1 Introduction

No exercises.

2 The Solow Model

Exercise 1. *A decrease in the investment rate.*

A decrease in the investment rate causes the $s\tilde{y}$ curve to shift down: at any given level of $\tilde{k}$, the investment-technology ratio is lower at the new rate of saving/investment.

Assuming the economy began in steady state, the capital-technology ratio is now higher than is consistent with the reduced saving rate, so it declines gradually, as shown in Figure 1.

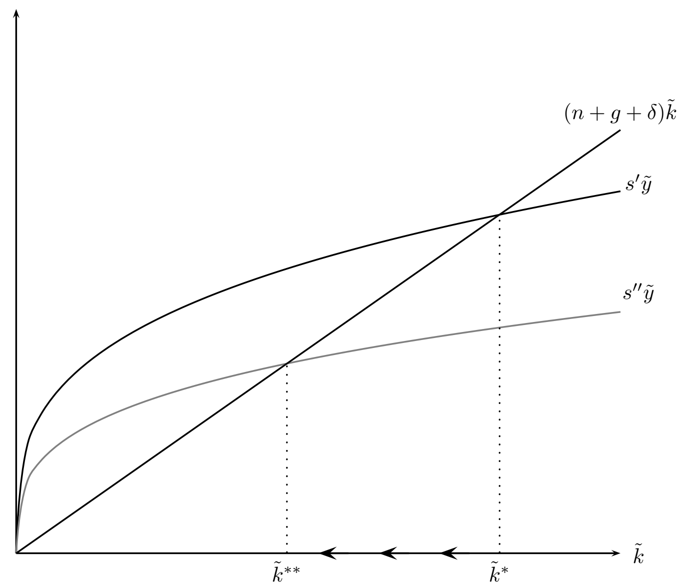

Figure 1: A Decrease in the Investment Rate

The log of output per worker y evolves as in Figure 2, and the dynamics of the growth rate are shown in Figure 3. Recall that $\log \tilde{y} = \alpha \log \tilde{k}$ and $\dot{\tilde{k}}/\tilde{k} = s''\tilde{k}^{\alpha-1} - (n+g+\delta)$.

The policy permanently reduces the level of output per worker, but the growth rate per worker is only temporarily reduced and will return to g in the long run.

Exercise 2. *An increase in the labor force.*

The key to this question is to recognize that the initial outcome of a sudden increase in the labor force is to reduce the capital-labor ratio since $k \equiv K/L$ and K is fixed at

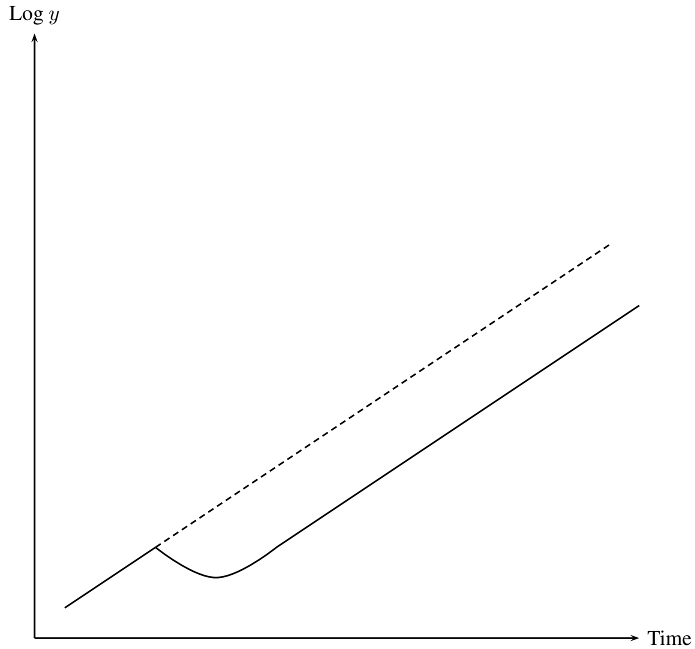

Figure 2: $y(t)$

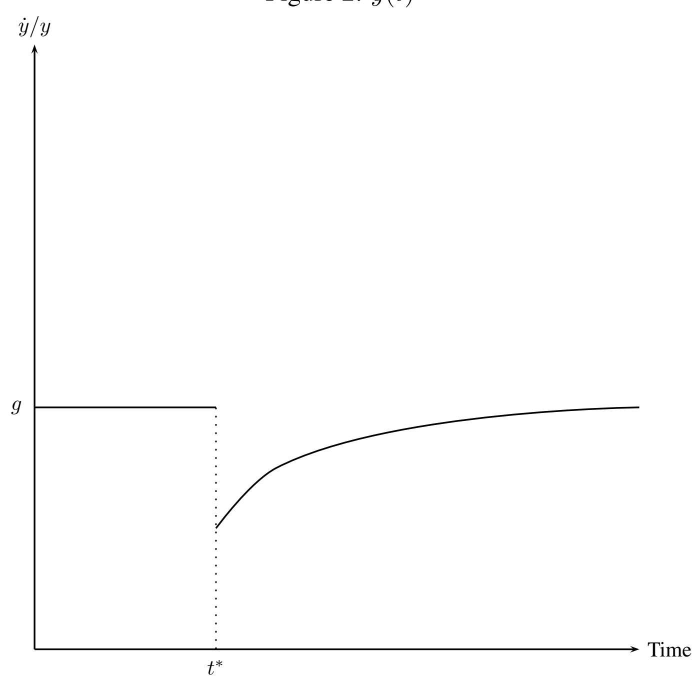

Figure 3: Growth Rate of Output per Worker

a moment in time. Assuming the economy was in steady state prior to the increase in labor force, k falls from k^* to some new level k_1. Notice that this is a movement *along* the sy and $(n+\delta)k$ curves rather than a *shift* of either schedule: both curves are plotted as functions of k, so that a change in k is a movement along the curves. (For this reason, it is somewhat tricky to put this question first!)

At k_1, $sy > (n+\delta)k_1$, so that $\dot{k} > 0$, and the economy evolves according to the usual Solow dynamics, as shown in Figure 4.

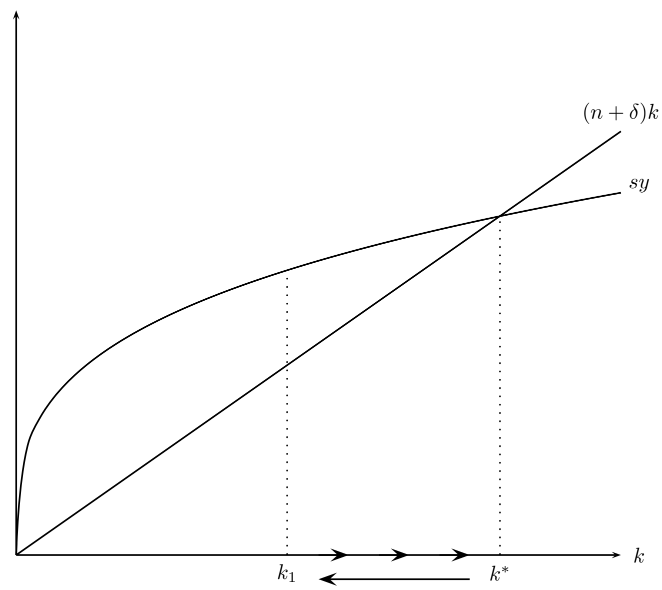

Figure 4: An Increase in the Labor Force

In the short run, per capita output and capital drop in response to an enlarged flow of workers. Then these two variables start to grow (at a decreasing rate), until in the long run, the per capita capital returns to the original level, k^*.

Exercise 3. *An income tax.*

Assume that the government throws away the resources it receives in taxes. This would mean that an income tax reduces the total amount available for investing and shifts the investment curve down as shown in Figure 5.

The tax policy permanently reduces the level of output per worker, but the growth rate per worker is only temporarily lowered. Notice that this experiment has basically the same results as that in Exercise 2.

For further thought: what happens if, instead of throwing away the resources it collects, the government uses all of its tax revenue to undertake investment?

Exercise 4. *Manna falls faster.*

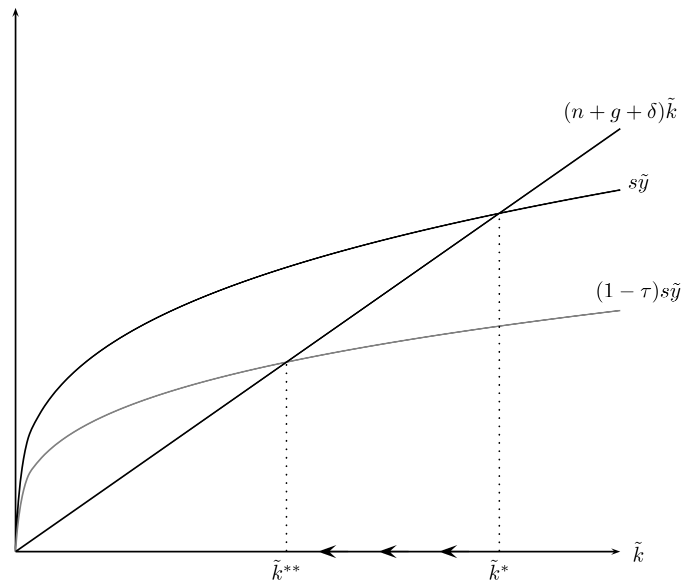

Figure 5: An Income Tax

Figure 6 shows the Solow diagram for this question. It turns out, however, that it's easier to answer this question using the transition dynamics version of the diagram, as shown in Figure 7. When g rises to g', $\dot{\tilde{k}}/\tilde{k}$ turns negative and $\dot{A}/A = g'$, the new steady-state growth rate.

To see what this implies about the growth rate of y, recall that

$$\frac{\dot{y}}{y} = \frac{\dot{\tilde{y}}}{\tilde{y}} + \frac{\dot{A}}{A} = \alpha\frac{\dot{\tilde{k}}}{\tilde{k}} + g'.$$

In order to determine what happens to the growth rate of y at the moment of change in g, we have to determine what happens to $\dot{\tilde{k}}/\tilde{k}$ at that moment. As can be seen in Figure 7, or by algebra, this growth rate falls to $g - g' < 0$. It is the negative of the difference between the two horizontal lines.

Substituting into the equation above, we see that $\dot{y}/y$ immediately after the increase in g (suppose this occurs at time $t = 0$) is given by

$$\left.\frac{\dot{y}}{y}\right|_{t=0} = \alpha(g - g') + g' = (1-\alpha)g' + \alpha g > 0.$$

Notice that this value, which is a weighted average of g' and g, is strictly less than g'.

After time $t = 0$, $\dot{y}/y$ rises up to g' (which can be seen by looking at the dynamics implied by Figure 6). Therefore, we know that the dynamics of the growth rate of output per worker look like those shown in Figure 8.