INSTRUCTOR'S SOLUTIONS MANUAL

PRESTRESSED CONCRETE

A Fundamental Approach

Fifth Edition Update
ACI, AASHTO, IBC 2009 Codes Version

EDWARD G. NAWY

Prentice Hall

Upper Saddle River Boston Columbus San Francisco New York

Indianapolis London Toronto Sydney Singapore Tokyo Montreal

Dubai Madrid Hong Kong Mexico City Munich Paris Amsterdam Cape Town

Vice President and Editorial Director, ECS: *Marcia J. Horton*
Senior Editor: *Holly Stark*
Associate Editor: *Dee Bernhard*
Editorial Assistant: *Keri Rand*
Director of Team-Based Project Management: *Vince O'Brien*
Senior Managing Editor: *Scott Disanno*
Senior Operations Supervisor: *Alan Fischer*
Operations Specialist: *Lisa McDowell*
Cover Designer: *Bruce Kenselaar*

About the Cover: The new I-35W bridge, Minneapolis, Minnesota. Designed for the Minnesota Department of Transportation by FIGG, this new bridge incorporates aesthetics selected by the community using a theme of "Arches–Water–Reflection" to complement the site across the Mississippi River. Curved, 70′ tall concrete piers meet the sweeping parabolic arch of the 504′ precast, prestressed concrete main span over the river to create a modern bridge. The new 10-lane interstate bridge was constructed by Flatiron-Manson, JV and opened to traffic on September 18, 2008. The bridge was designed and built in 11 months. The bridge incorporates the first use of LED highway lighting, the first major use in the United States of nanotechnology cement that cleans the air (gateway sculptures) and "smart bridge" technology with 323 sensors embedded throughout the concrete to provide valuable data for the future. The photograph of the new I-35W bridge is courtesy of FIGG.

Prentice Hall
is an imprint of

PEARSON

www.pearsonhighered.com

10 9 8 7 6 5 4 3 2 1

ISBN-13: 978-0-13-608151-7
ISBN-10: 0-13-608151-7

TABLE OF CONTENTS

ACKNOWLEDGMENTS

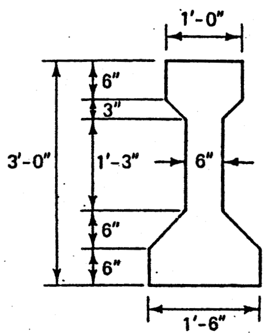
Grateful acknowledgment to Joe Davis, PhD, Rutgers University, for his input to the development of this edition's Solutions Manual. Thanks are also due to Engineer Anand A. Bhatt, MS, Rutgers, for his input to the development of the previous edition's Manual, part of which, not affected by the ACI 318 Code changes, is retained in this edition.

1.1. An AASHTO prestressed simply supported I beam has a span of 34 ft (10.4 m) and is 36 in. (91.4 cm) deep. Its cross section is shown in Figure 14.18. It is subjected to a live-load intensity $W_L = 3600$ plf (52.6 kN/m). Determine the required $\frac{1}{2}$-in.-diameter, stress-relieved, seven-wire strands to resist the applied gravity load and the self-weight of the beam, assuming that the tendon eccentricity at midspan is $e_c = 13.12$ in. (333 mm). Maximum permissible stresses are as follows:

$$f'_c = 6000 \text{ psi } (41.4 \text{ MPa})$$
$$f_c = 0.45 f'_c = 2700 \text{ psi } (26.7 \text{ MPa})$$
$$f_t = 12\sqrt{f'_c} = 930 \text{ psi } (6.4 \text{ MPa})$$
$$f_{pu} = 270{,}000 \text{ psi } (1862 \text{ MPa})$$
$$f_{pi} = 189{,}000 \text{ psi } (1303 \text{ MPa})$$
$$f_{pe} = 145{,}000 \text{ psi } (1000 \text{ MPa})$$

The section properties, given these stresses, are

$$A_c = 369 \text{ in.}^2$$
$$I_g = 50{,}979 \text{ in.}^4$$
$$r^2 = \frac{I_c}{A_c} = 138 \text{ in.}^2$$
$$c_b = 15.83 \text{ in.}$$
$$S_b = 3220 \text{ in.}^3$$
$$S^t = 2527 \text{ in.}^3$$
$$W_D = 384 \text{ plf}$$
$$W_L = 3600 \text{ plf}$$



Solve the problem by each of the following methods:

(a) Basic concept
(b) C-line
(c) Load balancing

SOLUTION:—

1. SOLUTION USING THE P-I METHOD:—

STRESS DATA:

Span = 34 ft
W_L = 3600 plf
f'_c = 6000 psi.
$f_c = 0.45 f'_c$ = 2,700 psi.
$f_t = 12\sqrt{f'_c}$ = 930 psi
f_{pu} = 270,000 psi
f_{pi} = 189,000 psi
f_{pe} = 145,000 psi

SECTION PROPERTIES:

$A_c = 369$ in^2
$I_g = 50{,}979$ in^4
$r^2 = I_g/A_c = 138$ in^2
$C_b = 15.83$ in
$C^t = 20.17$ in
$e_c = 13.12$ in
$S_t = 2527$ in^3
$S_b = 3{,}220$ in^3
$W_d = 384$ plf.

a) BASIC CONCEPT:–

Assume that 10 $\frac{1}{2}''$ dia. seven wire strand tendons are used to prestress

i) Initial Conditions at Prestressing:–

$$A_{ps} = 10(0.153) = 1.53 \text{ in}^2$$

$$P_i = A_{ps}.f_{pi} = 1.53(189{,}000) = 289{,}170 \text{ lb.}$$

$$P_e = 1.53(145{,}000) = 221{,}850 \text{ lb.}$$

The midspan self-weight dead-load moment is

$$M_D = \frac{W_D.\ell^2}{8} = \frac{384(34)^2}{8} \times 12 = 665{,}856 \text{ in-lb.}$$

$$f^t = \frac{-P_i}{A_c}\left(1 - \frac{e.C_t}{r^2}\right) - \frac{M_D}{S_t} = -\frac{289{,}170}{369}\left(1 - \frac{13.12(20)}{138}\right) - \frac{665{,}856}{2527}$$

$$\therefore f^t = 456 \text{ psi (C)}$$

$$f_b = -\frac{P_i}{A_c}\left(1 + \frac{e.C_b}{r^2}\right) + \frac{M_D}{S_b} = -\frac{289{,}170}{369}\left(1 + \frac{13.12(25.83)}{138}\right) - \frac{665{,}856}{3220}$$

$$\therefore f_b = -1{,}756 \text{ psi} = < f_{ci} = -2{,}880 \text{ psi allowed.}$$

ii) FINAL Conditions at Service Load:–

The midspan moment due to live load is:

$$M_L = \frac{w.l^2}{8} = \frac{3600(34)^2}{8} \times 12 = 6{,}242{,}400 \text{ in-lb}$$

$$M_T = 665{,}856 + 6{,}242{,}400 = 6{,}908{,}256 \text{ in-lb.}$$

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{e.C_t}{r^2}\right) - \frac{M_T}{S^t} = -\frac{221{,}850}{369}\left(1 - \frac{13.12(20.17)}{138}\right) - \frac{6{,}908{,}256}{2{,}527}$$

$$\boxed{f^t = -2{,}183 \text{ psi } (C)} < f_c = 2700 \text{ psi}$$

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{e \cdot C_b}{r^2}\right) + \frac{M_T}{S_b} = -$$

$$= -\frac{221{,}850}{369}\left(1 + \frac{13.12(15.83)}{138}\right) + \frac{6{,}908{,}256}{3220}$$

$$= 639 \text{ psi } (T) < f_t = 930 \text{ psi} \quad \therefore \underline{\text{O.k.}}$$

b) <u>C-LINE METHOD:</u>

$$P_e = 221{,}850 \text{ lb.}$$

$$M_T = 6{,}908{,}256 \text{ in-lb.}$$

$$a = \frac{M_T}{P_e} = 31.1 \text{ in}$$

$$e' = a - e = 31.1 - 13.12 = 18.02 \text{ in}$$

$$f^t = -\frac{P_e}{A_c}\left(1 + \frac{e' \cdot C_t}{r^2}\right) = \frac{-221{,}850}{369}\left(1 + \frac{18.02 \times 20.17}{138}\right)$$

$$= -2{,}183 \text{ psi } (C)$$

$$f_b = -\frac{P_e}{A_c}\left(1 - \frac{e' \cdot C_b}{r^2}\right) = \frac{-221{,}850}{369}\left(1 - \frac{18.02 \times 20.17}{138}\right)$$

$$= 639 \text{ psi } (T)$$

c) <u>LOAD BALANCING METHOD:</u> =

$$P' = P_e = 221{,}850 \text{ lb.}$$

$$a = 13.12 \text{ in} = e = 1.09 \text{ ft}$$

$$W_b = \frac{8 \cdot P' \cdot a}{l^2} = \frac{8 \times 221{,}850 \times 1.09}{(34)^2} = 1{,}678.5 \text{ plf}$$

$$W_T = 384 + 3600 = 3984 \text{ plf}$$

$$W_{ub} = 3984 - 1678.59 = 2305.41 \text{ plf.}$$

$$M_{ub} = \frac{W_{ub} \cdot \ell^2}{8} = \frac{2305.41(34)^2}{8} \times 12 = 3,997,581 \text{ in-lb}$$

$$f^t = -\frac{P'}{A_c} - \frac{M_{ub}}{S^t} = -\frac{221,850}{369} - \frac{3997,581}{2527} = -2183 \text{ psi } (C)$$

$$f_b = \frac{-P'}{A_c} + \frac{M_{ub}}{S_b} = -\frac{221,850}{369} + \frac{3,997,581}{3220} = 639 \text{ psi } (T)$$

$$< f_t = 930 \text{ psi} \;\therefore \underline{\underline{ok}}$$

2) S. I. SYSTEM:-

a) Basic Concept:-

Assume that ten 12.7 mm dia seven wire strand tendons are used to prestres

i) Initial Conditions at Prestressing:-

$A_{ps} = 10(99) = 990 \text{ mm}^2$

$P_i = A_{ps} \cdot f_{pi} = 990(1303) = 1290 \text{ KN}$

$P_e = 990(1000) = 990 \text{ KN}$

The midspan self-weight dead load Moment

$$M_D = \frac{W_D l^2}{8} = \frac{5.60(10.4)^2}{8} = 75.7 \text{ KN-m}.$$

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{e.c^t}{r^2}\right) - \frac{M_D}{S^t}$$

$$= -\frac{1290}{2381}\left(1 - \frac{33.3(51.2)}{891}\right) - \frac{75.7 \times 10^2}{41410} = 3.1 \text{ MPa}$$

$$f_b = -\frac{P_i}{A_c}\left(1 + \frac{e.C_b}{r^2}\right) + \frac{M_D}{S_b} = \frac{-1290}{2381}\left(1 - \frac{33.3(51.2)}{891}\right) + \frac{75.7 \times 10^2}{52766}$$

$= -12.1 \text{ MPa} < -19.9 \text{ MPa}$ $\therefore$ O.K.

Final Conditions at service Load:-

The midspan moment due to live load is

$$M_L = \frac{52.6(10.4)^2}{8} = 711 \text{ KN-m}.$$

$M_T = 75.7 + 711 = 787 \text{ KN-m}.$

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{e c^t}{r^2}\right) - \frac{M_T}{S^t}$$

$$= -\frac{990}{2381}\left(1-\frac{33.3\times 51.2}{891}\right)-\frac{787\times 10^2}{41410}$$

$$= -15.1 \text{ MPa (C)} < f_c = 18.6 \text{ MPa} \quad \therefore \underline{\text{o.k.}}$$

$$f_b = \frac{-990}{2381}\left(1+\frac{33.3\,(40.2)}{891}\right)-\frac{787\times 10^2}{52766}$$

$$= 4.5 \text{ MPa (T)} < f_t = 6.4 \text{ MPa (T)} \quad \therefore \underline{\underline{\text{ok.}}}$$

b) <u>C-LINE METHOD:</u>—

$P_e = 990 \text{ kN}$

$M_T = 787 \text{ kN-m}$

$$a = \frac{787}{990} = 0.795 \text{ m}$$

$e' = a - e = 79.44 - 33.3 = 46.19 \text{ cm.}$

$$f^t = \frac{-990 \text{ kN}}{2381 \text{ cm}^2}\left(1+\frac{46.19\times 51.2}{891}\right) \approx 15.1 \text{ MPa(C)}$$

$$f_b = \frac{-990 \text{ kN}}{2381 \text{ cm}^2}\left(1-\frac{46.19\times 40.2}{891}\right) = 4.5 \text{ MPa (T)}$$

c) LOAD BALANCING METHOD:—

$P' = P_e = 990 \text{ kN}$

$a = 33.3 \text{ cm} = e$

$$W_b = \frac{8P'.a}{l^2} = \frac{8(990)\times\frac{33.3}{100}}{(10.4)^2} = 24.38 \text{ kN/m}$$

$W_{ub} = 52.6 + 5.6 - 24.38 = 33.82 \text{ kN/m.}$

$$M_{ub} = \frac{W_{ub}.xl^2}{8} = \frac{33.82\times(10.4)^2}{8} = 457.25 \text{ kN-m}$$

$$f^t = -\frac{P'}{A_c} - \frac{M_{4b}}{S^t} = \frac{-990}{2381} - \frac{457.25 \times 10^2}{41410} \simeq 15.1\,\text{MPa (c)}$$

$$f_b = -\frac{P'}{A_c} + \frac{M_{4b}}{S_b} = \frac{-990}{2381} + \frac{457.25 \times 10^2}{52766} = 4.5\,\text{MPa (T)}$$

$$< f_t = 6.4\,\text{MPa}$$

$\therefore$ ok

1.3 A simply supported pretensioned pretopped double T-beam for a floor has a span of 70 ft (21.3 m) and the geometrical dimensions shown in Figure P1.3. It is subjected to a gravity live-load intensity $W_L = 480$ plf (7 kN/m), and the prestressing tendon has an eccentricity at midspan of $e_c = 19.96$ in. (494 mm). Compute the concrete extreme fiber stresses in this beam at transfer and at service load, and verify whether they are within the permissible limits. Assume that all permissible stresses and materials used are the same as in example 1.1. The section properties are:

Section Properties
Untopped

A_c = 1185 in.^{2}
I_g = 109,621 in.^{4}
C_b = 25.65 in.
C_t = 8.35 in.
S_b = 4274 in.^{3}
S^t = 13,128 in.^{3}
W_D = 1234 plf
82 psf
V/S = 2.45 in.

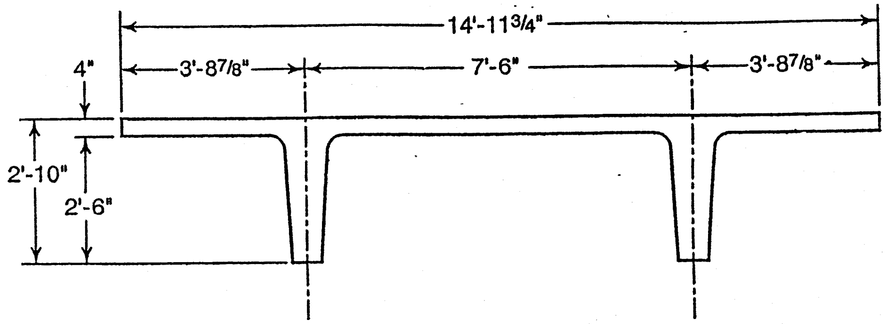


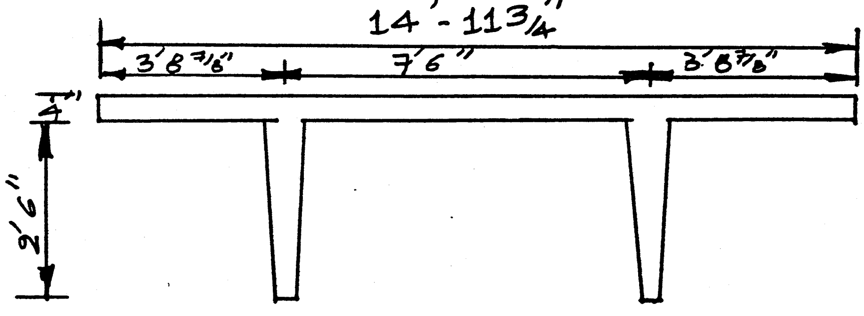
Figure P1.3.

Design the prestressing steel needed using ½-in.-dia stress-relieved seven-wire strands. Use the three methods of analysis discussed in this chapter in your solution.

SOLOTION:—

$l = 70$ ft
$W_L = 480$ lb/ft
$e_c = 19.96$ in
$f_c' = 6000$ psi
$f_c = 2700$ psi
$f_t = 12\sqrt{f_c'} = 930$ psi.
$f_{pu} = 270{,}000$ psi.
$f_{pi} = 189{,}000$ psi.
$f_{pe} = 145{,}000$ psi.

$A_c = 1185$ in^2
$I_g = 109{,}621$ in^4
$C_b = 25.65$ in
$C^t = 8.35$ in
$r^2 = I/A = 92.5$ in^2
$S_b = 4274$ in^3
$S^t = 13{,}124$ in^3
$W_D = 1234$ lb/ft
$V/S = 2.45$ in



pretopped

Assume $16 - \frac{1}{2}$" dia seven wire tendons are used.

Initial Conditions @ Prestressing:

$A_{ps} = 16 \times 0.153 = 2.45 \text{ in}^2$

$P_i = A_{ps} \cdot f_{pi} = 2.45 \times 189{,}000 = 463{,}050 \text{ lb}.$

$P_e = A_{ps} \cdot f_{pe} = 2.45 \times 145{,}000 = 355{,}250 \text{ lb}.$

Midspan self-wt. D.L. moment

$$= \frac{wl^2}{8} = \frac{1234(70)^2}{8} \times 12 = 9{,}069{,}900 \text{ in-lb}$$

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{e \cdot c_t}{r^2}\right) - \frac{M_D}{S^t} = -\frac{463{,}050}{1185}\left(1 - \frac{19.96 \times 8.35}{92.5}\right) - \frac{9{,}069{,}900}{13{,}128}$$

$$= 313.31 - 690.88 = -377.57 \text{ psi (C)}$$

$$f_b = -\frac{463{,}050}{1{,}185}\left(1 + \frac{19.96 \times 25.65}{92.5}\right) + \frac{9{,}069{,}900}{4274}$$

$$= -431.4 \text{ psi} \quad (C)$$

Assuming $f'_{ci} = 4800$ psi

$f_{ci} = 0.6 f'_{ci} = 2880$ psi.

then $-431.4 < -377.57 < f_{ci} = -2880$ psi

$\therefore$ o.k.

FINAL Condition @ Service Load:=

$$M_L = \frac{480(70)^2}{8} \times 12 = 3{,}528{,}000 \text{ in-lb}.$$

$$M_T = 3{,}528{,}000 + 9{,}069{,}900 = 12{,}597{,}900 \text{ in-lb}.$$

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{e \cdot c_t}{r^2}\right) - \frac{M_T}{S^t} = -\frac{355{,}250}{1185}\left(1 - \frac{19.96 \times 8.35}{92.5}\right) - \frac{12{,}597{,}900}{13{,}128}$$

$$= 240.4 - 959.6 = -719.2 \text{ psi (C)} < f_c$$

$$f_b = -\frac{355{,}250}{1185}\left(1 + \frac{19.96 \times 25.65}{92.5}\right) + \frac{12{,}597{,}900}{4274}$$

$$= -1959 + 2966 = 1007 \text{ psi (T)} > f_t = 930 \text{ psi}$$

$\therefore$ N.G.

Try increasing to 20 - $\frac{1}{2}$" dia SHS tendons.

$$A_{ps} = 0.153(20) = 3.06 \text{ in}^2$$

$$P_i = 3.06 \times 189{,}000 = 578{,}340 \text{ lb.}$$

$$P_e = 3.06 \times 145{,}000 = 443700 \text{ lb.}$$

Initial Conditions @ Prestress:-

$$f^t = -\frac{578{,}340}{1184}\left(1 - \frac{19.96(8.35)}{92.5}\right) - \frac{9{,}069{,}900}{13{,}128} = -299 \text{ psi (C)}$$

$$f_b = -\frac{578{,}340}{1184}\left(1 + \frac{19.96(25.65)}{92.5}\right) + \frac{9{,}069{,}900}{4{,}274} = -1070 \text{ psi (C)}$$

Final Conditions @ Service Load:-

$$f^t = -\frac{443{,}700}{1184}\left(1 - \frac{19.96 \times 8.35}{92.5}\right) - \frac{12{,}597{,}900}{13{,}128} = -659 \text{ psi (C)}$$

$$f_b = -\frac{443{,}700}{1184}\left(1 + \frac{19.96 \times .}{92.5}\right) - \frac{12{,}597{,}900}{4274} = 498 \text{ psi (T)} < f_t = 930 \text{ psi}$$

$\therefore$ O.K.

1.4 A T-shaped simply supported beam has the cross section shown in Figure P1.4. It has a span of 36 ft (11 m), is loaded with a gravity live-load unit intensity $W_L = 2{,}500$ plf (36.5 kN/), and is prestressed with twelve $\frac{1}{2}$-in.-dia (twelve 12.7-mm-dia) seven-wire stress-relieved strands. Compute the concrete fiber stresses at service load by each of the following methods:

(a) Basic concept
(b) C-line
(c) Load balancing

Assume that the tendon eccentricity at midspan is $e_c = 9.6$ in. (244 mm). Then given that

$$f'_c = 5{,}000 \text{ psi (34.5 MPa)}$$

$$f_t = 12\sqrt{f'_c} = 849 \text{ psi (5.9 MPa)}$$

$$f_{pe} = 165.000 \text{ psi (1,138 MPa)}$$

the section properties are as follows:

$A_c = 504$ in^2 $\quad W_D = 525$ plf

$I_c = 37{,}059$ in.^{4} $\quad e_c = 9.6$ in.

$r^2 = I_c/A_c = 73.5$ in.^{2} $\quad A_{ps} =$ twelve $\frac{1}{2}$-in.-dia, seven-wire stress-relieved strands

$c_b = 12.43$ in.

$S_b = 2{,}981$ in.^{3}

$S^t = 2{,}109$ in.^{3}

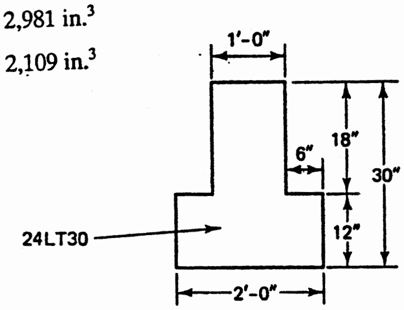


Figure P1.4.

SOLUTION :=

a) BASIC CONCEPT METHOD :=

$A_{ps} = 12 \times 0.153 = 1.836$ in^2

$P_e = A_{ps} \cdot f_{pe} = 1.836 \times 165{,}000 = 302{,}940$ lb

$M_D = 525 \left(\frac{36^2}{8}\right) \times 12 = 1{,}020{,}600$ in-lb.

$M_L = 2500 \left(\frac{36^2}{8}\right) \times 12 = 4{,}860{,}000$ in-lb.

$M_T = M_D + M_L = 5{,}880{,}600$ in-lb.

CONCRETE FIBER STRESSES AT SERVICE LOAD

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{e \cdot c^t}{r^2}\right) - \frac{M_T}{S^t}$$

$$= -\frac{302{,}940}{504}\left(1 - \frac{9.6 \times 17.57}{73.5}\right) - \frac{5{,}880{,}600}{2{,}109}$$

$$= -2{,}010 \text{ psi (comp.)} < f_c = 0.45 \times 5000 \quad \therefore \text{ o.k.}$$

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{e \cdot c_b}{r^2}\right) + \frac{M_T}{S_b}$$

$$= -\frac{302{,}940}{504}\left(1 + \frac{9.6 \times 12.43}{73.5}\right) + \frac{5{,}880{,}600}{2{,}981}$$

$$= 396 \text{ psi (T)} < f_t = 849 \text{ psi} \quad \therefore \text{ o.k.}$$

b) C-LINE METHOD:—

$$e' = a - e \quad \text{where } a = M_T / P_e = \frac{5{,}880{,}600}{302{,}940} = 19.41''$$

$$\therefore e' = 19.41 - 9.6 = 9.81''$$

CONCRETE FIBER STRESSES AT SERVICE LOAD:

$$f^t = -\frac{P_e}{A_c}\left(1 + \frac{e' \cdot c^t}{r^2}\right) = -\frac{302{,}940}{504}\left(1 + \frac{9.81 \times 17.57}{73.5}\right)$$

$$= -2{,}010 \text{ psi (C)} < f_c = 2250 \text{ psi} \quad \therefore \text{ o.k.}$$

$$f_b = -\frac{P_e}{A_c}\left(1 - \frac{e' \cdot c_b}{r^2}\right) = -\frac{302{,}940}{504}\left(1 - \frac{9.81 \times 12.43}{73.5}\right)$$

$$= 396 \text{ psi (T)} < f_t = 849 \text{ psi.}$$

C. LOAD BALANCING METHOD:–

$P' = P_e = 302{,}940$ lbs.

$a' = e = 9.6'' = 0.8$ ft;

Balancing load $W_b = \dfrac{8P.a'}{l^2} = \dfrac{8 \times 302{,}940 \times 0.8}{(36)^2}$

$= 1{,}496$ plf.

Unbalanced load $= W_D + W_L - W_b$

$= 525 + 2500 - 1496 = 1{,}529$ plf

$\therefore$ Unbalanced moment, $M_{ub} = 1529 \dfrac{(36)^2}{8} \times 12$

$= 2{,}972{,}376$ in-lb.

CONCRETE FIBER STRESSES AT SERVICE LOAD:

$f^t = -\dfrac{P'}{A_c} - \dfrac{M_{ub}}{S^t} = -\dfrac{302{,}940}{504} - \dfrac{2{,}972{,}376}{2{,}109}$

$= -2{,}010$ psi (comp) $< f_c$ $\therefore$ o.k.

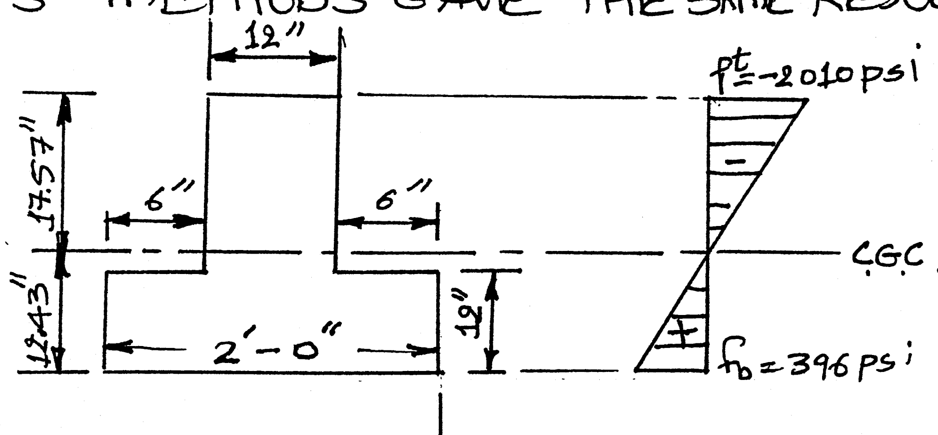
$f_b = -\dfrac{P'}{A_c} + \dfrac{M_{ub}}{S_b} = -\dfrac{302{,}940}{504} + \dfrac{2{,}972{,}376}{2{,}981}$

$= 396$ psi (T) $< f_t = 849$ psi.

$\therefore$ o.k.

CONCLUSION:

ALL 3 METHODS GAVE THE SAME RESULTS.



1.5 Solve problem 1.4 if $f'_c = 7{,}000$ psi (48.3 MPa) and $f_{pe} = 160{,}000$ psi (1,103 MPa).

SOLUTION :=

a) BASIC CONCEPT METHOD :=

$P_e = 1.836 \times 160{,}000 = 293{,}760$ lbs.

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{e.c^t}{r^2}\right) - \frac{M_T}{S^t}$$

$$= -\frac{293{,}760}{504}\left(1 - \frac{9.6 \times 17.57}{73.5}\right) - \frac{5{,}880{,}600}{2104}$$

$= -2{,}034$ psi $< f_c = 3150$ psi.

∴ O.K.

$$f_b = -\frac{293{,}760}{504}\left(1 + \frac{9.6 \times 12.43}{73.5}\right) + \frac{5{,}880{,}600}{2{,}481}$$

$= 443.6$ psi $< f_t = 1{,}004$ psi ∴ O.K.

b) C-LINE METHOD :=

$$a = \frac{M_T}{P_e} = \frac{5{,}880{,}600}{293{,}760} = 20.02 \text{ in}$$

$e' = a - e = 20.02 - 9.60 = 10.42$ in

$$f^t = -\frac{P_e}{A_c}\left(1 + \frac{e'.c^t}{r^2}\right)$$

$$= -\frac{293{,}760}{504}\left(1 + \frac{10.42 \times 17.57}{73.5}\right) = -2{,}034 \text{ psi}$$

$< f_c = 3{,}150$ psi

$$f_b = \frac{-293{,}760}{504}\left(1 - \frac{10.42 \times 12.43}{73.5}\right) = 444 \text{ psi}$$

< 1004 psi.

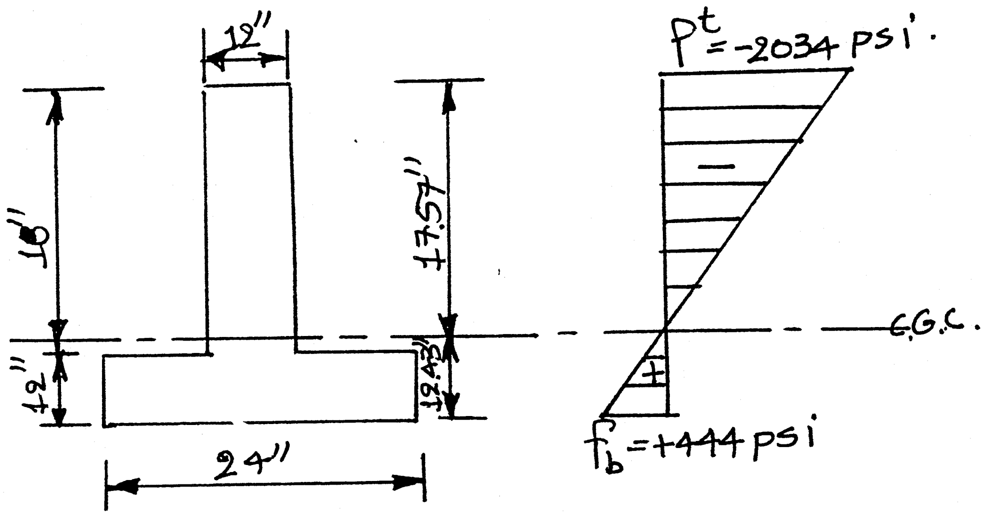
c) LOAD BALANCING METHOD:—

$$M_{ub} = 5,880,600 - 293,760 \times 9.6$$
$$= 3,060,504 \text{ in-lb.}$$

$$f^t = -\frac{Pe}{A_c} - \frac{M_{ub}}{S^t} = -\frac{293,760}{504} - \frac{3,060,504}{2,109}$$
$$= -2,034 \text{ psi} < 3,150 \text{ psi.}$$

$$f_b = -\frac{293,760}{504} + \frac{3,060,504}{2,981} = 444 \text{ psi}$$
$$< 1,004 \text{ psi}$$

$\therefore$ OK.



BEAM CROSS SECTION   STRESS DISTRIBUTION

3.1 A simply supported pretensioned beam has a span of 75 ft (22.9 m) and the cross section shown in Figure P3.1. It is subjected to a uniform gravitational live-load intensity $W_L = 1{,}200$ plf (17.5 kN/m) in addition to its self-weight and is prestressed with 20 stress-relieved $\frac{1}{2}$in. dia (12.7 mm dia) 7-wire strands. Compute the total prestress losses by the step-by-step method, and compare them with the values obtained by the lump-sum method. Take the following values as given:

$f'_c = 6{,}000$ psi (41.4 MPa), normal-weight concrete
$f'_{ci} = 4{,}500$ psi (31 MPa)
$f_{pu} = 270{,}000$ psi (1,862 MPa)
$f_{pi} = 0.70 f_{pu}$
Relaxation time $t = 5$ years
$e_c = 19$ in. (483 mm)
Relative humidity $RH = 75\%$
$V/S = 3.0$ in. (7.62 cm)

Assume SD load = 30% LL.

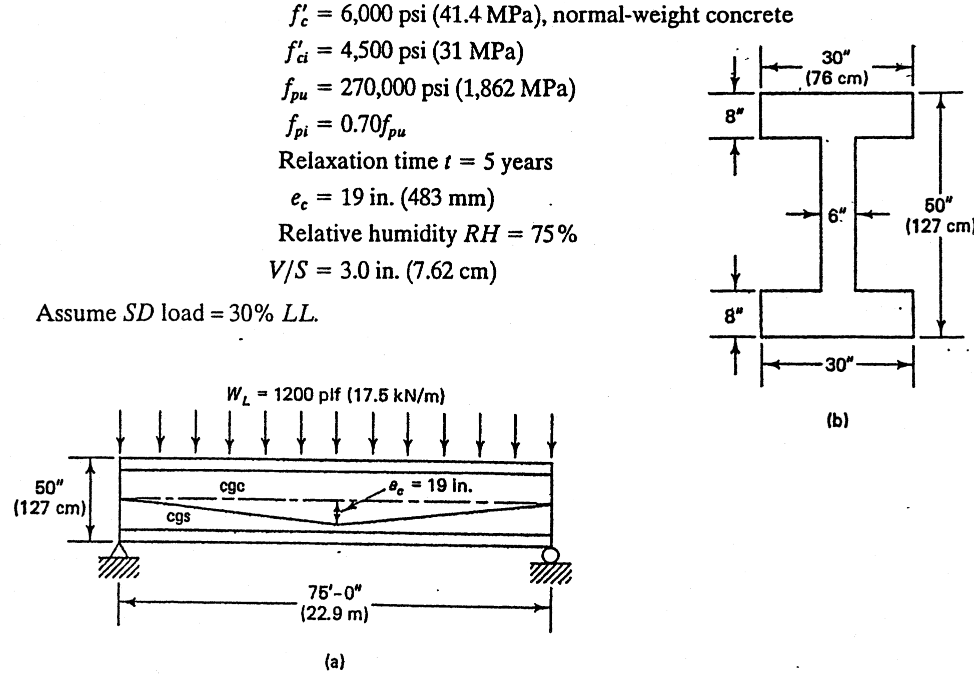


Figure P3.1 (a) Elevation. (b) Section.

SOLUTION:—

$$A_c = 2(30 \times 8) + 34 \times 6 = 684 \text{ in}^2$$

$$I_c = 2\left[\frac{30 \times (8)^3}{12} + 30 \times 8 \times (21)^2\right] + \frac{6(34)^3}{12}$$

$$= 233{,}892 \text{ in}^4$$

$$c_b = c^t = 25'' \quad , \quad r^2 = \frac{I_c}{A_c} = 342 \text{ in}^2$$

$$S_b = S^t = I_c / c_b = 9{,}356 \text{ in}^2$$

$$W_D = \frac{684}{144} \times 0.15 = 0.7125 \text{ K/ft.}$$

COMPUTATION OF LOSSES STEP BY STEP METHO

1) ANCHORAGE LOSS = 0

2) LOSS DUE TO ELASTIC SHORTENING

$$P_F = 0.9 P_i = 0.9 \times 3.06 \times (0.70 \times 270{,}000) = 520{,}506 \text{ lb.}$$

$$M_D = \frac{712.5(75)^2}{8} \times 12 = 6{,}011{,}719 \text{ in-lb.}$$

$$f_{SC} = -\frac{P_i}{A_c}\left(1 + \frac{e^2}{r^2}\right) + \frac{M_D e}{I_c}$$

$$= -\frac{520{,}506}{684}\left(1 + \frac{(19)^2}{(34)^2}\right) + \frac{6{,}011{,}719 \times 19}{233{,}892}$$

$$= -1{,}076 \text{ psi.}$$

$$n = \frac{E_{PS}}{E_{ci}} = \frac{28 \times 10^6}{2.7 \times 10^6} \simeq 10.4$$

$$\Delta f_{PES} = n \cdot f_{CS} = 10.4 \times 1{,}076 = 11{,}190 \text{ psi.}$$

3) <u>CREEP LOSS</u>: (CR)

$f_{SCD} = 0$, $K_{CR} = 2 \times 0.8 = 1.6$

$$\Delta f_{PCR} = K_{CR}(f_{SC} - f_{SCD})\frac{E_{PS}}{E_c} = 1.6(0. - 1076)\frac{28 \times 10^6}{3.2 \times 10^6}$$

$$= 15{,}064 \text{ psi.}$$

4) <u>SHRINKAGE LOSS</u> (SH):

$K_{SH} = 1$

$$\Delta f_{PSH} = 8.2 \times 10^{-6} \times 1 \times 28 \times 10^6 (1 - 0.006 \times 3)(100 - 75)$$

$$= 4{,}707 \text{ psi.}$$

5) <u>RELAXATION OF STEEL</u> (R):

$$f_{pi}' = 0.7 \times 270{,}000 - (\Delta f_{PES} + \Delta f_{PCR} + \Delta f_{PSH})$$

$$\therefore f_{pi}' = 189{,}000 - (11{,}190 + 15{,}064 + 4{,}707)$$

$$= 158{,}039 \text{ psi.}$$

$$t = 15 \times 365 \times 24 = 131{,}400 \text{ hrs.}$$

$$\Delta f_{PR} = f'_{Pi}\left(\frac{\log t}{10}\right)\left(\frac{f_{pi}'}{f_{py}} - 0.55\right)$$

$$= 158{,}039\left(\frac{\log 131{,}400}{10}\right)\left(\frac{158{,}039}{230{,}000} - 0.55\right)$$

$$= 11{,}093 \text{ psi.}$$

STRESS LEVEL	STEEL STRESS (psi)	%
AFTER TENSIONING	189,000	100
ELASTIC SHORTENING LOSS	−11,190	−5.9
CREEP LOSS	−15,064	−8.0
SHRINKAGE LOSS	−4,707	−2.5
RELAXATION LOSS	−11,093	−5.9
TOTAL	146,946	77.7

% of total loss = 100 − 77.7 = 22.3 %

LUMPSUM METHOD:—

FROM TABLE 3.1,

$\Delta P_T = 33{,}000$

$\therefore\ f_{pe} = 189{,}000 - 33{,}000 = 156{,}000$ psi.

% Difference between two methods.

$$= \frac{(156{,}000 - 146{,}946)}{189{,}000} = 4.8\ \%$$

3.2 Compute, by the detailed step-by-step method, the total losses in prestress of the 10 ft (3.28 m)-wide flanged double T-beam in Example 1.1 which has a span of 64 ft (19.5 m) for a steel relaxation period of 7 years. Use $RH = 70\%$ and $V/S = 3.5$ in. (8.9 cm), and solve for both pretensioned and post-tensioned prestressing conditions. Assume SD load $= 30\%$ LL. In the post-tensioned case, assume that the total jacking stress prior to the friction and anchorage seating losses is 189,000 psi.

SOLUTION :—

1) PRETENSIONING CONDITIONS:—

(i) ELASTIC SHORTENING LOSS (ES):

$$P_J = 0.9 P_i = 0.9 \times 12 \times 0.153 \times 190{,}000 = 313{,}956 \text{ psi.}$$

$$M_D = 494 (64)^2 \times \frac{12}{8} = 3{,}035{,}136 \text{ in-lb.}$$

$$f_{cs} = -\frac{313{,}956}{474}\left(1 + \frac{(14.6)^2}{45.44}\right) + \frac{3{,}035{,}136 \times 14.4}{21{,}450}$$

$$= -1{,}704 \text{ psi.}$$

$$n = \frac{28 \times 10^6}{3.2 \times 10^6} = 8.75$$

$$\therefore \Delta f_{PES} = n f_{cs} = 8.75 \times 1{,}704 = 14{,}910 \text{ psi.}$$

(ii) CREEP LOSS (CR):

Assume $W_{SD} = 200$ plf

$$M_{SD} = \frac{200 \times (64)^2}{8} \times 12 = 1{,}228{,}800 \text{ in-lb.}$$

$$f_{scD} = \frac{M_{SD} \cdot e}{I_c} = \frac{1{,}228{,}800 \times 14.6}{21{,}450} = 83.6 \text{ psi.}$$

$$K_{CR} = 2 \times 0.8 = 1.60$$

$$\Delta f_{PCR} = K_{CR}(f_{cs} - f_{scD}) \frac{E_{ps}}{E_c} = 1.6(1{,}704 - 836)\frac{28 \times 10^6}{2.8 \times 10^6}$$

$$= 13{,}888 \text{ psi.}$$

(iii) SHRINKAGE LOSS (SH):

$K_{SH} = 1$

$$\Delta f_{PSH} = 8.2 \times 10^{-6} \times 1 \times 28 \times 10^{6} (1 - 0.006 \times 3.5)(100 - 70)$$
$$= 5{,}442 \text{ psi.}$$

(iv) RELAXATION OF STEEL (R):

$$f'_{pi} = 190{,}000 - (14{,}910 + 13{,}888 + 5{,}442)$$
$$= 155{,}760 \text{ psi.}$$

$$t = 7 \times 365 \times 24 = 61{,}320 \text{ HRS.}$$

$$\Delta f_{PR} = f'_{pi} \left(\frac{\log t}{10}\right)\left(\frac{f'_{pi}}{f_{py}} - 0.55\right)$$
$$= 155{,}760 \left(\frac{\log .61320}{10}\right)\left(\frac{155{,}760}{220{,}000} - 0.55\right)$$
$$= 11{,}782 \text{ psi.}$$

(v) INCREASE OF STRESSE DUE TO 2" TOPPING:

$$f_{SD} = n \cdot f_{CSD} = \frac{28 \times 10^{6}}{2.8 \times 10^{6}} \times 836.4 = 8{,}364 \text{ psi.}$$

SUMMARY OF STRESSES:-

STRESS LEVEL	STEEL STRESS (PSI)	%
AFTER TENSIONING	190,000	100
ELASTIC SHORTENING LOSS	- 14,910	-7.8
CREEP LOSS	- 13,888	-7.3
SHRINKAGE LOSS	- 5,442	-2.9
RELAXATION LOSS	- 11,782	- 6.2
INCREASE IN STRESS (TOPPING)	+ 8,364	+4.4
TOTAL	152,342	80.2%

% OF TOTAL LOSS = 100 - 80.2% ≈ 20%

LUMPSUM METHOD:—

From table 3.1, $\Delta P_T = 33,000$ psi.

$\therefore f_{pe} = 190,000 - 33,000 = 157,000$ psi.

% Difference in the Above 2 methods

$$= \frac{(157,000 - 152,342)}{190,000} \times 100 \simeq 2.4\%$$

2. POST TENSIONING CONDITIONS:

(i) FRICTION LOSS:

$$\alpha = \frac{8e}{L} = \frac{8 \times 14.6}{64 \times 12} = 0.152$$

from table 3.7, $\mu = 0.25$ & $K = 0.001$

$$\therefore \Delta f_{PF} = -f_{pi}(\mu\alpha + KL)$$

$$= -190,000\ (0.25 \times 0.152 + 0.001 \times 64)$$

$$= -19,380 \text{ psi.}$$

$$f_{pr} = 190,000 - 19,380 = 170,620 \text{ psi.}$$

(ii) ELASTIC SHORTENING (SH):

$$\Delta f_{pSH} = 0$$

(iv) CREEP LOSS (CR):

$$f_{scD} = 836.4 \text{ psi}$$

$$f_f = 0.9 \times 12 \times 0.153 \times 170,620 = 281,932 \text{ psi.}$$

$$\Delta f_{pCR} = K_{CR}(f_{cs} - f_{csD})\frac{E_{ps}}{E_c}$$

$$= -\frac{281,432}{474}\left(1 + \frac{(14.6)^2}{45.44}\right) + \frac{3,035,136 \times 14.6}{21,450}$$

$$= -1,319 \text{ psi.}$$

$$\therefore \Delta f_{pCR} = 1.6 \times 0.8\ (1,319 - 836)\frac{28}{2.8} = 6,182 \text{ psi.}$$

iv) SHRINKAGE LOSS (SH):

$K_{SH} = 0.45$ (table 3.6)

$$\Delta f_{pSH} = 8.2\times10^{-6}\times 0.45\times 28\times10^{6}(1-0.06\times35)(100-70)$$
$$= 2,449 \text{ psi.}$$

(V) RELAXATION LOSS (R):

$$\text{Net } f_{pi}' = 170,620 - (6182+2449) = 161,989 \text{ psi}$$

$$f_{pR} = -161,989\left(\frac{\log 61320}{10}\right)\left(\frac{161,989}{280,000} - 0.55\right)$$
$$= -14,449 \text{ psi.}$$

vi) INCREASE IN STRESS DUE TO TOPPING:

$f_{SD} = 8,364$ psi (As before)

SUMMARY OF STRESSES:

STRESS LEVEL	STEEL STRESS (psi)	%
AFTER TENSIONING	190,000	100
FRICTION LOSS	−19,380	−10.0
ELASTIC SHORTENING LOSS	0	0
CREEP LOSS	−6,182	−3.3
SHRINKAGE LOSS	−2,449	−1.8
RELAXATION LOSS	−14,449	−7.6
INCREASE IN STRESS (TOPPING)	+8,364	+4.4
TOTAL	155,904	82%

% TOTAL LOSS = 100 − 82 = 18%

From table 3.2, Lumpsum loss = 35,000 psi

$\therefore f_{pe} = 190{,}000 - 35{,}000 = 155{,}000$ psi

% Difference by the above 2 methods

$$= \frac{(155{,}904 - 155{,}000)}{190{,}000} \times 100 = 0.48\%$$

3.3 Compute, by the detailed step-by-step method, the total losses of prestress in the AASHTO 36-in. (91.4 cm)-deep beam used in Problem 1.1 and which has a span of 34 ft (10.4 m) for both the pre-tensioned and the post-tensioned case. Use all the data of Problem 1.1 in your solution, and assume that the relative humidity $RH = 70\%$ and the volume-to-surface ratio $V/S = 3.2$. Determine the steel relaxation losses at the end of the first year after erection and at the end of 4 years.

SOLUTION :=

1) PRETENSIONING CONDITIONS:

a) STAGE I : AFTER 1 YEAR:

(i) ELASTIC SHORTENING:

$$P_i = 8 \times 0.153 \times 180{,}000 = 220{,}320 \text{ psi.}$$

$$M_D = \left[\frac{384\,(34)^2}{8}\right] 12 = 665{,}856 \text{ in-lb.}$$

$$P_f = 0.9 \times P_i = 198{,}288 \text{ psi}$$

$$f_{cs} = -\frac{198{,}288}{369}\left(1 + \frac{(13.12)^2}{138}\right) + \frac{665{,}856 \times 13.12}{50{,}979}$$

$$= -1{,}036 \text{ psi.}$$

$$\therefore \Delta f_{PES} = \frac{28 \times 10^6}{2.7 \times 10^6} \times 1{,}036 = 10{,}744 \text{ psi.}$$

(ii) CREEP LOSS (CR):

$$f_{scD} = 0$$

$$\Delta f_{PCR} = K_{CR}(f_{cs} - f_{scD})\frac{E_{ps}}{E_c}$$

$$= 1(1{,}036)\frac{28}{3.2} = 9{,}065 \text{ psi.}$$

(iii) SHRINKAGE LOSS (SH):

$$\Delta f_{PSH} = 8.2 \times 10^{-6} \times 1 \times 28 \times 10^6 (1 - 0.06 \times 3.2)(100 - 70)$$

$$= 5{,}566 \text{ psi.}$$

(iv) RELAXATION OF STEEL LOSS (R):

$$t = 1 \times 365 \times 24 = 8760 \text{ HRS.}$$