INSTRUCTOR'S SOLUTIONS MANUAL

PROBABILITY AND STATISTICAL INFERENCE

TENTH EDITION

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Pearson

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Publishing as Pearson, 330 Hudson Street, NY NY 10013

Pearson

ISBN-13: 978-0-13-518948-1
ISBN-10: 0-13-518948-9

Contents

Preface

This solutions manual provides answers for the even-numbered exercises in *Probability and Statistical Inference*, tenth edition, by Robert V. Hogg, Elliot A. Tanis, and Dale L. Zimmerman. Complete solutions are given for most of these exercises. You, the instructor, may decide how many of these solutions and answers you want to make available to your students. Note that the answers for the odd-numbered exercises are given in the textbook. Our hope is that this solutions manual will be helpful to each of you in your teaching.

All of the figures in this manual were generated using *Maple*, a computer algebra system. Most of the figures were generated and many of the solutions, especially those involving data, were solved using procedures that were written by Zaven Karian from Denison University. We thank him for providing these. These procedures are available free of charge for your use. They are available for down load at http://www.math.hope.edu/tanis/. Short descriptions of these procedures are provided on the "Maple Card." Complete descriptions of these procedures are given in *Probability and Statistics: Explorations with MAPLE*, second edition, 1999, written by Zaven Karian and Elliot Tanis, published by Prentice Hall (ISBN 0-13-021536-8). You can download a slightly revised edition of this manual at http://www.math.hope.edu/tanis/MapleManual.pdf.

We also want to acknowledge the many suggestions/corrections that were made by our accuracy checker, Kyle Siegrist.

If you find an error or wish to make a suggestion, please send them to dale-zimmerman@uiowa.edu. These **errata** will be posted on http://homepage.divms.uiowa.edu/∼dzimmer/.

E.A.T.
D.L.Z.

Chapter 1

Probability

1.1 Properties of Probability

1.1-2 Sketch a figure and fill in the probabilities of each of the disjoint sets.

Let $A = \{\text{insure more than one car}\}$, $P(A) = 0.85$.

Let $B = \{\text{insure a sports car}\}$, $P(B) = 0.23$.

Let $C = \{\text{insure exactly one car}\}$, $P(C) = 0.15$.

It is also given that $P(A \cap B) = 0.17$. Since $A \cap C = \phi$, $P(A \cap C) = 0$. It follows that $P(A \cap B \cap C') = 0.17$. Thus $P(A' \cap B \cap C') = 0.06$ and $P(B' \cap C) = 0.09$.

1.1-4 **(a)** $S = \{$HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, HTHT, THTH, THHT, HTTT, THTT, TTHT, TTTH, TTTT$\}$;

(b) **(i)** 5/16, **(ii)** 0, **(iii)** 11/16, **(iv)** 4/16, **(v)** 4/16, **(vi)** 9/16, **(vii)** 4/16.

1.1-6 **(a)** $P(A \cup B) = 0.5 + 0.6 - 0.4 = 0.7$;

(b)

$$\begin{aligned} A &= (A \cap B') \cup (A \cap B) \\ P(A) &= P(A \cap B') + P(A \cap B) \\ 0.5 &= P(A \cap B') + 0.4 \\ P(A \cap B') &= 0.1; \end{aligned}$$

(c) $P(A' \cup B') = P[(A \cap B)'] = 1 - P(A \cap B) = 1 - 0.4 = 0.6$.

1.1-8 Let A =$\{$lab work done$\}$, B =$\{$referral to a specialist$\}$,

$P(A) = 0.41$, $P(B) = 0.53$, $P([A \cup B]') = 0.21$.

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ 0.79 &= 0.41 + 0.53 - P(A \cap B) \\ P(A \cap B) &= 0.41 + 0.53 - 0.79 \;=\; 0.15. \end{aligned}$$

1.1-10

$$\begin{aligned} A \cup B \cup C &= A \cup (B \cup C) \\ P(A \cup B \cup C) &= P(A) + P(B \cup C) - P[A \cap (B \cup C)] \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P[(A \cap B) \cup (A \cap C)] \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) \\ &\quad + P(A \cap B \cap C). \end{aligned}$$

1.1-12 **(a)** 1/3; **(b)** 2/3; **(c)** 0; **(d)** 1/2.

1.1-14 $P(A) = \dfrac{2[r - r(\sqrt{3}/2)]}{2r} = 1 - \dfrac{\sqrt{3}}{2}.$

1.1-16 Note that the respective probabilities are $p_0,\ p_1 = p_0/4,\ p_2 = p_0/4^2, \cdots$.

$$\begin{aligned}\sum_{k=0}^{\infty} \frac{p_0}{4^k} &= 1 \\ \frac{p_0}{1 - 1/4} &= 1 \\ p_0 &= \frac{3}{4}\end{aligned}$$

$$1 - p_0 - p_1 = 1 - \frac{15}{16} = \frac{1}{16}.$$

1.2 Methods of Enumeration

1.2-2 **(a)** $(4)(5)(2) = 40$; **(b)** $(2)(2)(2) = 8$.

1.2-4 **(a)** $4\binom{6}{3} = 80$;

(b) $4(2^6) = 256$;

(c) $\dfrac{(4-1+3)!}{(4-1)!3!} = 20$.

1.2-6 $S =\{$ DDD, DDFD, DFDD, FDDD, DDFFD, DFDFD, FDDFD, DFFDD, FDFDD, FFDDD, FFF, FFDF, FDFF, DFFF FFDDF, FDFDF, DFFDF, FDDFF, DFDFF, DDFFF $\}$ so there are 20 possibilities. Note that the winning player (2 choices) must win the last set and two of the previous sets, so the number of outcomes is

$$2\left[\binom{2}{2} + \binom{3}{2} + \binom{4}{2}\right] = 20.$$

1.2-8 $3 \cdot 3 \cdot 2^{12} = 36{,}864$.

1.2-10
$$\begin{aligned}\binom{n-1}{r} + \binom{n-1}{r-1} &= \frac{(n-1)!}{r!(n-1-r)!} + \frac{(n-1)!}{(r-1)!(n-r)!} \\ &= \frac{(n-r)(n-1)! + r(n-1)!}{r!(n-r)!} = \frac{n!}{r!(n-r)!} = \binom{n}{r}.\end{aligned}$$

1.2-12
$$\begin{aligned}0 &= (1-1)^n = \sum_{r=0}^{n} \binom{n}{r}(-1)^r(1)^{n-r} = \sum_{r=0}^{n}(-1)^r\binom{n}{r}. \\ 2^n &= (1+1)^n = \sum_{r=0}^{n} \binom{n}{r}(1)^r(1)^{n-r} = \sum_{r=0}^{n}\binom{n}{r}.\end{aligned}$$

1.2-14 $\dbinom{5-1+29}{29} = \dfrac{33!}{29!4!} = 40{,}920$.

1.2-16 **(a)** $\dfrac{\dbinom{19}{3}\dbinom{52-19}{6}}{\dbinom{52}{9}} = \dfrac{102{,}486}{351{,}325} = 0.2917$;

(b) $\dfrac{\binom{19}{3}\binom{10}{2}\binom{7}{1}\binom{3}{0}\binom{5}{1}\binom{2}{0}\binom{6}{2}}{\binom{52}{9}} = \dfrac{7{,}695}{1{,}236{,}664} = 0.00622.$

1.2-18 **(a)** $P(A) = \sum_{n=1}^{5}(1/2)^n = 1 - (1/2)^5$;

(b) $P(B) = \sum_{n=1}^{10}(1/2)^n = 1 - (1/2)^{10}$;

(c) $P(A \cup B) = P(B) = 1 - (1/2)^{10}$;

(d) $P(A \cap B) = P(A) = 1 - (1/2)^5$;

(e) $P(C) = P(B) - P(A) = (1/2)^5 - (1/2)^{10}$;

(f) $P(B') = 1 - P(B) = (1/2)^{10}$.

1.3 Conditional Probability

1.3-2 **(a)** $\dfrac{1041}{1456}$;

(b) $\dfrac{392}{633}$;

(c) $\dfrac{649}{823}$.

(d) The proportion of women who favor a gun law is greater than the proportion of men who favor a gun law.

1.3-4 **(a)** $P(\text{HH}) = \dfrac{13}{52} \cdot \dfrac{12}{51} = \dfrac{1}{17}$;

(b) $P(\text{HC}) = \dfrac{13}{52} \cdot \dfrac{13}{51} = \dfrac{13}{204}$;

(c) $P(\text{Non-Ace Heart, Ace}) + P(\text{Ace of Hearts, Non-Heart Ace})$

$$= \frac{12}{52} \cdot \frac{4}{51} + \frac{1}{52} \cdot \frac{3}{51} = \frac{51}{52 \cdot 51} = \frac{1}{52}.$$

1.3-6 Let H ={died from heart disease}; P ={at least one parent had heart disease}.

$$P(H \mid P') = \frac{N(H \cap P')}{N(P')} = \frac{110}{648}.$$

1.3-8 **(a)** $\dfrac{3}{20} \cdot \dfrac{2}{19} \cdot \dfrac{1}{18} = \dfrac{1}{1140}$;

(b) $\dfrac{\binom{3}{2}\binom{17}{1}}{\binom{20}{3}} \cdot \dfrac{1}{17} = \dfrac{1}{380}$;

(c) $\displaystyle\sum_{k=1}^{9} \frac{\binom{3}{2}\binom{17}{2k-2}}{\binom{20}{2k}} \cdot \frac{1}{20-2k} = \frac{35}{76} = 0.4605$;

(d) Draw second. The probability of winning is $1 - 0.4605 = 0.5395$.

1.3-10 **(a)** $P(A) = \dfrac{52}{52} \cdot \dfrac{51}{52} \cdot \dfrac{50}{52} \cdot \dfrac{49}{52} \cdot \dfrac{48}{52} \cdot \dfrac{47}{52} = \dfrac{8{,}808{,}975}{11{,}881{,}376} = 0.74141$;

(b) $P(A') = 1 - P(A) = 0.25859$.

1.3-12 **(a)** It doesn't matter because $P(B_1) = \dfrac{1}{18},\ P(B_5) = \dfrac{1}{18},\ P(B_{18}) = \dfrac{1}{18}$;

(b) $P(B) = \dfrac{2}{18} = \dfrac{1}{9}$ on each draw.

1.3-14 **(a)** $5 \cdot 4 \cdot 3 = 60$;

(b) $5 \cdot 5 \cdot 5 = 125$.

1.3-16 $\dfrac{3}{5} \cdot \dfrac{5}{8} + \dfrac{2}{5} \cdot \dfrac{4}{8} = \dfrac{23}{40}$.

1.4 Independent Events

1.4-2 **(a)**
$$\begin{aligned} P(A \cap B) &= P(A)P(B) = (0.3)(0.6) = 0.18; \\ P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.3 + 0.6 - 0.18 \\ &= 0.72; \end{aligned}$$

(b) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0}{0.6} = 0$.

1.4-4 Proof of **(b)**:
$$\begin{aligned} P(A' \cap B) &= P(B)P(A'|B) \\ &= P(B)[1 - P(A|B)] \\ &= P(B)[1 - P(A)] \\ &= P(B)P(A'). \end{aligned}$$

Proof of **(c)**:
$$\begin{aligned} P(A' \cap B') &= P[(A \cup B)'] \\ &= 1 - P(A \cup B) \\ &= 1 - P(A) - P(B) + P(A \cap B) \\ &= 1 - P(A) - P(B) + P(A)P(B) \\ &= [1 - P(A)][1 - P(B)] \\ &= P(A')P(B'). \end{aligned}$$

1.4-6
$$\begin{aligned} P[A \cap (B \cap C)] &= P[A \cap B \cap C] \\ &= P(A)P(B)P(C) \\ &= P(A)P(B \cap C). \end{aligned}$$

$$\begin{aligned} P[A \cap (B \cup C)] &= P[(A \cap B) \cup (A \cap C)] \\ &= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \\ &= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C) \\ &= P(A)[P(B) + P(C) - P(B \cap C)] \\ &= P(A)P(B \cup C). \end{aligned}$$

$$\begin{aligned} P[A' \cap (B \cap C')] &= P(A' \cap C' \cap B) \\ &= P(B)[P(A' \cap C') \mid B] \\ &= P(B)[1 - P(A \cup C \mid B)] \\ &= P(B)[1 - P(A \cup C)] \\ &= P(B)P[(A \cup C)'] \\ &= P(B)P(A' \cap C') \\ &= P(B)P(A')P(C') \\ &= P(A')P(B)P(C') \\ &= P(A')P(B \cap C'). \end{aligned}$$

$$\begin{aligned}
P[A' \cap B' \cap C'] &= P[(A \cup B \cup C)'] \\
&= 1 - P(A \cup B \cup C) \\
&= 1 - P(A) - P(B) - P(C) + P(A)P(B) + P(A)P(C) + \\
&\qquad P(B)P(C) - PA)P(B)P(C) \\
&= [1 - P(A)][1 - P(B)][1 - P(C)] \\
&= P(A')P(B')P(C').
\end{aligned}$$

1.4-8 $\dfrac{1}{6} \cdot \dfrac{2}{6} \cdot \dfrac{3}{6} + \dfrac{1}{6} \cdot \dfrac{4}{6} \cdot \dfrac{3}{6} + \dfrac{5}{6} \cdot \dfrac{2}{6} \cdot \dfrac{3}{6} = \dfrac{2}{9}$.

1.4-10 **(a)** $\dfrac{3}{4} \cdot \dfrac{3}{4} = \dfrac{9}{16}$;

(b) $\dfrac{1}{4} \cdot \dfrac{3}{4} + \dfrac{3}{4} \cdot \dfrac{2}{4} = \dfrac{9}{16}$;

(c) $\dfrac{2}{4} \cdot \dfrac{1}{4} + \dfrac{2}{4} \cdot \dfrac{4}{4} = \dfrac{10}{16}$.

1.4-12 **(a)** $\left(\dfrac{1}{2}\right)^3 \left(\dfrac{1}{2}\right)^2$;

(b) $\left(\dfrac{1}{2}\right)^3 \left(\dfrac{1}{2}\right)^2$;

(c) $\left(\dfrac{1}{2}\right)^3 \left(\dfrac{1}{2}\right)^2$;

(d) $\dfrac{5!}{3!\,2!} \left(\dfrac{1}{2}\right)^3 \left(\dfrac{1}{2}\right)^2$.

1.4-14 **(a)** $1 - (0.4)^3 = 1 - 0.064 = 0.936$;

(b) $1 - (0.4)^8 = 1 - 0.00065536 = 0.99934464$.

1.4-16 **(a)** $\displaystyle\sum_{k=0}^{\infty} \frac{1}{5}\left(\frac{4}{5}\right)^{2k} = \frac{5}{9}$;

(b) $\dfrac{1}{5} + \dfrac{4}{5} \cdot \dfrac{3}{4} \cdot \dfrac{1}{3} + \dfrac{4}{5} \cdot \dfrac{3}{4} \cdot \dfrac{2}{3} \cdot \dfrac{1}{2} \cdot \dfrac{1}{1} = \dfrac{3}{5}$.

1.4-18 **(a)** 7; **(b)** $(1/2)^7$; **(c)** 63; **(d)** No! $(1/2)^{63} = 1/9{,}223{,}372{,}036{,}854{,}775{,}808$.

1.4-20 No. The equations that must hold are

$$(1 - p_1)(1 - p_2) = p_1(1 - p_2) + p_2(1 - p_1) = p_1 p_2.$$

There are no real solutions.

1.5 Bayes' Theorem

1.5-2 **(a)**
$$\begin{aligned}
P(G) &= P(A \cap G) + P(B \cap G) \\
&= P(A)P(G \,|\, A) + P(B)P(G \,|\, B) \\
&= (0.40)(0.85) + (0.60)(0.75) = 0.79;
\end{aligned}$$

(b)
$$\begin{aligned}
P(A \,|\, G) &= \frac{P(A \cap G)}{P(G)} \\
&= \frac{(0.40)(0.85)}{0.79} = 0.43.
\end{aligned}$$

1.5-4 Let event B denote an accident and let A_1 be the event that age of the driver is 16–25. Then

$$\begin{aligned} P(A_1 \mid B) &= \frac{(0.1)(0.05)}{(0.1)(0.05)+(0.55)(0.02)+(0.20)(0.03)+(0.15)(0.04)} \\ &= \frac{50}{50+110+60+60} = \frac{50}{280} = 0.179. \end{aligned}$$

1.5-6 Let B be the event that the policyholder dies. Let A_1, A_2, A_3 be the events that the deceased is standard, preferred and ultra-preferred, respectively. Then

$$\begin{aligned} P(A_1 \mid B) &= \frac{(0.60)(0.01)}{(0.60)(0.01)+(0.30)(0.008)+(0.10)(0.007)} \\ &= \frac{60}{60+24+7} = \frac{60}{91} = 0.659; \\ P(A_2 \mid B) &= \frac{24}{91} = 0.264; \\ P(A_3 \mid B) &= \frac{7}{91} = 0.077. \end{aligned}$$

1.5-8 Let A be the event that the tablet is under warranty.

$$\begin{aligned} P(B_1 \mid A) &= \frac{(0.40)(0.10)}{(0.40)(0.10)+(0.30)(0.05)+(0.20)(0.03)+(0.10)(0.02)} \\ &= \frac{40}{40+15+6+2} = \frac{40}{63} = 0.635; \\ P(B_2 \mid A) &= \frac{15}{63} = 0.238; \\ P(B_3 \mid A) &= \frac{6}{63} = 0.095; \\ P(B_4 \mid A) &= \frac{2}{63} = 0.032. \end{aligned}$$

1.5-10 **(a)** $P(D^+) = (0.02)(0.92) + (0.98)(0.05) = 0.0184 + 0.0490 = 0.0674;$

(b) $P(A^- \mid D^+) = \dfrac{0.0490}{0.0674} = 0.727;\ P(A^+ \mid D^+) = \dfrac{0.0184}{0.0674} = 0.273;$

(c) $P(A^- \mid D^-) = \dfrac{(0.98)(0.95)}{(0.02)(0.08)+(0.98)(0.95)} = \dfrac{9310}{16+9310} = 0.998;$

$P(A^+ \mid D^-) = 0.002;$

(d) Yes, particularly those in part (b).

1.5-12 Let $D = \{\text{defective roll}\}$. Then

$$\begin{aligned} P(I \mid D) &= \frac{P(I \cap D)}{P(D)} \\ &= \frac{P(I)\cdot P(D \mid I)}{P(I)\cdot P(D \mid I) + P(II)\cdot P(D \mid II)} \\ &= \frac{(0.60)(0.03)}{(0.60)(0.03)+(0.40)(0.01)} \\ &= \frac{0.018}{0.018+0.004} = \frac{0.018}{0.022} = 0.818. \end{aligned}$$

Chapter 2

Discrete Distributions

2.1 Random Variables of the Discrete Type

2.1-2 **(a)**

$$f(x) = \begin{cases} 0.6, & x = 1, \\ 0.3, & x = 5, \\ 0.1, & x = 10; \end{cases}$$

(b)

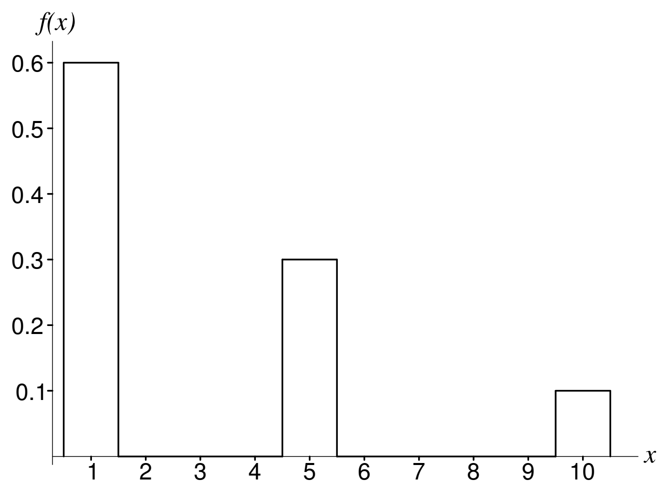

Figure 2.1–2: A probability histogram

2.1-4 **(a)**

$$\begin{aligned}\sum_{x=1}^{9} \log_{10}\left(\frac{x+1}{x}\right) &= \sum_{x=1}^{9} [\log_{10}(x+1) - \log_{10} x] \\ &= \log_{10} 2 - \log_{10} 1 + \log_{10} 3 - \log_{10} 2 + \cdots + \log_{10} 10 - \log_{10} 9 \\ &= \log_{10} 10 = 1;\end{aligned}$$

(b)

$$F(x) = \begin{cases} 0, & x < 1, \\ \log_{10} n, & n-1 \le x < n, \qquad n = 2, 3, \ldots, 9, \\ 1, & 9 \le x. \end{cases}$$

2.1-6 **(a)** $f(x) = \dfrac{1}{10}, \qquad x = 0, 1, 2, \cdots, 9;$

(b)
$\mathcal{N}(\{0\})/150 = 11/150 = 0.073;$ $\mathcal{N}(\{5\})/150 = 13/150 = 0.087;$
$\mathcal{N}(\{1\})/150 = 14/150 = 0.093;$ $\mathcal{N}(\{6\})/150 = 22/150 = 0.147;$
$\mathcal{N}(\{2\})/150 = 13/150 = 0.087;$ $\mathcal{N}(\{7\})/150 = 16/150 = 0.107;$
$\mathcal{N}(\{3\})/150 = 12/150 = 0.080;$ $\mathcal{N}(\{8\})/150 = 18/150 = 0.120;$
$\mathcal{N}(\{4\})/150 = 16/150 = 0.107;$ $\mathcal{N}(\{9\})/150 = 15/150 = 0.100.$

(c)

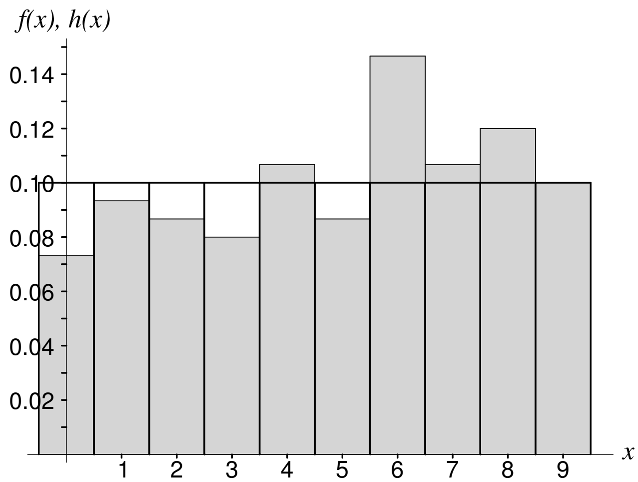

Figure 2.1–6: Michigan daily lottery digits

2.1-8 **(a)** $f(x) = \dfrac{6 - |7 - x|}{36}, \ x = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12;$

(b)

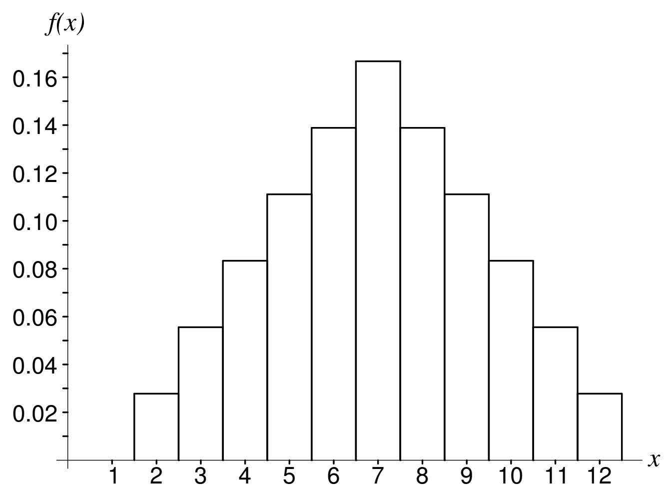

Figure 2.1–8: Probability histogram for the sum of a pair of dice

2.1-10 **(a)** The space of W is $S = \{0, 1, 2, 3, 4, 5, 6, 7\}$.

$P(W = 0) = P(X = 0, Y = 0) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$, assuming independence.

$P(W = 1) = P(X = 0, Y = 1) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$,

$P(W = 2) = P(X = 2, Y = 0) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$,

$P(W = 3) = P(X = 2, Y = 1) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$,

$P(W = 4) = P(X = 0, Y = 4) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$,

$P(W = 5) = P(X = 0, Y = 5) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$,

$P(W = 6) = P(X = 2, Y = 4) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$,

$P(W = 7) = P(X = 2, Y = 5) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$.

That is, $f(w) = P(W = w) = \frac{1}{8}, \quad w \in S$.

(b)

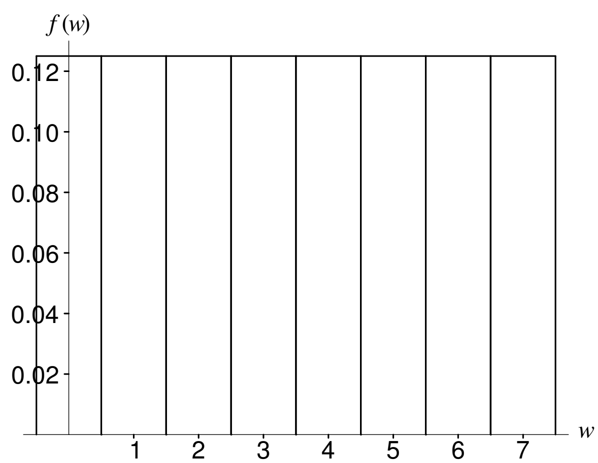

Figure 2.1–10: Probability histogram of sum of two special dice

2.1-12 Let x equal the number of orange balls and $144 - x$ the number of blue balls. Then

$$\begin{aligned}
\frac{x}{144} \cdot \frac{x-1}{143} + \frac{144-x}{144} \cdot \frac{143-x}{143} &= \frac{x}{144} \cdot \frac{144-x}{143} + \frac{144-x}{144} \cdot \frac{x}{143} \\
x^2 - x + 144 \cdot 143 - 144x - 143x + x^2 &= 2 \cdot 144x - 2 \cdot x^2 \\
x^2 - 144x + 5{,}148 &= 0 \\
(x - 78)(x - 66) &= 0
\end{aligned}$$

Thus there are 78 orange balls and 66 blue balls.

2.2 Mathematical Expectation

2.2-2 $E(X) = (-1)\left(\frac{4}{9}\right) + (0)\left(\frac{1}{9}\right) + (1)\left(\frac{4}{9}\right) = 0$;

$E(X^2) = (-1)^2\left(\frac{4}{9}\right) + (0)^2\left(\frac{1}{9}\right) + (1)^2\left(\frac{4}{9}\right) = \frac{8}{9}$;