

Chapter 2 – Problem Solutions

2.2.1

$$P = \gamma \cdot h; \text{ where } \gamma = (1.03)(9810 \text{ N/m}^3) = 1.01 \times 10^4 \text{ N/m}^3$$

(using the specific weight of water at standard conditions since water gets very cold at great depths)

$$P = \gamma \cdot h = (1.01 \times 10^4 \text{ N/m}^3)(730 \text{ m})$$

$$\mathbf{P = 7.37 \times 10^6 \text{ N/m}^2 = 1,070 \text{ psi}}$$

The pressure given is gage pressure. To get absolute pressure, atmospheric pressure must be added.

2.2.2

a) The force exerted on the tank bottom is equal to the weight of the water body.

$$F = W = m \cdot g = [\rho \cdot (\text{Vol})] (g)$$

$$F = [1.94 \text{ slugs/ft}^3 (\pi \cdot (5 \text{ ft})^2 \cdot 3 \text{ ft})] (32.2 \text{ ft/sec}^2)$$

$$\mathbf{F = 1.47 \times 10^4 \text{ lbs}}$$

(Note: 1 slug = 1 lb·sec²/ft)

b) The force exerted on the tank bottom is equal to the pressure on the bottom times the area of the bottom.

$$P = \gamma \cdot h = (62.3 \text{ lb/ft}^3)(3 \text{ ft}) = 187 \text{ lb/ft}^2$$

$$F = P \cdot A = (187 \text{ lb/ft}^2)(\pi \cdot (5 \text{ ft})^2)$$

$$\mathbf{F = 1.47 \times 10^4 \text{ lbs}}$$

2.2.3

$$\gamma_{\text{water}} \text{ at } 30^\circ\text{C} = 9.77 \text{ kN/m}^3$$

$$P_{\text{vapor}} \text{ at } 30^\circ\text{C} = 4.24 \text{ kN/m}^2$$

$$P_{\text{atm}} = P_{\text{column}} + P_{\text{vapor}}$$

$$P_{\text{atm}} = (9.8 \text{ m})(9.77 \text{ kN/m}^3) + (4.24 \text{ kN/m}^2)$$

$$\mathbf{P_{\text{atm}} = 95.7 \text{ kN/m}^2 + 4.24 \text{ kN/m}^2 = 99.9 \text{ kN/m}^2}$$

2.2.3 (cont.)

The percentage error if the direct reading is used and the vapor pressure is ignored is:

$$\text{Error} = (P_{\text{atm}} - P_{\text{column}})/(P_{\text{atm}})$$

$$\text{Error} = (99.9 \text{ kN/m}^2 - 95.7 \text{ kN/m}^2)/(99.9 \text{ kN/m}^2)$$

$$\mathbf{\text{Error} = 0.0420 = 4.20\%}$$

2.2.4

The atm. pressure found in problem 2.2.3 is 99.9 kN/m²

$$P_{\text{atm}} = (\gamma_{\text{Hg}})(h)$$

$$h = P_{\text{atm}}/\gamma_{\text{Hg}} = (99.9 \text{ kN/m}^2) / [(13.6)(9.77 \text{ kN/m}^3)]$$

$$\mathbf{h = 0.752 \text{ m} = 75.2 \text{ cm} = 2.47 \text{ ft}}$$

2.2.5

The force exerted on the tank bottom is equal to the pressure on the bottom times the area of the bottom.

$$P = \gamma \cdot h = (9.79 \text{ kN/m}^3)(6 \text{ m}) = 58.7 \text{ kN/m}^2$$

$$F = P \cdot A = (58.7 \text{ kN/m}^2)(36 \text{ m}^2)$$

$$\mathbf{F = 2,110 \text{ N}}$$

The force exerted on the sides of the tank may be found in like manner (pressure times the area). However, the pressure is not uniform on the tank sides since $P = \gamma \cdot h$. Therefore, the average pressure is required. Since the pressure is a linear relationship, the average pressure occurs at half the depth. Now,

$$P_{\text{avg}} = \gamma \cdot h_{\text{avg}} = (9.79 \text{ kN/m}^3)(3 \text{ m}) = 29.4 \text{ kN/m}^2$$

$$F = P_{\text{avg}} \cdot A = (29.4 \text{ kN/m}^2)(36 \text{ m}^2)$$

$$\mathbf{F = 1,060 \text{ N}}$$

Obviously, the force on the bottom is greater than the force on the sides by a factor of two.

2.2.6



$$W_{\text{total}} = (\text{Vol}_{\text{container}})(\gamma_{\text{water}}) + (\text{Vol}_{\text{pipe}})(\gamma_{\text{water}})$$

$$W_{\text{total}} = (3 \text{ ft})^3(62.3 \text{ lb/ft}^3) + [(\pi)(0.50 \text{ ft})^2(30 \text{ ft})](62.3 \text{ lb/ft}^3)$$

$$W_{\text{total}} = 1680 \text{ lb} + 1470 \text{ lb} = \mathbf{3,150 \text{ lb}}$$

$$P_{\text{bottom}} = \gamma h = (62.3 \text{ lb/ft}^3)(33 \text{ ft}) = 2060 \text{ lb/ft}^2$$

$$F_{\text{bottom}} = (2060 \text{ lb/ft}^2)(9 \text{ ft}^2) = \mathbf{18,500 \text{ lb}}$$

Note: The weight of the water is not equal to the force on the bottom. Why? (Hint: Draw a free body diagram of the 3 ft x 3 ft x 3 ft water body labeling all forces (vertical) acting on it. Don't forget the pressure from the container top. Now, to determine the side force:

$$P_{\text{avg}} = \gamma \cdot h_{\text{avg}} = (62.3 \text{ lb/ft}^3)(31.5 \text{ ft}) = 1960 \text{ lb/ft}^2$$

$$F = P_{\text{avg}} \cdot A = (1960 \text{ lb/ft}^2)(9 \text{ ft}^2)$$

$$\mathbf{F = 17,600 \text{ lb}}$$

2.2.7

$$P_{\text{bottom}} = P_{\text{gage}} + (\gamma_{\text{liquid}})(1.4 \text{ m}); \text{ and}$$

$$\gamma_{\text{liquid}} = (\text{SG})(\gamma_{\text{water}}) = (0.80)(9790 \text{ N/m}^3) = 7830 \text{ N/m}^3$$

$$\therefore P_{\text{bottom}} = 4.50 \times 10^4 \text{ N/m}^2 + (7830 \text{ N/m}^3)(1.4 \text{ m})$$

$$\mathbf{P_{\text{bottom}} = 5.60 \times 10^4 \text{ N/m}^2}$$

The pressure at the bottom of the liquid column can be determined two different ways which must be equal. Hence,

$$(h)(\gamma_{\text{liquid}}) = P_{\text{gage}} + (\gamma_{\text{liquid}})(1 \text{ m})$$

$$h = (P_{\text{gage}})/(\gamma_{\text{liquid}}) + 1 \text{ m}$$

$$\mathbf{h = (4.50 \times 10^4 \text{ N/m}^2)/7830 \text{ N/m}^3 + 1 \text{ m} = 6.75 \text{ m}}$$

2.2.8

$$\gamma_{\text{seawater}} = (\text{SG})(\gamma_{\text{water}}) = (1.03)(9790 \text{ N/m}^3)$$

$$\gamma_{\text{seawater}} = 1.01 \times 10^4 \text{ N/m}^3$$

$$P_{\text{tank}} = (\gamma_{\text{water}})(\Delta h) = (1.01 \times 10^4 \text{ N/m}^3)(6 \text{ m})$$

$$\mathbf{P_{\text{tank}} = 6.06 \times 10^4 \text{ N/m}^2 \text{ (Pascals)} = 8.79 \text{ psi}}$$

2.2.9

$$\gamma_{\text{oil}} = (\text{SG})(\gamma_{\text{water}}) = (0.85)(62.3 \text{ lb/ft}^3) = 53.0 \text{ lb/ft}^3$$

$$P_{10\text{ft}} = P_{\text{air}} + (\gamma_{\text{oil}})(10 \text{ ft})$$

$$P_{\text{air}} = P_{10\text{ft}} - (\gamma_{\text{oil}})(10 \text{ ft})$$

$$P_{\text{air}} = 23.7 \text{ psi} (144 \text{ in}^2/\text{ft}^2) - (53.0 \text{ lb/ft}^3)(10 \text{ ft})$$

$$\mathbf{P_{\text{air}} = 2.88 \times 10^4 \text{ lb/ft}^2 \text{ (20.0 psi); Gage pressure}}$$

$$P_{\text{abs}} = P_{\text{gage}} + P_{\text{atm}} = 20.0 \text{ psi} + 14.7 \text{ psi}$$

$$\mathbf{P_{\text{abs}} = 34.7 \text{ psi} (5.00 \times 10^4 \text{ lb/ft}^2); \text{ Absolute pressure}}$$

2.2.10

The mechanical advantage in the lever increases the input force delivered to the hydraulic jack. Thus,

$$F_{\text{input}} = (9)(50 \text{ N}) = 450 \text{ N}$$

The pressure developed in the system is:

$$P_{\text{input}} = F/A = (450 \text{ N})/(25 \text{ cm}^2) = 18 \text{ N/cm}^2$$

$$\mathbf{P_{\text{input}} = 180 \text{ kN/m}^2}$$

From Pascal's law, the pressure at the input piston should equal the pressure at the two output pistons.

\therefore The force exerted on each output piston is:

$$P_{\text{input}} = P_{\text{output}} \text{ equates to: } 18 \text{ N/cm}^2 = F_{\text{output}}/250 \text{ cm}^2$$

$$F_{\text{output}} = (18 \text{ N/cm}^2)(250 \text{ cm}^2)$$

$$\mathbf{F_{\text{output}} = 4.50 \text{ kN}}$$

2.4.1

Since the line passing through points 7 and 8 represents an equal pressure surface;

$$P_7 = P_8 \quad \text{or} \quad (h_{\text{water}})(\gamma_{\text{water}}) = (h_{\text{oil}})(\gamma_{\text{oil}})$$

However; $(h_{\text{oil}})(\gamma_{\text{oil}}) = (h_{\text{oil}})(\gamma_{\text{water}})(SG_{\text{oil}})$, thus

$$h_{\text{oil}} = (h_{\text{water}})/(SG_{\text{oil}}) = (52.3 \text{ cm})/(0.85) = \mathbf{61.5 \text{ cm}}$$

2.4.2

A surface of equal pressure surface can be drawn at the mercury-water meniscus. Therefore,

$$(3 \text{ ft})(\gamma_{\text{water}}) = (h)(\gamma_{\text{Hg}})$$

$$h = (3 \text{ ft})(\gamma_{\text{water}}/\gamma_{\text{Hg}}) = (3 \text{ ft}) / (SG_{\text{Hg}}) = (3 \text{ ft}) / (13.6)$$

$$h = \mathbf{0.221 \text{ ft} = 2.65 \text{ in.}}$$

2.4.3

The pressure at the bottom registered by the gage is equal to the pressure due to the liquid heights. Thus,

$$(h_{\text{Hg}})(SG_{\text{Hg}})(\gamma_{\text{water}}) = (4h)(\gamma_{\text{water}}) + (h)(SG_{\text{oil}})(\gamma_{\text{water}})$$

$$h = (h_{\text{Hg}})(SG_{\text{Hg}})/(4 + SG_{\text{oil}})$$

$$h = (26.3 \text{ cm})(13.6)/(4 + 0.82) = \mathbf{74.2 \text{ cm}}$$

2.4.4

A surface of equal pressure can be drawn at the mercury-water meniscus. Therefore,

$$P_A + (y)(\gamma_{\text{water}}) = (h)(\gamma_{\text{Hg}})$$

$$P_A + (0.034 \text{ m})(\gamma_{\text{water}}) = (0.026 \text{ m})(\gamma_{\text{Hg}})$$

$$P_A = (0.026 \text{ m})(13.6)(9,790 \text{ N/m}^3) - (0.034 \text{ m})(9,790 \text{ N/m}^3)$$

$$P_A = \mathbf{3,130 \text{ N/m}^2 \text{ (Pascals)} = 3.13 \text{ kN/m}^2}$$

2.4.5

A surface of equal pressure can be drawn at the mercury-water meniscus. Therefore,

$$P_{\text{pipe}} + (2 \text{ ft})(\gamma_{\text{water}}) = (h)(\gamma_{\text{Hg}})$$

$$(16.8 \text{ lb/in}^2)(144 \text{ in}^2/\text{ft}^2) + (2 \text{ ft})(\gamma_{\text{water}}) = (h)(\gamma_{\text{Hg}})$$

$$(2.42 \times 10^3 \text{ lb/ft}^2) + (2 \text{ ft})(62.3 \text{ lb/ft}^3) = (h)(13.6)(62.3 \text{ lb/ft}^3)$$

$$h = \mathbf{3.00 \text{ ft (manometer is correct)}}$$

2.4.6

Using the “swim through” technique, start at the end of the manometer which is open to the atmosphere and thus equal to zero gage pressure. Then “swim through” the manometer, adding pressure when “swimming” down and subtracting when “swimming” up until you reach the pipe. The computations are below:

$$0 - (0.66 \text{ m})(\gamma_{\text{CT}}) + [(0.66 + y + 0.58)\text{m}](\gamma_{\text{air}}) - (0.58 \text{ m})(\gamma_{\text{oil}}) = P_{\text{pipe}}$$

The specific weight of air is negligible when compared to fluids, so that term in the equation can be dropped.

$$P_{\text{pipe}} = 0 - (0.66 \text{ m})(SG_{\text{CT}})(\gamma) - (0.58 \text{ m})(SG_{\text{oil}})(\gamma)$$

$$P_{\text{pipe}} = 0 - (0.66 \text{ m})(1.60)(9790 \text{ N/m}^3) - (0.58 \text{ m})(0.82)(9790 \text{ N/m}^3)$$

$$P_{\text{pipe}} = -15.0 \text{ kN/m}^2 \quad \text{Pressure can be converted to}$$

height (head) of any liquid through $P = \gamma \cdot h$. Thus,

$$h_{\text{pipe}} = (-15,000 \text{ N/m}^2)/(9790 \text{ N/m}^3) = \mathbf{-1.53 \text{ m of water}}$$

2.4.7

A surface of equal pressure surface can be drawn at the mercury-water meniscus. Therefore,

$$P + (h_1)(\gamma) = (h_2)(\gamma_{\text{Hg}}) = (h_2)(SG_{\text{Hg}})(\gamma)$$

$$P + (0.575 \text{ ft})(62.3 \text{ lb/ft}^3) = (2.00 \text{ ft})(13.6)(62.3 \text{ lb/ft}^3)$$

$$P = \mathbf{1,660 \text{ lb/ft}^2 = 11.5 \text{ psi}}$$

2.4.8

A surface of equal pressure surface can be drawn at the mercury-water meniscus. Therefore,

$$P_{\text{pipe}} + (h_1)(\gamma) = (h_2)(\gamma_{\text{Hg}}) = (h_2)(SG_{\text{Hg}})(\gamma)$$

$$P_{\text{pipe}} + (0.20 \text{ m})(9790 \text{ N/m}^3) = (0.67 \text{ m})(13.6)(9790 \text{ N/m}^3)$$

$$P_{\text{pipe}} = 8.72 \times 10^4 \text{ N/m}^2 \text{ (Pascals)} = 87.2 \text{ KPa}$$

When the manometer reading rises or falls, mass balance must be preserve in the system. Therefore,

$$\text{Vol}_{\text{res}} = \text{Vol}_{\text{tube}} \quad \text{or} \quad A_{\text{res}} \cdot h_1 = A_{\text{tube}} \cdot h_2$$

$$h_1 = h_2 (A_{\text{tube}}/A_{\text{res}}) = h_2 [(D_{\text{tube}})^2/(D_{\text{res}})^2]$$

$$h_1 = (10 \text{ cm})[(0.5 \text{ cm})^2/(5 \text{ cm})^2] = 0.1 \text{ cm}$$

2.4.9

Using the “swim through” technique, start at pipe *A* and “swim through” the manometer, adding pressure when “swimming” down and subtracting when “swimming” up until you reach pipe *B*. The computations are:

$$P_A + (5.33 \text{ ft})(\gamma) - (1.67 \text{ ft})(\gamma_{\text{Hg}}) - (1.0 \text{ ft})(\gamma_{\text{oil}}) = P_B$$

$$P_A - P_B = (62.3 \text{ lb/ft}^3) [(1.0 \text{ ft})(0.82) - (5.33 \text{ ft}) + (1.67 \text{ ft})(13.6)]$$

$$P_A - P_B = 1,130 \text{ lb/ft}^2 = 7.85 \text{ psi}$$

2.4.10

Using the “swim through” technique, start at P_2 and “swim through” the manometer, adding pressure when “swimming” down and subtracting when “swimming” up until you reach P_1 . The computations are:

$$P_2 + (\Delta h)(\rho_1 \cdot g) + (y)(\rho_1 \cdot g) + (h)(\rho_2 \cdot g) - (h)(\rho_1 \cdot g) - (y)(\rho_1 \cdot g) = P_1$$

where *y* is the vertical elevation difference between the fluid surface in the left hand reservoir and the interface between the two fluids on the right side of the U-tube.

$$P_1 - P_2 = (\Delta h)(\rho_1 \cdot g) + (h)(\rho_2 \cdot g) - (h)(\rho_1 \cdot g)$$

2.4.10 – cont.

When the manometer reading (*h*) rises or falls, mass balance must be preserve in the system. Therefore,

$$\text{Vol}_{\text{res}} = \text{Vol}_{\text{tube}} \quad \text{or} \quad A_{\text{res}} \cdot (\Delta h) = A_{\text{tube}} \cdot h$$

$$\Delta h = h (A_{\text{tube}}/A_{\text{res}}) = h [(d_2)^2/(d_1)^2]; \text{ substituting yields}$$

$$P_1 - P_2 = h [(d_2)^2/(d_1)^2] (\rho_1 \cdot g) + (h)(\rho_2 \cdot g) - (h)(\rho_1 \cdot g)$$

$$P_1 - P_2 = h \cdot g [\rho_2 - \rho_1 + \rho_1 \{(d_2)^2/(d_1)^2\}]$$

$$P_1 - P_2 = h \cdot g [\rho_2 - \rho_1 \{1 - (d_2)^2/(d_1)^2\}]$$

2.4.11

Using the “swim through” technique, start at both ends of the manometers which are open to the atmosphere and thus equal to zero gage pressure. Then “swim through” the manometer, adding pressure when “swimming” down and subtracting when “swimming” up until you reach the pipes in order to determine P_A and P_B . The computations are below:

$$0 + (23)(13.6)(\gamma) - (44)(\gamma) = P_A; \quad P_A = 269 \cdot \gamma$$

$$0 + (46)(0.8)(\gamma) + (20)(13.6)(\gamma) - (40)(\gamma) = P_B$$

$$P_B = 269 \cdot \gamma; \quad \text{Therefore, } P_A = P_B \text{ and } h = 0$$

2.4.12

Using the “swim through” technique, start at the sealed right tank where the pressure is known. Then “swim through” the tanks and pipes, adding pressure when “swimming” down and subtracting when “swimming” up until you reach the left tank where the pressure is not known. The computations are as follows:

$$20 \text{ kN/m}^2 + (4.5 \text{ m})(9.79 \text{ kN/m}^3) - (2.5 \text{ m})(1.6)(9.79 \text{ kN/m}^3) - (5 \text{ m})(0.8)(9.79 \text{ kN/m}^3) = P_{\text{left}}$$

$$P_{\text{left}} = -14.3 \text{ kN/m}^2 \text{ (or } -14.3 \text{ kPa)}$$

$$P_B = (-14.3 \text{ kN/m}^2) / [(SG_{\text{Hg}})(\gamma)]$$

$$P_B = (-14.3 \text{ kN/m}^2) / [(13.6)(9.79 \text{ kN/m}^3)]$$

$$P_B = -0.107 \text{ m} = 10.7 \text{ cm (Hg)}$$

2.5.1

$$F = \gamma \cdot \bar{h} \cdot A = (9790 \text{ N/m}^3)[(3\text{m})/(3)] \cdot [6 \text{ m}^2]$$

$$\mathbf{F = 5.87 \times 10^4 \text{ N = 58.7 kN}}$$

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(4\text{m})(3\text{m})^3/36]}{[(4\text{m})(3\text{m})/2](1.00\text{m})} + 1.00\text{m}$$

$$y_p = \mathbf{1.50 \text{ m}}$$
 (depth to center of pressure)

2.5.2

$$F = \gamma \cdot \bar{h} \cdot A = (62.3 \text{ lb/ft}^3)[(30 \text{ ft})/2] \cdot [(30 \text{ ft})(1 \text{ ft})]$$

$$\mathbf{F = 2.80 \times 10^4 \text{ lbs per foot of length}}$$

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(1\text{ft})(30\text{ft})^3/12]}{[(30\text{ft})(1\text{ft})](15\text{ft})} + 15\text{ft}$$

$$y_p = \mathbf{20.0 \text{ ft}}$$
 (depth to the center of pressure)

In summing moments about the toe of the dam ($\sum M_A$), the weight acts to stabilize the dam (called a righting moment) and the hydrostatic force tends to tip it over (overturning moment).

$$M = (Wt.)[(2/3)(10)] - F(10\text{ft}) =$$

$$[1/2 (10 \text{ ft})(30 \text{ ft}) \cdot (1 \text{ ft})](2.67)(62.3 \text{ lb/ft}^3) \cdot (6.67 \text{ ft}) -$$

$$(2.80 \times 10^4 \text{ lbs}) \cdot (10 \text{ ft}) = -1.14 \times 10^5 \text{ ft-lbs}$$

$$\mathbf{M = 1.14 \times 10^5 \text{ ft-lbs (overturning; dam is unsafe)}}$$

2.5.3

$$F = \gamma \cdot \bar{h} \cdot A = (9790 \text{ N/m}^3)(1 \text{ m})(\pi)(0.5 \text{ m})^2$$

$$\mathbf{F = 7.69 \times 10^3 \text{ N = 7.69 kN}}$$
 ; $\bar{y} = \bar{h} / \sin 45^\circ$;

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[\pi(1\text{m})^4/64]}{[\pi(1\text{m})^2/4](1.414\text{m})} + 1.414\text{m}$$

$$y_p = \mathbf{1.46 \text{ m}}$$
 (distance from water surface to the center of pressure along the incline).

2.5.4

$$F_{\text{square}} = \gamma \cdot \bar{h} \cdot A = \gamma(L/2)(L^2) = (\gamma/2) \cdot L^3$$

$$F_{\text{tri}} = \gamma \cdot \bar{h} \cdot A = \gamma(L+H/3)(LH/2) = (\gamma/2)[L^2H + LH^2/3]$$

Setting the two forces equal: $F_{\text{square}} = F_{\text{tri}}$;

$$(\gamma/2) \cdot L^3 = (\gamma/2)[L^2H + LH^2/3]$$

$L^2 - HL - H^2/3 = 0$; divide by H^2 and solve quadratic

$$(L/H)^2 - (L/H) - 1/3 = 0; L/H = 1.26 \text{ or } \mathbf{H/L = 0.791}$$

2.5.5

$$F_{\text{left}} = \gamma \cdot \bar{h} \cdot A = (9790 \text{ N/m}^3)(0.5 \text{ m})[(1.41\text{m})(3\text{m})]$$

$$F_{\text{left}} = 20.7 \text{ kN}$$
 (where A is “wet” surface area)

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(3\text{m})(1.41\text{m})^3/12]}{[(3\text{m})(1.41\text{m})](0.705\text{m})} + 0.705\text{m}$$

$$y_p = 0.940 \text{ m}$$
 (inclined distance to center of pressure)

Location of this force from the hinge (moment arm):

$$Y' = 2 \text{ m} - 1.41 \text{ m} + 0.940 \text{ m} = 1.53 \text{ m}$$

$$F_{\text{right}} = \gamma \cdot \bar{h} \cdot A = (9790 \text{ N/m}^3)(h/2 \text{ m})[(h/\cos 45^\circ)(3\text{m})]$$

$$F_{\text{right}} = 20.8 \cdot h^2 \text{ kN}$$

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(3)(1.41 \cdot h)^3/12]}{[(3)(1.41 \cdot h)](0.705 \cdot h)} + 0.705 \cdot h$$

$$y_p = (0.940 \cdot h)\text{m};$$
 Moment arm of force from hinge:

$$Y'' = 2 \text{ m} - (h/\sin 45^\circ)\text{m} + (0.940 \cdot h)\text{m} = 2\text{m} - (0.474 \cdot h)\text{m}$$

The force due to the gate weight: $W = 20.0 \text{ kN}$
 Moment arm of this force from hinge: $X = 0.707 \text{ m}$

Summing moments about the hinge yields: $\sum M_{\text{hinge}} = 0$

$$(20.8 \cdot h^2)[2\text{m} - (0.474 \cdot h)] - 20.7(1.53) - 20(0.707) = 0$$

$$\mathbf{h = 1.25 \text{ m}}$$
 (gate opens when depth exceeds 1.25 m)

2.5.6

$$F = \gamma \cdot \bar{h} \cdot A = (62.3 \text{ lb/ft}^3)[(7 \text{ ft})] \cdot [\pi(6 \text{ ft})^2/4]$$

$$F = 1.23 \times 10^4 \text{ lbs}$$

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[\pi(6 \text{ ft})^4 / 64]}{[\pi(6 \text{ ft})^2 / 4](7 \text{ ft})} + 7 \text{ ft}$$

$$y_p = 7.32 \text{ ft} \quad (\text{depth to the center of pressure})$$

Thus, summing moments: $\sum M_{\text{hinge}} = 0$

$$P(3 \text{ ft}) - (1.23 \times 10^4 \text{ lbs})(0.32 \text{ ft}) = 0$$

$$P = 1.31 \times 10^3 \text{ lbs}$$

2.5.7

In order for the balance to be maintained at $h = 4$ feet, the center of pressure should be at the pivot point (i.e., the force at the bottom check block is zero). As the water rises above $h = 4$ feet, the center of pressure will rise above the pivot point and open the gate. Below $h = 4$ feet, the center of pressure will be lower than the pivot point and the gate will remain closed. Thus, for a unit width of gate, the center of pressure is

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(1 \text{ ft})(10 \text{ ft})^3 / 12]}{[(1 \text{ ft})(10 \text{ ft})](9 \text{ ft})} + 9 \text{ ft}$$

$$y_p = 9.93 \text{ ft} \quad (\text{vertical distance from water surface to the center of pressure})$$

Thus, the horizontal axis of rotation (0-0') should be 14 ft – 9.93 ft = 4.07 ft above the bottom of the gate.

2.5.8

$$F = \gamma \cdot \bar{h} \cdot A = (9790 \text{ N/m}^3)(2.5 \text{ m})[(\pi)\{(1.5)^2 - (0.5)^2\} \text{ m}^2]$$

$$F = 1.54 \times 10^5 \text{ N} = 154 \text{ kN}$$

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[\pi(3 \text{ m})^4 / 64 - \pi(1 \text{ m})^4 / 64]}{[\pi(3 \text{ m})^2 / 4 - \pi(1 \text{ m})^2 / 4](2.5 \text{ m})} + 2.5 \text{ m}$$

$$y_p = 2.75 \text{ m} \quad (\text{below the water surface})$$

2.5.9

$$F = \gamma \cdot \bar{h} \cdot A = (9790 \text{ N/m}^3)(2.5 \text{ m})[(\pi)(1.5 \text{ m})^2 - (1.0 \text{ m})^2]$$

$$F = 1.49 \times 10^5 \text{ N} = 149 \text{ kN}$$

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[\pi(3 \text{ m})^4 / 64 - (1 \text{ m})(1 \text{ m})^3 / 12]}{[\pi(1.5 \text{ m})^2 - (1 \text{ m})^2](2.5 \text{ m})} + 2.5 \text{ m}$$

$$y_p = 2.76 \text{ m} \quad (\text{below the water surface})$$

2.5.10

$$F = \gamma \cdot \bar{h} \cdot A = (9790 \text{ N/m}^3)(2.5 \text{ m})[(5/\cos 30^\circ)(3 \text{ m})]$$

$$F = 4.24 \times 10^5 \text{ N} = 424 \text{ kN}$$

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(3 \text{ m})(5.77 \text{ m})^3 / 12]}{[(3 \text{ m})(5.77 \text{ m})](2.89 \text{ m})} + 2.89 \text{ m}$$

$$y_p = 3.85 \text{ m} \quad (\text{inclined depth to center of pressure})$$

Summing moments about the base of the dam; $\sum M = 0$

$$(424 \text{ kN})(5.77 \text{ m} - 3.85 \text{ m}) - (F_{AB})(5.77 \text{ m}/2) = 0$$

$$F_{AB} = 282 \text{ kN}$$

2.5.11

$$F = \gamma \cdot \bar{h} \cdot A = (62.3 \text{ lb/ft}^3)[(d/2) \text{ ft}] \cdot [(d/\cos 30^\circ) \text{ ft}](8 \text{ ft})$$

$$F = 288 \cdot d^2 \text{ lbs}$$

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(8)(d/\cos 30^\circ)^3 / 12]}{[(8)(d/\cos 30^\circ)](d/2 \cos 30^\circ)} + (d/2 \cos 30^\circ)$$

$$y_p = [(0.192 \cdot d) + 0.577 \cdot d] \text{ ft} = 0.769 \cdot d \quad (\text{inclined depth})$$

Thus, summing moments: $\sum M_{\text{hinge}} = 0$

$$(288 \cdot d^2)[(d/\cos 30^\circ) - 0.769d] - (5,000)(15) = 0$$

$$d = 8.77 \text{ ft} \quad \text{A depth greater than this will make}$$

the gate open, and anything less will make it close.

2.5.12

The force on the side of the gate can be found as:

$$F = \gamma \cdot \bar{h} \cdot A = (9790 \text{ N/m}^3)[(h/2)m][(h \text{ m})(1 \text{ m})]$$

$$F = (4.90 \times 10^3)h^2 \text{ N (per meter of gate width)}$$

$$Y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(1)(h)^3/12]}{[(1)(h)][(h/2)]} + h/2 = (2/3)h \text{ m}$$

The force on the bottom of the gate can be found as:

$$F = p \cdot A = (9790 \text{ N/m}^3)(h)[(1\text{m})(1\text{m})]$$

$$F = (9.79 \times 10^3)h \text{ N (per meter of gate width)}$$

This force is located 0.50 m from the hinge.
Summing moments about the hinge; $\sum M_h = 0$

$$[(4.90 \times 10^3)h^2][h - (2/3)h] - [(9.79 \times 10^3)h](0.5) = 0$$

$$h = 1.73 \text{ m}$$

2.5.13

The total force from fluids A and B can be found as:

$$F_A = \gamma \cdot \bar{h} \cdot A = (\gamma_A)(h_A) \cdot [\pi(d)^2/4]$$

$$F_B = \gamma \cdot \bar{h} \cdot A = (\gamma_B)(h_B) \cdot [\pi(d)^2/4]$$

For equilibrium, forces must be equal, opposite, and collinear.

$$F_A = F_B; \quad (\gamma_A)(h_A) \cdot [\pi(d)^2/4] = (\gamma_B)(h_B) \cdot [\pi(d)^2/4]$$

$$h_A = [(\gamma_B)/(\gamma_A)](h_B)$$

2.5.14

$$F = \gamma \cdot \bar{h} \cdot A = (62.3 \text{ lb/ft}^3)[(30 \text{ ft})][(10 \text{ ft})(6 \text{ ft})]$$

$$F = 1.12 \times 10^5 \text{ lbs (Horizontal force on gate)}$$

Thus, summing vertical forces: $\sum F_y = 0$

$$T_{\text{up}} - W - F(C_{\text{friction}}) = 0$$

$$T = 3 \text{ tons (2000 lbs/1 ton)} + (1.12 \times 10^5 \text{ lbs})(0.2)$$

$$T = 2.84 \times 10^4 \text{ lbs (lifting force required)}$$

2.6.1

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of the curved gate.

$$F_H = \gamma \cdot \bar{h} \cdot A = (9790 \text{ N/m}^3)(5\text{m})[(10 \text{ m})(2 \text{ m})]$$

$$F_H = 9.79 \times 10^5 \text{ N} = 979 \text{ kN}$$

Now obtain the vertical component of the total hydrostatic pressure force by determining the weight of the water column above the curved gate.

$$F_V = \gamma \cdot Vol = (9790 \text{ N/m}^3)[(4\text{m})(2\text{m}) + \pi/4(2\text{m})^2](10 \text{ m})$$

$$F_V = 1.09 \times 10^6 \text{ N} = 1090 \text{ kN}; \quad \text{The total force is}$$

$$F = [(979 \text{ kN})^2 + (1090 \text{ kN})^2]^{1/2} = 1470 \text{ kN}$$

$$\theta = \tan^{-1}(F_V/F_H) = 48.1^\circ \quad \swarrow$$

Since all hydrostatic pressures pass through point A (i.e., they are all normal to the surface upon which they act), then the resultant must also pass through point A.

2.6.2

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of the viewing port.

$$F_H = \gamma \cdot \bar{h} \cdot A = (1.03 \cdot 9790 \text{ N/m}^3)(4\text{m})[\pi(1\text{m})^2] = 127 \text{ kN}$$

Now obtain the net vertical component of the total hydrostatic pressure force by combining the weight of the water column above the top of the viewing port (which produces a downward force) and the upward force on the bottom of the viewing port (equivalent to the weight of the water above it). The difference in the two columns of water is the weight of water in a hemispherical volume (the viewing port) acting upwards.

$$F_V = \gamma \cdot Vol = (1.03 \cdot 9790 \text{ N/m}^3)[(1/2)(4/3)\pi(1\text{m})^3] = 21.1 \text{ kN}$$

$$F = [(127 \text{ kN})^2 + (21.1 \text{ kN})^2]^{1/2} = 129 \text{ kN}$$

$$\theta = \tan^{-1}(F_V/F_H) = 9.43^\circ \quad \nwarrow$$

The resultant force will pass through the center of the hemisphere since all pressures pass through this point.

2.6.3

The vertical component of the total hydrostatic pressure force is equal to the weight of the water column above it to the free surface. In this case, it is the imaginary or displaced weight of water above the shell since the pressure is from below).

$$F_V = \gamma \cdot Vol = (62.3 \text{ lb/ft}^3)[(1/2)(4/3)(\pi)(3.0 \text{ ft})^3] = 3,520 \text{ lb}$$

The weight must be equal to this; thus **W = 3,520 lb**

2.6.4

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of curved surface AB (both sides).

$$F_H = (\gamma \cdot \bar{h} \cdot A)_{right} - (\gamma \cdot \bar{h} \cdot A)_{left} = (\gamma \cdot A)(\bar{h}_{right} - \bar{h}_{left}) \\ = (9790 \text{ N/m}^3)[(1.75 \text{ m})(1 \text{ m})] (3.875 \text{ m} - 0.875 \text{ m})$$

$$F_H = 5.14 \times 10^4 \text{ N} = \mathbf{51.4 \text{ kN (towards the barge)}}$$

Now obtain the resultant vertical component of the total hydrostatic pressure force subtracting the weight of the water column above the curved surface (the water in the barge) from the displaced weight for the case of the water on the outside of the barge.

$$F_V = (\gamma \cdot Vol)_{displaced} - (\gamma \cdot Vol)_{leaked} \\ = (9790 \text{ N/m}^3)[(1.75 \text{ m})(1 \text{ m})(3 \text{ m})]$$

$$F_V = 5.14 \times 10^4 \text{ N} = \mathbf{51.4 \text{ kN (upwards)}}$$

2.6.5

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of the curved gate. The height of the vertical projection is $(R)(\sin 45^\circ) = 8.49 \text{ m}$. Thus,

$$F_H = \gamma \cdot \bar{h} \cdot A = (9790 \text{ N/m}^3)(4.25 \text{ m})[(10 \text{ m})(8.49 \text{ m})]$$

$$F_H = 3.53 \times 10^6 \text{ N} = 3,530 \text{ kN}$$

Now obtain the vertical component of the total hydrostatic pressure force by determining the weight of the water column above the curved gate.

2.6.5 (continued)

The volume of water above the gate is:

$$Vol = (A_{rectangle} - A_{triangle} - A_{arc})(\text{length}) \\ Vol = [(12 \text{ m})(8.49 \text{ m}) - (1/2)(8.49 \text{ m})(8.49 \text{ m}) - (\pi/8)(12 \text{ m})^2](10 \text{ m}) \\ Vol = 92.9 \text{ m}^3$$

$$F_V = \gamma \cdot Vol = (9790 \text{ N/m}^3)(92.9 \text{ m}^3)$$

$$F_V = 9.09 \times 10^5 \text{ N} = 909 \text{ kN}; \quad \text{The total force is}$$

$$F = [(3,530 \text{ kN})^2 + (909 \text{ kN})^2]^{1/2} = \mathbf{3650 \text{ kN}}$$

$$\theta = \tan^{-1}(F_V/F_H) = \mathbf{14.4^\circ} \quad \rightarrow \nearrow$$

Since all hydrostatic pressures pass through point O (i.e., they are all normal to the surface upon which they act), then the resultant must also pass through point O.

2.6.6

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of the curved gate. Thus,

$$F_H = \gamma \cdot \bar{h} \cdot A = (62.3 \text{ lb/ft}^3)(7.0 \text{ ft})[(8.0 \text{ ft})(1.0 \text{ ft})]$$

$$F_H = 3.49 \times 10^3 \text{ lb} = 3,490 \text{ lb (per unit length of gate)}$$

Obtain the vertical component of the total hydrostatic pressure force by determining the imaginary (displaced) weight of the water column above the curved gate. The volume of displaced water above the gate is:

$$Vol = (A_{rectangle} + A_{arc} - A_{triangle})(\text{length})$$

$$Vol = [(4 \text{ ft})(3 \text{ ft}) + (53.1^\circ/360^\circ)\pi(10 \text{ ft})^2 - (1/2)(8 \text{ ft})(6 \text{ ft})](1 \text{ ft})$$

$$Vol = 34.3 \text{ ft}^3$$

$$F_V = \gamma \cdot Vol = (62.3 \text{ lb/ft}^3)(34.3 \text{ ft}^3)$$

$$F_V = 2.14 \times 10^3 \text{ lb} = 2,140 \text{ lb}; \quad \text{The total force is}$$

$$F = [(3,490 \text{ lb})^2 + (2,140 \text{ lb})^2]^{1/2} = \mathbf{4,090 \text{ lb}}$$

$$\theta = \tan^{-1}(F_V/F_H) = \mathbf{31.5^\circ} \quad \nearrow$$

The resultant force will pass through the center of the gate radius since all pressures pass through this point.

2.6.7

The force on the end of the cylinder is:

$$F = \gamma \cdot \bar{h} \cdot A = (0.9)(62.3 \text{ lb/ft}^3)(10 \text{ ft})[\pi(2 \text{ ft})^2]$$

$$\mathbf{F = 7,050 \text{ lb}}$$

The force on the side of the cylinder is:

$$F_H = \gamma \cdot \bar{h} \cdot A = (0.9)(62.3 \text{ lb/ft}^3)(10.0 \text{ ft})[(10 \text{ ft})(4 \text{ ft})]$$

$$F_H = 22,400 \text{ lb}$$

Based on the same theory as Example 2.6, the vertical force is downward and equal to the weight of the water in half of the tank. Thus,

$$F_V = \gamma \cdot Vol = (0.9)(62.3 \text{ lb/ft}^3)[\pi(2 \text{ ft})^2/2](10 \text{ ft})$$

$F_V = 3,520 \text{ lb}$ (acting downward); The total force is

$$\mathbf{F = [(22,400 \text{ lb})^2 + (3,520 \text{ lb})^2]^{1/2} = 22,700 \text{ lb}}$$

$$\theta = \tan^{-1} (F_V/F_H) = 8.93^\circ \quad \rightarrow$$

The resultant force will pass through the center of the tank since all pressures pass are normal to the tank wall and thus pass through this point.

2.6.8

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of the curved surface ABC.

$$F_H = \gamma \cdot \bar{h} \cdot A = (\gamma)(R)[(2R)(1)] = \mathbf{2(\gamma)(R)^2}$$

Now obtain the vertical component of the total hydrostatic pressure force by determining the weight of the water above the curved surface ABC. The volume of water above the curved surface is:

$$Vol = (A_{\text{quarter circle}} + A_{\text{rectangle}} - A_{\text{quarter circle}})(\text{unit length})$$

$$Vol = (A_{\text{rectangle}})(\text{unit length}) = (2R)(R)(1) = 2(R)^2$$

$$F_V = \gamma \cdot Vol = \gamma[2(R)^2] = \mathbf{2(\gamma)(R)^2}$$

2.6.9

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of the projecting surface. Thus,

$$F_H = \gamma \cdot \bar{h} \cdot A = (62.3 \text{ lb/ft}^3)(8.0 \text{ ft})[(12.0 \text{ ft})(1.0 \text{ ft})]$$

$$\mathbf{F_H = 5,980 \text{ lb (per unit length of surface)}}$$

$$Y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(1 \text{ ft})(12 \text{ ft})^3 / 12]}{[(1 \text{ ft})(12 \text{ ft})](8 \text{ ft})} + 8 \text{ ft} = \mathbf{9.50 \text{ ft}}$$

The vertical component of the total hydrostatic pressure force is equal to the weight of the water displaced by the quadrant and the triangle. In parts (and using Table 2.1 to locate the forces), we have

$$F_{V\text{Triangle}} = \gamma \cdot Vol = (62.3 \text{ lb/ft}^3)[(1/2)(8 \text{ ft})(4 \text{ ft})](1 \text{ ft})$$

$$\mathbf{F_{V\text{Triangle}} = 1000 \text{ lb upwards 1.33 ft from wall}}$$

$$F_{V\text{Quadrant}} = \gamma \cdot Vol = (62.3 \text{ lb/ft}^3)[(\pi/4)(4 \text{ ft})^2](1 \text{ ft})$$

$$\mathbf{F_{V\text{Quadrant}} = 780 \text{ lb upwards 1.70 ft from wall}}$$

2.6.10

There are four vertical forces at work on the cone plug. The cone unplugs when $\sum F_y = 0$.

The pressure force of fluid A on top of the plug (down):

$$F_{A\text{Top}} = \gamma \cdot Vol = (9790 \text{ N/m}^3)[\pi(0.15 \text{ m})^2(0.3 \text{ m})] = 208 \text{ N}$$

The pressure force of fluid A on the cone sides (up):

$$F_{A\text{Sides}} = \gamma \cdot Vol = [\pi(0.15 \text{ m})^2(0.3 \text{ m}) + (\pi/3)(0.15 \text{ m})^2(0.3 \text{ m}) - (\pi/3)(0.05 \text{ m})^2(0.1 \text{ m})](9790 \text{ N/m}^3) = 274 \text{ N}$$

The pressure force of fluid B on the cone bottom (up):

$$F_{B\text{Bottom}} = \gamma \cdot Vol = (0.8)(9790 \text{ N/m}^3)[\pi(0.05 \text{ m})^2(1.5 \text{ m}) + (\pi/3)(0.05 \text{ m})^2(0.1 \text{ m})] = 94.3 \text{ N}$$

$$\sum F_y = 208 - 274 - 94.3 + (\gamma_{\text{cone}})(\pi/3)(0.15 \text{ m})^2(0.3 \text{ m}) = 0;$$

$$\gamma_{\text{cone}} = \mathbf{22,700 \text{ N/m}^3}; \mathbf{S.G. = 2.32}$$

2.6.11

Everything is the same as in problem 2.6.10 except:

The equivalent depth of oil based on the air pressure is:

$$h = P/\gamma = (8,500 \text{ N/m}^2)/[(0.8)(9790 \text{ N/m}^3)] = 1.09 \text{ m}$$

The pressure force of fluid B on the cone bottom (up):

$$F_{B\text{bottom}} = \gamma \cdot Vol = (0.8)(9790 \text{ N/m}^3) [\pi(0.05\text{m})^2(1.09\text{m}) + (\pi/3)(0.05\text{m})^2(0.1\text{m})] = 69.1 \text{ N}$$

$$\Sigma F_y = 208 - 274 - 69.1 + (\gamma_{\text{cone}})(\pi/3)(0.15\text{m})^2(0.3\text{m}) = 0;$$

$$\gamma_{\text{cone}} = \mathbf{19,100 \text{ N/m}^3}; \text{ S.G.} = \mathbf{1.95}$$

2.6.12

The horizontal component of the hydrostatic pressure force due to fluids A and B are found as follows:

$$F_{HAL\text{left}} = \gamma \cdot \bar{h} \cdot A = (0.8)(9790 \text{ N/m}^3)(6 \text{ m})[(1 \text{ m})(1.41 \text{ m})]$$

$$F_{HAL\text{left}} = 66.3 \text{ kN}$$

$$F_{HAR\text{right}} = \gamma \cdot \bar{h} \cdot A = (0.8)(9790 \text{ N/m}^3)(5.65 \text{ m})[(1 \text{ m})(0.707 \text{ m})]$$

$$F_{HAR\text{right}} = 31.3 \text{ kN}$$

$$F_{HBR\text{right}} = \gamma \cdot \bar{h} \cdot A = (1.5)(9790 \text{ N/m}^3)(5.35 \text{ m})[(1 \text{ m})(0.707 \text{ m})]$$

$$F_{HBR\text{right}} = 55.5 \text{ kN}; \text{ Thus, } \mathbf{F_H} = \mathbf{20.5 \text{ kN}} \quad \leftarrow$$

The vertical component of the total hydrostatic pressure force due to fluids A and B are found as follows:

$$F_{VAT\text{top}} = (0.8)(9790 \text{ N/m}^3)[(1.41 \text{ m})(1 \text{ m})(6 \text{ m}) - \pi/2(0.707 \text{ m})^2(1.0 \text{ m})]$$

$$F_{VAT\text{top}} = 60.1 \text{ kN}$$

$$F_{VAB\text{bottom}} = (0.8)(9790 \text{ N/m}^3)[(0.707 \text{ m})(1 \text{ m})(6 \text{ m}) + \pi/4(0.707 \text{ m})^2(1.0 \text{ m})]$$

$$F_{VAB\text{bottom}} = 36.3 \text{ kN}$$

$$F_{VBB\text{bottom}} = (1.5)(9790 \text{ N/m}^3)[(0.707 \text{ m})(1 \text{ m})(5 \text{ m}) + \pi/4(0.707 \text{ m})^2(1.0 \text{ m})]$$

$$F_{VBB\text{bottom}} = 57.7 \text{ kN}$$

$$W_{\text{Cylinder}} = (2.0)(9790 \text{ N/m}^3)[\pi(0.707 \text{ m})^2(1.0 \text{ m})] = 30.7 \text{ kN}$$

$$\text{Thus, } \mathbf{F_V} = \mathbf{3.2 \text{ kN}} \quad \uparrow$$

2.8.1

The buoyant force equals the weight reduction. Thus,

$$B = 301 \text{ N} - 253 \text{ N} = 48.0 \text{ N} \quad \text{In addition,}$$

$$B = \text{wt. of water displaced} = \gamma \cdot Vol = (9790 \text{ N/m}^3)(Vol)$$

$$\text{Thus, } Vol = 4.90 \times 10^{-3} \text{ m}^3 \text{ and}$$

$$\gamma_{\text{metal}} = W/Vol = \mathbf{6.14 \times 10^4 \text{ N/m}^3}$$

$$\text{S.G.} = (6.14 \times 10^4 \text{ N/m}^3)/9.79 \times 10^3 \text{ N/m}^3 = \mathbf{6.27}$$

2.8.2

For floating bodies, weight equals the buoyant force.

$W = B$; and using w & L for width & length of blocks

$$\gamma_A(H)(w)(L) + \gamma_B(1.5 \cdot H)(w)(L) = \gamma(2 \cdot H)(w)(L)$$

$$\gamma_A + (1.5\gamma_A)(1.5) = \gamma(2); \quad \gamma_A(1 + 2.25) = \gamma(2);$$

$$\gamma_A = \mathbf{0.615\gamma}; \text{ and since } \gamma_B = 1.5 \cdot \gamma_A = \mathbf{0.923\gamma}$$

2.8.3

When the sphere is lifted off the bottom, equilibrium in the y -direction occurs with $W = B$. Therefore,

$$W = \gamma_{\text{sphere}} [(4/3)\pi(0.15 \text{ m})^3] + \gamma_{\text{buoy}} [\pi(0.25 \text{ m})^2(2 \text{ m})]$$

$$W = (13.5\gamma)[0.0141 \text{ m}^3] + (0.45\gamma)[0.393 \text{ m}^3] = 0.367 \gamma$$

$$B = \gamma_{\text{sea}} [(4/3)\pi(0.15 \text{ m})^3] + \gamma_{\text{sea}} [\pi(0.25 \text{ m})^2(0.30 \text{ m} + h)]$$

$$B = 0.0145 \gamma + 0.0607 \gamma + 0.202 \cdot h \cdot \gamma$$

$$\text{Equating; } \mathbf{h} = \mathbf{1.45 \text{ m}}$$

2.8.4

Theoretically, the lake level will fall. When the anchor is in the boat, it is displacing a volume of water equal to its weight. When the anchor is thrown in the water, it is only displacing its volume. Since it has a specific gravity greater than 1.0, it will displace more water by weight than by volume.

2.8.5

When the anchor is lifted off the bottom, equilibrium in the y-direction occurs ($\sum F_y = 0$). Therefore,

$$T (\sin 60^\circ) + B = W; \quad \text{where } T = \text{anchor line tension}$$

B = buoyancy force, and W = anchor weight

$$B = (62.3 \text{ lb/ft}^3)\pi(0.75 \text{ ft})^2(1.2 \text{ ft}) = 132 \text{ lb}$$

$$W = (2.7)(62.3 \text{ lb/ft}^3)\pi(0.75 \text{ ft})^2(1.2 \text{ ft}) = 357 \text{ lb}$$

Substituting, **T = 260 lb**

2.8.6

Two forces act on the gate, the hydrostatic pressure and the buoy force. The hydrostatic pressure is

$$F = \gamma \cdot \bar{h} \cdot A = (9790 \text{ N/m}^3)(1.5\text{m})[(1\text{m}/\sin 45^\circ)^2]$$

F = 29.4 kN; acting normal to the gate surface.

The location of the force is

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y}$$

$$y_P = \frac{\left[\frac{(1\text{m}/\sin 45^\circ)(1\text{m}/\sin 45^\circ)^3}{12} \right]}{\left[\frac{(1\text{m}/\sin 45^\circ)(1\text{m}/\sin 45^\circ)}{1.5/\sin 45^\circ} \right]} + (1.5/\sin 45^\circ)$$

$y_P = 2.20 \text{ m}$; This is the distance down the incline

from the water surface.

The distance up the incline from the hinge is

$$y' = (2\text{m}/\sin 45^\circ) - 2.20 \text{ m} = 0.628 \text{ m}$$

The buoyant force (on half the sphere) is

$$B = \gamma \cdot \text{Vol} = (9790 \text{ N/m}^3)(1/2)(4/3)\pi(R)^3 = 20.5(R)^3 \text{ kN}$$

$\sum M_{\text{hinge}} = 0$, ignoring the weights (gate and buoy)

$$(29.4 \text{ kN})(0.628 \text{ m}) - [20.5(R)^3 \text{ kN}](1 \text{ m}) = 0$$

R = 0.966 m

2.8.7

Three forces act on the rod; the weight, buoyant force, and the hinge force. The buoyant force is

$$B = (62.3 \text{ lb/ft}^3)(0.5 \text{ ft})(0.5 \text{ ft})(7 \text{ ft}/\sin \theta) = 109 \text{ lb}/\sin \theta;$$

$B = 109 \text{ lb}/\sin \theta$; \uparrow The buoyant force acts at the center of the submerged portion. $W = 150 \text{ lb}$

$\sum M_{\text{hinge}} = 0$, and assuming the rod is homogeneous,

$$(109 \text{ lb}/\sin \theta)(3.5 \text{ ft}/\tan \theta) - (150 \text{ lb})[(6 \text{ ft})(\cos \theta)] = 0$$

Noting that $\tan \theta = (\sin \theta/\cos \theta)$ and dividing by $\cos \theta$

$$(\sin \theta)^2 = 0.424; \quad \sin \theta = 0.651; \quad \theta = 40.6^\circ$$

2.8.8

The center of gravity (G) is given as 1 m up from the bottom of the barge. The center of buoyancy (B) is 0.75 m up from the bottom since the draft is 1.5 m.

Therefore $GB = 0.25 \text{ m}$, and GM is found using

$$\overline{GM} = \overline{MB} \pm \overline{GB} = \frac{I_0}{\text{Vol}} \pm \overline{GB}; \quad \text{where } I_0 \text{ is the waterline}$$

moment of inertial about the tilting axis. Chopping off the barge at the waterline and looking down we have a rectangle which is 14 m by 6 m. Thus,

$$\overline{GM} = \frac{I_0}{\text{Vol}} \pm \overline{GB} = \frac{\left[\frac{(14\text{m})(6\text{m})^3}{12} \right]}{(14\text{m})(6\text{m})(1.5\text{m})} - 0.25\text{m} = 1.75 \text{ m};$$

Note: Vol is the submerged volume and a negative sign is used since G is located above the center of buoyancy.

$$M = W \cdot \overline{GM} \cdot \sin \theta$$

$$M = [(1.03)(9790 \text{ N/m}^3)(14 \text{ m})(6 \text{ m})(1.5 \text{ m})](1.75 \text{ m})(\sin 4^\circ)$$

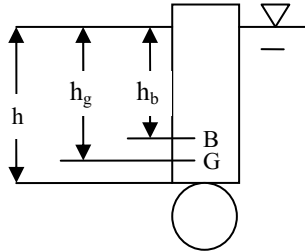
M = 155 kN·m (for a heel angle of 4°)

M = 309 kN·m (for a heel angle of 8°)

M = 462 kN·m (for a heel angle of 12°)

2.8.9

First determine how much the wooden pole is in the water. Summing forces in the y-direction, $W = B$



$$(\text{Vol}_s)(\text{SG}_s)(\gamma) + (\text{Vol}_p)(\text{SG}_p)(\gamma) = (\text{Vol}_s)(\gamma) + (\text{Vol}_p)(\gamma)$$

$$[(4/3)\pi(0.25\text{m})^3](1.4) + [\pi(0.125\text{m})^2(2\text{m})](0.62) =$$

$$[(4/3)\pi(0.25\text{m})^3] + [\pi(0.125\text{m})^2(h)];$$

$$0.0916\text{m}^3 + 0.0609\text{m}^3 = 0.0654\text{m}^3 + (0.0491\text{m}^2)h$$

$h = 1.77\text{ m}$; Find “B” using the principle of moments.

$$[(\text{Vol}_s)(\gamma) + (\text{Vol}_p)(\gamma)](h_b) = (\text{Vol}_s)(\gamma)(h + 0.25\text{m}) + (\text{Vol}_p)(\gamma)(h/2)$$

$$[(0.0654\text{m}^3 + (0.0491\text{m}^3)(1.77\text{m})](h_b) = (0.0654\text{m}^3)(1.77\text{m} + 0.25\text{m}) + [(0.0491\text{m}^3)(1.77\text{m})](1.77\text{m}/2)$$

$h_b = 1.37\text{ m}$; Find “G” using the principle of moments.

$$(W)(h_g + 0.23\text{m}) = (W_s)(2.0\text{ m} + 0.25\text{ m}) + (W_p)(2.0\text{m}/2)$$

$$[(0.0916\text{m}^3 + 0.0609\text{m}^3)](h_g + 0.23\text{m}) = (0.0916\text{m}^3)(2.25\text{m}) + (0.0609\text{m}^3)(1.00\text{m})$$

$h_g = 1.52\text{ m}$; $GB = h_g - h_b = 0.15\text{ m}$

$$MB = I_o/\text{Vol}$$

$$MB = [(1/64)\pi(0.25\text{m})^4] / [(0.0654\text{m}^3 + (0.0491\text{m}^3)(1.77\text{m})]$$

$$MB = 1.26 \times 10^{-3}\text{ m}$$

$$\mathbf{GM = MB + GB = 1.26 \times 10^{-3}\text{ m} + 0.15\text{ m} = \mathbf{0.151\text{ m}}$$

2.8.10

If the metacenter is at the same position as the center of gravity, then $GM = 0$ and the righting moment is

$$M = W \cdot \overline{GM} \cdot \sin \theta = 0. \text{ With no righting moment, the}$$

block will not be stable.

2.8.11

The center of gravity (G) is estimated as 17 ft up from the bottom of the tube based on the depth of the water inside it. The center of buoyancy (B) is 21 feet from the bottom since 42 feet is in the water.

Therefore $GB = 4.0\text{ ft}$, and GM is found using

$$\overline{GM} = \overline{MB} \pm \overline{GB} = \frac{I_o}{\text{Vol}} \pm \overline{GB}; \text{ where } I_o \text{ is the waterline}$$

moment of inertial about the tilting axis. Chopping off the tube at the waterline and looking down we have a circle with a 36 ft diameter. Thus,

$$\overline{GM} = \frac{I_o}{\text{Vol}} \pm \overline{GB} = \frac{[\pi(36\text{ft})^4 / 64]}{(\pi/4)(36\text{ft})^2(42\text{ft})} + 4.0\text{ft} = \mathbf{5.93\text{ ft}};$$

Note: Vol is the submerged volume and a positive sign is used since G is located below the center of buoyancy.

$$M = W \cdot \overline{GM} \cdot \sin \theta$$

$$M = [(1.02)(62.3\text{ lb/ft}^3)(\pi/4)(36\text{ft})^2(42\text{ft})](5.93\text{ft})(\sin 4^\circ)$$

$$\mathbf{M = 1.12 \times 10^6\text{ ft}\cdot\text{lb}} \text{ (for a heel angle of } 4^\circ)$$

2.8.12

The center of gravity (G) is roughly 1.7 m up from the bottom if the load is equally distributed. The center of buoyancy (B) is 1.4 m from the bottom since the draft is 2.0 m. Therefore $GB = 0.3\text{ m}$, and GM is found using

$$\overline{GM} = \frac{I_o}{\text{Vol}} \pm \overline{GB} = \frac{[(12\text{m})(4.8\text{m})^3 / 12]}{(12\text{m})(4.8\text{m})(2\text{m})} - 0.3\text{m} = 0.660\text{ m};$$

Note: Vol is the submerged volume and a negative sign is used since G is located above the center of buoyancy.

$$M = W \cdot \overline{GM} \cdot \sin \theta$$

$$M = [(1.03)(9790\text{ N/m}^3)(12\text{m})(4.8\text{m})(2\text{m})](0.660\text{m})(\sin 15^\circ)$$

$M = 198\text{ kN}\cdot\text{m}$; The distance G can be moved is

$$\mathbf{d = GM (\sin \theta) = (0.66\text{ m})(\sin 15^\circ) = \mathbf{0.171\text{ m}}$$