Chapter 2
Applying Time Value Concepts

■ Chapter Overview

Albert Einstein, the renowned physicist whose theories of relativity formed the theoretical base for the utilization of atomic energy, called the time value of money principle one of the strongest forces on earth. Chapter 2 discusses the importance of the time value of money. The concepts of simple and compound interest are introduced in the chapter. Simple interest refers to interest on a loan computed as a percentage of the loan amount. Compound interest refers to the process of earning interest on interest.

In addition, chapter 2 also discusses the time value of money as it is applied to two types of cash flows: a single dollar amount (or lump sum) and an annuity. An annuity is a stream of equal payments paid over equal intervals of time. The use of present and future value tables and formulas to aid calculations is explained in the chapter. In addition, the chapter explains how to use a financial calculator to make time value calculations. Example calculations show the inputs required using the TI BA II Plus calculator.

In discussing the future and present value of an annuity, the chapter differentiates between an ordinary annuity, for which payments occur at the end of the period, and an annuity due, for which payments occur at the beginning of the period. Annuities are illustrated through the use of timelines. As with the single dollar calculations, present and future value of an annuity tables are provided within the chapter, as are instructions for using a financial calculator. Throughout the chapter, practical uses for each type of calculation are described.

The chapter concludes with a discussion on how to convert a nominal interest rate to an effective interest rate and vice versa. The nominal interest rate is the stated, or quoted, rate of interest. It is the rate of interest that is used in time value of money calculations. The effective interest rate is the actual rate of interest that you earn, or pay, over a period of time. Effective interest rates can be compared with each other; whereas nominal interest rates cannot be directly compared in situations where the compounding period between interest rates is different.

■ Chapter Objectives

The objectives of this chapter are to:

- calculate the future value of a dollar amount that you save today,
- calculate the present value of a dollar amount that will be received in the future,
- calculate the future value of an annuity, and
- calculate the present value of an annuity.

■ Teaching Tips

1. The classic example of the power of compound interest is to ask students whether they would rather have $500,000 right now or one cent that you would double each day for the next thirty days. Many students will choose the $500,000 at first. However, the one cent would grow to $10,737,417.65 at the end of thirty days. While this represents a 100% daily interest rate which cannot be obtained, it is a powerful example. This chapter concerns the "time" value of money and it is a good idea to emphasize time over deposit amounts or rates of return. Have the students calculate, either using the tables or with a financial calculator, a single sum at a given interest rate, changing only the length of time of the investment. For instance, a one-time $5,000 investment at 10% compounded annually would return $87,247 after 30 years, but only $54,174 after 25 years. Only 5 years difference in time amounts to a difference of $33,073, or an average loss of $6,614 per year. After 20 years the investment would return only $33, 637, and after 10 years it would return only $12,969. Emphasize that all of this is based on the initial amount of $5,000 with no additional investment by the investor.

2. It is sometimes said that those who understand compound interest collect it while those who don't understand it pay it. Discuss the fact that while compound interest works to our advantage when we save and invest, it works to our detriment when we are in debt. Suppose you bought a $2,000 home entertainment system on a credit card that charges19.99% annual interest, compounded daily. Using minimum payments of 3% of the outstanding balance for each month, it will take 15 years and 3 months to pay off the debt. You would have paid $2,238.13 in interest, making the total payments $4,238.13, the true cost of the entertainment system when purchased on credit. The moral of this example is to never pay minimum payments on high interest credit cards. Paying $60 a month would pay off the same debt in 4 years and 2 months, and reduce the interest paid to $942.82. Paying $100 a month would pay the debt off in 2 years 1 month, and would cost only $452.92 in interest. Make the calculations before making the purchase to ensure that you can make payments that will minimize the amount of interest you will pay. <u>Note to instructor:</u> Calculations for this teaching tip were made using Credit Canada's debt calculator available at *http://www.creditcanada.com/debtCalc.asp*.

3. Demonstrate the difference between simple and compound interest. If you deposit $2,000 per year for 40 years and earn 10% compounded annually, but withdraw the interest and spend it, the $2,000 deposit annually would be worth $80,000 in 40 years. By allowing the interest to compound with the deposits, the investment would be worth $885,185.

4. Online/Team exercise—review of TVM problems. Generally, students will have a variety of backgrounds on this topic. The use of a team exercise gives those with some expertise a chance to help those with little or no background. It is a good review for those with the expertise and makes the others more comfortable with peer help.

■ Answers to End-of-Chapter Review Questions

1. The time value of money is a powerful principle that can be used to explain how money grows over time. When you spend money, you incur an opportunity cost of what you could have done with that money had you not spent it. For example, if you spent $2000 on a vacation rather than saving it, you would have incurred an opportunity cost of the alternative ways that you could have used the money. You can use the time value of money to compute the actual cost of the opportunity.

2. Interest is the rent charged for the use of money. Depending on whether you have borrowed or loaned money, you will either pay or receive interest, respectively. Simple interest is interest on a

loan computed as a percentage of the loan amount, or principal. The interest earned or paid is not reinvested. Simple interest is measured by multiplying the principal, the interest rate applied to the principal, and the loan's time to maturity (in years). Compound interest refers to the process of earning interest on interest.

3. For simple problems a time value of money table may be used to calculate the future or present value of a single dollar amount. Other methods that may be used to solve time value of money problems include time value of money formulas and financial calculators.

4. The time value of money is most commonly applied to two types of cash flows: a single dollar amount (also referred to as a lump sum) and an annuity. An annuity refers to the payment of a series of equal cash flow payments at equal intervals of time.

5. The inputs required when calculating the future value, FV, of a single dollar amount using a formula are the present future value of an investment (PV), the annual interest rate, i, (expressed as a decimal), the number of compounding periods per year (n), and time, t, (in years).

6. The future value interest factor (*FVIF*), is a factor multiplied by today's savings to determine how the savings will accumulate over time. The factor is determined based on an annual interest rate where the number of compounding periods is one. The formula for determining the future value of a single dollar amount when using the future value interest factor is:

$$FV = PV \times FVIF_{i,n}$$

In order to find the correct future value interest factor, you must know the interest rate and the number of years the money is invested.

7. Clear the existing TVM values in the calculator's TVM worksheet by entering 2ND CLR TVM.

8. A cash inflow (for example, income received from an investment) should be entered as a positive number. A cash outflow (for example, an investment amount) should be entered as a negative number. The +/- key on the TI BA II Plus is used to convert a positive number to a negative number, and vice versa.

9. There are 12 compounding periods in a year when an investment compounds interest monthly. An investment that compounds interest quarterly has 4 compounding periods. An investment that compounds interest daily has 365 compounding periods.

10. Discounting is the process of obtaining present values.

11. Suppose you need $20 000 to purchase a car in 3 years. You may want to determine how much money you need to invest today to achieve the $20 000 in three years. Another instance where determining the present value is useful would be if you want to pay off a loan today that will, for example, be paid over 3 years. In this case, you want to know the present value of these future payments.

12. The formula for the present value of a single dollar amount is:

$$PV = \frac{FV}{(1+\frac{i}{n})^{nt}}$$

13. The present value interest factor is a factor multiplied by the future value to determine the present value of that amount. The formula for determining the present value of a single dollar amount when using the present value interest factor is:

$$PV = FV \times PVIF_{i,n}$$

14. An annuity refers to the payment of a series of equal cash flow payments at equal intervals of time. An ordinary annuity is a stream of equal payments that are received or paid at equal intervals in time at the end of a period. An annuity due is a series of equal cash flow payments that occur at the beginning of each period. Thus, an annuity due differs from an ordinary annuity in that the payments occur at the beginning instead of the end of the period. The most important thing to note about an annuity is that if the payment changes over time, the payment stream does not reflect an annuity.

15. The formula used to determine the future value of an annuity is:

$$FV = PMT \text{ x } [\frac{(1+i)^n - 1}{i}]$$

16. The future value interest factor for an annuity, FVIFA, is a factor multiplied by the periodic savings level (annuity) to determine how the savings will accumulate over time. The formula for the future value interest factor for an annuity, when using a table, is:

$$FVA = PMT \times FVIFA_{i,n}$$

17. An annuity formula or table will provide the future value for an ordinary annuity. In order to adjust your calculation for an annuity due, you would multiply the annuity payment generated by multiplying the value from the table by $(1 + i)$.

18. The formula used to determine the present value of an annuity is:

$$PV = PMT \text{ x } \left[\frac{1 - [\frac{1}{(1+i)^n}]}{i}\right]$$

19. The present value interest factor for an annuity, PVIFA, is a factor multiplied by a periodic savings level (annuity) to determine the present value of the annuity. The formula for the present value interest factor for an annuity, when using a table, is:

$$PVA = PMT \times PVIFA_{i,n}$$

20. 60.

21. The nominal interest rate is the stated, or quoted, rate of interest. It is also known as the annual percentage rate (APR). The effective interest rate is the actual rate of interest that you earn, or pay, over a period of time. It is also known as the effective yield (EY). When comparing two or more interest rates, the nominal interest rate is not useful because it does not take into account the effect of compounding. In order to make objective investment decisions regarding loan costs or investment returns over different compounding frequencies, the effective interest rate has to be determined. The effective interest rate allows for the comparison of two or more interest rates because it reflects the effect of compound interest.

22. The present value of a single sum

23. The future value of an annuity

24. The future value of a single sum

25. The present value of an annuity

■ Answers to Financial Planning Problems

1. P/Y = 1, C/Y = 1, N = 5, I/Y = 4, PV = -1000, PMT = 0, **FV = ?**

   Rodney will have $1,216.65 in five years to put down on his car.

2. P/Y = 12, C/Y = 1, N = 60, I/Y = 3, PV = 0, PMT = -50, **FV = ?**

   Michelle's balance in five years will be $3,229.05.

3. Jessica: P/Y = 1, C/Y = 1, N = 10, I/Y = 10, PV = 0, PMT = -2000, **FV = ?**

   Jessica will have $31,874.85 after 10 years.

   Jessica: P/Y = 1, C/Y = 1, N = 30, I/Y = 10, PV = 31874.85, PMT = 0, **FV = ?**

   Jessica will have $556,197.07 at retirement. She contributed $20,000 in total.

   Joshua: P/Y = 1, C/Y = 1, N = 30, I/Y = 10, PV = 0, PMT = -2000, **FV = ?**

   Joshua will have $328,988.05 at retirement. He contributed $60,000 in total.

4. P/Y = 1, C/Y = 12, N = 3, I/Y = 4, **PV = ?**, PMT = 0, FV = 2000

   Cheryl must deposit $1,774.19 now in order to have the money she needs in three years.

5. P/Y = 12, C/Y = 2, N = 4, I/Y = 8, PV = 0, **PMT = ?**, FV = 7000

   Amy and Vince must save $124.56 each month to have the money they need.

6. P/Y = 12, C/Y = 1, N = 300, I/Y = 12, PV = 0, PMT = -400, **FV = ?**

   Judith's employer contributes $200 per month.
   She will have $674,482.60 in her retirement plan at retirement

7. P/Y = 12, C/Y = 4, N = 360, I/Y = 11, PV = 0, PMT = -300, **FV = ?**

   Stacey will not reach her retirement goal since she will only be able to accumulate $823,358.63 by the time she retires in 30 years.

8. P/Y = 1, C/Y = 1, N = 18, I/Y = 7, **PV = ?**, PMT = 0, FV = 10000

   Juan must deposit $2,958.64 now in order to achieve his goal.

9. P/Y = 1, C/Y = 12, N = 20, I/Y = 4, PV = 0, PMT = -100, **FV = ?**

   $3,000.82 will be in the account in 20 years.

10. P/Y = 1, C/Y = 4, N = 3, I/Y = 9, PV = -3000, PMT = 0, **FV = ?**

    Earl will have $3,918.15 to spend on his trip to Belize.

11. Lump-sum payment: P/Y = 1, C/Y = 12, N = 20, I/Y = 8, PV = 312950, PMT = 0, **FV = ?**

    The lump-sum payment will be worth $1,541,842.93 after 20 years.

    Annual payment: P/Y = 1, C/Y = 1, N = 20, I/Y = 6, PV = 0, PMT = 50000, **FV = ?**

    The annual payment will be worth $1,839,279.56 after 20 years.
    Jesse should choose the annual payment.

12. P/Y = 1, C/Y = 4, N = 20, I/Y = 7, **PV = ?**, PMT = 0, FV = 6000000

    The cash option payout would be $1,497,607.85.

13. P/Y = 52, C/Y = 12, N = 260, I/Y = 10, PV = 0, PMT = -10, **FV = ?**

    She will have $3,366.34 in five years.

14. Invest It: P/Y = 1, C/Y = 4, N = 3, I/Y = 5, PV = -1000, PMT = 0, **FV = ?**

    Investing the income tax refund will give him $1,160.75 at the end of three years.

    Purchase Stereo: P/Y = 12, C/Y = 365, N = 36, I/Y = 4, PV = 0, PMT = -30, **FV = ?**

    Purchasing the stereo and investing $30 per month will give him $1,145.56 at the end of three years.

15. P/Y = 12, C/Y = 52, N = 36, I/Y = 10, PV = 0, PMT = -75, **FV = ?**

    You will have $3,135.17.

16. The equivalent effective interest rate is 8.71%.

■ Suggested answers to Ethical Dilemma questions

(a) This question will hopefully spark a lively discussion between those students who believe that a salesperson's first obligation is to sell products or services and those students who believe that a salesperson's first obligation is to assist the customer.

(b) Two hundred dollars per month at 6 percent compounded annually will grow to $194,902.59 in 30 years. Two hundred and forty dollars per month at 6 percent compounded annually will grow to $162,309.35 in 25 years. Therefore, Herb is incorrect in his calculation.

■ Answers to Questions in The Sampson Family: A Continuing Case

1. *Savings Accumulated Over the Next 12 Years (Based on Plan to Save $300 per month)*

Amount Saved Per Month	$300	$300
Interest Rate	5%	7%
Years	12	12
Future Value of Savings	$59,029	$67,408

Savings Accumulated Over the Next 12 Years (Based on Plan to Save $400 per month)

Amount Saved Per Year	$400	$400
Interest Rate	5%	7%
Years	12	12
Future Value of Savings	$78,705	$89,878

2. If the Sampsons save $300 per month, the higher interest rate would result in an extra accumulation in savings of more than $8,000. If they save $400 per month, the higher interest rate would result in an extra accumulation in savings of more than $11,000 per year.

3. Using a 5% interest rate, saving $400 per month instead of $300 would increase their total savings by more than $19,000. Using a 7% interest rate, saving $400 per month instead of $300 would increase their total savings by more than $22,000.

4. To achieve a goal of $70,000 over 12 years, they would need to save an annual amount (annuity) as determined below:

P/Y = 1, C/Y = 12, N = 12, I/Y = 5, PV = 0, **PMT = ?**, FV = 70000

Thus, they would have to invest $4,368 by the end of each year to accumulate $70,000 in twelve years.

■ Answers to Myth or Fact Margin Questions

<table>
<tr><th>Page</th><th>Myth or Fact</th></tr>
<tr><td>23</td><td>The interest rate that you are quoted on an investment or loan represents the amount of interest that you will earn or pay.</td></tr>
<tr><td colspan="2">Myth. The interest rate quoted, i.e. the nominal rate of interest, may not be the same as the interest earned or paid, i.e. the effective or real rate of interest. It is important to know the number of compounding periods associated with a loan or investment.</td></tr>
<tr><td>31</td><td>All financial calculators calculate the time value of money in the same manner.</td></tr>
<tr><td colspan="2">Myth. Financial calculators produced by different manufacturers will involve different steps when performing a time value of money calculation.</td></tr>
<tr><td>33</td><td>Future value interest factors (FVIF) and a financial calculator will generate different answers to a question.</td></tr>
<tr><td colspan="2">Fact. Due to rounding error, each method will provide a slightly different answer.</td></tr>
</table>