CHAPTER 2 UNDERSTANDING VARIABLES AND SOLVING EQUATIONS

2.1 Introduction to Variables

2.1 Margin Exercises

1. $c + \underline{15}$

The expression is $\underline{c + 15}$.

The variable is $\underline{c}$. It represents the class limit.

The constant is $\underline{15}$.

2. **(a)** Evaluate the expression $c + 3$ when c is 25.

$$\begin{array}{ll} c+3 & \textit{Replace c with 25.} \\ \underbrace{\underline{25}+3} & \\ 28 & \end{array}$$

Order $\underline{28}$ books.

(b) Evaluate the expression $c + 3$ when c is 60.

$$\begin{array}{ll} c+3 & \textit{Replace c with 60.} \\ \underbrace{\underline{60}+3} & \\ 63 & \end{array}$$

Order 63 books.

3. **(a)** Evaluate the expression $4s$ when s is 3 feet.

$$\begin{array}{ll} 4s & \textit{Replace s with 3 feet.} \\ 4 \cdot \underline{3 \text{ feet}} & \\ \underline{12 \text{ feet}} & \end{array}$$

The perimeter of the square table is 12 feet.

(b) Evaluate the expression $4s$ when s is 7 miles.

$$\begin{array}{ll} 4s & \textit{Replace s with 7 miles.} \\ 4 \cdot 7 \text{ miles} & \\ 28 \text{ miles} & \end{array}$$

The perimeter of the square park is 28 miles.

4. Evaluate the expression $100 + \frac{a}{2}$ when a is 40.

$$\begin{array}{ll} 100 + \frac{a}{2} & \textit{Replace a with 40.} \\ 100 + \frac{40}{2} & \textit{Divide.} \\ 100 + 20 & \textit{Add.} \\ 120 & \end{array}$$

The approximate systolic blood pressure is 120.

5. **(a)** Evaluate the expression $\frac{t}{g}$ when t is 532 and g is 4.

$$\begin{array}{ll} \frac{t}{g} & \textit{Replace t with } \underline{532} \textit{ and g with } \underline{4}. \\ \frac{532}{4} & \textit{Divide.} \\ 133 & \end{array}$$

Your average score is $\underline{133}$.

(b)

Value of x	Value of y	Expression $x - y$
16	10	$16 - 10$ is 6
100	5	$100 - 5$ is 95
3	7	$3 - 7$ is -4
8	0	$8 - 0$ is 8

6. **(a)** Multiplying any number a by 0 gives a product of 0.

$$\begin{array}{ccccc} \text{Any number} & \text{times} & \text{zero} & & \\ \downarrow & \downarrow & \downarrow & & \\ a & \cdot & \underline{0} & = & \underline{0} \end{array}$$

(b) Changing the grouping of addends (a, b, c) does not change the sum.

$$(a + b) + c = a + (b + c)$$

7. **(a)** x^5 can be written as $\underbrace{x \cdot \underline{x} \cdot \underline{x} \cdot \underline{x} \cdot \underline{x}}_{\underline{x \text{ is used as a factor 5 times.}}}$

(b) $4a^2b^2$ can be written as $4 \cdot \underline{a} \cdot \underline{a} \cdot \underline{b} \cdot \underline{b}$

(c) $-10xy^3$ can be written as $-10 \cdot x \cdot y \cdot y \cdot y$

(d) s^4tu^2 can be written as $s \cdot s \cdot s \cdot s \cdot t \cdot u \cdot u$

8. **(a)** y^3 means

$$\begin{array}{ll} y \cdot y \cdot y & \textit{Replace y with } -5. \\ \underbrace{-5 \cdot \underline{(-5)}} \cdot (-5) & \textit{Multiply left to right.} \\ \underbrace{\underline{25} \cdot (-5)} & \\ \underline{-125} & \end{array}$$

(b) r^2s^2 means

$$\begin{array}{ll} r \cdot r \cdot s \cdot s & \textit{Replace r with 6 and s with 3.} \\ \underbrace{6 \cdot \underline{6}} \cdot 3 \cdot \underline{3} & \textit{Multiply left to right.} \\ \underbrace{\underline{36} \cdot 3} \cdot 3 & \\ \underbrace{\underline{108} \cdot 3} & \\ 324 & \end{array}$$

(c) $10xy^2$ means

$$\begin{array}{ll} 10 \cdot x \cdot y \cdot y & \textit{Replace x with 4 and y with } -3. \\ \underbrace{10 \cdot \underline{4}} \cdot (-3) \cdot (-3) & \textit{Multiply left to right.} \\ \underbrace{\underline{40} \cdot (-3)} \cdot (-3) & \\ \underbrace{\underline{-120} \cdot (-3)} & \\ 360 & \end{array}$$

(d) $-3c^4$ means

$$\begin{array}{ll} -3 \cdot c \cdot c \cdot c \cdot c & \textit{Replace c with 2.} \\ \underbrace{-3 \cdot \underline{2}} \cdot 2 \cdot 2 \cdot 2 & \textit{Multiply left to right.} \\ \underbrace{\underline{-6} \cdot 2} \cdot 2 \cdot 2 & \\ \underbrace{\underline{-12} \cdot 2} \cdot 2 & \\ \underbrace{\underline{-24} \cdot 2} & \\ -48 & \end{array}$$

2.1 Section Exercises

1. $c+4$ c is the variable;
4 is the constant.

2. $d+6$ d is the variable;
6 is the constant.

3. $-3+m$ m is the variable;
-3 is the constant.

4. $-4+n$ n is the variable;
-4 is the constant.

5. $5h$ h is the variable;
5 is the coefficient.

6. $3s$ s is the variable;
3 is the coefficient.

7. $2c-10$ c is the variable;
2 is the coefficient.
10 is the constant.

8. $6b-1$ b is the variable;
6 is the coefficient.
1 is the constant.

9. $x-y$ ▪ Both x and y are variables.

10. xy ▪ Both x and y are variables.

11. $-6g+9$ g is the variable;
-6 is the coefficient;
9 is the constant.

12. $-10k+15$ k is the variable;
-10 is the coefficient;
15 is the constant.

13. Expression (rule) for ordering robes: $g+10$

(a) Evaluate the expression when there are 654 graduates.

$g+10$ *Replace g with 654.*
$\underbrace{654+10}$ *Follow the rule and add.*
664 robes must be ordered.

(b) Evaluate the expression when there are 208 graduates.

$g+10$ *Replace g with 208.*
$\underbrace{208+10}$ *Follow the rule and add.*
218 robes must be ordered.

(c) Evaluate the expression when there are 95 graduates.

$g+10$ *Replace g with 95.*
$\underbrace{95+10}$ *Follow the rule and add.*
105 robes must be ordered.

14. Expression (rule) for degrees: $c+37$

(a) $45+37$ is 82 degrees.

(b) $33+37$ is 70 degrees.

(c) $58+37$ is 95 degrees.

15. Expression (rule) for finding perimeter of an equilateral triangle of side length s: $3s$

(a) Evaluate the expression when s, the side length, is 11 inches.

$3s$ *Replace s with 11.*
$\underbrace{3 \cdot 11}$ *Follow the rule and multiply.*
33 inches is the perimeter.

(b) Evaluate the expression when s, the side length, is 3 feet.

$3s$ *Replace s with 3.*
$\underbrace{3 \cdot 3}$ *Follow the rule and multiply.*
9 feet is the perimeter.

16. Expression (rule) for perimeter: $5s$

(a) $5 \cdot 25$ meters is 125 meters.

(b) $5 \cdot 8$ inches is 40 inches.

17. Expression (rule) for ordering brushes: $3c-5$

(a) Evaluate the expression when c, the class size, is 12.

$3c-5$ *Replace c with 12.*
$\underbrace{3 \cdot 12}-5$ *Multiply before subtracting.*
$\underbrace{36-5}$
31 brushes must be ordered.

(b) Evaluate the expression when c, the class size, is 16.

$3c-5$ *Replace c with 16.*
$\underbrace{3 \cdot 16}-5$ *Multiply before subtracting.*
$\underbrace{48-5}$
43 brushes must be ordered.

18. Expression (rule) for ordering doughnuts: $2n-4$

(a) $2 \cdot 13-4$ is 22 doughnuts must be ordered.

(b) $2 \cdot 18-4$ is 32 doughnuts must be ordered.

19. Expression (rule) for average test score, where p is the total points and t is the number of tests: p/t

(a) Evaluate the expression when p, the total points, is 332 and t, the number of tests, is 4.

$\frac{p}{t}$ *Replace p with 332 and t with 4.*
$\frac{332}{4}$ *Follow the rule and divide.*
83 points is the average test score.

(b) Evaluate the expression when p, the total points, is 637 and t, the number of tests, is 7.

$\frac{p}{t}$ *Replace p with 637 and t with 7.*

$\frac{637}{7}$ *Follow the rule and divide.*

91 points is the average test score.

20. Expression (rule) for buses: $\frac{p}{b}$

(a) $\frac{176}{44}$ is 4 buses.

(b) $\frac{72}{36}$ is 2 buses.

21.

Value of x	Expression $x+x+x+x$	Expression $4x$
12	$12+12+12+12$ is 48	$4 \cdot 12$ is 48
0	$0+0+0+0$ is 0	$4 \cdot 0$ is 0
-5	$-5+(-5)+(-5)+(-5)$ is -20	$4 \cdot (-5)$ is -20

22.

Value of y	Expression $3y$	Expression $y+2y$
10	$3(10)$ is 30	$10+2(10)$ is $10+20$, or 30
-3	$3(-3)$ is -9	$-3+2(-3)$ is $-3+(-6)$, or -9
0	$3(0)$ is 0	$0+2(0)$ is $0+0$, or 0

23.

Value of x	Value of y	Expression $-2x+y$
-4	5	$-2(-4)+5$ is $8+5$, or 13
-6	-2	$-2(-6)+(-2)$ is $12+(-2)$, or 10
0	-8	$-2(0)+(-8)$ is $0+(-8)$, or -8

24.

Value of x	Value of y	Expression $-2xy$
-4	5	$-2 \cdot (-4) \cdot 5$ is 40
-6	-2	$-2 \cdot (-6) \cdot (-2)$ is -24
0	-8	$-2 \cdot 0 \cdot (-8)$ is 0

25. A variable is a letter that represents the part of a rule that varies or changes depending on the situation. An expression expresses, or tells, the rule for doing something. For example, $c+5$ is an expression, and c is the variable.

26. The number part in a multiplication expression is the coefficient. For example, 4 is the coefficient in $4s$. A constant is a number that is added or subtracted in an expression. It does not vary. For example, 5 is the constant in $c+5$.

27. Multiplying a number by 1 leaves the number unchanged. Let b represent "a number."

$$b \cdot 1 = b \quad \text{or} \quad 1 \cdot b = b$$

28. Adding 0 to any number leaves the number unchanged. Let b represent "any number."

$$b + 0 = b \quad \text{or} \quad 0 + b = b$$

29. Any number divided by 0 is undefined. Let b represent "any number."

$\frac{b}{0}$ is undefined or $b \div 0$ is undefined.

30. Multiplication distributes over addition. Let a, b, and c represent variables.

$$a(b+c) = a \cdot b + a \cdot c$$

31. c^6 written without exponents is

$$c \cdot c \cdot c \cdot c \cdot c \cdot c$$

32. d^7 written without exponents is

$$d \cdot d \cdot d \cdot d \cdot d \cdot d \cdot d$$

33. x^4y^3 written without exponents is

$$x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y$$

34. c^2d^5 written without exponents is

$$c \cdot c \cdot d \cdot d \cdot d \cdot d \cdot d$$

35. $-3a^3b$ can be written as $-3 \cdot a \cdot a \cdot a \cdot b$.
The exponent 3 applies only to the base a.

36. $-8m^2n$ can be written as $-8 \cdot m \cdot m \cdot n$.
The exponent 2 applies only to the base m.

37. $9xy^2$ can be written as $9 \cdot x \cdot y \cdot y$.
The exponent 2 applies only to the base y.

38. $5ab^4$ can be written as $5 \cdot a \cdot b \cdot b \cdot b \cdot b$.
The exponent 4 applies only to the base b.

39. $-2c^5d$ can be written as $-2 \cdot c \cdot c \cdot c \cdot c \cdot c \cdot d$.
The exponent 5 applies only to the base c.

40. $-4x^3y$ can be written as $-4 \cdot x \cdot x \cdot x \cdot y$.
The exponent 3 applies only to the base x.

41. a^3bc^2 can be written as $a \cdot a \cdot a \cdot b \cdot c \cdot c$.
The exponent 3 applies only to the base a.
The exponent 2 applies only to the base c.

42. x^2yz^6 can be written as $x \cdot x \cdot y \cdot z \cdot z \cdot z \cdot z \cdot z \cdot z$.
The exponent 2 applies only to the base x.
The exponent 6 applies only to the base z.

43. Evaluate t^2 when t is -4.

t^2 means

$t \cdot t$ *Replace t with −4.*

$\underbrace{-4 \cdot (-4)}_{16}$ *Multiply.*

44. $r^2 = r \cdot r$ ▪ Replace r with -3.

$$-3 \cdot (-3) = 9$$

45. Evaluate rs^3 when r is -3 and s is 2.

rs^3 means

$r \cdot s \cdot s \cdot s$ *Replace r with −3 and s with 2.*

$\underbrace{-3 \cdot 2} \cdot 2 \cdot 2$ *Multiply left to right.*

$\underbrace{-6 \cdot 2} \cdot 2$

$\underbrace{-12 \cdot 2}$

-24

46. $s^4 t = s \cdot s \cdot s \cdot s \cdot t$ ▪ Replace s with 2 and t with -4.

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot (-4) = -64$$

47. Evaluate $3rs$ when r is -3 and s is 2.

$3rs$ means

$3 \cdot r \cdot s$ *Replace r with −3 and s with 2.*

$\underbrace{3 \cdot (-3)} \cdot 2$ *Multiply left to right.*

$\underbrace{-9 \cdot 2}$

-18

48. $6st$ ▪ Replace s with 2 and t with -4.

$$6 \cdot 2 \cdot (-4) = -48$$

49. Evaluate $-2s^2t^2$ when s is 2 and t is -4.

$-2s^2t^2$ means

$-2 \cdot s \cdot s \cdot t \cdot t$ *Replace s with 2 and t with −4.*

$\underbrace{-2 \cdot 2} \cdot 2 \cdot (-4) \cdot (-4)$ *Multiply left to right.*

$\underbrace{-4 \cdot 2} \cdot (-4) \cdot (-4)$

$\underbrace{-8 \cdot (-4)} \cdot (-4)$

$\underbrace{32 \cdot (-4)}$

-128

50. $-4rs^4 = -4 \cdot r \cdot s \cdot s \cdot s \cdot s$ ▪ Replace r with -3 and s with 2.

$$-4 \cdot (-3) \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 192$$

51. Evaluate $r^2s^5t^3$ when r is -3, s is 2, and t is -4, using a calculator.

$r^2s^5t^3$ *Replace r with −3, s with 2, and t with −4.*

$\underbrace{(-3)^2(2)^5(-4)^3}$ *Use the y^x key.*

$\underbrace{(9)(32)(-64)}$ *Multiply left to right.*

$\underbrace{(288)(-64)}$

$-18{,}432$

52. $r^3s^4t^2$ ▪ Use a calculator. Replace r with -3, s with 2, and t with -4.

$$(-3)^3(2)^4(-4)^2 = (-27)(16)(16) = -6912$$

53. Evaluate $-10r^5s^7$ when r is -3 and s is 2, using a calculator.

$-10r^5s^7$ *Replace r with −3 and s with 2.*

$-10\ \underbrace{(-3)^5}\ \underbrace{(2)^7}$ *Use the y^x key.*

$\underbrace{-10(-243)}(128)$ *Multiply left to right.*

$\underbrace{2430(128)}$

$311{,}040$

54. $-5s^6t^5$ ▪ Use a calculator. Replace s with 2 and t with -4.

$$-5(2)^6(-4)^5 = -5(64)(-1024) = 327{,}680$$

55. Evaluate $|xy| + |xyz|$ when x is 4, y is -2, and z is -6.

$|xy| + |xyz|$ *Replace x with 4, y with −2, and z with −6.*

$\underbrace{|4 \cdot (-2)|} + \left|\underbrace{4 \cdot (-2)} \cdot (-6)\right|$ *Multiply left to right within the abs. value bars.*

$|-8| + \left|\underbrace{-8 \cdot (-6)}\right|$

$|-8| + |48|$ *Evaluate the absolute values.*

$\underbrace{8 + 48}$ *Add.*

56

56. $x + |y^2| + |xz| = x + |y \cdot y| + |x \cdot z|$

Replace x with 4, y with -2, and z with -6.

$$\begin{aligned} 4 + |-2 \cdot (-2)| + |4 \cdot (-6)| &= 4 + |4| + |-24| \\ &= 4 + 4 + 24 \\ &= 32 \end{aligned}$$

57. Evaluate $\dfrac{z^2}{-3y + z}$ when z is -6 and y is -2.

$\dfrac{z^2}{-3y + z}$ *Replace z with −6 and y with −2.*

$\dfrac{(-6)^2}{-3(-2) + (-6)}$ *Follow the order of operations.*

$\dfrac{36}{0}$ *Numerator:* $(-6)^2 = -6 \cdot (-6) = 36$ *Denominator:* $-3(-2) + (-6) = 6 + (-6) = 0$

Undefined *Division by 0 is undefined.*

58. Evaluate $\frac{y^2}{x+2y}$ when x is 4 and y is -2.

$\frac{y^2}{x+2\bullet y}$ *Replace x with 4 and y with −2.*

$\frac{(-2)^2}{4+2\bullet(-2)}$ *Follow the order of operations.*

$\frac{4}{0}$ *Numerator:* $(-2)^2 = -2\bullet(-2) = 4$ *Denominator:* $4+2\bullet(-2) = 4+(-4) = 0$

Undefined *Division by 0 is undefined.*

Relating Concepts (Exercises 59–60)

59. **(a)** Evaluate $\frac{s}{5}$ when s is 15 seconds.

$\frac{s}{5}$ *Replace s with 15.*

$\frac{15}{5}$ *Divide.*

3 miles

(b) Evaluate $\frac{s}{5}$ when s is 10 seconds.

$\frac{s}{5}$ *Replace s with 10.*

$\frac{10}{5}$ *Divide.*

2 miles

(c) Evaluate $\frac{s}{5}$ when s is 5 seconds.

$\frac{s}{5}$ *Replace s with 5.*

$\frac{5}{5}$ *Divide.*

1 mile

60. **(a)** Using part (c) of Exercise 59, the distance covered in $2\frac{1}{2}$ seconds is half of the distance covered in 5 seconds, or $\frac{1}{2}$ mile.

(b) Using part (a) of Exercise 59, the time to cover $1\frac{1}{2}$ miles is half the time to cover 3 miles, or $7\frac{1}{2}$ seconds. Or, using parts (b) and (c), find the number halfway between 5 seconds and 10 seconds

(c) Using parts (a) and (b) of Exercise 59, find the number halfway between 10 seconds and 15 seconds; that is $12\frac{1}{2}$ seconds.

2.2 Simplifying Expressions

2.2 Margin Exercises

1. **(a)** $3b^2 + (-3b) + 3 + b^3 + b$

The like terms are $-3b$ and b since the variable parts match; both are b.

The coefficients are $\underline{-3}$ and $\underline{1}$.

(b) $-4xy + 4x^2y + (-4xy^2) + (-4) + 4$

The like terms are the constants, -4 and 4. There are no variable parts.

(c) $5r^2 + 2r + (-2r^2) + 5 + 5r^3$

The like terms are $5r^2$ and $-2r^2$ since the variable parts match; both are r^2.

The coefficients are 5 and -2.

(d) $-10 + (-x) + (-10x) + (-x^2) + (-10y)$

The like terms are $-x$ and $-10x$ since the variable parts match; both are x.

The coefficients are -1 and -10.

2. **(a)** $10b + 4b + 10b$ *These are like terms.*

$\downarrow \quad \downarrow \quad \downarrow$

$(10+4+10)b$ *Add the coefficients.*

$\underline{24}b$ *The variable part, b, stays the same.*

(b) $y^3 + 8y^3$ *These are like terms.*

$1y^3 + 8y^3$ *Rewrite y^3 as $1y^3$.*

$(1+8)y^3$ *Add the coefficients.*

$9y^3$ *The variable part, y^3, stays the same.*

(c) $-7n - n$ *These are like terms.*

$-7n - 1n$ *Rewrite n as 1n.*

$-7n + (-1n)$ *Change to addition.*

$[-7+(-1)]n$ *Add the coefficients.*

$-8n$ *The variable part, n, stays the same.*

(d) $3c - 5c - 4c$ *These are like terms.*

$3c + (-5c) + (-4c)$ *Change to addition.*

$[3+(-5)+(-4)]c$ *Add the coefficients.*

$-6c$ *The variable part, c, stays the same.*

(e) $-9xy + xy$ *These are like terms.*

$-9xy + 1xy$ *Rewrite xy as 1xy.*

$(-9+1)xy$ *Add the coefficients.*

$-8xy$ *The variable part, xy, stays the same.*

(f) $-4p^2 - 3p^2 + 8p^2$ *These are like terms.*

$-4p^2 + (-3p^2) + 8p^2$ *Change to addition.*

$[-4+(-3)+8]p^2$ *Add the coefficients.*

$1p^2$ or p^2 *The variable part, p^2, stays the same.*

(g) $ab - ab$ *These are like terms.*

$1ab - 1ab$ *Rewrite ab as 1ab.*

$1ab + (-1ab)$ *Change to addition.*

$[1+(-1)]ab$ *Add the coefficients.*

$0ab$ *Zero times anything is zero.*

0

3. **(a)** $3b^2+4d^2+7b^2$

$3b^2+7b^2+4d^2$ *Rewrite using the commutative property.*

$(3+7)b^2+4d^2$ *Combine $3b^2+7b^2$.*

$\underline{10}b^2+\underline{4d^2}$ *Add the coefficients.*

(b) $4a+b-6a+b$

$4a+b+(-6a)+b$ *Change to addition.*

$4a+(-6a)+b+b$ *Rewrite using the commutative property.*

$4a+(-6a)+1b+1b$ *Rewrite b as 1b.*

$[4+(-6)]a+(1+1)b$ *Add the coefficients of like terms.*

$-2a+2b$

(c) $-6x+5+6x+2$

$-6x+6x+5+2$ *Rewrite using the commutative property.*

$(-6+6)x+(5+2)$ *Add the coefficients of like terms.*

$0x+7$

$0+7$ *Zero times anything is zero.*

7

(d) $2y-7-y+7$

$2y+(-7)+(-y)+7$ *Change to addition.*

$2y+(-7)+(-1y)+7$ *Rewrite $-y$ as $-1y$.*

$2y+(-1y)+(-7)+7$ *Rewrite using the commutative prop.*

$[2+(-1)]y+(-7+7)$ *Add the coefficients of like terms.*

$1y+0$

$1y$ or y

(e) $-3x-5+12+10x$

$-3x+(-5)+12+10x$ *Change to addition.*

$-3x+10x+(-5)+12$ *Rewrite using the commutative prop.*

$(-3+10)x+(-5+12)$ *Add the coefficients of like terms.*

$7x+7$

4. **(a)** $7(4c)$ means $7\cdot(4\cdot c)$. Using the associative property, it can be rewritten as

$$\underbrace{\underbrace{(7\cdot 4)}\cdot c}$$
$$\underbrace{28\cdot c}$$
$$\underline{28c}$$

(b) $-3(5y^3)$ can be written as

$$\underbrace{(-3\cdot 5)}\cdot y^3$$
$$\underbrace{-15\cdot y^3}$$
$$-15y^3$$

(c) $20(-2a)$ can be written as

$$\underbrace{[20\cdot(-2)]}\cdot a$$
$$\underbrace{-40\cdot a}$$
$$-40a$$

(d) $-10(-x)$ *Rewrite $-x$ as $-1x$.*

$-10(-1x)$ can be written as

$$\underbrace{[-10\cdot(-1)]}\cdot x$$
$$\underbrace{10\cdot x}$$
$$10x$$

5. **(a)** $7(a+10)$ can be written as

$\underbrace{7\cdot a}+\underbrace{7\cdot 10}$

$\underline{7a}\ +\ \underline{70}$

(b) $3(x-3)$ can be written as

$\underbrace{3\cdot x}-\underbrace{3\cdot 3}$

$\underline{3x}\ -\ \underline{9}$

(c) $4(2y+6)$ can be written as

$4\cdot 2y+\underbrace{4\cdot 6}$

$\underbrace{4\cdot 2}\cdot y+24$

$\underbrace{8\cdot y}+24$

$8y+24$

(d) $-5(3b+2)$

$\underbrace{-5\cdot 3b}+\underbrace{(-5)\cdot 2}$

$-15b+(-10)$ *Multiply.*

$-15b-10$ *Change addition to subtraction.*

(e) $-8(c+4)$

$\underbrace{-8\cdot c}+\underbrace{-8\cdot 4}$

$-8c+(-32)$ *Multiply.*

$-8c-32$ *Change addition to subtraction.*

6. **(a)** $-4+5(y+1)$ *Distributive property*

$-4+5\cdot y+5\cdot 1$

$-4+\underline{5y}+\underline{5}$ *Rewrite using the commutative property.*

$\underbrace{-4+5}+5y$ *Combine constants.*

$1+5y$ or $\underline{5y+1}$

(b) $2(3w+4)-5$ *Distributive property*

$\underbrace{2\cdot 3w}+\underbrace{2\cdot 4}-5$ *Multiply.*

$6w+\underline{8-5}$ *Combine constants.*

$6w+3$

(c)

$5(6x-2)+3x$	*Distributive property*
$\underbrace{5\cdot 6x}-\underbrace{5\cdot 2}+3x$	*Multiply.*
$30x - 10 + 3x$	*Change to addition.*
$30x+(-10)+3x$	*Rewrite using the commutative property.*
$30x+3x+(-10)$	
$(30+3)x+(-10)$	*Add the coefficients of like terms.*
$33x+(-10)$ or $33x-10$	

(d)

$21+7(a^2-3)$	*Distributive property*
$21+7\cdot a^2-\underbrace{7\cdot 3}$	*Multiply.*
$21+7a^2 - 21$	*Change to addition.*
$21+7a^2+(-21)$	*Rewrite using the commutative property.*
$\underbrace{21+(-21)}+7a^2$	*Combine constants.*
$\underbrace{0+7a^2}$	
$7a^2$	

(e)

$-y+3(2y+5)-18$	*Distributive property*
$-y+\underbrace{3\cdot 2y}+\underbrace{3\cdot 5}-18$	*Rewrite $-y$ as $-1y$.*
$-1y+6y+15+(-18)$	*Change to addition.*
$\underbrace{(-1+6)}y+[15+(-18)]$	*Add the coefficients of like terms.*
$5y+(-3)$ or $5y-3$	

2.2 Section Exercises

1. $2b^2+2b+2b^3+b^2+6$ ■ $2b^2$ and b^2 are the only like terms in the expression. The variable parts match; both are b^2. The coefficients are $\underline{2}$ and $\underline{1}$.

2. $3x+x^3+3x^2+3+2x^3$ ■ x^3 and $2x^3$ are like terms. The variable parts match; both are x^3. The coefficients are 1 and 2.

3. $-x^2y+(-xy)+2xy+(-2xy^2)$ ■ $-xy$ and $2xy$ are the like terms in the expression. The variable parts match; both are xy. The coefficients are -1 and 2.

4. $ab^2+(-a^2b)+2ab+(-3a^2b)$ ■ $-a^2b$ and $-3a^2b$ are like terms. The variable parts match; both are a^2b. The coefficients are -1 and -3.

5. $7+7c+3+7c^3+(-4)$ ■ 7, 3, and -4 are like terms. There are no variable parts; constants are considered like terms.

6. $4d+(-5)+1+(-5d^2)+4$ ■ -5, 1, and 4 are like terms. There are no variable parts; constants are considered like terms.

7.

$6r+6r$	*These are like terms. Add the coefficients.*
$\downarrow \quad \downarrow$	
$\underbrace{(6+6)}r$	
$\underline{12}r$	*The variable part, r, stays the same.*

8.

$4t+10t$	*These are like terms. Add the coefficients.*
$\downarrow \quad \downarrow$	
$\underbrace{(4+10)}t$	
$\underline{14}t$	*The variable part, t, stays the same.*

9.

x^2+5x^2	*These are like terms. Rewrite x^2 as $1x^2$.*
$1x^2+5x^2$	*Add the coefficients.*
$(1+5)x^2$	
$6x^2$	*The variable part, x^2, stays the same.*

10.
$$9y^3+y^3=9y^3+1y^3 \\ =(9+1)y^3 \\ =10y^3$$

11.

$p-5p$	*These are like terms. Rewrite p as 1p.*
$1p-5p$	*Change to addition.*
$1p+(-5p)$	*Add the coefficients.*
$[1+(-5)]p$	
$-4p$	*The variable part, p, stays the same.*

12.
$$n-3n=1n+(-3n) \\ =[1+(-3)]n \\ =-2n$$

13.

$-2a^3-a^3$	*These are like terms. Rewrite a^3 as $1a^3$.*
$-2a^3-1a^3$	*Change to addition.*
$-2a^3+(-1a^3)$	*Add the coefficients.*
$[-2+(-1)]a^3$	
$-3a^3$	*The variable part, a^3, stays the same.*

14.
$$-10x^2-x^2=-10x^2-1x^2 \\ =-10x^2+(-1x^2) \\ =[-10+(-1)]x^2 \\ =-11x^2$$

15. $\underbrace{c-c}_{0}$ Any number minus itself is 0.

16. $\underbrace{b^2-b^2}_{0}$ Any number minus itself is 0.

17.

$9xy + xy - 9xy$	*These are like terms. Rewrite xy as 1xy.*
$9xy + 1xy - 9xy$	*Change to addition.*
$9xy + 1xy + (-9xy)$	*Add the coefficients.*
$[9 + 1 + (-9)]xy$	
$1xy$ or xy	*The variable part, xy, stays the same.*

18.
$$\begin{aligned} r^2s - 7r^2s + 7r^2s &= 1r^2s + (-7r^2s) + 7r^2s \\ &= [1 + (-7) + 7]r^2s \\ &= 1r^2s \quad \text{or} \quad r^2s \end{aligned}$$

19.

$5t^4 + 7t^4 - 6t^4$	*These are like terms. Change to addition.*
$5t^4 + 7t^4 + (-6t^4)$	*Add the coefficients.*
$[5 + 7 + (-6)]t^4$	
$6t^4$	*The variable part, t^4, stays the same.*

20.
$$\begin{aligned} 10mn - 9mn + 3mn &= 10mn + (-9mn) + 3mn \\ &= [10 + (-9) + 3]mn \\ &= 4mn \end{aligned}$$

21.

$y^2 + y^2 + y^2 + y^2$	*These are like terms. Write in the understood coefficients of 1.*
$1y^2 + 1y^2 + 1y^2 + 1y^2$	
$(1 + 1 + 1 + 1)y^2$	*Add the coefficients.*
$4y^2$	*The variable part, y^2, stays the same.*

22.
$$\begin{aligned} a + a + a &= 1a + 1a + 1a \\ &= (1 + 1 + 1)a \\ &= 3a \end{aligned}$$

23.

$-x - 6x - x$	*These are like terms. Rewrite −x as −1x and x as 1x.*
$-1x - 6x - 1x$	*Change to addition.*
$-1x + (-6x) + (-1x)$	*Add the coefficients.*
$[-1 + (-6) + (-1)]x$	
$-8x$	*The variable part, x, stays the same.*

24.
$$\begin{aligned} -y - y - 3y &= -1y - 1y - 3y \\ &= -1y + (-1y) + (-3y) \\ &= [-1 + (-1) + (-3)]y \\ &= -5y \end{aligned}$$

25.

$8a + 4b + 4a$	*Use the commutative property to rewrite the expression so that like terms are next to each other.*
$8a + 4a + 4b$	*Add the coefficients of like terms.*
$(8 + 4)a + 4b$	
$12a + 4b$	*The variable part, a, stays the same.*

26.
$$\begin{aligned} 6x + 5y + 4y &= 6x + (5 + 4)y \\ &= 6x + 9y \end{aligned}$$

27.

$6 + 8 + 7rs$	*Use the commutative property to put the constants at the end.*
$7rs + 6 + 8$	*Add the coefficients of like terms.*
$7rs + 14$	*The only like terms are constants.*

28.
$$\begin{aligned} 10 + 2c^2 + 15 &= 2c^2 + 10 + 15 \\ &= 2c^2 + 25 \end{aligned}$$

29.

$a + ab^2 + ab^2$	*Write in the understood coefficients of 1.*
$1a + 1ab^2 + 1ab^2$	*Add the coefficients of like terms.*
$1a + (1 + 1)ab^2$	
$1a + 2ab^2$,	*The variable part, ab^2, stays the same.*
or $a + 2ab^2$	

30.
$$\begin{aligned} n + mn + n &= 1n + 1mn + 1n \\ &= 1mn + 1n + 1n \\ &= 1mn + 2n \quad \text{or} \quad mn + 2n \end{aligned}$$

31.

$6x + y - 8x + y$	*Write in the understood coefficients of 1. Change to addition.*
$6x + 1y + (-8x) + 1y$	*Rewrite using the commutative property.*
$6x + (-8x) + 1y + 1y$	*Add the coefficients of like terms.*
$\underbrace{[6 + (-8)]x}_{-2x} + \underbrace{(1 + 1)y}_{2y}$	
$-2x + 2y$	

32.
$$\begin{aligned} d + 3c - 7c + 3d &= 1d + 3c + (-7c) + 3d \\ &= 3c + (-7c) + 1d + 3d \\ &= [3 + (-7)]c + (1 + 3)d \\ &= -4c + 4d \end{aligned}$$

33.

$8b^2 - a^2 - b^2 + a^2$	*Write in the understood coefficient of 1.*
$8b^2 - 1a^2 - 1b^2 + 1a^2$	*Change to addition.*
$8b^2 + (-1a^2) + (-1b^2) + 1a^2$	*Rewrite using the commutative property.*
$8b^2 + (-1b^2) + (-1a^2) + 1a^2$	*Add the coefficients of like terms.*
$\underbrace{[8 + (-1)]b^2}_{7b^2} + \underbrace{(-1 + 1)a^2}_{0 \cdot a^2}$	
$7b^2 + 0$	
$7b^2$	

34. $5ab - ab + 3a^2b - 4ab$
$= 5ab + (-1ab) + 3a^2b + (-4ab)$
$= 5ab + (-1ab) + (-4ab) + 3a^2b$
$= [5 + (-1) + (-4)]ab + 3a^2b$
$= 0ab + 3a^2b$
$= 3a^2b$

35. $-x^3 + 3x - 3x^2 + 2$ ▪ There are no like terms. The expression cannot be simplified.

36. $a^2b - 2ab - ab^3 + 3a^3b$ ▪ There are no like terms. The expression cannot be simplified.

37. $-9r + 6t - s - 5r + s + t - 6t + 5s - r$
Write in the understood coefficients of 1.
Change to addition.
$-9r + 6t + (-1s) + (-5r) + 1s + 1t$
$+ (-6t) + 5s + (-1r)$
Rewrite using the commutative property.
$-9r + (-5r) + (-1r) + (-1s) + 1s + 5s$
$+ 6t + 1t + (-6t)$
Add the coefficients of like terms.
$\underbrace{[-9 + (-5) + (-1)]\,r}_{-15r} + \underbrace{(-1 + 1 + 5)s}_{5s} + \underbrace{[6 + 1 + (-6)]\,t}_{1t}$
$-15r + 5s + t$

38. $-x - 3y + 4z + x - z + 5y - 8x - y$
$= -1x + (-3y) + 4z + 1x + (-1z) + 5y$
$+ (-8x) + (-1y)$
$= -1x + 1x + (-8x) + (-3y) + 5y + (-1y)$
$+ 4z + (-1z)$
$= [-1 + 1 + (-8)]x + [-3 + 5 + (-1)]y$
$+ [4 + (-1)]z$
$= -8x + 1y + 3z$ or $-8x + y + 3z$

39. By using the associative property, we can write $3(10a)$ as

$$(3 \cdot 10) \cdot a = 30 \cdot a = 30a.$$

So, $3(10a)$ simplifies to $30a$.

40. $8(4b) = (8 \cdot 4)b$
$= 32b$

41. By using the associative property, we can write $-4(2x^2)$ as

$$(-4 \cdot 2) \cdot x^2 = -8 \cdot x^2 = -8x^2.$$

So, $-4(2x^2)$ simplifies to $-8x^2$.

42. $-7(3b^3) = (-7 \cdot 3)b^3$
$= -21b^3$

43. By using the associative property, we can write $5(-4y^3)$ as

$$[5 \cdot (-4)] \cdot y^3 = -20 \cdot y^3 = -20y^3.$$

So, $5(-4y^3)$ simplifies to $-20y^3$.

44. $2(-6x) = [2 \cdot (-6)]x$
$= -12x$

45. By using the associative property, we can write $-9(-2cd)$ as

$$[-9 \cdot (-2)] \cdot c \cdot d = 18 \cdot c \cdot d = 18cd.$$

So, $-9(-2cd)$ simplifies to $18cd$.

46. $-6(-4rs) = [-6 \cdot (-4)]rs$
$= 24rs$

47. By using the associative property, we can write $7(3a^2bc)$ as

$$(7 \cdot 3) \cdot a^2 \cdot b \cdot c = 21 \cdot a^2 \cdot b \cdot c = 21a^2bc.$$

So, $7(3a^2bc)$ simplifies to $21a^2bc$.

48. $4(2xy^2z^2) = (4 \cdot 2)xy^2z^2$
$= 8xy^2z^2$

49. $-12(-w)$ *Write in the understood coefficient of −1.*
$-12(-1w)$ *Rewrite using the associative property.*
$[-12 \cdot (-1)]w$
$12 \cdot w$
$12w$

50. $-10(-k) = -10(-1k)$
$= [-10 \cdot (-1)]k$
$= 10k$

51. $6(b + 6)$ *Distributive property*
$\underbrace{6 \cdot b} + \underbrace{6 \cdot 6}$
$\underline{6b} + \underline{36}$

52. $5(a + 3)$ *Distributive property*
$\underbrace{5 \cdot a} + \underbrace{5 \cdot 3}$
$\underline{5a} + \underline{15}$

53. $7(x - 1)$ *Distributive property*
$7 \cdot x - 7 \cdot 1$
$7x - 7$

54. $4(y - 4) = 4 \cdot y - 4 \cdot 4$
$= 4y - 16$

55. $3(7t + 1)$ *Distributive property*
$3 \cdot 7t + 3 \cdot 1$
$21t + 3$

56. $8(2c + 5) = 8 \cdot 2c + 8 \cdot 5$
$= 16c + 40$

57. $-2(5r + 3)$ *Distributive property*
$-2 \cdot 5r + (-2) \cdot 3$
$-10r + (-6)$ *Change addition to subtraction of the opposite.*
$-10r - 6$

58. $-5(6z+2) = -5 \cdot 6z + (-5) \cdot 2$
$= -30z + (-10)$
or $-30z - 10$

59. $-9(k+4)$ *Distributive property*
$-9 \cdot k + (-9) \cdot 4$
$-9k + (-36)$ *Change addition to subtraction of the opposite.*
$-9k - 36$

60. $-3(p+7) = -3 \cdot p + (-3) \cdot 7$
$= -3p + (-21)$
or $-3p - 21$

61. $50(m-6)$ *Distributive property*
$50 \cdot m - 50 \cdot 6$
$50m - 300$

62. $25(n-1) = 25 \cdot n - 25 \cdot 1$
$= 25n - 25$

63. $10 + 2(4y+3)$ *Distributive property*
$10 + \underbrace{2 \cdot 4} \cdot y + \underbrace{2 \cdot 3}$
$10 \quad + \quad 8y \quad + \quad 6$ *Rewrite using the commutative property.*
$8y + 10 + 6$ *Combine like terms.*
$8y + 16$

64. $4 + 7(x^2+3) = 4 + 7 \cdot x^2 + \underbrace{7 \cdot 3}$
$= 4 + 7x^2 \quad + \quad 21$
$= 7x^2 + 25$

65. $6(a^2-2) + 15$ *Distributive property*
$6 \cdot a^2 - 6 \cdot 2 + 15$
$6a^2 - 12 + 15$ *Combine like terms.*
$6a^2 + 3$

66. $5(b-4) + 25 = 5 \cdot b - 5 \cdot 4 + 25$
$= 5b - 20 + 25$
$= 5b + (-20) + 25$
$= 5b + 5$

67. $2 + 9(m-4)$ *Distributive property*
$2 + 9 \cdot m - 9 \cdot 4$
$2 + 9m - 36$ *Change to addition.*
$2 + 9m + (-36)$ *Rewrite using the commutative property.*
$9m + 2 + (-36)$ *Add the coefficients of like terms.*
$9m + (-34)$ *Change addition to subtraction of the opposite.*
$9m - 34$

68. $6 + 3(n-8) = 6 + 3 \cdot n - 3 \cdot 8$
$= 6 + 3n - 24$
$= 6 + 3n + (-24)$
$= 3n + (-18)$ or $3n - 18$

69. $-5(k+5) + 5k$ *Distributive property*
$-5 \cdot k + (-5) \cdot 5 + 5k$
$-5k + (-25) + 5k$ *Rewrite using the commutative property.*
$-5k + 5k + (-25)$ *Add the coefficients of like terms.*
$\underbrace{(-5+5)}k + (-25)$
$0k + (-25)$ *Zero times any number is 0.*
$\underbrace{0 + (-25)}$ *Zero added to any number is the number*
-25

70. $-7(p+2) + 7p = -7 \cdot p + (-7) \cdot 2 + 7p$
$= -7p + (-14) + 7p$
$= 0p + (-14)$
$= 0 + (-14)$
$= -14$

71. $4(6x-3) + 12$ *Distributive property*
$4 \cdot 6x - 4 \cdot 3 + 12$
$24x - 12 + 12$ *Change to addition.*
$24x + (-12) + 12$ *Combine like terms. Any number plus its opposite is 0.*
$24x + 0$
$24x$

72. $6(3y-3) + 18 = 6 \cdot 3y - 6 \cdot 3 + 18$
$= 18y - 18 + 18$
$= 18y + (-18) + 18$
$= 18y$

73. $5 + 2(3n+4) - n$ *Distributive property*
$5 + 2 \cdot 3n + 2 \cdot 4 - n$ *Rewrite n as 1n.*
$5 + 6n + 8 - 1n$ *Change to addition.*
$5 + 6n + 8 + (-1n)$ *Rewrite using the commutative property.*
$5 + 8 + 6n + (-1n)$ *Add the coefficients of like terms.*
$(5+8) + [6 + (-1)]n$
$13 + 5n$ or $5n + 13$

74. $8 + 8(4z+5) - z = 8 + 8 \cdot 4z + 8 \cdot 5 - 1z$
$= 8 + 32z + 40 + (-1z)$
$= 31z + 48$

75. $-p+6(2p-1)+5$ *Distributive property*
$-p+6\cdot 2p-6\cdot 1+5$
$-p+12p-6+5$ *Rewrite $-p$ as $-1p$. Change to addition.*
$-1p+12p+(-6)+5$ *Add the coefficients of like terms.*
$(-1+12)p+(-6+5)$
$11p+(-1)$ *Change addition to subt. of the opposite.*
$11p-1$

76. $-k+3(4k-1)+2$
$=-1k+3\cdot 4k-3\cdot 1+2$
$=-1k+12k-3+2$
$=-1k+12k+(-3)+2$
$=11k+(-1)$ or $11k-1$

77. A simplified expression usually still has variables, but it is written in a simpler way. When evaluating an expression, the variables are all replaced by specific numbers and the final result is a numerical answer.

78. $5(3x+2)$ $\quad$ $5(2+3x)$
$=5\cdot 3x+5\cdot 2$ $\quad$ $=5\cdot 2+5\cdot 3x$
$=15x+10$ $\quad$ $=10+15x$
The answers are equivalent because of the commutative property of addition.

79. Like terms have matching variable parts, that is, matching letters and exponents. The coefficients do not have to match. Examples will vary. Possible examples: In $-6x+9+x$, the terms $-6x$ and x are like terms. In $4k+3-8k^2+10$, the terms 3 and 10 are like terms.

80. Add the coefficients of like terms. If no coefficient is shown, it is assumed to be 1. Keep the variable part the same. Examples will vary.

81. $\underbrace{-2x+7x}_{5x+8}+8$
Keep the variable part unchanged when combining like terms. As shown above, the correct answer is $5x+8$.

82. In the last step, do not change the sign of the first term. The correct answer is $-4a-5$.

83. $-4(3y)-5+2(5y+7)$ *Distributive prop.*
$-4\cdot 3y-5+2\cdot 5y+2\cdot 7$
$-12y-5+10y+14$ *Change subtraction to adding the opposite.*
$-12y+(-5)+10y+14$ *Group like terms and add the coefficients.*
$\underbrace{-12y+10y}_{-2y}+\underbrace{(-5)+14}_{9}$
$-2y+9$

84. $6(-3x)-9+3(-2x+6)$
$=-18x-9+3\cdot(-2x)+3\cdot 6$
$=-18x+(-9)+(-6x)+18$
$=-24x+9$

85. $-10+4(-3b+3)+2(6b-1)$
Distributive property
$-10+4\cdot(-3b)+4\cdot 3+2\cdot 6b-2\cdot 1$
$-10+(-12b)+12+12b-2$
Change to addition.
$-10+(-12b)+12+12b+(-2)$
Group like terms and add the coefficients.
$\underbrace{-12b+12b}_{0b}+\underbrace{-10+12+(-2)}_{0}$
0

86. $12+2(4a-4)+4(-2a-1)$
$=12+2\cdot 4a-2\cdot 4+4\cdot(-2a)-4\cdot 1$
$=12+8a-8+(-8a)-4$
$=12+8a+(-8)+(-8a)+(-4)$
$=8a+(-8a)+12+(-8)+(-4)$
$=0$

87. $-5(-x+2)+8(-x)+3(-2x-2)+16$
Distributive property
$-5\cdot(-x)+(-5)\cdot 2+8\cdot(-x)+3\cdot(-2x)-3\cdot 2+16$
$5x+(-10)+(-8x)+(-6x)-6+16$
Change to addition.
$5x+(-10)+(-8x)+(-6x)+(-6)+16$
Group like terms and add the coefficients.
$\underbrace{5x+(-8x)+(-6x)}_{-9x}+\underbrace{-10+(-6)+16}_{0}$
$-9x$

88. $-7(-y)+6(y-1)+3(-2y)+6-y$
$=7y+6y-6+(-6y)+6-y$
$=7y+6y+(-6)+(-6y)+6+(-1y)$
$=7y+6y+(-6y)+(-1y)+(-6)+6$
$=6y$

Summary Exercises *Variables and Expressions*

1. $-10-m$ or $-10-1m$ $\quad$ m is the variable; -1 is the coefficient; -10 is the constant.

2. $-8cd$ $\quad$ c and d are the variables; -8 is the coefficient.

3. $6+4x$ $\quad$ x is the variable; 4 is the coefficient; 6 is the constant.

4. Expression (rule) for finding the perimeter of an octagon of side length s: $8s$

(a) Evaluate the expression when s, the side length, is 4 yards.

$8s$ *Replace s with 4.*

$\underbrace{8 \cdot 4}$ *Follow the rule and multiply.*

32 yards is the perimeter.

(b) Evaluate the expression when s, the side length, is 15 inches.

$8s$ *Replace s with 15.*

$\underbrace{8 \cdot 15}$ *Follow the rule and multiply.*

120 inches is the perimeter.

5. Expression (rule) for finding the total cost of a car with down payment d, monthly payment m, and number of payments t: $d + mt$

(a) Evaluate the expression when the down payment is \$3000, the monthly payment is \$280, and the number of payments is 36.

$d + mt$ *Replace d with \$3000, m with \$280, and t with 36.*

$\underbrace{\$3000 + \$280 \cdot 36}$ *Multiply before adding.*

$\underbrace{\$3000 + \$10{,}080}$

\$13,080 is the total cost of the car.

(b) Evaluate the expression when the down payment is \$1750, the monthly payment is \$429, and the number of payments is 48.

$d + mt$ *Replace d with \$1750, m with \$429, and t with 48.*

$\underbrace{\$1750 + \$429 \cdot 48}$ *Multiply before adding.*

$\underbrace{\$1750 + \$20{,}592}$

\$22,342 is the total cost of the car.

6. ad^4 written without exponents is

$$a \cdot d \cdot d \cdot d \cdot d$$

7. b^3cd written without exponents is

$$b \cdot b \cdot b \cdot c \cdot d$$

8. $-7ab^5c^2$ written without exponents is

$$-7 \cdot a \cdot b \cdot b \cdot b \cdot b \cdot b \cdot c \cdot c$$

9. $w^4 = w \cdot w \cdot w \cdot w$ ▪ Replace w with 5.

$\underbrace{5 \cdot 5} \cdot 5 \cdot 5$ *Multiply left to right.*

$\underbrace{25 \cdot 5} \cdot 5$

$\underbrace{125 \cdot 5}$

625

10. $5xz$ ▪ Replace x with -2 and z with 0. If 0 is multiplied by any number, the result is 0. Thus, there is no need to make any calculations since the result is 0.

11. yz^2 ▪ Replace y with -6 and z with 0. If 0 is multiplied by any number, the result is 0. Thus, there is no need to make any calculations since the result is 0.

12. wxy ▪ Replace w with 5, x with -2, and y with -6.

$\underbrace{5 \cdot (-2)} \cdot (-6)$ *Multiply left to right.*

$\underbrace{-10 \cdot (-6)}$

60

13. $x^3 = x \cdot x \cdot x$ ▪ Replace x with -2.

$\underbrace{-2 \cdot (-2)} \cdot (-2)$ *Multiply left to right.*

$\underbrace{4 \cdot (-2)}$

-8

14. $-4wy$ ▪ Replace w with 5 and y with -6.

$\underbrace{-4 \cdot 5} \cdot (-6)$ *Multiply left to right.*

$\underbrace{-20 \cdot (-6)}$

120

15. $3xy^2 = 3 \cdot x \cdot y \cdot y$ ▪ Replace x with -2 and y with -6.

$\underbrace{3 \cdot (-2)} \cdot (-6) \cdot (-6)$ *Multiply left to right.*

$\underbrace{-6 \cdot (-6)} \cdot (-6)$

$\underbrace{36 \cdot (-6)}$

-216

16. $w^2x^5 = w \cdot w \cdot x \cdot x \cdot x \cdot x \cdot x$ ▪ Replace w with 5 and x with -2. Multiply left to right.

$\underbrace{5 \cdot 5} \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2)$

$\underbrace{25 \cdot (-2)} \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2)$

$\underbrace{-50 \cdot (-2)} \cdot (-2) \cdot (-2) \cdot (-2)$

$\underbrace{100 \cdot (-2)} \cdot (-2) \cdot (-2)$

$\underbrace{-200 \cdot (-2)} \cdot (-2)$

$\underbrace{400 \cdot (-2)}$

-800

17. $-7wx^4y^3$ ▪ Use a calculator. Replace w with 5, x with -2, and y with -6.

$$-7(5)(-2)^4(-6)^3 = -35(16)(-216) = 120{,}960$$

18.
$$\begin{aligned} 10b + 4b + 10b &= (10 + 4 + 10)b \\ &= 24b \end{aligned}$$

19.
$$\begin{aligned} &-3x - 5 + 12 + 10x \\ &= -3x + 10x + (-5) + 12 \\ &= (-3 + 10)x + (-5 + 12) \\ &= 7x + 7 \end{aligned}$$

20. $$\begin{aligned}-8(c+4) &= -8\cdot c + (-8)\cdot 4\\ &= -8c + (-32)\\ \text{or } &-8c - 32\end{aligned}$$

21. $$\begin{aligned}-9xy + 9xy &= (-9+9)xy\\ &= 0xy\\ &= 0\end{aligned}$$

22. $$\begin{aligned}-4(-3c^2d) &= [-4\cdot(-3)]\cdot c^2d\\ &= 12c^2d\end{aligned}$$

23. $$\begin{aligned}3f - 5f - 4f &= 3f + (-5f) + (-4f)\\ &= [3 + (-5) + (-4)]f\\ &= -6f\end{aligned}$$

24. $$\begin{aligned}2(3w+4) &= 2\cdot 3w + 2\cdot 4\\ &= (2\cdot 3)w + 2\cdot 4\\ &= 6w + 8\end{aligned}$$

25. $$\begin{aligned}-a - 6b - a &= -a + (-6b) + (-a)\\ &= -a + (-a) + (-6b)\\ &= -1a + (-1a) + (-6b)\\ &= [-1 + (-1)]\cdot a + (-6b)\\ &= -2a + (-6b)\\ \text{or } &-2a - 6b\end{aligned}$$

26. $$\begin{aligned}-10(-5x^3y^2) &= [-10\cdot(-5)]\cdot x^3y^2\\ &= 50x^3y^2\end{aligned}$$

27. $$\begin{aligned}&5r^3 + 2r^2 - 2r^2 + 5r^3\\ &= \underbrace{5r^3 + 5r^3}_{10r^3} + \underbrace{2r^2 + (-2r^2)}_{0}\\ &= 10r^3 + 0\\ &= 10r^3\end{aligned}$$

28. $$\begin{aligned}21 + 7(h^2 - 3) &= 21 + 7\cdot h^2 - 7\cdot 3\\ &= 21 + 7h^2 - 21\\ &= 7h^2\end{aligned}$$

29. $$\begin{aligned}-3(m+3) + 3m &= -3\cdot m + (-3)\cdot 3 + 3m\\ &= -3m + 3m + (-9)\\ &= (-3+3)\cdot m + (-9)\\ &= 0m + (-9)\\ &= 0 + (-9)\\ &= -9\end{aligned}$$

30. $$\begin{aligned}-4(8y-5) + 5 &= -4\cdot 8y - (-4)\cdot 5 + 5\\ &= (-4\cdot 8)\cdot y - (-20) + 5\\ &= -32y + 20 + 5\\ &= -32y + 25\end{aligned}$$

31. $$\begin{aligned}2 + 12(3x-1) &= 2 + 12\cdot 3x - 12\cdot 1\\ &= 2 + (12\cdot 3)\cdot x - 12\\ &= 2 + (-12) + 36x\\ &= -10 + 36x\\ \text{or } &36x - 10\end{aligned}$$

32. $$\begin{aligned}&-n + 5(4n-2) + 11\\ &= -n + 5\cdot 4n - 5\cdot 2 + 11\\ &= -n + (5\cdot 4)\cdot n - 10 + 11\\ &= -n + 20n + (-10) + 11\\ &= (-1+20)\cdot n + 1\\ &= 19n + 1\end{aligned}$$

33. **(a)** Simplifying the expression correctly:

$$\begin{aligned}6(n+2) &= 6\cdot n + 6\cdot 2\\ &= 6n + 12\end{aligned}$$

The student forgot to multiply $6\cdot 2$.

(b) Simplifying the expression correctly:

$$\begin{aligned}-5(-4a) &= [-5\cdot(-4)]\cdot a\\ &= 20a\end{aligned}$$

Two negative factors give a *positive* product.

(c) Simplifying the expression correctly:

$$\begin{aligned}3y + 2y - 10 &= (3+2)y - 10\\ &= 5y - 10\end{aligned}$$

Keep the variable part unchanged; that is, adding y's to y's gives an answer with y's, not y^2's.

34. In the last step, do not change the sign of the first term; keep $-7x$ as $-7x$. The correct answer is $-7x - 9$.

2.3 Solving Equations Using Addition

2.3 Margin Exercises

1. **(a)**
$c + 15 = 80$ *Given equation*
$95 + 15 \stackrel{?}{=} 80$ *Replace c with 95.*
$110 \neq 80$ *110 is more than 80.*
No, 95 is not the solution.
$65 + 15 \stackrel{?}{=} 80$ *Replace c with 65.*
$80 = 80$ Balances
Yes, 65 is the solution.
(No need to check 80 and 70.)

(b)
$28 = c - 4$ *Given equation*
$28 \stackrel{?}{=} 28 - 4$ *Replace c with 28.*
$28 \neq 24$
No, 28 is not the solution.
$28 \stackrel{?}{=} 20 - 4$ *Replace c with 20.*
$28 \neq 16$
No, 20 is not the solution.
$28 \stackrel{?}{=} 24 - 4$ *Replace c with 24.*
$28 \neq 20$
No, 24 is not the solution.
$28 \stackrel{?}{=} 32 - 4$ *Replace c with 32.*
$28 = 28$ Balances
Yes, 32 is the solution.

2. **(a)** Solve $12 = y + 5$ for y.

To get y by itself, add the opposite of 5, which is -5. To keep the balance, add -5 to *both* sides.

$$\begin{aligned} 12 &= y + 5 \\ -5 &\quad\;\; -5 \\ \hline 7 &= y + \underline{0} \\ 7 &= \underline{y} \end{aligned}$$
The solution is 7.

Check $12 = y + 5$ *Original equation*

$12 = \underbrace{7 + 5}$ *Replace y with 7.*

$12 = \underline{12}$ Balances, so solution is 7.

(b) Solve $b - 2 = -6$ for b.

Change to addition.

$$b + (-2) = -6$$

To get b by itself add the opposite of -2, which is 2, to both sides.

$$\begin{aligned} b + (-2) &= -6 \\ 2 &\quad\;\; 2 \\ \hline b + \quad 0 &= -4 \\ b &= -4 \end{aligned}$$
The solution is -4.

Check $b - 2 = -6$ *Original equation*

$-4 - 2 = -6$ *Replace b with −4.*

$-4 + (-2) = -6$

$-6 = -6$ Balances

3. **(a)** $2 - 8 = k - 2$ ▪ Rewrite both sides by changing subtraction to addition. Combine like terms.

$$\begin{aligned} 2 + (-8) &= k + (-2) \\ -6 &= k + (-2) \end{aligned}$$

To get k by itself add the opposite of -2, which is 2, to both sides.

$$\begin{aligned} -6 &= k + (-2) \\ 2 &\quad\quad\;\; 2 \\ \hline \underline{-4} &= \underbrace{k + 0} \\ \underline{-4} &= \underline{k} \end{aligned}$$
The solution is -4.

Check

$2 - 8 = k - 2$ *Original equation*

$2 - 8 = -4 - 2$ *Replace k with −4.*

$\underbrace{2 + (-8)} = \underbrace{-4 + (-2)}$

$\underline{-6} = \underline{-6}$ Balances

(b) $4r + 1 - 3r = -8 + 11$ ▪ Change to addition.

$$4r + 1 + (-3r) = -8 + 11$$

Rewrite the left side by using the commutative property.

$$\begin{aligned} 4r + (-3r) + 1 &= -8 + 11 && \textit{Combine like terms.} \\ 1r + 1 &= \quad 3 && \textit{To get r by itself,} \\ -1 &\quad\;\; -1 && \textit{add −1 to both sides.} \\ \hline 1r + 0 &= 2 \\ 1r &= 2 \\ \text{or} \quad r &= 2 && \text{The solution is 2.} \end{aligned}$$

Check

$4r + 1 - 3r = -8 + 11$

$4 \cdot 2 + 1 - 3 \cdot 2 = -8 + 11$ *Replace r with 2.*

$8 + 1 - 6 = 3$

$9 - 6 = 3$

$3 = 3$ Balances

2.3 Section Exercises

1. $n - 50 = 8$ ▪ Replace n with 58, 42, 60, and 8.

$n - 50 = 8$ *Given equation*

$\mathbf{58} - 50 \stackrel{?}{=} 8$ *Replace n with 58.*

$58 + (-50) \stackrel{?}{=} 8$

$8 = 8$

Yes, 58 is the solution.

(No need to check 42, 60, and 8.)

2. $r - 20 = 5$ ▪ Replace r with 5, 15, 30, and 25.

$\mathbf{5} - 20 \stackrel{?}{=} 5$

$5 + (-20) \stackrel{?}{=} 5$

$-15 \neq 5$

$\mathbf{15} - 20 \stackrel{?}{=} 5$

$15 + (-20) \stackrel{?}{=} 5$

$-5 \neq 5$

$\mathbf{30} - 20 \stackrel{?}{=} 5$

$30 + (-20) \stackrel{?}{=} 5$

$10 \neq 5$

$\mathbf{25} - 20 \stackrel{?}{=} 5$

$25 + (-20) \stackrel{?}{=} 5$

$5 = 5$

The check for 25 balances, so 25 is the solution.

3. $-6 = y + 10$ ▪ Replace y with -4, -16, 16, and -6.

$-6 = y + 10$ *Given equation*

$-6 \stackrel{?}{=} \mathbf{-4} + 10$ *Replace y with −4.*

$-6 \neq 6$

No, -4 is not the solution.

$-6 \stackrel{?}{=} \mathbf{-16} + 10$ *Replace y with −16.*

$-6 = -6$

Yes, -16 is the solution.

(No need to check 16 and -6.)

4. $-4 = x + 13$ ▪ Replace x with -4, 17, -17, and -9.

$-4 \stackrel{?}{=} \mathbf{-4} + 13$

$-4 \neq 9$

$-4 \stackrel{?}{=} \mathbf{17} + 13$

$-4 \neq 30$

$-4 \stackrel{?}{=} \mathbf{-17} + 13$

$-4 = -4$

-17 is the solution. (No need to check -9.)

5. **(a)** $m-8=1$ ■ Add 8 to both sides because $-8+8$ gives $m+0$ on the left side.

(b) $-7=w+5$ ■ Add -5 to both sides because $5+(-5)$ gives $w+0$ on the right side.

6. **(a)** $n+2=-9$ ■ Add -2 to both sides because $2+(-2)$ gives $n+0$ on the left side.

(b) $10=b-6$ ■ Add 6 to both sides because $-6+6$ gives $b+0$ on the right side.

7.
$$\begin{array}{rcll}
p+5 &=& 9 & \\
\underline{-5} && \underline{-5} & \textit{Add the opposite of 5, } -5\textit{, to both sides.} \\
\underbrace{p+0} &=& 4 & \\
\underline{p} &=& \underline{4} & \text{The solution is 4.}
\end{array}$$

Check
$$\begin{array}{rl}
p+5=9 & \\
4+5=9 & \textit{Replace p with 4.} \\
9=9 & \text{Balances}
\end{array}$$

8.
$$\begin{array}{rcll}
a+3 &=& 12 & \\
\underline{-3} && \underline{-3} & \textit{Add } -3 \textit{ to both sides.} \\
\underbrace{a+0} &=& 9 & \\
\underline{a} &=& \underline{9} & \text{The solution is 9.}
\end{array}$$

Check
$$\begin{array}{rl}
a+3=12 & \\
9+3=12 & \textit{Replace a with 9.} \\
12=12 & \text{Balances}
\end{array}$$

9.
$$\begin{array}{rcll}
8 &=& r-2 & \\
8 &=& r+(-2) & \textit{Change to addition.} \\
\underline{+2} && \underline{+2} & \textit{Add the opposite of } -2\textit{, 2, to both sides.} \\
10 &=& \underbrace{r+\quad 0} & \\
\underline{10} &=& \underline{r} & \text{The solution is 10.}
\end{array}$$

Check
$$\begin{array}{rl}
8=r-2 & \\
8=10-2 & \textit{Replace r with 10.} \\
8=8 & \text{Balances}
\end{array}$$

10.
$$\begin{array}{rcll}
3 &=& b-5 & \\
3 &=& b+(-5) & \textit{Change to addition.} \\
\underline{+5} && \underline{+5} & \textit{Add 5 to both sides.} \\
8 &=& \underbrace{b+\quad 0} & \\
\underline{8} &=& \underline{b} & \text{The solution is 8.}
\end{array}$$

Check
$$\begin{array}{rl}
3=b-5 & \\
3=8-5 & \textit{Replace b with 8.} \\
3=3 & \text{Balances}
\end{array}$$

11.
$$\begin{array}{rcll}
-5 &=& n+3 & \\
\underline{-3} && \underline{-3} & \textit{Add the opposite of 3, } -3\textit{, to both sides.} \\
-8 &=& n+0 & \\
-8 &=& n & \text{The solution is } -8.
\end{array}$$

Check
$$\begin{array}{rl}
-5=n+3 & \\
-5=-8+3 & \textit{Replace n with } -8. \\
-5=-5 & \text{Balances}
\end{array}$$

12.
$$\begin{array}{rcll}
-1 &=& a+8 & \\
\underline{-8} && \underline{-8} & \textit{Add } -8 \textit{ to both sides.} \\
-9 &=& a+0 & \\
-9 &=& a & \text{The solution is } -9.
\end{array}$$

Check
$$\begin{array}{rl}
-1=a+8 & \\
-1=-9+8 & \textit{Replace a with } -9. \\
-1=-1 & \text{Balances}
\end{array}$$

13.
$$\begin{array}{rcll}
-4+k &=& 14 & \\
\underline{4} && \underline{4} & \textit{Add the opposite of } -4\textit{, 4, to both sides.} \\
0+k &=& 18 & \\
k &=& 18 & \text{The solution is 18.}
\end{array}$$

Check
$$\begin{array}{rl}
-4+k=14 & \\
-4+18=14 & \textit{Replace k with 18.} \\
14=14 & \text{Balances}
\end{array}$$

14.
$$\begin{array}{rcll}
-9+y &=& 7 & \\
\underline{9} && \underline{9} & \textit{Add 9 to both sides.} \\
0+y &=& 16 & \\
y &=& 16 & \text{The solution is 16.}
\end{array}$$

Check
$$\begin{array}{rl}
-9+y=7 & \\
-9+16=7 & \textit{Replace y with 16.} \\
7=7 & \text{Balances}
\end{array}$$

15.
$$\begin{array}{rcll}
y-6 &=& 0 & \\
y+(-6) &=& 0 & \textit{Change to addition.} \\
\underline{6} && \underline{6} & \textit{Add the opposite of } -6\textit{, 6, to both sides.} \\
y+0 &=& 6 & \\
y &=& 6 & \text{The solution is 6.}
\end{array}$$

Check
$$\begin{array}{rl}
y-6=0 & \\
6-6=0 & \textit{Replace y with 6.} \\
0=0 & \text{Balances}
\end{array}$$

16.
$$\begin{array}{rcll}
k-15 &=& 0 & \textit{Change to addition.} \\
k+(-15) &=& 0 & \textit{Add 15 to both sides.} \\
\underline{15} && \underline{15} & \\
k+0 &=& 15 & \\
k &=& 15 & \text{The solution is 15.}
\end{array}$$

Check
$$\begin{array}{rl}
k-15=0 & \\
15-15=0 & \textit{Replace k with 15.} \\
0=0 & \text{Balances}
\end{array}$$

17.
$$\begin{array}{rcll}
7 &=& r+13 & \\
\underline{-13} && \underline{-13} & \textit{Add the opposite of 13, } -13\textit{, to both sides.} \\
-6 &=& r+0 & \\
-6 &=& r & \text{The solution is } -6.
\end{array}$$

Check
$$\begin{array}{rl}
7=r+13 & \\
7=-6+13 & \textit{Replace r with } -6. \\
7=7 & \text{Balances}
\end{array}$$

18.
$$\begin{array}{rcll} 12 &=& z+19 & \\ \underline{-19} & & \underline{\quad -19} & \textit{Add } -19 \textit{ to both sides.} \\ -7 &=& z+0 & \\ -7 &=& z & \text{The solution is } -7. \end{array}$$

Check $12 = z + 19$
$$\begin{array}{ll} 12 = -7 + 19 & \textit{Replace z with } -7. \\ 12 = 12 & \text{Balances} \end{array}$$

19.
$$\begin{array}{rcll} x-12 &=& -12 & \\ x+(-12) &=& -12 & \textit{Change to addition.} \\ \underline{\quad 12} & & \underline{12} & \textit{Add the opposite of } -12\textit{, 12, to both sides.} \\ x+0 &=& 0 & \\ x &=& 0 & \text{The solution is } 0. \end{array}$$

Check
$$\begin{array}{rl} x - 12 = -12 & \\ 0 - 12 = -12 & \textit{Replace x with 0.} \\ 0 + (-12) = -12 & \textit{Change to addition.} \\ -12 = -12 & \text{Balances} \end{array}$$

20.
$$\begin{array}{rcll} -3 &=& m-3 & \\ -3 &=& m+(-3) & \textit{Change to addition.} \\ \underline{3} & & \underline{\quad\quad 3} & \textit{Add 3 to both sides.} \\ 0 &=& m+\quad 0 & \\ 0 &=& m & \text{The solution is } 0. \end{array}$$

Check
$$\begin{array}{ll} -3 = m - 3 & \\ -3 = 0 - 3 & \textit{Replace m with 0.} \\ -3 = 0 + (-3) & \\ -3 = -3 & \text{Balances} \end{array}$$

21.
$$\begin{array}{rcll} -5 &=& -2+t & \\ \underline{2} & & \underline{2\quad} & \textit{Add the opposite of } -2\textit{, 2, to both sides.} \\ -3 &=& 0+t & \\ -3 &=& t & \text{The solution is } -3. \end{array}$$

Check
$$\begin{array}{ll} -5 = -2 + t & \\ -5 = -2 + (-3) & \textit{Replace t with } -3. \\ -5 = -5 & \text{Balances} \end{array}$$

22.
$$\begin{array}{rcll} -1 &=& -10+w & \\ \underline{10} & & \underline{10\quad} & \textit{Add 10 to both sides.} \\ 9 &=& 0+w & \\ 9 &=& w & \text{The solution is } 9. \end{array}$$

Check
$$\begin{array}{ll} -1 = -10 + w & \\ -1 = -10 + 9 & \textit{Replace w with 9.} \\ -1 = -1 & \text{Balances} \end{array}$$

23. $z - 5 = 3$ ▪ The given solution is -2.

Check
$$\begin{array}{rl} z - 5 = 3 & \\ -2 - 5 = 3 & \textit{Replace z with } -2. \\ -2 + (-5) = 3 & \textit{Change to addition.} \\ -7 \neq 3 & \text{Does not balance} \end{array}$$

Correct solution:
$$\begin{array}{rcll} z-5 &=& 3 & \\ z+(-5) &=& 3 & \textit{Change to addition.} \\ \underline{\quad 5} & & \underline{5} & \textit{Add the opposite of } -5\textit{, 5, to both sides.} \\ z+0 &=& 8 & \\ z &=& 8 & \text{The solution is } 8. \end{array}$$

Check
$$\begin{array}{rl} z - 5 = 3 & \\ 8 - 5 = 3 & \textit{Replace z with 8.} \\ 8 + (-5) = 3 & \textit{Change to addition.} \\ 3 = 3 & \text{Balances} \end{array}$$

24. $x - 9 = 4$ ▪ The given solution is 13.

Check
$$\begin{array}{rl} x - 9 = 4 & \\ 13 - 9 = 4 & \textit{Replace x with 13.} \\ 4 = 4 & \text{Balances} \end{array}$$

13 is the correct solution.

25. $7 + x = -11$ ▪ The given solution is -18.

Check
$$\begin{array}{rl} 7 + x = -11 & \\ 7 + (-18) = -11 & \textit{Replace x with } -18. \\ -11 = -11 & \text{Balances} \end{array}$$

-18 is the correct solution.

26. $2 + k = -7$ ▪ The given solution is -5.

Check
$$\begin{array}{rl} 2 + k = -7 & \\ 2 + (-5) = -7 & \textit{Replace k with } -5. \\ -3 \neq -7 & \text{Does not balance} \end{array}$$

Correct solution:
$$\begin{array}{rcll} 2+k &=& -7 & \\ \underline{-2\quad} & & \underline{-2} & \textit{Add the opposite of 2, } -2\textit{, to both sides.} \\ 0+k &=& -9 & \\ k &=& -9 & \text{The correct solution is } -9. \end{array}$$

Check
$$\begin{array}{rl} 2 + k = -7 & \\ 2 + (-9) = -7 & \textit{Replace k with } -9. \\ -7 = -7 & \text{Balances} \end{array}$$

27. $-10 = -10 + b$ ▪ The given solution is 10.

Check
$$\begin{array}{ll} -10 = -10 + b & \\ -10 = -10 + 10 & \textit{Replace b with 10.} \\ -10 \neq 0 & \text{Does not balance} \end{array}$$

Correct solution:
$$\begin{array}{rcll} -10 &=& -10+b & \\ \underline{10} & & \underline{10\quad} & \textit{Add the opposite of } -10\textit{, 10, to both sides.} \\ 0 &=& 0+b & \\ 0 &=& b & \text{The solution is } 0. \end{array}$$

Check $-10 = -10 + b$
$-10 = -10 + 0$ *Replace b with 0.*
$-10 = -10$ Balances

28. $0 = -14 + a$ ■ The given solution is 0.

Check $0 = -14 + a$
$0 = -14 + 0$ *Replace a with 0.*
$0 \neq -14$ Does not balance

Correct solution:

$$\begin{array}{rcll} 0 &=& -14 + a & \\ \underline{14} & & \underline{14\qquad} & \textit{Add the opposite of } -14\textit{, 14, to both sides.} \\ 14 &=& 0 + a & \\ 14 &=& a & \text{The correct solution is 14.} \end{array}$$

Check $0 = -14 + a$
$0 = -14 + 14$ *Replace a with 14.*
$0 = 0$ Balances

29.
$$\begin{array}{rcll} c - 4 &=& -8 + 10 & \\ c - 4 &=& 2 & \textit{Simplify the right side.} \\ c + (-4) &=& 2 & \textit{Change to addition.} \\ \underline{4} & & \underline{4} & \textit{Add 4 to both sides.} \\ c + 0 &=& 6 & \\ c &=& 6 & \text{The solution is 6.} \end{array}$$

Check $c - 4 = -8 + 10$
$6 - 4 = -8 + 10$ *Replace c with 6.*
$2 = 2$ Balances

30.
$$\begin{array}{rcll} b - 8 &=& 10 - 6 & \\ b - 8 &=& 4 & \\ \underline{8} & & \underline{8} & \\ b + 0 &=& 12 & \\ b &=& 12 & \text{The solution is 12.} \end{array}$$

Check $b - 8 = 10 - 6$
$12 - 8 = 10 - 6$ *Replace b with 12.*
$4 = 4$ Balances

31.
$$\begin{array}{rcll} -1 + 4 &=& y - 2 & \\ 3 &=& y - 2 & \textit{Simplify the left side.} \\ 3 &=& y + (-2) & \textit{Change to addition.} \\ \underline{2} & & \underline{\qquad 2} & \textit{Add 2 to both sides.} \\ 5 &=& y + \quad 0 & \\ 5 &=& y & \text{The solution is 5.} \end{array}$$

Check
$-1 + 4 = y - 2$
$-1 + 4 = 5 - 2$ *Replace y with 5.*
$-1 + 4 = 5 + (-2)$ *Change to addition.*
$3 = 3$ Balances

32.
$$\begin{array}{rcll} 2 + 3 &=& k - 4 & \\ 5 &=& k - 4 & \\ \underline{4} & & \underline{\quad 4} & \textit{Add 4 to both sides.} \\ 9 &=& k + 0 & \\ 9 &=& k & \text{The solution is 9.} \end{array}$$

Check $2 + 3 = k - 4$
$2 + 3 = 9 - 4$
$5 = 5$ Balances

33.
$$\begin{array}{rcll} 10 + b &=& -14 - 6 & \\ 10 + b &=& -14 + (-6) & \textit{Change to addition.} \\ 10 + b &=& -20 & \textit{Add.} \\ \underline{-10} & & \underline{-10} & \textit{Add } -10. \\ 0 + b &=& -30 & \\ b &=& -30 & \text{The solution is } -30. \end{array}$$

Check
$10 + b = -14 - 6$
$10 + (-30) = -14 + (-6)$ *Replace b with −30.*
$-20 = -20$ Balances

34.
$$\begin{array}{rcll} 1 + w &=& -8 - 8 & \\ 1 + w &=& -8 + (-8) & \\ 1 + w &=& -16 & \\ \underline{-1} & & \underline{-1} & \textit{Add } -1 \textit{ to both sides.} \\ 0 + w &=& -17 & \\ w &=& -17 & \text{The solution is } -17. \end{array}$$

Check
$1 + w = -8 - 8$
$1 + (-17) = -8 + (-8)$ *Replace w with −17.*
$-16 = -16$ Balances

35.
$$\begin{array}{rcll} t - 2 &=& 3 - 5 & \\ t + (-2) &=& 3 + (-5) & \textit{Change to addition.} \\ t + (-2) &=& -2 & \textit{Simplify the right side.} \\ \underline{2} & & \underline{2} & \textit{Add 2 to both sides.} \\ t + 0 &=& 0 & \\ t &=& 0 & \text{The solution is 0.} \end{array}$$

Check
$t - 2 = 3 - 5$
$0 - 2 = 3 - 5$ *Replace t with 0.*
$0 + (-2) = 3 + (-5)$ *Change to addition.*
$-2 = -2$ Balances

36.
$$\begin{array}{rcll} p - 8 &=& -10 + 2 & \\ p - 8 &=& -8 & \\ p + (-8) &=& -8 & \\ \underline{8} & & \underline{8} & \textit{Add 8 to both sides.} \\ p + 0 &=& 0 & \\ p &=& 0 & \text{The solution is 0.} \end{array}$$

Check $p - 8 = -10 + 2$
$0 - 8 = -10 + 2$ *Replace p with 0.*
$-8 = -8$ Balances

37.
$$\begin{aligned} 10z - 9z &= -15 + 8 \\ 10z + (-9z) &= -15 + 8 && \textit{Change to addition.} \\ 1z &= -7 && \textit{Combine like terms.} \\ z &= -7 && \textit{1z is the same as z.} \\ & && \text{The solution is } -7. \end{aligned}$$

Check
$$\begin{aligned} 10z - 9z &= -15 + 8 \\ 10 \cdot (-7) - 9 \cdot (-7) &= -15 + 8 && \textit{Replace z with } -7. \\ -70 - (-63) &= -7 \\ -70 + 63 &= -7 && \textit{Change to add.} \\ -7 &= -7 && \text{Balances} \end{aligned}$$

38.
$$\begin{aligned} 2r - r &= 5 - 10 \\ 2r + (-1r) &= 5 + (-10) \\ 1r &= -5 \\ r &= -5 && \text{The solution is } -5. \end{aligned}$$

Check
$$\begin{aligned} 2r - r &= 5 - 10 \\ 2 \cdot (-5) - (-5) &= 5 - 10 && \textit{Replace r with } -5. \\ -10 + 5 &= 5 + (-10) \\ -5 &= -5 && \text{Balances} \end{aligned}$$

39.
$$\begin{aligned} -5w + 2 + 6w &= -4 + 9 && \textit{Rearrange and combine like terms.} \\ \underbrace{-5w + 6w}_{1w} + 2 &= \underbrace{-4 + 9}_{5} \\ -2 &\quad -2 && \textit{Add } -2 \textit{ to both sides.} \\ \hline 1w + 0 &= 3 \\ w &= 3 && \text{The solution is 3.} \end{aligned}$$

Check
$$\begin{aligned} -5w + 2 + 6w &= -4 + 9 \\ -5 \cdot 3 + 2 + 6 \cdot 3 &= -4 + 9 && \textit{Replace w with 3.} \\ -15 + 2 + 18 &= -4 + 9 \\ 5 &= 5 && \text{Balances} \end{aligned}$$

40.
$$\begin{aligned} -2t + 4 + 3t &= 6 - 7 \\ 1t + 4 &= 6 + (-7) \\ t + 4 &= -1 \\ -4 &\quad -4 && \textit{Add } -4 \textit{ to both sides.} \\ \hline t + 0 &= -5 \\ t &= -5 && \text{The solution is } -5. \end{aligned}$$

Check
$$\begin{aligned} -2t + 4 + 3t &= 6 - 7 \\ -2(-5) + 4 + 3(-5) &= 6 - 7 && \textit{Let } t = -5. \\ 10 + 4 + (-15) &= 6 + (-7) \\ 14 + (-15) &= -1 \\ -1 &= -1 && \text{Balances} \end{aligned}$$

41.
$$\begin{aligned} -3 - 3 &= 4 - 3x + 4x && \textit{Change to addition.} \\ \underbrace{-3 + (-3)}_{-6} &= 4 + \underbrace{(-3x) + 4x}_{} && \textit{Combine like terms.} \\ -6 &= 4 + 1x \\ -4 &\quad -4 && \textit{Add } -4 \textit{ to both sides.} \\ \hline -10 &= 0 + 1x \\ -10 &= 1x && \textit{1x is the same as x.} \\ -10 &= x && \text{The solution is } -10. \end{aligned}$$

42.
$$\begin{aligned} -5 - 5 &= -2 - 6b + 7b \\ -5 + (-5) &= -2 + (-6b) + 7b \\ -10 &= -2 + 1b \\ -10 &= -2 + b \\ 2 &\quad 2 && \textit{Add 2.} \\ \hline -8 &= 0 + b \\ -8 &= b \end{aligned}$$

The solution is -8.

43.
$$\begin{aligned} -3 + 7 - 4 &= -2a + 3a \\ -3 + 7 + (-4) &= -2a + 3a && \textit{Change to addition.} \\ 0 &= 1a && \textit{Combine like terms.} \\ 0 &= a && \text{The solution is 0.} \end{aligned}$$

44.
$$\begin{aligned} 6 - 11 + 5 &= -8c + 9c \\ 6 + (-11) + 5 &= -8c + 9c && \textit{Change to addition.} \\ 0 &= 1c && \textit{Combine like terms.} \\ 0 &= c && \text{The solution is 0.} \end{aligned}$$

45.
$$\begin{aligned} y - 75 &= -100 \\ y + (-75) &= -100 && \textit{Change to addition.} \\ 75 &\quad 75 && \textit{Add 75 to both sides.} \\ \hline y + 0 &= -25 \\ y &= -25 && \text{The solution is } -25. \end{aligned}$$

46.
$$\begin{aligned} a - 200 &= -100 \\ a + (-200) &= -100 && \textit{Change to addition.} \\ 200 &\quad 200 && \textit{Add 200 to both sides.} \\ \hline a + 0 &= 100 \\ a &= 100 && \text{The solution is 100.} \end{aligned}$$

47.
$$\begin{aligned} -x + 3 + 2x &= 18 && \textit{Rearrange and combine like terms.} \\ 1x + 3 &= 18 \\ -3 &\quad -3 && \textit{Add } -3 \textit{ to both sides.} \\ \hline 1x + 0 &= 15 \\ x &= 15 && \text{The solution is 15.} \end{aligned}$$

48.
$$\begin{aligned} -s + 2s - 4 &= 13 \\ -1s + 2s + (-4) &= 13 \\ 1s + (-4) &\quad 13 \\ 4 &\quad 4 && \textit{Add 4 to both sides.} \\ \hline 1s + 0 &= 17 \\ s &= 17 && \text{The solution is 17.} \end{aligned}$$

49.
$$\begin{aligned} 82 &= -31 + k \\ 31 &\quad 31 && \textit{Add 31 to both sides.} \\ \hline 113 &= 0 + k \\ 113 &= k && \text{The solution is 113.} \end{aligned}$$

50.
$$\begin{aligned} -5 &= 72 + w \\ -72 &\quad -72 && \textit{Add } -72 \textit{ to both sides.} \\ \hline -77 &= 0 + w \\ -77 &= w && \text{The solution is } -77. \end{aligned}$$

51.
$$\begin{aligned}
-2+11 &= 2b-9-b \\
-2+11 &= 2b+(-9)+(-1b) && \textit{Change to addition.} \\
9 &= 1b+(-9) && \textit{Rearrange and combine like terms.} \\
\underline{9} &\quad \underline{9} && \textit{Add 9 to both sides.} \\
18 &= 1b+0 \\
18 &= b && \text{The solution is 18.}
\end{aligned}$$

52.
$$\begin{aligned}
-6+7 &= 2h-1-h \\
-6+7 &= 2h+(-1)+(-1h) \\
1 &= 1h+(-1) \\
\underline{1} &\quad \underline{1} && \textit{Add 1.} \\
2 &= 1h+\ 0 \\
2 &= h
\end{aligned}$$

The solution is 2.

53.
$$\begin{aligned}
r-6 &= 7-10-8 \\
r+(-6) &= 7+(-10)+(-8) && \textit{Change to addition.} \\
r+(-6) &= -11 && \textit{Combine like terms.} \\
\underline{6} &\quad \underline{6} && \textit{Add 6 to both sides.} \\
r+0 &= -5 \\
r &= -5 && \text{The solution is } -5.
\end{aligned}$$

54.
$$\begin{aligned}
m-5 &= 2-9+1 \\
m+(-5) &= 2+(-9)+1 && \textit{Change to addition.} \\
m+(-5) &= -6 && \textit{Combine like terms.} \\
\underline{5} &\quad \underline{5} && \textit{Add 5 to both sides.} \\
m+0 &= -1 \\
m &= -1 && \text{The solution is } -1.
\end{aligned}$$

55.
$$\begin{aligned}
-14 &= n+91 \\
\underline{-91} &\quad \underline{-91} && \textit{Add } {-91} \textit{ to both sides.} \\
-105 &= n+0 \\
-105 &= n && \text{The solution is } -105.
\end{aligned}$$

56.
$$\begin{aligned}
66 &= x-28 \\
66 &= x+(-28) && \textit{Change to addition.} \\
\underline{28} &\quad \underline{28} && \textit{Add 28 to both sides.} \\
94 &= x+\ 0 \\
94 &= x && \text{The solution is 94.}
\end{aligned}$$

57.
$$\begin{aligned}
-9+9 &= 5+h \\
0 &= 5+h && \textit{Combine like terms.} \\
\underline{-5} &\quad \underline{-5} && \textit{Add } {-5} \textit{ to both sides.} \\
-5 &= 0+h \\
-5 &= h && \text{The solution is } -5.
\end{aligned}$$

58.
$$\begin{aligned}
18-18 &= 6+p \\
0 &= 6+p \\
\underline{-6} &\quad \underline{-6} && \textit{Add } {-6} \textit{ to both sides.} \\
-6 &= 0+p \\
-6 &= p && \text{The solution is } -6.
\end{aligned}$$

59. No, the solution is -14, the number used to replace x in the original equation.

60. Check
$$\begin{aligned}
-3-6 &= n-5 \\
\underbrace{-3+(-6)}_{-9} &\ \underset{\neq}{=}\ \underbrace{-2+(-5)}_{-7} && \text{Does not balance}
\end{aligned}$$

To correct the errors, change $-3-6$ to $-3+(-6)$. Then, add 5 to both sides, not -5. The correct solution is -4.

61.
$$\begin{aligned}
g+10 &= 305 \\
\underline{-10} &\quad \underline{-10} && \textit{Add the opposite of 10, } {-10}\textit{, to both sides.} \\
g+0 &= 295 \\
g &= 295
\end{aligned}$$

There were 295 graduates this year.

62.
$$\begin{aligned}
g+10 &= 278 \\
\underline{-10} &\quad \underline{-10} && \textit{Add the opposite of 10, } {-10}\textit{, to both sides.} \\
g+0 &= 268 \\
g &= 268
\end{aligned}$$

There were 268 graduates last year.

63.
$$\begin{aligned}
92 &= c+37 \\
\underline{-37} &\quad \underline{-37} && \textit{Add } {-37} \textit{ to both sides.} \\
55 &= c+0 \\
55 &= c
\end{aligned}$$

When the temperature is 92 degrees, a field cricket chirps 55 times (in 15 seconds).

64.
$$\begin{aligned}
77 &= c+37 \\
\underline{-37} &\quad \underline{-37} && \textit{Add } {-37} \textit{ to both sides.} \\
40 &= c+0 \\
40 &= c
\end{aligned}$$

When the temperature is 77 degrees, a field cricket chirps 40 times (in 15 seconds).

65.
$$\begin{aligned}
p-65 &= 45 \\
p+(-65) &= 45 && \textit{Change to addition.} \\
\underline{65} &\quad \underline{65} && \textit{Add 65 to both sides.} \\
p+0 &= 110 \\
p &= 110
\end{aligned}$$

Ernesto's parking fees average \$110 per month in winter.

66.
$$\begin{aligned}
p-56 &= 98 \\
p+(-56) &= 98 && \textit{Change to addition.} \\
\underline{56} &\quad \underline{56} && \textit{Add 56 to both sides.} \\
p+0 &= 154 \\
p &= 154
\end{aligned}$$

Aimee's parking fees average \$154 per month in winter.

67.
$$\begin{aligned} -17-1+26-38 &= -3-m-8+2m \\ -17+(-1)+26+(-38) &= -3+(-1m)+(-8)+2m \end{aligned}$$
Change all subtractions to additions.
$$-17+(-1)+26+(-38) = -3+(-8)+(-1m)+2m$$
Commutative property
$$\begin{aligned} -30 &= -11+1m && \textit{Combine like terms.} \\ \underline{11} & \quad \underline{11} && \textit{Add 11 to both sides.} \\ -19 &= 1m \\ -19 &= m && \text{The solution is } -19. \end{aligned}$$

68.
$$\begin{aligned} 19-38-9+11 &= -t-6+2t-6 \\ 19+(-38)+(-9)+11 &= -1t+(-6)+2t+(-6) \\ -17 &= 1t+(-12) \\ \underline{12} & \quad \underline{12} \\ -5 &= 1t+0 \\ -5 &= t \quad \text{The solution is } -5. \end{aligned}$$

69. $-6x+2x+6+5x = |0-9|-|-6+5|$
$-6x+2x+5x+6 = |0+(-9)|-|-6+5|$
Change subtraction within absolute value to addition and rearrange the terms.
$1x+6 = |-9|-|-1|$
Simplify inside absolute value bars. Collect like terms.
$$\begin{aligned} 1x+6 &= 9-1 && \textit{Evaluate absolute values.} \\ 1x+6 &= 9+(-1) && \textit{Change to addition.} \\ 1x+6 &= 8 \\ \underline{-6} & \quad \underline{-6} && \textit{Add } -6 \textit{ to both sides.} \\ 1x+0 &= 2 \\ x &= 2 && \text{The solution is 2.} \end{aligned}$$

70.
$$\begin{aligned} -h-|-9-9|+8h-6h &= -12-|-5+0| \\ -h-|-9+(-9)|+8h+(-6h) &= -12-|-5| \\ -1h-|-18|+8h+(-6h) &= -12-5 \\ -1h-18+8h+(-6h) &= -12+(-5) \\ 1h-18 &= -17 \\ 1h+(-18) &= -17 \\ \underline{18} & \quad \underline{18} \\ h+0 &= 1 \\ h &= 1 \end{aligned}$$
The solution is 1.

Relating Concepts (Exercises 71–72)

71. **(a)** Equations will vary. Some possibilities are:
$$\begin{aligned} n-1 &= -3 \\ n+(-1) &= -3 && \textit{Change to addition.} \\ \underline{1} & \quad \underline{1} && \textit{Add 1 to both sides.} \\ n+0 &= -2 \\ n &= -2 && \text{The solution is } -2. \end{aligned}$$
$$\begin{aligned} 8 &= x+10 \\ \underline{-10} & \quad \underline{-10} && \textit{Add the opposite of 10, } -10\textit{, to both sides.} \\ -2 &= x+0 \\ -2 &= x && \text{The solution is } -2. \end{aligned}$$

(b) Equations will vary. Some possibilities are:
$$\begin{aligned} y+6 &= 6 \\ \underline{-6} & \quad \underline{-6} && \textit{Add the opposite of 6, } -6\textit{, to both sides.} \\ y+0 &= 0 \\ y &= 0 && \text{The solution is 0.} \end{aligned}$$
$$\begin{aligned} -5 &= -5+b \\ \underline{5} & \quad \underline{5} && \textit{Add the opposite of } -5\textit{, 5, to both sides.} \\ 0 &= 0+b \\ 0 &= b && \text{The solution is 0.} \end{aligned}$$

72. **(a)**
$$\begin{aligned} x+1 &= 1\tfrac{1}{2} \\ \underline{-1} & \quad \underline{-1} && \textit{Add the opposite of 1, } -1\textit{, to both sides.} \\ x+0 &= \tfrac{1}{2} \\ x &= \tfrac{1}{2} && \text{The solution is } \tfrac{1}{2}. \end{aligned}$$

(b)
$$\begin{aligned} \tfrac{1}{4} &= y-1 \\ \tfrac{1}{4} &= y+(-1) && \textit{Change to addition.} \\ \underline{1} & \quad \underline{1} && \textit{Add the opposite of } -1\textit{, 1, to both sides.} \\ 1\tfrac{1}{4} &= y+0 \\ 1\tfrac{1}{4} &= y \quad \text{or} \quad y = \tfrac{5}{4} && \text{The solution is } \tfrac{5}{4}. \end{aligned}$$

(c)
$$\begin{aligned} \$2.50+n &= \$3.35 \\ \underline{-\$2.50} & \quad \underline{-\$2.50} && \textit{Add the opposite of \$2.50, } -\$2.50\textit{, to both sides.} \\ \$0+n &= \$0.85 \\ n &= \$0.85 \end{aligned}$$
The solution is \$0.85.

(d) Equations will vary. Some possibilities are:
$a-\$7.32 = \$9.16$ The solution is \$16.48.
$5c-\$11.20 = 4c-\$2.00$ The solution is \$9.20.

2.4 Solving Equations Using Division

2.4 Margin Exercises

1. **(a)** Solve $4s = 44$.

Use division to undo multiplication. Divide *both* sides by the coefficient of the variable, which is 4.
$$\frac{4s}{4} = \frac{44}{4}$$
$$s = \underline{11}$$
The solution is 11.

Check $4s = 44$ *Original equation*
$\underbrace{4 \cdot \underline{11}} = 44$ *Replace s with $\underline{11}$.*
$\underline{44} = 44$ Balances

(b) $27 = -9p$
$\dfrac{27}{-9} = \dfrac{-9p}{-9}$ *Divide both sides by −9.*
$-3 = p$

The solution is -3.

Check $27 = -9p$
$27 = -9 \cdot (-3)$ *Replace p with −3.*
$27 = 27$ Balances

(c) $-40 = -5x$
$\dfrac{-40}{-5} = \dfrac{-5x}{-5}$ *Divide both sides by −5.*
$8 = x$

The solution is 8.

Check $-40 = -5x$
$-40 = -5 \cdot 8$ *Replace x with 8.*
$-40 = -40$ Balances

(d) $7t = -70$
$\dfrac{7t}{7} = \dfrac{-70}{7}$ *Divide both sides by 7.*
$t = -10$

The solution is -10.

Check $7t = -70$
$7 \cdot (-10) = -70$ *Replace t with −10.*
$-70 = -70$ Balances

2. **(a)** $-28 = -6n + 10n$
$-28 = 4n$ *Combine like terms.*
$\dfrac{-28}{4} = \dfrac{4n}{4}$ *Divide both sides by 4.*
$-7 = n$ The solution is -7.

Check $-28 = -6n + 10n$
$-28 = -6 \cdot (-7) + 10 \cdot (-7)$
Replace n with −7.
$-28 = 42 + (-70)$
$-28 = -28$ Balances

(b) $p - 14p = -2 + 18 - 3$
$1p + (-14p) = -2 + 18 + (-3)$
Change to addition. Rewrite p as 1p.
$-13p = 13$
Combine like terms.
$\dfrac{-13p}{-13} = \dfrac{13}{-13}$
Divide both sides by −13.
$p = -1$

The solution is -1.

Check $p - 14p = -2 + 18 - 3$
$-1 - 14(-1) = 16 - 3$
Replace p with −1.
$-1 - (-14) = 13$
$-1 + (+14) = 13$
$13 = 13$ Balances

3. **(a)** $-k = -12$
$-1k = -12$ *Write in the understood −1 as the coefficient of k.*
$\dfrac{-1k}{-1} = \dfrac{-12}{-1}$ *Divide both sides by −1.*
$k = 12$ The solution is 12.

Check $-k = -12$
$-1k = -12$
$-1 \cdot 12 = -12$ *Replace k with 12.*
$-12 = -12$ Balances

(b) $7 = -t$
$7 = -1t$ *Write −t as −1t.*
$\dfrac{7}{-1} = \dfrac{-1t}{-1}$ *Divide both sides by −1.*
$-7 = t$ The solution is -7.

Check $7 = -t$
$7 = -1t$
$7 = -1 \cdot (-7)$ *Replace t with −7.*
$7 = 7$ Balances

(c) $-m = -20$
$-1m = -20$ *Write −m as −1m.*
$\dfrac{-1m}{-1} = \dfrac{-20}{-1}$ *Divide both sides by −1.*
$m = 20$ The solution is 20.

Check $-m = -20$
$-1m = -20$
$-1 \cdot 20 = -20$ *Replace m with 20.*
$-20 = -20$ Balances

2.4 Section Exercises

1. $6z = 12$
$\dfrac{6z}{6} = \dfrac{12}{6}$ *Divide both sides by $\underline{6}$.*
$z = \underline{2}$ The solution is 2.

Check $6z = 12$
$\underbrace{6 \cdot 2} = 12$ *Replace z with 2.*
$\underline{12} = \underline{12}$ Balances

2. $8k = 24$
$\dfrac{8k}{8} = \dfrac{24}{\underline{8}}$
$k = \underline{3}$

Check $8k = 24$
$\underbrace{8 \cdot \underline{3}} = 24$
$\underline{24} = \underline{24}$
Balances

The solution is 3.

3. $48 = 12r$

$\dfrac{48}{12} = \dfrac{12r}{12}$ *Divide both sides by 12.*

$4 = r$ The solution is 4.

Check $48 = 12r$

$48 = 12 \cdot 4$ *Replace r with 4.*

$48 = 48$ Balances

4. $99 = 11m$

$\dfrac{99}{11} = \dfrac{11m}{11}$

$9 = m$

Check $99 = 11m$

$99 = 11 \cdot 9$

$99 = 99$

Balances

The solution is 9.

5. $3y = 0$

$\dfrac{3y}{3} = \dfrac{0}{3}$ *Divide both sides by 3.*

$y = 0$ The solution is 0.

Check $3y = 0$

$3 \cdot 0 = 0$ *Replace y with 0.*

$0 = 0$ Balances

6. $5a = 0$

$\dfrac{5a}{5} = \dfrac{0}{5}$

$a = 0$

Check $5a = 0$

$5 \cdot 0 = 0$

$0 = 0$

Balances

The solution is 0.

7. $-7k = 70$

$\dfrac{-7k}{-7} = \dfrac{70}{-7}$ *Divide both sides by −7.*

$k = -10$ The solution is -10.

Check $-7k = 70$

$-7 \cdot (-10) = 70$ *Replace k with −10.*

$70 = 70$ Balances

8. $-6y = 36$

$\dfrac{-6y}{-6} = \dfrac{36}{-6}$

$y = -6$

Check $-6y = 36$

$-6 \cdot (-6) = 36$

$36 = 36$

Balances

The solution is -6.

9. $-54 = -9r$

$\dfrac{-54}{-9} = \dfrac{-9r}{-9}$ *Divide both sides by −9.*

$6 = r$ The solution is 6.

Check $-54 = -9r$

$-54 = -9 \cdot 6$ *Replace r with 6.*

$-54 = -54$ Balances

10. $-36 = -4p$

$\dfrac{-36}{-4} = \dfrac{-4p}{-4}$

$9 = p$

Check $-36 = -4p$

$-36 = -4 \cdot 9$

$-36 = -36$

Balances

The solution is 9.

11. $-25 = 5b$

$\dfrac{-25}{5} = \dfrac{5b}{5}$ *Divide both sides by 5.*

$-5 = b$ The solution is -5.

Check $-25 = 5b$

$-25 = 5 \cdot (-5)$ *Replace b with −5.*

$-25 = -25$ Balances

12. $-70 = 10x$

$\dfrac{-70}{10} = \dfrac{10x}{10}$

$-7 = x$

Check $-70 = 10x$

$-70 = 10 \cdot (-7)$

$-70 = -70$

Balances

The solution is -7.

13. $2r = -7 + 13$

$2r = 6$ *Combine like terms.*

$\dfrac{2r}{2} = \dfrac{6}{2}$ *Divide both sides by 2.*

$r = 3$ The solution is 3.

Check $2r = -7 + 13$

$2 \cdot 3 = -7 + 13$ *Replace r with 3.*

$6 = 6$ Balances

14. $6y = 28 - 4$

$6y = 24$

$\dfrac{6y}{6} = \dfrac{24}{6}$

$y = 4$

Check $6y = 28 - 4$

$6 \cdot 4 = 28 - 4$

$24 = 24$

Balances

The solution is 4.

15. $-12 = 5p - p$

$-12 = 5p + (-p)$ *Change to addition.*

$-12 = 5p + (-1p)$ *Rewrite −p as −1p.*

$-12 = 4p$ *Combine like terms.*

$\dfrac{-12}{4} = \dfrac{4p}{4}$ *Divide both sides by 4.*

$-3 = p$ The solution is -3.

Check

$-12 = 5p - p$

$-12 = 5 \cdot (-3) - (-3)$ *Replace p with −3.*

$-12 = -15 - (-3)$

$-12 = -15 + 3$ *Change to addition.*

$-12 = -12$ Balances

16. $20 = z - 11z$

$20 = 1z + (-11z)$ *Change to addition.*

$20 = -10z$ *Combine like terms.*

$\dfrac{20}{-10} = \dfrac{-10z}{-10}$ *Divide both sides by −10.*

$-2 = z$ The solution is -2.

Check

$20 = z - 11z$

$20 = -2 - 11 \cdot (-2)$ *Replace z with −2.*

$20 = -2 - (-22)$

$20 = -2 + 22$ *Change to addition.*

$20 = 20$ Balances

17. $3 - 28 = 5a$ *Original equation*

$3 + (-28) = 5a$ *Change to addition.*

$-25 = 5a$ *Combine like terms.*

$\frac{-25}{5} = \frac{5a}{5}$ *Divide both sides by 5.*

$-5 = a$ The solution is -5.

18. $-55 + 7 = 8n$ *Original equation*

$-48 = 8n$ *Combine like terms.*

$\frac{-48}{8} = \frac{8n}{8}$ *Divide both sides by 8.*

$-6 = n$ The solution is -6.

19. $x - 9x = 80$ *Original equation*

$x + (-9x) = 80$ *Change to addition.*

$1x + (-9x) = 80$ *Rewrite x as 1x.*

$-8x = 80$ *Combine like terms.*

$\frac{-8x}{-8} = \frac{80}{-8}$ *Divide both sides by −8.*

$x = -10$ The solution is -10.

20. $4c - c = -27$ *Original equation*

$4c + (-c) = -27$ *Change to addition.*

$4c + (-1c) = -27$ *Rewrite −c as −1c.*

$3c = -27$ *Combine like terms.*

$\frac{3c}{3} = \frac{-27}{3}$ *Divide both sides by 3.*

$c = -9$ The solution is -9.

21. $13 - 13 = 2w - w$ *Original equation*

$13 + (-13) = 2w + (-w)$ *Change to addition.*

$13 + (-13) = 2w + (-1w)$ *Rewrite −w as −1w.*

$0 = 1w$ *Combine like terms. 1w is the same as w.*

$0 = w$ The solution is 0.

22. $-11 + 11 = 8t - 7t$ *Original equation*

$-11 + 11 = 8t + (-7t)$ *Change to addition.*

$0 = 1t$ *Combine like terms. 1t is the same as t.*

$0 = t$ The solution is 0.

23. $3t + 9t = 20 - 10 + 26$ *Original equation*

$3t + 9t = 20 + (-10) + 26$ *Change to addition.*

$12t = 36$ *Combine like terms.*

$\frac{12t}{12} = \frac{36}{12}$ *Divide both sides by 12.*

$t = 3$ The solution is 3.

24. $6m + 6m = 40 + 20 - 12$ *Original equation*

$6m + 6m = 60 + (-12)$ *Change to addition.*

$12m = 48$ *Combine like terms.*

$\frac{12m}{12} = \frac{48}{12}$ *Divide both sides by 12.*

$m = 4$ The solution is 4.

25. $0 = -9t$ *Original equation*

$\frac{0}{-9} = \frac{-9t}{-9}$ *Divide both sides by −9.*

$0 = t$ The solution is 0.

26. $-10 = 10b$ *Original equation*

$\frac{-10}{10} = \frac{10b}{10}$ *Divide both sides by 10.*

$-1 = b$ The solution is -1.

27. $-14m + 8m = 6 - 60$ *Original equation*

$-14m + 8m = 6 + (-60)$ *Change to addition.*

$-6m = -54$ *Combine like terms.*

$\frac{-6m}{-6} = \frac{-54}{-6}$ *Divide both sides by −6.*

$m = 9$ The solution is 9.

28. $7w - 14w = 1 - 50 + 49$ *Original eq.*

$7w + (-14w) = 1 + (-50) + 49$

$-7w = 0$ *Combine.*

$\frac{-7w}{-7} = \frac{0}{-7}$ *Divide both sides by −7.*

$w = 0$ The solution is 0.

29. $100 - 96 = 31y - 35y$ *Original equation*

$100 + (-96) = 31y + (-35y)$ *Change to addition.*

$4 = -4y$ *Combine like terms.*

$\frac{4}{-4} = \frac{-4y}{-4}$ *Divide both sides by −4.*

$-1 = y$ The solution is -1.

30. $150 - 139 = 20x - 9x$ *Original equation*

$150 + (-139) = 20x + (-9x)$ *Change to addition.*

$11 = 11x$ *Combine like terms.*

$\frac{11}{11} = \frac{11x}{11}$ *Divide both sides by 11.*

$1 = x$ The solution is 1.

31. $3(2z) = -30$ *Original equation*

$(3 \cdot 2) \cdot z = -30$ *To multiply on the left, use the associative property.*

$6z = -30$

$\frac{6z}{6} = \frac{-30}{6}$ *Divide both sides by 6.*

$z = -5$ The solution is -5.

32.
$$\begin{aligned} 2(4k) &= 16 && \textit{Original equation} \\ (2\cdot 4)\cdot k &= 16 && \textit{To multiply on the left, use the associative property.} \\ 8k &= 16 \\ \frac{8k}{8} &= \frac{16}{8} && \textit{Divide both sides by 8.} \\ k &= 2 && \text{The solution is 2.} \end{aligned}$$

33.
$$\begin{aligned} 50 &= -5(5p) && \textit{Original equation} \\ 50 &= (-5\cdot 5)\cdot p && \textit{To multiply on the right, use the associative prop.} \\ 50 &= -25p \\ \frac{50}{-25} &= \frac{-25p}{-25} && \textit{Divide both sides by } -25. \\ -2 &= p && \text{The solution is } -2. \end{aligned}$$

34.
$$\begin{aligned} 60 &= 4(-3a) && \textit{Original equation} \\ 60 &= [4\cdot(-3)]\cdot a && \textit{To multiply on the right, use the associative prop.} \\ 60 &= -12a \\ \frac{60}{-12} &= \frac{-12a}{-12} && \textit{Divide both sides by } -12. \\ -5 &= a && \text{The solution is } -5. \end{aligned}$$

35.
$$\begin{aligned} -2(-4k) &= 56 && \textit{Original equation} \\ [-2\cdot(-4)]\cdot k &= 56 && \textit{Associative property} \\ 8k &= 56 \\ \frac{8k}{8} &= \frac{56}{8} && \textit{Divide both sides by 8.} \\ k &= 7 && \text{The solution is 7.} \end{aligned}$$

36.
$$\begin{aligned} -5(4r) &= -80 && \textit{Original equation} \\ (-5\cdot 4)\cdot r &= -80 && \textit{Associative property} \\ -20r &= -80 \\ \frac{-20r}{-20} &= \frac{-80}{-20} && \textit{Divide both sides by } -20. \\ r &= 4 && \text{The solution is 4.} \end{aligned}$$

37.
$$\begin{aligned} -90 &= -10(-3b) && \textit{Original equation} \\ -90 &= [-10\cdot(-3)]\cdot b && \textit{Associative property} \\ -90 &= 30b \\ \frac{-90}{30} &= \frac{30b}{30} && \textit{Divide both sides by 30.} \\ -3 &= b && \text{The solution is } -3. \end{aligned}$$

38.
$$\begin{aligned} -90 &= -5(-2y) && \textit{Original equation} \\ -90 &= [-5\cdot(-2)]\cdot y && \textit{Associative property} \\ -90 &= 10y \\ \frac{-90}{10} &= \frac{10y}{10} && \textit{Divide both sides by 10.} \\ -9 &= y && \text{The solution is } -9. \end{aligned}$$

39.
$$\begin{aligned} -x &= 32 && \textit{Original equation} \\ -1x &= 32 && \textit{Write in the understood } -1. \\ \frac{-1x}{-1} &= \frac{32}{-1} && \textit{Divide both sides by } -1. \\ x &= -32 && \text{The solution is } -32. \end{aligned}$$

40.
$$\begin{aligned} -c &= 23 && \textit{Original equation} \\ -1c &= 23 && \textit{Write in the understood } -1. \\ \frac{-1c}{-1} &= \frac{23}{-1} && \textit{Divide both sides by } -1. \\ c &= -23 && \text{The solution is } -23. \end{aligned}$$

41.
$$\begin{aligned} -2 &= -w && \textit{Original equation} \\ -2 &= -1w && \textit{Write in the understood } -1. \\ \frac{-2}{-1} &= \frac{-1w}{-1} && \textit{Divide both sides by } -1. \\ 2 &= w && \text{The solution is 2.} \end{aligned}$$

42.
$$\begin{aligned} -75 &= -t && \textit{Original equation} \\ -75 &= -1t && \textit{Write in the understood } -1. \\ \frac{-75}{-1} &= \frac{-1t}{-1} && \textit{Divide both sides by } -1. \\ 75 &= t && \text{The solution is 75.} \end{aligned}$$

43.
$$\begin{aligned} -n &= -50 && \textit{Original equation} \\ -1n &= -50 && \textit{Write in the understood } -1. \\ \frac{-1n}{-1} &= \frac{-50}{-1} && \textit{Divide both sides by } -1. \\ n &= 50 && \text{The solution is 50.} \end{aligned}$$

44.
$$\begin{aligned} -x &= -1 && \textit{Original equation} \\ -1x &= -1 && \textit{Write in the understood } -1. \\ \frac{-1x}{-1} &= \frac{-1}{-1} && \textit{Divide both sides by } -1. \\ x &= 1 && \text{The solution is 1.} \end{aligned}$$

45.
$$\begin{aligned} 10 &= -p && \textit{Original equation} \\ 10 &= -1p && \textit{Write in the understood } -1. \\ \frac{10}{-1} &= \frac{-1p}{-1} && \textit{Divide both sides by } -1. \\ -10 &= p && \text{The solution is } -10. \end{aligned}$$

46.
$$\begin{aligned} 100 &= -k && \textit{Original equation} \\ 100 &= -1k && \textit{Write in the understood } -1. \\ \frac{100}{-1} &= \frac{-1k}{-1} && \textit{Divide both sides by } -1. \\ -100 &= k && \text{The solution is } -100. \end{aligned}$$

47. Each solution is the opposite of the number in the equation. So the rule is: When you change the sign of the variable from negative to positive, then change the number in the equation to its opposite. In $-x = 5$, the opposite of 5 is -5, so $x = -5$.

48. Equations will vary. Some possibilities are **(i)** $-5x = 20$ and **(ii)** $12 - 20 = 2x$.

(i)
$$\begin{aligned} -5x &= 20 \\ \frac{-5x}{-5} &= \frac{20}{-5} && \textit{Divide both sides by } -5. \\ x &= -4 && \text{The solution is } -4. \end{aligned}$$

(ii)

$$\begin{aligned} 12-20 &= 2x \\ 12+(-20) &= 2x && \textit{Change to addition.} \\ -8 &= 2x && \textit{Combine like terms.} \\ \frac{-8}{2} &= \frac{2x}{2} && \textit{Divide both sides by 2.} \\ -4 &= x && \text{The solution is } -4. \end{aligned}$$

49. Divide by the coefficient of x, which is 3, *not* by the opposite of 3.

$$\begin{aligned} 3x &= \underbrace{16-1} \\ 3x &= 15 \\ \frac{3x}{3} &= \frac{15}{3} \\ x &= 5 && \text{The correct solution is 5.} \end{aligned}$$

50. You can divide both sides of an equation by the same nonzero number and keep the equation balanced.

51.

$$\begin{aligned} 3s &= 45 \\ \frac{3s}{3} &= \frac{45}{3} && \textit{Divide both sides by 3.} \\ s &= 15 \end{aligned}$$

The length of one side is 15 feet.

52.

$$\begin{aligned} 3s &= 63 \\ \frac{3s}{3} &= \frac{63}{3} && \textit{Divide both sides by 3.} \\ s &= 21 \end{aligned}$$

The length of one side is 21 inches.

53.

$$\begin{aligned} 120 &= 5s \\ \frac{120}{5} &= \frac{5s}{5} && \textit{Divide both sides by 5.} \\ 24 &= s \end{aligned}$$

The length of one side is 24 meters.

54.

$$\begin{aligned} 335 &= 5s \\ \frac{335}{5} &= \frac{5s}{5} && \textit{Divide both sides by 5.} \\ 67 &= s \end{aligned}$$

The length of one side is 67 yards.

55.

$$\begin{aligned} 89-116 &= -4(-4y)-9(2y)+y \\ 89-116 &= [-4\cdot(-4)]\cdot y-(9\cdot 2)\cdot y+y \\ & \qquad \textit{Associative property} \\ 89+(-116) &= 16y+(-18y)+1y \\ & \qquad \textit{Change to addition.} \\ -27 &= -1y && \textit{Combine like terms.} \\ \frac{-27}{-1} &= \frac{-1y}{-1} && \textit{Divide both sides by } -1. \\ 27 &= y \end{aligned}$$

The solution is 27.

56.

$$\begin{aligned} 58-208 &= -b+8(-3b)+5(-5b) \\ 58-208 &= -b+[8\cdot(-3)]\cdot b+[5\cdot(-5)]\cdot b \\ & \qquad \textit{Associative property} \\ 58+(-208) &= -1b+(-24b)+(-25b) \\ & \qquad \textit{Change to addition.} \\ -150 &= -50b && \textit{Combine like terms.} \\ \frac{-150}{-50} &= \frac{-50b}{-50} && \textit{Divide both sides by } -50. \\ 3 &= b \end{aligned}$$

The solution is 3.

57.

$$\begin{aligned} -37(14x)+28(21x) &= |72-72|+|-166+96| \\ (-37\cdot 14)\cdot x+(28\cdot 21)\cdot x &= |0|+|-70| \\ & \qquad \textit{Assoc. prop. Simplify within the absolute values.} \\ -518x+588x &= 0+70 \\ & \qquad \textit{Simplify the absolute values.} \\ 70x &= 70 && \textit{Combine like terms.} \\ \frac{70x}{70} &= \frac{70}{70} && \textit{Divide both sides by 70.} \\ x &= 1 \end{aligned}$$

The solution is 1.

58.

$$\begin{aligned} 6a-10a-3(2a) &= |-25-25|-5(8) \\ 6a+(-10a)-6a &= |-25+(-25)|-40 \\ & \qquad \textit{Simplify within the absolute value.} \\ 6a+(-10a)+(-6a) &= |-50|-40 \\ -10a &= 50-40 \\ & \qquad \textit{Simplify the absolute value.} \\ -10a &= 10 && \textit{Combine like terms.} \\ \frac{-10a}{-10} &= \frac{10}{-10} && \textit{Divide both sides by } -10. \\ a &= -1 \end{aligned}$$

The solution is -1.

2.5 Solving Equations with Several Steps

2.5 Margin Exercises

1. (a)

$$\begin{aligned} 2r+7 &= 13 && \textit{To get 2r by itself,} \\ -7 \quad & \quad -7 && \textit{add } -7 \textit{ to both sides.} \\ \underbrace{2r+0} &= 6 \\ 2r &= 6 \\ \frac{2r}{2} &= \frac{6}{2} && \textit{To solve for r, divide both sides by the coefficient, 2.} \\ r &= \underline{3} && \text{The solution is 3.} \end{aligned}$$

Check

$$\begin{aligned} 2r+7 &= 13 \\ 2\cdot 3+7 &= 13 && \textit{Replace r with 3.} \\ \underbrace{6+7} &= 13 \\ \underline{13} &= 13 && \text{Balances} \end{aligned}$$

(b)

$$
\begin{aligned}
-10z - 9 &= 11 \\
-10z + (-9) &= 11 && \textit{Change to addition.} \\
\underline{9} & \underline{\;\,9\;\,} && \textit{Add 9 to both sides.} \\
-10z + 0 &= 20 \\
-10z &= 20 \\
\frac{-10z}{-10} &= \frac{20}{-10} && \textit{Divide both sides by } -10. \\
z &= -2 && \text{The solution is } -2.
\end{aligned}
$$

Check

$$
\begin{aligned}
-10z - 9 &= 11 \\
\underbrace{-10\cdot(-2)} - 9 &= 11 && \textit{Replace r with } -2. \\
\underbrace{20 \quad - 9} &= 11 \\
11 \quad &= 11 && \text{Balances}
\end{aligned}
$$

2. (a) Solve, keeping the variable on *left* side.

$$
\begin{aligned}
3y - 1 &= 2y + 7 \\
\underline{-2y} & \underline{-2y} && \textit{Add } -2y \textit{ to both sides.} \\
1y - 1 &= 0 + 7 \\
1y + (-1) &= 7 && \textit{Change to addition.} \\
\underline{1} & \underline{1} && \textit{Add 1 to both sides.} \\
1y + 0 &= 8 \\
1y &= 8 \\
\text{or } y &= 8 && \text{The solution is 8.}
\end{aligned}
$$

Solve, keeping the variable on the *right* side.

$$
\begin{aligned}
3y - 1 &= 2y + 7 \\
\underline{-3y} & \underline{-3y} && \textit{Add } -3y \textit{ to both sides.} \\
0 - 1 &= -1y + 7 \\
-1 &= -1y + 7 \\
\underline{-7} & \underline{-7} && \textit{Add } -7 \textit{ to both sides.} \\
-8 &= -1y + 0 \\
-8 &= -1y \\
\frac{-8}{-1} &= \frac{-1y}{-1} && \textit{Divide both sides by } -1. \\
8 &= y && \text{The solution is 8.}
\end{aligned}
$$

(b) Solve, keeping the variable on *left* side.

$$
\begin{aligned}
3p - 2 &= p - 6 \\
3p - 2 &= 1p - 6 && \textit{Rewrite p as 1p.} \\
\underline{-1p} & \underline{-1p} && \textit{Add } -1p \textit{ to both sides.} \\
2p - 2 &= 0 - 6 \\
2p + (-2) &= -6 && \textit{Change to addition.} \\
\underline{2} & \underline{2} && \textit{Add 2 to both sides.} \\
2p + 0 &= -4 \\
2p &= -4 \\
\frac{2p}{2} &= \frac{-4}{2} && \textit{Divide both sides by 2.} \\
p &= -2 && \text{The solution is } -2.
\end{aligned}
$$

Solve, keeping the variable on the *right* side.

$$
\begin{aligned}
3p - 2 &= p - 6 \\
3p - 2 &= 1p - 6 && \textit{Rewrite p as 1p.} \\
\underline{-3p} & \underline{-3p} && \textit{Add } -3p. \\
0 - 2 &= -2p - 6 \\
-2 &= -2p + (-6) \\
\underline{6} & \underline{6} && \textit{Add 6.} \\
4 &= -2p + 0 \\
4 &= -2p \\
\frac{4}{-2} &= \frac{-2p}{-2} && \textit{Divide both sides by } -2. \\
-2 &= p && \text{The solution is } -2.
\end{aligned}
$$

3. (a)

$$
\begin{aligned}
-12 &= 4(y - 1) \\
-12 &= \underbrace{4\cdot y} - \underbrace{4\cdot 1} && \textit{Distribute on the right.} \\
-12 &= 4y - 4 \\
-12 &= 4y + (-4) && \textit{Change to addition.} \\
\underline{4} & \underline{4} && \textit{Add 4 to both sides.} \\
-8 &= 4y + 0 \\
-8 &= 4y \\
\frac{-8}{4} &= \frac{4y}{4} && \textit{Divide both sides by } \underline{4}. \\
-2 &= y && \text{The solution is } -2.
\end{aligned}
$$

Check

$$
\begin{aligned}
-12 &= 4(y - 1) \\
-12 &= 4(-2 - 1) && \textit{Replace y with } -2. \\
-12 &= 4(-3) \\
-12 &= -12 && \text{Balances}
\end{aligned}
$$

(b)

$$
\begin{aligned}
5(m + 4) &= 20 \\
5\cdot m + 5\cdot 4 &= 20 && \textit{Distribute on the left.} \\
5m + 20 &= 20 \\
\underline{-20} & \underline{-20} && \textit{Add } -20 \textit{ to both sides.} \\
5m + 0 &= 0 \\
5m &= 0 \\
\frac{5m}{5} &= \frac{0}{5} && \textit{Divide both sides by 5.} \\
m &= 0 && \text{The solution is 0.}
\end{aligned}
$$

Check

$$
\begin{aligned}
5(m + 4) &= 20 \\
5(0 + 4) &= 20 && \textit{Replace m with 0.} \\
5(4) &= 20 \\
20 &= 20 && \text{Balances}
\end{aligned}
$$

(c)

$$
\begin{aligned}
6(t - 2) &= 18 \\
6\cdot t - 6\cdot 2 &= 18 && \textit{Distribute on the left.} \\
6t - 12 &= 18 \\
6t + (-12) &= 18 && \textit{Change to addition.} \\
\underline{12} & \underline{12} && \textit{Add 12 to both sides.} \\
6t + 0 &= 30 \\
6t &= 30 \\
\frac{6t}{6} &= \frac{30}{6} && \textit{Divide both sides by 6.} \\
t &= 5 && \text{The solution is 5.}
\end{aligned}
$$

Check $6(t-2)=18$
$6(5-2)=18$ *Replace t with 5.*
$6(3)=18$
$18=18$ Balances

4. **(a)**
$$\begin{aligned} 3(b+7) &= 2b-1 && \textit{Distribute.}\\ 3\cdot b+3\cdot 7 &= 2b-1\\ 3b+21 &= 2b+(-1) && \textit{Variables left}\\ -2b\quad & \quad -2b && \textit{Add } -2b.\\ 1b+21 &= 0+(-1)\\ 1b+21 &= -1\\ -21 & \quad -21 && \textit{Add } -21.\\ 1b+0 &= -22\\ 1b &= -22\\ \text{or } b &= -22 \end{aligned}$$

The solution is -22.

Check $3(b+7)=2b-1$
$3(-22+7)=2\cdot(-22)-1$
$3(-15)=-44-1$
$-45=-45$ Balances

(b)
$$\begin{aligned} 6-2n &= 14+4(n-5) && \textit{Distribute.}\\ 6-2n &= 14+4\cdot n-4\cdot 5\\ 6-2n &= 14+4n-20 && \textit{Add the opposite.}\\ 6+(-2n) &= 14+4n+(-20) && \textit{Combine like terms.}\\ 6+(-2n) &= -6+4n\\ 2n\quad & \quad 2n && \textit{Add } 2n.\\ 6+0 &= -6+6n\\ 6 &= -6+6n\\ 6\quad & \quad 6 && \textit{Add 6.}\\ 12 &= 0+6n\\ 12 &= 6n\\ \frac{12}{6} &= \frac{6n}{6} && \textit{Divide both sides by 6.}\\ 2 &= n \end{aligned}$$

The solution is 2.

Check $6-2n=14+4(n-5)$
$6-2\cdot 2=14+4(2-5)$ *Let n = 2.*
$6-4=14+4(-3)$
$2=14+(-12)$
$2=2$ Balances

2.5 Section Exercises

1.
$$\begin{aligned} 7p+5 &= 12 && \textit{To get 7p by itself,}\\ -5\quad & \quad -5 && \textit{add } -5 \textit{ to both sides.}\\ \underbrace{7p+0} &= 7\\ 7p &= 7\\ \frac{7p}{7} &= \frac{7}{7} && \textit{Divide both sides by 7.}\\ p &= 1 && \text{The solution is 1.} \end{aligned}$$

Check $7p+5=12$
$7(1)+5=12$ *Let p = 1.*
$7+5=12$
$\underline{12}=\underline{12}$ Balances

2.
$$\begin{aligned} 6k+3 &= 15\\ -3\quad & \quad -3\\ \underbrace{6k+0} &= 12\\ 6k &= 12\\ \frac{6k}{6} &= \frac{12}{6}\\ k &= \underline{2} \end{aligned}$$

Check $6k+3=15$
$6(2)+3=15$
$12+3=15$
$15=15$
Balances

The solution is 2.

3.
$$\begin{aligned} 2 &= 8y-6\\ 2 &= 8y+(-6) && \textit{Change to addition.}\\ 6\quad & \quad 6 && \textit{Add 6 to both sides.}\\ 8 &= \underbrace{8y+0}\\ 8 &= 8y\\ \frac{8}{8} &= \frac{8y}{8} && \textit{Divide both sides by 8.}\\ 1 &= y && \text{The solution is 1.} \end{aligned}$$

Check $2=8y-6$
$2=8(1)-6$ *Replace y with 1.*
$2=8-6$
$2=2$ Balances

4.
$$\begin{aligned} 10 &= 11p-12\\ 10 &= 11p+(-12)\\ 12\quad & \quad 12\\ 22 &= 11p+0\\ \frac{22}{11} &= \frac{11p}{11}\\ 2 &= p \end{aligned}$$

Check $10=11p-12$
$10=11(2)-12$
$10=22-12$
$10=10$
Balances

The solution is 2.

5.
$$\begin{aligned} 28 &= -9a+10 && \textit{To get } -9a \textit{ by itself,}\\ -10\quad & \quad -10 && \textit{add } -10 \textit{ to both sides.}\\ 18 &= -9a+0\\ 18 &= -9a\\ \frac{18}{-9} &= \frac{-9a}{-9} && \textit{Divide both sides by } -9.\\ -2 &= a && \text{The solution is } -2. \end{aligned}$$

Check $28=-9a+10$
$28=-9(-2)+10$ *Replace a with −2.*
$28=18+10$
$28=28$ Balances

6.
$$\begin{aligned} -4k+5 &= 5\\ -5\quad & \quad -5\\ -4k+0 &= 0\\ \frac{-4k}{-4} &= \frac{0}{-4}\\ k &= 0 \end{aligned}$$

Check $-4k+5=5$
$-4(0)+5=5$
$0+5=5$
$5=5$
Balances

The solution is 0.

7.

$$\begin{aligned} -3m+1 &= 1 && \textit{To get } -3m \textit{ by itself,} \\ -1 \quad & \quad -1 && \textit{add } -1 \textit{ to both sides.} \\ \hline -3m+0 &= 0 \\ -3m &= 0 \\ \frac{-3m}{-3} &= \frac{0}{-3} && \textit{Divide both sides by } -3. \\ m &= 0 && \text{The solution is } 0. \end{aligned}$$

Check

$$\begin{aligned} -3m+1 &= 1 \\ -3(0)+1 &= 1 && \textit{Replace m with 0.} \\ 0+1 &= 1 \\ 1 &= 1 && \text{Balances} \end{aligned}$$

8.

$$\begin{aligned} 75 &= -10w+25 \\ -25 \quad & \quad\quad -25 \\ \hline 50 &= -10w+0 \\ \frac{50}{-10} &= \frac{-10w}{-10} \\ -5 &= w \end{aligned}$$

The solution is -5.

Check

$$\begin{aligned} 75 &= -10w+25 \\ 75 &= -10(-5)+25 \\ 75 &= 50+25 \\ 75 &= 75 && \text{Balances} \end{aligned}$$

9.

$$\begin{aligned} -5x-4 &= 16 && \textit{Change to addition.} \\ -5x+(-4) &= 16 && \textit{To get } -5x \textit{ by itself,} \\ 4 \quad & \quad 4 && \textit{add 4 to both sides.} \\ \hline -5x+0 &= 20 \\ -5x &= 20 \\ \frac{-5x}{-5} &= \frac{20}{-5} && \textit{Divide both sides by } -5. \\ x &= -4 && \text{The solution is } -4. \end{aligned}$$

Check

$$\begin{aligned} -5x-4 &= 16 \\ -5(-4)-4 &= 16 && \textit{Replace x with } -4. \\ 20-4 &= 16 \\ 16 &= 16 && \text{Balances} \end{aligned}$$

10.

$$\begin{aligned} -12b-3 &= 21 \\ -12b+(-3) &= 21 \\ 3 \quad & \quad 3 \\ \hline -12b+0 &= 24 \\ \frac{-12b}{-12} &= \frac{24}{-12} \\ b &= -2 \end{aligned}$$

The solution is -2.

Check

$$\begin{aligned} -12b-3 &= 21 \\ -12(-2)-3 &= 21 \\ 24-3 &= 21 \\ 21 &= 21 \\ & \text{Balances} \end{aligned}$$

11. Solve, keeping the variable on the *left* side.

$$\begin{aligned} 6p-2 &= 4p+6 \\ 6p+(-2) &= 4p+6 && \textit{Change to addition.} \\ -4p \quad & \quad -4p && \textit{Add } -4p \textit{ to both sides.} \\ \hline 2p+(-2) &= 0+6 \\ 2p+(-2) &= 6 \\ 2 \quad & \quad 2 && \textit{Add 2 to both sides.} \\ \hline 2p+0 &= 8 \\ 2p &= 8 \\ \frac{2p}{2} &= \frac{8}{2} && \textit{Divide both sides by 2.} \\ p &= 4 && \text{The solution is } 4. \end{aligned}$$

Solve, keeping the variable on the *right* side.

$$\begin{aligned} 6p-2 &= 4p+6 \\ 6p+(-2) &= 4p+6 && \textit{Change to addition.} \\ -6p \quad & \quad -6p && \textit{Add } -6p \textit{ to both sides.} \\ \hline 0+(-2) &= -2p+6 \\ -2 &= -2p+6 \\ -6 \quad & \quad\quad -6 && \textit{Add } -6 \textit{ to both sides.} \\ \hline -8 &= -2p+0 \\ -8 &= -2p \\ \frac{-8}{-2} &= \frac{-2p}{-2} && \textit{Divide both sides by } -2. \\ 4 &= p && \text{The solution is } 4. \end{aligned}$$

Check

$$\begin{aligned} 6p-2 &= 4p+6 \\ 6(4)-2 &= 4(4)+6 \\ 24-2 &= 16+6 \\ 22 &= 22 && \text{Balances} \end{aligned}$$

12. **Left side:**

$$\begin{aligned} 5y-5 &= 2y+10 \\ -2y \quad & \quad -2y \\ \hline 3y-5 &= 0+10 \\ 3y+(-5) &= 10 \\ 5 \quad & \quad 5 \\ \hline 3y+0 &= 15 \\ \frac{3y}{3} &= \frac{15}{3} \\ y &= 5 \end{aligned}$$

Right side:

$$\begin{aligned} 5y-5 &= 2y+10 \\ -5y \quad & \quad -5y \\ \hline 0-5 &= -3y+10 \\ -5 &= -3y+10 \\ -10 \quad & \quad\quad -10 \\ \hline -15 &= -3y+0 \\ \frac{-15}{-3} &= \frac{-3y}{-3} \\ 5 &= y \end{aligned}$$

The solution is 5.

Check

$$\begin{aligned} 5y-5 &= 2y+10 \\ 5(5)-5 &= 2(5)+10 \\ 25-5 &= 10+10 \\ 20 &= 20 && \text{Balances} \end{aligned}$$

13. Solve, keeping the variable on the *left* side.

$$\begin{array}{rcll}
-2k-6 &=& 6k+10 & \\
-2k+(-6) &=& 6k+10 & \textit{Change to addition.} \\
-6k & & -6k & \textit{Add } -6k \textit{ to both sides.} \\ \hline
-8k+(-6) &=& 0+10 & \\
-8k+(-6) &=& 10 & \\
6 & & 6 & \textit{Add 6 to both sides.} \\ \hline
-8k+0 &=& 16 & \\
\dfrac{-8k}{-8} &=& \dfrac{16}{-8} & \textit{Divide both sides by } -8. \\
k &=& -2 & \text{The solution is } -2.
\end{array}$$

Solve, keeping the variable on the *right* side.

$$\begin{array}{rcll}
-2k-6 &=& 6k+10 & \\
-2k+(-6) &=& 6k+10 & \textit{Change to addition.} \\
2k & & 2k & \textit{Add } 2k \textit{ to both sides.} \\ \hline
0+(-6) &=& 8k+10 & \\
-6 &=& 8k+10 & \\
-10 & & -10 & \textit{Add } -10 \textit{ to both sides.} \\ \hline
-16 &=& 8k+0 & \\
\dfrac{-16}{8} &=& \dfrac{8k}{8} & \textit{Divide both sides by 8.} \\
-2 &=& k & \text{The solution is } -2.
\end{array}$$

Check

$$\begin{array}{rcll}
-2k-6 &=& 6k+10 & \\
-2(-2)-6 &=& 6(-2)+10 & \textit{Replace k with } -2. \\
4+(-6) &=& -12+10 & \\
-2 &=& -2 & \text{Balances}
\end{array}$$

14. **Left side:** **Right side:**

$$\begin{array}{rcl}
5x+4 &=& -3x-4 \\
3x & & 3x \\ \hline
8x+4 &=& 0-4 \\
8x+4 &=& -4 \\
-4 & & -4 \\ \hline
8x+0 &=& -8 \\
\dfrac{8x}{8} &=& \dfrac{-8}{8} \\
x &=& -1
\end{array}
\qquad
\begin{array}{rcl}
5x+4 &=& -3x-4 \\
-5x & & -5x \\ \hline
0+4 &=& -8x-4 \\
4 &=& -8x+(-4) \\
4 & & 4 \\ \hline
8 &=& -8x+0 \\
\dfrac{8}{-8} &=& \dfrac{-8x}{-8} \\
-1 &=& x
\end{array}$$

The solution is -1.

Check

$$\begin{array}{rcl}
5x+4 &=& -3x-4 \\
5(-1)+4 &=& -3(-1)-4 \\
-5+4 &=& 3+(-4) \\
-1 &=& -1
\end{array}$$

Balances

15. $-18+7a=2a+3+4$ simplifies to $-18+7a=2a+7$.

$$\begin{array}{rcll}
-18+7a &=& 2a+7 & \\
-2a & & -2a & \textit{Add } -2a \textit{ to both sides.} \\ \hline
-18+5a &=& 0+7 & \\
-18+5a &=& 7 & \\
18 & & 18 & \textit{Add 18 to both sides.} \\ \hline
0+5a &=& 25 & \\
5a &=& 25 & \\
\dfrac{5a}{5} &=& \dfrac{25}{5} & \textit{Divide both sides by 5.} \\
a &=& 5 &
\end{array}$$

The solution is 5.

Check

$$\begin{array}{rcll}
-18+7a &=& 2a+3+4 & \\
-18+7(5) &=& 2(5)+7 & \\
-18+35 &=& 10+7 & \\
17 &=& 17 & \text{Balances}
\end{array}$$

16. $-10+5r=-7-12-1$ simplifies to $-10+5r=-20$.

$$\begin{array}{rcll}
-10+5r &=& -20 & \\
10 & & 10 & \textit{Add 10 to both sides.} \\ \hline
0+5r &=& -10 & \\
5r &=& -10 & \\
\dfrac{5r}{5} &=& \dfrac{-10}{5} & \textit{Divide both sides by 5.} \\
r &=& -2 &
\end{array}$$

The solution is -2.

Check

$$\begin{array}{rcll}
-10+5r &=& -7-12-1 & \\
-10+5(-2) &=& -19-1 & \\
-10-10 &=& -20 & \\
-20 &=& -20 & \text{Balances}
\end{array}$$

17. Neither side can be simplified, so solve the equation.

$$\begin{array}{rcll}
-3t &=& 8t & \\
3t & & 3t & \textit{Add 3t to both sides.} \\ \hline
0 &=& 11t & \\
\dfrac{0}{11} &=& \dfrac{11t}{11} & \textit{Divide both sides by 11.} \\
0 &=& t & \text{The solution is 0.}
\end{array}$$

Check

$$\begin{array}{rcll}
-3t &=& 8t & \\
-3(0) &=& 8(0) & \textit{Replace t with 0.} \\
0 &=& 0 & \text{Balances}
\end{array}$$

18. Neither side can be simplified, so solve the equation.

$$\begin{array}{rcll}
15z &=& -9z & \\
\underline{9z} & & \underline{9z} & \textit{Add 9z to both sides.}\\
24z &=& 0 & \\
\dfrac{24z}{24} &=& \dfrac{0}{24} & \textit{Divide both sides by 24.}\\
z &=& 0 & \text{The solution is } 0.
\end{array}$$

Check
$$\begin{array}{rl}
15z = -9z & \\
15(0) = -9(0) & \textit{Replace z with 0.}\\
0 = 0 & \text{Balances}
\end{array}$$

19. $4+16-2=2-2b$ simplifies to $18=2-2b$.

$$\begin{array}{rcll}
18 &=& 2-2b & \\
\underline{-2} & & \underline{-2} & \textit{Add } -2 \textit{ to both sides.}\\
18-2 &=& 0-2b & \\
16 &=& -2b & \\
\dfrac{16}{-2} &=& \dfrac{-2b}{-2} & \textit{Divide both sides by } -2.\\
-8 &=& b &
\end{array}$$

The solution is 5.

Check
$$\begin{array}{rl}
4+16-2 = 2-2b & \\
20-2 = 2-2(-8) & \\
20+(-2) = 2+16 & \\
18 = 18 & \text{Balances}
\end{array}$$

20. $-9+2z=9z-1+13$ simplifies to $-9+2z=9z+12$.

$$\begin{array}{rcll}
-9+2z &=& 9z+12 & \\
\underline{-2z} & & \underline{-2z} & \textit{Add } -2z \textit{ to both sides.}\\
-9+0 &=& 7z+12 & \\
-9 &=& 7z+12 & \\
\underline{-12} & & \underline{-12} & \textit{Add } -12 \textit{ to both sides.}\\
-21 &=& 7z+0 & \\
-21 &=& 7z & \\
\dfrac{-21}{7} &=& \dfrac{7z}{7} & \textit{Divide both sides by 7.}\\
-3 &=& z &
\end{array}$$

The solution is -3.

Check
$$\begin{array}{rl}
-9+2z = 9z-1+13 & \\
-9+2(-3) = 9(-3)+12 & \\
-9-6 = -27+12 & \\
-15 = -15 & \text{Balances}
\end{array}$$

21.
$$\begin{array}{rcll}
8(w-2) &=& 32 & \\
8w-16 &=& 32 & \textit{Distribute.}\\
8w+(-16) &=& 32 & \textit{Change to addition.}\\
\underline{16} & & \underline{16} & \textit{Add 16 to both sides.}\\
8w+0 &=& 48 & \\
8w &=& 48 & \\
\dfrac{8w}{8} &=& \dfrac{48}{8} & \textit{Divide both sides by 8.}\\
w &=& 6 & \text{The solution is } 6.
\end{array}$$

22.
$$\begin{array}{rcll}
9(b-4) &=& 27 & \\
9b-36 &=& 27 & \textit{Distribute.}\\
9b+(-36) &=& 27 & \textit{Change to addition.}\\
\underline{36} & & \underline{36} & \textit{Add 36 to both sides.}\\
9b+0 &=& 63 & \\
\dfrac{9b}{9} &=& \dfrac{63}{9} & \textit{Divide both sides by 9.}\\
b &=& 7 & \text{The solution is } 7.
\end{array}$$

23.
$$\begin{array}{rcll}
-10 &=& 2(y+4) & \\
-10 &=& 2y+8 & \textit{Distribute.}\\
\underline{-8} & & \underline{-8} & \textit{Add } -8 \textit{ to both sides.}\\
-18 &=& 2y+0 & \\
-18 &=& 2y & \\
\dfrac{-18}{2} &=& \dfrac{2y}{2} & \textit{Divide both sides by 2.}\\
-9 &=& y & \text{The solution is } -9.
\end{array}$$

24.
$$\begin{array}{rcll}
-3 &=& 3(x+6) & \\
-3 &=& 3x+18 & \textit{Distribute.}\\
\underline{-18} & & \underline{-18} & \textit{Add } -18 \textit{ to both sides.}\\
-21 &=& 3x+0 & \\
\dfrac{-21}{3} &=& \dfrac{3x}{3} & \textit{Divide both sides by 3.}\\
-7 &=& x & \text{The solution is } -7.
\end{array}$$

25.
$$\begin{array}{rcll}
-4(t+2) &=& 12 & \\
-4t+(-8) &=& 12 & \textit{Distribute.}\\
\underline{8} & & \underline{8} & \textit{Add 8 to both sides.}\\
-4t+0 &=& 20 & \\
-4t &=& 20 & \\
\dfrac{-4t}{-4} &=& \dfrac{20}{-4} & \textit{Divide both sides by } -4.\\
t &=& -5 & \text{The solution is } -5.
\end{array}$$

26.
$$\begin{array}{rcll}
-5(k+3) &=& 25 & \\
-5k+(-15) &=& 25 & \textit{Distribute.}\\
\underline{15} & & \underline{15} & \textit{Add 15 to both sides.}\\
-5k+0 &=& 40 & \\
\dfrac{-5k}{-5} &=& \dfrac{40}{-5} & \textit{Divide both sides by } -5.\\
k &=& -8 & \text{The solution is } -8.
\end{array}$$

27.
$$
\begin{array}{rcll}
6(x-5) &=& -30 & \\
6x-30 &=& -30 & \textit{Distribute.}\\
6x+(-30) &=& -30 & \textit{Change to addition.}\\
30 & & 30 & \textit{Add 30 to both sides.}\\
\hline
6x+0 &=& 0 & \\
6x &=& 0 & \\
\dfrac{6x}{6} &=& \dfrac{0}{6} & \textit{Divide both sides by 6.}\\
x &=& 0 & \text{The solution is } 0.
\end{array}
$$

28.
$$
\begin{array}{rcll}
7(r-7) &=& -49 & \\
7r-49 &=& -49 & \textit{Distribute.}\\
7r+(-49) &=& -49 & \textit{Change to addition.}\\
49 & & 49 & \textit{Add 49 to both sides.}\\
\hline
7r+0 &=& 0 & \\
\dfrac{7r}{7} &=& \dfrac{0}{7} & \textit{Divide both sides by 7.}\\
r &=& 0 & \text{The solution is } 0.
\end{array}
$$

29.
$$
\begin{array}{rcll}
-12 &=& 12(h-2) & \\
-12 &=& 12h-24 & \textit{Distribute.}\\
-12 &=& 12h+(-24) & \textit{Change to addition.}\\
24 & & 24 & \textit{Add 24 to both sides.}\\
\hline
12 &=& 12h+0 & \\
12 &=& 12h & \\
\dfrac{12}{12} &=& \dfrac{12h}{12} & \textit{Divide both sides by 12.}\\
1 &=& h & \text{The solution is } 1.
\end{array}
$$

30.
$$
\begin{array}{rcll}
-11 &=& 11(c-3) & \\
-11 &=& 11c-33 & \textit{Distribute.}\\
-11 &=& 11c+(-33) & \textit{Change to addition.}\\
33 & & 33 & \textit{Add 33 to both sides.}\\
\hline
22 &=& 11c+0 & \\
\dfrac{22}{11} &=& \dfrac{11c}{11} & \textit{Divide both sides by 11.}\\
2 &=& c & \text{The solution is } 2.
\end{array}
$$

31.
$$
\begin{array}{rcll}
0 &=& -2(y+2) & \\
0 &=& -2y-4 & \textit{Distribute.}\\
0 &=& -2y+(-4) & \textit{Change to addition.}\\
4 & & 4 & \textit{Add 4 to both sides.}\\
\hline
4 &=& -2y+0 & \\
4 &=& -2y & \\
\dfrac{4}{-2} &=& \dfrac{-2y}{-2} & \textit{Divide both sides by } -2.\\
-2 &=& y & \text{The solution is } -2.
\end{array}
$$

32.
$$
\begin{array}{rcll}
0 &=& -9(b+1) & \\
0 &=& -9b-9 & \textit{Distribute.}\\
0 &=& -9b+(-9) & \textit{Change to addition.}\\
9 & & 9 & \textit{Add 9 to both sides.}\\
\hline
9 &=& -9b+0 & \\
\dfrac{9}{-9} &=& \dfrac{-9b}{-9} & \textit{Divide both sides by } -9.\\
-1 &=& b & \text{The solution is } -1.
\end{array}
$$

33.
$$
\begin{array}{rcll}
6m+18 &=& 0 & \\
-18 & & -18 & \textit{Add } -18 \textit{ to both sides.}\\
\hline
6m+0 &=& -18 & \\
6m &=& -18 & \\
\dfrac{6m}{6} &=& \dfrac{-18}{6} & \textit{Divide both sides by 6.}\\
m &=& -3 & \text{The solution is } -3.
\end{array}
$$

34.
$$
\begin{array}{rcll}
8p-40 &=& 0 & \\
8p+(-40) &=& 0 & \textit{Change to addition.}\\
40 & & 40 & \textit{Add 40 to both sides.}\\
\hline
8p+0 &=& 40 & \\
\dfrac{8p}{8} &=& \dfrac{40}{8} & \textit{Divide both sides by 8.}\\
p &=& 5 & \text{The solution is } 5.
\end{array}
$$

35.
$$
\begin{array}{rcll}
6 &=& 9w-12 & \\
6 &=& 9w+(-12) & \textit{Change to addition.}\\
12 & & 12 & \textit{Add 12 to both sides.}\\
\hline
18 &=& 9w+0 & \\
18 &=& 9w & \\
\dfrac{18}{9} &=& \dfrac{9w}{9} & \textit{Divide both sides by 9.}\\
2 &=& w & \text{The solution is } 2.
\end{array}
$$

36.
$$
\begin{array}{rcll}
8 &=& 8h+24 & \\
-24 & & -24 & \textit{Add } -24 \textit{ to both sides.}\\
\hline
-16 &=& 8h+0 & \\
\dfrac{-16}{8} &=& \dfrac{8h}{8} & \textit{Divide both sides by 8.}\\
-2 &=& h & \text{The solution is } -2.
\end{array}
$$

37.
$$
\begin{array}{rcll}
5x &=& 3x+10 & \\
-3x & & -3x & \textit{Add } -3x \textit{ to both sides.}\\
\hline
2x &=& 0+10 & \\
2x &=& 10 & \\
\dfrac{2x}{2} &=& \dfrac{10}{2} & \textit{Divide both sides by 2.}\\
x &=& 5 & \text{The solution is } 5.
\end{array}
$$

38.
$$
\begin{array}{rcll}
7n &=& -2n-36 & \\
7n &=& -2n+(-36) & \textit{Change to addition.}\\
2n & & 2n & \textit{Add } 2n \textit{ to both sides.}\\
\hline
9n &=& 0+(-36) & \\
\dfrac{9n}{9} &=& \dfrac{-36}{9} & \textit{Divide both sides by 9.}\\
n &=& -4 & \text{The solution is } -4.
\end{array}
$$

39.
$$\begin{array}{rcll}
2a+11 &=& 8a-7 & \\
2a+11 &=& 8a+(-7) & \textit{Change to addition.}\\
-2a & & -2a & \textit{Add } -2a.\\ \hline
0+11 &=& 6a+(-7) & \\
11 &=& 6a+(-7) & \\
7 & & 7 & \textit{Add 7 to both sides.}\\ \hline
18 &=& 6a+0 & \\
18 &=& 6a & \\
\frac{18}{6} &=& \frac{6a}{6} & \textit{Divide both sides by 6.}\\
3 &=& a & \text{The solution is 3.}
\end{array}$$

40.
$$\begin{array}{rcll}
r-10 &=& 10r+8 & \\
1r+(-10) &=& 10r+8 & \textit{Change to addition.}\\
-1r & & -1r & \textit{Add } -1r.\\ \hline
0+(-10) &=& 9r+8 & \\
-10 &=& 9r+8 & \\
-8 & & -8 & \textit{Add } -8.\\ \hline
-18 &=& 9r+0 & \\
\frac{-18}{9} &=& \frac{9r}{9} & \textit{Divide both sides by 9.}\\
-2 &=& r & \text{The solution is } -2.
\end{array}$$

41.
$$\begin{array}{rcll}
7-5b &=& 28+2b & \\
7+(-5b) &=& 28+2b & \textit{Change to addition.}\\
5b & & 5b & \textit{Add 5b to both sides.}\\ \hline
7+0 &=& 28+7b & \\
7 &=& 28+7b & \\
-28 & & -28 & \textit{Add } -28 \textit{ to both sides.}\\ \hline
-21 &=& 0+7b & \\
-21 &=& 7b & \\
\frac{-21}{7} &=& \frac{7b}{7} & \textit{Divide both sides by 7.}\\
-3 &=& b & \text{The solution is } -3.
\end{array}$$

42.
$$\begin{array}{rcll}
1-8t &=& -9-3t & \\
1+(-8t) &=& -9+(-3t) & \textit{Change to add.}\\
8t & & 8t & \textit{Add 8t.}\\ \hline
1+0 &=& -9+5t & \\
1 &=& -9+5t & \\
9 & & 9 & \textit{Add 9.}\\ \hline
10 &=& 0+5t & \\
\frac{10}{5} &=& \frac{5t}{5} & \textit{Divide both sides by 5.}\\
2 &=& t & \text{The solution is 2.}
\end{array}$$

43.
$$\begin{array}{rcll}
-20+2k &=& k-4k & \\
-20+2k &=& k+(-4k) & \textit{Change to addition.}\\
-20+2k &=& -3k & \textit{Combine like terms.}\\
-2k & & -2k & \textit{Add } -2k.\\ \hline
-20+0 &=& -5k & \\
-20 &=& -5k & \\
\frac{-20}{-5} &=& \frac{-5k}{-5} & \textit{Divide both sides by } -5.\\
4 &=& k & \text{The solution is 4.}
\end{array}$$

44.
$$\begin{array}{rcll}
6y-y &=& -16+y & \\
6y+(-1y) &=& -16+1y & \textit{Change to addition.}\\
5y &=& -16+1y & \textit{Combine like terms.}\\
-1y & & -1y & \textit{Add } -1y.\\ \hline
4y &=& -16+0 & \\
\frac{4y}{4} &=& \frac{-16}{4} & \textit{Divide both sides by 4.}\\
y &=& -4 & \text{The solution is } -4.
\end{array}$$

45.
$$\begin{array}{rcll}
10(c-6)+4 &=& 2+c-58 & \\
10c-60+4 &=& 2+c-58 & \textit{Distribute.}\\
10c+(-60)+4 &=& 2+c+(-58) & \textit{Change to add.}\\
10c+(-60)+4 &=& 2+(-58)+c & \textit{Group terms.}\\
10c+(-56) &=& -56+c & \textit{Combine terms.}\\
-c & & -c & \textit{Add } -c.\\ \hline
9c+(-56) &=& -56+0 & \\
9c+(-56) &=& -56 & \textit{Add 56.}\\
56 & & 56 & \\ \hline
9c+0 &=& 0 & \\
\frac{9c}{9} &=& \frac{0}{9} & \textit{Divide both sides by 9.}\\
c &=& 0 &
\end{array}$$

The solution is 0.

46.
$$\begin{array}{rcl}
8(z+7)-6 &=& z+60-10\\
8z+56-6 &=& z+60-10\\
8z+56+(-6) &=& z+60+(-10)\\
8z+50 &=& 1z+50\\
-1z & & -1z\\ \hline
7z+50 &=& 0+50\\
7z+50 &=& 50\\
-50 & & -50\\ \hline
7z+0 &=& 0\\
\frac{7z}{7} &=& \frac{0}{7}\\
z &=& 0
\end{array}$$

The solution is 0.

47.
$$\begin{array}{rcll}
-18+13y+3 &=& 3(5y-1)-2 & \\
-18+13y+3 &=& 15y-3-2 & \textit{Distribute.}\\
-18+13y+3 &=& 15y+(-3)+(-2) & \textit{Add the opposites.}\\
13y+(-18)+3 &=& 15y+(-3)+(-2) & \textit{Group like terms.}\\
13y+(-15) &=& 15y+(-5) & \textit{Combine like terms.}\\
-13y & & -13y & \textit{Add } -13y.\\ \hline
0+(-15) &=& 2y+(-5) & \\
-15 &=& 2y+(-5) & \\
5 & & 5 & \textit{Add 5.}\\ \hline
-10 &=& 2y+0 &
\end{array}$$

$$\begin{aligned} -10 &= 2y \\ \frac{-10}{2} &= \frac{2y}{2} \quad \textit{Divide by 2.} \\ -5 &= y \end{aligned}$$

The solution is -5.

48.
$$\begin{aligned} 3+5h-9 &= 4(3h+4)-1 \\ 3+5h+(-9) &= 12h+16+(-1) \\ 5h+(-6) &= 12h+15 \\ \underline{-5h\qquad} &\quad \underline{-5h\qquad} \\ 0+(-6) &= 7h+15 \\ -6 &= 7h+15 \\ \underline{-15\qquad} &\quad \underline{-15\qquad} \\ -21 &= 7h+0 \\ \frac{-21}{7} &= \frac{7h}{7} \\ -3 &= h \end{aligned}$$

The solution is -3.

49.
$$\begin{aligned} 6-4n+3n &= 20-35 \\ 6+(-4n)+3n &= 20+(-35) \quad \textit{Change to add.} \\ 6+(-1n) &= -15 \quad \textit{Combine terms.} \\ \underline{-6\qquad} &\quad \underline{-6\qquad} \quad \textit{Add } -6. \\ 0+(-1n) &= -21 \\ \frac{-1n}{-1} &= \frac{-21}{-1} \quad \textit{Divide both sides by } -1. \\ n &= 21 \end{aligned}$$

The solution is 21.

50.
$$\begin{aligned} -19+8 &= 6p-7p-5 \\ -19+8 &= 6p+(-7p)+(-5) \quad \textit{Change to add.} \\ -11 &= -1p+(-5) \quad \textit{Combine terms.} \\ \underline{5\qquad} &\quad \underline{5\qquad} \quad \textit{Add 5.} \\ -6 &= -1p+0 \\ \frac{-6}{-1} &= \frac{-1p}{-1} \quad \textit{Divide both sides by } -1. \\ 6 &= p \end{aligned}$$

The solution is 6.

51.
$$\begin{aligned} 6(c-2) &= 7(c-6) \\ 6c-12 &= 7c-42 \quad \textit{Distribute.} \\ 6c+(-12) &= 7c+(-42) \quad \textit{Change to add.} \\ \underline{-6c\qquad} &\quad \underline{-6c\qquad} \quad \textit{Add } -6c. \\ 0+(-12) &= 1c+(-42) \\ -12 &= 1c+(-42) \\ \underline{42\qquad} &\quad \underline{42\qquad} \quad \textit{Add 42.} \\ 30 &= 1c+0 \\ 30 &= c \end{aligned}$$

The solution is 30.

52.
$$\begin{aligned} -3(5+x) &= 4(x-2) \\ -15+(-3x) &= 4x-8 \\ \underline{3x\qquad} &\quad \underline{3x\qquad} \quad \textit{Add 3x.} \\ -15+0 &= 7x-8 \\ -15 &= 7x+(-8) \\ \underline{8\qquad} &\quad \underline{8\qquad} \quad \textit{Add 8.} \\ -7 &= 7x+0 \\ \frac{-7}{7} &= \frac{7x}{7} \quad \textit{Divide both sides by 7.} \\ -1 &= x \end{aligned}$$

The solution is -1.

53.
$$\begin{aligned} -5(2p+2)-7 &= 3(2p+5) \\ -10p+(-10)-7 &= 6p+15 \\ -10p+(-10)+(-7) &= 6p+15 \\ -10p+(-17) &= 6p+15 \\ \underline{-6p\qquad} &\quad \underline{-6p\qquad} \quad \textit{Add } -6p. \\ -16p+(-17) &= 0+15 \\ -16p+(-17) &= 15 \\ \underline{17\qquad} &\quad \underline{17\qquad} \quad \textit{Add 17.} \\ -16p+0 &= 32 \\ \frac{-16p}{-16} &= \frac{32}{-16} \quad \textit{Divide by } -16. \\ p &= -2 \end{aligned}$$

The solution is -2.

54.
$$\begin{aligned} 4(3m-6) &= 72+3(m-8) \\ 12m-24 &= 72+3m-24 \\ 12m+(-24) &= 72+3m+(-24) \\ 12m+(-24) &= 3m+48 \\ \underline{-3m\qquad} &\quad \underline{-3m\qquad} \\ 9m+(-24) &= 0+48 \\ 9m+(-24) &= 48 \\ \underline{24\qquad} &\quad \underline{24\qquad} \\ 9m+0 &= 72 \\ \frac{9m}{9} &= \frac{72}{9} \\ m &= 8 \end{aligned}$$

The solution is 8.

55.
$$\begin{aligned} 2(3b-2)-5b &= 4(b-1)+8b \\ 6b-4-5b &= 4b-4+8b \\ b-4 &= 12b-4 \\ \underline{4\qquad} &\quad \underline{4\qquad} \quad \textit{Add 4.} \\ b &= 12b \\ \underline{-b\qquad} &\quad \underline{-b\qquad} \quad \textit{Add } -b. \\ 0 &\quad 11b \\ \frac{0}{11} &= \frac{11b}{11} \\ 0 &= b \end{aligned}$$

The solution is 0.

56.
$$\begin{aligned}
-3(w+3)+10 &= -1(w+14)+w \\
-3w-9+10 &= -w-14+w \\
-3w+1 &= -14 \\
\underline{\quad -1} &\quad \underline{-1\quad} \quad \textit{Add } -1. \\
-3w &= -15 \\
\frac{-3w}{-3} &= \frac{-15}{-3} \\
w &= 5
\end{aligned}$$

The solution is 5.

57. The series of steps may vary. One possibility is:

$$\begin{array}{rcll}
-2t - \ \ 10 & = & 3t+5 & \textit{Change to addition.} \\
-2t+(-10) & = & 3t+5 & \textit{Add 2t to both sides} \\
\underline{2t\qquad\quad} & & \underline{2t\qquad} & \textit{(addition property).} \\
0+(-10) & = & 5t+5 & \textit{Add } -5 \textit{ to both sides} \\
\underline{-5} & = & \underline{-5} & \textit{(addition property).} \\
\frac{-15}{5} & = & \frac{5t}{5} & \textit{Divide both sides by 5} \\
 & & & \textit{(division property).} \\
-3 & = & t &
\end{array}$$

The solution is -3.

58. Multiplication distributes over both addition and subtraction. Examples will vary. Some possibilities are $3(2y+6)$ is $6y+18$ and $5(x-3)$ is $5x-15$.

59. **Check**
$$\begin{aligned}
-8+4a &= 2a+2 \\
-8+4(3) &= 2(3)+2 \\
-8+12 &= 6+2 \\
4 &\neq 8
\end{aligned}$$

The check does not balance, so 3 is not the correct solution. The student added $-2a$ to -8 on the left side, instead of adding $-2a$ to $4a$. The correct solution, obtained using $-8+2a=2$, $2a=10$, is $a=5$.

60. **Check**
$$\begin{aligned}
2(x+4) &= -16 \\
2(-10+4) &= -16 \\
2(-6) &= -16 \\
-12 &\neq -16
\end{aligned}$$

The check does not balance, so -10 is not the correct solution.

$$\begin{array}{rcll}
2(x+4) & = & -16 & \text{Student did not} \\
2x+8 & = & -16 & \text{distribute the 2} \\
\underline{-8} & & \underline{-8} & \text{over the 4.} \\
2x+0 & = & -24 & \\
\frac{2x}{2} & = & \frac{-24}{2} & \\
x & = & -12 &
\end{array}$$

The correct solution is -12.

Relating Concepts (Exercises 61–64)

61. **(a)** It must be negative, because the sum of two positive numbers is always positive.

(b) The sum of x and a positive number is negative, so x must be negative.

62. **(a)** It must be positive, because the sum of two negative numbers is always negative.

(b) The sum of d and a negative number is positive, so d must be positive.

63. **(a)** It must be positive. When the signs are the same, the product is positive, and when the signs are different, the product is negative.

(b) The product of n and a negative number is negative, so n must be positive.

64. **(a)** It must be negative also. When the signs are different, the product is negative, and when the signs match, the product is positive.

(b) The product of y and a negative number is positive, so y must be negative.

Chapter 2 Review Exercises

1. **(a)** In the expression $-3+4k$, k is the variable, 4 is the coefficient, and -3 is the constant term.

(b) The term that has 20 as the constant term and -9 as the coefficient is $-9y+20$.

2. **(a)** Evaluate $4c+10$ when c is 15.

$$4c+10$$
$$\underbrace{4 \cdot 15} + 10 \quad \textit{Replace c with 15.}$$
$$\underbrace{60+10}$$
$$70 \quad \text{Order 70 test tubes.}$$

(b) Evaluate $4c+10$ when c is 24.

$$4c+10$$
$$\underbrace{4 \cdot 24} + 10 \quad \textit{Replace c with 24.}$$
$$\underbrace{96+10}$$
$$106 \quad \text{Order 106 test tubes.}$$

3. **(a)** x^2y^4 means $x \cdot x \cdot y \cdot y \cdot y \cdot y$

(b) $5ab^3$ means $5 \cdot a \cdot b \cdot b \cdot b$

4. **(a)** n^2 means

$$n \cdot n$$
$$\underbrace{-3 \cdot (-3)} \quad \textit{Replace n with } -3.$$
$$9$$

(b) n^3 means

$$n \cdot n \cdot n$$
$$\underbrace{-3 \cdot (-3)} \cdot (-3) \quad \textit{Replace n with } -3.$$
$$\underbrace{9 \cdot (-3)}$$
$$-27$$

(c) $-4mp^2$ means

$$-4 \cdot m \cdot p \cdot p$$
$$\underbrace{-4 \cdot 2} \cdot 4 \cdot 4 \quad \textit{Replace m with 2 and p with 4.}$$
$$\underbrace{-8 \cdot 4} \cdot 4$$
$$\underbrace{-32 \cdot 4}$$
$$-128$$

(d) $5m^4n^2$ means

$$5 \cdot m \cdot m \cdot m \cdot m \cdot n \cdot n$$
$$\underbrace{5 \cdot 2} \cdot 2 \cdot 2 \cdot 2 \cdot (-3) \cdot (-3) \quad \textit{Replace m with 2 and n with } -3.$$
$$\underbrace{10 \cdot 2} \cdot 2 \cdot 2 \cdot (-3) \cdot (-3)$$
$$\underbrace{20 \cdot 2} \cdot 2 \cdot (-3) \cdot (-3)$$
$$\underbrace{40 \cdot 2} \cdot (-3) \cdot (-3)$$
$$\underbrace{80 \cdot (-3)} \cdot (-3)$$
$$\underbrace{-240 \cdot (-3)}$$
$$720$$

5. $ab + ab^2 + 2ab$

$\underline{1ab} + ab^2 + \underline{2ab}$ *Combine like terms.*

$3ab + ab^2$ or $ab^2 + 3ab$

6. $-3x + 2y - x - 7$

$-3x + 2y - 1x - 7$ *Rewrite x as 1x.*

$-3x + 2y + (-1x) + (-7)$ *Change to addition.*

$-4x + 2y - 7$ *Combine like terms.*

7. $-8(-2g^3)$ *Associative property*

$[-8 \cdot (-2)] \cdot g^3$

$16 \cdot g^3$

$16g^3$

8. $4(3r^2t)$ *Associative property*

$(4 \cdot 3) \cdot r^2t$

$12 \cdot r^2t$

$12r^2t$

9. $5(k + 2)$ *Distribute.*

$5 \cdot k + 5 \cdot 2$

$5k + 10$

10. $-2(3b + 4)$ *Distribute.*

$-2 \cdot 3b + (-2) \cdot 4$

$-6b + (-8)$ or $-6b - 8$

11. $3(2y - 4) + 12$ *Distribute.*

$\underbrace{3 \cdot 2y} - \underbrace{3 \cdot 4} + 12$

$6y - 12 + 12$

$6y + (-12) + 12$

$6y + 0$

$6y$

12. $-4 + 6(4x + 1) - 4x$ *Distribute.*

$-4 + 24x + 6 - 4x$

$-4 + 24x + 6 + (-4x)$

$2 + 20x$ or $20x + 2$

13. Expressions will vary. One possibility is $6a^3 + a^2 + 3a - 6$.

14.
$$\begin{aligned} 16 + n &= 5 \quad \textit{Add } -16 \textit{ to both sides.} \\ -16 & -16 \\ 0 + n &= -11 \\ n &= -11 \quad \text{The solution is } -11. \end{aligned}$$

Check $16 + n = 5$

$16 + (-11) = 5$ *Replace n with −11.*

$5 = 5$ Balances

15.
$$\begin{aligned} -4 + 2 &= 2a - 6 - a \\ -4 + 2 &= 2a + (-6) + (-1a) \\ -2 &= 1a + (-6) \\ 6 & 6 \\ 4 &= 1a + 0 \\ 4 &= a \end{aligned}$$

The solution is 4.

Check $-4 + 2 = 2a - 6 - a$

$-4 + 2 = 2(4) - 6 - 4$

$-2 = 8 + (-6) + (-4)$

$-2 = 2 + (-4)$

$-2 = -2$ Balances

16. $48 = -6m$

$\dfrac{48}{-6} = \dfrac{-6m}{-6}$ *Divide both sides by −6.*

$-8 = m$ The solution is -8.

17. $k - 5k = -40$

$1k - 5k = -40$

$1k + (-5k) = -40$

$-4k = -40$ *Combine like terms.*

$\dfrac{-4k}{-4} = \dfrac{-40}{-4}$ *Divide both sides by −4.*

$k = 10$ The solution is 10.

18. $\underbrace{-17 + 11 + 6}_{0} = 7t$

$0 = 7t$

$\dfrac{0}{7} = \dfrac{7t}{7}$ *Divide both sides by 7.*

$0 = t$ The solution is 0.

19. $-2p + 5p = 3 - 21$

$-2p + 5p = 3 + (-21)$

$3p = -18$

$\dfrac{3p}{3} = \dfrac{-18}{3}$ *Divide both sides by 3.*

$p = -6$ The solution is -6.

20.
$$\begin{aligned} -30 &= 3(-5r) \\ -30 &= -15r \\ \frac{-30}{-15} &= \frac{-15r}{-15} && \textit{Divide both sides by } -15. \\ 2 &= r && \text{The solution is 2.} \end{aligned}$$

21.
$$\begin{aligned} 12 &= -h \\ 12 &= -1h \\ \frac{12}{-1} &= \frac{-1h}{-1} && \textit{Divide both sides by } -1. \\ -12 &= h && \text{The solution is } -12. \end{aligned}$$

22.
$$\begin{array}{rcll} 12w-4 &=& 8w+12 & \\ 12w+(-4) &=& 8w+12 & \\ \underline{-8w} & & \underline{-8w} & \textit{Add } -8w \textit{ to both sides.} \\ 4w+(-4) &=& 0+12 & \\ 4w+(-4) &=& 12 & \\ \underline{4} & & \underline{4} & \textit{Add 4 to both sides.} \\ 4w+0 &=& 16 & \\ 4w &=& 16 & \\ \frac{4w}{4} &=& \frac{16}{4} & \textit{Divide both sides by 4.} \\ w &=& 4 & \end{array}$$

The solution is 4.

23.
$$\begin{array}{rcll} 0 &=& -4(c+2) & \\ 0 &=& -4 \cdot c + (-4) \cdot 2 & \textit{Distribute.} \\ 0 &=& -4c+(-8) & \\ \underline{8} & & \underline{8} & \textit{Add 8 to both sides.} \\ 8 &=& -4c+0 & \\ 8 &=& -4c & \\ \frac{8}{-4} &=& \frac{-4c}{-4} & \textit{Divide both sides by } -4. \\ -2 &=& c & \text{The solution is } -2. \end{array}$$

24.
$$\begin{array}{rcll} 34 &=& 2n+4 & \\ \underline{-4} & & \underline{-4} & \textit{Add } -4 \textit{ to both sides.} \\ 30 &=& 2n+0 & \\ 30 &=& 2n & \\ \frac{30}{2} &=& \frac{2n}{2} & \textit{Divide both sides by 2.} \\ 15 &=& n & \end{array}$$

The number of employees is 15.

25. **[2.5]**
$$\begin{array}{rcll} 12+7a &=& 4a-3 & \\ \underline{-4a} & & \underline{-4a} & \textit{Add } -4a \textit{ to both sides.} \\ 12+3a &=& 0-3 & \\ 12+3a &=& -3 & \\ \underline{-12} & & \underline{-12} & \\ 0+3a &=& -15 & \\ \frac{3a}{3} &=& \frac{-15}{3} & \textit{Divide both sides by 3.} \\ a &=& -5 & \text{The solution is } -5. \end{array}$$

26. **[2.5]**
$$\begin{array}{rcll} -2(p-3) &=& -14 & \\ -2p+6 &=& -14 & \textit{Distribute.} \\ \underline{-6} & & \underline{-6} & \textit{Add } -6 \textit{ to both sides.} \\ -2p+0 &=& -20 & \\ \frac{-2p}{-2} &=& \frac{-20}{-2} & \textit{Divide both sides by } -2. \\ p &=& 10 & \text{The solution is 10.} \end{array}$$

27. **[2.5]**
$$\begin{array}{rcll} 10y &=& 6y+20 & \\ \underline{-6y} & & \underline{-6y} & \textit{Add } -6y \textit{ to both sides.} \\ 4y &=& 0+20 & \\ \frac{4y}{4} &=& \frac{20}{4} & \textit{Divide both sides by 4.} \\ y &=& 5 & \text{The solution is 5.} \end{array}$$

28. **[2.5]**
$$\begin{array}{rcll} 2m-7m &=& 5-20 & \\ 2m+(-7m) &=& 5+(-20) & \textit{Add the opposites.} \\ -5m &=& -15 & \textit{Combine like terms.} \\ \frac{-5m}{-5} &=& \frac{-15}{-5} & \textit{Divide both sides by } -5. \\ m &=& 3 & \end{array}$$

The solution is 3.

29. **[2.5]**
$$\begin{array}{rcll} 20 &=& 3x-7 & \\ 20 &=& 3x+(-7) & \\ \underline{7} & & \underline{7} & \textit{Add 7 to both sides.} \\ 27 &=& 3x+0 & \\ \frac{27}{3} &=& \frac{3x}{3} & \textit{Divide both sides by 3.} \\ 9 &=& x & \text{The solution is 9.} \end{array}$$

30. **[2.5]**
$$\begin{array}{rcll} b+6 &=& 3b-8 & \\ \underline{-3b} & & \underline{-3b} & \textit{Add } -3b \textit{ to both sides.} \\ -2b+6 &=& 0-8 & \\ -2b+6 &=& -8 & \\ \underline{-6} & & \underline{-6} & \textit{Add } -6 \textit{ to both sides.} \\ -2b+0 &=& -14 & \\ \frac{-2b}{-2} &=& \frac{-14}{-2} & \textit{Divide both sides by } -2. \\ b &=& 7 & \text{The solution is 7.} \end{array}$$

31. **[2.3]**
$$\begin{array}{rcll} z+3 &=& 0 & \\ \underline{-3} & & \underline{-3} & \textit{Add } -3 \textit{ to both sides.} \\ z+0 &=& -3 & \\ z &=& -3 & \text{The solution is } -3. \end{array}$$

32. **[2.5]**

$$\begin{array}{rcll}
3(2n-1) &=& 3(n+3) & \\
6n-3 &=& 3n+9 & \textit{Distribute.} \\
-3n & & -3n & \textit{Add } -3n \textit{ to both sides.} \\
\hline
3n-3 &=& 0+9 & \\
3n-3 &=& 9 & \\
3 & & 3 & \textit{Add 3 to both sides.} \\
\hline
3n+0 &=& 12 & \\
\dfrac{3n}{3} &=& \dfrac{12}{3} & \textit{Divide both sides by 3.} \\
n &=& 4 & \text{The solution is } 4.
\end{array}$$

33. **[2.5]**

$$\begin{array}{rcll}
-4+46 &=& 7(-3t+6) & \\
-4+46 &=& -21t+42 & \textit{Distribute.} \\
42 &=& -21t+42 & \\
-42 & & -42 & \textit{Add } -42 \textit{ to both sides.} \\
\hline
0 &=& -21t+0 & \\
\dfrac{0}{-21} &=& \dfrac{-21t}{-21} & \textit{Divide both sides by } -21. \\
0 &=& t & \text{The solution is } 0.
\end{array}$$

34. **[2.5]**

$$\begin{array}{rcll}
6+10d-19 &=& 2(3d+4)-1 & \\
6+10d+(-19) &=& 6d+8-1 & \\
-13+10d &=& 6d+7 & \\
-6d & & -6d & \textit{Add } -6d. \\
\hline
-13+4d &=& 0+7 & \\
-13+4d &=& 7 & \\
13 & & 13 & \textit{Add 13 to both sides.} \\
\hline
0+4d &=& 20 & \\
\dfrac{4d}{4} &=& \dfrac{20}{4} & \textit{Divide by 4.} \\
d &=& 5 &
\end{array}$$

The solution is 5.

35. **[2.5]**

$$\begin{array}{rcll}
-4(3b+9) &=& 24+3(2b-8) & \\
-12b-36 &=& 24+6b-24 & \\
-12b+(-36) &=& 24+6b+(-24) & \\
-12b+(-36) &=& 6b & \\
12b & & 12b & \textit{Add 12b to both sides.} \\
\hline
0+(-36) &=& 18b & \\
\dfrac{-36}{18} &=& \dfrac{18b}{18} & \textit{Divide by 18.} \\
-2 &=& b &
\end{array}$$

The solution is -2.

Chapter 2 Test

1. In the expression $-7w+6$, -7 is the coefficient, w is the variable, and 6 is the constant term.

2. Evaluate the expression $3a+2c$ when a is 45 and c is 21.

$$\begin{gathered}
3a+2c \\
\underbrace{3 \cdot 45}+\underbrace{2 \cdot 21} \\
135 \;+\; 42 \\
177
\end{gathered}$$

Buy 177 hot dogs.

3. x^5y^3 means $x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y$

4. $4ab^4$ means $4 \cdot a \cdot b \cdot b \cdot b \cdot b$

5. $-2s^2t$ means

$$\begin{array}{ll}
-2 \cdot s \cdot s \cdot t & \\
\underbrace{-2 \cdot (-5)} \cdot (-5) \cdot 4 & \textit{Replace s with } -5 \textit{ and t with 4.} \\
\underbrace{10 \cdot (-5)} \cdot 4 & \\
\underbrace{-50 \cdot 4} & \\
-200 &
\end{array}$$

6.

$$\begin{gathered}
3w^3-8w^3+w^3 \\
3w^3-8w^3+1w^3 \\
\underbrace{3w^3+(-8w^3)}+1w^3 \\
\underbrace{-5w^3+1w^3} \\
-4w^3
\end{gathered}$$

7.

$$\begin{gathered}
xy-xy \\
1xy-1xy \\
(1-1)xy \\
0xy \\
0
\end{gathered}$$

8.

$$\begin{gathered}
-6c-5+7c+5 \\
-6c+(-5)+7c+5 \\
\underbrace{-6c+7c}+\underbrace{(-5)+5} \\
1c \quad + \quad 0 \\
1c \quad \text{or} \quad c
\end{gathered}$$

9. $3m^2-3m+3mn$
There are no like terms.
The expression cannot be simplified.

10.

$$\begin{array}{ll}
-10(4b^2) & \\
(-10 \cdot 4) \cdot b^2 & \textit{Associative property of multiplication} \\
-40b^2 &
\end{array}$$

11.
$$\begin{array}{ll} -5(-3k) & \\ [-5\cdot(-3)]\cdot k & \textit{Associative property of multiplication} \\ 15k & \end{array}$$

12.
$$\begin{array}{ll} 7(3t+4) & \\ 7(3t)+7(4) & \textit{Distributive property} \\ 21t+28 & \end{array}$$

13.
$$\begin{array}{ll} -4(a+6) & \\ -4\cdot a+(-4)\cdot 6 & \textit{Distributive property} \\ -4a+(-24) & \\ -4a-24 & \end{array}$$

14.
$$\begin{array}{ll} -8+6(x-2)+5 & \\ -8+6x-12+5 & \textit{Distributive property} \\ -8+6x+(-12)+5 & \\ 6x+(-15) & \textit{Combine like terms.} \\ \text{or } 6x-15 & \end{array}$$

15.
$$\begin{array}{ll} -9b-c-3+9+2c & \\ -9b-1c-3+9+2c & \\ -9b+(-1c)+(-3)+9+2c & \\ -9b+c+6 & \textit{Combine like terms.} \end{array}$$

16.
$$\begin{array}{rcll} -4 & = & x-9 & \\ 9 & & 9 & \textit{Add 9 to both sides.} \\ \hline 5 & = & x+0 & \\ 5 & = & x & \end{array}$$

The solution is 5.

Check
$$\begin{array}{ll} -4=x-9 & \\ -4=5-9 & \textit{Replace x with 5.} \\ -4=-4 & \text{Balances} \end{array}$$

17.
$$\begin{array}{ll} -7w=77 & \\ \dfrac{-7w}{-7}=\dfrac{77}{-7} & \textit{Divide both sides by } -7. \\ w=-11 & \end{array}$$

The solution is -11.

Check
$$\begin{array}{rll} -7w=77 & \\ -7\cdot(-11)=77 & \textit{Replace w with } -11. \\ 77=77 & \text{Balances} \end{array}$$

18.
$$\begin{array}{ll} -p=14 & \\ -1p=14 & \\ \dfrac{-1p}{-1}=\dfrac{14}{-1} & \textit{Divide both sides by } -1. \\ p=-14 & \end{array}$$

The solution is -14.

Check
$$\begin{array}{rll} -p=14 & \\ -1p=14 & \\ -1\cdot(-14)=14 & \textit{Replace p with } -14. \\ 14=14 & \text{Balances} \end{array}$$

19.
$$\begin{array}{rcll} -15 & = & -3(a+2) & \\ -15 & = & -3a-6 & \\ 6 & & 6 & \textit{Add 6 to both sides.} \\ \hline -9 & = & -3a & \\ \dfrac{-9}{-3} & = & \dfrac{-3a}{-3} & \textit{Divide both sides by } -3. \\ 3 & = & a & \end{array}$$

The solution is 3.

Check
$$\begin{array}{ll} -15=-3(a+2) & \\ -15=-3(3+2) & \textit{Replace a with 3.} \\ -15=-3(5) & \\ -15=-15 & \text{Balances} \end{array}$$

20.
$$\begin{array}{rcll} 6n+8-5n & = & -4+4 & \\ 6n+8+(-5n) & = & 0 & \\ n+8 & = & 0 & \\ -8 & & -8 & \textit{Add } -8. \\ \hline n & = & -8 & \end{array}$$

The solution is -8.

21.
$$\begin{array}{ll} 5-20=2m-3m & \\ 5+(-20)=2m+(-3m) & \\ -15=-1m & \\ \dfrac{-15}{-1}=\dfrac{-1m}{-1} & \textit{Divide both sides by } -1. \\ 15=m & \end{array}$$

The solution is 15.

22.
$$\begin{array}{rcll} -2x+2 & = & 5x+9 & \\ 2x & & 2x & \textit{Add 2x to both sides.} \\ \hline 2 & = & 7x+9 & \\ -9 & & -9 & \textit{Add } -9 \textit{ to both sides.} \\ \hline \dfrac{-7}{7} & = & \dfrac{7x}{7} & \textit{Divide both sides by 7.} \\ -1 & = & x & \end{array}$$

The solution is -1.

23.
$$\begin{array}{rcll} 3m-5 & = & 7m-13 & \\ -3m & & -3m & \textit{Add } -3m \textit{ to both sides.} \\ \hline 0-5 & = & 4m-13 & \\ -5 & = & 4m-13 & \\ 13 & & 13 & \textit{Add 13 to both sides.} \\ \hline \dfrac{8}{4} & = & \dfrac{4m}{4} & \textit{Divide both sides by 4.} \\ 2 & = & m & \end{array}$$

The solution is 2.

24.
$$\begin{aligned} 2+7b-44 &= -3b+12+9b \\ 7b-42 &= 6b+12 \\ -6b \quad & \quad -6b \qquad \textit{Add } -6b \textit{ to both sides.} \\ \hline 1b-42 &= 12 \\ 42 \quad & \quad 42 \qquad \textit{Add 42 to both sides.} \\ \hline 1b &= 54 \\ b &= 54 \end{aligned}$$

The solution is 54.

25.
$$\begin{aligned} 3c-24 &= 6(c-4) \\ 3c-24 &= 6c-24 \quad \textit{Distribute.} \\ -3c \quad & \quad -3c \\ \hline -24 &= 3c-24 \\ 24 \quad & \quad 24 \qquad \textit{Add 24 to both sides.} \\ \hline 0 &= 3c \\ \frac{0}{3} &= \frac{3c}{3} \qquad \textit{Divide both sides by 3.} \\ 0 &= c \end{aligned}$$

The solution is 0.

26. *Addition property of equality:* Start with a possible solution, for example, $x=-4$. Now add an abitrary number, say -5, to both sides, to give us the equation $x-5=-9$.

Division property of equality: Start with a possible solution, for example, $-4=y$. Now multiply both sides by an abitrary number, say 6, to give us the equation $-24=6y$.

Thus, equations will vary. Two possibilities are

$$x-5=-9 \quad \text{and} \quad -24=6y.$$

Solving:

$$\begin{aligned} x-5 &= -9 \\ 5 \quad & \quad 5 \qquad \textit{Add 5 to both sides.} \\ \hline x &= -4 \\ -24 &= 6y \\ \frac{-24}{6} &= \frac{6y}{6} \qquad \textit{Divide both sides by 6.} \\ -4 &= y \end{aligned}$$

Cumulative Review Exercises (Chapters 1–2)

1. 306,000,004,210 in words is three hundred six billion, four thousand, two hundred ten.

2. Eight hundred million, sixty-six thousand: 800,066,000

3. **(a)** -3 lies to the *right* of -10 on the number line, so $-3 > -10$.

(b) -1 lies to the *left* of 0 on the number line, so $-1 < 0$.

4. **(a)** $-6+2=2+(-6)$ ■ *Commutative property of addition:* Changing the order of the addends does not change the sum.

(b) $0 \cdot 25 = 0$ ■ *Multiplication property of zero:* Multiplying any number by 0 gives a product of 0.

(c) $5(-6+4)=5\cdot(-6)+5\cdot 4$ ■ *Distributive property:* Multiplication distributes over addition.

5. **(a)** $9047 \approx 9000$

Underline the hundreds place: $9\underline{0}47$

The next digit is 4 or less, so leave 0 as 0. Change 4 and 7 to 0.

(b) $289{,}610 \approx 290{,}000$

Underline the thousands place: $28\underline{9},610$

The next digit is 5 or more, so add 1 to 9, write the 0 and add 1 to the ten-thousands place. Change 6 and 1 to 0.

6.
$$\begin{aligned} &0-8 \\ &= 0+(-8) \quad \textit{Change to addition.} \\ &= -8 \end{aligned}$$

7.
$$\begin{aligned} &|-6|+|4| \\ &= 6+4 \quad \textit{−6 is 6 units from 0. 4 is 4 units from 0.} \\ &= 10 \end{aligned}$$

8.
$$\begin{aligned} &-3(-10) \\ &= 30 \quad \textit{Same sign, positive product} \end{aligned}$$

9.
$$\begin{aligned} &(-5)^2 \\ &= -5\cdot(-5) \\ &= 25 \quad \textit{Same sign, positive product} \end{aligned}$$

10.
$$\begin{aligned} &\frac{-42}{-6} \\ &= 7 \quad \textit{Same sign, positive quotient} \end{aligned}$$

11.
$$\begin{aligned} &-19+19 \\ &= 0 \quad \textit{Addition of a number and its opposite is zero.} \end{aligned}$$

12.
$$\begin{aligned} &(-4)^3 \\ &\underbrace{-4\cdot(-4)}\cdot(-4) \quad \textit{Exponent} \\ &\underbrace{16\cdot(-4)} \quad \textit{Multiply left to right.} \\ &-64 \end{aligned}$$

13. $\dfrac{-14}{0}$ is *undefined*. Division by 0 is undefined.

14.
$$\begin{aligned} &-5\cdot 12 \\ &= -60 \quad \textit{Different signs, negative product} \end{aligned}$$

15. $-20-20$
$=-20+(-20)$ *Change to addition.*
$=-40$

16. $\frac{45}{-5}=-9$ *Different signs, negative quotient*

17. $-50+25=-25$

18.
$-10+6(4-7)$
$-10+6[4+(-7)]$ *Change to addition.*
$-10+6(-3)$
$\underbrace{-10+(-18)}$ *Multiply.*
-18 *Add.*

19. $\frac{-20-3(-5)+16}{(-4)^2-3^3}$

Numerator:

$-20-3(-5)+16$
$-20-(-15)+16$ *Multiply.*
$\underbrace{-20+15}+16$ *Change to addition.*
$\underbrace{-5+16}$ *Add left to right.*
11

Denominator:

$(-4)^2-3^3$
$\underbrace{(-4)(-4)}-\underbrace{3\cdot3\cdot3}$ *Exponents*
$16 \quad - \quad 27$
$\underbrace{16+(-27)}$
-11

Last step is division: $\frac{11}{-11}=-1$

20. 22 days rounds to 20.
616 miles rounds to 600.
Average distance "per" day implies division.

Estimate: $\frac{600 \text{ miles}}{20 \text{ days}}=30$ miles per day

Exact: $\frac{616 \text{ miles}}{22 \text{ days}}=28$ miles per day

The average distance the tiger traveled each day was 28 miles.

21. -48 degrees rounds to -50.
"Rise" of 23 degrees rounds to 20.

A start temperature of -48 degrees followed by a rise of 23 degrees implies addition.

Estimate: $-50+20=-30$ degrees
Exact: $-48+23=-25$ degrees

The daytime temperature was -25 degrees.

22. 52 shares rounds to 50.
\$2132 rounds to \$2000.
\$8 stays \$8 (it's a single digit number).

Each stock dropped in value by \$8 and Doug owned 52 shares. Multiply to find out how much money he lost. Then, subtract this amount from the original total value.

Estimate: $\$2000-(50\cdot8)=\$1600$
Exact: $\$2132-(52\cdot8)=\$1716$

His shares are now worth \$1716.

23. \$758 rounds to \$800.
\$45 rounds to \$50.
12 months (in one year) rounds to 10.

Estimate: $10(\$800+\$50)$
$=10(\$850)=\$8500$

Exact: $12(\$758+\$45)=12(\$803)=\$9636$

She will spend \$9636 for rent and parking in one year.

24. $-4ab^3c^2$ means $-4\cdot a\cdot b\cdot b\cdot b\cdot c\cdot c$

25. $3xy^3$ means

$3\cdot x\cdot y\cdot y\cdot y$
$\underbrace{3\cdot(-5)}\cdot(-2)\cdot(-2)\cdot(-2)$ *Replace x with -5 and y with -2.*
$\underbrace{-15\cdot(-2)}\cdot(-2)\cdot(-2)$ *Multiply left to right.*
$\underbrace{30\cdot(-2)}\cdot(-2)$
$\underbrace{-60\cdot(-2)}$
120

26.
$3h-7h+5h$
$\underbrace{3h+(-7h)}+5h$ *Change to addition.*
$\underbrace{-4h+5h}$ *Combine like terms.*
$1h$ or h

27. c^2d-c^2d
$=1c^2d-1c^2d$ *Write the understood coefficients of* 1.
$=1c^2d+(-1c^2d)$ *Change to addition.*
$=[1+(-1)]c^2d$ *Combine like terms.*
$=0\cdot c^2d$
$=0$

28.
$4n^2-4n+6-8+n^2$
$4n^2+(-4n)+6+(-8)+n^2$
$\underbrace{4n^2+n^2}+(-4n)+\underbrace{6+(-8)}$
$5n^2 \quad +(-4n)+ \quad (-2)$
or $5n^2-4n-2$

29.
$$\begin{array}{ll} -10(3b^2) & \\ \underbrace{(-10 \cdot 3)\, b^2}_{-30b^2} & \textit{Associative property} \end{array}$$

30.
$$\begin{array}{ll} 7(4p-4) & \\ \underbrace{7 \cdot 4p}_{28p} - \underbrace{7 \cdot 4}_{28} & \textit{Distribute.} \end{array}$$

31.
$$\begin{array}{c} 3 + 5(-2w^2 - 3) + w^2 \\ 3 + (-10w^2) - 15 + w^2 \\ 3 + (-10w^2) + (-15) + w^2 \\ -9w^2 + (-12) \quad \text{or} \quad -9w^2 - 12 \end{array}$$

32.
$$\begin{array}{rcll} 3x &=& x - 8 & \\ \underline{-x} & & \underline{-x\quad} & \textit{Add } -x \textit{ to both sides.} \\ 2x &=& 0 - 8 & \\ 2x &=& -8 & \\ \dfrac{2x}{2} &=& \dfrac{-8}{2} & \textit{Divide both sides by 2.} \\ x &=& -4 & \end{array}$$

The solution is -4.

Check
$$\begin{array}{rll} 3x &= x - 8 & \\ 3(-4) &= -4 - 8 & \textit{Replace x with } -4. \\ -12 &= -4 + (-8) & \\ -12 &= -12 & \text{Balances} \end{array}$$

33.
$$\begin{array}{rcll} -44 &=& -2 + 7y & \\ \underline{2} & & \underline{2\qquad} & \textit{Add 2 to both sides.} \\ -42 &=& 0 + 7y & \\ -42 &=& 7y & \\ \dfrac{-42}{7} &=& \dfrac{7y}{7} & \textit{Divide both sides by 7.} \\ -6 &=& y & \end{array}$$

The solution is -6.

Check
$$\begin{array}{rll} -44 &= -2 + 7y & \\ -44 &= -2 + 7(-6) & \textit{Replace y with } -6. \\ -44 &= -2 + (-42) & \\ -44 &= -44 & \text{Balances} \end{array}$$

34.
$$\begin{array}{rll} 2k - 5k &= -21 & \\ 2k + (-5k) &= -21 & \\ -3k &= -21 & \\ \dfrac{-3k}{-3} &= \dfrac{-21}{-3} & \textit{Divide both sides by } -3. \\ k &= 7 & \end{array}$$

The solution is 7.

Check
$$\begin{array}{rll} 2k - 5k &= -21 & \\ 2(7) - 5(7) &= -21 & \textit{Replace k with 7.} \\ 14 - 35 &= -21 & \\ 14 + (-35) &= -21 & \\ -21 &= -21 & \text{Balances} \end{array}$$

35.
$$\begin{array}{rcll} m - 6 &=& -2m + 6 & \\ \underline{2m\qquad} & & \underline{2m\qquad} & \textit{Add 2m to both sides.} \\ 3m - 6 &=& 0 + 6 & \\ 3m - 6 &=& 6 & \\ \underline{6} & & \underline{6\qquad} & \textit{Add 6 to both sides.} \\ 3m + 0 &=& 12 & \\ \dfrac{3m}{3} &=& \dfrac{12}{3} & \textit{Divide both sides by 3.} \\ m &=& 4 & \end{array}$$

The solution is 4.

Check
$$\begin{array}{rll} m - 6 &= -2m + 6 & \\ 4 - 6 &= -2(4) + 6 & \textit{Replace m with 4.} \\ 4 + (-6) &= -8 + 6 & \\ -2 &= -2 & \text{Balances} \end{array}$$

36.
$$\begin{array}{rcll} 4 - 4x &=& 18 + 10x & \\ \underline{4x} & & \underline{\qquad 4x} & \textit{Add 4x to both sides.} \\ 4 + 0 &=& 18 + 14x & \\ 4 &=& 18 + 14x & \\ \underline{-18} & & \underline{-18\qquad} & \textit{Add } -18 \textit{ to both sides.} \\ -14 &=& 0 + 14x & \\ \dfrac{-14}{14} &=& \dfrac{14x}{14} & \textit{Divide both sides by 14.} \\ -1 &=& x & \text{The solution is } -1. \end{array}$$

37.
$$\begin{array}{rll} 18 &= -r & \\ 18 &= -1r & \\ \dfrac{18}{-1} &= \dfrac{-1r}{-1} & \textit{Divide both sides by } -1. \\ -18 &= r & \text{The solution is } -18. \end{array}$$

38.
$$\begin{array}{rcll} -8b - 11 + 7b &=& b - 1 & \\ -1b - 11 &=& 1b - 1 & \\ \underline{1b} & & \underline{1b} & \textit{Add 1b to both sides.} \\ 0b - 11 &=& 2b - 1 & \\ -11 &=& 2b - 1 & \\ \underline{1} & & \underline{1} & \textit{Add 1 to both sides.} \\ -10 &=& 2b + 0 & \\ \dfrac{-10}{2} &=& \dfrac{2b}{2} & \textit{Divide both sides by 2.} \\ -5 &=& b & \end{array}$$

The solution is -5.

39.

$$\begin{array}{rcll}
-2(t+1) & = & 4(1-2t) & \\
-2t+(-2) & = & 4-8t & \\
2 & & 2 & \textit{Add 2 to both sides.} \\
\hline
-2t+0 & = & 6-8t & \\
-2t & = & 6-8t & \\
8t & & 8t & \textit{Add 8t to both sides.} \\
\hline
6t & = & 6+0 & \\
\frac{6t}{6} & = & \frac{6}{6} & \textit{Divide both sides by 6.} \\
t & = & 1 &
\end{array}$$

The solution is 1.

40.

$$\begin{array}{rcll}
5+6y-23 & = & 5(2y+8)-10 & \\
5+6y+(-23) & = & 10y+40+(-10) & \\
6y+(-18) & = & 10y+30 & \\
18 & & 18 & \textit{Add 18.} \\
\hline
6y+0 & = & 10y+48 & \\
6y & = & 10y+48 & \\
-10y & & -10y & \textit{Add } -10y. \\
\hline
-4y & = & 0+48 & \\
\frac{-4y}{-4} & = & \frac{48}{-4} & \textit{Divide by } -4. \\
y & = & -12 &
\end{array}$$

The solution is -12.