CHAPTER 2
INTRODUCTION TO MANAGEMENT SCIENCE MODELS

TRUE/FALSE QUESTIONS

1. Linear programming models are a subset of constrained optimization models that require the assumptions of continuity of the variables, certainty of the coefficients, additivity of terms, and proportionality of costs, profits, and the use of resources to the value of the decision variables. (True, medium)

2. When specifying linear constraints, the modeler must take into account the unit specification of the decision variables so that the units represented by the left side of the constraints are consistent with the units represented by the right side of the constraints. (True, easy)

3. Minimization linear programming models may not involve "≤" constraints. (False, medium)

4. A linear programming problem with all "≤" functional constraints and nonnegative right hand side values will never be infeasible. (True, medium)

5. A non-binding constraint is always a redundant constraint. (False, difficult)

6. If two extreme points are optimal, then so is every point on the line segment connecting the two extreme points. (True, medium)

7. "Range of optimality" describes the impact of simultaneous changes in objective function values and right-hand-side values. (False, medium)

8. Linear programming and integer linear programming both yield a great amount of sensitivity analysis. (False, medium)

9. Binary linear programming allows for the possibility of mutually exclusive constraints. (True, medium)

10. Each decision variable must appear in at least two constraints. (False, one constraint, easy)

11. An extreme point is an optimal solution. (False, easy)

12. The difference between a boundary point and an extreme point is the number of constraints satisfied. (False, medium)

13. An optimal solution must have no slack on at least one constraint. (True, medium)

14. One of the reasons we cannot use sensitivity analysis for an integer linear program is that the shadow prices do not produce linear effects. That is, although in a linear program the shadow price for a resource represents the marginal improvement for each added unit of that resource, in an integer linear program, we cannot

assume that each added unit of a resource will produce the same marginal change. (True, medium)

15. The complementary slackness principle states that either there is zero slack on a constraint or the reduced cost is zero. (False, easy)

MULTIPLE CHOICE QUESTIONS

1. Compared with standard linear programming algorithms, those for treating integer linear programming problems usually are:

   a. less complex due to fewer possible solutions.
   b. less amenable to “what if” analysis.
   c. always heuristic.
   d. easier to formulate.

   (b, medium)

2. In the branch-and-bound technique for solving integer linear programming models with a maximization objective function:

   a. Once a feasible solution is found, it is optimal.
   b. If the solution generated at one stage is $X_1 = 5.7$, $X_2 = 3$ but gives an objective function value higher than that of the best integer solution found to date, X_2 must be either 0, 1, or 2 in the optimal solution.
   c. If the solution generated at one stage is $X_1 = 5.7$, $X_2 = 3$ but gives an objective function value higher than that of the best integer solution found to date, a new linear programming problem is solved with two constraints added: $X_1 \leq 6$ and $X_1 \geq 5$.
   d. If the solution generated at one stage is $X_1 = 5.7$, $X_2 = 3$ but gives an objective function value higher than that of the best integer solution found to date, two new linear programs are solved, one with the constraint $X_1 \geq 6$ added, the other with the constraint $X_1 \leq 5$ added.

   (d, medium)

3. In solving an integer linear programming problem, lifting the integer requirements and first solving the underlying standard linear programming problem:

   a. never yields an optimal solution.
   b. is not part of the branch-and-bound technique.
   c. is not part of the cutting plane technique.
   d. may yield an integer-valued solution.

   (d, medium)

4. When compared with standard linear programming, integer linear programming typically has:

   a. more feasible solution points to evaluate.
   b. fewer feasible solution points to evaluate.
   c. the same number of feasible solution points to evaluate.
   d. more linear constraints.

   (b, medium)

5. Which of the following is **not** a necessary linear programming assumption?

a. The decision variable values are discrete.
b. The parameters are specified with certainty.
c. Constant returns to scale in the linear constraints and the object function coefficients.
d. No interactions permitted between decision variables.

(a, easy)

6. The feasible region does not include:

a. interior points.
b. boundary points.
c. points at which at least one of the decision variables is zero.
d. points which violate at least one of the functional or non-negativity constraints.

(d, easy)

7. A "non-binding" constraint is:

a. redundant.
b. not satisfied with an equality at the optimal solution.
c. one having zero slack or surplus
d. never a non-negativity variable constraint.

(b, medium)

8. In a model with two decision variables, the restriction $3X_1 + 2X_2 \leq 6$ represents:

a. a straight line.
b. the region of infeasibility.
c. an extreme point.
d. a linear constraint.

(d, easy)

9. The effect of deleting a linear constraint from a linear programming model depends on whether or not that constraint:

a. is a "≤" or a "≥" constraint.
b. had negative coefficients.
c. is redundant.
d. is binding.

(d, medium)

10. An over-constrained linear programming problem results in what type of solution?

a. Unbounded.
b. Degenerate.
c. Infeasible.
d. Sub-optimal.

(c, medium)

11. The functional constraints of a linear model with nonnegative variables are $3X_1 + 5X_2 \leq 16$ and $4X_1 + X_2 \leq 10$. Which of the following points could **not** be an optimal solution for the model?

a. $X_1 = 2.5, X_2 = 0$
b. $X_1 = 0, X_2 = 3.2$
c. $X_1 = 1, X_2 = 2.25$
d. $X_1 = 2, X_2 = 2$

(c, difficult)

12. Which statement is **not** true if a maximization problem has an unbounded solution?

a. A data entry error has been made or a limiting constraint has been omitted.
b. The objective function value goes to $+\infty$.
c. The values of all decision variables go to $+\infty$.
d. The feasible region is unbounded.

(c, difficult)

13. Squire Leathers produces two sizes of wallets from cowhide. The first requires 60 squire inches of cowhide and the second requires 100 square inches. The company has 1000 square feet of cowhide. Part of the model is:

a. $60X_1 + 100X_2 \geq 144{,}000$
b. $60X_1 + 100X_2 \leq 144{,}000$
c. $60X_1 + 100X_2 = 144{,}000$
d. $60X_1 \leq 144{,}000$ and $100X_2 \leq 144{,}000$

(b, easy)

14. Banner Tools produces two styles of steel hammers with wooden handles. The first sells for \$6 and consists of .5 pounds of steel; the second sells for \$15 and consists of 1 pound of steel. Since steel costs the firm \$4 per pound and the handle, labor, and packaging costs amount to \$1 for either hammer, the profits coefficients are \$6 - .5(\$4) - \$1 = \$3 for the smaller hammer and \$15 – 1(\$4) - \$1 = \$10 for the larger hammer. Thus the objective function for this model is MAX $3X_1 + 10X_2$. Given that the shadow price for steel is \$2, which of the following statements is correct?

a. Banner should not buy more steel.
b. Banner should buy all the steel it can only if it can

purchase it for less than $2 per pound.

c. Banner should buy at least as much as the "ALLOWABLE INCREASE", but only if it can be purchased for less than $2 per pound.

d. Banner should buy at least as much as the "ALLOWABLE INCREASE", but only if it can be purchase for less than $6 per pound.

(d, difficult)

15. The principle of "complementary slackness" implies that:

a. if the reduced cost is not zero, than the value of the decision variable is zero.

b. if a decision variable is zero, then its reduced cost must be positive.

c. if a decision variable is zero, then its reduced cost must be non-zero.

d. if a decision variable is zero, then its reduced cost must be zero.

(a, difficult)

16. The objective function coefficients for X_1, X_2, and X_3 are 15, 32, and 48 respectively. Excel prints that their ranges of optimality are from 10 to 20, from 30 to 40, and from $-\infty$ to 50 respectively. If the objective function coefficients are changed to 14, 31, and 45, the optimal solution:

a. will not change.

b. may not change.

c. will definitely change.

d. may change.

(a, medium)

17. Excel Solver reports "Solver could not find a feasible solution." What is your best logical alternative?

a. Change the objective function.

b. Run Excel Solver again.

c. Relax a constraint.

d. Add a constraint.

(c, easy)

18. Dean Air uses a linear programming model to schedule flights, assign crews and meet passenger demand. The objective function is to minimize downtime. Constraints include a limit on pilot hours, meeting passenger demand, and scheduled downtime. Which of the following cannot be accomplished with sensitivity analysis?

a. Add a constraint of airport gate usage.
b. Change the objective to maximize profits.
c. Increase values for demand.
d. Drop the crew assignment constraint from the model.

(b, easy)

19. If the points (5, 5, 5) and (7, 9, 5) are both optimal solutions to a linear programming problem, what other point is also optimal?

a. (6,7,8)
b. (6,7,5)
c. (7,5,5)
d. (5,9,5)

(b, medium)

20. Which of the following constraints is redundant?

a. $X_1 + X_2 \leq 10$
b. $X_1 - X_2 \leq 10$
c. $X_1 + 3X_2 \leq 20$
d. $X_1,\ X_2 \geq 0$

(b, easy)

SHORT ANSWER QUESTIONS

1. What is a linear programming model?

(A model that seeks to optimize (maximize or minimize) a linear objective function subject to a set of linear constraints.) (easy)

2. Define the additivity assumption.

(The total value of the objective function equals the sum of the linear terms.) (easy)

3. Why would you not use linear programming to solve a problem with a single decision variable linear objective function and multiple constraints?

(The problem is solved by the most restrictive constraint.) (easy)

4. Compare the points that the Simplex method moves to with the points touched by the graphical method for solving a two variable linear program.

(The Simplex method moves from one extreme point to an adjacent extreme point until reaching the optimal point. In the graphical method, the objective function is first drawn through the feasible region, defining a set of interior points. Then the function is moved in a parallel fashion to a boundary point, followed by an extreme point, and then to the optimal point.) (medium)

5. What is wrong with this linear programming model?

$$
\begin{array}{lll}
\text{MAX} & 2X_1 + 3X_2 + X_3 & \\
\text{ST} & X_1 + 2X_2 & \geq 10 \\
 & X_3 & \leq 50 \\
 & X_1 + X_2 & < 65 \\
 & X_1, X_2, X_3 \geq 0 &
\end{array}
$$

(The third constraint is “less than” rather than “less than or equal to.”) (easy)

6. “Allowable Decrease” and “Allowable Increase” refer to what range?

(Range of optimality.) (easy)

7. Explain the different interpretations of shadow costs when the objective function is based on sunk costs versus included costs.

(For sunk costs, the shadow price equals the value of an extra unit of a resource. For included costs, the shadow price represents the premium value above the existing resource unit value.) (medium)

8. Explain the SUMPRODUCT function of Excel.

(SUMPRODUCT(A_1:A_n,B_1:B_n) = SUM(for I = 1 to n) (Ai * Bi)

That is, it is the sum of the products of two (or more) arrays. The arrays can be rows or columns.) (easy)

9. When running Excel, what does "The Set Cell values do not converge" mean?

(The solution is unbounded.) (easy)

10. What clue does Excel give to the possible existence of alternate optimal solutions?

(The allowable increase or the allowable decrease equals 0 for all decision variables in the optimal solution.) (easy)

11. What is the major difference between a product mix problem and a diet problem?

(The product mix problem maximizes an objective function subject to "≤" constraints. The diet problem minimizes an objective function subject to "≥" constraints.) (easy)

12. How do you designate variables as integers when using Excel Solver?

(Use the "ADD" button in the CONSTRAINTS section to bring up the Add Constraint dialogue box. Select "int" from the pull down menu in the middle input area.) (easy)

13. Ira Wax solved an integer linear programming problem by setting up a linear programming model without the integer constraints and rounding the solution. List the possible problems with this approach.

(The solution may be infeasible or suboptimal.) (easy)

FORMULATION/SOLUTION/ANALYSIS QUESTIONS

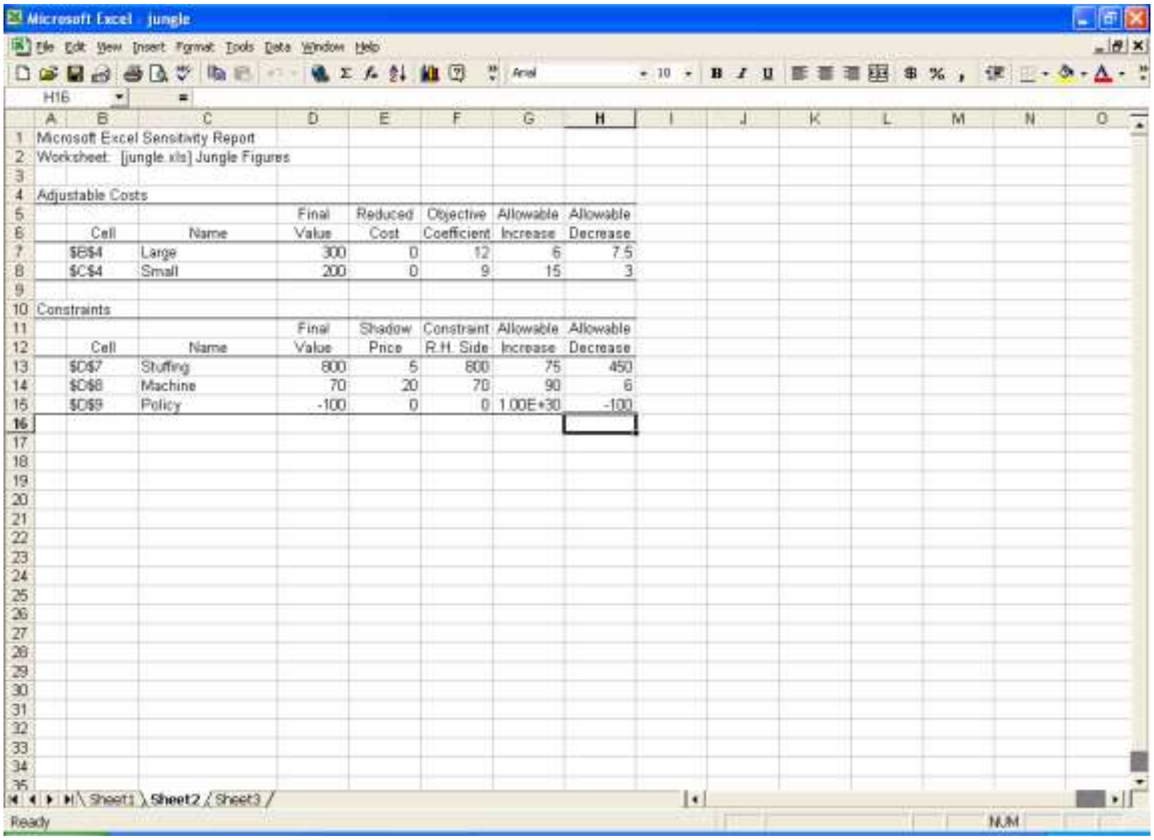
1. Office2000 produces expensive, quality 2-drawer and 4-drawer filing cabinets from solid oak for major corporations. A 2-drawer model utilizes 2 labor hours to produce and package for a net profit of $75. The 4-drawer model utilizes 3 labor hours to produce and package and nets a profit of $125. Each month Office2000 has 360 labor hours available and can obtain up to 200 2-drawer frames, 200 4-drawer frames, and 400 drawers from its oak supplier.
Formulate a linear programming model to maximize monthly profit from the manufacture of filing cabinets at Office2000.

(MAX $75X_1 + 125X_2$
ST $2X_1 + 3X_2 \leq 360$
$2X_1 + 4X_2 \leq 400$
$X_1 \leq 200$
$X_2 \leq 200$
$X_1, X_2 \geq 0$)
(easy)

2. Jungle Figures, Inc. produces two models of its stuffed giraffes, which it markets to high-end retail stores. The large giraffe requires 2 pounds of stuffing material and 6 minutes of machine time. The small giraffe requires 1 pound of stuffing material and 12 minutes of machine time since its tighter stitching pattern requires it to be stitched twice. There are 800 pounds of stuffing and 70 machine hours available each week. By adhering to a policy of not producing more than twice the number of large giraffes as small giraffes (this is a constraint), Jungle Figures has been able to sell all the giraffes it produces at $12 per large giraffe and $9 per small giraffe.
Formulate a linear programming model that maximizes Jungle Figures' weekly profit from the manufacturing of the giraffes.

(MAX $12X_1 + 9X_2$
ST $2X_1 + X_2 \leq 800$
$.1X_1 + .2X_2 \leq 70$
$X_1 - 2X_2 \leq 0$
$X_1, X_2 \geq 0$)
(medium)

3. Consider the following sensitivity report from Excel for the model in question 3, which has a profit of $5,400.



Microsoft Excel Sensitivity Report
Worksheet: [jungle.xls] Jungle Figures

Adjustable Costs

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	Large	300	0	12	6	7.5
C4	Small	200	0	9	15	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D7	Stuffing	800	5	800	75	450
D8	Machine	70	20	70	90	6
D9	Policy	-100	0	0	1.00E+30	-100

A. If Jungle Figures eliminates its policy of not producing more than twice the number of large giraffes as small giraffes, how will this affect the optimal solution?
(Not at all; it is not a binding constraint.) (easy)

B. Suppose 50 pounds of extra stuffing could be purchased for $200. Should Jungle do this? Would the optimal solution change?
(The shadow price of stuffing is 5. So 50 extra pounds will add $250 to gross profit. This is $50 more than the cost of the stuffing, and thus it should make the purchase. The optimal solution would change although the shadow prices would not.)
(medium)

C. Jungle is considering diverting 10 hours of machine time from the production of other products to produce giraffes. It figures it will lose $225 in gross profits from those products by this action. Should Jungle do this? Would the optimal solution change?
(No. An increase of 10 hours in the production of giraffes will only add $200 to gross profit, $25 less than the cost of diverting the hours. Jungle should keep the same solution.) (medium)

D. Suppose that demand is such that Jungle could effectively raise its prices to $15 on each giraffe (large or small). Would the optimal production schedule change?
(No. Using the 100% rule, the percent increase for large giraffes is 50% and for small giraffes 40%. Since the total percent change is less than 100%, the optimal solution would remain the same.)
(medium)

E. Suppose demand for small giraffes waned, and Jungle cut its process so that its profit on small giraffes was reduced by 50% to \$4.50. Would the optimal solution change?
(Yes. This is outside the range of optimality for the profit of small giraffes.) (easy)

4. The University of Iowa is experimenting with a blend of soil amendments to be used in an analysis of variance study of the response of tomatoes to various amounts of sunlight. To perform this study, all other elements must be controlled so that the only variable is the sunlight. The minimum requirements for calcium, phosphorous, and potassium are 125 pounds, 150 pounds, and 120 pounds respectively. The soil amendment mixture from Prairie Gold consist of 25% calcium, 25% phosphorous, 12½% potassium, and 37½% other ingredients. It costs \$0.40 per ounce. The mixture from Grinell Grow is 20% calcium, 25% phosphorous, 25% potassium, and 30% other ingredients. It sells for \$0.50 per ounce.
Formulate a linear programming model that will allow the University of Iowa to conduct this experiment using a minimum cost blend of the two soil amendment mixtures.

(Let X1, X2 = the number of *pounds* of Prairie Gold and Grinell Grow mixed.

$$\begin{aligned} \text{MIN} \quad & 6.40X_1 + 8.00X_2 \\ \text{ST} \quad & .25X_1 + .20X_2 \geq 125 \\ & .25X_1 + .25X_2 \geq 150 \\ & .125X_1 + .25X_2 \geq 120 \\ & X_1,\ X_2 \geq 0 \end{aligned}$$

) (medium)

14. A farmer has three possible cereal grain crops to grow in the coming planting season: barley, oats, and wheat. He has 240 acres of arable land: 160 are considered high grade and the remainder is low grade. He has \$42,000 in available capital and can hire virtually unlimited hours of field labor, locally, for \$6 per hour. Relevant crop factors are:

	Barley	Oats	Wheat
Price/bushel	\$3.20	\$3.00	\$2.60
Bushels per acre:			
High grade land	80	84	90
Low grade land	75	80	85
Labor hours/acre	20	16	15
Non-labor costs/acre	\$98	\$97	\$88

Formulate this as a linear programming model.

(First some analysis:

	Barley		Oats		Wheat	
	High	Low	High	Low	High	Low
Bushels/acre	80	75	84	80	90	85
X price	\$256	\$240	\$252	\$240	\$234	\$221
Labor cost/acre	120	120	96	96	90	90
Other costs/acre	98	98	97	97	88	88
Profit/acre	38	22	59	47	56	43

Formulation:

Let X_1 = # of acres of high grade land to plant in barley
X_2 = # of acres of low grade land to plant in barley
X_3 = # of acres of high grade land to plant in oats
X_4 = # of acres of low grade land to plant in oats
X_5 = # of acres of high grade land to plant in wheat
X_6 = # of acres of low grade land to plant in wheat

MAX $38X_1 + 22X_2 + 59X_3 + 47X_4 + 56X_5 + 43X_6$
ST $X_1 + X_3 + X_5 \le 160$
$X_2 + X_4 + X_6 \le 80$
$218X_1 + 218X_2 + 193X_3 + 193X_4 + 178X_5 + 178X_6 \le 42{,}000$
$X_1, X_2, X_3, X_4, X_5, X_6 \ge 0$) (medium)

15. The Hawaii Surfboard Company produces three different styles of surfboards (the Oahu, the Kauai, and the Maui) made from combinations of three different materials, A, B, and C. The Oahu model nets a $35 profit and requires 1 unit of A, 1 unit of B, and 1 unit of C. The Kauai model nets a $75 profit and requires 4 units of A and 1 unit of B. The Maui model nets a $100 profit and requires 1 unit of A, 4 units of B, and 1 unit of C. There are 200 units of A, 200 units of B, and 100 units of C available weekly.

A. Formulate a linear programming model to determine the optimal weekly profit.

(MAX $35X_1 + 75X_2 + 100X_3$
ST $X_1 + 4X_2 + X_3 \le 200$
$X_1 + X_2 + 4X_3 \le 200$
$X_1 + X_3 \le 100$
$X_1, X_2, X_3 \ge 0$) (easy)

B. How would the model change if Hawaii Surfboard wanted to produce an equal number of each model?
(Add the constraints X1 – X2 = 0 and X1 – X3 = 0 to the original formulation. This ensures an equal number of each model.) (medium)

16. The Carolina Chemical Company (CCC) makes two products, soap and shampoo, both in liquid form. The same two raw material inputs, glycerin and potash, and the same production line are used in the manufacture of each. Each production day, one ton of glycerin and two tons of potash are available. Each gallon of soap requires 8 pounds of glycerin and 32 pounds of potash, while a gallon of shampoo uses 12 pounds of glycerin and 19 pounds of potash.
CCC's production day is 8 full hours (480 minutes). Its production line can turn out 25 gallons of soap, or 20 gallons of shampoo, per hour. Only one product can be made at a time, but the line is easily changed over at negligible cost from one item to the other. Forecasts indicate that demand for soap and shampoo far exceeds production capacity. The profit per gallon of soap is $30 and per gallon of shampoo is $36.

A. Formulate a linear programming model that will maximize CCC's daily profit.

(MAX $30X_1 + 36X_2$

ST $8X_1 + 12X_2 \le 2000$ (Glycerin)

$32X_1 + 19X_2 \le 4000$ (Potash)

$.04X_1 + .05X_2 \le 8$ (Production hours)

$X_1, X_2 \ge 0$

Note: the most common student error is to write the third linear constraint as $25X_1 + 20X_2 \le 8$) (medium)

B. Suppose that the production of soap also yields a byproduct called Blyx. The manufacturing process yields 0.3 units of Blyx for every unit of soap. Up to 15 gallons of Blyx can be sold per day for canine shampoo at a profit of $10 per gallon. CCC must dispose of any excess of Blyx at a removal cost of $4 per gallon. Using X3 = the number of gallons of Blyx produced up to 15 gallons, and X4 = the number of gallons of Blyx produced over 15 gallons, modify the linear programming model for this problem.

(Change the objective function to: MAX $30X_1 + 36X_2 + 10X_3 - 4X_4$ and add the following constraints: $-.3X_1 + X_3 + X_4 = 0$

$X_3 \le 15$

$X_3, X_4 \ge 0$) (difficult)

17. Spee-D Delivery Service wants to blend two gasolines (R and S) for use in its trucks. For the upcoming period, at least 10,000 gallons are required. Spee-D has a 20,000 gallon storage facility. Any required quantities of R and S gasoline may be purchased from a local refinery and mixed.
R is 87 octane and costs $0.90 a gallon. S is 93 octane and costs $1.20 per gallon. Spee-D's fleet of 100 trucks requires an octane of no less than 89. Octane of the mixed output is linearly proportional to input octane. For example, equal parts of R and S would yield a 90 octane output. There is no gasoline loss in the blending process.
Formulate a linear programming model for this problem. Is a summation variable advisable here? Explain.

(MIN $.90X_1 + 1.20X_2$

ST $X_1 + X_2 \ge 10{,}000$ (Period's requirements)

$X_1 + X_2 \le 20{,}000$ (Storage capacity)

$2X_1 - 4X_2 \le 0$ (Derived from $87X_1 + 93X_2 \ge 89(X_1 + X_2)$)

$X_1, X_2 \ge 0$

A summation variable $X_3 = (X_1 + X_2)$ total gasoline to be blended, could be employed, but it is hardly necessary as it would appear in only one constraint.) (medium)

18. A small foundry has received an order for an iron alloy called Falloy which has the following specifications:

Content	Specification
Nickel	5%
Manganese	4%
Copper	1-2%
Impurities	no more than 3%
Iron	the balance

Falloy can be produced from the following inputs (balance to 100% consisting of iron).

	Input Composition				
	Ni	Mn	Cu	Other	Cost/ton
A	29.60	49.30	0.20	0.82	65
B	03.40	10.20	0.12	1.06	35
C	01.12	00.25	1.00	0.54	34
D	00.40	01.60	2.50	0.70	29
E	56.20	27.20	0.40	1.82	70

Formulate the linear programming problem that will indicate the blending "recipe" that will minimize the cost of this alloy.

$$
\begin{aligned}
(\ \text{MIN}\quad & 65X_1 + 35X_2 + 34X_3 + 29X_4 + 70X_5 \\
\text{ST}\quad & 29.6X_1 + 3.4X_2 + 1.12X_3 + 0.4X_4 + 56.2X_5 = 5.00 \text{ (Ni)} \\
& 49.3X_1 + 10.2X_2 + 0.25X_3 + 1.6X_4 + 27.2X_5 = 4.00 \text{ (Mn)} \\
& 0.2X_1 + 0.12X_2 + X_3 + 2.5X_4 + 0.4X_5 \geq 1.00 \text{ (Cu min)} \\
& 0.2X_1 + 0.12X_2 + X_3 + 2.5X_4 + 0.4X_5 \leq 2.00 \text{ (Cu max)} \\
& 0.82X_1 + 1.06X_2 + 0.54X_3 + 0.7X_4 + 1.82X_5 \leq 3.00 \text{ (Imp)} \\
& X_1 + X_2 + X_3 + X_4 + X_5 = 1.00 \text{ (100\%)} \\
& X_1, X_2, X_3, X_4, X_5 \geq 0\ )\quad \text{(medium)}
\end{aligned}
$$

19. Extel Industries produces integrated circuit components for use in the next generation of automobiles. In particular, it can make the AT50 unit that can control the air temperature in the car, the V35 unit that monitors vibration and makes adjustments accordingly, and the GM30 that regulates the engine so that the car provides maximum efficiency. The unit profits on these units are $84, $112, and $126, respectively. Each of these units uses different quantities of three computer chips (as shown below), which Extel purchases from a Taiwanese distributor.

	C101	H122	P043
AT50	1	1	1
V35	1	1	2
GM30	1	2	1

Each week Extel receives 350 C101's, 300 H122's, and 400 P043's. The purchase price for these chips represents a sunk cost to Extel.

A. Formulate a linear programming model to help Extel decide how many units of each product to produce weekly.

$$
\begin{aligned}
(\ \text{MAX}\quad & 84X_1 + 112X_2 + 126X_3 \\
\text{ST}\quad & X_1 + X_2 + X_3 \leq 350 \\
& X_1 + X_2 + 2X_3 \leq 300 \\
& X_1 + 2X_2 + X_3 \leq 400 \\
& X_1, X_2, X_3 \geq 0\ )\quad \text{(easy)}
\end{aligned}
$$

B. Explain why the constraint for the C101 chip is a redundant constraint.

(The constraint for H122 restricts $X_1 + X_2 + 2X_3$ to at most 300, thus $X_1 + X_2 + X_3$ must also be at most 300 which is less than 350.) (medium)

20. The Finast Filter Company makes two types of filters for domestic air purifiers: the standard (nicknamed the Cleaner) for ordinary use, and the Scrubber for industrial-strength problems. The Cleaners bring in a profit of $0.50 each, while the Scrubbers command a profit of $1.25 each.
Due to a higher piece-rate wage for the Scrubbers, the labor union was able to negotiate a requirement that each day the number of Scrubber filters produced will be at least as great as the number of Cleaners. The production line can turn out 25 Scrubbers per minute or 75 Cleaners per minute. Only one type of filter can be made at a time, but the production line is easily changed over at a negligible cost from one kind of filter production to the other.
FFC must produce at least 5,000 Cleaners today to fill a rush order. The production day is 8 full hours (480 minutes).
Formulate a linear programming model to determine how many of each filter should be produced today to maximize profit.

(MAX $.50X_1 + 1.25X_2$

ST $X_1 \geq 5{,}000$ (rush order)

$X_1 - X_2 \leq 0$ (union demand)

$X_1 + 3X_2 \leq 36{,}000$ (derived from $X_1/75 + X_2/25 \leq 480$)

$X_1, X_2 \geq 0$

A common student error is to specify separate production time constraints for each type filter. Another is to show the third linear constraint as $75X_1 + 25X_2 \leq 480$.) (medium)