# INSTRUCTOR'S RESOURCE MANUAL

# INTERMEDIATE ALGEBRA

THIRTEENTH EDITION

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# General First-Time Advice

*We asked the contributing professors for words of advice to instructors who are teaching this course for the first time or for the first time in a long while. Their responses can be found on the following pages.*

**David P. Bell**, *Florida Community College at Jacksonville*

1. So, you are considering teaching Intermediate Algebra in college for the first time! Before you even consider the syllabus, let's try to get a picture of your students. The most challenging students will be those who are essentially totally unprepared for college. These students went through high school attending class at least some of the time and waiting for the teacher to provide all the required topics and knowledge in class. These students rarely if ever studied or did homework. That certainly was a negative beginning, wasn't it. The fact is, if you prepare for these students, you will save time and effort on your part and more importantly for the students who do come prepared for college. For many of these students, you are their first and most important link to college level mathematics. These students need to be taught good study habits. They need to learn the value of timely performance of homework assignments and they need to learn to read the algebra textbook before each lesson. You still want to try mission impossible? That's great. Let's get started.

2. You will want to speak with an experienced faculty member and discuss just how and where to get started with the course. Many of those I work with use the review chapter to set the stage for the concept of student responsibility and ownership of their success. The Bittinger Intermediate Algebra text provides an excellent review of basic algebra in Chapter R. I skim the material in this chapter and stress the importance of this being review material. I state, "If the material and concepts in chapter R are new to you, or you do not remember any of them, stop, think, and come see me or visit a counselor. You may be in the wrong class! If the material is familiar, but you have not used it for some time, you have until the next class when we will test your knowledge of this material. You and I need to know if you have areas requiring extra review. Now is the time to make certain that you have every chance for success in the class!" Please don't use this statement as a class opener. You must try to gain some of the students' trust before you can be this honest with them.

3. I've set the stage and given you some disheartening details. Armed with this knowledge, you now set up the first class. You will see every expression on the faces of these students the first day of class, from fear to outright boredom, as you enter the classroom. This is your chance to get their attention and make that first impression. DO NOT pull out a stack of syllabi and start going over the details and requirements. Take this opportunity to open a discussion between the students and yourself. I have a group sheet that I pass out to get things started that includes room for four students and some general information. Notice that I have not mentioned algebra. The questions include: Why this college?; Where are you from?; What do you do?; What program are you in?; What do you expect from this class?; and e-mail and phone numbers. I then have the students break into groups and find students who reside in the same general area. This increases the chance that they will get together later to study. I give them plenty of time to gather the information knowing that they will each be introducing someone else in the class. The fear of math is now gone because I just told them that they each have to stand and introduce someone they most likely never met before. The icebreaker gives you the opportunity to get the roll and a feel for the class. Once the introductions are over, I introduce myself in the same format and we get down to business. Did I mention attendance? You can not overstress the need to read and understand the syllabus (The syllabus must contain all data pertinent to class management and student success for this course.) and the need to get started

NOW! Did I mention attendance? You should mention attendance as a requirement as often as you can stand it. I use the rest of this first session to review topics from the review chapter. This is the last time I will cover this material in class, but stress the need to visit you during office hours or visit school tutors if available. I require each student to send me an e-mail from their school provided e-mail account. Did I mention attendance? These students are very mobile and the e-mails, addresses, and phone numbers they provide the first week of class change rapidly. Their school e-mail does not change. Thus, you will be able to contact them should they not show up for class. The underlying theme you must get out and repeat often is that to be successful, they will need to read, do homework, and study. The more ways you find to get this message out, the better.

4. You should be able to procure an example of an elementary algebra final exam. You can provide this test to the students with a key and allow them to test themselves to see whether they are ready for this class. You may want to use the test as a quiz score or bonus just to emphasize to the student how important this material is. The important thing to keep in mind is "Now is the best time to remediate or relearn lost elementary algebra concepts." I have found that students who are made aware of possible problem areas will rise to the occasion and seek help early to increase their chances of success.

5. Once the first meeting is over, you always want to begin the day with something to make them think. I use quizzes, group problems, specific homework problems and anything else that I can to try to get their attention early. If they know the first few minutes are important, they will be less apt to be tardy. I try not to present the very examples that are in the text. If they read the textbook, presenting these often will create boredom that you do not want. I look for examples similar to those presented in the textbook. This provides the students with ample resources to complete the homework. Speaking of homework, try to assign mostly questions that have the answers in the textbook. Students can check the accuracy of their efforts. Some instructors collect homework so that the student sees a need to actually do it. I find that the students often just copy the solutions manual and turn it in as their own work. The contrast between the first test results and the homework is often a fantastic learning tool for the student. Spend a few minutes each day covering homework questions, but you control that time. Students would rather spend all day on homework than the new lesson.

6. Tests are the primary measurement tool you have to gauge student learning and comprehension. Create the test so that it checks understanding of the topics you chose to cover. I focus on open response and applications questions. Build the test so that the better students finish early. Set a time limit. These students need to learn time management and there is no time like the present. Speaking of time management, you might also want to talk about overall semester load, work, family, study time, etc, and see if there really is time to get it all done. The websites: http://www.askjeeves.com and http://www.purplemath.com have excellent interactive study skills and time management routines and assistance.

7. Timely return of test results is a must. Occasionally you will have students who score low for a number of silly errors. Let them know that they could have done much better if they had been a bit more careful. They like knowing that you care enough to notice.

8. The comprehensive final exam is your last chance to see if you have prepared these students well for college algebra or the next course in the sequence at your college. I often offer to replace each student's lowest test score with the final exam score. It provides a bit of extra motivation and if the student does well on the final, I can rest assured that they are as ready for college algebra as I can get them.

9. Throughout the course, try to use the class leaders to assist and motivate the students who are struggling. Everyone benefits from this exchange and the students get a sense of teamwork for their efforts.

10. Finally, on those occasions where you have given your best lecture with your best definitions and you know the students should understand, but you see confusion in their

eyes, you may get frustrated. The frustration comes from the belief that these students should have benefited from your presentation. Do your best to hide that frustration. The students will pick up on it very quickly and interpret that frustration as a personal issue. They feel you are frustrated with them and we know the frustration is directed at yourself. If you slip, just explain to the class your honesty with them will be a benefit later. Above all, keep an eye out for that light. You will see students suddenly perk up and profess "I see it now." It is so much fun seeing these people of all ages, colors, creeds, races, religions, etc. turn into knowledge hungry students, and they pay you too.
Best of luck.

## **Chris Bendixen,** *Lake Michigan College*

1. Generally the first day of class is one of the most important classes in the semester, not for the mathematical material, but for the "how to succeed" materials. The students need the syllabus for the first class meeting. Go over the syllabus in great detail, stressing that this is a contract between you (instructor) and them (students). Explain to the students how much material is generally covered in each class period. Thus, if they miss a class, they know about how much was covered. Do not talk in a dry monotone voice for the entire class. You need the students' attention. Show enthusiasm in your teaching. Make it appear that this is the best subject ever.

2. Next item is the class roster. Go over the roster, trying to correctly pronounce all students names.

3. I like to tell the students on day one that everybody has a 100% in the class, now. It is up to you (the student) to maintain this grade. To maintain the grade, you should do the following:

   Read the material before the material has been covered. It may not make sense at this point, but when the instructor goes over it, you will recognize the terminology. Also, after the material has been covered in class, go back over the reading. Amazingly, a lot more of the material will make sense.

   To be truly successful in math, you must do homework, homework, homework—as soon as the material has been covered. Do not wait until the day before the exam to work on the problems, stay on top of it. Related to this is odd answers are generally listed in the back part of the book. Tell the students to check the problems, and, if the answer is not correct, rework it until it is correct. DO not give up after the first attempt. Next, tell the students to take advantage of all the supplemental materials, such as: MathXL, InterAct Math, student solutions manuals, and additional internet math resources.

   I try to get the students involved in study groups with their peers. I volunteer to attend the initial meeting of the study group. After that, they are on their own.

   Always refresh yourself on prior materials. Math is a building block subject, you must be on top of all aspects of it throughout the entire course.

   Finally give the students a study guide for the exams. I personally give the students a sample exam. I also post the sample exam on the school's BlackBoard site.

4. I tell the class about special policies, like:
   - Attendance policy
   - Late exam policy
   - Cell phone policy (this is a must) make the students turn off their cell phones before class. If they go off during class, penalize the student. Cell phones are very disruptive to the flow of the class.
   - Grading policy
   - When to expect exams

## **Sandy Berry**, *Hinds Community College*

"What do you teach?" Throughout my 35-year career I have been asked that question many times. My response is that I try to teach students about mathematics. A teacher's focus should be on their students first. Treat each student with the utmost respect. Take time to get to know your students as people. Learn their names as soon as possible. Call them by name often. Remember what it was like the first time you were a student in a course that you knew would be difficult for you. Try to help your students understand that your purpose is to help them learn mathematics.

Individual learning styles or learning modes-visual, auditory, and/or kinesthetic-play an important role in student understanding and performance. Some students learn best through seeing problems worked out, others through hearing a thorough explanation, and others by engaging in hands-on activities. Most successful students utilize more than one mode of learning.

It is important that you as an instructor become aware of your own learning and teaching styles and work at developing techniques of presenting material that will address all learning styles.

Working through problems carefully and completely on the board while carefully telling students what you are doing and thinking will serve to accommodate most learners. Incorporating hands-on activities for kinesthetic learners will require some inventiveness on the part of the instructor. Brain research shows that learners will not develop appropriate neural networks for remembering how to do math problems without doing them and doing them correctly. So take time in class to let your students practice what you are demonstrating. Give them immediate feedback and help them correct their mistakes. Continually search for better ways of communicating with your students.

## **Deanna L. Dick**, *Alvin Community College*

1. Try to remember to stick to a schedule, approximately 50 minutes per section in the textbook.

2. Always allow more time for rational expressions and word problems.

3. Use fractions daily in class and the students will become much less afraid of them and will be more successful with them. Just be prepared—they may never like them!

4. Allow at most one-fourth of the class period for questions, and then move on to new material. Students are great at wasting time if they can get away with it and teachers are bad about wanting to answer all the questions. Make them come to your office hours. One-on-one explanations are always better anyway. If a class is persistent, answer the questions and then give the lecture in the last 10 minutes of class. They will never play the stall game again!

5. Try to point out when a problem requires previously learned material to finish solving it.

   Example: When working a problem like Exercise 34 in Section 5.5 (a rational equation) remind the students that they learned how to solve the resulting equation in Section 4.8. Sometimes it is hard for students to tie the sections together and to see how one problem relates to another.

6. On word problems-make your students identify the variable in words. Tell them to be specific. If they don't know what they are looking for, it will be difficult for them to come up with equations they can solve to find it!

   Example: $x$ = # of bull-riders in Cheyenne, Wyoming

   This is more descriptive than $x$ = # men, which could mean bull-riders or just citizens of the city. This is especially important when you are dealing with systems of equations that may talk about both ideas.

7. Have the students do the reviews and chapter tests when studying for their test.

8. Remember, most developmental math students don't read the textbook, though they should always be encouraged to do so. Therefore, helpful hints come primarily from the instructor. If the text gives a good hint, make sure you also give it in your lecture. Otherwise they may never see it.

9. Don't work the examples in the textbook. No college student wants to be read to. That is why they spent so much money on the textbook! Give them new examples to supplement what is in the text.

10. Don't panic or get upset if you make a mistake on a problem or get stuck on one. It happens to everyone eventually and can actually be encouraging to your students. Students love to correct their teachers, and the fact that they caught the mistake builds

their confidence. Also, they remember when they get stuck at home that we got stuck in class, which can make it a little less frustrating. After all, if we can solve a problem that gives us trouble with a little more time and thought, then that may be all they need as well.

11. Show your students how to use technology to check your results, but recognize that they will never understand it if they don't learn to do it by hand.

12. My first semester teaching, I taught a class that was almost all word problems, "finite math." It was an 8 A.M. class and I am not a morning person. I had a wonderful student who always asked questions and his favorite thing to ask was why I set up the problem this way instead of his way, and why his equation didn't work. Usually, I would erase the correct equation and write his in its place. As students often find out, sometimes it is harder to troubleshoot an equation than to start again from scratch. I wasted a lot of time, and confused students by focusing on the wrong equation. As a first time teacher I quickly learned to do the problem correctly in class, and trouble-shoot their home work after class.

## **Kathleen C. Ebert**, *Alfred State College*

I value five basic things in all my classes and here is my advice.

1. When you teach a course for the first few times, plan out the entire semester ahead. Additionally, until you get it down pat include many extra review days at the end. I am student centered and students always need much more time on a topic than I think. I can't always plan which topics they will struggle on or be sick during.

2. Summarize at the beginning of each class (what you've covered recently) and at the end of class (what we did today). It helps students see the big picture of how it all relates and comes together.

3. At the end of class I put 2-5 problems on the board (or on a handout) for them to work on before they leave. I walk around so I know who needs help and who doesn't; they can work together. Engaging the student is key.

4. I do not collect homework regularly but I do make sure they review for tests. I collect (or look through their notebooks) their review assignments and I count it as a quiz or homework grade.

5. Last but not least, I always make my students do test corrections. I do not believe in adding points to their tests for it but I do count them as a quiz grade (percent corrected for all problems they got wrong). I make them do these on a separate sheet of paper or on a new test (this makes it easier for me to grade). They can go to the lab, come to me, or find a friend that can help. They are able to continue to hand these in until all are correct. Often I make them write a sentence about why they got each one wrong. One other thing I occasionally do is offer extra credit. For extra credit, students complete a section in the book that we did not cover. They have to read the section, take good notes (that they could teach from), do the odd problems (showing all work and either present the information or take a quiz on it, or both. Somehow they find time. They practice all the important stuff, reading a math text, taking their own notes, learning for understanding. I usually wait with this offer until they start asking "is there anything I can do..." Depending on how much they do, and how well determines how much credit I give them.

## **Rosa Kavanaugh**, *Ozarks Technical College*

1. Since this is the second-level course in the traditional algebra sequence, students should have a basic familiarity with the introductory concepts of algebra. However, almost all students bring into this course a number of deficiencies in those basic concepts, and those students who are marginally placed into the course have some major holes in their understanding of the foundations of algebra. Part of what the instructor should be attempting to do in this course is to help students to identify and fill holes in their foundations as well as to learn the content that is new in this course.

2. The instructor who is familiar with common errors and misunderstandings can be more effective in helping the students to be successful in repairing and building upon this foundation. The text is a good reference for addressing many of these common errors. An instructor who chooses to include a discussion of such an error in the classroom should be very careful about what is written on the board. If the topic is avoiding "improper cancellation" in fractions, the instructor should either be careful to write

$$\frac{x^2+4x+3}{x+3} = \frac{x+4x+1}{x+1}$$

Wrong!

as the author does in Chapter 5 or should tell students to put down their pencils and simply watch. If the lesson is being presented on a dry-erase board, this is a very appropriate time to use a red marker for emphasis.

3. Mathematical language and notation are foreign and confusing for students who are seeing algebra for the first time. Much of the mathematical language has meaning in the more familiar English language. Although developmental mathematics students also have weak language skills, we can help them identify words by associating them with the English language. These associations can help our students make the connections that will give the mathematical language more meaning to them. A good example is the word "distribute." This is a good time to encourage some discussion of the meaning of the word in common everyday language and then make the connections to the specific mathematical meaning.

4. Perhaps the most troublesome counter-example to the notion that mathematical language has meaning in the English language is in the words "term" and "factor." In the English language, these words do not have the same specific meaning that they do in mathematics. When we say, "These were factors in our decision," that does not mean that those factors were multiplied. When we say, "These were the terms of our agreement," that does not mean that the terms were added. Yet these same words have those *very* precise meaning in mathematics. In the English language we use the word term in a very generic way. The word term should be used very deliberately and precisely in the mathematics classroom.

5. Students often confuse problem types on exams. One reason is that they tend to overlook the instructions as they do homework since all of the problems in a section lend to be the same type. I believe that the instructor should not only emphasize the meaning of the instructions but also write the instructions as part of the example. Students in developmental mathematics classes tend to include in their notes much of what is written on the board, but little of the words that the instructor says, but does not write. Thus it is important that the instructor not only say, "Solve" but also write:

   "Solve $3x+5=5(x-1)$.

   This kind of emphasis helps students avoid common confusion such as the difference between solve and simplify.

6 . Mathematics instructors themselves must model good mathematical notation. One of my notation "pet peeves" is improper use of equal signs. Occasionally, adjunct faculty insert errant equal signs as they perform intermediate steps of a problem.

7. Another more subtle but, I believe, equally serious error is the omission of equals signs between simplification steps. Perhaps one of the most common occurrences of this is in the demonstration of the factoring process. For example, I observe a number of faculty who write a factoring problem as:

$$3x^4-12y^4$$
$$3\left(x^4-4y^4\right)$$
$$3\left(x^2+2y^2\right)\left(x^2-2y^2\right)$$

   Omitting the equal signs between the steps is not only mathematically unsound but also eliminates the rationale for checking the result of a factoring problem by redistributing because they should be equal.

8. Correct mathematical language can also be an issue. Some instructors say "LCM" when they mean "LCD." Another common error is confusion of "expression" and "equation." If we are to convince our student that mathematics is a precise discipline with precise language, our own language must also model such precision.

9. Students often have difficulty memorizing formulae. Whenever there are formulae that students need to memorize, I give them a strategy for memorizing the formulae as they do their homework. I tell them that each time they use the formula, it is important that they write the formula first and then substitute the given info. Many students tend instead merely to write the result after substitution and miss the learning benefits of the writing process. I explain that the repeated writing of the original formula as they use it actually helps them to remember it.

10. Instructors should be prepared not only to describe to students what the process is but also to explain why the process works. This may not be an issue for some students but is particularly important to many adult learners. They may have heard the rules before but did not buy into the validity of the process. This stumbling block is sometimes an impediment to further learning in the course. Instructors need to be willing to provide such explanations during class if time permits. For example. some students never saw the validity of the rules for operations with signed numbers. Others are still confused about why division by zero is undefined. Settling these kinds of questions allows students to progress in topics where they were confused in the past.

11. The text contains a good introduction to the benefits of students' recognizing and using the learning objectives of each section. One other use of these objectives that we recommend to students is in preparing for exams. So many students at this level have never learned how to study. They tend to believe that the way to prepare for an exam is to work all of the several hundred problems assigned for homework. We find Bittinger's a, b, c, ... coding of learning objectives very helpful in helping students to identify that they do not have to rework all of the problems in a given section but should instead concentrate on one from each objective to identify areas of strength and weakness. Then they understand that if they have difficulty, they can return to the discussion correlating to that objective in the section/chapter.

## **Susan Leland**, *Montana Tech*

Using silly, catchy phrases helps students to remember common pitfalls and procedures in algebra. Some examples follow:

1. Bells and whistles: as in "Bells and whistles should go off in your mind every time you see a negative sign in front of parentheses! Change every sign inside." This can be used for many other situations also. I have students come back to see me two and three years after being in my class and they say they still hear "bells and whistles" when doing math.

2. The rules never change: as in "This equation has fractions in it—but the rules never change." Keep properties, theorems, procedures, and rules very generic so that they fit every conceivable situation. Students at this level need to do problems the same way every time. Try to avoid showing several different ways to do anyone type of problem—that just confuses many intermediate algebra students. Those students who already know another way to approach a given problem should be allowed to use a different, but correct, method, but don't burden the rest of the class with more than one method.

3. Details. details, details: as in missing signs, incorrect arithmetic, and copying problems incorrectly. If you can get students to concentrate on all the details, grades improve! My students hear me say this at least a million times in a semester—at least according to them!

4. Set the dumpster on fire, as in the following joke. A mathematician decided he wanted to change jobs. He went to the fire chief and said. "Chief, I'd like to become a fireman!" The fire chief said, "Great! I need to give you a little test." The chief took the mathematician into the alley where there was a dumpster, a hose on the ground, and a water spigot. The chief said to the mathematician, "What would you do if you came out here and found the dumpster on fire?" The mathematician immediately said, "I'd hook the hose up to the spigot, turn the water on, and put out the fire." The chief said, "Great! You'll make a super fireman! Now, what would you do if you came out here and found the dumpster wasn't on fire?" The

mathematician thought for a moment, then brightened and said, "I'd set it on fire!" The chief said, "What? That's terrible! Why in the world would you set the dumpster on fire?" The mathematician replied, "Because then I would have reduced it to a problem I already know how to solve!" This joke is applicable to so many topics in algebra, especially equation solving. Every equation in the book- systems, quadratic, rational, radical- eventually "reduces" to a linear equation, something we already know how to solve after Chapter One! So, "set the dumpster on fire" and get to a problem we already know how to do!

## **Michael Montaño**, *Riverside Community College*

1. Eye contact with your audience is essential when you are delivering a lecture.

2. When a class does not reply to a question, do not be too anxious and proceed to answer the question yourself. Use the "dead air" time to your advantage. A classroom does not need to be dynamic at all times. Silence allows students some time to organize and collect their thoughts.

3. Definitions are very important. Make sure that students understand every aspect of the definition.

4. Pattern recognition is a very powerful technique in the teaching of mathematics. Fitting the problem to the same format as the rule can be very useful.

## **Nancy Ressler**, *Oakton Community College*

1. The student *grapevine* is healthy and flourishing! If you habitually modify due dates for homework, exams, quizzes, projects and content presentation, students will expect that if they miss any of the above it is acceptable because you *probably* were going to make *another* change anyway. The matter of fairness is prominent. Adjusting for one student means that it is necessary to *adjust* for others. By doing so, you will have *twice* the course duties that are necessary for a seasoned educator.

2. Additionally, if you only address course expectations on the first class meeting, word will "get out" and *future* first class meetings will not be well attended since students will expect only a friendly discussion! They rationalize: they *can read* the syllabus by themselves later! On the first class meeting bring a tablet and sharpened pencils (usually each division or cluster office has supplies for faculty) for those students that have brought no supplies. Provide each with a pencil and a few sheets of paper as you begin your course lecture on day ONE! Ensuring good habits by a content focus on day ONE also allows students to experience your teaching methodology and style.

3. Students will complain about the amount of course content in most math courses. Faculty MUST prepare the students for success in future courses. This can only be achieved by covering all of the content described by the department's generic syllabus. You are hired to teach the c1ass ... not to cater to those that are better or less prepared. Individualized help can be provided during your office hours if a reasonable answer still leaves the student confused or if a better prepared student has future/later content questions.

4. Encourage students to visit with tutors. In high school, *needy* students or the labeled *nerds* frequented tutoring. In college, the bright and astute attend tutoring. The difference between A's and B's and B's and C's become apparent by tutor visits. Invite a college tutor to talk to your classes. Once the students meet the people they will be working with during the course it is easier for them to make the tutoring appointment!

## **Tomesa Smith**, *Wallace State Community College*

Fifteen years ago I began my teaching career at a community college. Considering the typically diverse nature of community college students, I decided to have students fill out an index card of information on the first day of class. On the card, I asked for:

> Name (how it will be written on test papers, first and last); Student Number; Major; Phone (in case calculators, purses, books, etc. are left

in the room); Email; Family; Job (approximate number of hours worked and approximate times); Math history (high school and college); Goal for class

Through the years, I have changed the format some, but I continue to utilize the cards because they have proven to be very helpful. While students complete their cards in class, I tell them some information about myself. After taking up the cards, I call roll by them immediately. This helps me to begin connecting faces with names and to ask for correct pronunciation if needed. I note these things on the card and then use the set as a deck of cards throughout the term. I call roll by them, assign groups by them, and document absences or unusual circumstances on them. I think students appreciate that I strive to learn their names, that I am concerned about them when they are absent and that I want them to achieve their goals.

I suppose through the years, I have incorporated other strategies that have become equally routine to me. I've always tried to start class on time. How can I expect students to be on time if I'm not? I also try to utilize every minute of class time. I never want students to leave my classroom feeling as though we could have accomplished more. Being organized is half the battle. I think students respect a teacher who is prepared and enthusiastic about each topic. I also try to foster respect in the classroom. I give students my attention when they are talking to me and expect them to give me their attention when I am talking to them. Often students will say that they did not want to ask a question because it was dumb. No questions are dumb ones, so I try to encourage students to ask me about anything they do not understand.

Aside from the actual lecturing and learning in the classroom, I have found that these tips have helped me to become a better instructor through the years. To teach a subject seems the best way to learn it, so repetition will help with the subject matter. To help with everything else, be prepared, punctual, respectful, cheerful, and willing to make adjustments along the way. If you can do these things, you are destined to be a great teacher.

## **Sharon Testone**, *Onondaga Community College*

1. Students enrolled in an intermediate algebra course at the community college level are often students who have just completed a developmental beginning algebra course or they are students who have had intermediate algebra in high school and just need to refresh their skills. These two groups of students are very different from each other. The former beginning algebra students may be struggling with this new material. Meanwhile, the students who just need a refresher may be bored with the material. It is the instructor's role to meet the needs of these diverse groups.

2. Group work can help to alleviate the problem of diverse abilities in an intermediate algebra course. The instructor should form the groups by including a mixture of students who never have had intermediate algebra and students who are just reviewing the material. This method will provide an avenue for students to truly help each other. Be sure to have individual students or a representative from the group show you at least one completed problem before the end of the class period. (Another alternative is to have a group representative put the solution on the board.) This technique helps to assure that students will be able to do their homework.

3. An important time management tip is to count the number of class periods, subtract the number of "testing days" and subtract at least two class periods for review at the end of the semester. This result is the number of instructional periods. Divide the number of instructional periods into the number of sections that need to be taught. This result gives the approximate number of sections that need to be taught each class period. This simple calculation will help new faculty avoid the pitfall of moving too quickly or too slowly through the course. Additionally, the faculty member will most likely complete all topics in the syllabus and not omit any essential material.

4. Always be on time to class and always end the class on time. Our students have very busy schedules and faculty needs to respect that fact.

5. Always be prepared for class and always hand back graded homework, quizzes, and tests the very next class period.

6. Before teaching the course for the first time, ask your department chairperson what the prerequisites are for this course. This information will assist you in gauging what knowledge the students should have when entering your classroom. They may not be completely prepared, but you will know what skills they are expected to possess and you will not spend much time reteaching material from the previous course. Additionally, determine what course(s) your students will enroll in after completing your course. Be sure that you prepare the students for those courses, but don't teach the topics from them.

7. Often intermediate algebra students do not complete their homework assignments and this leads to failure. One option is to require that students complete homework assignments in a separate notebook. On test days instructors can review the notebook (without actually grading it) to determine if students are completing their assignments. Another option is to collect homework daily or randomly and grade it.

8. Giving a five-minute quiz after reviewing homework questions at the beginning of the class period is often helpful for new instructors. The quiz results will let both the students and the instructor know how they are doing. If the whole class fails a quiz, then most likely the instructor needs to make improvements.

9. Prepare handouts with matching overheads or Power Point slides. Students at this level are often not good note takers and have difficulty listening and writing at the same time. Handouts that include key concepts, one or two worked-out examples, and two or three problems for the students to complete immediately are very useful.

10. Be sure to assign the synthesis problems in each chapter as group work. These problems are a little more challenging and they help the students increase their level of understanding.

## **Roy West**, *Robeson Community College*

Remember that the material being taught is developmental. A lot of times college instructors feel that most students just need some extra practice and they will catch on. This may be true for some classes. You as an instructor need to feel them out. I personally have found that nothing replaces working problems for them in class and showing the various situations that can occur. The responsible student will get the practice they need when they do the homework problems themselves at home. Use your class time wisely.

## **Rebecca Wyatt-Semple**, *Nash Community College*

1. Wish They Had Told Me ..

   Rule 1: For all instructors, whether full-time or adjunct: There is never enough time! Prioritize your schedule so that you don't get overwhelmed. If you come into class too tired to function, your value as an instructor will be minimal at best. Take care of yourself so that you will be able to take care of the students entrusted to you. We encourage students to set priorities in order to get their work done, so practice what you preach.

   Rule 2: You'll work harder than any of your students. You'll have to be creative in presentations, develop interesting assignments, challenge the most gifted students, and help the least gifted. You will be many things to your students, but you are not their buddy. You are a professional who should be clearly concerned with each student's welfare and progression in education.

   Rule 3: Use the text, and the supplementary items that are available, to your best advantage. Bittinger texts are extremely well written with excellent Study Tips for students, generous margins and margin exercises (Yes! Encourage students to write in their books), Chapter Summaries, Practice Tests, warnings and cautions, and Cumulative Reviews. Students can teach themselves from this text, especially if they take advantage of the Learning Resources such as the *Student's Solutions Manual,* videos, InterAct

Math®Tutorial Web site, and the on-line help available through MyMathLab and MathXL®.

Rule 4: The first week of class is critical. Set the pace of the course and let students know what is expected of them. Put your rules in writing and give them out with the course syllabus. The first day of class is the best time to have the students fill out an index card with information you may need: name, student ID number, e-mail, phone number, areas of interest, and intended major. Collect the cards and review them before the next scheduled meeting of the class. Try to learn the students' names as quickly as possible. Use the index cards in selecting problems to use in class and homework. For example, if you know that your class has a number of students interested in the sciences of medicine, problems can be selected to reflect their interest.

Rule 5: Know your school or department's policy on calculators in the classroom and their use on tests and exams. Find out which calculators are recommended for your course and which, if any, are banned. Talk to instructors who teach the courses following yours to see what calculator skills they expect students to have upon entry into their classes, and be certain that your students develop these skills before the end of the course. It is effective to split tests and exams in two parts: Part A covers basic concepts with minimal calculations and no calculators allowed, and Part B emphasizes problems in which the use of calculators would be beneficial.

Rule 6: If you assign homework, you'll have to grade homework. Students need a great deal of practice. You don't need to become bogged down in grading homework. Grade approximately one-fourth of the homework problems by random selection, or by some method of selection of your own. Students will not know ahead of time which problems are to be graded, and are responsible for all problems assigned. Let students know that you are practicing sampling, an acceptable statistical technique. If you fail to grade homework, or fail to give some incentive for doing it, most students will not do the practice necessary to internalize concepts. Let students know from day one what your policy on homework will be, and stick to your policy.

Rule 7: Use proper mathematical notation, draw neat graphs, and label the axes appropriately. Do what you want your students to do. Set high standards for your work and hold your students to these standards. It is far easier to develop good habits than to break bad habits. Being skilled in the reading and use of proper notation will allow your students to advance to higher levels of mathematics, read, and understand the texts.

Rule 8: Most students do not know how to read math texts. Teach them. Structure assignments so that they must read the text, make use of examples, and write brief discussions. Assign discussion problems as well as calculation problems. Have students "teach" a section to the class. Give group assignments in which a student must explain concepts to others in his group. Bittinger's math texts always have an abundance of discussion problems.

Rule 9: Prepare yourself for the question: "When will I ever need to know this stuff?" Make a mental list of situations that require algebraic thought. At the beginning of each chapter, think of ways the concepts of that chapter may be encountered in the "real world." If you can't think of many examples, do a little research or talk to other instructors in the math area or in other departments.

Rule 10: Multiple-choice tests should be used with caution. They are easy to grade, but you should not fall into the trap of always testing in this manner. Multiple-choice tests do not always reveal what students actually know, nor do they require critical thinking skills. Students, by their own admission, do a great deal of guessing. Mix multiple-choice questions with questions requiring that students show their work and thought processes.

# The Set of Real Numbers

## Learning Objectives:

a. Use roster notation and set-builder notation to name sets, and distinguish among various kinds of real numbers.
b. Determine which of two real numbers is greater and indicate which, using < and >; given an inequality like $a < b$, write another inequality with the same meaning; and determine whether an inequality like $-2 \le 3$ or $4 > 5$ is true.
c. Graph inequalities on the number line.
d. Find the absolute value of a real number.

## Examples:

1. Name all numbers from the set $\left\{-10, \sqrt{5}, -2\frac{3}{4}, 0, -5, 10\right\}$ that are:

    a) natural numbers  b) whole numbers  c) integers
    d) rational numbers  e) irrational numbers  f) real numbers

2. Use roster notation to name the set of positive integers less than 6.

3. Use set-builder notation to name the set of all real numbers greater than 2.

4. Use < or > for $\square$ to write a true sentence.

    a) $-3 \;\square\; 8$  b) $-\frac{4}{5} \;\square\; -\frac{1}{3}$  c) $10 \;\square\; -12$

5. Write a different inequality with the same meaning.
    a) $6 > x$  b) $-5 \le y$  c) $z < 2.7$

6. Graph each inequality.
    a) $x < 2$  b) $x \ge -3$

7. Find the absolute value.

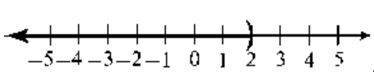
    a) $|-3|$  b) $|6|$  c) $\left|\frac{0}{-8}\right|$

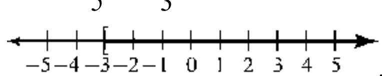
## Teaching Notes:

- Remind students that integers are rational numbers; any integer can be written as the ratio of itself and 1.
- Decimal numbers that terminate or repeat in a fixed block are both examples of rational numbers; ask students to give examples of both.
- The decimal form of an irrational number neither terminates nor repeats.
- Help students see the relationships among subsets of the real numbers.

*Answers*: *1a) 10, b) 0, 10, c)* $-10, -5, 0, 10$*, d)* $-10, -2\frac{3}{4}, -5, 0, 10$*, e)* $\sqrt{5}$*, f)* $-10, \sqrt{5}, -2\frac{3}{4}, 0, -5, 10$*;*

*2)* $\{1,2,3,4,5\}$*; 3)* $\{x \mid x > 2\}$*; 4a)* $-3 < 8$*, b)* $-\frac{4}{5} < -\frac{1}{3}$*, c)* $10 > -12$*; 5a)* $x < 6$*, b)* $y \ge -5$*, c)* $2.7 > z$*;*

*6a)* 

*, b)* 

*; 7a) 3, b) 6, c) 0*

# Operations with Real Numbers

## Learning Objectives:

a Add real numbers.
b Find the opposite, or additive inverse, of a number.
c Subtract real numbers.
d Multiply real numbers.
e Divide real numbers.

## Examples:

1. Find the opposite (additive inverse).

   a) $3$ b) $-\frac{3}{4}$ c) $0$ d) $-4y$

2. Add or subtract, as indicated.

   a) $-5+(-3)$ b) $5+(-3)$ c) $-5+3$ d) $-2.1+(-7.3)$

   e) $-\frac{2}{12}+\frac{3}{24}$ f) $\frac{1}{2}+\left(-\frac{1}{2}\right)$ g) $3-(-5)$ h) $-3-5$

   i) $-3-(-5)$ j) $5.4-9.2$ k) $-\frac{2}{3}-\frac{3}{4}$ l) $\frac{1}{8}-\left(-\frac{3}{4}\right)$

3. Multiply.

   a) $(3)(5)$ b) $(-3)(-5)$ c) $(-3)(5)$ d) $(3)(-5)$

   e) $-8(-2.4)$ f) $-\frac{2}{3}\cdot 15$ g) $-\frac{3}{8}\cdot\left(-\frac{2}{9}\right)$ h) $-6(-8)(-2)$

4. Find the reciprocal of each number, if possible.

   a) $12$ b) $\frac{5}{6}$ c) $-\frac{1}{2}$ d) $0$

5. Divide, if possible.

   a) $-8\div(-4)$ b) $0\div(-4)$ c) $3.2\div 0$ d) $\frac{-14}{-2}$

   e) $\frac{8}{2y-2y}$ f) $3.6\div(-1.2)$ g) $-\frac{3}{5}\div\frac{15}{20}$ h) $-250\div(-0.4)$

## Teaching Notes:

- Refer students to the boxes for ***Rules for Addition of Real Numbers, Subtracting by Adding the Opposite, and to Multiply or Divide Two Real Numbers***.
- Many students find working with fractions difficult. It may be useful to review finding the least common denominator.
- Remind students that only nonzero numbers have reciprocals.
- Refer students to the ***Sign Changes in Fraction Notation*** box in the text.

*Answers:* 1a) $-3$, b) $\frac{3}{4}$, c) 0, d) $4y$; 2a) $-8$, b) 2, c) $-2$, d) $-9.4$, e) $-\frac{1}{24}$, f) 0, g) 8, h) $-8$, i) 2, j) $-3.8$, k) $-\frac{17}{12}$, l) $\frac{7}{8}$; 3a) 15, b) 15, c) $-15$, d) $-15$, e) 19.2, f) $-10$, g) $\frac{1}{12}$, h) $-96$; 4a) $\frac{1}{12}$, b) $\frac{6}{5}$, c) $-2$, d) no reciprocal exists; 5a) 2, b) 0, c) not defined, d) 7, e) not defined, f) $-3$, g) $-\frac{4}{5}$, h) 625

# Exponential Notation and Order of Operations

**Learning Objectives:**

a Rewrite expressions with whole-number exponents, and evaluate exponential expressions.
b Rewrite expressions with or without negative integers as exponents.
c Simplify expressions using the rules for order of operations.

**Examples:**

1. Write exponential notation.
   a) $(4)(4)(4)(4)(4)$  b) $(-2)(-2)(-2)(-2)$  c) $x \cdot x \cdot x \cdot x \cdot x$

2. Evaluate.
   a) $5^2$  b) $2^3$  c) $3^4$  d) $(-5)^2$  e) $4^1$
   f) $(-2)^3$  g) $(-3)^0$  h) $(-3)^4$  i) $\left(-\frac{1}{2}\right)^2$  j) $(-1.2)^3$

3. Rewrite using a positive exponent. Evaluate, if possible.
   a) $\left(\frac{2}{5}\right)^{-2}$  b) $x^{-5}$  c) $\frac{1}{m^{-4}}$

4. Rewrite using a negative exponent.
   a) $\frac{1}{8^4}$  b) $\frac{1}{n^2}$  c) $\frac{1}{(-3)^5}$

5. Simplify.
   a) $5(2-4)+6$  b) $\left[-7-3(2-4)\right]^2$  c) $(9-6-1)^3 \div 4-9$
   d) $(-9)6^2-(-2)(-9)$  e) $\frac{3+(-5)^2+8\cdot 3}{4(6-5)}$  f) $\frac{|-2|+|-6+9|}{(-1)3^2-6}$

**Teaching Notes:**

- In Example 1b) some students answer $-2^4$ instead of $(-2)^4$. Many students think these are equal.
- Many students find Example 5 difficult and need a lot of practice.

*Answers:* *1a)* $4^5$*, b)* $(-2)^4$*, c)* $x^5$*; 2a) 25, b) 8, c) 81, d) 25, e) 4, f)* $-8$*, g) 1, h) 81, i)* $\frac{1}{4}$*, j)* $-1.728$*;*
*3a)* $\frac{25}{4}$*, b)* $\frac{1}{x^5}$*, c)* $m^4$*; 4a)* $8^{-4}$*, b)* $n^{-2}$*, c)* $(-3)^{-5}$*; 5a)* $-4$*, b) 1, c)* $-7$*, d)* $-342$*, e) 13, f)* $-\frac{1}{3}$

# Introduction to Algebraic Expressions

**Learning Objectives:**

a Translate a phrase to an algebraic expression.
b Evaluate an algebraic expression by substitution.

**Examples:**

1. Translate each phrase to an algebraic expression.
   a) 7 more than $x$

   b) 5 less than $y$

   c) twice $w$

   d) the product of 4 and $n$

   e) the sum of $x$ and $y$

2. Evaluate each expression.
   a) $x^2 + 5x$, when $x = -3$

   b) $4y - (2x)^2$, when $x = -2$ and $y = 4$

   c) $\frac{12p}{n}$, when $p = 5$ and $n = 10$

   d) $m^2 - 4(m - t)$, when $m = -3$ and $t = -1$

3. The area $A$ of a triangle with base $b$ and height $h$ is given by $A = \frac{1}{2}bh$. Find the area of a triangular garden with a base of 12 ft and a height of 8 ft.

**Teaching Note:**

- Many students have difficulty translating expressions containing "less than."

*Answers: 1a) $7 + x$, or $x + 7$, b) $y - 5$, c) $2w$, d) $4n$, e) $x + y$; 2a) $-6$, b) 0, c) 6, d) 17; 3) $A = 48\ ft^2$*

# Equivalent Algebraic Expressions

## Learning Objectives:

a Determine whether two expressions are equivalent by completing a table of values.
b Find equivalent fraction expressions by multiplying by 1, and simplify fraction expressions.
c Use the commutative laws and the associative laws to find equivalent expressions.
d Use the distributive laws to find equivalent expressions by multiplying and factoring.

## Examples:

1. Determine whether $4(x-3)$ is equivalent to $4x-3$ or to $4x-12$ by evaluating each expression for $x=3$ and $x=6$. Show your work.

2. Multiply by 1 to find an expression equivalent to $\frac{4}{5}$ with the denominator $20x$.

3. Simplify.
   a) $\frac{48y}{8y}$ b) $-\frac{120x}{80x}$

4. Name the law or property that justifies each statement.
   a) $(-5)+3=3+(-5)$ b) $(5+2)+4=5+(2+4)$ c) $4(5+2)=4\cdot 5+4\cdot 2$
   d) $\frac{3}{5}\cdot 1=\frac{3}{5}$ e) $-5\cdot(4\cdot 2)=(-5\cdot 4)\cdot 2$ f) $4\cdot 3=3\cdot 4$

5. Multiply.
   a) $3(x+2)$ b) $4(m-n)$ c) $-2(4x+5y)$
   d) $6k(x-y+z)$ e) $-8(3x-1)$ f) $\frac{1}{2}x(10y+24)$

6. Factor.
   a) $4x+yx$ b) $12y-12$ c) $15x-5y+20$
   d) $14k+49m+7$ e) $xy+xz-xw$ f) $\frac{3}{4}b+\frac{3}{4}ab$

## Teaching Notes:

- Remind students that the ***commutative laws*** deal with *order*; whereas, the ***associative laws*** deal with *grouping*.
- Remind students that parentheses do not always imply that the ***distributive laws*** are to be used (e.g., $(3a)(5)(6b)=90ab$, not $(3a)(5)+(3a)(6b)$ or $(3a)(5)\cdot(3a)(6b)$ ).

*Answers*: *1) $4x-12$; 2) $\frac{16x}{20x}$; 3a) 6, b) $-\frac{3}{2}$; 4a) commutative law of addition, b) associative law of addition, c) distributive law of multiplication over addition, d) identity property of 1, e) associative law of multiplication, f) commutative law of multiplication; 5a) $3x+6$, b) $4m-4n$, c) $-8x-10y$, d) $6kx-6ky+6kz$, e) $-24x+8$, f) $5xy+12x$; 6a) $x(4+y)$, b) $12(y-1)$, c) $5(3x-y+4)$, d) $7(2k+7m+1)$, e) $x(y+z-w)$, f) $\frac{3}{4}b(1+a)$*

# Simplifying Algebraic Expressions

**Learning Objectives:**

a Simplify an expression by collecting like terms.
b Simplify an expression by removing parentheses and collecting like terms.

**Examples:**

1. Collect like terms.
   a) $4x+5x$ b) $12y-y$ c) $3x-9-12x$
   d) $2.5y-8.6+3.4y-12.3$ e) $4x-5y+6x+8y$ f) $4x-8-7x+2$

2. Find an equivalent expression without parentheses.
   a) $-(-3x)$ b) $-(y+2)$ c) $-(m-8)$
   d) $-(5x-4y+3)$ e) $-(-6x+5y-4z+w)$

3. Simplify by removing parentheses and collecting like terms.
   a) $x+(3x+5)$ b) $4y-(2y+3)$
   c) $2k-(5k-4)$ d) $4m-3-(5-6m)$
   e) $7.3b+8.1-2(3.2b-0.4)$ f) $\frac{1}{2}(12x-4)-\frac{1}{6}(30x-y)$
   g) $-(9-t)+(3t-6)$ h) $5a-[3-2(4a-3)]$
   i) $4\{-3+2[5-6(4+1)]\}$ j) $[8(x+2)-3]+[2(x+5)+1]$
   k) $5\{[3(x-1)+2]-[4(2x+3)+1]\}$ l) $2y+\{3[2(4y-3)-(6y+4)]+3^2\}$

**Teaching Notes:**

- Some students forget to subtract each term in parentheses and only subtract the first term.
- Some students are overwhelmed by nested grouping symbols. Remind them to work methodically from the inside out.

*Answers:* 1a) $9x$, b) $11y$, c) $-9x-9$, d) $5.9y-20.9$, e) $10x+3y$, f) $-3x-6$; 2a) $3x$, b) $-y-2$, c) $-m+8$, d) $-5x+4y-3$, e) $6x-5y+4z-w$; 3a) $4x+5$, b) $2y-3$, c) $-3k+4$, d) $10m-8$, e) $0.9b+8.9$, f) $x+\frac{1}{6}y-2$, g) $4t-15$, h) $13a-9$, i) $-212$, j) $10x+24$, k) $-25x-70$, l) $8y-21$

# Properties of Exponents and Scientific Notation

## Learning Objectives:

a Use exponential notation in multiplication and division.
b Use exponential notation in raising a power to a power, and in raising a product or a quotient to a power.
c Convert between decimal notation and scientific notation, and use scientific notation with multiplication and division.

## Examples:

1. Multiply or divide. Simplify your answer. Write final answers with positive exponents only.

   a) $x^3 \cdot x^5$ b) $9^5 \cdot 9^{10}$ c) $(2x)(-3x^4)$

   d) $(-20x^2y^3)(6xy)$ e) $(2xy)^0(5x)$ f) $\dfrac{x^{12}}{x^3}$

   g) $\dfrac{x^5}{x^8}$ h) $\dfrac{4^{15}}{4^{19}}$ i) $\dfrac{15x^2y^5z}{-3x^2y^2}$

2. Simplify using the power rule. Write final answers with positive exponents only.

   a) $(x^6)^5$ b) $(3x^2y^6)^4$ c) $\left(\dfrac{x^2}{y^4z^9}\right)^5$

   d) $\left(\dfrac{2x^{-3}y}{x^{-4}y^3}\right)^{-2}$ e) $\dfrac{(m^3n^4)^{-1}}{(-3m^{-2}n^5)^2}$

3. Convert each number to the specified notation.

   a) Write in scientific notation: 125 3,442 0.022 0.00000453

   b) Write in standard notation: $2.04\times10^3$ $1.9902\times10^7$ $9.311\times10^{-4}$

4. Perform the calculations indicated.

   a) $(2.1\times10^{-3})(1.4\times10^{-5})$ b) $\dfrac{4.8\times10^{-6}}{1.2\times10^{-7}}$

## Teaching Notes:

- Students need a lot of repetition and practice in order to master these objectives.
- Students often move constants along with a variable that has a negative exponent. For example, a common mistake is to simplify $2x^{-3}$ to $\dfrac{1}{2x^3}$.

*Answers:* 1a) $x^8$, b) $9^{15}$, c) $-6x^5$, d) $-120x^3y^4$, e) $5x$, f) $x^9$, g) $\dfrac{1}{x^3}$, h) $\dfrac{1}{256}$, i) $-5y^3z$; 2a) $x^{30}$, b) $81x^8y^{24}$, c) $\dfrac{x^{10}}{y^{20}z^{45}}$, d) $\dfrac{y^4}{4x^2}$, e) $\dfrac{m}{9n^{14}}$; 3a) $1.25\times10^2$, $3.442\times10^3$, $2.2\times10^{-2}$, $4.53\times10^{-6}$, b) 2040, 19,902,000, 0.0009311; 4a) $2.94\times10^{-8}$, b) 40

# Solving Equations

**Learning Objectives:**

a Determine whether a given number is a solution of a given equation.
b Solve equations using the addition principle.
c Solve equations using the multiplication principle.
d Solve equations using the addition principle and the multiplication principle together, removing parentheses where appropriate.

**Examples:**

1. Determine whether the given number is a solution to the given equation. Justify your answer.
   a) Is $x = 3$ a solution to $4x + 2 = 18$?
   b) Is $x = 12$ a solution to $\frac{5}{6}x = 10$?

2. Solve using the addition principle. Check your solutions.
   a) $-12 + x = -2$ b) $13 + x = -35$ c) $y - 4.9 = 16$ d) $x + \frac{3}{4} = -\frac{2}{3}$

3. Solve using the multiplication principle. Check your solutions.
   a) $-5x = 70$ b) $5x = 45$ c) $\frac{1}{2}x = 8$ d) $-\frac{5}{6}x = 10$

4. Solve using the principles together. Check your solution.
   a) $10x + 5 = 17$ b) $2x - 3 = 7$
   c) $-14x - 10 = 8 - 5x$ d) $4(y + 5) = 24$
   e) $6x - 2 + 5x = 4x + 12$ f) $5 - x = 8 - 2(x - 1)$
   g) $\frac{x}{3} + 5 = \frac{3}{5}$ h) $3 - \frac{2}{3}(x + 2) = 1$
   i) $3(x + 2) = 5x + 6 - 2x$ j) $-5x + 7 = 3 - 9x + 4x$
   k) $2[3 - 4(5 - x)] - 6 = 7[2(4x - 3) + 1] - 8$

**Teaching Notes:**

- Encourage students to check their solutions.
- Encourage students to simplify each side of the equation as a first step.
- Some students prefer to always end with the variable on the left, while others prefer to arrive at a positive coefficient of the variable.
- Some students try to subtract the coefficient from the variable instead of eliminating it by dividing.
- Refer students to the ***Addition Principle, Multiplication Principle*** and ***Equation-Solving Procedure*** boxes in the textbook.

*Answers:* *1a) No, because $4 \cdot 3 + 2 \neq 18$, b) yes, because $\left(\frac{5}{6}\right) \cdot 12 = 10$; 2a) $x = 10$, b) $x = -48$, c) $y = 20.9$, d) $x = -\frac{17}{12}$; 3a) $x = -14$, b) $x = 9$, c) $x = 16$, d) $x = -12$; 4a) $x = \frac{6}{5}$, b) $x = 5$, c) $x = -2$, d) $y = 1$, e) $x = 2$, f) $x = 5$, g) $x = -\frac{66}{5}$, h) $x = 1$, i) all real numbers, j) no solution, k) $\frac{1}{16}$*

# Formulas and Applications

**Learning Objective:**

a Evaluate formulas and solve a formula for a specified letter.

**Examples:**

1. Solve for the given letter.

a) $6x+3y=5$, for $y$ b) $y=-\frac{2}{3}x+3$, for $x$ c) $V=lwh$, for $h$

d) $A=\frac{h}{2}(B+b)$, for $b$ e) $A=2\pi rh$, for $r$ f) $3(4ax+y)=3ax-y$, for $x$

2. Solve as indicated.

a) Solve for $C$: $F=\frac{9}{5}C+32$; then evaluate for $F$ = 25°. Round to the nearest tenth.

b) Solve for $h$: $V=\frac{1}{3}\pi r^2 h$; then evaluate for $V$ = 5.35, $r$ = 2, and use 3.14 for $\pi$. Round to the nearest hundredth.

3. Applications.

a) Suppose economists use as a model of the country's economy the equation $C=0.644D+5.822$, where $C$ and $D$ are in billions of dollars. Solve the equation for $D$, and use this result to determine the disposable income $D$ if the consumption $C$ is \$9.44 billion. Round your answer to the nearest tenth of a billion.

b) Some doctors use the formula $ND=1.1T$ to relate $N$ (the number of appointments scheduled in one day), $D$ (the duration of each appointment), and $T$ (the total number of minutes the doctor can use to see patients in one day). Solve the formula for $D$, and use this result to find the duration of each appointment if the doctor has 6 hours available for appointments and must see 25 patients per day.

**Teaching Notes:**

- Many students have difficulty with the problems in Example 1.
- Refer students to the box of steps for solving a formula for a given letter in the textbook.

*Answers*: 1a) $y=-2x+\frac{5}{3}$, b) $x=-\frac{3}{2}y+\frac{9}{2}$, c) $h=\frac{V}{lw}$, d) $b=\frac{2A}{h}-B$, e) $r=\frac{A}{2\pi h}$, f) $x=\frac{-4y}{9a}$;
2a) $C=\left(\frac{5}{9}\right)(F-32)$, –3.9℃, b) $h=\frac{3V}{\pi r^2}$, 1.28; 3a) $D=\frac{C-5.822}{0.644}$, \$5.6 billion,
b) $D=\frac{1.1T}{N}$, $D$ = 15.84 min

# Applications and Problem Solving

## Learning Objectives:

a Solve applied problems by translating to equations.
b Solve basic motion problems.

## Examples:

1. Use the **five steps for problem solving** to solve each of the following.

   a) ***Geometry*** The length of a rectangular room is 6 feet longer than twice the width. If the room's perimeter is 168 feet, what are the room's dimensions?

   b) ***Phone Service*** A promotional deal for long distance phone service charges a \$15 basic fee plus \$0.05 per minute for all calls. If Jerri's phone bill was \$75 under this promotional deal, how many minutes of phone calls did she make?

   c) ***Revenue*** The revenue of Company X quadrupled. Then it increased by \$1.6 million. The present revenue is \$24.4 million. What was the original revenue?

   d) ***Internet Service*** Shaun's internet provider charges its customers \$15 per month plus 3 cents per minute of online usage. Shaun received a bill from the provider covering a 5-month period and was charged a total of \$92.40. How many minutes did he spend online during that period?

   e) ***Balloon Altitude*** A hot air balloon spent several minutes ascending. It then stayed at a level altitude for three times as long as it had ascended. It took 5 minutes less to descend than it had to ascend. The entire trip took one hour and 30 minutes. For how long was the balloon at a level altitude?

   f) ***Investment*** Joe invested \$12,000 in certificates of deposit paying 4%. How much additional money should he invest in certificates paying 6% so that his total return for the investments will be \$792?

   g) ***Current*** Tara's motorboat travels at a speed of 12 mph in still water. Lamotte River flows at a speed of 2 mph. How long will it take Tara to travel 20 mi downstream? 20 mi upstream?

   h) ***Angle Measure*** The measure of one angle of a triangle is 4 degrees more than three times the measure of the second angle. The third angle measures 40 degrees. What are the measures of the two other angles of the triangle?

## Teaching Notes:

- Many students have difficulty with word problems.
- Some students need to see many problems in order to understand how an English sentence can be represented as an algebraic equation.
- Encourage students to draw and label diagrams when appropriate.
- Refer students to the ***Five Steps for Problem Solving*** box in the textbook.
- It is worth memorizing the formula $d = rt$.

*Answers:* 1a) $l = 58$ ft, $w = 26$ ft, b) 1200 min, c) \$5.7 million, d) 580 min, e) 57 min, f) \$5200, g) $1\frac{3}{7}$ hr, 2 hr, h) $34°$, $106°$

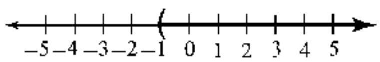
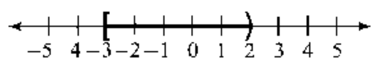
# Sets, Inequalities, and Interval Notation

## Learning Objectives:

a Determine whether a given number is a solution of an inequality.
b Write interval notation for the solution set or the graph of an inequality.
c Solve an inequality using the addition principle and the multiplication principle and then graph the inequality.
d Solve applied problems by translating to inequalities.

## Examples:

1. Determine whether the given numbers are solutions of the inequality.
   a) $3x-2\geq 4;\ 0,\ 2,\ -1,\ 4$
   b) $x-5<2x+1;\ 0,\ -4,\ -6,\ 1$

2. Write interval notation for the given set or graph.
   a) $\{x|x>1\}$ b) $\{x|x\leq -4\}$ c) $\{x|-2<x\leq 4\}$ d) $\{x|5>x\geq 3\}$
   e) 
   
   f) 
   

3. Solve and graph.
   a) $x-1<4$ b) $6x\leq -30$ c) $-5x>25$

4. Solve.
   a) $\frac{1}{2}x\geq 3$ b) $2x-6\leq 4$ c) $4-4x>16$
   d) $-6x+8>-7x+4$ e) $-25x+25\leq -5(4x-1)$ f) $3(2x+4)>2(x+2)$
   g) $\frac{1}{3}(x+2)<\frac{2}{5}(x+1)$

5. A student is taking a biology class in which there will be four exams. She has test scores of 68, 76, and 78 on the first three exams. What must she score on the last exam to earn a B (80 or better) in the course?

## Teaching Notes:

- Many students forget to reverse the inequality symbol when necessary.
- Some students do better if they move the variable in such a way that it has a positive coefficient whenever possible.
- Some students need frequent reminders that solving inequalities is the same as solving equations, with one important difference: reverse the inequality symbol when multiplying or dividing by a negative number.

*<u>Answers</u>: 1a) No, yes, no, yes, b) yes, yes, no, yes; 2a) $(1,\infty)$, b) $(-\infty,-4]$, c) $(-2,4]$, d) $[3,5)$, e) $(-1,\infty)$, f) $[-3,2)$; 3a) $\{x|x<5\}$, or $(-\infty,5)$, b) $\{x|x\leq -5\}$, or $(-\infty,-5]$, c) $\{x|x<-5\}$, or $(-\infty,-5)$, See graph answer pages.; 4a) $\{x|x\geq 6\}$, or $[6,\infty)$, b) $\{x|x\leq 5\}$, or $(-\infty,5]$, c) $\{x|x<-3\}$, or $(-\infty,-3)$, d) $\{x|x>-4\}$, or $(-4,\infty)$, e) $\{x|x\geq 4\}$, or $[4,\infty)$, f) $\{x|x>-2\}$, or $(-2,\infty)$, g) $\{x|x>4\}$, or $(4,\infty)$; 5) 98 or better, $\{S|S\geq 98\}$*

# Intersections, Unions, and Compound Inequalities

## Learning Objectives:

a Find the intersection of two sets. Solve and graph conjunctions of inequalities.
b Find the union of two sets. Solve and graph disjunctions of inequalities.
c Solve applied problems involving conjunctions and disjunctions of inequalities.

## Examples:

1. If $A=\{2,4,5,8,10\}$, $B=\{1,3,5,7,9\}$, $C=\{1,2,3,4\}$, and $D=\{3,4,5,6\}$, find the indicated union or intersection.
   a) $C \cap D$ b) $C \cup D$ c) $A \cup D$ d) $B \cap C$ e) $A \cap B$

2. Graph and write interval notation.
   a) $3 < x \textit{ and } x < 8$ b) $-5 < x \textit{ and } x \le 2$ c) $-2 \le x \le 5$

3. Graph and write interval notation.
   a) $x \ge 1 \textit{ or } x \le 0$ b) $x < 4 \textit{ or } x > \frac{13}{2}$ c) $x \le -4 \textit{ or } x \ge 3$

4. Solve for $x$ and graph your results.
   a) $x+1<7 \textit{ and } x>-2$ b) $x-1 \ge 5 \textit{ or } x-1<2.5$ c) $x<5 \textit{ and } x>8$

5. Solve.
   a) $2x-7 \ge -1 \textit{ and } 3x-5 \le 4$ b) $\frac{5x}{3}-5<\frac{5}{3} \textit{ and } 3x-\frac{1}{2}<-\frac{7}{2}$
   c) $4x+5<1 \textit{ or } 2x-1>-9$ d) $6x-7<11 \textit{ and } 2x+4>12$

6. ***Temperature*** The formula $F=1.8C+32$ can be used to convert Celsius temperatures to Fahrenheit temperatures. Joe is required to store his medicine at temperatures such that $32° < F \le 40°$. Find an inequality for the corresponding Celsius temperatures.

## Teaching Notes:

- Some students have trouble interpreting the meaning of an inequality when the variable comes second. For example, $3 < x$ means the same as $x > 3$.
- Some students need a lot of practice transitioning from the statement $x < 3 \textit{ and } x > -1$ to the statement $-1 < x < 3$.

*Answers:* 1a) $\{3,4\}$, b) $\{1,2,3,4,5,6\}$, c) $\{2,3,4,5,6,8,10\}$, d) $\{1,3\}$, e) $\varnothing$; 2a) $(3,8)$, b) $(-5,2]$, c) $[-2,5]$; 3a) $(-\infty,0]\cup[1,\infty)$, b) $(-\infty,4)\cup\left(\frac{13}{2},\infty\right)$, c) $(-\infty,-4]\cup[3,\infty)$, *See graph answer pages.;* 4a) $(-2,6)$, b) $(-\infty,3.5)\cup[6,\infty)$, c) $\varnothing$, *See graph answer pages.;* 5a) $\{3\}$, b) $(-\infty,-1)$, c) $(-\infty,\infty)$, d) $\varnothing$; 6) $0° < C \le 4\frac{4}{9}°$

# Absolute-Value Equations and Inequalities

## Learning Objectives:

a Simplify expressions containing absolute-value symbols.
b Find the distance between two points on the number line.
c Solve equations with absolute-value expressions.
d Solve equations with two absolute-value expressions.
e Solve inequalities with absolute-value expressions.

## Examples:

1. Simplify, leaving as little as possible inside absolute-value signs.

   a) $|4x|$ b) $|6x^2|$ c) $\left|\frac{-3}{y}\right|$ d) $\left|\frac{24x^5}{-6x}\right|$

2. Find the distance between the points on the number line.

   a) $-6,\ -14$ b) $-5,\ 8$ c) $18,\ 56$ d) $-2,\ 0$

3. Solve.

   a) $|x| = 6$ b) $|x| = -6$ c) $|3m| = 9.3$ d) $6|x| - 7 = 5$

   e) $|x+4| = 9$ f) $\left|\frac{x}{3} - 2\right| = 1$ g) $|5x| = 0$ h) $|2n+3| + 9 = 4$

   i) $2|x-1| + 15 = 20$

4. Solve.

   a) $|5x+9| = |x+4|$ b) $\left|\frac{1}{2}x + 3\right| = \left|\frac{2}{3}x - 1\right|$ c) $|x-4| = |4-x|$

5. Solve.

   a) $|x| \le 3$ b) $|x| \ge 3$ c) $|x| < -3$ d) $|x+3| < 7$

   e) $|x| + 4 \le 8$ f) $\left|\frac{x-3}{5}\right| < 1$ g) $|6-3x| < 4$ h) $|x-5| \ge 8$

   i) $|x| + 6 > 7$ j) $|9+4x| - 3 > -2$

## Teaching Notes:

- Most students need to see the solutions to 5a–d) on the number line in order to visualize the solution set.
- For the rest of the problems in 5 they should see the ***Solutions of Absolute-Value Equations and Inequalities*** summary box in the text.

*Answers*: 1a) $4|x|$, b) $6x^2$, c) $\frac{3}{|y|}$, d) $4x^4$; 2a) 8, b) 13, c) 38, d) 2; 3a) $\{-6,6\}$, b) $\varnothing$, c) $\{-3.1,3.1\}$, d) $\{-2,2\}$, e) $\{-13,5\}$, f) $\{3,9\}$, g) $\{0\}$, h) $\varnothing$, i) $\left\{-\frac{3}{2},\frac{7}{2}\right\}$; 4a) $\left\{-\frac{13}{6},-\frac{5}{4}\right\}$, b) $\left\{24,-\frac{12}{7}\right\}$, c) all real numbers; 5a) $\{x|-3 \le x \le 3\}$, or $[-3,3]$, b) $\{x|x \le -3 \text{ or } x \ge 3\}$, or $(-\infty,-3]\cup[3,\infty)$, c) $\varnothing$, d) $\{x|-10 < x < 4\}$, or $(-10,4)$, e) $\{x|-4 \le x \le 4\}$, or $[-4,4]$, f) $\{x|-2 < x < 8\}$, or $(-2,8)$, g) $\left\{x\middle|\frac{2}{3} < x < \frac{10}{3}\right\}$, or $\left(\frac{2}{3},\frac{10}{3}\right)$, h) $\{x|x \le -3 \text{ or } x \ge 13\}$, or $(\infty,-3]\cup[13,\infty)$, i) $\{x|x < -1 \text{ or } x > 1\}$, or $(-\infty,-1)\cup(1,\infty)$, j) $\left\{x\middle|x < -\frac{5}{2} \text{ or } x > -2\right\}$, or $\left(-\infty,-\frac{5}{2}\right)\cup(-2,\infty)$

# Graphs of Equations

## Learning Objectives:

a Plot points associated with ordered pairs of numbers.
b Determine whether an ordered pair of numbers is a solution of an equation.
c Graph linear equations using tables.
d Graph nonlinear equations using tables.

## Examples:

1. Fill in the missing information.
   a) The ordered pair $(x, y)$ that makes an equation true is called a ____________ to that equation.

   b) $(3,4)$ is a solution to $x + y = 7$ because ____________.

   c) Three more solutions to $x + y = 7$ are ____________.

   d) When the solutions to $x + y = 7$ are graphed on the Cartesian coordinate system the result is a ____________.

2. Determine whether the given point is a solution of the equation.
   a) $(-1,2)$; $y = 2x + 4$ b) $(-3,3)$; $x - y = 9$

3. Graph.
   a) $y = x + 1$ b) $2x + y = 3$ c) $y = \frac{2}{3}x - 3$

4. Graph.
   a) $y = x^2 + 3$ b) $y = \frac{4}{x}$ c) $y = |x| - 3$

## Teaching Notes:

- Some students find graphing very difficult.
- Some students do not realize they can choose any *x*-value at all, and solve for *y* to find an ordered pair that is a solution of an equation.

*Answers: 1a) solution, b) $3 + 4 = 7$, c) answers vary, d) line; 2a) Yes, b) no; 3-4) See graph answer pages.*

# Functions and Graphs

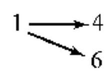
## Learning Objectives:

a Determine whether a correspondence is a function.
b Given a function described by an equation, find function values (outputs) for specified values (inputs).
c Draw the graph of a function.
d Determine whether a graph is that of a function using the vertical-line test.
e Solve applied problems involving functions and their graphs.

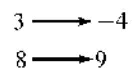
## Examples:

1. Determine whether each correspondence is a function.

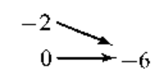
a) Domain Range




b) Domain Range




c) Domain Range




2. Given the following functions, find the indicated values.

a) $f(x)=4-2x$; find $f(-1), f(4), f\left(\frac{1}{2}\right)$

b) $f(x)=\frac{2}{5}x+2$; find $f(-5), f(2), f\left(\frac{1}{2}\right)$

c) $f(x)=-x^2+2x-3$; find $f(-1), f(2)$

d) $f(x)=x^3-4$; find $f(-1), f(10), f(-3)$

3. Graph each function.

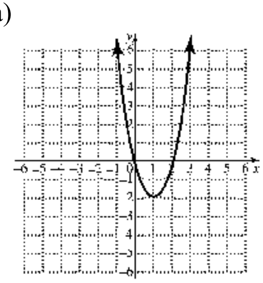
a) $f(x)=-2x+3$

b) $g(x)=-x^2+x+2$

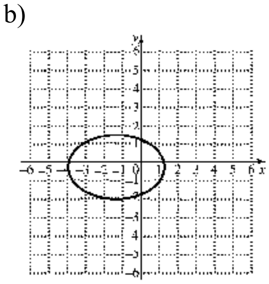
c) $f(x)=|x-2|$

4. Determine whether each of the following is the graph of a function.

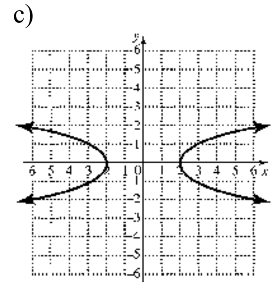
a)



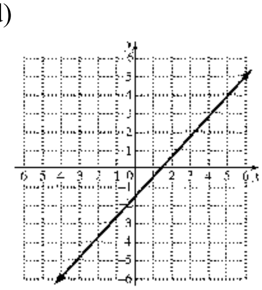
b)



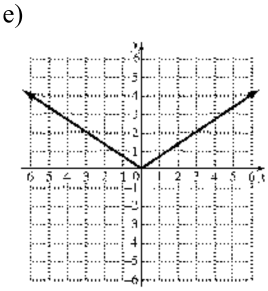
c)



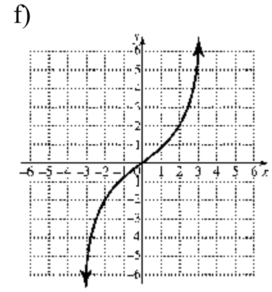
d)



e)



f)



5. Using the graph shown in Example 4e, find the output when the input is 3.

## Teaching Notes:

- Some students find it helpful to calculate ordered pairs for a linear equation and discuss why a linear equation is a function.
- Many students find function notation confusing at first.

*<u>Answers</u>: 1a) No, b) yes, c) yes; 2a)* $6,-4,3$ *, b)* $0,\frac{14}{5},\frac{11}{5}$ *, c)* $-6,-3$ *, d)* $-5,996,-31$ *; 3) See graph answer pages.; 4a) Function, b) not a function, c) not a function, d) function, e) function, f) function; 5) 2*

# Finding Domain and Range

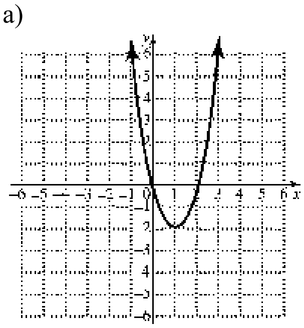
**Learning Objective:**

a Find the domain and the range of a function.

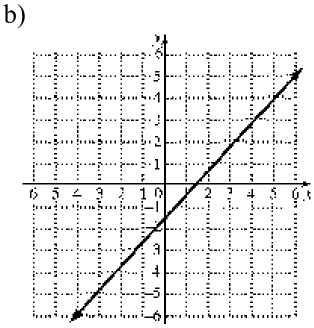
**Examples:**

1. The following graphs are functions. For each graph given, (i) estimate $f(1)$, (ii) determine the domain, (iii) estimate all $x$-values such that $f(x)=2$, and (iv) determine the range.

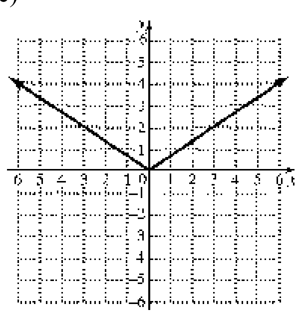
a)



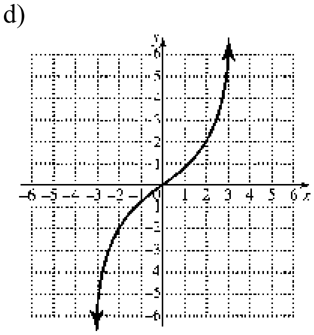
b)



c)



d)



2. Find the domain.

a) $f(x)=\dfrac{3}{x-4}$

b) $f(x)=-5x+3$

c) $f(x)=|x-5|$

d) $f(x)=\dfrac{x^3-1}{x}$

**Teaching Notes:**

- Students often mix up domain and range.
- Students sometimes think a numerator cannot be zero.

*Answers:* *1ai)* $-2$*, ii) all real numbers, iii)* $-0.4, 2.4$*, iv)* $[-2,\infty)$*, bi)* $-0.5$*, ii) all real numbers, iii) 3.3, iv) all real numbers, ci) 0.7, ii) all real numbers, iii)* $-3, 3$*, iv)* $[0,\infty)$*, di) 0.8, ii) all real numbers, iii) 2, iv) all real numbers; 2a)* $\{x \mid x \text{ is a real number and } x \neq 4\}$*, or* $(-\infty,4)\cup(4,\infty)$*, b) all real numbers, c) all real numbers, d)* $\{x \mid x \text{ is a real number and } x \neq 0\}$*, or* $(-\infty,0)\cup(0,\infty)$

# Linear Functions: Graphs and Slope

## Learning Objectives:

a Find the $y$-intercept of a line from the equation $y = mx + b$ or $f(x) = mx + b$.
b Given two points on a line, find the slope. Given a linear equation, derive the equivalent slope-intercept equation and determine the slope and the $y$-intercept.
c Solve applied problems involving slope.

## Examples:

1. Find the slope and the $y$-intercept.
   a) $y = 3x - 5$ b) $f(x) = \frac{1}{2}x$ c) $4x - 5y = 20$ d) $6x + 4 = 3y + 6x$

2. Find the slope, if possible, of the line containing the given pair of points.
   a) $(3,4)$ and $(5,8)$ b) $(-2,4)$ and $(-4,7)$ c) $(-1.5,4.5)$ and $(4.5,0)$

   d) $\left(2,\frac{3}{5}\right)$ and $\left(-7,\frac{3}{5}\right)$ e) $\left(\frac{5}{3},-2\right)$ and $\left(\frac{5}{3},-\frac{1}{6}\right)$

3. Solve.
   a) A river falls 48 ft vertically over a horizontal distance of 400 ft. Find its slope.

   b) A company's revenue is \$350,000. Two years earlier it was \$320,000. Find the rate of change.

## Teaching Notes:

- Some students need to see many numeric examples of $m = \frac{rise}{run}$ shown on a graph before trying to use the slope formula.
- Many students make sign errors with the slope formula.
- Some students consistently put the change in $x$ instead of the change in $y$ in the numerator when calculating slope.

*Answers*: *1a) $m = 3$, y-intercept $(0,-5)$, b) $m = \frac{1}{2}$, y-intercept $(0,0)$, c) $m = \frac{4}{5}$, y-intercept $(0,-4)$, d) $m = 0$, y-intercept $\left(0,\frac{4}{3}\right)$; 2a) 2, b) $-\frac{3}{2}$, c) $-0.75$, d) 0, e) undefined; 3a) $-\frac{3}{25}$, or $-0.12$, b) \$15,000 per year*

# More on Graphing Linear Equations

**Learning Objectives:**

a Graph linear equations using intercepts.
b Given a linear equation in slope-intercept form, use the slope and the $y$-intercept to graph the line.
c Graph linear equations of the form $x = a$ or $y = b$.
d Given the equations of two lines, determine whether their graphs are parallel or whether they are perpendicular.

**Examples:**

1. Find the $x$-intercept and the $y$-intercept line. Then graph the line.
   a) $2x + 4y = 8$ b) $-3x + 5y = 6$ c) $2x + 6y + 3 = 3$

2. Graph the line, using the slope and the $y$-intercept.
   a) $y = 2x - 2$ b) $g(x) = -\frac{2}{3}x + 3$

3. Graph and, if possible, determine the slope.
   a) $x = 3$ b) $2x - 10 = 0$ c) $6 - 3y = 0$

4. Given the equations of two lines, determine whether they are *parallel, perpendicular,* or *neither.*
   a) $2x - y = 6$ and $x + 2y = -5$ b) $x + 2y = 5$ and $x + 2y = 9$
   c) $4x - 3y = 11$ and $6x - y = 13$

**Teaching Note:**

- Avoid using the terminology "no slope." Instead, refer to zero slope and undefined slope.

*Answers:* *1a) x-intercept: $(4,0)$, y-intercept: $(0,2)$, b) x-intercept: $(-2,0)$, y-intercept: $\left(0,\frac{6}{5}\right)$, c) x-intercept: $(0,0)$, y-intercept: $(0,0)$, See graph answer pages.; 2) See graph answer pages.; 3a) Not defined, b) not defined, c) 0, See graph answer pages.; 4a) Perpendicular, b) parallel, c) neither*

# Finding Equations of Lines; Applications

**Learning Objectives:**

a Find an equation of a line when the slope and the $y$-intercept are given.
b Find an equation of a line when the slope and a point are given.
c Find an equation of a line when two points are given.
d Given a line and a point not on the given line, find an equation of the line parallel to the line and containing the point, and find an equation of the line perpendicular to the line and containing the point.
e Solve applied problems involving linear functions.

**Examples:**

1. Find an equation of the line having the given slope and $y$-intercept.

   a) slope = 2, $(0,-5)$ b) slope = $-\frac{3}{4}$, $(0,2)$

2. Find an equation of the line having the given slope and containing the given point.

   a) slope = $\frac{3}{2}$, $(-5,-4)$ b) slope = $-\frac{1}{2}$, $(2,-3)$

3. Find an equation of the line containing the given pair of points.

   a) $(3,4)$ and $(5,0)$ b) $(2,3)$ and $(-1,5)$ c) $\left(\frac{1}{2},-4\right)$ and $\left(\frac{7}{2},-6\right)$

4. Find an equation of the line with the given characteristics.
   a) parallel to $3x-y=4$, through $(0,-3)$ b) perpendicular to $2y=-5x$, and through $(2,4)$

5. A lawn aerator rents for \$25, plus \$3 per hour. Let $x$ represent the number of hours the aerator was rented and $y$ the total rental cost. Write an equation in the form $y=mx+b$ to represent this situation. How many hours was the aerator rented if the total cost was \$38.50?

**Teaching Note:**

- Many students have trouble digesting all of the information in this section and need to spend extra time working on it.

*Answers:* 1a) $y=2x-5$, b) $y=-\frac{3}{4}x+2$; 2a) $y=\frac{3}{2}x+\frac{7}{2}$, b) $y=-\frac{1}{2}x-2$; 3a) $y=-2x+10$, b) $y=-\frac{2}{3}x+\frac{13}{3}$, c) $y=-\frac{2}{3}x-\frac{11}{3}$; 4a) $y=3x-3$, b) $y=\frac{2}{5}x+\frac{16}{5}$; 5) $y=3x+25$, $4\frac{1}{2}$ hours

# Systems of Equations in Two Variables

**Learning Objective:**

a Solve a system of two linear equations or two functions by graphing and determine whether a system is consistent or inconsistent and whether the equations in a system are dependent or independent.

**Examples:**

1. Perform a check to see whether the given ordered pair is a solution to the system of equations.

a) $\begin{aligned} x+y&=4 \\ x-y&=2 \end{aligned}$; $(3,1)$

b) $\begin{aligned} x+y&=4 \\ x-y&=2 \end{aligned}$; $(2,2)$

c) $\begin{aligned} 2x-3(y-6)&=8 \\ 6y-2&=x+6 \end{aligned}$; $\left(-4,\dfrac{2}{3}\right)$

2. Solve each system of equations graphically. Then classify the system as consistent or inconsistent and the equations as dependent or independent.

a) $\begin{aligned} x+y&=4 \\ x-y&=2 \end{aligned}$

b) $\begin{aligned} 6x+2y&=10 \\ 2x-y&=5 \end{aligned}$

c) $\begin{aligned} y&=-x+3 \\ 2x+2y&=-1 \end{aligned}$

d) $\begin{aligned} x+y&=4 \\ 2x+2y&=8 \end{aligned}$

**Teaching Notes:**

- Some students mistakenly think that an infinite number of solutions means all ordered pairs are solutions.
- Students often struggle with the terms consistent, inconsistent, dependent, and independent.

*Answers:* *1a) Yes, b) no, c) yes; 2a) $(3,1)$, consistent, independent, b) $(2,-1)$, consistent, independent, c) no solution, inconsistent, independent, d) infinitely many solutions, consistent, dependent, See graph answer pages.*

# Solving by Substitution

## Learning Objectives:

a Solve systems of equations in two variables by the substitution method.
b Solve applied problems by solving systems of two equations using substitution.

## Examples:

1. Solve the system of equations by the substitution method. Check your answers.

a) $\begin{aligned} x+y&=4 \\ x-y&=2 \end{aligned}$

b) $\begin{aligned} 3x+y&=5 \\ y&=2x-5 \end{aligned}$

c) $\begin{aligned} x-\frac{1}{2}y&=2 \\ x-3y&=3 \end{aligned}$

d) $\begin{aligned} y&=-3x+4 \\ 6x+2y-12&=0 \end{aligned}$

2. Two complementary angles are such that one angle is $4^\circ$ more than the other. Find the measure of each angle.

## Teaching Notes:

- Some students have trouble with the substitution method when fractions are involved.
- Many students have trouble drawing a conclusion of "no solution" or "infinite number of solutions" from the results of the algebraic methods.

*Answers*: *1a)* $(3,1)$, *b)* $(2,-1)$, *c)* $\left(\frac{9}{5},-\frac{2}{5}\right)$, *d) no solution; 2)* $43^\circ$ *and* $47^\circ$