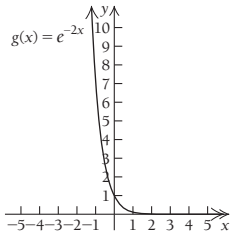
INSTRUCTOR ANSWERS: CHAPTER 2

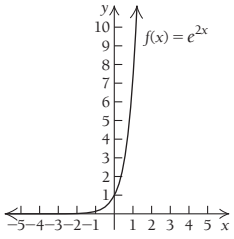
Exercise Set 2.1, p. 203

1.



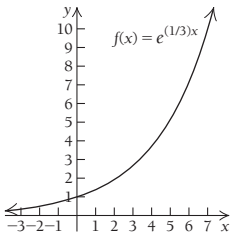
Domain: $(-\infty, \infty)$; range: $(0, \infty)$; y-intercept: $(0, 1)$; decreasing

2.



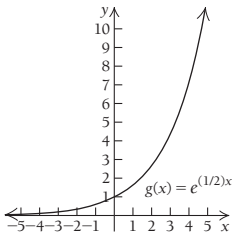
Domain: $(-\infty, \infty)$; range: $(0, \infty)$; y-intercept: $(0, 1)$; increasing

3.



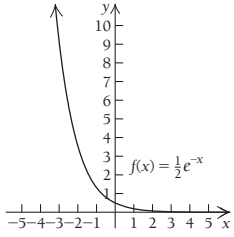
Domain: $(-\infty, \infty)$; range: $(0, \infty)$; y-intercept: $(0, 1)$; increasing

4.



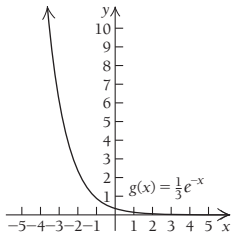
Domain: $(-\infty, \infty)$; range: $(0, \infty)$; y-intercept: $(0, 1)$; increasing

5.



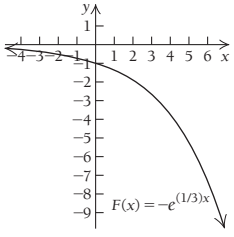
Domain: $(-\infty, \infty)$; range: $(0, \infty)$; y-intercept: $(0, \frac{1}{2})$; decreasing

6.



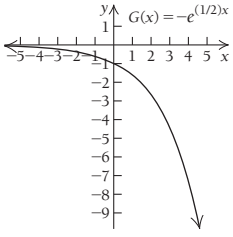
Domain: $(-\infty, \infty)$; range: $(0, \infty)$; y-intercept: $(0, \frac{1}{3})$; decreasing

7.



Domain: $(-\infty, \infty)$; range: $(-\infty, 0)$; y-intercept: $(0, -1)$; decreasing

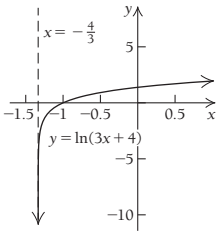
8.



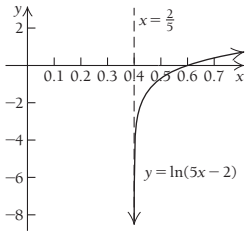
Domain: $(-\infty, \infty)$; range: $(-\infty, 0)$; y-intercept: $(0, -1)$; decreasing

9. **(a)** \$1698.12; **(b)** \$1701.14; **(c)** \$1701.41; **(d)** \$1701.42; **(e)** 22 yr **10.** **(a)** \$2941.92; **(b)** \$2947.93; **(c)** \$2948.46; **(d)** \$2948.48; **(e)** 25.2 yr **11.** **(a)** \$85,587.46; **(b)** \$85,818.59; **(c)** \$85,839.54; **(d)** \$85,840.26; **(e)** 15.4 yr **12.** **(a)** \$117,174.27; **(b)** \$117,368.37; **(c)** \$117,385.71; **(d)** \$117,386.30; **(e)** 30.3 yr **13.** **(a)** \$304.45; **(b)** \$305.00; **(c)** \$305.04; **(d)** \$305.05; **(e)** 34.8 yr **14.** **(a)** \$1920.34; **(b)** \$1924.85; **(c)** \$1925.25; **(d)** \$1925.27; **(e)** 33.3 yr **15.** **(a)** \$10,621.26; **(b)** \$10,633.19; **(c)** \$10,634.27; **(d)** \$10,634.30; **(e)** 16.9 yr **16.** **(a)** \$12,331.48; **(b)** \$12,337.14; **(c)** \$12,337.65; **(d)** \$12,337.66; **(e)** 18.7 yr **17.** 0.693; $e^{0.693} = 1.9997\ldots \approx 2$ **18.** 2.197; $e^{2.197} = 8.9979\ldots \approx 9$ **19.** 1.504; $e^{1.504} = 4.4996\ldots \approx 4.5$ **20.** 1.203; $e^{1.203} = 3.33009\ldots \approx 3.33$ **21.** -2.957; $e^{-2.957} = 0.05197\ldots \approx 0.052$ **22.** -2.273; $e^{-2.273} = 0.1030027\ldots \approx 0.103$ **23.** -0.288; $e^{-0.288} = 0.74976\ldots \approx 0.75$ **24.** 0.182; $e^{0.182} = 1.1996\ldots \approx 1.2$ **25.** 5; $e^5 = e^5$ **26.** 8; $e^8 = e^8$ **27.** 4.382 **28.** 2.9957 **29.** -1.6094 **30.** 0.2231 **31.** 2.3863 **32.** 2.6094 **33.** 4 **34.** 3 **35.** -0.2231 **36.** -1.3863 **37.** 0.3863 **38.** -0.6094 **39.** 2.079 **40.** 2.303 **41.** 2.267 **42.** 3.454 **43.** 4.605 **44.** 2.303 **45.** 140.671 **46.** -9.902 **47.** 0.549 **48.** 0.380 **49.** 24.414 **50.** 20.996

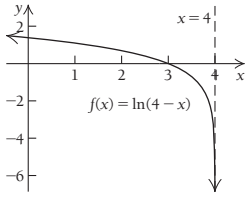
51. $\left(-\frac{4}{3}, \infty\right)$



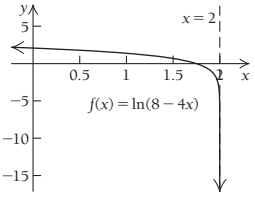
52. $\left(\frac{2}{5}, \infty\right)$



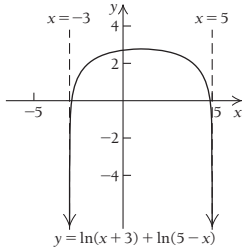
53. $(-\infty, 4)$



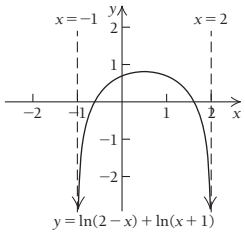
54. $(-\infty, 2)$



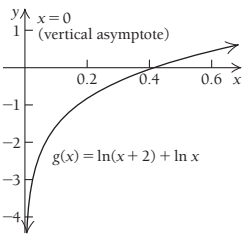
55. $(-3, 5)$



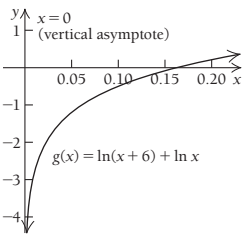
56. $(-1, 2)$



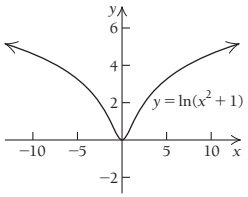
57. $(0, \infty)$



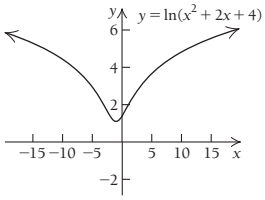
58. $(0, \infty)$



59. $(-\infty, \infty)$



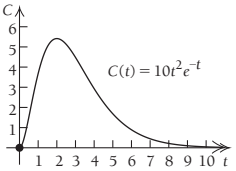
60. $(-\infty, \infty)$



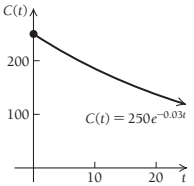
61. $e^{10} \approx 22{,}026.466$ **62.** $e^{6.5} \approx 665.142$
63. $e^{4.5} \approx 90.017$ **64.** $e^{5/3} \approx 5.294$
65. $\frac{1}{2}(e^{1.5} - 1) \approx 1.741$ **66.** $\frac{1}{3}(e^{-1.4} + 2) \approx 0.749$ **67.** 4
68. 4 **69. (a)** 2018: \$222.7 billion, 2020: \$268.7 billion; **(b)** in 2.8 yr; **(c)** in 7.37 yr **70. (a)** \$48.9 billion; **(b)** in 3.2 yr, or 2017; **(c)** in 6.7 yr, or 2020
71. (a) $A(t) = 10{,}000e^{0.0288t}$; **(b)** \$11,548.84; **(c)** in 14.1 yr; **(d)** in 24.1 yr **72. (a)** $A(t) = 25{,}000e^{0.0403t}$; **(b)** \$37,407.67; **(c)** in 11.7 yr; **(d)** in 17.2 yr **73. (a)** \$52.53; **(b)** \$25; **(c)** after 30.1 weeks **74. (a)** \$103.60; **(b)** \$60; **(c)** after 74.4 weeks **75. (a)** \$594.43; **(b)** about 2136 thousand units **76. (a)** \$529.04; **(b)** about 629 thousand units **77. (a)** $A(t) = 45{,}000e^{0.03t}$; **(b)** about 60,744; **(c)** in 6.7 yr; **(d)** in 23.1 yr
78. (a) $A(t) = 2500e^{0.025t}$; **(b)** about 2905; **(c)** in 18.8 yr; **(d)** in 27.7 yr **79. (a)** $A(t) = 2e^{0.045t}$; **(b)** 3.14 mm^2; **(c)** after 71.5 hr; **(d)** after 15.4 hr
80. (a) $Q(t) = 50e^{0.55t}$; **(b)** about 191,400; **(c)** in 18 weeks; **(d)** in 1.26 weeks **81. (a)** 75°C; **(b)** 58.4°C; **(c)** in 95.5 min; **(d)** 30, meaning that the coffee will decrease in temperature toward 30°C **82. (a)** 100°C; **(b)** 64.2°C; **(c)** in 35.8 min; **(d)** 22, meaning that the water will cool toward 22°C
83. (a) 3.87 mg; **(b)** 6.26 days; **(c)** 2.7 days; **(d)** 0, which means that the sample will eventually decay toward zero.
84. (a) 12,054 people; **(b)** after 4.1 yr, or in 2021; **(c)** after 13.5 yr, in 2030; **(d)** 0, which means that the population will eventually approach 0 as a limit. **85.** $x = 0, x = \ln 4$
86. $x = \ln 3, x = \ln 7$ **87.** $x = \ln 4, x = \ln 8$
88. $x = \ln 3, x = \ln 4$ **89.** $x = \ln 4$ **90.** $x = \ln 5$
91. $x = e^{e^2-1} \approx 595.29$ **92.** $x = e^{(e^{-1}-2)/3} \approx 0.58$
93. $x = e, x = e^3$ **94.** $x = e^5, x = e^{-3}$ **95.** 6.077%; 11.4 yr
96. 1520.18%; 0.05 yr **97.** 3.45 hr **98. (a)** $A(t) = 7500e^{t/35}$; **(b)** 2.857% **99.** Answers may vary. **100.** 10 yr **101.** 20 yr
102. Left to the student **103.** Answers may vary.
104. $x = 0.619, x = 1.512$ **105.** $x = 0.933$
106. $x = 1.314$ **107.** $x = 0.27$

Exercise Set 2.2, p. 210

1. $2e^{2x}$ **2.** $3e^{3x}$ **3.** $15e^{5x}$ **4.** $8e^{4x}$ **5.** $3x^2 - 10e^{2x}$
6. $5x^4 - 12e^{6x}$ **7.** $5x^4e^{2x} + 2x^5e^{2x}$ **8.** $7x^6e^{4x} + 4x^7e^{4x}$
9. $\dfrac{2e^{2x}(x-2)}{x^5}$ **10.** $\dfrac{3e^{3x}(x-2)}{x^7}$ **11.** x^2e^x **12.** $e^x(x^2 + 5x - 6)$
13. $(8 - 2x)e^{-x^2+8x}$ **14.** $(7 - 2x)e^{-x^2+7x}$ **15.** $\dfrac{e^x}{2\sqrt{e^x - 1}}$
16. $\dfrac{e^x}{2\sqrt{e^x + 1}}$ **17.** $3x^2 - xe^x$ **18.** $-2xe^{-2x} + e^{-2x} - e^{-x} + 3x^2$
19. $(8x^3 - 22x^2 - 13x + 3)e^{x^2-7x}$
20. $(10x^3 - 36x^2 + 42x - 8)e^{x^2-4x}$ **21.** $\dfrac{1 - 2t}{e^{2t}}$
22. $\dfrac{2t - 3t^2}{e^{3t}}$ **23.** $-\dfrac{2t^3 + 4t^2 - 2t - 2}{e^{t^2}}$
24. $-\dfrac{12t^5 - 60t^3 - 3t^2 + 5}{e^{4t^3}}$ **25.** $4e^{2x}$ **26.** $9e^{-3x}$
27. $\frac{1}{4}e^{(-1/2)x}$ **28.** $\frac{4}{9}e^{(2/3)x}$ **29.** $2e^{x^2}(2x^2 + 1)$
30. $3xe^{x^3}(3x^3 + 2)$ **31.** $4e^{2x+1}$ **32.** $16e^{3-4x}$
33. $\dfrac{e^{\sqrt{x}}(\sqrt{x} - 1)}{4x\sqrt{x}}$ **34.** $\dfrac{e^{\sqrt[3]{t}}(\sqrt[3]{t} - 2)}{9t\sqrt[3]{t^2}}$ **35.** $e^x(x + 2)$
36. $e^x(x^2 + 4x + 2)$ **37.** $3e^{3t}(6t + 13)$ **38.** $4e^{-2t}(5t - 9)$
39. $8e^{2x}(2e^{2x} + 1)$ **40.** $144e^{4x}(24e^{4x} + 27e^{8x} + 4)$
41. $e^{5t}(25t^2 + 70t + 97)$ **42.** $4e^{(1-2t)}(4t^3 - 12t^2 + 5t + 1)$
43. $\dfrac{3e^{3x}(3x^2 - 4x + 2)}{x^4}$ **44.** $\dfrac{4e^{8x}(16x^2 - 16x + 5)}{3x^6}$
45. $\dfrac{9e^{3t}(e^{3t} - 2)}{4(e^{3t} - 1)^{3/2}}$ **46.** $\dfrac{20e^{-4t}(2e^{4t} - 5)}{\sqrt{1 - 5e^{-4t}}(e^{4t} - 5)}$
47. (a) $dC/dx = 50e^{-x}$; **(b)** \$50 million/yr; **(c)** \$916,000/yr; **(d)** 100 and 0 **48. (a)** $dC/dx = 40e^{-x}$; **(b)** \$14.715 million/yr; **(c)** \$270,000/yr; **(d)** 200 and 0 **49. (a)** \$29,289.59; **(b)** \$1596.28/yr **50. (a)** \$11,532.87; **(b)** −\$961.84/yr
51. (a) −\$2355.35/yr; **(b)** \$20,019 **52. (a)** \$2.66/week; **(b)** \$75.35 **53. (a)** 0, 3.7, 5.4, 4.5, 0.05 (all in ppm);
(b)

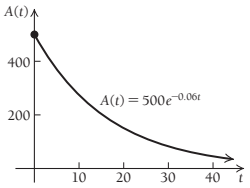


(c) $C(5) = 1.68$ ppm, $C'(5) = -1.01$ ppm/hr. After 5 hr, the concentration of the medication is about 1.68 ppm, and it is decreasing by about 1.01 ppm/hr. **54. (a)** 242.6, 215.2, 174.4 (all in ppm);
(b)



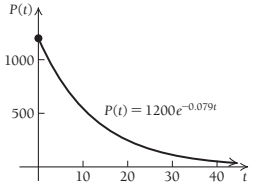
(c) $C(8) = 196.7$ ppm, $C'(8) = -5.9$ ppm/min. After 8 min, the concentration of medication is about 196.7 ppm, and it is decreasing by about 5.9 ppm/min. **55. (a)** 443.5, 370.4, 274.4 (all in mm^2);

(b)



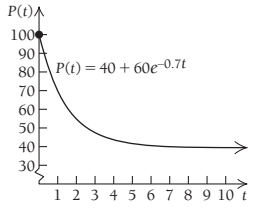
(c) $A(12) = 243.4$ mm^2, $A'(12) = -14.6$ mm^2/hr. After 12 hr, the area of the colony is about 243.4 mm^2, and it is decreasing by about 14.6 mm^2/hr. **56. (a)** 1025 fish, 747 fish, 367 fish;

(b)



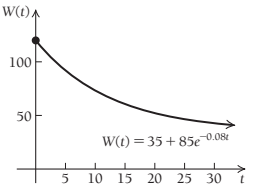
(c) $P(20) = 247$ fish, $P'(20) = -19.5$ fish/week. After 20 weeks, the population of the invasive species is about 247 fish, and it is decreasing by about 19.5 fish/week. **57. (a)** 100%, 69.8%, 54.8%, 40.9%, 40.1%; **(b)** 40%;

(c)



(d) $P'(t) = -42e^{-0.7t}$; **(e)** answers may vary.
58. (a) 97 words, 73 words, 52 words; **(b)** 35;

(c)



(d) $W(7) = 84$ words, $W'(7) = -3.88$ words/day. After 7 days, the student retained 84 words but was forgetting them at a rate of about 3.88 words per day.

59. $\dfrac{xe^{(1/2)x}}{2\sqrt{x-1}}$ **60.** $\dfrac{-e^{-x}(x^3 + x^2 + x - 1)}{(1 + x^2)^2}$

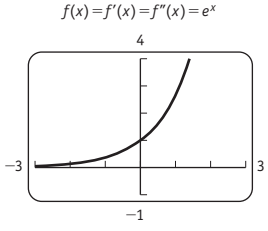
61. $\dfrac{4}{(e^x + e^{-x})^2}$ **62.** $e^{e^x + x}$ **63.** ex^{e-1}

64. (a) $f'(x) = -3e^{-3x}, f''(x) = 9e^{-3x}, f'''(x) = -27e^{-3x}$;

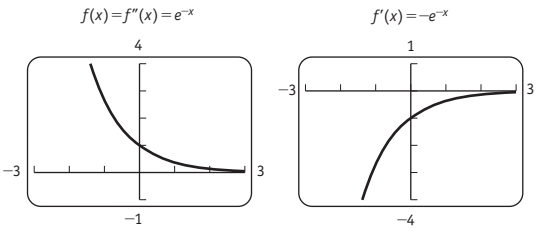
(b) $f^{(10)}(x) = (-3)^{10}e^{-3x} = 59{,}049e^{-3x}$ **65.** $g^{(7)}(x) = 128e^{2x}$

66. $y = 3x + 1$ **67.** $y = -8x + 2$ **68.** At $x = 0$, we have $(0, a)$ and $m = ab$, so $y - a = ab(x - 0)$, which simplifies to $y = abx + a$. **69.** $(1, e)$

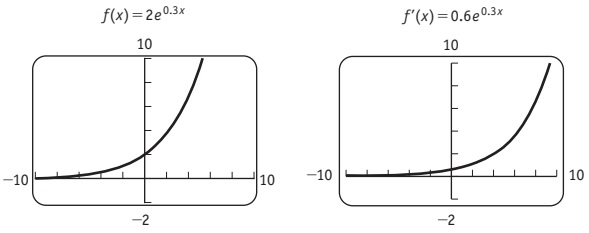
70.

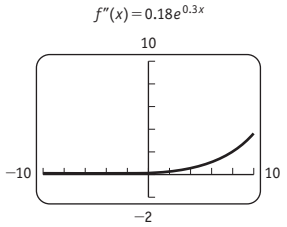


71.

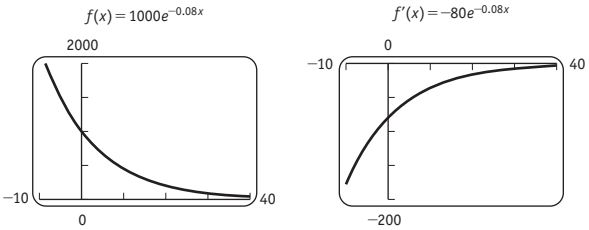


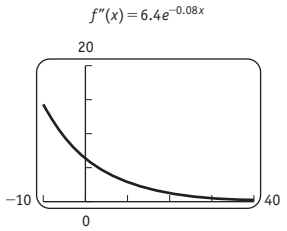
72.



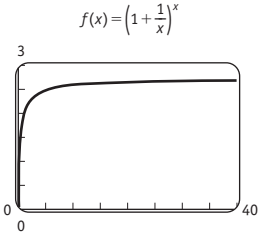


73.





74.



75. (a) 1.0517, 1.005, 1.0005, 0.95163, 0.999502, 0.99995; **(b)** 1

Exercise Set 2.3, p. 216

1. $-\dfrac{9}{x}$ **2.** $-\dfrac{8}{x}$ **3.** $\dfrac{1}{x}$ **4.** $\dfrac{1}{x}$ **5.** $\dfrac{1}{x}$ **6.** $\dfrac{1}{x}$

7. $x^5(1 + 6\ln x)$ **8.** $x^3(1 + 4\ln x)$ **9.** $\dfrac{1 - 5\ln x}{x^6}$

10. $\dfrac{1 - 4\ln x}{x^5}$ **11.** $\dfrac{2}{x}$ **12.** $\dfrac{4}{x}$ **13.** $\dfrac{6x + 2}{3x^2 + 2x - 1}$

14. $\dfrac{14x+5}{7x^2+5x+2}$ **15.** $\dfrac{2x}{x^2+5}-\dfrac{1}{x}$ **16.** $\dfrac{2x}{x^2-7}-\dfrac{1}{x}$

17. $\dfrac{4(\ln x)^3}{x}$ **18.** $\dfrac{3(\ln x)^2}{x}$ **19.** $\dfrac{2}{x}+\dfrac{3x^2}{x^3+1}-2$

20. $\dfrac{4}{x}+3+\dfrac{10x+5}{x^2+x+1}$ **21.** $y=0.732x-0.990$

22. $y=1.778x-1.358$ **23.** $y=4.329x-6.75$

24. $y=5.659x-3.579$ **25. (a)** 2000; **(b)** $N'(a)=\dfrac{500}{a}$, $N'(10)=50$; **(c)** \$7390; **(d)** answers may vary. **26. (a)** 1000; **(b)** $N'(a)=\dfrac{200}{a}$; $N'(10)=20$; **(c)** \$12,180; **(d)** answers may vary.

27. (a) $R(x)=53.5x-8x\ln x$; **(b)** $R'(x)=45.5-8\ln x$; **(c)** 5400 **28. (a)** $P'(x)=1.7-0.3\ln x$; **(b)** 0.197, which means that, when 150,000 candles are sold, profit is increasing by about 0.197 thousand dollars (\$197) per unit; **(c)** about 23,700 units **29. (a)** 78%; **(b)** 53.9%; **(c)** 29.7%; **(d)** $S'(t)=-\dfrac{15}{t+1}$; **(e)** $S'(4)=-3$, which means that, after 4 months, the average score is decreasing by 3 percentage points per month; $S'(24)=-0.6$, which means that, after 24 months, the average score is decreasing by 0.6 percentage point per month. **30. (a)** 2.45 ft/s; **(b)** 3.40 ft/s; **(c)** $v'(660)=0.000561$ and $v'(8550)=0.0000433$, which indicate that as population increases, walking speed increases; **(d)** about 450,000 **31. (a)** $t(15)=20.27$ min and $t'(15)=5$ min/°C, which mean that it takes 20.27 min to warm the water to 15°C, at which time the time needed for the water to warm by 1°C is changing by 5 min/degree; **(b)** 20.48°C **32. (a)** $t(60)=216.25$ min and $t'(60)=15.625$ min/°F, which mean that it takes 216.25 min to warm a frozen package of smoked salmon to 60°F, and at that moment, the time needed for the package to warm 1°F is changing by 15.625 min/°F; **(b)** 70.43°F **33. (a)** $t(5000)=22.34$ months, $t'(p)=0.0152$ month/bird. It takes about 22.34 months for the population to reach 5000 birds, and the time needed for the population to increase by one bird is about 0.0152 month/bird (about one new bird born every 10.9 hr); **(b)** 6570 birds **34. (a)** $t(12{,}000)=30.65$ weeks, $t'(12{,}000)=0.01$ week/firefly. It takes about 30.65 weeks for the population of fireflies to reach 12,000, and the time needed for the population to increase by one firefly is about 0.01 week/firefly (about one new firefly every 1.68 hr). **(b)** 8800 fireflies

35. $\dfrac{1}{x}$ **36.** $\dfrac{t(3t+4)}{(t+1)^2}+2\ln(t+1)$ **37.** $\dfrac{1}{(w-1)^2}-\dfrac{1}{w^2}$

38. $-\dfrac{2}{x^2}-\dfrac{9}{(3x-1)^2}$ **39. (a)** $T(t)=25-15e^{-0.02t}$; **(b)** 25, meaning that the water's temperature approaches 25°C as a limit. **40. (a)** $T(t)=76-38e^{-0.004t}$; **(b)** 76, meaning that the package's temperature approaches 76°F as a limit. **41. (a)** $p(t)=7500-4500e^{-0.0263t}$; **(b)** 7500, meaning that the population of birds approaches 7500 as a limit. **42. (a)** $p(t)=18{,}000-10{,}000e^{-0.0167t}$; **(b)** 18,000, meaning that the population of fireflies approaches 18,000 as a limit. **43.** (1.35, 0.3) **44.** (0.01832, −4.0) and (2.187, 0.782) **45.** $a=e^{-1}$ **46.** $a=-1$

Technology Connection, p. 221

1. 1.85 trillion **2.** 2.1 quadrillion **3.** 8.52×10^{22} (85.2 sextillion) **4.** Left to the student

Exercise Set 2.4, p. 226

1. $f(x)=ce^{4x}$ **2.** $g(x)=ce^{6x}$ **3.** $A(t)=ce^{-9t}$

4. $P(t)=ce^{-3t}$ **5.** $Q(t)=ce^{kt}$ **6.** $R(t)=ce^{kt}$

7. (a) $N(t)=453{,}000e^{0.039t}$; **(b)** 782,000; **(c)** about 30,499 applications/yr **8. (a)** $N(t)=50e^{0.10t}$; **(b)** 369; **(c)** 36.9 franchises/year **9. (a)** $P(t)=P_0e^{0.059t}$; **(b)** \$1060.78, \$1125.24; **(c)** \$62.59/yr, \$66.39/yr **10. (a)** $P(t)=P_0e^{0.043t}$; **(b)** \$20,878.76, \$21,796.13; **(c)** \$897.79/yr, \$937.23/yr

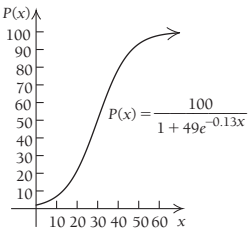
11. (a) $G(t)=11.8e^{0.085t}$; **(b)** 27.6 billion gallons; **(c)** 2.347 billion gallons/yr **12. (a)** $A(t)=75e^{0.292t}$; **(b)** 775.48 billion; **(c)** 226.4 billion/yr **13. (a)** $k=0.151$ (or 15.1%), $V(t)=30{,}000e^{0.151t}$; **(b)** \$2,486,000,000; **(c)** \$375,000,000/yr; **(d)** 78 yr **14. (a)** $I(t)=52{,}840e^{0.0219t}$; **(b)** \$70,244; **(c)** \$1538/yr **15. (a)** $F(t)=2.77e^{0.055314t}$; **(b)** \$4.08 trillion; **(c)** after 23.2 yr, or in 2036

16. (a) $c(t)=100e^{0.026114t}$; **(b)** \$284.21; **(c)** \$7.42/yr

17. (a) $y=7.384736154(1.472630104)^x$, $y=7.384736154e^{0.3870499885x}$; **(b)** 38.7%; **(c)** 163.3 EB; **(d)** after 8.5 yr; **(e)** about 77 EB/yr **18. (a)** $y=7e^{0.44365x}$; **(b)** 100.3 EB, 379.5 EB; **(c)** after 7.6 yr; **(d)** 1.56 yr; **(e)** answers may vary. **19.** About \$11 billion **20.** $R(t)=1.265e^{0.1283t}$; about \$188 billion **21. (a)** $S(t)=4e^{0.0451t}$; **(b)** 4.5%/yr; **(c)** 60 cents, 66 cents; **(d)** \$10,000; **(e)** answers may vary.

22. 10.89%; \$11,698,000 **23. (a)** 2%; **(b)** 3.8%, 7%, 21.6%, 50.2%, 93.1%, 98%; **(c)** $P'(x)=\dfrac{637e^{-0.13x}}{(1+49e^{-0.13x})^2}$;

(d)



24. (a) $c(t)=0.05e^{0.064t}$; **(b)** \$2.82; **(c)** \$0.18/yr

25. (a) $V(t)=0.10e^{0.227t}$; **(b)** about \$12,132,700; **(c)** after 86 yr, or in 2024 **26.** After 77.8 yr, in 2016 **27.** After 80.8 yr, in 2019 **28. (a)** 2.23%; **(b)** answers may vary.

29. (a) 40, 185, 199; **(b)** $P'(t)=\dfrac{61{,}100e^{-0.0982t}}{(17+183e^{-0.0982t})^2}$

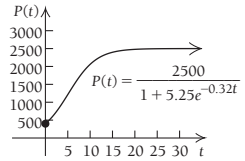
(c) 3.13 people/yr, 1.34 people/yr, 0.13 people/yr; **(d)** 200

30. (a) 20, 46, 119, 146; **(b)** $P'(t)=\dfrac{83{,}460e^{-0.214t}}{(20+130e^{-0.0214t})^2}$;

(c) 3.71 tortoises/yr, 6.86 tortoises/yr, 5.28 tortoises/yr, 0.93 tortoise/yr; **(d)** 150 **31. (a)** 400, 520, 1214, 2059, 2396, 2478;

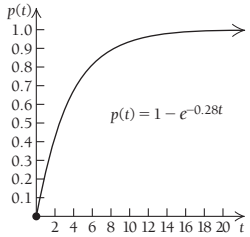
(b) $P'(t)=\dfrac{4200e^{-0.32t}}{(1+5.25e^{-0.32t})^2}$;

(c)



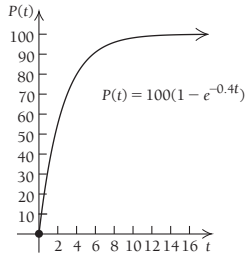
32. $N(t) = 48{,}869e^{0.03644t}$, where $t_0 = 1930$; exponential growth rate = 3.64% **33. (a)** 0.244, 0.429, 0.753, 0.954, 0.989, 0.996; **(b)** $p'(t) = 0.28e^{-0.28t}$;

(c)



34. (a) 11.8, 221.4, 547.2; **(b)** 4.4 new cases per week; **(c)** answers may vary. **35. (a)** 0%, 33%, 55%, 70%, 86%, 99.2%, 99.8%; **(b)** 2.43, which means that after 7 months, the percentage of physicians prescribing the new drug is increasing by 2.43 percentage points per month;

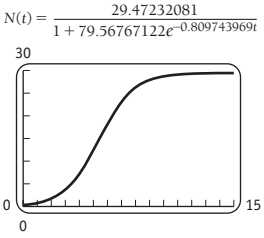
(c)



36. (a) $N(t) = \dfrac{29.47232081}{1 + 79.56767122e^{-0.809743969t}}$;

(b) 29 students;

(c)



(d) $N'(t) = \dfrac{1898.885181e^{-0.809743969t}}{(1 + 79.56767122e^{-0.809743969t})^2}$;

(e) answers may vary. **37–49**. Answers may vary.

50. $P'(t) = \dfrac{L^2kCe^{-Lkt}}{(1 + Ce^{-Lkt})^2}$. Also,

$$kP(L - P) = k\left(\frac{L}{1 + Ce^{-Lkt}}\right)\left(L - \frac{L}{1 + Ce^{-Lkt}}\right)$$

$$= k\left(\frac{L}{1 + Ce^{-Lkt}}\right)\left(\frac{L(1 + Ce^{-Lkt})}{1 + Ce^{-Lkt}} - \frac{L}{1 + Ce^{-Lkt}}\right).$$

Simplifying, we get

$$k\left(\frac{L}{1 + Ce^{-Lkt}}\right)\left(\frac{L + LCe^{-Lkt}}{1 + Ce^{-Lkt}} - \frac{L}{1 + Ce^{-Lkt}}\right)$$

$$= k\left(\frac{L}{1 + Ce^{-Lkt}}\right)\left(\frac{LCe^{-Lkt}}{1 + Ce^{-Lkt}}\right) = P'(t).$$

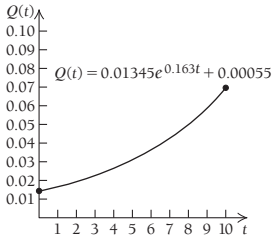
51. $P'(t) = -kCe^{-kt}$, and $k(L - P) = k\big(L - (L + Ce^{-kt})\big)$

$$= k(-Ce^{-kt}) = P'(t)$$

Exercise Set 2.5, p. 237

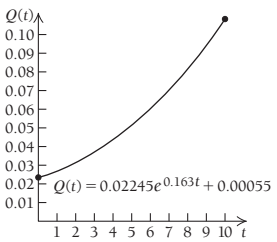
1. (a) $N(t) = N_0e^{-0.096t}$; **(b)** 341 g; **(c)** −32.7 g/day; **(d)** 7.2 days **2. (a)** $N(t) = N_0e^{-0.00012097t}$; **(b)** 182 g; **(c)** −0.022 g/yr; **(d)** 5730 yr **3. (a)** $N(t) = N_0e^{-0.000081547t}$; **(b)** 53.1 mg; **(c)** −0.00433 mg/yr; **(d)** 8500 yr **4. (a)** $N(t) = N_0e^{-0.0262t}$; **(b)** 51.9 mg; **(c)** −1.36 mg/sec; **(d)** 26.5 sec; **(e)** 176 sec **5. (a)** $N(t) = N_0e^{-0.2572t}$; **(b)** 1.65 mg; **(c)** −0.425 mg/day; **(d)** 2.69 days; **(e)** 7.38 days **6. (a)** $N(t) = N_0e^{-0.025t}$; **(b)** 9.45 mg; **(c)** −0.236 mg/day; **(d)** 27.7 days; **(e)** 119.83 days **7. (a)** $P(t) = 116{,}646e^{-0.0187t}$; **(b)** 60,621; **(c)** −1134 people/yr; **(d)** 37 yr **8. (a)** $P(t) = 92{,}124e^{-0.0272t}$; **(b)** 23,645; **(c)** −643 people/yr; **(d)** 25.5 yr **9. (a)** $A(t) = A_0e^{-kt}$; **(b)** 11 hr **10. (a)** $A(t) = A_0e^{-kt}$; **(b)** 9 hr **11. (a)** $A(t) = 35e^{-0.034t}$; **(b)** 47.34 min **12. (a)** $A(t) = 4e^{-0.0666t}$; **(b)** 34.6 hr **13. (a)** $P(t) = 5000e^{-0.116t}$; **(b)** 10.38 yr **14. (a)** $P(t) = 10{,}000e^{-0.099t}$; **(b)** 9.26 yr **15.** 5.78 yr **16.** 12.05 months **17.** 86.64 months **18.** 17.77 yr **19.** 23.1 yr **20.** 39.61 days **21.** 36.48 weeks **22.** 138.63 weeks **23.** 23.1%/min **24.** 0.0433%/yr **25.** 22 yr **26.** 25 yr **27.** 42.9 g **28.** 9.9 g **29.** 4223 yr **30.** 19,034 yr **31.** 25 days **32.** 7575 yr **33.** 3965 yr **34.** $9035.83 **35.** $13,858.23 **36.** $12,098.12 **37.** $6,788,463 **38.** $10,579,378 **39.** $42,863.76 **40.** $35,677.60 **41. (a)** $40,000; **(b)** $5413.41; **(c)** answers may vary. **42. (a)** $18,000; **(b)** $15,034.86; **(c)** answers may vary. **43. (a)** $23,500; **(b)** $14,541; **(c)** answers may vary. **44. (a)** $30,000; **(b)** $5936.96; **(c)** left to the student **45. (a)** $2600; **(b)** $2171.70; **(c)** left to the student **46. (a)** $y = 34{,}001.78697(0.6702977719)^x$, $V(t) = 34{,}001.78679e^{-0.4000332297t}$; **(b)** $2067.17, $622.56; **(c)** 8.8 yr; **(d)** 1.7 yr; **(e)** answers may vary. **47. (a)** 0.022, 0.031, 0.069;

(b)



48. (a) 0.037, 0.051, 0.115;

(b)



(c) $\frac{5}{3}$ **49. (a)** $N(t) = 5{,}650{,}000e^{-0.0153t}$; **(b)** 1,790,000; **(c)** in about 2063 **50. (a)** $B(t) = 64.6e^{-0.0068t}$; **(b)** 58.3 lb; **(c)** 2172 **51. (a)** $P(t) = 3.81e^{-0.00623t}$; **(b)** 3.36 million; **(c)** in 2028 **52. (a)** $P(t) = 2.37e^{-0.012t}$; **(b)** 1.86 million; **(c)** in 2025 **53. (a)** 60; **(b)** 0.01740; **(c)** 90°F; **(d)** 235 min; **(e)** answers may vary. **54. (a)** 27; **(b)** 0.05878; **(c)** 83°F; **(d)** 28.7 min; **(e)** answers may vary. **55.** At about 8 p.m. on the previous evening **56.** At about 7 p.m. **57. (a)** 112 lb; **(b)** −1 lb/day **58. (a)** 145 lb; **(b)** −1.2 lb/day **59. (a)** 14.0 lb/in^2; **(b)** 5.4 lb/in^2; **(c)** 0 ft; **(d)** answers may vary. **60. (a)** 11.2 W; **(b)** 173 days; **(c)** 402 days; **(d)** 50 W; **(e)** answers may vary. **61. (a)** $N(t) = 69{,}895e^{-0.0336t}$; **(b)** 8138, 7115; **(c)** after 78 yr, or in 2034 **62.** (a) or (c) **63.** (a) **64.** (e) **65.** (c) **66.** (f) **67.** (b) or (d) **68.** (d) **69.** (f) **70.** (a) **71.** (b) **72. (a)** 12.05 weeks; **(b)** 24.1 weeks **73. (a)** 4.27 yr; **(b)** 5.64 yr **74.** Answers may vary. **75.** Answers may vary. **76.** Answers may vary. **77.** Answers may vary. **78. (a)** Answers may vary; **(b)** 25%; **(c)** 6.25%

Exercise Set 2.6, p. 248

1. $P(t) = 450 \cdot 2^{t/5}$; $P(t) = 450e^{0.1386t}$
2. $P(t) = 1300 \cdot 2^{t/25}$; $P(t) = 1300e^{0.02773t}$
3. $P(t) = 5000 \cdot 3^{t/8}$; $P(t) = 5000e^{0.1373t}$
4. $P(t) = 6 \cdot 3^{t/9}$; $P(t) = 6e^{0.1221t}$
5. $P(t) = 100 \cdot 7^{t/12}$; $P(t) = 100e^{0.1622t}$
6. $P(t) = 35{,}000 \cdot 5^{t/20}$; $P(t) = 35{,}000e^{0.08047t}$
7. $P(t) = 1200(0.9)^{t/2}$; $P(t) = 1200e^{-0.05268t}$
8. $P(t) = 100{,}000(0.75)^{t/36}$; $P(t) = 100{,}000e^{-0.00799t}$
9. $P(t) = 60{,}000(0.6)^{t/8}$; $P(t) = 60{,}000e^{-0.0639t}$
10. $P(t) = 25{,}000(0.925)^{t/9}$; $P(t) = 25{,}000e^{-0.008662t}$
11. $P(t) = 6500 \cdot 1.5^{t/6}$; $P(t) = 6500e^{0.06758t}$
12. $P(t) = 75 \cdot 2.25^{t/2}$; $P(t) = 75e^{0.4055t}$ **13.** $\ln 6 \cdot 6^x$
14. $\ln 7 \cdot 7^x$ **15.** $\ln 15 \cdot 15^t$ **16.** $\ln 20 \cdot 20^t$
17. $\ln 12.5 \cdot 12.5^x$ **18.** $\ln\left(\frac{3}{4}\right) \cdot \left(\frac{3}{4}\right)^x$ **19.** $3 \ln 5 \cdot 5^x$
20. $24 \ln 9 \cdot 9^x$ **21.** $4x^3 \cdot \ln 7 \cdot 7^{x^4+2}$ **22.** $2x \cdot \ln 4 \cdot 4^{x^2+5}$
23. $100 \ln 0.52 \cdot 0.52^t$ **24.** $3500 \ln 0.038 \cdot 0.038^t$
25. $\frac{1}{x \ln 6}$ **26.** $\frac{1}{x \ln 13}$ **27.** $\frac{3}{x \ln 4}$ **28.** $\frac{7}{x \ln 11}$
29. $\frac{3}{(3x-1)\ln 2}$ **30.** $-\frac{4}{(5-4x)\ln 3}$ **31.** $\frac{10x+5}{(x^2+x)\ln 6}$
32. $\frac{16-24x^2}{(2x-x^3)\ln 3}$ **33.** $\frac{4^x}{x \ln 5} + \ln 4 \cdot 4^x \cdot \log_5 x$
34. $\frac{7^x}{x \ln 12} + \ln 7 \cdot 7^x \cdot \log_{12} x$ **35.** $3^x x(x \ln 3 + 2)$
36. $2 \cdot 5^x x^3(x \ln 5 + 4)$ **37.** $x^2\left(\frac{1}{\ln 7} + 3 \log_7 x\right)$
38. $x^5\left(\frac{1}{\ln 4} + 6 \log_4 x\right)$ **39.** $\frac{9^x((2x+1)\ln 9 - 2)}{(2x+1)^2}$
40. $\frac{3 \log_6 x - \frac{3x+2}{x \ln 6}}{(\log_6 x)^2}$ **41. (a)** $P(t) = 10{,}000 \cdot 2^{t/9}$; **(b)** $P(t) = 10{,}000e^{0.07702t}$; **(c)** 7.702%/yr; **(d)** 80,000 people; **(e)** by 3081 people/yr **42. (a)** $A(t) = 5000 \cdot 2^{t/8}$; **(b)** $A(t) = 5000e^{0.08664t}$; **(c)** 8.664%/yr; **(d)** $20,000; **(e)** by $3465.74/yr **43. (a)** $F(t) = 50 \cdot 3^{t/7}$; **(b)** $F(t) = 50e^{0.1569t}$; **(c)** 15.69%/month; **(d)** 450 followers; **(e)** by 212 people/month **44. (a)** $M(t) = 0.005 \cdot 5^{t/36}$; **(b)** $M(t) = 0.005e^{0.04471t}$; **(c)** 4.471%/hr; **(d)** 0.125 g; **(e)** by 0.0279 mg/hr **45. (a)** $V(t) = 100(0.5)^{t/5}$; **(b)** $V(t) = 100e^{-0.1386t}$; **(c)** −13.86%/day; **(d)** $12.50; **(e)** by −$3.47/day **46. (a)** $M(t) = 50(0.8)^{t/2}$; **(b)** $M(t) = 50e^{-0.1116t}$; **(c)** −11.16%/week; **(d)** 32 g; **(e)** by −2.856 g/week **47. (a)** $A(t) = 50{,}000 \cdot 1.2^{t/4}$; **(b)** $A(8) = 72{,}000$, which means that after 8 yr, the account is worth $72,000; **(c)** $A'(8) = 3281.79$, which means that after 8 years, the account is growing by $3281.79/yr
48. (a) $V(t) = 2500 \cdot 1.35^{t/10}$; **(b)** $V(20) = 4556.25$, which means that after 20 yr, the autograph is worth about $4556.25; **(c)** $V'(20) = 136.74$, which means that after 20 yr, the value of the autograph is increasing by $136.74/yr **49. (a)** After 5 yr, the value of the machine is $1703.94; **(b)** after 5 yr, the value is changing by −$380.22/yr; **(c)** after 3.11 yr **50. (a)** After 3 yr, there are 30,138.45 lb still is use; **(b)** after 3 yr, the amount in use is changing by −21,254 lb/yr; **(c)** after 3.27 yr **51. (a)** After 4 yr, there are 5408.51 lb of glass still in use; **(b)** after 4 yr, the amount of glass in use is changing by −5818.87 lb/yr; **(c)** after 2.78 yr
52. (a) After 10 yr, 0.817, or 81.7%, of the crop is nonheirloom; **(b)** in 10 yr, the percentage of the crop that is nonheirloom is changing by −1.65 percentage points/yr; **(c)** 34.31 yr
53. (a) $I(7) = I_0 10^7$; **(b)** $I(8) = I_0 10^8$; **(c)** a magnitude 8 quake is 10 times more powerful than a magnitude 7 quake; **(d)** $I'(R) = I_0 10^R(\ln 10)$; **(e)** answers may vary.
54. (a) $I(100) = I_0 10^{10}$; **(b)** $I(10) = I_0 10$; **(c)** the sound in part (a) is 10^9 times as powerful as the sound in part (b); **(d)** $I'(L) = I_0 10^{0.1L}(\ln 10)(0.1)$; **(e)** answers may vary.
55. (a) $R'(I) = \frac{1}{I \ln 10}$; **(b)** answers may vary.
56. (a) $L'(I) = \frac{10}{I \ln 10}$; **(b)** answers may vary.
57. (a) $P(t) = 35{,}000 \cdot 2^{t/16.233}$; **(b)** $P(t) = 35{,}000 \cdot 4^{t/32.466}$; **(c)** with base 4, the value of T is double that with base 2; **(d)** 48.699 **58. (a)** $M(t) = 25\left(\frac{1}{2}\right)^{t/6.359}$; **(b)** $M(t) = 25\left(\frac{1}{4}\right)^{t/12.718}$; **(c)** with base $\frac{1}{4}$, the value of T is double that with base $\frac{1}{2}$; **(d)** 19.077
59. (a) $P(t) = 100{,}000\left(\frac{1}{3}\right)^{t/33.291}$; **(b)** $P(t) = 100{,}000\left(\frac{1}{9}\right)^{t/66.582}$; **(c)** with base $\frac{1}{9}$, the value of T is double that with base $\frac{1}{3}$; **(d)** 99.873
60. (a) $A(t) = 2500 \cdot 3^{t/43.0828}$; **(b)** $A(t) = 2500 \cdot 9^{t/86.1657}$; **(c)** with base 9, the value of T is double that with base 3; **(d)** 129.2484 **61. (a)** 7.925 yr; **(b)** 800%
62. (a) 4.42 months; **(b)** 300% **63. (a)** 37.6 hr; **(b)** 75%
64. (a) 2 yr; **(b)** 64% **65.** $\ln a$ **66.** $\ln 3$ **67.** $\ln 7$ **68.** 1

Chapter 2 Review Exercises, p. 255

1. (b) **2.** (e) **3.** (f) **4.** (c) **5.** (a) **6.** (d)
7. False **8.** False **9.** False **10.** False **11.** True
12. True **13.** False **14.** True **15.** False **16.** False

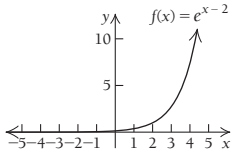
17. $\frac{1}{x}$ **18.** e^x **19.** $\frac{4x^3}{x^4+5}$ **20.** $\frac{e^{2\sqrt{x}}}{\sqrt{x}}$ **21.** $\frac{1}{2x}$

22. $3x^4e^{3x}+4x^3e^{3x}$ **23.** $\frac{1-3\ln x}{x^4}$ **24.** $\frac{e^{x^2}}{x}+2xe^{x^2}(\ln 4x)$

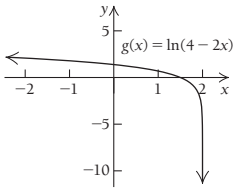
25. $4e^{4x}-\frac{1}{x}$ **26.** $\frac{1-x}{e^x}$ **27.** $(\ln 9)9^x$ **28.** $\frac{1}{x\ln 2}$

29. $\frac{2\cdot 3^x}{(2x+1)\ln 4}+3^x(\ln 3)(\log_4(2x+1))$

30. No x-intercept, y-intercept at $(0, e^{-2}) \approx (0, 0.135)$, domain: $\{x|-\infty < x < \infty\}$, range: $\{y|0 < y < \infty\}$



31. x-intercept at $\left(\frac{3}{2}, 0\right)$, y-intercept at $(0, \ln 4)$, domain: $\{x|-\infty < x < 2\}$, range: $\{y|-\infty < y < \infty\}$

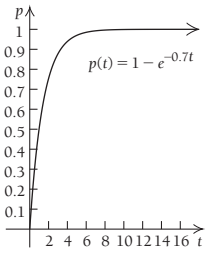


32. 2.639 **33.** −1.253 **34.** 3.332 **35.** 1.253 **36.** 0.973
37. −1.386 **38.** $Q(t) = 25e^{7t}$ **39.** **(a)** $r = 0.0433$, or 4.33%; **(b)** $P(t) = 4000e^{0.0433t}$; **(c)** 5416; **(d)** 235 people/yr
40. \$411.51/yr **41.** **(a)** $C(t) = 15.81e^{0.0206t}$; **(b)** \$34.59; **(c)** \$0.71/yr **42.** **(a)** $C(t) = 2.69e^{0.0039t}$; **(b)** \$3.14; **(c)** \$0.012/yr **43.** **(a)** $P(t) = 120 - 40e^{-0.0406t}$; **(b)** 0.721, meaning that the stock is increasing in value by about \$0.72/week after 20 weeks; **(c)** after about 34 weeks

44. **(a)** $N(t) = \frac{1000}{1+49e^{-0.0604t}}$; **(b)** after about 64.4 hr

45. **(a)** $N(t) = 60e^{0.12t}$; **(b)** 157; **(c)** about 19 franchises/yr
46. **(a)** $N(t) = 24e^{0.07t}$; **(b)** about 74; **(c)** about 5.15 franchises/yr
47. **(a)** $N(t) = 10e^{-0.13t}$; **(b)** 6.77 mg; **(c)** −0.88 mg/yr
48. **(a)** −18.2%/day; **(b)** $N(t) = 50e^{-0.182t}$; **(c)** −3.05 mg/day
49. **(a)** $A(t) = 800e^{-0.07t}$; **(b)** 197 g; **(c)** −13.81 g/day
50. **(a)** 0.5, 0.75, 0.97, 0.999; **(b)** $p'(t) = 0.7e^{-0.7t}$; **(c)** answers may vary;

(d)

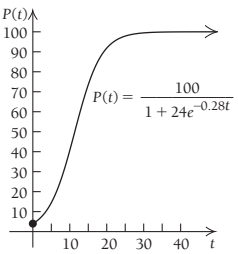


51. \$186,373.98 **52.** **(a)** $P(t) = 2500\cdot 2^{t/6.5}$; **(b)** $P(t) = 2500e^{0.1066t}$; **(c)** 10.66%; **(d)** \$10,000
53. **(a)** $P(t) = 15{,}000\cdot 1.25^{t/5}$; **(b)** 23,438 people; **(c)** 1045.99, meaning that Oak Fork was growing by about 1046 people/yr in 2000 **54.** $\ln 4\cdot 4^{x^3+2x+1}[6x+\ln 4(9x^4+12x^2+4)]$
55. 19.02 yr **56.** 18.68 yr **57.** 0
58. **(a)** $y = 989.9661965(1.033223777)^x$, $y = 989.9661965e^{0.0326837949t}$, 3.27%; **(b)** \$1616.4 billion; **(c)** after about 21.5 yr; **(d)** 21.2 yr

Chapter 2 Test, p. 257

1. $6e^{3x}$ **2.** $\frac{4(\ln x)^3}{x}$ **3.** $-2xe^{-x^2}$ **4.** $\frac{1}{x}$ **5.** e^x-15x^2
6. $\frac{3e^x}{x}+3e^x\ln x$ **7.** $(\ln 7)7^x+(\ln 3)3^x$ **8.** $\frac{1}{x\ln 14}$
9. **(a)** 9.5427; **(b)** 0.2746 **10.** 2.302 **11.** 3.218
12. −0.916 **13.** $M(t) = 2e^{6t}$ **14.** 23.1%/hr
15. **(a)** $A(t) = 10{,}000e^{0.06931t}$; **(b)** \$13,194.83; **(c)** \$914.53/yr
16. **(a)** $C(t) = 3.22e^{0.0028t}$; **(b)** \$3.39; **(c)** \$0.009/yr
17. **(a)** $A(t) = 3e^{-0.1t}$; **(b)** 1.1 cc; **(c)** −0.11 cc/hr; **(d)** after 6.9 hr **18.** About 16.47 centuries, or 1647 yr
19. **(a)** $A(t) = 14e^{-0.04077t}$; **(b)** 1.21 mg; **(c)** −0.049 mg/s
20. **(a)** 4%; **(b)** 5.2%, 14.5%, 40.7%, 91.8%, 99.5%;
(c) $P'(t) = \frac{672e^{-0.28t}}{(1+24e^{-0.28t})^2}$; **(d)** answers may vary;

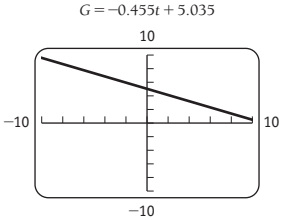
(e)



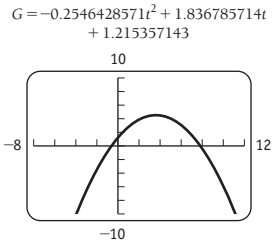
21. **(a)** $A(t) = 10{,}000\cdot 2^{t/8.25}$; **(b)** $A(t) = 10{,}000e^{0.084t}$; **(c)** 8.4%; **(d)** 2301.70, which means that the value of Andres's account is growing by about \$2301.70 per year after 12 yr
22. **(a)** $P(t) = 7500\cdot(0.8)^{t/3}$; **(b)** 9.32 yr; **(c)** −307.68, which means that after 8 yr, the population is decreasing by about 308 people/yr **23.** $(\ln x)^2$ **24.** $\frac{24-18x^2}{(3x^2+4)^2\ln 3}$
25. 2 **26.** **(a)** $y = 740336.2908(1.073657297)^x$, $y = 740336.2908e^{0.071070855t}$; **(b)** \$7.2 million; **(c)** 101 yr; **(d)** about 9.8 yr

Extended Technology Application, p. 259

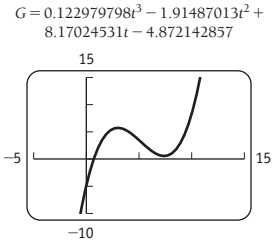
1. Linear: $G = -0.455t + 5.035$



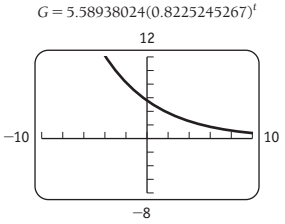
Quadratic: $G = -0.2546428571t^2 + 1.836785714t + 1.215357143$



Cubic: $G = 0.122979798t^3 - 1.91487013t^2 + 8.17024531t - 4.872142857$



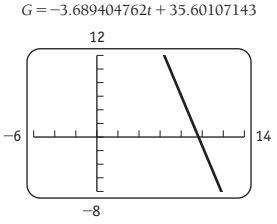
Exponential: $G = 5.58938024(0.8225245267)^t$



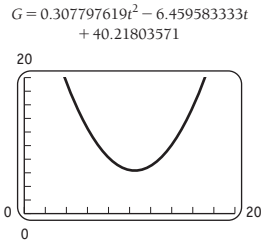
The cubic function fits well, but rises at higher values of t, which is not appropriate since weekly revenues are expected to fall as t increases. The exponential function also fits well and is better suited for forecasting future revenue. **2.** Week 9: \$0.96 million; week 10: \$0.79 million; week 11: \$0.65 million; week 12: \$0.54 million; week 13: \$0.44 million **3.** Week 9: \$24.86 million; week 10: \$25.65 million; week 11: \$26.3 million; week 12: \$26.84 million; week 13: \$27.28 million **4.** $R(t) = \dfrac{23.38497984}{1 + 23.62209015e^{-1.055833743t}}$. The limiting value is about \$23.4 million, which seems too low. **5.** $R(t) = \dfrac{24.31102164}{1 + 8.994071065e^{-0.7582056951t}}$. The limiting value is about \$24.3 million, which agrees better with the actual data.

6. Left to the student

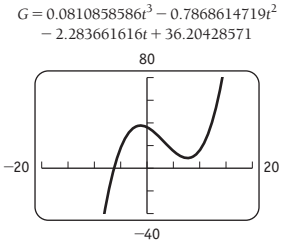
7. Linear: $G = -3.689404762t + 35.60107143$



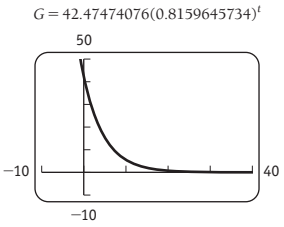
Quadratic: $G = 0.307797619t^2 - 6.459583333t + 40.21803571$



Cubic: $G = 0.0810858586t^3 - 0.7868614719t^2 - 2.283661616t + 36.20428571$



Exponential: $G = 42.47474076(0.8159645734)^t$



The exponential function is an excellent fit. **8.** Week 9: \$6.81 million; week 10: \$5.56 million; week 11: \$4.53 million; week 12: \$3.7 million; week 13: \$3.02 million; week 14: \$2.46 million **9.** $R(t) = \dfrac{153.2872607}{1 + 5.563261043e^{-0.6327045804t}}$. The limiting value is about \$153.29 million, which seems too low.

10. $R(t) = \dfrac{169.4506888}{1 + 3.215356463e^{-0.4138102809t}}$. The limiting value is about \$169.45 million. This seems to be a better estimation.

11. In the 9th week **12.** Answers may vary.