CHAPTER 2

Tools of the Trade

This chapter introduces some of the necessary terminology and reviews some of the basic techniques the student is assumed to have at the start of a typical QM course. Inevitably, students' numerical ability will be varied and I tend to use this material as self-study for students rather than formally teach it in a class context. I find it helpful to encourage students to work on this material in small self-support groups so that they can assist one another in the basics. I do tend, however, to have a tutorial focused on a number of the end-of-chapter exercises to ensure students have the basic skills required.

In particular, I try to ensure that students are comfortable with, and competent in, dealing with proportions, percentages, fractions and decimals. I also ensure they can formulate simple models using symbols and can accurately draw simple graphs. I tend to encourage students to draw the first few graphs by hand to check whether they understand the principles of co-ordinates and can extract information directly from the graph they have produced. The simple break-even model introduced in this chapter is easy for students to grasp and useful in highlighting the necessary principles.

Other concepts that I try to ensure they understand from the beginning are the difference between real and money terms and the concept of a simple index.

The Cap Gemini Ernst and Young (CGEY) case on pp. 42–43 should help students appreciate that these basic Tools of the Trade are of practical use.

As I mention in the text itself, there is another text I have written for Financial Times Management: *Foundation Quantitative Methods for Business* (1996, ISBN 0273607650) which is aimed primarily at the student at the undergraduate (or equivalent) level for those with little, or no, real experience of business or management. In that text I have a much expanded *Tools of the Trade* chapter which includes self-assessment tasks for the student to complete to help them assess their numerical ability in a number of key areas. Students who are less comfortable with quantitative data and basic arithmetic could be encouraged to use this as remedial material.

Figures are given at the end-of-the-chapter solutions.

Solutions

1. With the original equations the break-even solution was derived as:

Q = 15000

Taking each part in turn, the new break-even level is derived as follows:

(a) Overheads increase by 15 per cent

This will affect C but not R. Since we know that the C equation comprises an element for fixed costs (45000) and one for variable costs (6.99Q) an increase in overheads will affect only fixed costs. The new C equation will be:

$C = 51750 + 6.99Q$

giving:

$F = R - C$

$= 9.99Q - (51750 + 6.99Q) = -51750 + 3Q$

Break-even occurs when F = 0

$F = -51750 + 3Q = 0$

or $3Q = 51750$

or Q = 17250 as the new break-even level of output.

Students should be encouraged to understand why Q has increased as overheads increase: as the firm's costs have increased but its selling price has not, it needs to sell more to achieve the same profit levels. We could also derive the new solution by determining that the change in fixed costs is £6750 (51750 − 45000). Since each item sold generates a profit of £3, the company must sell an extra 2250 units (6750 ⁄ 3) to generate the same profit it had before the overhead increase. The student should realise that this increase of 2250 units will apply at any level of production not just at break-even. If they're not sure, get them to calculate profit, F, before and after the overhead increase for two or three arbitrary production levels.

(b) Costs increase by £1.50 per item

Again, the C equation is affected, not R, but this time it is the variable cost element of the C equation which changes. We now have:

$C = 45000 + 8.49Q$

and require:

$F = 9.99Q - (45000 + 8.49Q) = 0$

Solving gives:

$Q = 45000 / 1.50 = 30000$

as the new break-even level of output.

(c) Selling price increases to £11.99

Now R changes while C does not, giving:

F = 11.99Q − (45000 + 6.99Q) = 0

and solving gives:

Q = 45000 ⁄ 5 = 9000

as the new break-even level of output.

Although not part of the question, it can also be worthwhile asking students to consider how the relevant graph of C and R would change in each case. This will allow an early introduction of the concepts of intercept and slope of linear equations and their business implications. The three graphs, with each showing the original C and R equations together with the new, are shown in Figs. 2X1A−2X1C. In Fig. 2X1A, the new C equation causes an upward shift in the C line equal to the increase in overheads. At any level for Q, costs have increased by a constant amount. In Fig. 2X1B, the new C line increases its slope/gradient as the variable costs increase but has the same intercept or constant value as overheads have not changed. In Fig. 2X1C, it is the R line which changes again with the slope/gradient increasing with the increase in selling price.

2. Although the use of logarithms is introduced in this chapter they are not used again extensively until Chapter 15 and the tutor may wish to delay their coverage until then.

This exercise is simply to give students some practice at the use of logarithmic calculations. Table 2X2 shows the relevant calculations. The amount at the start of each year is shown in the second column, followed by the interest earned on that amount at 8 per cent. These two added together then give the amount at the end of the year. So, at the start of year 1 £15000 is invested, earns £12000 interest and the savings fund has a value of £16200 at the end of year 1. The value of the fund at the end of a year is then the value of the fund at the start of the following year. So, by the end of the 10-year period the fund is worth £32384. The logarithms of the Amount at end figures are shown in the final column (base 10).

Table 2X2 Investment over time

Year	Amount at start	Interest	Amount at end	Log of amount at end
1	15000	1200	16200	4.2095
2	16200	1296	17496	4.2429
3	17496	1400	18896	4.2764
4	18896	1512	20407	4.3098
5	20407	1633	22040	4.3432
6	22040	1763	23803	4.3766
7	23803	1904	25707	4.4101
8	25707	2057	27764	4.4435
9	27764	2221	29985	4.4769
10	29985	2399	32384	4.5103

Fig. 2X2 shows the graph of both the actual monetary series and its log equivalent. Note that this graph uses both the Y axes to show both series together (this type of graph is discussed in more detail in Chapter 3). From the graph we see that the actual monetary value of the fund is not only increasing each year, it is increasing by an ever-increasing amount (thanks to the principle of compound interest). This can be related back directly to the Interest column in the table. However, the graph of the log of the amount values is clearly linear. This might puzzle some students, but what the log graph is actually showing is that *the rate of change* in the amount is constant over time (at 8 per cent). Students should be asked to consider the implications of a log graph where the log values produced an upward curve (like that of the actual amount values) or a downward curve. The use of log graphs where management interest lies primarily in growth rates rather than actual values could also be discussed: examples from sales, market penetration/market share, change in customer numbers, etc. could be used to illustrate this further.

The exercise is readily extended to look at the effect of different rates of interest on the final fund value and the impact of inflation on fund value (allowing a discussion of real versus money value terms).

3. (a) We have a linear equation:

$Q_d = 1000 - 5P$

Given it is linear, in order to graph it we need two sets of co-ordinates (normally taken as those corresponding to the two ends of the X axis scale). Here the variable on the vertical, Y, axis will be Q_d and price, P, will be on the horizontal or X axis.

We are told that price (P) will vary from 0 to 200. So, using P = 0 we have:

$Q_d = 1000 - 5P$

$Q_d = 1000 - 5(0) = 1000$

That is, when P = 0 $Q_d = 1000$

Similarly, when P = 200

$Q_d = 1000 - 5P$

$Q_d = 1000 - 5(200) = 1000 - 1000 = 0$

Our two sets of co-ordinates then for plotting on the graph are:

$P = 0, Q_d = 1000$

$P = 200, Q_d = 0$

We then have a graph as in Fig. 2X3A.

If you are drawing the graph manually it is clearly logical to have the X scale from 0 to 200 and the Y scale from 0 to 1000 with suitable 'tick' marks to make the scales easy to read. If you are using a spreadsheet then scales, etc. will automatically be calculated, although it is still worthwhile encouraging students to override the automatic settings from time to time to see the effect this has on the diagram.

I also tend to encourage students to produce one or two graphs at this stage manually rather than relying totally on a spreadsheet package. This allows me – and the student – to assess their grasp of plotting co-ordinates and their ability to plot and read information from a graph accurately.

(b) Clearly, from the graph the equation is linear and slopes downwards as we look at it from left to right. That is, as P increases (from 0 to 200), Q_d decreases (from 1000 to 0). In the context of the demand for some product this clearly makes sense: the higher the price charged then, normally, the lower the demand for that product.

Some discussion of the realism of a linear demand equation, as opposed to a non-linear one, is often helpful in encouraging students to connect the implications of the mathematics to the real world and in appreciating the implications of the difference between linear and non-linear relationships.

(c) Revenue is defined as quantity sold times price. Quantity sold will effectively be Q_d so we have:

$R = Q_d \times P$

But $Q_d = 1000 - 5P$ so we have:

$R = Q_d \times P = (1000 - 5P) \times P = 1000P - 5P^2$

Take care that students multiply both parts of the equation for Q_d by P. The use of brackets helps remind them to do this. So we have:

$R = 1000P - 5P^2$

as the revenue equation.

(d) The revenue equation involves a term to the power 2 (P^2) and is not a linear equation. In order to draw a graph, we have to determine more than two sets of points. The text suggests between 10 and 12. Using P from 0 to 200 again, we can work out R for values of P at 0, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200. These would be:

Table 2X3 Values for P and Q

P	Q
0	0
20	18000
40	32000
60	42000
80	48000
100	50000
120	48000
140	42000
160	18000
180	32000
200	0

It would make sense to have a Y axis from 0 to 50000 and the corresponding graph is shown in Fig. 2X3D. It is important that students should be able to assess the business implications of the graph (and its function) and not simply draw the graph. R starts at zero (where P is zero also). We already know from the demand equation that when the firm charges a price of zero it will 'sell' 1000 units. However, its revenue will be zero since it is effectively giving these units away for nothing and revenue shows the income it receives from sales. From the graph we see that revenue gradually climbs up to a maximum of £50000 and then gradually falls away again to zero (since at a price of 200 we know the firm will sell nothing and hence its revenue will also be nothing).

If students are drawing this type of graph manually, ensure that they join the points together (for each P, R combination) with a smooth line and that the co-ordinates are correctly plotted. Technically, this type of equation/graph is known as a *quadratic* and is commonly used in business and economic modelling because of its shape and the implied consumer behaviour.

(e) From the graph we already know that revenue will be at its maximum at £50000. This corresponds to a price of £100.

4. (a) This question requires us to derive relevant equations for this problem and then use these equations to help answer the questions posed. We shall refer to the two alternatives as A and B.

For A we have:

a charge of £45 per day

For B we have:

a charge of £30 per day plus 5p per mile.

In both cases we require the car for four days. The cost (C) of the two alternatives will then be:

$C_A = 180$ (four days at £45)

$C_B = 120 + 0.05M$

where M is the number of miles we would travel.

It is clear that if our mileage is low then C_B is likely to be less than C_A. Equally, if our mileage is high, the reverse will be true. What is not clear, however, is the exact mileage that would make one cheaper than the other. It will help, though, if we can calculate what we can refer to as the break-even mileage: the mileage covered such that both companies would cost the same. This would mean that:

$C_A = C_B$

but substituting the two equations we get:

$C_A = C_B$

$180 = 120 + 0.05M$

We need to collect all the numerical terms on one side of this equation and the unknown (M) on the other. Using the idea that we can alter the left-hand side of an equation (LHS) as long as we also alter the right-hand side (RHS) in the same way, let us subtract 120 from both sides:

$180 - 120 = 120 + 0.05M - 120$

to give:

$60 = 0.05M$

Let us now divide both sides by 0.05:

$$\frac{60}{0.05} = \frac{0.05\,M}{0.05}$$

to give:

$1200 = M$ (since any number divided by itself equals 1) that is, the break-even mileage is 1200 miles. If I travel exactly 1200 miles, the two companies will charge me exactly the same for hire of the car. We can easily check (and you should do until you're more used to this type of calculation):

C_A = £180 (since there is no mileage charge)

C_B = £120 + 0.05(M) = 120 + 0.05(1200) = 120 + 60 = £180

So, if my mileage is likely to be less than 1200, company B would be cheaper. If I expect to do more than 1200 miles, company A would be cheaper.

Some students typically encounter difficulty in manipulating equations to derive the numerical solution to some unknown variable. It is worthwhile encouraging them to adopt the method illustrated even though there may be quicker ways of finding a solution.

(b) Fig. 2X4B shows the corresponding graph.

We note that C_A is a horizontal line since it remains unchanged no matter what value M takes. We also see that the two equations intersect (or cross) when M is 1200. We also see that C_B is always below C_A when M is less than 1200 and always above C_A when M is above 1200.

(c) Imposition of VAT at 17.5 per cent will clearly affect both costs and, potentially, the break-even mileage solution. We now have:

C_A = 211.50 $\quad$ (180 × 1.175)

C_B = 141 + 0.05M

Re-solving the two equations for break-even gives M = 1410 (211.5, 141 divided by 0.05).

5. From the information given we can put together an equation to represent the cost, C, of a flight. We know that there is a fixed cost per flight of £25000 and a cost per passenger of £75. Denoting passengers as P we have:

C = 25000 + 75P

(a) Using the equation, break-even occurs when revenue equals cost. Revenue here would be:

R = 225P

and C = 25000 + 75P

We require R = C:

R = C

225P = 25000 + 75P

150P = 25000

P = 25000 ⁄ 150 = 166.7

as the number of passengers required to break-even (which we would need to round to 167 to give a sensible result).

(b) The flights have a capacity of 200 passengers. If only 80 per cent of seats are sold this equates to 160 passengers. From the cost equation, C, we know that costs will be:

C = 25000 + 75P = 25000 + 75(160) = 25000 + 12000 = 37000

To break-even, the company must generate revenue equal to its costs. With 160 seats to sell it must charge a price for each of £37000 ⁄ 160 or £231.25.

(c) With only 190 seats available, the number of seats sold will now be 152 (80 per cent of 190). However, the firm now has extra revenue of £5000, which can be offset against the costs. We have:

$C = 25000 + 75P = 25000 + 75(152) = 36400$

but only have to recover £31400 of this from seat income since £5000 is received from the cargo contract. So, the break-even ticket price should be £31400 ⁄ 152 = £206.58.

6. No answer is given here as the data collected by students will vary depending on source.

7. We have:

$F = -100 + 100Q - 5Q^2$

$C = 100 + 2Q^2$

(a) To obtain an equation for revenue, R, we must add costs, C, to profit, F:

$R = F + C$

$R = (-100 + 100Q - 5Q^2) + (100 + 2Q^2)$

Note the use of brackets to keep the arithmetic clear. Multiplying out the C term we have:

$R = (-100 + 100Q - 5Q^2) + 100 + 2Q^2$

$R = 100Q - 3Q^2$

For students not used to this type of manipulation, encourage them where you can to check their result. Here, for example, we can reverse the calculations since $F = R - C$. By subtracting the C equation (which we know to be correct) from the R equation we have obtained we should, if our answer is correct, obtain the F equation.

The graph for the profit equation is shown in Fig. 2X7B, plotted from Q = 0 to 20 in units of 1 (000). The profit equation (taking the standard quadratic shape) starts with negative values, increases to break-even, climbs to a maximum and then starts to fall away. It will be worthwhile for students to consider the implications of this general shape in terms of business and management (some may have encountered the principles already in economics). Fig. 2X7C, not specifically required by this exercise, shows all three equations together and can be used to help students see the inter-relationships between the equations.

(b) From Fig. 2X7B the profit maximising level of output can be seen to be when Q=10 (000).

(c) When X = 10, from the C equation we can confirm that the firm's costs will be 300 and profit, F, 400.

Fig. 2X1A Overheads increase by 15%

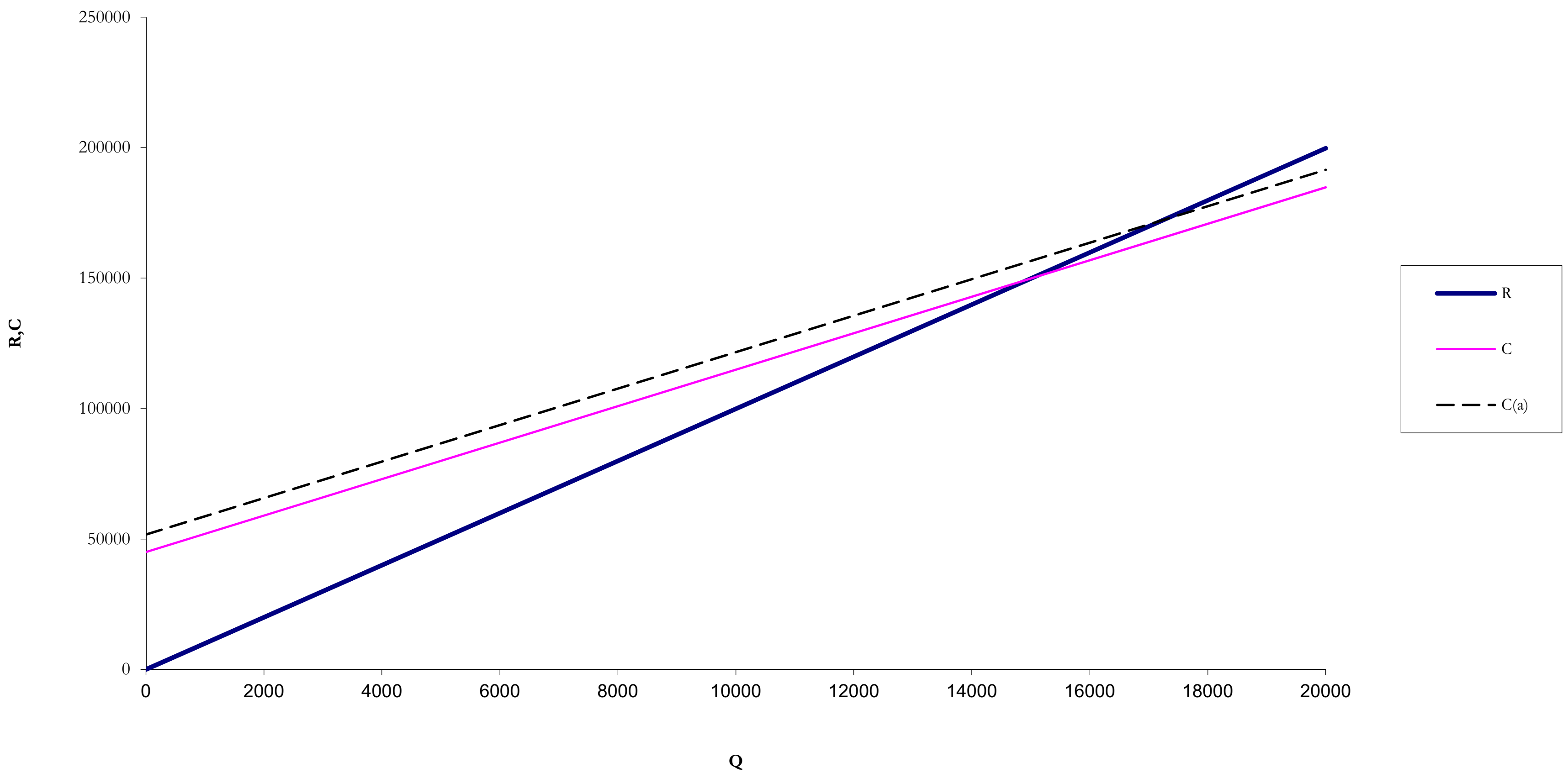

Fig. 2X1B Cost increase by £1.50 per item

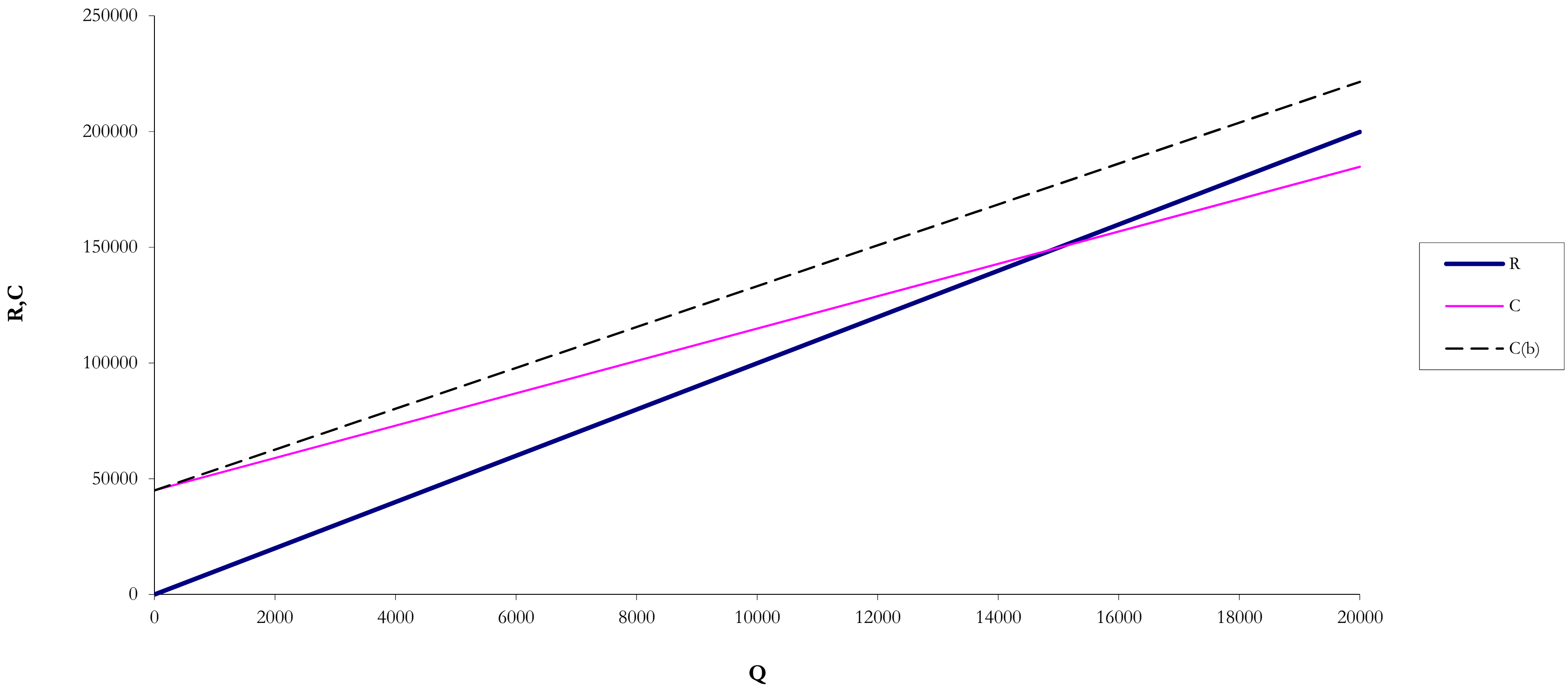

Fig. 2X1C Price increases to £11.99

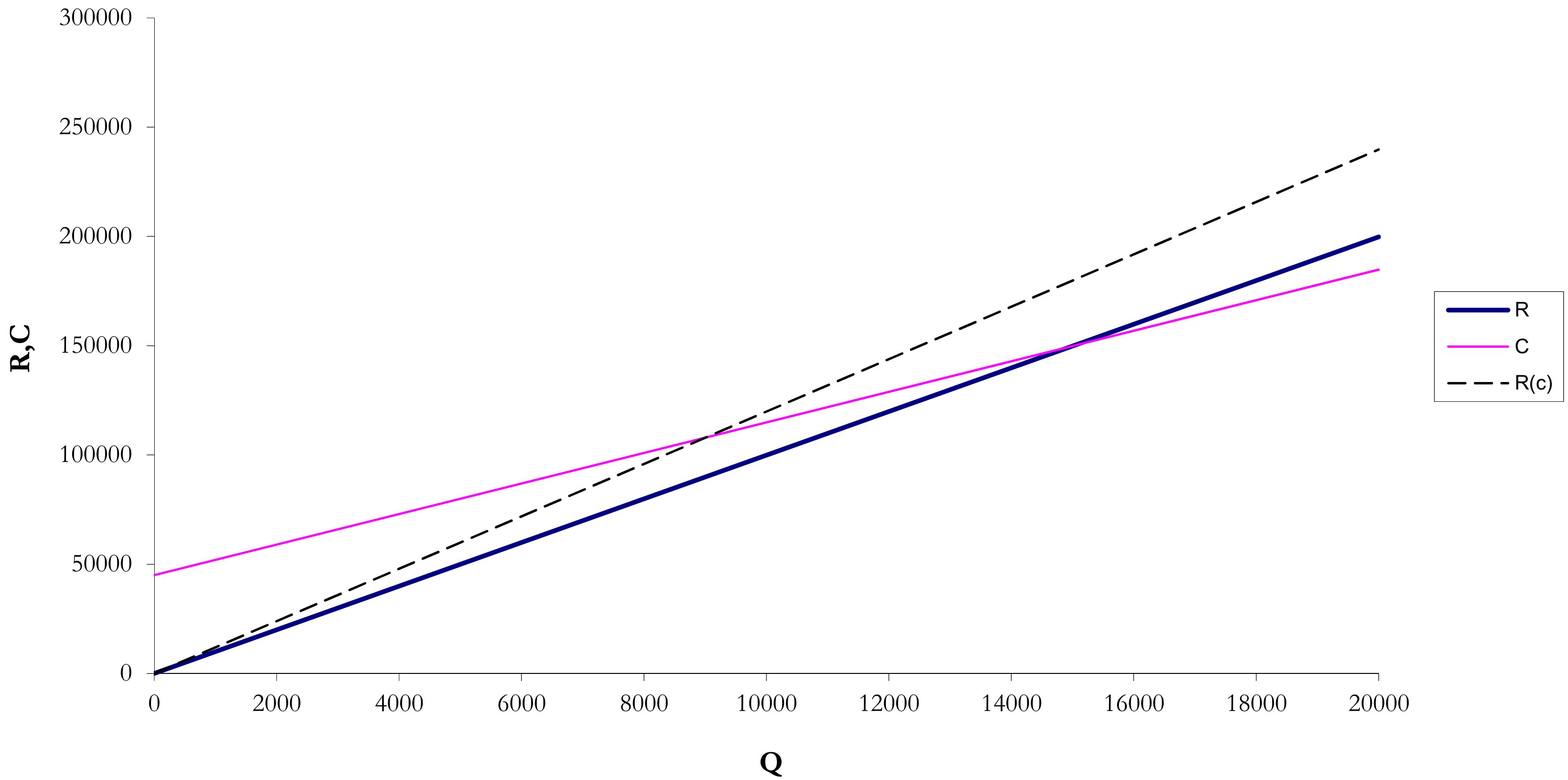

Fig. 2X2 Monetary series and the log equivalent

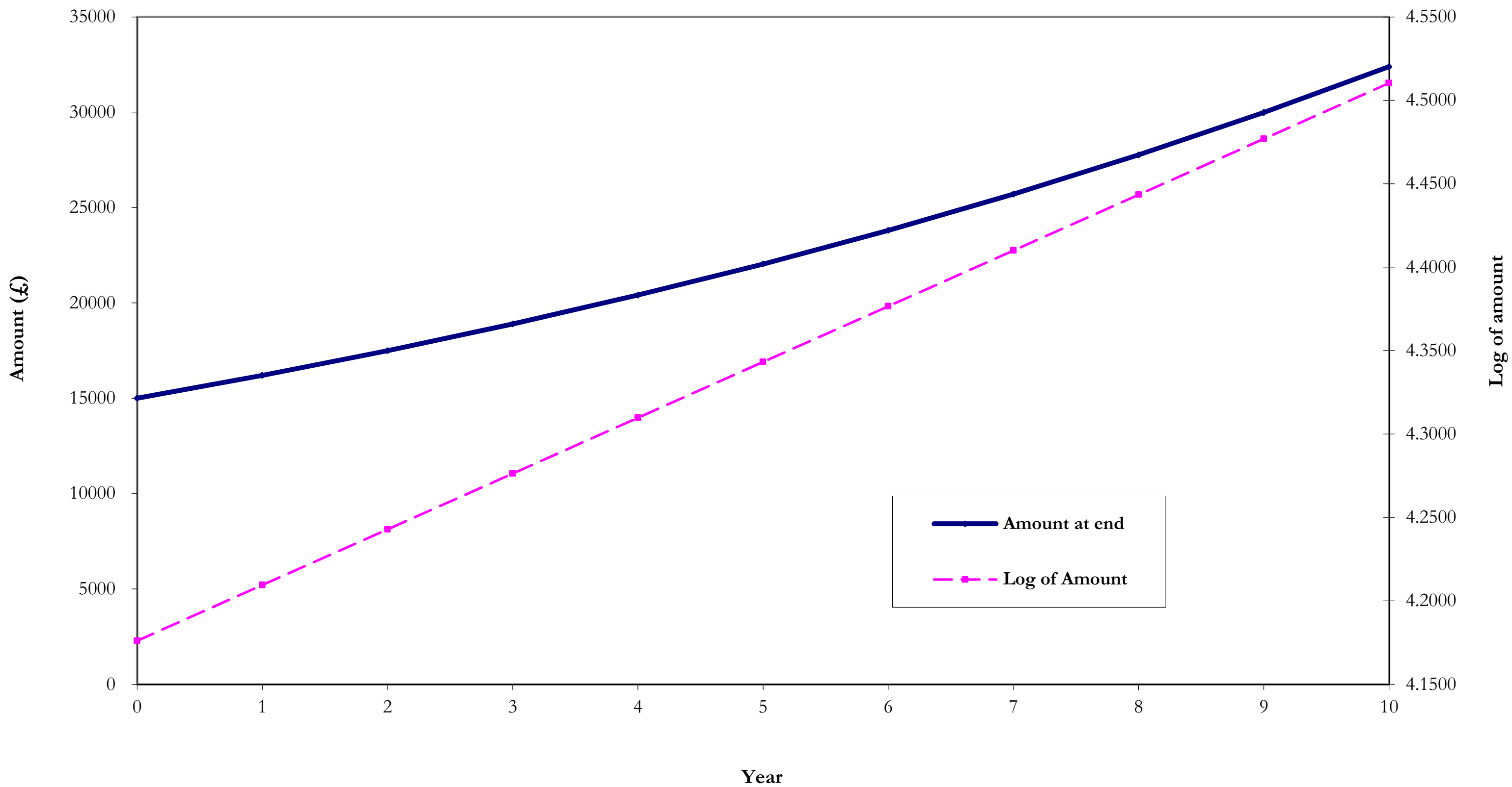

Fig. 2X3A $Q_d = 1000 - 5p$

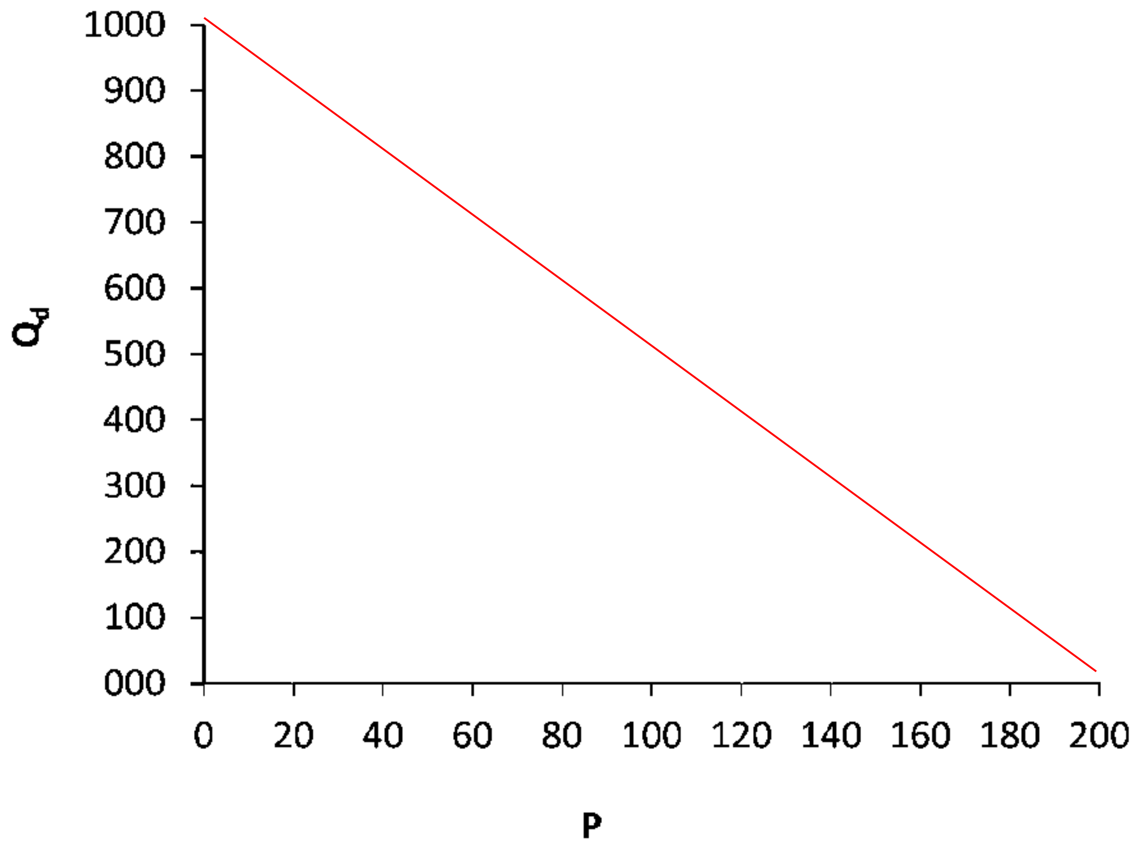

Fig. 2X3D Revenue equation

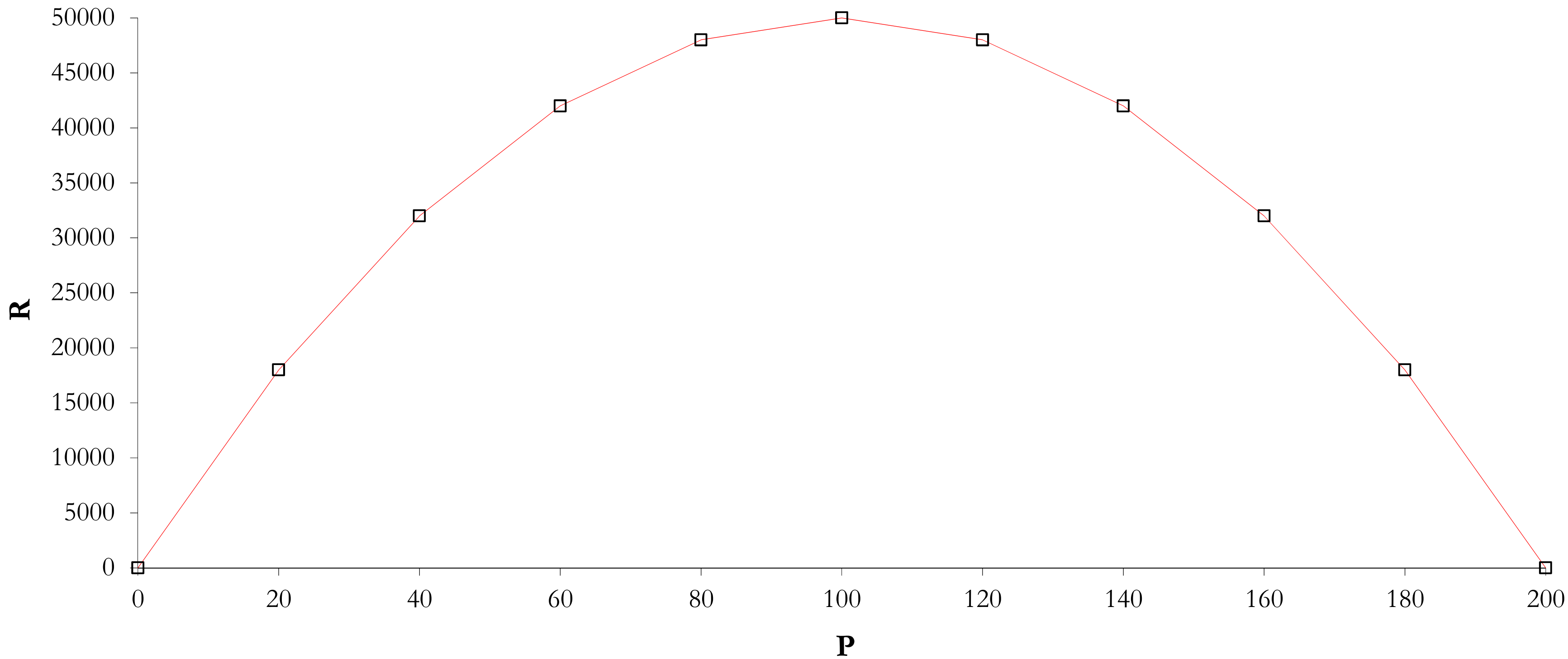

Fig. 2X4B Hire costs

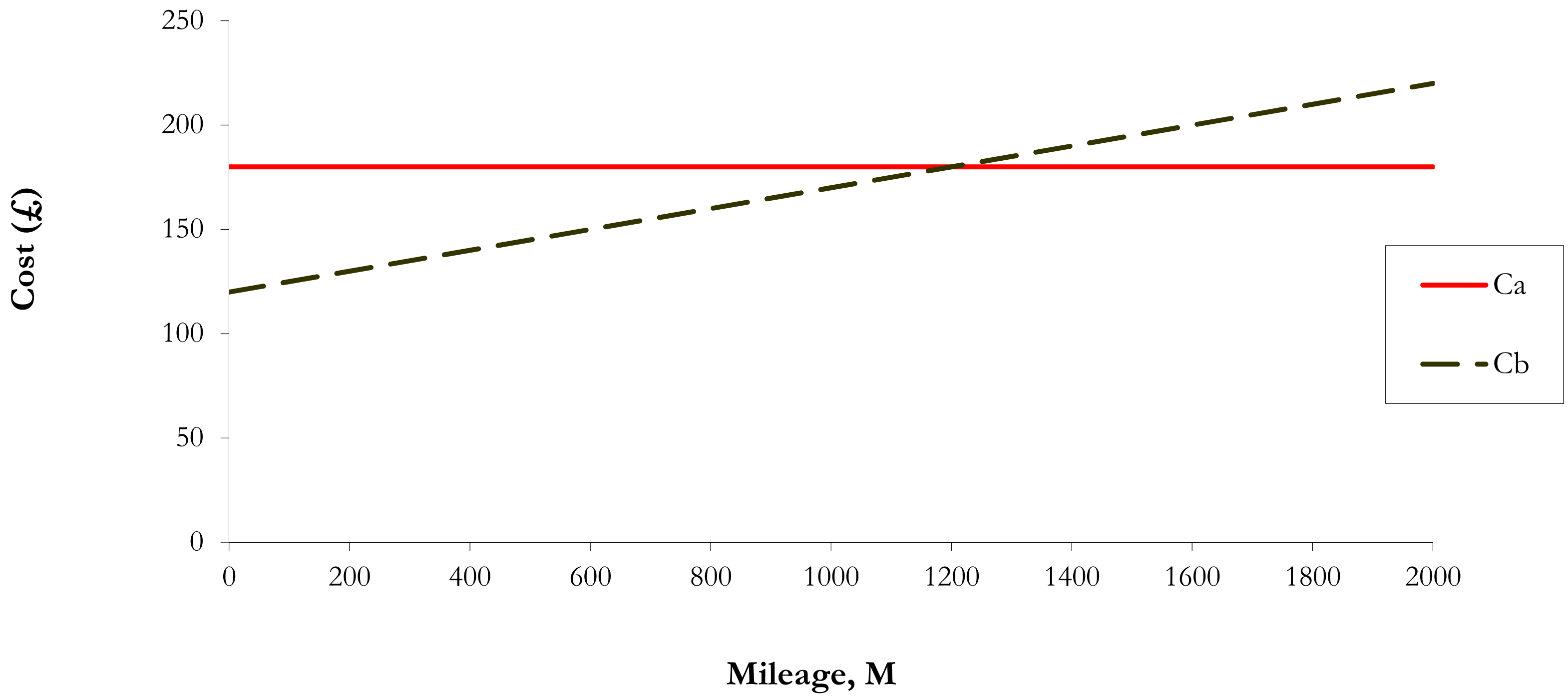

Fig. 2X7B Profit equation

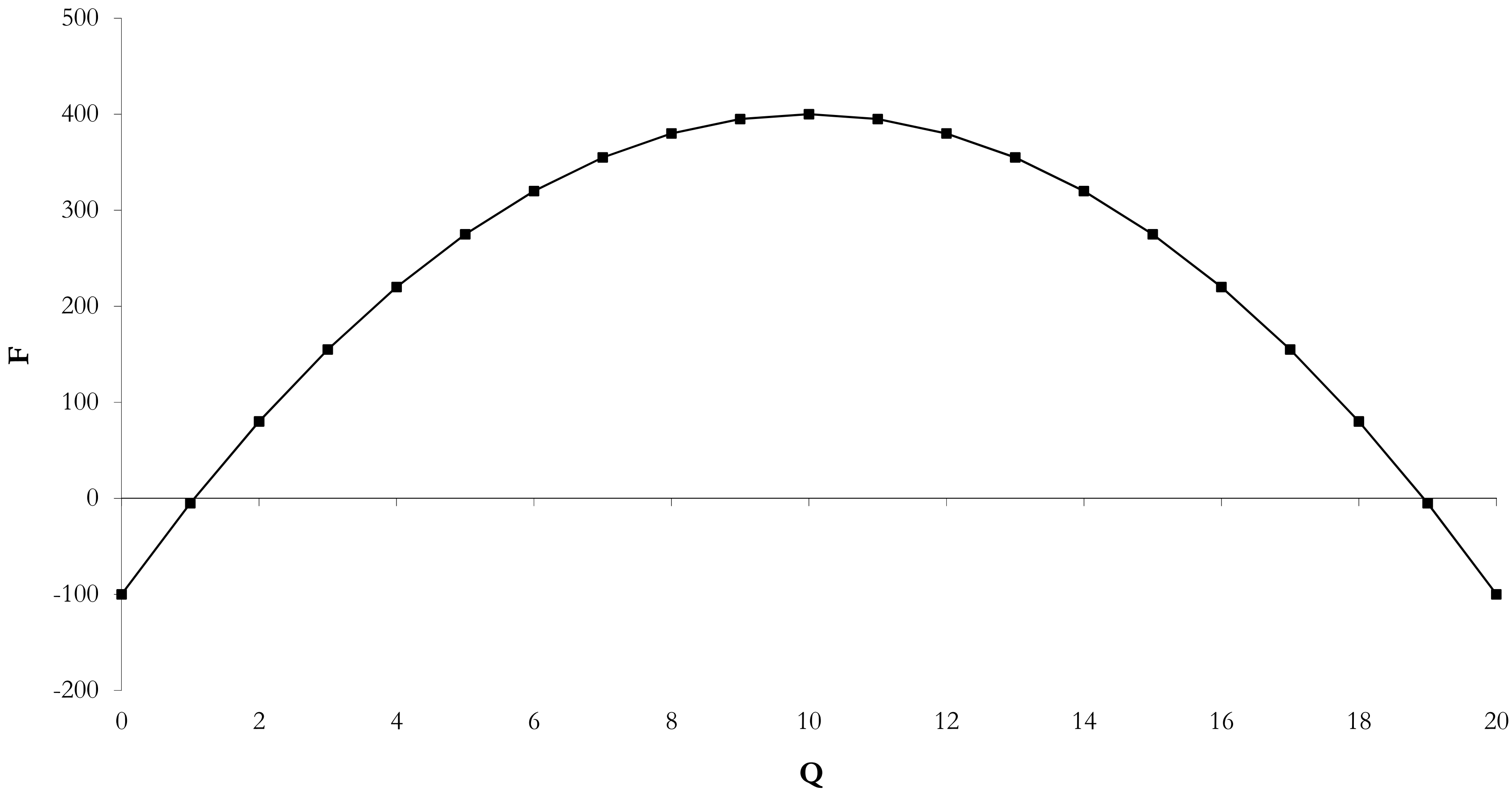

Fig. 2X7C Profit, cost and revenue