Chapter 2

PLANE ANALYTIC GEOMETRY

2.1 Basic Definitions

1. Given: $(x_1, y_1) = (3, 8)$; $(x_2, y_2) = (-1, -2)$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1-3)^2 + (-2-8)^2} \\ &= \sqrt{(-4)^2 + (-10)^2} \\ &= \sqrt{16+100} = \sqrt{116} = \sqrt{4 \times 29} = 2\sqrt{29} \end{aligned}$$

2. Given: $(x_1, y_1) = (-1, 3)$; $(x_2, y_2) = (-8, -4)$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-8-(-1))^2 + (-4-3)^2} \\ &= \sqrt{(-7)^2 + (-7)^2} \\ &= \sqrt{49+49} = \sqrt{98} \\ &= \sqrt{2 \times 49} = 7\sqrt{2} \end{aligned}$$

3. Given: $(x_1, y_1) = (4, -5)$; $(x_2, y_2) = (4, -8)$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4-4)^2 + (-8-(-5))^2} \\ &= \sqrt{0^2 + (-3)^2} \qquad \text{vertical line} \\ &= \sqrt{9} = 3 \end{aligned}$$

4. Given: $(x_1, y_1) = (-3, 7)$; $(x_2, y_2) = (2, 10)$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2-(-3))^2 + (10-7)^2} \\ &= \sqrt{(5)^2 + (3)^2} = \sqrt{25+9} \\ &= \sqrt{34} \end{aligned}$$

5. Given: $(x_1, y_1) = (-12, 20)$; $(x_2, y_2) = (32, -13)$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(32+12)^2 + (-13-20)^2} \\ &= \sqrt{(44)^2 + (-33)^2} \\ &= \sqrt{1936+1089} = \sqrt{3025} = 55 \end{aligned}$$

6. Given: $(x_1, y_1) = (23, -9)$; $(x_2, y_2) = (-25, 11)$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-25-23)^2 + (11-(-9))^2} \\ &= \sqrt{(48)^2 + (20)^2} \\ &= \sqrt{2304+400} = \sqrt{2704} \\ &= 52 \end{aligned}$$

7. $d = \sqrt{(\sqrt{32} - (-\sqrt{50}))^2 + (-\sqrt{18} - \sqrt{8})^2}$
$= \sqrt{212} = 2\sqrt{53}$

8. $d = \sqrt{(e - (-2e))^2 + (-\pi - (-\pi))^2}$
$d = \sqrt{(3e)^2}$
$d = 3e$
$d \approx 8.15$

9. Given: $(x_1, y_1) = (1.22, -3.45)$;
$(x_2, y_2) = (-1.07, -5.16)$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1.07-1.22)^2 + (-5.16-(-3.45))^2} \\ &= \sqrt{(-2.29)^2 + (-5.16+3.45)^2} \\ &= \sqrt{(-2.29)^2 + (-1.71)^2} = \sqrt{8.1682} = 2.86 \end{aligned}$$

10. Given: $(x_1, y_1) = (-5.6, 2.3)$; $(x_2, y_2) = (8.2, -7.5)$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(8.2-(-5.6))^2 + (-7.5-2.3)^2} \\ &= \sqrt{(13.8)^2 + (-9.8)^2} \\ &= \sqrt{190.44+96.04} \\ &= \sqrt{286.48} = 16.9 \end{aligned}$$

11. Given: $(x_1, y_1) = (3, 8)$; $(x_2, y_2) = (-1, -2)$

$$m = \frac{y_2 - y_1)}{x_2 - x_1} = \frac{-2-8}{-1-3} = \frac{-10}{-4} = \frac{10}{4} = \frac{5}{2}$$

12. Given: $(x_1, y_1) = (-1, 3)$; $(x_2, y_2) = (-8, -4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-3}{-8-(-1)} = \frac{-7}{-7} = 1$$

13. Given: $(x_1, y_1) = (4, -5)$; $(x_2, y_2) = (4. -8)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8-(-5)}{4-4}$$

Since $x_2 - x_1 = 4 - 4 = 0$, the slope is undefined.

14. Given: $(x_1, y_1) = (-3, 7)$; $(x_2, y_2) = (2, 10)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-7}{2-(-3)} = \frac{3}{5}$$

15. Given: $(x_1, y_1) = (-12, 20)$; $(x_2, y_2) = (32, -13)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-13-20}{32-(-12)} = \frac{-33}{44} = -\frac{3}{4}$$

16. Given: $(x_1, y_1) = (23, -9)$; $(x_2, y_2) = (-25, 11)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11-(-9)}{-25-23} = \frac{20}{-48} = -\frac{5}{12}$$

17. Given: $(x_1, y_1) = (\sqrt{32}, -\sqrt{18})$;
$(x_2, y_2) = (-\sqrt{50}, \sqrt{8})$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\sqrt{8} - (-\sqrt{18})}{-\sqrt{50} - \sqrt{32}} = \frac{-5}{9}$$

18. $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-\pi - (-\pi)}{-2e - e}$$

$m = 0$

19. Given: $(x_1, y_1) = (1.22, -3.45)$;
$(x_2, y_2) = (-1.07, -5.16)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5.16 - (-3.45)}{-1.07 - 1.22}$$
$$= \frac{-1.71}{-2.29} = 0.747$$

20. Given: $(x_1, y_1) = (-5.6, 2.3)$; $(x_2 - y_2) = (8.2, -7.5)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7.5 - 2.3}{8.2 - (-5.6)} = \frac{-9.8}{13.8} = -0.71$$

21. Given: $\alpha = 30°$; $m = \tan\alpha, 0° < \alpha < 180°$

$$\tan 30° = \frac{\sqrt{3}}{3} \text{ or } \frac{1}{3}\sqrt{3}$$

22. Given: $\alpha = 62.5°$; $\tan\alpha$; $0° < \alpha < 180°$
$m = \tan 62.5° = 1.92$

23. Given: $\alpha = 132.7°$, $m = \tan\alpha$; $0° < \alpha < 180°$
$m = \tan 132.7° = -1.084$

24. Given: $\alpha = 135°$; $m = \tan\alpha; 0° < \alpha < 180°$
$m = \tan 135° = -1$

25. Given: $m = 0.364$; $m = \tan\alpha$; $0.364 = \tan\alpha$;
$\alpha = 20.0°$

26. Given: $m = 0.824$; $m = \tan\alpha$
$0.824 = \tan\alpha$; $\alpha = 39.5°$

27. Given: $m = -6.691$; $0° < \alpha < 180°$; (negative quadrant two)
$m = \tan\alpha$
$-6.691 = \tan\alpha$; $\alpha = 180° - 81.500°$; $\alpha = 98.50°$

28. Given: $m = -1.428$; $m = \tan\alpha$ (α in quadrant two)
$-1.428 = \tan\alpha$; $\alpha = (180° - 55°) = 125°$

29. Given: $(x, y) = (6, -1)$; $(x_1, y_1) = (4, 3)$
$(x_2, y_2) = (-5, 2)$; $(x_3, y_3) = (-7, 6)$

$$m_1 = \frac{y - y_1}{x - x_1} = \frac{-1 - 3}{6 - 4} = \frac{-4}{2} = -2$$
$$m_2 = \frac{y_2 - y_3}{x_2 - x_3} = \frac{2 - 6}{-5 - (-7)} = \frac{-4}{-5 + 7}$$
$$= \frac{-4}{2} = -2$$

$m_3 = m_2$ for all parallel lines.

30. Given: $(x_1, y_1) = (-3, 9)$; $(x_2, y_2) = (4, 4)$
$(x_3, y_3) = (9, -1)$; $(x_4, y_4) = (4, -8)$

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 9}{4 - (-3)}$$
$$= \frac{-5}{7} = -\frac{5}{7}$$
$$m_2 = \frac{y_4 - y_3}{x_4 - x_3} = \frac{-8 - (-1)}{4 - 9}$$
$$= \frac{-7}{-5} = \frac{7}{5}$$

m_1 is negative reciprocal of m_2; therefore, lines are perpendicular.

31. Given: $(x_1, y_1) = (-1, -4)$; $(x_2, y_2) = (2, 3)$ line 1
$(x_3, y_3) = (-5, 2)$; $(x_4, y_4) = (-19, 8)$ line 2

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-4)}{2 - (-1)} = \frac{7}{3}$$
$$m_2 = \frac{y_4 - y_3}{x_4 - x_3} = \frac{8 - 2}{-19 - (-5)}$$
$$= \frac{6}{-14} = -\frac{3}{7}$$

m_1 is negative reciprocal of m_2; therefore, lines are perpendicular.

32. Given: $(x_1, y_1) = (-1, -2)$; $(x_2, y_2) = (3, 6)$
$(x_3, y_3) = (2, -6)$; $(x_4, y_4) = (5, 0)$

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-2)}{3 - (-1)} = \frac{8}{4} = 2$$
$$m_2 = \frac{y_4 - y_3}{x_4 - x_3} = \frac{0 - (-6)}{5 - 2} = \frac{6}{3} = 2$$

$m_1 = m_2$; therefore, lines are parallel.

33. Given: distance between $(-1,3)$ and $(11,k)$ is 13.

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$
$$13=\sqrt{(-1-11)^2+(3-k)^2}$$
$$=\sqrt{(-12)^2+(3-k)^2}$$
$$=\sqrt{144+(3-k)^2}$$
$$169=144+(3-k)^2;$$
$$(3-k)^2=25;\ 3-k=\pm5$$
$$-k=-3\pm5;\ k=-2,\ 8$$

34. Given: $(x_1,y_1)=(k,0)$; $(x_2,y_2)=(0,2k)$

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2};\ d=10$$
$$10=\sqrt{(0-k)^2+(2k-0)^2}=\sqrt{(-k)^2+(2k)^2}$$
$$=\sqrt{k^2+4k^2}=\sqrt{5k^2}$$
$$100=5k^2;\ k^2=20;\ k=\pm\sqrt{20}$$
$$k=\pm\sqrt{4\times5}=\pm2\sqrt{5}$$

35. Given $(6,-1),(3,k)$; $(-3,-7)$ on same line, therefore slope constant between points.
$(x_1,y_1)=(6,-1)$; $(x_2,y_2)=(-3,-7)$
Slope between these two points is:

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-7-(-1)}{-3-6}=\frac{-6}{-9}=\frac{2}{3}$$

Therefore, slope between $(6,-1)$ and $(3,k)$ must be $\frac{2}{3}$.

$(x_1,y_1)=(6,-1)$; $(x_3,y_3)=(3,k)$

$$m=\frac{y_3-y_1}{x_3-x_1};\ \frac{2}{3}=\frac{k-(-1)}{3-6};\ \frac{2}{3}=\frac{k+1}{-3};$$
$$-6=3(k+1)=3k+3;\ 3k=-9;\ k=-3$$

36. Given: $A(6,-1)=(x_1,y_1)$; $B(3,k)=(x_2,y_2)$; $C(-3-7)=(x_3,y_3)$
By Pythagorus:

(hypotenuse)^2 = (opposite)^2 + (adjacent)^2
Hypotenuse is opposite $(3,k)$.

$$(x_3-x_1)^2+(y_3-y_1)^2$$
$$=(x_2-x_1)^2+(y_2-y_1)^2+(x_3-x_2)^2+(y_3-y_2)^2$$
$$(-3-6)^2+(-7-(-1))^2$$
$$=(3-6)^2+(k-(-1))^2+(-3-3)^2+(-7-k)^2$$
$$(-9)^2+(-6)^2=(-3)^2+(k+1)^2+(-6)^2+(-7-k)^2$$
$$81+36=9+(k+1)^2+36+(-1(7+k))^2$$
$$81+36-9-36=k^2+2k+1+49+14k+k^2$$
$$72=2k^2+16k+50;\ 2k^2+16k-22=0$$
$$k^2+8k-11=0$$

Completing the square.

$$k^2+8k+16=11+16;\ (k+4)^2=27;\ k+4=\pm\sqrt{27}$$
$$k=-4\pm\sqrt{27}$$
$$k=-4\pm3\sqrt{3}$$

37. $d_1=\sqrt{(9-7)^2+[4-(-2)]^2}$

$$=\sqrt{2^2+6^2}=\sqrt{40}=2\sqrt{10}$$
$$d_2=\sqrt{(9-3)^2+(4-2)^2}=\sqrt{6^2+2^2}$$
$$=\sqrt{40}=2\sqrt{10}$$

$d_1=d_2$ so the triangle is isosceles.

38. Given: $A(-1,3)=(x_1,y_1)$; $B(3,5)=(x_2,y_2)$; $C(5,1)=(x_3,y_3)$

Therefore, by inspection (3, 5) appears to be a right angle. Therefore, if slopes for AB and BC are negative reciprocals, we have a right triangle.

$$m_1=\frac{y_2-y_1}{x_2-x_1}=\frac{5-3}{3-(-1)}=\frac{2}{4}=\frac{1}{2}$$
$$m_2=\frac{y_3-y_2}{x_3-x_2}=\frac{1-5}{5-3}=\frac{-4}{2}=-2$$

m_1 is negative reciprocal of m_2, therefore a right triangle.

39. Given: $A(-5,-4)$, $B(7,1)$, $C(10,5)$, $D(-2,0)$.
By inspection, if slopes of AD and BC are equal and slope of AB and DC are equal, then a parallelogram is formed.
Let $(x_1,y_1)=(-5-4)$; $(x_2,y_2)=(7,1)$
$(x_3,y_3)=(10,5)$; $(x_4,y_4)=(-2,0)$

$$\text{Slope AD}=\frac{0-(-4)}{-2-(-5)}=\frac{4}{3}$$
$$\text{Slope BC}=\frac{5-1}{10-7}=\frac{4}{3};\ \frac{4}{3}=\frac{4}{3}$$
$$\text{Slope AB}=\frac{1-(-4)}{7-(-5)}=\frac{5}{12}$$
$$\text{Slope DC}=\frac{5-0}{10-(-2)}=\frac{5}{12};\ \frac{5}{12}=\frac{5}{12}$$

Therefore, a parallelogram is formed.

40. Given: $A(-5,6)=(x_1,y_1)$
$B(0,8)=(x_2,y_2)$
$C(-3,1)=(x_3,y_3)$
$D(2,3)=(x_4,y_4)$

We have a square if four sides are of equal length and four angles are right angles.

$$d_1=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(0-(-5))^2+(8-6)^2}$$
$$=\sqrt{(5)^2+(2)^2}=\sqrt{25+4}=\sqrt{29}$$
$$d_2=\sqrt{(x_3-x_1)^2+(y_3-y_1)^2}$$
$$=\sqrt{(-3-(-5))^2+(1-6)^2}$$
$$=\sqrt{(2)^2+(-5)^2}=\sqrt{4+25}$$
$$=\sqrt{29}$$

$$\begin{aligned} d_3 &= \sqrt{(x_4-x_2)^2+(y_4-y_2)^2} \\ &= \sqrt{(2-0)^2+(3-8)^2} \\ &= \sqrt{(2)^2+(-5)^2} = \sqrt{4+25} \\ &= \sqrt{29} \\ d_4 &= \sqrt{(x_4-x_3)^2+(y_4-y_3)^2} \\ &= \sqrt{2-(-3))^2+(3-1)^2} \\ &= \sqrt{(5)^2+(2)^2} = \sqrt{25+4} \\ &= \sqrt{29} \end{aligned}$$

$$m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{2}{5};\ m_2 = \frac{y_3-y_1}{x_3-x_1} = \frac{-5}{2} = -\frac{5}{2}$$

$$m_3 = \frac{y_4-y_2}{x_4-x_2} = \frac{-5}{2} = -\frac{5}{2};\ m_4 = \frac{y_4-y_3}{x_4-x_3} = \frac{2}{5}$$

Above conditions satisfied; therefore, we have a square.

41. $$\begin{aligned} d_1 &= \sqrt{(3-5)^2+(-1-3)^2} = \sqrt{(-2)^2+(-4)^2} \\ &= \sqrt{4+16} = \sqrt{20} \end{aligned}$$

$$m_1 = \frac{y-y_1}{x-x_1} = \frac{5-3}{3-(-1)} = \frac{5-3}{3+1} = \frac{2}{4} = \frac{1}{2}$$

$$\begin{aligned} d_2 &= \sqrt{(5-1)^2+(3-5)^2} = \sqrt{(4)^2+(-2)^2} \\ &= \sqrt{16+4} = \sqrt{20} \end{aligned}$$

$$m_2 = \frac{y-y_1}{x-x_1} = \frac{5-1}{3-5} = \frac{4}{-2} = -2$$

$$m_1 = \frac{-1}{m_2},\ m_1 \perp m_2$$

$$\begin{aligned} A &= \frac{1}{2}d_1d_2 = \frac{1}{2}\sqrt{20}\sqrt{20} \\ &= \frac{1}{2}(20) = 10 \end{aligned}$$

42. Area of square = (side)^2
Area = $(\sqrt{29})^2 = 29$ square units

43. Given A(2, 3); B(4, 9); C(−2, 7)
The perimeter $p = d_1 + d_2 + d_3$
Let $(x_1, y_1) = (2, 3)$; $(x_2, y_2) = (4, 9)$; $(x_3, y_3) = (-2, 7)$

$$\begin{aligned} d_1 &= \sqrt{(4-2)^2+(9-3)^2} \\ &= \sqrt{2^2+6^2} \\ &= \sqrt{40} = 2\sqrt{10} \\ d_2 &= \sqrt{(-2-2)^2+(7-3)^2} \\ &= \sqrt{(-4)^2+4^2} \\ &= \sqrt{32} = 4\sqrt{2} \\ d_3 &= \sqrt{(-2-4)^2+(7-9)^2} \\ &= \sqrt{(-6)^2+(-2)^2} = \sqrt{40} \\ &= 2\sqrt{10} \end{aligned}$$

Perimeter $= 2\sqrt{10} + 4\sqrt{2} + 2\sqrt{10}$
$p = 4\sqrt{10} + 4\sqrt{2} = 4(\sqrt{10} + \sqrt{2}) = 18.3$

44. Given: $A(-5,-4) = (x_1, y_1)$
$B(7,1) = (x_2, y_2)$
$C(10,5) = (x_3, y_3)$
$D(-2,0) = (x_4, y_4)$

Perimeter $= 2d_1 + 2d_2$

$$\begin{aligned} d_1 &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(7-(-5))^2+(1-(-4))^2} \\ &= \sqrt{(12)^2+(5)^2} \\ &= \sqrt{169} \\ &= 13 \\ d_2 &= \sqrt{(x_4-x_1)^2+(y_4-y_1)^2} \\ &= \sqrt{(-2)-(-5))^2+(0-(-4))^2} \\ &= \sqrt{(3)^2+(4)^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

Perimeter $= 2(13+5) = 2(18) = 36$ units

45. $$\left(\frac{-4+6}{2}, \frac{9+1}{2}\right) = \left(\frac{2}{2}, \frac{10}{2}\right) = (1, 5)$$

46. Given: $(x_1, y_1) = (-1, 6)$; $(x_2, y_2) = (-13, -8)$

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-1+(-13)}{2}, \frac{6+(-8)}{2}\right) = (-7, -1)$$

47. Given: A(−12.4, 25.7); B(6.8, −17.3)
Midpoint A and B

$$x = \frac{x_1+x_2}{2} = \frac{-12.4+6.8}{2} = -2.8$$

$$y = \frac{y_1+y_2}{2} = \frac{25.7+(-17.3)}{2} = 4.2$$

(−2.8, 4.2)

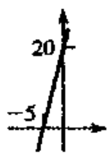
48. Given: $(x_1 y_1) = (2.6, 5.3)$; $(x_2, y_2) = (-4.2, -2.7)$

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{2.6+(-4.2)}{2}, \frac{5.3+(-2.7)}{2}\right) = (-0.8, 1.3)$$

2.2 The Straight Line

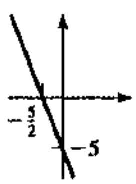
1. Given: $m = 4$; $(x_1, y_1) = (-3, 8)$

$y - y_1 = m(x - x_1)$
$y - 8 = 4[x - (-3)] = 4(x+3) = 4x + 12$
$y = 4x + 20$ or $4x - y + 20 = 0$



2. Given: $(x_1, y_1) = (-2, -1)$; $m = -2$

$y - y_1 = m(x - x_1)$
$y - (-1) = -2(x - (-2))$
$y + 1 = -2(x+2)$
$y + 1 = -2x - 4$
$y = -2x - 5$ or $2x + y + 5 = 0$



3. Given: $(x_1, y_1) = (2, -5)$; $(x_2, y_2) = (4, 2)$

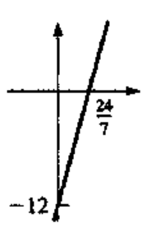
$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{y - (-5)}{x - 2} = \frac{2 - (-5)}{4 - 2}$$

$$\frac{y+5}{x-2} = \frac{7}{2};$$

$2(y+5) = 7(x-2)$; $2y + 10 = 7x - 14$; $y = \frac{7}{2}x - 12$

or $7x - 2y - 24 = 0$

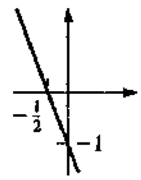


4. Given: $(x_1, y_1) = (-3, 5)$; $(x_2, y_2) = (-2, 3)$

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

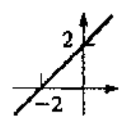
$$\frac{y - 5}{x - (-3)} = \frac{3 - 5}{-2 - (-3)} = \frac{-2}{1}$$

$y - 5 = -2(x+3)$; $y - 5 = -2x - 6$; $y = -2x - 1$
or $2x + y + 1 = 0$

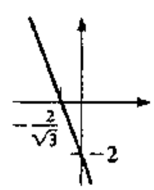


5. Given: $(x_1, y_1) = (1, 3)$; $\alpha = 45°$

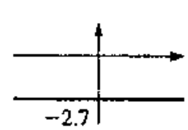
$m = \tan\alpha = \tan 45° = 1$
$y - y_1 = m(x - x_1)$
$y - 3 = 1(x - 1) = x - 1$; $y = x + 2$
or $x - y + 2 = 0$



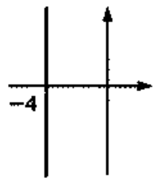
6. $\alpha = 120°$; $m = \tan\alpha$
$m = \tan 120°$
$m = -\sqrt{3}$ $\quad (x_1, y_1) = (0, -2)$
$y - y_1 = m(x - x_1)$
$y - (-2) = -\sqrt{3}(x - 0)$
$y + 2 = -\sqrt{3}x$
$y = -\sqrt{3}x - 2$ or $\sqrt{3}x + y + 2 = 0$



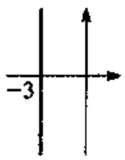
7. $m = 0$
$y - y_1 = m(x - x_1)$
$y - (-2.7) = 0(x - 5.3)$
$y + 2.7 = 0$
$y = -2.7$



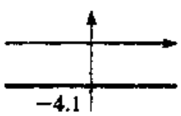
8. Slope is undefined
Therefore, $x = -4$



9. Parallel to y-axis and
3 units left of y-axis.
$x = -3$



10. $m = 0$; $(x_1, y_1) = (0, -4.1)$
Therefore, $y = -4.1$



11. $(x_1, y_1) = (0, -6)$; $(x_2, y_2) = (4, 0)$

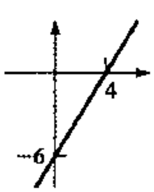
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-6)}{4 - 0}$$
$$= \frac{6}{4} = \frac{3}{2}$$

$$y - y_1 = m(x - x_1)$$

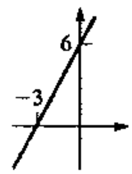
$$y - (-6) = \frac{3}{2}(x - 0)$$

$$y + 6 = \frac{3}{2}x$$

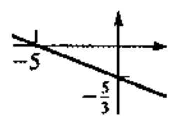
$$y = \frac{3}{2}x - 6 \text{ or } 3x - 2y - 12 = 0$$



12. $(x_1, y_1) = (-3, 0)$; $m = 2$
$y - y_1 = m(x - x_1)$
$y - 0 = 2(x - (-3))$
$y = 2(x + 3)$
$y = 2x + 6$ or $2x - y + 6 = 0$



13. Perpendicular to line with slope 3;
$(x_1, y_1) = (1, -2)$
$y - y_1 = (x - x_1); y - (-2) = -\frac{1}{3}(x - 1)$
$y + 2 = -\frac{1}{3}x + \frac{1}{3}; y = -\frac{1}{3}x - \frac{5}{3}$
or $-\frac{1}{3}x - y - \frac{5}{3} = 0; x + 3y + 5 = 0$

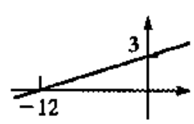


14. Perpendicular to line with $m = -4$
Therefore, slope of line $= \dfrac{1}{4}$
$(x_1, y_1) = (0, 3)$
$y - y_1 = m(x - x_1)$

$$y - 3 = \frac{1}{4}(x - 0)$$

$$y - 3 = \frac{1}{4}x$$

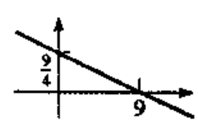
$$y = \frac{1}{4}x + 3 \text{ or } x - 4y + 12 = 0$$



15. Parallel to line with $m = -\dfrac{1}{4}$
Therefore, slope of line $= -\dfrac{1}{4}$
$(x_1, y_1) = (1, 2)$
$y - y_1 = m(x - x_1)$

$$y - 2 = -\frac{1}{4}(x - 1)$$

$x + 4y - 9 = 0$

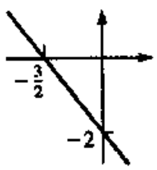


16. Parallel to a line through $(7,-1)$ and $(4,3)$; y-intercept is -2

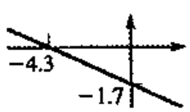
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-1)}{4-7} = \frac{4}{-3} = -\frac{4}{3}$$

y-intercept is $b = -2$

$$y = mx + b;\ y = -\frac{4}{3}x - 2;\ 4x + 3y + 6 = 0$$



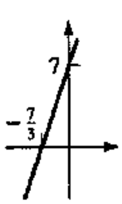
17. Given: $6.0x - 2.4y - 3.9 = 0$
or $y = 2.5x - 1.625$
Perpendicular to this line
Therefore, slope of desired line
$= -\frac{1}{2.5} = -0.4 = m$
$(x_1, y_1) = (7.5, -4.7)$
$y - y_1 = m(x - x_1)$
$y - (-4.7) = -0.4(x - 7.5)$
$y + 4.7 = -0.4x + 3$
$y = -0.4x - 1.7$ or $0.4x + y + 1.7 = 0$



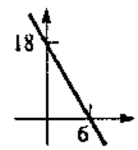
18. Given: $2y - 6x - 5 = 0$ or $y = 3x + \frac{5}{2}$

Parallel to this line
Therefore, slope of desired line $= 3 = m$

$(x_1, y_1) = (-4, -5)$
$y - y_1 = m(x - x_1)$
$y - (-5) = 3(x - (-4))$
$y + 5 = 3(x + 4)$
$y + 5 = 3x + 12$
$y = 3x + 7$ or $3x - y + 7 = 0$



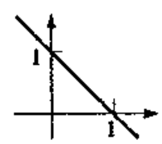
19. $5x - y = 6$ and $x + y = 12$ intersect at $(3, 9)$
$y - y_1 = m(x - x_1)$
$y - 9 = -3(x - 3)$
$y - 9 = -3x + 9$
$3x + y - 18 = 0$



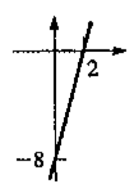
20. $2x + y - 3 = 0$ and $x - y - 3 = 0$ intersect at $(2, -1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-(-1)}{4-2} = -1$$

$y - y_1 = m(x - x_1)$
$y - (-1) = -1(x - 2)$
$y + 1 = -x + 2$
$x + y - 1 = 0$

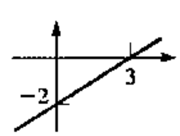


21. Given: $4x - y = 8, y = 4x - 8, m = 4, b = -8$
When $x = 0, y = -8$
$y = 0, x = 2$



22. Given: $2x - 3y - 6 = 0, y = \frac{2}{3}x - 2, m = \frac{2}{3}, b = -2$

When $x = 0, y = -2$
$y = 0, x = 3$

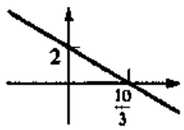


23. Given: $3x + 5y - 10 = 0, y = \frac{-3}{5}x + 2,$

$m = \frac{-3}{5}, b = 2$

When $x = 0, y = 2$

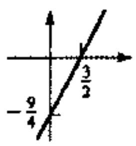
$y = 0, x = \frac{10}{3}$



24. Given: $4y = 6x - 9, y = \frac{3}{2}x - \frac{9}{4}, m = \frac{3}{2}, b = \frac{-9}{4}$

When $x = 0, y = -\frac{9}{4}$

$y = 0, x = \frac{9}{6} = \frac{3}{2}$



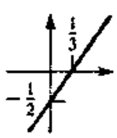
25. Given: $3x - 2y - 1 = 0$

$3x - 2y - 1 = 0; -2y = -3x + 1$

$y = \frac{-3}{-2}x + \frac{1}{-2}; y = \frac{3}{2}x - \frac{1}{2}$

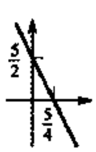
Slope $= \frac{3}{2} = m;$

y-intercept $= -\frac{1}{2} = b$

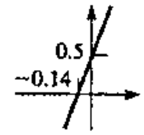


26. Given: $4x+2y-5 = 0; 2y = -4x+5; y = -2x+\frac{5}{2}$

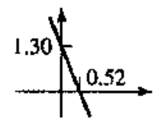
Slope $= -2$; y-intercept $= \frac{5}{2}$



27. Given: $11.2x - 3.2y + 1.6 = 0; 3.2y = 11.2x + 1.6$
$y = 3.5x + 0.5$
$m = 3.5$; y-intercept $= 0.5$



28. Given: $11.5x + 4.60y - 5.98 = 0$
$4.60y = -11.5x + 5.98; y = -2.50x + 1.30$
$m = -2.50$; y-intercept $= 1.30$



29. Given: $4x - ky = 6 \parallel 6x + 3y + 2 = 0$
$6x + 3y + 2 = 0; 3y = -6x - 2$
$y = \frac{-6}{3}x - \frac{2}{3}; y = -2x - \frac{2}{3}$; slope is -2

$4x - ky = 6; -ky = -4x + 6$

$y = \frac{-4}{-k}x + \frac{6}{-k}; y = \frac{4}{k}x - \frac{6}{k}$; slope is $\frac{4}{k}$

Since the lines are parallel, the slopes are equal.

$\frac{4}{k} = -2; 4 = -2k; k = -2$

30. Given: $4x - ky = 6 \perp$ to $6x + 3y + 2 = 0$

$6x + 3y + 2 = 0;\ 3y = -6x - 2;\ y = -2x - \frac{2}{3}$

$m_1 = -2$

Therefore, slope of other line $= \frac{1}{2} = m$

$4x - ky = 6;\ ky = 4x - 6;\ y = \frac{4}{k}x - \frac{6}{k}, \frac{4}{k} = \frac{1}{2}$

Therefore, $k = 8$

31. Given: $3x - y - 9 = 0 \perp$ to $kx + 3y = 5$
Slope of line $3x - y - 9$ is 3

Therefore, slope of line $kx + 3y = 5$ is $-\frac{1}{3}$

Slope of line $kx + 3y = 5$ is $-\frac{k}{3}$

Therefore, $k = 1$

32. Slope of line $3x - y - 9 = 0$ is 3
Therefore, slope of line $kx + 3y = 5$ is also 3

Therefore, $-\frac{k}{3} = 3, k = -9$

33. $3x - 2y + 5 = 0; -2y = -3x - 5;$

$y = \frac{-3}{-2}x + \frac{-5}{-2}; y = \frac{3}{2}x + \frac{5}{2};$

slope $= \frac{3}{2} = m_1$

$4y = 6x - 1; y = \frac{6}{4}x - \frac{1}{4};$

$y = \frac{3}{2}x - \frac{1}{4}$; slope $= \frac{3}{2} = m_2$

$m_1 = m_2$ for all parallel lines.

34. $8x - 4y + 1 = 0 \Rightarrow y = 2x + \frac{1}{4}, m_1 = 2$

$4x + 2y - 3 = 0 \Rightarrow y = -2x + \frac{3}{2}, m_2 = -2$

Neither perpendicular or parallel

35. Line one: $6x - 3y - 2 = 0$; $m_1 = 2$; negative reciprocal

Line two: $x + 2y - 4 = 0$; $m_2 = -\frac{1}{2}$

Therefore, lines are perpendicular.

36. Line one: $3y - 2x = 4$; $m_1 = \frac{2}{3}$; $m_1 = m_2$

Line two: $6x - 9y = 5$; $m_2 = \frac{6}{9} = \frac{2}{3}$

Therefore, lines are parallel.

37. $5x + 2y = 3 \Rightarrow y = \frac{-5}{2} \cdot x + \frac{3}{2}$

$10y = 7 - 4x \Rightarrow y = \frac{-4}{10}x + \frac{7}{10}$

$m_1 m_2 = \frac{-5}{2}\left(\frac{-4}{10}\right) = 1 \neq -1$

$m_1 \neq m_2$

Lines are neither perpendicular nor parallel.

38. $48y - 36x = 71 \Rightarrow y = \frac{3}{4}x + \frac{71}{48}, m_1 = \frac{3}{4}$

$52x = 17 - 39y \Rightarrow y = -\frac{4}{3}x + \frac{17}{39}, m_2 = -\frac{4}{3}$

$m_1 m_2 = \frac{3}{4}\left(\frac{-4}{3}\right) = -1$, perpendicular

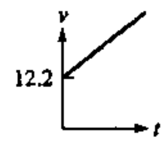
39. Line one: $4.5x - 1.8y = 1.7$; $m_1 = 2.5$; negative reciprocal
Line two: $2.4x + 6.0y = 0.3$; $m_2 = -0.4$
Therefore, lines are perpendicular.

40. $3.5y = 4.3 - 1.5x \Rightarrow y = -\frac{3}{7}x + \frac{43}{35}, m_1 = \frac{-3}{7}$

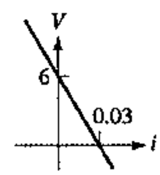
$3.6x + 8.4y = 1.7 \Rightarrow y = -\frac{3}{7}x + \frac{17}{84}, m_2 = -\frac{3}{7}$

Therefore lines are parallel

41. $v = v_0 + at$
$35.4 = 12.2 + a(4.50)$
$4.50a = 35.4 - 12.2$
$a = 5.16 \text{ ft/s}^2$
$v = 12.2 + 5.16t$



42. $9.17 \text{ mA} = 9.17 \times 10^{-3} \text{ A}$
$V = E - iR$
$4.35 = 6.00 - 9.17 \times 10^{-3} \text{ R}$
$R = 180\ \Omega$
Therefore, $V = 6.00 - 180i$



43. $v = mT + b, 343 = 0.607(20°) + b \Rightarrow b = 331$
$v = 0.607T + 331$

44. $0.20x + 0.30y = 20$; therefore, $2x + 3y = 200$

45. $50x + 60y = 12,200; 5x + 6y = 1220$

46. $x = 0$; $T = 3\ °\text{C}$
$T = kx + T_1$
$3 = 0 + T_1$
$T = kx + 3$
$x = 15$ cm; $T = 23\ °\text{C}$
$23° = k(15) + 3$
$k = \frac{20}{15} = \frac{4}{3}\ °\text{C/cm}$, slope = change in temperature in °C per cm.
Therefore, $T = \frac{4}{3}x + 3$

47. $0.72x + 0.90y = 135,000$

$y = \frac{135,000 - 0.72x}{0.90}; y = 150,000 - 0.80x$

48. $l = w + 10$
$p = 2w + 2l$
$p = 2w + 2(w + 10)$
$p = 4w + 20$, slope $= 4$. Therefore each one cm change in width produces a 4 cm change in perimeter.

49. $m = \tan(180° - 0.0032°)$
$b = 24\mu\text{m} = 24 \times 10^{-6}\text{ m} = 2.4 \times 10^{-5}\text{ m}$
$m = -5.6 \times 10^{-5}$
$y = mx + b = -5.6 \times 10^{-5}x + 2.4 \times 10^{-5}$
$y = (-5.6x + 2.4)10^{-5}$

50. $m = \tan\alpha$
$m = \tan(90° - 16.5°)$
$m = \tan 73.5°$
$m = 3.38$

Therefore, $y = 3.38x$

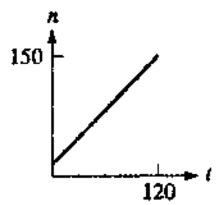
51. Start is 6:30 = 0 min
Therefore, $(t_1n_1) = (30, 45)$; $(t_2n_2) = (90, 115)$

$$m = \frac{n_2 - n_1}{t_2 - t_1} = \frac{115 - 45}{90 - 30} = \frac{7}{6}$$

$$n - n_1 = m(t - t_1)$$

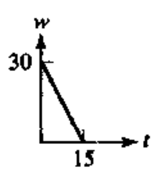
$$n - 45 = \frac{7}{6}(t - 30)$$

$$n - 45 = \frac{7}{6}t - 35;\ n = \frac{7}{6}t + 10$$



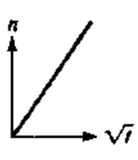
Therefore, n at 6:30 $(t = 0) = 10$; at 8:30 $(t = 120) = 150$

52. $t = 0$ month: $w = 30$ mg
Rate of growth is -2 mg/month
Therefore, $w = -2t + 30$



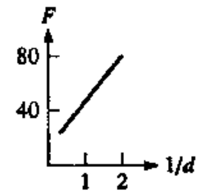
53. $n = 1200\sqrt{t} + 0$
$m = 1200$
$b = 0$

t	$\sqrt{t}$	h
0	0	0
1	1	1200
4	2	2400



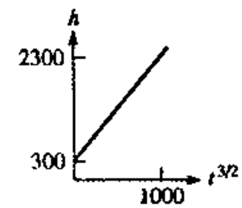
54. $F = \dfrac{40}{d} = 40\left(\dfrac{1}{d}\right)$

d	1	2	$\frac{1}{2}$
$\frac{1}{d}$	1	$\frac{1}{2}$	2
F	40	20	80



55. $h = 300 + 2t^{3/2}$
Let $t_1 = t^{3/2}$
$h = 300 + 2t_1$

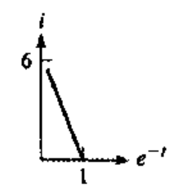
t	0	100
t_1	0	1000
h	300	2300



$0 \le t < 100$

56. $i = 6(1 - e^{-t})$
Let $e^{-t} = t_1$
$i = 6 - 6t_1$

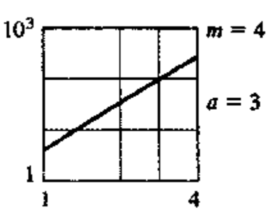
t_1	0	1
i	6	0



(e^{-t} never equals zero)

57. Slope is found by measuring between points. The vertical displacement and the horizontal displacement between the extreme points is in a 1 to 2 ratio; $m = \frac{1}{2}$.
Since the graph is linear, the log equation is of the form $\log y = m\log x + \log a$, where a is the intercept $(1, a)$.
$y = ax^n$; $y = 3x^4$
$a = 3, n = 4$

x	y
1.0	3.0
1.1	4.4
1.2	6.2
1.3	8.6



$\log y = \log a + n\log x$
$\log y = \log 3 + 4\log x$
Verify

(1) Slope is $\dfrac{\log y - \log a}{\log x} = 4.$

Vertical and horizontal meaasures in millimeter between points are shown. Each slope is 4.

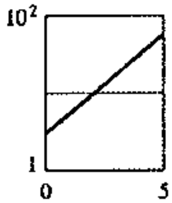
(2) The intercept is $a = 3$.
The line crosses the vertical axis at $x = 1.0$, $y = 3.0$.

58. $y = a(b^x)$
$\log y = \log a + x \log b$; straight line form
Given: $y = 3(2^x)$
Intercept: $x = 0$
$\log a = \log y$; $a = y$

Slope: $\log b = \dfrac{\log y - \log a}{x}$;

$\log b = m$

x	y
0	3
1	6
2	12
3	24
4	48
5	96



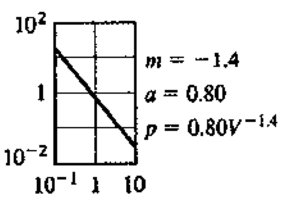
$\log y = \log 3 + x \log 2$
Intercept on graph is $3 = a$.
Slope: use $(3, 24)$

$$m = \log b = \frac{\log 24 - \log 3}{3} = \frac{\log 8}{3} = 0.3010\ldots$$

$b = 2$
$m = 0.3010\ldots$

59. Log paper, therefore $y = ax^n$;
$p = aV^n$; $\log p = \log a + n \log V$
From graph: When $V = 1, a = 0.8$
Slope is negative, therefore, $n < 0$.
Using typical log paper and measuring horizontal and vertical distances:

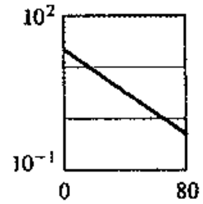
$$m = \frac{-7.2\text{ cm}}{5.1\text{ cm}} = -1.4$$



60. Semilog paper, therefore, $y = a(b^x)$
$v = a(b^t)$; $\log v = \log a + t \log b$
$t = 0$; $v = 40$, therefore, $a = 40$
Use point $(20, 15)$ to solve for b.

$$\log 15 = \log 40 + 20 \log b;\ \log b = \frac{\log 15 - \log 40}{20}$$

$\log b = -0.0213 = m$; $b = 0.952$
Therefore, $v = 40(0.952)^t$



2.3 The Circle

1. $(x-2)^2 + (y-1)^2 = 25$
Center at $(2, 1)$, radius is 5.

2. $(x-3)^2 + (y+4)^2 = 49$
Center at $(3, -4)$, radius is 7.

3. $(x+1)^2 + y^2 = 4$
Center at $(-1, 0)$, radius is 2.

4. $x^2 + (y-6)^2 = 64$
Center at $(0, 6)$, radius is 8.

5. $(x-h)^2 + (y-k)^2 = r^2; C(0,0), r = 3$
$(x-0)^2 + (y-0)^2 = 3^2; x^2 + y^2 = 9$

6. $(x-h)^2 + (y-k)^2 = r^2\ \ C(0,\ 0), r = 1$
$(x-0)^2 + (y-0)^2 = 1^2;\ x^2 + y^2 = 1$

7. $(x-2)^2 + (y-2)^2 = 4^2$; C(2, 2), $r = 4$
$x^2 + y^2 - 4x - 4y - 8 = 0$

8. $(x-0)^2 + (y-2)^2 = 2^2$; C(0, 2), $r = 2$
$x^2 + (y-2)^2 = 4;\ x^2 + y^2 - 4y = 0$

9. $(x-h)^2 + (y-k)^2 = r^2; C(-2, 5), r = \sqrt{5}$
$[x - (-2)]^2 + (y-5)^2 = (\sqrt{5})^2;$
$(x+2)^2 + (y-5)^2 = \sqrt{25}$
$x^2 + 4x + 4 + y^2 - 10y + 25 = 5$
$x^2 + y^2 + 4x - 10y + 4 + 25 - 5 = 0;$
$x^2 + y^2 + 4x - 10y + 24 = 0$

10. $(x-(-3))^2+(y-(-5))^2=(2\sqrt{3})^2$;
$C(-3,-5)$, $r=2\sqrt{3}$
$(x+3)^2+(y+5)^2=12$;
$x^2+y^2+6x+10y+22=0$

11. $(x-12)^2+(y-(-15))^2=18^2$; $C(12,-15)$, $r=18$
$(x-12)^2+(y+15)^2=324$;
$x^2+y^2-24x+30y+45=0$

12. $\left(x-\frac{3}{2}\right)^2+(y-(-2))^2=\left(\frac{5}{2}\right)^2$;

$C\left(\frac{3}{2},-2\right), r=\frac{5}{2}$

$\left(x-\frac{3}{2}\right)^2+(y+2)^2=\frac{25}{4}$;

$4x^2+4y^2-12x+16y=0$

13. $C(2, 1)$, passes through $(4,-1)$
$r^2=(2-4)^2+(1+1)^2=(-2)^2+(2)^2=8$
$(x-h)^2+(y-k)^2=r^2$;
$(x-2)^2+(y-1)^2=8$
$x^2-4x+4+y^2-2y+1=8$;
$x^2+y^2-4x-2y-3=0$

14. $r^2=(-2-(-1))^2+(3-4)^2$;
$r^2=(-1)^2+(-1)^2=2$;
$r=\sqrt{2}$; $C(-1,4)$, passes through $(-2,3)$.
$(x-h)^2+(y-k)^2=r^2$; $(x-(-1))^2+(y-4)^2=2$
$(x+1)^2+(y-4)^2=2$
$x^2+y^2+2x-8y+15=0$

15. Touches x-axis at $(-3,0)$
Therefore, radius is 5.

$(x-(-3))^2+(y-5)^2=5^2$
$(x+3)^2+(y-5)^2=25$
$x^2+y^2+6x-10y+9=0$

16. Touches x-axis at $(0,-4)$
Therefore, radius is 2.

$(x-2)^2+(y-(-4))^2=2^2$
$(x-2)^2+(y+4)^2=4$
$x^2+y^2-4x+8y+16=0$

17. The center is (2,2) and radius is 2.
$(x-h)^2+(y-k)^2=r^2$
$(x-2)^2+(y-2)^2=2^2$
$x^2-4x+4+y^2-4y+4=4$
$x^2+y^2-4x-4y+4=0$

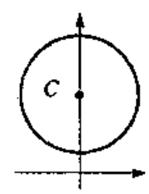
18. Center at $(-4,4)$.
$(x-(-4))^2+(y-4)^2=4^2$
$(x+4)^2+(y-4)^2=16$
$x^2+y^2+8x-8y+16=0$

19. Center on line $5x=2y$; $y=\frac{5}{2}x$; $r=5$

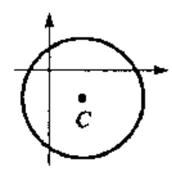
Tangent to x-axis; center at $(2,5)$ or $(-2,-5)$
$(x-2)^2+(y-5)^2=5^2$; $x^2+y^2-4x-10y+4=0$
$(x+2)^2+(y+5)^2=5^2$; $x^2+y^2+4x+10y+4=0$

20. Ends of diameter: $(3,8),(-3,0)$
Diameter $=\sqrt{6^2+8^2}=\sqrt{100}=10$; $r=5$
Therefore, halfway point (center) is at $(0,4)$.
$(x-0)^2+(y-4)^2=(5)^2$
$x^2+(y-4)^2=25$
$x^2+y^2-8y-9=0$

21. $x^2+(y-3)^2=4$ is the same as
$(x-0)^2+(y-3)^2=2^2$, so
Therefore, $h=0, k=3, r=2$
$C(0,3)$



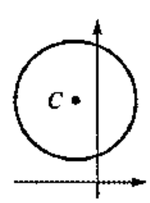
22. $(x-2)^2+(y+3)^2=49$
$(x-h)^2+(y-k)^2=r^2$
Therefore, $h=2, k=-3, r=7$, $C(2,-3)$



23. $4(x+1)^2+4(y-5)^2=81$

$(x+1)^2+(y-5)^2=\frac{81}{4}$

$h=-1, k=5, r=\sqrt{\frac{81}{4}}=\frac{9}{2}$; $C(-1,5)$

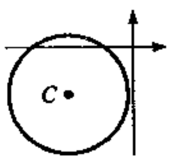


24. $2(x+4)^2 + 2(y+3)^2 = 25$

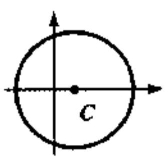
$$(x+4)^2 + (y+3)^2 = \frac{25}{2}$$

$h = -4, k = -3$, $C(-4, -3)$;

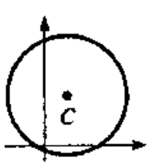
$$r = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$



25. $x^2 + y^2 - 2x - 8 = 0$
$x^2 - 2x + 1 + y^2 = 9$
$(x-1)^2 + (y-0)^2 = 9$
$h = 1, k = 0, r = 3$
$C(1,0)$



26. $x^2 + y^2 - 4x - 6y - 12 = 0$
$x^2 - 4x + y^2 - 6y = 12$
$x^2 - 4x + 4 + y^2 - 6y + 9 = 12 + 13$
$(x-2)^2 + (y-3)^3 = 25$
$h = 2, k = 3, r = 5$; $C(2, 3)$

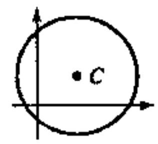


27. $x^2 + y^2 - 4.20x - 2.60y - 3.51 = 0$

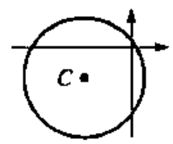
Complete the square by dividing the coefficient of x and y by 2 and squaring.

$x^2 - 4.20x + 4.41 + y^2 - 2.60y + 1.69$
$= 3.51 + 4.41 + 1.69$
$(x - 2.10)^2 + (y - 1.30)^2 = 9.61$
$[x - (-2.10)]^2 + (y - 1.30)^2 = 3.1^2$

Center is $(2.10, 1.30)$; radius is 3.1.



28. $x^2 + y^2 + 22x + 14y = 26$
$x^2 + 22x + 121 + y^2 + 14y + 49 = 26 + 121 + 49$
$(x - (-11))^2 + (y - (-7))^2 = 196 = 14^2$
$C(-11, -7), r = 14$

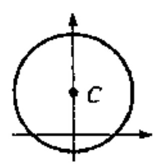


29. $4x^2 + 4y^2 - 16y = 9$
$4(x-0)^2 + 4(y^2 - 4y + 4) = 9 + 16$

$$(x-0)^2 + (y-2) = \frac{25}{4}$$

$$h = 0, k = 2, r = \frac{5}{2}$$

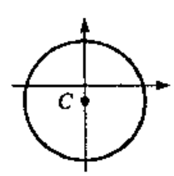
$C(0, 2)$



30. $9x^2 + 9y^2 + 18y = 7$
$9 \cdot x^2 + 9(y^2 + 2y + 1) = 7 + 9 = 16$

$$(x-0)^2 + (y-(-1))^2 = \frac{16}{9} = \left(\frac{4}{3}\right)^2$$

$$C(0, -1), r = \frac{4}{3}$$



31. $2x^2 - 4x + 2y^2 - 8y = 1$

$$x^2 - 2x + y^2 - 4y = \frac{1}{2}$$

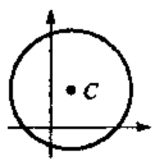
$$x^2 - 2x + 1 + y^2 - 4y + 4 = \frac{1}{2} + 5$$

$$(x-1)^2 + (y-2)^2 = \frac{1}{2} + 5$$

$$(x-1)^2 + (y-2)^2 = \frac{11}{2}$$

$h = 1, k = 2$, C(1, 2)

$$r = \sqrt{\frac{11}{2}} = \frac{\sqrt{22}}{2}$$



32. $3x^2 - 12x + 3y^2 = -4$

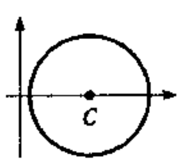
$$x^2 - 4x + y^2 = -\frac{4}{3}$$

$$x^2 - 4x + 4 + y^2 = -\frac{4}{3} + 4$$

$$(x-2)^2 + (y-0)^2 = \frac{8}{3}$$

$h = 2, k = 0$, C(2, 0)

$$r = \sqrt{\frac{8}{3}} = \frac{2\sqrt{6}}{3}$$



33. $(-x)^2 + y^2 = 100$; $x^2 + y^2 = 100$
Symmetrical to y-axis
$x^2 + (-y)^2 = 100$; $x^2 + y^2 = 100$
Symmetrical to x-axis
$(-x)^2 + (-y)^2 = 100$; $x^2 + y^2 = 100$
Symmetrical to origin

34. $x^2 - 4x + y^2 - 5 = 0$; $(-x)^2 - 4(-x) + (-y)^2 - 5 = 0$
$x^2 + 4x + y^2 - 5 = 0$
$(-y)^2 = y^2$
$4x \neq -4x$; therefore, symmetrical to x-axis.

35. Replace x with $-x$ and y with $-y$:

$$3(-x)^2 + 3(-y^2) + 24(-y) = 8$$

$3x^2 + 3y^2 - 24y = 8$, not symmetrical with respect to origin
Replace x with $-x$:

$$3(-x^2) + 3y^2 + 24y = 8$$

$3x^2 + 3y^2 + 24y = 8$, symmetrical with respect to y-axis
Replace y with $-y$:

$$3x^2 + 3(-y)^2 + 24(-y) = 8$$

$3x^2 + 3y^2 - 24y = 8$, not symmetrical with respect to x-axis.

36. Replace x with $-x$ and y with $-y$:

$$5(-x)^2 + 5(-y)^2 - 10(-x) + 20(-y) = 3$$

$5x^2 + 5y^2 + 10x - 20y = 3$, not symmetrical with respect to origin
Replace x with $-x$:

$$5(-x)^2 + 5y^2 - 10(-x) + 20y = 3$$

$5x^2 + 5y^2 + 10x + 20y = 3$, not symmetrical with respect to y-axis.
Replace y with $-y$:

$$5x^2 + 5(-y)^2 - 10x + 20(-y) = 3$$

$5x^2 + 5y^2 - 10x - 20y = 3$, not symmetrical with respect to x-axis.

37. Find all points for which $y = 0$.
$x^2 - 6x + (0)^2 - 7 = 0$; $x^2 - 6x - 7 = 0 \Rightarrow$
$(x+1)(x-7) = 0$; $x = -1$ or $x = 7$;
$(-1, 0)$ and $(7, 0)$

38. Points of intersection occur at same coordinates on both circle and line.
Therefore, substitute $y = x - 1$ into equation of circle.

$x^2 + y^2 - x - 3y = 0$; $x^2 + (x-1)^2 - x - 3(x-1) = 0$
$x^2 + x^2 - 2x + 1 - x - 3x + 3 = 0$; $2x^2 - 6x + 4 = 0$
$x^2 - 3x + 2 = 0$; $(x-2)(x-1) = 0$

Therefore, $x = 2$ and $x = 1$.
Substituting values into $y = x - 1$ gives:

$x = 2, y = 1$; $x = 1, y = 0$
$(2, 1), (1, 0)$ are intersection points.

39. $d_2 = 2d_1$

By Pythagorus:

$$d_2 = \sqrt{(2-x)^2 + (4-y)^2}$$
$$d_1 = \sqrt{(x-0)^2 + (y-0)^2}$$
$$d_2 = 2d_1$$
$$\sqrt{(2-x)^2 + (4-y)^2} = 2\sqrt{x^2+y^2}$$
$$(2-x)^2 + (4-y)^2 = 4(x^2+y^2)$$
$$4 - 4x + x^2 + 16 - 8y + y^2 = 4x^2 + 4y^2$$
$$3x^2 + 4x + 3y^2 + 8y - 20 = 0$$

This is the equation of a circle.

40. $m_1 = -\dfrac{1}{m_2}$

$$m_1 m_2 = -1$$

$$m_1 = \frac{y-0}{x-2} = \frac{y}{x-2}$$

$$m_2 = \frac{y-0}{x-(-2)} = \frac{y}{x+2}$$

$$\left(\frac{y}{x-2}\right)\left(\frac{y}{x+2}\right) = -1$$

$$y^2 = -(x-2)(x+2)$$
$$y^2 = -(x^2-4) = -x^2 + 4$$
$$x^2 + y^2 = 4$$

This is the equation of a circle.

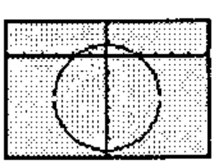
41. $x^2 + y^2 + 5y - 4 = 0$
$y^2 + 5y + (x^2 - 4) = 0$; solve for y

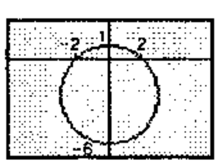
$$y = \frac{-5 \pm \sqrt{5^2 - 4(x^2-4)}}{2} = \frac{-5 \pm \sqrt{41 - 4x^2}}{2}$$
$$= -2.5 \pm \sqrt{10.25 - x^2}$$

Set the range for

$x_{\min} = -6, x_{\max} = 6, y_{\min} = -6, y_{\max} = 2$

$$y_1 = -2.5 + \sqrt{10.25 - x^2}$$
$$y_2 = -2.5 - \sqrt{10.25 - x^2}$$





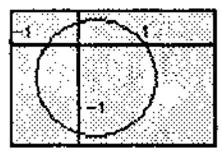
42. $2x^2 + 2y^2 + 2y - x - 1 = 0$; solve for y
$2y^2 + 2y + (2x^2 - x - 1) = 0$

$$y = \frac{-2 \pm \sqrt{2^2 - 4(2)(2x^2 - x - 1)}}{2(2)}$$
$$= \frac{-2 \pm \sqrt{12 + 8x - 16x^2}}{4}$$
$$= -0.5 \pm \sqrt{0.75 + 0.5x - x^2}$$

Set range:

$x_{\min} = -1, x_{\max} = 2, y_{\min} = -1.5, y_{\max} = 0.5$

$$y_1 = -0.5 + \sqrt{0.75 + 0.5x - x^2}$$
$$y_2 = -0.5 - \sqrt{0.75 + 0.5x - x^2}$$



43. $x^2 + y^2 = 14.5$ is a circle with center $(0,0)$,
$r = \sqrt{14.5}$

$x^2 + y^2 - 19.6y + 86 = 0$
$x^2 + y^2 - 19.6y + 96.04 = -86 + 96.04 = 10.04$
$(x-0)^2 + (y-9.8)^2 = 10.04$, circle with center $(0, 9.8)$,
$r = \sqrt{10.04}$
distance between circles $= 9.8 - \sqrt{10.04} - \sqrt{14.5}$
$= 2.82$ in.

44. $x^2 + y^2 = 42.5$; C(0, 0), $r = \sqrt{42.5} = 6.52$
$x^2 + y^2 + 3.06y - 1.24 = 0$; C(0, 1.53), $r = 1.89$

The least distance is d, the distance between the two points as the straight line $x = 0$ (y-axis), cutting the two circles at $(0, x_1)$ and $(0, x_2)$.
$x_1 = 6.52$ and $x_2 = 1.53 + 1.89 = 3.42$

Therefore, $d = 6.52 - 3.42 = 3.10$ in

45. 60 Hz = 60 cycles/s = 37.7 m/s; $(h, k) = (0, 0)$
60 cycles = 37.7 m; 1 cycle = 0.628 m
$r = 0.628 \text{ m} \div 2\pi; r = 0.10$
$x^2 + y^2 = (0.10)^2; x^2 + y^2 = 0.0100$

46. $r_1 = 3960$ mi; $r_2 = r_1 + 22{,}500$ mi; $r_2 = 26{,}460$ mi

By Pythagorus:

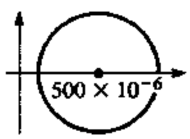
$v^2 = v_H^2 + v_V^2$

$w = \dfrac{1\text{ r}}{24\text{ h}} = \dfrac{2\pi\text{ rad}}{24\text{ h}}$

$v = rw = 26{,}460\left(\dfrac{2\pi}{24}\right) = 6927.2$ mi/h (constant)

Therefore, $v_H^2 + v_V^2 = v^2 = (6927.2)^2 = 4.80\times 10^7$

47.



Center of circle is at (500, 0).
Radius of circle is 300.

$(x-h)^2 + (y-k)^2 = r^2$
$(x-500)^2 + (y-0)^2 = 400^2$
$x^2 - 1000x + 25\times 10^4 + y^2 = 16\times 10^4$
$x^2 + y^2 - 1000x + 9\times 10^4 = 0$
$x^2 + y^2 - 1000\times 10^{-6} + 9\times 10^{-8} = 0$
or $(x - 500\times 10^{-6})^2 + y^2 = (400\times 10^{-6})^2$
$= 0.16\times 10^{-6}$

48. Area of window is made up of area of top semicircle plus area of bottom rectangle.

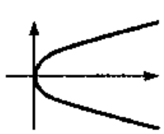
$x^2 + y^2 - 3.00y + 1.25 = 0$
$(x-0)^2 + y^2 - 3.00y + (1.500)^2 = -1.25 + (1.500)^2$
$(x-0)^2 + (y-1.50)^2 = 1$
$h = 0, k = 1.50, r = 1$

Area of semicircle $= \dfrac{\pi r^2}{2} = \dfrac{\pi(1)^2}{2} = \dfrac{\pi}{2}$ m^2

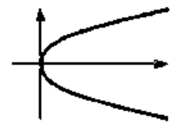
Area of rectangle $= lw = 1.50\times 2(1) = 3.00$ m^2
Total area is 4.57 m^2.

2.4 The Parabola

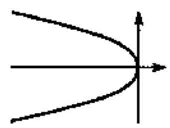
1. $y^2 = 4x$
$y^2 = 4px$
$y^2 = 4x = 4(1)x; p = 1$
$F(1,0)$; directrix $x = -1$



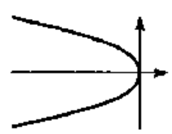
2. $y^2 = 16x$; $y^2 = 4px$
$4p = 16$; $p = 4$
$F(4,0)$; directrix $x = -4$



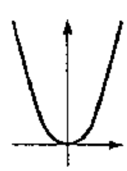
3. $y^2 = -4x$; $y^2 = 4px$
$4p = -4$; $p = -1$
$F(-1,0)$; directrix $x = 1$



4. $y^2 = -16x$
$4p = -16$; $p = -4$
$F(-4,0)$; directrix $x = 4$



5. $x^2 = 8y$
$x^2 = 4py$
$x^2 = 8y = 4(2)y; p = 2$
$F(0,2)$; directrix $y = -2$

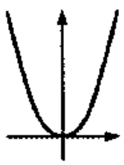


6. $x^2 = 10y$

$x^2 = 4py$

$4p = 10;\ p = \frac{10}{4} = \frac{5}{2}$

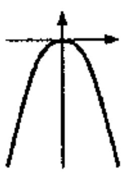
$F\left(0, \frac{5}{2}\right)$; directrix $y = -\frac{5}{2}$



7. $x^2 = -4y$

$4p = -4;\ p = -1$

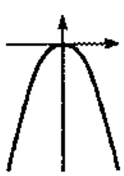
$F(0,-1)$; directrix $y = 1$



8. $x^2 = -12y$

$4p = -12;\ p = -3$

$F(0,-3)$; directrix $y = 3$

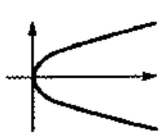


9. $2y^2 = 5x$

$y^2 = \frac{5}{2}x = 4px$

$p = \frac{5}{8}$

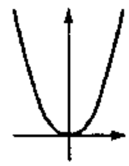
$F(\frac{5}{8},0)$; directrix $x = -\frac{5}{8}$



10. $3x^2 = 8y,\ x^2 = \frac{8}{3}y$

$4p = \frac{8}{3};\ p = \frac{2}{3}$

$F\left(0, \frac{2}{3}\right)$; directrix $y = -\frac{2}{3}$

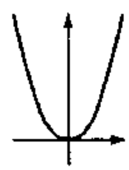


11. $y = 0.48x^2$

$x^2 = \frac{y}{0.48} = \frac{100}{48}y$

$4p = \frac{100}{48};\ p = \frac{25}{48}$

$F\left(0, \frac{25}{48}\right)$; directrix $y = -\frac{25}{48}$



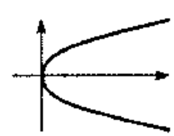
12. $x = 7.6y^2$

$y^2 = \frac{x}{7.6} = \frac{10}{76}x$

$4p = \frac{10}{76};\ p = \frac{10}{4(76)} = \frac{5}{152}$

$F\left(\frac{5}{152}, 0\right)$;

directric $x = -\frac{5}{152}$



13. $F(3,0)$; directrix $x = -3$; $p = 3$

$y^2 = 4px$

$y^2 = 4(3)x$;

$y^2 = 12x$

14. $F(-2,0)$; directrix $x = 2$, therefore $p = -2$

Therefore, $y^2 = -8x$

15. $F(0,4)$; $V(0,0)$, therefore, $p = 4$
Therefore, $x^2 = 16y$

16. $F(-3,0)$; $V(0,0)$, therefore $p = -3$
Therefore, $y^2 = -12x$

17. $V(0,0)$, directrix $y = -0.16$
$F(0,0.16)$, $p = 0.16$
$x^2 = 4py = 4(0.16)y$
$x^2 = 0.64y$

18. $V(0,0)$; directrix $y = 2.3$, therefore $p = -2.3$
Therefore, $x^2 = -9.2y$

19. $V(0,0)$
Therefore, $x^2 = 4py$

$(-1)^2 = 4p(8)$; $1 = 32p$; $p = \dfrac{1}{32}$

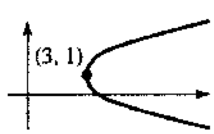
Therefore, $x^2 = \dfrac{1}{8}y$

20. $V(0,0)$

$y^2 = 4px$; $(-1)^2 = 4p(2)$; $p = \dfrac{1}{8}$

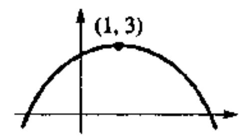
Therefore, $y^2 = \dfrac{1}{2}x$

21. $F(6,1)$; directrix $x = 0$; $V(3,1)$
$d_1 = d_2$
$d_1 = x$
$d_2 = \sqrt{(x-6)^2 + (y-1)^2}$
$x = \sqrt{(x-6)^2 + (y-1)^2}$
$x^2 = (x-6)^2 + (y-1)^2$
$x^2 = x^2 - 12x + 36 + y^2 - 2y + 1$
$0 = -12x + 36 + y^2 - 2y + 1$
$y^2 - 2y - 12x + 37 = 0$



22. $F(1,1)$; directrix $y = 5$
Therefore, V is (1, 3).

$d_1 = d_2$
$d_2 = 5 - y$
$d_1 = \sqrt{(x-1)^2 + (y-1)^2}$
$\sqrt{(x-1)^2 + (y-1)^2} = 5 - y$
$(x-1)^2 + (y-1)^2 = (5-y)^2$
$x^2 - 2x + 1 + y^2 - 2y + 1 = 25 - 10y + y^2$
$x^2 - 2x + 2 + 8y - 25 = 0$
$x^2 - 2x + 8y - 23 = 0$

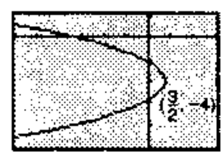


23. $y^2 + 2x + 8y + 13 = 0$; solve for y
$y^2 + 8y + (2x + 13) = 0$

$$y = \frac{-8 \pm \sqrt{8^2 - 4(2x+13)}}{2}$$

$$y = \frac{-8 \pm \sqrt{12 - 8x}}{2}$$

$y_1 = -4 + \sqrt{3 - 2x}$, $y_2 = -4 - \sqrt{3 - 2x}$

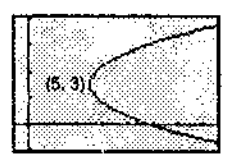


24. $y^2 - 2x - 6y + 19 = 0$; solve for y
$y^2 = -6y + (-2x - 19) = 0$

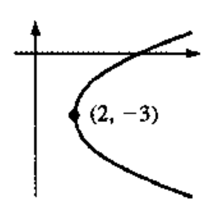
$$y = \frac{6 \pm \sqrt{(-6)^2 - 4(-2x+19)}}{2}$$

$$= \frac{6 \pm \sqrt{8x - 40}}{2}$$

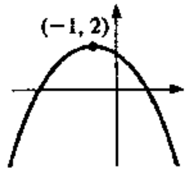
$y_1 = 3 + \sqrt{2x - 10}$; $y_2 = 3 - \sqrt{2x - 10}$



25. Vertex at $(2, -3)$
Focus is 2 units from vertex at $(4, -3)$.
$(y+3)^2 = 8(x-2)$



26. $V(h,k)$; $(x-h)^2 = 4p(y-k)$
(h,k) is $(-1,2)$; $p=-3$
$(x+1)^2 = -12(y-2)$



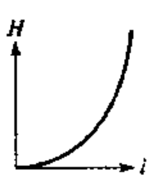
27. $y^2 = 4px$
When $x = p$; $y^2 = 4p^2$; $y = 2p$
Therefore, latus rectum intersects parabola at $(p, 2p)$.
Therefore, length of latus rectum is $2(2p) = 4p$.

28. $x^2 = 8y$
$4p = 8$; $p = 2$
$F(0,2)$; directrix $y=-2$
Center is circle of (0, 1).
Radius is 1.
$(x-h)^2 + (y-k)^2 = r^2$
$(x-0)^2 + (y-1)^2 = 1$
$x^2 + y^2 - 2y + 1 = 1$
$x^2 + y^2 - 2y = 0$

29. Let the vertex of the parabola be at the origin. $x^2 = 4py$. A point on the parabola will be (2100, 300). Substitute this into the equation and solve for p.
$2100^2 = 4p(300)$; $1200p = 4,410,000$; $p = 3675$
$x^2 = 4(3675)y = 14,700y$

30. Equation of parabolas is $x^2 = -4py$.
Curve passes through $(3.7, -5.6)$.
$(3.7)^2 = -4p(-5.6)$
$p = 0.611$
Therefore, $x^2 = -2.44y$

31. $H = Ri^2$; $R = 6.0\ \Omega$
Therefore, $H = 6.0i^2$; H vs i
of the form $y = 4px$
$4p = 6.0$; $p = \frac{3}{2}$
H and $i > 0$



32. Place origin at top.
Therefore, equation of curve is $x^2 = -4py$.
$(2.10, -5)$ lies on curve.
Therefore, $(2.10)^2 = -4p(-5)$
$p = 0.2205$
Therefore, $x^2 = -0.822y$
Substitute $y = -2.50$; $x = 1.485$
Therefore, length of bar = 2.97 ft

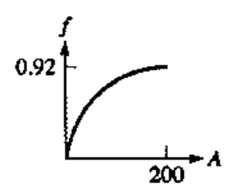
33. $y^2 = 4px$
$(1.20)^2 = 4p(0.00625)$
$p = 57.6$ m

34. $x = v_0 t$, therefore $t = \frac{x}{v_0}$
$y = \frac{1}{2}gt^2$
Substitute: $y = \frac{1}{2}g\left(\frac{x}{v_0}\right) = \frac{1}{2}g\frac{x^2}{v_0^2} = \frac{g}{2v_0^2}x^2$

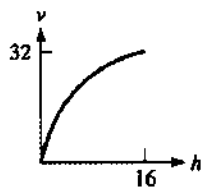
35. x-value of focus occurs where $y = 0$.
$0 = -12.0x + 3.6$
Therefore, $x = 0.3$
Focus $(0.3, 0)$
Therefore, $p = 0.3$
Equation is $y^2 = 4px$.
Therefore, $y^2 = 4(0.3)x = 1.2x$

36. Equation of curve is $x^2 = 4py$
Substitute $(100, 6.0)$; $100^2 = 4p(6)$; $4p = \frac{10^4}{6}$
Therefore, $x^2 = \frac{10^4}{6}y$
Substitute $x = 50.0$; $50.0^2 = \frac{10^4}{6}y$; $y = 1.5$ ft
Therefore, the wire is 1.5 + 30.0 = 31.5 ft above the ground, 50.0 ft from either pole by symmetry.

37. The graph is parabolic since it can be transformed into the form $f^2 = 4pA$.
$f = 0.065\sqrt{A}$
$= 0.065\sqrt{200}$
$= 0.92$



38. $v = 8\sqrt{h}$; v versus h
$v^2 = 64h$ is a parabola of form $y^2 = 4px$.



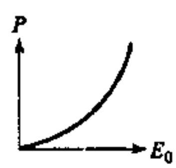
39. Path of ship channel is a parabola with focus at $(0, -2)$ and vertex $(0, 0)$ and directrix at $y = 2$. Therefore, $p = -2$. Parabola of the type $x^2 = -4py$; therefore, $x^2 = -8y$. If positions of island and shoreline are interchanged, the equation would be $x^2 = 8y$. If a parabola of the form $y^2 = 4px$ is assumed, the path would be $y^2 = 8x$ or $y^2 = -8x$.

40. $P_{max} = \dfrac{kE_0^2}{R_i}$, $P_{max} = 10$ W; $E_0 = 2.0$ V;

$R_i = 0.10\ \Omega$

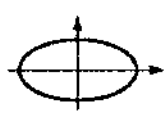
$10 = \dfrac{k(2.0)^2}{0.10}$; $k = 0.25$; $P_{max} = \dfrac{0.25E_0^2}{R_i}$

For constant R_i, $P = 2.5E_0^2$

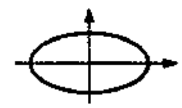


2.5 The Ellipse

1. $\dfrac{x^2}{4} + \dfrac{y^2}{1} = 1$
$a^2 = 4, b^2 = 1$
$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$
$c^2 = a^2 - b^2$
$c^2 = 4 - 1 = 3, c = \sqrt{3}$
$V(\pm 2, 0), F(\pm\sqrt{3}, 0)$,
y-intercepts $(0, \pm 1)$

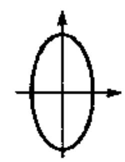


2. $\dfrac{x^2}{100} + \dfrac{y^2}{64} = 1$
$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$
$a^2 = 100$; $a = 10$
$b^2 = 64$; $b = 8$
$c^2 = a^2 - b^2$
$c^2 = 100 - 64 = 36$
$c = 6$



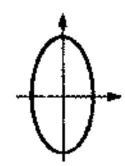
$V(\pm 10, 0), F(\pm 6, 0)$, y-intercepts $(0, 8)$

3. $\dfrac{x^2}{25} + \dfrac{y^2}{36} = 1$
$a^2 = 36$; $a = 6$
$b^2 = 25$; $b = 5$
$c^2 = 36 - 25 = 11$
$c = \sqrt{11}$



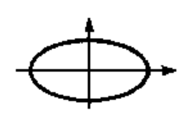
$V(0, \pm 6)$, $F(0, \pm\sqrt{11})$, x-intercepts $(\pm 5, 0)$

4. $\dfrac{x^2}{49} + \dfrac{y^2}{81} = 1$
$a^2 = 81$; $a = 9$
$b^2 = 49$; $b = 7$
$c^2 = 81 - 49 = 32$
$c = \sqrt{32} = 4\sqrt{2}$

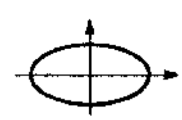


$V(0, \pm 9)$, $F(0, \pm 4\sqrt{2})$, x-intercepts $(\pm 7, 0)$

5. $4x^2 + 9y^2 = 36$
$\dfrac{4x^2}{36} + \dfrac{9y^2}{36} = 1$
$\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$
$a^2 = 9, b^2 = 4$
$c^2 = 9 - 4 = 5; c = \sqrt{5}$
$V(\pm 3, 0), F(\pm\sqrt{5}, 0)$,
y-intercepts $(0, \pm 2)$



6. $x^2 + 36y^2 = 144$
$\dfrac{x^2}{144} + \dfrac{36y^2}{144} = 1$
$a = 12, b = \dfrac{12}{6} = 2$

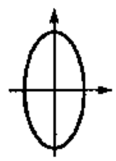


$c^2 = 144 - \dfrac{144}{36} = \dfrac{144(36 - 1)}{36}$
$c = 2\sqrt{35}$
$V(\pm 12, 0)$, $F(\pm 2\sqrt{35}, 0)$, y-intercepts $(\pm 2, 0)$

7. $49x^2 + 4y^2 = 196$

$\frac{49}{196}x^2 + \frac{4}{196}y^2 = 1$

$\frac{x^2}{196/49} + \frac{y^2}{196/4} = 1$



$a^2 = \frac{196}{4}$; $a = 7$;

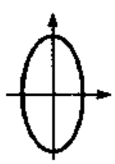
$b^2 = \frac{196}{49}$; $b = 2$

$c^2 = \frac{196}{4} - \frac{196}{49}$, $c = 3\sqrt{5}$

$V(0, \pm 7)$, $F(0, \pm 3\sqrt{5})$, x-intercepts $(\pm 2, 0)$

8. $25x^2 + y^2 = 25$

$\frac{x^2}{1} + \frac{y^2}{25} = 1$



$a^2 = 25$; $a = 5$

$b^2 = 1$; $b = 1$

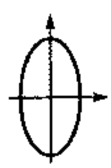
$c^2 = 25 - 1 = 24$

$c = 2\sqrt{6}$

$V(0, \pm 5)$, $F(0, \pm 2\sqrt{6})$, x-intercepts $(\pm 1, 0)$

9. $8x^2 + y^2 = -16$

$\frac{8x^2}{16} + \frac{y^2}{16} = 1$



$\frac{x^2}{2} + \frac{y^2}{16} = 1$

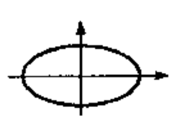
$\frac{y^2}{16} + \frac{x^2}{2} = 1$

$a^2 = 16, b^2 = 2, c^2 = 16 - 2 = 14$

$V(0, \pm 4), F(0, \pm\sqrt{14})$, x-intercepts $(\pm\sqrt{2}, 0)$

10. $2x^2 + 3y^2 = 600$

$\frac{x^2}{300} + \frac{y^2}{200} = 1$



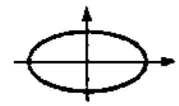
$a = \sqrt{300}, b = \sqrt{200}, c = 10$

$V(\pm\sqrt{300}, 0)$, $F(\pm 10, 0)$, y-intercepts $(0, \pm\sqrt{200})$

11. $4x^2 + 25y^2 = 0.25 = \frac{1}{4}$

$16x^2 + 100y^2 = 1$

$\frac{x^2}{1/16} + \frac{y^2}{1/100} = 1$



$a = \frac{1}{4} = 0.25$

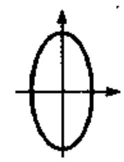
$b = \frac{1}{10} = 0.1$

$c = \frac{\sqrt{84}}{40} = 0.23$

$V(\pm 0.25, 0)$, $F(\pm 0.23, 0)$, y-intercepts $(0, \pm 0.1)$

12. $9x^2 + 4y^2 = 0.09 = \frac{9}{100}$

$\frac{900}{9}x^2 + \frac{400}{9}y^2 = 1$



$\frac{x^2}{1/100} + \frac{y^2}{9/400} = 1$

$a = \frac{3}{20}, b = \frac{1}{10}, c = \frac{\sqrt{5}}{20}$

$V\left(0, \pm\frac{3}{20}\right)$, $F\left(0, \pm\frac{\sqrt{5}}{20}\right)$, x-intercepts $\left(\pm\frac{1}{10}, 0\right)$

13. $V(15, 0)$; $F(9, 0)$

$a = 15, a^2 = 225$;

$c = 9, c^2 = 81; a^2 - c^2 = b^2$

$b^2 = 144; \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$;

$\frac{x^2}{225} + \frac{y^2}{144} = 1$

$144x^2 + 225y^2 = 32,400$

14. Minor axis 8, therefore $b = 4$

$V(0, -5)$, therefore $a = 5$

$c^2 = 25 - 16 = 9$

$c = 3$

$\frac{y^2}{25} + \frac{x^2}{16} = 1$ or $25x^2 + 16y^2 = 400$

15. $F(0, 2)$, therefore $c = 2$

Major axis 6, therefore $a = 3$

$c^2 = a^2 - b^2$

$b^2 = 9 - 4 = 5$

$b = \sqrt{5}$

$\frac{y^2}{9} + \frac{x^2}{5} = 1$ or $9x^2 + 5y^2 = 45$

16. $2a + 2b = 18, F(3,0) \Rightarrow c = 3$

$a + b = 9$

$b = 9 - a$

$a^2 = b^2 + c^2 = b^2 + 3^2$

$a^2 = (9 - a)^2 + 9$

$a^2 = 81 - 18a + a^2 + 9$

$18a = 90$

$a = 5$

$b = 9 - 5 = 4$

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

17. Vertex $(8,0)$; (x,y) is $(2,3)$; $a^2 = 8^2 = 64$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; \ \frac{x^2}{64} + \frac{y^2}{b^2} = 1$$

$$\frac{(2)^2}{64} + \frac{(3)^2}{b^2} = 1; \ \frac{9}{b^2} = \frac{16}{16} - \frac{1}{16};$$

$$15b^2 = 144; \ b^2 = \frac{144}{15}$$

$$\frac{x^2}{64} + \frac{15y^2}{144} = 1; \ 144x^2 + 960y^2 = 9216$$

$3x^2 + 20y^2 = 192$

18. $F(0,2), c = 2$; $(x,y) = (-1, \sqrt{3})$

$a^2 = b^2 + c^2$; $a^2 = b^2 + 4$

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1; \ \frac{(\sqrt{3})^2}{a^2} + \frac{(-1)^2}{b^2} = 1; \ \frac{3}{a^2} + \frac{1}{b^2} = 1$$

$$\frac{3}{b^2 + 4} + \frac{1}{b^2} = 1; \ 3b^2 + b^2 + 4 = b^4 + 4b^2$$

$b^4 = 4, b^2 = 2, a^2 = 2 + 4 = 6$

Therefore, $\frac{y^2}{6} + \frac{x^2}{2} = 1$ or $3x^2 + y^2 = 6$

19. $(x_1, y_1) = (2,2), (x_2, y_2) = (1,4)$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Substitute: $\frac{4}{b^2} + \frac{4}{a^2} = 1$

Therefore, $4a^2 + 4b^2 = a^2b^2$

$$\frac{1}{b^2} + \frac{16}{a^2} = 1$$

Therefore, $a^2 + 16b^2 = a^2b^2$

$a^2 + 16b^2 = a^2b^2$

$16b^2 = a^2b^2 - a^2 = a^2(b^2 - 1)$

Therefore, $a^2 = \frac{16b^2}{b^2 - 1}$

Substitute:

$$4a^2 + 4b^2 = a^2b^2; \ \frac{64b^2}{b^2 - 1} + 4b^2 = \frac{16b^4}{b^2 - 1}$$

$64b^2 + 4b^4 - 4b^2 = 16b^4$; $-12b^4 + 60b^2 = 0$

$12b^2(-b^2 + 5) = 0$

$b^2 = 5$

Therefore, $a^2 = \frac{16(5)}{4} = 20$

Therefore, $\frac{y^2}{20} + \frac{x^2}{5} = 1$

or: $5y^2 + 20x^2 = 100$; $4x^2 + y^2 = 20$

20. $(x_1, y_1 0 = (-2, 2), (x_2, y_2) = (1, \sqrt{6})$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Substitute: $\frac{4}{a^2} + \frac{4}{b^2} = 1$

Therefore, $4b^2 + 4a^2 = a^2b^2$

$$\frac{1}{a^2} + \frac{6}{b^2} = 1$$

Therefore, $b^2 + 6a^2 = a^2b^2$

$b^2 + 6a^2 = a^2b^2$

$b^2 = a^2b^2 - 6a^2 = a^2(b^2 - 6)$

Therefore, $a^2 = \frac{b^2}{b^2 - 6}$

Substitute:

$4b^2 + 4a^2 = a^2b^2$;

$$4b^2 + \frac{4b^2}{b^2 - 6} = \frac{b^4}{b^2 - 6}$$

$4b^4 - 24b^2 + 4b^2 = b^4$;

$3b^4 - 20b^2 = 0$

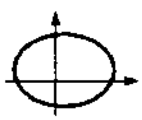
$$b^2(3b^2 - 20) = 0; \ b^2 = \frac{20}{3}$$

$$\text{Therefore, } a^2 = \frac{\frac{20}{3}}{\frac{20}{3} - 6} = 10$$

Therefore, $\frac{x^2}{10} + \frac{y^2}{\frac{20}{3}} = 1$ or $2x^2 + 3y^2 = 20$

21. $F(-2,1)$ and $(4,1)$, major axis 10

$$\sqrt{[x-(-2)]^2+(y-1)^2}+\sqrt{(x-4)^2+(y-1)^2}=10$$
$$\sqrt{(x+2)^2+(y-1)^2}=10-\sqrt{(x-4)^2+(y-1)^2}$$
$$(x+2)^2+(y-1)^2=100-20\sqrt{(x-4)^2+(y-1)^2}+(x-4)^2+(y-1)^2$$
$$x^2+4x+4+y^2-2y+1=100-20\sqrt{(x-4)^2+(y-1)^2}+x^2-8x+16+y^2-2y+1$$
$$x^2+4x+4+y^2-2y+1-100-x^2+8x-16-y^2+2y-1=-20\sqrt{(x-4)^2+(y-1)^2}$$
$$12x-112=-20\sqrt{(x-4)^2+(y-1)^2}$$
$$3x-28=-5\sqrt{(x-4)^2+(y-1)^2}$$
$$(3x-28)^2=25[(x-4)^2+(y-1)^2]$$
$$9x^2-168x+784=25(x^2-8x+16+y^2-2y+1)$$
$$9x^2-168x+784=25x^2-200x+400+25y^2-50y+25$$
$$-16x^2-25y^2+32x+50y+359=0$$
$$16x^2+25y^2-32x-50y-359=0$$



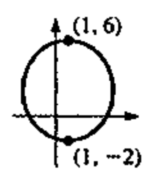
22. $\sqrt{(x-1)^2+(y-4)^2}+\sqrt{(x-1)^2+(y-0)^2}=$ constant. Let $(x,y)=(4,4)$

$$\sqrt{(4-1)^2+(4-4)^2}+\sqrt{(4-1)^2+(4-0)^2}=\text{constant}$$
$$\sqrt{3^2+0^2}+\sqrt{3^2+4^2}=\text{constant}$$
$$3+5=8=\text{constant}$$
$$\sqrt{(x-1)^2+(y-4)^2}+\sqrt{(x-1)^2+y^2}=8$$
$$\sqrt{(x-1)^2+(y-4)^2}=8-\sqrt{(x-1)^2+y^2}$$
$$x^2-2x+1+y^2-8y+16=64-16\sqrt{(x-1)^2+y^2}+x^2-2x+1+y^2$$
$$-8y-48=-16\sqrt{(x-1)^2+y^2}$$
$$y+6=2\sqrt{(x-1)^2+y^2}$$
$$y^2+12y+36=4(x^2-2x+1+y^2)$$
$$y^2+12y+36=4x^2-8x+4+4y^2$$
$$0=4x^2+3y^2-8x-12y-32$$
$$4x^2+3y^2-8x-12y-32=0$$

x	y_1	y_2
1	6	−2
4	4	0
0	5.83	−1.83
−2.46	2	2
4.46	2	2

To sketch, solve $3y^2-12y+4x^2-8x-32=0$ for y using quadratic formula,

$$y_1=\frac{12+\sqrt{144-4(3)(4x^2-8x-32)}}{2(3)}=\frac{12+\sqrt{144-12(4x^2-8x-32)}}{6}$$
$$y_2=\frac{12-\sqrt{144-12(4x^2-8x-32)}}{6}$$



23. $4x^2 + 3y^2 + 16x - 18y + 31 = 0$; solve for y

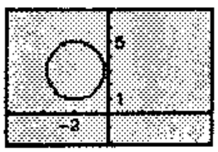
$3y^2 - 18y + (4x^2 + 16x + 31) = 0$

$$y = \frac{18 \pm \sqrt{(-18)^2 - 4(3)(4x^2 + 16x + 31)}}{2(3)}$$

$$= \frac{18 + \sqrt{-48x^2 - 192x - 48}}{2}$$

$$y_1 = 3 + \frac{\sqrt{-12x^2 - 48x - 12}}{3}$$

$$y_2 = 3 - \frac{\sqrt{-12x^2 - 48x - 12}}{3}$$



24. $4x^2 + 8y^2 + 4x - 24y + 1 = 0$; solve for y

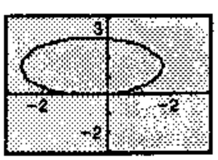
$8y^2 - 24y + (4x^2 + 4x + 1) = 0$

$$y = \frac{24 \pm \sqrt{(-24)^2 - 4(8)(4x^2 + 4x + 1)}}{2(8)}$$

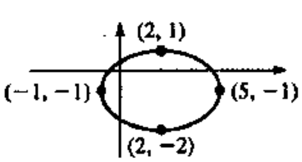
$$= \frac{24 \pm \sqrt{-128x^2 - 128x + 544}}{16}$$

$$y_1 = 1.5 + \frac{\sqrt{-8x^2 - 8x + 34}}{4}$$

$$y_2 = 1.5 - \frac{\sqrt{-8x^2 - 8x + 34}}{4}$$

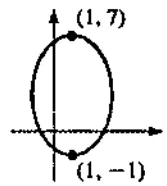


25. $2a = 6; a = 3; 2b = 4; b = 2$; $(h, k) = (2, -1)$



26. Center (h, k); $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$

Major axis $= 8$; minor axis $= 6$; $(h, k) = (1, 3)$



27. $x^2 + y^2 = 1, k > 0$

Therefore, $\frac{x^2}{1} + \frac{y^2}{\frac{1}{k}} = 1$

$\frac{1}{k} = a^2 = 1$

Therefore, $\sqrt{\frac{1}{k}} = a > 1 \Rightarrow \frac{1}{k} > 1$

Therefore, $0 < k < 1$

28. $x^2 + k^2y^2 = 25$

$\frac{x^2}{25} + \frac{k^2y^2}{25} = 1$

Therefore, $\frac{x^2}{25} + \frac{y^2}{\frac{25}{k^2}} = 1$

$F(3, 0)$, therefore, $c = 3$

Equation of form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$a = 5$, therefore $b = 4$

Therefore, $\frac{25}{k^2} = 16$; $k^2 = \frac{25}{16}$; $k = \pm\frac{5}{4}$

29. Given: $2x^2 + 3y^2 - 8x - 4 = 0$

$2x^2 + 3(-y)^2 - 8x - 4 = 2x^2 + 3y^2 - 8x - 4$

30. $5x^2 + y^2 - 3y - 7 = 5(-x)^2 + y^2 - 3y - 7$

31. Eccentricity $e = \frac{c}{a}$

$x^2 + 9y^2 = 81$; $\frac{x^2}{81} + \frac{9y^2}{81} = 1$; $\frac{x^2}{81} + \frac{y^2}{\frac{81}{9}} = 1$

$a^2 = 81$; $a = 9$; $b^2 = \frac{81}{9}$; $b = 3$

$c^2 = a^2 - b^2$; $c = \sqrt{81 - 9} = \sqrt{72} = 6\sqrt{2}$

Therefore, $e = \frac{6\sqrt{2}}{9}$; $e = \frac{2\sqrt{2}}{3} \approx 0.943$

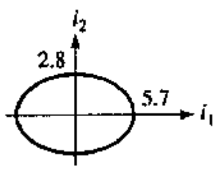
32. $2a = 4.6 + 2.8 = 7.4;\ a = 3.7$
$2c = 4.6 - 2.8 = 1.8;\ c = 0.9$

Therefore, $e = \frac{c}{a} = \frac{0.9}{3.7} = 0.24$

33. If the two vertices of each base are fixed at $(-3,0)$ and $(3,0)$, and the sum of the two leg lengths is also fixed, the third vertex lies on an ellipse. The base is 6 cm, so
$d_1 + d_2 = 14\text{ cm} - 6\text{ cm} = 8\text{ cm}$
$(-3,0)$ and $(3,0)$ are foci $(-c,0)$ and $(c,0)$
$d_1 + d_2 = 2a = 8; a = 4$
$a^2 - c^2 = b^2$
$4^2 - 3^2 = b^2$
$b^2 = 7, a^2 = 16$

The equation is $\frac{x^2}{16} + \frac{y^2}{7} = 1$, or $7x^2 + 16y^2 = 112$

34. $P = Ri^2$
$P_T = R_1 i_1^2 + R_2 i_2^2$
$64 = 2i_1^2 + 8i_2^2$
$\frac{2}{64} i_1^2 + \frac{8}{64} i_2^2 = 1$
$\frac{i_1^2}{32} + \frac{i_2^2}{8} = 1$
$i_1^2 + 4i_2^2 = 32$
$a^2 = 32;\ a = \sqrt{32} \approx 5.7$
$b^2 = 8;\ b = \sqrt{8} \approx 2.8$



35. $36x^2 + 225y^2 = 8100$

Two people must be separated by a distance $= 2c$.

$\frac{36x^2}{8100} + \frac{225y^2}{8100} = 1$

$\frac{x^2}{225} + \frac{y^2}{36} = 1$

$a^2 = 225;\ a = 15$
$b^2 = 36;\ b = 6$
$c^2 = a^2 - b^2 = 225 - 36 = 189;\ c = 13.748$
$2c = 27.5$ m

36. $a = 4.20, b = 0.60$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, therefore $\frac{x^2}{(4.20)^2} + \frac{y^2}{(0.60)^2} = 1$

$\frac{x^2}{17.64} + \frac{y^2}{0.36} = 1$

$0.36x^2 + 17.64y^2 = 6.3504;\ x^2 + 49y^2 = 17.6$

37. $a = \frac{64}{2} = 32, b = 18$

Let the center be at the origin. The equation of the ellipse is

$\frac{x^2}{32^2} + \frac{y^2}{18^2} = 1;\ \frac{x^2}{1024} + \frac{y^2}{324} = 1$

If $x = 22, \frac{22^2}{1024} + \frac{y^2}{324} = 1$

$y = 13$ ft

38. $a = 2.25;\ b = 1.60;\ p = \pi(a+b);\ p = \pi(2.25+1.60);$
$p = 12.1$ ft

39. $9x^2 + 20y^2 = 180;\ \frac{9x^2}{180} + \frac{20y^2}{180} = 1;\ \frac{x^2}{20} + \frac{y^2}{9} = 1$

$a = \sqrt{20} = 2\sqrt{5} \approx 4.5;\ b = 3;$
$V =$ area of end $\times$ length
$V = \pi ab \times 20.0$
$V = \pi(2\sqrt{5})(3)(20.0) = 843\text{ ft}^3$

40. $\cos 28° = \frac{6.80}{2a}$, therefore $a = \frac{3.40}{\cos 28°} = 3.851$ mm

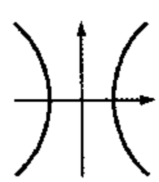
$b = 3.40$ mm

Laser beam is circular in cross section and appears as an ellipse on plane surface when incident at 62°. One diameter is elongated; other diameter, at right angles to first, is in true length.
Ellipse of minor axis = 6.80 mm, major axis = 7.702 mm.

$A = \pi ab$
$A = \pi(3.851)(3.40)$
$A = 41.1\text{ mm}^2$

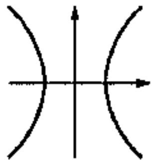
2.6 The Hyperbola

1. $\frac{x^2}{25} - \frac{y^2}{144} = 1$
$a^2 = 25; a = 5$
$b^2 = 144; b = 12$
$c^2 = a^2 + b^2$
$c^2 = 169; c = 13$
$V(\pm 5, 0), F(\pm 13, 0)$



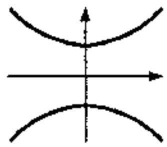
2. $\dfrac{x^2}{16} - \dfrac{y^2}{4} = 1$

$a^2 = 16;\ a = 4$
$b^2 = 4;\ b = 2$
$c^2 = a^2 + b^2$
$c^2 = 20;\ c = 2\sqrt{5}$
$V(\pm 4, 0), F(\pm 2\sqrt{5}, 0)$



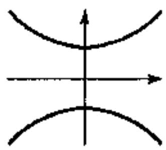
3. $\dfrac{y^2}{9} - \dfrac{x^2}{1} = 1$

$a^2 = 9;\ a = 3$
$b^2 = 1;\ b = 1$
$c^2 = 10;\ c = \sqrt{10}$
$V(0, \pm 3), F(0, \pm\sqrt{10})$



4. $\dfrac{y^2}{2} - \dfrac{x^2}{2} = 1$

$a^2 = 2;\ a = \sqrt{2}$
$b^2 = 2;\ b = \sqrt{2}$
$c^2 = 4;\ c = 2$
$V(0, \pm\sqrt{2}), F(0, \pm 2)$

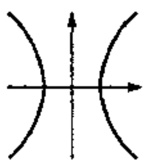


5. $4x^2 - y^2 = 4$

$\dfrac{4x^2}{4} - \dfrac{y^2}{4} = 1;$

$\dfrac{x^2}{1} - \dfrac{y^2}{4} = 1$

$a^2 = 1; b^2 = 4; c^2 = 5$
$V(\pm 1, 0); F(\pm\sqrt{5}, 0)$

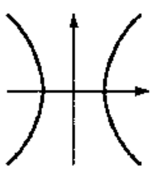


6. $x^2 - 9y^2 = 81$

$\dfrac{x^2}{81} - \dfrac{9y^2}{81} = 1$

$\dfrac{x^2}{81} - \dfrac{y^2}{9} = 1$

$a^2 = 81;\ a = 9$
$b^2 = 9;\ b = 3$
$c^2 = 90;\ c = 3\sqrt{10}$
$V(\pm 9, 0), F(\pm 3\sqrt{10}, 0)$

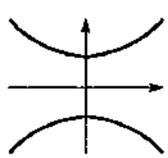


7. $2y^2 - 5x^2 = 10$

$\dfrac{2y^2}{10} - \dfrac{5x^2}{10} = 1$

$\dfrac{y^2}{5} - \dfrac{x^2}{2} = 1$

$a^2 = 5;\ a = \sqrt{5}$
$b^2 = 2;\ b = \sqrt{2}$
$c^2 = 7;\ c = \sqrt{7}$
$V(0, \pm\sqrt{5}), F(0, \pm\sqrt{7})$

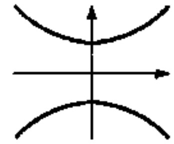


8. $3y^2 - 2x^2 = 300$

$\dfrac{3y^2}{300} - \dfrac{2x^2}{300} = 1$

$\dfrac{y^2}{100} - \dfrac{x^2}{150} = 1$

$a^2 = 100;\ a = 10$
$b^2 = 150;\ b = 5\sqrt{6}$
$c^2 = 250;\ c = 5\sqrt{10}$
$V(0, \pm 10), F(0, \pm 5\sqrt{10})$



9. $4x^2 - y^2 + 4 = 0$

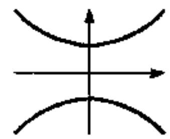
$4x^2 - y^2 = -4$

$\dfrac{4x^2}{-4} - \dfrac{y^2}{-4} = \dfrac{-4}{-4}$

$-x^2 + \dfrac{y^2}{4} = 1$

$\dfrac{y^2}{4} - \dfrac{x^2}{1} = 1$

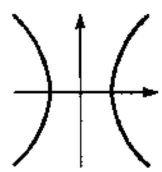
$a^2 = 4; b^2 = 1; c^2 = 5$
$V(0, \pm 2), F(0, \pm\sqrt{5})$



10. $9x^2 - y^2 = 9$

$\dfrac{x^2}{1} - \dfrac{y^2}{9} = 1$

$a = 1;\ b = 3;$
$c^2 = 10;\ c = \sqrt{10}$
$V(\pm 1, 0), F(\pm\sqrt{10}, 0)$

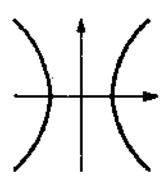


11. $4x^2 - y^2 = 0.64$

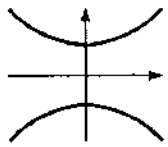
$\dfrac{4x^2}{0.64} - \dfrac{y^2}{0.64} = 1$

$\dfrac{x^2}{0.16} - \dfrac{y^2}{0.64} = 1$

$a^2 = 0.16;\ a = 0.40$
$b^2 = 0.64;\ b = 0.80$
$c^2 = 0.80;\ c = 0.89$
$V(\pm 0.4, 0), F(\pm 0.89, 0)$



12. $9y^2 - x^2 = 0.36$



$\frac{9y^2}{0.36} - \frac{x^2}{0.36} = 1$

$\frac{y^2}{0.04} - \frac{x^2}{0.36} = 1$

$a^2 = 0.04;\ a = 0.2$

$b^2 = 0.36;\ b = 0.6$

$c^2 = 0.40;\ c^2 = \frac{40}{100}$

$c = \frac{2}{10}\sqrt{10};\ c = \frac{1}{5}\sqrt{10}$

$V(0, \pm 0.2), F\left(0, \pm\frac{\sqrt{10}}{5}\right)$

13. $V(3,0); F(5,0)$

$a = 3; c = 5; a^2 = 9; c^2 = 25$

$b^2 = c^2 - a^2 = 25 - 9 = 16$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1; \frac{x^2}{9} - \frac{y^2}{16} = 1;$

$16x^2 - 9y^2 = 144$

14. $V(0,1);\ F(0, \sqrt{3})$

$a = 1;\ c = \sqrt{3};\ c^2 = a^2 + b^2;\ b^2 = 3 - 1 = 2;$
$b = \sqrt{2}$

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1;\ \frac{y^2}{1} - \frac{x^2}{2} = 1$

$2y^2 - x^2 = 2$

15. Conjugate axis $= 12$; $V(0, 10)$

$a = 10;\ 2b = 12;\ b = 6$

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1;\ \frac{y^2}{100} - \frac{x^2}{36} = 1$

$9y^2 - 25x^2 = 900$

16. focus $(10,0) \Rightarrow c = 10$

$a + b = 14, b = 14 - a$

$$a^2 + b^2 = c^2 = 10^2$$
$$a^2 + (14 - a)^2 = 100$$
$$a^2 + 196 - 28a + a^2 = 100$$
$$2a^2 - 28a + 96 = 0$$
$$a^2 - 14a + 48 = 0$$
$$(a - 6)(a - 8) = 0$$
$$a - 6 = 0\ , \qquad a - 8 = 0$$
$$a = 6 \qquad a = 8$$
$$b = 14 - 6 = 8 \qquad b = 14 - 8 = 6$$

$\frac{x^2}{36} - \frac{y^2}{64} = 1$ or $\frac{x^2}{64} - \frac{y^2}{36} = 1$

17. (x, y) is $(2,3)$; $F(2,0), (-2,0); c = \pm 2, c^2 = 4$

$d_1 = \sqrt{(2 - [-2])^2 + (3 - 0)^2}$
$= \sqrt{4^2 + 3^2} = \sqrt{16 + 9}$
$= \sqrt{25} = 5$

$d_2 = \sqrt{(2 - 2)^2 + (3 - 0)^2}$
$= \sqrt{0 + 9} = \sqrt{9} = 3$

$d_1 - d_2 = 2a; 5 - 3 = 2a; 2 = 2a; 1 = a; a^2 = 1$

$c^2 = 4; b^2 = c^2 - a^2 = 3$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1; \frac{x^2}{1} - \frac{y^2}{3} = 1$

$3x^2 - y^2 = 3$

18. $(x_1, y_1) = (8, \sqrt{3}), V(4,0), a = 4$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1;\ \frac{8^2}{16} - \frac{3}{b^2} = 1;\ 4 - \frac{3}{b^2} = 1;$

$\frac{3}{b^2} = 3;\ b^2 = 1;\ b = 1$

Therefore, $\frac{x^2}{16} - \frac{y^2}{1} = 1;\ x^2 - 16y^2 = 16$

19. $(x_1, y_1) = (5,4), (x_2, y_2) = \left(3, \frac{4}{5}\sqrt{5}\right)$

Use: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Solve for a^2 and b^2:

$\frac{25}{a^2} - \frac{16}{b^2} = 1;\ 25b^2 - 16a^2 = a^2b^2$

$25b^2 - a^2b^2 = 16a^2;\ b^2(25 - a^2) = 16a^2$

$b^2 = \frac{16a^2}{25 - a^2}$

Substitute:

$$\frac{9}{a^2} - \frac{\frac{16}{5}}{b^2} = 1;\ \frac{9}{a^2} - \frac{\frac{16}{5}}{\frac{16a^2}{25} - a^2} = 1$$

$\frac{9}{a^2} - \frac{25 - a^2}{5a^2} = 1;\ 45 - 25 + a^2 = 5a^2;\ 4a^2 = 20$

$a^2 = 5$

Therefore, $b^2 = \frac{16(5)}{25 - 5} = 4$

Therefore, $\frac{x^2}{5} - \frac{y^2}{4} = 1;\ 4x^2 - 5y^2 = 20$

20. $(x_1, y_1) = (1,2), (x_2, y_2) = (2, 2\sqrt{2})$

Use: $\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$

Solve for a^2 and b^2:

$\dfrac{4}{a^2} - \dfrac{1}{b^2} = 1$; $4b^2 - a^2 = a^2b^2$; $4b^2 - a^2b^2 = a^2$

$b^2(4 - a^2) = a^2$; $b^2 = \dfrac{a^2}{4 - a^2}$

Substitute:

$\dfrac{8}{a^2} - \dfrac{4}{b^2} = 1$; $\dfrac{8}{a^2} - \dfrac{4}{\dfrac{a^2}{4} - a^2} = 1$

$\dfrac{8}{a^2} - \dfrac{(4 - a^2)4}{a^2} = 1$; $8 - 16 + 4a^2 = a^2$; $3a^2$

$a^2 = \dfrac{3}{8}$

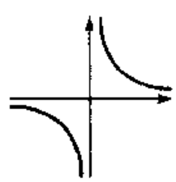
Therefore, $b^2 = \dfrac{\frac{8}{3}}{4 - \frac{8}{3}}$; $b^2 = 2$

Therefore, $\dfrac{y^2}{\frac{8}{3}} - \dfrac{x^2}{2} = 1$; $3y^2 - 4x^2 = 8$

21. $xy = 2$; $y = \dfrac{2}{x}$

x	y
$\frac{1}{2}$	4
1	2
2	1
4	$\frac{1}{2}$
8	$\frac{1}{4}$

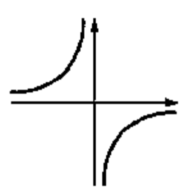
x	y
$-\frac{1}{2}$	-4
-1	-2
-2	-1
-4	$-\frac{1}{2}$
-8	$-\frac{1}{4}$



22. $xy = -4$; $y = -\dfrac{4}{x}$

x	y
$\frac{1}{2}$	-8
1	-4
2	-2
4	-1
8	$-\frac{1}{2}$

x	y
$-\frac{1}{2}$	8
-1	4
-2	2
-4	1
-8	$\frac{1}{2}$



23. $F(1,2)$ and $F(11,2)$

Transverse axis $= 8 = 2a$

$d_1 = \sqrt{(x-1)^2 + (y-2)^2}$
$d_2 = \sqrt{(x-11)^2 + (y-2)^2}$
$d_1 - d_2 = 2a$
$\sqrt{(x-1)^2 + (y-2)^2} - \sqrt{(x-11)^2 + (y-2)^2} = 8$
$\sqrt{(x-1)^2 + (y-2)^2} = 8 + \sqrt{(x-11)^2 + (y-2)^2}$

Square both sides.

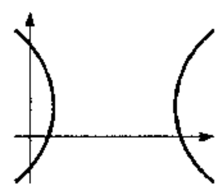
$(x-1)^2 + (y-2)^2$
$= 64 + 16\sqrt{(x-11)^2 + (y-2)^2} + (x-11)^2 + (y-2)^2$

Expand, gather like terms, keep radical to right.

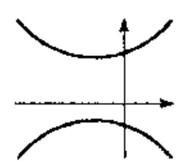
$20x - 184 = 16\sqrt{(x-11)^2 + (y-2)^2}$ Factor.
$5x - 46 = 4\sqrt{(x-11)^2 + (y-2)^2}$ Square both sides.
$25x^2 - 460x + 2116 = 16[(x-11)^2 + (y-2)^2]$

Expand, gather terms.

$25x^2 - 460x + 2116$
$= 16(x^2 - 22x + 121 + y^2 - 4y + 4)$
$25x^2 - 460x + 2116$
$= 16x^2 - 352x + 1936 + 16y^2 - 64y + 64$
$9x^2 - 16y^2 - 108x + 64y + 116 = 0$



24. $V(-2,4)$; $V(-2,-2)$
Conjugate axis $= 4$
Therefore, $2b = 4$; $b = 2$
Transverse axis $= 6$
$2a = 6$; $a = 3$
Therefore, $c = \sqrt{13}$
Center of transeve axis $(-2, 1)$
$F_1(-2, 1 + \sqrt{13})$;
$F_2(-2, 1 - \sqrt{13})$



$d_2 - d_1 = 6$
$\sqrt{(x+2)^2 + (y-1+\sqrt{13})^2} - \sqrt{(x+2)^2 + (y-1-\sqrt{13})^2}$
$= 6$
$\sqrt{(x+2)^2 + (y-1+\sqrt{13})^2}$
$= 6 + \sqrt{(x+2)^2 + (y-1-\sqrt{13})^2}$

Square both sides.

$(x+12)^2 + (y-1-\sqrt{13})^2$
$= 36 + 12\sqrt{(x+2)^2 + (y-1-\sqrt{13})^2}$
$+ (x+2)^2 + (y-1-\sqrt{13})^2$

Expand trinomials and cancel like terms.

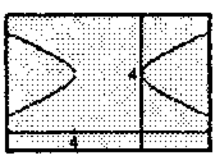
$$4\sqrt{13}y - 4\sqrt{13} - 36 = 12\sqrt{(x+2)^2 + (y-1-\sqrt{13})^2}$$

Square both sides.

$$52y^2 - 104y - 72\sqrt{13}y + 52 + 72\sqrt{13} + 32y$$
$$= 36(x^2+4x+4+y^2-2y-2\sqrt{13}y+2\sqrt{13}+14);$$
$$52y^2 - 104y - 72\sqrt{13}y + 376 + 72\sqrt{13}$$
$$= 36x^2 + 144x + 144 + 36y^2 - 72y + 72\sqrt{13}y$$
$$+ 72\sqrt{13} + 504$$
$$16y^2 - 36x^2 - 32y - 144x - 272 = 0$$
$$4y^2 - 9x^2 - 8y - 36x - 68 = 0$$

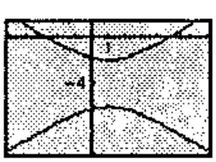
25. $x^2 - 4y^2 + 4x + 32y - 64 = 0$; solve for y

$$4y^2 - 34y + (-x^2 - 4x + 64) = 0$$
$$y = \frac{32 \pm \sqrt{(-32)^2 - 4(4)(-x^2-4x+64)}}{2(4)}$$
$$= \frac{32 \pm \sqrt{16x^2+64x}}{8}$$
$$y_1 = 4 + 0.5\sqrt{x^2+4x}, y_2 = 4 - 0.5\sqrt{x^2+4x}$$

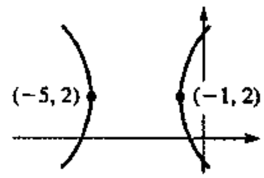


26. $5y^2 - 4x^2 + 8x + 40y + 56 = 0$; solve for y

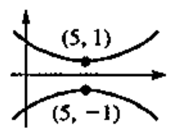
$$5y^2 + 40y + (-4x^2 + 8x + 56) = 0$$
$$y = \frac{-40 \pm \sqrt{40^2 - 4(5)(-4x^2+8x+56)}}{2(5)}$$
$$= \frac{-40 \pm \sqrt{80x^2 - 160x + 480}}{10}$$
$$y_1 = -4 + 0.4\sqrt{5x^2 - 10x + 30}$$
$$y_2 = -4 - 0.4\sqrt{5x^2 - 10x + 30}$$



27. Transverse axis = 4; conjugate axis = 6; $(h, k) = (-3, 2)$



28. Transverse axis = 2; conjugate axis = 8; $(h, k) = (5, 0)$



29. $V(0,1), F(0,\sqrt{3}); c^2 = a^2 + b^2$ where $c = \sqrt{3}$ and $a = 1; b^2 = \sqrt{3}^2 - 1^2 = 2$

$$\frac{y^2}{1^2} - \frac{x^2}{\sqrt{2}^2} = 1$$

The transverse axis of the first equation is length $2a = 2\sqrt{1}$ along the y-axis. Its conjugate axis is length $2b = 2\sqrt{2}$ along the x-axis.
The transverse axis of the conjugate hyperbola is length $2\sqrt{2}$ along the x-axis, and its conjugate axis is length $2\sqrt{1}$ along the y-axis.

The equation, then, is $\dfrac{x^2}{\sqrt{2}^2} - \dfrac{y^2}{\sqrt{1}^2} = 1$

$$\frac{x^2}{2} - \frac{y^2}{1} = 1 \text{ or } x^2 - 2y^2 = 2$$

30. $e = \dfrac{c}{a}$

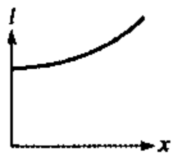
$$2x^2 - 3y^2 = 24;\ \frac{2}{24}x^2 - \frac{3}{24}y^2 = 1;\ \frac{x^2}{12} - \frac{y^2}{8} = 1$$

Therefore, $a^2 = 12$; $a = 2\sqrt{3}$; $b^2 = 8$; $b = 2\sqrt{2}$

$$c^2 = a^2 + b^2;\ c = 12 + 8 = 20;\ c = 2\sqrt{5}$$

$$e = \frac{2\sqrt{5}}{2\sqrt{3}} = \frac{\sqrt{5}}{\sqrt{3}} = \frac{\sqrt{15}}{3} \approx 1.29$$

31.



By Pythagorus: $l^2 = 2000^2 + x^2$
Therefore, $l^2 - x^2 = 2000^2$ is the equation of a hyperbola.

$$\frac{l^2}{2000^2} - \frac{x^2}{2000^2} = 1$$

$a = 2000$, therefore transverse axis $= 4000$ m
$b = 2000$, therefore transverse axis $= 4000$ m

32. $\pi R^2 - 2\pi r^2 = 24$

$$\frac{\pi R^2}{24} - \frac{2\pi r^2}{24} = 1$$

$$\frac{R^2}{24/\pi} - \frac{r^2}{12} = 1$$ is the equation of a hyperbola.

$$a^2 = \frac{24}{\pi};\ b^2 = 12$$

33. $V = iR$ (Ohm's law)
$6.00 = iR$

Therefore, $i = \dfrac{6.00}{R}$

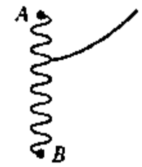
R	i
0.5	12
1	6
2	3
3	2
4	1.5
6	1
9	0.7
12	0.5

34. Foci at $(-3.5, 0)$; $(3.5, 0)$
Vertices at $(-2.8, 0)$; $(2.8, 0)$
Therefore, $a = 2.8$; $c = 3.5$
$c^2 = a^2 + b^2$; $b^2 = (3.5)^2 - (2.8)^2$; $b^2 = 4.41$

Equation of type:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1;\ \frac{x^2}{(2.8)^2} - \frac{y^2}{4.41} = 1;\ \frac{x^2}{7.84} - \frac{y^2}{4.41} = 1$$

35.



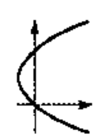
$d_1 - d_2 =$ constant
Let t_1 be time for signal to go from B to ship.
The t_2 is time for signal to go from A to ship.
Where $t_2 = t_1 - 1.20$ ms
$V = \dfrac{s}{t}$, therefore, $s = Vt$
Therefore, $d_1 = 300t_1$ and $d_2 = 300(t_1 - 1.20)$
$d_1 - d_2 = 2a$
$300t_1 - 300(t_1 - 1.20) =$ constant $= 360$ km $= 2a$
Therefore, the ship could lie anywhere on the hyperbolic arc so sketched.
Foci at $(\pm 300, 0)$, therefore $c = 300$
Vertices at $(\pm 180, 0)$, therefore $a = 180$; therefore $b = 240$

36. Foci at $(\pm 2, 0)$, therefore $c = 2$
$d_1 - d_2 = 2 = 2a$, therefore $a = 1$
$b^2 = 4 - 1 = 3$

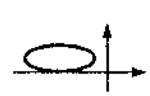
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1;\ \frac{x^2}{1} - \frac{y^2}{3} = 1;\ 3x^2 - y^2 = 3$$

2.7 Translation of Axes

1. $(y-2)^2 = 4(x+1)$; parabola
$y - 2 = y'; x + 1 = x'; y'^2 = 4x'$
Origin O′ at $(h, k) = (-1, 2)$
$y'^2 = 4(1)x'; p = 1;$
Focus $(-1+p, 2)$ or $(0, 2)$
Directrix $x' = -p; x + 1 = -1; x = -2$
Vertex $(-1, 2)$

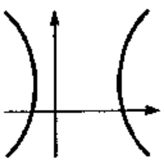


2. $\dfrac{(x+4)^2}{4} + \dfrac{(y-1)^2}{1} = 1$; ellipse
Center $(-4, 1)$; $a = 2$; $b = 1$



3. $\frac{(x-1)^2}{4} - \frac{(y-2)^2}{9} = 1$; hyperbola

Center $(1, 2)$; $a = 2$; $b = 3$



4. $(y+5)^2 = -8(x-2)$; parabola

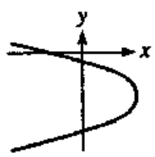
$y + 5 = y'$; $x - 2 = x'$

$y'^2 = -8x'$

Origin $0'$ at $(h, k) = (2, -5)$

$y'^2 = 4(-2)x'$; $p = -2$

Vertex $(2, -5)$, focus $(0, -5)$, directrix $x = 4$



5. $\frac{(x+1)^2}{1} + \frac{y^2}{9} = 1$; ellipse

$x' = x + 1; x - h = x + 1; h = -1$

$y' = y - 0; y - k = y - 0; k = 0$

$\frac{x'^2}{1} - \frac{y'^2}{9} = 1; \frac{y'^2}{9} + \frac{x'^2}{1} = 1$

Center (h, k) at $(-1, 0)$

$a^2 = 9; a = 3; b^2 = 1; b = 1;$

$c^2 = 8; c = 2\sqrt{2}$

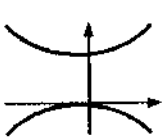
6. $\frac{(y-4)^2}{16} - \frac{(x+2)^2}{4} = 1$;

eq. (21-33), hyperbola

Center $(-2, 4)$

$a = 4$

$b = 2$



7. $(x+3)^2 = -12(y-1)$; eq. (21-29), parabola

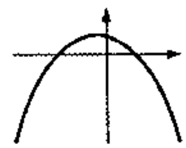
$x' = x + 3$; $y' = y - 1$

$x'^2 = -12y'$

Origin at O′ at $(h, k) = (-3, 1)$

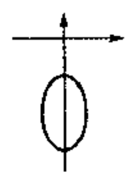
$x'^2 = 4(3y')$; therefore $p = 3$

Vertex $(-3, 1)$, focus $(-3, -2)$, directrix $y = 4$



8. $\frac{x^2}{0.16} + \frac{(y+1)^2}{0.25} = 1$, $\frac{(x-0)}{0.4^2} + \frac{(y-(-1))^2}{0.5^2} = 1$,
ellipse

Center $(0, -1)$, $a = 0.5, b = 0.4$



9. Parabola: $V(-1, 3); p = 4$; parallel to x-axis;

vertex (h, k) at $(-1, 3)$

$y'^2 = 4px'; (y-k)^2 = 4p(x-h)$

$(y-3)^2 = 4(4)[x-(-1)]; (y-3)^2 = 16(x+1)$

$y^2 - 6y + 9 = 16x + 16; y^2 - 6y - 16x + 9 - 16 = 0$

$y^2 - 6y - 16x - 7 = 0$

10. Parabola: $V(2, -1)$, directrix $y = 3$

Focus at $(2, -5)$; $p = 4$

$(x')^2 = -4py'$; parallel to y-axis.

$(x-2)^2 = -16(y+1)$

or $x^2 - 4x + 16y + 20 = 0$

11. $F(12, 0), V(6, 0), p = 6$

$$(y-k)^2 = 4p(x-h)^2$$
$$(y-0)^2 = 4(6)(x-6)$$
$$y^2 = 24(x-6)$$

12. Parabola: $F(2, 4)$, directrix $x = 6$

Therefore, vertex at $V(4, 4)$; $p = 2$

$(y')^2 = -4px'$; parallel to x-axis

$(y-4)^2 = -8(x-4)$

or $y^2 - 8y + 8x - 16 = 0$

13. Ellipse: center $(-2, 2)$; foci $(-5, 2), (1, 2)$; vertices $(-7, 2), (3, 2)$
(h, k) at $(-2, 2)$; $c = 3$; $c^2 = 9$; $a = 5$; $a^2 = 25$;
$b^2 = a^2 - c^2 = 16$

$$\frac{x'^2}{a^2} + \frac{y'^2}{b^2} = 1; \frac{[x - (-2)]^2}{25} + \frac{(y-2)^2}{16} = 1$$
$$\frac{(x+2)^2}{25} + \frac{(y-2)^2}{16} = 1; 16(x+2)^2 + 25(y-2)^2 = 400$$
$$16(x^2 + 4x + 4) + 25(y^2 - 4y + 4) = 400$$
$$16x^2 + 64x + 64 + 25y^2 - 100y + 100 - 400 = 0$$
$$16x^2 + 25y^2 + 64x - 100y - 236 = 0$$

14. Ellipse: center $(0, 3)$, $F(12, 3)$, major axis $= 26$
$2a = 26$; $a = 13$; $c = 12$;
$a^2 = b^2 + c^2$; $b^2 = 13^2 - 12^2 = 25$; $b = 5$

Therefore, $(h, k) = (0, 3)$; major axis parallel to x-axis.

$$\frac{x^2}{a^2} + \frac{(y-k)^2}{b^2} = 1; \frac{x^2}{169} + \frac{(y-3)^2}{25} = 1$$
or $25x^2 + 169y^2 - 1014y - 2704 = 0$

15. Ellipse: $V(-2, -3), V(-2, 5)$, end minor axis $(0, 1)$
$a = 4$; $b = 2$; therefore, $c = \sqrt{12}$
$(h - k) = (-2, 1)$
Major axis parallel to y-axis.

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$
$$\frac{(y-1)^2}{16} + \frac{(x+2)^2}{4} = 1$$
or $4x^2 + y^2 + 16x - 2y + 1 = 0$

16. Ellipse: $F(1, -2), F(1, 10)$, minor axis $= 5$
Therefore, $2c = 12$; $c = 6$

$b = \frac{5}{2}$; center $(1, 4)$

$$a^2 = b^2 + c^2$$
$$a^2 = (2.5)^2 + 36 = \left(\frac{5}{2}\right)^2 + 36$$
$$a^2 = \frac{25}{4} + 36 = \frac{169}{4}$$

Major axis parallel to y-axis.

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1; (h, k) = (1, 4)$$
$$\frac{(y-4)^2}{\frac{169}{4}} + \frac{(x-1)^2}{\frac{25}{4}} = 1$$
$$\frac{4(y-4)^2}{169} + \frac{4(x-1)^2}{25} = 1$$
$$676x^2 + 100y^2 - 1352x - 800y - 1949 = 0$$

17. Hyperbola: center $(h, k) = (-1, 2)$; $F_1 = (-1, 4)$ and
$V_1 = (-1, 1)$
$c^2 = a^2 + b^2$ where $c = \sqrt{(-1+1)^2 + (4-2)^2} = 2$ is the distance between F_1 and (h, k).
Substituting known values:

$$\frac{(y-2)^2}{a^2} - \frac{(x+1)^2}{b^2} = 1 \quad (h, k) \text{ substituted}$$
$$\frac{(1-2)^2}{a^2} - \frac{(-1+1)^2}{b^2} = 1 \quad V_1 \text{ substituted}$$
$$\frac{1}{a^2} - \frac{0}{b^2} = 1; a^2 = 1$$

Since $c^2 = a^2 + b^2$; $2^2 = 1^2 + b^2$; $b^2 = 3$

$$\frac{(y-2)^2}{1} - \frac{(x+1)^2}{3} = 1; 3(y-2)^2 - (x+1)^2 = 3$$
or
$$x^2 - 3y^2 + 2x + 12y - 8 = 0$$

18. Hyperbola: $F(2, 1)$; $F(8, 1)$
Conjugate axis $= 6$
Therefore, $2c = 6$; $c = 3$
$2b = 6$; $b = 3$
$c = b$; center $(h, k) = (5, 1)$
$a^2 = 18$; $a = 3\sqrt{2}$
No hyperbola.
c must be $> b$.
$2a$ must be less than 6.
Therefore, no hyperbola.

19. Hyperbola: $V(2, 1), V(-4, 1), F(-6, 1)$
Center: $(h, k) = (-1, 1)$
$2a = 6$; $a = 3$; $c = 5$
Therefore, $b^2 = c^2 - a^2 = 25 - 9 = 16$
Transverse axis parallel to x-axis.

Therefore, $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$$\frac{(x+1)^2}{9} - \frac{(y-1)^2}{16} = 1$$
or $16x^2 - 9y^2 + 32x + 18y - 137 = 0$

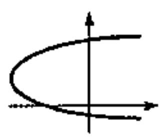
20. Hyperbola; $C(1, -4), F(1, 1)$
Transverse axis $= 8$; $2a = 8$; $a = 4$
$(h, k) = (1, -4)$; $c = 5$
$b^2 = 25 - 16 = 9$
Transverse axis parallel to y-axis.

Therefore, $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$$\frac{(y+4)^2}{16} - \frac{(x-1)^2}{9} = 1$$
or $9y^2 - 16x^2 + 32x + 72y - 16 = 0$

21. $x^2+2x-4y-3=0$; $x^2+2x-3=4y$
$x^2+2x=4y+3$; $x^2+2x+1=4y+3+1$
$(x+1)^2=4y+4$; $(x+1)^2=4(y+1)$
Parabola, $p=1$; $x-h=x+1$; $h=-1$;
$y-k=y+1$; $k=-1$
Vertex is $(-1,-1)$

22. $y^2-2x-2y-9=0$
$y^2-2y-2x=9$
$y^2-2y+1=2x+10$
$(y-1)^2=2(x+5)$
$y'^2=2(x')$; parabola
$y-k=y-1$; $k=1$
$x-h=x+5$; $h=-5$
$4p=2$; $p=\frac{1}{2}$
Vertex: $(-5,1)$



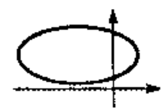
23. $4x^2+9y^2+24x=0$
$4x^2+24x+9y^2=0$
$x^2+6x+\frac{9}{4}y^2=0$
$x^2+6x+9+\frac{9}{4}y^2=9$
$(x+3)^2+\frac{9}{4}y^2=0$
$\frac{(x+3)^2}{9}+\frac{y^2}{4}=1$; ellipse
$x-h=3$; $h=-3$; $a=3$
$y-k=y$; $k=0$; $b=2$
Center: $(-3,0)$

24. $2x^2+9y^2+8x-72y+134=0$
$2x^2+8x+9y^2-72y=-134$
$2(x^2+4x)+9(y^2-8y)=-134$

$$\frac{(x^2+4x+4)}{\frac{1}{2}}+\frac{(y^2-8y+16)}{\frac{1}{9}}=-134+8+144$$

$$\frac{(x+2)^2}{\frac{1}{2}}+\frac{(y-4)^2}{\frac{1}{9}}=18$$

$\frac{(x+2)^2}{9}+\frac{(y-4)^2}{2}=1$; ellipse
Center: $(-2,4)$



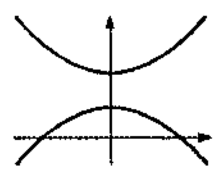
25. $9x^2-y^2+8y-7=0$
$9x^2-(y^2-8y+7+9)=-9$
$9x^2-(y-4)^2=-9$

$$\frac{9x^2}{-9}+\frac{(y-4)^2}{-9}=1$$

$$-x^2+\frac{(y-4)^2}{9}=1$$

$$\frac{(y-4)^2}{9}-\frac{x^2}{1}=1$$

Hyperbola, (h,k) is $(0,4)$; $a=3$; $b=1$



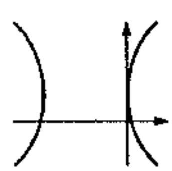
26. $5x^2-4y^2+20x+8y=4$
$5x^2+20x-4y^2+8y=4$
$5(x^2+4x)-4(y^2-2y)=4$

$$\frac{(x^2+4x+4)}{\frac{1}{5}}-\frac{(y^2-2y+1)}{\frac{1}{4}}=4+20-4$$

$$\frac{(x+2)^2}{\frac{1}{5}}-\frac{(y-1)^2}{\frac{1}{4}}=20$$

$$\frac{(x^2+2)^2}{4}-\frac{(y-1)^2}{5}=1$$

Hyperbola, center $(-2,1)$



27. $2x^2 - 4x = 9y - 2$

$$x^2 - 2x = \frac{9}{2}y - 1$$

$$(x^2 - 2x + 1) = \frac{9}{2}y - 1 + 1$$

$$(x-1)^2 = \frac{9}{2}y$$

$$(x')^2 = \frac{9}{2}y'$$

A parabola, parallel to y-axis

$x - h = x - 1$; $h = 1$

$y - k = y$; $k = 0$

Vertex $(1, 0)$; $4p = \frac{9}{2}$; $p = \frac{9}{8}$

28. $0.04x^2 + 0.16y^2 = 0.0y$; $0.04x^2 + 0.16y^2 - 0.01y = 0$

$$x^2 + 4y^2 - \frac{1}{4}y = 0$$

$$x^2 + 4\left(y^2 - \frac{1}{16}y\right) = 0$$

$$x^2 + 4\left(y^2 - \frac{1}{16}y + \left(\frac{1}{32}\right)^2\right) = \left(\frac{1}{32}\right)^2 = 4$$

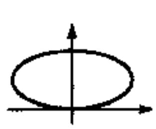
$$x^2 + \frac{\left(y - \frac{1}{32}\right)^2}{\frac{1}{4}} = \frac{1}{256};$$

$$256x^2 + 256\frac{\left(y - \frac{1}{32}\right)^2}{\frac{1}{4}} = 1$$

$$\frac{x^2}{\frac{1}{256}} + \frac{\left(y - \frac{1}{32}\right)^2}{\frac{1}{1024}} = 1$$

$a^2 = \frac{1}{256}$; $b^2 = \frac{1}{1024}$

An ellipse. Center $\left(0, \frac{1}{32}\right)$ or $(0, 0.03)$



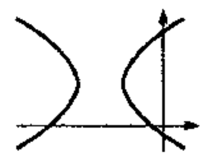
29. $4x^2 - y^2 + 32x + 10y + 35 = 0$

$4(x^2 + 8x) - (y^2 - 10y) = -35$

$4(x^2 + 8x + 16) - (y^2 - 10y + 25) = -35 + 64 - 25$

$$\frac{(x+4)^2}{1^2} - \frac{(y-5)^2}{2^2} = 1, \text{ hyperbola}$$

$C(-4, 5)$

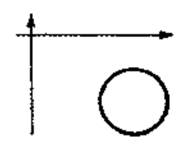


30. $2x^2 + 2y^2 - 24x + 16y + 95 = 0$

$2(x^2 - 12x + 36) + 2(y^2 + 8y + 16) = -95 + 72 + 32 = 9$

$(x-6)^2 + (y+4)^2 = \frac{9}{2}$; circle,

center $(6, -4)$, $r = \frac{3}{\sqrt{2}}$



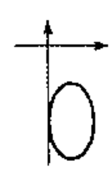
31. $9x^2 + 4y^2 - 12x + 16y + 16 = 0$

$9x^2 - 12x + 4y^2 + 16y + 16 = 0$

$$9\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) + 4(y^2 + 4y + 4) = -16 + 16 + 4$$

$$\frac{\left(x - \frac{2}{3}\right)^2}{\frac{4}{9}} + \frac{(y - (-2))^2}{1} = 1; \text{ ellipse,}$$

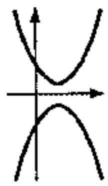
center $\left(\frac{2}{3}, -2\right)$



32. $5x^2 - 3y^2 - 40x + 95 = 0$
$5(x^2 - 8x + 16) - 3y^2 = -95 + 80$
$= -15$

$\frac{y^2}{5} - \frac{(x-4)^2}{3} = 1$, hyperbola,

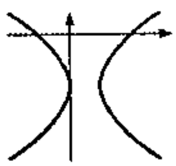
center $(4, 0)$



33. $7x^2 - y^2 - 14x - 16y - 64 = 0$
$7x^2 - 14x - y^2 - 16y - 64 = 0$
$7(x^2 - 2x + 1) - (y^2 + 16y + 64) = 64 + 7 - 64$

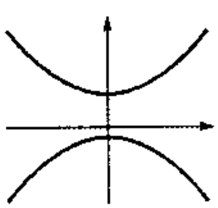
$\frac{(x-1)^2}{1^2} - \frac{(y+8)^2}{\sqrt{7}^2} = 1$

hyperbola, $C(1, -8)$



34. $5x^2 - 2y^2 + 12y + 18 = 0$
$5x^2 - 2(y^2 - 6y + 9) = -18 - 18$
$2(y-3)^2 - 5x^2 = 36$; hyperbola,

center $(0, 3)$



35. $9x^2 + 9y^2 - 6x - 24y + 14 = 0$

$9\left(x^2 - \frac{2}{3}x + \frac{1}{9}\right) + 9\left(y^2 - \frac{8}{3}y + \frac{16}{9}\right)$

$= -14 + 1 + 16 = 3$

$\left(x - \frac{1}{3}\right)^2 + \left(y - \frac{4}{3}\right)^2 = \frac{1}{3}$; circle,

center $\left(\frac{1}{3}, \frac{4}{3}\right)$, $r = \frac{1}{\sqrt{3}}$

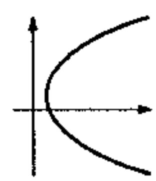
36. $4y^2 - 15x - 12y + 29 = 0$

$4\left(y^2 - 3y + \frac{9}{4}\right)^2 = 15x - 29 + 9$
$= 15x - 20$

$4\left(y - \frac{3}{2}\right)^2 = 15\left(x - \frac{4}{3}\right)$

$\left(y - \frac{3}{2}\right)^2 = \frac{15}{4}\left(x - \frac{4}{3}\right)$; parabola,

vertex $\left(\frac{4}{3}, \frac{3}{2}\right)$



37. Hyperbola: asymptotes: $x - y = -1$ or $x + 1 = y$, and $x + y = -3$ or $y = -x - 3$; vertices $(3, -1)$ and $(-7, -1)$. The center is at the point of interaction of the asymptotes. The equations for the asymptotes are solved simultaneously by adding, $2y = -2; y = -1; -1 = x + 1; x = -2$. Therefore, the coordinates of the center are $(-2, -1)$. Since the slopes are 1 and $-1, a = b$, where a is the distance from the center $(-2, -1)$ to the vertex $(3, -1); a = 5, b = 5$.

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1;$$

$$\frac{[x-(-2)]^2}{25} - \frac{[y-(-1)]^2}{25} = 1$$

$$\frac{(x+2)^2}{25} - \frac{(y+1)^2}{25} = 1$$

$x^2 + 4x + 4 - (y^2 + 2y + 1) = 25;$
$x^2 + 4x + 4 - 2y - 1 = 25$
$x^2 - y^2 + 4x - 2y - 22 = 0$

38. $x^2 + y^2 + 4x - 5 = 0$
$x^2 + 4x + 4 + y^2 = 5 + 4$
$(x+2)^2 + y^2 = 9$
Center $(-2, 0)$; $r = 3$
Ellipse: $c = 3$; $b = 3$
Therefore, $a = \sqrt{18} = 3\sqrt{2}$
Center $(h, k) = (-2, 0)$

$$\frac{(x+2)^2}{18} + \frac{y^2}{9} = 1$$

$x^2 + 4x + 4 + 2y^2 = 18$
$x^2 + 2y^2 + 4x - 14 = 0$

39. $y^2 = 4x$ (second parabola)
$4p = 4$; $p = 1$
Therefore, focus of second is at $(1, 0)$.
Vertex of second is at $(0, 0)$.
Therefore, focus of first is $(0, 0)$.
Vertex of first is $(1, 0)$; $p = -1$
$y'^2 = 4px'$
$(y-k)^2 = 4p(x-h)$
$(y-0)^2 = 4(-1)(x-1)$
$y^2 = -4x + 4$
$y^2 + 4x - 4 = 0$

40. $4y^2 - x^2 - 6x - 2y - 14 = 0$; $4y^2 - 2y - x^2 - 6x = 14$

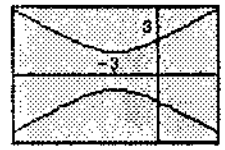
$$4\left(y^2 - \frac{1}{2}y\right) - (x^2 + 6x) = 14$$

$$4\left(y^2 - \frac{1}{2}y + \frac{1}{16}\right) - (x^2 + 6x + 9) = 14 + \frac{1}{4} - 9$$

$$\frac{\left(y - \frac{1}{4}\right)^2}{\frac{1}{4}} - (x+3)^2 = \frac{21}{4};$$

$$\frac{\left(y - \frac{1}{4}\right)^2}{\frac{21}{16}} - \frac{(x+3)^2}{\frac{21}{4}} = 1$$

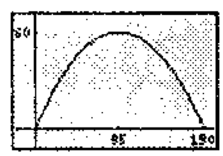
Hyperbola



41. $(x-h)^2 = 4p(y-k)$
$(x-95)^2 = 4p(y-60)$
Solve for $4p$ using $(x, y) = (0, 0)$.
$(-95)^2 = 4p(-60)$

$$4p = \frac{95^2}{-60}$$

$$(x-95)^2 = \frac{95^2}{-60}(y-60)$$



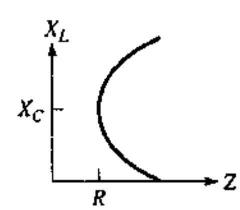
42. $|Z| = \sqrt{R^2 + (X_L - X_c)^2}$; X_c constant, R constant
$Z^2 = R^2 + (X_L - X_c)^2$
$(X_L - X_c)^2 = Z^2 - R^2$; $Z^2 - (X_L - X_c)^2 = R^2$

$$\frac{Z^2}{R^2} - \frac{(X_L - X_c)^2}{R^2} = 1$$

Hyperbola

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$h = 0$, $k = X_c$, center $(0, X_c)$
a and $b = R$



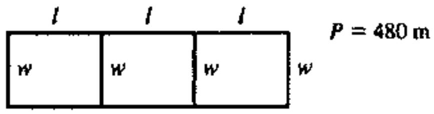
43. First ellipse:
$a = 4, b = 3$, therefore $c = \sqrt{7}$
Center $(0, 0)$

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1;\ \frac{y^2}{16} + \frac{x^2}{9.0} = 1$$

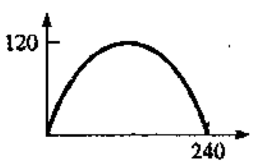
Second ellipse:
$a = 4, b = 3$, therefore $c = \sqrt{7}$
Center $(7.0, 0.0)$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1;\ \frac{(x-7)^2}{16} + \frac{y^2}{9.0} = 1$$

44.



$P = 2l + 2w = 480$; $l + w = 240$; $l = 240 - w$
$A = lw$; $A = (240 - w)(w)$; $A = 240w - w^2$ is a parabola, concave down.



A is greatest at $w = 120$ m (position of axis of parabola).

2.8 The Second-Degree Equation

1. $x^2 + 2y^2 - 2 = 0$
$A \neq C$, they have the same sign, and $B = 0$; ellipse

2. $x^2 - y = 0$; $y = x^2$
$A \neq 0$; $B = 0$; $C = 0$; parabola

3. $2x^2 - y^2 - 1 = 0$
A and C have different signs, $B = 0$; hyperbola

4. $y(y + x^2) = 4$
$y^2 + yx^2 = 4$, not second degree.

5. $2x^2 + 2y^2 - 3y - 1 = 0$
$A = C$; $B = 0$; circle

6. $x^2 - 3x = y - 2y^3$, not second degree.

7. $2.2x^2 - x - y = 1.6$
$A \neq 0$; $C = 0$; $B = 0$; parabola

8. $2x^2 + 4y^2 - y - 2x = 4$
$A \neq C$; $B = 0$; ellipse

9. $x^2 = y^2 - 1$; $x^2 - y^2 + 1 = 0$
A and C have different signs;
$B = 0$; hyperbola

10. $32x^2 = 21y - 47y^2$; $32x^2 + 47y^2 - 21y = 0$
$A \neq C$; $B = 0$; ellipse

11. $3.6x^2 = 1.1y - 3.6y^2$; $3.6x^2 + 3.6y^2 - 1.1y = 0$
$A = C$; $B = 0$; circle

12. $y = 3 - 6x^2$; $6x^2 + y - 3 = 0$
$A \neq 0$; $B = 0$; $C = 0$; parabola

13. $y(3 - 2x) = x(5 - 2y)$
$3y - 2xy = 5x - 2xy$

$$y = \frac{5}{3}x, \text{ line}$$

14. $x(13 - 5x) = 5y^2$; $13x - 5x^2 = 5y^2$;
$5y^2 + 5x^2 - 13x = 0$
$A = C$; $B = 0$; circle

15. $2xy + x - 3y = 6$
$A = 0$; $B \neq 0$; $C = 0$; hyperbola

16. $(y + 1)^2 = x^2 + y^2 - 1$; $y^2 + 2y + 1 = x^2 + y^2 - 1$
$x^2 - 2y - 2 = 0$
$A \neq 0$; $B = 0$; $C = 0$; parabola

17. $2x(x - y) = y(3 - y - 2x)$;
$2x^2 - 2xy = 3y - y^2 - 2xy$
$2x^2 - 2xy + 2xy + y^2 - 3y = 0$;
$2x^2 + y^2 - 3y = 0$
$A \neq 0$, same sign; $B = 0$; ellipse

18. $2x^2 = x(x - 1) + 4y^2$; $2x^2 = x^2 - x + 4y^2$
$x^2 - 4y^2 + x = 0$
A and C are different signs, $B = 0$; hyperbola

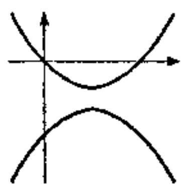
19. $x(y + 3x) = x^2 + xy - y^2 + 1$;
$xy + 3x^2 = x^2 + xy - y^2 + 1$
$2x^2 + y^2 - 1 = 0$
$A \neq C$; $B = 0$; ellipse

20. $4x(x - 1) = 2x^2 - 2y^2 + 3$;
$4x^2 - 4x = 2x^2 - 2y^2 + 32x^2 + 2y^2 - 4x - 3 = 0$
$A = C$; $B = 0$; circle

21. $x^2 = 8(y - x - 2)$
$x^2 = 8y - 8x - 16$
$x^2 + 8x - 8y + 16 = 0$
$A \neq 0$; $B = 0$;
$C = 0$; parabola
$x^2 + 8x - 8y + 16 = 0$
$x^2 + 8x + 16 = 8y$
$(x+4)^2 = 4(2)y; p = 2$
Vertex $(-4, 0)$, focus $(-4, 2)$

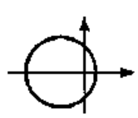
22. $x^2 = 6x - 4y^2 - 1$
$x^2 + 4y^2 - 6x + 1 = 0$
$A \neq C$; $B = 0$; ellipse
$x^2 - 6x + 4y^2 = -1$
$x^2 - 6x + 9 + 4y^2 = -1 + 9$
$(x-3)^2 + 4y^2 = 8$
$$\frac{(x-3)^2}{8} + \frac{y^2}{2} = 1$$
$(h, k) = (3, 0)$
$a = 2\sqrt{2}, b = \sqrt{2}$, therefore $c = \sqrt{6}$
$V(3 \pm 2\sqrt{2}, 0)$

23. $y^2 = 2(x^2 - 2x - 2y)$
$y^2 = 2x^2 - 4x - 4y$
$y^2 - 2x^2 + 4y + 4x = 0$
A and C are different signs.
$B = 0$; hyperbola
$y^2 + 4y - 2x^2 + 4x = 0$



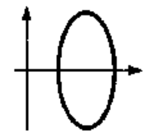
$y^2 + 4y + 4 - 2(x^2 - 2x + 1) = 4 - 2$
$(y+2)^2 - 2(x-1)^2 = 2$
$$\frac{(y+2)^2}{2} - (x-1)^2 = 1$$
$(h, k) = (1, -2)$
$a^2 = 2$; $b^2 = 1$; $c^2 = 3$
$C(1, -2), V(1, -2 \pm \sqrt{2})$

24. $4x^2 + 4 = 9 - 8x - 4y^2$
$4x^2 + 4y^2 + 8x - 5 = 0$
$A = C$; $B = 0$; circle
$4x^2 + 8x + 4y^2 = 5$



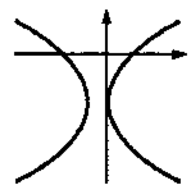
$$x^2 + 2x + y^2 = \frac{5}{4}$$
$$(x+1)^2 + y^2 = \frac{5}{4} + 1$$
$$(x+1)^2 + y^2 = \frac{9}{4}$$
$$(h, k) = (-1, 0);\ r = \frac{3}{2}$$

25. $y^2 + 42 = 2x(10 - x)$
$y^2 + 42 = 20x - 2x^2$
$y^2 + 2x^2 - 20x + 42 = 0$; ellipse



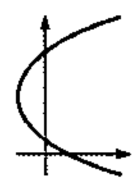
$$\frac{y^2}{2} + x^2 - 10x = -21$$
$$\frac{y^2}{2} + x^2 - 10x + 25 = -21 + 25$$
$$\frac{y^2}{2} + (x-5)^2 = 4$$
$$\frac{y^2}{8} + \frac{(x-5)^2}{4} = 1$$
(h, k) at $(5, 0), V(5, \pm 2\sqrt{2})$
$a = \sqrt{8} = 2\sqrt{2}; b = 2$

26. $x^2 - 4y = y^2 + 4(1 - x)$
$x^2 - 4y - y^2 - 4 + 4x = 0$
$x^2 - y^2 + 4x - 4y - 4 = 0$
A and C are different signs.
$B = 0$; hyperbola
$x^2 + 4x - y^2 - 4y = 4$



$x^2 + 4x + 4 - (y^2 + 4y + 4) = 4 + 4 - 4$
$(x+2)^2 - (y+2)^2 = 4$
$$\frac{(x+2)^2}{4} - \frac{(y+2)^2}{4} = 1$$
$(h - k) = (-2, -2)$
$a^2 = 4, b^2 = 4, c^2 = 8$
$C(-2, -2)$; $V(0, -2)$; $V(-4, -2)$

27. $4(y^2 - 4x - 2) = 5(4y - 5)$
$4y^2 - 16x - 8 = 20y - 25$
$4y^2 - 20y - 16x + 17 = 0$
$A = 0, B = 0, C \neq 0$; parabola
$4y^2 - 20y = 16x - 17$



$$y^2 - 5y = 4x - \frac{17}{4}$$
$$\left(y^2 - 5y + \frac{25}{4}\right) = 4x - \frac{17}{4} + \frac{25}{4}$$
$$\left(y - \frac{5}{2}\right)^2 = 4x + 2$$
$$\left(y - \frac{5}{2}\right)^2 = 4\left(x + \frac{1}{2}\right)$$
$(y')^2 = 4p(x')$
$$y - k = y - \frac{5}{2};\ k = \frac{5}{2}$$
$$x - h = x + \frac{1}{2};\ h = -\frac{1}{2}$$
$4p = 4$; $p = 1$

$V\left(-\frac{1}{2},\frac{5}{2}\right), F\left(\frac{1}{2},\frac{5}{2}\right)$

Directrix $x = -\frac{3}{2}$

28. $2(2x^2 - y) = 8 - y^2$

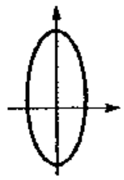
$4x^2 - 2y - 8 + y^2 = 0$

$4x^2 + y^2 - 2y - 8 = 0$

$A \neq C$; $B = 0$; ellipse

$4x^2 + y^2 - 2y + 1 = 8 + 1$

$4x^2 + (y-1)^2 = 9$



$$\frac{x^2}{\frac{9}{4}} + \frac{(y-1)^2}{9} = 1$$

$$\frac{(y-1)^2}{9} + \frac{x^2}{\frac{9}{4}} = 1$$

$(h,k) = (0,1)$

$a^2 = 9, b^2 = \frac{9}{4}, c^2 = \frac{27}{4}$

$C(0,1)$; $V(0,4), V(0,-2)$

29. $x^2 + 2y^2 - 4x + 12y + 14 = 0$; solve for y

$x^2 + 0xy + 2y^2 - 4x + 12y + 14 = 0$

$A \neq C$, they have the same sign, and $B = 0$; ellipse

Solve for y.

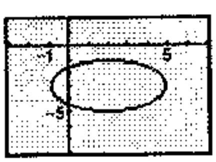
$2y^2 + 12 + (x^2 - 4x + 14) = 0$

$$y = \frac{-12 \pm \sqrt{12^2 - 4(2)(x^2 - 4x + 14)}}{2(2)}$$

$$= \frac{-12 \pm \sqrt{-8x^2 + 32x + 32}}{4}$$

$y_1 = -3 + 0.5\sqrt{-2x^2 + 8x + 8}$

$y_2 = -3 - 0.5\sqrt{-2x^2 + 8x + 8}$



30. $4y^2 - x^2 + 40y - 4x + 60 = 0$

A and C have different signs; $B = 0$; hyperbola; solve for y.

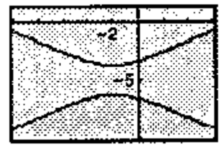
$4y^2 + 40y + (-x^2 - 4x + 60) = 0$

$$y = \frac{-40 \pm \sqrt{40^2 - 4(4)(-x^2 - 4x + 60)}}{2(4)}$$

$$= \frac{-40 \pm \sqrt{16x^2 + 64x + 640}}{8}$$

$y_1 = -5 + 0.5\sqrt{x^2 + 4x + 10}$

$y_2 = -5 - 0.5\sqrt{x^2 + 4x + 10}$



31. $x^2 + 6xy + 9y^2 - 2x + 14y - 10 = 0$; solve for y.

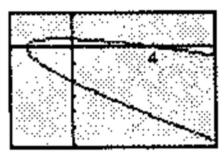
$9y^2 + (14 + 6x)y + (x^2 - 2x - 10) = 0$

$$y = \frac{-(14+6x) \pm \sqrt{(14+6x)^2 - 4(9)(x^2 - 2x - 10)}}{2(9)}$$

$$= \frac{-(14+6x) \pm \sqrt{240x + 556}}{18}$$

$$y_1 = \frac{-7 - 3x + \sqrt{60x + 139}}{9}$$

$$y_2 = \frac{-7 - 3x - \sqrt{60x + 139}}{9}$$



32. $x^2 - xy + y^2 - 6 = 0$; solve for y

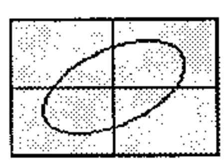
$y^2 - xy + (x^2 - 6) = 0$

$$y = \frac{x \pm \sqrt{(-x)^2 - 4(x^2 - 6)}}{2}$$

$$= \frac{x \pm \sqrt{24 - 3x^2}}{2}$$

$y_1 = 0.5(x + \sqrt{24 - 3x^2})$

$y_2 = 0.5(x - \sqrt{24 - 3x^2})$



33. **(a)** If $k = 1$, $x^2 + ky^2 = a^2$; $x^2 + (1)y^2 = a^2$
$x^2 + y^2 = a^2$(circle)

(b) If $k < 0$, $x^2 + ky^2 = a^2$; $x^2 - |k|y^2 = a^2$
$\frac{x^2}{a^2} - \frac{y^2}{a^2/|k|} = 1$ (hyperbola)

(c) If $k > 0$ $(k \neq 1)$, $x^2 + ky^2 = a^2$
$\frac{x^2}{a^2} + \frac{y^2}{a^2/k} = 1$ (ellipse)

34. $\frac{x^2}{4-C} - \frac{y^2}{C} = 1$

(a) $C < 0$; $\frac{x^2}{4+C} - \frac{y^2}{-C} = 1$; $\frac{x^2}{4+C} + \frac{y^2}{C}$

Let $\frac{1}{4+C} = k_1$ and $\frac{1}{C} = k_2$
$k_1x^2 + k_2y^2 - 1 = 0$
Eq. (21-34): $k_1 \neq k_2$, $B = 0$; therefore ellipse

(b) $0 < C < 4$; $\frac{x^2}{4-C} - \frac{y^2}{C} = 1$

Therefore $4 - C > 0$; let $\frac{1}{4-C} = k_1 > 0$

$C > 0$; let $\frac{1}{C} = k_2 > 0$
$k_1x^2 - k_2y^2 = 1$
Eq. (21-34): k_1 and k_2 different signs, $B = 0$
hyperbola

35. $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$
$A = C \neq 0$; $B = D = E = F = 0$
Let $A = C = k$
$kx^2 + ky^2 = 0$; $y^2 = -x^2$; $y = \sqrt{-x^2}$
Valid in real number system for $x = 0$ only. For $x < 0$ and $0 < x$ we have imaginary number.
Solution is $(0, 0)$ origin.

36. $\frac{x^2}{4-C} - \frac{y^2}{C} = 1$; $C > 4$; $4 - C < 0$

$\frac{y^2}{C} = \frac{x^2}{4-C} - 1$; $y^2 = C\left(\frac{x^2}{4-C} - 1\right)$

$y = \sqrt{C\left(\frac{x^2}{4-C} - 1\right)}$; $\frac{x^2}{4-C} < 0$ for all $x, C > 4$

Therefore, $C\left(\frac{x^2}{4-C} - 1\right) < 0$ for all x.

No real solution (root of negative numbers).

37. $x^2 + y^2 = (x+3)^2$
$x^2 + y^2 = x^2 + 6x + 9$
$y^2 = 6x + 9 = 6\left(x + \frac{3}{2}\right)$
Therefore a parabola.

38. $A_T = \pi(r-2)^2 + \pi r^2$; $A_T = \pi(r^2 - 2r + 4) + \pi r^2$
$A_T = \pi r^2 - 2\pi r + 4\pi + \pi r^2$; $A_T = 2\pi r^2 - 2\pi r + 4\pi$
$A_T - 2\pi r^2 + 2\pi r - 4\pi = 0$
Coefficient of $A_T^2 = 0$; coefficient of $r^2 \neq 0$; no Ar term
Therefore a parabola.

39. (a) Beam is perpendicular to floor. We have a circle.

(b) Beam is not perpendicular to floor. We have an ellipse.

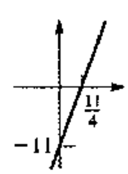
* See conic section diagrams, Fig. 2-90, p. 63 of text.

40. Shape of curve on lake is that of a hyperbola.

* See conic section diagrams, Fig. 2-90, p. 63 of text.

Chapter 2 Review Exercises

1. Given: straight line; (x_1, y_1) is $(1, -7)$; $m = 4$
$y - y_1 = m(x - x_1)$; $y - (-7) = 4(x - 1)$
$y + 7 = 4x - 4$; $y = 4x - 4 - 7$
$y = 4x - 11$ or $4x - y - 11 = 0$

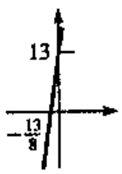


2. Given: straight line; $(x_1, y_1) = (-1, 5)$ and $(x_2, y_2) = (-2, -3)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-5}{-2-(-1)}$
$= 8$

$y - y_1 = m(x - x_1)$

$$\begin{aligned} y - 5 &= 8(x - (-1)) \\ &= 8(x+1) \\ &= 8x + 8 \\ 8x - y + 13 &= 0 \end{aligned}$$

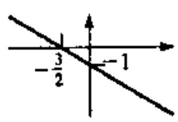


3. Given: straight line; perpendicular to $3x - 2y + 8 = 0$ with y-intercept of $(0, -1)$

$$\begin{aligned} 3x - 2y + 8 &= 0 \\ 2y &= 3x + 8 \\ y &= \frac{3}{2}x + 4, \\ m &= \frac{3}{2} \text{ from which } m = -\frac{2}{3} \end{aligned}$$

for given line.

$$\begin{aligned} y = mx + b &= -\frac{2}{3}x + (-1) \\ &= -\frac{2}{3}x - 1 \\ 3y &= -2x - 3 \\ 2x + 3y + 3 &= 0 \end{aligned}$$

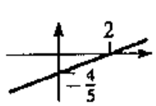


4. Given: straight line; parallel to $2x - 5y + 1 = 0$ with x-intercept $(2, 0)$.

$$\begin{aligned} 2x - 5y + 1 &= 0 \\ 5y &= 2x + 1 \\ y &= \frac{2}{5}x + \frac{1}{5} \text{ from which} \end{aligned}$$

$m = \frac{2}{5}$ for the given line.

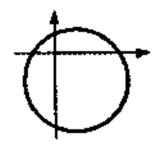
$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 0 &= \frac{2}{5}(x - 2) \\ 5y &= 2x - 4 \\ 2x - 5y - 4 &= 0 \end{aligned}$$



5. Given: circle; (h, k) is $(1, -2)$; (x, y) is $(4, -3)$

$$\begin{aligned} r &= \sqrt{(1-4)^2 + [-2 - (-3)]^2} \\ &= \sqrt{(-3)^2 + (-2+3)^2} \\ &= \sqrt{9+1} = \sqrt{10} \end{aligned}$$

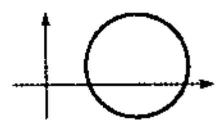
$$\begin{aligned} &(x-h)^2 + (y-k)^2 = r^2; \\ &(x-1)^2 + (y+2)^2 = \sqrt{10^2} \\ &x^2 - 2x + 1 + y^2 + 4y + 4 = 10 \\ &x^2 + y^2 - 2x + 4y + 1 + 4 - 10 = 0 \\ &x^2 + y^2 - 2x + 4y + 1 + 4 - 10 = 0; \\ &x^2 + y^2 - 2x + 4y - 5 = 0 \end{aligned}$$



6. Given: circle; tangent to line $x = 3$, center $(5, 1)$

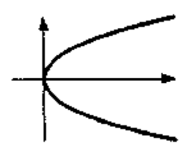
$(x-h)^2 + (y-k)^2 = r^2$ and letting $r = 2$,

$$\begin{aligned} &(x-5)^2 + (y-1)^2 = 2^2 \\ &x^2 - 10x + 25 + y^2 - 2y + 1 = 4 \\ &x^2 + y^2 - 10x - 2y + 22 = 0 \end{aligned}$$



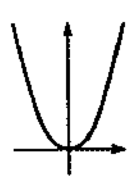
7. Given: parabola; focus $(3, 0)$, vertex $(0, 0)$

$$\begin{aligned} (y-k)^2 &= 4p(x-h) \\ (y-0)^2 &= 4(3)(x-0) \\ y^2 &= 12x \end{aligned}$$



8. Given: parabola; directrix, $y = -5$, vertex $(0, 0)$

$$\begin{aligned} (x-h)^2 &= 4p(y-k) \\ (x-0)^2 &= 4(5)(y-0) \\ x^2 &= 20y \end{aligned}$$



9. Given: ellipse; vertex $(10,0)$; focus $(8,0)$; (h,k) is $(0,0)$

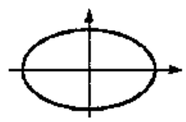
$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$
$$\frac{x^2}{100}+\frac{y^2}{36}=1$$
$$9x^2+25y^2=900$$

$a=10$ $\qquad c=8$

$b^2=a^2-c^2$

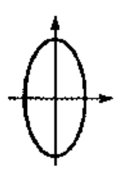
$b^2=100-64$

$b^2=36$



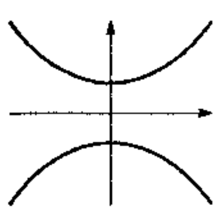
10. Given: ellipse; center $(0,0)$, passes through $(0,3)$ and $(2,1)$.

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$
$$\frac{(x-0)^2}{a^2}+\frac{(y-0)^2}{3^2}=1$$
$$\frac{x^2}{a^2}+\frac{y^2}{9}=1,\text{ letting } (x,y)=(2,1)$$
$$\frac{2^2}{a^2}+\frac{1^2}{9}=1$$
$$\frac{1}{a^2}=\frac{2}{9}$$
$$\frac{2x^2}{9}+\frac{y^2}{9}=1$$
$$2x^2+y^2=9$$



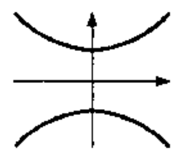
11. Given: hyperbola, $V(0,13), C(0,0)$, conj. axis 24.

$$\frac{(y-h)^2}{a^2}-\frac{(x-k)^2}{b^2}=1$$
$$\frac{(y-0)^2}{13^2}-\frac{(x-0)^2}{12^2}=1$$
$$144y^2-169x^2=24{,}336$$



12. Given: hyperbola; $F(0,10), F(0,-10), V(0,8)$

$$\frac{(y-h)^2}{a^2}-\frac{(x-k)^2}{b^2}=1$$
$$\frac{(y-0)^2}{8^2}-\frac{x^2}{b^2}=1$$
$$c^2=a^2+b^2$$
$$10^2=8^2+b^2$$
$$36=b^2$$
$$\frac{y^2}{64}-\frac{x^2}{36}=1$$
$$36y^2-64x^2=2304$$
$$9y^2-16x^2=576$$



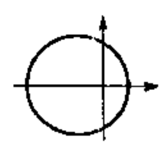
13. Given: $x^2+y^2+6x-7=0$

$(x^2+6x)+(y^2)=7$; $(x^2+6x+9)+y^2=7+9$

$(x+3)^2+(y+0)^2=16$

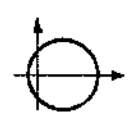
$[x-(-3)]^2+(y-0)^2=4^2$

center $(h,k)=(-3,0)$; radius $r=4$



14. Given:
$$x^2+y^2-4x+2y-20=0$$
$$x^2-4x+y^2+2y=20$$
$$x^2-4x+4+y^2+2y+1=25$$
$$(x-2)^2+(y+1)^2=5^2$$

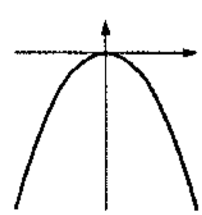
center $(h,k)=(2,-1)$; radius $r=5$



15. Given: $x^2=-20y$

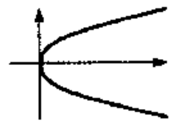
$x^2=4(-5)y$, $p=-5$

focus $(0,-5)$. directrix $y=5$



16. Given: $y^2 = 24x = 4(6)x, p = 6$

focus $(6,0)$, directrix, $x = -6$



17. Given: $16x^2 + y^2 = 16$

$$\frac{16x^2}{16} + \frac{y^2}{16} = 1;\ \frac{x^2}{1^2} + \frac{y^2}{4^2} = 1;\ \frac{y^2}{4^2} + \frac{x^2}{1^2} = 1$$

$a = 4, b = 1, c = \sqrt{16-1} = \sqrt{15}$

vertices $(0,a)$, $(0,-a)$ or $(0,4)$, $(0,-4)$

foci $(0,c)$, $(0,-c)$ or $(0,\sqrt{15})$, $(0,-\sqrt{15})$

18. Given: $2y^2 - 9x^2 = 18$

$$\frac{y^2}{9} - \frac{x^2}{2} = 1$$

$$\frac{y^2}{3^2} - \frac{x^2}{\sqrt{2}^2} = 1$$

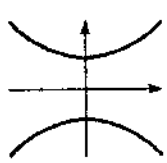
$c^2 = a^2 + b^2 = 9 + 2$

$= 11$

$c = \sqrt{11}$

vertices: $(0,3), (0,-3)$.

foci: $(0,\sqrt{11})$, $(0,-\sqrt{11})$



19. Given: $2x^2 - 5y^2 = 0.25$

$$\frac{x^2}{(\frac{1}{2\sqrt{2}})^2} - \frac{y^2}{(\frac{1}{2\sqrt{5}})^2} = 1$$

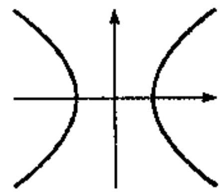
$$c^2 = a^2 + b^2 = \frac{1}{8} + \frac{1}{20}$$

$$= \frac{7}{40}$$

$$c = \frac{\sqrt{70}}{20}$$

vertices: $\left(\frac{1}{2\sqrt{2}}, 0\right), \left(\frac{-1}{2\sqrt{2}}, 0\right)$

foci: $\left(\frac{\sqrt{70}}{20}, 0\right), \left(\frac{-\sqrt{70}}{20}, 0\right)$



20. Given: $2x^2 + 25y^2 = 800$

$$\frac{x^2}{400} + \frac{y^2}{32} = 1$$

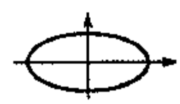
$$\frac{x^2}{20^2} + \frac{y^2}{(4\sqrt{2})^2} = 1$$

$c^2 = a^2 + b^2 = 400 + 32 = 432$

$c = 4\sqrt{23}$

vertices: $(20,0), (-20,0)$

foci: $(4\sqrt{23},0), (-4\sqrt{23},0)$



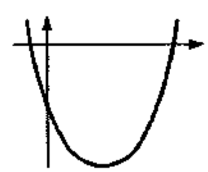
21. Given: $x^2 - 8x - 4y - 16 = 0$

$x^2 - 8x = 4y + 16$; $x^2 - 8x + 16 = 4y + 16 + 16$

$(x-4)^2 = 4y + 32$; $(x-4)^2 = 4(y+8)$

$(x-4)^2 = 4(1)(y+8)$; $p = 1$

vertex (h,k) is $(4,-8)$; focus is $(4,-7)$



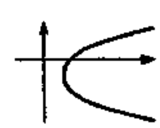
22. Given: $y^2 - 4x + 4y + 24 = 0$

$$y^2 + 4y + 4 = 4x - 24 + 4$$

$$(y+2)^2 = 4x - 20$$

$$(y+2)^2 = 4(1)(x-5), p = 1$$

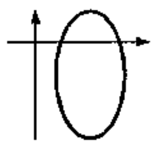
vertex: $(5,-2)$, directrix: $x = 4$



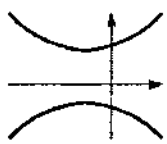
23. Given:

$$4x^2 + y^2 - 16x + 2y + 13 = 0$$
$$4x^2 - 16x + y^2 + 2y = -13$$
$$4(x^2 - 4x + 4) + y^2 + 2y + 1 = -13 + 16 + 1$$
$$4(x-2)^2 + (y+1)^2 = 4$$
$$\frac{(x-2)^2}{1} + \frac{(y+1)^2}{4} = 1$$

center: $(2, -1)$



24.
$$x^2 - 2y^2 + 4x + 4y = 6$$
$$x^2 + 4x + 4 - 2(y^2 - 2y + 1) = -4$$
$$\frac{(y-1)^2}{2} - \frac{(x+2)^2}{4} = 1, \text{ center } (-2, 1)$$



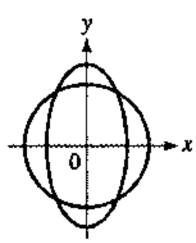
25. $x^2 + y^2 = 9$ circle; center $(0,0)$; radius 3

$x^2 + y^2 = 3^2$

$4x^2 + y^2 = 16$ ellipse; centered at $(0,0)$ $(\pm 2, 0)$ and $(0, \pm 4)$ vertices

$$\frac{x^2}{4} + \frac{y^2}{16} = 1$$
$$\frac{x^2}{2^2} + \frac{y^2}{4^2} = 1$$

four real solutions



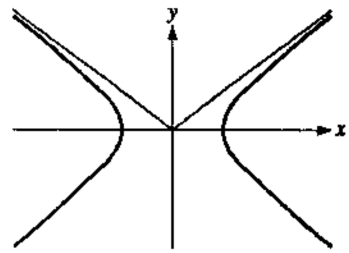
26. $y = |x|$ absolute value function

$y = x, x \geq 0$

$y = -x, x < 0$

$x^2 - y^2 = 1$ hyperbola with $y = \pm 1$ asymptotes.

There are no points of intersection and thus no real solutions.



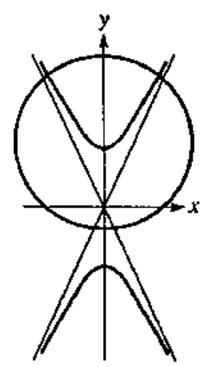
27.
$$x^2 + y^2 - 4y - 5 = 0$$
$$x^2 + y^2 - 4y + 4 = 5 + 4$$
$$x^2 + (y-2)^2 = 9 = 3^2$$

is a circle with center $(0, 2)$ and radius 3

$$y^2 - 4x^2 - 4 = 0$$
$$y^2 - 4x^2 = 4$$
$$\frac{y^2}{4} - \frac{x^2}{1} = 1$$

is an hyperbola with center $(0,0)$ and asymptotes $y = \pm 2x$

There are 2 points of intersection and thus 2 real solutions.



28.
$$x^2 - 4y^2 + 2x - 3 = 0$$
$$x^2 + 2x + 1 - 4y^2 = 3 + 1$$
$$(x+1)^2 - 4y^2 = 4$$
$$\frac{(x+1)^2}{4} - \frac{y^2}{1} = 1,$$

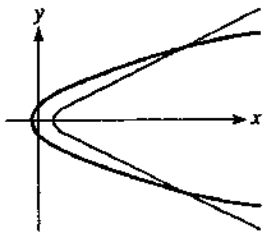
hyperbola with center $(-1, 0)$ and asymptotes

$$y = \pm\frac{1}{2}(x+1)$$

$$y^2 - 4x - 4 = 0$$
$$y^2 = 4x + 4$$
$$y^2 = 4(x+1),$$

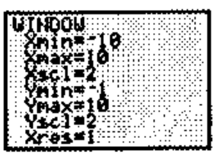
parabola with vertex $(-1, 0)$

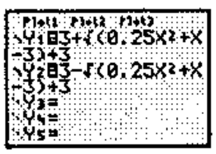
From the graphs there are 2 intersection points and thus 2 real solutions.

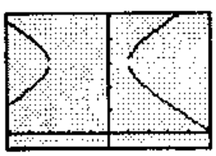


29. $x^2 - 4y^2 + 4x + 24y - 48 = 0$. Solve for y by completing the square.

$$y^2 - 6y = 0.25x^2 + x - 12$$
$$y^2 - 6y + 9 = 0.25x^2 + x - 3$$
$$(y-3)^2 = 0.25x^2 + x - 3$$
$$y = \pm\sqrt{0.25x^2 + x - 3} + 3$$



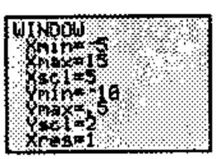


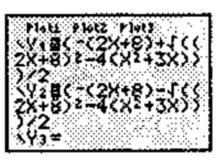


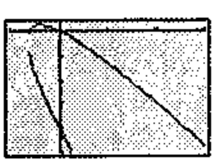
30. $x^2 + 2xy + y^2 - 3x + 8y = 0$. Solve for y with quadratic formula.

$$y^2 + (2x+8)y + (x^2 - 3x) = 0$$

$$y = \frac{-(2x+8) \pm \sqrt{(2x+8)^2 - 4(1)(x^2+3x)}}{2(1)}$$







31. $y^2 - 4y + 6x - 8 = 0$. Solve for y with quadratic formula.

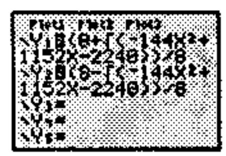
$$y = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(6x-8)}}{2}$$
$$= \frac{4 \pm \sqrt{16 - 4(6x-8)}}{2}$$
$$y = 2 \pm \sqrt{12 - 6x}$$

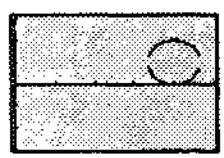
32. $9x^2 + 4y^2 - 72x - 8y + 144 = 0$. Solve for y with quadratic formula.

$$4y^2 - 8y + 9x^2 - 72x + 144 = 0$$

$$y = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(9x^2 - 72x + 144)}}{2(4)}$$

$$y = \frac{8 \pm \sqrt{-144x^2 + 1152x - 2240}}{8}$$





33. (a) The slope of the line between $(-3, 11)$ and $(2, -1)$ is $-\frac{12}{5}$. The slope of the line between $(14, 4)$ and $(2, -1)$ is $\frac{5}{12}$. The product of the slopes is negative one which shows that the line segments form a right triangle.

(b) The distance between $(-3, 11)$ and $(2, -1)$ is 13. The distance between $(14, 4)$ and $(2, -1)$ is 13. The distance between $(-3, 11)$ and $(14, 4)$ is $\sqrt{338}$. Since $13^2 + 13^2 = 338$ the line segments form a right triangle by the Pythagorean Theorem.

34. Let $A = (-2,5), B = (3,-1), C = (2,-4)$.

$$m_{AB} = \frac{-1-5}{3-(-2)} = -\frac{6}{5}$$

The equation of the altitude from C to AB is

$$y+4 = \frac{5}{6}(x-2)$$
$$6y+24 = 5x-10$$

(1) $5x - 6y = 34$

$$m_{BC} = \frac{-4-(-1)}{2-3} = 3$$

The equation of the altitude from A to BC is

$$y-5 = -\frac{1}{3}(x+2)$$
$$3y-15 = -x-2$$

(2) $x + 3y = 13$

$$m_{AC} = \frac{-4-5}{2(-2)} = -\frac{9}{4}$$

The equation of the altitude from B to AC is

$$y+1 = \frac{4}{9}(x-3)$$
$$9y+9 = 4x-12$$

(3) $4x - 9y = 21$

Solving (1) and (2) gives $\left(\frac{60}{7}, \frac{31}{21}\right)$ and since this is a solution of (3), the altitudes meet at $\left(\frac{60}{7}, \frac{31}{21}\right)$.

35. The line $y = x$ intersects the ellipse $7x^2 + 2y^2 = 18$ when $7x^2 + 2x^2 = 18$ or $x = \pm\sqrt{2}$. In first quadrant the intersection point is $(\sqrt{2}, \sqrt{2})$. The area of the square is

$$A = 4(\sqrt{2})^2 = 8$$

36. The equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ or

$$x^2 = a^2\left(1 - \frac{y^2}{b^2}\right)$$

(slope of PA)(slope of PB)

$$= \left(\frac{y-0}{x-a}\right)\left(\frac{y-0}{x+a}\right) = \frac{y^2}{x^2-a^2}$$
$$= \frac{y^2}{a^2\left(1-\frac{y^2}{b^2}\right) - a^2}$$
$$= \frac{y^2}{a^2 - \frac{a^2y^2}{b^2} - a^2}$$
$$= \frac{y^2}{-\frac{a^2y^2}{b^2}}$$
$$= -\frac{b^2}{a^2}$$

37. Given: focus $(3,1)$; directrix $y = -3$; vertex $(3,-1)$ is (h,k); $p = 2$
By definition, $d_1 = d_2$ where d_1 is from (x,y) to $(x,-3)$, a point on the
directrix, and d_2 is from (x,y) to$(3,1)$, the focus.

$d_1 = \sqrt{(x-x)^2 + [y-(-3)]^2}$;
$d_2 = \sqrt{(x-3)^2 + (y-1)^2}$
$\sqrt{0^2 + (y+3)^2} = \sqrt{(x-3)^2 + (y-1)^2}$;
$0 + (y+3)^2 = (x-3)^2 + (y-1)^2$
$y^2 + 6y + 9 = x^2 - 6x + 9 + y^2 - 2y + 1$
$0 = x^2 + y^2 - y^2 - 6x - 2y - 6y + 9 + 1 - 9$;
$0 = x^2 - 6x - 8y + 1$
By translation, $(x-h)^2 = 4(p)(y-k)$

$(x-3)^2 = 4(2)[y-(-1)]$; $(x-3)^2 = 8(y+1)$
$x^2 - 6x + 9 = 8y + 8$; $x^2 - 6x - 8y + 9 - 8 = 0$
$x^2 - 6x - 8y + 1 = 0$

38. Rewrite $x^2 - ky^2 = 1$ as $\frac{x^2}{1} + \frac{y^2}{-\frac{1}{k}} = 1$.

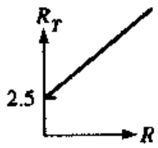
For an ellipse $-\frac{1}{k} > 0$ or $\frac{1}{k} < 0$ which implies $k < 0$. (1)

For the vertices to be on y-axis $-\frac{1}{k} > 1$ or

$\frac{1}{k} < -1$ and using (1) this may be written as

$1 > -k \iff -1 < k$. (2)
Combining (1) and (2) gives $-1 < k < 0$

39. $R_T = R + 2.5$



40. $a = \dfrac{v - v_o}{t - t_0} = 20$

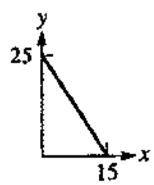
$$\frac{v-5}{t-0} = 20$$

$$v = 20t + 5$$

41. $2500x + 1500y = 37{,}500$

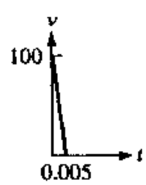
$$1500y = -2500x + 37{,}500$$

$$y = -\frac{5}{3}x + 25$$



42. $v(t) = 100 - 20{,}000t$

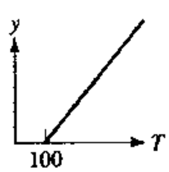
t(h)	v(mi/h)
0	100
0.001	80
0.002	60
0.004	20
0.005	0



43. $y = 2.010(50.00)(T - 100)$

$y = 100.5T - 10{,}050$

T(°C)	y(kJ)
100	0
200	10,500
500	40,200
1000	90,450



44. $(0, 27)$ and $(2500, 12)$ are points on the graph of T vs. h.

$$m = \frac{12 - 27}{2500 - 0} = -0.006$$

The equation is

$$T - 12 = -0.006(h - 2500)$$
$$T - 12 = -0.006h + 15$$
$$T = -0.006h + 27$$

45. $A = \pi r^2 = \pi(490(\tan 7°))^2$

$A = 11{,}000 \text{ ft}^2$

46. $c = 2\pi r = 191$

$$= \frac{191}{2\pi}$$

center $\left(0, 2.0 + \dfrac{191}{2\pi}\right)$

$$(x-0)^2 + \left(y - \left(2.0 + \frac{191}{2\pi}\right)\right)^2 = \left(\frac{191}{2\pi}\right)^2$$

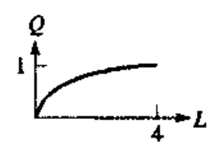
$$x^2 + y^2 - 2\left(2.0 + \frac{191}{2\pi}\right)y + \left(2.0 + \frac{191}{2\pi}\right)^2 = \left(\frac{191}{2\pi}\right)^2$$

$$x^2 + y^2 - 64.8y + 126 = 0$$

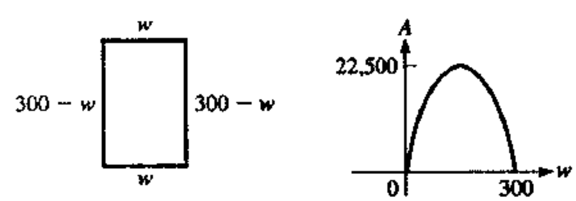
47. The equation of the parabola is $y^2 = 4px$ and $(50, 40)$ is on the graph, $40^2 = 4p(50)$ gives $4p = 32$ from which $y^2 = 32x$.

48. $Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{1000}\sqrt{\dfrac{L}{4.00 \times 10^{-6}}} = \dfrac{1}{2}\sqrt{L}$

$$Q(L) = \frac{1}{2.00}\sqrt{L}$$



49.

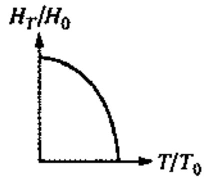


$$A = w(300 - w) = 300w - w^2$$
$$-A = w^2 - 300w$$
$$-A + 150^2 = w^2 - 300w + 150^2 = (w - 150)^2$$
$$-1(A - 150^2) = (w - 150)^2$$
$$(w - 150)^2 = -1(A - 150^2)$$

Parabola with $(h, k) = (150, 150^2)$
$= (150, 22{,}500)$

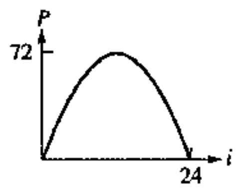
50. $\dfrac{H_T}{H_o} = 1 - \left(\dfrac{T}{T_o}\right)^2$

$\frac{T}{T_o}$	$\frac{H_T}{H_o}$
0	1
0.25	0.9375
0.50	0.75
0.75	0.4375
1	0



51. $P = 12.0i - 0.500i^2$

i(A)	P(W)
0.00	0.00
6.00	54.0
12.0	72.0
18.0	54.0
24.0	0.00



52. $A = \pi ab = \pi(94)(78)$
$= 2.3 \times 10^4 \text{ m}^2$

53. $a = \dfrac{120}{2} = 60;\ c = 60 - 15 = 45;$

$$b = \sqrt{a^2 - b^2} = \sqrt{60^2 - 45^2} = \sqrt{1575}$$
$$A = \pi ab = \pi(60)\sqrt{1575} = 7500 \text{ ft}^2$$

54. $\dfrac{(d-10)^2}{10^2} + \dfrac{(f-0)^2}{1^2} = 1$

$$\frac{(d-10)^2}{100} + \frac{f^2}{1} = 1, 0 \le f \le 1 \text{ and } 0 < d \le 10$$

55. If the pins are on the x-axis and centered, the equation of the ellipse is

$$\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1.$$

$$a^2 = b^2 + c^2$$
$$c^2 = a^2 - b^2$$
$$c^2 = 25 - 9$$
$$c^2 = 16$$
$$c = 4$$

The coordinates of the pin art $(\pm 4, 0)$, so the pins should be 8 cm apart. The length of the string, l, is

$$l = 2a + 2c$$
$$l = 2(5) + 2(4)$$
$$l = 18 \text{ cm}$$

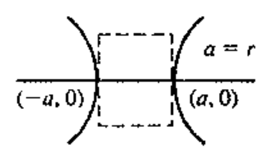
56. The length of the major axis is $70+2(1080)+190 = 2420$, from which $a = 2420/2 = 1210$. Since $70+1080 = 1150$ is less than 1210, the distance from the center to focus is $1210 - 1150 = 60$. From

$$a^2 = b^2 + c^2$$
$$1210^2 = b^2 + 60^2$$
$$b^2 = 1210^2 - 60^2.$$

The equation of the ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
$$\frac{x^2}{1210^2} + \frac{y^2}{1210^2 - 60^2} = 1$$
$$\frac{x^2}{1.464 \times 10^6} + \frac{y^2}{1.461 \times 10^6} = 1$$

57.



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$
$y = 40$ when $x = 40$
$y = 100$ when $x = 50$

$$\begin{cases} \dfrac{40^2}{a^2} - \dfrac{40^2}{b^2} = 1 \text{ or } 40^2b^2 - 40^2a^2 = a^2b^2 \\ \dfrac{50^2}{a^2} - \dfrac{100^2}{b^2} = 1 \text{ or } 50^2b^2 - 100^2a^2 = a^2b^2 \end{cases}$$

$$\begin{matrix} M \text{ by } 100^2 \\ M \text{ by } -40^2 \end{matrix} \left\{ \begin{matrix} 40^2b^2 - 40^2a^2 = a^2b^2 \\ 50^2b^2 - 100^2a^2 = a^2b^2 \end{matrix} \right\}$$

$$\left\{ \begin{matrix} 100^2 40^2 b^2 - 100^2 40^2 a^2 = 100^2 a^2 b^2 \\ -40^2 50^2 b^2 + 40^2 100^2 a^2 = -40^2 a^2 b^2 \end{matrix} \right\}$$

$$\text{Add} \left\{ \begin{matrix} 16 \times 10^6 b^2 - 1.6 \times 10^7 a^2 = 10 \times 10^3 a^2 b^2 \\ 4 \times 10^6 b^2 + 1.6 \times 10^7 a^2 = -1.6 \times 10^3 a^2 b^2 \end{matrix} \right.$$

$$12 \times 10^6 b^2 = 8.4 \times 10^3 a^2 b^2$$
$$12 \times 10^6 = 8.4 \times 10^3 a^2$$

$$a^2 = \frac{12 \times 10^6}{8.4 \times 10^3} = 1.42 \times 10^3$$
$$a = 37.8 \text{ ft}$$

58. Let (x, y) be the point where the earthquake occurred, $(0, p)$ be Palo Alto, $(0, -p)$ be Pasadena where $p = \frac{510}{2}$, then

$\sqrt{(x-0)^2 + (y+p)^2} = 5.0t, t =$ time to reach Caltech

$$\sqrt{(x-0)^2 + (y-p)^2} = 5.0(t+36) = 5.0t + 180$$

$$\sqrt{x^2 + (y-p)^2} - \sqrt{x^2 + (y+p)^2} = 180$$
$$\sqrt{x^2 + (y-p)^2} = 180 + \sqrt{x^2 + (y+p)^2}$$
$$x^2 + y^2 - 2py + p^2 = 180^2 + 360\sqrt{x^2 + (y+p)^2} + x^2 + y^2 + 2py + p^2$$
$$-4py - 180^2 = 360\sqrt{x^2 + (y+p)^2}$$
$$16p^2y^2 + 8(180^2)py + 180^4 = 360^2(x^2 + y^2 + 2py + p^2)$$
$$16p^2y^2 + 8(180^2)py + 180^4 = 360^2x^2 + 360^2y^2 + 2(360^2)py + 360^2p^2$$
$$(16p^2 - 360^2)y^2 - 360^2x^2 = 360^2p^2 - 180^4$$

$$\frac{y^2}{\frac{360^2p^2 - 180^4}{16p^2 - 360^2}} - \frac{x^2}{\frac{360^2p^2 - 180^4}{16p^2 - 360^2}} = 1, \text{ letting } p = \frac{510}{2}$$

$$\frac{y^2}{8100} - \frac{x^2}{57{,}000} = 1$$

(Palo Alto is at upper focus and Pasadena at lower focus.)

59. Let the coordinates of the top of the hyperbolic arch be $(0, -a)$ where $a > 0$. The coordinates of the top right corner of the building and the arch are $(9, -a-3)$. The focus coordinates $(0, -a-3)$. The equation of the arch is

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \text{ where}$$

$$b^2 = (-a-3)^2 - a^2$$

$$\frac{y^2}{a^2} - \frac{x^2}{(-a-3)^2 - a^2} = 1$$

$$\frac{y^2}{a^2} - \frac{x^2}{a^2 + 6a + 9 - a^2} = 1$$

(1) $$\frac{y^2}{a^2} - \frac{x^2}{6a+9} = 1 \text{ and letting}$$

$$(x, y) = (9, -a-3)$$

$$\frac{a^2 + 6a + 9}{a^2} - \frac{81}{6a+9} = 1$$

$$(a^2 + 6a + 9)(6a + 9) - 81a^2 = a^2(6a + 9)$$
$$6a^3 + 9a^2 + 36a^2 + 54a + 54a + 81 - 81a^2 = 6a^3 + 9a^2$$

$-45a^2 + 108a + 81 = 0$ from which $a = 3$

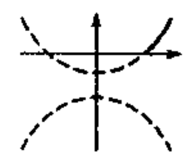
(1) is now $\frac{y^2}{9} - \frac{x^2}{27} = 1$ or $3y^2 - x^2 = 27$.

60. Let PM be the distance from the pulley to the person, then

$$10 - h + PM = 60$$
$$PM = 50 + h \text{ and}$$
$$(50 + h)^2 = 6^2 + x^2.$$

Since $h \geq 0, x \geq \sqrt{2464}$.

$$50 + h = \sqrt{36 + x^2}$$
$$h = \sqrt{36 + x^2} - 50$$



61. Let $P(x, y)$ be the coordinates of the recorder in a coordinate system with origin at the target. Let rifle be at $(0, r)$, then

$\sqrt{x^2 + (y-r)^2} = v_s(t_0 + t_1)$ where t_0 is the time for bullet to reach target and t_1 is the time for sound to reach detector from the target.

$$\sqrt{x^2 + y^2} = v_s t_1$$
$$\sqrt{x^2 + (y-r)^2} - \sqrt{x^2 + y^2} = v_s t_0 = \text{constant} = 2a$$
$$\sqrt{x^2 + (y-r)^2} = 2a + \sqrt{x^2 + y^2}$$
$$x^2 + y^2 - 2ry + r^2 = 4a^2 + 4a\sqrt{x^2 + y^2} + x^2 + y^2$$
$$-2ry + (r^2 - 4a^2) = 4a\sqrt{x^2 + y^2}$$
$$4r^2y^2 - 4r(r^2 - 4a^2)y + (r^2 - 4a^2) = 16a^2(x^2 + y^2)$$
$$(4r^2 - 16a^2)y^2 - 4r(r^2 - 4a^2)y - 16a^2x^2 = 4a^2 - r^2$$

which has the form $Ay^2 - By - dx^2 = e$

$$y^2 - \frac{B}{A}y + \frac{B^2}{4A^2} - \frac{d}{A}x^2 = \frac{e}{A} + \frac{B^2}{4A^2}$$

$$\left(y - \frac{B}{2A}\right)^2 - Dx^2 = F$$

$$\frac{\left(y - \frac{B}{2A}\right)^2}{F} - \frac{x^2}{\frac{F}{D}} = 1,$$

which is a hyperbola.

Summary: The distance from the rifle to P minus the distance from the target to P = constant which is related to the distance from the rifle to the target.