PREFACE

I once facilitated a mandatory workshop on the teaching of calculus that was attended by a diverse group of professors and teaching assistants. Before the seminar began, I asked for written answers to the following question: "Ideally, what would you like to get out of the next two days' activities?"

Their responses formed a collection of contradictory expectations. Some were cautionary: "You can tell me *what* to teach, but don't tell me *how*." Some wanted help with group work and cooperative learning, while others just wanted a general idea of what it meant to teach calculus "with modern pedagogy." And many wanted specifics: "How can you teach the Chain Rule 'reform-style'?" "How much homework should I assign?"

This Instructor's Guide tries to address the issues brought out by the above comments. The overall goal was not to write a reference book for a shelf, but to provide a user-friendly source of suggestions and activities for any teacher of calculus within a typical calculus curriculum. Instructors that have used previous editions of this book have said that it saved them a great deal of time, and helped them to teach a more student-oriented course. They have also reported that their classes have become more "fun," but agreed that this unfortunate by-product of an engaged student population can't always be avoided.

This guide should be used together with *Essential Calculus, Early Transcendentals, Second Edition* as a source of both supplementary and complementary material. Depending on individual preference, instructors can choose from occasionally glancing through the Guide for content ideas and alternate approaches, or using the material from the Instructor's Guide as a major component in planning their day-to-day classes as well as to set homework assignments and reading quizzes. There are student activities and worksheets, sample exam questions, and examples for every section.

Some of the continuing debates about changes in calculus content and pedagogy are rendered moot by adopting the principle that the instruction of any topic in calculus can be enhanced by using a wider range of approaches. This guide includes some conceptual and geometric problems in topics as mundane as rules for differentiation, and as traditional as ε-δ limits. Whether a class consists of a straight lecture or an hour of group work, the materials provided are meant to help.

I value reactions from all my colleagues who are teaching calculus from this guide, both to correct any errors and to suggest additional material for future editions. I am especially interested in which particular parts of the guide are the most and the least useful. Please email any feedback to `calculus@dougshaw.com`.

PREFACE

This guide could not have been completed without the help of many people. I especially want to thank Jim Stewart for his continuing belief in this project and trust in me. My editor, Liza Neustaetter, has been wonderful. John Samons has been a big help on short notice, and I appreciate it. Previous versions of this guide have benefitted from the input of Virge Cornelius, Tom Hull, Joe Mercer, Melissa Pfohl, Michael Prophet, and Suzanne Riehl. Over the years, I've had students read through the guide and offer suggestions from their perspective. Thanks go to Kate Degner, Ken Doss, Job Evers, Slade Hovick, Patricia Kloeckner, Jordan Meyer, Ben Nicholson, Paul Schou, Laura Waechter, and Cody Wilson. Further thanks go to James Stewart, John Hall, Robert Hesse, Harvey Keynes, Michael Lawler, and Dan O'Loughlin for their contributions to earlier incarnations of this guide. The book's typesetter and proofreader, Andy Bulman-Fleming, again went above and beyond the call of duty, both in his work on the book, and in keeping me humble by regularly trouncing me at online Scrabble as we produced it. The talents of these people and others at Cengage have truly helped to make writing this guide a learning experience.

This book is dedicated to sarah-marie belcastro.

Doug Shaw

CONTENTS

HOW TO USE THE INSTRUCTOR'S GUIDE

For each section of *Essential Calculus, Early Transcendentals, Second Edition*, this *Instructor's Guide* provides information on the items listed below.

1. **Suggested Time and Emphasis** Here are suggestions for the amount of time to spend in a class of "average" students, and whether or not the material is essential to the rest of the course. If a section is labeled optional, the time range given is the amount of time for the material in the event that it is covered.
2. **Points to Stress** This is a short summary of the main topics to be covered. The stress is on the big ideas, rather than specific details.
3. **Quiz Questions** Some instructors have reported that they like to open or close class by handing out a single question, either as a quiz or to start a discussion. Two types are included:
   - **Text Question** This question is designed for students who have done the reading, but haven't yet seen the material in class. These questions can be used to help ensure that the students are reading the textbook carefully.
   - **Drill Question** These questions are designed to be straightforward "right down the middle" questions for students who have tried, but not necessarily mastered, the material.
4. **Materials for Lecture** These suggestions are meant to work along with the text to create a classroom atmosphere of experimentation and inquiry. They have a theoretical bent to help the students understand the material at a deep conceptual level. In a course with a "lecture and discussion" format, these ideas can be used during the lectures.
5. **Workshop/Discussion** These suggestions are interesting examples and applications aimed at motivating the material and helping the students master it. In a course with a "lecture and discussion" format, these ideas can be used during the discussions.
6. **Group Work** One of the main difficulties instructors have in presenting group work to their classes is that of choosing an appropriate group task. Suggestions for implementation and answers to the group activities are provided first, followed by photocopy-ready handouts on separate pages. The guide's main philosophy of group work is that there should be a solid introduction to each exercise ("What are we supposed to do?") and good closure before class is dismissed ("Why did we just do that?")
7. *Tools for Enriching Calculus* is a companion to the text, intended to enrich and complement its contents. You can find it in Enhanced WebAssign, CourseMate, and PowerLecture. Marginal notes in the main text direct students to TEC modules where appropriate. When a TEC module relates to an *Instructor's Guide* item, it is referenced there as well.
8. **Homework Problems** For each section, a set of essential **Core Exercises** (a bare minimum set of homework problems) is provided. Using this core set as a base, a **Sample Assignment** is suggested, and each exercise in that assignment is classified as **Descriptive**, **Algebraic**, **Numeric**, and/or **Graphic**.
   - **Descriptive:** The student is required to translate mathematical concepts into everyday terms, or vice-versa.

- **Algebraic:** The student is required to use algebraic and/or symbolic manipulation and computation.
- **Numeric:** The student is required to work with numerical data, or provide a numerical approximation.
- **Graphic:** The student is required to explain or understand information in graphic form.

In addition, TEC includes "homework hints" for representative exercises (usually odd-numbered) for each section of the text. These hints are constructed so as not to reveal any more of the actual solution than is minimally necessary to make progress toward the solution. Also available is a *Student Solutions Manual* which presents complete solutions to all of the odd-numbered exercises in the text.

9. **Sample Exam Questions** I recommend that tests in a calculus course have a mix of routine and non-routine questions. The sample exam questions provided are meant to inspire the "non-routine" portion of a calculus test. We do not recommend that calculus tests be composed entirely of questions from this section. One strategy is to announce to the students that one-third of the text questions will be based on homework, one-third will be based on in-class group work, and one-third will not be immediately familiar.
10. **Web Resources** Useful resources can be found on the website for *Essential Calculus, Early Transcendentals, Second Edition* (`http://www.stewartcalculus.com`).

HOW TO IMPLEMENT THE PROJECTS

One exciting yet intimidating aspect of teaching a calculus course is projects. An extended assignment gives students the chance to take a focused problem or project and explore it in-depth — making conjectures, discussing them, eventually drawing conclusions and writing them up in a clear, precise format. *Essential Calculus, Early Transcendentals, Second Edition* has many possible projects throughout its chapters. Here are some tips on ensuring that your students have a successful experience.

Time Students should have two to three weeks to work on any extended out-of-class assignment. This is not because they will need all this time to complete them! But a fifteen-to-twenty-day deadline allows the students to be flexible in structuring their time wisely, and allows the instructors to apply fairly strict standards in grading the work.

Groups Students usually work in teams and are expected to have team meetings. The main problem students have in setting up these meetings is scheduling. Four randomly selected undergraduates will probably find it very hard to get together for more than a few hours, which may not be sufficient. One way to help your students is to clearly specify a minimum number of meetings, and have one or all group members turn in summaries of what was accomplished at each meeting. On a commuter campus, a good first grouping might be by location.

Studies have shown that the optimal group size is three people, followed by four, then two. I advocate groups of four whenever possible. That way, if someone doesn't show up to a team meeting, there are still three people there to discuss the problems.

Before the first project, students should discuss the different roles that are assumed in a team. Who will be responsible for keeping people informed of where and when they meet? Who will be responsible for making sure that the final copy of the report is all together when it is supposed to be? These types of jobs can be assigned within the team, or by the teacher at the outset.

Tell the students that you will be grading on both content and presentation. They should gear their work toward an audience that is bright, but not necessarily up-to-speed on this problem. For example, they can think of themselves as professional mathematicians writing for a manager, or as research assistants writing for a professor who is not necessarily a mathematician.

If the students are expected to put some effort into the project, it is important to let them know that some effort was put into the grading. Both form and content should be commented on, and recognition of good aspects of their work should be included along with criticism.

One way to help ensure cooperation is to let the students know that there will be an exam question based on the project. If every member of the group does well on that particular question, then they can all get a bonus, either on the exam or on the project grade.

Providing assistance Make sure that the students know when you are available to help them, and what kind of help you are willing to provide. Students may be required to hand in a rough draft ten days before the due date, to give them a little more structure and to make sure they have a solid week to write up the assignment.

Individual Accountability It is important that the students are individually accountable for the output of their group. Giving each student a different grade is a dangerous solution, because it does not necessarily encourage the students to discuss the material, and may actually discourage their working together. A better alternative might be to create a feedback form. If the students are given a copy of the feedback form ahead of time, and they know that their future group placement will be based on what they do in their present group, then they are given an incentive to work hard. One surprising result is that when a group consists of students who were previously slackers, that group often does quite well. The exam question idea discussed earlier also gives individuals an incentive to keep up with their colleagues.

HOW TO USE THE REVIEW SECTIONS

Review sections for chapters of a calculus book are often assigned to students the weekend before a test, but never graded. Students realize that they won't be evaluated on this work and often skip the exercises, instead reworking previously done homework problems or quizzes, if they study at all. A more useful activity for students is to use the review sections in *Essential Calculus, Early Transcendentals, Second Edition* to discover their precise areas of difficulty. Implemented carefully, these are a useful resource for the students, particularly for helping them to retain the skills and concepts they've learned. To encourage more student usage, try the following alternatives:

1. Instead of giving a review session where you reiterate previous lectures, make notes of the types of problems students had difficulty with during the quarter and assign students to work on these exercises in the review sections and go over them at the end of class.
2. Use the review section problems to create a game. For instance, break students into groups and have a contest where the group that correctly answers the most randomly picked review questions "wins". One fun technique is to create a math "bingo" game. Give each group a 5×5 grid with answers to review problems. If you laminate the cards and give the students dry-erase markers, then you can use them year after year. Randomly pick review problems, and write the questions on the board. Make sure that for a group to win, they must not only have the correct answers to the problems, but be able to give sound explanations as to how they got the answers.
3. A very "low-maintenance" way to give students an incentive to look at a review section is to use one of the problems, verbatim, for an exam question, and make no secret of your intention to do so. It is important that students have an opportunity to get answers to any questions they have on the review problems before the exam is given; otherwise, this technique loses a great deal of its value.

TIPS ON IN-CLASS GROUP WORK

This *Instructor's Guide* gives classroom-tested group work activities for every section of *Essential Calculus, Early Transcendentals, Second Edition*. One reason for the popularity of in-class group work is that *it is effective*. When students are engaged in doing mathematics, and talking about mathematics with others, they tend to learn better and retain the material longer. Think back to your own career: didn't you learn a lot of mathematics when you began teaching it to other people? Many skeptics experiment by trying group work for one semester, and then they get hooked. Pick a group activity from the guide that you like, make some photocopies, and dive in!

1. **Mechanics** Books and seminars on in-class group work abound. I have conducted many such seminars myself. What follows are some tips to give you a good start:
   (a) **Do it on the first day.**
   The sources all agree on this one. If you want your students to believe that group work is an important part of the course, you have to start them on the first day. My rule of thumb is "at least three times the first week, and then at least once a week thereafter." I mention this first because it is the most important.
   (b) **Make students move.**
   Ideally, students should be eye-to-eye and knee-to-knee. If this isn't possible, do the best you can. But it is important to have them move. If your groups are randomly selected, then they will have to get up and sit in a different chair. If your groups are organized by where they are seated in the classroom, make them move their chairs so they face each other. There needs to be a "break" between sitting-and-writing mode and talking-to-colleagues mode.
   (c) **Use the ideal group size.**
   Research has shown that the ideal group size is three students, with four-student groups next. I like to use groups of four: if one of them is absent (physically or otherwise), the group still has three participating members.
   (d) **Fixed versus random groups.**
   There is a lot of disagreement here. Fixed groups allow each student to find her or his niche, and allow you to be thoughtful when you assign groups or reassign them after exams. Random groups allow students to have the experience of working with a variety of people. I believe the best thing to do is to try both methods, and see which works best for you and your students.
   (e) **Should students hand in their work?**
   The advantage of handing in group works is accountability. My philosophy is that I want the group work to have obvious, intrinsic benefit. I try to make the experience such that it is obvious to the student that they get a lot out of participating, so I don't need the threat of "I'm grading this" to get them to focus. I sometimes have the students hand in the group work, but only as a last resort.

2. **Closure** As stated above, I want my students to understand the value of working together actively in their groups. Once you win this battle, you will find that a lot of motivation and discipline problems simply go away. I've found the best way to ensure that the students understand why they've done an activity is to tell them. The students should leave the room having seen the solutions and knowing why they did that particular activity. You can have the students present answers or present them yourself, whatever suits your teaching style. I've had success with having groups write their results on transparencies and present them to the class (after I've checked their accuracy).

   Here is another way to think about closure: Once in a while, give a future homework problem out as a group work. When the students realize that participating fully in the group work helps them in the homework, they get a very solid feeling about the whole process.
3. **Introduction** The most important part of a group activity, in my opinion, is closure. The second most important is the introduction. A big killer of group work is that awful time between you telling your students they can start, and the first move of pencil on paper—the "what on earth do we do now?" moment. A good introduction should be focused on getting them past that moment. You don't want to give too much away, but you also don't want to throw them into the deep end of the swimming pool. In some classes, you will have to say very little, and in some you may have to do the first problem with them. Experiment with your introductions, but never neglect them.
4. **Help when you are needed** Some group work methods involve giving absolutely no help when the students are working. Again, you will have to find what is best for you. If you give help too freely, the students have no incentive to talk to each other. If you are too stingy, the students can wind up frustrated. When a student asks me for help, I first ask the group what they think, and if it is clear they are all stuck at the same point, I give a hint.
5. **Make understanding a goal in itself** Convey to the students (again, directness is a virtue here) that their goal is not just to get the answer written down, but to ensure that every student in their group understands the answer. Their work is not done until they are sure that every one of their colleagues can leave the room knowing how to do the problem. You don't have to sell every single student on this idea for it to work.
6. **Bring it back when you can** Many of the group works in this guide foreshadow future material. When you are lecturing, try to make reference to past group works when it is appropriate. You will find that your students more easily recall a particular problem they discussed with their friends than a particular statement that you made during a lecture.

The above is just the tip of the iceberg. There are plenty of resources available, both online and in print. Don't be intimidated by the literature—start it on the first day of the next semester, and once you are into it, you may naturally want to read what other people have to say!

1 FUNCTIONS AND LIMITS

1.1 FUNCTIONS AND THEIR REPRESENTATIONS

SUGGESTED TIME AND EMPHASIS

2 classes Essential material

POINTS TO STRESS

1. Definition of function, including piecewise functions.
2. Understanding the interplay between the four ways of representing a function (verbally, numerically, visually, algebraically) perhaps using the concepts of increasing and decreasing functions as an example.
3. Finding the domain and range of a function, regardless of representation.
4. Investigating even and odd functions.

QUIZ QUESTIONS

- **TEXT QUESTION** Fill in the blanks: $|x| = \begin{cases} \_\_\_ & \text{if } x \geq 0 \\ \_\_\_ & \text{if } x < 0 \end{cases}$

  ANSWER x, $-x$
- **DRILL QUESTION** What is the domain of the function $f(x) = \sqrt{1 - \sqrt{x}}$?

  ANSWER $0 \leq x \leq 1$

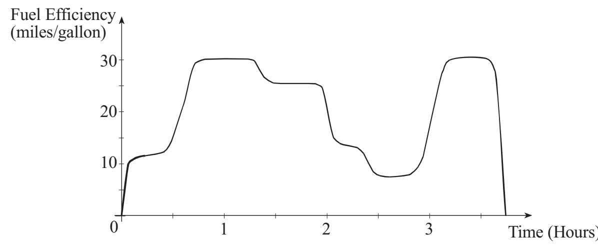
MATERIALS FOR LECTURE

- Draw a graph of electrical power consumption in the classroom versus time on a typical weekday, pointing out important features throughout, and using the vocabulary of this section as much as possible.
- In 1984, United States President Ronald Reagan proposed a plan to change the United States personal income tax system. According to his plan, the income tax would be 15% on the first \$19,300 earned, 25% on the next \$18,800, and 35% on all income above and beyond that. Describe this situation to the class, and have them graph (marginal) tax rate and tax owed versus income for incomes ranging from \$0 to \$80,000. Then have them try to come up with equations describing this situation.
- In the year 2000, Presidential candidate Steve Forbes proposed a "flat tax" model: 0% on the first \$36,000 and 17% on the rest. Have the students do the same analysis, and compare the two models. As an extension, perhaps have the students look at a current tax table and draw similar graphs.
- Let $f(x)$ be the leftmost nonzero digit of x. So $f(386.6) = 3$ and $f(0.000451) = 4$. Have the students try to find the domain and range of f.

  ANSWER The domain seems to be all real numbers except zero, and the range seems to be the set of integers from 1 through 9. Although the graph of this "function" cannot be drawn, ask the students to verify that it passes the Vertical Line Test. It turns out that it does not (and in fact is not even a function), for a subtle reason. For example, let $x = \frac{1}{5}$. If we write x as 0.2, then $f(x) = 2$, but if we write x as $0.1\overline{9}$, then $f(x) = 1$. Therefore, f is not a function.

WORKSHOP/DISCUSSION

- Draw a graph of fuel efficiency versus time on a trip, such as the one below. Lead a discussion of what could have happened on the trip.



- Present graphs of even and odd functions, such as $\sin x$, $\cos x + x^2$, and $\cos(\sin x)$, and check with the standard algebraic tests.
- Start with a table of values for the function $f(x) = \frac{1}{4}x^2 + x$:

x	0	1	2	3	4
$f(x)$	0	1.25	3	5.25	8

  First, have the class describe the behavior of the function in words, trying to elicit the information that the function is increasing, and that its rate of increase is also increasing. Then, have them try to extrapolate the function in both directions, debating whether or not the function is always positive and increasing. Plot the points and connect the dots, then have them try to concoct a formula (not necessarily expecting them to succeed).

- Discuss the domain and range of a function such as $f(x) = \begin{cases} \sqrt{x} & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$

  Also talk about why f is neither increasing nor decreasing for $x > 0$. Stress that when dealing with new sorts of functions, it becomes important to know the precise mathematical definitions of such terms.

- Define "difference quotient" as done in Exercises 21–24. Define $f(x) = x^3$, and show that $\dfrac{f(a+h) - f(a)}{h} = 3a^2 + 3ah + h^2$. This example both reviews algebra skills and foreshadows future calculations.

GROUP WORK 1: EVERY PICTURE TELLS A STORY

Put the students in groups of four, and have them work on the exercise. If there are questions, encourage them to ask each other before asking you. After going through the correct matching with them, have each group tell their story to the class and see if it fits the remaining graph.

ANSWERS

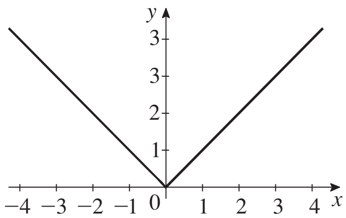
1. (b) **2.** (a) **3.** (c) **4.** The roast beef was cooked in the morning and put in the refrigerator in the afternoon.

GROUP WORK 2: A CHAIN OF FUNCTIONS

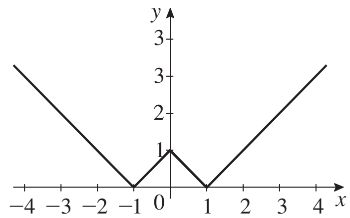
It is recommended that students not be allowed to use graphing technology to do this activity. The intention is to give them an opportunity to practice working with absolute values and order of operations, and to reinforce the idea of looking for mathematical patterns.

ANSWERS

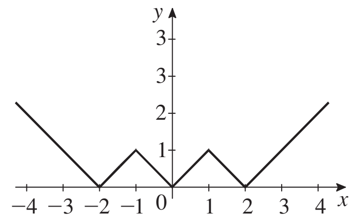
1.



$|x|$

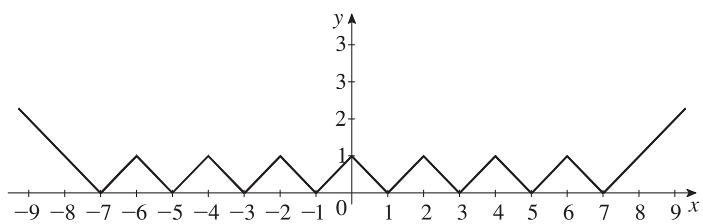


$|1 - |x||$



$|1 - |1 - |x|||$

2. (a) 1 (b)



$|1 - |1 - |1 - |1 - |1 - |1 - |1 - |x||||||||$

GROUP WORK 3: FINDING A FORMULA

Make sure that the students know the equation of a circle with radius r, and that they remember the notation for piecewise-defined functions. Split the students into groups of four. In each group, have half of the students work on each problem first, and then have them check each other's work. If the students find these problems difficult, have them work together on each problem.

ANSWERS

1. $f(x) = \begin{cases} -x - 2 & \text{if } x \le -2 \\ x + 2 & \text{if } -2 < x \le 0 \\ 2 & \text{if } x > 0 \end{cases}$ **2.** $g(x) = \begin{cases} x + 4 & \text{if } x \le -2 \\ 2 & \text{if } -2 < x \le 0 \\ \sqrt{4 - x^2} & \text{if } 0 < x \le 2 \\ x - 2 & \text{if } x > 2 \end{cases}$

HOMEWORK PROBLEMS

CORE EXERCISES 6, 10, 18, 27, 34, 36, 50, 55, 65

SAMPLE ASSIGNMENT 6, 10, 14, 18, 22, 27, 32, 34, 36, 44, 47, 50, 52, 55, 58, 60, 62, 65

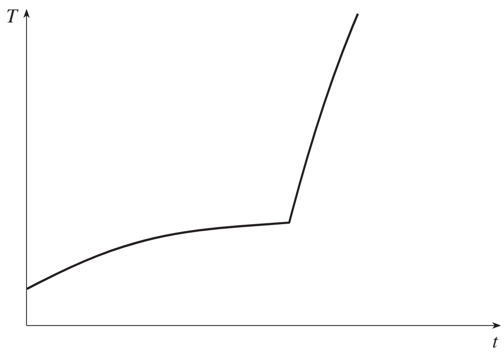
EXERCISE	D	A	N	G
6				×
10	×			×
14				×
18				×
22		×		
27		×		

EXERCISE	D	A	N	G
32		×		×
34		×		×
36		×		×
44		×		
47		×		
50		×		

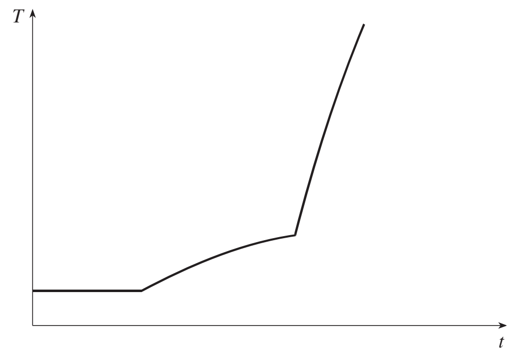
EXERCISE	D	A	N	G
52		×		×
55				×
58				×
60		×		
62		×		
65	×			

GROUP WORK 1, SECTION 1.1
Every Picture Tells a Story

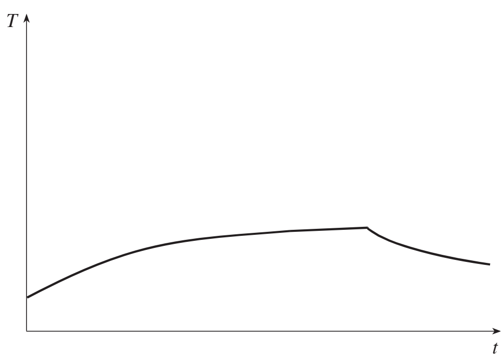
One of the skills you will be learning in this course is the ability to take a description of a real-world occurrence, and translate it into mathematics. Conversely, given a mathematical description of a phenomenon, you will learn how to describe what is happening in plain language. Here follow four graphs of temperature versus time and three stories. Match the stories with the graphs. When finished, write a similar story that would correspond to the final graph.



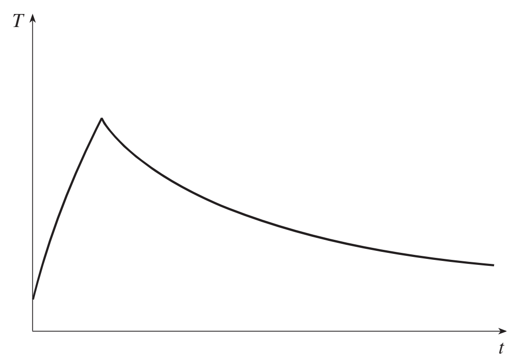
Graph 1



Graph 2



Graph 3



Graph 4

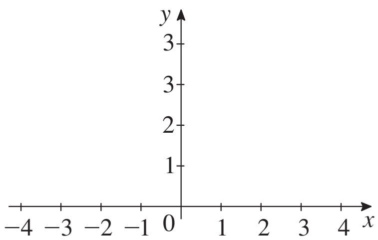
(a) I took my roast beef out of the freezer at noon, and left it on the counter to thaw. Then I cooked it in the oven when I got home.

(b) I took my roast beef out of the freezer this morning, and left it on the counter to thaw. Then I cooked it in the oven when I got home.

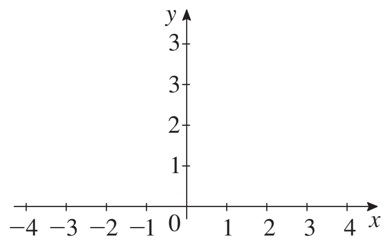
(c) I took my roast beef out of the freezer this morning, and left it on the counter to thaw. I forgot about it, and went out for Chinese food on my way home from work. I put it in the refrigerator when I finally got home.

GROUP WORK 2, SECTION 1.1
A Chain of Functions

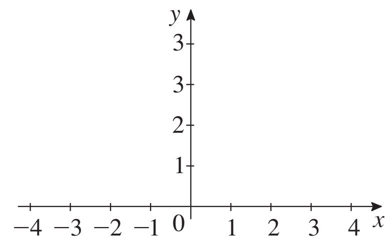
1. Sketch graphs of the following three functions.



$f_1(x) = |x|$



$f_2(x) = |1 - |x||$

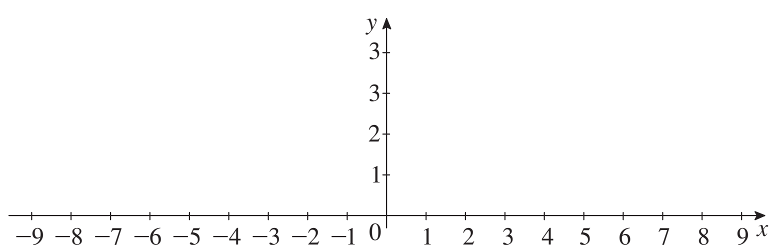


$f_3(x) = |1 - |1 - |x|||$

2. Continuing with the pattern, we get $f_8(x) = |1 - |1 - |1 - |1 - |1 - |1 - |1 - |x||||||||$.

(a) Compute $f(0)$.

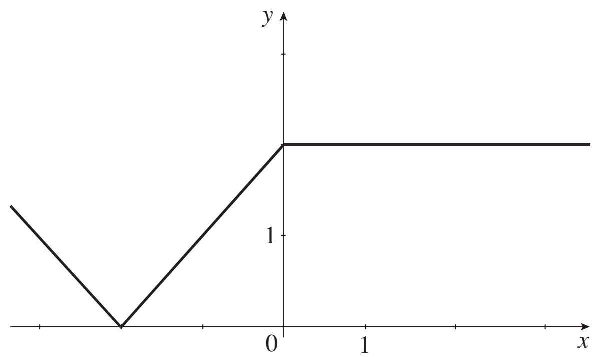
(b) Using your graphs from Part 1 as a guide, sketch the graph of $f_8(x)$.



GROUP WORK 3, SECTION 1.1
Finding a Formula

Find formulas for the following functions:

1.



2.