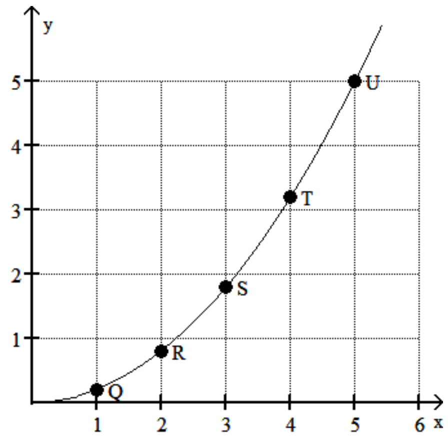
Exam

Name___________________________________

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the slopes of UQ, UR, US, and UT to estimate the rate of change of y at the specified value of x.

1) x = 5 1) _______

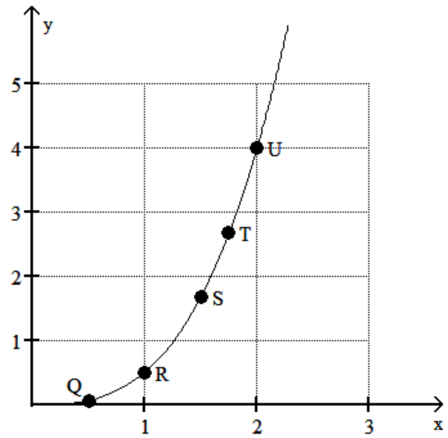


A) 0 B) 5 C) 2 D) 1

Answer: C

Explanation: A)
B)
C)
D)

2) x = 2 2) _______

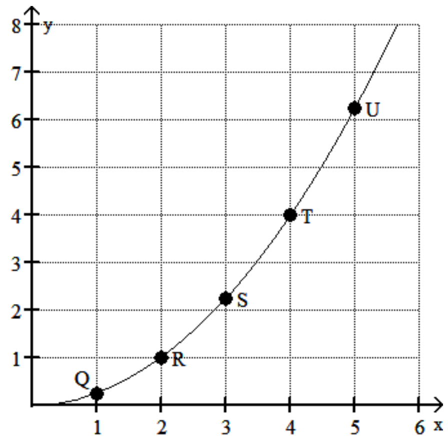


A) 4 B) 0 C) 3 D) 6

Answer: D

Explanation: A)
B)
C)
D)

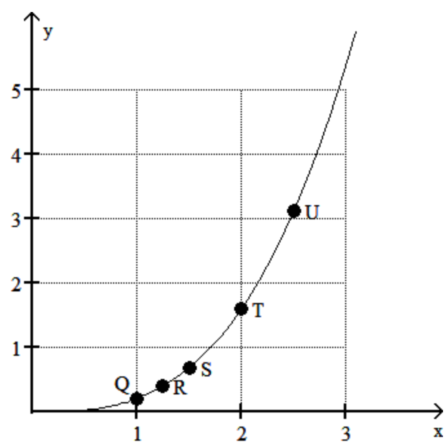
3) x = 5 3) ________



A) $\frac{5}{4}$ B) $\frac{5}{2}$ C) $\frac{25}{4}$ D) 0

Answer: B
Explanation: A)
B)
C)
D)

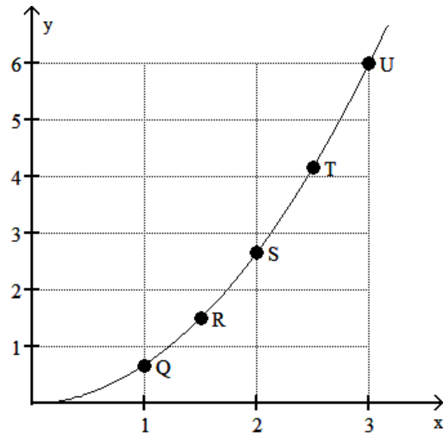
4) x = 2.5 4) ________



A) 3.75 B) 7.5 C) 0 D) 1.25

Answer: A
Explanation: A)
B)
C)
D)

5) x = 3 5) ______



A) 0 B) 4 C) 2 D) 6

Answer: B

Explanation: A)
B)
C)
D)

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

6) Give an example of a function f(x) that is continuous at all values of x except at x = 7, where it has a removable discontinuity. Explain how you know that f is discontinuous at x = 7 and how you know the discontinuity is removable. 6) ______________

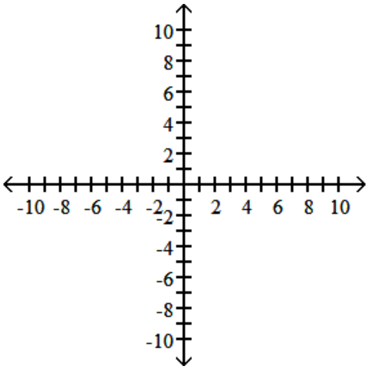
Answer: Let $f(x) = \frac{\sin(x - 7)}{(x - 7)}$ be defined for all $x \neq 7$. The function f is continuous for all $x \neq 7$.

The function is not defined at x = 7 because division by zero is undefined; hence f is not continuous at x = 7. This discontinuity is removable because

$\lim_{x \to 7} \frac{\sin(x - 7)}{x - 7} = 1$. (We can extend the function to x = 7 by defining its value to be 1.)

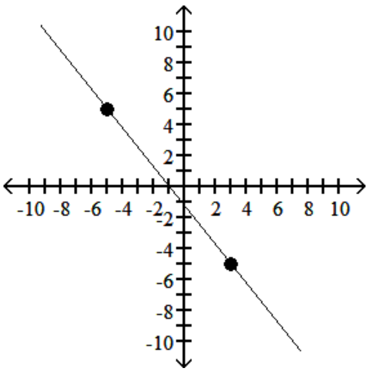
Explanation:

7) A function y = f(x) is continuous on [-5, 3]. It is known to be positive at x = -5 and negative at x = 3. What, if anything, does this indicate about the equation f(x) = 0? Illustrate with a sketch. 7) ______________



Answer: The Intermediate Value Theorem implies that there is at least one solution to f(x) = 0 on the interval [-5, 3].

Possible graph:



Explanation:

8) Use the formal definitions of limits to prove $\lim_{x \to 0} \frac{4}{|x|} = \infty$ 8) ______________

Answer: Given B > 0, we want to find $\delta > 0$ such that $0 < |x - 0| < \delta$ implies $\frac{4}{|x|} > B$.

Now, $\frac{4}{|x|} > B$ if and only if $|x| < \frac{4}{B}$.

Thus, choosing $\delta = 4/B$ (or any smaller positive number), we see that

$|x| < \delta$ implies $\frac{4}{|x|} > \frac{4}{|\delta|} \geq B$.

Therefore, by definition $\lim_{x \to 0} \frac{4}{|x|} = \infty$

Explanation:

9) It can be shown that the inequalities $1 - \frac{x^2}{6} < \frac{x \sin(x)}{2 - 2\cos(x)} < 1$ hold for all values of x close to zero. What, if anything, does this tell you about $\frac{x \sin(x)}{2 - 2\cos(x)}$? Explain. 9) ______________

Answer: Answers may vary. One possibility: $\lim_{x \to 0} 1 - \frac{x^2}{6} = \lim_{x \to 0} 1 = 1$. According to the squeeze theorem, the function $\frac{x \sin(x)}{2 - 2\cos(x)}$, which is squeezed between $1 - \frac{x^2}{6}$ and 1, must also approach 1 as x approaches 0. Thus, $\lim_{x \to 0} \frac{x \sin(x)}{2 - 2\cos(x)} = 1$.

Explanation:

10) Use the Intermediate Value Theorem to prove that $10x^4 - 7x^3 - 4x - 10 = 0$ has a solution between -1 and 0. 10) ______________

Answer: Let $f(x) = 10x^4 - 7x^3 - 4x - 10$ and let $y_0 = 0$. $f(-1) = 11$ and $f(0) = -10$. Since f is continuous on [-1, 0] and since $y_0 = 0$ is between f(-1) and f(0), by the Intermediate Value Theorem, there exists a c in the interval (-1, 0) with the property that f(c) = 0. Such a c is a solution to the equation $10x^4 - 7x^3 - 4x - 10 = 0$.

Explanation:

Prove the limit statement

11) $\lim_{x \to 5} (5x - 3) = 22$ 11) ______________

Answer:

Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon/5$. Then $0 < |x - 5| < \delta$ implies that

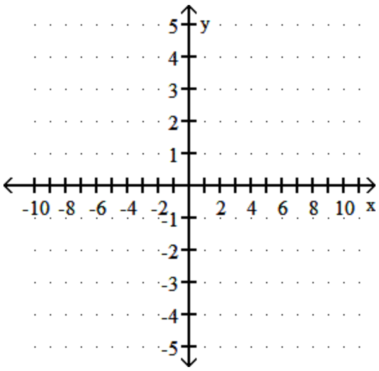
$$|(5x - 3) - 22| = |5x - 25| = |5(x - 5)| = 5|x - 5| < 5\delta = \varepsilon$$

Thus, $0 < |x - 5| < \delta$ implies that $|(5x - 3) - 22| < \varepsilon$

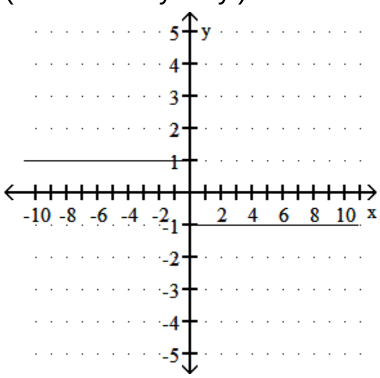
Explanation:

Find a function that satisfies the given conditions and sketch its graph.

12) $\lim_{x \to -\infty} f(x) = 1$, $\lim_{x \to 0^+} f(x) = -1$, $\lim_{x \to \infty} f(x) = -1$, $\lim_{x \to 0^-} f(x) = 1$ 12) ______________



Answer: (Answers may vary.) Possible answer: $f(x) = \begin{cases} 1, & x < 0 \\ -1, & x > 0 \end{cases}$



Explanation:

Provide an appropriate response.

13) Give an example of a function f(x) that is continuous for all values of x except x = 6, where it has a nonremovable discontinuity. Explain how you know that f is discontinuous at x = 6 and why the discontinuity is nonremovable. 13) ______________

Answer: Let $f(x) = \dfrac{1}{(x - 6)^2}$, for all $x \neq 6$. The function f is continuous for all $x \neq 6$, and

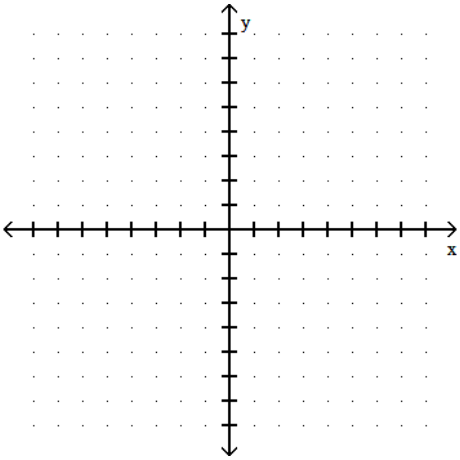
$\lim_{x \to 6} \dfrac{1}{(x - 6)^2} = \infty$. As f is unbounded as x approaches 6, f is discontinuous at x = 6,

and, moreover, this discontinuity is nonremovable.

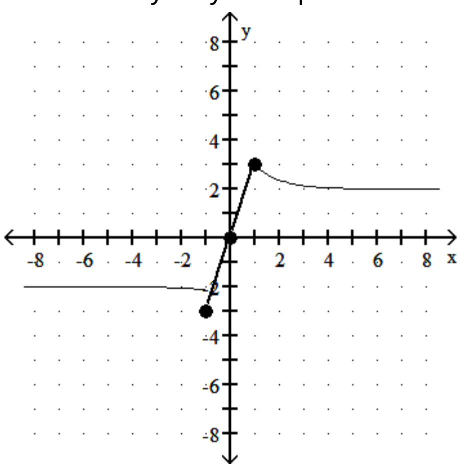
Explanation:

Sketch the graph of a function y = f(x) that satisfies the given conditions.

14) $f(0) = 0$, $f(1) = 3$, $f(-1) = -3$, $\lim\limits_{x \to \infty} f(x) = -2$, $\lim\limits_{x \to \infty} f(x) = 2$. 14) ______________



Answer: Answers may vary. One possible answer:



Explanation:

Provide an appropriate response.

15) Use the formal definitions of limits to prove $\lim\limits_{x \to 0^+} \frac{5}{x} = \infty$ 15) ______________

Answer: Given B > 0, we want to find $\delta > 0$ such that $x_0 < x < x_0 + \delta$ implies $\frac{5}{x} > B$.

Now, $\frac{5}{x} > B$ if and only if $x < \frac{5}{B}$.

We know $x_0 = 0$. Thus, choosing $\delta = 5/B$ (or any smaller positive number), we see that

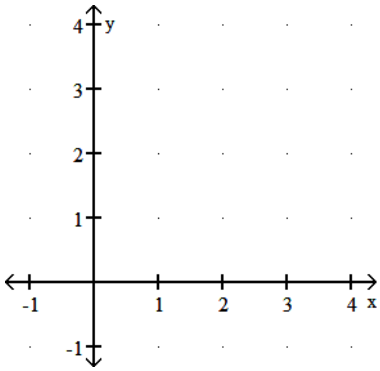
$x < \delta$ implies $\frac{5}{x} > \frac{5}{\delta} \geq B$.

Therefore, by definition $\lim\limits_{x \to 0^+} \frac{5}{x} = \infty$

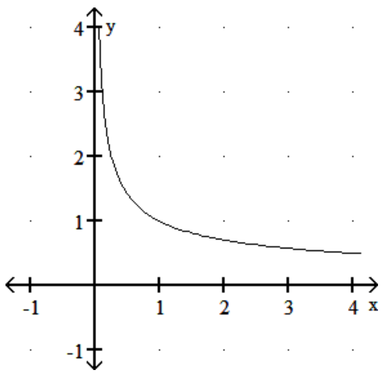
Explanation:

Find a function that satisfies the given conditions and sketch its graph.

16) $\lim_{x \to \infty} f(x) = 0$, $\lim_{x \to 0^+} f(x) = \infty$. 16) ______________



Answer: (Answers may vary.) Possible answer: $f(x) = \frac{1}{\sqrt{x}}$.



Explanation:

Provide an appropriate response.

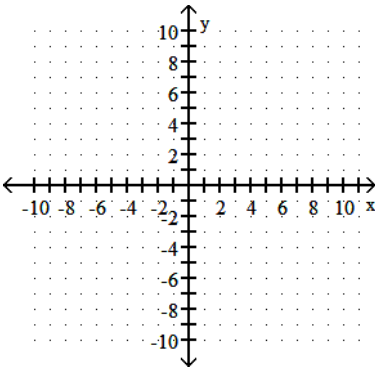
17) If functions $f(x)$ and $g(x)$ are continuous for $0 \le x \le 4$, could $\frac{f(x)}{g(x)}$ possibly be discontinuous at a point of [0,4]? Provide an example. 17) ______________

Answer: Yes, if $f(x) = 1$ and $g(x) = x - 2$, then $h(x) = \frac{1}{x - 2}$ is discontinuous at $x = 2$.

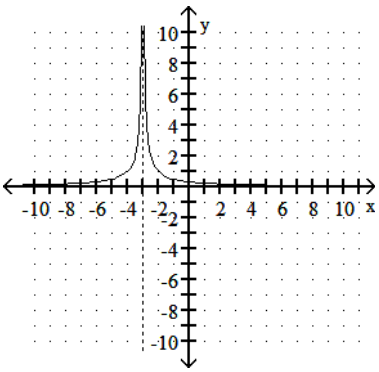
Explanation:

Find a function that satisfies the given conditions and sketch its graph.

18) $\lim_{x \to \pm\infty} f(x) = 0$, $\lim_{x \to 3^-} f(x) = \infty$, $\lim_{x \to 3^+} f(x) = \infty$. 18) ______________



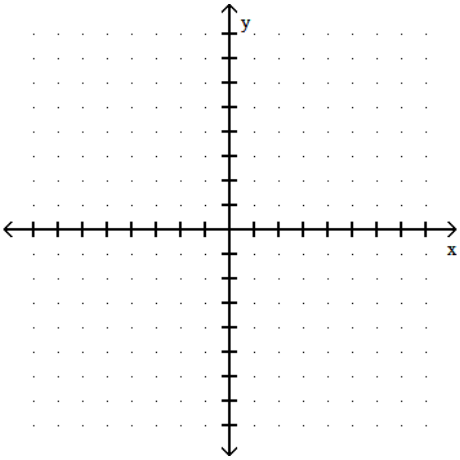
Answer: (Answers may vary.) Possible answer: $f(x) = \frac{1}{|x + 3|}$.



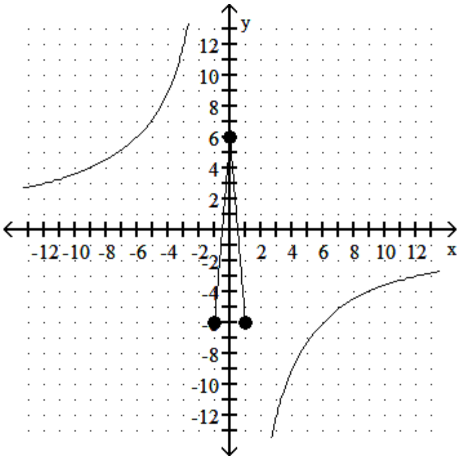
Explanation:

Sketch the graph of a function y = f(x) that satisfies the given conditions.

19) $f(0) = 6$, $f(1) = -6$, $f(-1) = -6$, $\lim_{x \to \pm\infty} f(x) = 0$. 19) ____________



Answer: Answers may vary. One possible answer:



Explanation:

Provide an appropriate response.

20) Use the Intermediate Value Theorem to prove that $x(x - 3)^2 = 3$ has a solution between 2 and 4. 20) ____________

Answer: Let $f(x) = x(x - 3)^2$ and let $y_0 = 3$. $f(2) = 2$ and $f(4) = 4$. Since f is continuous on [2, 4] and since $y_0 = 3$ is between f(2) and f(4), by the Intermediate Value Theorem, there exists a c in the interval (2, 4) with the property that f(c) = 3. Such a c is a solution to the equation $x(x - 3)^2 = 3$.

Explanation:

Prove the limit statement

21) $\lim_{x \to 6} \frac{3x^2 - 14x - 24}{x - 6} = 22$ 21) ______________

Answer: Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon/3$. Then $0 < |x - 6| < \delta$ implies that

$$\begin{aligned}\left|\frac{3x^2 - 14x - 24}{x - 6} - 22\right| &= \left|\frac{(x - 6)(3x + 4)}{x - 6} - 22\right| \\ &= |(3x + 4) - 22| \quad \text{for } x \neq 6 \\ &= |3x - 18| \\ &= |3(x - 6)| \\ &= 3|x - 6| < 3\delta = \varepsilon\end{aligned}$$

Thus, $0 < |x - 6| < \delta$ implies that $\left|\frac{3x^2 - 14x - 24}{x - 6} - 22\right| < \varepsilon$

Explanation:

Provide an appropriate response.

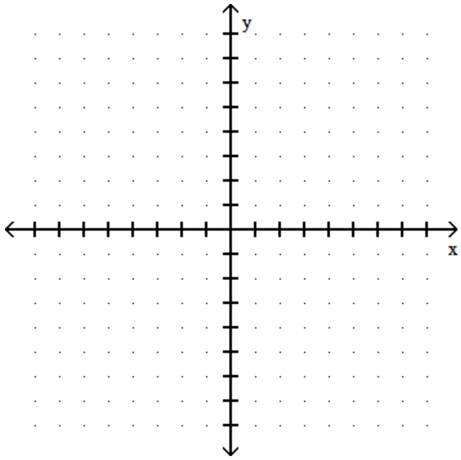
22) If $f(x) = 2x^3 - 5x + 5$, show that there is at least one value of c for which f(x) equals π. 22) ______________

Answer: Notice that $f(0) = 5$ and $f(1) = 2$. As f is continuous on [0,1], the Intermediate Value Theorem implies that there is a number c such that $f(c) = \pi$.

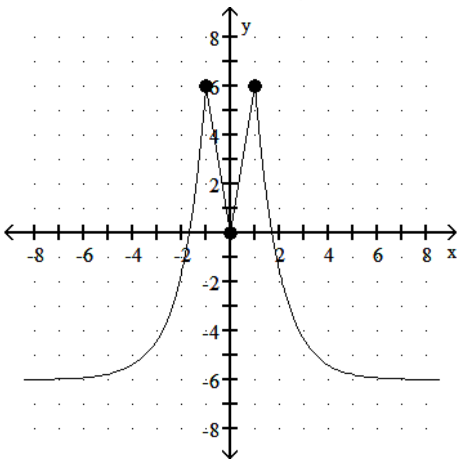
Explanation:

Sketch the graph of a function y = f(x) that satisfies the given conditions.

23) $f(0) = 0$, $f(1) = 6$, $f(-1) = 6$, $\lim\limits_{x \to \pm\infty} f(x) = -6$. 23) ____________

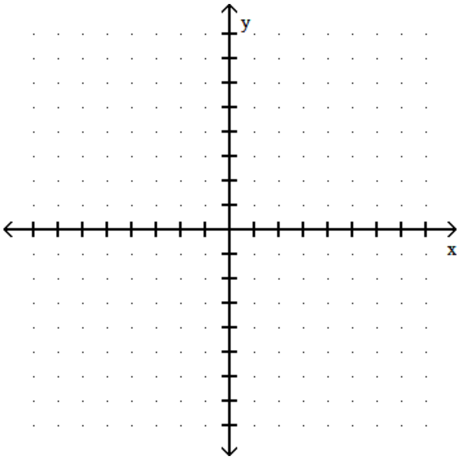


Answer: Answers may vary. One possible answer:

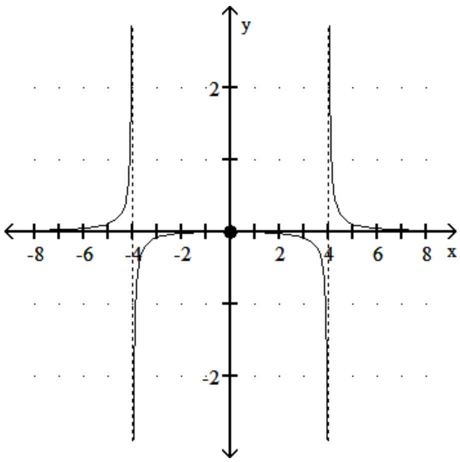


Explanation:

24) $f(0) = 0$, $\lim\limits_{x \to \pm\infty} f(x) = 0$, $\lim\limits_{x \to -4^-} f(x) = -\infty$, $\lim\limits_{x \to 4^+} f(x) = -\infty$, $\lim\limits_{x \to -4^+} f(x) = \infty$, $\lim\limits_{x \to 4^-} f(x) = \infty$. 24) ______________



Answer: Answers may vary. One possible answer:



Explanation:

Provide an appropriate response.

25) Use the Intermediate Value Theorem to prove that $3x^3 - 7x^2 - 9x + 6 = 0$ has a solution between 3 and 4. 25) ______________

Answer: Let $f(x) = 3x^3 - 7x^2 - 9x + 6$ and let $y_0 = 0$. $f(3) = -3$ and $f(4) = 50$. Since f is continuous on [3, 4] and since $y_0 = 0$ is between f(3) and f(4), by the Intermediate Value Theorem, there exists a c in the interval (3 , 4) with the property that f(c) = 0. Such a c is a solution to the equation $3x^3 - 7x^2 - 9x + 6 = 0$.

Explanation:

26) Use the Intermediate Value Theorem to prove that 4 sin x = x has a solution between $\frac{\pi}{2}$ and π. 26) ______________

Answer: Let $f(x) = \frac{\sin x}{x}$ and let $y_0 = \frac{1}{4}$. $f\left(\frac{\pi}{2}\right) \approx 0.6366$ and $f(\pi) = 0$. Since f is continuous on $\left[\frac{\pi}{2}, \pi\right]$ and since $y_0 = \frac{1}{4}$ is between $f\left(\frac{\pi}{2}\right)$ and $f(\pi)$, by the Intermediate Value Theorem, there exists a c in the interval $\left(\frac{\pi}{2}, \pi\right)$, with the property that $f(c) = \frac{1}{4}$. Such a c is a solution to the equation 4 sin x = x.

Explanation:

Prove the limit statement

27) $\lim\limits_{x \to 5} \frac{1}{x} = \frac{1}{5}$ 27) ______________

Answer: Let $\varepsilon > 0$ be given. Choose $\delta = \min\{5/2, 25\varepsilon/2\}$. Then $0 < |x - 5| < \delta$ implies that

$$\left|\frac{1}{x} - \frac{1}{5}\right| = \left|\frac{5 - x}{5x}\right|$$

$$= \frac{1}{|x|} \cdot \frac{1}{5} \cdot |x - 5|$$

$$< \frac{1}{5/2} \cdot \frac{1}{5} \cdot \frac{25\varepsilon}{2} = \varepsilon$$

Thus, $0 < |x - 5| < \delta$ implies that $\left|\frac{1}{x} - \frac{1}{5}\right| < \varepsilon$

Explanation:

Provide an appropriate response.

28) Explain why the following five statements ask for the same information. 28) ______________

(a) Find the roots of $f(x) = 4x^3 - 1x - 5$.

(b) Find the x-coordinate of the points where the curve $y = 4x^3$ crosses the line $y = 1x + 5$.

(c) Find all the values of x for which $4x^3 - 1x = 5$.

(d) Find the x-coordinates of the points where the cubic curve $y = 4x^3 - 1x$ crosses the line $y = 5$.

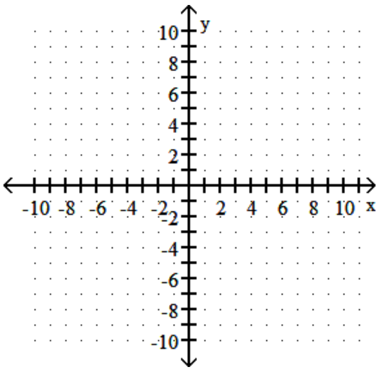
(e) Solve the equation $4x^3 - 1x - 5 = 0$.

Answer: The roots of f(x) are the solutions to the equation f(x) = 0. Statement (b) is asking for the solution to the equation $4x^3 = 1x + 5$. Statement (d) is asking for the solution to the equation $4x^3 - 1x = 5$. These three equations are equivalent to the equations in statements (c) and (e). As five equations are equivalent, their solutions are the same.

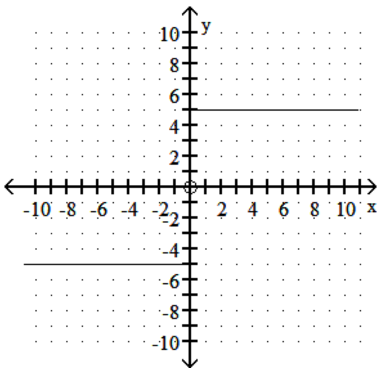
Explanation:

Find a function that satisfies the given conditions and sketch its graph.

29) $\lim\limits_{x \to \infty} g(x) = -5$, $\lim\limits_{x \to \infty} g(x) = 5$, $\lim\limits_{x \to 0^+} g(x) = 5$, $\lim\limits_{x \to 0^-} g(x) = -5$. 29) ______________

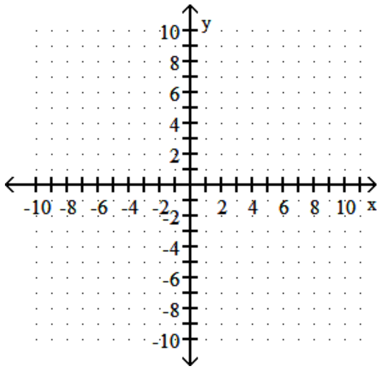


Answer: (Answers may vary.) Possible answer: $f(x) = \begin{cases} 5, & x > 0 \\ -5, & x < 0 \end{cases}$

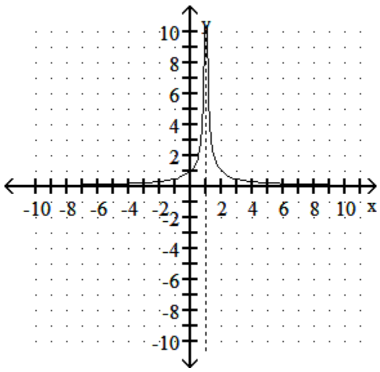


Explanation:

30) $\lim_{x \to \pm\infty} f(x) = 0$, $\lim_{x \to 1^-} f(x) = \infty$, $\lim_{x \to 1^+} f(x) = \infty$. 30) ______________



Answer: (Answers may vary.) Possible answer: $f(x) = \frac{1}{|x - 1|}$.



Explanation:

Prove the limit statement

31) $\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = 4$ 31) ______________

Answer: Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon$. Then $0 < |x - 2| < \delta$ implies that

$$\left|\frac{x^2 - 4}{x - 2} - 4\right| = \left|\frac{(x - 2)(x + 2)}{x - 2} - 4\right|$$

$$= |(x + 2) - 4| \quad \text{for } x \neq 2$$

$$= |x - 2| < \delta = \varepsilon$$

Thus, $0 < |x - 2| < \delta$ implies that $\left|\frac{x^2 - 4}{x - 2} - 4\right| < \varepsilon$

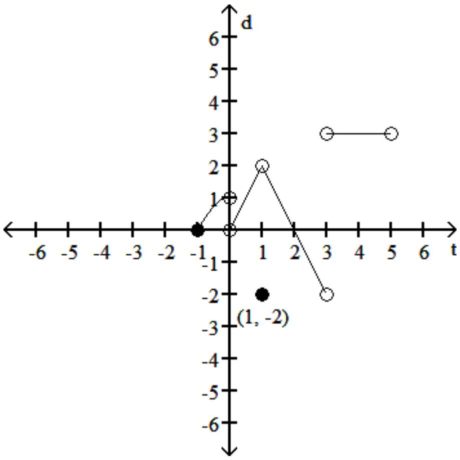
Explanation:

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Answer the question.

32) Does $\lim_{x \to 1} f(x)$ exist? 32) ______

$$f(x) = \begin{cases} -x^2 + 1, & -1 \le x < 0 \\ 2x, & 0 < x < 1 \\ -2, & x = 1 \\ -2x + 4 & 1 < x < 3 \\ 3, & 3 < x < 5 \end{cases}$$



A) No B) Yes

Answer: B

Explanation: A)

B)

Provide an appropriate response.

33) Use a calculator to graph the function f to see whether it appears to have a continuous extension to the origin. If it does, use Trace and Zoom to find a good candidate for the extended function's value at x = 0. If the function does not appear to have a continuous extension, can it be extended to be continuous at the origin from the right or from the left? If so, what do you think the extended function's value(s) should be? 33) ______

$f(x) = \dfrac{4 \sin x}{|x|}$

A) continuous extension exists from the right; f(0) = 4
continuous extension exists from the left; f(0) = -4

B) continuous extension exists at origin; f(0) = 0

C) continuous extension exists at origin; f(0) = 4

D) continuous extension exists from the right; f(0) = 1
continuous extension exists from the left; f(0) = -1

Answer: A

Explanation: A)

B)

C)

D)

Divide numerator and denominator by the highest power of x in the denominator to find the limit.

34) $\lim_{x \to -\infty} \dfrac{\sqrt[3]{x} - 5x + 7}{-7x + x^{2/3} + 2}$ 34) ______

A) $\frac{7}{5}$ B) $\frac{5}{7}$ C) 0 D) $-\infty$

Answer: B
Explanation: A)
B)
C)
D)

A function f(x), a point x_0, the limit of f(x) as x approaches x_0, and a positive number ε is given. Find a number δ > 0 such that for all x, $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

35) $f(x) = 3x^2$, $L = 243$, $x_0 = 9$, and $\varepsilon = 0.5$ 35) ______

A) 0.00925 B) 9.00925 C) 0.00926 D) 8.99074

Answer: A
Explanation: A)
B)
C)
D)

Find the limit.

36) $\lim_{x \to 1^-} \dfrac{4}{x^2 - 1}$ 36) ______

A) 1 B) $-\infty$ C) 0 D) ∞

Answer: B
Explanation: A)
B)
C)
D)

Divide numerator and denominator by the highest power of x in the denominator to find the limit.

37) $\lim_{x \to \infty} \dfrac{7x + 5}{\sqrt{5x^2 + 1}}$ 37) ______

A) ∞ B) 0 C) $\frac{7}{\sqrt{5}}$ D) $\frac{7}{5}$

Answer: C
Explanation: A)
B)
C)
D)

Evaluate $\lim_{h \to 0} \dfrac{f(x_0 + h) - f(x_0)}{h}$ for the given x_0 and function f.

38) $f(x) = \dfrac{x}{3} + 4$ for $x_0 = 4$ 38) ______

A) $\dfrac{1}{3}$ B) $\dfrac{16}{3}$ C) Does not exist D) $\dfrac{4}{3}$

Answer: A
Explanation: A)
B)
C)
D)

Provide an appropriate response.

39) If $\lim_{x \to 0} f(x) = L$, which of the following expressions are true? 39) ______

I. $\lim_{x \to 0^-} f(x)$ does not exist.

II. $\lim_{x \to 0^+} f(x)$ does not exist.

III. $\lim_{x \to 0^-} f(x) = L$

IV. $\lim_{x \to 0^+} f(x) = L$

A) I and IV only B) I and II only C) III and IV only D) II and III only

Answer: C
Explanation: A)
B)
C)
D)

Use a CAS to plot the function near the point x_0 being approached. From your plot guess the value of the limit.

40) $\lim_{x \to 0} \dfrac{\sqrt{6 + 6x} - \sqrt{6}}{x}$ 40) ______

A) $\dfrac{1}{2}$ B) 0 C) $\sqrt{6}$ D) $\dfrac{\sqrt{6}}{2}$

Answer: D
Explanation: A)
B)
C)
D)

Find the limit using $\lim_{x=0} \frac{\sin x}{x} = 1$.

41) $\lim_{x \to 0} \frac{x}{\sin 3x}$ 41) _______

A) 1 B) does not exist C) $\frac{1}{3}$ D) 3

Answer: C
Explanation: A)
B)
C)
D)

Find the limit.

42) $\lim_{x \to \infty} \frac{2x^3 - 4x^2 + 3x}{-x^3 - 2x + 5}$ 42) _______

A) -2 B) ∞ C) 2 D) $\frac{3}{2}$

Answer: A
Explanation: A)
B)
C)
D)

Evaluate $\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$ for the given x_0 and function f.

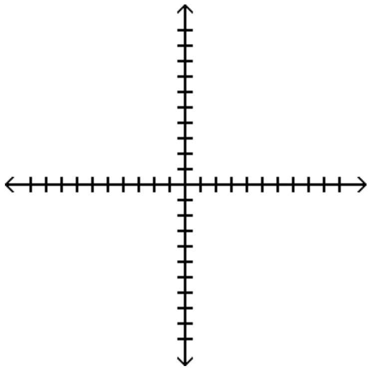
43) $f(x) = 3x^2$ for $x_0 = 9$ 43) _______

A) 27 B) 243 C) 54 D) Does not exist

Answer: C
Explanation: A)
B)
C)
D)

Determine the limit by sketching an appropriate graph.

44) $\lim_{x \to 7^+} f(x)$, where $f(x) = \begin{cases} -2x - 7 & \text{for } x < 7 \\ 2x - 6 & \text{for } x \geq 7 \end{cases}$ 44) ______



A) -6 B) -5 C) -21 D) 8

Answer: D
Explanation: A)
B)
C)
D)

Determine if the given function can be extended to a continuous function at x = 0. If so, approximate the extended function's value at x = 0 (rounded to four decimal places if necessary). If not, determine whether the function can be continuously extended from the left or from the right and provide the values of the extended functions at x = 0. Otherwise write "no continuous extension."

45) $f(x) = \frac{\cos 2x}{|2x|}$ 45) ______

A) f(0) = 2 only from the right
B) f(0) = 2
C) f(0) = 2 only from the left
D) No continuous extension

Answer: D
Explanation: A)
B)
C)
D)

Find the limit using $\lim_{x=0} \frac{\sin x}{x} = 1$.

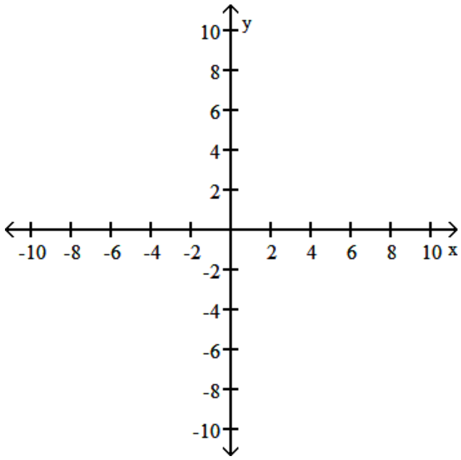
46) $\lim_{x \to 0} \frac{\sin x \cos 4x}{x + x \cos 5x}$ 46) ______

A) 0 B) does not exist C) $\frac{4}{5}$ D) $\frac{1}{2}$

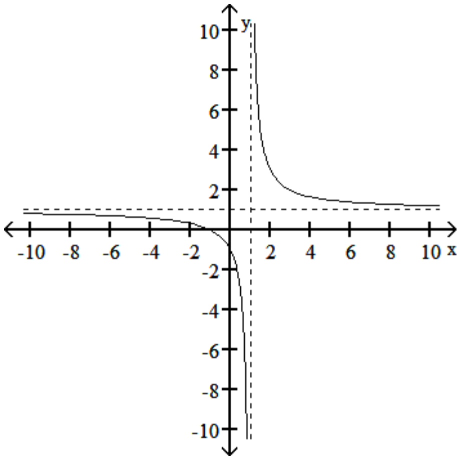
Answer: D
Explanation: A)
B)
C)
D)

Graph the rational function. Include the graphs and equations of the asymptotes.

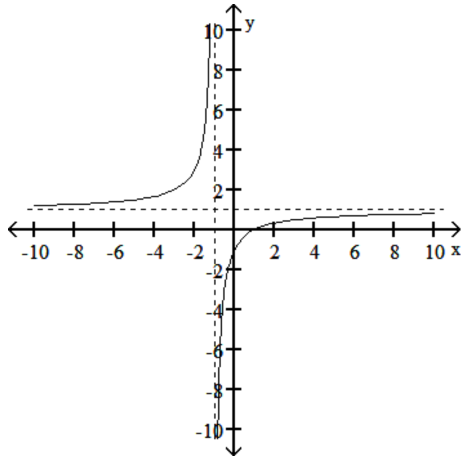
47) $f(x) = \frac{x - 1}{x + 1}$ 47) ______



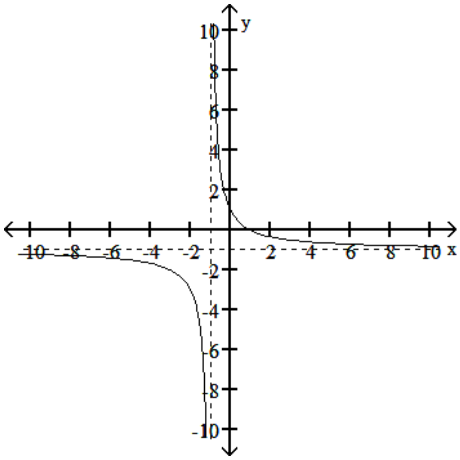
A) asymptotes: x = 1, y = 1



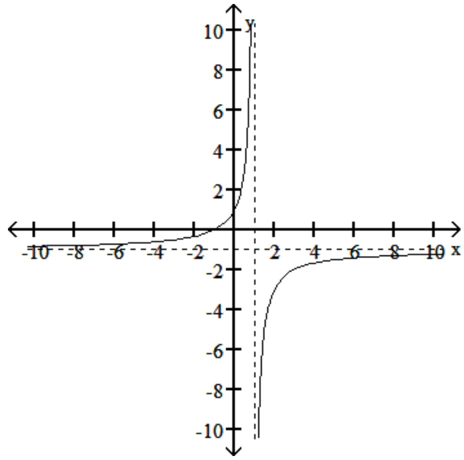
B) asymptotes: x = -1, y = 1



C) asymptotes: x = -1, y = -1



D) asymptotes: x = 1, y = -1



Answer: B

Explanation:
A)
B)
C)
D)

Find the limit, if it exists.

48) $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x} - 1}{x}$ 48) ______

A) 1/4 B) Does not exist C) 1/2 D) 0

Answer: C

Explanation: A)
B)
C)
D)

Use the graph to find a $\delta > 0$ such that for all x, $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

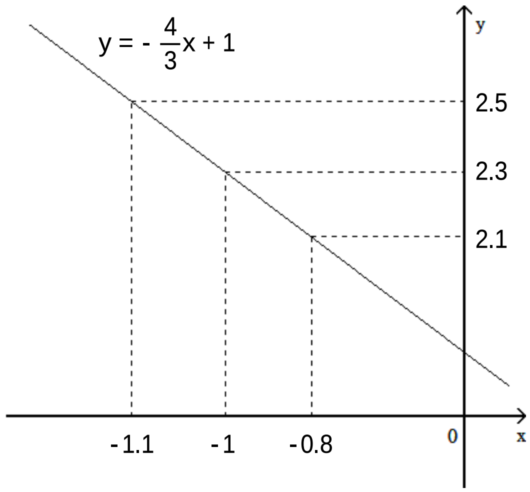
49) 49) ______

$f(x) = -\frac{4}{3}x + 1$

$x_0 = -1$

$L = 2.3$

$\varepsilon = 0.2$



NOT TO SCALE

A) 0.3 B) 3.3 C) 0.1 D) -0.3

Answer: C

Explanation: A)
B)
C)
D)

Evaluate $\lim\limits_{h \to 0} \dfrac{f(x_0 + h) - f(x_0)}{h}$ for the given x_0 and function f.

50) $f(x) = \dfrac{5}{x}$ for $x_0 = 6$ 50) ______

A) Does not exist B) $-\dfrac{5}{36}$ C) -30 D) $\dfrac{5}{6}$

Answer: B

Explanation: A)
B)
C)
D)

A function f(x), a point x_0, the limit of f(x) as x approaches x_0, and a positive number ε is given. Find a number $\delta > 0$ such that for all x, $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

51) $f(x) = mx + b$, $m > 0$, $L = (m/8) + b$, $x_0 = 1/8$, and $\varepsilon = c > 0$ 51) ______

A) $\delta = \frac{8}{m}$ B) $\delta = \frac{c}{m}$ C) $\delta = \frac{1}{8} + \frac{c}{m}$ D) $\delta = \frac{c}{8}$

Answer: B
Explanation: A)
B)
C)
D)

Find the limit.

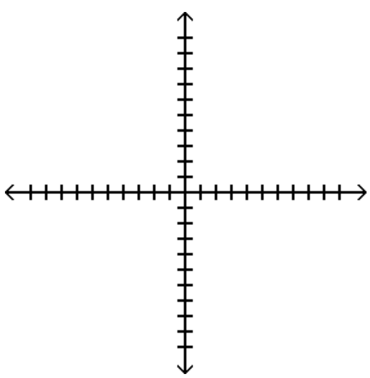
52) $\lim\limits_{x \to 5^-} \frac{\sqrt{3x}(x - 5)}{|x - 5|}$ 52) ______

A) Does not exist B) $-\sqrt{15}$ C) $\sqrt{15}$ D) 0

Answer: B
Explanation: A)
B)
C)
D)

Determine the limit by sketching an appropriate graph.

53) $\lim\limits_{x \to 3^-} f(x)$, where $f(x) = \begin{cases} -4x + 1 & \text{for } x < 3 \\ 2x + 2 & \text{for } x \geq 3 \end{cases}$ 53) ______



A) 3 B) -11 C) 8 D) 2

Answer: B
Explanation: A)
B)
C)
D)

Provide an appropriate response.

54) Given $\varepsilon > 0$, find an interval $I = (5 - \delta, 5)$, $\delta > 0$, such that if x lies in I, then $\sqrt{5 - x} < \varepsilon$. What limit is being verified and what is its value? 54) ______

A) $\lim_{x \to 5^+} \sqrt{5 - x} = 0$

B) $\lim_{x \to 5^-} \sqrt{x} = 5$

C) $\lim_{x \to 0^-} \sqrt{5 - x} = 0$

D) $\lim_{x \to 5^-} \sqrt{5 - x} = 0$

Answer: D

Explanation: A)
B)
C)
D)

Find the intervals on which the function is continuous.

55) $y = \frac{\sin(3\theta)}{4\theta}$ 55) ______

A) discontinuous only when $\theta = \frac{\pi}{2}$

B) discontinuous only when $\theta = 0$

C) discontinuous only when $\theta = \pi$

D) continuous everywhere

Answer: B

Explanation: A)
B)
C)
D)

Find the limit if it exists.

56) $\lim_{x \to 11} \sqrt{10}$ 56) ______

A) 10

B) 11

C) $\sqrt{10}$

D) $\sqrt{11}$

Answer: C

Explanation: A)
B)
C)
D)

Use a CAS to plot the function near the point x_0 being approached. From your plot guess the value of the limit.

57) $\lim_{x \to 0} \frac{\sqrt{36 + 2x} - 6}{x}$ 57) ______

A) $\frac{1}{12}$

B) $\frac{1}{6}$

C) 36

D) $\frac{1}{3}$

Answer: B

Explanation: A)
B)
C)
D)

Use the table of values of f to estimate the limit.

58) Let $f(x) = x^2 + 8x - 2$, find $\lim_{x \to 2} f(x)$. 58) ______

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)						

A)

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	16.810	17.880	17.988	18.012	18.120	19.210

; limit = 18.0

B)

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	5.043	5.364	5.396	5.404	5.436	5.763

; limit = 5.40

C)

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	5.043	5.364	5.396	5.404	5.436	5.763

; limit = ∞

D)

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	16.692	17.592	17.689	17.710	17.808	18.789

; limit = 17.70

Answer: A

Explanation: A)
B)
C)
D)

Find the limit, if it exists.

59) $\lim_{x \to 7} \frac{|7 - x|}{7 - x}$ 59) ______

A) 1 B) 0 C) -1 D) Does not exist

Answer: D

Explanation: A)
B)
C)
D)

Give an appropriate answer.

60) Let $\lim_{x \to 7} f(x) = -8$ and $\lim_{x \to 7} g(x) = 5$. Find $\lim_{x \to 7} [f(x) - g(x)]$. 60) ______

A) -3 B) -8 C) -13 D) 7

Answer: C

Explanation: A)
B)
C)
D)

Find the slope of the curve at the given point P and an equation of the tangent line at P.

61) $y = x^2 + 11x - 15$, P(1, -3) 61) ______

A) slope is -39; y = -39x - 80
B) slope is 13; y = 13x - 16
C) slope is $\frac{1}{20}$; $y = \frac{x}{20} + \frac{1}{5}$
D) slope is $-\frac{4}{25}$; $y = -\frac{4x}{25} + \frac{8}{5}$

Answer: B
Explanation: A)
B)
C)
D)

Determine if the given function can be extended to a continuous function at x = 0. If so, approximate the extended function's value at x = 0 (rounded to four decimal places if necessary). If not, determine whether the function can be continuously extended from the left or from the right and provide the values of the extended functions at x = 0. Otherwise write "no continuous extension."

62) $f(x) = (1 + 2x)^{1/x}$ 62) ______

A) f(0) = 7.3891
B) f(0) = 5.4366
C) f(0) = 2.7183
D) No continuous extension

Answer: A
Explanation: A)
B)
C)
D)

Provide an appropriate response.

63) Write the formal notation for the principle "the limit of a quotient is the quotient of the limits" and include a statement of any restrictions on the principle. 63) ______

A) If $\lim_{x \to a} g(x) = M$ and $\lim_{x \to a} f(x) = L$, then $\lim_{x \to a} \frac{g(x)}{f(x)} = \frac{\lim_{x \to a} g(x)}{\lim_{x \to a} f(x)} = \frac{M}{L}$, provided that $L \neq 0$.

B) If $\lim_{x \to a} g(x) = M$ and $\lim_{x \to a} f(x) = L$, then $\lim_{x \to a} \frac{g(x)}{f(x)} = \frac{\lim_{x \to a} g(x)}{\lim_{x \to a} f(x)} = \frac{M}{L}$, provided that $f(a) \neq 0$.

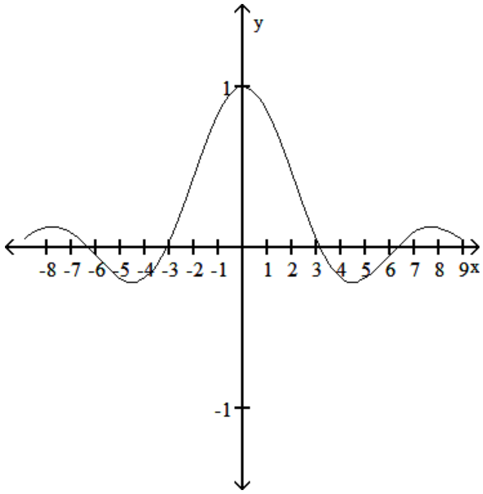
C) $\lim_{x \to a} \frac{g(x)}{f(x)} = \frac{g(a)}{f(a)}$, provided that $f(a) \neq 0$.

D) $\lim_{x \to a} \frac{g(x)}{f(x)} = \frac{g(a)}{f(a)}$.

Answer: A
Explanation: A)
B)
C)
D)

Use the graph to evaluate the limit.

64) $\lim_{x \to 0} f(x)$ 64) ______



A) -1 B) 1 C) does not exist D) 0

Answer: B

Explanation: A)
B)
C)
D)

Find the limit.

65) If $\lim_{x \to 0} \frac{f(x)}{x} = 1$, find $\lim_{x \to 0} f(x)$. 65) ______

A) 2 B) 1 C) 0 D) Does not exist

Answer: C

Explanation: A)
B)
C)
D)

66) $\lim_{x \to 1} \frac{x}{3x + 2}$ 66) ______

A) 0 B) $-\frac{1}{5}$ C) 1 D) does not exist

Answer: C

Explanation: A)
B)
C)
D)

Find the intervals on which the function is continuous.

67) $y = \frac{3}{|x| + 1} - \frac{x^2}{3}$ 67) ______

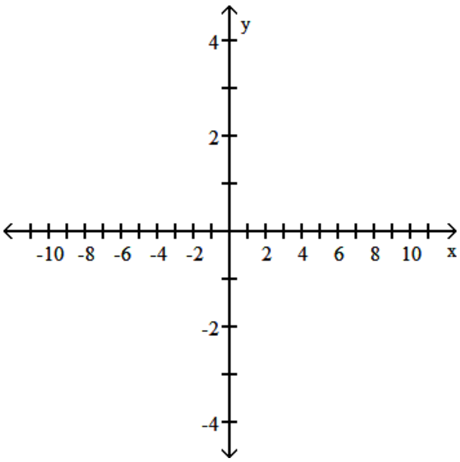
A) continuous everywhere
B) discontinuous only when x = -3 or x = -1
C) discontinuous only when x = -1
D) discontinuous only when x = -4

Answer: A

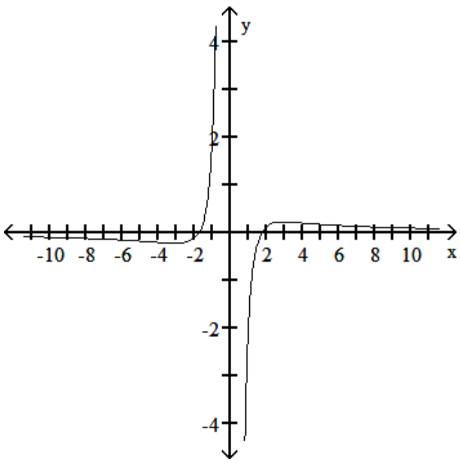
Explanation:
A)
B)
C)
D)

Graph the rational function. Include the graphs and equations of the asymptotes.

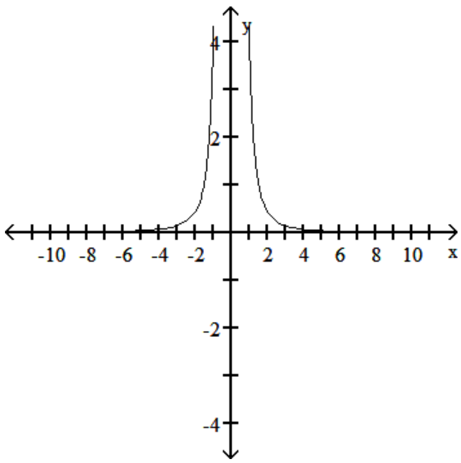
68) $f(x) = \frac{x^2 + 3}{x^3}$ 68) ______



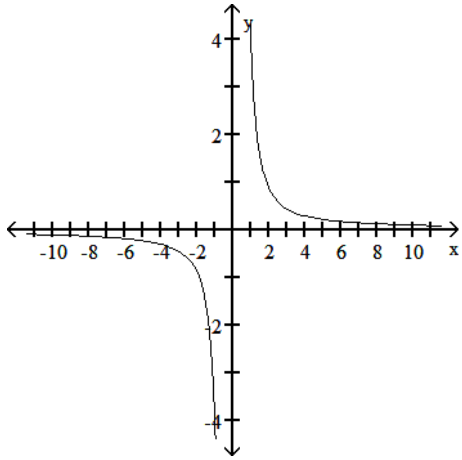
A) asymptotes: x = 0, y = 0



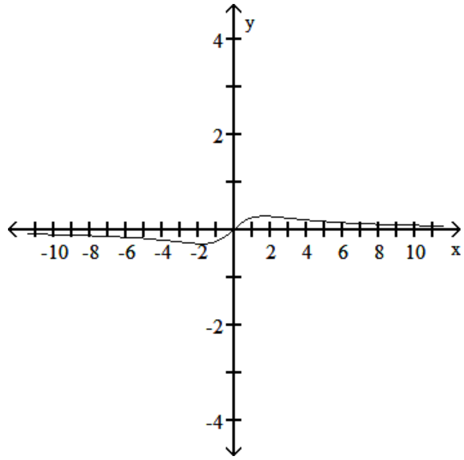
B) asymptotes: x = 0, y = 0



C) asymptotes: x = 0, y = 0



D) asymptote: y = 0



Answer: C

Explanation: A)
B)
C)
D)

Find numbers a and b, or k, so that f is continuous at every point.

69) 69) ______

$$f(x) = \begin{cases} x^2, & \text{if } x \le 8 \\ kx, & \text{if } x > 8 \end{cases}$$

A) $k = \frac{1}{8}$ B) k = 64 C) k = 8 D) Impossible

Answer: C

Explanation: A)
B)
C)
D)

Find the limit if it exists.

70) $\lim_{x \to 3} (4x - 4)$ 70) ______

A) -16 B) 8 C) 16 D) -8

Answer: A

Explanation: A)
B)
C)
D)

Find the limit.

71) $\lim\limits_{x \to 5} \sqrt{x^2 + 10x + 25}$ 71) _______

A) does not exist B) 100 C) ±10 D) 10

Answer: D

Explanation: A)
B)
C)
D)

72) $\lim\limits_{x \to 7^+} \frac{1}{x - 7}$ 72) _______

A) ∞ B) -1 C) 0 D) $-\infty$

Answer: A

Explanation: A)
B)
C)
D)

Use the table to estimate the rate of change of y at the specified value of x.

73) x = 1. 73) _______

x	y
0	0
0.2	0.02
0.4	0.08
0.6	0.18
0.8	0.32
1.0	0.5
1.2	0.72
1.4	0.98

A) 0.5 B) 1.5 C) 2 D) 1

Answer: D

Explanation: A)
B)
C)
D)

Divide numerator and denominator by the highest power of x in the denominator to find the limit.

74) $\lim\limits_{x \to \infty} \sqrt{\frac{49x^2}{2 + 4x^2}}$ 74) _______

A) does not exist B) $\frac{7}{2}$ C) $\frac{49}{2}$ D) $\frac{49}{4}$

Answer: B

Explanation: A)
B)
C)
D)

Find the limit.

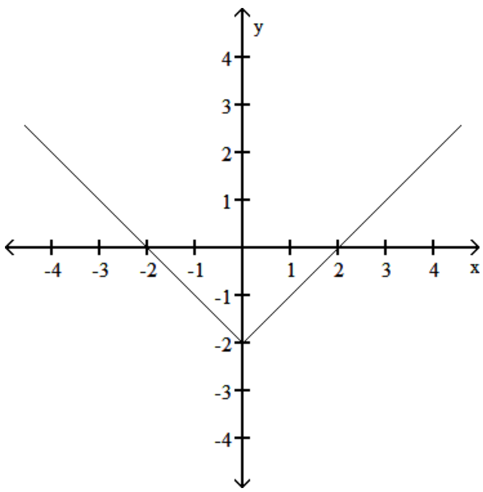
75) $\lim_{x \to 1^+} \left(\frac{x}{x+9}\right)\left(\frac{-3x+8}{x^2+9x}\right)$ 75) ______

A) Does not exist B) $\frac{1}{11}$ C) $\frac{11}{82}$ D) $\frac{11}{64}$

Answer: D
Explanation: A)
B)
C)
D)

Use the graph to evaluate the limit.

76) $\lim_{x \to 0} f(x)$ 76) ______



A) 0 B) does not exist C) -2 D) 2

Answer: C
Explanation: A)
B)
C)
D)

Solve the problem.

77) Ohm's Law for electrical circuits is stated V = RI, where V is a constant voltage, R is the resistance in ohms and I is the current in amperes. Your firm has been asked to supply the resistors for a circuit in which V will be 12 volts and I is to be 3 ± 0.1 amperes. In what interval does R have to lie for I to be within 0.1 amps of the target value $I_0 = 3$? 77) ______

A) $\left(\frac{31}{120}, \frac{29}{120}\right)$ B) $\left(\frac{120}{29}, \frac{120}{31}\right)$ C) $\left(\frac{10}{29}, \frac{10}{31}\right)$ D) $\left(\frac{120}{31}, \frac{120}{29}\right)$

Answer: D
Explanation: A)
B)
C)
D)

Find the limit.

78) $\lim_{x \to 1^-} \frac{x^2 - 5x + 4}{x^3 - x}$ 78) ______

A) ∞ B) 0 C) $-\frac{3}{2}$ D) $-\infty$

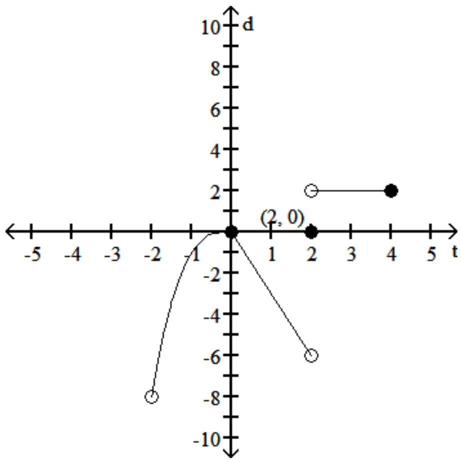
Answer: C

Explanation: A)
B)
C)
D)

Answer the question.

79) Is f continuous on (-2, 4]? 79) ______

$$f(x) = \begin{cases} x^3, & -2 < x \le 0 \\ -3x, & 0 \le x < 2 \\ 2, & 2 < x \le 4 \\ 0, & x = 2 \end{cases}$$



A) Yes B) No

Answer: B

Explanation: A)
B)

Provide an appropriate response.

80) Which of the following statements defines $\lim_{x \to (x_0)^+} f(x) = \infty$? 80) ______

I. For every positive real number B there exists a corresponding $\delta > 0$ such that $f(x) > B$ whenever $x_0 - \delta < x < x_0 + \delta$.

II. For every positive real number B there exists a corresponding $\delta > 0$ such that $f(x) > B$ whenever $x_0 < x < x_0 + \delta$.

III. For every positive real number B there exists a corresponding $\delta > 0$ such that $f(x) > B$ whenever $x_0 - \delta < x < x_0$.

A) I B) III C) II D) None

Answer: C

Explanation: A)
B)
C)
D)

Find the average rate of change of the function over the given interval.

81) $y = 4x^2, \left[0, \frac{7}{4}\right]$ 81) ______

A) $\frac{1}{3}$ B) $-\frac{3}{10}$ C) 7 D) 2

Answer: C

Explanation: A)
B)
C)
D)

Provide an appropriate response.

82) Which of the following statements defines $\lim_{x \to x_0} f(x) = \infty$? 82) ______

I. For every positive real number B there exists a corresponding $\delta > 0$ such that $f(x) > B$ whenever $x_0 - \delta < x < x_0 + \delta$.

II. For every positive real number B there exists a corresponding $\delta > 0$ such that $f(x) > B$ whenever $x_0 < x < x_0 + \delta$.

III. For every positive real number B there exists a corresponding $\delta > 0$ such that $f(x) > B$ whenever $x_0 - \delta < x < x_0$.

A) III B) I C) II D) None

Answer: B

Explanation: A)
B)
C)
D)

Find the limit, if it exists.

83) $\lim_{x \to 1} \frac{2x - 7}{4x + 5}$ 83) ______

A) $-\frac{5}{9}$ B) $-\frac{7}{5}$ C) Does not exist D) $-\frac{1}{2}$

Answer: A

Explanation: A)
B)
C)
D)

84) $\lim_{x \to 14} \frac{1}{x - 14}$ 84) ______

A) 14 B) 0 C) Does not exist D) 28

Answer: C

Explanation: A)
B)
C)
D)

Provide an appropriate response.

85) Which of the following statements defines $\lim_{x \to (x_0)^-} f(x) = \infty$? 85) ______

I. For every positive real number B there exists a corresponding $\delta > 0$ such that $f(x) > B$ whenever $x_0 - \delta < x < x_0 + \delta$.

II. For every positive real number B there exists a corresponding $\delta > 0$ such that $f(x) > B$ whenever $x_0 < x < x_0 + \delta$.

III. For every positive real number B there exists a corresponding $\delta > 0$ such that $f(x) > B$ whenever $x_0 - \delta < x < x_0$.

A) II B) III C) I D) None

Answer: B

Explanation: A)
B)
C)
D)

Solve the problem.

86) To what new value should f(2) be changed to remove the discontinuity? 86) ______

$$f(x) = \begin{cases} 2x - 3, & x < 2 \\ 3 & x = 2 \\ x - 1, & x > 2 \end{cases}$$

A) -6 B) 0 C) -7 D) 1

Answer: D

Explanation: A)
B)
C)
D)

Use a CAS to plot the function near the point x_0 being approached. From your plot guess the value of the limit.

87) $\lim_{x \to 0} \frac{\sqrt{1 - x} - 1}{x}$ 87) ______

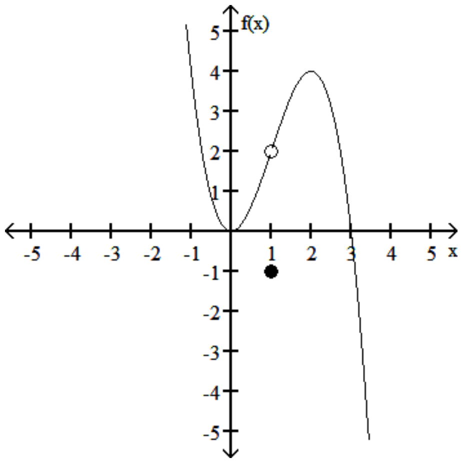
A) 1 B) $\frac{1}{2}$ C) 2 D) $-\frac{1}{2}$

Answer: D

Explanation: A)
B)
C)
D)

For the function f whose graph is given, determine the limit.

88) Find $\lim_{x \to 1^-} f(x)$. 88) ______



A) does not exist B) 2 C) -1 D) $\frac{1}{2}$

Answer: B

Explanation: A)
B)
C)
D)

Find the limit if it exists.

89) $\lim_{x \to 1} 9x(x + 4)(x - 4)$ 89) ______

A) -225 B) 135 C) -81 D) -135

Answer: B

Explanation: A)
B)
C)
D)

Find the limit using $\lim_{x=0} \frac{\sin x}{x} = 1$.

90) $\lim_{x \to 0} \frac{\sin 3x \cot 4x}{\cot 5x}$ 90) ______

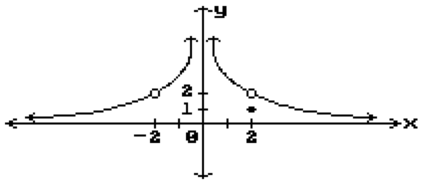
A) 0 B) $\frac{15}{4}$ C) does not exist D) $\frac{12}{5}$

Answer: A

Explanation: A)
B)
C)
D)

Find all points where the function is discontinuous.

91)



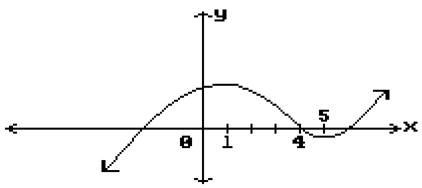
A) x = -2, x = 0
B) x = -2, x = 0, x = 2
C) x = 0, x = 2
D) x = 2

91) ______

Answer: B

Explanation:
A)
B)
C)
D)

92)



A) x = 1, x = 5
B) x = 1, x = 4, x = 5
C) None
D) x = 4

92) ______

Answer: C

Explanation:
A)
B)
C)
D)

Divide numerator and denominator by the highest power of x in the denominator to find the limit.

93) $\lim_{t \to \infty} \dfrac{\sqrt{9t^2 - 27}}{t - 3}$

93) ______

A) does not exist
B) 3
C) 27
D) 9

Answer: B

Explanation:
A)
B)
C)
D)

Use the table of values of f to estimate the limit.

94) Let $f(x) = x^2 - 5$, find $\lim_{x \to 0} f(x)$. 94) ______

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)						

A)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-2.9910	-2.9999	-3.0000	-3.0000	-2.9999	-2.9910

; limit = -3.0

B)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-1.4970	-1.4999	-1.5000	-1.5000	-1.4999	-1.4970

; limit = -15.0

C)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-1.4970	-1.4999	-1.5000	-1.5000	-1.4999	-1.4970

; limit = ∞

D)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-4.9900	-4.9999	-5.0000	-5.0000	-4.9999	-4.9900

; limit = -5.0

Answer: D

Explanation: A)
B)
C)
D)

Find the limit using $\lim_{x=0} \frac{\sin x}{x} = 1$.

95) $\lim_{x \to 0} \frac{\sin(\sin x)}{\sin x}$ 95) ______

A) 1 B) does not exist C) 0 D) -1

Answer: A

Explanation: A)
B)
C)
D)

Find the limit L for the given function f, the point x_0, and the positive number ε. Then find a number $\delta > 0$ such that, for all x, $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

96) $f(x) = \frac{x^2 + 4x + -21}{x + 7}$, $x_0 = -7$, $\varepsilon = 0.03$ 96) ______

A) L = 0; $\delta = 0.03$
B) L = 4; $\delta = 0.04$
C) L = -10; $\delta = 0.03$
D) L = -6; $\delta = 0.04$

Answer: C

Explanation: A)
B)
C)
D)

Find the intervals on which the function is continuous.

97) $y = \sqrt{x^2 - 2}$ 97) ______

A) continuous everywhere

B) continuous on the interval $[-\sqrt{2}, \sqrt{2}]$

C) continuous on the intervals $(-\infty, -\sqrt{2}]$ and $[\sqrt{2}, \infty)$

D) continuous on the interval $[\sqrt{2}, \infty)$

Answer: C

Explanation: A)

B)

C)

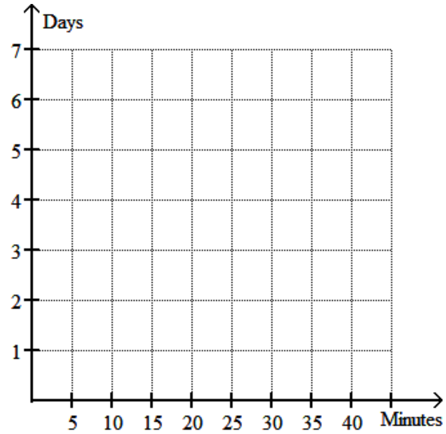
D)

Solve the problem.

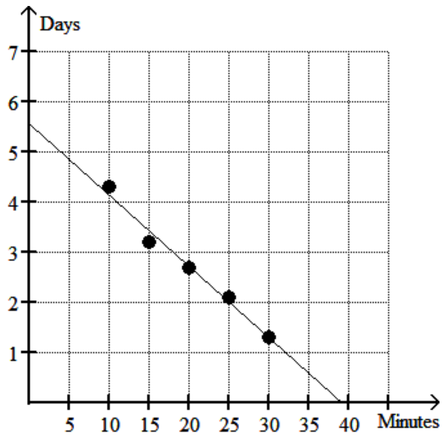
98) When exposed to ethylene gas, green bananas will ripen at an accelerated rate. The number of days for ripening becomes shorter for longer exposure times. Assume that the table below gives average ripening times of bananas for several different ethylene exposure times. 98) ______

Exposure time (minutes)	Ripening Time (days)
10	4.3
15	3.2
20	2.7
25	2.1
30	1.3

Plot the data and then find a line approximating the data. With the aid of this line, determine the rate of change of ripening time with respect to exposure time. Round your answer to two significant digits.

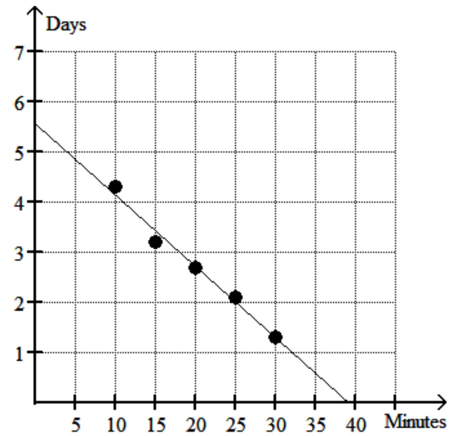


A)



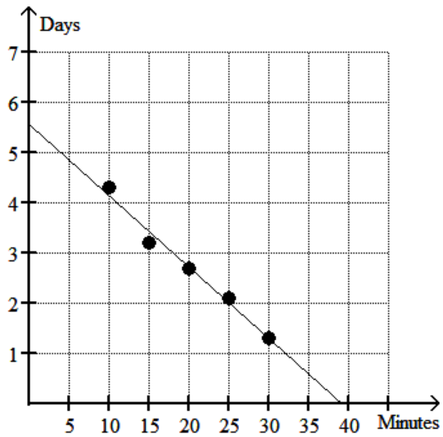
38 minutes

B)



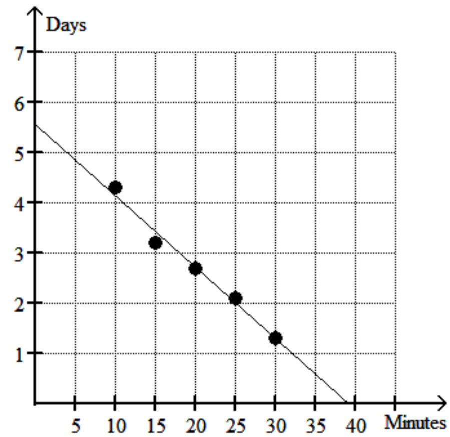
5.6 days

C)



-6.7 days per minute

D)



-0.14 day per minute

Answer: D

Explanation: A)
B)
C)
D)

Find the limit using $\lim\limits_{x=0} \frac{\sin x}{x} = 1$.

99) $\lim\limits_{x \to 0} \frac{\tan 4x}{x}$ 99) ______

A) does not exist B) 1 C) 4 D) $\frac{1}{4}$

Answer: C

Explanation: A)
B)
C)
D)

Find the limit, if it exists.

100) $\lim\limits_{x \to 0} \frac{x^3 - 6x + 8}{x - 2}$ 100) ______

A) Does not exist B) -4 C) 0 D) 4

Answer: B

Explanation: A)
B)
C)
D)

Find the limit using $\lim\limits_{x=0} \frac{\sin x}{x} = 1$.

101) $\lim\limits_{x \to 0} \frac{\sin 4x}{\sin 5x}$ 101) ______

A) 0 B) $\frac{5}{4}$ C) does not exist D) $\frac{4}{5}$

Answer: D

Explanation: A)
B)
C)
D)

Find the limit.

102) $\lim\limits_{x \to -2^+} \frac{1}{x + 2}$ 102) ______

A) 0 B) ∞ C) $-\infty$ D) -1

Answer: B

Explanation: A)
B)
C)
D)

103) $\lim\limits_{x \to \infty} \sqrt{x^2 + 12x} - x$ 103) ______

A) 12 B) ∞ C) 6 D) 0

Answer: C

Explanation: A)
B)
C)
D)

Provide an appropriate response.

104) If $\lim_{x \to 0^-} f(x) = 1$ and f(x) is an odd function, which of the following statements are true? 104) ______

I. $\lim_{x \to 0} f(x) = 1$

II. $\lim_{x \to 0^+} f(x) = -1$

III. $\lim_{x \to 0} f(x)$ does not exist.

A) I, II, and III B) I and III only C) I and II only D) II and III only

Answer: D

Explanation: A)
B)
C)
D)

Find the limit.

105) $\lim_{x \to 0} \frac{1}{x^{2/3}}$ 105) ______

A) $-\infty$ B) 0 C) ∞ D) 2/3

Answer: C

Explanation: A)
B)
C)
D)

Find the limit using $\lim_{x=0} \frac{\sin x}{x} = 1$.

106) $\lim_{x \to 0} 6x^2(\cot 3x)(\csc 2x)$ 106) ______

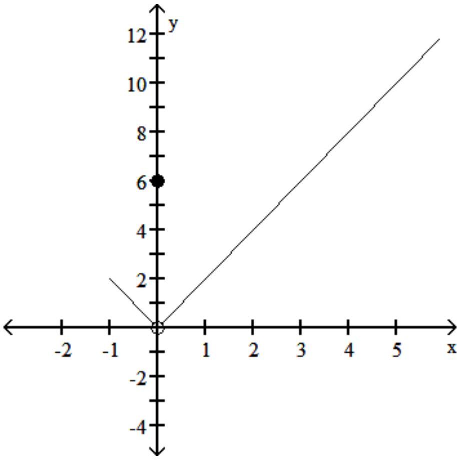
A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) does not exist D) 1

Answer: D

Explanation: A)
B)
C)
D)

Use the graph to estimate the specified limit.

107) Find $\lim_{x \to 0} f(x)$ 107) _____



A) does not exist B) 6 C) -1 D) 0

Answer: D

Explanation: A)
B)
C)
D)

Find the limit if it exists.

108) $\lim_{x \to 3} (x + 3128)^{3/5}$ 108) _____

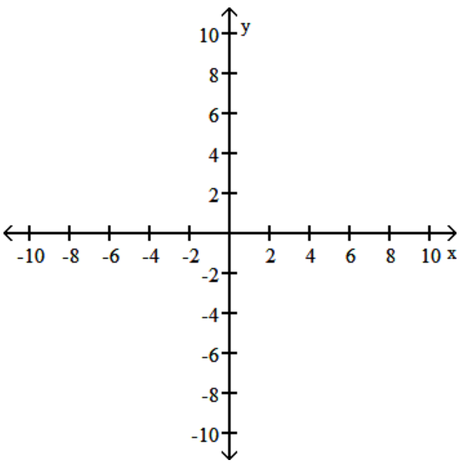
A) 25 B) 625 C) -125 D) 125

Answer: D

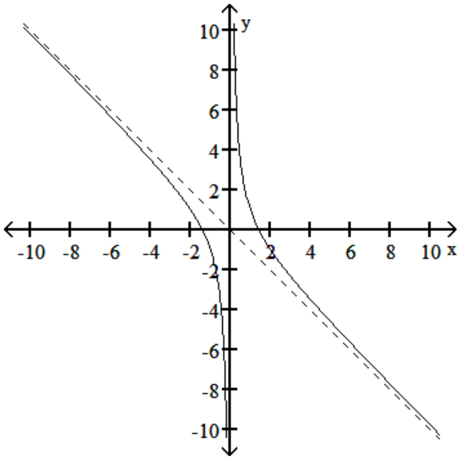
Explanation: A)
B)
C)
D)

Graph the rational function. Include the graphs and equations of the asymptotes.

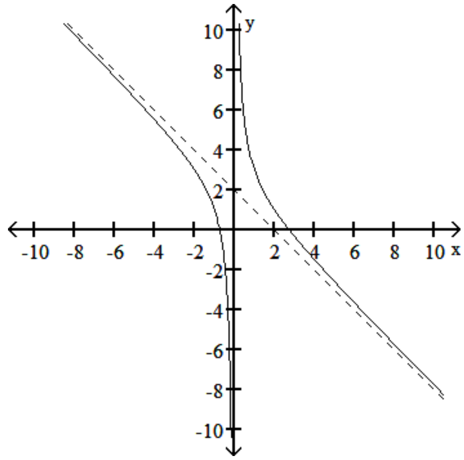
109) $f(x) = \dfrac{2 - 2x - x^2}{x}$ 109) _____



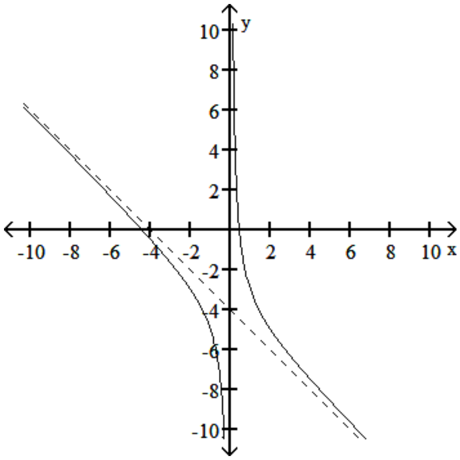
A) asymptotes: x = 0, y = -x



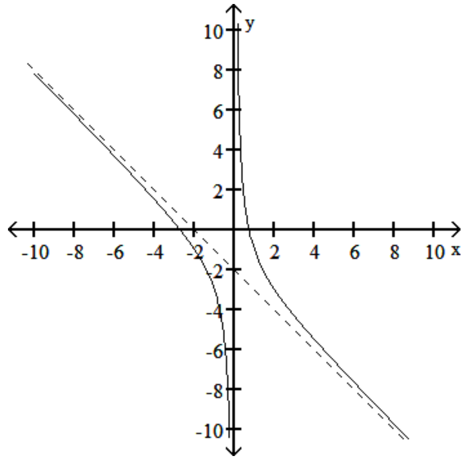
B) asymptotes: x = 0, y = -x + 2



C) asymptotes: x = 0, y = -x - 4



D) asymptotes: x = 0, y = -x - 2



Answer: D

Explanation: A)
B)
C)
D)

Use the table to estimate the rate of change of y at the specified value of x.

110) x = 1. 110) ______

x	y
0	0
0.2	0.01
0.4	0.04
0.6	0.09
0.8	0.16
1.0	0.25
1.2	0.36
1.4	0.49

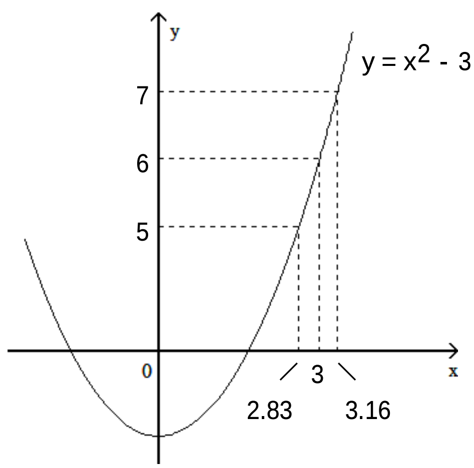
A) 0.5 B) 1 C) 2 D) 1.5

Answer: A

Explanation: A)
B)
C)
D)

Use the graph to find a $\delta > 0$ such that for all x, $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

111) 111) ______



$f(x) = x^2 - 3$
$x_0 = 3$
$L = 6$
$\varepsilon = 1$

NOT TO SCALE

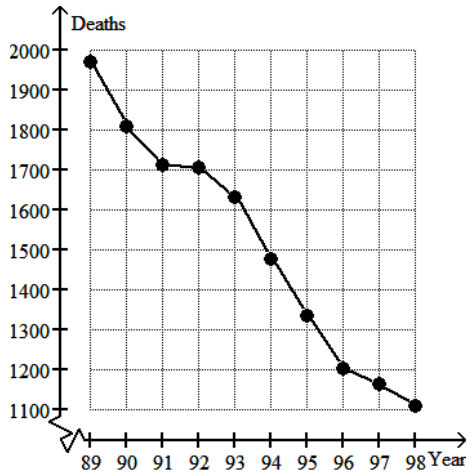
A) 0.16 B) 0.33 C) 0.17 D) 3

Answer: A

Explanation: A)
B)
C)
D)

Solve the problem.

112) The graph below shows the number of tuberculosis deaths in the United States from 1989 to 1998. 112) ______



Estimate the average rate of change in tuberculosis deaths from 1993 to 1995.

A) About -80 deaths per year
B) About -300 deaths per year
C) About -150 deaths per year
D) About -1 deaths per year

Answer: C

Explanation: A)
B)
C)
D)

Find the average rate of change of the function over the given interval.

113) $h(t) = \sin(4t), \left[0, \frac{\pi}{8}\right]$ 113) ______

A) $\frac{4}{\pi}$
B) $\frac{8}{\pi}$
C) $-\frac{8}{\pi}$
D) $\frac{\pi}{8}$

Answer: B

Explanation: A)
B)
C)
D)

Provide an appropriate response.

114) The inequality $1 - \frac{x^2}{2} < \frac{\sin x}{x} < 1$ holds when x is measured in radians and $|x| < 1$. 114) ______

Find $\lim_{x \to 0} \frac{\sin x}{x}$ if it exists.

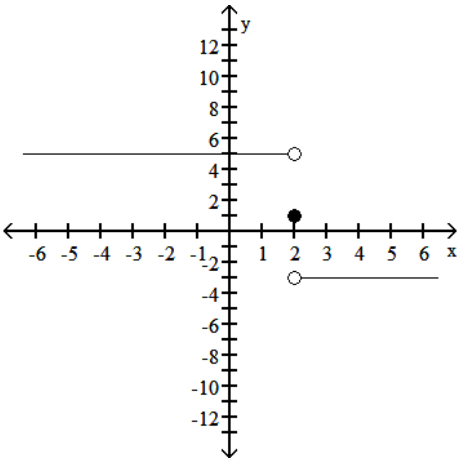
A) 1
B) does not exist
C) 0.0007
D) 0

Answer: A

Explanation: A)
B)
C)
D)

Use the graph to estimate the specified limit.

115) Find $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$ 115) ______



A) does not exist; does not exist B) 1; 1

C) -3; 5 D) 5; -3

Answer: D

Explanation: A)

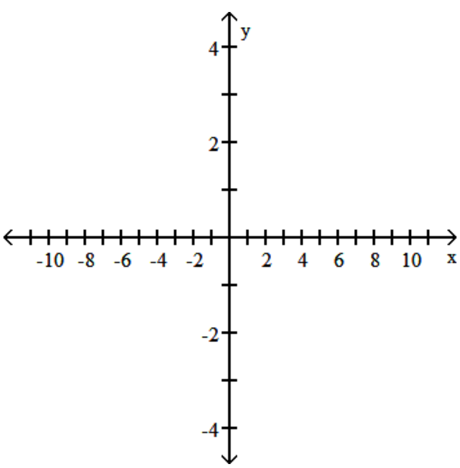
B)

C)

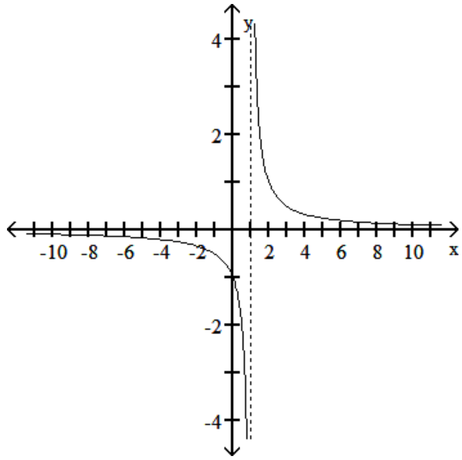
D)

Graph the rational function. Include the graphs and equations of the asymptotes.

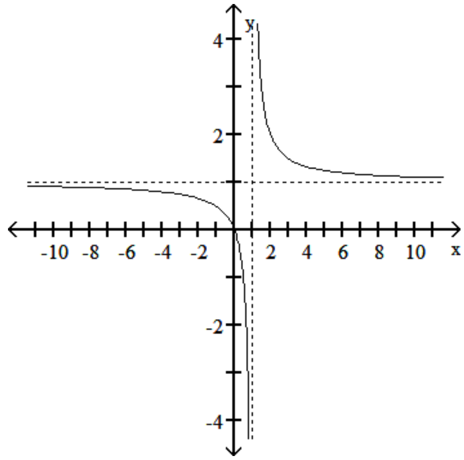
116) $f(x) = \frac{x}{x - 1}$ 116) ______



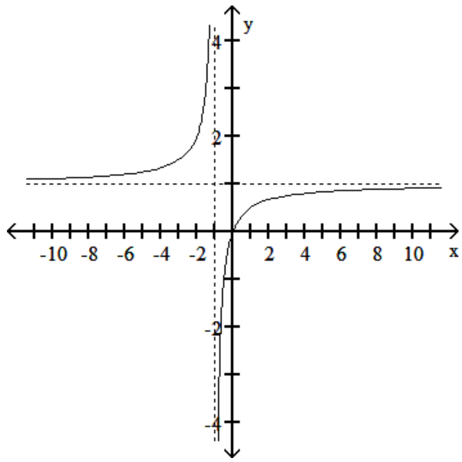
A) asymptotes: $x = 1$, $y = 0$



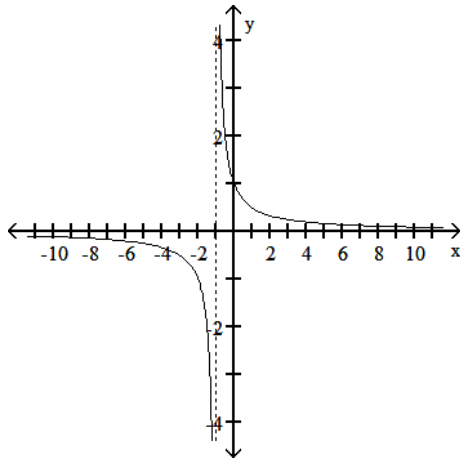
B) asymptotes: $x = 1$, $y = 1$



C) asymptotes: $x = -1$, $y = 1$



D) asymptotes: $x = -1$, $y = 0$



Answer: B

Explanation: A)
B)
C)
D)

Evaluate $\lim_{h \to 0} \dfrac{f(x_0 + h) - f(x_0)}{h}$ for the given x_0 and function f.

117) $f(x) = 4x + 5$ for $x_0 = 2$ 117) ______

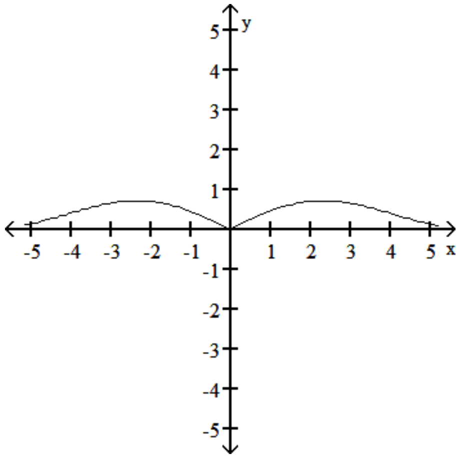
A) 4 B) Does not exist C) 13 D) 8

Answer: A

Explanation: A)
B)
C)
D)

For the function f whose graph is given, determine the limit.

118) Find $\lim_{x \to 0} f(x)$. 118) ______



A) -1 B) 0 C) 1 D) does not exist

Answer: B

Explanation: A)
B)
C)
D)

A function f(x), a point x_0, the limit of f(x) as x approaches x_0, and a positive number ε is given. Find a number $\delta > 0$ such that for all x, $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

119) $f(x) = 5x + 2$, $L = 17$, $x_0 = 3$, and $\varepsilon = 0.01$ 119) ______

A) 0.003333 B) 0.002 C) 0.004 D) 0.01

Answer: B

Explanation: A)
B)
C)
D)

Use the table to estimate the rate of change of y at the specified value of x.

120) x = 1. 120) ______

x	y
0.900	-0.05263
0.990	-0.00503
0.999	-0.0005
1.000	0.0000
1.001	0.0005
1.010	0.00498
1.100	0.04762

A) 0.5 B) 0 C) -0.5 D) 1

Answer: A

Explanation: A)
B)
C)
D)

Find the limit, if it exists.

121) $\lim_{x \to -6} \frac{x^2 + 10x + 24}{x + 6}$ 121) ______

A) Does not exist B) -2 C) 120 D) 10

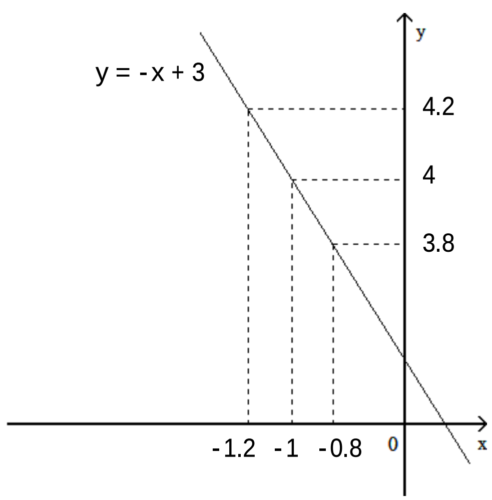
Answer: B

Explanation: A)
B)
C)
D)

Use the graph to find a $\delta > 0$ such that for all x, $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

122) 122) ______

$f(x) = -x + 3$
$x_0 = -1$
$L = 4$
$\varepsilon = 0.2$



NOT TO SCALE

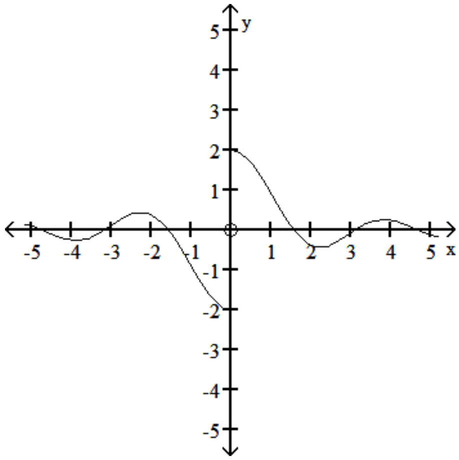
A) 0.2 B) 0.4 C) 5 D) -0.2

Answer: A

Explanation: A)
B)
C)
D)

For the function f whose graph is given, determine the limit.

123) Find $\lim_{x \to 0} f(x)$. 123) ______



A) -2 B) 0 C) does not exist D) 2

Answer: C

Explanation:
A)
B)
C)
D)

Give an appropriate answer.

124) Let $\lim_{x \to 5} f(x) = 7$ and $\lim_{x \to 5} g(x) = 10$. Find $\lim_{x \to 5} [f(x) \cdot g(x)]$. 124) ______

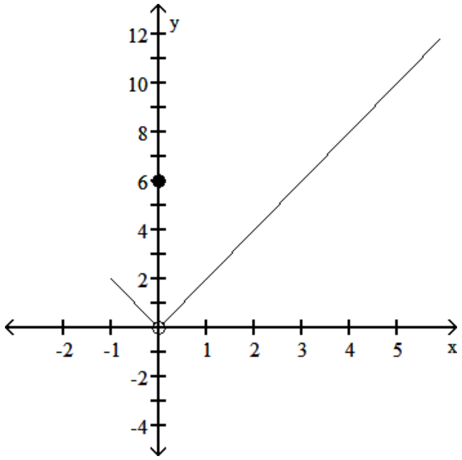
A) 5 B) 70 C) 10 D) 17

Answer: B

Explanation:
A)
B)
C)
D)

Use the graph to evaluate the limit.

125) $\lim_{x \to 0} f(x)$ 125) ______



A) does not exist B) -1 C) 0 D) 6

Answer: C

Explanation: A)
B)
C)
D)

Find the average rate of change of the function over the given interval.

126) $g(t) = 4 + \tan t$, $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ 126) ______

A) $\frac{4}{\pi}$ B) 0 C) $-\frac{4}{\pi}$ D) $-\frac{3}{2}$

Answer: A

Explanation: A)
B)
C)
D)

Find the limit and determine if the function is continuous at the point being approached.

127) $\lim_{x \to 2\pi} \sin\left(\frac{-3\pi}{2} \cos(\tan x)\right)$ 127) ______

A) 1; yes B) does not exist; yes
C) does not exist; no D) 1; no

Answer: A

Explanation: A)
B)
C)
D)

Find the limit.

128) $\lim_{x \to 0} (3 \sin x - 1)$ 128) ______

A) -1 B) 0 C) 3 D) 3 - 1

Answer: B

Explanation: A)
B)
C)
D)

129) $\lim_{x \to 0^+} (1 + \csc x)$ 129) ______

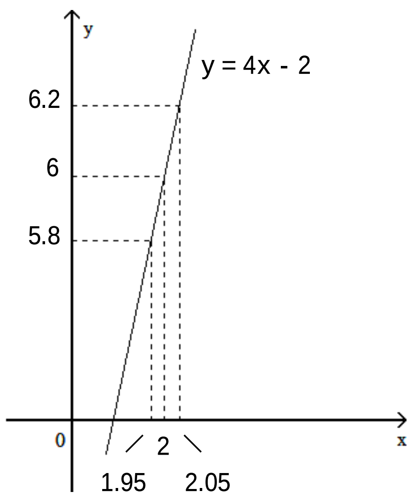
A) 0 B) 1 C) ∞ D) Does not exist

Answer: C

Explanation: A)
B)
C)
D)

Use the graph to find a $\delta > 0$ such that for all x, $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

130) 130) ______



$f(x) = 4x - 2$
$x_0 = 2$
$L = 6$
$\varepsilon = 0.2$

NOT TO SCALE

A) 4 B) 0.1 C) 0.5 D) 0.05

Answer: D

Explanation: A)
B)
C)
D)

Solve the problem.

131) Identify the incorrect statements about limits. 131) ______

I. The number L is the limit of f(x) as x approaches x_0 if f(x) gets closer to L as x approaches x_0.

II. The number L is the limit of f(x) as x approaches x_0 if, for any $\varepsilon > 0$, there corresponds a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - x_0| < \delta$.

III. The number L is the limit of f(x) as x approaches x_0 if, given any $\varepsilon > 0$, there exists a value of x for which $|f(x) - L| < \varepsilon$.

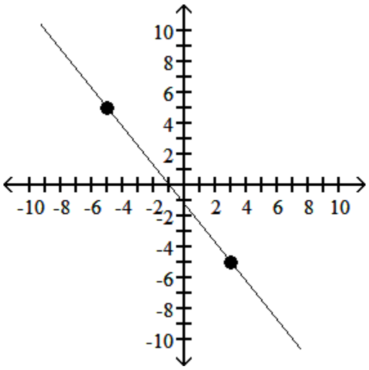
A) I and III B) II and III C) I and II D) I, II, and III

Answer: A

Explanation:
A)
B)
C)
D)

1) C
2) D
3) B
4) A
5) B
6) Let $f(x) = \frac{\sin(x - 7)}{(x - 7)}$ be defined for all $x \neq 7$. The function f is continuous for all $x \neq 7$. The function is not defined at $x = 7$ because division by zero is undefined; hence f is not continuous at $x = 7$. This discontinuity is removable because $\lim_{x \to 7} \frac{\sin(x - 7)}{x - 7} = 1$. (We can extend the function to $x = 7$ by defining its value to be 1.)
7) The Intermediate Value Theorem implies that there is at least one solution to $f(x) = 0$ on the interval $[-5, 3]$.

Possible graph:



8) Given $B > 0$, we want to find $\delta > 0$ such that $0 < |x - 0| < \delta$ implies $\frac{4}{|x|} > B$.

Now, $\frac{4}{|x|} > B$ if and only if $|x| < \frac{4}{B}$.
Thus, choosing $\delta = 4/B$ (or any smaller positive number), we see that
$|x| < \delta$ implies $\frac{4}{|x|} > \frac{4}{|\delta|} \geq B$.

Therefore, by definition $\lim_{x \to 0} \frac{4}{|x|} = \infty$

9) Answers may vary. One possibility: $\lim_{x \to 0} 1 - \frac{x^2}{6} = \lim_{x \to 0} 1 = 1$. According to the squeeze theorem, the function $\frac{x \sin(x)}{2 - 2\cos(x)}$, which is squeezed between $1 - \frac{x^2}{6}$ and 1, must also approach 1 as x approaches 0. Thus, $\lim_{x \to 0} \frac{x \sin(x)}{2 - 2\cos(x)} = 1$.

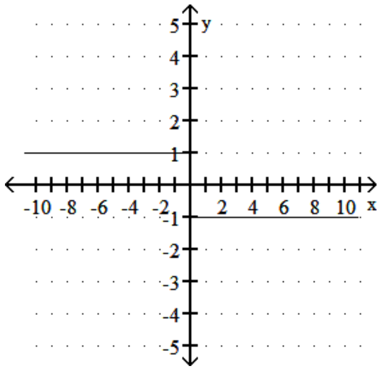
10) Let $f(x) = 10x^4 - 7x^3 - 4x - 10$ and let $y_0 = 0$. $f(-1) = 11$ and $f(0) = -10$. Since f is continuous on $[-1, 0]$ and since $y_0 = 0$ is between $f(-1)$ and $f(0)$, by the Intermediate Value Theorem, there exists a c in the interval $(-1, 0)$ with the property that $f(c) = 0$. Such a c is a solution to the equation $10x^4 - 7x^3 - 4x - 10 = 0$.

11)
Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon/5$. Then $0 < |x - 5| < \delta$ implies that

$$\begin{aligned}|(5x - 3) - 22| &= |5x - 25| \\ &= |5(x - 5)| \\ &= 5|x - 5| < 5\delta = \varepsilon\end{aligned}$$

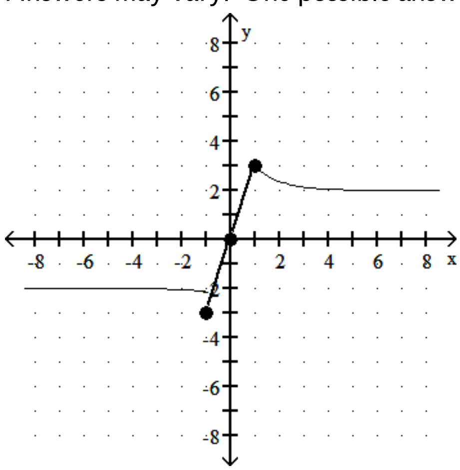
Thus, $0 < |x - 5| < \delta$ implies that $|(5x - 3) - 22| < \varepsilon$

12) (Answers may vary.) Possible answer: $f(x) = \begin{cases} 1, & x < 0 \\ -1, & x > 0 \end{cases}$



13) Let $f(x) = \dfrac{1}{(x - 6)^2}$, for all $x \neq 6$. The function f is continuous for all $x \neq 6$, and $\lim\limits_{x \to 6} \dfrac{1}{(x - 6)^2} = \infty$. As f is unbounded as x approaches 6, f is discontinuous at x = 6, and, moreover, this discontinuity is nonremovable.

14) Answers may vary. One possible answer:



15) Given $B > 0$, we want to find $\delta > 0$ such that $x_0 < x < x_0 + \delta$ implies $\dfrac{5}{x} > B$.

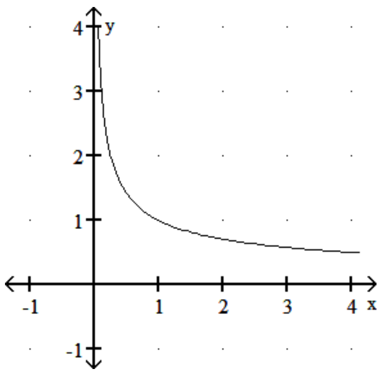
Now, $\dfrac{5}{x} > B$ if and only if $x < \dfrac{5}{B}$.

We know $x_0 = 0$. Thus, choosing $\delta = 5/B$ (or any smaller positive number), we see that

$x < \delta$ implies $\dfrac{5}{x} > \dfrac{5}{\delta} \geq B$.

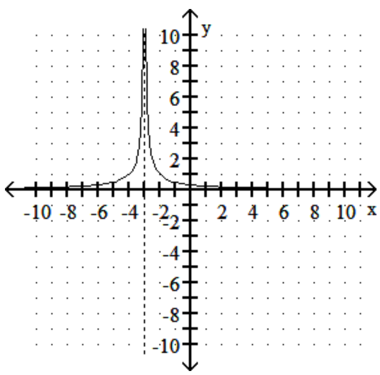
Therefore, by definition $\lim\limits_{x \to 0^+} \dfrac{5}{x} = \infty$

16) (Answers may vary.) Possible answer: $f(x) = \frac{1}{\sqrt{x}}$.

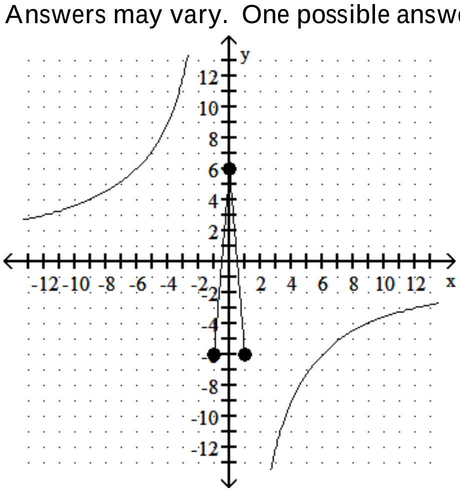


17) Yes, if $f(x) = 1$ and $g(x) = x - 2$, then $h(x) = \frac{1}{x - 2}$ is discontinuous at $x = 2$.

18) (Answers may vary.) Possible answer: $f(x) = \frac{1}{|x + 3|}$.



19) Answers may vary. One possible answer:



20) Let $f(x) = x(x - 3)^2$ and let $y_0 = 3$. $f(2) = 2$ and $f(4) = 4$. Since f is continuous on [2, 4] and since $y_0 = 3$ is between f(2) and f(4), by the Intermediate Value Theorem, there exists a c in the interval (2, 4) with the property that f(c) = 3. Such a c is a solution to the equation $x(x - 3)^2 = 3$.

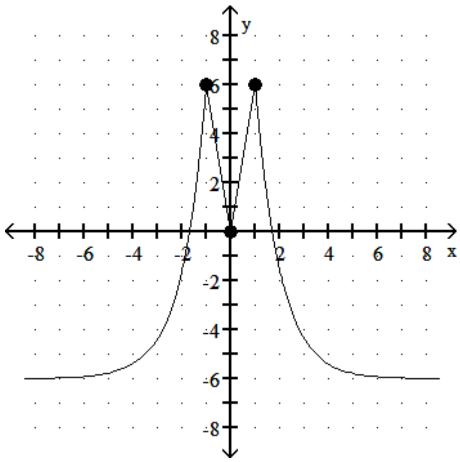
21) Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon/3$. Then $0 < |x - 6| < \delta$ implies that

$$\left|\frac{3x^2 - 14x - 24}{x - 6} - 22\right| = \left|\frac{(x - 6)(3x + 4)}{x - 6} - 22\right|$$
$$= |(3x + 4) - 22| \quad \text{for } x \neq 6$$
$$= |3x - 18|$$
$$= |3(x - 6)|$$
$$= 3|x - 6| < 3\delta = \varepsilon$$

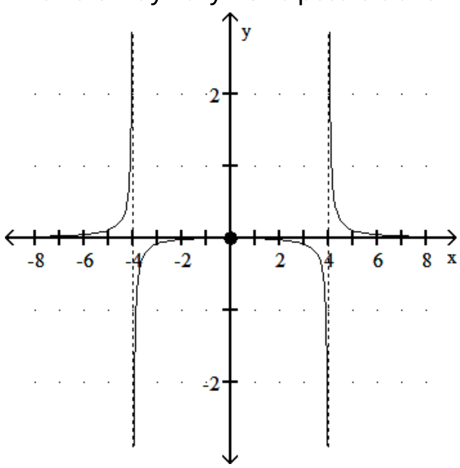
Thus, $0 < |x - 6| < \delta$ implies that $\left|\frac{3x^2 - 14x - 24}{x - 6} - 22\right| < \varepsilon$

22) Notice that f(0) = 5 and f(1) = 2. As f is continuous on [0,1], the Intermediate Value Theorem implies that there is a number c such that f(c) = π.

23) Answers may vary. One possible answer:



24) Answers may vary. One possible answer:



25) Let $f(x) = 3x^3 - 7x^2 - 9x + 6$ and let $y_0 = 0$. f(3) = -3 and f(4) = 50. Since f is continuous on [3, 4] and since $y_0 = 0$ is between f(3) and f(4), by the Intermediate Value Theorem, there exists a c in the interval (3 , 4) with the property that f(c) = 0. Such a c is a solution to the equation $3x^3 - 7x^2 - 9x + 6 = 0$.

26) Let $f(x) = \frac{\sin x}{x}$ and let $y_0 = \frac{1}{4}$. $f\left(\frac{\pi}{2}\right) \approx 0.6366$ and $f(\pi) = 0$. Since f is continuous on $\left[\frac{\pi}{2}, \pi\right]$ and since $y_0 = \frac{1}{4}$ is between $f\left(\frac{\pi}{2}\right)$ and $f(\pi)$, by the Intermediate Value Theorem, there exists a c in the interval $\left(\frac{\pi}{2}, \pi\right)$, with the property that $f(c) = \frac{1}{4}$. Such a c is a solution to the equation 4 sin x = x.

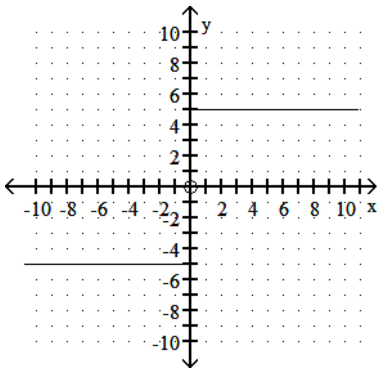
27) Let $\varepsilon > 0$ be given. Choose $\delta = \min\{5/2, 25\varepsilon/2\}$. Then $0 < |x - 5| < \delta$ implies that

$$\left|\frac{1}{x} - \frac{1}{5}\right| = \left|\frac{5 - x}{5x}\right|$$
$$= \frac{1}{|x|} \cdot \frac{1}{5} \cdot |x - 5|$$
$$< \frac{1}{5/2} \cdot \frac{1}{5} \cdot \frac{25\varepsilon}{2} = \varepsilon$$

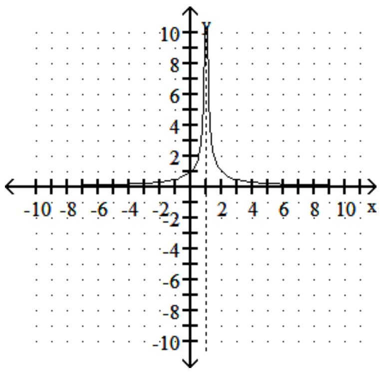
Thus, $0 < |x - 5| < \delta$ implies that $\left|\frac{1}{x} - \frac{1}{5}\right| < \varepsilon$

28) The roots of f(x) are the solutions to the equation f(x) = 0. Statement (b) is asking for the solution to the equation $4x^3 = 1x + 5$. Statement (d) is asking for the solution to the equation $4x^3 - 1x = 5$. These three equations are equivalent to the equations in statements (c) and (e). As five equations are equivalent, their solutions are the same.

29) (Answers may vary.) Possible answer: $f(x) = \begin{cases} 5, & x > 0 \\ -5, & x < 0 \end{cases}$



30) (Answers may vary.) Possible answer: $f(x) = \frac{1}{|x - 1|}$.



31) Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon$. Then $0 < |x - 2| < \delta$ implies that

$$\left|\frac{x^2 - 4}{x - 2} - 4\right| = \left|\frac{(x - 2)(x + 2)}{x - 2} - 4\right|$$
$$= |(x + 2) - 4| \quad \text{for } x \neq 2$$
$$= |x - 2| < \delta = \varepsilon$$

Thus, $0 < |x - 2| < \delta$ implies that $\left|\frac{x^2 - 4}{x - 2} - 4\right| < \varepsilon$

32) B
33) A

34) B
35) A
36) B
37) C
38) A
39) C
40) D
41) C
42) A
43) C
44) D
45) D
46) D
47) B
48) C
49) C
50) B
51) B
52) B
53) B
54) D
55) B
56) C
57) B
58) A
59) D
60) C
61) B
62) A
63) A
64) B
65) C
66) C
67) A
68) C
69) C
70) A
71) D
72) A
73) D
74) B
75) D
76) C
77) D
78) C
79) B
80) C
81) C
82) B
83) A

84) C
85) B
86) D
87) D
88) B
89) B
90) A
91) B
92) C
93) B
94) D
95) A
96) C
97) C
98) D
99) C
100) B
101) D
102) B
103) C
104) D
105) C
106) D
107) D
108) D
109) D
110) A
111) A
112) C
113) B
114) A
115) D
116) B
117) A
118) B
119) B
120) A
121) B
122) A
123) C
124) B
125) C
126) A
127) A
128) B
129) C
130) D
131) A