CHAPTER 2 Demand Theory

Lecture Notes

1. Introduction

- Objectives
  - ➢ To explain the importance of market demand in managerial decision making
  - ➢ To understand the many factors that influence the demand for a product
  - ➢ To measure and analyze the sensitivity of demand to changes in factors affecting demand. The tool used for this type of sensitivity analysis is *demand elasticity.*
    - ∗ *Elasticity:* Measures the percentage change in one factor given a small (marginal) percentage change in another factor
    - ∗ *Elasticity:* Measures the sensitivity of one factor to another
    - ∗ *Demand elasticity:* Measures the percentage change in quantity demanded of a product given a small (marginal) percentage change in another factor that affects the demand for the product
  - ➢ Explain the role of managers in influencing and predicting market demand.
    - ∗ Managers can influence demand by controlling, among other things, advertising, product quality, and distribution strategies.
    - ∗ Managers cannot control, but need to understand, elements of the competitive environment that influence demand, including the availability of substitute or complement goods, their pricing, and the advertising strategies employed by their sellers.
    - ∗ Managers cannot control, but need to understand how, the macroeconomic environment influences demand, including interest rates, taxes, and both local and global levels of economic activity such as the level of income in the economy.

2. The Market Demand Curve

- *Market Demand Schedule:* A table showing the total quantity of the good purchased at each price during a given time period
- *Market Demand Curve:* A plot of the market demand schedule on a graph
- *Example* (Table 2.1): Demand schedule for tablets
- It shows the total quantity of tablets demanded at each price, not the quantity demanded from a particular firm.
  - *Convention:* Price is on the vertical axis and quantity is on the horizontal axis.
  - *Example* (Figure 2.1): Demand curve for tablets
- Characteristics of the Market Demand Curve
  - Quantity demanded is for output of the entire market or the industry, not of a single firm.
  - For most products and services, the market demand curve slopes downward and to the right.
  - *Example:* The quantity of tablets demanded increases as the price of tablets falls.
  - Quantity demanded is defined with regard to a particular time period.
- Determinants of the position and shape of the market demand curve—Some of the important factors include
  - Consumer tastes or preferences
    - An increase in consumer tastes shifts the demand curve to the right.
    - A decrease in consumer tastes shifts the demand curve to the left.
  - Consumer income (or more specifically per capita disposable income)
    - Normal and inferior goods
    - *Example* (Figure 2.3): Increase in income causes an increase in demand for tablets; that is, tablets are a normal good.
  - Population size in the market

STRATEGY SESSION:
The Customer Is Always Right—Wrong!

Discussion Questions

1. Like retail technology stores, clothing stores have their angels and devils. How do you think the devils prey on clothing stores, and how could their behavior be discouraged? How do you think angels could be encouraged to shop at a particular clothing store?

   Answer: Devils buy clothes, wear them, and then return them for a refund. Stores can refuse to provide refunds on returns and, instead, provide a credit for future purchases or only allow exchanges. Angels buy lots of clothes on impulse. Stores could offer quantity discounts or a "shoppers club" with special notification of sales.

2. Some electronics stores refuse to allow customers to return or exchange products, instead requiring them to deal directly with the manufacturer. What are the pros and cons of this approach with regard to the stores' objective of encouraging angels and discouraging devils?

3. Industry and Firm Demand Functions

- *Market Demand Function:* The relationship between the quantity demanded of a product and the various factors that influence this quantity
  - Quantity demanded of good X: $Q = f(\text{factors}) = f(P, P\_r, I, T, N, A, \ldots)$
  - Factors include
    - price of X: P
    - incomes of consumers: I
    - tastes of consumers: T
    - prices of related goods in consumption: P_r
    - population size: N
    - advertising expenditures: A
    - general demand function:

$$Q = f(\text{factors}) = f(P, P\_r, I, T, N, A, \text{ other factors})$$

  - *Example* (equation 2.1): A *linear* demand function:

$$Q = b_1P + b_2I + b_3S + b_4A$$

    - Assumes that population is constant
    - P = price of tablets
    - I = per capita disposable income
    - S = average price of software
    - A = amount spent on advertising
    - b_1, b_2, b_3, and b_4 are parameters that are estimated using statistical methods, namely, *regression analysis*.
  - *Parameters:* Constant or variable terms used in the function that helps managers determine the specific form of the function but not its general nature
    - *Example:* $Q = -2{,}000P + 70I - 375S + 0.0001A$
  - Relationship between the market demand function and the market demand curve
    - The market demand curve shows the relationship between Q and P when all other variables are held constant at specific values.
    - The market demand function does not explicitly hold any values constant.
  - *Example* (equation 2.3): Suppose $I = 13{,}000$, $S = 400$, and $A = 50$ million. Then
  - $Q = -700P + 200(13{,}000) - 500(400) + 0.01(50{,}000{,}000)$
    That is

* $Q = 2{,}900{,}000 - 700P$ (direct demand function)

Solving for P in terms of Q gives the inverse demand function:

$P = 4{,}143 - 0.001429Q$ (graphed in Figure 2.4)

- *Example*: Shifting the demand curve
  Suppose the price of software falls from $400 to $200. Then

  $Q = -700P + 200(13{,}000) - 500(200) + 0.01(50{,}000{,}000)$
  $Q = 3{,}000{,}000 - 700P$

  * Solving for P gives

  $P = 4{,}286 - 0.001429Q$ (graphed in Figure 2.4)

  Note: Same slope; only the vertical intercept changes.

- The Firm's Demand Curve
  - Negative slope with regard to own price
    * Slope may not be the same as that of the market demand curve
  - Represents a portion of market demand
    * Market share
    * Responds to same market and macroeconomic factors as the market demand curve

4. The Own-Price Elasticity of Demand

- *Own-Price Elasticity of Demand:* More simply referred to as the price elasticity of demand, it is the concept that managers use to measure the percentage change in quantity demanded of their firm's products resulting from a 1% change in the products' own prices.
  - The elasticity of a function is the percentage change in the dependent (Y) variable in response to a 1% change in the independent (X) variable.
  - The price elasticity of a demand function is the percentage change in quantity demanded in response to a 1% change in price.
  - Formula:

$$\eta = \left(\frac{P}{Q}\right)\frac{\Delta Q}{\Delta P}$$

  - Along a demand function, it is given by

$$\eta = \left(\frac{P}{Q}\right)\frac{\partial Q}{\partial P}$$

  - Price elasticity of demand generally is different at different prices and on different markets.

- Terminology
  - Price elasticity of demand is symbolized by the Greek letter eta (η) (not a general convention).
  - *Fact:* $0 \geq \eta \geq -\infty$. That is, price elasticity of demand is always non-positive.
  - When $|\eta| > 1$, demand is elastic.
  - When $|\eta| < 1$, demand is inelastic.
  - When $|\eta| = 1$ or $\eta = -1$, demand is unitary.

Limiting cases:
  - When $\eta = 0$, demand is perfectly inelastic and the demand curve is vertical.
    - Quantity demanded is the same at all prices.
  - When $\eta = -\infty$, demand is perfectly elastic and the demand curve is horizontal.
    - Price is the same for all quantities demanded.
    - If price rises, quantity demanded falls to zero.
    - If price falls, quantity demanded increases without limit.
- Linear Demand Curves
  - The slope of a linear demand curve is constant.
  - If the demand curve is neither vertical nor horizontal, the price elasticity will vary depending on price level.
    - At the midpoint of a linear demand curve, $\eta = -1$, with η approaching zero as price approaches the vertical intercept.
    - At prices above the midpoint, demand is elastic, with η approaching negative infinity as quantity approaches zero or as price approaches the vertical intercept.
    - At prices below the midpoint, demand is inelastic, with η approaching 0 as price approaching 0.
  - Given a demand curve defined as $P = a - bQ$, the price elasticity of demand is: $\eta = \left(\frac{-1}{b}\right)\frac{a - bQ}{Q} = -P/(a - P)$

5. Point and Arc Elasticities

- The point price elasticity formula should be used when working with an estimated demand curve or when the change in price is very small. It is written as

$$\eta = \left(\frac{P}{Q}\right)\frac{\partial Q}{\partial P}$$

  - Calculated value of price elasticity for small changes in prices will differ depending on whether P and Q are the starting values or the ending values after the price change. The change will be small if the change is small.

    - *Example:* $P_1 = 99.95$, $P_2 = 100.00$, $Q_1 = 20{,}002$, and $Q_2 = 20{,}000$
    - $\eta = [(20{,}002 - 20{,}000)/(99.95 - 100)][99.95/20{,}002] = -0.1999$
    - $\eta = [(20{,}000 - 20{,}002)/(100 - 99.95][100/200{,}000] = -0.22$
  - If the price change is large, then the direction of change will influence the calculated elasticity.
    - *Example:* $P_1 = 5$, $P_2 = 4$, $Q_1 = 3$, and $Q_2 = 40$
    - $\eta = [(40 - 3)/(4 - 5)][5/3] = -61.67$
    - $\eta = [(3 - 40)/(5 - 4)][4/40] = -3.70$
  - This problem is corrected by using the arc or midpoint formula.
- *Recommendation:* The midpoint or arc elasticity formula should be used to estimate the price elasticity of demand from a demand schedule where price differences are not very small. It is given by

$$\eta = \left(\frac{\Delta Q}{\Delta P}\right)\left(\frac{P_1 + P_2}{Q_1 + Q_2}\right)$$

  - *Example:* $P_1 = 5$, $P_2 = 4$, $Q_1 = 3$, and $Q_2 = 40$
  - $\eta = [(40 - 3)/(4 - 5)][(5 + 4)/(3 + 40)] = -7.74$

6. Using the Demand Function to Calculate the Price Elasticity of Demand

Whenever the demand function is specified, one usually uses the point price elasticity of demand.

- *Example:* Given
  - $Q = -700P + 200I - 500S + 0.01A$
  - Q = Quantity demanded of tablet computers
  - Price = $P = 3{,}000$
  - Income = $I = 13{,}000$
  - Software = $S = 400$
  - Advertising = $A = 50{,}000{,}000$
- Derive the demand curve
  - $Q = -700P + (200)(13{,}000) - (500)(400) + (0.01)(50{,}000{,}000)$
  - $Q = 2{,}900{,}000 - 700P$
- Determine Q
  - $Q = 2{,}900{,}000 - (700)(3{,}000) = 800{,}000.$

    We have $\dfrac{\partial Q}{\partial P} = -700$. Thus
- $\eta = (-700)(3{,}000/800{,}000) = -2.62$
- For $P = 2{,}000$, $Q = 2{,}900{,}000 - (700)(2{,}000) = 1{,}500{,}000$, so $\eta = (-700)(2{,}000/1{,}500{,}000) = -0.93$

7. The Effect of Price Elasticity on the Firm's Revenue

- Derivation of relationship between marginal revenue ($\Delta TR/\Delta Q$ or dTR/dQ) and the price elasticity of demand
  - Total revenue: $TR = PQ$
  - Consider quantity as a function of price: $Q = f(P)$. Then, differentiating TR with respect to P, using the product rule for derivative, gives

$$dTR/dP = Q(dP/dP) + P(dQ/dP)$$

Dividing by Q gives

$$(1/Q)(dTR/dP) = (\mathrm{d}P/dP) + (P/Q)(dQ/dP) = 1 + \eta$$

- $1/Q$ is positive.
- *Implications:*
  - *Case 1:* If $\eta = -1$, that is, unitary elastic demand, then $1 + \eta = 0$, and $dTR/dP = 0$, so total revenue is at a maximum and a change in P will have no effect on total revenue.
  - *Case 2:* If $\eta > -1$ (inelastic demand), then $1 + \eta > 0$ and $dTR/dP > 0$, so an increase in P (and consequent decrease in Q) will increase total revenue.
  - *Case 3:* If $\eta < -1$ (elastic demand), then $1 + \eta < 0$, and $dTR/dP < 0$, so an increase in P (and consequent decrease in Q) will reduce total revenue.
  - *To summarize:*
- If the price elasticity is unitary, any price change will cause an equal and opposite percentage change in quantity demanded. Total revenue will remain constant.
- If the price elasticity is in the inelastic range, then a 1% change in P will cause less than a 1% change in quantity in the opposite direction. Therefore, total revenue will change in the same direction as price.
- If the price elasticity is in the inelastic range, then a 1% change in P will cause more than a 1% change in quantity in the opposite direction. Therefore, total revenue will change in the opposite direction from price.

PROBLEM SOLVED:
Price Elasticity of Demand: Philip Morris

Discussion Questions

1. The decline in total revenue from cigarette sales in 1993 is attributed to Philip Morris's cut in the price of cigarettes. Are there other factors that might have contributed to this decline in revenue?

   Answer: The price elasticity of demand assumes that "all other things" are held constant. Changes in taxes, consumer income, or attitudes toward

tobacco during this period might have reduced demand, while the price cut increased quantity demanded. If this was the case, then the true price elasticity would likely be closer to −1.

8. Funding Public Transit

- Given
  - ➢ Price (fare) elasticity of demand for public transit in the United States is estimated to be about −0.3.

  Facts:
  - ➢ All public transit systems in the United States lose money.
  - ➢ Public transit systems are funded by federal, state, and local governments, all of which have budget issues.
- Which transit systems have the most difficult time getting public funding?
  - ➢ Revenue from sales will increase if fares are increased because demand is inelastic.
  - ➢ Costs will likely decrease if fares are increased because quantity demanded (ridership) will fall.
  - ➢ Implication: Managers of public transit will therefore increase fares if they do not receive enough public funds to balance their budgets.

9. Determinants of the Own-Price Elasticity of Demand

- Number and Similarity of Available Substitutes
  Fact: A product with many close substitutes generally has elastic demand.
- Product's Price Relative to a Consumer's Total Budget
  - ➢ *Facts:*
    - ∗ Products for which the typical consumer spends only a very small fraction of her income are quite elastic.
      - ◦ *Examples:* Thimbles, rubber bands, salt
    - ∗ Products that command a larger percentage of the consumer's total budget tend to be more price elastic.
      - ◦ *Examples:* Kitchen appliances, automobiles
- Time Period Available for Adjustment to a Price Change
  - ➢ *Fact:* For nondurable goods, demand is likely to be more elastic over a long period relative to a short period.
  - ➢ *Rationale:* The longer the time period, the easier it is for consumers to substitute one good for another.

10. The Strategic Use of the Price Elasticity of Demand

- Managers can change the price elasticity of demand for their products.
- A Useful Tool: *Product differentiation*

- Differentiation strategies convince consumers the product is unique; hence it has fewer substitutes.
- *Caution:* Differentiation is not effective if consumers do not perceive it.
- *Example:* Strategic pricing of first-class ($\eta = -0.45$), regular economy ($\eta = -1.30$), and excursion ($\eta = -1.83$) airline tickets between the United States and Europe
  - First-class prices should be relatively high because demand is inelastic.
  - Regular economy and excursion prices should be relatively low because demand is elastic.
  - If consumers perceive that a product has fewer substitutes, then their price elasticity of demand for the product will decrease (become less elastic) in absolute value.
  - Differentiation strategies do not require actual differences in products, only a perceived difference.

STRATEGY SESSION:
Elasticity in Use

Discussion Questions

1. Suppose that a manufacturer sells a product through an upscale boutique and, with a different brand name, through a discount retailer. The elasticity of demand at the boutique is −1.2, and at the discount retailer it is −2.6. If the optimal price at the boutique is \$85, what price (P_D) should be charged at the discount retailer?

   Answer: $85(1 - 1/1.2) = P_D(1 - 1/2.6)$ so $P_D = \$23.02$

2. A consulting firm charges \$250 per hour to Fortune 500 companies. The estimated elasticity of demand for consulting services is −3.1. The firm is planning to spin off a subsidiary firm that will work with smaller businesses. The estimated elasticity of demand for these firms is −7.3. What price per hour (P_S), to the nearest dollar, should be charged by the subsidiary?

   Answer: $250(1 - 1/3.1) = P_S(1 - 1/7.3)$ so $P_S = \$200$

11. Total Revenue, Marginal Revenue, and Price Elasticity

- A firm's total revenue (TR) is equal to the total amount of money consumers spend on the product in a given time period.
  - Linear demand curve: $P = a - bQ$
  - Corresponding total revenue curve: $TR = PQ = aQ - bQ^2$
- *Marginal Revenue:* The incremental revenue earned from selling an additional unit of output
  - $MR = dTR/dQ = d(aQ - bQ^2)/dQ = a - 2bQ$
    - $\eta = (-1/b)[(a - bQ)/Q] = -a/(bQ) + 1$

- *Case 1:* If $Q = a/2b$, then $\eta = -1$, unitary elastic demand.
- *Case 2:* If $Q > a/2b$, then $\eta > -1$, and demand is inelastic.
- *Case 3:* If $Q < a/2b$, then $\eta < -1$, and demand is elastic.

➢ $MR = dTR/dQ = d(PQ)/dQ = P(dQ/dQ) + Q(dP/dQ) = P[1 + (Q/P)(dP/dQ)$. Thus $MR = P(1 + 1/\eta)$

- *Case 1:* $|\eta| > 1$ (elastic) implies $MR > 0$.
- *Case 2:* $|\eta| < 1$ (inelastic) implies $MR < 0$.
- *Case 3:* $|\eta| = 1$ (unitary elastic) implies $MR = 0$.

12. The Income Elasticity of Demand

- *Income Elasticity of Demand* (η_I): Measures the percentage change in quantity demanded (Q) resulting from a 1% change in consumer income (I)
  - ➢ Income can be defined as aggregate consumer income or as per capita disposable income, depending on circumstances.
  - ➢ $\eta_I = \left(\frac{\Delta Q}{\Delta I}\right)\left(\frac{I}{Q}\right)$
  - ➢ For a demand function, it is given by

$$\eta = \left(\frac{I}{Q}\right)\frac{\partial Q}{\partial I}$$

  - ➢ $\eta_I > 0$ for normal goods because $\frac{\partial Q}{\partial I} > 0$ for normal goods.
    - For most products, $\eta_I > 0$; that is, most goods are normal, since increases in aggregate income are associated with increases in aggregate consumer spending.
  - ➢ $\eta_I < 0$ for inferior goods because $\frac{\partial Q}{\partial I} < 0$ for inferior goods.
    - *Examples:* Hamburgers and public transportation
    - *Fact:* When the economy is expanding, products with high-income elasticities will enjoy a significant increase in sales and managers must prepare for probable significant increase in sales.
- Strategic Management and the Income Elasticity of Demand
  - ➢ The demand for a product that has an income elasticity of demand that is large in absolute value will vary widely with changes in income caused by economic growth and recessions.
  - ➢ Managers can lessen the impact of economic changes on such products by limiting fixed costs so that changes in production capacity can be made quickly.
  - ➢ Managers can forecast demand for products using the income elasticity of demand combined with forecasts of aggregate income.

PROBLEM SOLVED:
Income Elasticity of Demand

Discussion Questions

1. Suppose that a market demand function is defined as $Q = 20{,}000 - 8P + 0.1I$ and suppose that $P = 2{,}000$ and $I = 20{,}000$. What is the income elasticity of demand at this point?

   Answer:
   Here, we have
   $\frac{\partial Q}{\partial I} = 0.1$ (a normal good). Then we have

   $$Q = 6{,}000, \text{ so that } \eta_I = 0.1(20{,}000/6{,}000) = -1/3$$

2. If the income elasticity of demand for a product is unitary, then a 1% change in income will change demand in the same direction by 1%. If price remains constant, then spending on the product will change by 1% and, consequently, spending on the product will be the same percentage of income after the income change as it was before. If the income elasticity of demand is greater than one, then spending will increase as a percentage of income as income increases. If it is less than one, spending will decrease as a percentage of income as income increases. How do you think the percentage of income spent on jewelry, food, clothing, housing, and automobiles responds to a 1% increase in income?

13. Cross-Price Elasticities of Demand

- *Cross-Price Elasticity of Demand* (η_{XY}): The percentage change in quantity demanded of one good (Q_X) resulting from a 1% change in the price of a related good (P_Y)
  - Income can be defined as aggregate consumer income or as per capita disposable income, depending on circumstances.
  - $$\eta_{XY} = \left(\frac{\Delta Q_X}{\Delta P_Y}\right)\left(\frac{P_Y}{Q_X}\right)$$
  - $\eta_{XY} > 0$ if the two products are substitutes.
    - *Example:* Wheat and corn
  - $\eta_{XY} < 0$ if the two products are complements.
    - *Example*: Computers and computer software
  - $\eta_{XY} = 0$ if the two products are independent or unrelated.
    - *Example*: Butter and airline tickets
  - *Example*: A linear demand function
    - Given: $Q_X = 1{,}000 - 0.2P_X + 0.5P_Y + 0.04I$, $Q_X = 2{,}000$, and $P_Y = 500$
    - $\eta_{XY} = 0.5(500/2{,}000) = 0.125$ so the two products are substitutes.

- Strategic management and the cross-price elasticity of demand
  - ➢ Managers can use information about the cross-price elasticity of demand to predict the effect of competitors' pricing strategies on the demand for their product.
  - ➢ Antitrust authorities use the cross-price elasticity of demand to determine the likely effect of mergers on the degree of competition in an industry.
    - ∗ A high positive cross-price elasticity, indicating that the two goods are strong substitutes, suggests that a merger would significantly reduce competition in the industry.
    - ∗ A high negative cross-price elasticity of demand, indicating that the two goods are strong complements, suggests that a merger might give the merged firm excessive control over the supply chain. The merged firm may refuse to sell the intermediate product to other producers.

14. The Advertising Elasticity of Demand

- *Advertising Elasticity of Demand* (η_A): The percentage change in quantity demanded (Q) resulting from a 1% change in advertising expenditure (A)
  - ➢ $\eta_A = \left(\frac{\Delta Q}{\Delta A}\right)\left(\frac{A}{Q}\right)$
  - ➢ *Example calculation:* A linear demand function
    - ∗ Suppose: $Q = 500 - 0.5P + 0.01I + 0.82A$ and suppose, A/Q, the amount of advertising per unit of output, is \$2. Then
    - ∗ $\eta_A = 0.82(2) = 1.64$, indicating that a 1% increase in income results in a 1.64% increase in the quantity demanded.

15. The Constant-Elasticity and Unitary Demand Function

- *Constant-Elasticity Demand Function:* Mathematical form of a demand function that always yields that same elasticity, regardless of the product's price and consumers' income and other factors that influence demand
  - ➢ *Example:* A multiplicative demand function

$$Q = 200P^{-0.3}I^2$$

  - ➢ Price elasticity of demand = − 0.3
  - ➢ Income elasticity of demand = 2.0
  - ➢ More generally, an example of a constant elasticity demand function is

$$Q = aP^{-b}\mathrm{I}^c$$

    where a, b, c are parameters to be estimated.
  - ➢ Price elasticity of demand = $-b$
  - ➢ Income elasticity of demand = c

- ➢ *Fact:* For a multiplicative demand function, the exponents represent elasticities.

- Unitary elastic demand function and total revenue (TR)
  - ➢ TR = PQ so if TR is constant, $Q = (TR)(P^{-1})$
  - ➢ Price elasticity of demand = -1
  - ➢ More generally, a unitary elastic demand curve is given by the demand function

$$Q = mP^{-1} \text{ or } Q = m/P$$

  - ➢ Its graph: A rectangular hyperbola

Chapter 2: Problem Solutions

1. The Dolan Corporation, a maker of small engines, determines that in 2012 the demand curve for its product is

$$P = 2{,}000 - 50Q$$

where P is the price (in dollars) of an engine and Q is the number of engines sold per month.

a. To sell 20 engines per month, what price would Dolan have to charge?
b. If managers set a price of \$500, how many engines will Dolan sell per month?
c. What is the price elasticity of demand if price equals \$500?
d. At what price, if any, will the demand for Dolan's engines be of unitary elasticity?

SOLUTION:

a. For $Q = 20$, $P = 2{,}000 - 50(20) = \$1{,}000$.
b. For $P = 500$, we have $500 = 2{,}000 - 50Q$. Solving for Q gives $50Q = 1{,}500$, that is, $Q = 30$.
c. $\eta = (\partial Q/\partial P)(P/Q) = (-1/50)(500/30) = -1/3$.
d. For $P = \$1{,}000$, $\eta = (\partial Q/\partial P)(P/Q) = (-1/50)(1{,}000/20) = -1$. Alternatively, set $(-1/50)(2{,}000 - 50Q)/Q = -1$. Solve for Q and then P.

2. The Johnson Robot Company's marketing managers estimate that the demand curve for the company's robots in 2012 is

$$P = 3{,}000 - 40Q$$

where P is the price of a robot and Q is the number sold per month.

a. Derive the marginal revenue curve for the firm.
b. At what prices is the demand for the firm's product price elastic?
c. If the firm wants to maximize its dollar sales volume, what price should it charge?

SOLUTION:

a. $TR = PQ = (3{,}000 - 40Q)Q = 3{,}000Q - 40Q^2$. Hence,

$$MR = dTR/dQ = 3{,}000 - 80Q.$$

b. Recall: For a linear demand curve $P = a - bQ$, demand is elastic if $P > a/2$. Hence, we have $P \geq \$1{,}500$.

c. Revenue is maximized at $MR = 0$. That is, $3{,}000 - 80Q = 0$. Solving for Q gives $Q = 3{,}000/80 = 37.5$ and therefore $P = 3{,}000 - 40(37.5)$, that is $P = \$1{,}500$.
Alternatively, one can use the fact that for a linear demand curve $P = a - bQ$, revenue is maximized at $Q = a/2b$ or $P = a/2$.

3. After a careful statistical analysis, the Chidester Company concludes the demand function for its product is

$$Q = 500 - 3P + 2P_r + 0.1I$$

where Q is the quantity demanded of its product, P is the price of its product, P_r is the price of its rival's product, and I is per capita disposable income (in dollars). At present, $P = \$10$, $P_r = \$20$, and $I = \$6{,}000$.

a. What is the price elasticity of demand for the firm's product?
b. What is the income elasticity of demand for the firm's product?
c. What is the cross-price elasticity of demand between its product and its rival's product?
d. What is the implicit assumption regarding the population in the market?

SOLUTION:

a. $\eta = (\partial Q/\partial P)(P/Q) = (-3)(10/1{,}110) = -3/111$
b. $\eta_I = (\partial Q/\partial I)(I/Q) = (0.1)(6{,}000/1{,}110) = 60/111$
c. $\eta_{\text{cross}} = (\partial Q/\partial P_r)(P_r/Q) = (2)(20/1{,}110) = 4/111$
d. The model for market demand implicitly assumes that the population is constant.

4. The Haas Corporation's executive vice president circulates a memo to the firm's top management in which he argues for a reduction in the price of the firm's product. He says such a price cut will increase the firm's sales and profits.

a. The firm's marketing manager responds with a memo pointing out that the price elasticity of demand for the firm's product is about -0.5. Why is this fact relevant?
b. The firm's president concurs with the opinion of the executive vice president. Is she correct?

SOLUTION:

a. Whether total revenue will go up or down when the product price is lowered and more units are sold depends on whether the quantity of units sold increases by a greater percentage than the price is reduced by. That is, it depends on whether the demand is elastic or inelastic. The nature of the price elasticity of demand can be used as a test to determine the effect of a price change on total revenue. This is the so-called total revenue test.
b. Assuming that the marketing manager is correct that the demand elasticity is −0.5, then demand is price inelastic and therefore a price reduction will cause the number of units sold to increase by a smaller percentage than price has fallen, and both the president and executive vice president will have egg on their faces when total revenues decline after the price is reduced.

5. Managers of the Hanover Manufacturing Company believe the demand curve for its product is

$$P = 5 - Q$$

where P is the price of its product (in dollars) and Q is the number of millions of units of its product sold per day. It is currently charging $1 per unit for its product.

a. Evaluate the wisdom of the firm's pricing policy.
b. A marketing specialist says that the price elasticity of demand for the firm's product is −1.0. Is this correct?

SOLUTION:

a. We have $TR = (5 - Q)Q = 5Q - Q^2$. Therefore,

$$MR = 5 - 2Q.$$

At $P = 1$, $Q = 4$ and $MR = -3$ and total revenue decreases. The price is too low; increasing the price and selling fewer units would increase revenues.

b. No, while $dQ/dP = -1$, $(dQ/dP)(P/Q) = -1/4$ at $P = 1$.

6. On the basis of historical data, Richard Tennant has concluded, "The consumption of cigarettes is . . . [relatively] insensitive to changes in price. . . . In contrast, the demand for individual brands is highly elastic in its response to price. . . . In 1918, for example, Lucky Strike was sold for a short time at a higher retail price than Camel or Chesterfield and rapidly lost half its business."

a. Explain why the demand for a particular brand is more elastic than the demand for all cigarettes. If Lucky Strike raised its price by 1% in 1918, was the price elasticity of demand for its product greater than −2?

b. Do you think that the demand curve for cigarettes is the same now as it was in 1918? If not, describe in detail the factors that have shifted the demand curve and whether each has shifted it to the left or right.

SOLUTION:

a. As we define a product more narrowly, consumers have better substitutes (whose prices are held constant) as the price of the good under consideration varies. This makes the demand for a good more elastic the more narrowly the good is defined. We are not told how much Lucky Strike was priced above Camel and Chesterfield, but assuming that the margin was less than 25%, we can conclude that the cross-price elasticity was greater than 2. This isn't exactly right; we must also assume that Lucky Strike's fall in sales resulted from a reduction in the price of Camels and Chesterfields for the cross-price elasticities as explained to students.
b. Population, per capita income, and subsidized health care have all increased; this probably caused the demand curve for cigarettes to shift out, or to the right. Public health education and general education have increased; this probably shifted the demand curve in, or to the left.

7. According to S. Sackrin of the U.S. Department of Agriculture, the price elasticity of demand for cigarettes is between −0.3 and −0.4, and the income elasticity of demand is about 0.5.
   a. Suppose the federal government, influenced by findings that link cigarettes and cancer, were to impose a tax on cigarettes that increased their price by 15%. What effect would this have on cigarette consumption?
   b. Suppose a brokerage house advised you to buy cigarette stocks because if incomes were to rise by 50% in the next decade, cigarette sales would be bound to spurt enormously. What would be your reaction to this advice?

   SOLUTION:

   a. Cigarette consumption would fall by between 4.5 and 6.0%.
   b. Assuming that the prices of cigarettes were to remain constant, a 50% increase in income would cause sales of cigarettes to increase 25%. The weighted average of all income elasticities equals 1, so consumption of noncigarette items would increase by more than 50% and certainly more than the 25% performance of cigarettes. I would not follow the broker's advice.

8. A survey of major U.S. firms estimates on average, the advertising elasticity of demand was only about 0.003. Doesn't this indicate that firms spend too much on advertising?

SOLUTION:

No. The fact that the elasticity of demand with respect to advertising is relatively small (0.003) does not necessarily mean that an additional dollar spent on advertising would not be profitable or that the last dollar spent was not profitable. This number indicates that a 1% increase in advertising expenditure results in a 0.003% increase in the quantity demanded of the product.

9. The McCauley Company hires a marketing consultant to estimate the demand function for its product. The consultant concludes that this demand function is

$$Q = 100P^{-3.1}I^{2.3}A^{0.1}$$

where Q is the quantity demanded per capita per month, P is the product's price (in dollars), I is per capita disposable income (in dollars), and A is the firm's advertising expenditures (in thousands of dollars).

a. What is the price elasticity of demand?
b. Will price increases result in increases or decreases in the amount spent on McCauley's product?
c. What is the income elasticity of demand?
d. What is the advertising elasticity of demand?
e. If the population in the market increases by 10%, what is the effect on the quantity demanded if P, I, and A are held constant?

SOLUTION:

Observation: This is a multiplicative demand function.
Therefore, the exponents represent elasticities.

a. The price elasticity of demand is −3.1.
b. An increase in price will cause revenues to fall because the demand is elastic.
c. The income elasticity of demand is 2.3.
d. The advertising elasticity of demand is 0.1.
e. If P, I, and A are held constant, per capita consumption, Q, is constant. Notice that if N represents the population size, aggregate market demand is given

$$Q_T = 100NP^{-3.1}I^{2.3}A^{0.1}$$

indicating that the population elasticity is 1.

Therefore, a 10% increase in population gives rise to a 10% increase in the quantity demanded.

10. The Schmidt Corporation estimates that its demand function is

$$Q = 400 - 3P + 4I + 0.6A$$

where Q is the quantity demanded per month, P is the product's price (in dollars), I is per capita disposable income (in thousands of dollars), and A is the firm's advertising expenditures (in thousands of dollars per month). Population is assumed to be constant.

a. During the next decade, per capita disposable income is expected to increase by \$5,000. What effect will this have on the firm's sales?
b. If Schmidt wants to raise its price enough to offset the effect of the increase in per capita disposable income, by how much must it raise its price?
c. If Schmidt raises its price by this amount, will it increase or decrease the price elasticity of demand? Explain. Make sure your answers reflect the fact that elasticity is a negative number.

SOLUTION:

Observation: This is a linear demand function.

a. We have $= \dfrac{\partial Q}{\partial I} = 4$, meaning that a \$1,000 increase in per capita disposable income results in a 4-unit increase in the quantity demanded of the product. Therefore, if per capita disposable income is expected to increase by \$5,000, then the firm's sales will increase by 20 units per month.
b. Here, several variables are changing, not just one. Write

   $\Delta Q = -3\,\Delta P + 4\,\Delta I + 0.6\,\Delta A$

   $= -3\,\Delta P + 4\,\Delta I$, assuming A is constant in the analysis. Now, offsetting the effect of the increase in per capita disposable income means that the combined effects of a price increase and a per capita disposable income increase leave quantity demanded unchanged. Hence,

   $-3\,\Delta P + 4\,\Delta I = 0$. Solving for ΔP gives

   $\Delta P = 20/3 = \$6.67$. That is, price should be raised by \$6.67 per unit.
c. There are two possible interpretations, both leading to more elastic demand. You could assume that the question is asking if demand is more elastic after both the income and price have increased. Since $\dfrac{\partial Q}{\partial P}$ is unchanged and P/Q has increased, the demand will be more elastic. Alternatively, you might assume that the question is asking, as we increase the price to choke off the anticipated increase in the quantity demanded after income has gone up, does the demand become more or less elastic? This is, of course, just moving up a linear demand curve, which implies an increasingly elastic demand.