CHAPTER 2—ECONOMIC OPTIMIZATION

MULTIPLE CHOICE

1. An equation is:
   a. an analytical expressions of functional relationships.
   b. a visual representation of data.
   c. a table of electronically stored data.
   d. a list of economic data.

   ANS: A

2. Inflection is:
   a. a line that touches but does not intersect a given curve.
   b. a point of maximum slope.
   c. a measure of the steepness of a line.
   d. an activity level that generates highest profit.

   ANS: B

3. The breakeven level of output occurs where:
   a. marginal cost equals average cost.
   b. marginal profit equals zero.
   c. total profit equals zero.
   d. marginal cost equals marginal revenue.

   ANS: C

4. Incremental profit is:
   a. the change in profit that results from a unitary change in output.
   b. total revenue minus total cost.
   c. the change in profit caused by a given managerial decision.
   d. the change in profits earned by the firm over a brief period of time.

   ANS: C

5. The incremental profit earned from the production and sale of a new product will be higher if:
   a. the costs of materials needed to produce the new product increase.
   b. excess capacity can be used to produce the new product.
   c. existing facilities used to produce the new product must be modified.
   d. the revenues earned from existing products decrease.

   ANS: B

6. Which of the following short run strategies should a manager select to obtain the highest degree of sales penetration?
   a. maximize revenues.
   b. minimize average costs.
   c. minimize total costs.
   d. maximize profits.

   ANS: A

7. If total revenue increases at a constant rate as output increases, marginal revenue:

a. is greater than average revenue.
b. is less than average revenue.
c. is greater than average revenue at low levels of output and less than average revenue at high levels of output.
d. equals average revenue.

ANS: D

8. The comprehensive impact resulting from a decision is the:
a. gain or loss associated with a given managerial decision.
b. change in total cost.
c. change in total profit.
d. incremental change.

ANS: D

9. Total revenue is maximized at the point where:
a. marginal revenue equals zero.
b. marginal cost equals zero.
c. marginal revenue equals marginal cost.
d. marginal profit equals zero.

ANS: A

10. If P = \$1,000 - \$4Q:
a. MR = \$1,000 - \$4Q
b. MR = \$1,000 - \$8Q
c. MR = \$1,000Q - \$4
d. MR = \$250 - \$0.25P

ANS: B

11. Total cost minimization occurs at the point where:
a. MC = 0
b. MC = AC
c. AC = 0
d. Q = 0

ANS: D

12. Average cost minimization occurs at the point where:
a. MC = 0
b. MC = AC
c. AC = 0
d. Q = 0

ANS: B

13. The slope of a straight line from the origin to the total profit curve indicates:
a. marginal profit at that point.
b. an inflection point.
c. average profit at that point.
d. total profit at that point.

ANS: C

14. The optimal output decision:
    a. minimizes the marginal cost of production.
    b. minimizes production costs.
    c. is most consistent with managerial objectives.
    d. minimizes the average cost of production.

    ANS: C

15. Marginal profit equals:
    a. the change in total profit following a one-unit change in output.
    b. the change in total profit following a managerial decision.
    c. average revenue minus average cost.
    d. total revenue minus total cost.

    ANS: A

16. Profit per unit is rising when marginal profit is:
    a. greater than average profit per unit.
    b. less than average profit per unit.
    c. equal to average profit per unit.
    d. positive.

    ANS: A

17. Marginal cost is rising when marginal cost is:
    a. positive.
    b. less than average cost.
    c. greater than average cost.
    d. none of these.

    ANS: B

18. Marginal profit equals average profit when:
    a. marginal profit is maximized.
    b. average profit is maximized.
    c. marginal profit equals marginal cost.
    d. the profit minimizing output is produced.

    ANS: B

19. Total revenue increases at a constant rate as output increases when average revenue:
    a. increases as output increases.
    b. increases and then decreases as output increases.
    c. exceeds price.
    d. is constant.

    ANS: D

20. The optimal decision produces:
    a. maximum revenue.
    b. maximum profits.
    c. minimum average costs.
    d. a result consistent with managerial objectives.

    ANS: D

21. If average profit increases with output marginal profit must be:
    a. decreasing.
    b. greater than average profit.
    c. less than average profit.
    d. increasing.

    ANS: B

22. At the profit-maximizing level of output:
    a. marginal profit equals zero.
    b. marginal profit is less than average profit.
    c. marginal profit exceeds average profit.
    d. marginal cost equals average cost.

    ANS: A

23. When marginal profit equals zero:
    a. the firm can increase profits by increasing output.
    b. the firm can increase profits by decreasing output.
    c. marginal revenue equals average revenue.
    d. profit is maximized.

    ANS: D

24. If profit is to rise as output expands, then marginal profit must be:
    a. falling.
    b. constant.
    c. positive.
    d. rising.

    ANS: C

25. An optimal decision:
    a. minimizes output cost.
    b. maximizes profits.
    c. produces the result most consistent with decision maker objectives.
    d. maximizes product quality.

    ANS: C

PROBLEM

1. **Marginal Analysis**. Consider the price (P) and output (Q) data in the following table.

Q	**P**	**TR**	**MR**	**AR**
0	$35			
1	30			
2	25			
3	20			
4	15			
5	10			
6	5			
7	0			

A. Calculate the related total revenue (TR), marginal revenue (MR), and average revenue (AR) figures.

B. At what output level is revenue maximized?

ANS:

A.

Q	P	TR=P×Q	MR=∂TR/∂Q	AR=TR/Q=P
0	$35	$ 0	--	--
1	30	30	$30	$30
2	25	50	20	25
3	20	60	10	20
4	15	60	0	15
5	10	50	-10	10
6	5	30	-20	5
7	0	0	-30	0

B. Revenue is maximized at an output level 4, where MR = 0.

2. **Marginal Analysis**. Evaluate the price (P) and the output (Q) data in the following table.

Q	P	TR	MR	AR
0	$80			
1	70			
2	60			
3	50			
4	40			
5	30			
6	20			
7	10			
8	0			

A. Compute the related total revenue (TR), marginal revenue (MR), and average revenue (AR) figures:

B. At what output level is revenue maximized?

ANS:

A.

Q	P	TR=P×Q	MR=∂TR/∂Q	AR=TR/Q=P
0	$80	$ 0	--	--
1	70	70	$70	$70
2	60	120	50	60
3	50	150	30	50
4	40	160	10	40
5	30	150	-10	30
6	20	120	-30	20
7	10	70	-50	10
8	0	0	-70	0

B. Revenue is maximized at an output level slightly greater than 4, where MR = 0.

3. **Revenue Maximization**. Assume the following output (Q) and price (P) data.

Q	P	TR	MR	AR
0	$50			
1	45			
2	40			
3	35			
4	30			
5	25			
6	20			
7	15			
8	10			
9	5			
10	0			

A. At what output level is revenue maximized?

B. Why is marginal revenue less than average revenue at each price level?

ANS:

A. Notice the following figures for total revenue and marginal revenue.

Q	P	TR=P×Q	MR=∂TR/∂Q	AR=TR/Q=P
0	$50	$ 0	--	--
1	45	45	$45	$45
2	40	80	35	40
3	35	105	25	35
4	30	120	15	30
5	25	125	5	25
6	20	120	-5	20
7	15	105	-15	15
8	10	80	-25	10
9	5	45	-35	5
10	0	0	-45	0

Revenue is maximized at an output level of 5.

B. At every price level, price must be cut by $5 in order to increase sales by an additional unit. This means that the "benefit" of added sales from new customers is only gained at the "cost" of some loss in revenue from current customers. Thus, the net increase in revenue from added sales is always less than the change in gross revenue. Therefore, marginal revenue is always less than average revenue (or price).

4. **Profit Maximization**. Fill in the missing data for price (P), total revenue (TR), marginal revenue (MR), total cost (TC), marginal cost (MC), profit (π), and marginal profit ($M\pi$) in the following table.

Q	P	TR=P×Q	MR=∂TR/∂Q	TC	MC=∂TC/∂Q	π=TR-TC	Mπ=∂π/∂Q
0	$200	$ 0	--	$ 0	--	$ 0	--
1	180	180	$180	100	$100	80	$ 80
2		320		175			65
3		420	100	240	65	180	
4	120		60		55	185	5
5	100	500		350	55	150	-35
6	80	480	-20	400			-70
7	60		-60	450	50	-30	-110

8		320	-100		55	-185	-155
9	20	180		570	65		-205
10	10		-80	750	180	-650	-260

A. At what output (Q) level is profit maximized?

B. At what output (Q) level is revenue maximized?

C. Discuss any differences in your answers to parts A and B.

ANS:

A. Profit increases so long as MR > MC and Mπ > 0. In this problem, profit is maximized at Q = 4 where π = \$185 (and TR = \$480).

Q	**P**	**TR=P×Q**	**MR=∂TR/∂Q**	**TC**	**MC=∂TC/∂Q**	**π=TR-TC**	**Mπ=∂π/∂Q**
0	$200	$ 0	--	$ 0	--	$ 0	--
1	180	180	$180	100	$100	80	$ 80
2	160	320	140	175	75	145	65
3	140	420	100	240	65	180	35
4	120	480	60	295	55	185	5
5	100	500	20	350	55	150	-35
6	80	480	-20	400	50	80	-70
7	60	420	-60	450	50	-30	-110
8	40	320	-100	505	55	-185	-155
9	20	180	-140	570	65	-390	-205
10	10	100	-80	750	180	-650	-260

B. Total Revenue increases so long as MR > 0. In this problem, revenue is maximized at Q = 5 where TR = \$500 (and π = \$150).

C. Given a downward sloping demand curve and MC > 0, as is typically the case, profits will be maximized at an output level that is less than the revenue maximizing level. Revenue maximization requires lower prices and greater output than would be true with profit maximization. The potential long-run advantage of a revenue maximizing strategy is that it might generate rapid market expansion and long-run benefits in terms of customer loyalty and future unit cost reductions. The cost is, of course, measured in terms of lost profits in the short-run (here the loss is \$35 in profits).

5. **Profit Maximization**. Fill in the missing data for price (P), total revenue (TR), marginal revenue (MR), total cost (TC), marginal cost (MC), profit (π), and marginal profit (Mπ) in the following table.

Q	**P**	**TR**	**MR**	**TC**	**MC**	**π**	**Mπ**
0	$160	$ 0	$ --	$ 0	$ --	$ 0	$ --
1	150	150	150	25	25	125	125
2	140			55	30		100
3		390			35	300	75
4	120		90	130		350	
5	110	550		175			25
6		600	50		55	370	
7		630		290	60		-30
8	80	640		355		285	
9		630			75		-85
10		600		525		75	

A. At what output (Q) level is profit maximized?

B. At what output (Q) level is revenue maximized?

C. Discuss any differences in your answers to parts A and B.

ANS:

A. Profit increases so long as MR > MC and Mπ > 0. In this problem, profit is maximized at Q = 5 where π = $375 (and TR = $550).

Q	P	TR=P×Q	MR=∂TR/∂Q	TC	MC=∂TC/∂Q	π=TR-TC	Mπ=∂π/∂Q
0	$160	$ 0	--	$ 0	--	$ 0	--
1	150	150	$150	25	$25	125	$125
2	140	280	130	55	30	225	100
3	130	390	110	90	35	300	75
4	120	480	90	130	40	350	50
5	110	550	70	175	45	375	25
6	110	600	50	230	55	370	-5
7	90	640	30	290	60	340	-30
8	80	640	10	355	65	285	-55
9	70	630	-10	430	75	200	-85
10	60	600	-30	525	95	75	-125

B. Total Revenue increases so long as MR > 0. In this problem, revenue is maximized at Q = 8 where TR = $640 (and π = $285).

C. Given a downward sloping demand curve and MC > 0, as is typically the case, profits will be maximized at an output level that is less than the revenue maximizing level. Revenue maximization requires lower prices and greater output than would be true with profit maximization. The potential long-run advantage of a revenue maximizing strategy is that it might generate rapid market expansion and long-run benefits in terms of customer loyalty and future unit cost reductions. The cost is, of course, measured in terms of lost profits in the short-run (here the loss is $90 in profits).

6. **Profit Maximization**. Fill in the missing data for price (P), total revenue (TR), marginal revenue (MR), total cost (TC), marginal cost (MC), profit (π), and marginal profit (Mπ) in the following table.

Q	P	TR	MR	TC	MC	π	Mπ
0	$230	$ 0	$ --	$ 0	$ --	$ 0	$ --
1	210			10		200	
2		380			20		150
3	170		130		30	450	
4		600		100	40		50
5	130				60	490	-10
6		660		160		430	
7		630	-30	310			-110
8	70		-70	400	90	160	-160

A. At what output (Q) level is profit maximized?

B. At what output (Q) level is revenue maximized?

C. Discuss any differences in your answers to parts A and B.

ANS:

A. Profit increases so long as MR > MC and Mπ > 0. In this problem, profit is maximized at Q = 4 where π = 500 (and TR = $600).

Q	P	TR=P×Q	MR=∂TR/∂Q	TC	MC=∂TC/∂Q	π=TR-TC	Mπ=∂π/∂Q
0	$230	$ 0	--	$ 0	--	$ 0	--
1	210	210	$210	10	$10	200	$200
2	190	380	170	30	20	350	150
3	170	510	130	60	30	450	100
4	150	600	90	100	40	500	50
5	130	650	50	160	60	490	-10
6	110	660	10	230	70	430	-60
7	90	630	-30	310	80	320	-110
8	70	560	-70	400	90	160	-160

B. Total Revenue increases so long as MR > 0. In this problem, total revenue is maximized at Q = 6 where TR = $660 (and π = $430).

C. Given a downward sloping demand curve and MC > 0, as is typically the case, profits will be maximized at an output level that is less than the revenue maximizing level. Revenue maximization requires lower prices and greater output than would be true with profit maximization. The potential long-run advantage of a revenue maximizing strategy is that it might generate rapid market expansion and long-run benefits in terms of customer loyalty and future unit cost reductions. The cost is, of course, measured in terms of lost profits in the short-run (here the loss is $130 in profits).

7. **Profit Maximization**. Fill in the missing data for price (P), total revenue (TR), marginal revenue (MR), total cost (TC), marginal cost (MC), profit (π), and marginal profit (Mπ) in the following table.

Q	P	TR	MR	TC	MC	π	Mπ
0	$50	$ 0	$ --	$ 10	$ --	$ -10	$ --
1	45	45	45	60	50	-15	-5
2	40		35	115		-35	
3	35			175	60		-35
4		120	15		65	-120	-50
5	25		5	310			-65
6	20		-5		75		-80

A. At what output (Q) level is profit maximized (or losses minimized)? Explain.

B. At what output (Q) level is revenue maximized?

ANS:

A. At every output level given, profit is negative. In this problem, profit is maximized (loss is minimized) at Q = 0 where π = -$10 (and TR = 0).

Q	P	TR=P×Q	MR=∂TR/∂Q	TC	MC=∂TC/∂Q	π=TR-TC	Mπ=∂π/∂Q
0	$50	$ 0	--	$ 10	--	$ -10	--
1	45	45	$45	60	$50	-15	$ -5
2	40	80	35	115	55	-35	-20

3	35	105	25	175	60	-70	-35
4	30	120	15	240	65	-120	-50
5	25	125	5	310	70	-185	-65
6	20	120	-5	385	75	-265	-80

B. Total Revenue increases so long as MR > 0. In this problem, total revenue is maximized at Q = 5 where TR = \$125 (and π = -\$185).

8. **Marginal Analysis**. Characterize each of the following statements as true or false, and explain your answer.

A. Given a downward-sloping demand curve and positive marginal costs, profit-maximizing firms will always sell less output and at higher prices than will revenue-maximizing firms.

B. Profits will be maximized when marginal revenue equals marginal cost.

C. Total profit is the difference between total revenue and total cost and will always exceed zero at the profit-maximizing activity level.

D. Marginal cost must be less than average cost at the average cost minimizing output level.

E. The demand curve will be downward sloping if marginal revenue is less than price.

ANS:

A. True. Profit maximization involves setting marginal revenue equal to marginal cost. Revenue maximization involves setting marginal revenue equal to zero. Given a downward sloping demand curve and positive marginal costs, revenue maximizing firms will charge lower prices and offer greater quantities of output than will firms that seek to maximize profits.

B. True. Profits are maximized when marginal revenue equals marginal cost. Profits equal zero at the breakeven point where total revenue equals total cost.

C. False. High fixed costs or depressed demand conditions can give rise to zero or negative profits at the profit-maximizing activity level. Profit maximization only ensures that profits are as high as possible, or that losses are minimized, subject to demand and cost conditions.

D. False. Average cost falls as output expands so long as marginal cost is less than average cost. Thus, average cost is minimized at the point where average and marginal costs are equal.

E. True. The demand curve is the average revenue curve. Because price (average revenue) is falling along a downward sloping demand curve, marginal revenue is less than average revenue.

9. **Optimization**. Describe each of the following statements as true or false, and explain your answer.

A. To maximize the value of the firm, management should always produce the level of output that maximizes short run profit.

B. Average profit equals the slope of the line tangent to the total product function at each level of output.

C. Marginal profit equals zero at the profit maximizing level of output.

D. To maximize profit, total revenue must also be maximized.

E. Marginal cost equals average cost at the average cost minimizing level of output.

ANS:

A. False. Value can be maximized by producing a level of output higher than that which maximizes profits in the short run if the long run future profits derived from greater market penetration and scale advantages are sufficient to overcome the disadvantage of lost short run profits.

B. False. Average profit is represented by the slope of the ray running from the origin to the total product function at each level of output.

C. True. Marginal profit equals the slope of the line tangent to the total profit function at each level of output. The slope of the line tangent to the total profit function at its maximum point equals zero. Thus, marginal profit equals zero at the profit maximizing level of output.

D. False. Total revenue is maximized at a level of output greater than the level of output that maximizes profit because the level of output at which MR = 0 is greater than the level of output at which MR = MC > 0 when MR is decreasing.

E. True. Marginal cost equals average cost at the average cost minimizing level of output.

10. **Marginal Analysis: Tables**. Bree Van De Camp is a regional sales representative for Snappy Tools, Inc., and sells hand tools to auto mechanics in New England states. Van De Camp's goal is to maximize total monthly commission income, which is figured at 6.25% of gross sales. In reviewing experience over the past year, Van De Camp found the following relations between days spent in each state and weekly sales generated.

Days	Maine Sales	New Hampshire Sales	Vermont Sales
0	$ 4,000	$ 3,000	$1,900
1	10,000	7,000	5,200
2	15,000	10,600	7,400
3	19,000	13,800	8,600
4	22,000	16,600	9,200
5	24,000	19,000	9,600
6	25,000	21,000	9,800

A. Construct a table showing Van De Camp's marginal sales per day in each state.

B. If Van De Camp is limited to 6 selling days per week, how should they be spent?

C. Calculate Van De Camp's maximum weekly commission income.

ANS:

A.

	Maine Marginal	New Hampshire Marginal	Vermont Marginal

Days	Sales	Sales	Sales
0	--	--	--
1	$6,000	$4,000	$3,300
2	5,000	3,600	2,200
3	4,000	3,200	1,200
4	3,000	2,800	600
5	2,000	2,400	400
6	1,000	2,000	200

B. The maximum commission income is earned by allocating 6 selling days on the basis of obtaining the largest marginal sales for each additional day of selling activity. Using the data in part A, and with 6 days to spend per week, 3 days should be spent in Maine, 2 days in New Hampshire, and 1 day should be spent in Vermont.

C. Given this time allocation, Van De Camp's maximum commission income is:

State	Sales	
Maine (3)	$19,000	
New Hampshire (2)	10,600	
Vermont (1)	5,200	
Total	$34,800	
× Commission rate	0.0625	
	$2,175	per week

11. **Marginal Analysis: Tables**. Susan Mayer is a sales representative for the Desperate Insurance Company, and sells life insurance policies to individuals in the Phoenix area. Mayer's goal is to maximize total monthly commission income, which is figured at 10% of gross sales. In reviewing monthly experience over the past year, Mayer found the following relations between days spent in each city and monthly sales generated.

Days	Phoenix Sales	Scottsdale Sales	Tempe Sales
0	$ 5,000	$ 7,500	$ 2,500
1	15,000	15,000	6,500
2	23,000	21,500	9,500
3	29,000	27,000	11,500
4	33,000	31,500	12,500
5	35,000	35,000	12,500
6	35,000	37,500	12,500
7	35,000	39,000	12,500

A. Construct a table showing Mayer's marginal sales per day in each city.

B. If administrative duties limit Mayer to only 10 selling days per month, how should she spend them?

C. Calculate Mayer's maximum monthly commission income.

ANS:

A.

	Phoenix Marginal	Scottsdale Marginal	Tempe Marginal

Days	Sales	Sales	Sales
0	---	---	---
1	$10,000	$7,500	$4,000
2	8,000	6,500	3,000
3	6,000	5,500	2,000
4	4,000	4,500	1,000
5	2,000	3,500	0
6	0	2,500	0
7	0	1,500	0

B. The maximum commission income is earned by allocating 10 selling days on the basis of obtaining the largest marginal sales for each additional day of selling activity. Using the data in part A, and with 10 days to spend per month, 4 days should be spent in Phoenix, 5 days in Scottsdale, and 1 day should be spent in Tempe.

C. Given this time allocation, Mayer's maximum commission income is:

City	Sales	
Phoenix (4)	$33,000	
Scottsdale (5)	35,000	
Tempe (1)	6,500	
Total	$74,500	
× Commission rate	0.10	
	$ 7,450	per month

12. **Marginal Analysis: Tables**. Lynette Scavo is a telemarketing manager for Laser Supply, Inc., which sells replacement chemicals to businesses with copy machines. Scavo's goal is to maximize total monthly commission income, which is figured at 5% of gross sales of per telemarketer. In reviewing monthly experience over the past year, Scavo found the following relations between worker-hours spent in each market segment and monthly sales generated.

Businesses with less than 250 employees		Businesses with 250-500 employees		Businesses with over 500 employees	
Worker-hours	Gross Sales	Worker-hours	Gross Sales	Worker-hours	Gross Sales
0	$18,000	0	$15,000	0	$21,000
100	25,500	100	24,000	100	27,000
200	32,100	200	31,500	200	31,500
300	37,800	300	37,500	300	34,500
400	42,600	400	42,000	400	36,900
500	46,500	500	45,000	500	37,700
600	49,500	600	46,500	600	40,200
700	51,600	700	46,500	700	41,100

A. Construct a table showing Scavo's marginal sales per 100 worker-hours in each market segment.

B. Scavo employs telemarketers for 1,000 worker-hours per month, how should their hours be allocated among market segments?

C. Calculate Scavo's maximum monthly commission income.

ANS:

A.

Businesses with less than 250 employees		Businesses with 250-500 employees		Businesses with over 500 employees	
Worker-hours	Marginal Sales	Worker-hours	Marginal Sales	Worker-hours	Marginal Sales
0	--	0	--	0	--
100	$7,500	100	$9,000	100	$6,000
200	6,600	200	7,500	200	4,500
300	5,700	300	6,000	300	3,000
400	4,800	400	4,500	400	2,400
500	3,900	500	3,000	500	1,800
600	3,000	600	1,500	600	1,500
700	2,100	700	0	700	900

B. The maximum commission income is earned by allocating worker-hours on the basis of obtaining the largest marginal sales for each additional worker-hour of selling activity. Using the data in part A, 400 worker-hours should be spent calling businesses with less than 250 employees, 400 worker-hours calling businesses with 250-500 employees, and 200 worker-hours should be spent calling business with over 500 employees.

C. Given this time allocation, Scavo's maximum commission income is:

Business	Sales	
Less than 250 employees	$ 42,600	
250-500 employees	42,000	
Over 500 employees	31,500	
Total	$116,100	
× Commission rate	0.05	
	$ 5,805	per month

13. **Marginal Analysis: Tables**. Gabrielle Solis is a regional sales representative for Specialty Books, Inc., and sells textbooks to universities in Midwestern states. Solis goal is to maximize total monthly commission income, which is figured at 10% of gross sales. In reviewing monthly experience over the past year, Solis found the following relations between days spent in each state and monthly sales generated:

Kansas		Oklahoma		Nebraska	
Days	Gross Sales	Days	Gross Sales	Days	Gross Sales
0	$ 8,000	0	$ 2,000	0	$ 4,000
1	16,000	1	6,000	1	14,000
2	22,400	2	9,200	2	22,000
3	27,200	3	11,600	3	28,000
4	31,600	4	13,200	4	32,400
5	34,000	5	14,000	5	35,600
6	35,200	6	14,400	6	37,600
7	35,600	7	14,400	7	38,400

A. Construct a table showing Solis marginal sales per day in each state.

B. If administrative duties limit Solis to only 15 selling days per month, how should he spend them?

C. Calculate Solis maximum monthly commission income.

ANS:

A.

Kansas		Oklahoma		Nebraska	
Days	**Marginal Sales**	**Days**	**Marginal Sales**	**Days**	**Marginal Sales**
0	--	0	--	0	--
1	$8,000	1	$4,000	1	$10,000
2	6,400	2	3,200	2	8,000
3	4,800	3	2,400	3	6,000
4	4,400	4	1,600	4	4,400
5	2,400	5	800	5	3,200
6	1,200	6	400	6	2,000
7	400	7	0	7	800

B. The maximum commission income is earned by allocating selling days on the basis of obtaining the largest marginal sales for each additional day of selling activity. Using the data in part A, 5 days should be spent in Kansas, 4 days in Oklahoma, and 6 days should be spent in Nebraska.

C. Given this time allocation, Solis' maximum commission income is:

State	**Sales**	
Kansas	$34,000	
Oklahoma	13,200	
Nebraska	37,600	
Total	$84,800	
× Commission rate	0.10	
	$ 8,480	per month

14. **Profit Maximization: Equations**. Woodland Instruments, Inc. operates in the highly competitive electronics industry. Prices for its R2-D2 control switches are stable at $100 each. This means that P = MR = $100 in this market. Engineering estimates indicate that relevant total and marginal cost relations for the R2-D2 model are:

$TC = \$500,000 + \$25Q + \$0.0025Q^2$

$MC = \partial TC/\partial Q = \$25 + \$0.005Q$

A. Calculate the output level that will maximize R2-D2 profit.

B. Calculate this maximum profit.

ANS:

A. To find the profit-maximizing level of output, set MR = MC and solve for Q:

$$\begin{aligned} MR &= MC \\ \$100 &= \$25 + \$0.005Q \\ 0.005Q &= 75 \end{aligned}$$

$$Q = 15{,}000$$

(*Note:* Profits are decreasing for Q > 15,000.)

B. The total revenue function for Woodland is:

$TR = P \times Q = \$100Q$

Then, total profit is:

$$\begin{aligned} \pi &= TR - TC \\ &= \$100Q - \$500{,}000 - \$25Q - \$0.0025Q^2 \\ &= -\$0.0025Q^2 + \$75Q - \$500{,}000 \\ &= -\$0.0025(15{,}000^2) + \$75(15{,}000) - \$500{,}000 \\ &= \$62{,}500 \end{aligned}$$

15. **Profit Maximization: Equations**. Austin Heating & Air Conditioning, Inc., offers heating and air conditioning system inspections in the Austin, Texas, market. Prices are stable at \$50 per unit. This means that P = MR = \$50 in this market. Total cost (TC) and marginal cost (MC) relations are:

$TC = \$1{,}000{,}000 + \$10Q + \$0.00025Q^2$

$MC = \partial TC/\partial Q = \$10 + \$0.0005Q$

A. Calculate the output level that will maximize profit.

B. Calculate this maximum profit.

ANS:

A. To find the profit-maximizing level of output, set MR = MC and solve for Q:

$$\begin{aligned} MR &= MC \\ \$50 &= \$10 + \$0.0005Q \\ 0.0005Q &= 40 \\ Q &= 80{,}000 \end{aligned}$$

(*Note:* Profits are decreasing for Q > 80,000.)

B. The total revenue function is:

$TR = PQ = \$50Q$

Total profit is:

$$\begin{aligned} \pi &= TR - TC \\ &= \$50Q - \$1{,}000{,}000 - \$10Q - \$0.00025Q^2 \\ &= -\$0.00025Q^2 + \$40Q - \$1{,}000{,}000 \\ &= -\$0.00025(80{,}000^2) + \$40(80{,}000) - \$1{,}000{,}000 \\ &= \$600{,}000 \end{aligned}$$

16. **Profit Maximization: Equations**. Jewelry.com is a small but rapidly growing Internet retailer. A popular product is its standard 14k white gold diamond anniversary rings (1/4 ct. tw.) that retail for \$250. Prices are stable, so P = MR = \$250 in this market. Total and marginal cost relations for this product are:

$TC = \$3{,}250{,}000 + \$70Q + \$0.002Q^2$

$MC = \partial TC/\partial Q = \$70 + \$0.004Q$

A. Calculate the output level that will maximize profit.

B. Calculate this maximum profit.

ANS:

A. To find the profit-maximizing level of output, set MR = MC and solve for Q:

$$\begin{aligned} MR &= MC \\ \$250 &= \$70 + \$0.004Q \\ 0.004Q &= 180 \\ Q &= 45{,}000 \end{aligned}$$

(*Note:* Profits are decreasing for Q > 45,000.)

B. The total revenue function is:

$TR = PQ = \$250Q$

Total profit is:

$$\begin{aligned} \pi &= TR - TC \\ &= \$250Q - \$3{,}250{,}000 - \$70Q - \$0.002Q^2 \\ &= -\$0.002Q^2 + \$180Q - \$3{,}250{,}000 \\ &= -\$0.002(45{,}000^2) + \$180(45{,}000) - \$3{,}250{,}000 \\ &= \$800{,}000 \end{aligned}$$

17. **Profit Maximization: Equations**. VirusSoft, Inc., operates in the highly competitive virus detection and protection software industry. Prices for its basic software are stable at \$30 each. This means that P = MR = \$30 in this market. Engineering estimates indicate that relevant total and marginal cost relations for this product are:

$TC = \$750{,}000 + \$20Q + \$0.00002Q^2$

$MC = \partial TC/\partial Q = \$20 + \$0.00004Q$

A. Calculate the output level that will maximize profit.

B. Calculate this maximum profit.

ANS:

A. To find the profit-maximizing level of output we set MR = MC and solve for Q:

$$\begin{aligned} MR &= MC \\ \$30 &= \$20 + \$0.00004Q \\ 0.00004Q &= 10 \\ Q &= 250{,}000 \end{aligned}$$

(*Note:* Profits are decreasing for Q > 250,000.)

B. The total revenue function is:

TR = PQ = $30Q

Then, total profit is:

$$\begin{aligned} \pi &= TR - TC \\ &= \$30Q - \$750{,}000 - \$20Q - \$0.00002Q^2 \\ &= -\$0.00002Q^2 + \$10Q - \$750{,}000 \\ &= -\$0.00002(250{,}000^2) + \$10(250{,}000) - \$750{,}000 \\ &= \$500{,}000 \end{aligned}$$

18. **Profit Maximization: Equations**. Lone Star Insurance offers mail-order automobile insurance to preferred-risk drivers in the state of Texas. The company is the low-cost provider of insurance in this market with fixed costs of $18 million per year, plus variable costs of $750 for each driver insured on an annual basis. Annual demand and marginal revenue relations for the company are:

$P = \$1{,}500 - \$0.005Q$

$MR = \partial TR/\partial Q = \$1{,}500 - \$0.01Q$

A. Calculate the profit-maximizing activity level.

B. Calculate the company's optimal profit and return-on-sales levels.

ANS:

A. Set MR = MC and solve for Q to find the profit-maximizing activity level:

$$\begin{aligned} MR &= MC \\ \$1{,}500 - \$0.01Q &= \$750 \\ 0.01Q &= 750 \\ Q &= 75{,}000 \end{aligned}$$

B.

$$\begin{aligned} \pi &= PQ - TC \\ &= \$1{,}500(75{,}000) - \$0.005(75{,}000^2) - \$18{,}000{,}000 - \$750(75{,}000) \\ &= \$10{,}125{,}000 \end{aligned}$$

$$\begin{aligned} TR &= PQ \\ &= \$1{,}500(75{,}000) - \$0.005(75{,}000^2) \\ &= \$84{,}375{,}000 \end{aligned}$$

$$\begin{aligned} \text{Return on Sales} &= \pi/TR \\ &= \$10{,}125{,}000/\$84{,}375{,}000 \\ &= 12\% \end{aligned}$$

19. **Profit Maximization: Equations**. Dot.com Products, Inc., offers storage containers for fine china on the Internet. The company is the low-cost retailer of these quilted boxes with fixed costs of $480,000 per year, plus variable costs of $30 for each box. Annual demand and marginal revenue relations for the company are:

$P = \$70 - \$0.0005Q$

$MR = \partial TR/\partial Q = \$70 - \$0.001Q$

A. Calculate the profit-maximizing activity level.

B. Calculate the company's optimal profit and return-on-sales levels.

ANS:

A. Set MR = MC and solve for Q to find the profit-maximizing activity level:

$$\begin{aligned} MR &= MC \\ \$70 - \$0.001Q &= \$30 \\ 0.001Q &= 40 \\ Q &= 40{,}000 \end{aligned}$$

B.

$$\begin{aligned} \pi &= PQ - TC \\ &= \$70(40{,}000) - \$0.0005(40{,}000^2) - \$480{,}000 - \$30(40{,}000) \\ &= \$320{,}000 \end{aligned}$$

$$\begin{aligned} TR &= PQ \\ &= \$70(40{,}000) - \$0.0005(40{,}000^2) \\ &= \$2{,}000{,}000 \end{aligned}$$

$$\begin{aligned} \text{Return on Sales} &= \pi/TR \\ &= \$320{,}000/\$2{,}000{,}000 \\ &= 16\% \end{aligned}$$

20. **Profit Maximization: Equations**. Steam Cleanin, Inc., offers professional carpet cleaning to home owners in Huntsville, Alabama. The company is the low-cost provider in this market with fixed costs of $168,750 per year, plus variable costs of $10 per room of carpet cleaning. Annual demand and marginal revenue relations for the company are:

$P = \$40 - \$0.001Q$

$MR = \partial TR/\partial Q = \$40 - \$0.002Q$

A. Calculate the profit-maximizing activity level.

B. Calculate the company's optimal profit and return-on-sales levels.

ANS:

A. Set MR = MC and solve for Q to find the profit-maximizing activity level:

$$\begin{aligned} MR &= MC \\ \$40 - \$0.002Q &= \$10 \end{aligned}$$

$$0.002Q = 30$$
$$Q = 15{,}000$$

B.
$$\pi = PQ - TC$$
$$= \$40(15{,}000) - \$0.001(15{,}000^2) - \$168{,}750 - \$10(15{,}000)$$
$$= \$56{,}250$$

$$TR = PQ$$
$$= \$40(15{,}000) - \$0.001(15{,}000^2)$$
$$= \$375{,}000$$

$$\text{Return on Sales} = \pi/TR$$
$$= \$56{,}250/\$375{,}000$$
$$= 15\%$$

21. **Optimal Profit**. Hardwood Cutters offers seasoned, split fireplace logs to consumers in Toledo, Ohio. The company is the low-cost provider of firewood in this market with fixed costs of $10,000 per year, plus variable costs of $25 for each cord of firewood. Annual demand and marginal revenue relations for the company are:

$P = \$225 - \$0.125Q$

$MR = \partial TR/\partial Q = \$225 - \$0.25Q$

A. Calculate the profit-maximizing activity level.

B. Calculate the company's optimal profit and return-on-sales levels.

ANS:

A. Set MR = MC and solve for Q to find the profit-maximizing activity level:

$$MR = MC$$
$$\$225 - \$0.25Q = \$25$$
$$0.25Q = 200$$
$$Q = 800$$

B.
$$\pi = PQ - TC$$
$$= \$225(800) - \$0.125(800^2) - \$10{,}000 - \$25(800)$$
$$= \$70{,}000$$

$$TR = PQ$$
$$= \$225(800) - \$0.125(800^2)$$
$$= \$100{,}000$$

$$\text{Return on Sales} = \pi/TR$$
$$= \$70{,}000/\$100{,}000$$
$$= 70\%$$

22. **Not-for-Profit Analysis**. The Indigent Care Center, Inc., is a private, not-for-profit, medical treatment center located in Denver, Colorado. An important issue facing Dr. Kerry Weaver, ICC's administrative director, is the determination of an appropriate patient load (level of output). To efficiently employ scarce ICC resources, the board of directors has instructed Weaver to maximize ICC operating surplus, defined as revenues minus operating costs. They have also asked Weaver to determine the effects of two proposals for meeting new state health care regulations. Plan A involves an increase in costs of \$100 per patient, whereas plan B involves a \$20,000 increase in fixed expenses. In her calculations, Weaver has been asked to assume that a \$3,000 fee will be received from the state for each patient treated, irrespective of whether plan A or plan B is adopted.

In the calculations for determining an optimal patient level, Weaver regards price as fixed; therefore, P = MR = \$3,000. Prior to considering the effects of the new regulations, Weaver projects total and marginal cost relations of:

$$TC = \$75,000 + \$2,000Q + \$2.5Q^2$$

$$MC = \partial TC/\partial Q = \$2,000 + \$5Q$$

where Q is the number of ICC patients.

A. Before considering the effects of the proposed regulations, calculate ICC's optimal patient and operating surplus levels.

B. Calculate these levels under plan A.

C. Calculate these levels under plan B.

ANS:

A. Set MR = MC, and solve for Q to find the operating surplus (profit) maximizing activity level:

$$\begin{aligned} MR &= MC \\ \$3,000 &= \$2,000 + \$5Q \\ 5Q &= 1,000 \\ Q &= 200 \end{aligned}$$

$$\begin{aligned} \text{Surplus} &= PQ - TC \\ &= \$3,000(200) - \$75,000 - \$2,000(200) - \$2.5(200^2) \\ &= \$25,000 \end{aligned}$$

B. When operating costs increase by \$100 per patient, the marginal cost function and optimal activity level are both affected. Under plan A we Set MR = MC + \$100, and solve for Q to find the new operating surplus (profit) maximizing activity level.

$$\begin{aligned} MR &= MC + \$100 \\ \$3,000 &= \$2,000 + \$5Q + \$100 \\ 5Q &= 900 \\ Q &= 180 \end{aligned}$$

$$\begin{aligned} \text{Surplus} &= PQ - TC - \text{Plan A cost} \\ &= \$3,000(180) - \$75,000 - \$2,000(180) - \$2.5(180^2) - \$100(180) \\ &= \$6,000 \end{aligned}$$

C. When operating costs increase by a flat \$20,000, the marginal cost function and operating surplus (profit) maximizing activity level are unaffected. As in part A, Q = 200.

The new operating surplus (profit) level is:

$$
\begin{aligned}
\text{Operating Surplus} &= PQ - TC - \text{Plan B cost} \\
&= \$25{,}000 - \$20{,}000 \\
&= \$5{,}000
\end{aligned}
$$

Here, the ICC would be slightly better off under plan A. In general, a fixed-sum increase in costs will decrease the operating surplus (profit) by a like amount, but have no influence on price nor activity levels in the short-run. In the long-run, however, both price and activity levels will be affected if cost increases depress the operating surplus (profit) below a normal (or required) rate of return.

23. **Average Cost Minimization**. Commercial Recording, Inc., is a manufacturer and distributor of reel-to-reel recording decks for commercial recording studios. Revenue and cost relations are:
$TR = \$3{,}000Q - \$0.5Q^2$

$MR = \partial TR/\partial Q = \$3{,}000 - \$1Q$

$TC = \$100{,}000 + \$1{,}500Q + \$0.1Q^2$

$MC = \partial TC/\partial Q = \$1{,}500 + \$0.2Q$

A. Calculate output, marginal cost, average cost, price, and profit at the average cost-minimizing activity level.

B. Calculate these values at the profit-maximizing activity level.

C. Compare and discuss your answers to parts A and B.

ANS:

A. To find the average cost-minimizing level of output, set MC = AC and solve for Q:

$$\$1{,}500 + \$0.2Q = \frac{\$100{,}000 + \$1{,}500Q + \$0.1Q^2}{Q}$$

$$1{,}500 + 0.2Q = \frac{100{,}000}{Q} + 1{,}500 + 0.1Q$$

$$0.1Q = \frac{100{,}000}{Q}$$

$$Q^2 = \frac{100{,}000}{0.1}$$

$$Q = \sqrt{\frac{100{,}000}{0.1}}$$

$$Q = 1{,}000$$

And,

$MC = \$1,500 + \$0.2(1,000)$
$= \$1,700$

$$AC = \frac{\$100,000}{1,000} + \$1,500 + \$0.1(1,000)$$
$= \$1,700$

$P = TR/Q$
$= (\$3,000Q - \$0.5Q^2)/Q$
$= \$3,000 - \$0.5Q$
$= \$3,000 - \$0.5(1,000)$
$= \$2,500$

$\pi = (P - AC)Q$
$= (\$2,500 - \$1,700)1,000$
$= \$800,000$

(*Note:* Average cost is rising for Q > 1,000.)

B. To find the profit-maximizing level of output, Set MR = MC and solve for Q:

MR = MC
$\$3,000 - \$1Q = \$1,500 + \$0.2Q$
$1.2Q = 1,500$
$Q = 1,250$

And

$MC = \$1,500 + \$0.2(1,250)$
$= \$1,750$

$$AC = \frac{\$100,000}{1,250} + \$1,500 + \$0.1(1,250)$$
$= \$1,705$

$P = \$3,000 - \$0.5(1,250)$
$= \$2,375$

$\pi = (P - AC)Q$
$= (\$2,375 - \$1,705)1,250$
$= \$837,500$

(*Note:* Profit is falling for Q > 1,250.)

C. Average cost is minimized when MC = AC = \$1,700. Given P = \$2,500, a \$800 profit per unit of output is earned when Q = 1,000. Total profit π = \$800,000.

Profit is maximized when Q = 1,250 because MR = MC = \$1,750 at that activity level. Because MC = \$1,750 > AC = \$1,705, average cost is rising. Given P = \$2,375 and AC = \$1,750, a \$670 profit per unit of output is earned when Q = 1,250. Total profit π = \$837,500.

Total profit is higher at the Q = 1,250 activity level because the modest $5(= $1,705 - $1,700) increase in average cost is more than offset by the 250 unit expansion in sales from Q = 1,000 to Q = 1,250 and the resulting increase in total revenues.

24. **Average Cost Minimization**. Better Buys, Inc., is a leading discount retailer of wide-screen digital and cable-ready plasma HDTVs. Revenue and cost relations for a popular 55-inch model are:

$TR = \$4{,}500Q - \$0.1Q^2$

$MR = \partial TR/\partial Q = \$4{,}500 - \$0.2Q$

$TC = \$2{,}000{,}000 + \$1{,}500Q + \$0.5Q^2$

$MC = \partial TC/\partial Q = \$1{,}500 + \$1Q.$

A. Calculate output, marginal cost, average cost, price, and profit at the average cost-minimizing activity level.

B. Calculate these values at the profit-maximizing activity level.

C. Compare and discuss your answers to parts A and B.

ANS:

A. To find the average cost-minimizing level of output, set MC = AC and solve for Q:

$$MC = AC$$

$$\$1{,}500 + \$1Q = \frac{\$2{,}000{,}000 + \$1{,}500Q + \$0.5Q^2}{Q}$$

$$1{,}500 + Q = \frac{2{,}000{,}000}{Q} + 1{,}500 + 0.5Q$$

$$0.5Q = \frac{2{,}000{,}000}{Q}$$

$$Q^2 = \frac{2{,}000{,}000}{0.5}$$

$$Q = \sqrt{\frac{2{,}000{,}000}{0.5}}$$

$$= 2{,}000$$

And,

$$MC = \$1{,}500 + \$1(2{,}000)$$
$$= \$3{,}500$$

$$AC = \frac{\$2{,}000{,}000}{2{,}000} + \$1{,}500 + \$0.5(2{,}000)$$
$$= \$3{,}500$$

$P = TR/Q$
$= (\$4{,}500Q - \$0.1Q^2)/Q$
$= \$4{,}500 - \$0.1Q$
$= \$4{,}500 - \$0.1(2{,}000)$
$= \$4{,}300$

$\pi = (P - AC)Q$
$= (\$4{,}300 - \$3{,}500)2{,}000$
$= \$1{,}600{,}000$

(*Note:* Average cost is rising for Q > 2,000.)

B. To find the profit-maximizing level of output, Set MR = MC and solve for Q:

$MR = MC$
$\$4{,}500 - \$0.2Q = \$1{,}500 + \$1Q$
$1.2Q = 3{,}000$
$Q = 2{,}500$

And

$MC = \$1{,}500 + \$1(2{,}500)$
$= \$4{,}000$

$$AC = \frac{2{,}000{,}000}{2{,}500} + \$1{,}500 + \$0.5(2{,}500)$$
$= \$3{,}550$

$P = \$4{,}500 - \$0.1(2{,}500)$
$= \$4{,}250$

$\pi = (P - AC)Q$
$= (\$4{,}250 - \$3{,}550)2{,}500$
$= \$1{,}750{,}000$

(*Note:* Profit is falling for Q > 2,500.)

C. Average cost is minimized when MC = AC = \$3,500. Given P = \$4,300, a \$800 profit per unit of output is earned when Q = 2,000. Total profit π = \$1.6 million.

Profit is maximized when Q = 2,500 because MR = MC = \$4,000 at that activity level. Because MC = \$4,000 > AC = \$3,550, average cost is rising. Given P = \$4,250 and AC = \$3,550, a \$700 profit per unit of output is earned when Q = 2,500. Total profit π = \$1.75 million.

Total profit is higher at the Q = 2,500 activity level because the modest \$50(= \$3,550 - \$3,500) increase in average cost is more than offset by the 500 unit expansion in sales from Q = 2,000 to Q = 2,500 and the resulting increase in total revenues.

25. **Revenue Maximization**. Restaurant Marketing Services, Inc., offers affinity card marketing and monitoring systems to fine dining establishments nationwide. Fixed costs are \$600,000 per year. Sponsoring restaurants are paid \$60 for each card sold, and card printing and distribution costs are \$3 per card. This means that RMS's marginal costs are \$63 per card. Based on recent sales experience, the estimated demand curve and marginal revenue relations for are:

$P = \$130 - \$0.000125Q$

$MR = \partial TR/\partial Q = \$130 - \$0.00025Q$

A. Calculate output, price, total revenue, and total profit at the revenue-maximizing activity level.

B. Calculate output, price, total revenue, and total profit at the profit-maximizing activity level.

C. Compare and discuss your answers to parts A and B.

ANS:

A. To find the revenue-maximizing level of output, set MR = 0 and solve for Q:

$$\begin{aligned} MR &= 0 \\ \$130 - \$0.00025Q &= 0 \\ 0.00025Q &= 130 \\ Q &= 520{,}000 \end{aligned}$$

$$\begin{aligned} P &= \$130 - \$0.000125Q \\ &= \$130 - \$0.000125(520{,}000) \\ &= \$65 \end{aligned}$$

$$\begin{aligned} TR &= PQ \\ &= \$65(520{,}000) \\ &= \$33{,}800{,}000 \end{aligned}$$

$$\begin{aligned} \pi &= TR - TC \\ &= \$33{,}800{,}000 - \$600{,}000 - \$63(520{,}000) \\ &= \$440{,}000 \end{aligned}$$

(*Note:* Revenue is falling when Q > 520,000.)

B. To find the profit-maximizing level of output, Set MR = MC and solve for Q:

$$\begin{aligned} MR &= MC \\ \$130 - \$0.00025Q &= \$63 \\ 0.00025Q &= 67 \\ Q &= 268{,}000 \end{aligned}$$

$$\begin{aligned} P &= \$130 - \$0.000125(268{,}000) \\ &= \$96.50 \end{aligned}$$

$$\begin{aligned} TR &= \$96.50(268{,}000) \\ &= \$25{,}862{,}000 \end{aligned}$$

$$\pi = \$25,862,000 - \$600,000 - \$63(268,000)$$
$$= \$8,378,000$$

(*Note:* Profit is decreasing for Q > 268,000.)

C. Revenue maximization is achieved when MR = 0. Profit maximization requires MR = MC. These output levels will only be the same if MC = 0. This would be highly unusual. In this problem, as is typical, MC > 0 and profit maximization occurs at an activity level with lower output and revenue, but higher prices and profits, than the revenue-maximizing activity level.