# INSTRUCTOR'S SOLUTIONS MANUAL

EDGAR REYES
*Southeastern Louisiana University*

# TRIGONOMETRY

## FIFTH EDITION

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Pearson

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ISBN-13: 978-0-13-520744-4
ISBN-10: 0-13-520744-4

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## For Thought

**1.** False, the point $(2, -3)$ is in Quadrant IV.

**2.** False, the point $(4, 0)$ does not belong to any quadrant.

**3.** False, since the distance is $\sqrt{(a-c)^2 + (b-d)^2}$.

**4.** False, since $Ax + By = C$ is a linear equation.

**5.** True

**6.** False, since $\sqrt{7^2 + 9^2} = \sqrt{130} \approx 11.4$

**7.** True

**8.** True

**9.** True

**10.** False, since the radius is 3.

## P.1 Exercises

**1.** ordered

**2.** Cartesian

**3.** $x$-axis

**4.** origin

**5.** Pythagorean theorem

**6.** circle

**7.** linear equation

**8.** $y$-intercept

**9.** $(4, 1)$, Quadrant I

**10.** $(-3, 2)$, Quadrant II

**11.** $(1, 0)$, $x$-axis

**12.** $(-1, -5)$, Quadrant III

**13.** $(5, -1)$, Quadrant IV

**14.** $(0, -3)$, $y$-axis

**15.** $(-4, -2)$, Quadrant III

**16.** $(-2, 0)$, $x$-axis

**17.** $(-2, 4)$, Quadrant II

**18.** $(1, 5)$, Quadrant I

**19.** $c = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$

**20.** Since $a^2 + a^2 = \sqrt{2}^2$, we get $2a^2 = 2$ or $a^2 = 1$. Then $a = 1$.

**21.** Since $b^2 + 2^2 = 3^2$, we get $b^2 + 4 = 9$ or $b^2 = 5$. Then $b = \sqrt{5}$.

**22.** Since $b^2 + \left(\frac{1}{2}\right)^2 = 1^2$, we get $b^2 + \frac{1}{4} = 1$ or $b^2 = \frac{3}{4}$. Thus, $b = \frac{\sqrt{3}}{2}$.

**23.** Since $a^2 + 3^2 = 5^2$, we get $a^2 + 9 = 25$ or $a^2 = 16$. Then $a = 4$.

**24.** $c = \sqrt{3^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13}$

**25.** $\sqrt{4 \cdot 7} = 2\sqrt{7}$

**26.** $\sqrt{25 \cdot 2} = 5\sqrt{2}$

**27.** $\frac{\sqrt{5}}{\sqrt{9}} = \frac{\sqrt{5}}{3}$

**28.** $\frac{\sqrt{3}}{\sqrt{16}} = \frac{\sqrt{3}}{4}$

**29.** $\frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$

**30.** $\frac{\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15}}{5}$

**31.** $\frac{2\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{15}}{5}$

**32.** $\frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$

**33.** $\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

**34.** $\frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

**35.** $\frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$

**36.** $\frac{\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10}}{2}$

**37.** Distance is $\sqrt{(4-1)^2 + (7-3)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$, midpoint is $(2.5, 5)$

**38.** Distance is $\sqrt{144 + 25} = 13$, midpoint is $(3, 0.5)$

**39.** Distance is $\sqrt{(-1-1)^2 + (-2-0)^2} = \sqrt{4 + 4} = 2\sqrt{2}$, midpoint is $(0, -1)$

**40.** Distance is $\sqrt{4+4} = 2\sqrt{2}$, midpoint is $(0,1)$

**41.** Distance is $\sqrt{\left(\frac{\sqrt{2}}{2}-0\right)^2 + \left(\frac{\sqrt{2}}{2}-0\right)^2} =$ $\sqrt{\frac{2}{4}+\frac{2}{4}} = \sqrt{1} = 1$, midpoint is $\left(\frac{\sqrt{2}/2+0}{2}, \frac{\sqrt{2}/2+0}{2}\right) = \left(\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}\right)$

**42.** Distance is $\sqrt{\left(\sqrt{3}-0\right)^2 + (1-0)^2} =$ $\sqrt{3+1} = 2$, midpoint is $\left(\frac{\sqrt{3}+0}{2}, \frac{1+0}{2}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

**43.** Distance is $\sqrt{\left(\sqrt{18}-\sqrt{8}\right)^2 + \left(\sqrt{12}-\sqrt{27}\right)^2} =$ $\sqrt{(3\sqrt{2}-2\sqrt{2})^2 + (2\sqrt{3}-3\sqrt{3})^2} =$ $\sqrt{(\sqrt{2})^2 + (-\sqrt{3})^2} = \sqrt{5}$,
midpoint is $\left(\frac{\sqrt{18}+\sqrt{8}}{2}, \frac{\sqrt{12}+\sqrt{27}}{2}\right) =$ $\left(\frac{3\sqrt{2}+2\sqrt{2}}{2}, \frac{2\sqrt{3}+3\sqrt{3}}{2}\right) = \left(\frac{5\sqrt{2}}{2}, \frac{5\sqrt{3}}{2}\right)$

**44.** Distance is $\sqrt{\left(\sqrt{72}-\sqrt{50}\right)^2 + \left(\sqrt{45}-\sqrt{20}\right)^2} =$ $\sqrt{(6\sqrt{2}-5\sqrt{2})^2 + (3\sqrt{5}-2\sqrt{5})^2} =$ $\sqrt{(\sqrt{2})^2 + (\sqrt{5})^2} = \sqrt{7}$,
midpoint is $\left(\frac{\sqrt{72}+\sqrt{50}}{2}, \frac{\sqrt{45}+\sqrt{20}}{2}\right) =$ $\left(\frac{6\sqrt{2}+5\sqrt{2}}{2}, \frac{3\sqrt{5}+2\sqrt{5}}{2}\right) = \left(\frac{11\sqrt{2}}{2}, \frac{5\sqrt{5}}{2}\right)$

**45.** Distance is $\sqrt{(1.2+3.8)^2 + (4.4+2.2)^2} =$ $\sqrt{25+49} = \sqrt{74}$, midpoint is $(-1.3, 1.3)$

**46.** Distance is $\sqrt{49+81} = \sqrt{130}$,
midpoint is $(1.2, -3)$

**47.** Distance is $\frac{\sqrt{\pi^2+4}}{2}$, midpoint is $\left(\frac{3\pi}{4}, \frac{1}{2}\right)$

**48.** Distance is $\frac{\sqrt{\pi^2+4}}{2}$, midpoint is $\left(\frac{\pi}{4}, \frac{1}{2}\right)$

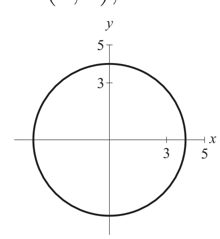
**49.** Distance is $\sqrt{(2\pi-\pi)^2 + (0-0)^2} = \sqrt{\pi^2} = \pi$,
midpoint is $\left(\frac{2\pi+\pi}{2}, \frac{0+0}{2}\right) = \left(\frac{3\pi}{2}, 0\right)$

**50.** Distance is $\sqrt{\left(\pi-\frac{\pi}{2}\right)^2 + (1-1)^2} = \sqrt{\frac{\pi^2}{4}} =$ $\frac{\pi}{2}$, midpoint is $\left(\frac{\pi+\pi/2}{2}, \frac{1+1}{2}\right) = \left(\frac{3\pi}{4}, 1\right)$

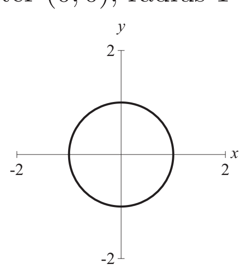
**51.** Distance is $\sqrt{\left(\frac{\pi}{2}-\frac{\pi}{3}\right)^2 + \left(-\frac{1}{3}-\frac{1}{2}\right)^2} =$ $\sqrt{\frac{\pi^2}{36}+\frac{25}{36}} = \frac{\sqrt{\pi^2+25}}{6}$,
midpoint is $\left(\frac{\frac{\pi}{3}+\frac{\pi}{2}}{2}, \frac{\frac{1}{2}-\frac{1}{3}}{2}\right) = \left(\frac{5\pi}{12}, \frac{1}{12}\right)$

**52.** Distance is $\sqrt{\left(\pi-\frac{2\pi}{3}\right)^2 + \left(-1+\frac{1}{2}\right)^2} =$ $\sqrt{\frac{\pi^2}{9}+\frac{1}{4}} = \frac{\sqrt{4\pi^2+9}}{6}$,
midpoint is $\left(\frac{\frac{2\pi}{3}+\pi}{2}, \frac{-\frac{1}{2}-1}{2}\right) = \left(\frac{5\pi}{6}, -\frac{3}{4}\right)$

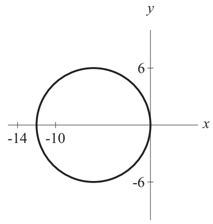
**53.** Center$(0,0)$, radius 4



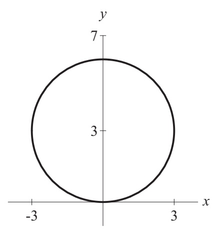

**54.** Center $(0,0)$, radius 1



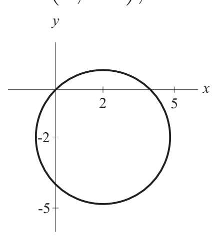
**55.** Center $(-6, 0)$, radius 6



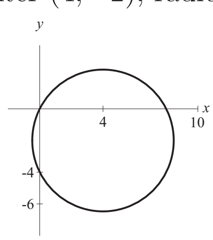

**56.** Center $(0, 3)$, radius 3




**57.** Center $(2, -2)$, radius $2\sqrt{2}$




**58.** Center $(4, -2)$, radius $2\sqrt{5}$




**59.** $x^2 + y^2 = 7$

**60.** $x^2 + y^2 = 12$ since $(2\sqrt{3})^2 = 12$

**61.** $(x + 2)^2 + (y - 5)^2 = 1/4$

**62.** $(x + 1)^2 + (y + 6)^2 = 1/9$

**63.** The distance between $(3, 5)$ and the origin is $\sqrt{34}$ which is the radius. The standard equation is $(x - 3)^2 + (y - 5)^2 = 34$.

**64.** The distance between $(-3, 9)$ and the origin is $\sqrt{90}$ which is the radius. The standard equation is $(x + 3)^2 + (y - 9)^2 = 90$.

**65.** Note, the distance between $(\sqrt{2}/2, \sqrt{2}/2)$ and the origin is 1. Thus, the radius is 1. The standard equation is $x^2 + y^2 = 1$.

**66.** Note, the distance between $(\sqrt{3}/2, 1/2)$ and the origin is 1. Thus, the radius is 1. The standard equation is $x^2 + y^2 = 1$.

**67.** The radius is $\sqrt{(-1-0)^2 + (2-0)^2} = \sqrt{5}$. The standard equation is $(x+1)^2 + (y-2)^2 = 5$.

**68.** Since the center is $(0, 0)$ and the radius is 2, the standard equation is $x^2 + y^2 = 4$.

**69.** Note, the center is $(1, 3)$ and the radius is 2. The standard equation is $(x-1)^2 + (y-3)^2 = 4$.

**70.** The radius is $\sqrt{(2-0)^2 + (2-0)^2} = \sqrt{8}$. The standard equation is $(x - 2)^2 + (y - 2)^2 = 8$.

**71.** We solve for $a$.

$$\begin{aligned} a^2 + \left(\frac{3}{5}\right)^2 &= 1 \\ a^2 &= 1 - \frac{9}{25} \\ a^2 &= \frac{16}{25} \\ a &= \pm\frac{4}{5} \end{aligned}$$

**72.** We solve for $a$.

$$\begin{aligned} a^2 + \left(-\frac{1}{2}\right)^2 &= 1 \\ a^2 &= 1 - \frac{1}{4} \\ a^2 &= \frac{3}{4} \\ a &= \pm\frac{\sqrt{3}}{2} \end{aligned}$$

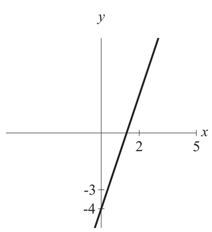
**73.** We solve for $a$.

$$\begin{aligned} \left(-\frac{2}{5}\right)^2 + a^2 &= 1 \\ a^2 &= 1 - \frac{4}{25} \\ a^2 &= \frac{21}{25} \\ a &= \pm\frac{\sqrt{21}}{5} \end{aligned}$$

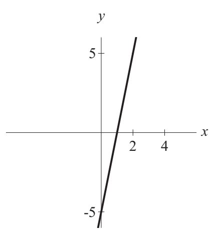
**74.** Solve for $a$:

$$\begin{aligned} \left(\frac{2}{3}\right)^2 + a^2 &= 1 \\ a^2 &= 1 - \frac{4}{9} \\ a^2 &= \frac{5}{9} \\ a &= \pm\frac{\sqrt{5}}{3} \end{aligned}$$

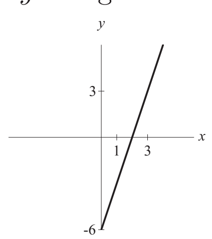
**75.** $y = 3x - 4$ goes through $(0, -4)$, $\left(\frac{4}{3}, 0\right)$.



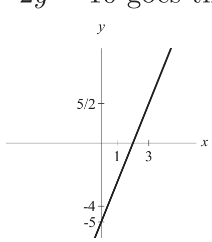

**76.** $y = 5x - 5$ goes through $(0, -5), (1, 0)$.



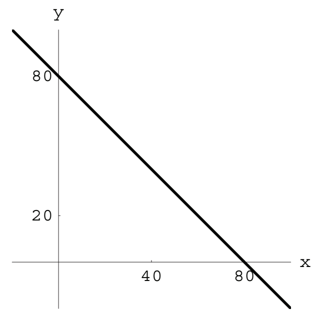

**77.** $3x - y = 6$ goes through $(0, -6)$, $(2, 0)$.



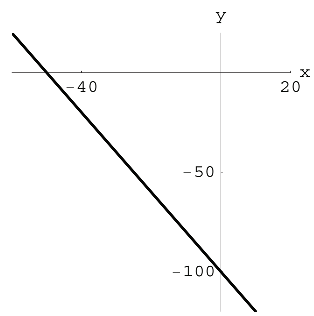

**78.** $5x - 2y = 10$ goes through $(0, -5)$, $(2, 0)$.



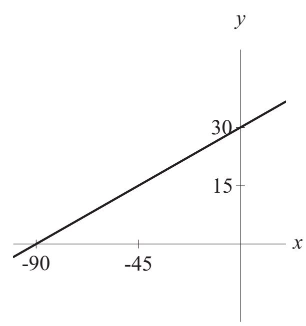

**79.** $x + y = 80$ goes through $(0, 80)$, $(80, 0)$.



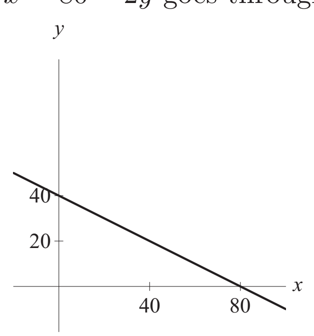

**80.** $2x + y = -100$ goes through $(0, -100)$, $(-50, 0)$.



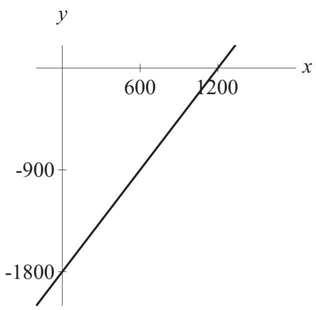

**81.** $x = 3y - 90$ goes through $(0, 30), (-90, 0)$.



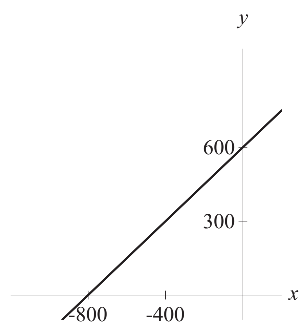

**82.** $x = 80 - 2y$ goes through $(0, 40), (80, 0)$.



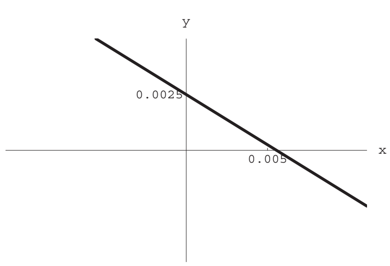
**83.** $\frac{1}{2}x - \frac{1}{3}y = 600$ goes through $(0, -1800)$, $(1200, 0)$.



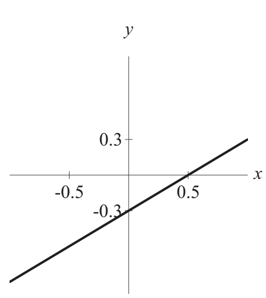

**84.** $\frac{2}{3}y - \frac{1}{2}x = 400$ goes through $(0, 600)$, $(-800, 0)$.



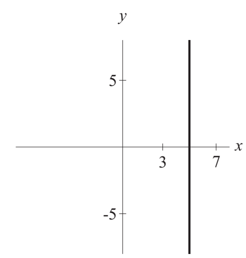

**85.** Intercepts are $(0, 0.0025), (0.005, 0)$.



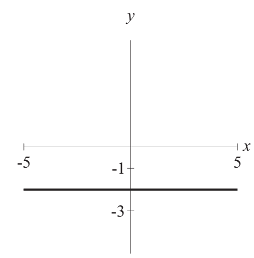

**86.** Intercepts are $(0, -0.3), (0.5, 0)$.



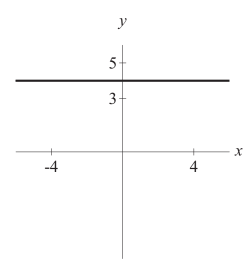

**87.** $x = 5$



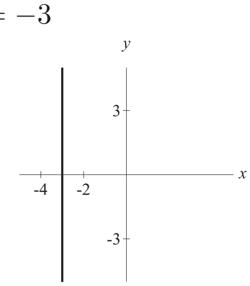

**88.** $y = -2$



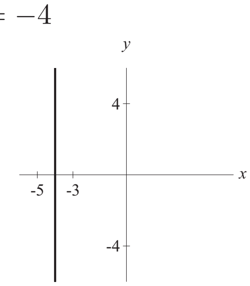

**89.** $y = 4$



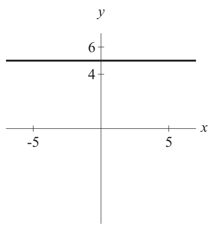

**90.** $x = -3$



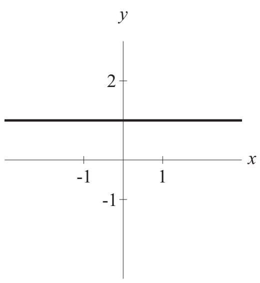

**91.** $x = -4$



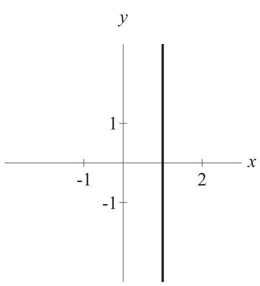
**92.** $y = 5$



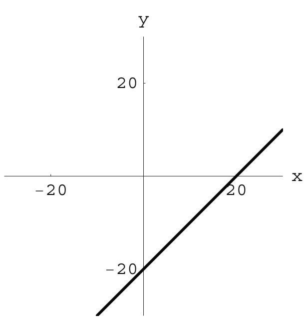

**93.** Solving for $y$, we have $y = 1$.



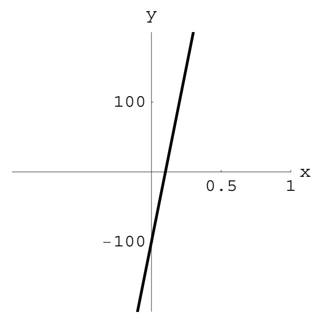

**94.** Solving for $x$, we get $x = 1$ .



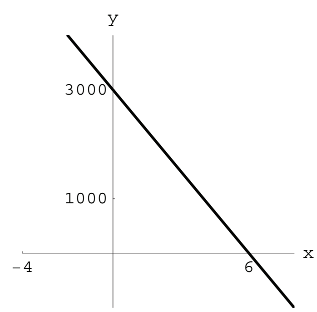

**95.** $y = x - 20$ goes through $(0, -20)$, $(20, 0)$.



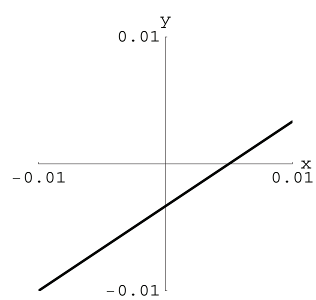

**96.** $y = 999x - 100$ goes through $(0, -100)$, $(100/999, 0)$.




**97.** $y = 3000 - 500x$ goes through $(0, 3000)$, $(6, 0)$.




**98.** $y = \dfrac{1}{300}(200x - 1)$ goes through $(0, -1/300)$, $(1/200, 0)$.




**99.** The hypotenuse is $\sqrt{6^2 + 8^2} = \sqrt{100} = 10$.

**100.** The other leg is $\sqrt{10^2 - 4^2} = \sqrt{84} = 2\sqrt{21}$ ft.

**101.** a) Let $r$ be the radius of the smaller circle. Consider the right triangle with vertices at the origin, another vertex at the center of a smaller circle, and a third vertex at the center of the circle of radius 1. By the Pythagorean Theorem, we obtain

$$\begin{aligned} 1 + (2 - r)^2 &= (1 + r)^2 \\ 5 - 4r &= 1 + 2r \end{aligned}$$

$$\begin{aligned} 4 &= 6r \\ r &= \frac{2}{3}. \end{aligned}$$

The diameter of the smaller circle is $2r = \frac{4}{3}$.

b) The smallest circles are centered at $(\pm r, 0)$ or $(\pm 4/3, 0)$. The equations of the circles are

$$\left(x - \frac{4}{3}\right)^2 + y^2 = \frac{4}{9}$$

and

$$\left(x + \frac{4}{3}\right)^2 + y^2 = \frac{4}{9}$$

**102.** Draw a right triangle with vertices at the centers of the circles, and another vertex at a point of intersection of the two circles. The legs of the right triangle are 5 and 12. By the Pythagorean theorem, the hypotenuse is $\sqrt{5^2 + 12^2} = 13$.

**103.** Let $C(h, k)$ and $r$ be the center and radius of the smallest circle, respectively. Then $k = -r$. We consider two right triangles each of which has a vertex at $C$.

The right triangles have sides that are perpendicular to the coordinate axes. Also, one side of each right triangle passes through the center of a larger circle.

Applying the Pythagorean Theorem, we list a system of equations

$$\begin{aligned} (r+1)^2 &= h^2 + (1-r)^2 \\ (2-r)^2 &= h^2 + r^2. \end{aligned}$$

The solutions are $r = 1/2$, $h = \sqrt{2}$, and $k = -r = -1/2$.

The equation of the smallest circle is

$$(x - \sqrt{2})^2 + (y + 1/2)^2 = 1/4.$$

**104.** We apply symmetry to the centers of the remaining three circles. From the answer or equation in Exercise 103, the equations of the remaining circles are

$$\begin{aligned} (x - \sqrt{2})^2 + (y - 1/2)^2 &= 1/4 \\ (x + \sqrt{2})^2 + (y - 1/2)^2 &= 1/4 \\ (x + \sqrt{2})^2 + (y + 1/2)^2 &= 1/4. \end{aligned}$$

**105.** The midpoint of $(0, 20.8)$ and $(48, 27.4)$ is

$$\left(\frac{0+48}{2}, \frac{20.8+27.4}{2}\right) = (24, 24.1).$$

In 1994 $(= 1970 + 24)$, the median age at first marriage was 24.1 years.

**106.** a) If $h = 0$, then $0 = 0.229n + 5.203$. Then $n = -5.203/0.229 \approx -22.72$. The $n$-intercept is

$$(-22.72, 0).$$

There were no unmarried couples in 1977 $(\approx 2000 - 22.7)$. Nonsense.

b) If $n = 0$, then $h = 0.229(0) + 5.203 = 5.203$. The $h$-intercept is $(0, 5.203)$. In 2000, there where 5,203,000 unmarried-couple households.

**107.** The distance between $(10, 0)$ and $(0, 0)$ is 10. The distance between $(1, 3)$ and the origin is $\sqrt{10}$. If two points have integer coordinates, then the distance between them is of the form $\sqrt{s^2 + t^2}$ where $s^2, t^2 \in \{0, 1, 2^2, 3^2, 4^2, ...\} = \{0, 1, 4, 9, 16, ...\}$.

Note, there are no numbers $s^2$ and $t^2$ in $\{0, 1, 4, 9, 16, ...\}$ satisfying $s^2 + t^2 = 19$. Thus, one cannot find two points with integer coordinates whose distance between them is $\sqrt{19}$.

**108.** One can assume the vertices of the right triangle are $A(0, 0)$, $B(1, \sqrt{3})$, and $C(1, 0)$.

The midpoint of the hypotenuse $AB$ is $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. The distance between the midpoint and $C$ is $\sqrt{\left(\frac{1}{2} - 1\right)^2 + \left(\frac{\sqrt{3}}{2} - 0\right)^2} = 1$, which is also the distance from the midpoint to $A$, and the distance from the midpoint to $B$.

**111.** On day 1, break off a 1-dollar piece and pay the gardener.

On day 2, break of a 2-dollar and pay the gardener. The gardener will give you back your change which is a 1-dollar piece.

On day 3, you pay the gardener with the 1-dollar piece you received as change from the previous day.

On day 4, pay the gardener with the 4-dollar bar. The gardener will give you back your change which will consist of a 1-dollar piece and a 2-dollar piece.

On day 5, you pay the gardener with the 1-dollar piece you received as change from the previous day.

On day 6, pay the gardener with the 2-dollar piece you received as change from day 4. The gardener will give you back your change which is a 1-dollar piece.

On day 7, pay the gardener with the 1-dollar piece you received as change from day 6.

**112.** Let $\triangle ABC$ be a right triangle with vertices at $A(2,7)$, $B(0,-3)$, and $C(6,1)$. Notice, the midpoints of the sides of $\triangle ABC$ are $(3,-1)$, $(4,4)$ and $(1,2)$. The area of $\triangle ABC$ is

$$\begin{aligned} \frac{1}{2}\overline{AC} \times \overline{BC} &= \frac{1}{2}\sqrt{4^2+6^2}\sqrt{6^2+4^2} \\ &= \frac{1}{2}(52) = 26. \end{aligned}$$

## P.1 Pop Quiz

**1.** The distance is $\sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$.

**2.** Center $(3,-5)$, radius 9

**3.** Completing the square, we find

$$\begin{aligned} (x^2+4x+4)+(y^2-10y+25) &= -28+4+25 \\ (x+2)^2+(y-5)^2 &= 1. \end{aligned}$$

The center is $(-2,5)$ and the radius is 1.

**4.** The distance between $(3,4)$ and the origin is 5, which is the radius. The circle is given by $(x-3)^2+(y-4)^2 = 25$.

**5.** By setting $x = 0$ and $y = 0$ in $2x - 3y = 12$ we find $-3y = 12$ and $2x = 12$, respectively. Since $y = -4$ and $x = 6$ are the solutions of the two equations, the intercepts are $(0,-4)$ and $(6,0)$.

**6.** $(5,-1)$

## For Thought

**1** True, since the number of gallons purchased is 20 divided by the price per gallon.

**2.** False, since a student's exam grade is a function of the student's preparation. If two classmates had the same IQ and only one prepared then the one who prepared will most likely achieve a higher grade.

**3.** False, since $\{(1,2),(1,3)\}$ is not a function.

**4.** True

**5.** True

**6.** True

**7.** False, the domain is the set of all real numbers.

**8.** True

**9.** True, since $f(0) = \dfrac{0-2}{0+2} = -1$.

**10.** True, since if $a - 5 = 0$ then $a = 5$.

## P.2 Exercises

**1.** function

**2.** independent, dependent

**3.** domain, range

**4.** parabola

**5.** function

**6.** function

**7.** Note, $b = 2\pi a$ is equivalent to $a = \dfrac{b}{2\pi}$. Then $a$ is a function of $b$, and $b$ is a function of $a$.

**8.** Note, $b = 2(5+a)$ is equivalent to $a = \dfrac{b-10}{2}$. Then $a$ is a function of $b$, and $b$ is a function of $a$.

**9.** $a$ is a function of $b$ since a given denomination has a unique length. Since a dollar bill and a five-dollar bill have the same length, then $b$ is not a function of $a$.

**10.** Since different U.S. coins have different diameters, then $a$ is a function of $b$, and $b$ is a function of $a$.

**11.** Since an item has only one price, $b$ is a function of $a$. Since two items may have the same price, $a$ is not a function of $b$.

**12.** $a$ is not a function of $b$ since there may be two students with the same semester grades but different final exams scores. $b$ is not a function of $a$ since there may be identical final exam scores with different semester grades.

**13.** $a$ is not a function of $b$ since it is possible that two different students can obtain the same final exam score but the times spent on studying are different.

$b$ is not a function of $a$ since it is possible that two different students can spend the same time studying but obtain different final exam scores.

**14.** $a$ is not a function of $b$ since it is possible that two adult males can have the same shoe size but have different ages.

$b$ is not a function of $a$ since it is possible for two adults with the same age to have different shoe sizes.

**15.** Since 1 in $\approx$ 2.54 cm, $a$ is a function of $b$ and $b$ is a function of $a$.

**16.** Since there is only one cost for mailing a first class letter, then $a$ is a function of $b$. Since two letters with different weights each under 1/2-ounce cost 47 cents to mail first class, $b$ is not a function of $a$.

**17.** Since $b = a^3$ and $a = \sqrt[3]{b}$, we get that $b$ is a function of $a$, and $a$ is a function of $b$.

**18.** Since $b = a^4$ and $a = \pm\sqrt[4]{b}$, we get that $b$ is a function of $a$, but $a$ is not a function of $b$.

**19.** Since $b = |a|$, we get $b$ is a function of $a$. Since $a = \pm b$, we find $a$ is not a function of $b$.

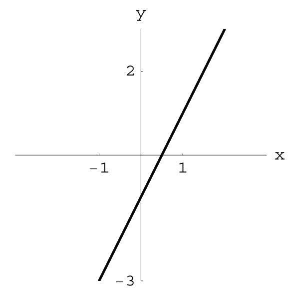
**20.** Note, $b = \sqrt{a}$ since $a \geq 0$, and $a = b^2$. Thus, $b$ is a function of $a$, and $a$ is a function of $b$.

**21.** $A = s^2$ **22.** $s = \sqrt{A}$ **23.** $s = \dfrac{\sqrt{2}d}{2}$

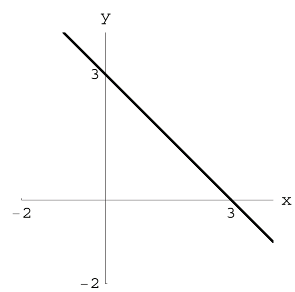
**24.** $d = s\sqrt{2}$ **25.** $P = 4s$ **26.** $s = P/4$

**27.** $A = P^2/16$ **28.** $d = \sqrt{2A}$

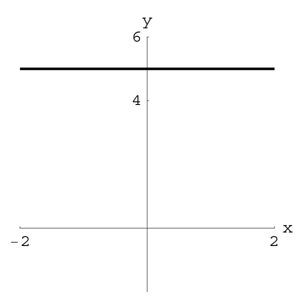
**29.** $y = 2x - 1$ has domain $(-\infty, \infty)$ and range $(-\infty, \infty)$, some points are $(0, -1)$ and $(1, 1)$



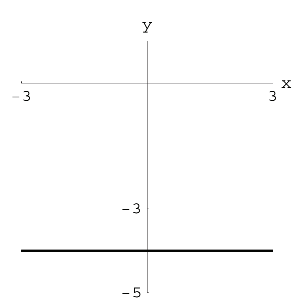

**30.** $y = -x + 3$ has domain $(-\infty, \infty)$ and range $(-\infty, \infty)$, some points are $(0, 3)$ and $(3, 0)$



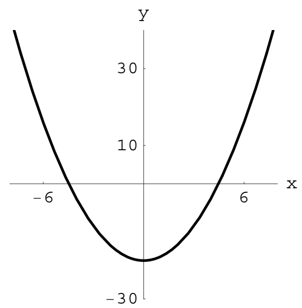

**31.** $y = 5$ has domain $(-\infty, \infty)$ and range $\{5\}$, some points are $(0, 5)$ and $(1, 5)$



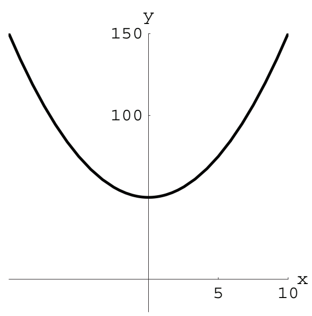

**32.** $y = -4$ has domain $(-\infty, \infty)$ and range $\{-4\}$, some points are $(0, -4)$ and $(1, -4)$



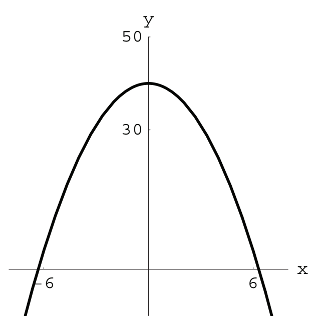
**33.** $y = x^2 - 20$ has domain $(-\infty, \infty)$ and range $[-20, \infty)$, some points are $(0, -20)$ and $(6, 16)$



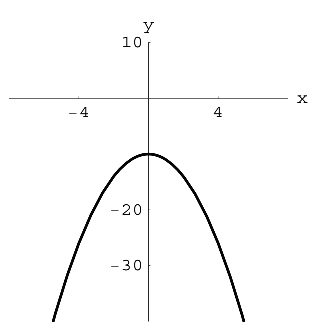

**34.** $y = x^2 + 50$ has domain $(-\infty, \infty)$ and range $[50, \infty)$, some points are $(0, 50)$ and $(5, 75)$



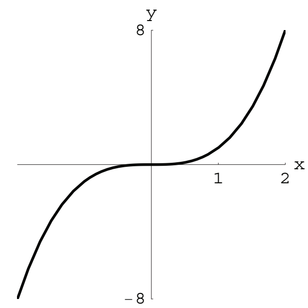

**35.** $y = 40 - x^2$ has domain $(-\infty, \infty)$ and range $(-\infty, 40]$, some points are $(0, 40)$ and $(6, 4)$



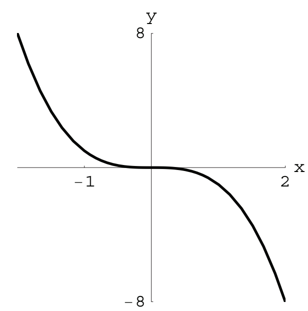

**36.** $y = -10 - x^2$ has domain $(-\infty, \infty)$ and range $(-\infty, -10]$, some points are $(0, -10)$ and $(4, -26)$



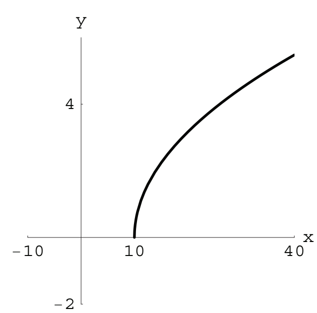

**37.** $y = x^3$ has domain $(-\infty, \infty)$ and range $(-\infty, \infty)$, some points are $(0, 0)$ and $(2, 8)$



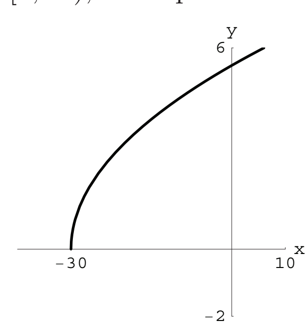

**38.** $y = -x^3$ has domain $(-\infty, \infty)$ and range $(-\infty, \infty)$, some points are $(0, 0)$ and $(0, -8)$



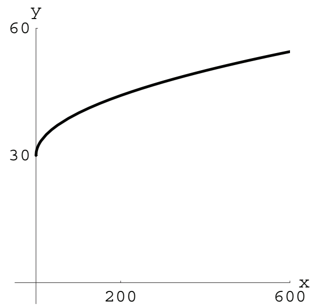

**39.** $y = \sqrt{x - 10}$ has domain $[10, \infty)$ and range $[0, \infty)$, some points are $(10, 0)$ and $(14, 2)$



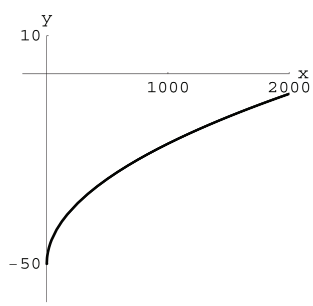

**40.** $y = \sqrt{x + 30}$ has domain $[-30, \infty)$ and range $[0, \infty)$, some points are $(-30, 0)$ and $(-26, 2)$



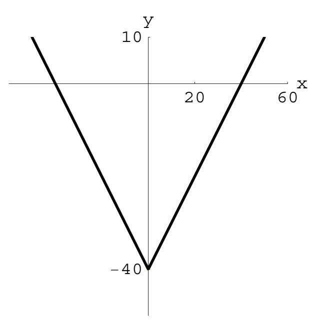
**41.** $y = \sqrt{x} + 30$ has domain $[0, \infty)$ and range $[30, \infty)$, some points are $(0, 30)$ and $(400, 50)$



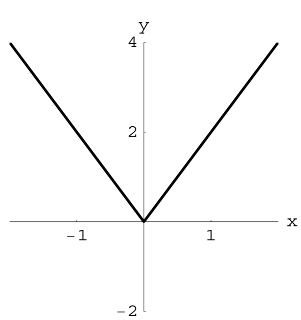

**42.** $y = \sqrt{x} - 50$ has domain $[0, \infty)$ and range $[-50, \infty)$, some points are $(0, -50)$ and $(900, -20)$



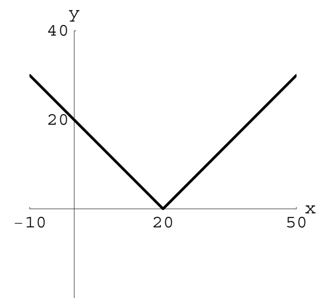

**43.** $y = |x| - 40$ has domain $(-\infty, \infty)$ and range $[-40, \infty)$, some points are $(0, -40)$ and $(40, 0)$



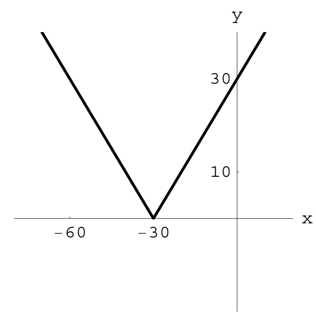

**44.** $y = 2|x|$ has domain $(-\infty, \infty)$ and range $[0, \infty)$, some points are $(0, 0)$ and $(1, 2)$




**45.** $y = |x - 20|$ has domain $(-\infty, \infty)$ and range $[0, \infty)$, some points are $(0, 20)$ and $(20, 0)$




**46.** $y = |x + 30|$ has domain $(-\infty, \infty)$ and range $[0, \infty)$, some points are $(0, 30)$ and $(-30, 0)$




**47.** $3 \cdot 4 - 2 = 10$ **48.** $3(16) + 4 = 52$

**49.** $-4 - 2 = -6$ **50.** $-8 - 2 = -10$

**51.** $|8| = 8$ **52.** $|-1| = 1$

**53.** $4 + (-6) = -2$ **54.** $24 \cdot 6 = 144$

**55.** $80 - 2 = 78$ **56.** $2/2 = 1$

**57.** $3a^2 - a$ **58.** $4b - 2$

**59.** $f(-x) = 3(-x)^2 - (-x) = 3x^2 + x$

**60.** $g(-x) = 4(-x) - 2 = -4x - 2$

**61.** Factoring, we get $x(3x - 1) + 0$. So $x = 0, 1/3$.

**62.** Since $4x - 2 = 3$, we get $x = 5/4$.

**63.** Since $|a + 3| = 4$ is equivalent to $a + 3 = 4$ or $a + 3 = -4$, we have $a = 1, -7$.

**64.** Since $3t^2 - t - 10 = (t - 2)(3t + 5) = 0$, we find $t = 2, -5/3$.

**65.** $C = 353n$

**66.** $P = 580n$

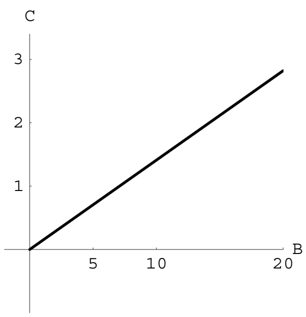
**67.** $C = 35n + 50$

**68.** $C = 2.50 + 0.50n$

**69.** We find

$$C = \frac{4B}{\sqrt[3]{D}} = \frac{4(12 + 11/12)}{\sqrt[3]{22,800}} \approx 1.822$$

and a sketch of the graph of $C = \dfrac{4B}{\sqrt[3]{22,800}}$ is given below.




**70.** Solving for $B$, we get

$$\begin{aligned} \frac{4B}{\sqrt[3]{22,800}} &< 2 \\ B &< \frac{\sqrt[3]{22,800}}{2} \\ B &< 14 \text{ ft}, 2 \text{ in.} \end{aligned}$$

Then the maximum displacement is 14 ft, 2 in.

With $D$ fixed, we get that $C$ becomes larger as the beam $B$ becomes larger. Thus, a boat is more likely to capsize as the beam gets larger.

**71.** Let $N = 2$, $B = 3.498$, and $S = 4.250$. Then

$$\begin{aligned} D &= \frac{\pi}{4}B^2 \cdot S \cdot N \\ &= 81.686 \text{ in.}^3 \end{aligned}$$

Then $D \approx 81.7$ in.$^3$.

**72.** Let $N = 2$, $B = 3.518$, and $S = 4.250$. Then

$$\begin{aligned} D &= \frac{\pi}{4}B^2 \cdot S \cdot N \\ &= 82.622 \text{ in.}^3 \end{aligned}$$

Using the unrounded answer to Exercise 71, the difference in the displacement is

$$82.622 - 81.686 \approx 0.94 \text{ in.}^3.$$

**73.** Solving for $B$,

$$\begin{aligned} D &= \frac{\pi}{4}B^2 \cdot S \cdot N \\ \frac{4D}{\pi S \cdot N} &= B^2 \\ B &= 2\sqrt{\frac{D}{\pi S \cdot N}}. \end{aligned}$$

**74.** Solving for $V$,

$$\begin{aligned} CR &= 1 + \frac{\pi B^2 \cdot S}{4V} \\ CR - 1 &= \frac{\pi B^2 \cdot S}{4V} \\ \frac{1}{CR - 1} &= \frac{4V}{\pi B^2 \cdot S} \\ V &= \frac{\pi B^2 \cdot S}{4(CR - 1)}. \end{aligned}$$

**75.** Pythagorean, legs, hypotenuse

**76.** circle, radius, center

**77** . $\sqrt{(2+3)^2 + (-4+6)^2} = \sqrt{29}$

**78.** $\left(\dfrac{4-6}{2}, \dfrac{-8+16}{2}\right) = (-1, 4)$

**79.** If we replace $x = 0$ in $4x - 6y = 40$, then $-6y = 40$ or $y = -20/3$. The $y$-intercept is $(0, -\frac{20}{3})$.

If we replace $y = 0$ in $4x - 6y = 40$, then $4x = 40$ or $x = 10$. The $x$-intercept is $(10, 0)$.

**80.** The diagonal is $\sqrt{3^2 + 7^2} = \sqrt{58}$ ft.

**81.** Rewriting the equation, we find

$$\begin{aligned} \frac{1}{3^3} \cdot 3^{100} \cdot \frac{1}{3^4} \cdot 3^{2x} &= \frac{1}{3} \cdot 3^x \\ \frac{1}{3^3} \cdot 3^{100} \cdot \frac{1}{3^4} \cdot 3^{2x} &= 3^{x-1} \\ 3^{2x+93} &= 3^{x-1} \\ 2x + 93 &= x - 1 \\ x &= -94. \end{aligned}$$

**82.** First, $9 = (a+b)^2 = (a^2+b^2)+2ab = 89+2ab$. Then $ab = -40$. Thus,

$$\begin{aligned} a^3 + b^3 &= (a + b)(a^2 + b^2 - ab) \\ &= 3(89 + 40) \\ &= 387. \end{aligned}$$

## P.2 Pop Quiz

**1.** Yes, since $r \geq 0$ and $r = \sqrt{A/\pi}$

**2.** Since $A = s^2$ and $s \geq 0$, we obtain $s = \sqrt{A}$.

**3.** No, since $b = \pm a$.

**4.** $[1, \infty)$

**5.** $[2, \infty)$

**6.** $f(3) = 3(3) + 6 = 15$

**7.** We find

$$\begin{aligned} 2a - 4 &= 10 \\ 2a &= 14 \\ a &= 7. \end{aligned}$$

## For Thought

**1.** False, it is a reflection in the y-axis.

**2.** False, the graph of $y = x^2 - 4$ is shifted down 4 units from the graph of $y = x^2$.

**3.** False, rather it is a left translation.

**4.** True

**5.** True

**6.** False, the down shift should come after the reflection.

**7.** True

**8.** False, since the domains are different.

**9.** True

**10.** True, since $f(-x) = -f(x)$ where $f(x) = x^3$.

## P.3 Exercises

**1.** rigid

**2.** nonrigid

**3.** reflection

**4.** upward translation, downward translation

**5.** right, left

**6.** stretching, shrinking

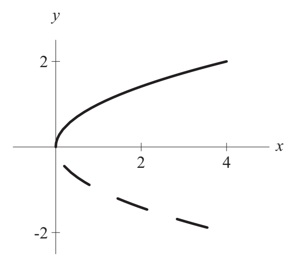
**7.** odd

**8.** even

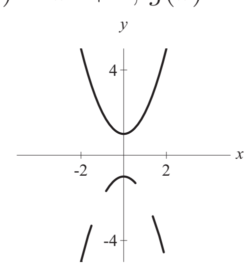
**9.** transformation

**10.** family

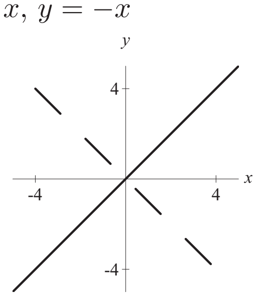
**11.** $f(x) = \sqrt{x}$, $g(x) = -\sqrt{x}$



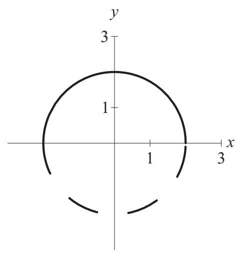

**12.** $f(x) = x^2 + 1$, $g(x) = -x^2 - 1$



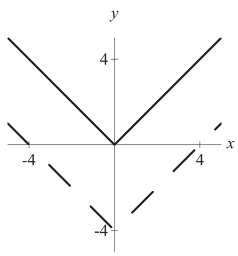

**13.** $y = x$, $y = -x$



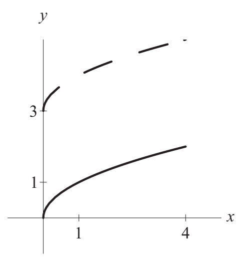
**14.** $y = \sqrt{4 - x^2}$, $y = -\sqrt{4 - x^2}$



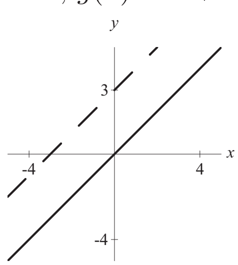

**15.** $f(x) = |x|$, $g(x) = |x| - 4$



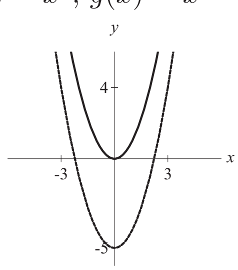

**16.** $f(x) = \sqrt{x}$, $g(x) = \sqrt{x} + 3$



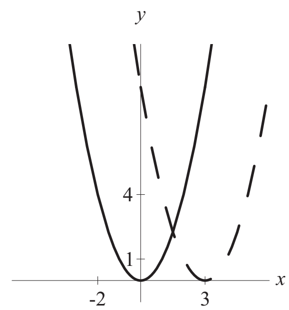

**17.** $f(x) = x$, $g(x) = x + 3$



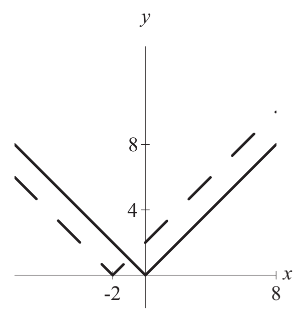

**18.** $f(x) = x^2$, $g(x) = x^2 - 5$



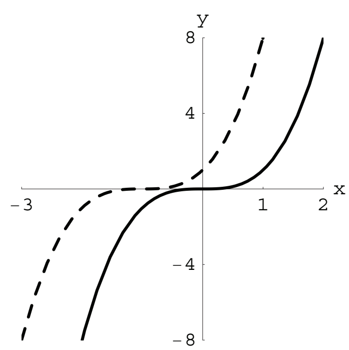

**19.** $y = x^2$, $y = (x - 3)^2$



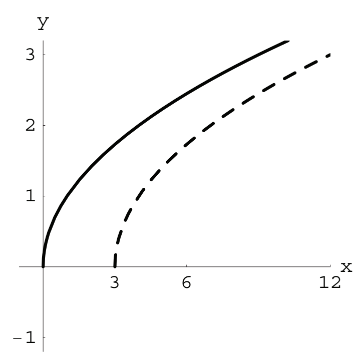

**20.** $y = |x|$, $y = |x + 2|$



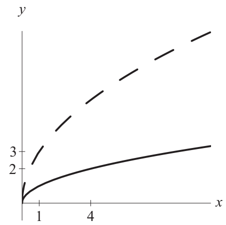

**21.** $f(x) = x^3$, $g(x) = (x + 1)^3$



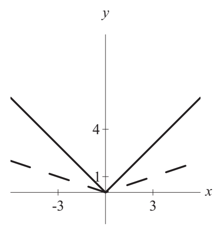

**22.** $f(x) = \sqrt{x}$, $g(x) = \sqrt{x - 3}$



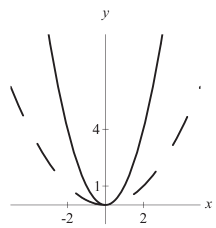
**23.** $y = \sqrt{x}, y = 3\sqrt{x}$



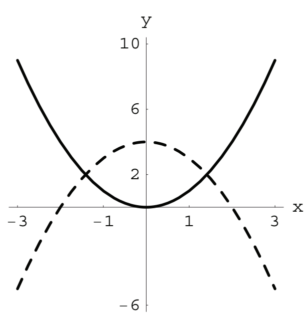

**24.** $y = |x|, y = \dfrac{1}{3}|x|$




**25.** $y = x^2, y = \dfrac{1}{4}x^2$




**26.** $y = x^2$, $y = 4 - x^2$




**27.** g **28.** h **29.** b **30.** d

**31.** c **32.** a **33.** f **34.** e

**35.** $y = (x - 10)^2 + 4$

**36.** $y = \sqrt{x + 5} - 12$

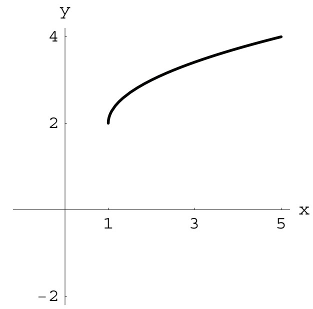
**37.** $y = -3|x - 7| + 9$

**38.** $y = -2(x + 6) - 8$ or $y = -2x - 20$

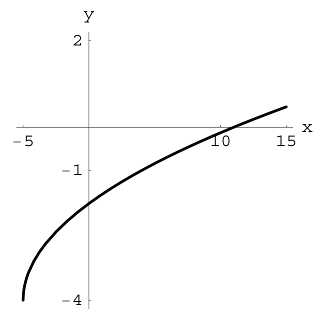
**39.** $y = -(3\sqrt{x} + 5)$ or $y = -3\sqrt{x} - 5$

**40.** $y = -\left((x - 13)^2 - 6\right)$ or $y = -(x - 13)^2 + 6$

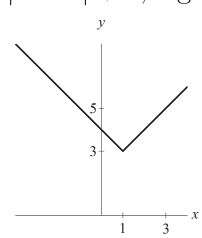
**41.** $y = \sqrt{x - 1} + 2$; right by 1, up by 2



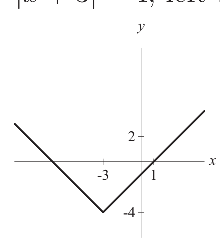

**42.** $y = \sqrt{x + 5} - 4$; left by 5, down by 4



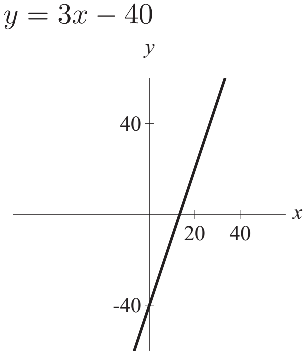

**43.** $y = |x - 1| + 3$; right by 1, up by 3



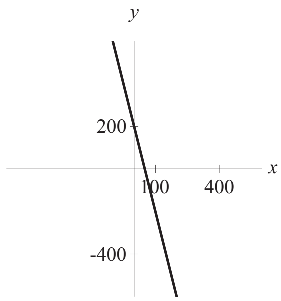

**44.** $y = |x + 3| - 4$; left by 3, down by 4



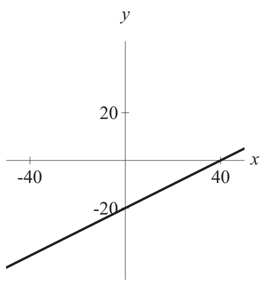

**45.** $y = 3x - 40$



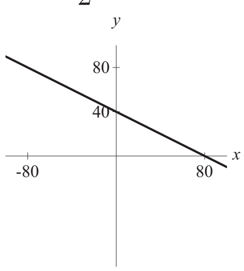
**46.** $y = -4x + 200$



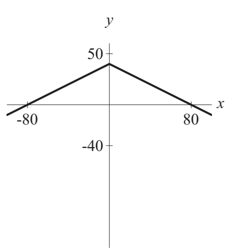

**47.** $y = \frac{1}{2}x - 20$



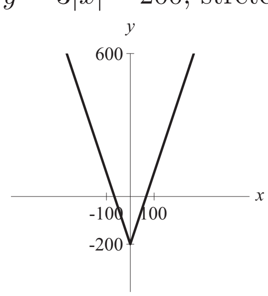

**48.** $y = -\frac{1}{2}x + 40$



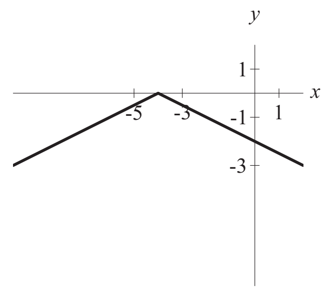

**49.** $y = -\frac{1}{2}|x| + 40$, shrink by 1/2, reflect about $x$-axis, up by 40



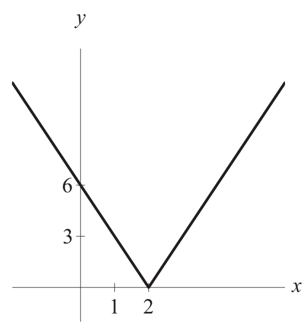

**50.** $y = 3|x| - 200$, stretch by 3, down by 200



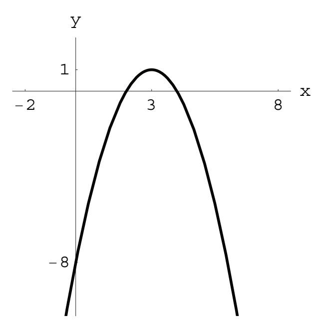

**51.** $y = -\frac{1}{2}|x + 4|$, left by 4, reflect about $x$-axis, shrink by 1/2



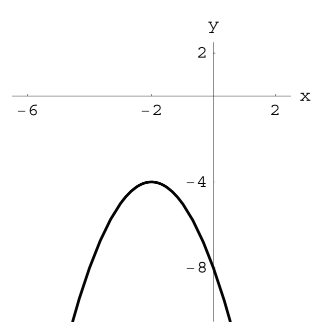

**52.** $y = 3|x - 2|$, right by 2, stretch by 3



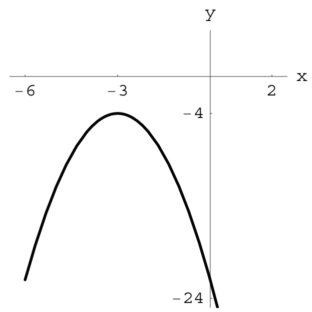

**53.** $y = -(x - 3)^2 + 1$; right by 3, reflect about $x$-axis, up by 1



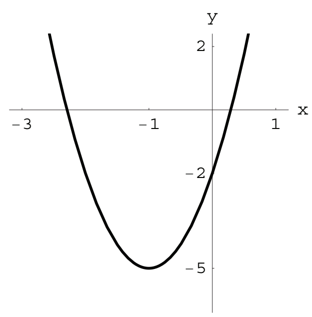

**54.** $y = -(x + 2)^2 - 4$; left by 2, reflect about $x$-axis, down by 4



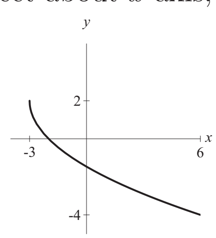
**55.** $y = -2(x + 3)^2 - 4$; left by 3, stretch by 2, reflect about $x$-axis, down by 4



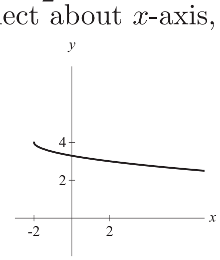

**56.** $y = 3(x + 1)^2 - 5$; left by 1, stretch by 3, down by 5




**57.** $y = -2\sqrt{x + 3} + 2$, left by 3, stretch by 2, reflect about $x$-axis, up by 2




**58.** $y = -\frac{1}{2}\sqrt{x + 2} + 4$, left by 2, shrink by 1/2, reflect about $x$-axis, up by 4




**59.** Symmetric about $y$-axis, even function since $f(-x) = f(x)$

**60.** Symmetric about $y$-axis, even function since $f(-x) = f(x)$

**61.** No symmetry, neither even nor odd since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

**62.** Symmetric about the origin, odd function since $f(-x) = -f(x)$

**63.** Neither symmetry, neither even nor odd since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

**64.** Neither symmetry, neither even nor odd since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

**65.** No symmetry, not an even or odd function since $f(-x) = -f(x)$ and $f(-x) \neq -f(x)$

**66.** Symmetric about the $y$-axis, even function since $f(-x) = f(x)$

**67.** Symmetric about the origin, odd function since $f(-x) = -f(x)$

**68.** Symmetric about the origin, odd function since $f(-x) = -f(x)$

**69.** No symmetry, not an even or odd function since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

**70.** No symmetry, not an even or odd function since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

**71.** No symmetry, not an even or odd function since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

**72.** Symmetric about the $y$-axis, even function since $f(-x) = f(x)$

**73.** Neither symmetry, not an even or odd function since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

**74.** Symmetric about the $y$-axis, even function since $f(-x) = f(x)$

**75.** Symmetric about the $y$-axis, even function since $f(-x) = f(x)$

**76.** Symmetric about the $y$-axis, even function since $f(-x) = f(x)$

since $f(-x) = f(x)$

since $f(-x) = f(x)$

**77.** e **78.** a **79.** g **80.** h

**81.** b **82.** d **83.** c **84.** f

**85.** $N(x) = x + 2000$

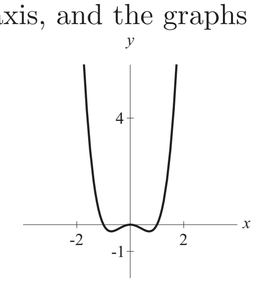
**86.** $N(x) = 1.05x + 3000$. Yes, if the merit increase is followed by the cost of living raise then the new salary becomes higher and is $N'(x) = 1.05(x + 3000) = 1.05x + 3150$.

**87.** If inflation rate is less than 50%, then $1 - \sqrt{x} < \dfrac{1}{2}$. This simplifies to $\dfrac{1}{2} < \sqrt{x}$. After squaring we have $\dfrac{1}{4} < x$ and so $x > 25\%$.

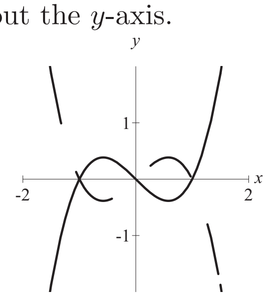
**88.** If production is at least 28 windows, then $1.75\sqrt{x} \geq 28$. They need at least $x = \left(\dfrac{28}{1.75}\right)^2 = 256$ hrs.

**89.**

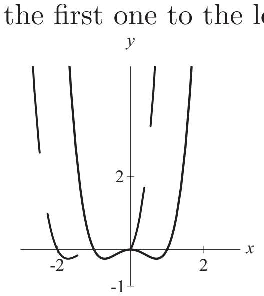
**(a)** Both functions are symmetric about the $y$-axis, and the graphs are identical.



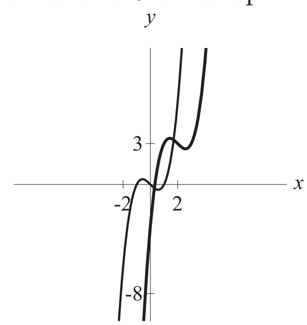

**(b)** One graph is a reflection of the other about the $y$-axis, and both are symmetric about the $y$-axis.



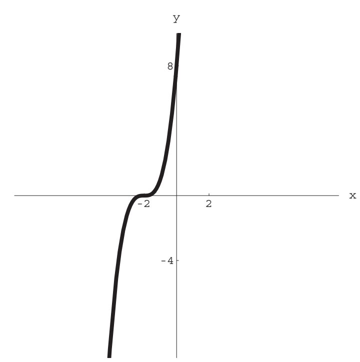

**(c)** The second graph is obtained by translating the first one to the left by 1 unit.




**(d)** The second graph is obtained by translating the first one to the right by 2 units and 3 units up.




**90.** The graph of $y = x^3 + 6x^2 + 12x + 8$ or equivalently $y = (x + 2)^3$ can be obtained by shifting the graph of $y = x^3$ to the left by 2 units.




**91.** $x^2 + y^2 = 1$

**92.** $y = 5$

**93.** $x = 4$

**94.** If $A$ is the area and $s$ is the length of a side, then $A = s^2$.

**95.** $f(6) = 2(36) - 3(6) = 72 - 18 = 54$
$f(-x) = 2(-x)^2 - 3(-x) = 2x^2 + 3x$

**96.** Solve for $a$:

$$\begin{aligned} 2a^2 + 1 &= 9 \\ 2a^2 &= 8 \\ a^2 &= 4 \\ a &= \pm 2. \end{aligned}$$

**97.** Draw the numbers in $\{1, 2, 3, ..., 999\}$ that can be written in the form

$$(2n + 1) \cdot 2^{2m}$$

for $n, m \geq 0$. These means that you will have drawn 666 numbers. Then the 667th number you will draw has the form

$$(2n + 1) \cdot 2^{2m+1}$$

which must be twice one of the first

666 numbers.

**98.** Let $A(x_1, 2)$ and $B(x_2, 2)$ be the other two opposite vertices where $x_1 < x_2$. Since the sides at $B$ are perpendicular, the slopes are negative reciprocals, i.e.,

$$\frac{3}{3 - x_2} = -\frac{x_2 - 1}{3}.$$

Solving, $x_2 = 2 + \sqrt{10}$. Since the distance $AB$ is the same as the distance between $(1, -1)$ and $(3, 5)$, we obtain

$$\begin{aligned} x_2 - x_1 &= \sqrt{40} = 2\sqrt{10} \\ x_1 &= x_2 - 2\sqrt{10} \\ x_1 &= (2 + \sqrt{10}) - 2\sqrt{10} \\ x_1 &= 2 - \sqrt{10}. \end{aligned}$$

Then the other vertices are $(2 \pm \sqrt{10}, 2)$.

## P.3 Pop Quiz

**1.** $y = \sqrt{x} + 8$

**2.** $y = (x - 9)^2$

**3.** $y = (-x)^3$ or $y = -x^3$

**4.** Domain $[1, \infty)$, range $(-\infty, 5]$

**5.** $y = -3(x - 6)^2 + 4$

**6.** Even function

## For Thought

**1** True, since $A = P^2/16$.

**2.** False, rather $y = (x - 1)^2 = x^2 - 2x + 1$.

**3.** False, rather $(f \circ g)(x) = \sqrt{x - 2}$.

**4.** True

**5.** False, since $(h \circ g)(x) = x^2 - 9$.

**6.** False, rather $f^{-1}(x) = \frac{1}{2}x - \frac{1}{2}$.

**7.** True

**8.** False; $g^{-1}$ does not exist since the graph of $g$ which is a parabola fails the horizontal line test.

**9.** False, since $y = x^2$ is a function which does not have an inverse function.

**10.** True

## P.4 Exercises

**1.** composition

**2.** one-to-one

**3.** invertible

**4.** inverse

**5.** switch-and-solve

**6.** symmetric

**7.** $y = 2(3x + 1) - 3 = 6x - 1$

**8.** $y = -4(-3x - 2) - 1 = 12x + 7$

**9.** $y = (x^2 + 6x + 9) - 2 = x^2 + 6x + 7$

**10.** $y = 3(x^2 - 2x + 1) - 3 = 3x^2 - 6x$

**11.** $y = 3 \cdot \frac{x + 1}{3} - 1 = x + 1 - 1 = x$

**12.** $y = 2\left(\frac{1}{2}x - \frac{5}{2}\right) + 5 = x - 5 + 5 = x$

**13.** $y = 2 \cdot \frac{x + 1}{2} - 1 = x + 1 - 1 = x$

**14.** $y = \frac{(5x - 1) + 1}{5} = \frac{5x}{5} = x$

**15.** $f(2) = 5$ **16.** $g(-4) = 17$

**17.** $f(2) = 5$ **18.** $h(-22) = -7$

**19.** $f(17) = 3(17) - 1 = 50$

**20.** $g\left(-\frac{8}{3}\right) = \frac{64}{9} + 1 = \frac{73}{9}$

**21.** $3(x^2 + 1) - 1 = 3x^2 + 2$

**22.** $(3x - 1)^2 + 1 = 9x^2 - 6x + 2$

**23.** $(3x - 1)^2 + 1 = 9x^2 - 6x + 2$

**24.** $3(x^2 + 1) - 1 = 3x^2 + 2$ **25.** $\frac{x^2+2}{3}$

**26.** $\frac{x^2 + 2x + 1}{9} + 1 = \frac{1}{9}x^2 + \frac{2}{9}x + \frac{10}{9}$

**27.** $F = g \circ h$ **28.** $G = g \circ f$

**29.** $H = h \circ g$ **30.** $M = f \circ g$

**31.** $N = f \circ h$ **32.** $R = h \circ f$

**33.** Interchange $x$ and $y$ then solve for $y$.

$$\begin{aligned} x &= 3y - 7 \\ \frac{x+7}{3} &= y \\ \frac{x+7}{3} &= f^{-1}(x) \end{aligned}$$

**34.** Interchange $x$ and $y$ then solve for $y$.

$$\begin{aligned} x &= -2y + 5 \\ 2y &= 5 - x \\ f^{-1}(x) &= \frac{5-x}{2} \\ f^{-1}(x) &= \frac{x-5}{-2} \end{aligned}$$

**35.** Interchange $x$ and $y$ then solve for $y$.

$$\begin{aligned} x &= 2 + \sqrt{y-3} \quad \text{for } x \geq 2 \\ (x-2)^2 &= y - 3 \quad \text{for } x \geq 2 \\ f^{-1}(x) &= (x-2)^2 + 3 \quad \text{for } x \geq 2 \end{aligned}$$

**36.** Interchange $x$ and $y$ then solve for $y$.

$$\begin{aligned} x &= \sqrt{3y-1} \quad \text{for } x \geq 0 \\ x^2 + 1 &= 3y \quad \text{for } x \geq 0 \\ f^{-1}(x) &= \frac{x^2+1}{3} \quad \text{for } x \geq 0 \end{aligned}$$

**37.** Interchange $x$ and $y$ then solve for $y$.

$$\begin{aligned} x &= -y - 9 \\ y &= -x - 9 \\ f^{-1}(x) &= -x - 9 \end{aligned}$$

**38.** Interchange $x$ and $y$ then solve for $y$.

$$\begin{aligned} x &= -y + 3 \\ y &= -x + 3 \\ f^{-1}(x) &= -x + 3 \end{aligned}$$

**39.** Interchange $x$ and $y$ then solve for $y$.

$$\begin{aligned} x &= -\frac{1}{y} \\ xy &= -1 \\ f^{-1}(x) &= -\frac{1}{x} \end{aligned}$$

**40.** Clearly $f^{-1}(x) = x$

**41.** Interchange $x$ and $y$ then solve for $y$.

$$\begin{aligned} x &= \sqrt[3]{y-9} + 5 \\ x - 5 &= \sqrt[3]{y-9} \\ (x-5)^3 &= y - 9 \\ f^{-1}(x) &= (x-5)^3 + 9 \end{aligned}$$

**42.** Interchange $x$ and $y$ then solve for $y$.

$$\begin{aligned} x &= \sqrt[3]{\frac{y}{2}} + 5 \\ x - 5 &= \sqrt[3]{\frac{y}{2}} \\ (x-5)^3 &= \frac{y}{2} \\ f^{-1}(x) &= 2(x-5)^3 \end{aligned}$$

**43.** Interchange $x$ and $y$ then solve for $y$.

$$\begin{aligned} x &= (y-2)^2 \quad x \geq 0 \\ \sqrt{x} &= y - 2 \\ f^{-1}(x) &= \sqrt{x} + 2 \end{aligned}$$

**44.** Interchange $x$ and $y$ then solve for $y$.

$$\begin{aligned} x &= (y+3)^2 \quad x \geq 0 \\ \sqrt{x} &= y + 3 \\ f^{-1}(x) &= \sqrt{x} - 3 \end{aligned}$$

**45.** $(f^{-1} \circ f)(x) = \frac{1}{2}(2x-1) + \frac{1}{2} = x$ and
$(f \circ f^{-1})(x) = 2\left(\frac{1}{2}x + \frac{1}{2}\right) - 1 = x$

**46.** $(f^{-1} \circ f)(x) = 3\left(\frac{x+1}{3}\right) - 1 = x$ and
$(f \circ f^{-1})(x) = \frac{(3x-1)+1}{3} = x$

**47.** $(f^{-1} \circ f)(x) = 0.25(4x+4) - 1 = x$ and
$(f \circ f^{-1})(x) = 4(0.25x - 1) + 4 = x$

**48.** $(f^{-1} \circ f)(x) = -0.2(20 - 5x) + 4 = x$ and
$(f \circ f^{-1})(x) = 20 - 5(-0.2x + 4) = x$

**49.** We obtain

$$\begin{aligned} (f^{-1} \circ f)(x) &= \frac{4 - \left(\sqrt[3]{4-3x}\right)^3}{3} \\ &= \frac{4-(4-3x)}{3} \\ &= \frac{3x}{3} \\ (f^{-1} \circ f)(x) &= x \end{aligned}$$

and

$$\begin{aligned} (f \circ f^{-1})(x) &= \sqrt[3]{4 - 3\left(\frac{4-x^3}{3}\right)} \\ &= \sqrt[3]{4-(4-x^3)} \\ &= \sqrt[3]{x^3} \\ (f \circ f^{-1})(x) &= x. \end{aligned}$$

**50.** We obtain

$$\begin{aligned} (f^{-1} \circ f)(x) &= \sqrt[3]{(x^3+5)-5} \\ &= \sqrt[3]{x^3} \\ (f^{-1} \circ f)(x) &= x \end{aligned}$$

and

$$\begin{aligned} (f \circ f^{-1})(x) &= \left(\sqrt[3]{x-5}\right)^3 + 5 \\ &= (x-5)+5 \\ (f \circ f^{-1})(x) &= x. \end{aligned}$$

**51.** We obtain

$$\begin{aligned} (f^{-1} \circ f)(x) &= \sqrt[5]{\left(\sqrt[3]{x^5-1}\right)^3 + 1} \\ &= \sqrt[5]{x^5 - 1 + 1} \\ &= \sqrt[5]{x^5} \\ (f^{-1} \circ f)(x) &= x \end{aligned}$$

and

$$\begin{aligned} (f \circ f^{-1})(x) &= \sqrt[3]{\left(\sqrt[5]{x^3+1}\right)^5 - 1} \\ &= \sqrt[3]{x^3 + 1 - 1} \\ &= \sqrt[3]{x^3} \\ (f \circ f^{-1})(x) &= x. \end{aligned}$$

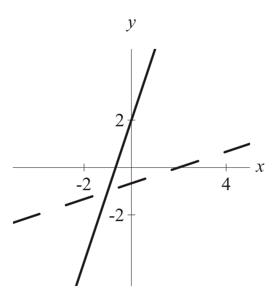
**52.** We find

$$\begin{aligned} (f^{-1} \circ f)(x) &= \left((x^{3/5} - 3) + 3\right)^{5/3} \\ &= \left(x^{3/5}\right)^{5/3} \\ (f^{-1} \circ f)(x) &= x \end{aligned}$$

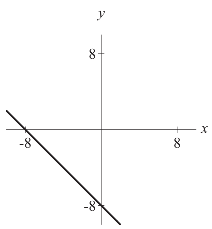
and

$$\begin{aligned} (f \circ f^{-1})(x) &= \left((x+3)^{5/3}\right)^{3/5} - 3 \\ &= (x-3) - 3 \\ (f \circ f^{-1})(x) &= x. \end{aligned}$$

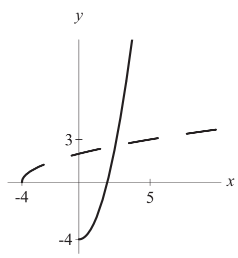
**53.** $f^{-1}(x) = \frac{x-2}{3}$



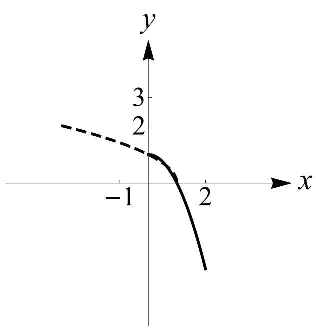
**54.** $f^{-1}(x) = -x - 8$



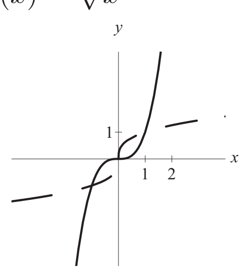

**55.** $f^{-1}(x) = \sqrt{x + 4}$



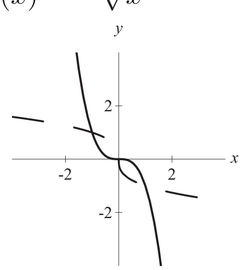

**56.** $f^{-1}(x) = \sqrt{1 - x}$



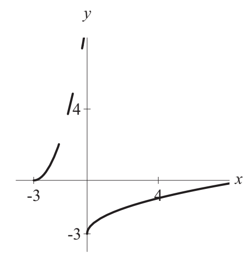

**57.** $f^{-1}(x) = \sqrt[3]{x}$



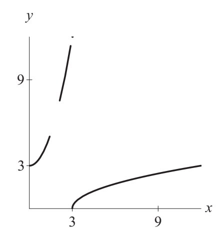

**58.** $f^{-1}(x) = -\sqrt[3]{x}$




**59.** $f^{-1}(x) = (x + 3)^2$ for $x \geq -3$




**60.** $f^{-1}(x) = x^2 + 3$ for $x \geq 0$




**61.** The inverse is the composition of subtracting 2 and dividing by 5, i.e., $f^{-1}(x) = \dfrac{x - 2}{5}$

**62.** The inverse is the composition of adding 1 and dividing by 0.5, i.e., $f^{-1}(x) = \dfrac{x + 1}{0.5} = 2x + 2$

**63.** The inverse is the composition of adding 88 and dividing by 2, i.e.,

$$f^{-1}(x) = \frac{x + 88}{2} = \frac{1}{2}x + 44$$

**64.** The inverse is the composition of subtracting 99 and dividing by 3, i.e.,

$$f^{-1}(x) = \frac{x - 99}{3} = \frac{1}{3}x - 33$$

**65.** The inverse is the composition of subtracting 4 and dividing by −3, i.e.,

$$f^{-1}(x) = \frac{x - 4}{-3} = -\frac{1}{3}x + \frac{4}{3}$$

**66.** The inverse is the composition of adding 5 and dividing by −2, i.e.,

$$f^{-1}(x) = \frac{x - 5}{-2} = -\frac{1}{2}x + \frac{5}{2}$$

**67.** The inverse is the composition of adding 9 and multiplying by 2, i.e.,

$$f^{-1}(x) = 2(x + 9) = 2x + 18$$

**68.** The inverse is the composition of subtracting 6 and multiplying by 3, i.e.,

$$f^{-1}(x) = 3(x - 6) = 3x - 18$$

**69.** The inverse is the composition of subtracting 3 and taking the multiplicative inverse, i.e.,

$f^{-1}(x) = \dfrac{1}{x-3}$

**70.** The inverse is the composition of taking the multiplicative inverse and subtracting 3, i.e.,

$f^{-1}(x) = \dfrac{1}{x} - 3$

**71.** The inverse is the composition of adding 9 and raising an expression to the third power, i.e.,

$$f^{-1}(x) = (x+9)^3.$$

**72.** The inverse is the composition of raising an expression to the third power and adding 9, i.e., $f^{-1}(x) = x^3 + 9$.

**73.** The inverse is the composition of adding 7, dividing by 2, and taking the cube root, i.e.,

$$f^{-1}(x) = \sqrt[3]{\frac{x+7}{2}}.$$

**74.** The inverse is the composition of subtracting by 4, multiplying by $-1$, and taking the cube root, i.e., $f^{-1}(x) = \sqrt[3]{-(x-4)} = \sqrt[3]{4-x}$

**75.** $A = d^2/2$ **76.** $P = 4\sqrt{A}$

**77.** Let $x$ be the number of ice cream bars sold. Then $W(x) = (0.20)(0.40x+200) = 0.08x+40$.

**78.** Let $x$ be the number of radios manufactured. Then $P(x) = (0.80)(0.90x) = 0.72x$.

**79.** $C = 1.08P$ expresses the total cost as a function of the purchase price; and $P = C/1.08$ is the purchase price as a function of the total cost.

**80.** $V(x) = x^3, S(x) = \sqrt[3]{x}$

**81.**

$$r = \sqrt{5.625 \times 10^{-5} - \frac{V}{500}}$$

where $0 \le V \le 0.028125$

**82.** Solving for $I$ in $E = 0.4I + 10,000$, we obtain $\dfrac{E - 10,000}{0.4} = I$ or equivalently

$$I = \frac{5}{2}E - 25,000.$$

If $E = I$, then $I = 0.4I + 10,000$ or $0.6I = 10,000$. Thus, if $E = I$ then $I = \dfrac{10,000}{0.6}$ or $I = \$16,666.67$.

**83.** When $V = \$18,000$, the depreciation rate is

$$1 - \left(\frac{18,000}{50,000}\right)^{1/5} \approx 0.1848$$

or the depreciation rate is 18.48%.

Solving for $V$, we obtain

$$\begin{aligned} \left(\frac{V}{50,000}\right)^{1/5} &= 1 - r \\ \frac{V}{50,000} &= (1-r)^5 \\ V &= 50,000(1-r)^5. \end{aligned}$$

**84.** When $P = \$23,580$, the annual growth rate is

$$\left(\frac{23,580}{10,000}\right)^{1/10} - 1 \approx 0.0896$$

or the annual growth rate is 8.96%.

Solving for $P$, we obtain

$$\begin{aligned} \left(\frac{P}{10,000}\right)^{1/10} &= r + 1 \\ \frac{P}{10,000} &= (r+1)^{10} \\ P &= 10,000(r+1)^{10}. \end{aligned}$$

**85.** Since $g^{-1}(x) = \dfrac{x+5}{3}$ and $f^{-1}(x) = \dfrac{x-1}{2}$, we have

$$g^{-1} \circ f^{-1}(x) = \frac{\frac{x-1}{2} + 5}{3} = \frac{x+9}{6}.$$

Likewise, since $(f \circ g)(x) = 6x - 9$, we get

$$(f \circ g)^{-1}(x) = \frac{x+9}{6}.$$

Hence, $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

**86.** Since $\left(f \circ g \circ g^{-1} \circ f^{-1}\right)(x) = \left(f \circ \left(g \circ g^{-1}\right)\right)\left(f^{-1}(x)\right) = f(f^{-1}(x)) = x$ and the range of one function is the domain of the other function, we have $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

**87.** $x^2 + (y-1)^2 = 9$

**88.** $\left(\dfrac{\pi/3 + \pi/2}{2}, \dfrac{1+1}{2}\right) = \left(\dfrac{5\pi/6}{2}, 1\right) = \left(\dfrac{5\pi}{12}, 1\right)$

**89.** function

**90.** reflection

**91.** Domain $(-\infty, \infty)$, range $[1, \infty)$

**92.** Since $x - 2 \geq 0$, the domain $[2, \infty)$.

Note, the range of $-\sqrt{x}$ is $(-\infty, 0]$. Then the range of $y = -\sqrt{x-2} + 3$ is $(-\infty, 3]$.

**93.** Note, if $a > b > 0$ and $n > 0$, then

$$\frac{a}{n} + \frac{b}{n+1} > \frac{b}{n} + \frac{a}{n+1}.$$

Thus, the arrangement with the largest sum is

$$\frac{2025}{1} + \frac{2024}{2} + ... + \frac{2}{2024} + \frac{1}{2025}.$$

**94.** If $f(x) = mx + b$, then

$$(f \circ f \circ f)(x) = m^3x + b(m^2 + m + 1).$$

Since

$$(f \circ f \circ f)(x) = 27x + 26$$

we obtain $m^3 = 27$ or $m = 3$.

Also, $b(m^2 + m + 1) = 26$. If we substitute $m = 3$, then $13b = 26$ or $m = 2$.

Thus, $f(x) = 3x + 2$. The $y$-intercept is $(0, 2)$.

## P.4 Pop Quiz

**1.** $A = \pi r^2 = \pi(d/2)^2$ or $A = \dfrac{\pi d^2}{4}$

**2.** $y = 3(4x - 5) - 1 = 12x - 16$

**3.** $(f \circ g)(5) = f(3) = 9$

**4.** $M = g \circ h \circ f$

**5.** Interchange $x$ and $y$ then solve for $y$.

$$\begin{aligned} 2y - 1 &= x \\ y &= \frac{x+1}{2} \\ f^{-1}(x) &= \frac{x+1}{2} \end{aligned}$$

**6.** Interchange $x$ and $y$ then solve for $y$.

$$\begin{aligned} \sqrt[3]{y+1} - 4 &= x \\ y + 1 &= (x+4)^3 \\ y &= (x+4)^3 - 1 \\ g^{-1}(x) &= (x+4)^3 - 1 \end{aligned}$$

## Chapter P Review Exercises

**1.** $\sqrt{49 \cdot 2} = 7\sqrt{2}$

**2.** $\sqrt{100 \cdot 2} = 10\sqrt{2}$

**3.** $\dfrac{3}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{3\sqrt{5}}{5}$

**4.** $\dfrac{4}{\sqrt{13}} \cdot \dfrac{\sqrt{13}}{\sqrt{13}} = \dfrac{4\sqrt{13}}{13}$

**5.** $\dfrac{\sqrt{5}}{\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{\sqrt{30}}{6}$

**6.** $\dfrac{\sqrt{3}}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{6}}{4}$

**7.** $\dfrac{8}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{8\sqrt{2}}{2} = 4\sqrt{2}$

**8.** $\dfrac{9}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{9\sqrt{3}}{3} = 3\sqrt{3}$

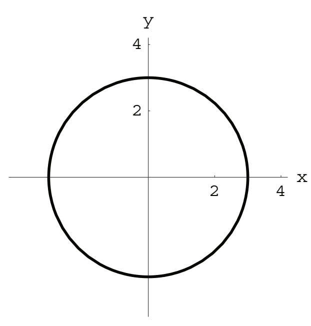
**9.** The distance is $\sqrt{(-3-2)^2+(5-(-6))^2} = \sqrt{(-5)^2+11^2} = \sqrt{25+121} = \sqrt{146}$. The midpoint is $\left(\frac{-3+2}{2}, \frac{5-6}{2}\right) = \left(-\frac{1}{2}, -\frac{1}{2}\right)$.

**10.** The distance is $\sqrt{(-1-(-2))^2+(1-(-3))^2} = \sqrt{1^2+4^2} = \sqrt{17}$. The midpoint is $\left(\frac{-1-2}{2}, \frac{1-3}{2}\right) = \left(-\frac{3}{2}, -1\right)$.

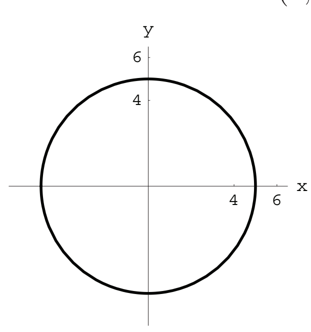
**11.** The distance is $\sqrt{\left(\pi-\frac{\pi}{2}\right)^2+(1-1)^2} = \sqrt{\left(\frac{\pi}{2}\right)^2+0} = \frac{\pi}{2}$. The midpoint is $\left(\frac{\pi/2+\pi}{2}, \frac{1+1}{2}\right) = \left(\frac{3\pi}{4}, 1\right)$.

**12.** The distance is $\sqrt{\left(\frac{\pi}{2}-\frac{\pi}{3}\right)^2+(2-2)^2} = \sqrt{\left(\frac{\pi}{6}\right)^2+0} = \frac{\pi}{6}$. The midpoint is $\left(\frac{\pi/2+\pi/3}{2}, \frac{2+2}{2}\right) = \left(\frac{5\pi}{12}, 2\right)$.

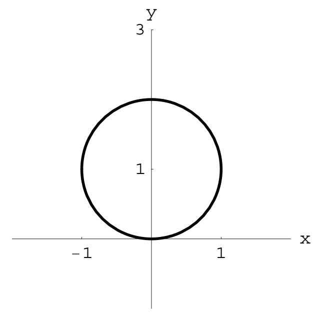
**13.** Circle with center at $(0,0)$ and radius 3



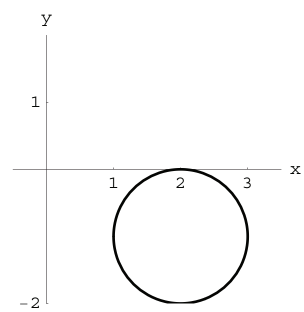

**14.** Circle with center at $(0,0)$ and radius 5



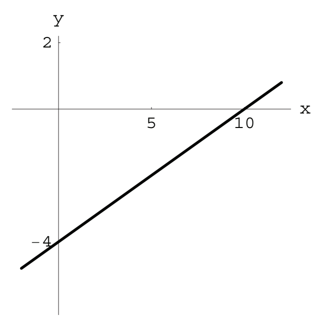

**15.** Circle with center at $(0,1)$ and radius 1



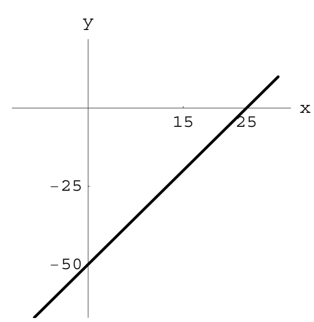

**16.** Circle with center at $(2,-1)$ and radius 1



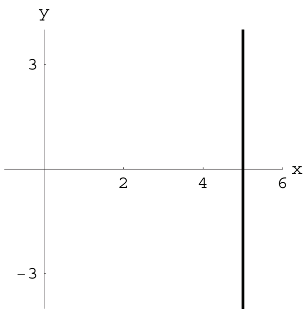

**17.** The line through the points $(10,0)$ and $(0,-4)$.



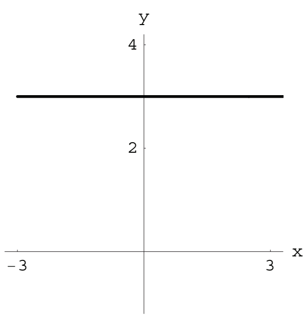

**18.** The line through the points $(25,0)$ and $(0,-50)$.



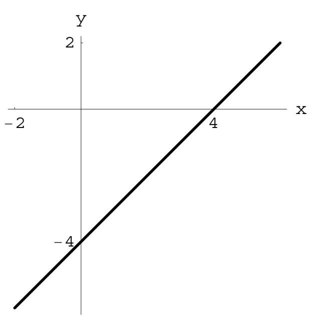
**19.** The vertical line through the point $(5, 0)$.



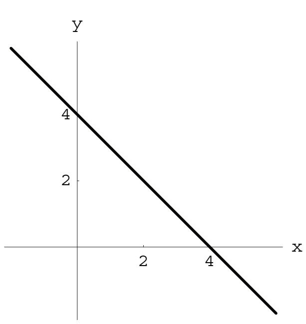

**20.** The horizontal line through the point $(0, 3)$.



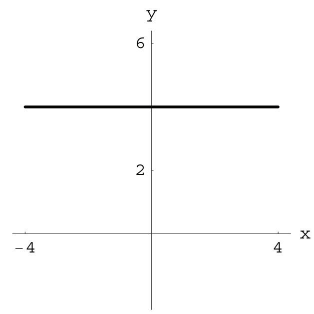

**21.** Domain $(-\infty, \infty)$ and range $(-\infty, \infty)$



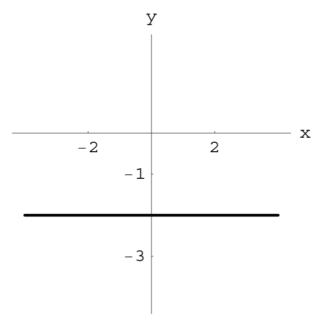

**22.** Domain $(-\infty, \infty)$ and range $(-\infty, \infty)$



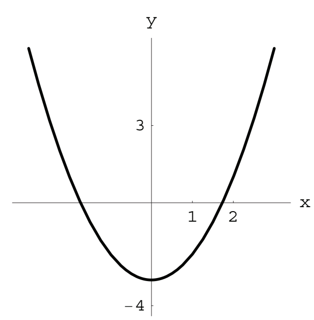

**23.** Domain $(-\infty, \infty)$ and range $\{4\}$. The horizontal line through the point $(0, 4)$.




**24.** Domain $(-\infty, \infty)$ and range $\{-2\}$




**25.** Domain $(-\infty, \infty)$ and range $[-3, \infty)$




**26.** Domain $(-\infty, \infty)$ and range $(-\infty, 6]$