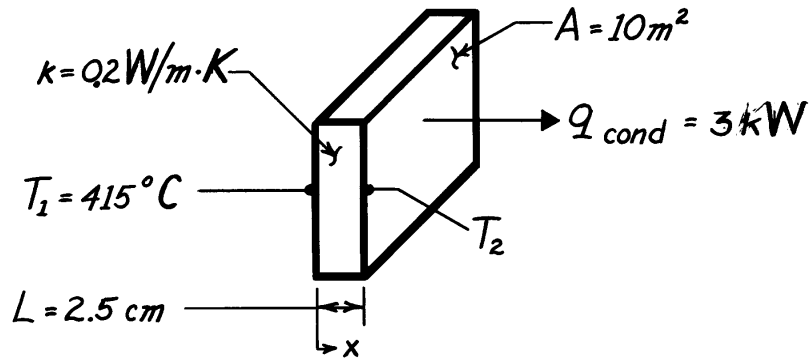


### PROBLEM 1.1

**KNOWN:** Heat rate,  $q$ , through one-dimensional wall of area  $A$ , thickness  $L$ , thermal conductivity  $k$  and inner temperature,  $T_1$ .

**FIND:** The outer temperature of the wall,  $T_2$ .

**SCHEMATIC:**



**ASSUMPTIONS:** (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

**ANALYSIS:** The rate equation for conduction through the wall is given by Fourier's law,

$$q_{\text{cond}} = q_x = q_x'' \cdot A = -k \frac{dT}{dx} \cdot A = kA \frac{T_1 - T_2}{L}.$$

Solving for  $T_2$  gives

$$T_2 = T_1 - \frac{q_{\text{cond}} L}{kA}.$$

Substituting numerical values, find

$$T_2 = 415^\circ\text{C} - \frac{3000\text{W} \times 0.025\text{m}}{0.2\text{W/m}\cdot\text{K} \times 10\text{m}^2}$$

$$T_2 = 415^\circ\text{C} - 37.5^\circ\text{C}$$

$$T_2 = 378^\circ\text{C}.$$

<

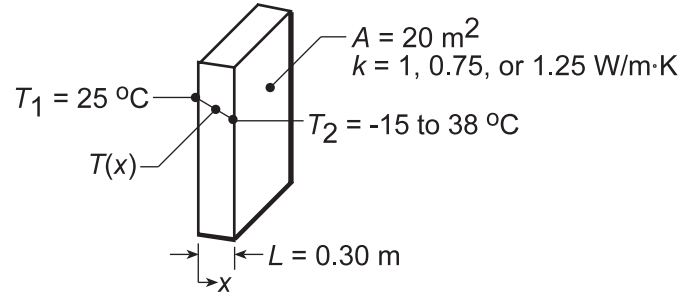
**COMMENTS:** Note direction of heat flow and fact that  $T_2$  must be less than  $T_1$ .

## PROBLEM 1.2

**KNOWN:** Inner surface temperature and thermal conductivity of a concrete wall.

**FIND:** Heat loss by conduction through the wall as a function of ambient air temperatures ranging from -15 to 38°C.

**SCHEMATIC:**



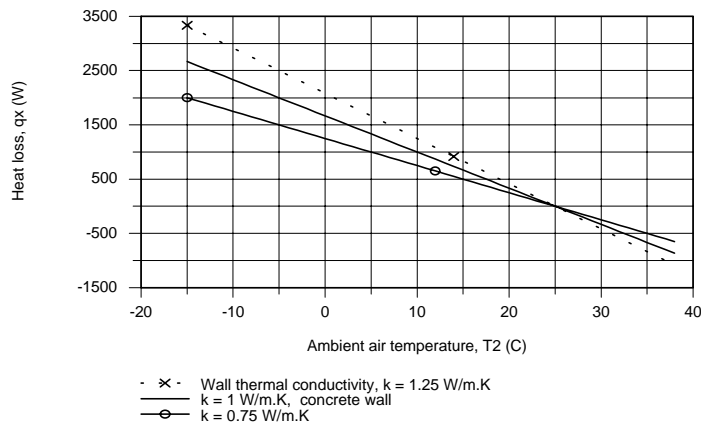
**ASSUMPTIONS:** (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties, (4) Outside wall temperature is that of the ambient air.

**ANALYSIS:** From Fourier's law, it is evident that the gradient,  $dT/dx = -q''_x/k$ , is a constant, and hence the temperature distribution is linear, if  $q''_x$  and  $k$  are each constant. The heat flux must be constant under one-dimensional, steady-state conditions; and  $k$  is approximately constant if it depends only weakly on temperature. The heat flux and heat rate when the outside wall temperature is  $T_2 = -15^\circ\text{C}$  are

$$q''_x = -k \frac{dT}{dx} = k \frac{T_1 - T_2}{L} = 1 \text{ W/m} \cdot \text{K} \frac{25^\circ\text{C} - (-15^\circ\text{C})}{0.30 \text{ m}} = 133.3 \text{ W/m}^2. \quad (1)$$

$$q_x = q''_x \times A = 133.3 \text{ W/m}^2 \times 20 \text{ m}^2 = 2667 \text{ W}. \quad (2) <$$

Combining Eqs. (1) and (2), the heat rate  $q_x$  can be determined for the range of ambient temperature,  $-15 \leq T_2 \leq 38^\circ\text{C}$ , with different wall thermal conductivities,  $k$ .



For the concrete wall,  $k = 1 \text{ W/m} \cdot \text{K}$ , the heat loss varies linearly from +2667 W to -867 W and is zero when the inside and ambient temperatures are the same. The magnitude of the heat rate increases with increasing thermal conductivity.

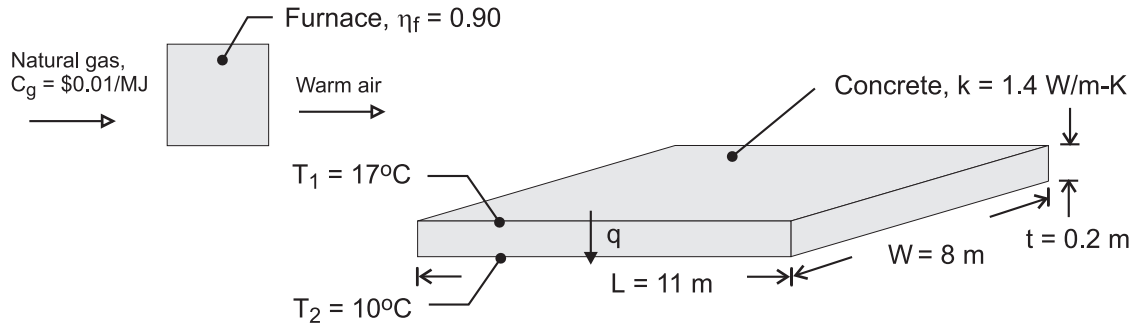
**COMMENTS:** Without steady-state conditions and constant  $k$ , the temperature distribution in a plane wall would not be linear.

### PROBLEM 1.3

**KNOWN:** Dimensions, thermal conductivity and surface temperatures of a concrete slab. Efficiency of gas furnace and cost of natural gas.

**FIND:** Daily cost of heat loss.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady state, (2) One-dimensional conduction, (3) Constant properties.

**ANALYSIS:** The rate of heat loss by conduction through the slab is

$$q = k (LW) \frac{T_1 - T_2}{t} = 1.4 \text{ W/m} \cdot \text{K} (11 \text{ m} \times 8 \text{ m}) \frac{7^\circ\text{C}}{0.20 \text{ m}} = 4312 \text{ W} \quad <$$

The daily cost of natural gas that must be combusted to compensate for the heat loss is

$$C_d = \frac{q C_g}{\eta_f} (\Delta t) = \frac{4312 \text{ W} \times \$0.01/\text{MJ}}{0.9 \times 10^6 \text{ J/MJ}} (24 \text{ h/d} \times 3600 \text{ s/h}) = \$4.14/\text{d} \quad <$$

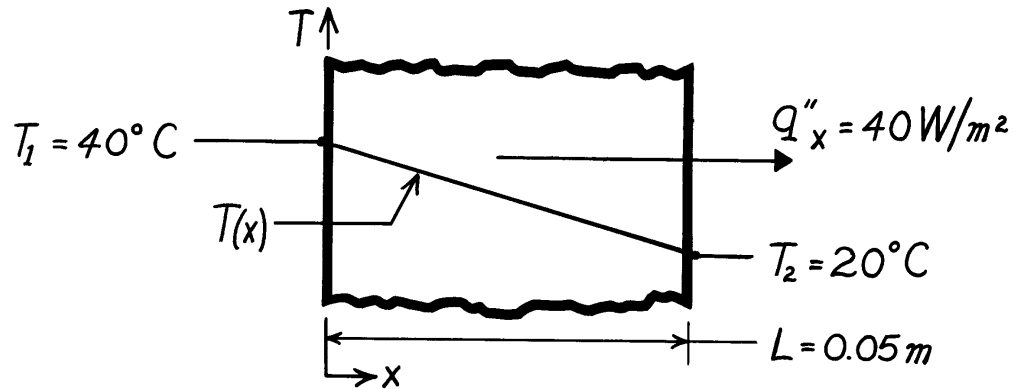
**COMMENTS:** The loss could be reduced by installing a floor covering with a layer of insulation between it and the concrete.

### PROBLEM 1.4

**KNOWN:** Heat flux and surface temperatures associated with a wood slab of prescribed thickness.

**FIND:** Thermal conductivity,  $k$ , of the wood.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

**ANALYSIS:** Subject to the foregoing assumptions, the thermal conductivity may be determined from Fourier's law, Eq. 1.2. Rearranging,

$$k = q''_x \frac{L}{T_1 - T_2} = 40 \frac{\text{W}}{\text{m}^2} \frac{0.05 \text{ m}}{(40 - 20)^\circ \text{C}}$$

$$k = 0.10 \text{ W/m} \cdot \text{K}$$

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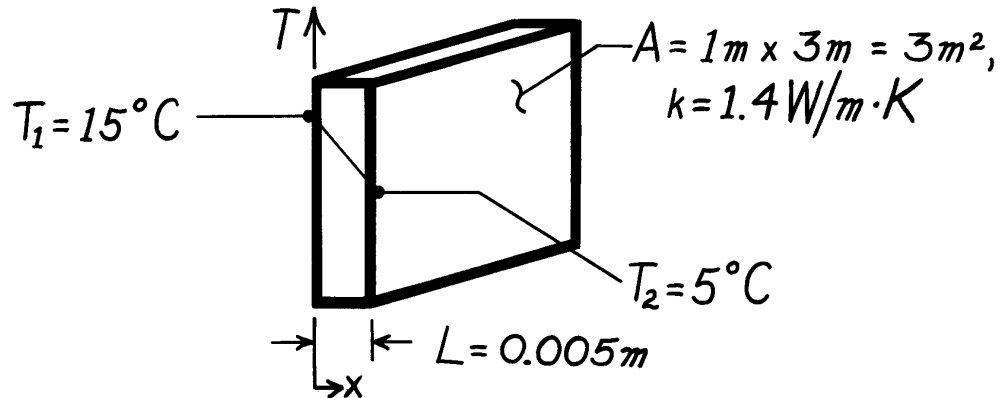
**COMMENTS:** Note that the  $^\circ \text{C}$  or K temperature units may be used interchangeably when evaluating a temperature difference.

### PROBLEM 1.5

**KNOWN:** Inner and outer surface temperatures of a glass window of prescribed dimensions.

**FIND:** Heat loss through window.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) One-dimensional conduction in the  $x$ -direction, (2) Steady-state conditions, (3) Constant properties.

**ANALYSIS:** Subject to the foregoing conditions the heat flux may be computed from Fourier's law, Eq. 1.2.

$$q_x'' = k \frac{T_1 - T_2}{L}$$
$$q_x'' = 1.4 \frac{\text{W}}{\text{m}\cdot\text{K}} \frac{(15-5)^\circ\text{C}}{0.005\text{m}}$$
$$q_x'' = 2800 \text{ W/m}^2.$$

Since the heat flux is uniform over the surface, the heat loss (rate) is

$$q = q_x'' \times A$$
$$q = 2800 \text{ W/m}^2 \times 3\text{m}^2$$
$$q = 8400 \text{ W.}$$

<

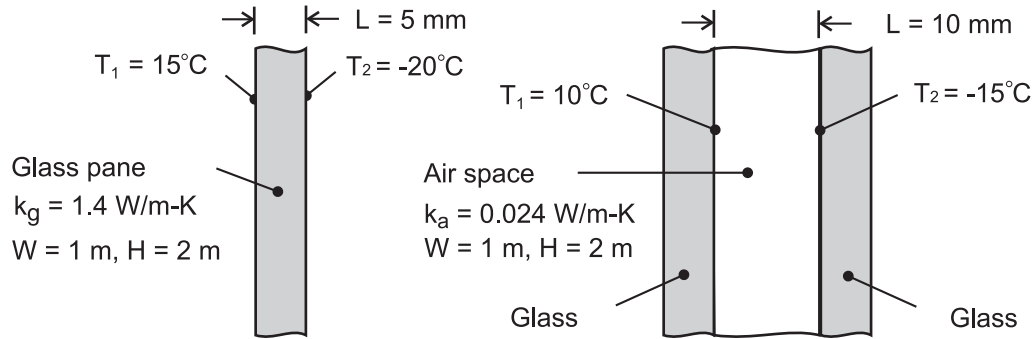
**COMMENTS:** A linear temperature distribution exists in the glass for the prescribed conditions.

## PROBLEM 1.6

**KNOWN:** Width, height, thickness and thermal conductivity of a single pane window and the air space of a double pane window. Representative winter surface temperatures of single pane and air space.

**FIND:** Heat loss through single and double pane windows.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) One-dimensional conduction through glass or air, (2) Steady-state conditions, (3) Enclosed air of double pane window is stagnant (negligible buoyancy induced motion).

**ANALYSIS:** From Fourier's law, the heat losses are

$$\text{Single Pane: } q_g = k_g A \frac{T_1 - T_2}{L} = 1.4 \text{ W/m} \cdot \text{K} \left( 2 \text{ m}^2 \right) \frac{35 \text{ }^\circ\text{C}}{0.005 \text{ m}} = 19,600 \text{ W}$$

$$\text{Double Pane: } q_a = k_a A \frac{T_1 - T_2}{L} = 0.024 \left( 2 \text{ m}^2 \right) \frac{25 \text{ }^\circ\text{C}}{0.010 \text{ m}} = 120 \text{ W}$$

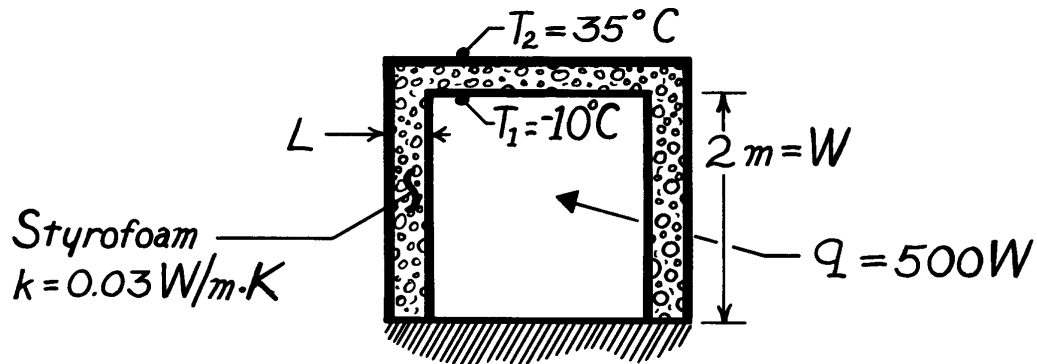
**COMMENTS:** Losses associated with a single pane are unacceptable and would remain excessive, even if the thickness of the glass were doubled to match that of the air space. The principal advantage of the double pane construction resides with the low thermal conductivity of air (~ 60 times smaller than that of glass). For a fixed ambient outside air temperature, use of the double pane construction would also increase the surface temperature of the glass exposed to the room (inside) air.

### PROBLEM 1.7

**KNOWN:** Dimensions of freezer compartment. Inner and outer surface temperatures.

**FIND:** Thickness of styrofoam insulation needed to maintain heat load below prescribed value.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Perfectly insulated bottom, (2) One-dimensional conduction through 5 walls of area  $A = 4m^2$ , (3) Steady-state conditions, (4) Constant properties.

**ANALYSIS:** Using Fourier's law, Eq. 1.2, the heat rate is

$$q = q'' \cdot A = k \frac{\Delta T}{L} A_{\text{total}}$$

Solving for  $L$  and recognizing that  $A_{\text{total}} = 5 \times W^2$ , find

$$L = \frac{5 k \Delta T W^2}{q}$$

$$L = \frac{5 \times 0.03 \text{ W/m} \cdot \text{K} [35 - (-10)]^\circ \text{C} (4\text{m}^2)}{500 \text{ W}}$$

$$L = 0.054\text{m} = 54\text{mm.}$$

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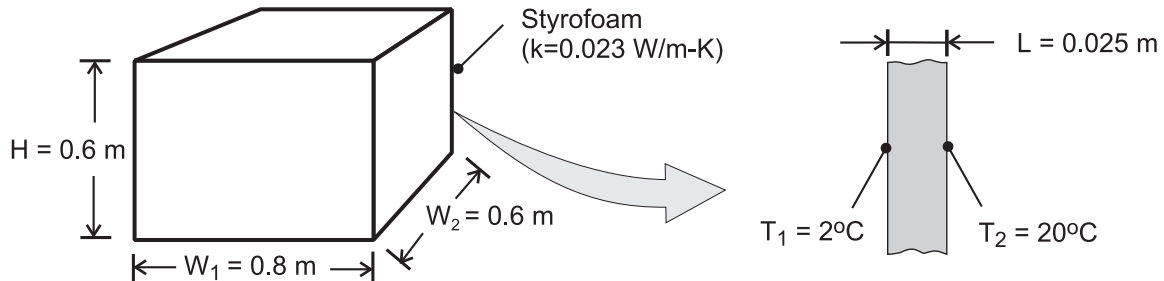
**COMMENTS:** The corners will cause local departures from one-dimensional conduction and a slightly larger heat loss.

### PROBLEM 1.8

**KNOWN:** Dimensions and thermal conductivity of food/beverage container. Inner and outer surface temperatures.

**FIND:** Heat flux through container wall and total heat load.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state conditions, (2) Negligible heat transfer through bottom wall, (3) Uniform surface temperatures and one-dimensional conduction through remaining walls.

**ANALYSIS:** From Fourier's law, Eq. 1.2, the heat flux is

$$q'' = k \frac{T_2 - T_1}{L} = \frac{0.023 \text{ W/m} \cdot \text{K} (20 - 2)^\circ \text{C}}{0.025 \text{ m}} = 16.6 \text{ W/m}^2 \quad <$$

Since the flux is uniform over each of the five walls through which heat is transferred, the heat load is

$$q = q'' \times A_{\text{total}} = q'' [H(2W_1 + 2W_2) + W_1 \times W_2]$$
$$q = 16.6 \text{ W/m}^2 [0.6 \text{ m}(1.6 \text{ m} + 1.2 \text{ m}) + (0.8 \text{ m} \times 0.6 \text{ m})] = 35.9 \text{ W} \quad <$$

**COMMENTS:** The corners and edges of the container create local departures from one-dimensional conduction, which increase the heat load. However, for  $H, W_1, W_2 \gg L$ , the effect is negligible.

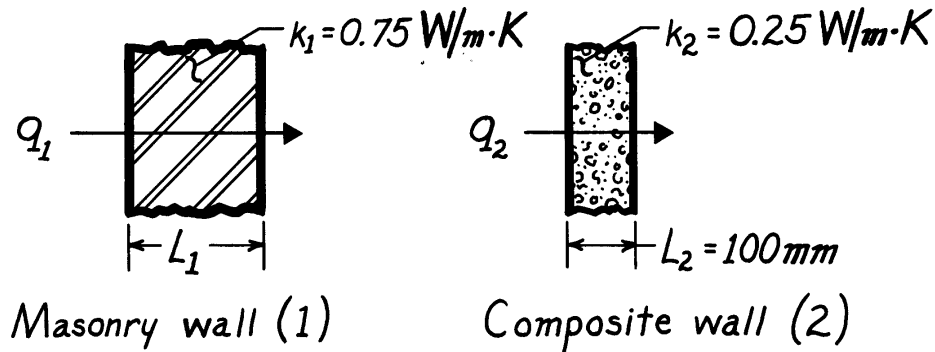


### PROBLEM 1.9

**KNOWN:** Masonry wall of known thermal conductivity has a heat rate which is 80% of that through a composite wall of prescribed thermal conductivity and thickness.

**FIND:** Thickness of masonry wall.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Both walls subjected to same surface temperatures, (2) One-dimensional conduction, (3) Steady-state conditions, (4) Constant properties.

**ANALYSIS:** For steady-state conditions, the conduction heat flux through a one-dimensional wall follows from Fourier's law, Eq. 1.2,

$$q'' = k \frac{\Delta T}{L}$$

where  $\Delta T$  represents the difference in surface temperatures. Since  $\Delta T$  is the same for both walls, it follows that

$$L_1 = L_2 \frac{k_1}{k_2} \cdot \frac{q_2''}{q_1''}$$

With the heat fluxes related as

$$q_1'' = 0.8 q_2''$$

$$L_1 = 100 \text{ mm} \frac{0.75 \text{ W/m}\cdot\text{K}}{0.25 \text{ W/m}\cdot\text{K}} \times \frac{1}{0.8} = 375 \text{ mm.} \quad <$$

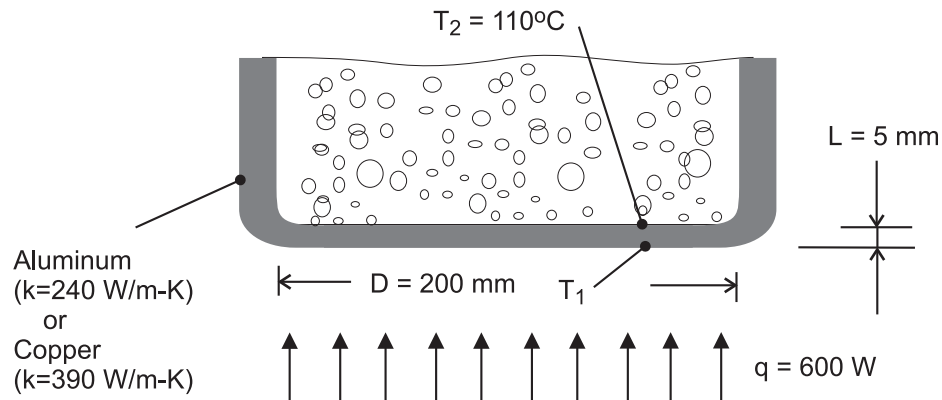
**COMMENTS:** Not knowing the temperature difference across the walls, we cannot find the value of the heat rate.

### PROBLEM 1.10

**KNOWN:** Thickness, diameter and inner surface temperature of bottom of pan used to boil water. Rate of heat transfer to the pan.

**FIND:** Outer surface temperature of pan for an aluminum and a copper bottom.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) One-dimensional, steady-state conduction through bottom of pan.

**ANALYSIS:** From Fourier's law, the rate of heat transfer by conduction through the bottom of the pan is

$$q = kA \frac{T_1 - T_2}{L}$$

Hence,

$$T_1 = T_2 + \frac{qL}{kA}$$

where  $A = \pi D^2 / 4 = \pi (0.2 \text{ m})^2 / 4 = 0.0314 \text{ m}^2$ .

$$\text{Aluminum: } T_1 = 110^\circ\text{C} + \frac{600 \text{ W} (0.005 \text{ m})}{240 \text{ W/m}\cdot\text{K} (0.0314 \text{ m}^2)} = 110.40^\circ\text{C}$$

$$\text{Copper: } T_1 = 110^\circ\text{C} + \frac{600 \text{ W} (0.005 \text{ m})}{390 \text{ W/m}\cdot\text{K} (0.0314 \text{ m}^2)} = 110.25^\circ\text{C}$$

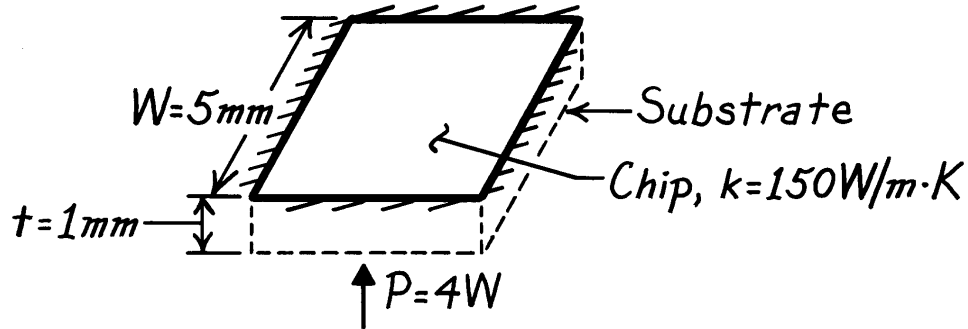
**COMMENTS:** Although the temperature drop across the bottom is slightly larger for aluminum (due to its smaller thermal conductivity), it is sufficiently small to be negligible for both materials. To a good approximation, the bottom may be considered *isothermal* at  $T \approx 110^\circ\text{C}$ , which is a desirable feature of pots and pans.

### PROBLEM 1.11

**KNOWN:** Dimensions and thermal conductivity of a chip. Power dissipated on one surface.

**FIND:** Temperature drop across the chip.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state conditions, (2) Constant properties, (3) Uniform heat dissipation, (4) Negligible heat loss from back and sides, (5) One-dimensional conduction in chip.

**ANALYSIS:** All of the electrical power dissipated at the back surface of the chip is transferred by conduction through the chip. Hence, from Fourier's law,

$$P = q = kA \frac{\Delta T}{t}$$

or

$$\Delta T = \frac{t \cdot P}{kW^2} = \frac{0.001\text{ m} \times 4\text{ W}}{150\text{ W/m}\cdot\text{K} (0.005\text{ m})^2}$$

$$\Delta T = 1.1^\circ\text{ C.}$$

<

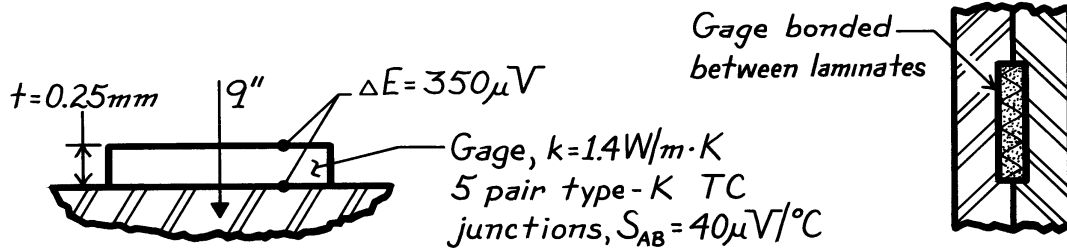
**COMMENTS:** For fixed  $P$ , the temperature drop across the chip decreases with increasing  $k$  and  $W$ , as well as with decreasing  $t$ .

## PROBLEM 1.12

**KNOWN:** Heat flux gage with thin-film thermocouples on upper and lower surfaces; output voltage, calibration constant, thickness and thermal conductivity of gage.

**FIND:** (a) Heat flux, (b) Precaution when sandwiching gage between two materials.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state conditions, (2) One-dimensional heat conduction in gage, (3) Constant properties.

**ANALYSIS:** (a) Fourier's law applied to the gage can be written as

$$q'' = k \frac{\Delta T}{\Delta x}$$

and the gradient can be expressed as

$$\frac{\Delta T}{\Delta x} = \frac{\Delta E / N}{S_{AB} t}$$

where  $N$  is the number of differentially connected thermocouple junctions,  $S_{AB}$  is the Seebeck coefficient for type K thermocouples (A-chromel and B-alumel), and  $\Delta x = t$  is the gage thickness. Hence,

$$q'' = \frac{k \Delta E}{N S_{AB} t}$$

$$q'' = \frac{1.4 \text{ W/m}\cdot\text{K} \times 350 \times 10^{-6} \text{ V}}{5 \times 40 \times 10^{-6} \text{ V}/^\circ\text{C} \times 0.25 \times 10^{-3} \text{ m}} = 9800 \text{ W/m}^2. \quad <$$

(b) The major precaution to be taken with this type of gage is to match its thermal conductivity with that of the material on which it is installed. If the gage is bonded between laminates (see sketch above) and its thermal conductivity is significantly different from that of the laminates, one dimensional heat flow will be disturbed and the gage will read incorrectly.

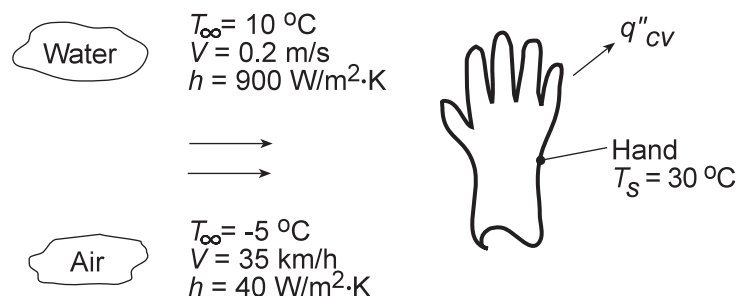
**COMMENTS:** If the thermal conductivity of the gage is lower than that of the laminates, will it indicate heat fluxes that are systematically high or low?

### PROBLEM 1.13

**KNOWN:** Hand experiencing convection heat transfer with moving air and water.

**FIND:** Determine which condition feels colder. Contrast these results with a heat loss of  $30 \text{ W/m}^2$  under normal room conditions.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Temperature is uniform over the hand's surface, (2) Convection coefficient is uniform over the hand, and (3) Negligible radiation exchange between hand and surroundings in the case of air flow.

**ANALYSIS:** The hand will feel colder for the condition which results in the larger heat loss. The heat loss can be determined from Newton's law of cooling, Eq. 1.3a, written as

$$q'' = h(T_s - T_{\infty})$$

For the air stream:

$$q''_{\text{air}} = 40 \text{ W/m}^2 \cdot \text{K} [30 - (-5)] \text{ K} = 1,400 \text{ W/m}^2 \quad <$$

For the water stream:

$$q''_{\text{water}} = 900 \text{ W/m}^2 \cdot \text{K} (30 - 10) \text{ K} = 18,000 \text{ W/m}^2 \quad <$$

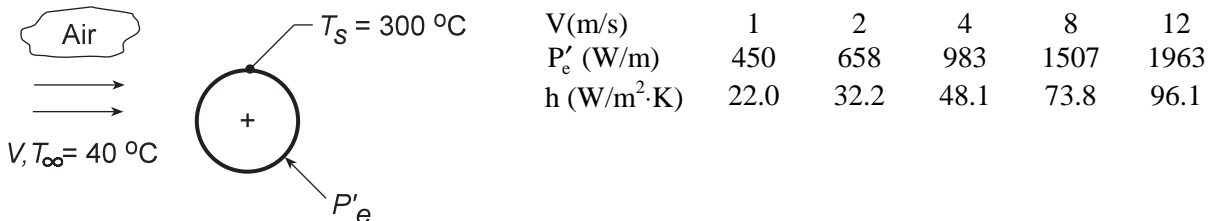
**COMMENTS:** The heat loss for the hand in the water stream is an order of magnitude larger than when in the air stream for the given temperature and convection coefficient conditions. In contrast, the heat loss in a normal room environment is only  $30 \text{ W/m}^2$  which is a factor of 400 times less than the loss in the air stream. In the room environment, the hand would feel comfortable; in the air and water streams, as you probably know from experience, the hand would feel uncomfortably cold since the heat loss is excessively high.

## PROBLEM 1.14

**KNOWN:** Power required to maintain the surface temperature of a long, 25-mm diameter cylinder with an imbedded electrical heater for different air velocities.

**FIND:** (a) Determine the convection coefficient for each of the air velocity conditions and display the results graphically, and (b) Assuming that the convection coefficient depends upon air velocity as  $h = CV^n$ , determine the parameters C and n.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Temperature is uniform over the cylinder surface, (2) Negligible radiation exchange between the cylinder surface and the surroundings, (3) Steady-state conditions.

**ANALYSIS:** (a) From an overall energy balance on the cylinder, the power dissipated by the electrical heater is transferred by convection to the air stream. Using Newton's law of cooling on a per unit length basis,

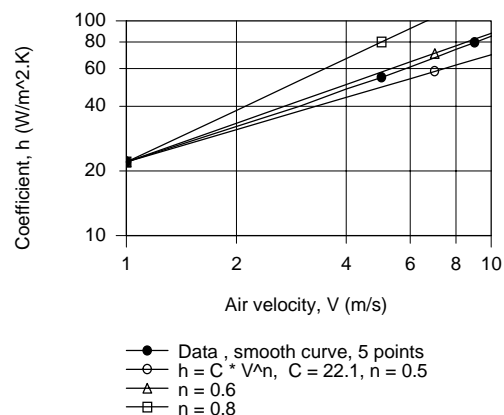
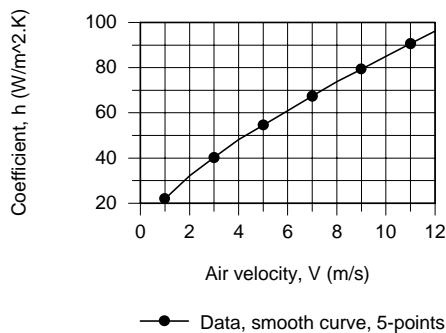
$$P'_e = h(\pi D)(T_s - T_\infty)$$

where  $P'_e$  is the electrical power dissipated per unit length of the cylinder. For the  $V = 1$  m/s condition, using the data from the table above, find

$$h = 450 \text{ W/m} / \pi \times 0.025 \text{ m} (300 - 40)^\circ \text{C} = 22.0 \text{ W/m}^2 \cdot \text{K}$$

Repeating the calculations, find the convection coefficients for the remaining conditions which are tabulated above and plotted below. Note that h is not linear with respect to the air velocity.

(b) To determine the (C,n) parameters, we plotted h vs. V on log-log coordinates. Choosing  $C = 22.12 \text{ W/m}^2 \cdot \text{K}(\text{s/m})^n$ , assuming a match at  $V = 1$ , we can readily find the exponent n from the slope of the h vs. V curve. From the trials with  $n = 0.8, 0.6$  and  $0.5$ , we recognize that  $n = 0.6$  is a reasonable choice. Hence,  $C = 22.12$  and  $n = 0.6$ .

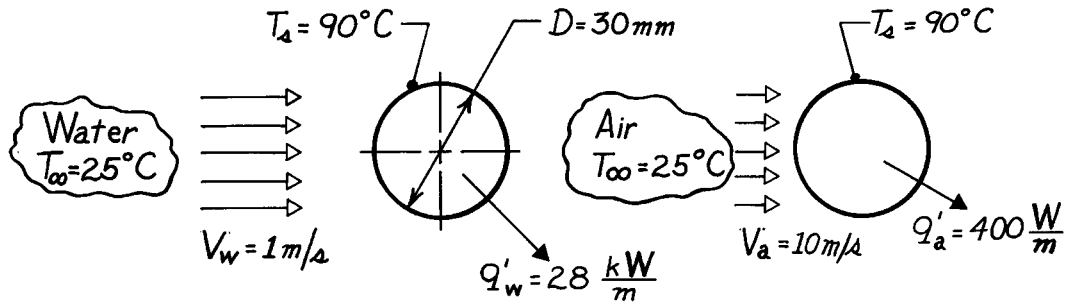


### PROBLEM 1.15

**KNOWN:** Long, 30mm-diameter cylinder with embedded electrical heater; power required to maintain a specified surface temperature for water and air flows.

**FIND:** Convection coefficients for the water and air flow convection processes,  $h_w$  and  $h_a$ , respectively.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Flow is cross-wise over cylinder which is very long in the direction normal to flow.

**ANALYSIS:** The convection heat rate from the cylinder per unit length of the cylinder has the form

$$q' = h(\pi D) (T_s - T_\infty)$$

and solving for the heat transfer convection coefficient, find

$$h = \frac{q'}{\pi D (T_s - T_\infty)}$$

Substituting numerical values for the water and air situations:

$$\text{Water} \quad h_w = \frac{28 \times 10^3 \text{ W/m}}{\pi \times 0.030\text{ m} (90-25)^\circ \text{ C}} = 4,570 \text{ W/m}^2 \cdot \text{K} <$$

$$\text{Air} \quad h_a = \frac{400 \text{ W/m}}{\pi \times 0.030\text{ m} (90-25)^\circ \text{ C}} = 65 \text{ W/m}^2 \cdot \text{K} <$$

**COMMENTS:** Note that the air velocity is 10 times that of the water flow, yet

$$h_w \approx 70 \times h_a.$$

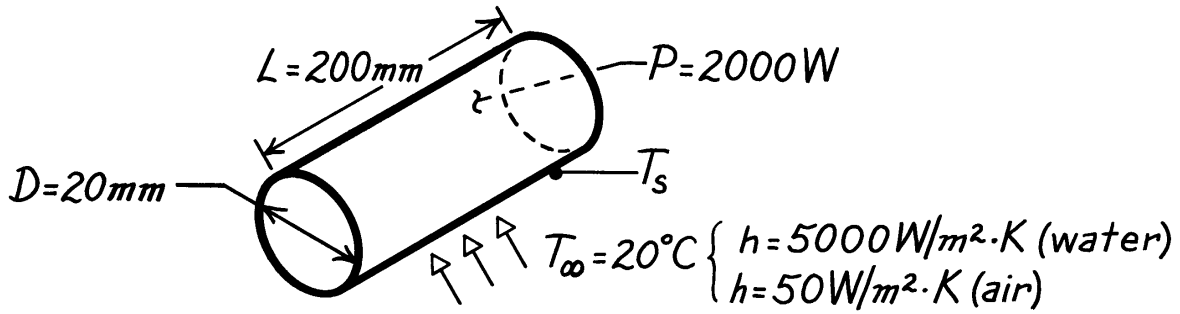
These values for the convection coefficient are typical for forced convection heat transfer with liquids and gases. See Table 1.1.

### PROBLEM 1.16

**KNOWN:** Dimensions of a cartridge heater. Heater power. Convection coefficients in air and water at a prescribed temperature.

**FIND:** Heater surface temperatures in water and air.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state conditions, (2) All of the electrical power is transferred to the fluid by convection, (3) Negligible heat transfer from ends.

**ANALYSIS:** With  $P = q_{\text{conv}}$ , Newton's law of cooling yields

$$P = hA(T_s - T_\infty) = h\pi DL(T_s - T_\infty)$$
$$T_s = T_\infty + \frac{P}{h\pi DL}$$

In water,

$$T_s = 20^\circ\text{C} + \frac{2000\text{ W}}{5000\text{ W/m}^2 \cdot \text{K} \times \pi \times 0.02\text{ m} \times 0.200\text{ m}}$$

$$T_s = 20^\circ\text{C} + 31.8^\circ\text{C} = 51.8^\circ\text{C} \quad <$$

In air,

$$T_s = 20^\circ\text{C} + \frac{2000\text{ W}}{50\text{ W/m}^2 \cdot \text{K} \times \pi \times 0.02\text{ m} \times 0.200\text{ m}}$$

$$T_s = 20^\circ\text{C} + 3183^\circ\text{C} = 3203^\circ\text{C} \quad <$$

**COMMENTS:** (1) Air is much less effective than water as a heat transfer fluid. Hence, the cartridge temperature is much higher in air, so high, in fact, that the cartridge would melt.

(2) In air, the high cartridge temperature would render radiation significant.

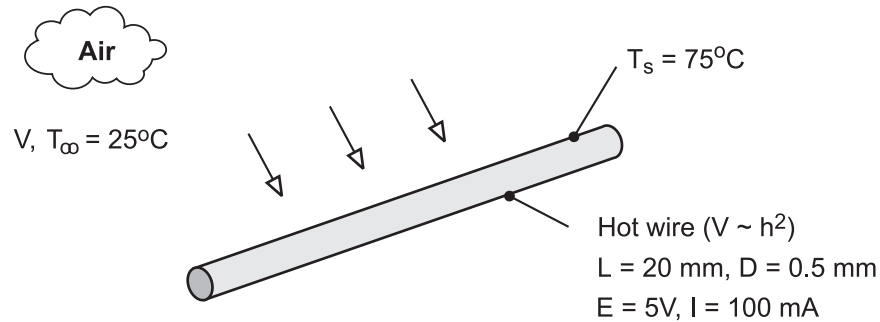


### PROBLEM 1.17

**KNOWN:** Length, diameter and calibration of a hot wire anemometer. Temperature of air stream. Current, voltage drop and surface temperature of wire for a particular application.

**FIND:** Air velocity

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state conditions, (2) Negligible heat transfer from the wire by natural convection or radiation.

**ANALYSIS:** If all of the electric energy is transferred by convection to the air, the following equality must be satisfied

$$P_{\text{elec}} = EI = hA(T_s - T_\infty)$$

where  $A = \pi DL = \pi(0.0005\text{m} \times 0.02\text{m}) = 3.14 \times 10^{-5} \text{m}^2$ .

Hence,

$$h = \frac{EI}{A(T_s - T_\infty)} = \frac{5\text{V} \times 0.1\text{A}}{3.14 \times 10^{-5} \text{m}^2 (50^\circ\text{C})} = 318 \text{ W/m}^2 \cdot \text{K}$$

$$V = 6.25 \times 10^{-5} h^2 = 6.25 \times 10^{-5} (318 \text{ W/m}^2 \cdot \text{K})^2 = 6.3 \text{ m/s} \quad <$$

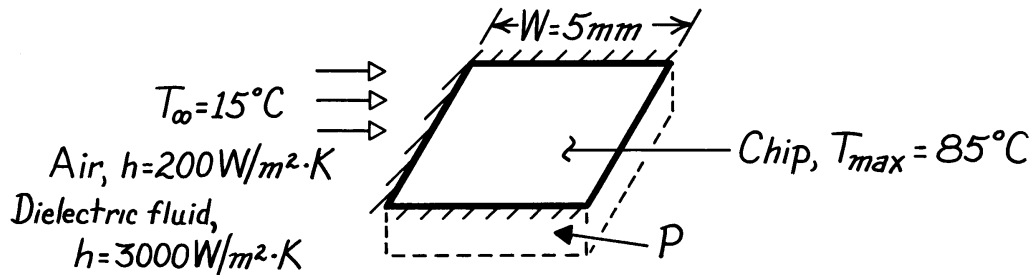
**COMMENTS:** The convection coefficient is sufficiently large to render buoyancy (natural convection) and radiation effects negligible.

### PROBLEM 1.18

**KNOWN:** Chip width and maximum allowable temperature. Coolant conditions.

**FIND:** Maximum allowable chip power for air and liquid coolants.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state conditions, (2) Negligible heat transfer from sides and bottom, (3) Chip is at a uniform temperature (isothermal), (4) Negligible heat transfer by radiation in air.

**ANALYSIS:** All of the electrical power dissipated in the chip is transferred by convection to the coolant. Hence,

$$P = q$$

and from Newton's law of cooling,

$$P = hA(T - T_\infty) = hW^2(T - T_\infty).$$

In *air*,

$$P_{\max} = 200 \text{ W/m}^2 \cdot \text{K} (0.005 \text{ m})^2 (85 - 15)^\circ \text{C} = 0.35 \text{ W.} \quad <$$

In the *dielectric liquid*

$$P_{\max} = 3000 \text{ W/m}^2 \cdot \text{K} (0.005 \text{ m})^2 (85 - 15)^\circ \text{C} = 5.25 \text{ W.} \quad <$$

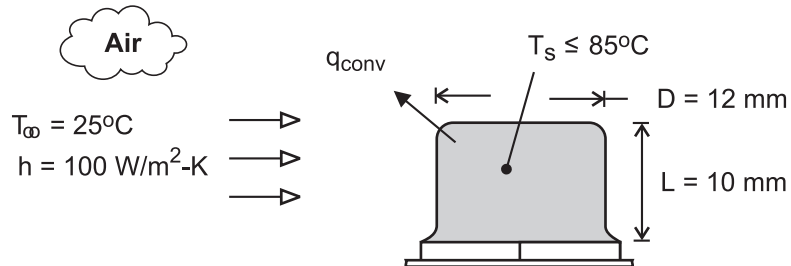
**COMMENTS:** Relative to liquids, air is a poor heat transfer fluid. Hence, in air the chip can dissipate far less energy than in the dielectric liquid.

### PROBLEM 1.19

**KNOWN:** Length, diameter and maximum allowable surface temperature of a power transistor. Temperature and convection coefficient for air cooling.

**FIND:** Maximum allowable power dissipation.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state conditions, (2) Negligible heat transfer through base of transistor, (3) Negligible heat transfer by radiation from surface of transistor.

**ANALYSIS:** Subject to the foregoing assumptions, the power dissipated by the transistor is equivalent to the rate at which heat is transferred by convection to the air. Hence,

$$P_{\text{elec}} = q_{\text{conv}} = hA(T_s - T_{\infty})$$

$$\text{where } A = \pi \left( DL + \frac{D^2}{4} \right) = \pi \left[ 0.012\text{m} \times 0.01\text{m} + \frac{(0.012\text{m})^2}{4} \right] = 4.90 \times 10^{-4} \text{m}^2.$$

For a maximum allowable surface temperature of 85°C, the power is

$$P_{\text{elec}} = 100 \text{ W/m}^2 \cdot \text{K} \left( 4.90 \times 10^{-4} \text{m}^2 \right) (85 - 25)^{\circ} \text{C} = 2.94 \text{ W} \quad <$$

**COMMENTS:** (1) For the prescribed surface temperature and convection coefficient, radiation will be negligible relative to convection. However, conduction through the base could be significant, thereby permitting operation at a larger power.

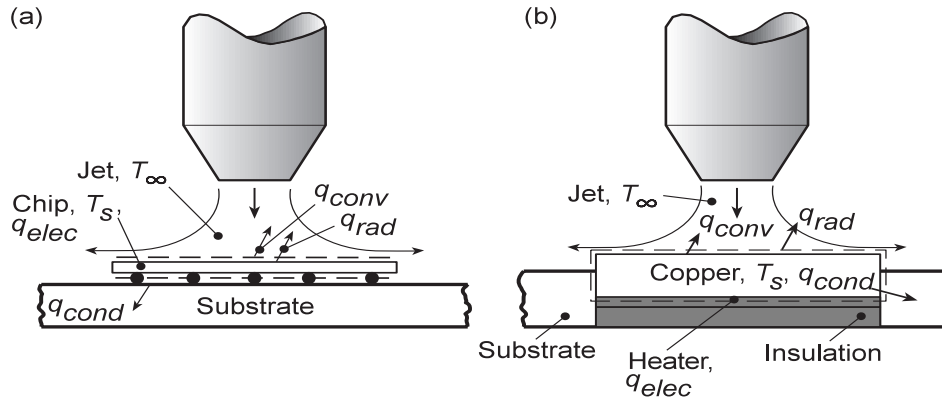
(2) The *local* convection coefficient varies over the surface, and *hot spots* could exist if there are locations at which the local value of  $h$  is substantially smaller than the prescribed average value.

## PROBLEM 1.20

**KNOWN:** Air jet impingement is an effective means of cooling logic chips.

**FIND:** Procedure for measuring convection coefficients associated with a 10 mm × 10 mm chip.

**SCHEMATIC:**



**ASSUMPTIONS:** Steady-state conditions.

**ANALYSIS:** One approach would be to use the actual chip-substrate system, Case (a), to perform the measurements. In this case, the electric power dissipated in the chip would be transferred from the chip by radiation and conduction (to the substrate), as well as by convection to the jet. An energy balance for the chip yields  $q_{elec} = q_{conv} + q_{cond} + q_{rad}$ . Hence, with  $q_{conv} = hA(T_s - T_{\infty})$ , where  $A = 100 \text{ mm}^2$  is the surface area of the chip,

$$h = \frac{q_{elec} - q_{cond} - q_{rad}}{A(T_s - T_{\infty})} \quad (1)$$

While the electric power ( $q_{elec}$ ) and the jet ( $T_{\infty}$ ) and surface ( $T_s$ ) temperatures may be measured, losses from the chip by conduction and radiation would have to be estimated. Unless the losses are negligible (an unlikely condition), the accuracy of the procedure could be compromised by uncertainties associated with determining the conduction and radiation losses.

A second approach, Case (b), could involve fabrication of a heater assembly for which the conduction and radiation losses are controlled and minimized. A 10 mm × 10 mm copper block ( $k \sim 400 \text{ W/m}\cdot\text{K}$ ) could be inserted in a poorly conducting substrate ( $k < 0.1 \text{ W/m}\cdot\text{K}$ ) and a patch heater could be applied to the back of the block and insulated from below. If conduction to both the substrate and insulation could thereby be rendered negligible, heat would be transferred almost exclusively through the block. If radiation were rendered negligible by applying a low emissivity coating ( $\epsilon < 0.1$ ) to the surface of the copper block, virtually all of the heat would be transferred by convection to the jet. Hence,  $q_{cond}$  and  $q_{rad}$  may be neglected in equation (1), and the expression may be used to accurately determine  $h$  from the known ( $A$ ) and measured ( $q_{elec}$ ,  $T_s$ ,  $T_{\infty}$ ) quantities.

**COMMENTS:** Since convection coefficients associated with gas flows are generally small, concurrent heat transfer by radiation and/or conduction must often be considered. However, jet impingement is one of the more effective means of transferring heat by convection and convection coefficients well in excess of  $100 \text{ W/m}^2\cdot\text{K}$  may be achieved.