, 4). So $\frac{3+x}{2} = 6 \Rightarrow x = 9$ and $\frac{2+y}{2} = 4 \Rightarrow y = 6$.

68. Let $P(0, 0)$, $Q(a, 0)$, $R(a + b, c)$, and $S(b, c)$ be the vertices of the parallelogram.

$PQ = RS = \sqrt{(a-0)^2 + (0-0)^2} = |a|.$

$QR = PS = \sqrt{((a+b)-a)^2 + (c-0)^2}$
$= \sqrt{b^2 + c^2}.$

The sum of the squares of the lengths of the sides $= 2(a^2 + b^2 + c^2).$

$d(P, R) = \sqrt{(a+b)^2 + c^2}.$

$d(Q, S) = \sqrt{(a-b)^2 + (0-c)^2}.$

The sum of the squares of the lengths of the diagonals is

$\left((a+b)^2 + c^2\right) + \left((a-b)^2 + c^2\right) =$
$a^2 + 2ab + b^2 + c^2 + a^2 - 2ab + b^2 + c^2 =$
$2a^2 + 2b^2 + 2c^2 = 2(a^2 + b^2 + c^2).$

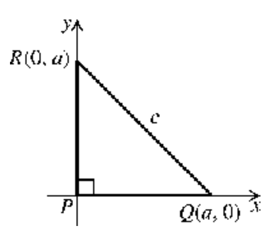
69. Let $P(0, 0)$, $Q(a, 0)$, and $R(0, b)$ be the vertices of the right triangle. The midpoint M of the hypotenuse is $\left(\frac{a}{2}, \frac{b}{2}\right)$.

$$d(Q, M) = \sqrt{\left(a - \frac{a}{2}\right)^2 + \left(0 - \frac{b}{2}\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2} = \frac{\sqrt{a^2 + b^2}}{2}$$

$$d(R, M) = \sqrt{\left(0 - \frac{a}{2}\right)^2 + \left(b - \frac{b}{2}\right)^2} = \sqrt{\left(-\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2} = \frac{\sqrt{a^2 + b^2}}{2}$$

$$d(P, M) = \sqrt{\left(0 - \frac{a}{2}\right)^2 + \left(0 - \frac{b}{2}\right)^2} = \sqrt{\left(-\frac{a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2} = \frac{\sqrt{a^2 + b^2}}{2}$$

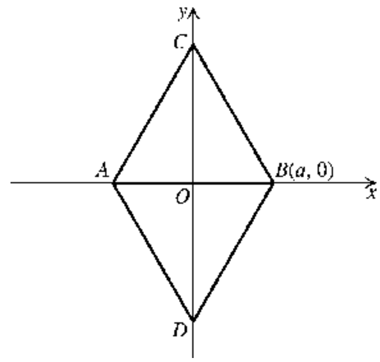
70. Let $P(0, 0)$, $Q(a, 0)$, and $R(0, a)$ be the vertices of the triangle.



Using the Pythagorean theorem, we have

$c^2 = a^2 + a^2 \Rightarrow c^2 = 2a^2 \Rightarrow c = \sqrt{2}a \Rightarrow$
$a = \frac{1}{\sqrt{2}}c = \frac{\sqrt{2}}{2}c$

71. Since ABC is an equilateral triangle and O is the midpoint of AB, then the coordinates of A are $(-a, 0)$.

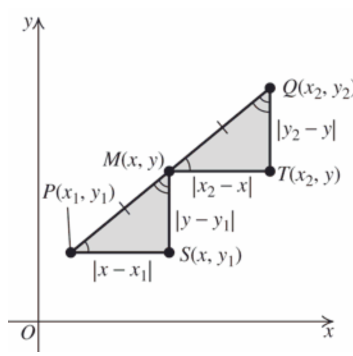


$AB = AC = AB = 2a$. Using triangle BOC and the Pythagorean theorem, we have

$BC^2 = OB^2 + OC^2 \Rightarrow (2a)^2 = a^2 + OC^2 \Rightarrow$
$4a^2 = a^2 + OC^2 \Rightarrow 3a^2 = OC^2 \Rightarrow OC = \sqrt{3}a$

Thus, the coordinates of C are $\left(0, \sqrt{3}a\right)$ and the coordinates of D are $\left(0, -\sqrt{3}a\right)$.

72.



To show that M is the midpoint of the line segment PQ, we need to show that the distance between M and Q is the same as the distance between M and P and that this distance is half the distance from P to Q.

$PQ = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$

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(*continued*)

$$MP = \sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2}$$
$$= \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2}$$
$$= \sqrt{\frac{(x_2 - x_1)^2}{4} + \frac{(y_2 - y_1)^2}{4}}$$
$$= \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Thus, we have $MP = \frac{1}{2}PQ$.

Similarly, we can show that $MQ = \frac{1}{2}PQ$.

Thus, M is the midpoint of PQ, and
$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

2.1 Critical Thinking/Discussion/Writing

73. a. y-axis

b. x-axis

74. a. The union of the x- and y-axes

b. The plane without the x- and y-axes

75. a. Quadrants I and III

b. Quadrants II and IV

76. a. The origin

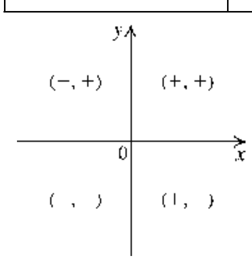
b. The plane without the origin

77. a. Right half-plane

b. Upper half-plane

78. Let (x, y) be the point.

The point lies in	if
Quadrant I	$x > 0$ and $y > 0$
Quadrant II	$x < 0$ and $y > 0$
Quadrant III	$x < 0$ and $y < 0$
Quadrant IV	$x > 0$ and $y < 0$



2.1 Getting Ready for the Next Section

79. a. $x^2 + y^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

b. $x^2 + y^2 = \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} + \frac{2}{4} = 1$

80. a. $(x-1)^2 + (y+2)^2 = [(-1)-1]^2 + (1+2)^2$
$= (-2)^2 + 3^2 = 4 + 9 = 13$

b. $(x-1)^2 + (y+2)^2 = (4-1)^2 + (2+2)^2$
$= 3^2 + 4^2 = 9 + 16 = 25$

81. a. $\frac{x}{|x|} + \frac{|y|}{y} = \frac{2}{|2|} + \frac{|-3|}{-3} = \frac{2}{2} + \frac{3}{-3} = 1 - 1 = 0$

b. $\frac{x}{|x|} + \frac{|y|}{y} = \frac{-4}{|-4|} + \frac{|3|}{3} = \frac{-4}{4} + \frac{3}{3} = -1 + 1 = 0$

82. a. $\frac{|x|}{x} + \frac{|y|}{y} = \frac{|-1|}{-1} + \frac{|-2|}{-2} = \frac{1}{-1} + \frac{2}{-2}$
$= -1 + (-1) = -2$

b. $\frac{|x|}{x} + \frac{|y|}{y} = \frac{|3|}{3} + \frac{|2|}{2} = \frac{3}{3} + \frac{2}{2} = 1 + 1 = 2$

83. $x^2 - 6x + \left(\frac{-6}{2}\right)^2 = x^2 - 6x + 3^2$
$= x^2 - 6x + \underline{9}$

84. $x^2 - 8x + \left(\frac{-8}{2}\right)^2 = x^2 - 8x + (-4)^2$
$= x^2 - 8x + \underline{16}$

85. $y^2 + 3y = y^2 + 3y + \left(\frac{3}{2}\right)^2 = y^2 + 3y + \underline{\frac{9}{4}}$

86. $y^2 + 5y + \left(\frac{5}{2}\right)^2 = y^2 + 5y + \underline{\frac{25}{4}}$

87. $x^2 - ax + \left(\frac{-a}{2}\right)^2 = x^2 - ax + \underline{\frac{a^2}{4}}$

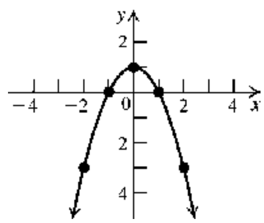
88. $x^2 + xy + \left(\frac{y}{2}\right)^2 = x^2 + xy + \underline{\frac{y^2}{4}}$

2.2 Graphs of Equations

2.2 Practice Problems

1. $y=-x^2+1$

x	$y=-x^2+1$	(x, y)
−2	$y=-(-2)^2+1$	(−2, −3)
−1	$y=-(-1)^2+1$	(−1, 0)
0	$y=-(0)^2+1$	(0, 1)
1	$y=-(1)^2+1$	(1, 0)
2	$y=-(2)^2+1$	(2, −3)



2. To find the x-intercept, let $y = 0$, and solve the equation for x: $0=2x^2+3x-2 \Rightarrow$ $0=(2x-1)(x+2) \Rightarrow x=\frac{1}{2}$ or $x=-2$. To find the y-intercept, let $x = 0$, and solve the equation for y:
$y=2(0)^2+3(0)-2 \Rightarrow y=-2.$
The x-intercepts are $\frac{1}{2}$ and −2; the y-intercept is −2.

3. To test for symmetry about the y-axis, replace x with $-x$ to determine if $(-x, y)$ satisfies the equation.
$(-x)^2-y^2=1 \Rightarrow x^2-y^2=1$, which is the same as the original equation. So the graph is symmetric about the y-axis.

4. x-axis: $x^2=(-y)^3 \Rightarrow x^2=-y^3$, which is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
y-axis: $(-x)^2=y^3 \Rightarrow x^2=y^3$, which is the same as the original equation, so the equation is symmetric with respect to the y-axis.
origin: $(-x)^2=(-y)^3 \Rightarrow x^2=-y^3$, which is not the same as the original equation, so the equation is not symmetric with respect to the origin.

5. $y=-t^4+77t^2+324$

a. First, find the intercepts. If $t = 0$, then $y = 324$, so the y-intercept is (0, 324). If $y = 0$, then we have

$$0=-t^4+77t^2+324$$
$$t^4-77t^2-324=0$$
$$(t^2-81)(t^2+4)=0$$
$$(t+9)(t-9)(t^2+4)=0 \Rightarrow t=-9, 9, \pm 2i$$

So, the t-intercepts are (−9, 0) and (9, 0). Next, check for symmetry.
t-axis: $-y=-t^4+77t^2+324$ is not the same as the original equation, so the equation is not symmetric with respect to the t-axis.
y-axis: $y=-(-t)^4+77(-t)^2+324 \Rightarrow$ $y=-t^4+77t^2+324$, which is the same as the original equation. So the graph is symmetric with respect to the y-axis.
origin: $-y=-(-t)^4+77(-t)^2+324 \Rightarrow$ $-y=-t^4+77t^2+324$, which is not the same as the original equation. So the graph is not symmetric with respect to the origin.
Now, make a table of values. Since the graph is symmetric with respect to the y-axis, if (t, y) is on the graph, then so is $(-t, y)$. However, the graph pertaining to the physical aspects of the problem consists only of those values for $t \geq 0$.

t	$y=-t^4+77t^2+324$	(t, y)
0	324	(0, 324)
1	400	(1, 400)
2	616	(2, 616)
3	936	(3, 936)
4	1300	(4, 1300)
5	1624	(5, 1624)
6	1800	(6, 1800)
7	1696	(7, 1696)
8	1156	(8, 1156)
9	0	(9, 0)

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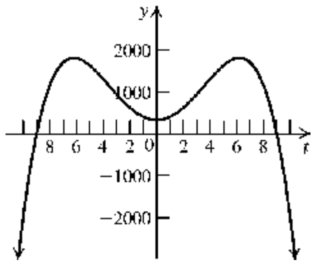
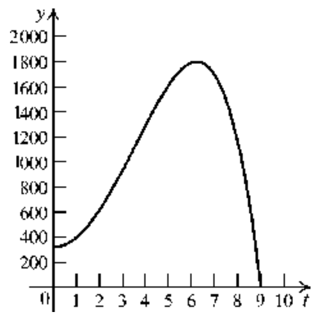


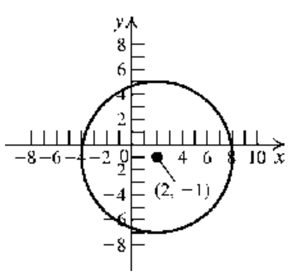
b.



c. The population becomes extinct after 9 years.

6. The standard form of the equation of a circle is $(x-h)^2+(y-k)^2=r^2$
$(h, k) = (3, -6)$ and $r = 10$
The equation of the circle is
$(x-3)^2+(y+6)^2=100.$

7. $(x-2)^2+(y+1)^2=36 \Rightarrow (h,k)=(2,-1),\ r=6$
This is the equation of a circle with center $(2, -1)$ and radius 6.



8. $x^2+y^2+4x-6y-12=0 \Rightarrow$
$x^2+4x+y^2-6y=12$
Now complete the square:
$x^2+4x+4+y^2-6y+9=12+4+9 \Rightarrow$
$(x+2)^2+(y-3)^2=25$
This is a circle with center $(-2, 3)$ and radius 5.

2.2 Concepts and Vocabulary

1. The graph of an equation in two variables, such as *x* and *y*, is the set of all ordered pairs (*a*, *b*) that satisfy the equation.

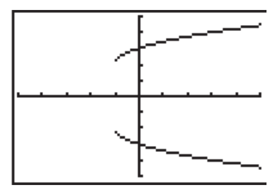
2. If (–2, 4) is a point on a graph that is symmetric with respect to the *y*-axis, then the point (2, 4) is also on the graph.

3. If (0, −5) is a point of a graph, then −5 is a *y*- intercept of the graph.

4. An equation in standard form of a circle with center (1, 0) and radius 2 is $(x-1)^2+y^2=4$.

5. False. The equation of a circle has both an x^2-term and a y^2-term. The given equation does not have a y^2-term.

6. False. The graph below is an example of a graph that is symmetric about the *x*-axis, but does not have an *x*-intercept.



7. False. The center of the circle with equation $(x+3)^2+(y+4)^2=9$ is $(-3,-4)$.

8. True

2.2 Building Skills

In exercises 9−14, to determine if a point lies on the graph of the equation, substitute the point's coordinates into the equation to see if the resulting statement is true.

9. on the graph: (–3, –4), (1, 0), (4, 3); not on the graph: (2, 3)

10. on the graph: (–1, 1), (1, 4), $\left(-\frac{5}{3},0\right)$; not on the graph: (0, 2)

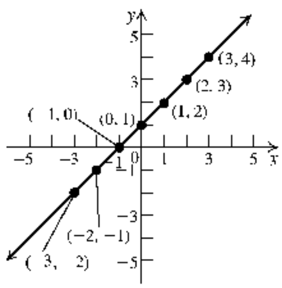
11. on the graph: (3, 2), (0, 1), (8, 3); not on the graph: (8, –3)

12. on the graph: (1, 1), $\left(2,\frac{1}{2}\right)$; not on the graph: (0, 0), $\left(-3,\frac{1}{3}\right)$

13. on the graph: $(1, 0)$, $\left(2, \sqrt{3}\right), \left(2, -\sqrt{3}\right)$; not on the graph: $(0, -1)$

14. Each point is on the graph.

15.



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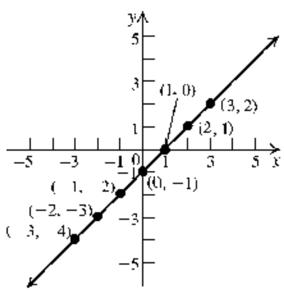

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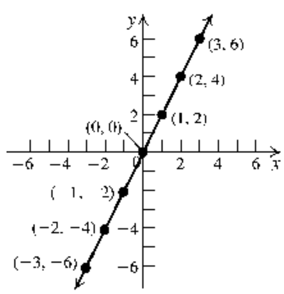

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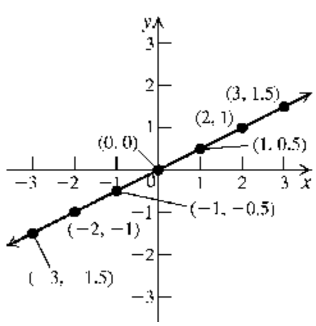

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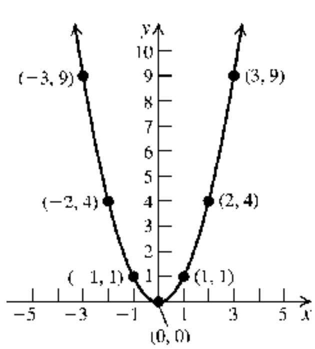

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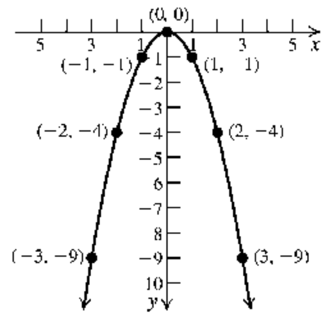

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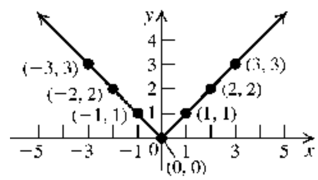

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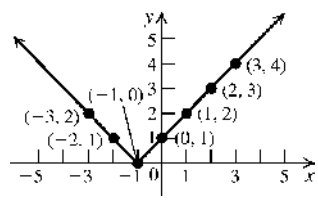

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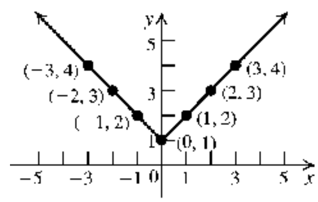

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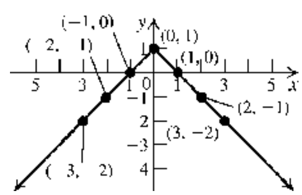

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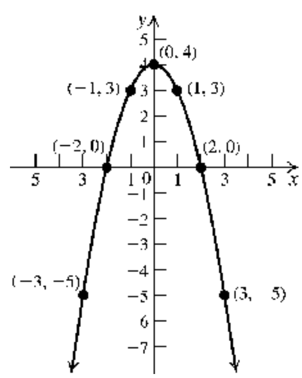

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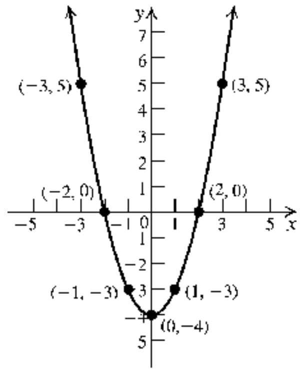

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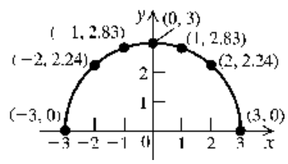

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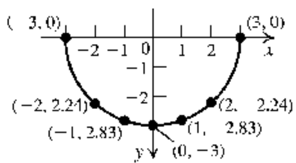

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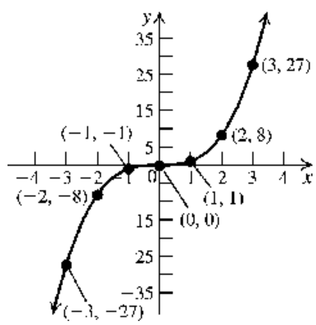

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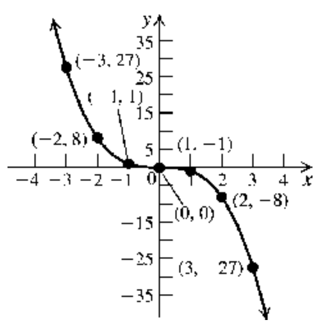

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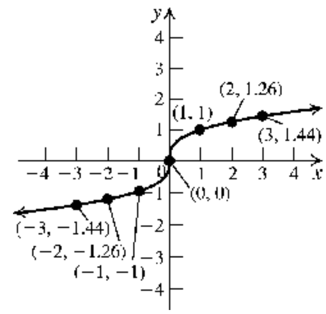
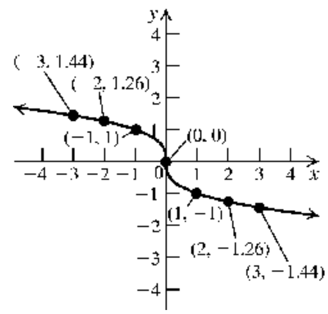


32.



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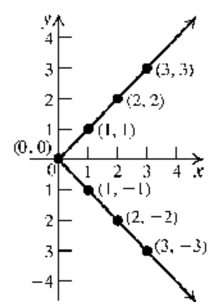

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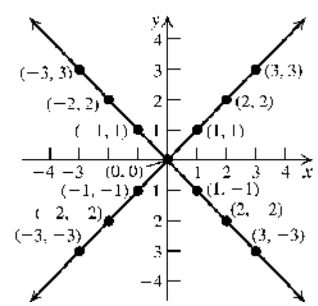

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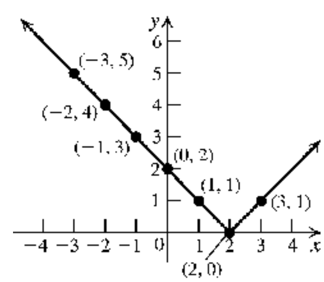

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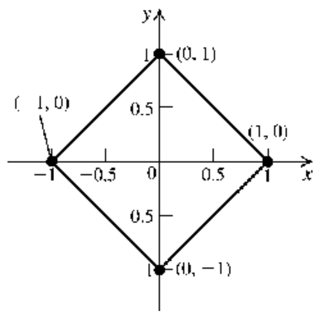


For exercises 37–46, read the answers directly from the given graphs.

37. x-intercepts: –1, 1
y-intercepts: none
symmetries: y-axis

38. x-intercepts: none
y-intercepts: –1, 1
symmetries: x-axis

39. x-intercepts: $-\pi$, 0, π
y-intercepts: 0
symmetries: origin

40. x-intercepts: $-\frac{\pi}{2}, \frac{\pi}{2}$
y-intercepts: 2
symmetries: y-axis

41. x-intercepts: –3, 3
y-intercepts: –2, 2
symmetries: x-axis, y-axis, origin

42. x-intercepts: –2, 2
y-intercepts: –3, 3
symmetries: x-axis, y-axis, origin

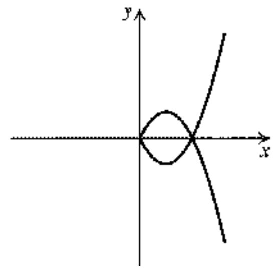
43. x-intercepts: –2, 0, 2
y-intercepts: 0
symmetries: origin

44. x-intercepts: –2, 0, 2
y-intercepts: 0
symmetries: origin

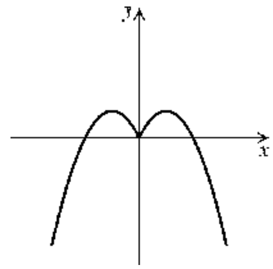
45. x-intercepts: –2, 0, 2
y-intercepts: 0, 3
symmetries: y-axis

46. x-intercepts: 0, 3
y-intercepts: –2, 0, 2
symmetries: x-axis

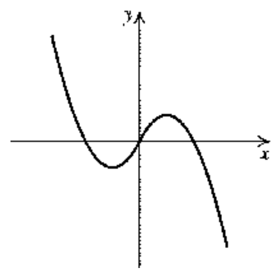
47.



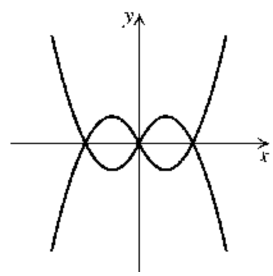
48.



49.



50.



51. To find the x-intercept, let $y = 0$, and solve the equation for x: $3x + 4(0) = 12 \Rightarrow x = 4$. To find the y-intercept, let $x = 0$, and solve the equation for y: $3(0) + 4y = 12 \Rightarrow y = 3$. The x-intercept is 4; the y-intercept is 3.

52. To find the x-intercept, let $y = 0$, and solve the equation for x: $2x + 3(0) = 5 \Rightarrow x = \frac{5}{2}$. To find the y-intercept, let $x = 0$, and solve the equation for y: $2(0) + 3y = 5 \Rightarrow y = \frac{5}{3}$. The x-intercept is $5/2$; the y-intercept is $5/3$.

53. To find the x-intercept, let $y = 0$, and solve the equation for x: $\frac{x}{5}+\frac{0}{3}=1 \Rightarrow x=5$. To find the y-intercept, let $x = 0$, and solve the equation for y: $\frac{0}{5}+\frac{y}{3}=1 \Rightarrow y=3$. The x-intercept is 5; the y-intercept is 3.

54. To find the x-intercept, let $y = 0$, and solve the equation for x: $\frac{x}{2}-\frac{0}{3}=1 \Rightarrow x=2$. To find the y-intercept, let $x = 0$, and solve the equation for y: $\frac{0}{2}-\frac{y}{3}=1 \Rightarrow y=-3$. The x-intercept is 2; the y-intercept is -3.

55. To find the x-intercept, let $y = 0$, and solve the equation for x: $0=\frac{x+2}{x-1} \Rightarrow x=-2$. To find the y-intercept, let $x = 0$, and solve the equation for y: $y=\frac{0+2}{0-1}=-2$. The x-intercept is –2; the y-intercept is –2.

56. To find the x-intercept, let $y = 0$, and solve the equation for x: $x=\frac{0-2}{0+1} \Rightarrow x=-2$. To find the y-intercept, let $x = 0$, and solve the equation for y: $0=\frac{y-2}{y+1} \Rightarrow y=2$. The x-intercept is –2; the y-intercept is 2.

57. To find the x-intercept, let $y = 0$, and solve the equation for x: $0=x^2-6x+8 \Rightarrow x=4$ or $x=2$. To find the y-intercept, let $x = 0$, and solve the equation for y: $y=0^2-6(0)+8 \Rightarrow y=8$. The x-intercepts are 2 and 4; the y-intercept is 8.

58. To find the x-intercept, let $y = 0$, and solve the equation for x: $x=0^2-5(0)+6 \Rightarrow x=6$. To find the y-intercept, let $x = 0$, and solve the equation for y: $0=y^2-5y+6 \Rightarrow y=2$ or $y=3$. The x-intercept is 6; the y-intercepts are 2 and 3.

59. To find the x-intercept, let $y = 0$, and solve the equation for x: $x^2+0^2=4 \Rightarrow x=\pm 2$. To find the y-intercept, let $x = 0$, and solve the equation for y: $0^2+y^2=4 \Rightarrow y=\pm 2$. The x-intercepts are –2 and 2; the y-intercepts are –2 and 2.

60. To find the x-intercept, let $y = 0$, and solve the equation for x:
$(x-1)^2+0^2=9 \Rightarrow x-1=\pm 3 \Rightarrow$
$x=-2$ or $x=4$
To find the y-intercept, let $x = 0$, and solve the equation for y:
$(0-1)^2+y^2=9 \Rightarrow 1+y^2=9 \Rightarrow y^2=8 \Rightarrow$
$y=\pm\sqrt{8}=\pm 2\sqrt{2}$
The x-intercepts are –2 and 4; the y-intercepts are $\pm 2\sqrt{2}$.

61. To find the x-intercept, let $y = 0$, and solve the equation for x: $0=\sqrt{9-x^2} \Rightarrow x=\pm 3$. To find the y-intercept, let $x = 0$, and solve the equation for y: $y=\sqrt{9-0^2} \Rightarrow y=3$. The x-intercepts are –3 and 3; the y-intercept is 3.

62. To find the x-intercept, let $y = 0$, and solve the equation for x: $0=\sqrt{x^2-1} \Rightarrow x=\pm 1$. To find the y-intercept, let $x = 0$, and solve the equation for y: $y=\sqrt{0^2-1} \Rightarrow$ no solution. The x-intercepts are –1 and 1; there is no y-intercept.

63. To find the x-intercept, let $y = 0$, and solve the equation for x: $x(0)=1 \Rightarrow$ no solution. To find the y-intercept, let $x = 0$, and solve the equation for y: $(0)y=1 \Rightarrow$ no solution. There is no x-intercept; there is no y-intercept.

64. To find the x-intercept, let $y = 0$, and solve the equation for x: $0=x^2+1 \Rightarrow x^2=-1 \Rightarrow$ there is no real solution. To find the y-intercept, let $x = 0$, and solve the equation for y: $y=0^2+1 \Rightarrow y=1$. There is no x-intercept; the y-intercept is 1.

In exercises 65–74, to test for symmetry with respect to the x-axis, replace y with $-y$ to determine if $(x, -y)$ satisfies the equation. To test for symmetry with respect to the y-axis, replace x with $-x$ to determine if $(-x, y)$ satisfies the equation. To test for symmetry with respect to the origin, replace x with $-x$ and y with $-y$ to determine if $(-x, -y)$ satisfies the equation.

65. $-y = x^2 + 1$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
$y = (-x)^2 + 1 \Rightarrow y = x^2 + 1$, so the equation is symmetric with respect to the y-axis.
$-y = (-x)^2 + 1 \Rightarrow -y = x^2 + 1$, is not the same as the original equation, so the equation is not symmetric with respect to the origin.

66. $x = (-y)^2 + 1 \Rightarrow x = y^2 + 1$**,** so the equation is symmetric with respect to the x-axis.
$-x = y^2 + 1$ is not the same as the original equation, so the equation is not symmetric with respect to the y-axis.
$-x = (-y)^2 + 1 \Rightarrow -x = y^2 + 1$ is not the same as the original equation, so the equation is not symmetric with respect to the origin.

67. $-y = x^3 + x$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
$y = (-x)^3 - x \Rightarrow y = -x^3 - x \Rightarrow$
$y = -(x^3 + x)$ is not the same as the original equation, so the equation is not symmetric with respect to the y-axis.
$-y = (-x)^3 - x \Rightarrow -y = -x^3 - x \Rightarrow$
$-y = -(x^3 + x) \Rightarrow y = x^3 + x,$ so the equation is symmetric with respect to the origin.

68. $-y = 2x^3 - x$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
$y = 2(-x)^3 - (-x) \Rightarrow y = -2x^3 + x \Rightarrow$
$y = -2(x^3 - x)$ is not the same as the original equation, so the equation is not symmetric with respect to the y-axis.
$-y = 2(-x)^3 - (-x) \Rightarrow -y = -2x^3 + x \Rightarrow$
$-y = -2(x^3 - x) \Rightarrow y = 2x^3 - x$, so the equation is symmetric with respect to the origin.

69. $-y = 5x^4 + 2x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
$y = 5(-x)^4 + 2(-x)^2 \Rightarrow y = 5x^4 + 2x^2$, so the equation is symmetric with respect to the y-axis.
$-y = 5(-x)^4 + 2(-x) \Rightarrow -y = 5x^4 + 2x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the origin.

70. $-y = -3x^6 + 2x^4 + x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
$y = -3(-x)^6 + 2(-x)^4 + (-x)^2 \Rightarrow$
$y = -3x^6 + 2x^4 + x^2$, so the equation is symmetric with respect to the y-axis.
$-y = -3(-x)^6 + 2(-x)^4 + (-x)^2 \Rightarrow$
$-y = -3x^6 + 2x^4 + x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the origin.

71. $-y = -3x^5 + 2x^3$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
$y = -3(-x)^5 + 2(-x)^3 \Rightarrow y = 3x^5 - 2x^3$ is not the same as the original equation, so the equation is not symmetric with respect to the y-axis.
$-y = -3(-x)^5 + 2(-x)^3 \Rightarrow -y = 3x^5 - 2x^3 \Rightarrow$
$-y = -(-3x^5 + 2x^3) \Rightarrow y = -3x^5 + 2x^3$, so the equation is symmetric with respect to the origin.

72. $-y = 2x^2 - |x|$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
$y = 2(-x)^2 - |-x| \Rightarrow y = 2x^2 - |x|$, so the equation is symmetric with respect to the y-axis.
$-y = 2(-x)^2 - |-x| \Rightarrow -y = 2x^2 - |x|$ is not the same as the original equation, so the equation is not symmetric with respect to the origin.

73. $x^2(-y)^2 + 2x(-y) = 1 \Rightarrow x^2y^2 - 2xy = 1$ is not the same as the original equation, so the equation is not symmetric with respect to the *x*-axis.
$(-x)^2 y^2 + 2(-x)y = 1 \Rightarrow x^2y^2 - 2xy = 1$ is not the same as the original equation, so the equation is not symmetric with respect to the *y*-axis.
$(-x)^2(-y)^2 + 2(-x)(-y) = 1 \Rightarrow$
$x^2y^2 + 2xy = 1$, so the equation is symmetric with respect to the origin.

74. $x^2 + (-y)^2 = 16 \Rightarrow x^2 + y^2 = 16$, so the equation is symmetric with respect to the *x*-axis.
$(-x)^2 + y^2 = 16 \Rightarrow x^2 + y^2 = 16$, so the equation is symmetric with respect to the *y*-axis.
$(-x)^2 + (-y)^2 = 16 \Rightarrow x^2 + y^2 = 16$, so the equation is symmetric with respect to the origin.

For exercises 75–78, use the standard form of the equation of a circle, $(x-h)^2 + (y-k)^2 = r^2$.

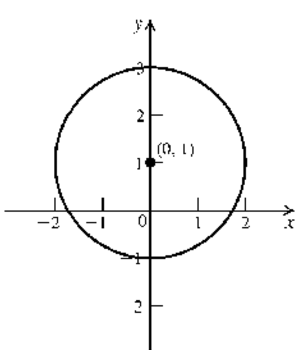
75. Center (2, 3); radius = 6

76. Center (–1, 3); radius = 4

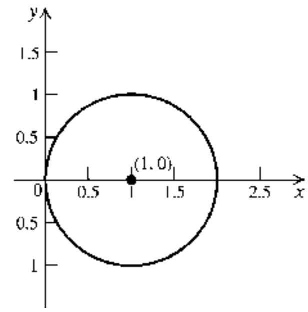
77. Center (–2, –3); radius = $\sqrt{11}$

78. Center $\left(\frac{1}{2}, -\frac{3}{2}\right)$; radius = $\frac{\sqrt{3}}{2}$

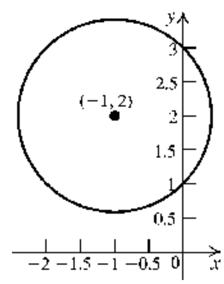
79. $x^2 + (y-1)^2 = 4$



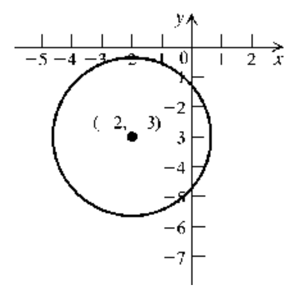
80. $(x-1)^2 + y^2 = 1$



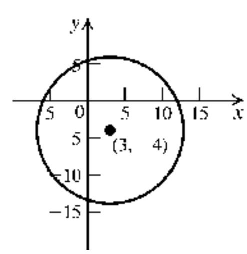
81. $(x+1)^2 + (y-2)^2 = 2$



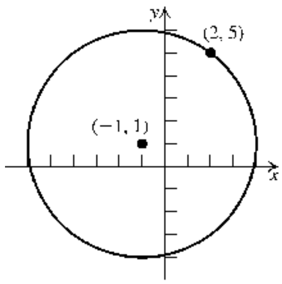
82. $(x+2)^2 + (y+3)^2 = 7$



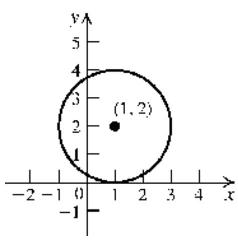
83. Find the radius by using the distance formula:
$d = \sqrt{(-1-3)^2 + (5-(-4))^2} = \sqrt{97}$.
The equation of the circle is
$(x-3)^2 + (y+4)^2 = 97$.



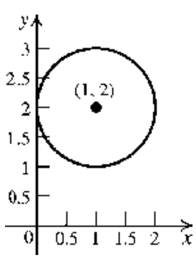
84. Find the radius by using the distance formula:
$d = \sqrt{(-1-2)^2 + (1-5)^2} = \sqrt{25} = 5$. The equation of the circle is
$(x+1)^2 + (y-1)^2 = 25.$



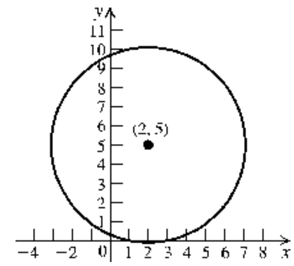
85. The circle touches the x-axis, so the radius is 2. The equation of the circle is
$(x-1)^2 + (y-2)^2 = 4.$



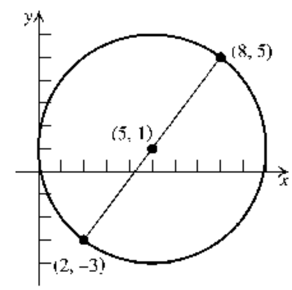
86. The circle touches the y-axis, so the radius is 1. The equation of the circle is
$(x-1)^2 + (y-2)^2 = 1$



87. Find the diameter by using the distance formula:
$d = \sqrt{(-3-7)^2 + (6-4)^2} = \sqrt{104} = 2\sqrt{26}$.
So the radius is $\sqrt{26}$. Use the midpoint formula to find the center:
$M = \left(\frac{7+(-3)}{2}, \frac{4+6}{2}\right) = (2,5)$. The equation of the circle is $(x-2)^2 + (y-5)^2 = 26$.



88. Find the center by finding the midpoint of the diameter: $C = \left(\frac{2+8}{2}, \frac{-3+5}{2}\right) = (5, 1)$
Find the length of the radius by finding the length of the diameter and dividing that by 2.
$d = \sqrt{(2-8)^2 + (-3-5)^2} = \sqrt{100} = 10$
Thus, the length of the radius is 5, and the equation of the circle is
$(x-5)^2 + (y-1)^2 = 25.$



89. a. $x^2 + y^2 - 2x - 2y - 4 = 0 \Rightarrow$
$x^2 - 2x + y^2 - 2y = 4$
Now complete the square:
$x^2 - 2x + 1 + y^2 - 2y + 1 = 4 + 1 + 1 \Rightarrow$
$(x-1)^2 + (y-1)^2 = 6$. This is a circle with center (1, 1) and radius $\sqrt{6}$.

b. To find the x-intercepts, let $y = 0$ and solve for x:
$(x-1)^2 + (0-1)^2 = 6 \Rightarrow (x-1)^2 + 1 = 6 \Rightarrow$
$(x-1)^2 = 5 \Rightarrow x - 1 = \pm\sqrt{5} \Rightarrow x = 1 \pm \sqrt{5}$
Thus, the x-intercepts are $\left(1+\sqrt{5}, 0\right)$ and $\left(1-\sqrt{5}, 0\right)$.
To find the y-intercepts, let $x = 0$ and solve for y:
$(0-1)^2 + (y-1)^2 = 6 \Rightarrow 1 + (y-1)^2 = 6 \Rightarrow$
$(y-1)^2 = 5 \Rightarrow y - 1 = \pm\sqrt{5} \Rightarrow y = 1 \pm \sqrt{5}$
Thus, the y-intercepts are $\left(0, 1+\sqrt{5}\right)$ and $\left(0, 1-\sqrt{5}\right)$.

90. a. $x^2 + y^2 - 4x - 2y - 15 = 0 \Rightarrow$

$x^2 - 4x + y^2 - 2y = 15$

Now complete the square:

$x^2 - 4x + 4 + y^2 - 2y + 1 = 15 + 4 + 1 \Rightarrow$

$(x-2)^2 + (y-1)^2 = 20.$ This is a circle with center (2, 1) and radius $2\sqrt{5}$.

b. To find the x-intercepts, let $y = 0$ and solve for x: $(x-2)^2 + (0-1)^2 = 20 \Rightarrow$

$(x-2)^2 + 1 = 20 \Rightarrow (x-2)^2 = 19 \Rightarrow$

$x - 2 = \pm\sqrt{19} \Rightarrow x = 2 \pm \sqrt{19}$

Thus, the x-intercepts are $\left(2+\sqrt{19},\ 0\right)$ and $\left(2-\sqrt{19},\ 0\right)$.

To find the y-intercepts, let $x = 0$ and solve for y: $(0-2)^2 + (y-1)^2 = 20 \Rightarrow$

$4 + (y-1)^2 = 20 \Rightarrow (y-1)^2 = 16 \Rightarrow$

$y - 1 = \pm 4 \Rightarrow y = -3, 5$

Thus, the y-intercepts are (0, −3) and (0, 5).

91. a. $2x^2 + 2y^2 + 4y = 0 \Rightarrow$

$2(x^2 + y^2 + 2y) = 0 \Rightarrow x^2 + y^2 + 2y = 0.$

Now complete the square:

$x^2 + y^2 + 2y + 1 = 0 + 1 \Rightarrow x^2 + (y+1)^2 = 1.$

This is a circle with center (0, –1) and radius 1.

b. To find the x-intercepts, let $y = 0$ and solve for x: $x^2 + (0+1)^2 = 1 \Rightarrow x^2 = 0 \Rightarrow x = 0$

Thus, the x-intercept is (0, 0).

To find the y-intercepts, let $x = 0$ and solve for y:

$0^2 + (y+1)^2 = 1 \Rightarrow y + 1 = \pm 1 \Rightarrow y = 0, -2$

Thus, the y-intercepts are (0, 0) and (0, −2).

92. a. $3x^2 + 3y^2 + 6x = 0 \Rightarrow$

$3(x^2 + y^2 + 2x) = 0 \Rightarrow x^2 + 2x + y^2 = 0.$

Now complete the square:

$x^2 + 2x + 1 + y^2 = 0 + 1 \Rightarrow (x+1)^2 + y^2 = 1.$

This is a circle with center (–1, 0) and radius 1.

b. To find the x-intercepts, let $y = 0$ and solve for x:

$(x+1)^2 + 0^2 = 1 \Rightarrow x + 1 = \pm 1 \Rightarrow x = 0, -2$

Thus, the x-intercepts are (0, 0) and (−2, 0).

To find the y-intercepts, let $x = 0$ and solve for y: $(0+1)^2 + y^2 = 1 \Rightarrow y^2 = 0 \Rightarrow y = 0$

Thus, the y-intercept is (0, 0).

93. a. $x^2 + y^2 - x = 0 \Rightarrow x^2 - x + y^2 = 0.$

Now complete the square:

$x^2 - x + \frac{1}{4} + y^2 = 0 + \frac{1}{4} \Rightarrow$

$\left(x - \frac{1}{2}\right)^2 + y^2 = \frac{1}{4}.$ This is a circle with center $\left(\frac{1}{2}, 0\right)$ and radius $\frac{1}{2}$.

b. To find the x-intercepts, let $y = 0$ and solve for x: $\left(x - \frac{1}{2}\right)^2 + 0^2 = \frac{1}{4} \Rightarrow x - \frac{1}{2} = \pm\frac{1}{2} \Rightarrow$

$x = 0, 1.$ Thus, the x-intercepts are (0, 0) and (1, 0). To find the y-intercepts, let $x = 0$ and solve for y:

$\left(0 - \frac{1}{2}\right)^2 + y^2 = \frac{1}{4} \Rightarrow y^2 + \frac{1}{4} = \frac{1}{4} \Rightarrow$

$y^2 = 0 \Rightarrow y = 0.$

Thus, the y-intercept is (0, 0).

94. a. $x^2 + y^2 + 1 = 0 \Rightarrow x^2 + y^2 = -1.$ The radius cannot be negative, so there is no graph.

b. There are no intercepts.

2.2 Applying the Concepts

95. The distance from $P(x, y)$ to the x-axis is $|x|$ while the distance from P to the y-axis is $|y|$. So the equation of the graph is $|x| = |y|$.

96. The distance from $P(x, y)$ to (1, 2) is $\sqrt{(x-1)^2 + (y-2)^2}$ while the distance from P to (3, –4) is $\sqrt{(x-3)^2 + (y+4)^2}$.

So the equation of the graph is

$\sqrt{(x-1)^2 + (y-2)^2} = \sqrt{(x-3)^2 + (y+4)^2} \Rightarrow$

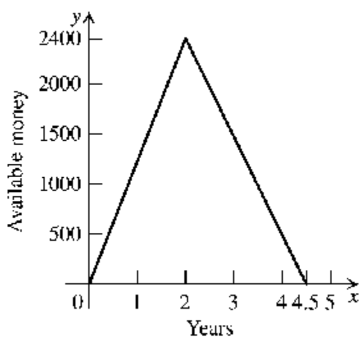
$(x-1)^2 + (y-2)^2 = (x-3)^2 + (y+4)^2 \Rightarrow$

(*continued on next page*)

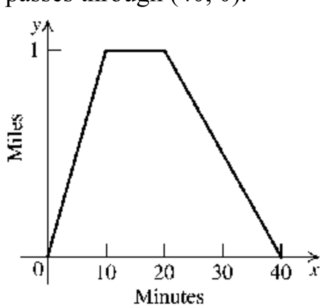
(*continued*)

$$x^2 - 2x + 1 + y^2 - 4y + 4 =$$
$$x^2 - 6x + 9 + y^2 + 8y + 16 \Rightarrow$$
$$-2x - 4y + 5 = -6x + 8y + 25 \Rightarrow$$
$$4x - 20 = 12y \Rightarrow y = \frac{1}{3}x - \frac{5}{3}.$$

97. If you save \$100 each month, it will take 24 months (or two years) to save \$2400. So, the graph starts at (0, 0) and increases to (2, 2400). It will take another 30 months (or 2.5) years to withdraw \$80 per month until the \$2400 is gone. Thus, the graph passes through (4.5, 0).



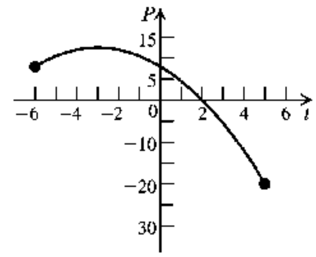
98. If you jog at 6 mph for 10 minutes, then you have traveled $6\left(\frac{1}{6}\right) = 1$ mile. So the graph starts at (0, 0) and increases to (10, 1). Resting for 10 minutes takes the graph to (20, 1). It will take 20 minutes to walk one mile at 3 mph back to the starting point. Thus, the graph passes through (40, 0).



99. a. July 2018 is represented by $t = 0$, so March 2018 is represented by $t = -4$. The monthly profit for March is determined by $P = -0.5(-4)^2 - 3(-4) + 8 = \$12$ million.

b. July 2018 is represented by $t = 0$, so October 2018 is represented by $t = 3$. So the monthly profit for October is determined by $P = -0.5(3)^2 - 3(3) + 8 = -\$5.5$ million. This is a loss.

c.



Because $t = 0$ represents July 2018, $t = -6$ represents January 2018, and $t = 5$ represents December 2018.

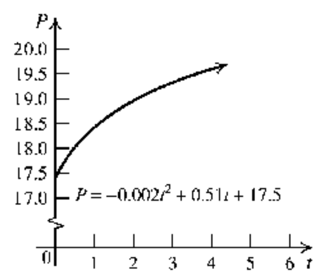
d. To find the t-intercept, set $P = 0$ and solve for t: $0 = -0.5t^2 - 3t + 8 \Rightarrow$

$$t = \frac{3 \pm \sqrt{(-3)^2 - 4(-0.5)(8)}}{2(-0.5)} = \frac{3 \pm \sqrt{25}}{-1}$$
$$= 2 \text{ or } -8$$

The t-intercepts represent the months with no profit and no loss. In this case, $t = -8$ makes no sense in terms of the problem, so we disregard this solution. $t = 2$ represents Sept 2018.

e. To find the P-intercept, set $t = 0$ and solve for P: $P = -0.5(0)^2 - 3(0) + 8 \Rightarrow P = 8.$ The P-intercept represents the profit in July 2018.

100. a.

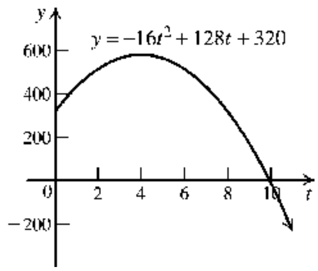


b. To find the P-intercept, set $t = 0$ and solve for P: $P = -0.002(0)^2 + 0.51(0) + 17.5 \Rightarrow$ $P = 17.5.$ The P-intercept represents the number of female college students (in millions) in 2005.

101. a.

t	Height = $-16t^2 + 128t + 320$
0	320 feet
1	432 feet
2	512 feet
3	560 feet
4	576 feet
5	560 feet
6	512 feet

b.



c. $0 \le t \le 10$

d. To find the *t*-intercept, set $y = 0$ and solve for *t*:

$0 = -16t^2 + 128t + 320 \Rightarrow$

$0 = -16(t^2 - 8t - 20) \Rightarrow$

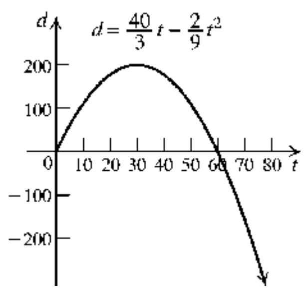
$0 = (t - 10)(t + 2) \Rightarrow t = 10$ or $t = -2$.

The graph does not apply if $t < 0$, so the *t*-intercept is 10. This represents the time when the object hits the ground. To find the *y*-intercept, set $t = 0$ and solve for *y*:

$y = -16(0)^2 + 128(0) + 320 \Rightarrow y = 320.$

This represents the height of the building.

102. a.



b. $0 \le t \le 60$

c. The total time of the experiment is 60 minutes or 1 hour.

2.2 Beyond the Basics

103. $x^2 + y^2 - 4x + 2y - 20 = 0 \Rightarrow$

$x^2 - 4x + y^2 + 2y = 20 \Rightarrow$

$x^2 - 4x + 4 + y^2 + 2y + 1 = 20 + 4 + 1 \Rightarrow$

$(x-2)^2 + (y+1)^2 = 25$

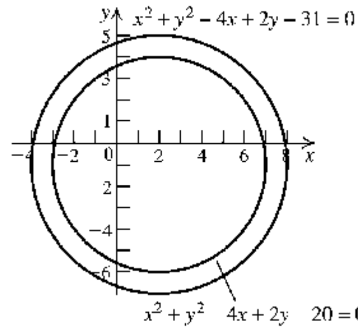
So this is the graph of a circle with center $(2, -1)$ and radius 5. The area of this circle is 25π. $x^2 + y^2 - 4x + 2y - 31 = 0 \Rightarrow$

$x^2 - 4x + y^2 + 2y = 31 \Rightarrow$

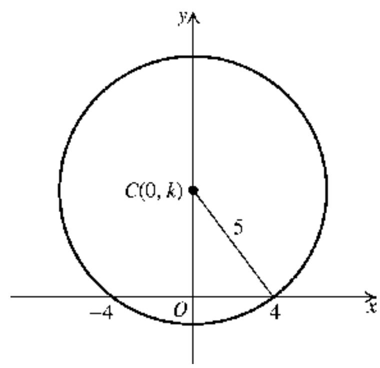
$x^2 - 4x + 4 + y^2 + 2y + 1 = 31 + 4 + 1 \Rightarrow$

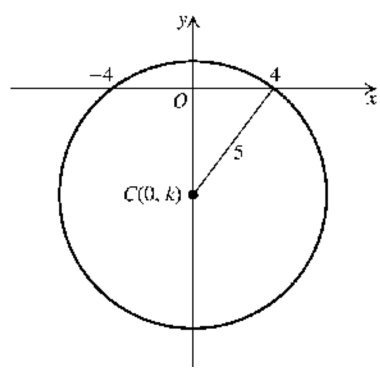
$(x-2)^2 + (y+1)^2 = 36$

So, this is the graph of a circle with center $(2, -1)$ and radius 6. The area of this circle is 36π. Both circles have the same center, so the area of the region bounded by the two circles equals $36\pi - 25\pi = 11\pi$.



104. Using the hint, we know that the center of the circle will have coordinates $(0, k)$.





Use the Pythagorean theorem to find *k*.

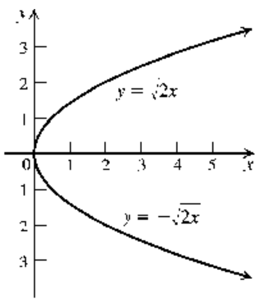
$k^2 + 4^2 = 5^2 \Rightarrow k^2 + 16 = 25 \Rightarrow k^2 = 9 \Rightarrow$
$k = \pm 3$

The equations of the circles are

$x^2 + (y \pm 3)^2 = 5^2.$

2.2 Critical Thinking/Discussion/Writing

105. The graph of $y^2 = 2x$ is the union of the graphs of $y = \sqrt{2x}$ and $y = -\sqrt{2x}$.



106. Let (x, y) be a point on the graph. The graph is symmetric with regard to the x-axis, so the point $(x, -y)$ is also on the graph. Because the graph is symmetric with regard to the y-axis, the point $(-x, y)$ is also on the graph. Therefore the point $(-x, -y)$ is on the graph, and the graph is symmetric with respect to the origin. The graph of $y = x^3$ is an example of a graph that is symmetric with respect to the origin but is not symmetric with respect to the x- and y-axes.

107. a. First find the radius of the circle:

$$d(A,B) = \sqrt{(6-0)^2 + (8-1)^2} = \sqrt{85} \Rightarrow$$
$$r = \frac{\sqrt{85}}{2}.$$

The center of the circle is

$$\left(\frac{6+0}{2}, \frac{1+8}{2}\right) = \left(3, \frac{9}{2}\right).$$

So the equation of the circle is

$$(x-3)^2 + \left(y - \frac{9}{2}\right)^2 = \frac{85}{4}.$$

To find the x-intercepts, set $y = 0$, and solve for x:

$$(x-3)^2 + \left(0 - \frac{9}{2}\right)^2 = \frac{85}{4} \Rightarrow$$
$$(x-3)^2 + \frac{81}{4} = \frac{85}{4} \Rightarrow x^2 - 6x + 9 = 1 \Rightarrow$$
$$x^2 - 6x + 8 = 0$$

The x-intercepts are the roots of this equation.

b. First find the radius of the circle:

$$d(A,B) = \sqrt{(a-0)^2 + (b-1)^2} = \sqrt{a^2 + (b-1)^2} \Rightarrow$$
$$r = \frac{\sqrt{a^2 + (b-1)^2}}{2}.$$

The center of the circle is

$$\left(\frac{a+0}{2}, \frac{b+1}{2}\right) = \left(\frac{a}{2}, \frac{b+1}{2}\right)$$

So the equation of the circle is

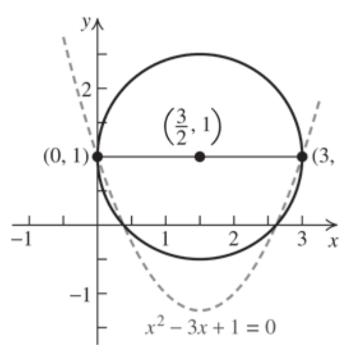
$$\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b+1}{2}\right)^2 = \frac{a^2 + (b-1)^2}{4}.$$

To find the x-intercepts, set $y = 0$ and solve for x:

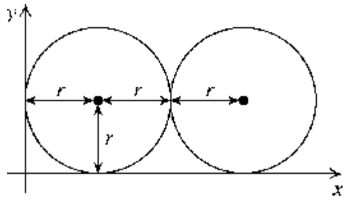
$$\left(x - \frac{a}{2}\right)^2 + \left(0 - \frac{b+1}{2}\right)^2 = \frac{a^2 + (b-1)^2}{4}$$
$$x^2 - ax + \frac{a^2}{4} + \frac{(b+1)^2}{4} = \frac{a^2 + (b-1)^2}{4}$$
$$4x^2 - 4ax + a^2 + b^2 + 2b + 1 = a^2 + b^2 - 2b + 1$$
$$4x^2 - 4ax + 4b = 0$$
$$x^2 - ax + b = 0$$

The x-intercepts are the roots of this equation.

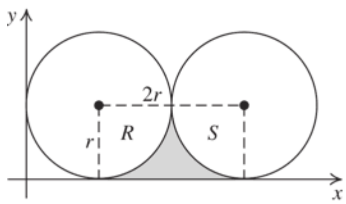
c. $a = 3$ and $b = 1$. Approximate the roots of the equation by drawing a circle whose diameter has endpoints $A(0, 1)$ and $B(3, 1)$. The center of the circle is $\left(\frac{3}{2}, 1\right)$ and the radius is $\frac{3}{2}$. The roots are approximately (0.4, 0) and (2.6, 0).



108. a. The coordinates of the center of each circle are (r, r) and $(3r, r)$.



b. To find the area of the shaded region, first find the area of the rectangle shown in the figure below, and then subtract the sum of the areas of the two sectors, A and B.



$$A_{\text{rect}} = r(2r) = 2r^2$$
$$A_{\text{sector } R} = A_{\text{sector } S} = \frac{1}{4}\pi r^2$$
$$A_{\text{shaded region}} = 2r^2 - \left(\frac{1}{4}\pi r^2 + \frac{1}{4}\pi r^2\right) = 2r^2 - \frac{1}{2}\pi r^2 = \left(2 - \frac{\pi}{2}\right)r^2$$

2.2 Getting Ready for the Next Section

109. $\frac{5-3}{6-2} = \frac{2}{4} = \frac{1}{2}$

110. $\frac{1-2}{-2-2} = \frac{-1}{-4} = \frac{1}{4}$

111. $\frac{2-(-3)}{3-13} = \frac{5}{-10} = -\frac{1}{2}$

112. $\frac{3-1}{-2-(-6)} = \frac{2}{4} = \frac{1}{2}$

113. $\frac{\frac{1}{2}-\frac{1}{4}}{\frac{3}{8}-\left(-\frac{1}{4}\right)} = \frac{\frac{1}{4}}{\frac{5}{8}} = \frac{1}{4}\cdot\frac{8}{5} = \frac{2}{5}$

114. $\frac{\frac{3}{4}-1}{\frac{1}{2}-\frac{1}{6}} = \frac{-\frac{1}{4}}{\frac{1}{3}} = \left(-\frac{1}{4}\right)(3) = -\frac{3}{4}$

115. $-\frac{1}{2}$ **116.** $\frac{1}{3}$

117. $\frac{3}{2}$ **118.** $-\frac{3}{4}$

119. $-\frac{2+\frac{3}{4}}{1-\frac{1}{2}} = -\frac{\frac{11}{4}}{\frac{1}{2}} = -\frac{11}{4}\cdot 2 = -\frac{11}{2}$

120. $-\frac{\frac{5}{6}-\left(-\frac{3}{4}\right)}{\frac{2}{3}-\frac{1}{4}} = -\frac{\frac{19}{12}}{\frac{5}{12}} = -\frac{19}{5}$

121. $2x+3y=6 \Rightarrow 3y=-2x+6 \Rightarrow y=-\frac{2}{3}x+2$

122. $\frac{x}{2}-\frac{y}{5}=3 \Rightarrow -\frac{y}{5}=-\frac{x}{2}+3 \Rightarrow y=\frac{5}{2}x-15$

123. $y-2-\frac{2}{3}(x+1)=0 \Rightarrow y-2=\frac{2}{3}(x+1) \Rightarrow$
$y=\frac{2}{3}x+\frac{2}{3}+2=\frac{2}{3}x+\frac{8}{3}$

124. $0.1x+0.2y=0 \Rightarrow 0.2y=-0.1x \Rightarrow y=-0.5x$

2.3 Lines

2.3 Practice Problems

1. $m = \frac{-3-5}{-6-(-7)} = -\frac{8}{13}$

A slope of $-\frac{8}{13}$ means that the value of y decreases 8 units for every 13 units increase in x.

2. $P(-2,-3)$, $m=-\frac{2}{3}$

$$y-(-3) = -\frac{2}{3}[x-(-2)]$$
$$y+3 = -\frac{2}{3}(x+2)$$
$$y+3 = -\frac{2}{3}x-\frac{4}{3} \Rightarrow y = -\frac{2}{3}x-\frac{13}{3}$$

3. $m = \dfrac{-4-6}{-3-(-1)} = \dfrac{-10}{-2} = 5$

Use either point to determine the equation of the line. Using (−3, −4), we have

$y-(-4) = 5[x-(-3)] \Rightarrow y+4 = 5(x+3) \Rightarrow$
$y+4 = 5x+15 \Rightarrow y = 5x+11$

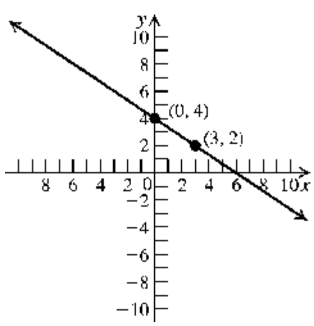
Using (−1, 6), we have

$y-6 = 5[x-(-1)] \Rightarrow y-6 = 5(x+1) \Rightarrow$
$y-6 = 5x+5 \Rightarrow y = 5x+11$

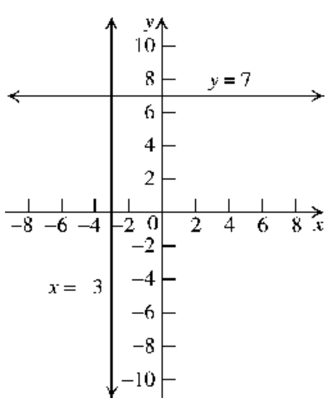
4. $y - y_1 = m(x-x_1) \Rightarrow y-(-3) = 2(x-0)$
point-slope form

$y-(-3) = 2(x-0) \Rightarrow$
$y+3 = 2x \Rightarrow y = 2x-3$

5. The slope is $-\dfrac{2}{3}$ and the y-intercept is 4. The line goes through (0, 4), so locate a second point by moving two units down and three units right. Thus, the line goes through (3, 2).



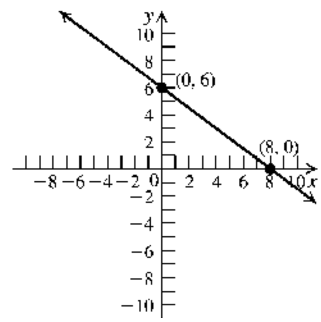
6. $x = -3$. The slope is undefined, and there is no y-intercept. The x-intercept is −3.
$y = 7$. The slope is 0, and the y-intercept is 7.



7. First, solve for y to write the equation in slope-intercept form:

$3x+4y = 24 \Rightarrow 4y = -3x+24 \Rightarrow$
$y = -\dfrac{3}{4}x+6$. The slope is $-\dfrac{3}{4}$, and the y-intercept is 6. Find the x-intercept by setting $y = 0$ and solving the equation for x:

$0 = -\dfrac{3}{4}x+6 \Rightarrow 6 = \dfrac{3}{4}x \Rightarrow 8 = x$. Thus, the graph passes through the points (0, 6) and (8, 0).



8. Use the equation $H = 2.6x+65$.

$H_1 = 2.6(43)+65 = 176.8$
$H_2 = 2.6(44)+65 = 179.4$

The person is between 176.8 cm and 179.4 cm tall, or 1.768 m and 1.794 m.

9. a. Parallel lines have the same slope, so the slope of the line is $m = \dfrac{3-7}{2-5} = \dfrac{-4}{-3} = \dfrac{4}{3}$.
Using the point-slope form, we have

$y-5 = \dfrac{4}{3}[x-(-2)] \Rightarrow 3y-15 = 4(x+2) \Rightarrow$
$3y-15 = 4x+8 \Rightarrow 4x-3y+23 = 0$

b. The slopes of perpendicular lines are negative reciprocals. Write the equation $4x+5y+1=0$ in slope-intercept form to find its slope: $4x+5y+1=0 \Rightarrow$
$5y = -4x-1 \Rightarrow y = -\dfrac{4}{5}x-\dfrac{1}{5}$. The slope of a line perpendicular to this lines is $\dfrac{5}{4}$.
Using the point-slope form, we have

$y-(-4) = \dfrac{5}{4}(x-3) \Rightarrow 4(y+4) = 5(x-3) \Rightarrow$
$4y+16 = 5x-15 \Rightarrow 5x-4y-31 = 0$

10. Because 2016 is 10 years after 2006, set $x = 10$. Then $y = 0.44(10) + 6.70 = 11.1$ There were 11.1 million registered motorcycles in the U.S. in 2016.

2.3 Concepts and Vocabulary

1. The slope of a horizontal line is $\underline{\ 0\ }$; the slope of a vertical line is <u>undefined</u>.

2. The slope of the line passing through the points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ is given by the formula $m = \underline{\dfrac{y_2 - y_1}{x_2 - x_1}}$.

3. Every line parallel to the line $y = 3x - 2$ has slope, m, equal to $\underline{\ 3\ }$.

4. Every line perpendicular to the line $y = 3x - 2$ has slope, m, equal to $\underline{-\frac{1}{3}}$.

5. False. The slope of the line $y = -\frac{1}{4}x + 5$ is equal to $-\frac{1}{4}$.

6. False. The y-intercept of the line $y = 2x - 3$ is equal to –3.

7. True

8. True

2.3 Building Skills

9. $m = \dfrac{7-3}{4-1} = \dfrac{4}{3}$; the graph is rising.

10. $m = \dfrac{0-4}{2-0} = \dfrac{-4}{2} = -2$; the graph is falling.

11. $m = \dfrac{-2-(-2)}{-2-6} = \dfrac{0}{-8} = 0$; the graph is horizontal.

12. $m = \dfrac{7-(-4)}{-3-(-3)} = \dfrac{11}{0} \Rightarrow$ slope is undefined; the graph is vertical.

13. $m = \dfrac{-3.5-2}{3-0.5} = \dfrac{-5.5}{2.5} = -2.2$; the graph is falling.

14. $m = \dfrac{-3-(-2)}{2-3} = \dfrac{-1}{-1} = 1$; the graph is rising.

15. $m = \dfrac{5-1}{\left(1+\sqrt{2}\right)-\sqrt{2}} = \dfrac{4}{1} = 4$; the graph is rising.

16. $m = \dfrac{3\sqrt{3}-0}{\left(1+\sqrt{3}\right)-\left(1-\sqrt{3}\right)} = \dfrac{3\sqrt{3}}{2\sqrt{3}} = \dfrac{3}{2}$; the graph is rising.

17. ℓ_3

18. ℓ_2

19. ℓ_4

20. ℓ_1

21. ℓ_1 passes through the points (2, 3) and (−5, 4). $m_{\ell_1} = \dfrac{-4-3}{-5-2} = \dfrac{-7}{-7} = 1.$

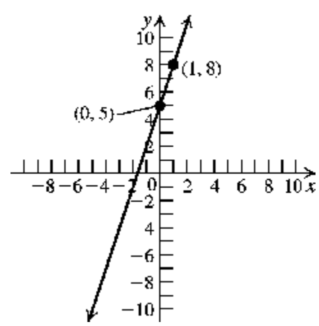
22. ℓ_2 is a horizontal line, so it has slope 0.

23. ℓ_3 passes through the points (2, 3) and (0, −1). $m_{\ell_3} = \dfrac{-1-3}{0-2} = 2.$

24. ℓ_4 passes through the points (−3, 3) and (0, −1). $m_{\ell_4} = \dfrac{-1-3}{0-(-3)} = -\dfrac{4}{3}.$

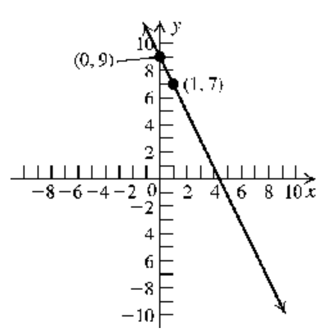
25. $(0, 5)$; $m = 3$

$y = 3x + 5$

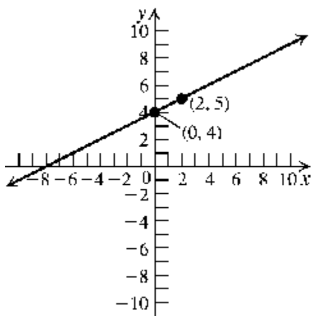


26. $(0, 9)$; $m = -2$

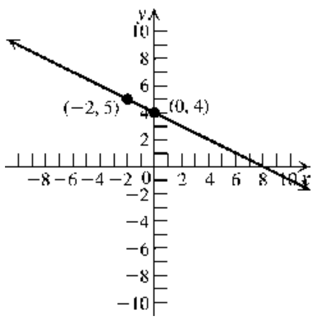
$y = -2x + 9$



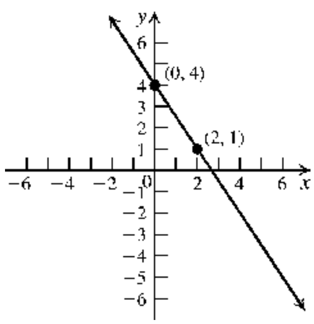
27. $y = \frac{1}{2}x + 4$



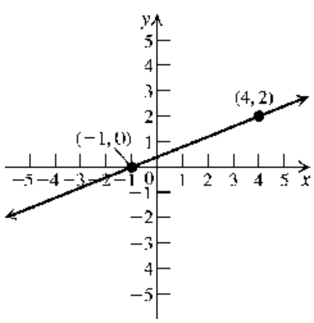
28. $y = -\frac{1}{2}x + 4$



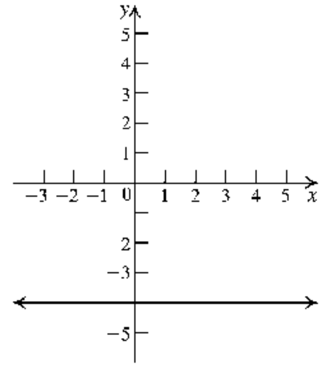
29. $y - 1 = -\frac{3}{2}(x - 2) \Rightarrow y - 1 = -\frac{3}{2}x + 3 \Rightarrow$
$y = -\frac{3}{2}x + 4$



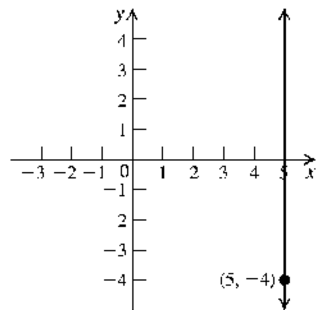
30. $y = \frac{2}{5}(x + 1) \Rightarrow y = \frac{2}{5}x + \frac{2}{5}$



31. $y + 4 = 0(x - 5) \Rightarrow y + 4 = 0 \Rightarrow y = -4$



32. Because the slope is undefined, the graph is vertical. The equation is $x = 5$.



33. $m = \frac{0 - 1}{1 - 0} = -1$. The y-intercept is (0, 1), so the equation is $y = -x + 1$.

34. $m = \frac{3 - 1}{1 - 0} = 2$. The y-intercept is (0, 1), so the equation is $y = 2x + 1$.

35. $m = \frac{3 - 3}{3 - (-1)} = 0$ Because the slope = 0, the line is horizontal. Its equation is $y = 3$.

36. $m = \frac{7 - 1}{2 - (-5)} = \frac{6}{7}$. Now write the equation in point-slope form, and then solve for y to write the equation in slope-intercept form.
$y - 1 = \frac{6}{7}(x + 5) \Rightarrow y - 1 = \frac{6}{7}x + \frac{30}{7} \Rightarrow$
$y = \frac{6}{7}x + \frac{37}{7}$

37. $m = \dfrac{1-(-1)}{1-(-2)} = \dfrac{2}{3}$. Now write the equation in point-slope form, and then solve for y to write the equation in slope-intercept form.

$y+1 = \dfrac{2}{3}(x+2) \Rightarrow y+1 = \dfrac{2}{3}x + \dfrac{4}{3} \Rightarrow$

$y = \dfrac{2}{3}x + \dfrac{1}{3}$

38. $m = \dfrac{-9-(-3)}{6-(-1)} = -\dfrac{6}{7}$. Now write the equation in point-slope form, and then solve for y to write the equation in slope-intercept form.

$y+3 = -\dfrac{6}{7}(x+1) \Rightarrow y+3 = -\dfrac{6}{7}x - \dfrac{6}{7} \Rightarrow$

$y = -\dfrac{6}{7}x - \dfrac{27}{7}$

39. $m = \dfrac{2-\frac{1}{4}}{0-\frac{1}{2}} = \dfrac{\frac{7}{4}}{-\frac{1}{2}} = -\dfrac{7}{2}$. Now write the equation in point-slope form, and then solve for y to write the equation in slope-intercept form. $y-2 = -\dfrac{7}{2}x \Rightarrow y = -\dfrac{7}{2}x + 2$

40. $m = \dfrac{3-(-7)}{4-4} = \dfrac{10}{0} \Rightarrow$ the slope is undefined. So the graph is a vertical line. The equation is $x = 4$.

41. $x = 5$ **42.** $y = 1.5$

43. $y = 0$ **44.** $x = 0$

45. $y = 14$ **46.** $y = 2x + 5$

47. $y = -\dfrac{2}{3}x - 4$ **48.** $y = -6x - 3$

49. $m = \dfrac{4-0}{0-(-3)} = \dfrac{4}{3}$; $y = \dfrac{4}{3}x + 4$

50. $m = \dfrac{-2-0}{0-(-5)} = -\dfrac{2}{5}$; $y = -\dfrac{2}{5}x - 2$

51. $y = 7$ **52.** $x = 4$

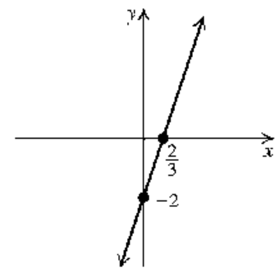
53. $y = -5$ **54.** $x = -3$

55. $y = 3x - 2$

The slope is 3 and the y-intercept is (0, –2).

$0 = 3x - 2 \Rightarrow 3x = 2 \Rightarrow x = \dfrac{2}{3}$

The x-intercept is $\left(\frac{2}{3}, 0\right)$.

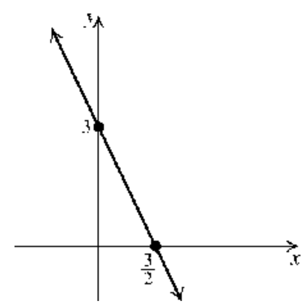


56. $y = -2x + 3$

The slope is –2 and the y-intercept is (0, 3).

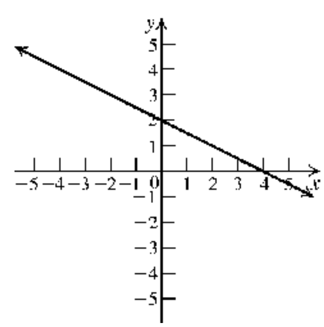
$0 = -2x + 3 \Rightarrow 2x = 3 \Rightarrow x = \dfrac{3}{2}$

The x-intercept is $\left(\frac{3}{2}, 0\right)$.



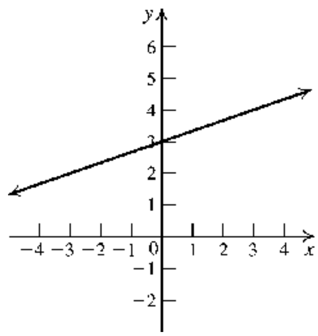
57. $x + 2y - 4 = 0 \Rightarrow 2y = -x + 4 \Rightarrow y = -\dfrac{1}{2}x + 2.$

The slope is $-1/2$, and the y-intercept is (0, 2). To find the x-intercept, set $y = 0$ and solve for x: $x + 2(0) - 4 = 0 \Rightarrow x = 4$.



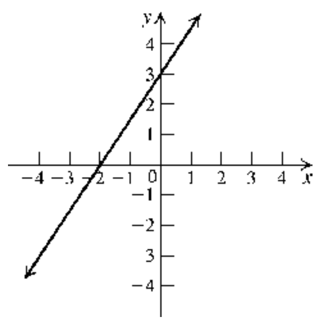
58. $x = 3y - 9 \Rightarrow x + 9 = 3y \Rightarrow y = \frac{1}{3}x + 3$

The slope is $1/3$, and the y-intercept is (0, 3). To find the x-intercept, set $y = 0$ and solve for x: $x = 3(0) - 9 \Rightarrow x = -9$.



59. $3x - 2y + 6 = 0 \Rightarrow 3x + 6 = 2y \Rightarrow \frac{3}{2}x + 3 = y$.

The slope is $3/2$, and the y-intercept is (0, 3). To find the x-intercept, set $y = 0$ and solve for x: $3x - 2(0) + 6 = 0 \Rightarrow 3x = -6 \Rightarrow x = -2$.

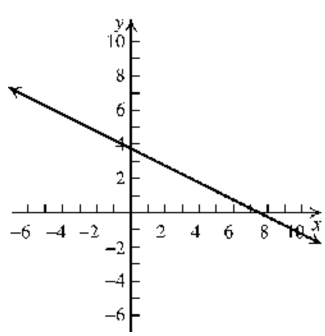


60. $2x = -4y + 15 \Rightarrow 2x - 15 = -4y \Rightarrow$

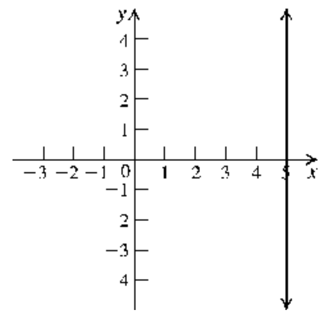
$-\frac{1}{2}x + \frac{15}{4} = y$. The slope is $-1/2$, and the y-intercept is $15/4$. To find the x-intercept, set

$y = 0$ and solve for x: $2x = -4(0) + 15 \Rightarrow$

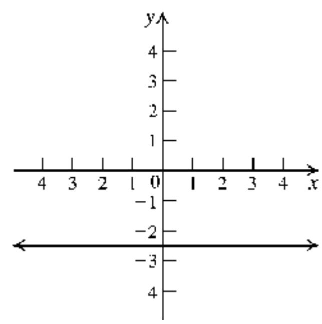
$x = 15/2$.



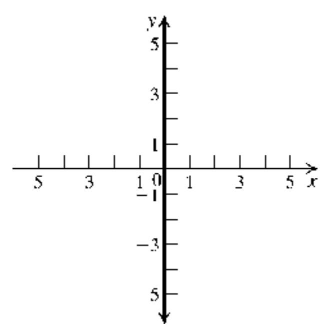
61. $x - 5 = 0 \Rightarrow x = 5$. The slope is undefined, and there is no y-intercept. The x-intercept is 5.



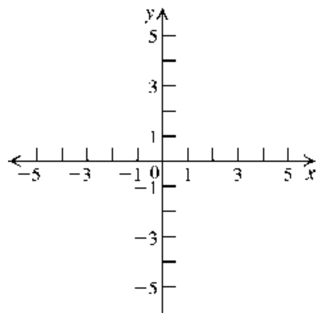
62. $2y + 5 = 0 \Rightarrow y = -\frac{5}{2}$. The slope is 0, and the y-intercept is $-5/2$. This is a horizontal line, so there is no x-intercept.



63. $x = 0$. The slope is undefined, and the y-intercepts are the y-axis. This is a vertical line whose x-intercept is 0.



64. $y = 0$. The slope is 0, and the x-intercepts are the x-axis. This is a horizontal line whose y-intercept is 0.



For exercises 65–68, the two-interecepts form of the equation of a line is $\frac{x}{a} + \frac{y}{b} = 1.$

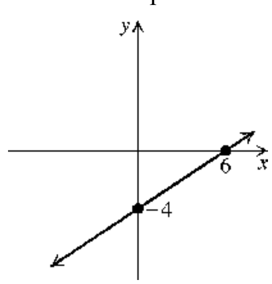
65. $\frac{x}{4} + \frac{y}{3} = 1$

66. $-\frac{x}{3} + \frac{y}{2} = 1$

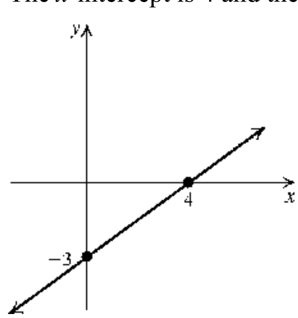
67. $2x + 3y = 6 \Rightarrow \frac{2x}{6} + \frac{3y}{6} = \frac{6}{6} \Rightarrow \frac{x}{3} + \frac{y}{2} = 1;$
x-intercept = 3; y-intercept = 2

68. $3x - 4y + 12 = 0 \Rightarrow 3x - 4y = -12 \Rightarrow$
$\frac{3x}{-12} - \frac{4y}{-12} = \frac{-12}{-12} \Rightarrow -\frac{x}{4} + \frac{y}{3} = 1;$
x-intercept = –4; y-intercept = 3

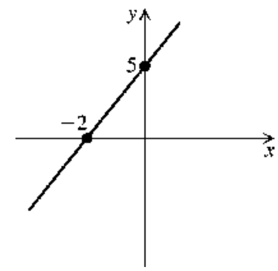
69. $2x - 3y = 12 \Rightarrow \frac{2x}{12} - \frac{3y}{12} = 1 \Rightarrow \frac{x}{6} - \frac{y}{4} = 1$
The x-intercept is 6 and the y-intercept is –4.



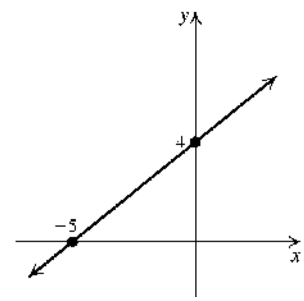
70. $3x - 4y = 12 \Rightarrow \frac{3x}{12} - \frac{4y}{12} = 1 \Rightarrow \frac{x}{4} - \frac{y}{3} = 1$
The x-intercept is 4 and the y-intercept is –3.



71. $-5x + 2y = 10 \Rightarrow -\frac{5x}{10} + \frac{2y}{10} = 1 \Rightarrow -\frac{x}{2} + \frac{y}{5} = 1$
The x-intercept is –2 and the y-intercept is 5.



72. $-4x + 5y = 20 \Rightarrow -\frac{4x}{20} + \frac{5y}{20} = 1 \Rightarrow -\frac{x}{5} + \frac{y}{4} = 1$
The x-intercept is –5 and the y-intercept is 4.



73. $m = \frac{9-4}{7-2} = \frac{5}{5} = 1.$ The equation of the line through (2, 4) and (7, 9) is $y - 4 = 1(x - 2) \Rightarrow$ $y = x + 2$. Check to see if (–1, 1) satisfies the equation by substituting $x = -1$ and $y = 1$: $1 = -1 + 2 \Rightarrow 1 = 1.$ So (–1, 1) lies on the line.

74. $m = \frac{-3-2}{2-7} = \frac{-5}{-5} = 1.$ The equation of the line through (7, 2) and (2, –3) is
$y - 2 = 1(x - 7) \Rightarrow \quad y = x - 5.$ Check to see if (5, 1) satisfies the equation by substituting x = 5 and y = 1:
$1 = 5 - 5 \Rightarrow 1 \neq 0.$
So (5, 1) does not lie on the line.

75. The given line passes through the points (0, 3) and (4, 0), so its slope is $-\frac{3}{4}$. Any line parallel to this line will have the same slope. The line that passes through the origin and is parallel to the given line has equation
$y = -\frac{3}{4}x.$

76. From exercise 75, the slope of the given line is $-\frac{3}{4}$. Any line perpendicular to this line will have slope $\frac{4}{3}$. The line that passes through the origin and is perpendicular to the given line has equation $y = \frac{4}{3}x$.

77. The red line passes through the points (–2, 0) and (0, 3), so its slope is $\frac{3}{2}$. The blue line passes through (4, 2) and has the same slope, so its equation is
$y - 2 = \frac{3}{2}(x - 4) \Rightarrow 2y - 4 = 3x - 12 \Rightarrow$
$2y = 3x - 8 \Rightarrow y = \frac{3}{2}x - 4$

78. The red line passes through the points (–2, 0) and (0, 3), so its slope is $\frac{3}{2}$. The green line passes through (4, 2) and has slope $-\frac{2}{3}$, so its equation is
$y - 2 = -\frac{2}{3}(x - 4) \Rightarrow 3y - 6 = -2x + 8 \Rightarrow$
$3y = -2x + 14 \Rightarrow y = -\frac{2}{3}x + \frac{14}{3}$

79. The slope of $y = 3x - 1$ is 3. The slope of $y = 3x + 2$ is also 3. The lines are parallel.

80. The slope of $y = 2x + 2$ is 2. The slope of $y = -2x + 2$ is –2. The lines are neither parallel nor perpendicular.

81. The slope of $y = 2x - 4$ is 2. The slope of $y = -\frac{1}{2}x + 4$ is $-\frac{1}{2}$. The lines are perpendicular.

82. The slope of $y = 3x + 1$ is 3. The slope of $y = \frac{1}{3}x - 1$ is $\frac{1}{3}$. The lines are neither parallel nor perpendicular.

83. The slope of $3x + 8y = 7$ is $-3/8$, while the slope of $5x - 7y = 0$ is $5/7$. The lines are neither parallel nor perpendicular.

84. The slope of $10x + 2y = 3$ is –5. The slope of $5x + y = -1$ is also –5, so the lines are parallel.

85. The slope of $x = 4y + 8$ is $1/4$. The slope of $y = -4x + 1$ is -4, so the lines are perpendicular.

86. The slope of $y = 3x + 1$ is 3. The slope of $6y + 2x = 0$ is $-1/3$. The lines are perpendicular.

87. Both lines are vertical lines. The lines are parallel.

88. The slope of $2x + 3y = 7$ is $-2/3$, while $y = 2$ is a horizontal line. The lines are neither parallel nor perpendicular.

89. The equation of the line through (2, –3) with slope 3 is
$y + 3 = 3(x - 2) \Rightarrow y + 3 = 3x - 6 \Rightarrow$
$y = 3x - 9.$

90. The equation of the line through (–1, 3) with slope –2 is
$y - 3 = -2(x - (-1)) \Rightarrow y - 3 = -2(x + 1) \Rightarrow$
$y - 3 = -2x - 2 \Rightarrow y = -2x + 1.$

91. A line perpendicular to a line with slope $-\frac{1}{2}$ has slope 2. The equation of the line through (–1, 2) with slope 2 is $y - 2 = 2(x - (-1)) \Rightarrow$
$y - 2 = 2(x + 1) \Rightarrow y - 2 = 2x + 2 \Rightarrow$
$y = 2x + 4.$

92. A line perpendicular to a line with slope $\frac{1}{3}$ has slope –3. The equation of the line through (2, –1) with slope –3 is
$y - (-1) = -3(x - 2) \Rightarrow$
$y + 1 = -3(x - 2) \Rightarrow y + 1 = -3x + 6 \Rightarrow$
$y = -3x + 5.$

93. The slope of the line joining (1, –2) and (–3, 2) is $\dfrac{2 - (-2)}{-3 - 1} = -1.$ The equation of the line through (–2, –5) with slope –1 is
$y - (-5) = -(x - (-2)) \Rightarrow y + 5 = -(x + 2) \Rightarrow$
$y + 5 = -x - 2 \Rightarrow y = -x - 7.$

94. The slope of the line joining (–2, 1) and (3, 5) is $\dfrac{5 - 1}{3 - (-2)} = \dfrac{4}{5}.$
The equation of the line through (1, 2) with slope $\frac{4}{5}$ is
$y - 2 = \dfrac{4}{5}(x - 1) \Rightarrow 5(y - 2) = 4(x - 1) \Rightarrow$
$5y - 10 = 4x - 4 \Rightarrow 5y = 4x + 6 \Rightarrow$
$y = \dfrac{4}{5}x + \dfrac{6}{5}.$

95. The slope of the line joining (–3, 2) and (–4, –1) is

$$\frac{-1-2}{-4-(-3)} = 3.$$

A line perpendicular to this line has slope $-\frac{1}{3}$. The equation of the line through (1, –2) with slope $-\frac{1}{3}$ is

$$y-(-2) = -\frac{1}{3}(x-1) \Rightarrow 3(y+2) = -(x-1) \Rightarrow$$
$$3y+6 = -x+1 \Rightarrow 3y = -x-5 \Rightarrow$$
$$y = -\frac{1}{3}x - \frac{5}{3}.$$

96. The slope of the line joining (2, 1) and (4, –1) is

$$\frac{-1-1}{4-2} = -1.$$

A line perpendicular to this line has slope 1 The equation of the line through (–1, 2) with slope 1 is

$$y-2 = x-(-1) \Rightarrow y-2 = x+1 \Rightarrow y = x+3.$$

97. The slope of the line $y = 6x+5$ is 6. The lines are parallel, so the slope of the new line is also 6. The equation of the line with slope 6 and y-intercept 4 is $y = 6x+4$.

98. The slope of the line $y = -\frac{1}{2}x+5$ is $-\frac{1}{2}$. The lines are parallel, so the slope of the new line is also $-\frac{1}{2}$. The equation of the line with slope $-\frac{1}{2}$ and y-intercept 2 is $y = -\frac{1}{2}x+2$.

99. The slope of the line $y = 6x+5$ is 6. The lines are perpendicular, so the slope of the new line is $-\frac{1}{6}$. The equation of the line with slope $-\frac{1}{6}$ and y-intercept 4 is $y = -\frac{1}{6}x+4$.

100. The slope of the line $y = -\frac{1}{2}x+5$ is $-\frac{1}{2}$. The lines are perpendicular, so the slope of the new line is 2. The equation of the line with slope 2 and y-intercept –4 is $y = 2x-4$.

101. The slope of $x + y = 1$ is −1. The lines are parallel, so they have the same slope. The equation of the line through (1, 1) with slope −1 is $y-1 = -(x-1) \Rightarrow y-1 = -x+1 \Rightarrow$ $y = -x+2$.

102. The slope of $-2x+3y = 7$ is $2/3$. The lines are parallel, so they have the same slope. The equation of the line through (1, 0) with slope $2/3$ is $y-0 = \frac{2}{3}(x-1) \Rightarrow y = \frac{2}{3}x - \frac{2}{3}$.

103. The slope of $3x-9y = 18$ is $1/3$. The lines are perpendicular, so the slope of the new line is -3. The equation of the line through (–2, 4) with slope -3 is $y-4 = -3(x-(-2)) \Rightarrow$ $y-4 = -3x-6 \Rightarrow y = -3x-2.$

104. The slope of $-2x+y = 14$ is 2. The lines are perpendicular, so the slope of the new line is $-1/2$. The equation of the line through (0, 2) with slope $-1/2$ is $y = -\frac{1}{2}x+2$.

2.3 Applying the Concepts

105. a. The y-intercept represents the initial expenses.

b. The x-intercept represents the point at which the teacher breaks even, i.e., the expenses equal the income.

c. The teacher's profit if there are 16 students in the class is \$640.

d. The slope of the line is

$$\frac{640-(-750)}{16-0} = \frac{1390}{16} = \frac{695}{8}$$

The equation of the line is

$$P = \frac{695}{8}n - 750.$$

106. a. The y-intercept represents the initial prepaid amount.

b. The x-intercept represents the total number of minutes the cellphone can be used.

c. The slope of the line is

$$\frac{0-15}{75-0} = -\frac{15}{75} = -\frac{1}{5}.$$

The equation of the line is $P = -\frac{1}{5}t + 15$.

d. The cost per minute is $\$\frac{1}{5} = 20¢$.

107. slope $= \frac{\text{rise}}{\text{run}} \Rightarrow \frac{4}{40} = \frac{1}{10}$

108. 4 miles = 21,120 feet.
$|\text{slope}| = \frac{\text{rise}}{\text{run}} \Rightarrow \frac{2000}{21{,}120} = \frac{25}{264}$

109. 8 in. in two weeks $\Rightarrow$ the plant grows 4 in. per week. John wants to trim the hedge when it grows 6 in., so he should trim it every $\frac{6}{4} = 1.5$ weeks ≈ 10 days.

110. $\frac{2\text{ min.}}{5\text{ in.}} = \frac{x\text{ min.}}{31\text{ in.}} \Rightarrow x = \frac{2 \cdot 31}{5} = 12.4$ min.
The water will overflow in about 12 min.

111. a. x = the number of weeks; y = the amount of money in the account after x weeks; $y = 7x + 130$

b. The slope is the amount of money deposited each week; the y-intercept is the initial deposit.

112. a. x = the number of sessions of golf; y = the yearly payment to the club; $y = 35x + 1000$

b. The slope is the cost per golf session; the y-intercept is the yearly membership fee.

113. a. x = the number of months owed to pay off the refrigerator; y = the amount owed; $y = -15x + 600$

b. The slope is the amount that the balance due changes per month; the y-intercept is the initial amount owed.

114. a. x = the number of rupees; y = the number of dollars equal to x rupees.
$y = \frac{1}{50.5}x = 0.019802x$

b. The slope is the number of dollars per rupee. The y-intercept is the number of dollars for 0 rupees.

115. a. x = the number of years after 2010; y = the life expectancy of a female born in the year 2010 + x; $y = 0.17x + 80.8$

b. The slope is the rate of increase in life expectancy; the y-intercept is the current life expectancy.

116. a. $v = -1400(2) + 14{,}000 = \$11{,}200$

b. $v = -1400(6) + 14{,}000 = \$5600$
To find when the tractor will have no value, set $v = 0$ and solve the equation for t:
$0 = -1400t + 14{,}000 \Rightarrow t = 10$ years

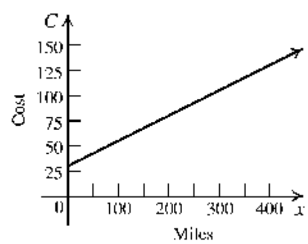
117. There are 30 days in June. For the first 13 days, you used data at a rate of $\frac{435}{13} \approx 33.5$ MB/day. At the same rate, you will use $33.5(17) = 569.5$ MB for the rest of the month.
435 + 569.5 = 1004.5
So, you don't need to buy extra data. You will have about 20 MB left.

118. For the first three hours, you traveled at $\frac{195}{3} = 65$ mph.
$d = rt \Rightarrow 520 - 195 = 65t \Rightarrow 325 = 65t \Rightarrow t = 5$
You will arrive at your destination five hours after 12 pm or 5 pm.

119. $y = 5x + 40{,}000$

120. a. $C = 0.25x + 30$

b.



c. $y = 0.25(60) + 30 = \$45$

d. $47.75 = 0.25x + 30 \Rightarrow x = 71$ miles

121. a. The two points are (100, 212) and (0, 32). So the slope is $\frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}$.
The equation is
$F - 32 = \frac{9}{5}(C - 0) \Rightarrow F = \frac{9}{5}C + 32$

b. One degree Celsius change in the temperature equals $9/5$ degrees change in degrees Fahrenheit.

c.

C	$F = \frac{9}{5}C + 32$
40ºC	104°F
25º C	77°F
–5ºC	23°F
–10ºC	14°F

d. $100°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 37.78°\text{C}$

$90°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 32.22°\text{C}$

$75°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 23.89°\text{C}$

$-10°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = -23.33°\text{C}$

$-20°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = -28.89°\text{C}$

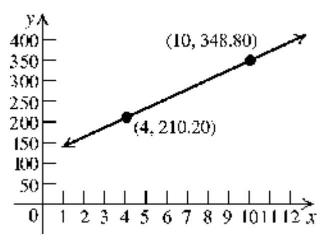
e. $97.6°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 36.44°\text{C}$;

$99.6°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 37.56°\text{C}$

f. Let x = °F = °C. Then $x = \frac{9}{5}x + 32 \Rightarrow$

$-\frac{4}{5}x = 32 \Rightarrow x = -40$. At –40°, °F = °C.

122. a. The two points are (4, 210.20) and (10, 348.80). So the slope is $\frac{348.80 - 210.20}{10 - 4} = \frac{138.6}{6} = 23.1$. The equation is $y - 348.8 = 23.1(x - 10) \Rightarrow$ $y = 23.1x + 117.8$

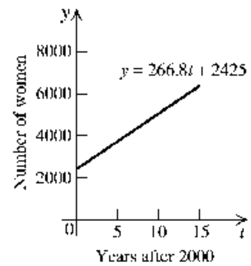


b. The slope represents the cost of producing one modem. The y-intercept represents the fixed cost.

c. $y = 23.1(12) + 117.8 \Rightarrow y = \$395$

123. a. The year 2005 is represented by $t = 0$, and the year 2011 is represented by $t = 6$. The points are (0, 2425) and (6, 4026). So the slope is $\frac{4026 - 2425}{6} \approx 266.8$ The equation is $y - 2425 = 266.8(t - 0) \Rightarrow$ $y = 266.8t + 2425$

b.



c. The year 2008 is represented by $t = 3$. So $y = 266.8(3) + 2425 \Rightarrow y = 3225.4$. Note that there cannot be a fraction of a person, so. there were 3225 women prisoners in 2008.

d. The year 2017 is represented by $t = 12$. So $y = 266.8(12) + 2425 \Rightarrow y = 5626.6$. There will be 5627 women prisoners in 2017.

124. a. The two points are (5, 5.73) and (8, 6.27). The slope is $\frac{6.27 - 5.73}{8 - 5} = \frac{0.54}{3} = 0.18$. The equation is $V - 5.73 = 0.18(x - 5) \Rightarrow$ $V = 0.18x + 4.83$.

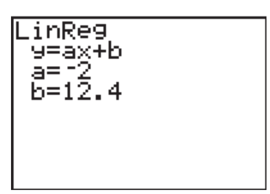
b. The slope represents the monthly change in the number of viewers. The V-intercept represents the number of viewers when the show first started.

c. $V = 0.18(11) + 4.83 \Rightarrow V = 6.81$ million

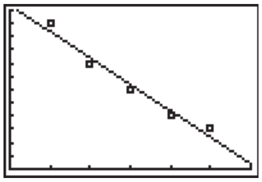
125. The independent variable t represents the number of years after 2000, with $t = 0$ representing 2000. The two points are (0, 11.7) and (5, 12.7). So the slope is $\frac{12.7 - 11.7}{5} = 0.2$. The equation is $p - 11.7 = 0.2(t - 0) \Rightarrow p = 0.2t + 11.7$. The year 2010 is represented by $t = 10$. $p = 0.2(10) + 11.7 \Rightarrow p = 13.7\%$.

126. The year 2004 is represented by $t = 0$, so the year 2009 is represented by $t = 5$. The two points are (0, 82.7) and (5, 84.2). So the slope is $\frac{84.2 - 82.7}{5} = \frac{1.5}{5} = 0.3$. The equation is $y = 0.3t + 82.7$.

127. a.



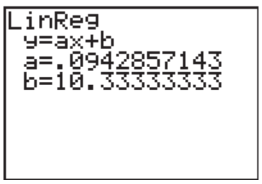
$y = -2x + 12.4$

b. 

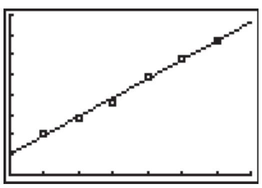
[0, 6, 1] by [0, 12, 1]

c. The price in the table is given as the number of nickels. 35¢ = 7 nickels, so let $x = 7.\ \ y = -2(7) + 12.4 = -1.6$

Thus, no newspapers will be sold if the price per copy is 35¢. Note that this is also clear from the graph, which appears to cross the *x*-axis at approximately $x = 6$.

128. a. 

$y \approx 0.09x + 10.3$

b. 

[0, 700, 100] by [0, 80, 10]

c. The advertising expenses in the table are given as thousands of dollars, so let $x = 700.\ \ y \approx 0.09(700) + 10.3 = 73.3$

Sales are given in thousands, so approximately $73.3 \times 1000 = 73{,}300$ computers will be sold.

2.3 Beyond the Basics

129. $3 = \dfrac{c-3}{1-(-2)} \Rightarrow 9 = c - 3 \Rightarrow c = 12$

130. The *y*-intercept is –4, so its coordinates are (0, –4). Substitute $x = 0$, $y = -4$ into the equation and solve for *c*.

$3x - cy - 2 = 0 \Rightarrow 3(0) - c(-4) - 2 = 0 \Rightarrow$

$4c - 2 = 0 \Rightarrow 4c = 2 \Rightarrow c = \dfrac{1}{2}$

131. a. Let $A = (0, 1)$, $B = (1, 3)$, $C = (-1, -1)$.

$m_{AB} = \dfrac{3-1}{1-0} = 2; m_{BC} = \dfrac{-1-3}{-1-1} = \dfrac{-4}{-2} = 2$

$m_{AC} = \dfrac{-1-1}{-1-0} = 2$

The slopes of the three segments are the same, so the points are collinear.

b. $d(A,B) = \sqrt{(1-0)^2 + (3-1)^2} = \sqrt{5}$

$d(B,C) = \sqrt{(-1-1)^2 + (-1-3)^2} = 2\sqrt{5}$

$d(A,C) = \sqrt{(-1-0)^2 + (-1-1)^2} = \sqrt{5}$

Because $d(B, C) = d(A, B) + d(A, C)$, the three points are collinear.

132. a. Let $A = (1, 0.5)$, $B = (2, 0)$, $C = (0.5, 0.75)$.

$m_{AB} = \dfrac{0-0.5}{2-1} = -0.5; m_{BC} = \dfrac{0.75-0}{0.5-2} = -0.5$

$m_{AC} = \dfrac{0.75-0.5}{0.5-1} = -0.5$

The slopes of the three segments are the same, so the points are collinear.

b. $d(A,B) = \sqrt{(1-2)^2 + \left(\dfrac{1}{2}-0\right)^2} = \sqrt{\dfrac{5}{4}} = \dfrac{\sqrt{5}}{2}$

$d(B,C) = \sqrt{\left(\dfrac{1}{2}-2\right)^2 + \left(\dfrac{3}{4}-0\right)^2}$

$= \sqrt{\dfrac{45}{16}} = \dfrac{3\sqrt{5}}{4}$

$d(A,C) = \sqrt{\left(\dfrac{1}{2}-1\right)^2 + \left(\dfrac{3}{4}-\dfrac{1}{2}\right)^2}$

$= \sqrt{\dfrac{5}{16}} = \dfrac{\sqrt{5}}{4}$

Because $d(B, C) = d(A, B) + d(A, C)$, the three points are collinear.

133. a. $m_{AB} = \dfrac{4-1}{-1-1} = -\dfrac{3}{2}; m_{BC} = \dfrac{8-4}{5-(-1)} = \dfrac{2}{3}$.

The product of the slopes = –1, so $AB \perp BC$.

b. $d(A,B) = \sqrt{(-1-1)^2 + (4-1)^2} = \sqrt{13}$

$d(B,C) = \sqrt{(5-(-1))^2 + (8-4)^2} = \sqrt{52}$

$d(A,C) = \sqrt{(5-1)^2 + (8-1)^2} = \sqrt{65}$

$(d(A,B))^2 + (d(B,C))^2 = (d(A,C))^2$, so the triangle is a right triangle.

134. $m_{AB} = \frac{2-(-1)}{1-(-4)} = \frac{3}{5}; m_{BC} = \frac{1-2}{3-1} = -\frac{1}{2}$

$m_{CD} = \frac{-2-1}{-2-3} = \frac{3}{5}; m_{AD} = \frac{-2-(-1)}{-2-(-4)} = -\frac{1}{2}$

So, $AB \parallel CD$ and $BC \parallel AD$, and $ABCD$ is a parallelogram.

For exercises 135 and 136, refer to the figures accompanying the exercises in your text.

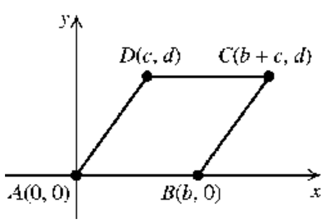
135. $\overline{AD}$ and $\overline{BC}$ are parallel because they lie on parallel lines l_1 and l_2. $\overline{AB}$ and $\overline{CD}$ are parallel because they are parallel to the x-axis. Therefore, $ABCD$ is a parallelogram. $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$ because opposite sides of a parallelogram are congruent. $\triangle ABD \cong \triangle CDB$ by SSS. Then $m_1 = \frac{\text{rise}}{\text{run}} = \frac{BD}{CD}$ and $m_2 = \frac{\text{rise}}{\text{run}} = \frac{BD}{AB}$. Since $AB = CD$, $m_1 = \frac{BD}{CD} = \frac{BD}{AB} = m_2$.

136. $\triangle OKA \sim \triangle BLO$ because $OL = AK = d$ and $BL = OK = c$. Then, $m_1 = \frac{\text{rise}}{\text{run}} = \frac{d}{c}$ and

$m_2 = \frac{\text{rise}}{\text{run}} = \frac{c}{-d} = -\frac{c}{d}$.

$m_1 \cdot m_2 = \frac{d}{c}\left(-\frac{c}{d}\right) = -1.$

137. Let the quadrilateral $ABCD$ be such that $AB \cong CD$ and $AB \parallel CD$. Locate the points as shown in the figure.



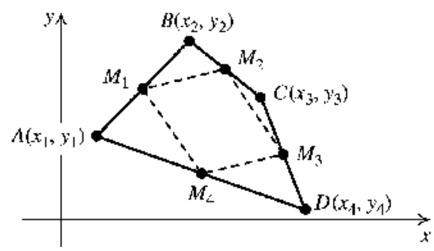
Because $AB \parallel CD$, the y-coordinates of C and D are equal. Because $AB \cong CD$, the x-coordinates of the points are as shown in the figure. The slope of AD is d/c. The slope of BC is $\frac{d-0}{b+c-b} = \frac{d}{c}$. So $AD \parallel BC$.

$d(A,D) = \sqrt{d^2 + c^2}$.

$d(B,C) = \sqrt{d^2 + ((b+c)-b)^2} = \sqrt{d^2 + c^2}$.

So $AD \cong BC$.

138. Let $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ and $D(x_4, y_4)$ be the vertices of the quadrilateral.



Then the midpoint M_1 of AB is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$; the midpoint M_2 of BC is $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$; the midpoint M_3 of CD is $\left(\frac{x_3+x_4}{2}, \frac{y_3+y_4}{2}\right)$; and the midpoint M_4 of AD is $\left(\frac{x_1+x_4}{2}, \frac{y_1+y_4}{2}\right)$.

The slope of M_1M_2 is

$$\frac{\frac{y_1+y_2}{2} - \frac{y_2+y_3}{2}}{\frac{x_1+x_2}{2} - \frac{x_2+x_3}{2}} = \frac{y_1-y_3}{x_1-x_3}.$$

The slope of M_2M_3 is

$$\frac{\frac{y_2+y_3}{2} - \frac{y_3+y_4}{2}}{\frac{x_2+x_3}{2} - \frac{x_3+x_4}{2}} = \frac{y_2-y_4}{x_2-x_4}.$$

The slope of M_3M_4 is

$$\frac{\frac{y_3+y_4}{2} - \frac{y_1+y_4}{2}}{\frac{x_3+x_4}{2} - \frac{x_1+x_4}{2}} = \frac{y_3-y_1}{x_3-x_1} = \frac{y_1-y_3}{x_1-x_3}.$$

The slope of M_1M_4 is

$$\frac{\frac{y_1+y_2}{2} - \frac{y_1+y_4}{2}}{\frac{x_1+x_2}{2} - \frac{x_1+x_4}{2}} = \frac{y_2-y_4}{x_2-x_4}.$$

So $M_1M_2 \parallel M_3M_4$ and $M_2M_3 \parallel M_1M_4$, and $M_1M_2M_3M_4$ is a parallelogram.

139. Let (x, y) be the coordinates of point B. Then

$d(A,B) = 12.5 = \sqrt{(x-2)^2 + (y-2)^2} \Rightarrow$

$(x-2)^2 + (y-2)^2 = 156.25$ and

$m_{AB} = \frac{4}{3} = \frac{y-2}{x-2} \Rightarrow 4(x-2) = 3(y-2) \Rightarrow$

$y = \frac{4}{3}x - \frac{2}{3}$. Substitute this into the first equation and solve for x:

$$(x-2)^2 + \left(\left(\frac{4}{3}x - \frac{2}{3}\right) - 2\right)^2 = 156.25$$

$$(x-2)^2 + \left(\frac{4}{3}x - \frac{8}{3}\right)^2 = 156.25$$

$$x^2 - 4x + 4 + \frac{16}{9}x^2 - \frac{64}{9}x + \frac{64}{9} = 156.25$$

$$9x^2 - 36x + 36 + 16x^2 - 64x + 64 = 1406.25$$

$$25x^2 - 100x - 1306.25 = 0$$

Solve this equation using the quadratic formula:

$$x = \frac{100 \pm \sqrt{100^2 - 4(25)(-1306.25)}}{2(25)}$$

$$= \frac{100 \pm \sqrt{10,000 + 130,625}}{50}$$

$$= \frac{100 \pm \sqrt{140,625}}{50} = \frac{100 \pm 375}{50}$$

$= 9.5$ or -5.5

Now find y by substituting the x-values into the slope formula: $\frac{4}{3} = \frac{y-2}{9.5-2} \Rightarrow y = 12$ or

$\frac{4}{3} = \frac{y-2}{-5.5-2} \Rightarrow y = -8.$ So the coordinates of B are (9.5, 12) or (–5.5, –8).

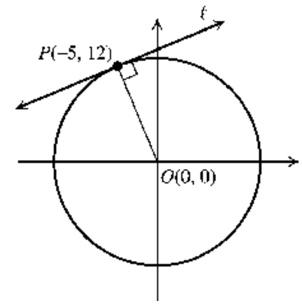
140. Let (x, y) be a point on the circle with (x_1, y_1) and (x_2, y_2) as the endpoints of a diameter. Then the line that passes through (x, y) and (x_1, y_1) is perpendicular to the line that passes through (x, y) and (x_2, y_2), and their slopes are negative reciprocals. So

$$\frac{y-y_1}{x-x_1} = -\frac{x-x_2}{y-y_2} \Rightarrow$$

$$(y-y_1)(y-y_2) = -(x-x_1)(x-x_2) \Rightarrow$$

$$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0.$$

141.



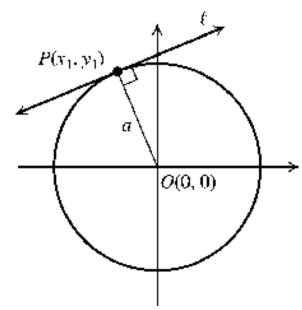
$$m_{\overline{OP}} = \frac{12-0}{-5-0} = -\frac{12}{5}$$

Since the tangent line ℓ is perpendicular to $\overline{OP}$, the slope of ℓ is the negative reciprocal of $-\frac{12}{5}$ or $\frac{5}{12}$. Using the point-slope form, we have

$$y - 12 = \frac{5}{12}\left[x - (-5)\right] \Rightarrow y - 12 = \frac{5}{12}(x+5) \Rightarrow$$

$$y - 12 = \frac{5}{12}x + \frac{25}{12} \Rightarrow y = \frac{5}{12}x + \frac{169}{12}$$

142.



$$m_{\overline{OP}} = \frac{y_1 - 0}{x_1 - 0} = \frac{y_1}{x_1}$$

Since the tangent line ℓ is perpendicular to $\overline{OP}$, the slope of ℓ is the negative reciprocal of $\frac{y_1}{x_1}$ or $-\frac{x_1}{y_1}$.

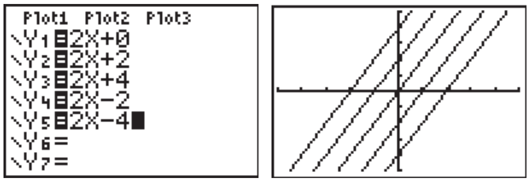
Using the point-slope form, we have

$$y - y_1 = -\frac{x_1}{y_1}(x - x_1) \Rightarrow$$

$$yy_1 - y_1^2 = -xx_1 + x_1^2 \Rightarrow xx_1 + yy_1 = x_1^2 + y_1^2$$

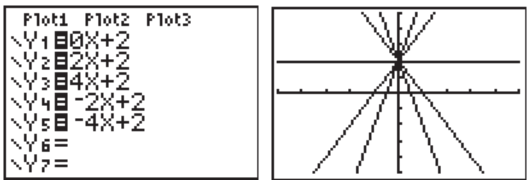
Since the equation of the circle is $x^2 + y^2 = a^2$, we substitute a^2 for $x_1^2 + y_1^2$ to obtain $xx_1 + yy_1 = a^2$.

143.



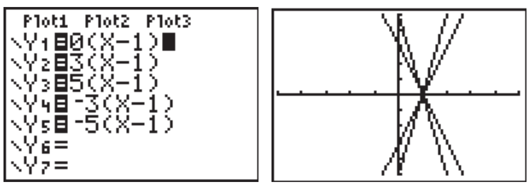
The family of lines has slope 2. The lines have different y-intercepts.

144.



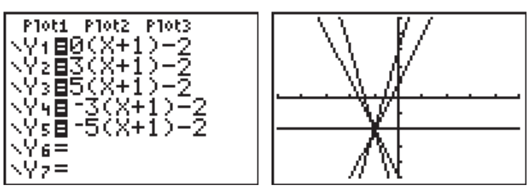
The family of lines has y-intercept 2. The lines have different slopes.

145.



The lines pass through (1, 0). The lines have different slopes.

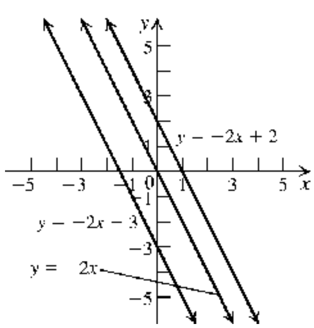
146.



The lines pass through (−1, −2). The lines have different slopes.

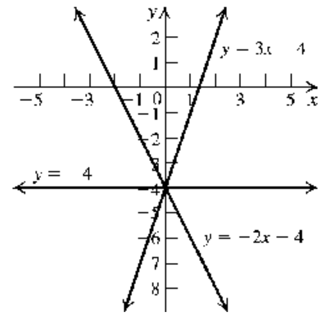
2.3 Critical Thinking/Discussion/Writing

147. a.



This is a family of lines parallel to the line $y = -2x$. They all have slope –2.

b.



This is a family of lines that passes through the point (0, –4). Their y-intercept is –4.

148. $\left.\begin{array}{l} y = m_1x + b_1 \\ y = m_2x + b_2 \end{array}\right\} \Rightarrow m_1x + b_1 = m_2x + b_2 \Rightarrow$

$m_1x - m_2x = b_2 - b_1 \Rightarrow x(m_1 - m_2) = b_2 - b_1 \Rightarrow$

$x = \dfrac{b_2 - b_1}{m_1 - m_2}$

a. If $m_1 > m_2 > 0$ and $b_1 > b_2$, then

$x = \dfrac{b_2 - b_1}{m_1 - m_2} = -\dfrac{b_1 - b_2}{m_1 - m_2}.$

b. If $m_1 > m_2 > 0$ and $b_1 < b_2$, then

$x = \dfrac{b_2 - b_1}{m_1 - m_2}.$

c. If $m_1 < m_2 < 0$ and $b_1 > b_2$, then

$x = \dfrac{b_2 - b_1}{m_1 - m_2} = \dfrac{b_1 - b_2}{m_2 - m_1}.$

d. If $m_1 < m_2 < 0$ and $b_1 < b_2$, then

$x = \dfrac{b_2 - b_1}{m_1 - m_2} = -\dfrac{b_2 - b_1}{m_2 - m_1}.$

2.3 Getting Ready for the Next Section

149. $x^2 - 4 = 0 \Rightarrow (x + 2)(x - 2) = 0 \Rightarrow$
$x + 2 = 0 \Rightarrow x = -2$ or
$x - 2 = 0 \Rightarrow x = 2$
Solution: {–2, 2}

150. $1 - x^2 = 0 \Rightarrow (1 + x)(1 - x) = 0 \Rightarrow$
$1 + x = 0 \Rightarrow x = -1$ or
$1 - x = 0 \Rightarrow x = 1$
Solution: {–1, 1}

151. $x^2 - x - 2 = 0$
$(x + 1)(x - 2) = 0$
$x + 1 = 0 \Rightarrow x = -1$ or $x - 2 = 0 \Rightarrow x = 2$
Solution: {–1, 2}

152. $x^2 + 2x - 3 = 0$
$(x+3)(x-1) = 0$
$x + 3 = 0 \Rightarrow x = -3$
$x - 1 = 0 \Rightarrow x = 1$
Solution: {–3, 1}

153. $(3(a+h)+1)-(3a+1) = (3a+3h+1)-(3a+1)$
$= 3h$

154. $\left(2(a+h)^2+1\right)-\left(2a^2+1\right)$
$= \left(2\left(a^2+2ah+h^2\right)+1\right)-\left(2a^2+1\right)$
$= \left(2a^2+4ah+2h^2+1\right)-\left(2a^2+1\right)$
$= 4ah+2h^2$

155. $\dfrac{-(a+h)^2+a^2}{h} = \dfrac{-\left(a^2+2ah+h^2\right)+a^2}{h}$
$= \dfrac{-2ah-h^2}{h} = \dfrac{h(-2a-h)}{h}$
$= -2a-h$

156. $\dfrac{1}{h}\left(\dfrac{1}{a+h}-\dfrac{1}{a}\right) = \dfrac{1}{h}\left(\dfrac{a-(a+h)}{a(a+h)}\right) = \dfrac{-h}{h(a(a+h))}$
$= -\dfrac{1}{a(a+h)}$

157. $(x-1)(x-3) < 0$
Solve the associated equation:
$(x-1)(x-3) = 0 \Rightarrow x = 1$ or $x = 3$.
So, the intervals are
$(-\infty, 1), (1, 3)$, and $(1, \infty)$.

Interval	Test point	Value of $(x-1)(x-3)$	Result
$(-\infty, 1)$	0	3	+
$(1, 3)$	2	–2	–
$(3, \infty)$	4	3	+

The solution set is $(1, 3)$.

158. $x^2 - 2x - 3 \geq 0$
$x^2 - 2x - 3 \geq 0 \Rightarrow (x-3)(x+1) \geq 0$
Now solve the associated equation:
$(x-3)(x+1) = 0 \Rightarrow x = 3$ or $x = -1$.
So, the intervals are
$(-\infty, -1], [-1, 3]$, and $[3, \infty)$.

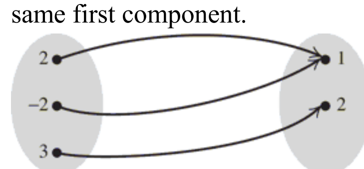
Interval	Test point	Value of $x^2 - 2x - 3$	Result
$(-\infty, -1]$	–2	5	+
$[-1, 3]$	0	–3	–
$[3, \infty)$	5	12	+

The solution set is $(-\infty, -1] \cup [3, \infty)$.

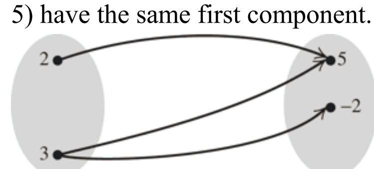
2.4 Functions

2.4 Practice Problems

1. a. The domain of R is $\{2, -2, 3\}$ and its range is $\{1, 2\}$. The relation R is a function because no two ordered pairs in R have the same first component.



b. The domain of S is $\{2, 3\}$ and its range is $\{5, -2\}$. The relation S is not a function because the ordered paired (3, −2) and (3, 5) have the same first component.



2. Solve each equation for y.

a. $2x^2 - y^2 = 1 \Rightarrow 2x^2 - 1 = y^2 \Rightarrow$
$\pm\sqrt{2x^2-1} = y$; not a function

b. $x - 2y = 5 \Rightarrow x - 5 = 2y \Rightarrow \dfrac{1}{2}(x-5) = y$;
a function

3. $g(x) = -2x^2 + 5x$

a. $g(0) = -2(0)^2 + 5(0) = 0$

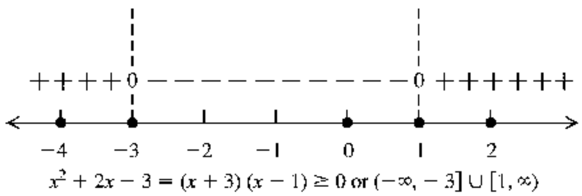
b. $g(-1) = -2(-1)^2 + 5(-1) = -7$

c. $g(x+h) = -2(x+h)^2 + 5(x+h)$
$= -2\left(x^2+2xh+h^2\right)+5x+5h$
$= -2x^2-4hx+5x-2h^2+5h$

4. $A_{TLMS} = (\text{length})(\text{height}) = (|3-1|)(22)$
$= (2)(22) = 44$ sq. units

5. a. $f(x) = \frac{1}{\sqrt{1-x}}$ is not defined when $1 - x = 0 \Rightarrow x = 1$ or when $1 - x < 0 \Rightarrow 1 < x.$. Thus, the domain of *f* is $(-\infty, 1)$.

b. $g(x) = \sqrt{x^2 + 2x - 3}$ is not defined when $x^2 + 2x - 3 < 0.$
Use the test point method to see that $x^2 + 2x - 3 < 0$ on the interval (–3, 1). Thus, the domain of *g* is $(-\infty, -3] \cup [1, \infty)$.



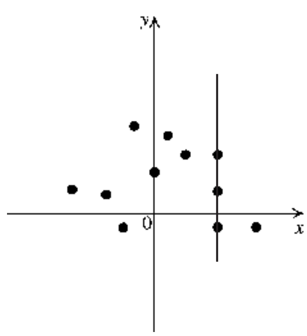
6. $f(x) = x^2$, domain $X = [-3, 3]$

a. $f(x) = 10 \Rightarrow x^2 = 10 \Rightarrow x = \pm\sqrt{10} \approx \pm 3.16$
Since $\sqrt{10} > 3$ and $-\sqrt{10} < -3$, neither solution is in the interval $X = [-3, 3]$. Therefore, 10 is not in the range of *f*.

b. $f(x) = 4 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$
Since $-3 < -2 < 2 < 3$, 4 is in the range of *f*.

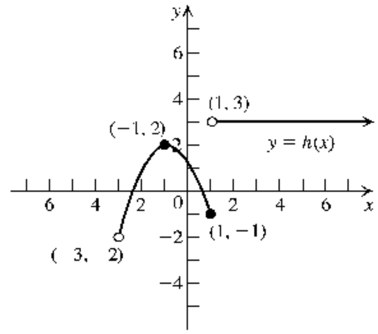
c. The range of *f* is the interval [0, 9] because for each number *y* in this interval, the number $x = \sqrt{y}$ is in the interval [−3, 3].

7.



The graph is not a function because a vertical line can be drawn through three points, as shown.

8.



Domain: $(-3, \infty)$; range: $(-2, 2] \cup \{3\}$

9. $y = f(x) = x^2 + 4x - 5$

a. Check whether the ordered pair (2, 7) satisfies the equation:
$$7 \stackrel{?}{=} 2^2 + 4(2) - 5$$
$$7 = 7 \checkmark$$
The point (2, 7) is on the graph.

b. Let $y = -8$, then solve for *x*:
$-8 = x^2 + 4x - 5 \Rightarrow 0 = x^2 + 4x + 3 \Rightarrow$
$0 = (x+3)(x+1) \Rightarrow x = -3$ or $x = -1$
The points (−3, −8) and (−1, −8) lie on the graph.

c. Let $x = 0$, then solve for *y*:
$y = 0^2 + 4(0) - 5 = -5$
The *y*-intercept is −5.

d. Let $y = 0$, then solve for *x*:
$0 = x^2 + 4x - 5 \Rightarrow 0 = (x+5)(x-1) \Rightarrow$
$x = -5$ or $x = 1$
The *x*-intercepts are −5 and 1.

10. The range of *C*(*t*) is [6, 12).
$C(11) = \frac{1}{2}C(10) + 6 = \frac{1}{2}(11.989) + 6 \approx 11.995$.

11. From Example 11, we have $AP = \sqrt{500^2 + x^2}$ and $\overline{PD} = 1200 - x$ feet. If *c* = the cost on land, the total cost *C* is given by $C = 1.3c(PD) + c(AP)$
$$= 1.3c\sqrt{500^2 + x^2} + c(1200 - x)$$
$$= c\left[1.3\sqrt{500^2 + x^2} + 1200 - x\right]$$

12. a. $C(x) = 1200x + 100{,}000$

b. $R(x) = 2500x$

c. $P(x) = R(x) - C(x)$
$= 2500x - (1200x + 100,000)$
$= 1300x - 100,000$

d. The break-even point occurs when $C(x) = R(x)$.
$1200x + 100,000 = 2500x$
$100,000 = 1300x \Rightarrow x \approx 77$
Metro needs 77 shows to break even.

2.4 Basic Concepts and Skills

1. In the functional notation $y = f(x)$, x is the independent variable.
2. If $f(-2) = 7$, then -2 is in the domain of the function f, and 7 is in the range of f.
3. If the point $(9, -14)$ is on the graph of a function f, then $f(9) = -14$.
4. If (3, 7) and (3, 0) ar both points on a graph, then the graph cannot be the graph of a function.
5. False.
6. False. For example, if $f(x) = \frac{1}{x}$, then $a = 1$ and $b = -1$ are both in the domain of f. However, $a + b = 0$ is not in the domain of f.
7. True. $-x = 7$ and the square root function is defined for all positive numbers.
8. False. The domain of f is all real x for $x > -2$. Values of $x \le -2$ make the fraction undefined.

2.4 Building Skills

9. Domain: $\{a, b, c\}$; range: $\{d, e\}$; function
10. Domain: $\{a, b, c\}$; range: $\{d, e, f\}$; function
11. Domain: $\{a, b, c\}$; range: $\{1, 2\}$; function
12. Domain: $\{1, 2, 3\}$; range: $\{a, b, c, d\}$; not a function
13. Domain: $\{0, 3, 8\}$; range: $\{-3, -2, -1, 1, 2\}$; not a function
14. Domain: $\{-3, -1, 0, 1, 2, 3\}$; range: $\{-8, -3, 0, 1\}$; function
15. $x + y = 2 \Rightarrow y = -x + 2$; a function
16. $x = y - 1 \Rightarrow y = x + 1$; a function
17. $y = \frac{1}{x}$; a function
18. $xy = -1 \Rightarrow y = -\frac{1}{x}$; a function
19. $y^2 = x^2 \Rightarrow y = \pm\sqrt{x^2} \Rightarrow y = \pm x$; not a function
20. $x = |y| \Rightarrow y = x$ or $y = -x$; not a function
21. $y = \frac{1}{\sqrt{2x-5}}$; a function
22. $y = \frac{1}{\sqrt{x^2-1}}$; a function
23. $2 - y = 3x \Rightarrow y = 2 - 3x$; a function
24. $3x - 5y = 15 \Rightarrow y = \frac{3}{5}x - 3$; a function
25. $x + y^2 = 8 \Rightarrow y = \pm\sqrt{8-x}$; not a function
26. $x = y^2 \Rightarrow y = \sqrt{x}$ or $y = -\sqrt{x}$; not a function
27. $x^2 + y^3 = 5 \Rightarrow y = \sqrt[3]{5 - x^2}$; a function
28. $x + y^3 = 8 \Rightarrow y = \sqrt[3]{8-x}$; a function

In exercises 29–32, $f(x) = x^2 - 3x + 1$, $g(x) = \frac{2}{\sqrt{x}}$, and $h(x) = \sqrt{2-x}$.

29. $f(0) = 0^2 - 3(0) + 1 = 1$
$g(0) = \frac{2}{\sqrt{0}} \Rightarrow g(0)$ is undefined
$h(0) = \sqrt{2-0} = \sqrt{2}$
$f(a) = a^2 - 3a + 1$
$f(-x) = (-x)^2 - 3(-x) + 1 = x^2 + 3x + 1$

30. $f(1) = 1^2 - 3(1) + 1 = -1; g(1) = \frac{2}{\sqrt{1}} = 2;$
$h(1) = \sqrt{2-1} = 1; g(a) = \frac{2}{\sqrt{a}};$
$g(x^2) = \frac{2}{\sqrt{x^2}} = \frac{2}{|x|}$

31. $f(-1) = (-1)^2 - 3(-1) + 1 = 5;$
$g(-1) = \frac{2}{\sqrt{-1}} \Rightarrow g(-1)$ is undefined;
$h(-1) = \sqrt{2-(-1)} = \sqrt{3}; h(c) = \sqrt{2-c};$
$h(-x) = \sqrt{2-(-x)} = \sqrt{2+x}$

32. $f(4) = 4^2 - 3(4) + 1 = 5; g(4) = \frac{2}{\sqrt{4}} = 1;$

$h(4) = \sqrt{2-4} = \sqrt{-2} \Rightarrow h(4)$ is undefined;

$g(2+k) = \frac{2}{\sqrt{2+k}};$

$$f(a+k) = (a+k)^2 - 3(a+k) + 1 = a^2 + 2ak + k^2 - 3a - 3k + 1$$

33. a. $f(0) = \frac{2(0)}{\sqrt{4-0^2}} = 0$

b. $f(1) = \frac{2(1)}{\sqrt{4-1^2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

c. $f(2) = \frac{2(2)}{\sqrt{4-2^2}} = \frac{4}{0} \Rightarrow f(2)$ is undefined

d. $f(-2) = \frac{2(-2)}{\sqrt{4-(-2)^2}} = \frac{-4}{0} \Rightarrow f(-2)$ is undefined

e. $f(-x) = \frac{2(-x)}{\sqrt{4-(-x)^2}} = \frac{-2x}{\sqrt{4-x^2}}$

34. a. $g(0) = 2(0) + \sqrt{0^2 - 4} \Rightarrow g(0)$ is undefined

b. $g(1) = 2(1) + \sqrt{1^2 - 4} \Rightarrow g(1)$ is undefined

c. $g(2) = 2(2) + \sqrt{2^2 - 4} = 4$

d. $g(-3) = 2(-3) + \sqrt{(-3)^2 - 4} = -6 + \sqrt{5}$

e. $g(-x) = 2(-x) + \sqrt{(-x)^2 - 4} = -2x + \sqrt{x^2 - 4}$

35. The width of each rectangle is 1. The height of the left rectangle is $f(1) = 1^2 + 2 = 3.$ The height of the right rectangle is $f(2) = 2^2 + 2 = 6.$

$$A = (1)(f(1)) + (1)(f(2)) = 1(3) + (1)(6) = 9 \text{ sq. units}$$

36. The width of each rectangle is 1. The height of the left rectangle is $f(0) = 0^2 + 2 = 2.$ The height of the right rectangle is $f(1) = 1^2 + 2 = 3.$

$$A = (1)(f(0)) + (1)(f(1)) = 1(2) + (1)(3) = 5 \text{ sq. units}$$

37. $(-\infty, \infty)$

38. $(-\infty, \infty)$

39. The denominator is not defined for $x = 9$. The domain is $(-\infty, 9) \cup (9, \infty)$

40. The denominator is not defined for $x = -9$. The domain is $(-\infty, -9) \cup (-9, \infty)$

41. The denominator is not defined for $x = -1$ or $x = 1$. The domain is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty).$

42. The denominator is not defined for $x = -2$ or $x = 2$. The domain is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty).$

43. The numerator is not defined for $x < 3$, and the denominator is not defined for $x = -2$. The domain is $[3, \infty)$

44. The denominator is not defined for $x \geq 4$. The domain is $(-\infty, 4)$

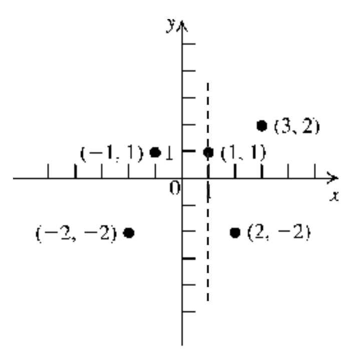
45. The denominator equals 0 if $x = -1$ or $x = -2$. The domain is $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$.

46. The denominator equals 0 if $x = -2$ or $x = -3$. The domain is $(-\infty, -3) \cup (-3, -2) \cup (-2, \infty)$.

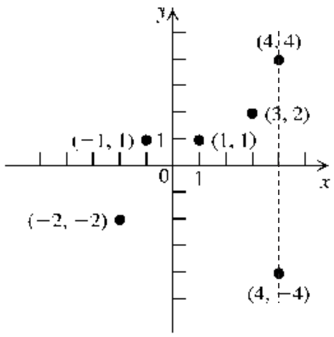
47. The denominator is not defined for $x = 0$. The domain is $(-\infty, 0) \cup (0, \infty)$

48. The denominator is defined for all values of x. The domain is $(-\infty, \infty)$ **.**

49. a function

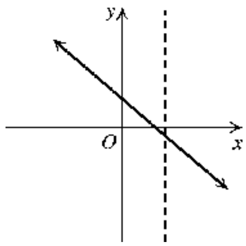


50.



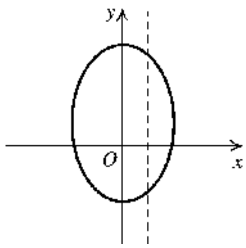
not a function

51.



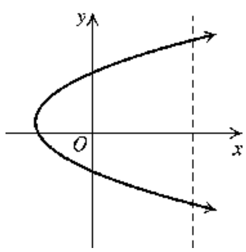
a function

52.



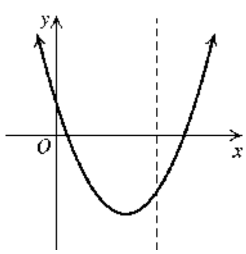
not a function

53.



not a function

54.



a function

55. $f(-4) = -2; f(-1) = 1; f(3) = 5; f(5) = 7$

56. $g(-2) = 5; g(1) = -4; g(3) = 0; g(4) = 5$

57. $h(-2) = -5; h(-1) = 4; h(0) = 3; h(1) = 4$

58. $f(-1) = 4; f(0) = 0; f(1) = -4$

59. $h(x) = 7$, so solve the equation

$7 = x^2 - x + 1$.

$x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0 \Rightarrow x = -2$ or $x = 3$.

60. $H(x) = 7$, so solve the equation

$7 = x^2 + x + 8$.

$$x^2 + x + 1 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1 - 4(1)(1)}}{2(1)} \Rightarrow$$

$$x = \frac{-1 \pm \sqrt{-3}}{2} \Rightarrow \text{there is no real solution.}$$

61. a. $1 = -2(1+1)^2 + 7 \Rightarrow 1 = -1$, which is false. Therefore, (1, 1) does not lie on the graph of *f*.

b. $1 = -2(x+1)^2 + 7 \Rightarrow 2(x+1)^2 = 6 \Rightarrow (x+1)^2 = 3 \Rightarrow x + 1 = \pm\sqrt{3} \Rightarrow x = -1 \pm \sqrt{3}$

The points $\left(-1-\sqrt{3}, 1\right)$ and $\left(-1+\sqrt{3}, 1\right)$ lie on the graph of *f*.

c. $y = -2(0+1)^2 + 7 \Rightarrow y = 5$

The *y*-intercept is (0, 5).

d. $0 = -2(x+1)^2 + 7 \Rightarrow -7 = -2(x+1)^2 \Rightarrow$

$$\frac{7}{2} = (x+1)^2 \Rightarrow \pm\sqrt{\frac{7}{2}} = \pm\frac{\sqrt{14}}{2} = x + 1 \Rightarrow$$

$$x = -1 \pm \frac{\sqrt{14}}{2}$$

The *x*-intercepts are $\left(-1 - \frac{\sqrt{14}}{2}, 0\right)$ and $\left(-1 + \frac{\sqrt{14}}{2}, 0\right)$.

62. a. $10 = -3(-2)^2 - 12(-2) \Rightarrow 10 = 12$, which is false. Therefore, (−2, 10) does not lie on the graph of *f*.

b. $f(x) = 12$, so solve the equation

$-3x^2 - 12x = 12$.

$-3x^2 - 12x = 12 \Rightarrow x^2 + 4x = -4 \Rightarrow$

$x^2 + 4x + 4 = 0 \Rightarrow (x+2)^2 = 0 \Rightarrow$

$x + 2 = 0 \Rightarrow x = -2$

c. $y = -3(0)^2 - 12(0) \Rightarrow y = 0$

The *y*-intercept is (0, 0).

d. $0 = -3x^2 - 12x \Rightarrow 0 = -3x(x+4) \Rightarrow$
$x = 0$ or $x = -4$
The x-intercepts are (0, 0) and (−4, 0).

63. Domain: [−3, 2]; range: [−3, 3]

64. Domain: [−1, 3]; range: [−2, 4]

65. Domain: $[-4, \infty)$; range: [−2, 3]

66. Domain: $(-\infty, 4]$; range: [−1, 3]

67. Domain: $[-3, \infty)$; range: $[-1, 4] \cup \{-3\}$

68. Domain: $(-\infty, -1) \cup [1, 4)$
Range: (−2, 4]

69. Domain: $(-\infty, 4] \cup [-2, 2] \cup [4, \infty)$
Range: $[-2, 2] \cup \{3\}$

70. Domain: $(-\infty, -2) \cup [-1, \infty)$
Range: $(-\infty, \infty)$

71. $[-9, \infty)$ **72.** $[-1, 7]$

73. −3, 4, 7, 9 **74.** 6

75. $f(-7) = 4,\ f(1) = 5,\ f(5) = 2$

76. $f(-4) = 4,\ f(-1) = 7,\ f(3) = 3$

77. $\{-3.75, -2.25, 3\} \cup [12, \infty)$

78. $\varnothing$

79. $[-9, \infty)$ **80.** $\{-4\} \cup [-2, 6]$

81. $g(-4) = -1,\ g(1) = 3,\ g(3) = 4$

82. $|g(-5) - g(5)| = |-2 - 6| = 8$

83. $[-9, -5)$ **84.** $[5, \infty)$

2.4 Applying the Concepts

85. A function because there is only one high temperature per day.

86. A function because there is only one cost of a first-class stamp on January 1 each year.

87. Not a function because there are several states that begin with N (i.e., New York, New Jersey, New Mexico, Nevada, North Carolina, North Dakota); there are also several states that begin with T and S.

88. Not a function because people with a different name may have the same birthday.

89. $A(x) = x^2; A(4) = 16;$ $A(4)$ represents the area of a tile with side 4.

90. $V(x) = x^3; V(3) = 27$ in.^{3}; $V(3)$ represents the volume of a cube with edge 3.

91. It is a function. $S(x) = 6x^2; S(3) = 54$

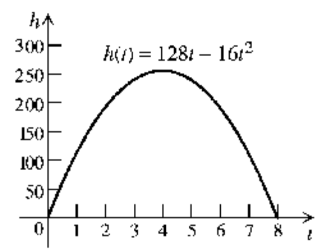
92. $f(x) = \dfrac{x}{39.37}; f(59) \approx 1.5$ meters

93. a. The domain is [0, 8].

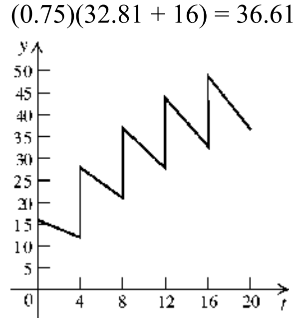
b. $h(2) = 128(2) - 16(2^2) = 192$
$h(4) = 128(4) - 16(4^2) = 256$
$h(6) = 128(6) - 16(6^2) = 192$

c. $0 = 128t - 16t^2 \Rightarrow 0 = 16t(8 - t) \Rightarrow$
$t = 0$ or $t = 8$. It will take 8 seconds for the stone to hit the ground.

d.



94. After 4 hours, there are (0.75)(16) = 12 ml of the drug.
After 8 hours, there are (0.75)(12 + 16) = 21 ml. After 12 hours, there are (0.75)(21 + 16) = 27.75 ml.
After 16 hours, there are (0.75)(27.75 + 16) = 32.81 ml.
After 20 hours, there are (0.75)(32.81 + 16) = 36.61 ml.

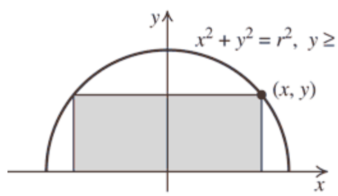


95. $x + y = 28 \Rightarrow y = 28 - x$
$P = x(28 - x) = 28x - x^2$

96. $P = 60 = 2(x + y) \Rightarrow 30 = x + y \Rightarrow y = 30 - x$
$A = x(30 - x) = 30x - x^2$

97. Note that the length of the base = the width of the base = x.
$V = lwh = x^2h = 64 \Rightarrow h = \frac{64}{x^2}$
$S = 2lw + 2lh + 2wh$
$= 2x^2 + 2x\left(\frac{64}{x^2}\right) + 2x\left(\frac{64}{x^2}\right)$
$= 2x^2 + \frac{128}{x} + \frac{128}{x} = 2x^2 + \frac{256}{x}$

98.



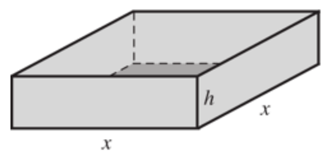
a. $x^2 + y^2 = r^2 \Rightarrow y^2 = r^2 - x^2 \Rightarrow$
$y = \sqrt{r^2 - x^2}$
The length of the rectangle is $2x$ and its height is $y = \sqrt{r^2 - x^2}$.
$P = 2l + 2w = 2(2x) + 2\sqrt{r^2 - x^2}$
$= 4x + 2\sqrt{r^2 - x^2}$

b. $A = lw = 2x\sqrt{r^2 - x^2}$

99. The piece with length x is formed into a circle, so $C = x = 2\pi r \Rightarrow r = \frac{x}{2\pi}$. Thus, the area of the circle is $A = \pi r^2 = \pi\left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi}$.
The piece with length $20 - x$ is formed into a square, so $P = 20 - x = 4s \Rightarrow s = \frac{1}{4}(20 - x)$.
Thus, the area of the square is
$s^2 = \left[\frac{1}{4}(20 - x)\right]^2 = \frac{1}{16}(20 - x)^2$.
The sum of the areas is $A = \frac{x^2}{4\pi} + \frac{1}{16}(20 - x)^2$

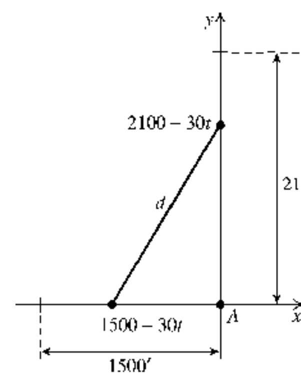
100. The volume of the tank is $V = 64 = \pi r^2 h$, so $h = \frac{64}{\pi r^2}$. The top is open, so the surface area is given by
$\pi r^2 + 2\pi rh = \pi r^2 + 2\pi r\left(\frac{64}{\pi r^2}\right)$
$= \pi r^2 + \frac{128}{r}$.

101. The volume of the pool is
$V = 288 = x^2 h \Rightarrow h = \frac{288}{x^2}$.



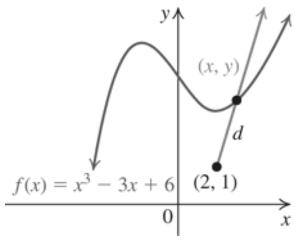
The total area to be tiled is
$4xh = 4x\left(\frac{288}{x^2}\right) = \frac{1152}{x}$
The cost of the tile is $6\left(\frac{1152}{x}\right) = \frac{6912}{x}$.
The area of the bottom of the pool is x^2, so the cost of the cement is $2x^2$. Therefore, the total cost is $C = 2x^2 + \frac{6912}{x}$.

102.



Using the Pythagorean theorem, we have
$d^2 = (1500 - 30t)^2 + (2100 - 30t)^2 \Rightarrow$
$d = \left[(1500 - 30t)^2 + (2100 - 30t)^2\right]^{1/2}$

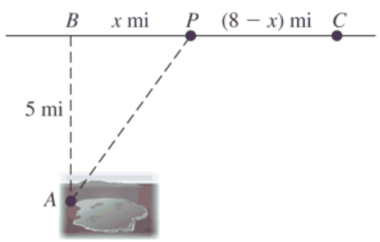
103.



Using the distance formula we have

$$d = \sqrt{(x-2)^2 + (y-1)^2}$$
$$= \sqrt{(x-2)^2 + \left[\left(x^3 - 3x + 6\right) - 1\right]^2}$$
$$= \sqrt{(x-2)^2 + \left(x^3 - 3x + 5\right)^2}$$
$$= \left[(x-2)^2 + \left(x^3 - 3x + 5\right)^2\right]^{1/2}$$

104.

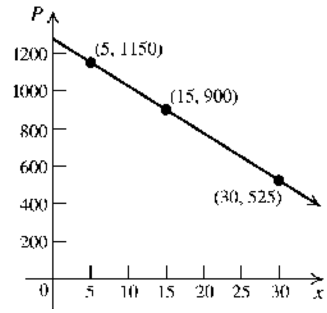


The distance from A to P is $\sqrt{x^2 + 5^2} = \sqrt{x^2 + 25}$ mi. At 4 mi/hr, it will take Julio $\frac{\sqrt{x^2+25}}{4}$ hr to row that distance. The distance from P to C is $(8 - x)$ mi, so it will take Julio $\frac{8-x}{5}$ hr to walk that distance. The total time it will take him to travel is

$$T = \frac{\sqrt{x^2+25}}{4} + \frac{8-x}{5}.$$

105. a. $p(5) = 1275 - 25(5) = 1150$. If 5000 TVs can be sold, the price per TV is $1150.
$p(15) = 1275 - 25(15) = 900$. If 15,000 TVs can be sold, the price per TV is $900.
$p(30) = 1275 - 25(30) = 525$. If 30,000 TVs can be sold, the price per TV is $525.

b.

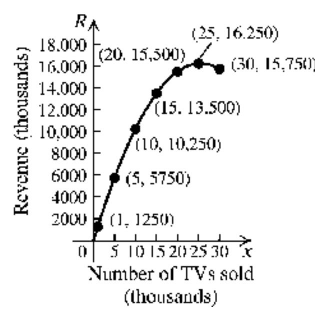


c. $650 = 1275 - 25x \Rightarrow -625 = -25x \Rightarrow x = 25$
25,000 TVs can be sold at $650 per TV.

106. a. $R(x) = (1275 - 25x)x = 1275x - 25x^2$
domain [1, 30]

b. $R(1) = 1275(1) - 25(1^2) = 1250$
$R(5) = 1275(5) - 25(5^2) = 5750$
$R(10) = 1275(10) - 25(10^2) = 10,250$
$R(15) = 1275(15) - 25(15^2) = 13,500$
$R(20) = 1275(20) - 25(20^2) = 15,500$
$R(25) = 1275(25) - 25(25^2) = 16,250$
$R(30) = 1275(30) - 25(30^2) = 15,750$
This is the amount of revenue (in thousands of dollars) for the given number of TVs sold (in thousands).

c.



d. $4700 = 1275x - 25x^2 \Rightarrow$
$x^2 - 51x + 188 = 0 \Rightarrow$
$$\frac{51 \pm \sqrt{51^2 - 4(1)(188)}}{2(1)} = x \Rightarrow$$
$x = 4$ or $x = 47$
47 is not in the domain, so 4000 TVs must be sold in order to generate revenue of 4.7 million dollars.

107. a. $C(x) = 5.5x + 75,000$

b. $R(x) = 0.6(15)x = 9x$

c. $P(x) = R(x) - C(x) = 9x - (5.5x + 75,000)$
$= 3.5x - 75,000$

d. The break-even point is when the profit is zero: $3.5x - 75,000 = 0 \Rightarrow x = 21,429$

e. $P(46,000) = 3.5(46,000) - 75,000 = \$86,000$

The company's profit is \$86,000 when 46,000 copies are sold.

108. a. $C(x) = 0.5x + 500,000; R(x) = 5x$. The break-even point is when the profit is zero (when the revenue equals the cost): $5x = 0.5x + 500,000 \Rightarrow 4.5x = 500,000 \Rightarrow$ $x = 111,111.11$. Because a fraction of a CD cannot be sold, 111,111 CD's must be sold.

b.
$$P(x) = R(x) - C(x)$$
$$750,000 = 5x - (0.5x + 500,000)$$
$$1,250,000 = 4.5x \Rightarrow x = 277,778$$

The company must sell 277,778 CDs in order to make a profit of \$750,000.

2.4 Beyond the Basics

109. $x = \dfrac{2}{y-4} \Rightarrow xy - 4x = 2 \Rightarrow xy = 2 + 4x \Rightarrow$

$y = \dfrac{4x+2}{x} \Rightarrow f(x) = \dfrac{4x+2}{x};$

Domain: $(-\infty, 0) \cup (0, \infty)$. $f(4) = \dfrac{9}{2}$.

110. $xy - 3 = 2y \Rightarrow 2y - xy = -3 \Rightarrow$

$y(2-x) = -3 \Rightarrow y = -\dfrac{3}{2-x} \Rightarrow f(x) = \dfrac{3}{x-2}$

Domain: $(-\infty, 2) \cup (2, \infty)$. $f(4) = \dfrac{3}{2}$

111. $\left(x^2+1\right)y + x = 2 \Rightarrow y = \dfrac{2-x}{x^2+1} \Rightarrow$

$f(x) = \dfrac{2-x}{x^2+1}$; Domain: $(-\infty, \infty)$; $f(4) = -\dfrac{2}{17}$

112. $yx^2 - \sqrt{x} = -2y \Rightarrow yx^2 + 2y = \sqrt{x} \Rightarrow$

$y\left(x^2+2\right) = \sqrt{x} \Rightarrow y = \dfrac{\sqrt{x}}{x^2+2} \Rightarrow f(x) = \dfrac{\sqrt{x}}{x^2+2}$

Domain: $[0, \infty)$; $f(4) = \dfrac{1}{9}$

113. $f(x) \neq g(x)$ because they have different domains.

114. $f(x) \neq g(x)$ because they have different domains.

115. $f(x) \neq g(x)$ because they have different domains. $g(x)$ is not defined for $x = -1$, while $f(x)$ is defined for all real numbers.

116. $f(x) \neq g(x)$ because they have different domains. $g(x)$ is not defined for $x = 3$, while $f(x)$ is not defined for $x = 3$ or $x = -2$.

117. $f(x) = g(x)$ because $f(3) = 10 = g(3)$ and $f(5) = 26 = g(5)$.

118. $f(x) \neq g(x)$ because $f(2) = 16$ while $g(2) = 13$.

119. $f(2) = 15 = a(2^2) + 2a - 3 \Rightarrow 15 = 6a - 3 \Rightarrow$ $a = 3$.

120. $g(6) = 28 = 6^2 + 6b + b^2 \Rightarrow b^2 + 6b + 8 = 0 \Rightarrow$ $(b+2)(b+4) = 0 \Rightarrow b = -2$ or $b = -4$.

121. $h(6) = 0 = \dfrac{3(6)+2a}{2(6)-b} \Rightarrow 0 = 18 + 2a \Rightarrow a = -9$

$h(3)$ is undefined $\Rightarrow \dfrac{3(3)+2(-9)}{2(3)-b}$ has a zero in the denominator. So $6 - b = 0 \Rightarrow b = 6$.

122. $f(x) = 2x - 3 \Rightarrow f\left(x^2\right) = 2x^2 - 3$

$\left(f(x)\right)^2 = (2x-3)^2 = 4x^2 - 12x + 9$

123. $g(x) = x^2 - \dfrac{1}{x^2} \Rightarrow g\left(\dfrac{1}{x}\right) = \dfrac{1}{x^2} - \dfrac{1}{\frac{1}{x^2}} = \dfrac{1}{x^2} - x^2$

$g(x) + g\left(\dfrac{1}{x}\right) = \left(x^2 - \dfrac{1}{x^2}\right) + \left(\dfrac{1}{x^2} - x^2\right) = 0$

124. $f(x) = \dfrac{x-1}{x+1} \Rightarrow f\left(\dfrac{x-1}{x+1}\right) = \dfrac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1}$

$$= \frac{\frac{(x-1)-(x+1)}{x+1}}{\frac{(x-1)+(x+1)}{x+1}} = -\frac{2}{2x} = -\frac{1}{x}$$

125. $f(x)=\dfrac{x+3}{4x-5}\Rightarrow$

$$f(t)=\dfrac{\dfrac{3+5x}{4x-1}+3}{4\left(\dfrac{3+5x}{4x-1}\right)-5}=\dfrac{\dfrac{(3+5x)+3(4x-1)}{4x-1}}{\dfrac{(12+20x)-(5(4x-1))}{4x-1}}$$
$$=\dfrac{(3+5x)+(12x-3)}{(12+20x)-(20x-5)}=\dfrac{17x}{17}=x$$

2.4 Critical Thinking/Discussion/Writing

126. Answers may vary. Sample answers are given

a. $y=\sqrt{x-2}$ **b.** $y=\dfrac{1}{\sqrt{x-2}}$

c. $y=\sqrt{2-x}$ **d.** $y=\dfrac{1}{\sqrt{2-x}}$

127. a. $ax^2+bx+c=0$

b. $y=c$

c. The equation will have no x-intercepts if $b^2-4ac<0$.

d. It is not possible for the equation to have no y-intercepts because $y=f(x)$.

128.a. $f(x)=|x|$ **b.** $f(x)=0$

c. $f(x)=x$

d. $f(x)=\sqrt{-x^2}$ (Note: the point is the origin.)

e. $f(x)=1$

f. A vertical line is not a function.

129.a. {(*a*, 1), (*b*, 1)} {(*a*, 2), (*b*, 1)}
{(*a*, 1), (*b*, 2)} {(*a*, 2), (*b*, 2)}
{(*a*, 1), (*b*, 3)} {(*a*, 2), (*b*, 3)}

{(*a*, 3), (*b*, 1)}
{(*a*, 3), (*b*, 2)}
{(*a*, 3), (*b*, 3)}
There are nine functions from *X* to *Y*.

b. {(1, *a*)}, {(2, *a*)}, {(3, *a*)}
{(1, *a*)}, {(2, *a*)}, {(3, *b*)}
{(1, *a*)}, {(2, *b*}, {(3, *a*)}
{(1, *b*)}, {(2, *a*)}, {(3, *a*)}
{(1, *b*)}, {(2, *a*)}, {(3, *a*)}
{(1, *b*)}, {(2, *b*}, {(3, *a*)}
{(1, *b*)}, {(2, *a*)}, {(3, *b*)}
{(1, *b*)}, {(2, *b*)}, {(3, *b*)}
There are eight functions from *Y* to *X*.

130. If a set *X* has *m* elements and a set of *Y* has *n* elements, there are n^m functions that can be defined from *X* to *Y*. This is true since a function assigns each element of *X* to an element of *Y*. There are *m* possibilities for each element of *X*, so there are $\underbrace{n\cdot n\cdot n\cdot\cdots\cdot n}_{m}=n^m$ possible functions.

2.4 Getting Ready for the Next Section

131. $2x-4<12\Rightarrow 2x<16\Rightarrow x<8$
The solution set is $(-\infty, 8)$.

132. $5x+9\le 7(x+1)\Rightarrow 5x+9\le 7x+7\Rightarrow$
$2\le 2x\Rightarrow 1\le x$ or $x\ge 1$
The solution set is $[1, \infty)$.

133. $x^2>0$
Solve the associated equation:
$x^2=0\Rightarrow x=0.$
So, the intervals are $(-\infty, 0)$ and $(0,\infty)$.

Interval	Test point	Value of x^2	Result
$(-\infty, 0)$	–1	1	+
$(0,\infty)$	1	1	+

The solution set is $(-\infty, 0)\cup(0, \infty)$

134. $(3-x)(x+5)\ge 0$
Solve the associated equation:
$(3-x)(x+5)=0\Rightarrow x=3$ or $x=-5$.
So, the intervals are
$(-\infty, -5]$, $[-5, 3]$, and $[3,\infty)$.

Interval	Test point	Value of $(3-x)(x+5)$	Result
$(-\infty, -5]$	–10	–65	–
$[-5, 3]$	0	15	+
$[3,\infty)$	5	–20	–

The solution set is $[-5, 3]$.

For exercises 135–140, $f(x) = 3 - 2x^2$, $g(x) = \sqrt{x+3}$, and $h(x) = \dfrac{2}{x^2+1}$.

135. $f(0) = 3 - 2(0)^2 = 3$
$g(0) = \sqrt{0+3} = \sqrt{3}$
$h(0) = \dfrac{2}{0^2+1} = 2$

136. $f(1) = 3 - 2(1)^2 = 1$
$g(1) = \sqrt{1+3} = \sqrt{4} = 2$
$h(1) = \dfrac{2}{1^2+1} = 1$

137. $f(-2) = 3 - 2(-2)^2 = 3 - 8 = -5$
$g(-2) = \sqrt{-2+3} = \sqrt{1} = 1$
$h(-2) = \dfrac{2}{(-2)^2+1} = \dfrac{2}{5}$

138.
$$f(x+1) = 3 - 2(x+1)^2 = 3 - 2\left(x^2+2x+1\right) = 3 - 2x^2 - 4x - 2 = -2x^2 - 4x + 1$$
$$g(x+1) = \sqrt{(x+1)+3} = \sqrt{x+4}$$
$$h(x+1) = \frac{2}{(x+1)^2+1} = \frac{2}{\left(x^2+2x+1\right)+1} = \frac{2}{x^2+2x+2}$$

139. $f(-x) = 3 - 2(-x)^2 = 3 - x^2$
$g(-x) = \sqrt{-x+3}$
$h(-x) = \dfrac{2}{(-x)^2+1} = \dfrac{2}{x^2+1}$

140. $-f(x) = -\left(3 - 2x^2\right) = 2x^2 - 3$
$-g(x) = -\sqrt{x+3}$
$-h(x) = -\dfrac{2}{x^2+1}$

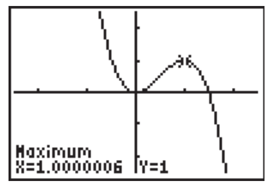
2.5 Properties of Functions

2.5 Practice Problems

1. The function is decreasing on $(0, 3)$, $(12, 13)$, and $(15, 24)$; increasing on $(3, 12)$ and $(13, 15)$

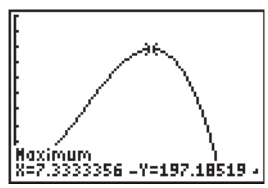
2. Relative maxima of 3640 at $x = 12$ and 4070 at $x = 15$; relative minima of 40 at $x = 3$ and 3490 at $x = 13$.

3.



Relative minimum of 0 at $x = 0$
Relative maximum of 1 at $x = 1$

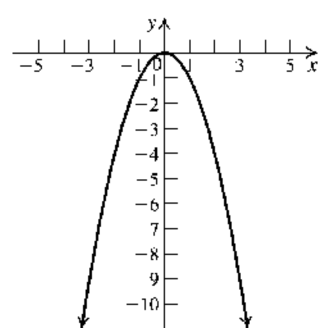
4. $v = (11 - r)r^2$



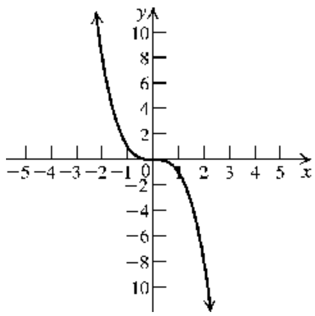
[0, 13, 1] by [0, 250, 25]

Mrs. Osborn's windpipe should be contracted to a radius of 7.33 mm for maximizing the airflow velocity.

5. $f(x) = -x^2$
Replace x with $-x$:
$f(-x) = -(-x)^2 = -x^2 = f(x)$
Thus, the function is even.



6. $f(x) = -x^3$
Replace x with $-x$:
$f(-x) = -(-x)^3 = x^3 = -f(x)$
Thus, the function is odd.



7. a. $g(-x) = 3(-x)^4 - 5(-x)^2$
$= 3x^4 - 5x^2 = f(x) \Rightarrow$
$g(x)$ is even.

b. $f(-x) = 4(-x)^5 + 2(-x)^3 = -4x^5 - 2x^3$
$= -\left(4x^5 + 2x^3\right) = -f(x) \Rightarrow$
$f(x)$ is odd.

c. $h(-x) = 2(-x) + 1 = -2x + 1$
$\neq h(x)$
$\neq h(-x) \Rightarrow h$ is neither even nor odd.

8. $f(x) = 1 - x^2;\ a = 2, b = 4$
$f(2) = 1 - 2^2 = -3;\ f(4) = 1 - 4^2 = -15$
$$\frac{f(b) - f(a)}{b - a} = \frac{-15 - (-3)}{4 - 2} = \frac{-12}{2} = -6$$
The average rate of change is −6.

9. $f(t) = 1 - t;\ a = 2,\ b = x,\ x \neq 2$
$f(a) = f(2) = 1 - 2 = -1$
$f(b) = f(x) = 1 - x$
$$\frac{f(b) - f(a)}{b - a} = \frac{(1 - x) - (-1)}{x - 2} = \frac{2 - x}{x - 2} = \frac{-1(x - 2)}{x - 2} = -1$$
The average rate of change is −1.

10. $f(x) = 100x^2 - 800x + 2000$
$f(0) = 100(0)^2 - 800(0) + 2000 = 2000$
$f(3) = 100(3)^2 - 800(3) + 2000 = 500$
$$\frac{f(3) - f(0)}{3 - 0} = \frac{500 - 2000}{3} = \frac{-1500}{3} = -500$$
The average rate of change is –500, so the number of bacteria per cubic centimeter decreases by 500 each day after adding the chlorine.

11. $f(x) = -x^2 + x - 3$
$f(x + h) = -(x + h)^2 + (x + h) - 3$
$= -x^2 - 2xh - h^2 + x + h - 3$
$$\frac{f(x + h) - f(x)}{h} = \frac{\left(-x^2 - 2xh - h^2 + x + h - 3\right) - \left(-x^2 + x - 3\right)}{h} = \frac{-2xh - h^2 + h}{h} = -2x - h + 1$$

2.5 Concepts and Vocabulary

1. A function *f* is decreasing if $x_1 < x_2$ implies that $f(x_1) > f(x_2)$.

2. $f(a)$ is a relative maximum of *f* if there is an interval (x_1, x_2) containing *a* such that $f(a) \geq f(x)$ for every *x* in the interval (x_1, x_2).

3. A function *f* is even if *f*(−*x*) = *f*(*x*) for all *x* in the domain of *f*.

4. The average rate of change of *f* as *x* changes from *x* = *a* to *x* = *b* is $\frac{f(b) - f(a)}{b - a}$, $a \neq b$.

5. True

6. False. A relative maximum or minimum could occur at an endpoint of the domain of the function.

7. True

8. False. The graph of an odd function is symmetric with respect to the origin.

2.5 Building Skills

9. Increasing on $(-\infty, \infty)$

10. Decreasing on $(-\infty, \infty)$

11. Increasing on $(-\infty, 2)$, decreasing on $(2, \infty)$

12. Decreasing on $(-\infty, 3)$, increasing on $(3, \infty)$

13. Increasing on $(-\infty, -2)$, constant on $(-2, 2)$, increasing on $(2, \infty)$

14. Decreasing on $(-\infty, -1)$, constant on $(-1, 4)$, decreasing on $(4, \infty)$

15. Increasing on $(-\infty, -3)$ and $\left(-\frac{1}{2}, 2\right)$, decreasing on $\left(-3, -\frac{1}{2}\right)$ and $(2, \infty)$

16. Increasing on $(-3, -1)$, $(0, 1)$, and $(2, \infty)$. Decreasing on $(-\infty, -3)$, $(-1, 0)$, and $(1, 2)$.

17. No relative extrema

18. No relative extrema

19. (2, 10) is a relative maximum point and a turning point.

20. (3, 2) is a relative minimum point and a turning point.

21. Any point on (x, 2) is a relative maximum and a relative minimum point on the interval (–2, 2). Relative maximum at (−2, 2); relative minimum at (2, 2). None of these points are turning points.

22. Any point on (x, 3) is a relative maximum and a relative minimum point on the interval (–1, 4). Relative maximum at (4, 3); relative minimum at (−1, 3). None of these points are turning points.

23. (–3, 4) and (2, 5) are relative maxima points and turning points. $\left(-\frac{1}{2}, -2\right)$ is a relative minimum and a turning point.

24. (–3, –2), (0, 0), and (2, –3) are relative minimum points and turning points. (–1, 1) and (1, 2) are relative maximum points and turning points.

For exercises 25−34, recall that the graph of an even function is symmetric about the y-axis, and the graph of an odd function is symmetric about the origin.

25. The graph is symmetric with respect to the origin. The function is odd.

26. The graph is symmetric with respect to the origin. The function is odd.

27. The graph has no symmetries, so the function is neither odd nor even.

28. The graph has no symmetries, so the function is neither odd nor even.

29. The graph is symmetric with respect to the origin. The function is odd.

30. The graph is symmetric with respect to the origin. The function is odd.

31. The graph is symmetric with respect to the y-axis. The function is even.

32. The graph is symmetric with respect to the y-axis. The function is even.

33. The graph is symmetric with respect to the origin. The function is odd.

34. The graph is symmetric with respect to the origin. The function is odd.

For exercises 35−48, $f(-x) = f(x) \Rightarrow f(x)$ is even and $f(-x) = -f(x) \Rightarrow f(x)$ is odd.

35. $f(-x) = 2(-x)^4 + 4 = 2x^4 + 4 = f(x) \Rightarrow$ $f(x)$ is even.

36. $g(-x) = 3(-x)^4 - 5 = 3x^4 - 5 = g(x) \Rightarrow$ $g(x)$ is even.

37. $f(-x) = 5(-x)^3 - 3(-x) = -5x^3 + 3x$
$= -(5x^3 - 3x) = -f(x) \Rightarrow$
$f(x)$ is odd.

38. $g(-x) = 2(-x)^3 + 4(-x)$
$= -2x^3 - 4x = -g(x) \Rightarrow$
$g(x)$ is odd.

39. $f(-x) = 2(-x) + 4 = -2x + 4$
$\neq -f(x) \neq f(x) \Rightarrow$
$f(x)$ is neither even nor odd.

40. $g(-x) = 3(-x) + 7 = -3x + 7$
$\neq -g(x) \neq g(x) \Rightarrow$
$g(x)$ is neither even nor odd.

41. $f(-x) = \dfrac{1}{(-x)^2 + 4} = \dfrac{1}{x^2 + 4} = f(x) \Rightarrow$
$f(x)$ is even.

42. $g(-x) = \dfrac{(-x)^2 + 2}{(-x)^4 + 1} = \dfrac{x^2 + 2}{x^4 + 1} = g(x) \Rightarrow$
$g(x)$ is even.

43. $f(-x) = \dfrac{(-x)^3}{(-x)^2 + 1} = -\dfrac{x^3}{x^2 + 1} = -f(x) \Rightarrow$
$f(x)$ is odd.

44. $g(-x)=\dfrac{(-x)^4+3}{2(-x)^3-3(-x)}=\dfrac{x^4+3}{-2x^3+3x}$
$=-\dfrac{x^4+3}{2x^3-3x}=-f(x)\Rightarrow f(x)$ is odd.

45. $f(-x)=\dfrac{-x}{(-x)^5-3(-x)^3}=\dfrac{-x}{-x^5+3x^3}$
$=\dfrac{(-1)(-x)}{(-1)\left(-x^5+3x^3\right)}=\dfrac{x}{x^5-3x^3}=f(x)$

Thus, $f(x)$ is even.

46. $g(-x)=\dfrac{(-x)^3+2(-x)}{2(-x)^5-3(-x)}=\dfrac{-x^3-2x}{-2x^5+3x}$
$=\dfrac{(-1)\left(-x^3-2x\right)}{(-1)\left(-2x^5+3x\right)}=\dfrac{x^3+2x}{2x^5-3x}=f(x)$

Thus, $f(x)$ is even.

47. $f(-x)=\dfrac{(-x)^2-2(-x)}{5(-x)^4+4(-x)^2+7}=\dfrac{x^2+2x}{5x^4+4x^2+7}$
$\neq -f(x)\neq f(x)$

Thus, $f(x)$ is neither even nor odd.

48. $g(-x)=\dfrac{3(-x)^2+7}{(-x)-3}=\dfrac{3x^2+7}{-x-3}\neq -g(x)\neq g(x)$

Thus, $g(x)$ is neither even nor odd.

49. a. domain: $(-\infty,\infty)$; range: $(-\infty, 3]$

b. x-intercepts: $(-3, 0)$, $(3, 0)$
y-intercept: $(0, 3)$

c. increasing on $(-\infty, 0)$, decreasing on $(0, \infty)$

d. relative maximum at $(0, 3)$

e. even

50. a. domain: $(-\infty,\infty)$; range: $(-\infty,\infty)$

b. x-intercepts: $(-4, 0)$, $(0, 0)$, $(4, 0)$
y-intercept: $(0, 0)$

c. decreasing on $(-\infty, -2)$ and $(2, \infty)$, increasing on $(-2, 2)$

d. relative maximum at $(2,3)$; relative minimum at $(-2, -3)$

e. odd

51. a. domain: $(-3, 4)$; range: $[-2, 2]$

b. x-intercept: $(1, 0)$; y-intercept: $(0, -1)$

c. constant on $(-3, -1)$ and $(3, 4)$
increasing on $(-1, 3)$

d. Since the function is constant on $(-3, -1)$, any point $(x, -2)$ is both a relative maximum and a relative minimum on that interval. Since the function is constant on $(3, 4)$, any point $(x, 2)$ is both a relative maximum and a relative minimum on that interval.

e. neither even not odd

52. a. domain: $(-3, 3)$; range: $\{-2, 0, 2\}$

b. x-intercept: $(0, 0)$; y-intercept: $(0, 0)$

c. constant on $(-3, 0)$ and $(0, 3)$

d. Since the function is constant on $(-3, 0)$, any point $(x, 2)$ is both a relative maximum and a relative minimum on that interval. Since the function is constant on $(0, 3)$, any point
$(x, -2)$ is both a relative maximum and a relative minimum on that interval.

e. odd

53. a. domain: $(-2, 4)$; range: $[-2, 3]$

b. x-intercept: $(0, 0)$; y-intercept: $(0, 0)$

c. decreasing on $(-2, -1)$ and $(3, 4)$
increasing on $(-1, 3)$

d. relative maximum: $(3, 3)$
relative minimum: $(-1, -2)$

e. neither even nor odd

54. a. domain: $(-\infty, \infty)$
range: $(-\infty, \infty)$

b. x-intercepts: $(2, 0)$, $(3, 0)$
y-intercept: $(0, 3)$

c. decreasing on $(-\infty, 0)\cup(0, 3)\cup(3, \infty)$

d. no relative minimum
relative maximum: $(0, 3)$

e. neither even nor odd

55. a. domain: $(-\infty,\infty)$; range: $(0, \infty)$

b. no x-intercept; y-intercept: $(0, 1)$

c. increasing on $(-\infty, \infty)$

d. no relative minimum or relative maximum

e. neither even nor odd

56. a. domain: $(-\infty, 0)\cup(0, \infty)$
range: $(-\infty, \infty)$

b. x-intercepts: (−1.5, 0), (1.5, 0)
no y-intercept

c. decreasing on $(-\infty, 0)$
increasing on $(0, \infty)$

d. no relative minimum or relative maximum

e. even

57. $f(x) = -2x + 7;\ a = -1,\ b = 3$
$f(3) = -2(3) + 7 = 1;\ f(-1) = -2(-1) + 7 = 9$

$$\text{average rate of change} = \frac{f(3) - f(-1)}{3 - (-1)} = \frac{1-9}{4} = -2$$

58. $f(x) = 4x - 9;\ a = -2,\ b = 2$
$f(2) = 4(2) - 9 = -1;\ f(-2) = 4(-2) - 9 = -17$

$$\text{average rate of change} = \frac{f(2) - f(-2)}{2 - (-2)} = \frac{-1-(-17)}{4} = 4$$

59. $f(x) = 3x + c;\ a = 1,\ b = 5$
$f(5) = 3\cdot 5 + c = 15 + c;\ f(1) = 3\cdot 1 + c = 3 + c$

$$\text{average rate of change} = \frac{f(5) - f(1)}{5 - 1} = \frac{15 + c - (3 + c)}{4} = \frac{12}{4} = 3$$

60. $f(x) = mx + c;\ a = -1,\ b = 7$
$f(7) = 7m + c;\ f(-1) = -m + c$

$$\text{average rate of change} = \frac{f(7) - f(-1)}{7 - (-1)} = \frac{7m + c - (-m + c)}{8} = \frac{8m}{8} = m$$

61. $h(x) = x^2 - 1;\ a = -2,\ b = 0$
$h(0) = 0^2 - 1 = -1;\ h(-2) = (-2)^2 - 1 = 3$

$$\text{average rate of change} = \frac{h(0) - h(-2)}{0 - (-2)} = \frac{-1-3}{2} = -2$$

62. $h(x) = 2 - x^2;\ a = 3,\ b = 4$
$h(4) = 2 - 4^2 = -14;\ h(3) = 2 - 3^2 = -7$

$$\text{average rate of change} = \frac{h(4) - h(3)}{4 - 3} = \frac{-14-(-7)}{1} = -7$$

63. $f(x) = (3 - x)^2;\ a = 1,\ b = 3$
$f(4) = (3 - 3)^2 = 0;\ f(1) = (3 - 1)^2 = 4$

$$\text{average rate of change} = \frac{f(3) - f(1)}{3 - 1} = \frac{0-4}{2} = -2$$

64. $f(x) = (x - 2)^2;\ a = -1,\ b = 5$
$f(5) = (5 - 2)^2 = 9;\ f(-1) = (-1 - 2)^2 = 9$

$$\text{average rate of change} = \frac{f(5) - f(-1)}{5 - (-1)} = \frac{9-9}{6} = 0$$

65. $g(x) = x^3;\ a = -1,\ b = 3$
$g(3) = 3^3 = 27;\ g(-1) = (-1)^3 = -1$

$$\text{average rate of change} = \frac{g(3) - g(-1)}{3 - (-1)} = \frac{27-(-1)}{4} = 7$$

66. $g(x) = -x^3;\ a = -1,\ b = 3$
$g(3) = -3^3 = -27;\ g(-1) = -(-1)^3 = 1$

$$\text{average rate of change} = \frac{g(3) - g(-1)}{3 - (-1)} = \frac{-27-1}{4} = -7$$

67. $h(x) = \dfrac{1}{x};\ a = 2,\ b = 6$
$h(2) = \dfrac{1}{2};\ h(6) = \dfrac{1}{6}$

$$\text{average rate of change} = \frac{h(6) - h(2)}{6 - 2} = \frac{\frac{1}{6} - \frac{1}{2}}{4} = -\frac{1}{12}$$

68. $h(x)=\dfrac{4}{x+3};\ a=-2,\ b=4$

$h(4)=\dfrac{4}{4+3}=\dfrac{4}{7};\ h(-2)=\dfrac{4}{-2+3}=4$

$$\text{average rate of change}=\frac{h(4)-h(-2)}{4-(-2)}=\frac{\frac{4}{7}-4}{6}=-\frac{4}{7}$$

69. $f(x+h)=x+h$

$f(x+h)-f(x)=x+h-x=h$

$$\frac{f(x+h)-f(x)}{h}=\frac{h}{h}=1$$

70. $f(x+h)=3(x+h)+2=3x+3h+2$

$f(x+h)-f(x)=3x+3h+2-(3x+2)=3h$

$$\frac{f(x+h)-f(x)}{h}=\frac{3h}{h}=3$$

71. $f\left(x+h\right)=-2\left(x+h\right)+3=-2x-2h+3$

$f\left(x+h\right)-f\left(x\right)=-2x-2h+3-\left(-2x+3\right)=-2h$

$$\frac{f(x+h)-f(x)}{h}=\frac{-2h}{h}=-2$$

72. $f\left(x+h\right)=-5\left(x+h\right)-6=-5x-5h-6$

$f\left(x+h\right)-f\left(x\right)=-5x-5h-6-\left(-5x-6\right)=-5h$

$$\frac{f(x+h)-f(x)}{h}=\frac{-5h}{h}=-5$$

73. $f\left(x+h\right)=m\left(x+h\right)+b=mx+mh+b$

$f\left(x+h\right)-f\left(x\right)=mx+mh+b-\left(mx+b\right)=mh$

$$\frac{f(x+h)-f(x)}{h}=\frac{mh}{h}=m$$

74. $f\left(x+h\right)=-2a\left(x+h\right)+c=-2ax-2ah+c$

$f\left(x+h\right)-f\left(x\right)=-2ax-2ah+c-\left(-2ax+c\right)=-2ah$

$$\frac{f(x+h)-f(x)}{h}=\frac{-2ah}{h}=-2a$$

75. $f(x+h)=(x+h)^2=x^2+2xh+h^2$

$f(x+h)-f(x)=x^2+2xh+h^2-x^2=2xh+h^2$

$$\frac{f(x+h)-f(x)}{h}=\frac{2xh+h^2}{h}=2x+h$$

76. $f(x+h)=(x+h)^2-(x+h)$
$=x^2+2xh+h^2-x-h$
$=x^2+2xh-x+h^2-h$

$f(x+h)-f(x)$
$=x^2+2xh-x+h^2-h-(x^2-x)$
$=2xh+h^2-h$

$$\frac{f(x+h)-f(x)}{h}=\frac{2xh+h^2-h}{h}=2x+h-1$$

77. $f(x+h)=2(x+h)^2+3(x+h)$
$=2x^2+4xh+2h^2+3x+3h$
$=2x^2+4xh+3x+2h^2+3h$

$f(x+h)-f(x)$
$=2x^2+4xh+3x+2h^2+3h-(2x^2+3x)$
$=4xh+2h^2+3h$

$$\frac{f(x+h)-f(x)}{h}=\frac{4xh+2h^2+3h}{h}=4x+2h+3$$

78. $f(x+h)=3(x+h)^2-2(x+h)+5$
$=3x^2+6xh+3h^2-2x-2h+5$
$=3x^2+6xh-2x+3h^2-2h+5$

$f(x+h)-f(x)=3x^2+6xh-2x+3h^2-2h+5-(3x^2-2x+5)$
$=6xh+3h^2-2h$

$$\frac{f(x+h)-f(x)}{h}=\frac{6xh+3h^2-2h}{h}=6x+3h-2$$

79. $f(x+h)=4$

$f(x+h)-f(x)=4-4=0$

$$\frac{f(x+h)-f(x)}{h}=\frac{0}{h}=0$$

80. $f(x+h)=-3$

$f(x+h)-f(x)=-3-(-3)=0$

$$\frac{f(x+h)-f(x)}{h}=\frac{0}{h}=0$$

81. $f(x+h)=\dfrac{1}{x+h}$

$$f(x+h)-f(x)=\frac{1}{x+h}-\frac{1}{x}=\frac{x}{x(x+h)}-\frac{x+h}{x(x+h)}=-\frac{h}{x(x+h)}$$

$$\frac{f(x+h)-f(x)}{h}=\frac{-\frac{h}{x(x+h)}}{h}=-\frac{1}{x(x+h)}$$

82. $f(x+h)=-\dfrac{1}{x+h}$

$$f(x+h)-f(x)=-\frac{1}{x+h}-\left(-\frac{1}{x}\right)=-\frac{x}{x(x+h)}+\frac{x+h}{x(x+h)}=\frac{h}{x(x+h)}$$

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{h}{x(x+h)}}{h}=\frac{1}{x(x+h)}$$

2.5 Applying the Concepts

83. a. Increasing: (2006, 2009), (2011, 2012), (2013, 2014)
Decreasing: (2009, 2011), (2012, 2013)

b. Relative maxima: 251.1 at $x = 2009$, 293.2 at $x = 2012$
Relative minima: 21.5 at $x = 2011$, 187.0 at $x = 2013$

84. a. Increasing: (Jan., June), (July, Sept.)
Decreasing: (June, July), (Sept., Dec.)

b. Relative maxima: 185 in June, 185 in Sept.
Relative minima: 132 in July

85. domain: $[0, \infty)$

The particle's motion is tracked indefinitely from time $t = 0$.

86. range: $[-7, 5]$

The particle takes on all velocities between –7 an 5. Note that a negative velocity indicates that the particle is moving backward.

87. The graph is above the t-axis on the intervals (0, 9) and (21, 24). This means that the particle was moving forward between 0 and 9 seconds and between 21 and 24 seconds.

88. The graph is below the t-axis on the interval (11, 19). This means that the particle is moving backward between 11 and 19 seconds.

89. The function is increasing on (0, 3), (5, 6), (16, 19), and (21, 23). However, the speed $|v|$ of the particle is increasing on (0, 3), (5, 6), (11, 15), and (21, 23). Note that the particle is moving forward on (0, 3), (5, 6), and (21, 23), and moving backward on (11, 15).

90. The function is decreasing on (6, 9), (11, 15), and (23, 24). However, the speed $|v|$ of the particle is decreasing on (6, 9), (16, 19), and (23, 24). Note that the particle is moving forward on (6, 9) and (23, 24), and moving backward on (16, 19).

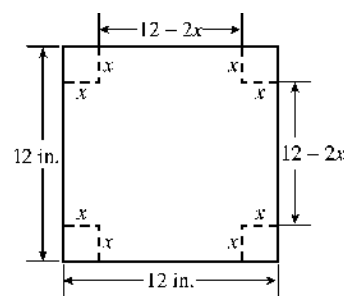
91. The maximum speed is between times $t = 15$ and $t = 16$.

92. The minimum speed is 0 on the intervals (9, 11), (19,21), and (24, ∞).

93. The particle is moving forward with increasing velocity.

94. The particle is moving backward with decreasing speed.

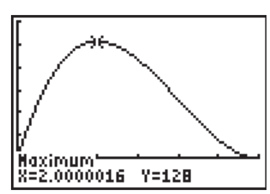
95.



a. $V = lwh = (12-2x)(12-2x)x$
$= (144-48x+4x^2)x$
$= 4x^3-48x^2+144x$

b. The length of the squares in the corners must be greater than 0 and less than 6, so the domain of V is (0, 6).

c.

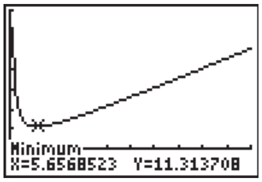


[0, 6, 1] by [−25, 150, 25]
range: [0, 128]

d. V is at its maximum when $x = 2$.

96. a. Let x = one of the numbers. Then $\frac{32}{x}$ = the other number. The sum of the numbers is $S = x + \frac{32}{x}$.

b.



[0, 50, 10] by [−10, 70, 10]

The minimum value of approximately 11.31 occurs at $x \approx 5.66$.

97. a. $C(x) = 210x + 10{,}500$

b. $C(50) = 210(50) + 10{,}500 = \$21{,}000$

It costs $21,000 to produce 50 notebooks per day.

c. average cost = $\frac{\$21{,}000}{50} = \$420$

d. $\frac{210x + 10{,}500}{x} = 315$

$210x + 10{,}500 = 315x$

$10{,}500 = 105x \Rightarrow x = 100$

The average cost per notebook will be $315 when 100 notebooks are produced.

98. $f(x) = -2x^2 + 3x + 4$

$f(1) = -2(1)^2 + 3(1) + 4 = 5$

$f(3) = -2(3)^2 + 3(3) + 4 = -5$

The secant passes through the points (1, 5) and (3, −5).

$m = \frac{-5-5}{3-1} = \frac{-10}{2} = -5$

The equation of the secant is

$y - 5 = -5(x-1) \Rightarrow y - 5 = -5x + 5 \Rightarrow$
$y = -5x + 10$

99. average rate of increase $= \frac{f(2014) - f(2000)}{2014 - 2000}$

$= \frac{20.2 - 15.3}{14} = 0.35$

The average rate of increase was 0.35 million, or 350,000, students per year.

100. $f(t) = \frac{60}{t} - 5$

$f(5) = \frac{60}{5} - 5 = 7;\quad f(1) = \frac{60}{1} - 5 = 55$

average rate of decrease $= \frac{f(5) - f(1)}{5-1}$

$= \frac{7-55}{4} = -12$

The average rate of decrease is 12 gallons per minute.

101. a. $f(0) = 0^2 + 3(0) + 4 = 4$

The particle is 4 ft to the right from the origin.

b. $f(4) = 4^2 + 3(4) + 4 = 32$

The particle started 4 ft from the origin, so it traveled 32 − 4 = 28 ft in four seconds.

c. $f(3) = 3^2 + 3(3) + 4 = 22$

The particle started 4 ft from the origin, so it traveled 22 − 4 = 18 ft in three seconds. The average velocity is $18/3 = 6$ ft/sec

d. $f(2) = 2^2 + 3(2) + 4 = 14$

$f(5) = 5^2 + 3(5) + 4 = 44$

The particle traveled 44 − 14 = 30 ft between the second and fifth seconds. The average velocity is $30/(5-2) = 10$ ft/sec

102. a. $P(0) = 0.01(0)^2 + 0.2(0) + 50 = 50$

$P(4) = 0.01(4)^2 + 0.2(4) + 50 = 50.96$

The population of Sardonia was 50 million in 2000 and 50.96 million in 2004.

b. $P(10) = 0.01(10)^2 + 0.2(10) + 50 = 53$

The average rate of growth from 2000 to 2010 was $\frac{53-50}{10} = 0.3$ million per year.

2.5 Beyond the Basics

103. $f(x)=\frac{x-1}{x+1}$

$f(2x)=\frac{2x-1}{2x+1}$

$$\frac{3f(x)+1}{f(x)+3}=\frac{3\left(\frac{x-1}{x+1}\right)+1}{\frac{x-1}{x+1}+3}=\frac{\frac{3x-3}{x+1}+1}{\frac{x-1+3(x+1)}{x+1}}$$

$$=\frac{\frac{3x-3+x+1}{x+1}}{\frac{x-1+3(x+1)}{x+1}}=\frac{4x-2}{4x+2}$$

$$=\frac{2(2x-1)}{2(2x+1)}=\frac{2x-1}{2x+1}=f(2x)$$

104. $f(x)=0$ is both even and odd.

105. In order to find the relative maximum, first observe that the relative maximum of $-(x+1)^2\le 0$. Then $-(x+1)^2\le 0\Rightarrow$ $(x+1)^2\ge 0\Rightarrow x\ge -1$.
Thus, the x-coordinate of the relative maximum is -1. $f(-1)=-(-1+1)^2+5=5$
The relative maximum is $(-1, 5)$.
There is no relative minimum.

106. $f(x)=\begin{cases} x+10 & \text{if } x<-5 \\ 5 & \text{if } -5\le x\le 5 \\ -x & \text{if } x>5 \end{cases}$

The point (0, 5) is a relative maximum, but not a turning point.

107. $f(x)=\sqrt{x}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$=\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$=\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$=\frac{1}{\sqrt{x+h}+\sqrt{x}}$$

108. $f(x)=\sqrt{x-1}$

$$\frac{f(x+h)-f(x)}{h}$$

$$=\frac{\sqrt{x+h-1}-\sqrt{x-1}}{h}\cdot\frac{\sqrt{x+h-1}+\sqrt{x-1}}{\sqrt{x+h-1}+\sqrt{x-1}}$$

$$=\frac{x+h-1-(x-1)}{h\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}$$

$$=\frac{h}{h\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}$$

$$=\frac{1}{\sqrt{x+h-1}+\sqrt{x-1}}$$

109. $f(x)=-\frac{1}{\sqrt{x}}$

$$\frac{f(x+h)-f(x)}{h}$$

$$=\frac{-\frac{1}{\sqrt{x+h}}+\frac{1}{\sqrt{x}}}{h}=\frac{\frac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x}\sqrt{x+h}}}{h}$$

$$=\frac{\frac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x(x+h)}}}{h}=\frac{\sqrt{x+h}-\sqrt{x}}{h\sqrt{x(x+h)}}$$

$$=\frac{\sqrt{x+h}-\sqrt{x}}{h\sqrt{x(x+h)}}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$=\frac{(x+h)-x}{h\sqrt{x(x+h)}\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$=\frac{1}{\sqrt{x(x+h)}\left(\sqrt{x}+\sqrt{x+h}\right)}$$

110. $f(x)=\frac{1}{x^2}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$$

$$=\frac{\frac{x^2-(x+h)^2}{x^2(x+h)^2}}{h}$$

$$=\frac{x^2-(x+h)^2}{hx^2(x+h)^2}$$

$$=\frac{x^2-\left(x^2+2xh+h^2\right)}{hx^2(x+h)^2}$$

$$=\frac{-2xh-h^2}{hx^2(x+h)^2}=-\frac{2x+h}{x^2(x+h)^2}$$

2.5 Critical Thinking/Discussion/Writing

111. f has a relative maximum at $x = a$ if there is an interval $[a, x_1)$ with $a < x_1 < b$ such that $f(a) \ge f(x)$, or $f(x) \le f(a)$, for every x in the interval $(x_1, b]$.

112. f has a relative minimum at $x = b$ if there is x_1 in $[a, b]$ such that $f(x) \ge f(b)$ for every x in the interval $(x_1, b]$.

113. Answers will vary. Sample answers are given.

a. $f(x) = x$ on the interval $[-1, 1]$

b. $f(x) = \begin{cases} x & \text{if } 0 \le x < 1 \\ 0 & \text{if } x = 1 \end{cases}$

c. $f(x) = \begin{cases} x & \text{if } 0 < x \le 1 \\ 1 & \text{if } x = 0 \end{cases}$

d. $f(x) = \begin{cases} 0 & \text{if } x = 0 \text{ or } x = 1 \\ 1 & \text{if } 0 < x < 1 \text{ and } x \text{ is rational} \\ -1 & \text{if } 0 < x < 1 \text{ and } x \text{ is irrational} \end{cases}$

2.5 Getting Ready for the Next Section

114. $m = \dfrac{-2-3}{4-(-1)} = \dfrac{-5}{5} = -1$

$y - 3 = -(x - (-1)) \Rightarrow y - 3 = -(x+1) \Rightarrow$
$y - 3 = -x - 1 \Rightarrow y = -x + 2$

115. $m = \dfrac{-1-2}{7-6} = \dfrac{-3}{1} = -3$

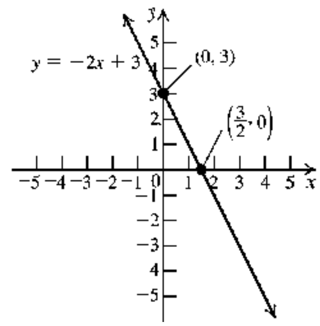
$y - 2 = -3(x - 6) \Rightarrow y - 2 = -3x + 18 \Rightarrow$
$y = -3x + 20$

116. $m = \dfrac{-3-(-5)}{6-3} = \dfrac{2}{3}$

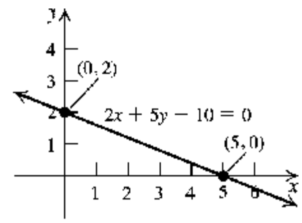
$y - (-5) = \dfrac{2}{3}(x-3) \Rightarrow y + 5 = \dfrac{2}{3}x - 2 \Rightarrow$

$y = \dfrac{2}{3}x - 7$

117.



118.



119. If $x = -3$, then $|x+1| = \underline{2}$.

120. $|2x - 8| = 0 \Rightarrow 2x - 8 = 0 \Rightarrow 2x = 8 \Rightarrow x = 4$

121. $f(x) = x^{3/2}$

(i) $f(2) = 2^{3/2} = \left(\sqrt{2}\right)^3 = \sqrt{8} = 2\sqrt{2}$

(ii) $f(4) = 4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$

(iii) $f(-4) = (-4)^{3/2} = \left(\sqrt{-4}\right)^3 = (2i)^3 = -8i$

122. $f(x) = x^{2/3}$

(i) $f(2) = 2^{2/3} = 4^{1/3} = \sqrt[3]{4}$

(ii) $f(8) = 8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$

(iii) $f(-8) = (-8)^{2/3} = \left(\sqrt[3]{-8}\right)^2 = (-2)^2 = 4$

2.6 A Library of Functions

2.6 Practice Problems

1. Because $g(-2) = 2$ and $g(1) = 8$, the line passes through the points $(-2, 2)$ and $(1, 8)$.

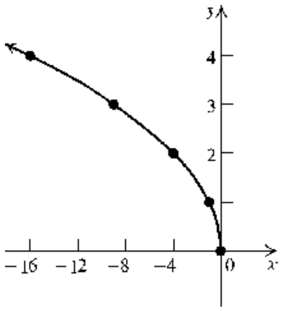
$m = \dfrac{8-2}{1-(-2)} = \dfrac{6}{3} = 2$

Use the point-slope form:

$y - 8 = 2(x - 1) \Rightarrow y - 8 = 2x - 2 \Rightarrow$
$y = 2x + 6 \Rightarrow g(x) = 2x + 6$

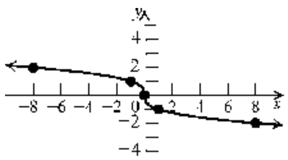
2. Using the formula
Shark length = (0.96)(tooth height) − 0.22, gives:
Shark length = (0.96)(16.4) − 0.22 = 15.524 m

3.



Domain: $(-\infty, 0]$; range: $[0, \infty)$

4.



Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

5. $f(x) = \begin{cases} x^2 & \text{if } x \le -1 \\ 2x & \text{if } x > -1 \end{cases}$

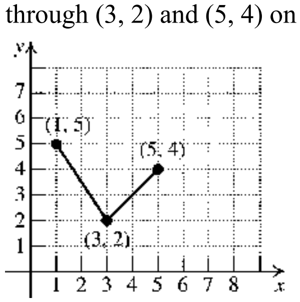
$f(-2) = (-2)^2 = 4$; $f(3) = 2(3) = 6$

6. a. $f(x) = \begin{cases} 50 + 4(x - 55) & 56 \le x < 75 \\ 200 + 5(x - 75) & x \ge 75 \end{cases}$

b. The fine for driving 60 mph is $50 + 4(60 - 55) = \$70$.

c. The fine for driving 90 mph is $200 + 5(90 - 75) = \$275$.

7. The graph of *f* is made up of two parts: a line segment passing through (1, 5) and (3, 2) on the interval [1, 3], and a line segment passing through (3, 2) and (5, 4) on the interval [3, 5].



For the first line segment:

$m = \frac{2-5}{3-1} = -\frac{3}{2}$

$y - 5 = -\frac{3}{2}(x - 1) \Rightarrow 2y - 10 = -3(x - 1) \Rightarrow$
$2y - 10 = -3x + 3 \Rightarrow 2y = -3x + 13 \Rightarrow$
$y = -\frac{3}{2}x + \frac{13}{2}$

For the second line segment:

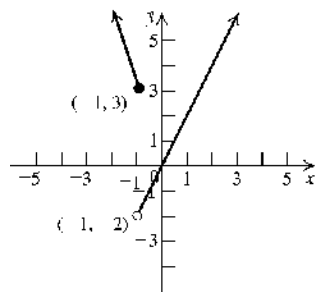
$m = \frac{4-2}{5-3} = 1$

$y - 4 = x - 5 \Rightarrow y = x - 1$

The piecewise function is

$g(x) = \begin{cases} -\frac{3}{2}x + \frac{13}{2} & \text{if } 1 \le x \le 3 \\ x - 1 & \text{if } 3 < x \le 5 \end{cases}$

8. $f(x) = \begin{cases} -3x & \text{if } x \le -1 \\ 2x & \text{if } x > -1 \end{cases}$



Graph $f(x) = -3x$ on the interval $(-\infty, -1]$, and graph $f(x) = 2x$ on the interval $(-1, \infty)$.

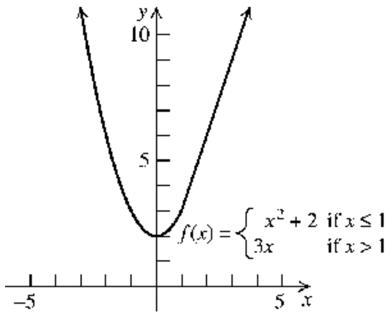
9. $f(x) = [\![x]\!]$
$f(-3.4) = -4$; $f(4.7) = 4$

2.6 Concepts and Vocabulary

1. The graph of the linear function $f(x) = b$ is a <u>horizontal</u> line.

2. The absolute value function can be expressed as a piecewise function by writing
$f(x) = |x| = \underline{\begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \ge 0 \end{cases}}$.

3. The graph of the function
$$f(x)=\begin{cases} x^2+2 & \text{if } x\le 1\\ ax & \text{if } x>1\end{cases}$$
will have a break at $x = 1$ unless a = 3.



4. The line that is the graph of $f(x)=-2x+3$ has slope –2.

5. True. The equation of the graph of a vertical line has the format $x = a$.

6. False. The absolute value function,
$$f(x)=|x|=\begin{cases} -x & \text{if } x<0\\ x & \text{if } x\ge 0\end{cases}$$ is an example of a piecewise function with domain $(-\infty,\ \infty)$.

7. True

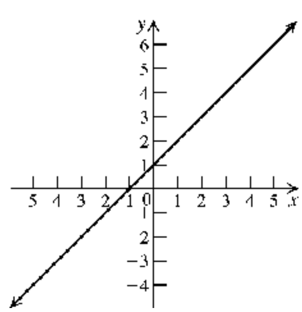
8. False. The function is constant on [0, 1), [1, 2), and [2, 3).

2.6 Building Skills

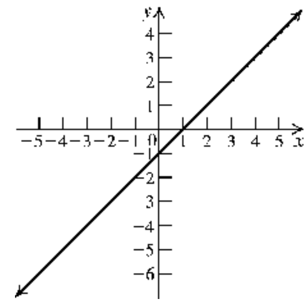
In exercises 9−18, first find the slope of the line using the two points given. Then substitute the coordinates of one of the points into the slope-intercept form of the equation to solve for b.

9. The two points are (0, 1) and (–1, 0).
$$m=\frac{0-1}{-1-0}=1.\ \ 1=1(0)+b\Rightarrow b=1.$$

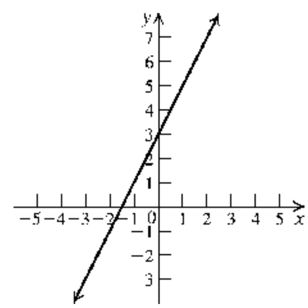
$f(x)=x+1$



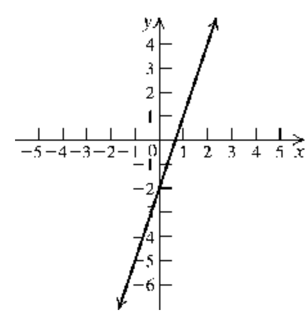
10. The two points are (1, 0) and (2, 1).
$$m=\frac{1-0}{2-1}=1.\ \ 0=1+b\Rightarrow b=-1.$$
$f(x)=x-1$



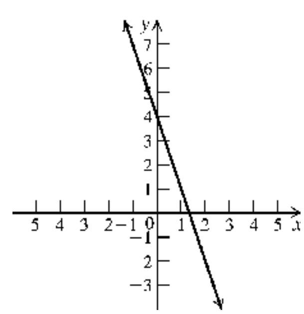
11. The two points are (–1, 1) and (2, 7).
$$m=\frac{7-1}{2-(-1)}=2.\ \ 1=2(-1)+b\Rightarrow 3=b.$$
$f(x)=2x+3$



12. The two points are (–1, –5) and (2, 4).
$$m=\frac{4-(-5)}{2-(-1)}=3.\ \ 4=3(2)+b\Rightarrow b=-2.$$
$f(x)=3x-2.$



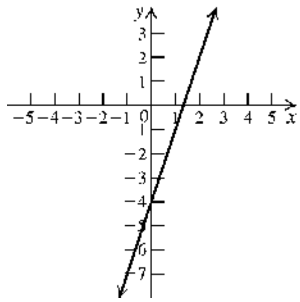
13. The two points are (1, 1) and (2, –2).
$$m=\frac{-2-1}{2-1}=-3.\ \ 1=-3(1)+b\Rightarrow b=4.$$
$f(x)=-3x+4.$



14. The two points are (1, –1) and (3, 5).

$m = \frac{5-(-1)}{3-1} = 3. \quad -1 = 3(1) + b \Rightarrow b = -4.$

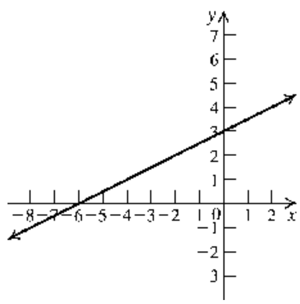
$f(x) = 3x - 4.$



15. The two points are (–2, 2) and (2, 4).

$m = \frac{4-2}{2-(-2)} = \frac{1}{2}. \quad 4 = \frac{1}{2}(2) + b \Rightarrow b = 3.$

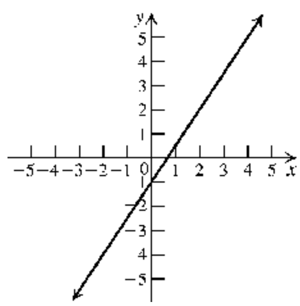
$f(x) = \frac{1}{2}x + 3.$



16. The two points are (2, 2) and (4, 5).

$m = \frac{5-2}{4-2} = \frac{3}{2}. \quad 2 = \frac{3}{2}(2) + b \Rightarrow b = -1.$

$f(x) = \frac{3}{2}x - 1.$

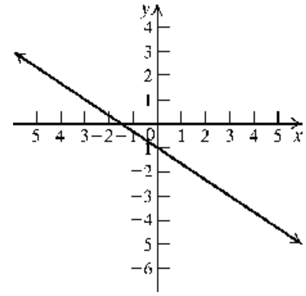


17. The two points are (0, –1) and (3, –3).

$m = \frac{-3-(-1)}{3-0} = -\frac{2}{3}.$

$-1 = -\frac{2}{3}(0) + b \Rightarrow b = -1.$

$f(x) = -\frac{2}{3}x - 1.$

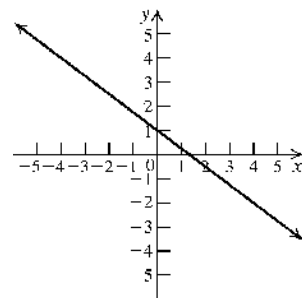


18. The two points are (1, 1/4) and (4, –2).

$m = \frac{-2-1/4}{4-1} = \frac{-9/4}{3} = -\frac{3}{4}.$

$-2 = -\frac{3}{4}(4) + b \Rightarrow b = 1.$

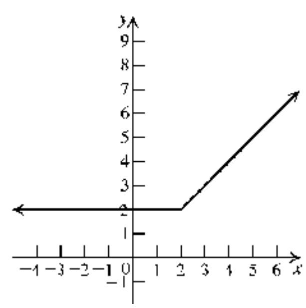
$f(x) = -\frac{3}{4}x + 1.$



19. $f(x) = \begin{cases} x & \text{if } x \ge 2 \\ 2 & \text{if } x < 2 \end{cases}$

a. $f(1) = 2; \; f(2) = 2; \; f(3) = 3$

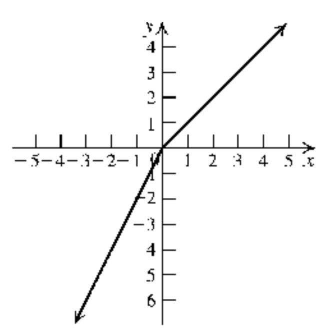
b.



20. $g(x) = \begin{cases} 2x & \text{if } x < 0 \\ x & \text{if } x \ge 0 \end{cases}$

a. $g(-1) = -2; \; g(0) = 0; \; g(1) = 1$

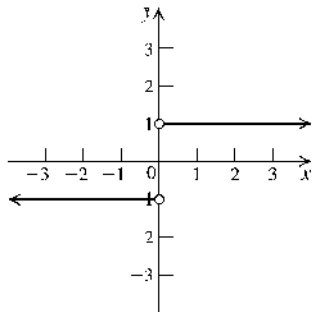
b.



21. $g(x)=\begin{cases}1 & \text{if } x>0\\ -1 & \text{if } x<0\end{cases}$

a. $f(-15)=-1;\ f(12)=1$

b.



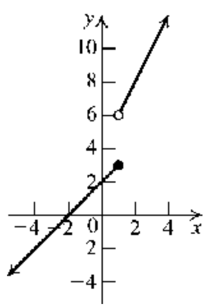
c. domain: $(-\infty,\ 0)\cup(0,\ \infty)$

range: $\{-1,\ 1\}$

22. $g(x)=\begin{cases}2x+4 & \text{if } x>1\\ x+2 & \text{if } x\le 1\end{cases}$

a. $g(-3)=-1;\ g(1)=3;\ g(3)=10$

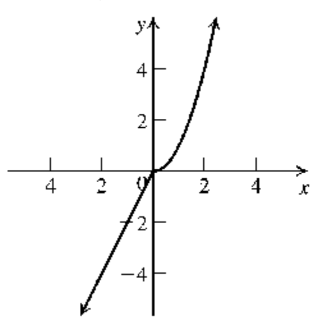
b.



c. domain: $(-\infty,\ \infty)$

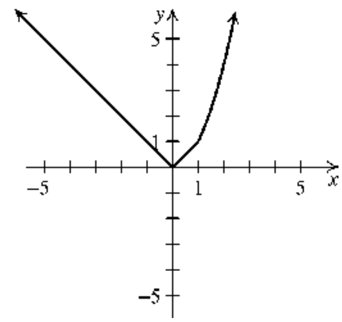
range: $(-\infty,\ 3]\cup(6,\ \infty)$

23. $f(x)=\begin{cases}2x & \text{if } x<0\\ x^2 & \text{if } x\ge 0\end{cases}$



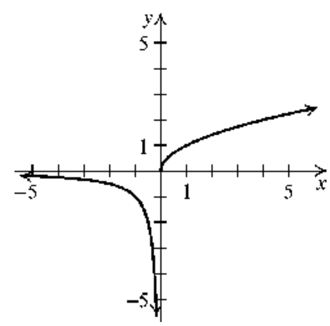
Domain: $(-\infty,\ \infty)$; range: $(-\infty,\ \infty)$

24. $f(x)=\begin{cases}|x| & \text{if } x<1\\ x^2 & \text{if } x\ge 1\end{cases}$



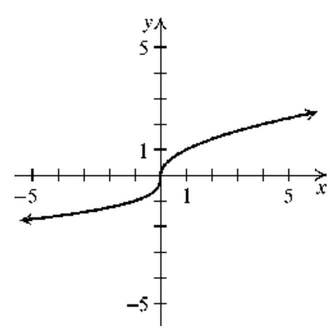
Domain: $(-\infty,\ \infty)$; range: $[0,\ \infty)$

25. $g(x)=\begin{cases}\dfrac{1}{x} & \text{if } x<0\\ \sqrt{x} & \text{if } x\ge 0\end{cases}$



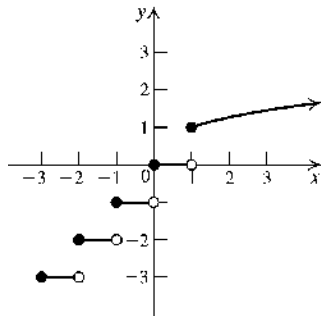
Domain: $(-\infty,\ \infty)$; range: $(-\infty,\ \infty)$

26. $h(x)=\begin{cases}\sqrt[3]{x} & \text{if } x<1\\ \sqrt{x} & \text{if } x\ge 1\end{cases}$



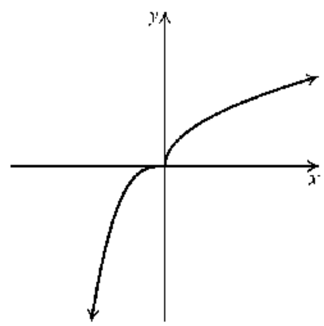
Domain: $(-\infty,\ \infty)$; range: $(-\infty,\ \infty)$

27. $f(x)=\begin{cases}[\![x]\!] & \text{if } x<1\\ \sqrt[3]{x} & \text{if } x\ge 1\end{cases}$



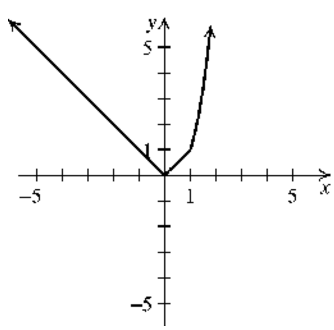
Domain: $(-\infty,\infty)$;
range: $\{\ldots,-3,-2,-1,0\}\cup[1,\infty)$

28. $g(x)=\begin{cases}x^3 & \text{if } x<0\\ \sqrt{x} & \text{if } x\ge 0\end{cases}$



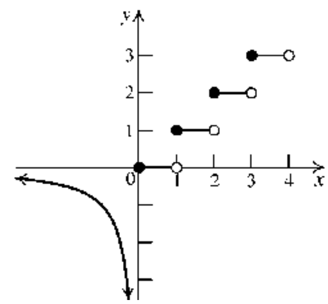
Domain: $(-\infty,\infty)$; range: $(-\infty,\infty)$

29. $g(x)=\begin{cases}|x| & \text{if } x<1\\ x^3 & \text{if } x\ge 1\end{cases}$



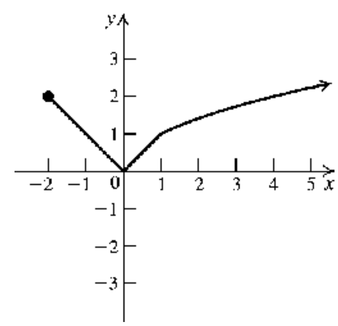
Domain: $(-\infty,\infty)$; range: $[0,\infty)$

30. $f(x)=\begin{cases}\dfrac{1}{x} & \text{if } x<0\\ [\![x]\!] & \text{if } x\ge 0\end{cases}$



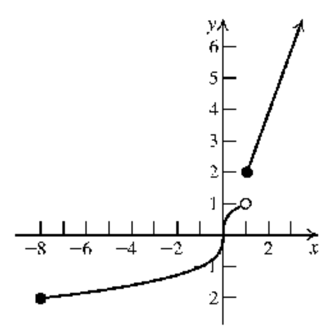
Domain: $(-\infty,\infty)$; range:
$(-\infty,0]\cup\{1,2,3,\ldots\}$

31. $f(x)=\begin{cases}|x| & \text{if } -2\le x<1\\ \sqrt{x} & \text{if } x\ge 1\end{cases}$



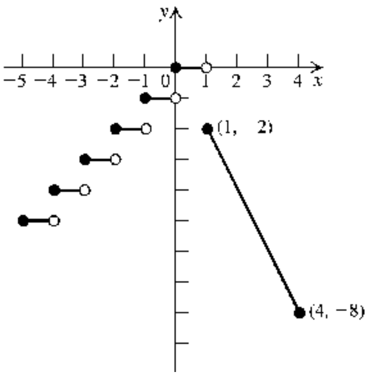
Domain: $[-2,\infty)$; range: $[0,\infty)$

32. $g(x)=\begin{cases}\sqrt[3]{x} & \text{if } -8\le x<1\\ 2x & \text{if } x\ge 1\end{cases}$



Domain: $[-8,\infty)$; range: $[-2,1)\cup[2,\infty)$

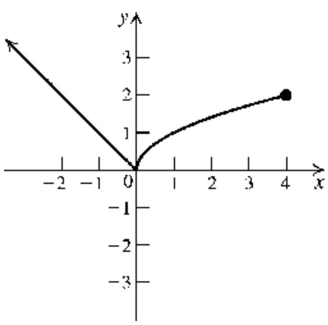
33. $f(x)=\begin{cases}[\![x]\!] & \text{if } x<1\\ -2x & \text{if } 1\le x\le 4\end{cases}$



Domain: $(-\infty, 4]$;

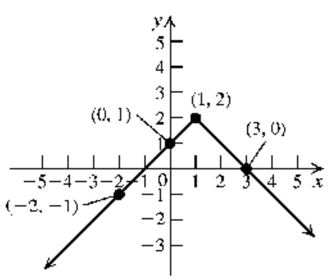
range: $\{\ldots, -3, -2, -1, 0\}\cup[-8, -2]$

34. $h(x)=\begin{cases}|x| & \text{if } x<0\\ \sqrt{x} & \text{if } 0\le x\le 4\end{cases}$



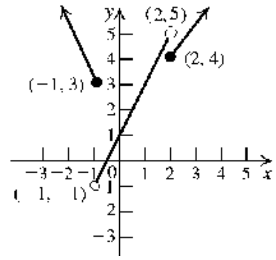
Domain: $(-\infty, 4]$; range: $[0, \infty)$

35. $f(x)=\begin{cases}2x+3 & \text{if } x<-2\\ x+1 & -2\le x<1\\ -x+3 & \text{if } x\ge 1\end{cases}$



Domain: $(-\infty, \infty)$; range: $(-\infty, 2]$

36. $f(x)=\begin{cases}-2x+1 & \text{if } x\le -1\\ 2x+1 & -1<x<2\\ x+2 & \text{if } x\ge 2\end{cases}$



Domain: $(-\infty, \infty)$; range: $(-1, \infty)$

37. The graph of *f* is made up of two parts.
For $x < 2$, the graph is made up of the half-line passing through the points $(-1, 0)$ and $(2, 3)$.

$m=\dfrac{3-0}{2-(-1)}=\dfrac{3}{3}=1$

$y-0=x-(-1)\Rightarrow y=x+1$

For $x\ge 2$, the graph is a half-line passing through the points $(2, 3)$ and $(3, 0)$.

$m=\dfrac{0-3}{3-2}=-3$

$y-0=-3(x-3)\Rightarrow y=-3x+9$

Combining the two parts, we have

$f(x)=\begin{cases}x+1 & \text{if } x<-2\\ -3x+9 & \text{if } x\ge 2\end{cases}$

38. The graph of *f* is made up of two parts.
For $x < 2$, the graph is made up of the half-line passing through the points $(2, -1)$ and $(0, 3)$.

$m=\dfrac{3-(-1)}{0-2}=\dfrac{4}{-2}=-2$

$y=-2x+3$

For $x\ge 2$, the graph is a half-line passing through the points $(2, -1)$ and $(4, 0)$.

$m=\dfrac{-1-0}{2-4}=\dfrac{1}{2}$

$y-0=\dfrac{1}{2}(x-4)\Rightarrow y=\dfrac{1}{2}x-2$

Combining the two parts, we have

$f(x)=\begin{cases}-2x+3 & \text{if } x<-2\\ \dfrac{1}{2}x-2 & \text{if } x\ge 2\end{cases}$

39. The graph of *f* is made up of three parts.
For $x < -2$, the graph is the half-line passing through the points $(-2, 2)$ and $(-3, 0)$.

$m=\dfrac{0-2}{-3-(-2)}=\dfrac{-2}{-1}=2$

$y-0=2(x-(-3))\Rightarrow y=2(x+3)\Rightarrow$

$y=2x+6$

(*continued on next page*)

(*continued*)

For $-2 \le x < 2$, the graph is a horizontal line segment passing through the points (−2, 4) and (2, 4), so the equation is $y = 4$.
For $x \ge 2$, the graph is the half-line passing through the points (2, 1) and (3, 0).

$$m = \frac{0-1}{3-2} = -1$$

$$y - 0 = -(x-3) \Rightarrow y = -x + 3$$

Combining the three parts, we have

$$f(x) = \begin{cases} 2x+6 & \text{if } x < -2 \\ 4 & \text{if } -2 \le x < 2 \\ -x+3 & \text{if } x \ge 2 \end{cases}$$

40. The graph of *f* is made up of four parts.
For $x \le -2$, the graph is the half-line passing through the points (−2, 0) and (−4, 3).

$$m = \frac{3-0}{-4-(-2)} = -\frac{3}{2}$$

$$y - 0 = -\frac{3}{2}(x-(-2)) \Rightarrow y = -\frac{3}{2}(x+2) \Rightarrow$$

$$y = -\frac{3}{2}x - 3$$

For $-2 < x \le 0$, the graph is a line segment passing through the points (−2, 0) and (0, 3).

$$m = \frac{3-0}{0-(-2)} = \frac{3}{2}$$

$$y = \frac{3}{2}x + 3$$

For $0 < x \le 2$, the graph is a line segment passing through the points (0, 3) and (2, 0).

$$m = \frac{0-3}{2-0} = -\frac{3}{2}$$

$$y = -\frac{3}{2}x + 3$$

For $x \ge 2$, the graph is the half-line passing through the points (2, 0) and (4, 3).

$$m = \frac{3-0}{4-2} = \frac{3}{2}$$

$$y - 0 = \frac{3}{2}(x-2) \Rightarrow y = \frac{3}{2}x - 3$$

Combining the four parts, we have

$$f(x) = \begin{cases} y = -\frac{3}{2}x - 3 & \text{if } x \le -2 \\ y = \frac{3}{2}x + 3 & \text{if } -2 < x \le 0 \\ y = -\frac{3}{2}x + 3 & \text{if } 0 < x \le 2 \\ y = \frac{3}{2}x - 3 & \text{if } x > 2 \end{cases}$$

2.6 Applying the Concepts

41. a. $f(x) = \dfrac{x}{33.81}$
Domain: $[0, \infty)$; range: $[0, \infty)$.

b. $f(3) = \dfrac{3}{33.81} \approx 0.0887$
This means that 3 oz ≈ 0.0887 liter.

c. $f(12) = \dfrac{12}{33.81} \approx 0.3549$ liter.

42. a. $B(0) = -1.8(0) + 212 = 212.$
The y-intercept is 212. This means that water boils at 212°F at sea level.
$0 = -1.8h + 212 \Rightarrow h \approx 117.8$
The h-intercept is approximately 117.80. This means that water boils at 0°F at approximately 117,800 feet above sea level.

b. Domain: closed interval from 0 to the end of the atmosphere, in thousands of feet.

c. $98.6 = -1.8h + 212 \Rightarrow h = 63.$ Water boils at 98.6°F at 63,000 feet. It is dangerous because 98.6°F is the temperature of human blood.

43. a. $P(0) = \dfrac{1}{33}(0) + 1 = 1.$ The y-intercept is 1.
This means that the pressure at sea level ($d = 0$) is 1 atm.
$0 = \dfrac{1}{33}d + 1 \Rightarrow d = -33.$
d can't be negative, so there is no d-intercept.

b. $P(0) = 1$ atm; $P(10) = \dfrac{1}{33}(10) + 1 \approx 1.3$ atm;
$P(33) = \dfrac{1}{33}(33) + 1 = 2$ atm;
$P(100) = \dfrac{1}{33}(100) + 1 \approx 4.03$ atm.

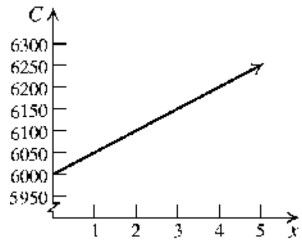
c. $5 = \dfrac{1}{33}d + 1 \Rightarrow d = 132$ feet
The pressure is 5 atm at 132 feet.

44. a. $V(90) = 1055 + 1.1(90) = 1154$ ft/sec
The speed of sound at 90ºF is 1154 feet per second.

b. $1100 = 1055 + 1.1T \Rightarrow T \approx 40.91$°F
The speed of sound is 1100 feet per second at approximately 40.91ºF.

45. a. $C(x) = 50x + 6000$

b. The y-intercept is the fixed overhead cost.



c. $11,500 = 50x + 6000 \Rightarrow 110$

110 printers were manufactures on a day when the total cost was $11,500.

46. a. The rate of change (slope) is 100. Find the y-intercept by using the point (10, 750): $750 = 100(10) + b \Rightarrow b = -250$. The equation is $f(p) = 100p - 250$.

b. $f(15) = 100(15) - 250 = 1250$

When the price is $15 per unit, there are 1250 units.

c. $1750 = 100p - 250 \Rightarrow p = \$20$.

1750 units can be supplied at $20 per unit.

47. a. $R = 900 - 30x$

b. $R(6) = 900 - 30(6) = 720$

If you move in 6 days after the first of the month, the rent is $720.

c. $600 = 900 - 30x \Rightarrow x = 10$

You moved in ten days after first of the month.

48. a. Let $t = 0$ represent the year 2009. The rate of change (slope) is $\frac{995 - 976}{0 - 2} = -9.5$. The y-intercept is 995, so the equation is $f(t) = -9.5t + 995$.

b. $f(4) = -9.5(4) + 995 = 957$

The average SAT score will be 957 in 2013.

c. $-9.5t + 995 = 900 \Rightarrow -9.5t = -95 \Rightarrow t = 10$

2009 + 10 = 2019.

The average SAT score will be 900 in 2019.

49. The rate of change (slope) is $\frac{100 - 40}{20 - 80} = -1$.

Use the point (20, 100) to find the equation of the line: $100 = -20 + b \Rightarrow b = 120$. The equation of the line is $y = -x + 120$. Now solve $50 = -x + 120 \Rightarrow x = 70$.

Age 70 corresponds to 50% capacity.

50. a. $y = \frac{2}{25}(5)(60) = 24$

The dosage for a five-year-old child is 24 mg.

b. $60 = \frac{2}{25}(60)a \Rightarrow a = 12.5$

A child would have to be 12.5 years old to be prescribed an adult dosage.

51. a. The rate of change (slope) is $\frac{50 - 30}{420 - 150} = \frac{2}{27}$.

The equation of the line is

$y - 30 = \frac{2}{27}(x - 150) \Rightarrow$

$y = \frac{2}{27}(x - 150) + 30$.

b. $y = \frac{2}{27}(350 - 150) + 30 \Rightarrow y = \frac{1210}{27} \approx 44.8$

There can't be a fractional number of deaths, so round up. There will be about 45 deaths when x = 350 milligrams per cubic meter.

c. $70 = \frac{2}{27}(x - 150) + 30 \Rightarrow x = 690$

If the number of deaths per month is 70, the concentration of sulfur dioxide in the air is 690 mg/m^3.

52. a. The rate of change is $\frac{1}{3}$. The y-intercept is $\frac{47}{12}$, so the equation is

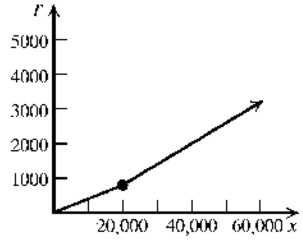
$y = L(S) = \frac{1}{3}S + \frac{47}{12}$.

b. $L(4) = \frac{1}{3}(4) + \frac{47}{12} = 5.25$

A child's size 4 shoe has insole length 5.25 inches.

c. $\frac{61}{10} = \frac{1}{3}x + \frac{47}{12} \Rightarrow x = 6.55 \approx 6.5$
A child whose insole length is 6.1 inches wears a size 6.5 shoe.

53. a.



b.(i) $T(12,000) = 0.04(12,000) = \$480$

(ii) $T(20,000) = 800 + 0.06(20,000 - 20,000) = \$800$

(iii) $T(50,000) = 800 + 0.06(50,000 - 20,000) = \$2600$

c. (i) $600 = 0.04x \Rightarrow x = \$15,000$

(ii) $1200 = 0.04x \Rightarrow x = \$30,000$, which is outside of the domain. Try $1200 = 800 + 0.06(x - 20,000) \Rightarrow x \approx \$26,667$

(iii) $2300 = 800 + 0.06(x - 20,000) \Rightarrow x = \$45,000$

54. a. $$f(x) = \begin{cases} 0.1x & \text{if } 0 < x \le 9225 \\ 922.50 + 0.15(x - 9225) & \text{if } 9225 < x \le 37,450 \\ 5156.25 + 0.25(x - 37,450) & \text{if } 37,450 < x \le 90,750, \\ 18,481.25 + 0.28(x - 90,750) & \text{if } 90,750 < x \le 189,300 \\ 46,075.25 + 0.33(x - 189,300) & \text{if } 189,300 < x \le 411,500 \\ 119,401.25 + 0.35(x - 411,500) & \text{if } 411,500 < x \le 413,200 \\ 119,996.25 + 0.396(x - 413,200) & \text{if } 413,200 < x \end{cases}$$

b. (i) $f(35,000) = 922.50 + 0.15(35,000 - 9225) = \$4788.75 \approx \$4789$

(ii) $f(100,000) = 18,481.25 + 0.28(100,000 - 90,750) = \$21,071.25 \approx \$21,071$

(iii) $f(500,000) = 119,996.25 + 0.396(500,000 - 413,200) = \$154,369.05 \approx \$154,369$

c. (i) $3500 = 922.50 + 0.15(x - 9225) \Rightarrow 2577.5 = 0.15(x - 9225) \Rightarrow 17183.33 = x - 9225 \Rightarrow x \approx \$26,408$

(ii) $12,700 = 5156.25 + 0.25(x - 37,450) \Rightarrow 7543.75 = 0.25(x - 37,450) \Rightarrow 30,175 = x - 37,450 \Rightarrow x = \$67,625$

(iii) $35,000 = 18,481.25 + 0.28(x - 90,750) \Rightarrow 16518.75 = 0.28(x - 90,750) \Rightarrow 58995.54 = x - 90,750 \Rightarrow x \approx \$149,746$

2.6 Beyond the Basics

55. $2(3) - 1 = a - 3(3) \Rightarrow 5 = a - 9 \Rightarrow a = 14$

56. $1 - 3 = 3a + 3 \Rightarrow -2 = 3a + 3 \Rightarrow -5 = 3a \Rightarrow a = -\frac{5}{3}$

57. a. Domain: $(-\infty, \infty)$; range: $[0,1)$

b. The function is increasing on $(n, n + 1)$ for every integer n.

c. $f(-x) = -x - [\![-x]\!] \ne -f(x) \ne f(x)$, so the function is neither even nor odd.

58. a. Domain: $(-\infty, 0) \cup [1, \infty)$
range: $\left\{\frac{1}{n} : n \ne 0, n \text{ an integer}\right\}$

b. The function is constant on $(n, n + 1)$ for every nonzero integer n.

c. $f(-x) = \frac{1}{[\![-x]\!]} \ne -f(x) \ne f(x)$, so the function is neither even nor odd.

59. a. (i) $WCI(2) = 40$

(ii)
$WCI(16) = 91.4 + (91.4 - 40) \cdot \left(0.0203(16) - 0.304\sqrt{16} - 0.474\right) \approx 21$

(iii) $WCI(50) = 1.6(40) - 55 = 9$

b. **(i)** $-58 = 91.4 + (91.4 - T) \cdot (0.0203(36) - 0.304\sqrt{36} - 0.474$
$-58 = 91.4 + (91.4 - T)(-1.5672)$
$-58 = 91.4 - 143.24 + 1.5672T$
$-58 = -51.84 + 1.5672T \Rightarrow T \approx -4°\text{F}$

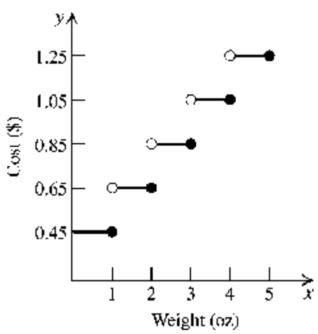
(ii) $-10 = 1.6T - 55 \Rightarrow T \approx 28°\text{F}$

60. a. $C(x) = 20\left(f(x) - 1\right) + 45 = 20\left(-[\![-x]\!] - 1\right) + 45$

b. $C(2.3) = 20\left(-[\![-2.3]\!] - 1\right) + 45 = 20\left[-(-3) - 1\right] + 45 = 20(2) + 45 = 85$

It will cost 85¢ to mail a first-class letter weighing 2.3 oz.

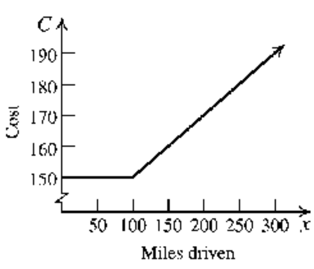
c.



61. $C(x) = 2[\![x]\!] + 4$

62. a. $C(x) = \begin{cases} 150 & \text{if } x \le 100 \\ 0.2[\![x - 100]\!] + 150 & \text{if } x > 100 \end{cases}$

b.



c. $190 = 0.2[\![x - 99]\!] + 150$
$40 = 0.2[\![x - 99]\!] \Rightarrow 200 = [\![x - 99]\!] \Rightarrow$
$x = 300$ miles

2.6 Critical Thinking/Discussion/Writing

63. D **64.** C

65. a. If f is even, then f is increasing on $(-\infty, -1)$ and decreasing on $(-1, 0)$.

b. If f is odd, then f is decreasing on $(-\infty, -1)$ and increasing on $(-1, 0)$.

66. a. If f is even, then f has a relative maximum at $x = -1$ and a relative minimum at $x = -3$.

b. If f is odd, then f has a relative minimum at $x = -1$ and a relative maximum at $x = -3$.

2.6 Getting Ready for the Next Section

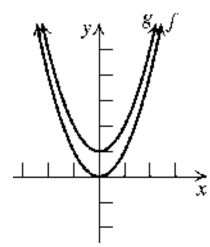
67. If we add 3 to each y-coordinate of the graph of f, we will obtain the graph of $y = f(x) + 3$.

68. If we subtract 2 from each x-coordinate of the graph of f, we will obtain the graph of $y = f(x + 2)$.

69. If we replace each x-coordinate with its opposite in the graph of f, we will obtain the graph of $y = f(-x)$.

70. If we replace each y-coordinate with its opposite in the graph of f, we will obtain the graph of $y = -f(x)$.

71.



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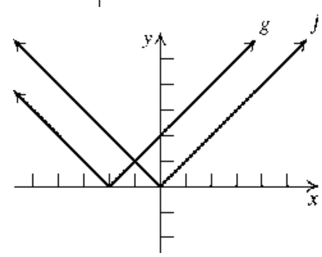

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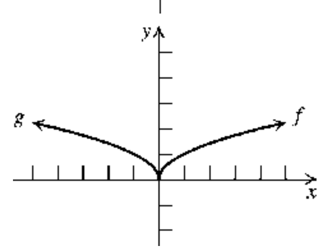
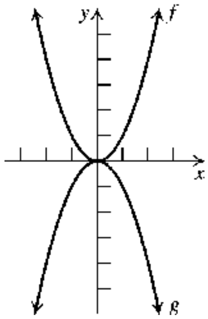


74.



For exercises 75–78, refer to section 1.3 in your text for help on completing the square.

75. $x^2 + 8x + \underline{16}$; $(x+4)^2$

76. $x^2 - 6x + \underline{9}$; $(x-3)^2$

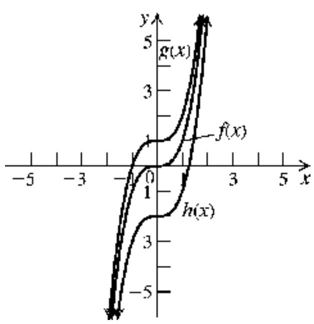
77. $x^2 - \frac{2}{3}x + \underline{\frac{1}{9}}$; $\left(x - \frac{1}{3}\right)^2$

78. $x^2 + \frac{4}{5}x + \underline{\frac{4}{25}}$; $\left(x + \frac{2}{5}\right)^2$

2.7 Transformations of Functions

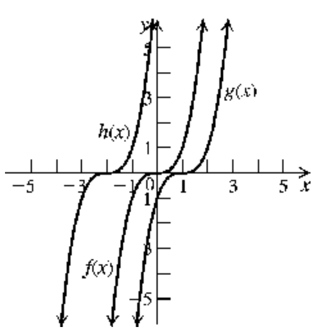
2.7 Practice Problems

1.



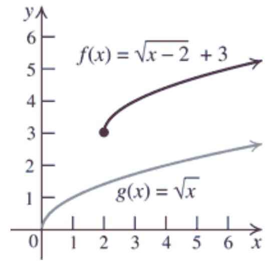
The graph of *g* is the graph of *f* shifted one unit up. The graph of *h* is the graph of *f* shifted two units down.

2.



The graph of *g* is the graph of *f* shifted one unit to the right. The graph of *h* is the graph of *f* shifted two units to the left.

3. The graph of $f(x) = \sqrt{x-2} + 3$ is the graph of $g(x) = \sqrt{x}$ shifted two units to the right and three units up.

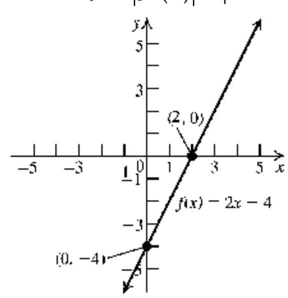


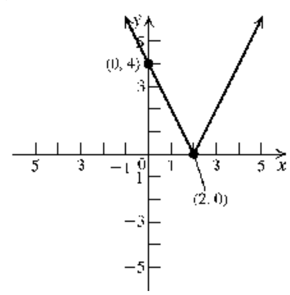
4. The graph of $y = -(x-1)^2 + 2$ can be obtained from the graph of $y = x^2$ by first shifting the graph of $y = x^2$ one unit to the right. Reflect the resulting graph about the *x*-axis, and then shift the graph two units up

5. The graph of $y = 2x - 4$ is obtained from the graph of $y = 2x$ by shifting it down by four units. We know that

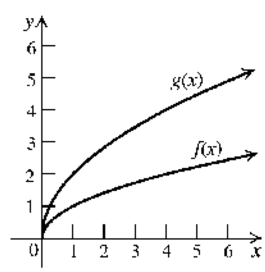
$$|y| = \begin{cases} y & \text{if } y \geq 0 \\ -y & \text{if } y < 0. \end{cases}$$

This means that the portion of the graph on or above the *x*-axis $(y \geq 0)$ is unchanged while the portion of the graph below the *x*-axis ($y <$ 0) is reflected above the *x*-axis. The graph of $y = |f(x)| = |2x - 4|$ is given on the right.



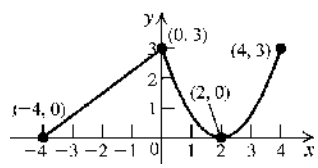


6.

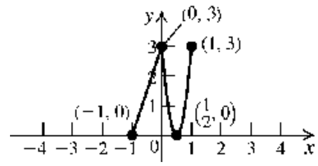


The graph of *g* is the graph of *f* stretched vertically by multiplying each of its *y*-coordinates by 2.

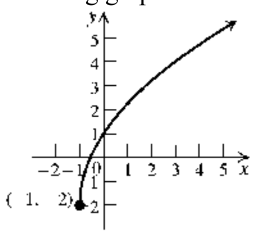
7. a.



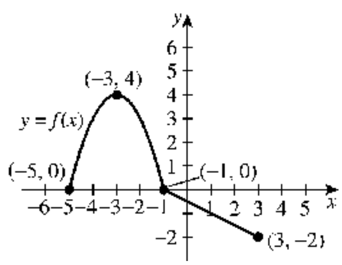
b.



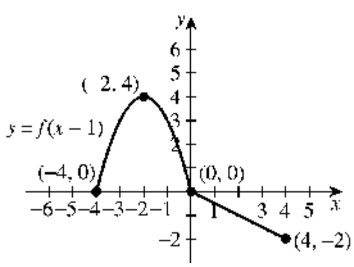
8. Start with the graph of $y = \sqrt{x}$. Shift the graph one unit to the left, then stretch the graph vertically by a factor of three. Shift the resulting graph down two units.



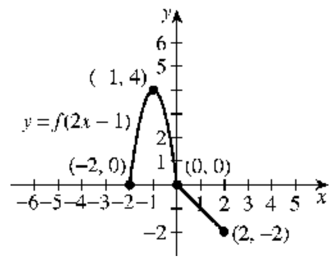
9.



Shift the graph one unit right to graph $y = f(x-1)$.

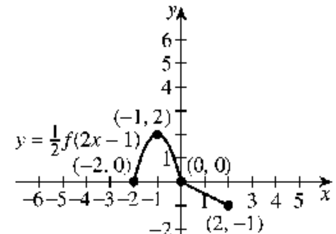


Compress horizontally by a factor of 2. Multiply each x-coordinate by $\frac{1}{2}$ to graph $y = f(2x-1)$.

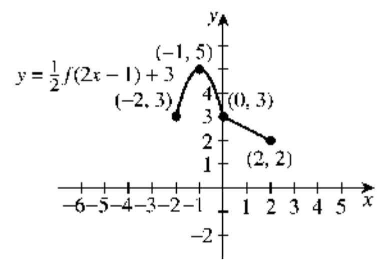


Compress vertically by a factor of $\frac{1}{2}$.

Multiply each y-coordinate by $\frac{1}{2}$ to graph $y = \frac{1}{2} f(2x-1)$.



Shift the graph up three units to graph $y = \frac{1}{2} f(2x-1) + 3$.



$$y = f(x) \to y = f(x-1) \to y = f(2x-1) \to$$
$$y = \frac{1}{2} f(2x-1) \Rightarrow y = \frac{1}{2} f(2x-1) + 3$$

2.7 Concepts and Vocabulary

1. The graph of $y = f(x) - 3$ is found by vertically shifting the graph of $y = f(x)$ three units down.

2. The graph of $y = f(x+5)$ is found by horizontally shifting the graph of $y = f(x)$ five units to the left.

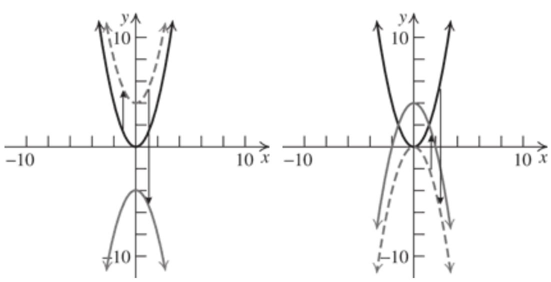
3. The graph of $y = f(bx)$ is a horizontal compression of the graph of $y = f(x)$ is b is greater than 1.

4. The graph of $y = f(-x)$ is found by reflecting the graph of $y = f(x)$ about the y-axis.

5. False. The graphs are the same if the function is an even function.

6. True

7. False. The graph on the left shows $y = x^2$ first shifted up two units and then reflected about the x-axis, while the graph on the right shows $y = x^2$ reflected about the x-axis and then shifted up two units.



8. True

2.7 Building Skills

9. a. The graph of g is the graph of f shifted two units up.

b. The graph of h is the graph of f shifted one unit down.

10. a. The graph of g is the graph of f shifted one unit up.

b. The graph of h is the graph of f shifted two units down.

11. a. The graph of g is the graph of f shifted one unit to the left.

b. The graph of h is the graph of f shifted two units to the right.

12. a. The graph of g is the graph of f shifted two units to the left.

b. The graph of h is the graph of f shifted three units to the right.

13. a. The graph of g is the graph of f shifted one unit left, then two units down.

b. The graph of h is the graph of f shifted one unit right, then three units up.

14. a. The graph of g is the graph of f reflected about the x-axis.

b. The graph of h is the graph of f reflected about the y-axis.

15. a. The graph of g is the graph of f reflected about the x-axis.

b. The graph of h is the graph of f reflected about the y-axis.

16. a. The graph of g is the graph of f stretched vertically by a factor of 2.

b. The graph of h is the graph of f compressed horizontally by a factor of 2.

17. a. The graph of g is the graph of f vertically stretched by a factor of 2.

b. The graph of h is the graph of f horizontally compressed by a factor of 2.

18. a. The graph of g is the graph of f shifted two units to the right, then one unit up.

b. The graph of h is the graph of f shifted one unit to the left, reflected about the x-axis, and then shifted two units up.

19. a. The graph of g is the graph of f reflected about the x-axis and then shifted one unit up.

b. The graph of h is the graph of f reflected about the y-axis and then shifted one unit up.

20. a. The graph of g is the graph of f shifted one unit to the right and then shifted two units up.

b. The graph of h is the graph of f stretched vertically by a factor of three and then shifted one unit down.

21. a. The graph of g is the graph of f shifted one unit up.

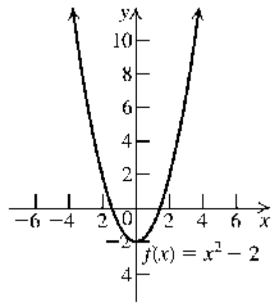
b. The graph of h is the graph of f shifted one unit to the left.

22. a. The graph of g is the graph of f shifted one unit left, vertically stretched by a factor of 2, reflected about the y-axis, and then shifted 4 units up.

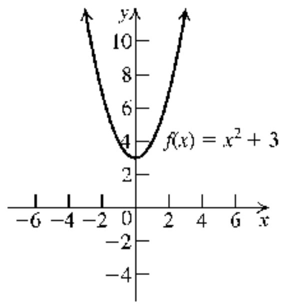
b. The graph of h is the graph of f shifted one unit to the right, reflected about the x-axis, and then shifted three units up.

23. e	**24.** c	**25.** g	**26.** h
27. i	**28.** a	**29.** b	**30.** k
31. l	**32.** f	**33.** d	**34.** J

35.



36.



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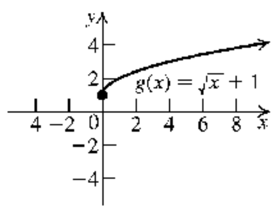
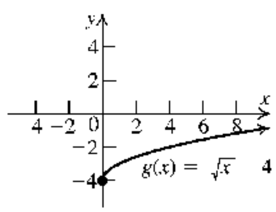
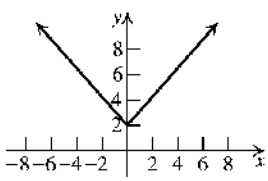


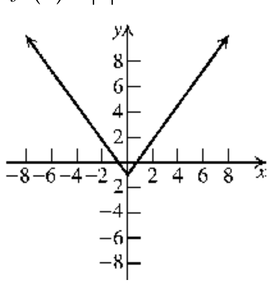
38.



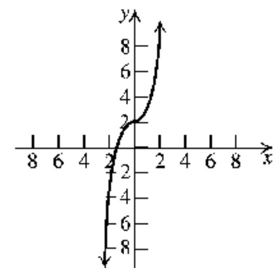
39. $f(x) = |x| + 2$



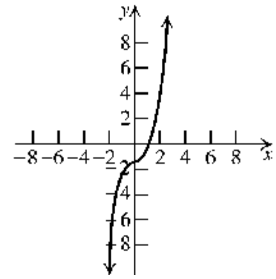
40. $f(x) = |x| - 1$



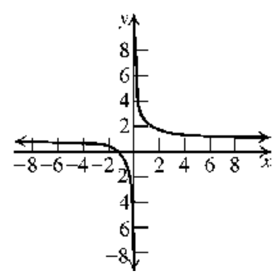
41. $f(x) = x^3 + 2$



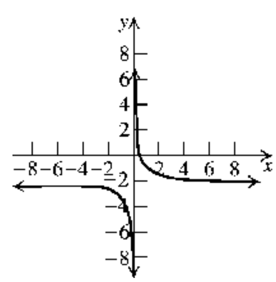
42. $f(x) = x^3 - 1$



43. $f(x) = \frac{1}{x} + 1$



44. $f(x) = \frac{1}{x} - 2$



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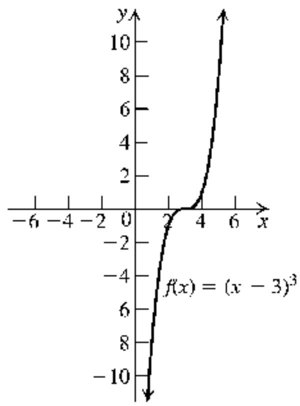

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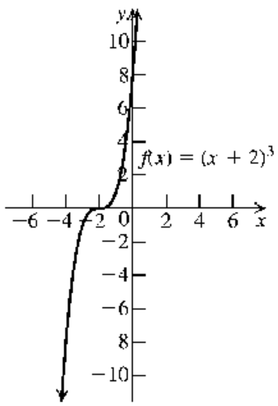
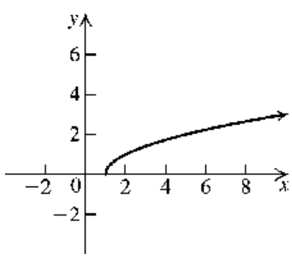
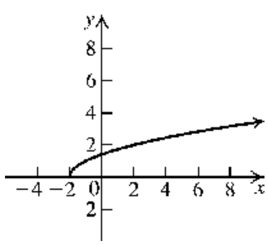


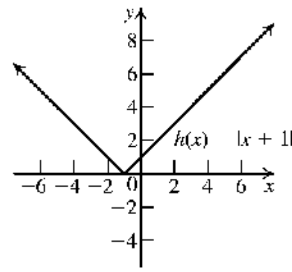
47. $f(x)=\sqrt{x-1}$



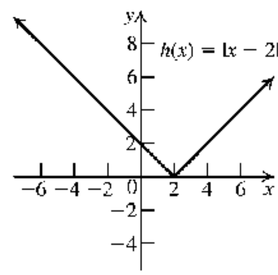
48. $f(x)=\sqrt{x+2}$



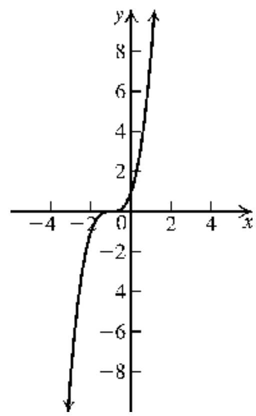
49.



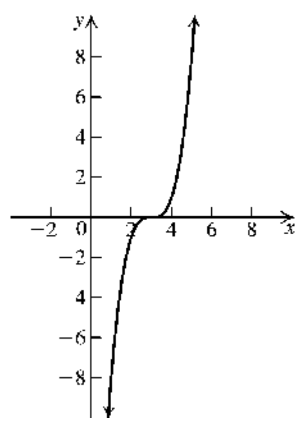
50.



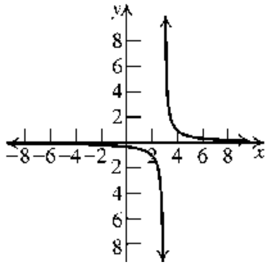
51. $f(x)=(x+1)^3$



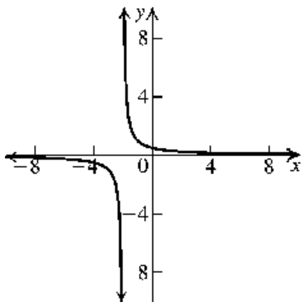
52. $f(x)=(x-3)^3$



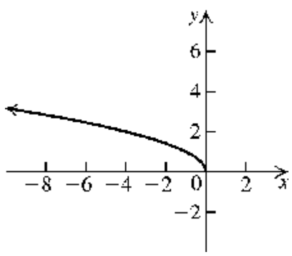
53. $f(x)=\frac{1}{x-3}$



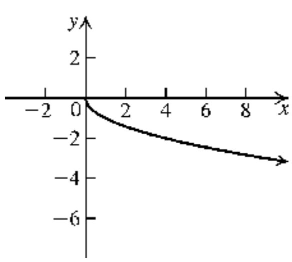
54. $f(x)=\frac{1}{x+2}$



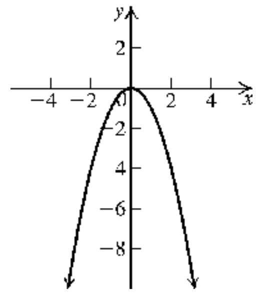
55. $f(x)=\sqrt{-x}$



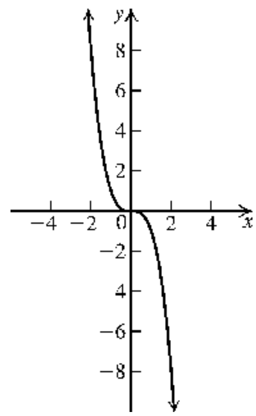
56. $f(x)=-\sqrt{x}$



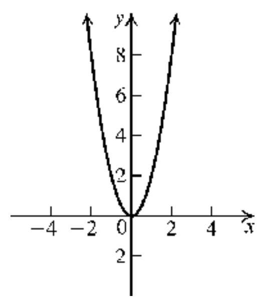
57. $f(x)=-x^2$



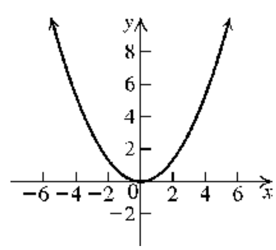
58. $f(x)=-x^3$



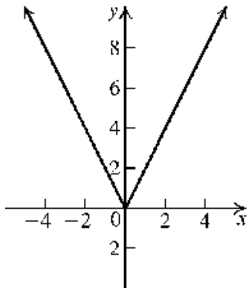
59. $f(x)=2x^2$



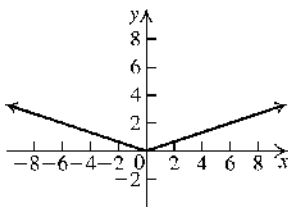
60. $f(x)=\frac{1}{3}x^2$



61. $f(x) = 2|x|$

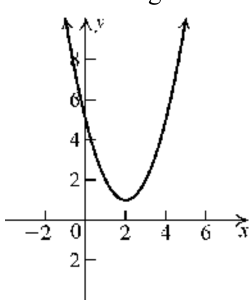


62. $f(x) = \frac{1}{3}|x|$



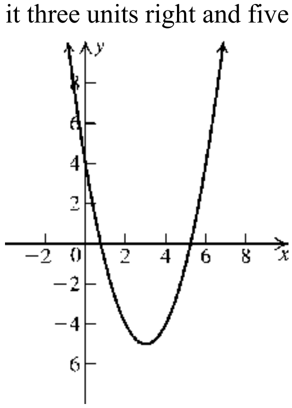
63. $f(x) = (x-2)^2 + 1$

Start with the graph of $f(x) = x^2$, then shift it two units right and one unit up.



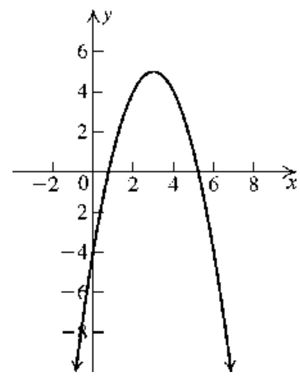
64. $f(x) = (x-3)^2 - 5$

Start with the graph of $f(x) = x^2$, then shift it three units right and five units down.



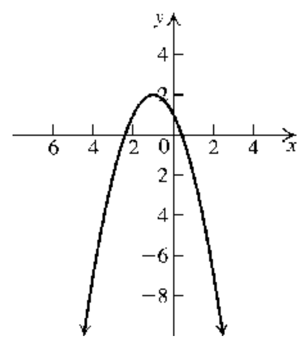
65. $f(x) = 5 - (x-3)^2$

Start with the graph of $f(x) = x^2$, then shift it three units right. Reflect the graph across the
x-axis. Shift it five units up.



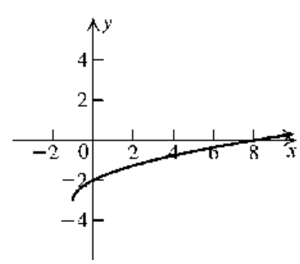
66. $f(x) = 2 - (x+1)^2$

Start with the graph of $f(x) = x^2$, then shift it one unit left. Reflect the graph across the x-axis. Shift it two units up.



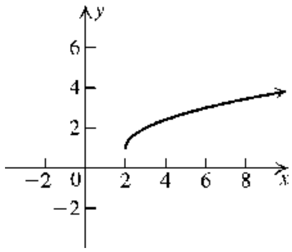
67. $f(x) = \sqrt{x+1} - 3$

Start with the graph of $f(x) = \sqrt{x}$, then shift it one unit left and three units down.



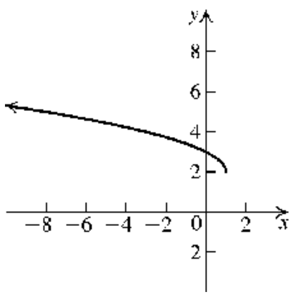
68. $f(x)=\sqrt{x-2}+1$

Start with the graph of $f(x)=\sqrt{x}$, then shift it two units right and one unit up



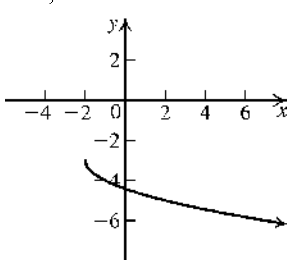
69. $f(x)=\sqrt{1-x}+2$

Start with the graph of $f(x)=\sqrt{x}$, then shift it one unit left. Reflect the graph across the y-axis, and then shift it two units up.



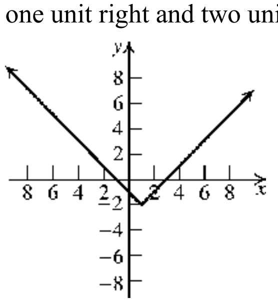
70. $f(x)=-\sqrt{x+2}-3$

Start with the graph of $f(x)=\sqrt{x}$, then shift it two units left. Reflect the graph across the x-axis, and then shift it three units down.



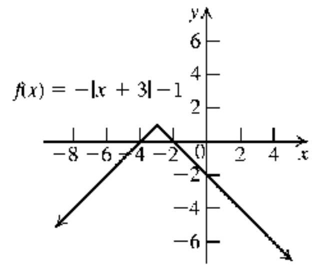
71. $f(x)=|x-1|-2$

Start with the graph of $f(x)=|x|$, then shift it one unit right and two units down.



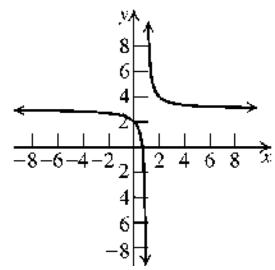
72. $f(x)=-|x+3|+1$

Start with the graph of $f(x)=|x|$, then shift it three units left. Reflect the graph across the x-axis, and then shift it one unit up.



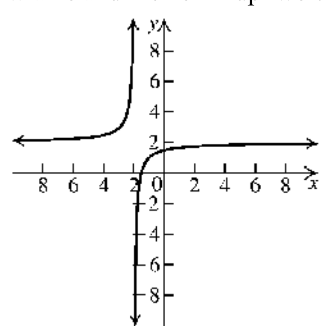
73. $f(x)=\dfrac{1}{x-1}+3$

Start with the graph of $f(x)=\frac{1}{x}$, then shift it one unit right and three units up.



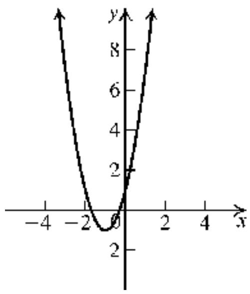
74. $f(x)=2-\dfrac{1}{x+2}$

Start with the graph of $f(x)=\frac{1}{x}$, then shift it two units left. Reflect the graph across the x-axis and then shift up two units up.



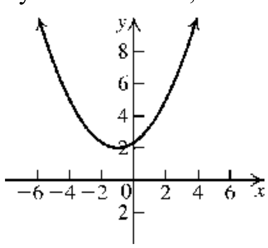
75. $f(x)=2(x+1)^2-1$

Start with the graph of $f(x)=x^2$, then shift it one unit left. Stretch the graph vertically by a factor of 2, then shift it one unit down.



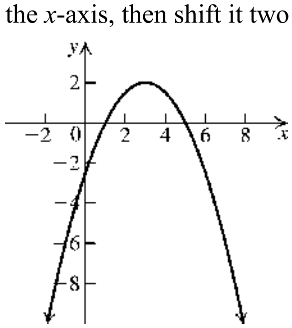
76. $f(x)=\frac{1}{3}(x+1)^2+2$

Start with the graph of $f(x)=x^2$, then shift it one unit left. Compress the graph vertically by a factor of 1/3, then shift it two units up.



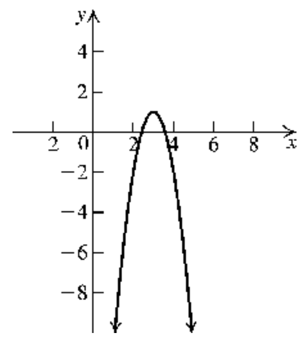
77. $f(x)=2-\frac{1}{2}(x-3)^2$

Start with the graph of $f(x)=x^2$, then shift it three units right. Compress the graph vertically by a factor of 1/2, reflect it across the x-axis, then shift it two units up.



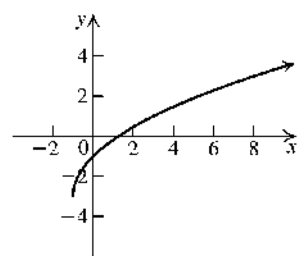
78. $f(x)=1-3(x-3)^2$

Start with the graph of $f(x)=x^2$, then shift it three units right. Stretch the graph vertically by a factor of 3, reflect it across the x-axis, then shift it one unit up.



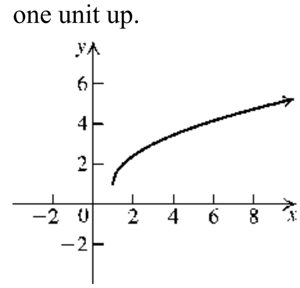
79. $f(x)=2\sqrt{x+1}-3$

Start with the graph of $f(x)=\sqrt{x}$, then shift it one unit left. Stretch the graph vertically by a factor of 2, and then shift it three units down.



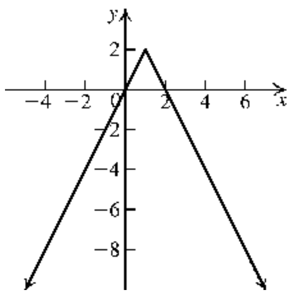
80. $f(x)=\sqrt{2x-2}+1$

Start with the graph of $f(x)=\sqrt{x}$, then shift it two units right. Compress the graph horizontally by a factor of 1/2, and then shift it one unit up.



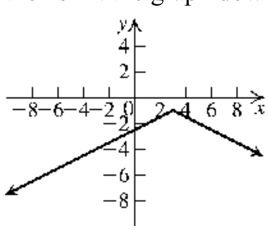
81. $f(x) = -2|x-1| + 2$

Start with the graph of $f(x) = |x|$, then shift it one unit right. Stretch the graph vertically by a factor of 2, then reflect it across the *x*-axis. Shift the graph up two units.



82. $f(x) = -\frac{1}{2}|3-x| - 1$

Start with the graph of $f(x) = |x|$, then shift it three units right. Compress the graph vertically by a factor of 1/2, then reflect it across the
y-axis. Reflect the graph across the *x*-axis, and then shift the graph down one unit.



83. $y = x^3 + 2$ **84.** $y = \sqrt{x+3}$

85. $y = -|x|$ **86.** $y = \sqrt{-x}$

87. $y = (x-3)^2 + 2$ **88.** $y = -(x+2)^2$

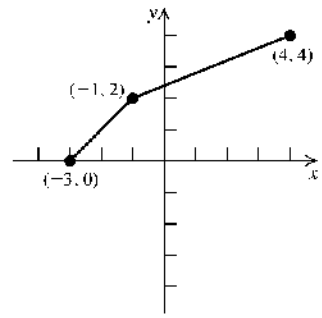
89. $y = -\sqrt{x+3} - 2$ **90.** $y = -\frac{1}{2}\left(\sqrt{x} - 2\right)$

91. $y = 3(-x+4)^3 + 2$

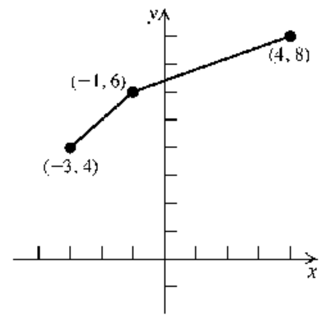
92. $y = -(-x+1)^3 + 1$

93. $y = -2|x-4| - 3$ **94.** $y = \frac{1}{2}|-x-2| - 3$

95.



96.



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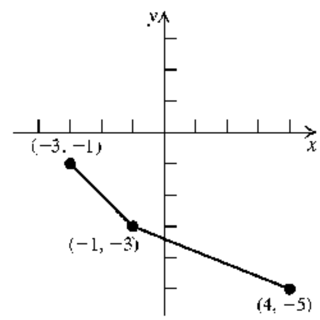

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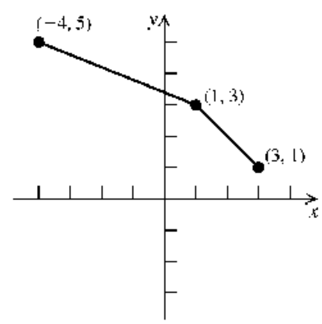

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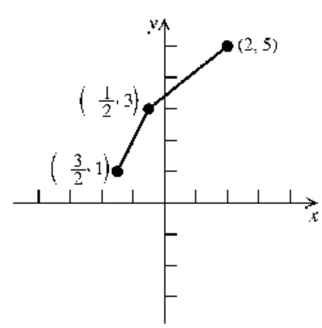
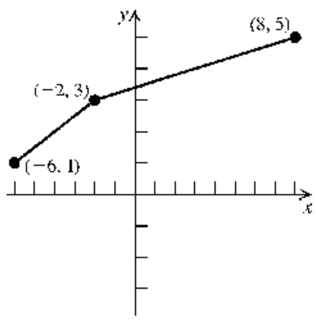
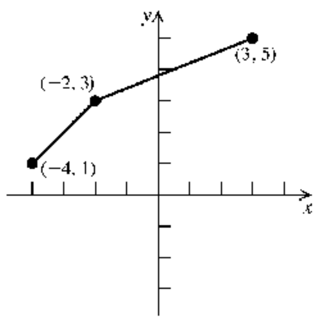


100.



101.



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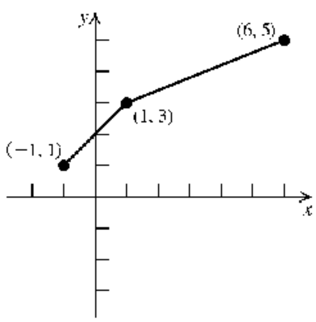
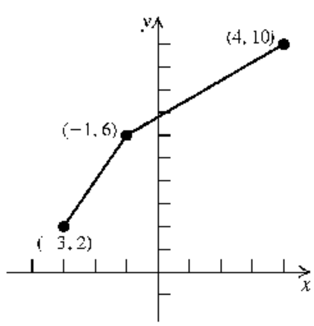
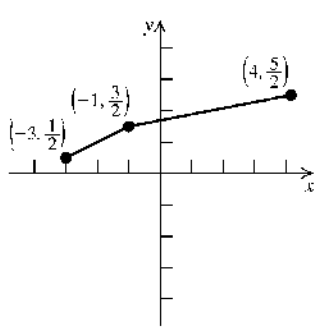


103.



104.



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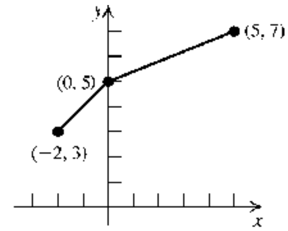

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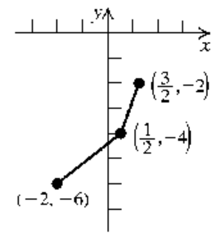
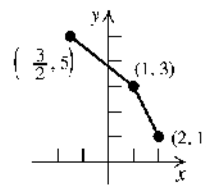
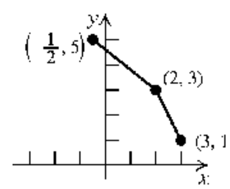


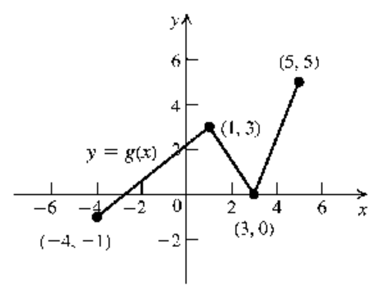
107.



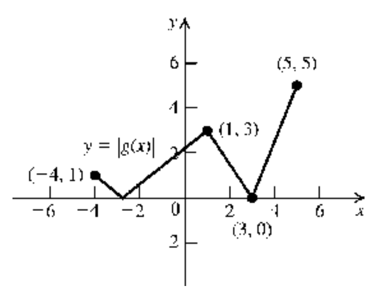
108.



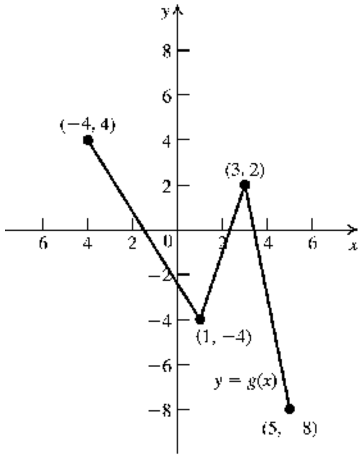
109. a.



b.



110. a.



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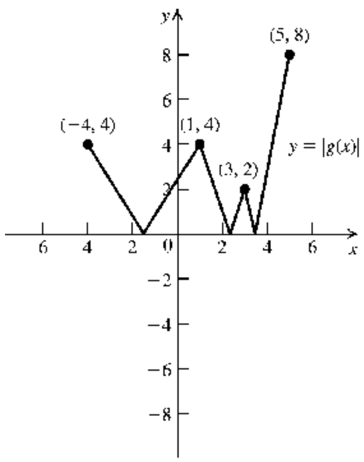
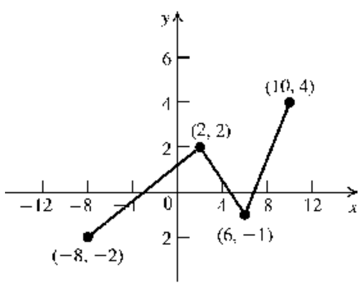


111. a.



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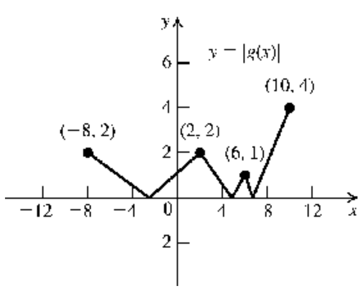
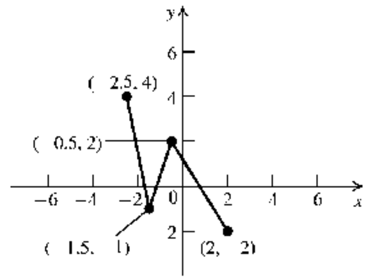


112. a.



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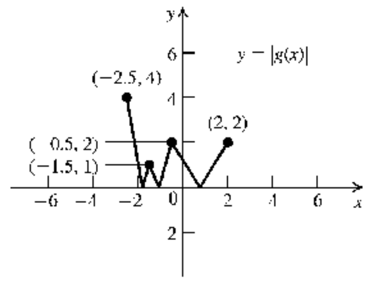
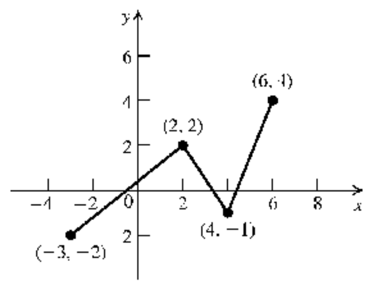


113. a.



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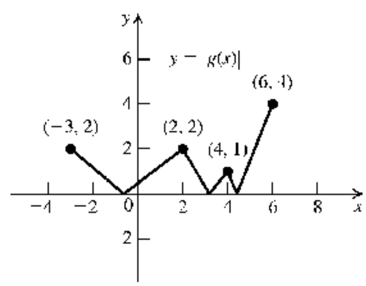
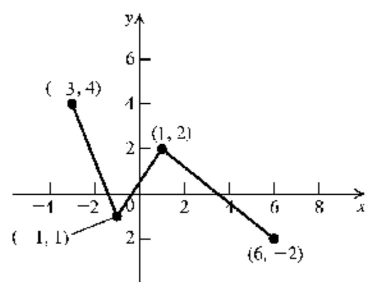


114. a.



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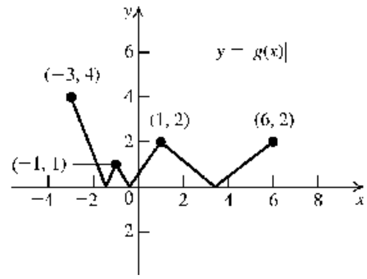
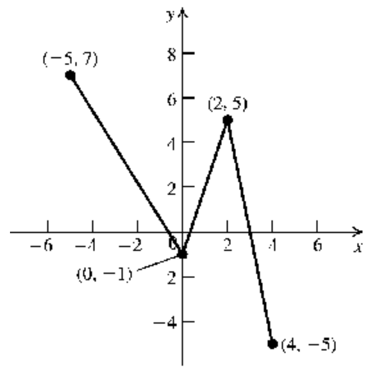


115. a.



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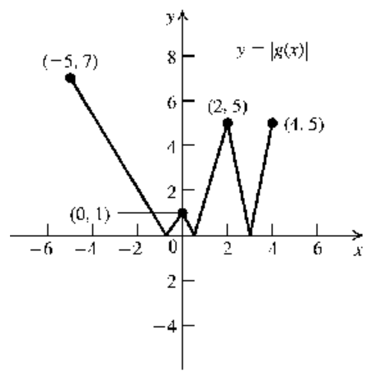
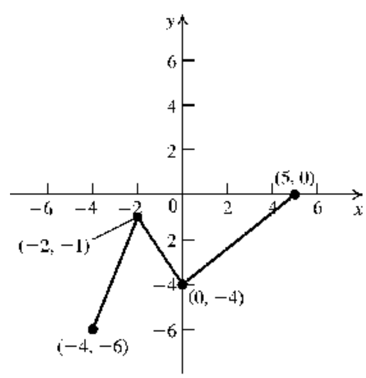


116. a.



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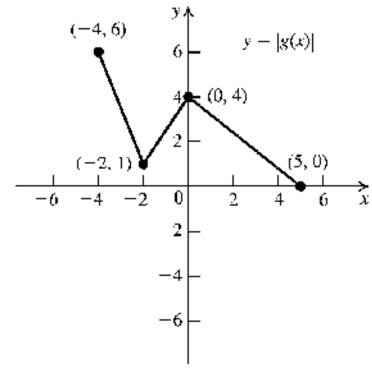
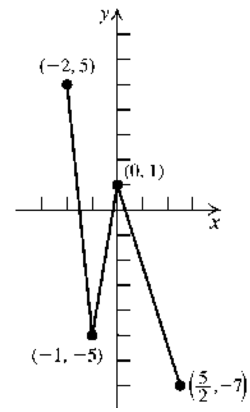


117. a.



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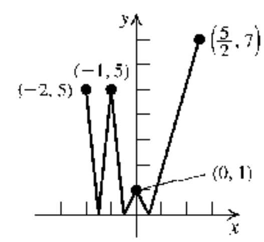
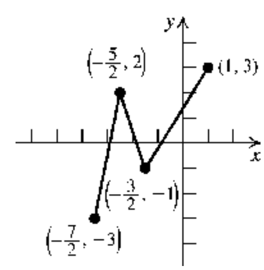
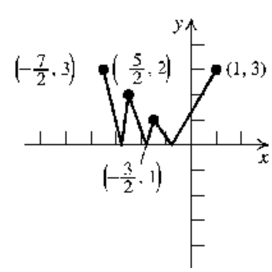


118. a.



b.



2.7 Applying the Concepts

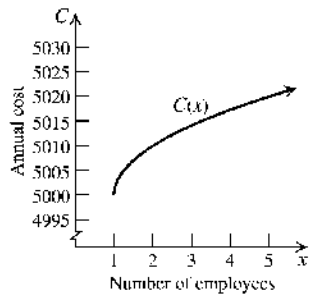
119. $g(x) = f(x) + 800$

120. $h(x) = 1.05f(x)$

121. $p(x) = 1.02\left(f(x) + 500\right)$

122. $g(x) = \begin{cases} 1.1f(x) & \text{if } f(x) < 30{,}000 \\ 1.02f(x) & \text{if } f(x) \geq 30{,}000 \end{cases}$

123. a. Shift one unit right, stretch vertically by a factor of 10, and shift 5000 units up.



b. $C(400) = 5000 + 10\sqrt{400 - 1} = \$5199.75$

124. a. For the center of the artery, R = 3 mm and r = 0.

$v = 1000\left(3^2 - 0^2\right) = 9000$ mm/minute

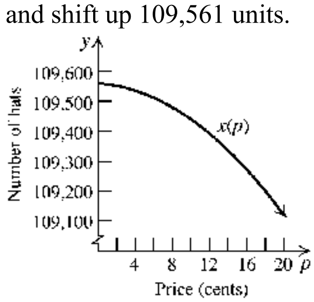
b. For the inner linings of the artery, R = 3 mm and r = 3 mm

$v = 1000\left(3^2 - 3^2\right) = 0$ mm/minute

c. Midway between the center and the inner linings, R = 3 mm and r = 1.5 mm

$v = 1000\left(3^2 - 1.5^2\right) = 6750$ mm/minute

125. a. Shift one unit left, reflect across the x-axis, and shift up 109,561 units.



b. $69{,}160 = 109{,}561 - (p+1)^2$

$40{,}401 = (p+1)^2$

$201 = p + 1 \Rightarrow p = 200¢ = \$2.00$

c. $0 = 109{,}561 - (p+1)^2$

$109{,}561 = (p+1)^2$

$331 = p + 1 \Rightarrow p = 330¢ = \$3.30$

126. Write $R(p)$ in the form $-3(p-h)^2 + k$:

$R(p) = -3p^2 + 600p = -3(p^2 - 200p)$

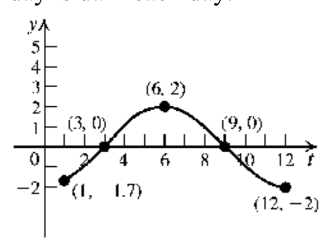
Complete the square

$= -3(p^2 - 200p + 10{,}000) + 30{,}000$

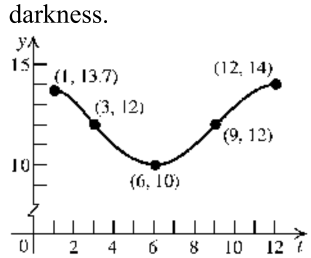
$= -3(p - 100)^2 + 30{,}000$

To graph this, shift $R(p)$ 100 units to the right, stretch by a factor of 3, reflect about the x-axis, and shift by 30,000 units up.

127. The first coordinate gives the month; the second coordinate gives the hours of daylight. From March to September, there is daylight more than half of the day each day. From September to March, more than half of the day is dark each day.



128. The graph shows the number of hours of darkness.

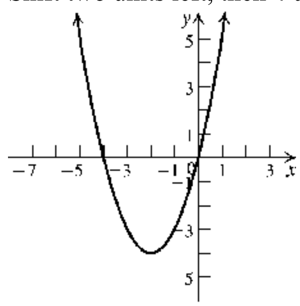


2.7 Beyond the Basics

129. The graph is shifted one unit right, then reflected about the x-axis, and finally reflected about the y-axis. The equation is $g(x) = -\sqrt{1-x}$.

130. The graph is shifted two units right and then reflected about the y-axis. The equation is $g(x) = f(-2-x)$.

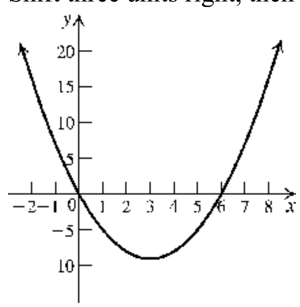
131. Shift two units left, then 4 units down.



132. $f(x) = x^2 - 6x = \left(x^2 - 6x + 9\right) - 9$

$= (x-3)^2 - 9$

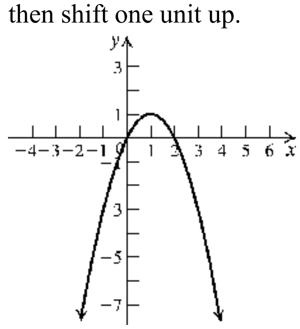
Shift three units right, then 9 units down.



133. $f(x) = -x^2 + 2x = -\left(x^2 - 2x + 1\right) + 1$

$= -(x-1)^2 + 1$

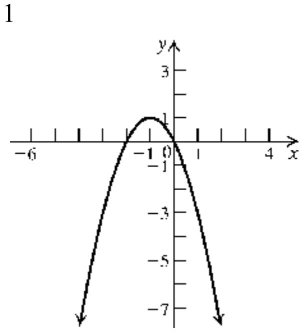
Shift one unit right, reflect about the x-axis, then shift one unit up.



134. $f(x) = -x^2 - 2x = -\left(x^2 + 2x + 1\right) + 1$

$= -(x+1)^2 + 1$

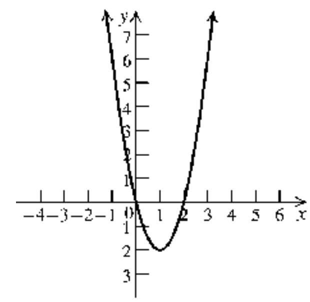
Shift one unit left, reflect about the x-axis, then shift one unit up.



135. $f(x) = 2x^2 - 4x = 2\left(x^2 - 2x + 1\right) - 2$

$= 2(x-1)^2 - 2$

Shift one unit right, stretch vertically by a factor of 2, then shift two units down.



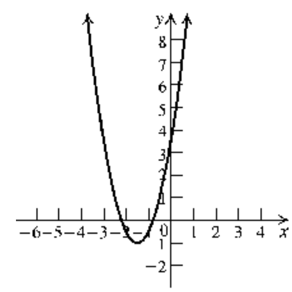
136. $f(x) = 2x^2 + 6x + 3.5$

$= 2\left(x^2 + 3x\right) + 3.5$

$= 2\left(x^2 + 3x + 2.25\right) + 3.5 - 2(2.25)$

$= 2(x + 1.5)^2 - 1$

Shift 1.5 units left, stretch vertically by a factor of 2, then shift one unit down.

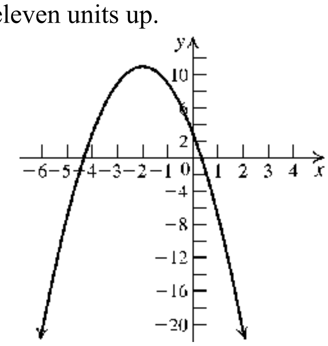


137. $f(x) = -2x^2 - 8x + 3 = -2\left(x^2 + 4x\right) + 3$

$= -2\left(x^2 + 4x + 4\right) + 3 + 2(4)$

$= -2(x+2)^2 + 11$

Shift two units left, stretch vertically by a factor of 2, reflect across the x-axis, then shift eleven units up.



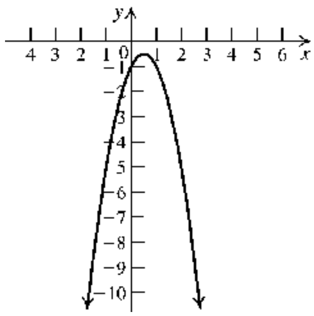
138. $f(x) = -2x^2 + 2x - 1 = -2\left(x^2 - x\right) - 1$

$= -2\left(x^2 - x + 0.25\right) - 1 + 2(0.25)$

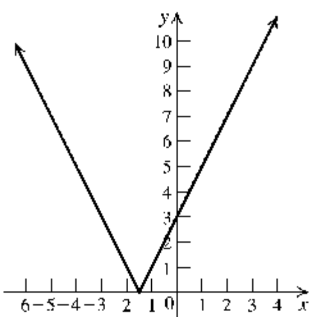
$= -2\left(x^2 - x + 0.25\right) - 0.5$

$= -2(x - 0.5)^2 - 0.5$

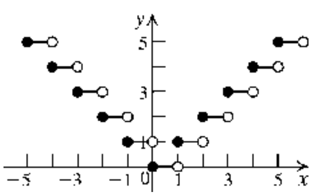
Shift 0.5 unit right, stretch vertically by a factor of 2, reflect across the *x*-axis, then shift 0.5 unit down.



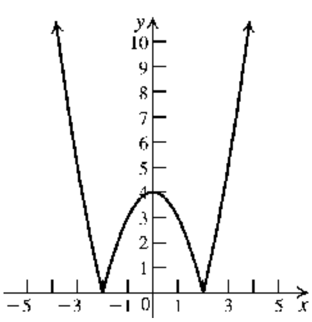
139.



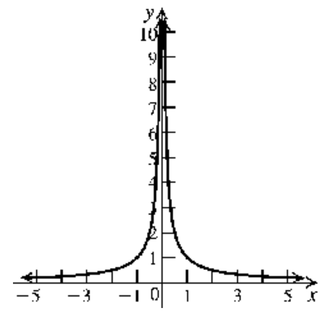
140.



141.



142.



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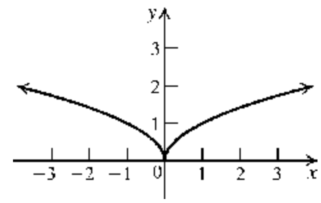
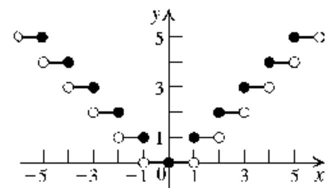


144.



2.7 Critical Thinking/Discussion/Writing

145. a. $y = f(x+2)$ is the graph of $y = f(x)$ shifted two units left. So the *x*-intercepts are –1 – 2 = –3, 0 – 2 = –2, and 2 – 2 = 0.

b. $y = f(x-2)$ is the graph of $y = f(x)$ shifted two units right. So the *x*-intercepts are –1 + 2 = 1, 0 + 2 = 2, and 2 + 2 = 4.

c. $y = -f(x)$ is the graph of $y = f(x)$ reflected across the *x*-axis. The *x*-intercepts are the same, –1, 0, 2.

d. $y = f(-x)$ is the graph of $y = f(x)$ reflected across the *y*-axis. The *x*-intercepts are the opposites, 1, 0, –2.

e. $y = f(2x)$ is the graph of $y = f(x)$ compressed horizontally by a factor of 1/2. The *x*-intercepts are $-\frac{1}{2}$, 0, 1.

f. $y = f\left(\frac{1}{2}x\right)$ is the graph of $y = f(x)$ stretched horizontally by a factor of 2. The *x*-intercepts are –2, 0, 4.

146. a. $y = f(x) + 2$ is the graph of $y = f(x)$ shifted two units up. The *y*-intercept is 2 + 2 = 4.

b. $y = f(x) - 2$ is the graph of $y = f(x)$ shifted two units down. The y-intercept is $2 - 2 = 0$.

c. $y = -f(x)$ is the graph of $y = f(x)$ reflected across the x-axis. The y-intercept is the opposite, –2.

d. $y = f(-x)$ is the graph of $y = f(x)$ reflected across the y-axis. The y-intercept is the same, 2.

e. $y = 2f(x)$ is the graph of $y = f(x)$ stretched vertically by a factor of 2. The y-intercept is 4.

f. $y = \frac{1}{2}f(x)$ is the graph of $y = f(x)$ compressed horizontally by a factor of 1/2. The y-intercept is 1.

147. a. $y = f(x+2)$ is the graph of $y = f(x)$ shifted two units left. The domain is $[-1-2,\ 3-2] = [-3,\ 1]$. The range is the same, $[-2,\ 1]$.

b. $y = f(x) - 2$ is the graph of $y = f(x)$ shifted two units down. The domain is the same, $[-1,\ 3]$. The range is $[-2-2,\ 1-2] = [-4,\ -1]$.

c. $y = -f(x)$ is the graph of $y = f(x)$ reflected across the x-axis. The domain is the same, $[-1,\ 3]$. The range is the opposite, $[-1,\ 2]$.

d. $y = f(-x)$ is the graph of $y = f(x)$ reflected across the y-axis. The domain is the opposite, $[-3,\ 1]$. The range is the same, $[-2,\ 1]$.

e. $y = 2f(x)$ is the graph of $y = f(x)$ stretched vertically by a factor of 2. The domain is the same, $[-1,\ 3]$. The range is $[2(-2),\ 2(1)] = [-4,\ 2]$.

f. $y = \frac{1}{2}f(x)$ is the graph of $y = f(x)$ compressed horizontally by a factor of 1/2. The domain is the same, $[-1,\ 3]$. The range is $\left[\frac{1}{2}(-2),\ \frac{1}{2}(1)\right] = \left[-1,\ \frac{1}{2}\right]$.

148. a. $y = f(x+2)$ is the graph of $y = f(x)$ shifted two units left. So the relative maximum is at $x = 1 - 2 = -1$, and the relative minimum is at $x = 2 - 2 = 0$.

b. $y = f(x) - 2$ is the graph of $y = f(x)$ shifted two units down. The locations of the relative maximum and minimum do not change. Relative maximum at $x = 1$, relative minimum at $x = 2$.

c. $y = -f(x)$ is the graph of $y = f(x)$ reflected across the x-axis. The relative maximum and relative minimum switch. The relative maximum occurs at $x = 2$, and the relative minimum occurs at $x = 1$.

d. $y = f(-x)$ is the graph of $y = f(x)$ reflected across the y-axis. The relative maximum and relative minimum occur at their opposites. The relative maximum occurs at $x = -1$, and the relative minimum occurs at $x = -2$.

e. $y = 2f(x)$ is the graph of $y = f(x)$ stretched vertically by a factor of 2. The locations of the relative maximum and minimum do not change. Relative maximum at $x = 1$, relative minimum at $x = 2$.

f. $y = \frac{1}{2}f(x)$ is the graph of $y = f(x)$ compressed horizontally by a factor of 1/2. The locations of the relative maximum and minimum do not change. Relative maximum at $x = 1$, relative minimum at $x = 2$.

2.7 Getting Ready for the Next Section

149. $(5x^2 + 5x + 7) + (x^2 + 9x - 4) = 6x^2 + 14x + 3$

150. $(x^2 + 2x) + (6x^3 - 2x + 5) = 6x^3 + x^2 + 5$

151. $(5x^2 + 6x - 2) - (3x^2 - 9x + 1) = 2x^2 + 15x - 3$

152. $(x^3 + 2) - (2x^3 + 5x - 3) = -x^3 - 5x + 5$

153. $(x-2)(x^2 + 2x + 4)$
$= x^3 + 2x^2 + 4x - 2x^2 - 4x - 8$
$= x^3 - 8$

154. $\left(x^2+x+1\right)\left(x^2-x+1\right)$
$= x^4 - x^3 + x^2 + x^3 - x^2 + x + x^2 - x + 1$
$= x^4 + x^2 + 1$

155. $f(x) = \dfrac{2x-3}{x^2-5x+6}$
The function is not defined when the denominator is zero.
$x^2 - 5x + 6 = 0 \Rightarrow (x-2)(x-3) = 0 \Rightarrow x = 2, 3$
The domain is $(-\infty, 2)\cup(2, 3)\cup(3, \infty)$.

156. $f(x) = \dfrac{x-2}{x^2-4}$
The function is not defined when the denominator is zero.
$x^2 - 4 = 0 \Rightarrow (x+2)(x-2) = 0 \Rightarrow x = -2, 2$
The domain is $(-\infty, -2)\cup(-2, 2)\cup(2, \infty)$.

157. $f(x) = \sqrt{2x-3}$
The function is defined only if $2x - 3 \geq 0$.
$2x - 3 \geq 0 \Rightarrow 2x \geq 3 \Rightarrow x \geq \dfrac{3}{2}$
The domain is $\left[\dfrac{3}{2}, \infty\right)$.

158. $f(x) = \dfrac{1}{\sqrt{5-2x}}$
The function is defined only if $5 - 2x > 0$.
$5 - 2x > 0 \Rightarrow -2x > -5 \Rightarrow x < \dfrac{5}{2}$
The domain is $\left(-\infty, \dfrac{5}{2}\right)$.

159. $\dfrac{x-1}{x-10} < 0$
First, solve $x - 1 = 0 \Rightarrow x = 1$ and $x - 10 = 0 \Rightarrow x = 10$.
So the intervals are $(-\infty, 1), (1, 10),$ and $(10, \infty)$.

Interval	Test point	Value of $\frac{x-1}{x-10}$	Result
$(-\infty, 1)$	0	$\frac{1}{10}$	+
$(1, 10)$	5	$-\frac{4}{5}$	–
$(10, \infty)$	15	$\frac{14}{5}$	+

Note that the fraction is undefined if $x = 10$.
The solution set is $(1, 10)$.

160. $\dfrac{-3}{(1-x)^2} > 0$
Set the denominator equal to zero and solve for x.
$(1-x)^2 = 0 \Rightarrow x = 1$
The intervals are $(-\infty, 1)$ and $(1, \infty)$.

Interval	Test point	Value of $\frac{-3}{(1-x)^2}$	Result
$(-\infty, 1)$	0	-3	–
$(1, \infty)$	2	-3	–

There is no solution. The solution set is $\varnothing$.

161. $\dfrac{-2x+8}{x^2+1} \leq 0$
Set the numerator and denominator equal to zero and solve for x.
$-2x + 8 = 0 \Rightarrow -2x = -8 \Rightarrow x = 4$
$x^2 + 1 = 0$ has no real solution.
The intervals are $(-\infty, 4]$ and $[4, \infty)$.

Interval	Test point	Value of $\frac{-2x+8}{x^2+1}$	Result
$(-\infty, 4]$	0	8	+
$[4, \infty)$	5	$-\frac{1}{13}$	–

The solution set is $[4, \infty)$.

162. $\dfrac{(x-3)(x-1)}{(x-5)(x+1)} \geq 0$
Set the numerator and denominator equal to zero and solve for x.
$(x-3)(x-1) = 0 \Rightarrow x = 3, 1$
$(x-5)(x+1) = 0 \Rightarrow x = 5, -1$
The intervals are $(-\infty, -1), (-1, 1]$, $[1, 3], [3, 5)$, and $(5, \infty)$.

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Interval	Test point	Value of $\frac{(x-3)(x-1)}{(x-5)(x+1)}$	Result
$(-\infty, -1)$	-2	$\frac{15}{7}$	+
$(-1, 1]$	0	$-\frac{3}{5}$	–
$[1, 3]$	2	$\frac{1}{9}$	+
$[3, 5)$	4	$-\frac{3}{5}$	–
$(5, \infty)$	6	$\frac{15}{7}$	+

The solution set is $(-\infty, -1) \cup [1, 3] \cup (5, \infty)$.

2.8 Combining Functions; Composite Functions

2.8 Practice Problems

1. $f(x) = 3x - 1,\ g(x) = x^2 + 2$

$$(f+g)(x) = f(x) + g(x) = 3x - 1 + x^2 + 2 = x^2 + 3x + 1$$
$$(f-g)(x) = f(x) - g(x) = (3x-1) - (x^2+2) = -x^2 + 3x - 3$$
$$(fg)(x) = f(x) \cdot g(x) = (3x-1)(x^2+2) = 3x^3 - x^2 + 6x - 2$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x-1}{x^2+2}$$

2. $f(x) = \sqrt{x-1},\ g(x) = \sqrt{3-x}$

The domain of *f* is $[1, \infty)$ and the domain of *g* is $(-\infty, 3]$. The intersection of D_f and D_g, $D_f \cap D_g = [1, 3]$.

The domain of *fg* is [1, 3].

The domain of $\frac{f}{g}$ is [1, 3).

The domain of $\frac{g}{f}$ is (1, 3].

3. $f(x) = -5x,\ g(x) = x^2 + 1$

a. $(f \circ g)(0) = f(g(0)) = f(0^2 + 1) = f(1) = -5$

b. $(g \circ f)(0) = g(f(0)) = g(-5 \cdot 0) = g(0) = 1$

4. $f(x) = 2 - x,\ g(x) = 2x^2 + 1$

a.
$$(g \circ f)(x) = g(f(x)) = g(2-x) = 2(2-x)^2 + 1 = 2(4 - 4x + x^2) + 1 = 8 - 8x + 2x^2 + 1 = 2x^2 - 8x + 9$$

b.
$$(f \circ g)(x) = f(g(x)) = f(2x^2+1) = 2 - (2x^2 + 1) = 1 - 2x^2$$

c.
$$(g \circ g)(x) = g(g(x)) = g(2x^2+1) = 2(2x^2+1)^2 + 1 = 2(4x^4 + 4x^2 + 1) + 1 = 8x^4 + 8x^2 + 3$$

5. $f(x) = \sqrt{x+1},\ g(x) = \frac{2}{x-3}$

Let $A = \{x \mid g(x) \text{ is defined}\}$.

$g(x)$ is not defined if $x = 3$, so $A = (-\infty, 3) \cup (3, \infty)$.

Let $B = \{x \mid f(g(x)) \text{ is defined}\}$.

$$f(g(x)) = \sqrt{\frac{2}{x-3} + 1} = \sqrt{\frac{2 + x - 3}{x-3}} = \sqrt{\frac{x-1}{x-3}}$$

$f(g(x))$ is not defined if $x = 3$ or if $\frac{x-1}{x-3} < 0$.

$x - 1 = 0 \Rightarrow x = 1$

Interval	Test point	Value of $\frac{x-1}{x-3}$	Result
$(-\infty, 1]$	0	$\frac{1}{3}$	+
$[1, 3)$	2	-1	–
$(3, \infty)$	4	3	+

$f(g(x))$ is not defined for $[1, 3)$, so $B = (-\infty, 1] \cup (3, \infty)$.

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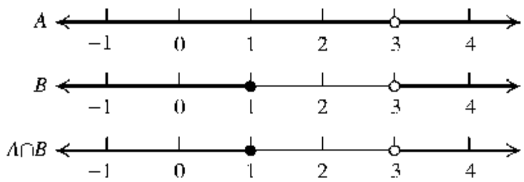


The domain of $f \circ g$ is $A \cap B = (-\infty, 1] \cup (3, \infty)$.

6. $f(x) = \sqrt{x};\quad g(x) = 3 - x$

$f \circ g = f(3 - x) = \sqrt{3 - x}$

$f \circ g$ is not defined if $3 - x < 0 \Rightarrow 3 < x$ or $x > 3$. $f \circ g$ is defined for $x < 3$. Thus, the domain of $f \circ g$ is $(-\infty, 3]$.

7. $f(x) = \sqrt{x - 1},\quad g(x) = \sqrt{4 - x^2}$

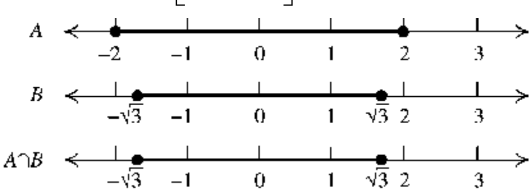
a. $(f \circ g)(x) = f(g(x)) = f\left(\sqrt{4 - x^2}\right)$
$= \sqrt{\sqrt{4 - x^2} - 1}$

The function $g(x) = \sqrt{4 - x^2}$ is defined for $4 - x^2 \ge 0 \Rightarrow x^2 \le 4 \Rightarrow -2 \le x \le 2$. So, $A = [-2, 2]$.

The function $f(g(x))$ is defined for

$\sqrt{4 - x^2} - 1 \ge 0 \Rightarrow \sqrt{4 - x^2} \ge 1 \Rightarrow$
$4 - x^2 \ge 1 \Rightarrow -x^2 \ge -3 \Rightarrow x^2 \le 3 \Rightarrow$
$-\sqrt{3} \le x \le \sqrt{3}$

So, $B = \left[-\sqrt{3}, \sqrt{3}\right]$.

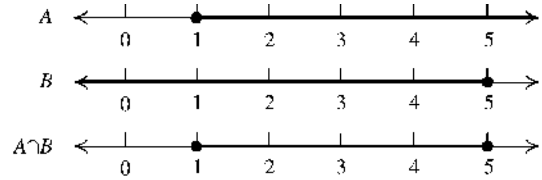


The domain of $f \circ g$ is $A \cap B = \left[-\sqrt{3}, \sqrt{3}\right]$.

b. $(g \circ f)(x) = g(f(x)) = \sqrt{4 - \left(\sqrt{x - 1}\right)^2}$
$= \sqrt{4 - (x - 1)} = \sqrt{5 - x}$

The function $f(x) = \sqrt{x - 1}$ is defined for $x - 1 \ge 0 \Rightarrow x \ge 1$. So, $A = [1, \infty)$.

The function $g(f(x))$ is defined for $5 - x \ge 0 \Rightarrow 5 \ge x$, or $x \le 5$. So, $B = (-\infty, 5]$.



The domain of $g \circ f$ is $A \cap B = [1, 5]$.

8. $H(x) = \dfrac{1}{\sqrt{2x^2 + 1}},\quad g(x) = \sqrt{2x^2 + 1}$

If $f(x) = \dfrac{1}{x}$, then

$H(x) = (f \circ g)(x) = f\left(\sqrt{2x^2 + 1}\right) = \dfrac{1}{\sqrt{2x^2 + 1}}$.

9. a. $A = f(g(t)) = f(g(3)) = f(3t)$
$= \pi(3t)^2 = 9\pi t^2$

b. $A = 9\pi t^2 = 9\pi(6)^2 = 324\pi$

The area covered by the oil slick is $324\pi \approx 1018$ square miles.

10. a. $r(x) = x - 4500$

b. $d(x) = x - 0.06x = 0.94x$

c. i. $(r \circ d)(x) = r(0.94x) = 0.94x - 4500$

ii. $(d \circ r)(x) = d(x - 4500)$
$= 0.94(x - 4500)$
$= 0.94x - 4230$

d. $(d \circ r)(x) - (r \circ d)(x)$
$= (0.94x - 4230) - (0.94x - 4500)$
$= 270$

2.8 Concepts and Vocabulary

1. $(f \cdot g)(x) = \underline{f(x) \cdot g(x)}$.

2. The domain of the function $f + g$ consists of those values of x that are <u>common</u> to the domains of f and g.

3. The composition of the function f with the function g is written as $f \circ g$ and is defined by $f \circ g(x) = \underline{f(g(x))}$.

4. The domain of the composite function $f \circ g$ consists of those values of x in the domain of g for which $g(x)$ is in the domain of f.

5. False. For example, if $f(x) = 2x$ and $g(x) = x^2$, then $(f \circ g)(x) = f(g(x)) = f(x^2) = 2x^2$, while $(g \circ f)(x) = g(f(x)) = g(2x) = 4x^2$.

6. True

7. False. The domain of $f \cdot g$ may include $g(x) = 0$, but the domain of $\frac{f}{g}$ cannot include $g(x) = 0$.

8. True

2.8 Building Skills

9. $(f+g)(-2) = f(-2) + g(-2) = 1 + 2 = 3$

10. $(f+g)(2) = f(2) + g(2) = -2 + (-1) = -3$

11. $(f-g)(4) = f(4) - g(4) = -2 - 1 = -3$

12. $(f-g)(-1) = f(-1) - g(-1) = 1 - (-4) = 5$

13. $(f \cdot g)(-1) = f(-1) \cdot g(-1) = 1 \cdot (-4) = -4$

14. $(f \cdot g)(2) = f(2) \cdot g(2) = -2 \cdot (-1) = 2$

15. $\left(\frac{f}{g}\right)(-2) = \frac{f(-2)}{g(-2)} = \frac{1}{2}$

16. $\left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)} = \frac{-2}{-1} = 2$

17. $(f \circ g)(1) = f(g(1)) = f(-2) = 1$

18. $(g \circ f)(1) = g(f(1)) = g(-2) = 2$

19. $(f \circ g)(-3) = f(g(-3)) = f(0) = 0$

20. $(g \circ f)(-3) = g(f(-3)) = g(1) = -2$

21. **a.** $(f+g)(-1) = f(-1) + g(-1)$
$= 2(-1) + -(-1) = -2 + 1 = -1$

b. $(f-g)(0) = f(0) - g(0) = 2(0) - (-0) = 0$

c. $(f \cdot g)(2) = f(2) \cdot g(2) = 2(2) \cdot (-2) = -8$

d. $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{2(1)}{-1} = -2$

22. **a.** $(f+g)(-1) = f(-1) + g(-1)$
$= \left(1 - (-1)^2\right) + (-1 + 1) = 0$

b. $(f-g)(0) = f(0) - g(0)$
$= (1 - 0^2) - (0 + 1) = 0$

c. $(f \cdot g)(2) = f(2) \cdot g(2)$
$= (1 - 2^2) \cdot (2 + 1) = -9$

d. $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{1 - 1^2}{1 + 1} = 0$

23. **a.** $(f+g)(-1) = f(-1) + g(-1)$
$= \frac{1}{\sqrt{-1+2}} + (2(-1) + 1) = 0$

b. $(f-g)(0) = f(0) - g(0)$
$= \frac{1}{\sqrt{0+2}} - (2(0) + 1) = \frac{\sqrt{2}}{2} - 1$

c. $(f \cdot g)(2) = f(2) \cdot g(2)$
$= \frac{1}{\sqrt{2+2}} \cdot (2(2) + 1) = \frac{5}{2}$

d. $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{\frac{1}{\sqrt{1+2}}}{2(1) + 1} = \frac{1}{3\sqrt{3}} = \frac{\sqrt{3}}{9}$

24. **a.** $(f+g)(-1) = f(-1) + g(-1)$
$= \frac{-1}{(-1)^2 - 6(-1) + 8} + (3 - (-1))$
$= -\frac{1}{15} + 4 = \frac{59}{15}$

b. $(f-g)(0) = f(0) - g(0)$
$= \frac{0}{0^2 - 6(0) + 8} - (3 - 0) = -3$

c. $(f \cdot g)(2) = f(2) \cdot g(2)$
$= \frac{2}{2^2 - 6(2) + 8} \cdot (3 - 2) = \frac{2}{0} \cdot 1 \Rightarrow$
the product does not exist.

d. $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{\frac{1}{1^2 - 6(1) + 8}}{3 - 1} = \frac{\frac{1}{3}}{2} = \frac{1}{6}$

25. a. $f+g=x^2+x-3$; domain: $(-\infty,\infty)$

b. $f-g=x-3-x^2=-x^2+x-3$;
domain: $(-\infty,\infty)$

c. $f\cdot g=(x-3)x^2=x^3-3x^2$;
domain: $(-\infty,\infty)$

d. $\frac{f}{g}=\frac{x-3}{x^2}$; domain: $(-\infty,0)\cup(0,\infty)$

e. $\frac{g}{f}=\frac{x^2}{x-3}$; domain: $(-\infty,3)\cup(3,\infty)$

26. a. $f+g=x^2+2x-1$; domain: $(-\infty,\infty)$

b. $f-g=2x-1-x^2=-x^2+2x-1$;
domain: $(-\infty,\infty)$

c. $f\cdot g=(2x-1)x^2=2x^3-x^2$;
domain: $(-\infty,\infty)$

d. $\frac{f}{g}=\frac{2x-1}{x^2}$; domain: $(-\infty,0)\cup(0,\infty)$

e. $\frac{g}{f}=\frac{x^2}{2x-1}$; domain: $\left(-\infty,\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$

27. a. $f+g=\left(x^3-1\right)+\left(2x^2+5\right)=x^3+2x^2+4$;
domain: $(-\infty,\infty)$

b. $f-g=\left(x^3-1\right)-\left(2x^2+5\right)=x^3-2x^2-6$;
domain: $(-\infty,\infty)$

c. $f\cdot g=\left(x^3-1\right)\left(2x^2+5\right)$
$=2x^5+5x^3-2x^2-5$;
domain: $(-\infty,\infty)$

d. $\frac{f}{g}=\frac{x^3-1}{2x^2+5}$; domain: $(-\infty,\infty)$

e. $\frac{g}{f}=\frac{2x^2+5}{x^3-1}$; domain: $(-\infty,1)\cup(1,\infty)$

28. a. $f+g=\left(x^2-4\right)+\left(x^2-6x+8\right)$
$=2x^2-6x+4$; domain: $(-\infty,\infty)$

b. $f-g=\left(x^2-4\right)-\left(x^2-6x+8\right)=6x-12$;
domain: $(-\infty,\infty)$

c. $f\cdot g=\left(x^2-4\right)\left(x^2-6x+8\right)$
$=x^4-6x^3+4x^2+24x-32$;
domain: $(-\infty,\infty)$

d. $\frac{f}{g}=\frac{x^2-4}{x^2-6x+8}=\frac{(x+2)(x-2)}{(x-2)(x-4)}=\frac{x+2}{x-4}$
The denominator of the original fraction = 0 if $x=2$ or $x=4$, so the domain is $(-\infty,2)\cup(2,4)\cup(4,\infty)$.

e. $\frac{g}{f}=\frac{x^2-6x+8}{x^2-4}=\frac{(x-2)(x-4)}{(x-2)(x+2)}=\frac{x-4}{x+2}$
The denominator of the original fraction = 0 if $x=2$ or $x=4$, so the domain is $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$.

29. a. $f+g=2x+\sqrt{x}-1$; domain: $[0,\infty)$

b. $f-g=2x-\sqrt{x}-1$; domain: $[0,\infty)$

c. $f\cdot g=(2x-1)\sqrt{x}=2x\sqrt{x}-\sqrt{x}$;
domain: $[0,\infty)$

d. $\frac{f}{g}=\frac{2x-1}{\sqrt{x}}$; domain: $(0,\infty)$

e. $\frac{g}{f}=\frac{\sqrt{x}}{2x-1}$; the numerator is defined only for $x\ge 0$, while the denominator = 0 when $x=\frac{1}{2}$, so the domain is $\left[0,\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$.

30. a. $f+g=x-1+\sqrt{x}$; domain: $[0,\infty)$

b. $f-g=x-1-\sqrt{x}$; domain: $[0,\infty)$

c. $f\cdot g=(x-1)\sqrt{x}$; domain: $[0,\infty)$

d. $\frac{f}{g}=\frac{x-1}{\sqrt{x}}$; domain: $(0,\infty)$

e. $\frac{g}{f}=\frac{\sqrt{x}}{x-1}$; the numerator is defined only for $x\ge 0$, while the denominator = 0 when $x=1$, so the domain is $[0,1)\cup(1,\infty)$.

31. a. $f+g=x-6+\sqrt{x-3}$; domain: $[3,\infty)$

b. $f-g=x-6-\sqrt{x-3}$; domain: $[3,\infty)$

c. $f\cdot g=(x-6)\sqrt{x-3}$; domain: $[3,\infty)$

d. $\dfrac{f}{g}=\dfrac{x-6}{\sqrt{x-3}}$; domain: $(3,\infty)$

e. $\dfrac{g}{f}=\dfrac{\sqrt{x-3}}{x-6}$; the numerator is defined only for $x \ge 3$, while the denominator = 0 when $x = 6$, so the domain is $[3, 6)\cup(6, \infty)$.

32. a. $f+g=x+2+\sqrt{1-x}$; domain: $(-\infty, 1]$

b. $f-g=x+2-\sqrt{1-x}$; domain: $(-\infty, 1]$

c. $f\cdot g=(x+2)\sqrt{1-x}$; domain: $(-\infty, 1]$

d. $\dfrac{f}{g}=\dfrac{x+2}{\sqrt{1-x}}$; domain: $(-\infty, 1)$

e. $\dfrac{g}{f}=\dfrac{\sqrt{1-x}}{x+2}$; the numerator is defined only for $x \le 1$, while the denominator = 0 when $x = -2$, so the domain is $(-\infty. -2)\cup(-2, 1]$.

33. a.
$$f+g=1-\frac{2}{x+1}+\frac{1}{x}=\frac{x(x+1)-2x+(x+1)}{x(x+1)}=\frac{x^2+x-2x+x+1}{x(x+1)}=\frac{x^2+1}{x(x+1)}$$
domain: $(-\infty, -1)\cup(-1, 0)\cup(0, \infty)$

b.
$$f-g=1-\frac{2}{x+1}-\frac{1}{x}=\frac{x(x+1)-2x-(x+1)}{x(x+1)}=\frac{x^2+x-2x-x-1}{x(x+1)}=\frac{x^2-2x-1}{x(x+1)}$$
domain: $(-\infty, -1)\cup(-1, 0)\cup(0, \infty)$

c.
$$f\cdot g=\left(1-\frac{2}{x+1}\right)\frac{1}{x}=\left(\frac{x+1-2}{x+1}\right)\frac{1}{x}=\left(\frac{x-1}{x+1}\right)\frac{1}{x}=\frac{x-1}{x(x+1)}$$
domain: $(-\infty, -1)\cup(-1, 0)\cup(0, \infty)$

d.
$$\frac{f}{g}=\frac{1-\frac{2}{x+1}}{\frac{1}{x}}=\left(1-\frac{2}{x+1}\right)(x)=\left(\frac{x+1-2}{x+1}\right)x=\frac{x(x-1)}{x+1}$$
domain: $(-\infty, -1)\cup(-1, 0)\cup(0, \infty)$

e.
$$\frac{g}{f}=\frac{\frac{1}{x}}{1-\frac{2}{x+1}}=\frac{\frac{1}{x}(x+1)}{\left(1-\frac{2}{x+1}\right)(x+1)}=\frac{\frac{x+1}{x}}{x+1-2}=\frac{\frac{x+1}{x}}{x-1}=\frac{x+1}{x(x-1)}$$
The denominator equals zero when $x = 0$ or $x = 1$, so the domain is $(-\infty, -1)\cup(-1, 0)\cup(0, 1)\cup(1, \infty)$.

34. a. $f+g=\left(1-\dfrac{1}{x}\right)+\dfrac{1}{x}=1$

Neither f nor g is defined for $x = 0$, so the domain is $(-\infty,0)\cup(0,\infty)$.

b. $f-g=\left(1-\dfrac{1}{x}\right)-\dfrac{1}{x}=1-\dfrac{2}{x}$; domain: $(-\infty,0)\cup(0,\infty)$.

c. $f\cdot g=\left(1-\dfrac{1}{x}\right)\left(\dfrac{1}{x}\right)=\dfrac{1}{x}-\dfrac{1}{x^2}=\dfrac{x-1}{x^2}$; domain: $(-\infty,0)\cup(0,\infty)$.

d.
$$\frac{f}{g}=\frac{1-\frac{1}{x}}{\frac{1}{x}}=\frac{\frac{x-1}{x}}{\frac{1}{x}}=x-1$$
Neither f nor g is defined for $x = 0$, so the domain is $(-\infty,0)\cup(0,\infty)$.

e.
$$\frac{g}{f}=\frac{\frac{1}{x}}{1-\frac{1}{x}}=\frac{\frac{1}{x}}{\frac{x-1}{x}}=\frac{1}{x-1}$$
Neither f nor g is defined for $x = 0$, and g/f is not defined for $x = 1$, so the domain is $(-\infty,0)\cup(0,1)\cup(1,\infty)$.

35. a. $f+g=\dfrac{2}{x+1}+\dfrac{x}{x+1}=\dfrac{2+x}{x+1}$

Neither f nor g is defined for $x = -1$, so the domain is $(-\infty,-1)\cup(-1,\infty)$.

b. $f-g=\dfrac{2}{x+1}-\dfrac{x}{x+1}=\dfrac{2-x}{x+1}$. domain: $(-\infty,-1)\cup(-1,\infty)$.

c. $f\cdot g=\left(\dfrac{2}{x+1}\right)\left(\dfrac{x}{x+1}\right)=\dfrac{2x}{(x+1)^2}$. domain: $(-\infty,-1)\cup(-1,\infty)$.

d. $\dfrac{f}{g}=\dfrac{\frac{2}{x+1}}{\frac{x}{x+1}}=\dfrac{2}{x}$. Neither f nor g is defined for $x=-1$, and f/g is not defined for $x=0$, so the domain is $(-\infty,-1)\cup(-1,\ 0)\cup(0,\infty)$.

e. $\dfrac{f}{g}=\dfrac{\frac{x}{x+1}}{\frac{2}{x+1}}=\dfrac{x}{2}$. Neither f nor g is defined for $x=-1$, so the domain is $(-\infty,-1)\cup(-1,\ \infty)$.

36. $f(x)=\dfrac{5x-1}{x+1}$; $g(x)=\dfrac{4x+10}{x+1}$

a. $f+g=\dfrac{5x-1}{x+1}+\dfrac{4x+10}{x+1}=\dfrac{9x+9}{x+1}$
$=\dfrac{9(x+1)}{x+1}=9$
Neither f nor g is defined for $x=-1$, so the domain is $(-\infty,-1)\cup(-1,\ \infty)$.

b. $f-g=\dfrac{5x-1}{x+1}-\dfrac{4x+10}{x+1}=\dfrac{x-11}{x+1}$
Neither f nor g is defined for $x=-1$, so the domain is $(-\infty,-1)\cup(-1,\ \infty)$.

c. $f\cdot g=\dfrac{5x-1}{x+1}\cdot\dfrac{4x+10}{x+1}=\dfrac{20x^2+46x-10}{x^2+2x+1}$
Neither f nor g is defined for $x=-1$, so the domain is $(-\infty,-1)\cup(-1,\ \infty)$.

d. $\dfrac{f}{g}=\dfrac{\frac{5x-1}{x+1}}{\frac{4x+10}{x+1}}=\dfrac{5x-1}{4x+10}$
Neither f nor g is defined for $x=-1$ and f/g is not defined for $x=-5/2$, so the domain is
$\left(-\infty,-\dfrac{5}{2}\right)\cup\left(-\dfrac{5}{2},\ -1\right)\cup(-1,\ \infty)$.

e. $\dfrac{g}{f}=\dfrac{\frac{4x+10}{x+1}}{\frac{5x-1}{x+1}}=\dfrac{4x+10}{5x-1}$
Neither f nor g is defined for $x=-1$ and g/f is not defined for $x=1/5$, so the domain is $(-\infty,-1)\cup\left(-1,\ \dfrac{1}{5}\right)\cup\left(\dfrac{1}{5},\ \infty\right)$.

37. $f(x)=\dfrac{x^2}{x+1}$; $g(x)=\dfrac{2x}{x^2-1}$

a. $f+g=\dfrac{x^2}{x+1}+\dfrac{2x}{x^2-1}=\dfrac{x^2(x-1)}{x^2-1}+\dfrac{2x}{x^2-1}$
$=\dfrac{x^3-x^2+2x}{x^2-1}$
f is not defined for $x=-1$, g is not defined for $x=\pm1$, and $f+g$ is not defined for either -1 or 1, so the domain is $(-\infty,-1)\cup(-1,\ 1)\cup(1,\ \infty)$.

b. $f-g=\dfrac{x^2}{x+1}-\dfrac{2x}{x^2-1}=\dfrac{x^2(x-1)}{x^2-1}-\dfrac{2x}{x^2-1}$
$=\dfrac{x^3-x^2-2x}{x^2-1}=\dfrac{x\left(x^2-x-2\right)}{x^2-1}$
$=\dfrac{x(x-2)(x+1)}{(x-1)(x+1)}=\dfrac{x^2-2x}{x-1}$
f is not defined for $x=-1$, g is not defined for $x=\pm1$, and $f-g$ is not defined for 1, so the domain is $(-\infty,-1)\cup(-1,\ 1)\cup(1,\ \infty)$.

c. $f\cdot g=\dfrac{x^2}{x+1}\cdot\dfrac{2x}{x^2-1}=\dfrac{2x^3}{x^3+x^2-x-1}$
f is not defined for $x=-1$, g is not defined for $x=\pm1$, and fg is not defined for either -1 or 1, so the domain is $(-\infty,-1)\cup(-1,\ 1)\cup(1,\ \infty)$.

d. $\dfrac{f}{g}=\dfrac{\frac{x^2}{x+1}}{\frac{2x}{x^2-1}}=\dfrac{x^2}{x+1}\cdot\dfrac{x^2-1}{2x}=\dfrac{x(x-1)}{2}$
f is not defined for $x=-1$, g is not defined for $x=\pm1$, and f/g is not defined for either -1, 0, or 1, so the domain is $(-\infty,-1)\cup(-1,\ 0)\cup(0,\ 1)\cup(1,\ \infty)$.

e. $\dfrac{g}{f}=\dfrac{\frac{2x}{x^2-1}}{\frac{x^2}{x+1}}=\dfrac{2x}{x^2-1}\cdot\dfrac{x+1}{x^2}=\dfrac{2}{x(x-1)}$
Neither f nor g is defined for $x=-1$ and g/f is not defined for $x=0$ or $x=1$, so the domain is $(-\infty,-1)\cup(-1,0)\cup(0,\ 1)\cup(1,\ \infty)$.

38. $f(x)=\dfrac{x-3}{x^2-25};\ g(x)=\dfrac{x-3}{x^2+9x+20}$

a. $f+g=\dfrac{x-3}{x^2-25}+\dfrac{x-3}{x^2+9x+20}$

$=\dfrac{x-3}{(x-5)(x+5)}+\dfrac{x-3}{(x+4)(x+5)}$

$=\dfrac{(x-3)(x+4)+(x-3)(x-5)}{(x-5)(x+5)(x+4)}$

$=\dfrac{x^2+x-12+x^2-8x+15}{x^3+4x^2-25x-100}$

$=\dfrac{2x^2-7x+3}{x^3+4x^2-25x-100}$

f is not defined for $x = -5$ and $x = 5$, *g* is not defined for $x = -4$ and $x = -5$, and $f + g$ is not defined for −5, 5 or −4, so the domain is $(-\infty,-5)\cup(-5,-4)\cup(-4,5)\cup(5,\infty)$.

b. $f-g=\dfrac{x-3}{x^2-25}-\dfrac{x-3}{x^2+9x+20}$

$=\dfrac{x-3}{(x-5)(x+5)}-\dfrac{x-3}{(x+4)(x+5)}$

$=\dfrac{(x-3)(x+4)-(x-3)(x-5)}{(x-5)(x+5)(x+4)}$

$=\dfrac{x^2+x-12-\left(x^2-8x+15\right)}{x^3+4x^2-25x-100}$

$=\dfrac{9x-27}{x^3+4x^2-25x-100}$

f is not defined for $x = -5$ and $x = 5$, *g* is not defined for $x = -4$ and $x = -5$, and $f + g$ is not defined for −5, 5, or −4, so the domain is $(-\infty,-5)\cup(-5,-4)\cup(-4,5)\cup(5,\infty)$.

c. $f\cdot g=\dfrac{x-3}{x^2-25}\cdot\dfrac{x-3}{x^2+9x+20}$

$=\dfrac{(x-3)^2}{\left(x^2-25\right)\left(x^2+9x+20\right)}$

$=\dfrac{x^2-6x+9}{x^4+9x^3-5x^2-225x-500}$

f is not defined for $x = -5$ and $x = 5$, *g* is not defined for $x = -4$ and $x = -5$, and *fg* is not defined for −5, 5, or −4, so the domain is $(-\infty,-5)\cup(-5,-4)\cup(-4,5)\cup(5,\infty)$.

d. $\dfrac{f}{g}=\dfrac{\frac{x-3}{x^2-25}}{\frac{x-3}{x^2+9x+20}}$

$=\dfrac{x-3}{(x-5)(x+5)}\cdot\dfrac{(x+5)(x+4)}{x-3}=\dfrac{x+4}{x-5}$

f is not defined for $x = -5$ and $x = 5$, *g* is not defined for $x = -4$ and $x = -5$, and f/g is not defined for $x = 5$, so the domain is $(-\infty,-5)\cup(-5,-4)\cup(-4,3)\cup(3,5)\cup(5,\infty)$.

e. $\dfrac{g}{f}=\dfrac{\frac{x-3}{x^2+9x+20}}{\frac{x-3}{x^2-25}}$

$=\dfrac{x-3}{(x+5)(x+4)}\cdot\dfrac{(x-5)(x+5)}{x-3}=\dfrac{x-5}{x+4}$

f is not defined for $x = -5$ and $x = 5$, *g* is not defined for $x = -4$ and $x = -5$, and g/f is not defined for $x = -4$, so the domain is $(-\infty,-5)\cup(-5,-4)\cup(-4,3)$
$\cup(3,5)\cup(5,\infty)$.

39. $f(x)=\sqrt{x-1};\ g(x)=\sqrt{5-x}$

a. $f\cdot g=\sqrt{x-1}\cdot\sqrt{5-x}$

f is not defined for $x < 1$, *g* is not defined for $x > 5$. The domain is [1, 5].

b. $\dfrac{f}{g}=\dfrac{\sqrt{x-1}}{\sqrt{5-x}}$

f is not defined for $x < 1$, *g* is not defined for $x > 5$. The denominator is zero when $x = 5$. The domain is [1, 5).

40. $f(x)=\sqrt{x-2};\ g(x)=\sqrt{x+2}$

a. $f\cdot g=\sqrt{x-2}\cdot\sqrt{x+2}=\sqrt{x^2-4}$

f is not defined for $x < 2$, *g* is not defined for $x < -2$. The domain is $[2,\infty)$.

b. $\dfrac{f}{g}=\dfrac{\sqrt{x-2}}{\sqrt{x+2}}$

f is not defined for $x < 2$, *g* is not defined for $x < -2$. The denominator is zero when $x = -2$. The domain is $[2,\infty)$.

41. $f(x)=\sqrt{x+2};\ g(x)=\sqrt{9-x^2}$

a. $f\cdot g=\sqrt{x+2}\cdot\sqrt{9-x^2}$
f is not defined for $x<-2$, *g* is defined for $[-3, 3]$. The domain is $[-2, 3]$.

b. $\dfrac{f}{g}=\dfrac{\sqrt{x+2}}{\sqrt{9-x^2}}$
f is not defined for $x<-2$, *g* is defined for $[-3, 3]$. The denominator is zero when $x=-3$ or $x=3$. The domain is $[-2, 3)$.

42. $f(x)=\sqrt{x^2-4};\ g(x)=\sqrt{25-x^2}$

a. $f\cdot g=\sqrt{x^2-4}\cdot\sqrt{25-x^2}$
f is defined for $x\le -2$ or $x\ge 2$, *g* is defined for $[-5, 5]$. The domain is $[-5,-2]\cup[2, 5]$.

b. $\dfrac{f}{g}=\dfrac{\sqrt{x^2-4}}{\sqrt{25-x^2}}$
f is defined for $x\le -2$ or $x\ge 2$, *g* is defined for $[-5, 5]$. The denominator is zero when $x=-5$ or $x=5$. The domain is $(-5,-2]\cup[2, 5)$.

43. $(g\circ f)(x)=2\left(x^2-1\right)+3=2x^2+1;$
$(g\circ f)(2)=2\left(2^2-1\right)+3=9;$
$(g\circ f)(-3)=2\left((-3)^2-1\right)+3=19$

44. $(g\circ f)(x)=3|x+1|^2-1=3\left|x^2+2x+1\right|-1;$
$(g\circ f)(2)=3|2+1|^2-1=26;$
$(g\circ f)(-3)=3|(-3)+1|^2-1=11$

45. $(f\circ g)(2)=2\left(2(2^2)-3\right)+1=11$

46. $(g\circ f)(2)=2(2(2)+1)^2-3=47$

47. $(f\circ g)(-3)=2\left(2(-3)^2-3\right)+1=31$

48. $(g\circ f)(-5)=2(2(-5)+1)^2-3=159$

49. $(f\circ g)(0)=2\left(2(0^2)-3\right)+1=-5$

50. $(g\circ f)\left(\dfrac{1}{2}\right)=2\left(2\left(\dfrac{1}{2}\right)+1\right)^2-3=5$

51. $(f\circ g)(-c)=2\left(2(-c)^2-3\right)+1=4c^2-5$

52. $(f\circ g)(c)=2\left(2c^2-3\right)+1=4c^2-5$

53. $(g\circ f)(a)=2(2a+1)^2-3$
$=2\left(4a^2+4a+1\right)-3$
$=8a^2+8a-1$

54. $(g\circ f)(-a)=2(2(-a)+1)^2-3$
$=2\left(4a^2-4a+1\right)-3$
$=8a^2-8a-1$

55. $(f\circ f)(1)=2(2(1)+1)+1=7$

56. $(g\circ g)(-1)=2\left(2(-1)^2-3\right)^2-3=-1$

57. $f(x)=\dfrac{1}{x};\ g(x)=10-5x$
$(f\circ g)(x)=\dfrac{1}{10-5x}$
The domain of $f\circ g$ is the set of all real numbers such that $10-5x\ne 0$, or $x\ne 2$. The domain of $f\circ g$ is $(-\infty, 2)\cup(2, \infty)$.

58. $f(x)=\dfrac{1}{x};\ g(x)=\sqrt{x}$
$(f\circ g)(x)=\dfrac{1}{\sqrt{x}}$
The domain of *f* is the set of all real numbers such that $x\ne 0$. The domain of $g(x)$ is $[0, \infty)$. Therefore, the domain of $f\circ g$ is $(0, \infty)$.

59. $f(x)=\sqrt{x};\ g(x)=2x-8$
$(f\circ g)(x)=\sqrt{2x-8}$
The domain of $f\circ g$ is the set of all real numbers such that $2x-8\ge 0$, or $x\ge 4$. The domain of $f\circ g$ is $[4, \infty)$.

60. $f(x)=\sqrt{x};\ g(x)=-x$
$(f\circ g)(x)=\sqrt{-x}$
The domain of $f\circ g$ is the set of all real numbers such that $-x\ge 0$, or $x\le 0$. The domain of $f\circ g$ is $(-\infty, 0]$.

61. $(f \circ g)(x) = \dfrac{2}{\frac{1}{x}+1} = \dfrac{2}{\frac{x+1}{x}} = \dfrac{2x}{x+1}$

The domain of *g* is $(-\infty,\ 0) \cup (0,\ \infty)$. Since –1 is not in the domain of *f*, we must exclude those values of *x* that make *g*(*x*) = –1.

$\dfrac{1}{x} = -1 \Rightarrow x = -1$

Thus, the domain of $f \circ g$ is $(-\infty,\ -1) \cup (-1,\ 0) \cup (0,\ \infty)$.

62. $(f \circ g)(x) = \dfrac{1}{\frac{2}{x+3} - 1} = \dfrac{1}{\frac{2-(x+3)}{x+3}}$
$= \dfrac{x+3}{-x-1} = -\dfrac{x+3}{x+1}.$

The domain of *g* is $(-\infty, -3) \cup (-3,\ \infty)$. Since 1 is not in the domain of *f*, we must exclude those values of *x* that make *g*(*x*) = 1.

$\dfrac{2}{x+3} = 1 \Rightarrow 2 = x+3 \Rightarrow x = -1$

Thus, the domain of $f \circ g$ is $(-\infty,\ -3) \cup (-3,\ -1) \cup (-1,\ \infty)$.

63. $(f \circ g)(x) = \sqrt{(2-3x)-3} = \sqrt{-1-3x}.$

The domain of *g* is $(-\infty, \infty)$. Since *f* is not defined for $(-\infty,\ 3)$, we must exclude those values of *x* that make *g*(*x*) < 3.

$2 - 3x < 3 \Rightarrow -3x < 1 \Rightarrow x > -\dfrac{1}{3}$

Thus, the domain of $f \circ g$ is $\left(-\infty, -\dfrac{1}{3}\right]$.

64. $(f \circ g)(x) = \dfrac{2+5x}{(2+5x)-1} = \dfrac{2+5x}{1+5x}$

The domain of *g* is $(-\infty, \infty)$. Since *f* is not defined for *x* = 1 we must exclude those values of *x* that make *g*(*x*) = 1.

$2 + 5x = 1 \Rightarrow 5x = -1 \Rightarrow x = -\dfrac{1}{5}$

Thus, the domain of $f \circ g$ is $\left(-\infty, -\dfrac{1}{5}\right) \cup \left(-\dfrac{1}{5}, \infty\right)$.

65. $(f \circ g)(x) = \left|x^2 - 1\right|$; domain: $(-\infty, \infty)$

66. $(f \circ g)(x) = 3|x-1| - 2$; domain: $(-\infty, \infty)$

67. a. $(f \circ g)(x) = 2(x+4) - 3 = 2x + 5$; domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = (2x-3) + 4 = 2x + 1$; domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = 2(2x-3) - 3 = 4x - 9$; domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = (x+4) + 4 = x + 8$; domain: $(-\infty, \infty)$

68. a. $(f \circ g)(x) = (3x-5) - 3 = 3x - 8$; domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 3(x-3) - 5 = 3x - 14$; domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = (x-3) - 3 = x - 6$; domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 3(3x-5) - 5 = 9x - 20$; domain: $(-\infty, \infty)$

69. a. $(f \circ g)(x) = 1 - 2(1+x^2) = -2x^2 - 1$; domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 1 + (1-2x)^2 = 4x^2 - 4x + 2$; domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = 1 - 2(1-2x) = 4x - 1$; domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 1 + (1+x^2)^2 = x^4 + 2x^2 + 2$; domain: $(-\infty, \infty)$

70. a. $(f \circ g)(x) = 2(2x^2) - 3 = 4x^2 - 3$; domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 2(2x-3)^2 = 8x^2 - 24x + 18$; domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = 2(2x-3) - 3 = 4x - 9$; domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 2\left(2x^2\right)^2 = 8x^4$; domain: $(-\infty, \infty)$

71. a. $(f \circ g)(x) = 2(2x-1)^2 + 3(2x-1)$
$= 2(4x^2 - 4x + 1) + 6x - 3$
$= 8x^2 - 2x - 1$; domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 2(2x^2 + 3x) - 1 = 4x^2 + 6x - 1$; domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = 2\left(2x^2+3x\right)^2 + 3\left(2x^2+3x\right)$
$= 2\left(4x^4+12x^3+9x^2\right)+6x^2+9x$
$= 8x^4+24x^3+24x^2+9x;$
domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 2(2x-1)-1 = 4x-3;$
domain: $(-\infty, \infty)$

72. a. $(f \circ g)(x) = (2x)^2 + 3(2x) = 4x^2 + 6x;$
domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 2(x^2+3x) = 2x^2+6x;$
domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = \left(x^2+3x\right)^2 + 3\left(x^2+3x\right)$
$= x^4+6x^3+9x^2+3x^2+9x$
$= x^4+6x^3+12x^2+9x;$
domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 2(2x) = 4x;$ domain: $(-\infty, \infty)$

73. a. $(f \circ g)(x) = \left(\sqrt{x}\right)^2 = x;$ domain: $[0, \infty)$

b. $(g \circ f)(x) = \sqrt{x^2} = |x|;$ domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = \left(x^2\right)^2 = x^4;$ domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = \sqrt{\sqrt{x}} = \sqrt[4]{x};$ domain: $[0, \infty)$

74. a. $(f \circ g)(x) = \left(\sqrt{x+2}\right)^2 + 2\sqrt{x+2}$
$= x+2+2\sqrt{x+2};$ domain: $[-2, \infty)$

b. $(g \circ f)(x) = \sqrt{x^2+2x+2};$ domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = (x^2+2x)^2 + 2(x^2+2x)$
$= x^4+4x^3+4x^2+2x^2+4x$
$= x^4+4x^3+6x^2+4x;$
domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = \sqrt{\sqrt{x+2}+2};$ domain: $[-2, \infty)$

75. a. $(f \circ g)(x) = \dfrac{1}{2\left(\dfrac{1}{x^2}\right)-1} = \dfrac{1}{\dfrac{2-x^2}{x^2}}$
$= \dfrac{x^2}{2-x^2} = -\dfrac{x^2}{x^2-2}.$

The domain of *g* is $(-\infty, 0) \cup (0, \infty)$. Since $\frac{1}{2}$ is not in the domain of *f*, we must find those values of *x* that make $g(x) = \frac{1}{2}$.

$\dfrac{1}{x^2} = \dfrac{1}{2} \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

Thus, the domain of $f \circ g$ is
$\left(-\infty, -\sqrt{2}\right) \cup \left(-\sqrt{2}, 0\right) \cup \left(0, \sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right).$

b. $(g \circ f) = \dfrac{1}{\left(\dfrac{1}{2x-1}\right)^2} = (2x-1)^2$

The domain of *f* is $\left(-\infty, \dfrac{1}{2}\right) \cup \left(\dfrac{1}{2}, \infty\right)$. Since 0 is not in the domain of *g*, we must find those values of *x* that make $f(x) = 0$. However, there are no such values, so the domain of $g \circ f$ is $\left(-\infty, \dfrac{1}{2}\right) \cup \left(\dfrac{1}{2}, \infty\right)$.

c. $(f \circ f)(x) = \dfrac{1}{2\left(\dfrac{1}{2x-1}\right)-1} = \dfrac{1}{\dfrac{2-2x+1}{2x-1}}$
$= \dfrac{2x-1}{3-2x} = -\dfrac{2x-1}{2x-3}$

The domain of *f* is $\left(-\infty, \dfrac{1}{2}\right) \cup \left(\dfrac{1}{2}, \infty\right)$.

$-\dfrac{2x-1}{2x-3}$ is defined for $\left(-\infty, \dfrac{3}{2}\right) \cup \left(\dfrac{3}{2}, \infty\right)$, so the domain of $f \circ f$ is
$\left(-\infty, \dfrac{1}{2}\right) \cup \left(\dfrac{1}{2}, \dfrac{3}{2}\right) \cup \left(\dfrac{3}{2}, \infty\right).$

d. $(g \circ g)(x) = \dfrac{1}{\left(\dfrac{1}{x^2}\right)} = x^4.$

The domain of *g* is $(-\infty, 0) \cup (0, \infty)$, while $g \circ g = x^4$ is defined for all real numbers. Thus, the domain of $g \circ g$ is $(-\infty, 0) \cup (0, \infty)$.

76. a. $(f \circ g)(x) = \dfrac{x}{x+1} - 1 = \dfrac{x-(x+1)}{x+1} = -\dfrac{1}{x+1}$

The domain of *g* is $(-\infty, -1) \cup (-1, \infty)$. Since *f* is defined for all real numbers, there are no values that must be excluded. Thus, the domain of $f \circ g$ is $(-\infty, -1) \cup (-1, \infty)$.

b. $(g \circ f)(x) = \dfrac{x-1}{(x-1)+1} = \dfrac{x-1}{x}$

The domain of f is all real numbers. Since g is not defined for $x = -1$, we must exclude those values of x that make $f(x) = -1$.

$x - 1 = -1 \Rightarrow x = 0$

Thus, the domain of $g \circ f$ is $(-\infty, 0) \cup (0, \infty)$.

c. $(f \circ f)(x) = (x-1) - 1 = x - 2;$
domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = \dfrac{\frac{x}{x+1}}{\frac{x}{x+1}+1} = \dfrac{\frac{x}{x+1}}{\frac{x+x+1}{x+1}} = \dfrac{x}{2x+1}$

The domain of g is $(-\infty, -1) \cup (-1, \infty)$,

while $\dfrac{x}{2x+1}$ is defined for

$\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$.

The domain of $g \circ g$ is

$(-\infty, -1) \cup \left(-1. -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$.

77. a. $(f \circ g)(x) = \sqrt{\sqrt{4-x} - 1}$; domain: $(-\infty, 3]$

b. $(g \circ f)(x) = \sqrt{4 - \sqrt{x-1}}$; domain: $[1, 17]$

c. $(f \circ f)(x) = \sqrt{\sqrt{x-1} - 1}$; domain: $[2, \infty)$

d. $(g \circ g)(x) = \sqrt{4 - \sqrt{4-x}}$; domain: $[-12, 4]$

78. a. $(f \circ g)(x) = \left(\sqrt{4-x^2}\right)^2 - 4 = -x^2$

domain: $[-2, 2]$

b. $(g \circ f)(x) = \sqrt{4 - \left(x^2 - 4\right)^2}$

domain: $\left[-\sqrt{6}, -\sqrt{2}\right] \cup \left[\sqrt{2}, \sqrt{6}\right]$

c.
$$\begin{aligned}(f \circ f)(x) &= \left(x^2 - 4\right)^2 - 4\\ &= \left(x^4 - 8x^2 + 16\right) - 4\\ &= x^4 - 8x^2 + 12\end{aligned}$$

domain: $(-\infty, \infty)$

d.
$$\begin{aligned}\sqrt{4 - \left(\sqrt{4-x^2}\right)^2} &= \sqrt{4 - \left(4 - x^2\right)}\\ &= \sqrt{4 - 4 + x^2} = \sqrt{x^2} = |x|\end{aligned}$$

domain: $[-2, 2]$

79. a.
$$\begin{aligned}(f \circ g)(x) &= \frac{1 - \frac{x+3}{x-4}}{\frac{x+3}{x-4} + 2} = \frac{\left(1 - \frac{x+3}{x-4}\right)(x-4)}{\left(\frac{x+3}{x-4} + 2\right)(x-4)}\\ &= \frac{(x-4) - (x+3)}{(x+3) + 2(x-4)} = -\frac{7}{3x-5}\end{aligned}$$

The domain of g is $(-\infty, 4) \cup (4, \infty)$. The denominator of $f \circ g$ is 0 when $x = \frac{5}{3}$, so the domain of $f \circ g$ is

$\left(-\infty, \frac{5}{3}\right) \cup \left(\frac{5}{3}, 4\right) \cup (4, \infty)$.

b.
$$\begin{aligned}(g \circ f)(x) &= \frac{\frac{1-x}{x+2} + 3}{\frac{1-x}{x+2} - 4} = \frac{\left(\frac{1-x}{x+2} + 3\right)(x+2)}{\left(\frac{1-x}{x+2} - 4\right)(x+2)}\\ &= \frac{(1-x) + 3(x+2)}{(1-x) - 4(x+2)} = \frac{2x+7}{-5x-7}\\ &= -\frac{2x+7}{5x+7}\end{aligned}$$

The domain of f is $(-\infty, -2) \cup (-2, \infty)$. The denominator of $g \circ f$ is 0 when $x = -\frac{7}{5}$, so, the domain of $g \circ f$ is

$(-\infty, -2) \cup \left(-2, -\frac{7}{5}\right) \cup \left(-\frac{7}{5}, \infty\right)$.

c.
$$\begin{aligned}(f \circ f)(x) &= \frac{1 - \frac{1-x}{x+2}}{\frac{1-x}{x+2} + 2} = \frac{\left(1 - \frac{1-x}{x+2}\right)(x+2)}{\left(\frac{1-x}{x+2} + 2\right)(x+2)}\\ &= \frac{(x+2) - (1-x)}{(1-x) + 2(x+2)} = \frac{2x+1}{x+5}\end{aligned}$$

The domain of f is $(-\infty, -2) \cup (-2, \infty)$. The denominator of $f \circ f$ is 0 when $x = -5$, so, the domain of $f \circ f$ is

$(-\infty, -5) \cup (-5, -2) \cup (-2, \infty)$.

d.
$$\begin{aligned}(g \circ g)(x) &= \frac{\frac{x+3}{x-4} + 3}{\frac{x+3}{x-4} - 4} = \frac{\left(\frac{x+3}{x-4} + 3\right)(x-4)}{\left(\frac{x+3}{x-4} - 4\right)(x-4)}\\ &= \frac{(x+3) + 3(x-4)}{(x+3) - 4(x-4)} = \frac{4x-9}{-3x+19}\\ &= -\frac{4x-9}{3x-19}\end{aligned}$$

The domain of g is $(-\infty, 4) \cup (4, \infty)$. The denominator of $g \circ g$ is 0 when $x = \frac{19}{3}$, so the domain of $g \circ g$ is

$(-\infty, 4) \cup \left(4, \frac{19}{3}\right) \cup \left(\frac{19}{3}, \infty\right)$.

80. a. $(f \circ g)(x) = \dfrac{\frac{x+1}{x-1}+2}{\frac{x+1}{x-1}-3} = \dfrac{\left(\frac{x+1}{x-1}+2\right)(x-1)}{\left(\frac{x+1}{x-1}-3\right)(x-1)}$
$= \dfrac{(x+1)+2(x-1)}{(x+1)-3(x-1)} = \dfrac{3x-1}{-2x+4}$
$= -\dfrac{3x-1}{2x-4}$

The domain of g is $(-\infty,1)\cup(1,\infty)$. The denominator of $f \circ g$ is 0 when $x = 2$, so the domain of $f \circ g$ is $(-\infty, 1)\cup(1, 2)\cup(2,\infty)$.

b. $(g \circ f)(x) = \dfrac{\frac{x+2}{x-3}+1}{\frac{x+2}{x-3}-1} = \dfrac{\left(\frac{x+2}{x-3}+1\right)(x-3)}{\left(\frac{x+2}{x-3}-1\right)(x-3)}$
$= \dfrac{(x+2)+(x-3)}{(x+2)-(x-3)} = \dfrac{2x-1}{5}$

The domain of f is $(-\infty, 3)\cup(3, \infty)$. The denominator of $g \circ f$ is never 0, so, the domain of $g \circ f$ is $(-\infty, 3)\cup(3, \infty)$.

c. $(f \circ f)(x) = \dfrac{\frac{x+2}{x-3}+2}{\frac{x+2}{x-3}-3} = \dfrac{\left(\frac{x+2}{x-3}+2\right)(x-3)}{\left(\frac{x+2}{x-3}-3\right)(x-3)}$
$= \dfrac{(x+2)+2(x-3)}{(x+2)-3(x-3)}$
$= \dfrac{3x-4}{-2x+11} = -\dfrac{3x-4}{2x-11}$

The domain of f is $(-\infty, 3)\cup(3, \infty)$. The denominator of $f \circ f$ is 0 when $x = \frac{11}{2}$, so, the domain of $f \circ f$ is $(-\infty, 3)\cup\left(3, \frac{11}{2}\right)\cup\left(\frac{11}{2},\infty\right)$.

d. $(g \circ g)(x) = \dfrac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1} = \dfrac{\left(\frac{x+1}{x-1}+1\right)(x-1)}{\left(\frac{x+1}{x-1}-1\right)(x-1)}$
$= \dfrac{(x+1)+(x-1)}{(x+1)-(x-1)} = \dfrac{2x}{2} = x$

The domain of g is $(-\infty,1)\cup(1,\infty)$. The denominator of $g \circ g$ is never 0 so the domain of $g \circ g$ is $(-\infty,1)\cup(1,\infty)$.

In exercises 81−90, sample answers are given. Other answers are possible.

81. $H(x) = \sqrt{x+2} \Rightarrow f(x) = \sqrt{x}, g(x) = x+2$

82. $H(x) = |3x+2| \Rightarrow f(x) = |x|, g(x) = 3x+2$

83. $H(x) = \left(x^2-3\right)^{10} \Rightarrow f(x) = x^{10}, g(x) = x^2-3$

84. $H(x) = \sqrt{3x^2}+5 \Rightarrow f(x) = \sqrt{x}+5, g(x) = 3x^2$

85. $H(x) = \dfrac{1}{3x-5} \Rightarrow f(x) = \dfrac{1}{x}, g(x) = 3x-5$

86. $H(x) = \dfrac{5}{2x+3} \Rightarrow f(x) = \dfrac{5}{x}, g(x) = 2x+3$

87. $H(x) = \sqrt[3]{x^2-7} \Rightarrow f(x) = \sqrt[3]{x}, g(x) = x^2-7$

88. $H(x) = \sqrt[4]{x^2+x+1} \Rightarrow f(x) = \sqrt[4]{x}$, $g(x) = x^2+x+1$

89. $H(x) = \dfrac{1}{\left|x^3-1\right|} \Rightarrow f(x) = \dfrac{1}{|x|}, g(x) = x^3-1$

90. $H(x) = \sqrt[3]{1+\sqrt{x}} \Rightarrow f(x) = \sqrt[3]{x}, g(x) = 1+\sqrt{x}$

2.8 Applying the Concepts

91. a. $f(x)$ is the cost function.

b. $g(x)$ is the revenue function.

c. $h(x)$ is the selling price of x shirts including sales tax.

d. $P(x)$ is the profit function.

92. a. $C(p) = C(5000-5p)$
$= 4(5000-5p)+12{,}000$
$= 20{,}000-20p+12{,}000$
$= 32{,}000-20p$

b. $R(p) = px = p(5000-5p) = 5000p-5p^2$

c. $P(p) = R(p)-C(p)$
$= 5000p-5p^2-(32{,}000-20p)$
$= -5p^2+5020p-32{,}000$

93. a. $P(x) = R(x)-C(x) = 25x-(350+5x)$
$= 20x-350$

b. $P(20) = 20(20)-350 = 50.$ This represents the profit when 20 radios are sold.

c. $P(x) = 20x-350; 500 = 20x-350 \Rightarrow x = 43$

d. $C = 350 + 5x \Rightarrow x = \dfrac{C - 350}{5} = x(C).$

$(R \circ x)(C) = 25\left(\dfrac{C - 350}{5}\right) = 5C - 1750.$

This function represents the revenue in terms of the cost C.

94. a. $g(x) = 0.04x$

b. $h(x)$ is the after tax selling price of merchandise worth x dollars.

c. $f(x) = 0.02h(x) + 3$

d. $T(x)$ represents the total price of merchandise worth x dollars, including the shipping and handling fee.

95. a. $f(x) = 0.7x$

b. $g(x) = x - 5$

c. $(g \circ f)(x) = 0.7x - 5$

d. $(f \circ g)(x) = 0.7(x - 5)$

e. $$\begin{aligned}(f \circ g) - (g \circ f) &= 0.7(x-5) - (0.7x - 5)\\ &= 0.7x - 3.5 - 0.7x + 5\\ &= \$1.50\end{aligned}$$

96. a. $f(x) = 0.8x$

b. $g(x) = 0.9x$

c. $(g \circ f)(x) = 0.9(0.8x) = 0.72x$

d. $(f \circ g)(x) = 0.8(0.9x) = 0.72x$

e. They are the same.

97. a. $f(x) = 1.1x; g(x) = x + 8$

b. $(f \circ g)(x) = 1.1(x + 8) = 1.1x + 8.8$

. This represents a final test score computed by first adding 8 points to the original score and then increasing the total by 10%.

c. $(g \circ f)(x) = 1.1x + 8$

This represents a final test score computed by first increasing the original score by 10% and then adding 8 points.

d. $(f \circ g)(70) = 1.1(70 + 8) = 85.8;$
$(g \circ f)(70) = 1.1(70) + 8 = 85.0;$

e. $(f \circ g)(x) \neq (g \circ f)(x)$

f. **(i)** $(f \circ g)(x) = 1.1x + 8.8 \geq 90 \Rightarrow x \geq 73.82$

(ii) $(g \circ f)(x) = 1.1x + 8 \geq 90 \Rightarrow x \geq 74.55$

98. a. $f(x)$ is a function that models 3% of an amount x.

b. $g(x)$ represents the amount of money that qualifies for a 3% bonus.

c. Her bonus is represented by $(f \circ g)(x)$.

d. $200 + 0.03(17,500 - 8000) = \$485$

e. $521 = 200 + 0.03(x - 8000) \Rightarrow x = \$18,700$

99. a. $f(x) = \pi x^2$

b. $g(x) = \pi(x + 30)^2$

c. $g(x) - f(x)$ represents the area between the fountain and the fence.

d. The circumference of the fence is $2\pi(x + 30)$.

$10.5(2\pi(x+30)) = 4200 \Rightarrow$
$\pi(x + 30) = 200 \Rightarrow$
$\pi x + 30\pi = 200 \Rightarrow \pi x = 200 - 30\pi.$

$$\begin{aligned}g(x) - f(x) &= \pi(x+30)^2 - \pi x^2\\ &= \pi(x^2 + 60x + 900) - \pi x^2\\ &= 60\pi x + 900\pi.\end{aligned}$$
Now substitute $200 - 30\pi$ for πx to compute the estimate:

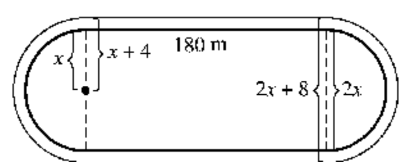
$$\begin{aligned}&1.75[60(200 - 30\pi) + 900\pi]\\ &\quad = 1.75(12,000 - 900\pi) \approx \$16,052.\end{aligned}$$

100. a. $$\begin{aligned}f(x) &= 180(2x + 8) + \pi(x+4)^2\\ &= 1440 + 360x + \pi(x+4)^2\end{aligned}$$

b. $g(x) = 2x(180) + \pi x^2 = 360x + \pi x^2$

c. $f(x) - g(x)$ represents the area of the track.

d.



(i) First find the radius of the inner track:

$900 = 2\pi x + 360 \Rightarrow \dfrac{270}{\pi} = x$. Use this value to compute $f(x) - g(x)$.

(*continued on next page*)

(*continued*)

$$f\left(\frac{270}{\pi}\right)-g\left(\frac{270}{\pi}\right)$$
$$=\left(1440+360\left(\frac{270}{\pi}\right)+\pi\left(\frac{270}{\pi}+4\right)^2\right)$$
$$-\left(360\left(\frac{270}{\pi}\right)+\pi\left(\frac{270}{\pi}\right)^2\right)$$
$$=1440+360\left(\frac{270}{\pi}\right)+\frac{270^2}{\pi}+2160+16\pi$$
$$-360\left(\frac{270}{\pi}\right)-\frac{270^2}{\pi}$$
$$=3600+16\pi\approx 3650.27 \text{ square meters}$$

(ii) The outer perimeter
$$=360+2\pi\left(\frac{270}{\pi}+4\right)\approx 925.13 \text{ meters}$$

101. a. $(f\circ g)(t)=\pi(2t+1)^2$

b. $A(t)=f(2t+1)=\pi(2t+1)^2$

c. They are the same.

102. a. $(f\circ g)(t)=\frac{4}{3}\pi(2t)^3=\frac{32}{3}\pi t^3$

b. $V(t)=\frac{4}{3}\pi(2t)^3=\frac{32}{3}\pi t^3$

c. They are the same.

2.8 Beyond the Basics

103. a. When you are looking for the domain of the sum of two functions that are given as sets, you are looking for the intersection of their domains. Since the x-values that f and g have in common are −2, 1, and 3, the domain of $f+g$ is {−2, 1, 3}. Now add the y-values.
$(f+g)(-2)=3+0=3$
$(f+g)(1)=2+(-2)=0$
$(f+g)(3)=0+2=2$
Thus, $f+g=\{(-2, 3), (1, 0), (3, 2)\}$.

b. When you are looking for the domain of the product of two functions that are given as sets, you are looking for the intersection of their domains. Since the x-values that f and g have in common are −2, 1, and 3, the domain of $f+g$ is {−2, 1, 3}.Now multiply the y-values.
$(fg)(-2)=3\cdot 0=0$
$(fg)(1)=2\cdot(-2)=-4$
$(fg)(3)=0\cdot 2=0$
Thus, $fg=\{(-2, 0), (1, -4), (3, 0)\}$.

c. When you are looking for the domain of the quotient of two functions that are given as sets, you are looking for the intersection of their domains and values of x that do not cause the denominator to equal zero. The x-values that f and g have in common are −2, 1, and 3; however, $g(-2)=0$, so the domain is {1, 3}. Now divide the y-values.
$\left(\frac{f}{g}\right)(1)=\frac{2}{-2}=-1$
$\left(\frac{f}{g}\right)(3)=\frac{0}{2}=0$
Thus, $\frac{f}{g}=\{(1, -1), (3, 0)\}$.

d. When you are looking for the domain of the composition of two functions that are given as sets, you are looking for values that come from the domain of the inside function and when you plug those values of x into the inside function, the output is in the domain of the outside function.
$f(g(-2))=f(0)$, which is undefined
$f(g(0))=f(2)=1,$
$f(g(1))=f(-2)=3,$
$f(g(3))=f(2)=1$
Thus, $f\circ g=\{(0, 1), (1, 3), (3, 1)\}$.

104. When you are looking for the domain of the sum of two functions, you are looking for the intersection of their domains. The domain of f is [−2, 3], while the domain of g is [−3, 3]. The intersection of the two domains is [−2, 3], so the domain of $f+g$ is [−2, 3].
For the interval [−2, 1],
$f+g=2x+(x+1)=3x+1$.
For the interval (1, 2)
$f+g=(x+1)+(x+1)=2x+2$.
For the interval [2, 3],
$f+g=(x+1)+(2x-1)=3x$.
Thus,
$$(f+g)(x)=\begin{cases}3x+1 & \text{if } -2\le x\le 1\\ 2x+2 & \text{if } 1<x<2\\ 3x & \text{if } 2\le x\le 3.\end{cases}$$

105. a. $f(-x) = h(-x) + h(-(-x)) = h(-x) + h(x)$
$= f(x) \Rightarrow f(x)$ is an even function.

b. $g(-x) = h(-x) - h(-(-x)) = h(-x) - h(x)$
$= -g(x) \Rightarrow g(x)$ is an odd function.

c. $\begin{cases} f(x) = h(x) + h(-x) \\ g(x) = h(x) - h(-x) \Rightarrow \end{cases}$
$f(x) + g(x) = 2h(x) \Rightarrow$
$h(x) = \dfrac{f(x)+g(x)}{2} = \dfrac{f(x)}{2} + \dfrac{g(x)}{2} \Rightarrow$
$h(x)$ is the sum of an even function and an odd function.

106. a. $h(x) = x^2 - 2x + 3 \Rightarrow f(x) = x^2$ (even),
$g(x) = -2x + 3$ (odd) or $f(x) = x^2 + 3$ (even),
$g(x) = -2x$ (odd)

b. $h(x) = [\![x]\!] + x \Rightarrow f(x) = \dfrac{[\![x]\!] + [\![-x]\!]}{2}$ (even),
$g(x) = x + \dfrac{[\![x]\!] - [\![-x]\!]}{2}$ (odd)

107. $f(x) = \sqrt{\dfrac{1-|x|}{2-|x|}}$

$f(x)$ is defined if $\dfrac{1-|x|}{2-|x|} \ge 0$ and $2-|x| \ne 0$.

$2 - |x| = 0 \Rightarrow 2 = |x| \Rightarrow x = \pm 2$

Thus, the values −2 and 2 are not in the domain of *f*.

$\dfrac{1-|x|}{2-|x|} \ge 0$ if $1-|x| \ge 0$ and $2-|x| > 0$, or if $1-|x| \le 0$ and $2-|x| < 0$.

Case 1: $1-|x| \ge 0$ and $2-|x| > 0$.

$1-|x| \ge 0 \Rightarrow 1 \ge |x| \Rightarrow -1 \le x \le 1$

$2-|x| > 0 \Rightarrow 2 > |x| \Rightarrow -2 < x < 2$

Thus, $1-|x| \ge 0$ and $2-|x| > 0 \Rightarrow -1 \le x \le 1$.

Case 2: $1-|x| \le 0$ and $2-|x| < 0$.

$1-|x| \le 0 \Rightarrow 1 \le |x| \Rightarrow (-\infty, -1] \cup [1, \infty)$

$2-|x| < 0 \Rightarrow 3 \le |x| \Rightarrow (-\infty, -2) \cup (2, \infty)$

Thus, $1-|x| \le 0$ and $2-|x| < 0 \Rightarrow$
$(-\infty, -2) \cup (2, \infty)$.

The domain of *f* is
$(-\infty, -2) \cup [-1, 1] \cup (2, \infty)$.

108. $f(x) = \begin{cases} -1 & \text{if } -2 \le x \le 0 \\ x-1 & \text{if } 0 < x \le 2 \end{cases}$

$f(|x|) = |x| - 1, \ -2 \le x \le 2$

$|f(x)| = \begin{cases} 1 & \text{if } -2 \le x \le 0 \\ |x-1| = 1-x & \text{if } 0 < x < 1 \\ x-1 & \text{if } 1 \le x \le 2 \end{cases}$

$g(x) = f(|x|) + |f(x)|$

If $-2 \le x \le 0$, then
$g(x) = |x| - 1 + 1 = |x| = -x$.

If $0 < x < 1$, then $g(x) = (1-x) + (x-1) = 0$.

If $1 \le x \le 2$, then
$g(x) = (x-1) + (x-1) = 2(x-1)$.

Writing *g* as a piecewise function, we have

$g(x) = \begin{cases} |x| = -x & \text{if } -2 \le x \le 0 \\ 0 & \text{if } 0 < x < 1 \\ 2(x-1) & \text{if } 1 \le x \le 2 \end{cases}$

2.8 Critical Thinking/Discussion/Writing

109. a. The domain of $f(x)$ is $(-\infty, 0) \cup [1, \infty)$.

b. The domain of $g(x)$ is [0, 2].

c. The domain of $f(x) + g(x)$ is [1, 2].

d. The domain of $\dfrac{f(x)}{g(x)}$ is [1, 2).

110. a. The domain of *f* is $(-\infty, 0)$. The domain of $f \circ f$ is $\varnothing$ because $f \circ f = \dfrac{1}{\sqrt{-\dfrac{1}{\sqrt{-x}}}}$ and the denominator is the square root of a negative number.

b. The domain of *f* is $(-\infty, 1)$. The domain of $f \circ f$ is $(-\infty, 0)$ because
$f \circ f = \dfrac{1}{\sqrt{1 - \dfrac{1}{\sqrt{1-x}}}}$ and the denominator must be greater than 0. If $x = 0$, then the denominator = 0.

111. a. The sum of two even functions is an even function.
$f(x) = f(-x)$ and $g(x) = g(-x) \Rightarrow$
$(f+g)(x) = f(x) + g(x) = f(-x) + g(-x)$
$= (f+g)(-x)$.

b. The sum of two odd functions is an odd function.
$f(-x)=-f(x)$ and $g(-x)=-g(x)\Rightarrow$
$(f+g)(-x)=f(-x)+g(-x)=-f(x)-g(x)$
$=-(f+g)(x).$

c. The sum of an even function and an odd function is neither even nor odd.
$f(x)$ even $\Rightarrow f(x)=f(-x)$ and $g(x)$ odd $\Rightarrow$
$g(-x)=-g(x)\Rightarrow f(-x)+g(-x)=$
$f(x)+(-g(x))$, which is neither even nor odd.

d. The product of two even functions is an even function.
$f(x)=f(-x)$ and $g(x)=g(-x)\Rightarrow$
$(f\cdot g)(x)=f(x)\cdot g(x)=f(-x)\cdot(g(-x))$
$=(f\cdot g)(-x).$

e. The product of two odd functions is an even function.
$f(-x)=-f(x)$ and $g(-x)=-g(x)\Rightarrow$
$(f\cdot g)(-x)=f(-x)\cdot g(-x)=-f(x)\cdot(-g(x))$
$=(f\cdot g)(x).$

f. The product of an even function and an odd function is an odd function.
$f(x)$ even $\Rightarrow f(x)=f(-x)$ and $g(x)$ odd $\Rightarrow$
$g(-x)=-g(x)\Rightarrow$
$f(-x)\cdot g(-x)=f(x)\cdot(-g(x))=-(f\cdot g)(x)$

112. a. $f(-x)=-f(x)$ and $g(-x)=-g(x)\Rightarrow$
$(f\circ g)(-x)=f(g(-x))=f(-g(x))=$
$-f(g(x))\Rightarrow(f\circ g)(x)$ is odd.

b. $f(x)=f(-x)$ and $g(x)=g(-x)\Rightarrow$
$(f\circ g)(-x)=f(g(-x))=f(g(x))\Rightarrow$
$(f\circ g)(x)$ is even.

c. $f(x)$ odd $\Rightarrow f(-x)=-f(x)$ and
$g(x)$ even $\Rightarrow g(x)=g(-x)\Rightarrow(f\circ g)(-x)$
$f(g(x))=f(g(-x))\Rightarrow(f\circ g)(x)$ is even.

d. $f(x)$ even $\Rightarrow f(x)=f(-x)$ and $g(x)$ odd $\Rightarrow$
$g(-x)=-g(x)\Rightarrow(f\circ g)(-x)=f(-g(x))$
$=f(g(x))=(f\circ g)(-x)\Rightarrow(f\circ g)(x)$ is even.

2.8 Getting Ready for the Next Section

113. a. Yes, R defines a function.

b. $S=\{(2,-3),(1,-1),(3,1),(1,2)\}$
No, S does not define a function since the first value 1 maps to two different second values, −1 and 2.

114. The slope of $PP'=\dfrac{2-5}{5-2}=-1,$ while the slope of $y=x$ is 1. Since the slopes are the negative reciprocals, the lines are perpendicular. The midpoint of PP' is $\left(\dfrac{2+5}{2},\dfrac{5+2}{2}\right)=\left(\dfrac{7}{2},\dfrac{7}{2}\right),$ which lies on the line $y=x$. Thus, $y=x$ is the perpendicular bisector of PP'.

115. $x=2y+3\Rightarrow x-3=2y\Rightarrow\dfrac{x-3}{2}=y$

116. $x=y^2+1,\ y\ge 0\Rightarrow x-1=y^2\Rightarrow\sqrt{x-1}=y$

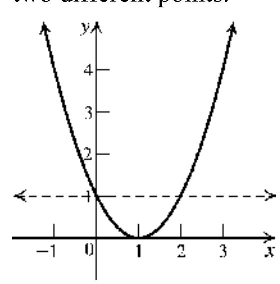
117. $x^2+y^2=4,\ x\le 0\Rightarrow x^2=4-y^2\Rightarrow$
$x=-\sqrt{4-y^2}$

118. $2x-\dfrac{1}{y}=3\Rightarrow-\dfrac{1}{y}=3-2x\Rightarrow\dfrac{1}{y}=-3+2x\Rightarrow$
$y=\dfrac{1}{2x-3}$

2.9 Inverse Functions

2.9 Practice Problems

1. $f(x)=(x-1)^2$ is not one-to-one because the horizontal line $y=1$ intersects the graph at two different points.



2. a. $f^{-1}(12)=-3$

b. $f(9)=4$

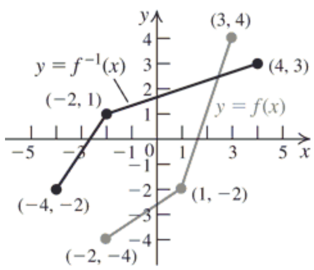
3. $f(x) = 3x - 1,\ g(x) = \frac{x+1}{3}$

$(f \circ g)(x) = f\left(\frac{x+1}{3}\right) = 3\left(\frac{x+1}{3}\right) - 1 = x$

$(g \circ f)(x) = g(3x-1) = \frac{3x-1+1}{3} = x$

Because $f(g(x)) = g(f(x)) = x$, the two functions are inverses.

4. The graph of f^{-1} is the reflection of the graph of *f* about the line $y = x$.



5. $f(x) = -2x + 3$ is a one-to-one function, so the function has an inverse. Interchange the variables and solve for *y*:

$f(x) = y = -2x + 3 \Rightarrow x = -2y + 3 \Rightarrow$

$\frac{x-3}{-2} = y \Rightarrow y = f^{-1}(x) = \frac{3-x}{2}.$

6. Interchange the variables and solve for *y*:

$f(x) = y = \frac{x}{x+3}, x \neq -3$

$x = \frac{y}{y+3} \Rightarrow xy + 3x = y \Rightarrow 3x = y - xy \Rightarrow$

$3x = y(1-x) \Rightarrow \frac{3x}{1-x} = y \Rightarrow$

$f^{-1}(x) = \frac{3x}{1-x},\ x \neq 1$

7. $f(x) = \frac{x}{x+3}$

The function is not defined if the denominator is zero, so the domain is $(-\infty, -3) \cup (-3, \infty)$.

The range of the function is the same as the domain of the inverse, thus the range is $(-\infty, 1) \cup (1, \infty)$.

8. *G* is one-to-one because the domain is restricted, so an inverse exists.

$G(x) = y = x^2 - 1, x \leq 0$. Interchange the variables and solve for y:

$x = y^2 - 1,\ y \leq 0 \Rightarrow y = G^{-1}(x) = -\sqrt{x+1}.$

9. From the text, we have $d = \frac{11p}{5} - 33$.

$d = \frac{11 \cdot 1650}{5} - 33 = 3597$

The bell was 3597 feet below the surface when the gauge failed.

2.9 Concepts and Vocabulary

1. If no horizontal line intersects the graph of a function *f* in more than one point, the *f* is a one-to-one function.

2. A function *f* is one-to-one if different *x*-values correspond to different *y*-values.

3. If $f(x) = 3x$, then $f^{-1}(x) = \frac{1}{3}x$.

4. The graphs of a function *f* and its inverse f^{-1} are symmetric about the line $y = x$.

5. True

6. True. For example, the inverse of $f(x) = x$ is $f^{-1}(x) = x$.

7. False. $f^{-1}(x)$ means the inverse of *f*.

8. True

2.9 Building Skills

9. One-to-one
10. Not one-to-one
11. Not one-to-one
12. One-to-one
13. Not one-to-one
14. Not one-to-one
15. One-to-one
16. Not one-to-one

17. $f(2) = 7 \Rightarrow f^{-1}(7) = 2$

18. $f^{-1}(4) = -7 \Rightarrow f(-7) = 4$

19. $f(-1) = 2 \Rightarrow f^{-1}(2) = -1$

20. $f^{-1}(-3) = 5 \Rightarrow f(5) = -3$

21. $f(a) = b \Rightarrow f^{-1}(b) = a$

22. $f^{-1}(c) = d \Rightarrow f(d) = c$

23. $(f^{-1} \circ f)(337) = f^{-1}(f(337)) = 337$

24. $(f \circ f^{-1})(25\pi) = f(f^{-1}(25\pi)) = 25\pi$

25. a. $f(3) = 2(3) - 3 = 3$

b. Using the result from part (a), $f^{-1}(3) = 3$.

c. $\left(f \circ f^{-1}\right)(19) = f\left(f^{-1}(19)\right) = 19$

d. $\left(f \circ f^{-1}\right)(5) = f\left(f^{-1}(5)\right) = 5$

26. a. $f(2) = 2^3 = 8$

b. Using the result from part (a), $f^{-1}(8) = 2$.

c. $\left(f \circ f^{-1}\right)(15) = f\left(f^{-1}(15)\right) = 15$

d. $\left(f \circ f^{-1}\right)(27) = f\left(f^{-1}(27)\right) = 27$

27. a. $f(1) = 1^3 + 1 = 2$

b. Using the result from part (a), $f^{-1}(2) = 1$.

c. $\left(f \circ f^{-1}\right)(269) = f\left(f^{-1}(269)\right) = 269$

28. a. $g(1) = \sqrt[3]{2\left(1^3\right) - 1} = \sqrt[3]{1} = 1$

b. Using the result from part (a), $g^{-1}(1) = 1$.

c. $\left(g^{-1} \circ g\right)(135) = g^{-1}\left(g(135)\right) = 135$

29. $f\left(g(x)\right) = 3\left(\frac{x-1}{3}\right) + 1 = x - 1 + 1 = x$

$g\left(f(x)\right) = \frac{(3x+1)-1}{3} = \frac{3x}{3} = x$

30. $f\left(g(x)\right) = 2 - 3\left(\frac{2-x}{3}\right) = 2 - 2 + x = x$

$g\left(f(x)\right) = \frac{2-(2-3x)}{3} = \frac{3x}{3} = x$

31. $f\left(g(x)\right) = \left(\sqrt[3]{x}\right)^3 = x$

$g\left(f(x)\right) = \sqrt[3]{x^3} = x$

32. $f\left(g(x)\right) = g\left(f(x)\right) = \frac{1}{\frac{1}{x}} = x$

33. $f\left(g(x)\right) = \frac{\frac{1+2x}{1-x} - 1}{\frac{1+2x}{1-x} + 2} = \frac{\frac{1+2x-(1-x)}{1-x}}{\frac{1+2x+2(1-x)}{1-x}}$

$= \frac{3x}{3} = x$

$g\left(f(x)\right) = \frac{1+2\left(\frac{x-1}{x+2}\right)}{1-\frac{x-1}{x+2}} = \frac{1+\frac{2x-2}{x+2}}{1-\frac{x-1}{x+2}}$

$= \frac{\frac{x+2+2x-2}{x+2}}{\frac{x+2-(x-1)}{x+2}} = \frac{3x}{3} = x$

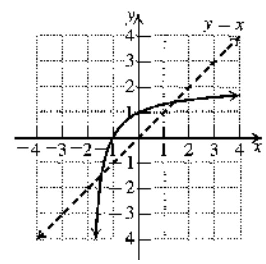
34. $f\left(g(x)\right) = \frac{3\left(\frac{x+2}{x-3}\right)+2}{\frac{x+2}{x-3}-1} = \frac{\frac{3x+6}{x-3}+\frac{2(x-3)}{x-3}}{\frac{x+2}{x-3}-\frac{1(x-3)}{x-3}}$

$= \frac{\frac{3x+6+2x-6}{x-3}}{\frac{x+2-x+3}{x-3}} = \frac{5x}{5} = x$

$g\left(f(x)\right) = \frac{\frac{3x+2}{x-1}+2}{\frac{3x+2}{x-1}-3} = \frac{\frac{3x+2}{x-1}+\frac{2(x-1)}{x-1}}{\frac{3x+2}{x-1}-\frac{3(x-1)}{x-1}}$

$= \frac{\frac{3x+2+2x-2}{x-1}}{\frac{3x+2-3x+3}{x-1}} = \frac{5x}{5} = x$

35.



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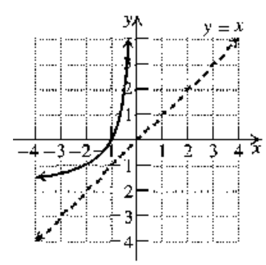

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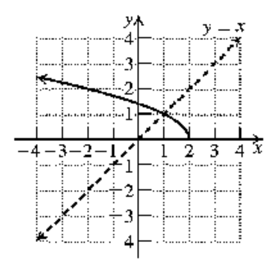

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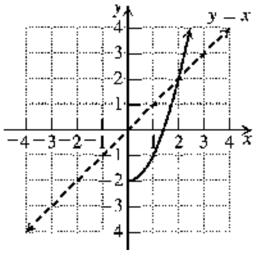

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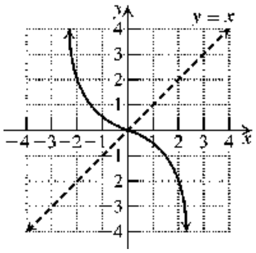

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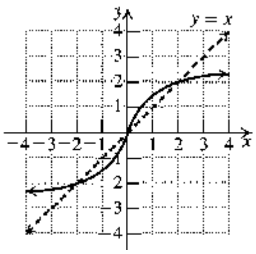

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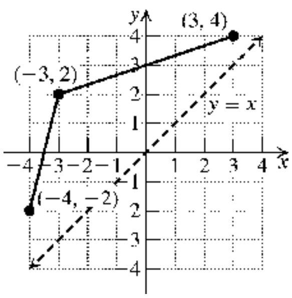

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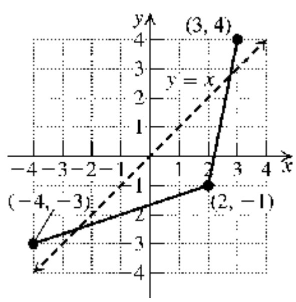
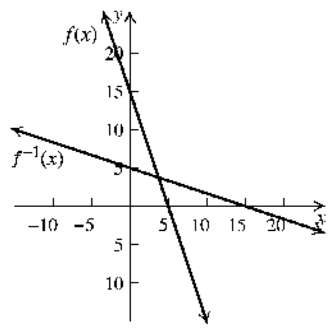


43. a. One-to-one

b. $f(x) = y = 15 - 3x$. Interchange the variables and solve for $y: x = 15 - 3y \Rightarrow$
$y = f^{-1}(x) = \dfrac{15 - x}{3} = 5 - \dfrac{1}{3}x.$

c.

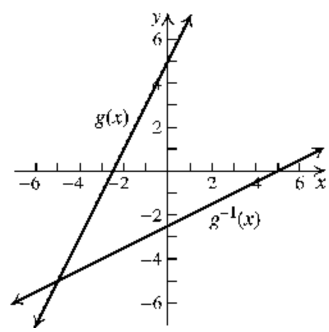


d. Domain of *f*: $(-\infty, \infty)$; *x*-intercept of *f*: 5; *y*-intercept of *f*: 15
domain of f^{-1}: $(-\infty, \infty)$; *x*-intercept of f^{-1}: 15; *y*-intercept of f^{-1}: 5

44. a. One-to-one

b. $g(x) = y = 2x + 5$. Interchange the variables and solve for $y: x = 2y + 5 \Rightarrow$
$y = g^{-1}(x) = \dfrac{x - 5}{2} = \dfrac{1}{2}x - \dfrac{5}{2}.$

c.



d. Domain of *g*: $(-\infty, \infty)$
x-intercept of *g*: $-\dfrac{5}{2}$
y-intercept of *g*: 5
domain of g^{-1}: $(-\infty, \infty)$; *x*-intercept of g^{-1}: 5; *y*-intercept of g^{-1}: $-\dfrac{5}{2}$

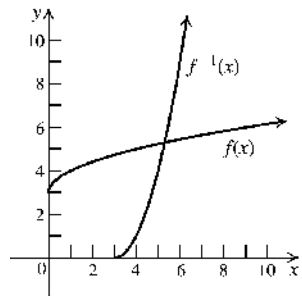
45. a. Not one-to-one

46. a. Not one-to-one

47. a. One-to-one

b. $f(x) = y = \sqrt{x} + 3$ Interchange the variables and solve for $y: x = \sqrt{y} + 3 \Rightarrow$
$x - 3 = \sqrt{y} \Rightarrow y = f^{-1}(x) = (x - 3)^2.$

c.

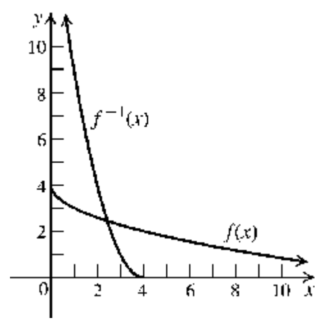


d. Domain of *f*: $[0, \infty)$; *x*-intercept of *f*: none;
y-intercept of *f*: 3
domain of f^{-1}: $[3, \infty)$; *x*-intercept of f^{-1}: 3; *y*-intercept of f^{-1}: none

48. a. One-to-one

b. $f(x) = y = 4 - \sqrt{x}$. Interchange the variables and solve for $y: x = 4 - \sqrt{y} \Rightarrow$
$-4 + x = -\sqrt{y} \Rightarrow y = f^{-1}(x) = (x-4)^2$

c.

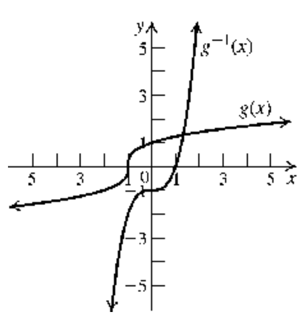


d. Domain of *f*: $[0, \infty)$; *x*-intercept of *f*: 16;
y-intercept of *f*: 4
domain of f^{-1}: $(-\infty, 4]$; *x*-intercept of f^{-1}: 4; *y*-intercept of f^{-1}: 16

49. a. One-to-one

b. $g(x) = y = \sqrt[3]{x+1}$. Interchange the variables and solve for $y: x = \sqrt[3]{y+1} \Rightarrow$
$x^3 = y + 1 \Rightarrow y = g^{-1}(x) = x^3 - 1$

c.

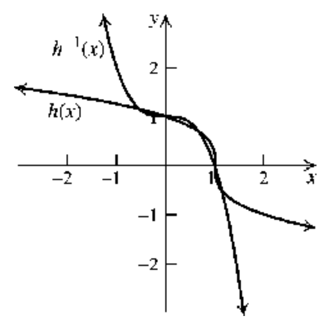


d. Domain of *g*: $(-\infty, \infty)$; *x*-intercept of *g*: -1;
y-intercept of *g*: 1
domain of g^{-1}: $(-\infty, \infty)$; *x*-intercept of g^{-1}: 1; *y*-intercept of g^{-1}: -1

50. a. One-to-one

b. $h(x) = y = \sqrt[3]{1-x}$. Interchange the variables and solve for $y: x = \sqrt[3]{1-y} \Rightarrow$
$x^3 = 1 - y \Rightarrow y = g^{-1}(x) = 1 - x^3$.

c.

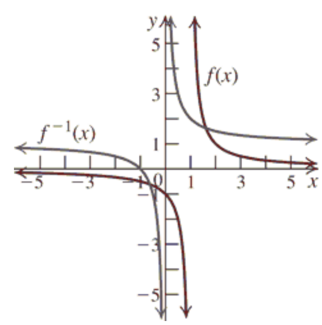


d. Domain of *h*: $(-\infty, \infty)$; *x*-intercept of *h*: 1;
y-intercept of *h*: 1
domain of h^{-1}: $(-\infty, \infty)$; *x*-intercept of h^{-1}: 1; *y*-intercept of h^{-1}: 1

51. a. One-to-one

b. $f(x) = y = \dfrac{1}{x-1}$. Interchange the variables and solve for $y: x = \dfrac{1}{y-1} \Rightarrow x(y-1) = 1 \Rightarrow$
$\dfrac{1}{x} = y - 1 \Rightarrow y = f^{-1}(x) = \dfrac{1}{x} + 1 = \dfrac{1+x}{x}$.

c.

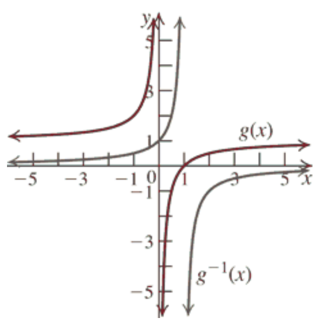


d. Domain of *f*: $(-\infty, 1) \cup (1, \infty)$
x-intercept of *f*: none; y-intercept of *f*: −1
domain of f^{-1}: $(-\infty, 0) \cup (0, \infty)$
x-intercept of f^{-1}: −1
y-intercept of f^{-1}: none

52. a. One-to-one

b. $g(x) = y = 1 - \frac{1}{x}$. Interchange the variables and solve for y: $x = 1 - \frac{1}{y} \Rightarrow x = \frac{y-1}{y} \Rightarrow$
$xy = y - 1 \Rightarrow xy - y = -1 \Rightarrow y(x-1) = -1 \Rightarrow$
$y = g^{-1}(x) = -\frac{1}{x-1} = \frac{1}{1-x}$.

c.

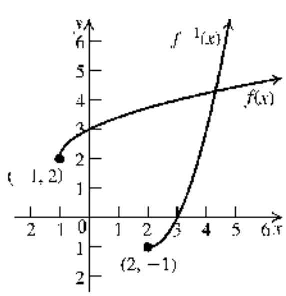


d. Domain of g: $(-\infty, 0) \cup (0, \infty)$
x-intercept of g: 1; y-intercept of g: none
domain of g^{-1}: $(-\infty, 1) \cup (1, \infty)$
x-intercept of g^{-1}: none
y-intercept of g^{-1}: 1

53. a. One-to-one

b. $f(x) = y = 2 + \sqrt{x+1}$. Interchange the variables and solve for y: $x = 2 + \sqrt{y+1} \Rightarrow$
$x - 2 = \sqrt{y+1} \Rightarrow (x-2)^2 = y + 1 \Rightarrow$
$y = f^{-1}(x) = (x-2)^2 - 1 = x^2 - 4x + 3$

c.

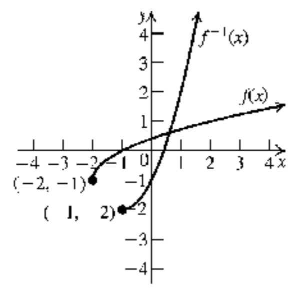


d. Domain of *f*: $[-1, \infty)$; x-intercept of *f*: none; y-intercept of *f*: 3
Domain of f^{-1}: $[2, \infty)$;
x-intercept of f^{-1}: 3
y-intercept of f^{-1}: none

54. a. One-to-one

b. $f(x) = y = -1 + \sqrt{x+2}$. Interchange the variables and solve for y:
$x = -1 + \sqrt{y+2} \Rightarrow x + 1 = \sqrt{y+2} \Rightarrow$
$(x+1)^2 = y + 2 \Rightarrow$
$y = f^{-1}(x) = (x+1)^2 - 2 = x^2 + 2x - 1$

c.



d. Domain of *f*: $[-2, \infty)$; x-intercept of *f*: −1;
y-intercept of *f*: $-1 + \sqrt{2}$
Domain of f^{-1}: $[-1, \infty)$
x-intercept of f^{-1}: $-1 + \sqrt{2}$
y-intercept of f^{-1}: −1

In exercises 55 and 56, use the fact that the range of *f* is the same as the domain of f^{-1}.

55. Domain: $(-\infty, -2) \cup (-2, \infty)$
Range: $(-\infty, 1) \cup (1, \infty)$

56. Domain: $(-\infty, 1) \cup (1, \infty)$
Range: $(-\infty, 3) \cup (3, \infty)$

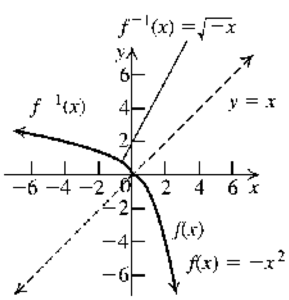
57. $f(x) = y = \frac{x+1}{x-2}$. Interchange the variables and solve for y: $x = \frac{y+1}{y-2} \Rightarrow xy - 2x = y + 1 \Rightarrow$
$xy - y = 2x + 1 \Rightarrow y(x-1) = 2x + 1 \Rightarrow$
$y = f^{-1}(x) = \frac{2x+1}{x-1}$.
Domain of *f*: $(-\infty, 2) \cup (2, \infty)$
Range of *f*: $(-\infty, 1) \cup (1, \infty)$.

58. $g(x) = y = \dfrac{x+2}{x+1}$. Interchange the variables and solve for $y: x = \dfrac{y+2}{y+1} \Rightarrow xy + x = y + 2 \Rightarrow$
$xy - y = -x + 2 \Rightarrow y(x-1) = -x + 2 \Rightarrow$
$y = g^{-1}(x) = \dfrac{-x+2}{x-1} = \dfrac{x-2}{1-x}$.
Domain of g: $(-\infty,-1) \cup (-1,\infty)$
Range of g: $(-\infty,1) \cup (1,\infty)$.

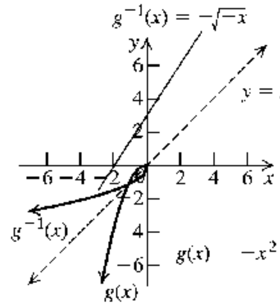
59. $f(x) = y = \dfrac{1-2x}{1+x}$. Interchange the variables and solve for $y: x = \dfrac{1-2y}{1+y} \Rightarrow$
$x + xy = 1 - 2y \Rightarrow xy + 2y = 1 - x \Rightarrow$
$y(x+2) = 1 - x \Rightarrow y = f^{-1}(x) = \dfrac{1-x}{x+2}$.
Domain of f: $(-\infty,-1) \cup (-1,\infty)$
Range of f: $(-\infty,-2) \cup (-2,\infty)$.

60. $h(x) = y = \dfrac{x-1}{x-3}$. Interchange the variables and solve for $y: x = \dfrac{y-1}{y-3} \Rightarrow xy - 3x = y - 1 \Rightarrow$
$xy - y = 3x - 1 \Rightarrow y(x-1) = 3x - 1 \Rightarrow$
$y = h^{-1}(x) = \dfrac{3x-1}{x-1}$.
Domain of h: $(-\infty,3) \cup (3,\infty)$
Range of h: $(-\infty,1) \cup (1,\infty)$.

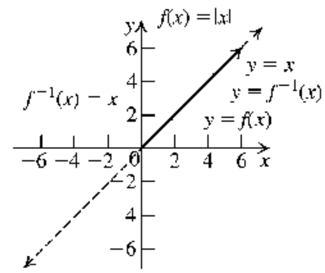
61. f is one-to-one since the domain is restricted, so an inverse exists.
$f(x) = y = -x^2, x \ge 0$. Interchange the variables and solve for y:
$x = -y^2 \Rightarrow y = \sqrt{-x},\ x \le 0$.



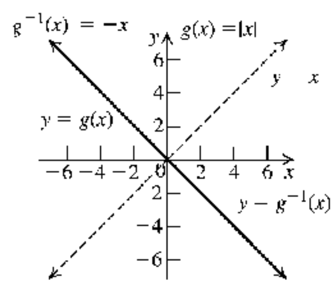
62. g is one-to-one since the domain is restricted, so an inverse exists.
$g(x) = y = -x^2,\ x \le 0$. Interchange the variables and solve for y:
$x = -y^2 \Rightarrow y = -\sqrt{-x},\ x \le 0$.



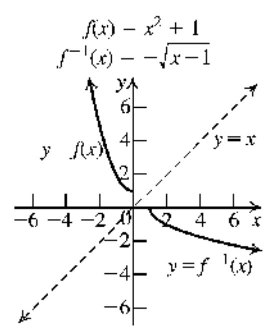
63. f is one-to-one since the domain is restricted, so an inverse exists.
$f(x) = y = |x| = x,\ x \ge 0$. Interchange the variables and solve for $y: y = x,\ x \ge 0$.



64. g is one-to-one since the domain is restricted, so an inverse exists.
$g(x) = y = |x| = -x,\ x \le 0$. Interchange the variables and solve for $y: y = -x,\ x \ge 0$.



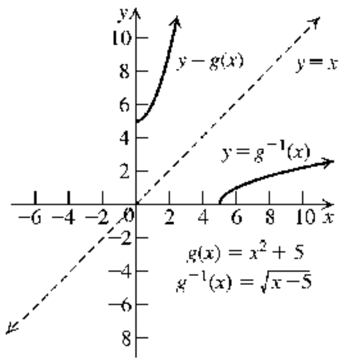
65. f is one-to-one since the domain is restricted, so an inverse exists.
$f(x) = y = x^2 + 1,\ x \le 0$. Interchange the variables and solve for y:
$x = y^2 + 1 \Rightarrow y = -\sqrt{x-1},\ x \ge 1$.



66. g is one-to-one since the domain is restricted, so an inverse exists.

$g(x) = y = x^2 + 5,\ x \ge 0$. Interchange the variables and solve for y:

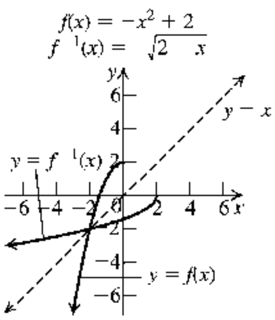
$x = y^2 + 5 \Rightarrow y = \sqrt{x-5},\ x \ge 5.$



67. f is one-to-one since the domain is restricted, so an inverse exists.

$f(x) = y = -x^2 + 2,\ x \le 0$. Interchange the variables and solve for y:

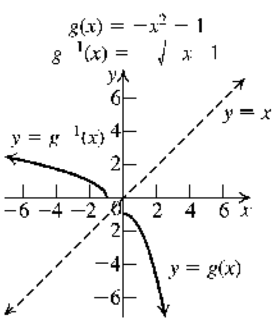
$x = -y^2 + 2 \Rightarrow y = -\sqrt{2-x},\ x \le 2.$



68. g is one-to-one since the domain is restricted, so an inverse exists.

$g(x) = y = -x^2 - 1,\ x \ge 0$. Interchange the variables and solve for y:

$x = -y^2 - 1 \Rightarrow y = \sqrt{-x-1},\ x \le -1.$



2.9 Applying the Concepts

69. a. $K(C) = C + 273 \Rightarrow$
$C(K) = K - 273 = K^{-1}(C).$
This represents the Celsius temperature corresponding to a given Kelvin temperature.

b. $C(300) = 300 - 273 = 27°\text{C}$

c. $K(22) = 22 + 273 = 295°\text{K}$

70. a. The two points are (212, 373) and (32, 273). The rate of change is
$\frac{373-273}{212-32} = \frac{100}{180} = \frac{5}{9}.$
$273 = \frac{5}{9}(32) + b \Rightarrow b = \frac{2297}{9} \Rightarrow$
$K(F) = \frac{5}{9}F + \frac{2297}{9}.$

b. $K = \frac{5}{9}F + \frac{2297}{9} \Rightarrow K - \frac{2297}{9} = \frac{5}{9}F \Rightarrow$
$9K - 2297 = 5F \Rightarrow F(K) = \frac{9}{5}K - \frac{2297}{5}$
This represents the Fahrenheit temperature corresponding to a given Kelvin temperature.

c. $K(98.6) = \frac{5}{9}(98.6) + \frac{2297}{9} = 310°\text{K}$

71. a.
$$F(K(C)) = \frac{9}{5}(C+273) - \frac{2297}{5} = \frac{9}{5}C + \frac{9(273)}{5} - \frac{2297}{5} = \frac{9}{5}C + \frac{160}{5} = \frac{9}{5}C + 32$$

b.
$$C(K(F)) = \frac{5}{9}F + \frac{2297}{9} - 273 = \frac{5}{9}F + \frac{2297-2457}{9} = \frac{5}{9}F - \frac{160}{9}$$

72.
$$F(C(x)) = \frac{9}{5}\left(\frac{5}{9}x - \frac{160}{9}\right) + 32 = x - 32 + 32 = x$$
$$C(F(x)) = \frac{5}{9}\left(\frac{9}{5}x + 32\right) - \frac{160}{9} = x + \frac{160}{9} - \frac{160}{9} = x$$
Therefore, F and C are inverses of each other.

73. a. $E(x) = 0.75x$, where x represents the number of dollars
$D(x) = 1.25x$, where x represents the number of euros.

b. $E(D(x)) = 0.75(1.25x) = 0.9375x \neq x.$
Therefore, the two functions are not inverses.

c. She loses money either way.

74. a. $w = 4 + 0.05x \Rightarrow w - 4 = 0.05x \Rightarrow$
$x = 20w - 80.$
This represents the food sales in terms of his hourly wage.

b. $x = 20(12) - 80 = \$160$

75. a. $7 = 4 + 0.05x \Rightarrow x = \$60$. This means that if food sales $\leq$ \$60, he will receive the minimum hourly wage. If food sales > \$60, his wages will be based on food sales.

$$w = \begin{cases} 4 + 0.05x & \text{if } x > 60 \\ 7 & \text{if } x \leq 60 \end{cases}$$

b. The function does not have an inverse because it is constant on (0, 60), and it is not one-to-one.

c. If the domain is restricted to $[60, \infty)$, the function has an inverse.

76. a. $T = 1.11\sqrt{l} \Rightarrow l = \left(\dfrac{T}{1.11}\right)^2$. This shows the length as the function of the period.

b. $l = \left(\dfrac{2}{1.11}\right)^2 \approx 3.2$ ft

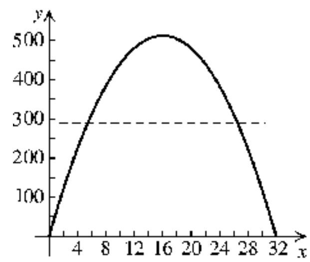
c. $T = 1.11\sqrt{70} \approx 9.3$ sec

77. a. $V = 8\sqrt{x} \Rightarrow \dfrac{V}{8} = \sqrt{x} \Rightarrow \dfrac{1}{64}V^2 = x = V^{-1}(x)$
This represents the height of the water in terms of the velocity.

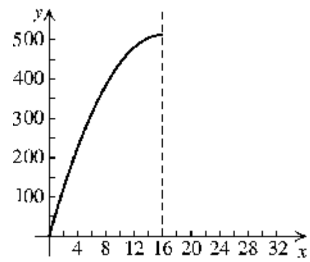
b. (i) $x = \dfrac{1}{64}\left(30^2\right) = 14.0625$ ft

(ii) $x = \dfrac{1}{64}\left(20^2\right) = 6.25$ ft

78. a. $y = 64x - 2x^2$ has no inverse because it is not one-to-one across its domain, [0, 32]. (It fails the horizontal line test.)



However, if the domain is restricted to [0, 16], the function is one-to-one, and it has an inverse.



$y = 64x - 2x^2 \Rightarrow 2x^2 - 64x + y = 0 \Rightarrow$

$$x = \frac{64 \pm \sqrt{64^2 - 8y}}{4} \Rightarrow$$

$$x = \frac{64 \pm \sqrt{4096 - 8y}}{4} = \frac{64 \pm 2\sqrt{1024 - 2y}}{4}$$

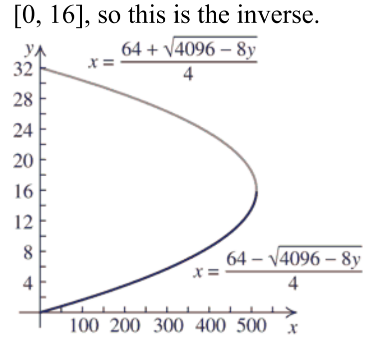
$$= \frac{32 \pm \sqrt{1024 - 2y}}{2}$$

$1024 - 2y \geq 0 \Rightarrow 0 \leq y \leq 512.$

(Because y is a number of feet, it cannot be negative.) This is the range of the original function. The domain of the original function is [0, 16], which is the range of the inverse.

The range of $x = \dfrac{32 + \sqrt{1024 - 2y}}{2}$ is [16, 32], so this is not the inverse.

The range of $x = \dfrac{32 - \sqrt{1024 - 2y}}{2}$, $0 \leq y \leq 512$, is [0, 16], so this is the inverse.



Note that the bottom half of the graph is the inverse.

b. **(i)** $x = \frac{64 - \sqrt{4096 - 8(32)}}{4} \approx 0.51$ ft

(ii) $x = \frac{64 - \sqrt{4096 - 8(256)}}{4} \approx 4.69$ ft

(iii) $x = \frac{64 - \sqrt{4096 - 8(512)}}{4} \approx 16$ ft

79. a. The function represents the amount she still owes after x months.

b. $y = 36,000 - 600x$. Interchange the variables and solve for y: $x = 36,000 - 600y \Rightarrow$

$600y = 36,000 - x \Rightarrow y = 60 - \frac{x}{600} \Rightarrow$

$f^{-1}(x) = 60 - \frac{1}{600}x.$

This represents the number of months that have passed from the first payment until the balance due is \$$x$.

c. $y = 60 - \frac{1}{600}(22,000) = 23.33 \approx 24$ months

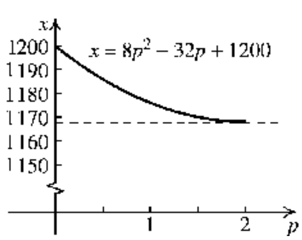
There are 24 months remaining.

80. a. To find the inverse, solve $x = 8p^2 - 32p + 1200$ for p:

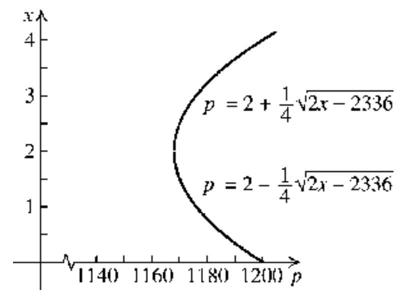
$8p^2 - 32p + 1200 - x = 0 \Rightarrow$

$$p = \frac{32 \pm \sqrt{(-32)^2 - 4(8)(1200 - x)}}{2(8)}$$
$$= \frac{32 \pm \sqrt{1024 - 38,400 + 32x}}{16}$$
$$= \frac{32 \pm \sqrt{32x - 37376}}{16} = \frac{32 \pm 4\sqrt{2x - 2336}}{16}$$
$$= 2 \pm \frac{1}{4}\sqrt{2x - 2336}$$

Because the domain of the original function is (0, 2], its range is [1168, 1200).



So the domain of the inverse is [1168, 1200), and its range is (0, 2]. The range of $p = 2 + \frac{1}{4}\sqrt{2x - 2336}$ is (2, 4], so it is not the inverse. The range of $p = 2 - \frac{1}{4}\sqrt{2x - 2336}, 1168 \le x < 1200,$ is (0, 2], so it is the inverse. This gives the price of computer chips in terms of the demand x.



Note that the bottom half of the graph is the inverse.

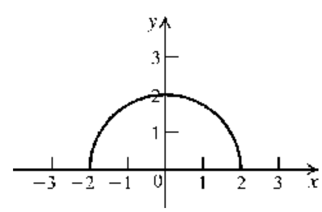
b. $p = 2 - \frac{1}{4}\sqrt{2(1180.5) - 2336} = \$0.75$

2.9 Beyond the Basics

81. $f(g(3)) = f(1) = 3, f(g(5)) = f(3) = 5,$ and $f(g(2)) = f(4) = 2 \Rightarrow f(g(x)) = x$ for each x.
$g(f(1)) = g(3) = 1, g(f(3)) = g(5) = 3,$ and $g(f(4)) = g(2) = 4 \Rightarrow g(f(x)) = x$ for each x.
So, f and g are inverses.

82. $f(g(-2)) = f(1) = -2, f(g(0)) = f(2) = 0,$ $f(g(-3)) = f(3) = -3,$ and $f(g(-2)) = f(1) = -2 \Rightarrow f(g(x)) = x$ for each x.
$g(f(1)) = g(-2) = 1, g(f(2)) = g(0) = 2,$ $g(f(3)) = g(-3) = 3,$ and $g(f(4)) = g(1) = 4$ $\Rightarrow g(f(x)) = x$ for each x.
So f and g are inverses.

83. a.



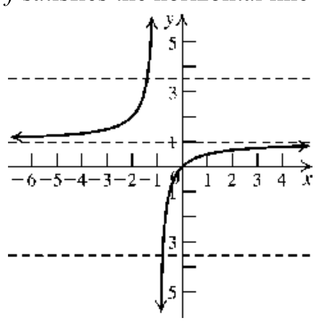
b. f is not one-to-one

c. Domain: [–2, 2]; range: [0, 2]

84. a. Domain: $(-\infty, 2) \cup [3, \infty)$. Note that the domain is not $(-\infty, 2) \cup (2, \infty)$ because $[\![x]\!] = 2$ for $2 \le x < 3$.

b. The function is not one-to-one. The function is constant on each interval $[n, n + 1)$, n an integer.

85. a. f satisfies the horizontal line test.



b. $y = 1 - \dfrac{1}{x+1}$. Interchange the variables and solve for y: $x = 1 - \dfrac{1}{y+1} \Rightarrow$

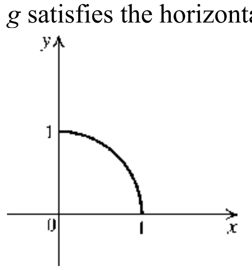
$\dfrac{1}{y+1} = 1 - x \Rightarrow 1 = y + 1 - xy - x \Rightarrow$

$xy - y = -x \Rightarrow y(x-1) = -x \Rightarrow$

$y = f^{-1}(x) = -\dfrac{x}{x-1} = \dfrac{x}{1-x}$

c. Domain of f: $(-\infty, -1) \cup (-1, \infty)$; range of f: $(-\infty, 1) \cup (1, \infty)$.

86. a. g satisfies the horizontal line test.



b. $y = \sqrt{1-x^2}$. Interchange the variables and solve for y: $x = \sqrt{1-y^2} \Rightarrow$

$x^2 = 1 - y^2 \Rightarrow y^2 = 1 - x^2 \Rightarrow$

$y = g^{-1}(x) = \sqrt{1-x^2}$

c. Domain of f = range of f: [0, 1]

87. a. (i) $f(x) = y = 2x - 1$. Interchange the variables and solve for y: $x = 2y - 1 \Rightarrow$

$y = f^{-1}(x) = \dfrac{1}{2}x + \dfrac{1}{2}$

(ii) $g(x) = y = 3x + 4$. Interchange the variables and solve for y: $x = 3y + 4 \Rightarrow$

$y = g^{-1}(x) = \dfrac{1}{3}x - \dfrac{4}{3}$

(iii) $(f \circ g)(x) = 2(3x+4) - 1 = 6x + 7$

(iv) $(g \circ f)(x) = 3(2x-1) + 4 = 6x + 1$

(v) $(f \circ g)(x) = y = 6x + 7$. Interchange the variables and solve for y:

$x = 6y + 7 \Rightarrow (f \circ g)^{-1}(x) = \dfrac{1}{6}x - \dfrac{7}{6}$

(vi) $(g \circ f)(x) = y = 6x + 1$. Interchange the variables and solve for y:

$x = 6y + 1 \Rightarrow (f \circ g)^{-1}(x) = \dfrac{1}{6}x - \dfrac{1}{6}$

(vii) $\left(f^{-1} \circ g^{-1}\right)(x) = \dfrac{1}{2}\left(\dfrac{1}{3}x - \dfrac{4}{3}\right) + \dfrac{1}{2}$

$= \dfrac{1}{6}x - \dfrac{2}{3} + \dfrac{1}{2} = \dfrac{1}{6}x - \dfrac{1}{6}$

(viii) $\left(g^{-1} \circ f^{-1}\right)(x) = \dfrac{1}{3}\left(\dfrac{1}{2}x + \dfrac{1}{2}\right) - \dfrac{4}{3}$

$= \dfrac{1}{6}x + \dfrac{1}{6} - \dfrac{4}{3} = \dfrac{1}{6}x - \dfrac{7}{6}$

b. (i) $(f \circ g)^{-1}(x) = \dfrac{1}{6}x - \dfrac{7}{6}$

$= \left(g^{-1} \circ f^{-1}\right)(x)$

(ii) $(g \circ f)^{-1}(x) = \dfrac{1}{6}x - \dfrac{1}{6}$

$= \left(f^{-1} \circ g^{-1}\right)(x)$

88. a. (i) $f(x) = y = 2x + 3$. Interchange the variables and solve for y: $x = 2y + 3 \Rightarrow$

$y = f^{-1}(x) = \dfrac{1}{2}x - \dfrac{3}{2}$

(ii) $g(x) = y = x^3 - 1$. Interchange the variables and solve for y: $x = y^3 - 1 \Rightarrow$

$y = g^{-1}(x) = \sqrt[3]{x+1}$

(iii) $(f \circ g)(x) = 2\left(x^3 - 1\right) + 3 = 2x^3 + 1$

(iv) $(g \circ f)(x) = (2x+3)^3 - 1$

$= 8x^3 + 36x^2 + 54x + 26$

(v) $(f \circ g)(x) = y = 2x^3 + 1$. Interchange the variables and solve for y:

$x = 2y^3 + 1 \Rightarrow (f \circ g)^{-1}(x) = \sqrt[3]{\frac{x-1}{2}}$

(vi) $(g \circ f)(x) = y$
$= 8x^3 + 36x^2 + 54x + 26$

Interchange the variables and solve for y:

$x = 8y^3 + 36y^2 + 54y + 26 \Rightarrow$
$x + 1 = 8y^3 + 36y^2 + 54y + 27 \Rightarrow$
$x + 1 = (2y+3)^3 \Rightarrow \sqrt[3]{x+1} = 2y + 3 \Rightarrow$
$y = (g \circ f)^{-1}(x) = \frac{1}{2}\sqrt[3]{x+1} - \frac{3}{2}$

(vii) $\left(f^{-1} \circ g^{-1}\right)(x) = \frac{1}{2}\left(\sqrt[3]{x+1}\right) - \frac{3}{2}$

(viii) $\left(g^{-1} \circ f^{-1}\right)(x) = \sqrt[3]{\frac{1}{2}x - \frac{3}{2} + 1}$
$= \sqrt[3]{\frac{1}{2}x - \frac{1}{2}} = \sqrt[3]{\frac{x-1}{2}}$

b. (i) $(f \circ g)^{-1}(x) = \sqrt[3]{\frac{1}{2}x - \frac{1}{2}} = \sqrt[3]{\frac{x-1}{2}}$
$= \left(g^{-1} \circ f^{-1}\right)(x)$

(ii) $(g \circ f)^{-1}(x) = \frac{1}{2}\left(\sqrt[3]{x+1}\right) - \frac{3}{2}$
$= \left(f^{-1} \circ g^{-1}\right)(x)$

2.9 Critical Thinking/Discussion/Writing

89. No. For example, $f(x) = x^3 - x$ is odd, but it does not have an inverse, because $f(0) = f(1)$, so it is not one-to-one.

90. Yes. The function $f = \{(0,1)\}$ is even, and it has an inverse: $f^{-1} = \{(1,0)\}$.

91. Yes, because increasing and decreasing functions are one-to-one.

92. a. $R = \{(-1,1), (0,0), (1,1)\}$

b. $R = \{(-1,1), (0,0), (1,2)\}$

2.9 Getting Ready for the Next Section

93. $x^2 - x - 12 = (x+3)(x-4)$

94. $x^2 - 5x + 6 = (x-2)(x-3)$

95. $x^2 + 2x - 8 = (x+4)(x-2)$

96. $x^2 + 7x + 10 = (x+2)(x+5)$

97. $x^2 - 7x + 12 = 0$
$(x-3)(x-4) = 0$
$x - 3 = 0 \Rightarrow x = 3$ | $x - 4 = 0 \Rightarrow x = 4$
Solution: {3, 4}

98. $x^2 - x - 6 = 0$
$(x+2)(x-3) = 0$
$x + 2 = 0 \Rightarrow x = -2$ | $x - 3 = 0 \Rightarrow x = 3$
Solution: {–2, 3}

99. $3x^2 + 7x + 2 = 0$
$(3x+1)(x+2) = 0$
$3x + 1 = 0 \Rightarrow x = -\frac{1}{3}$ | $x + 2 = 0 \Rightarrow x = -2$
Solution: $\left\{-2, -\frac{1}{3}\right\}$

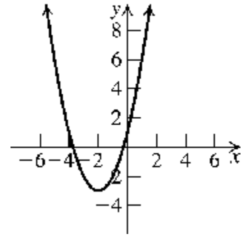
100. $x^2 - 4x + 1 = 0$
Use the quadratic formula.
$a = 1, b = -4, c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

Solution: $\left\{2 - \sqrt{3}, 2 + \sqrt{3}\right\}$

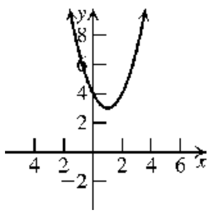
101. $y = (x+2)^2 - 3$

Start with the graph of $f(x) = x^2$, then shift it two units left and three units down.



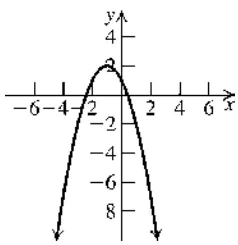
102. $y=(x-1)^2+3$

Start with the graph of $f(x)=x^2$, then shift it one unit right and three units up.



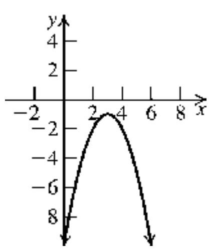
103. $y=-(x+1)^2+2$

Start with the graph of $f(x)=x^2$, then shift it one unit left. Reflect the graph across the x-axis and then shift it two units up



104. $y=-(x-3)^2-1$

Start with the graph of $f(x)=x^2$, then shift it three units right. Reflect the graph across the
the
x-axis and then shift it one unit down.



Chapter 2 Review Exercises

Building Skills

1. False. The midpoint is $\left(\frac{-3+3}{2},\frac{1+11}{2}\right)=(0,6)$.

2. False. The equation is a circle with center $(-2,-3)$ and radius $\sqrt{5}$.

3. True

4. False. A graph that is symmetric with respect to the origin is the graph of an odd function. A graph that is symmetric with respect to the y-axis is the graph of an even function.

5. False.
The slope is 4/3 and the y-intercept is 3.

6. False. The slope of a line that is perpendicular to a line with slope 2 is –1/2.

7. True

8. False. There is no graph because the radius cannot be negative.

9. a. $d(P,Q)=\sqrt{(-1-3)^2+(3-5)^2}=2\sqrt{5}$

b. $M=\left(\frac{3+(-1)}{2},\frac{5+3}{2}\right)=(1,4)$

c. $m=\frac{3-5}{-1-3}=\frac{1}{2}$

10. a. $d(P,Q)=\sqrt{(3-(-3))^2+(-1-5)^2}=6\sqrt{2}$

b. $M=\left(\frac{-3+3}{2},\frac{5+(-1)}{2}\right)=(0,2)$

c. $m=\frac{-1-5}{3-(-3)}=-1$

11. a. $d(P,Q)=\sqrt{(9-4)^2+(-8-(-3))^2}=5\sqrt{2}$

b. $M=\left(\frac{4+9}{2},\frac{-3+(-8)}{2}\right)=\left(\frac{13}{2},-\frac{11}{2}\right)$

c. $m=\frac{-8-(-3)}{9-4}=-1$

12. a. $d(P,Q)=\sqrt{(-7-2)^2+(-8-3)^2}=\sqrt{202}$

b. $M=\left(\frac{2+(-7)}{2},\frac{3+(-8)}{2}\right)=\left(-\frac{5}{2},-\frac{5}{2}\right)$

c. $m=\frac{-8-3}{-7-2}=\frac{11}{9}$

13. a. $D(P,Q)=\sqrt{(5-2)^2+(-2-(-7))^2}=\sqrt{34}$

b. $M=\left(\frac{2+5}{2},\frac{-7+(-2)}{2}\right)=\left(\frac{7}{2},-\frac{9}{2}\right)$

c. $m=\frac{-2-(-7)}{5-2}=\frac{5}{3}$

14. a. $d(P,Q)=\sqrt{(10-(-5))^2+(-3-4)^2}=\sqrt{274}$

b. $M=\left(\frac{-5+10}{2},\frac{4+(-3)}{2}\right)=\left(\frac{5}{2},\frac{1}{2}\right)$

c. $m=\frac{-3-4}{10-(-5)}=-\frac{7}{15}$

15. $d(A,B)=\sqrt{(-2-0)^2+(-3-5)^2}=\sqrt{68}$
$d(A,C)=\sqrt{(3-0)^2+(0-5)^2}=\sqrt{34}$
$d(B,C)=\sqrt{(3-(-2))^2+(0-(-3))^2}=\sqrt{34}$
Using the Pythagorean theorem, we have
$AC^2+BC^2=\left(\sqrt{34}\right)^2+\left(\sqrt{34}\right)^2$
$=68=\left(\sqrt{68}\right)^2=AB^2$
Alternatively, we can show that AC and CB are perpendicular using their slopes.
$m_{AC}=\frac{0-5}{3-0}=-\frac{5}{3}; m_{CB}=\frac{0-(-3)}{3-(-2)}=\frac{3}{5}$
$m_{AC}\cdot m_{CB}=-1\Rightarrow AC\perp CB$, so $\triangle ABC$ is a right triangle.

16. $d(A,B)=\sqrt{(4-1)^2+(8-2)^2}=3\sqrt{5}$
$d(C,D)=\sqrt{(10-7)^2+(5-(-1))^2}=3\sqrt{5}$
$d(A,C)=\sqrt{(7-1)^2+(-1-2)^2}=3\sqrt{5}$
$d(B,D)=\sqrt{(10-4)^2+(5-8)^2}=3\sqrt{5}$
The four sides are equal, so the quadrilateral is a rhombus.

17. $A=(-6,3), B=(4,5)$
$d(A,O)=\sqrt{(-6-0)^2+(3-0)^2}=\sqrt{45}$
$d(B,O)=\sqrt{(4-0)^2+(5-0)^2}=\sqrt{41}$
(4, 5) is closer to the origin.

18. $A=(-6,4), B=(5,10), C=(2,3)$
$d(A,C)=\sqrt{(2-(-6))^2+(3-4)^2}=\sqrt{65}$
$d(B,C)=\sqrt{(2-5)^2+(3-10)^2}=\sqrt{58}$
(5, 10) is closer to (2, 3).

19. $A=(-5,3), B=(4,7), C=(x,0)$
$d(A,C)=\sqrt{(x-(-5))^2+(0-3)^2}$
$=\sqrt{(x+5)^2+9}$
$d(B,C)=\sqrt{(x-4)^2+(0-7)^2}$
$=\sqrt{(x-4)^2+49}$
$d(A,C)=d(B,C)\Rightarrow$
$\sqrt{(x+5)^2+9}=\sqrt{(x-4)^2+49}$
$(x+5)^2+9=(x-4)^2+49$
$x^2+10x+34=x^2-8x+65$
$x=\frac{31}{18}\Rightarrow$ The point is $\left(\frac{31}{18},0\right)$.

20. $A=(-3,-2), B(2,-1), C(0,y)$
$d(A,C)=\sqrt{(0-(-3))^2+(y-(-2))^2}$
$=\sqrt{(y+2)^2+9}$
$d(B,C)=\sqrt{(0-(2))^2+(y-(-1))^2}$
$=\sqrt{(y+1)^2+4}$
$d(A,C)=d(B,C)\Rightarrow$
$\sqrt{(y+2)^2+9}=\sqrt{(y+1)^2+4}$
$(y+2)^2+9=(y+1)^2+4$
$y^2+4y+13=y^2+2y+5$
$y=-4\Rightarrow$ The point is $(0,-4)$.

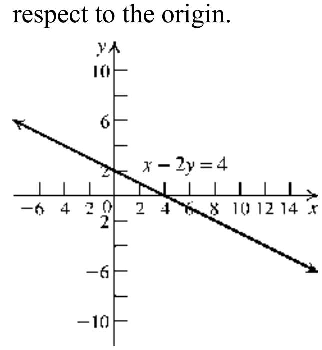
21. Not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.

22. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.

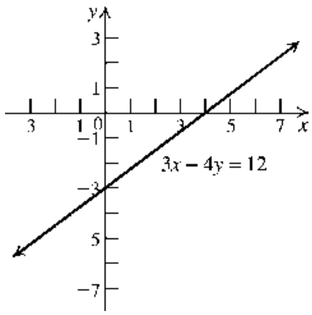
23. Symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.

24. Symmetric with respect to the x-axis; symmetric with respect to the y-axis; symmetric with respect to the origin.

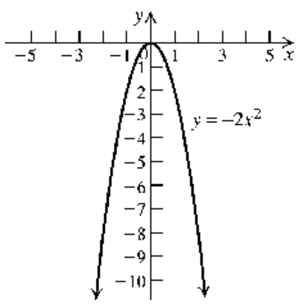
25. x-intercept: 4; y-intercept: 2; not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.



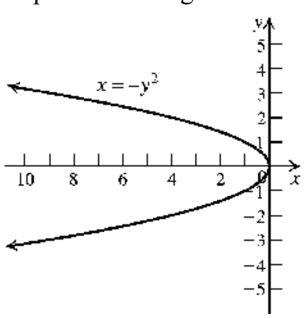
26. x-intercept: 4; y-intercept: –3; not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.



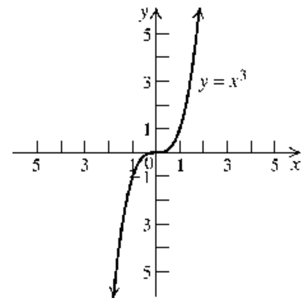
27. x-intercept: 0; y-intercept: 0; not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.



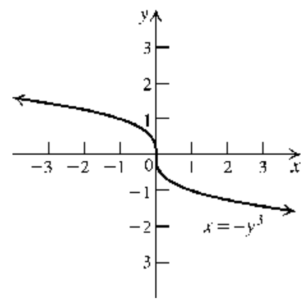
28. x-intercept: 0; y-intercept: 0; symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.



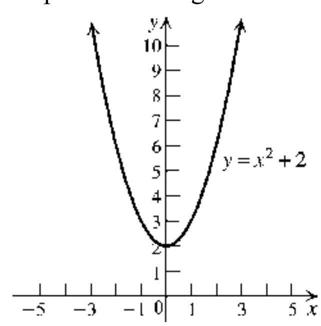
29. x-intercept: 0; y-intercept: 0; not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.



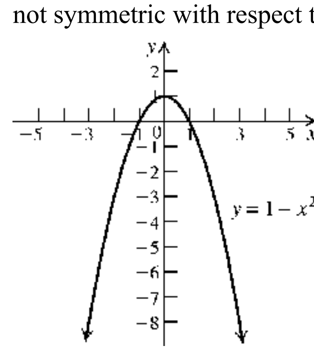
30. x-intercept: 0; y-intercept: 0; not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.



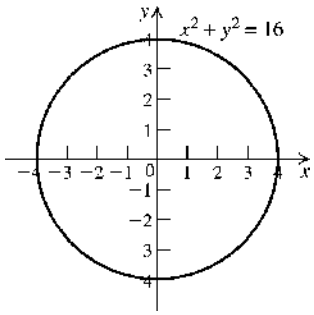
31. No x-intercept; y-intercept: 2; not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.



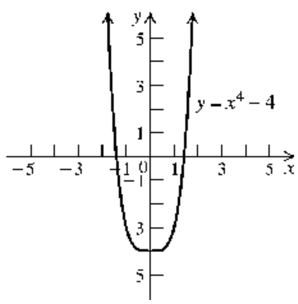
32. x-intercepts: –1, 1; y-intercept: 1;
not symmetric with respect to the x-axis;
symmetric with respect to the y-axis;
not symmetric with respect to the origin.



33. x-intercepts: –4, 4; y-intercepts: –4, 4;
symmetric with respect to the x-axis;
symmetric with respect to the y-axis;
symmetric with respect to the origin.



34. x-intercepts: $-\sqrt{2}, \sqrt{2}$; y-intercept: –4
not symmetric with respect to the x-axis
symmetric with respect to the y-axis
not symmetric with respect to the origin.

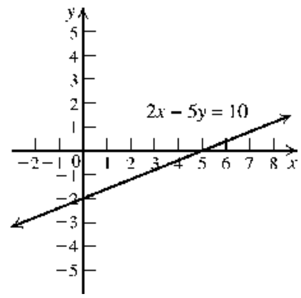


35. $(x-2)^2 + (y+3)^2 = 25$

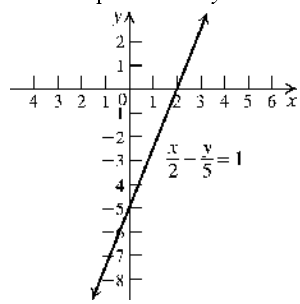
36. The center of the circle is the midpoint of the diameter. $M = \left(\frac{5+(-5)}{2} + \frac{2+4}{2}\right) = (0,3).$
The length of the radius is the distance from the center to one of the endpoints of the diameter $= \sqrt{(5-0)^2 + (2-3)^2} = \sqrt{26}.$ The equation of the circle is $x^2 + (y-3)^2 = 26.$

37. The radius is 2, so the equation of the circle is $(x+2)^2 + (y+5)^2 = 4.$

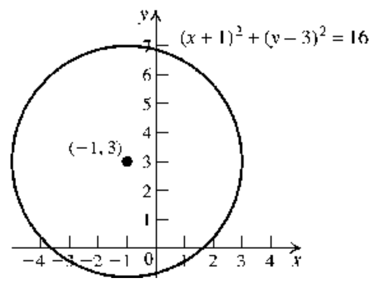
38. $2x - 5y = 10 \Rightarrow \frac{2}{5}x - 2 = y.$
Line with slope 2/5 and y-intercept –2.



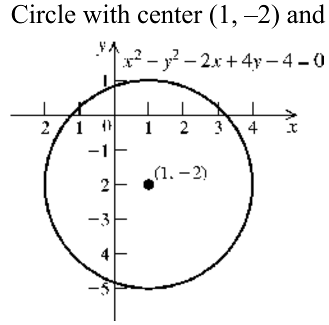
39. $\frac{x}{2} - \frac{y}{5} = 1 \Rightarrow 5x - 2y = 10 \Rightarrow \frac{5}{2}x - 5 = y.$ Line with slope 5/2 and y-intercept –5.



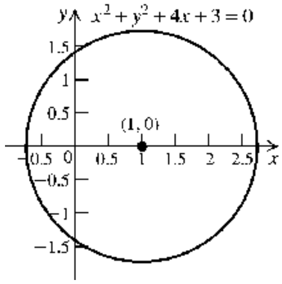
40. Circle with center (–1, 3) and radius 4.



41. $x^2 + y^2 - 2x + 4y - 4 = 0 \Rightarrow$
$x^2 - 2x + 1 + y^2 + 4y + 4 = 4 + 1 + 4 \Rightarrow$
$(x-1)^2 + (y+2)^2 = 9.$
Circle with center (1, –2) and radius 3.



42. $3x^2 + 3y^2 - 6x - 6 = 0 \Rightarrow x^2 - 2x + y^2 = 2 \Rightarrow$
$x^2 - 2x + 1 + y^2 = 2 + 1 \Rightarrow (x-1)^2 + y^2 = 3.$
Circle with center (1, 0) and radius $\sqrt{3}$.



43. $y - 2 = -2(x-1) \Rightarrow y = -2x + 4$

44. $m = \dfrac{5-0}{0-2} = -\dfrac{5}{2};\ y = -\dfrac{5}{2}x + 5$

45. $m = \dfrac{7-3}{-1-1} = -2; 3 = -2(1) + b \Rightarrow 5 = b \Rightarrow$
$y = -2x + 5$

46. $x = 1$

47. a. $y = 3x - 2 \Rightarrow m = 3;\ y = 3x + 2 \Rightarrow m = 3$
The slopes are equal, so the lines are parallel.

b. $3x - 5y + 7 \Rightarrow m = 3/5;$
$5x - 3y + 2 = 0 \Rightarrow m = 5/3$
The slopes are neither equal nor negative reciprocals, so the lines are neither parallel nor perpendicular.

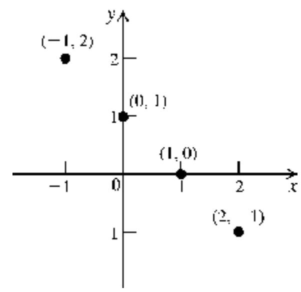
c. $ax + by + c = 0 \Rightarrow m = -a/b;$
$bx - ay + d = 0 \Rightarrow m = b/a$
The slopes are negative reciprocals, so the lines are perpendicular.

d. $y + 2 = \dfrac{1}{3}(x-3) \Rightarrow m = \dfrac{1}{3};$
$y - 5 = 3(x-3) \Rightarrow m = 3$
The slopes are neither equal nor negative reciprocals, so the lines are neither parallel nor perpendicular.

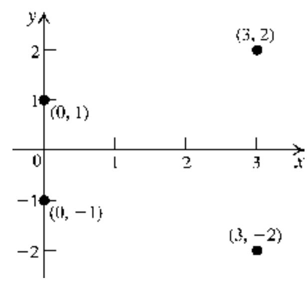
48. a. The equation with x-intercept 4 passes through the points (0, 2) and (4, 0), so its slope is $\dfrac{0-2}{4-0} = -\dfrac{1}{2}$. Thus, the slope of the line we are seeking is also $-\frac{1}{2}$. The line passes through (0, 1), so its equation is
$y - 1 = -\dfrac{1}{2}(x - 0) \Rightarrow y = -\dfrac{1}{2}x + 1.$

b. The slope of the line we are seeking is 2 and the line passes through the origin, so its equation is $y - 0 = 2(x - 0)$, or $y = 2x$.

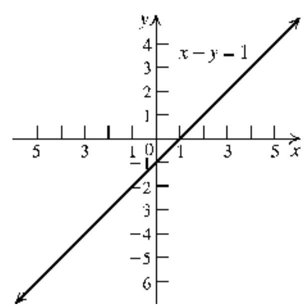
49. Domain: {–1, 0, 1, 2}; range: {–1, 0, 1, 2}.
This is a function.



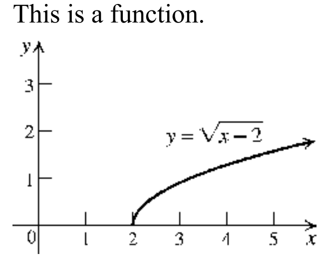
50. Domain: {0, 3}; range: {–2, –1, 1, 2}.
This is not a function.



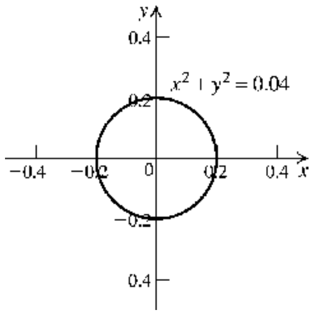
51. Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$.
This is a function.



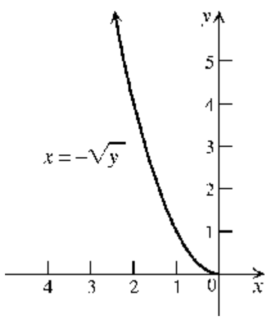
52. Domain: $[2, \infty)$; range: $[0, \infty)$.
This is a function.



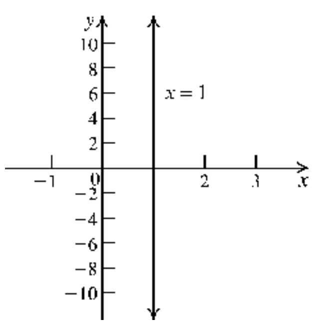
53. Domain: $[-0.2, 0.2]$; range: $[-0.2, 0.2]$.
This is not a function.



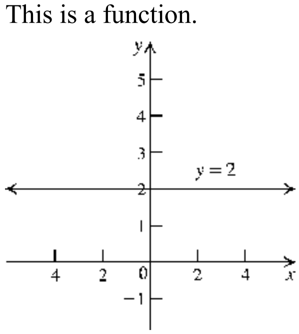
54. Domain: $(-\infty, 0]$; range: $[0, \infty)$.
This is a function.



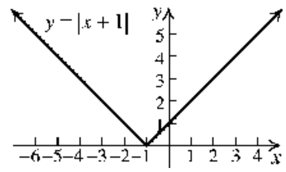
55. Domain: $\{1\}$; range: $(-\infty, \infty)$.
This is not a function.



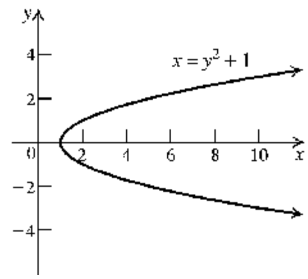
56. Domain: $(-\infty, \infty)$; range: $\{2\}$.
This is a function.



57. Domain: $(-\infty, \infty)$; range: $[0, \infty)$.
This is a function.



58. Domain: $[1, \infty)$; range: $(-\infty, \infty)$.
This is not a function.



59. $f(-2) = 3(-2) + 1 = -5$

60. $g(-2) = (-2)^2 - 2 = 2$

61. $f(x) = 4 \Rightarrow 3x + 1 = 4 \Rightarrow x = 1$

62. $g(x) = 2 \Rightarrow x^2 - 2 = 2 \Rightarrow x = \pm 2$

63. $(f + g)(1) = f(1) + g(1)$
$= (3(1) + 1) + (1^2 - 2) = 3$

64. $(f - g)(-1) = f(-1) - g(-1)$
$= (3(-1) + 1) - ((-1)^2 - 2) = -1$

65. $(f \cdot g)(-2) = f(-2) \cdot g(-2)$
$= (3(-2) + 1) \cdot ((-2)^2 - 2) = -10$

66. $(g \cdot f)(0) = g(0) \cdot f(0)$
$= (0^2 - 2) \cdot (3(0) + 1) = -2$

67. $(f \circ g)(3) = 3(3^2 - 2) + 1 = 22$

68. $(g \circ f)(-2) = (3(-2) + 1)^2 - 2 = 23$

69. $(f \circ g)(x) = 3(x^2 - 2) + 1 = 3x^2 - 5$

70. $(g \circ f)(x) = (3x + 1)^2 - 2 = 9x^2 + 6x - 1$

71. $(f \circ f)(x) = 3(3x + 1) + 1 = 9x + 4$

72. $(g \circ g)(x) = (x^2 - 2)^2 - 2 = x^4 - 4x^2 + 2$

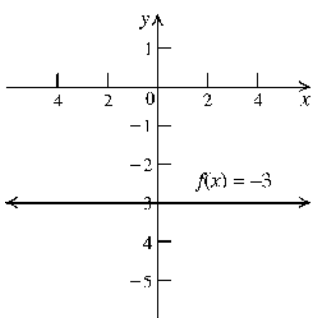
73. $f(a+h)=3(a+h)+1=3a+3h+1$

74. $g(a-h)=(a-h)^2-2=a^2-2ah+h^2-2$

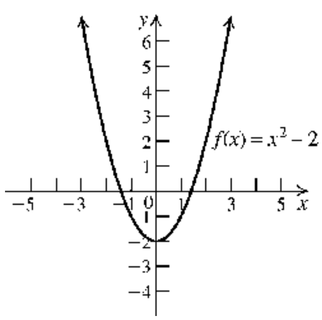
75. $$\frac{f(x+h)-f(x)}{h}=\frac{(3(x+h)+1)-(3x+1)}{h}=\frac{3x+3h+1-3x-1}{h}=\frac{3h}{h}=3$$

76. $$\frac{g(x+h)-g(x)}{h}=\frac{((x+h)^2-2)-(x^2-2)}{h}=\frac{x^2+2xh+h^2-2-x^2+2}{h}=\frac{h^2+2xh}{h}=h+2x$$

77. Domain: $(-\infty,\infty)$; range: $\{-3\}$. Constant on $(-\infty,\infty)$.

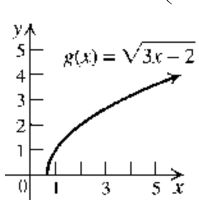


78. Domain: $(-\infty,\infty)$; range: $[-2,\infty)$. Decreasing on $(-\infty,0)$; increasing on $(0,\infty)$.

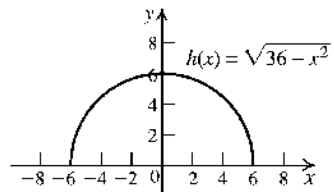


79. Domain: $\left[\frac{2}{3},\infty\right)$; range: $[0,\infty)$

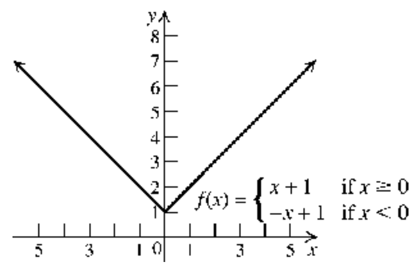
Increasing on $\left(\frac{2}{3},\infty\right)$.



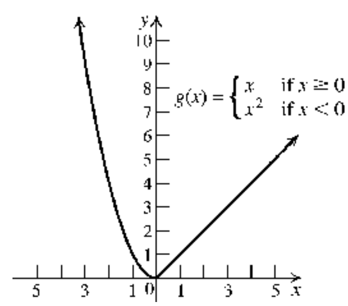
80. Domain: $[-6,6]$; range: $[0,6]$. Increasing on $(-6,0)$; decreasing on $(0,6)$.



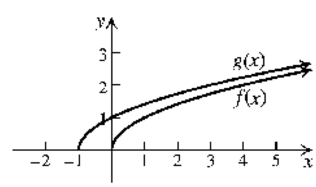
81. Domain: $(-\infty,\infty)$; range: $[1,\infty)$. Decreasing on $(-\infty,0)$; increasing on $(0,\infty)$.



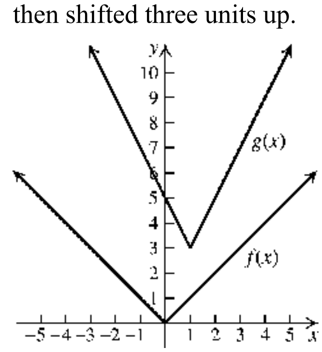
82. Domain: $(-\infty,\infty)$; range: $[0,\infty)$. Decreasing on $(-\infty,0)$; increasing on $(0,\infty)$.



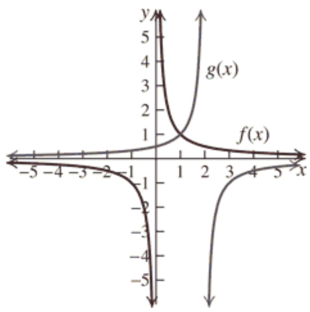
83. The graph of *g* is the graph of *f* shifted one unit left.



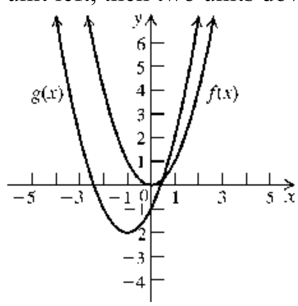
84. The graph of *g* is the graph of *f* shifted one unit right, stretched vertically by a factor of 2, then shifted three units up.



85. The graph of g is the graph of f shifted two units right, and then reflected in the x-axis.



86. The graph of g is the graph of f shifted one unit left, then two units down.



87. $f(-x) = (-x)^2 - (-x)^4 = x^2 - x^4 = f(x) \Rightarrow$ $f(x)$ is even. Not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.

88. $f(-x) = (-x)^3 + (-x) = -x^3 - x = -f(x) \Rightarrow$ $f(x)$ is odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.

89. $f(-x) = |-x| + 3 = |x| + 3 = f(x) \Rightarrow$ $f(x)$ is even. Not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.

90. $f(-x) = -3x + 5 \neq f(x)$ or $f(-x) \Rightarrow f(x)$ is neither even nor odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.

91. $f(-x) = \sqrt{-x} \neq f(x)$ or $f(-x) \Rightarrow f(x)$ is neither even nor odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.

92. $f(-x) = -\dfrac{2}{x} = -f(x) \Rightarrow f(x)$ is odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.

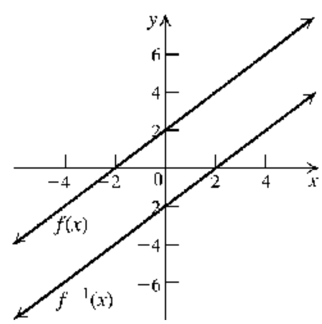
93. $f(x) = \sqrt{x^2 - 4} \Rightarrow f(x) = (g \circ h)(x)$ where $g(x) = \sqrt{x}$ and $h(x) = x^2 - 4$.

94. $g(x) = \left(x^2 - x + 2\right)^{50} \Rightarrow g(x) = (f \circ h)(x)$ where $f(x) = x^{50}$ and $h(x) = x^2 - x + 2$.

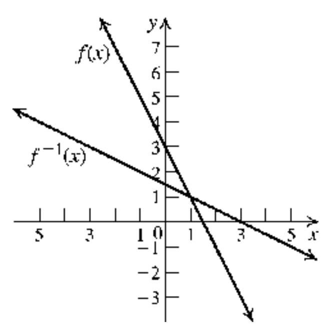
95. $h(x) = \sqrt{\dfrac{x-3}{2x+5}} \Rightarrow h(x) = (f \circ g)(x)$ where $f(x) = \sqrt{x}$ and $g(x) = \dfrac{x-3}{2x+5}$.

96. $H(x) = (2x-1)^3 + 5 \Rightarrow H(x) = (f \circ g)(x)$ where $f(x) = x^3 + 5$ and $g(x) = 2x - 1$.

97. $f(x)$ is one-to-one. $f(x) = y = x + 2$. Interchange the variables and solve for y: $x = y + 2 \Rightarrow y = x - 2 = f^{-1}(x)$.



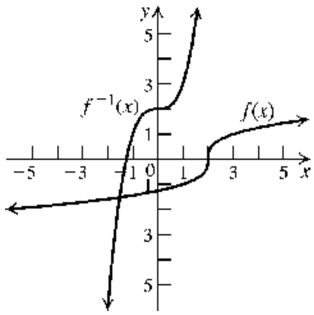
98. $f(x)$ is one-to-one. $f(x) = y = -2x + 3$. Interchange the variables and solve for y: $x = -2y + 3 \Rightarrow y = -\dfrac{1}{2}x + \dfrac{3}{2} = f^{-1}(x)$.



99. $f(x)$ is one-to-one. $f(x) = y = \sqrt[3]{x-2}$.

Interchange the variables and solve for y:

$x = \sqrt[3]{y-2} \Rightarrow y = x^3 + 2 = f^{-1}(x)$.

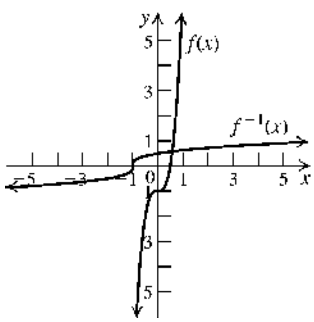


100. $f(x)$ is one-to-one. $f(x) = y = 8x^3 - 1$.

Interchange the variables and solve for y:

$x = 8y^3 - 1 \Rightarrow y = \sqrt[3]{\dfrac{x+1}{8}} \Rightarrow$

$y = \dfrac{1}{2}\sqrt[3]{x+1} = f^{-1}(x)$.



101. $f(x) = y = \dfrac{x-1}{x+2}, x \neq -2$.

Interchange the variables and solve for y.

$x = \dfrac{y-1}{y+2} \Rightarrow xy + 2x = y - 1 \Rightarrow$

$xy - y = -2x - 1 \Rightarrow y(x-1) = -2x - 1 \Rightarrow$

$y = \dfrac{-2x-1}{x-1} \Rightarrow y = f^{-1}(x) = \dfrac{2x+1}{1-x}$

Domain of f: $(-\infty, -2) \cup (-2, \infty)$

Range of f: $(-\infty, 1) \cup (1, \infty)$

102. $f(x) = y = \dfrac{2x+3}{x-1}, x \neq 1$.

Interchange the variables and solve for y.

$x = \dfrac{2y+3}{y-1} \Rightarrow xy - x = 2y + 3 \Rightarrow$

$xy - 2y = x + 3 \Rightarrow y(x-2) = x + 3 \Rightarrow$

$y = f^{-1}(x) = \dfrac{x+3}{x-2}$

Domain of f: $(-\infty, 1) \cup (1, \infty)$

Range of f: $(-\infty, 2) \cup (2, \infty)$

103. a. $A = (-3, -3), B = (-2, 0), C = (0, 1), D = (3, 4)$.

Find the equation of each segment:

$m_{AB} = \dfrac{0-(-3)}{-2-(-3)} = 3.0 = 3(-2) + b \Rightarrow b = 6$.

The equation of AB is $y = 3x + 6$.

$m_{BC} = \dfrac{1-0}{0-(-2)} = \dfrac{1}{2}; b = 1$.

The equation of BC is $y = \dfrac{1}{2}x + 1$.

$m_{CD} = \dfrac{4-1}{3-0} = 1; b = 1$.

The equation of CD is $y = x + 1$.

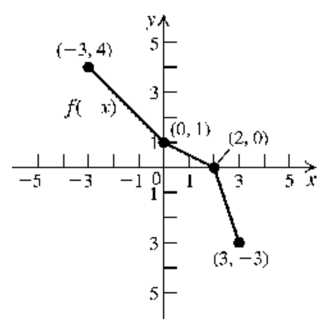
So,

$$f(x) = \begin{cases} 3x+6 & \text{if } -3 \le x \le -2 \\ \dfrac{1}{2}x + 1 & \text{if } -2 < x < 0 \\ x+1 & \text{if } 0 \le x \le 3 \end{cases}$$

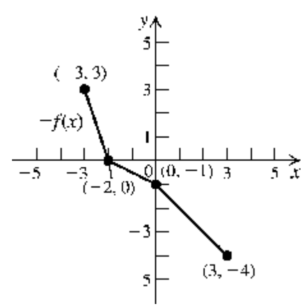
b. Domain: [–3,3]; range: [–3,4]

c. x-intercept: –2; y-intercept: 1

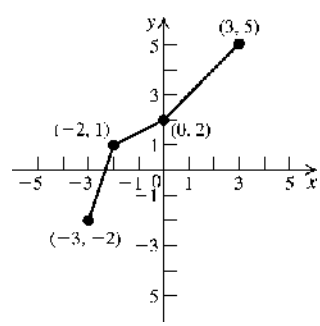
d.



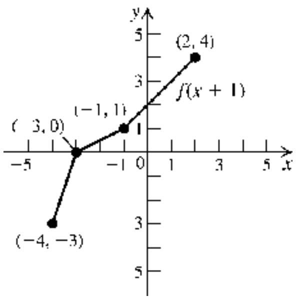
e.



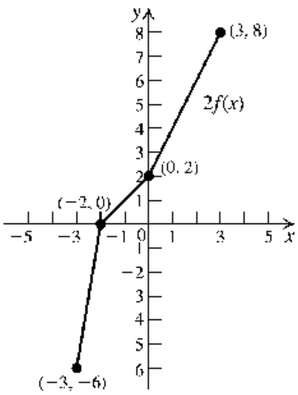
f.



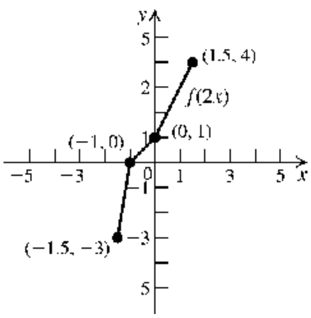
g.



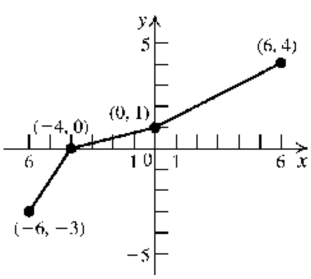
h.



i.

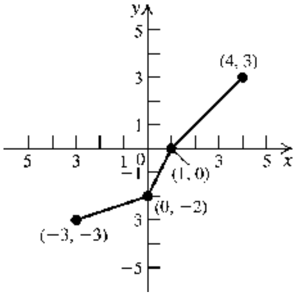


j.



k. f is one-to-one because it satisfies the horizontal line test.

l.



Applying the Concepts

104. a. rate of change (slope) $= \dfrac{25.95 - 19.2}{25 - 10} = 0.45.$
$19.2 = 0.45(10) + b \Rightarrow b = 14.7.$
The equation is $P = 0.45d + 14.7.$

b. The slope represents the amount of increase in pressure (in pounds per square inch) as the diver descends one foot deeper. The y-intercept represents the pressure at the surface of the sea.

c. $P = 0.45(160) + 14.7 = 86.7 \text{ lb/in.}^2$

d. $104.7 = 0.45d + 14.7 \Rightarrow 200$ feet

105. a. rate of change $(\text{slope}) = \dfrac{173{,}000 - 54{,}000}{223{,}000 - 87{,}000}$
$= 0.875$
$54{,}000 = 0.875(87{,}000) + b \Rightarrow$
$b = -22{,}125.$
The equation is $C = 0.875w - 22{,}125.$

b. The slope represents the cost to dispose of one pound of waste. The x-intercept represents the amount of waste that can be disposed with no cost. The y-intercept represents the fixed cost.

c. $C = 0.875(609{,}000) - 22{,}125 = \$510{,}750$

d. $1{,}000{,}000 = 0.875w - 22{,}125 \Rightarrow$
$w = 1{,}168{,}142.86$ pounds

106. a. At 60 mph = 1 mile per minute, so if the speedometer is correct, the number of minutes elapsed is equal to the number of miles driven.

b. The odometer is based on the speedometer, so if the speedometer is incorrect, so is the odometer.

107. a. $f(2) = 100 + 55(2) - 3(2)^2 = \$198.$
She started with \$100, so she won \$98.

b. She was winning at a rate of \$49/hour.

c. $0 = 100 + 55t - 3t^2 \Rightarrow (-t + 20)(3t + 5) \Rightarrow$
$t = 20,\ t = -5/3.$ Since t represents the amount of time, we reject $t = -5/3.$
Chloe will lose all her money after playing for 20 hours.

d. \$100/20 = \$5/hour.

108. If $100 < x \le 500$, then the sales price per case is \$4 – 0.2(4) = \$3.20. The first 100 cases cost \$400.

$$f(x)=\begin{cases}4x & \text{if } 0 \le x \le 100\\ 3.2x+80 & \text{if } 100 < x \le 500\\ 3x+180 & \text{if } x > 500\end{cases}$$

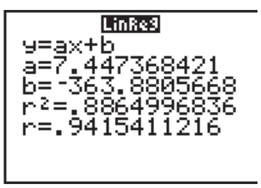
109.a. $(L \circ x)(t) = 0.5\sqrt{\left(1+0.002t^2\right)^2+4}$
$= 0.5\sqrt{0.000004t^4+0.004t^2+5}$

b. $(L \circ x)(5) = 0.5\sqrt{\left(1+0.002(5^2)\right)^2+4}$
$= 0.5\sqrt{(1.05)^2+4} = 0.5\sqrt{5.1025}$
≈ 1.13

110. a. Revenue = number of units × price per unit:
$x \bullet p = (5000+50t+10t^2)(10+0.5t)$
$= 5t^3+125t^2+3000t+50,000$

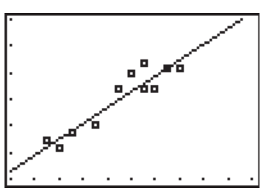
b. $p = 10+0.5t \Rightarrow t = 2p-20.$
$x(t) = x(2p-20)$
$= 5000+50(2p-20)+10(2p-20)^2$
$= 40p^2-700p+8000,$ which is the number of toys made at price p. The revenue is $p\left(40p^2-700p+8000\right) =$
$40p^3-700p^2+8000p.$

111. a.



$y \approx 7.4474x - 363.88$

b.



[70, 90, 2] by [150, 300, 25]

c. $y \approx 7.4474(76) - 363.88 \approx 202$
A player whose height is 76 inches weighs about 202 pounds.

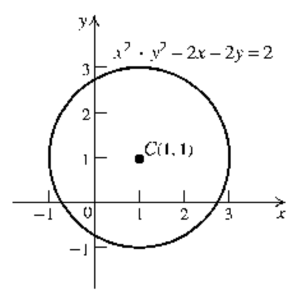
Chapter 2 Practice Test A

1. The endpoints of the diameter are (−2, 3) and (−4, 5), so the center of the circle is
$C = \left(\frac{-2+(-4)}{2}, \frac{3+5}{2}\right) = (-3, 4).$
The length of the diameter is
$\sqrt{\left(-4-(-2)\right)^2+(5-3)^2} = \sqrt{8} = 2\sqrt{2}.$
Therefore, the length of the radius is $\sqrt{2}$.
The equation of the circle is
$(x+3)^2+(y-4)^2 = 2.$

2. To test if the graph is symmetric with respect to the y-axis, replace x with $-x$:
$3(-x)+2(-x)y^2 = 1 \Rightarrow -3x-2xy^2 = 1,$ which is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. To test if the graph is symmetric with respect to the x-axis, replace y with $-y$:
$3x+2x(-y)^2 = 1 \Rightarrow 3x+2xy^2 = 1,$ which is the same as the original equation, so the graph is symmetric with respect to the x-axis. To test if the graph is symmetric with respect to the origin, replace x with $-x$ and y with $-y$:
$3(-x)+2(-x)(-y)^2 = 1 \Rightarrow -3x-2xy^2 = 1,$
which is not the same as the original equation, so the graph is not symmetric with respect to the origin.

3. $0 = x^2(x-3)(x+1) \Rightarrow x = 0$ or $x = 3$ or $x = -1$
$y = 0^2(0-3)(0+1) \Rightarrow y = 0.$ The x-intercepts are 0, 3, and –1; the y-intercept is 0.

4.



Intercepts:
$y^2-2y = 2 \Rightarrow y = 1 \pm \sqrt{3}$
$x^2-2x = 2 \Rightarrow x = 1 \pm \sqrt{3}$

5. $7 = -1(2)+b \Rightarrow 9 = b$
The equation is $y = -x + 9$.

6. $8x - 2y = 7 \Rightarrow y = 4x - \frac{7}{2} \Rightarrow$ the slope of the line is 4. $-1 = 4(2) + b \Rightarrow b = -9.$ So the equation is $y = 4x - 9.$

7. $$\begin{aligned}(fg)(2) &= f(2) \cdot g(2) \\ &= (-2(2)+1)(2^2 + 3(2) + 2) \\ &= (-3)(12) = -36\end{aligned}$$

8. $g(f(2)) = g(2(2) - 3) = g(1) = 1 - 2(1)^2 = -1$

9. $$\begin{aligned}(f \circ f)(x) &= (x^2 - 2x)^2 - 2(x^2 - 2x) \\ &= x^4 - 4x^3 + 4x^2 - 2x^2 + 4x \\ &= x^4 - 4x^3 + 2x^2 + 4x\end{aligned}$$

10. a. $f(-1) = (-1)^3 - 2 = -3$

b. $f(0) = 0^3 - 2 = -2$

c. $f(1) = 1 - 2(1)^2 = -1$

11. $1 - x > 0 \Rightarrow x < 1; x$ must also be greater than or equal to 0, so the domain is [0, 1).

12. $\frac{f(4) - f(1)}{4 - 1} = \frac{(2(4)+7) - (2(1)+7)}{3} = 2$

13. $f(-x) = 2(-x)^4 - \frac{3}{(-x)^2} = 2x^4 - \frac{3}{x^2} = f(x) \Rightarrow$ $f(x)$ is even.

14. Increasing on $(-\infty, 0)$ and $(2, \infty)$; decreasing on $(0, 2)$.

15. Shift the graph of $y = \sqrt{x}$ three units to the right, then stretch the graph vertically by a factor of 2, and then shift the resulting graph four units up.

16. $25 = 25 - (2t - 5)^2 \Rightarrow 0 = -(2t - 5)^2 \Rightarrow$
$0 = 2t - 5 \Rightarrow t = 5/2 = 2.5$ seconds

17. $f(2) = 7 \Rightarrow f^{-1}(7) = 2$

18. $f(x) = y = \frac{2x}{x - 1}$. Interchange the variables and solve for y: $x = \frac{2y}{y - 1} \Rightarrow$
$xy - x = 2y \Rightarrow xy - 2y = x \Rightarrow$
$y(x - 2) = x \Rightarrow y = f^{-1}(x) = \frac{x}{x - 2}$

19. $A(x) = 100x + 1000$

20. a. $C(230) = 0.25(230) + 30 = \$87.50$

b. $57.50 = 0.25m + 30 \Rightarrow m = 110$ miles

Chapter 2 Practice Test B

1. To test if the graph is symmetric with respect to the y-axis, replace x with $-x$:
$|-x| + 2|y| = 2 \Rightarrow |x| + 2|y| = 2,$ which is the same as the original equation, so the graph is symmetric with respect to the y-axis. To test if the graph is symmetric with respect to the x-axis, replace y with $-y$:
$|x| + 2|-y| = 2 \Rightarrow |x| + 2|y| = 2,$ which is the same as the original equation, so the graph is symmetric with respect to the x-axis. To test if the graph is symmetric with respect to the origin, replace x with $-x$, and y with $-y$:
$|-x| + 2|-y| = 2 \Rightarrow |x| + 2|y| = 2,$ which is the same as the original equation, so the graph is symmetric with respect to the origin. The answer is D.

2. $0 = x^2 - 9 \Rightarrow x = \pm 3; y = 0^2 - 9 \Rightarrow y = -9.$
The x-intercepts are ±3; the y-intercept is –9. The answer is B.

3. D **4.** D **5.** C

6. Suppose the coordinates of the second point are (a, b). Then $-\frac{1}{2} = \frac{b - 2}{a - 3}$. Substitute each of the points given into this equation to see which makes it true. The answer is C.

7. Find the slope of the original line:
$6x - 3y = 5 \Rightarrow y = 2x - \frac{5}{3}$. The slope is 2.
The equation of the line with slope 2, passing through (–1, 2) is $y - 2 = 2(x + 1)$.
The answer is D.

8. $(f \circ g)(x) = 3(2 - x^2) - 5 = 1 - 3x^2.$
The answer is B.

9. $$\begin{aligned}(f \circ f)(x) &= 2(2x^2 - x)^2 - (2x^2 - x) \\ &= 8x^4 - 8x^3 + x\end{aligned}$$
The answer is A.

10. $g(a - 1) = \frac{1 - (a - 1)}{1 + (a - 1)} = \frac{2 - a}{a}.$
The answer is C.

11. $1-x \ge 0 \Rightarrow x \le 1$; x must also be greater than or equal to 0, so the domain is [0, 1].
The answer is A.

12. $x^2+3x-4=6 \Rightarrow x^2+3x-10=0 \Rightarrow$
$(x+5)(x-2)=0 \Rightarrow x=-5,\ 2$
The answer is D.

13. A **14.** A **15.** B

16. D **17.** C

18. $f(x)=y=\dfrac{x}{3x+2}$. Interchange the variables and solve for y: $x=\dfrac{y}{3y+2} \Rightarrow$
$3xy+2x=y \Rightarrow 3xy-y=-2x \Rightarrow$
$y(3x-1)=-2x \Rightarrow y=f^{-1}(x)=-\dfrac{2x}{3x-1} \Rightarrow$
$f^{-1}(x)=\dfrac{2x}{1-3x}$
The answer is C.

19. $w=5x-190; w=5(70)-190=160.$
The answer is B.

20. $50=0.2m+25 \Rightarrow m=125.$ The answer is A.

Cumulative Review Exercises (Chapters P–2)

1. a. $\left(\dfrac{x^3}{y^2}\right)^2\left(\dfrac{y^2}{x^3}\right)^3=\left(\dfrac{x^6}{y^4}\right)\left(\dfrac{y^6}{x^9}\right)=\dfrac{y^2}{x^3}$

b. $\dfrac{x^{-1}y^{-1}}{x^{-1}+y^{-1}}=\dfrac{\frac{1}{x}\cdot\frac{1}{y}}{\frac{1}{x}+\frac{1}{y}}=\dfrac{\frac{1}{xy}}{\frac{y+x}{xy}}=\dfrac{1}{x+y}$

2. a. $2x^2+x-15=(2x-5)(x+3)$

b. $x^3-2x^2+4x-8=x^2(x-2)+4(x-2)$
$=(x^2+4)(x-2)$

3. a. $\sqrt{75}+\sqrt{108}-\sqrt{192}=5\sqrt{3}+6\sqrt{3}-8\sqrt{3}$
$=3\sqrt{3}$

b. $\dfrac{x-1}{x+1}-\dfrac{x-2}{x+2}=\dfrac{(x-1)(x+2)-(x-2)(x+1)}{(x+1)(x+2)}$
$=\dfrac{(x^2+x-2)-(x^2-x-2)}{(x+1)(x+2)}$
$=\dfrac{2x}{(x+1)(x+2)}$

4. a. $\dfrac{1}{2+\sqrt{3}}=\dfrac{1}{2+\sqrt{3}}\cdot\dfrac{2-\sqrt{3}}{2-\sqrt{3}}=\dfrac{2-\sqrt{3}}{4-3}=2-\sqrt{3}$

b. $\dfrac{1}{\sqrt{5}-2}=\dfrac{1}{\sqrt{5}-2}\cdot\dfrac{\sqrt{5}+2}{\sqrt{5}+2}=\dfrac{\sqrt{5}+2}{5-4}=\sqrt{5}+2$

5. a. $3x-7=5 \Rightarrow 3x=12 \Rightarrow x=4$

b. $\dfrac{1}{x-1}=\dfrac{3}{x-1} \Rightarrow$ There is no solution.

6. a. $x^2-3x=0 \Rightarrow x(x-3)=0 \Rightarrow$
$x=0$ or $x=3$

b. $x^2+3x-10=0 \Rightarrow (x+5)(x-2)=0 \Rightarrow$
$x=-5$ or $x=2$

7. a. $2x^2-x+3=0 \Rightarrow x=\dfrac{1\pm\sqrt{1-4(2)(3)}}{2(2)} \Rightarrow$
$x=\dfrac{1\pm\sqrt{-23}}{4} \Rightarrow x=\dfrac{1\pm i\sqrt{23}}{4}$

b. $4x^2-12x+9=0 \Rightarrow (2x-3)^2=0 \Rightarrow x=\dfrac{3}{2}$

8. a. $x-6\sqrt{x}+8=0 \Rightarrow \left(\sqrt{x}-4\right)\left(\sqrt{x}-2\right)=0 \Rightarrow$
$\sqrt{x}=4 \Rightarrow x=16$ or $\sqrt{x}=2 \Rightarrow x=4$

b. $\left(x-\dfrac{1}{x}\right)^2-10\left(x-\dfrac{1}{x}\right)+21=0.$
Let $u=x-\dfrac{1}{x}$.
$u^2-10u+21=0 \Rightarrow$
$(u-7)(u-3)=0 \Rightarrow u=7$ or $u=3$;
$x-\dfrac{1}{x}=7 \Rightarrow x^2-1=7x \Rightarrow$
$x^2-7x-1=0 \Rightarrow x=\dfrac{7\pm\sqrt{7^2-(4)(-1)}}{2} \Rightarrow$
$x=\dfrac{7\pm\sqrt{53}}{2}$; $x-\dfrac{1}{x}=3 \Rightarrow x^2-1=3x \Rightarrow$
$x^2-3x-1=0 \Rightarrow x=\dfrac{3\pm\sqrt{3^2-4(-1)}}{2} \Rightarrow$
$x=\dfrac{3\pm\sqrt{13}}{2}$
The solution set is
$\left\{\dfrac{7-\sqrt{53}}{2},\dfrac{7+\sqrt{53}}{2},\dfrac{3-\sqrt{13}}{2},\dfrac{3+\sqrt{13}}{2}\right\}.$

9. a. $\sqrt{3x-1} = 2x-1 \Rightarrow 3x-1 = (2x-1)^2 \Rightarrow$
$3x-1 = 4x^2-4x+1 \Rightarrow 4x^2-7x+2=0 \Rightarrow$
$x = \dfrac{7 \pm \sqrt{(-7)^2-4(4)(2)}}{2(4)} = \dfrac{7 \pm \sqrt{17}}{8}$. If
$x = \dfrac{7-\sqrt{17}}{8}, \sqrt{3\left(\dfrac{7-\sqrt{17}}{8}\right)-1} \approx 0.281$
while $2\left(\dfrac{7-\sqrt{17}}{8}\right)-1 \approx -0.281$, so the solution set is $\left\{\dfrac{7+\sqrt{17}}{8}\right\}$.

b.
$$\sqrt{1-x} = 2-\sqrt{2x+1}$$
$$\left(\sqrt{1-x}\right)^2 = \left(2-\sqrt{2x+1}\right)^2$$
$$1-x = 4-4\sqrt{2x+1}+2x+1$$
$$-4-3x = -4\sqrt{2x+1}$$
$$(-4-3x)^2 = \left(-4\sqrt{2x+1}\right)^2$$
$$16+24x+9x^2 = 16(2x+1)$$
$$16+24x+9x^2 = 32x+16$$
$$9x^2-8x=0 \Rightarrow x(9x-8)=0$$
$$x = 0 \text{ or } x = \frac{8}{9}.$$
Check to make sure that neither solution is extraneous. The solution set is $\{0, 8/9\}$.

10. a. $2x-5<11 \Rightarrow x<8 \Rightarrow (-\infty, 8)$

b. $-3x+4>-5 \Rightarrow x<3 \Rightarrow (-\infty, 3)$

11. a. $-3<2x-3<5 \Rightarrow 0<2x<8 \Rightarrow 0<x<4$.
The solution set is (0, 4).

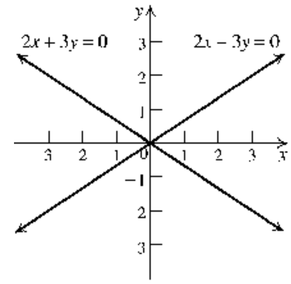
b. $5 \le 1-2x \le 7 \Rightarrow 4 \le -2x \le 6 \Rightarrow -2 \ge x \ge -3$.
The solution set is [–3, –2].

12. a. $|2x-1| \le 7 \Rightarrow -7 \le 2x-1 \le 7 \Rightarrow$
$-6 \le 2x \le 8 \Rightarrow -3 \le x \le 4$
The solution set is [–3, 4].

b. $|2x-3| \ge 5 \Rightarrow 2x-3 \ge 5 \Rightarrow x \ge 4$ or
$2x-3 \le -5 \Rightarrow x \le -1$.
The solution set is $(-\infty, -1] \cup [4, \infty)$.

13. $d(A,C) = \sqrt{(2-5)^2+(2-(-2))^2} = 5$
$d(B,C) = \sqrt{(2-6)^2+(2-5)^2} = 5$
Since the lengths of the two sides are equal, the triangle is isosceles.

14.



15. First, find the equation of the circle with center (2, –1) and radius determined by (2, –1) and (–3, –1):
$r = \sqrt{2-(-3))^2+(-1-(-1))^2} = 5$.
The equation is $(x-2)^2+(y+1)^2 = 5^2$. Now check to see if the other three points satisfy the equation:
$(2-2)^2+(4+1)^2 = 5^2 \Rightarrow 5^2 = 5^2$,
$(5-2)^2+(3+1)^2 = 5^2 \Rightarrow 3^2+4^2 = 5^2$ (true because 3, 4, 5 is a Pythagorean triple), and
$(6-2)^2+(2+1)^2 = 5^2 \Rightarrow 4^2+3^2 = 5^2$.
Since all the points satisfy the equation, they lie on the circle.

16. $x^2+y^2-6x+4y+9=0 \Rightarrow$
$x^2-6x+y^2+4y=-9$.
Now complete both squares:
$x^2-6x+9+y^2+4y+4 = -9+9+4 \Rightarrow$
$(x-3)^2+(y+2)^2 = 4$.
The center is (3, –2) and the radius is 2.

17. $y = -3x+5$

18. The x-intercept is 4, so (4, 0) satisfies the equation. To write the equation in slope-intercept form, find the y-intercept:
$0 = 2(4)+b \Rightarrow -8 = b$
The equation is $y = 2x-8$.

19. The slope of the perpendicular line is the negative reciprocal of the slope of the original line. The slope of the original line is 2, so the slope of the perpendicular is –1/2. Now find the y-intercept of the perpendicular:
$-1 = -\dfrac{1}{2}(2)+b \Rightarrow b = 0$. The equation of the perpendicular is $y = -\dfrac{1}{2}x$.

20. The slope of the parallel line is the same as the slope of the original line, 2. Now find the y-intercept of the parallel line: $-1 = 2(2) + b \Rightarrow$ $b = -5$. The equation of the parallel line is $y = 2x - 5$.

21. The slope of the perpendicular line is the negative reciprocal of the slope of the original line. The slope of the original line is $\frac{7-(-1)}{5-3} = 4$, so the slope of the perpendicular is –1/4. The perpendicular bisector passes through the midpoint of the original segment. The midpoint is $\left(\frac{3+5}{2}, \frac{-1+7}{2}\right) = (4, 3)$. Use this point and the slope to find the y-intercept:

$3 = -\frac{1}{4}(4) + b \Rightarrow b = 4$. The equation of the perpendicular bisector is $y = -\frac{1}{4}x + 4$.

22. The slope is undefined because the line is vertical. Because it passes through (5, 7), the equation of the line is $x = 5$.

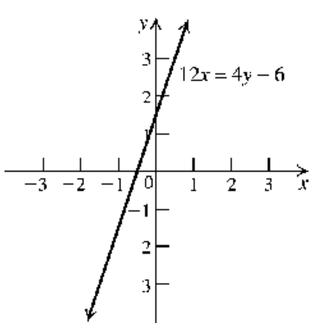
23. Use the slope formula to solve for x:

$2 = \frac{5-11}{x-5} \Rightarrow 2(x-5) = -6 \Rightarrow 2x - 10 = -6 \Rightarrow$
$x = 2$

24. The line through (x, 3) and (3, 7) has slope –2 because it is perpendicular to a line with slope 1/2. Use the slope formula to solve for x:

$-2 = \frac{3-7}{x-3} \Rightarrow -2(x-3) = -4 \Rightarrow x - 3 = 2 \Rightarrow$
$x = 5$

25.



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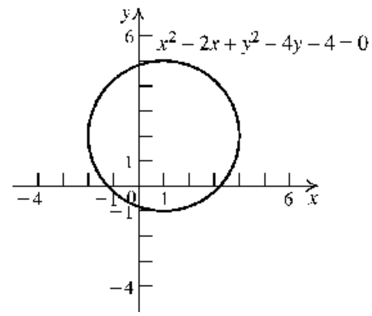

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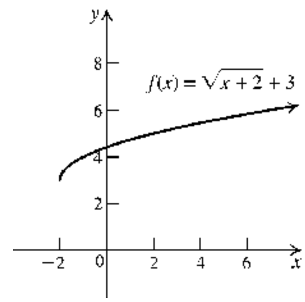

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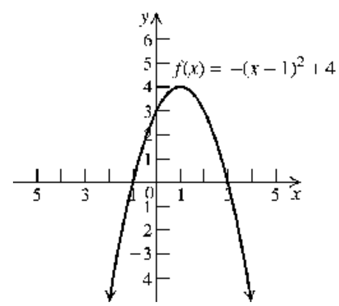


29. Let x = the number of books initially purchased, and $\frac{1650}{x}$ = the cost of each book. Then x – 16 = the number of books sold, and $\frac{1650}{x-16}$ = the selling price of each book. The profit = the selling price – the cost, so

$\frac{1650}{x-16} - \frac{1650}{x} = 10 \Rightarrow$
$1650x - 1650(x-16) = 10x(x-16) \Rightarrow$
$1650x - 1650x + 26,400 = 10x^2 - 160x \Rightarrow$
$10x^2 - 160x - 26,400 = 0 \Rightarrow$
$x^2 - 16x - 2640 = 0 \Rightarrow (x-60)(x+44) = 0 \Rightarrow$
$x = 60,\ x = -44$. Reject –44 because there cannot be a negative number of books. So she bought 60 books.

30. Let x = the monthly note on the 1.5 year lease, and $1.5(12)x = 18x$ = the total expense for the 1.5 year lease. Then $x - 250$ = the monthly note on the 2 year lease, and $2(12)(x - 250) = 24x - 6000$ the total expenses for the 2 year lease. Then
$18x + 24x - 6000 = 21{,}000 \Rightarrow$
$42x = 27{,}000 \Rightarrow x = 642.86.$ So the monthly note for the 1.5 year lease is \$642.86, and the monthly note for the 2 year lease is $\$642.86 - 250 = \$392.86$.

31. a. The domain of f is the set of all values of x which make $x + 1 \ge 0$ (because the square root of a negative number is not a real value.) So $x \ge -1$ or $[-1, \infty)$ in interval notation is the domain.

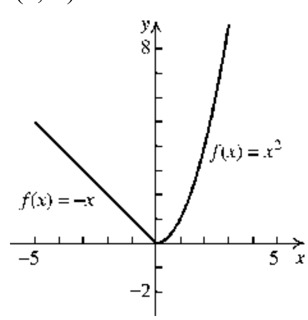
b. $y = \sqrt{0+1} - 3 \Rightarrow y = -2; 0 = \sqrt{x+1} - 3 \Rightarrow$
$3 = \sqrt{x+1} \Rightarrow 9 = x + 1 \Rightarrow 8 = x.$ The x-intercept is 8, and the y-intercept is –2.

c. $f(-1) = \sqrt{-1+1} - 3 = -3$

d. $f(x) > 0 \Rightarrow \sqrt{x+1} - 3 > 0 \Rightarrow \sqrt{x+1} > 3 \Rightarrow$
$x + 1 > 9 \Rightarrow x > 8$. In interval notation, this is $(8, \infty)$.

32. a. $f(-2) = -(-2) = 2; f(0) = 0^2 = 0;$
$f(2) = 2^2 = 4$

b. f decreases on $(-\infty, 0)$ and increases on $(0, \infty)$.



33. a. $(f \circ g)(x) = \dfrac{1}{\frac{2}{x} - 2} = \dfrac{1}{\frac{2-2x}{x}} = \dfrac{x}{2-2x}.$

Because 0 is not in the domain of g, it must be excluded from the domain of $(f \circ g)$.
Because 2 is not in the domain of f, any values of x for which $g(x) = 2$ must also be excluded from the domain of
$(f \circ g): \dfrac{2}{x} = 2 \Rightarrow x = 1,$ so 1 is excluded also. The domain of $(f \circ g)$ is
$(-\infty, 0) \cup (0, 1) \cup (1, \infty)$.

b. $(g \circ f)(x) = \dfrac{2}{\frac{1}{x-2}} = 2(x-2) = 2x - 4.$

Because 2 is not in the domain of f, it must be excluded from the domain of $(g \circ f)$.
Because 0 is not in the domain of g, any values of x for which $f(x) = 0$ must also be excluded from the domain of $(g \circ f)$.
However, there is no value for x which makes $f(x) = 0$. So the domain of $(g \circ f)$ is
$(-\infty, 2) \cup (2, \infty)$.