Discrete Mathematics with Applications, 4th Edition
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Test Bank Questions

Chapter 1

1. Fill in the blanks to rewrite the following statement with variables: Is there an integer with a remainder of 1 when it is divided by 4 and a remainder of 3 when it is divided by 7?

   (a) Is there an integer n such that n has _____?

   (b) Does there exist _____ such that if n is divided by 4 the remainder is 1 and if _____?

2. Fill in the blanks to rewrite the following statement with variables:

   Given any positive real number, there is a positive real number that is smaller.

   (a) Given any positive real number r, there is _____ s such that s is _____.

   (b) For any _____ , _____ such that $s < r$.

3. Rewrite the following statement less formally, without using variables:

   There is an integer n such that $1/n$ is also an integer.

4. Fill in the blanks to rewrite the following statement:

   For all objects T, if T is a triangle then T has three sides.

   (a) All triangles _____.

   (b) Every triangle _____ .

   (c) If an object is a triangle, then it _____.

   (d) If T _____ , then T _____.

   (e) For all triangles T, _____.

5. Fill in the blanks to rewrite the following statement:

   Every real number has an additive inverse.

   (a) All real numbers _____.

   (b) For any real number x, there is _____ for x.

   (c) For all real numbers x, there is real number y such that _____.

6. Fill in the blanks to rewrite the following statement:

   There is a positive integer that is less than or equal to every positive integer.

   (a) There is a positive integer m such that m is _____.

   (b) There is a _____ such that _____ every positive integer.

   (c) There is a positive integer m which satisfies the property that given any positive integer n, m is _____.

7. (a) Write in words how to read the following out loud $\{n \in \mathbf{Z} \mid n \text{ is a factor of } 9\}$.

   (b) Use the set-roster notation to indicate the elements in the set.

8. (a) Is $\{5\} \in \{1, 3, 5\}$?
   (b) Is $\{5\} \subseteq \{1, 3, 5\}$?
   (c) Is $\{5\} \in \{\{1\}, \{3\}, \{5\}\}$?
   (d) Is $\{5\} \subseteq \{\{1\}, \{3\}, \{5\}\}$?

9. Let $A = \{a, b, c\}$ and $B = \{u, v\}$. Write *a.* $A \times B$ and *b.* $B \times A$.

10. Let $A = \{3, 5, 7\}$ and $B = \{15, 16, 17, 18\}$, and define a relation R from A to B as follows: For all $(x, y) \in A \times B$,
$$(x, y) \in R \quad \Leftrightarrow \quad \frac{y}{x} \text{ is an integer.}$$
   (a) Is 3 R 15? Is 3 R 16? Is $(7, 17) \in R$? Is $(3, 18) \in R$?
   (b) Write R as a set of ordered pairs.
   (c) Write the domain and co-domain of R.
   (d) Draw an arrow diagram for R.
   (e) Is R a function from A to B? Explain.

11. Define a relation R from $\mathbf{R}$ to $\mathbf{R}$ as follows: For all $(x, y) \in \mathbf{R} \times \mathbf{R}$,$(x, y) \in R$ if, and only if, $x = y^2 + 1$.
   (a) Is $(2, 5) \in R$? Is $(5, 2) \in R$? Is (-3) R 10? Is 10 R (-3)?]
   (b) Draw the graph of R in the Cartesian plane.
   (c) Is R a function from A to B? Explain.

12. Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$. Define a function G: $A \to B$ as follows:
$$G = \{(1, b), (2, c), (3, b), (4, c)\}.$$
   (a) Find $G(2)$.
   (b) Draw an arrow diagram for G.

13. Define functions F and G from $\mathbf{R}$ to $\mathbf{R}$ by the following formulas:
$$F(x) = (x + 1)(x - 3) \quad \text{and} \quad G(x) = (x - 2)^2 - 7.$$
   Does $F = G$? Explain.

Chapter 2

1. Which of the following is a negation for "Jim is inside and Jan is at the pool."
   (a) Jim is inside or Jan is not at the pool.
   (b) Jim is inside or Jan is at the pool.
   (c) Jim is not inside or Jan is at the pool.
   (d) Jim is not inside and Jan is not at the pool.
   (e) Jim is not inside or Jan is not at the pool.

2. Which of the following is a negation for "Jim has grown or Joan has shrunk."

   (a) Jim has grown or Joan has shrunk.

   (b) Jim has grown or Joan has not shrunk.

   (c) Jim has not grown or Joan has not shrunk.

   (d) Jim has grown and Joan has shrunk.

   (e) Jim has not grown and Joan has not shrunk.

   (f) Jim has grown and Joan has not shrunk.

3. Write a negation for each of the following statements:

   (a) The variable S is undeclared and the data are out of order.

   (b) The variable S is undeclared or the data are out of order.

   (c) If Al was with Bob on the first, then Al is innocent.

   (d) $-5 \leq x < 2$ (where x is a particular real number)

4. Are the following statement forms logically equivalent: $p \vee q \rightarrow p$ and $p \vee (\sim p \wedge q)$? Include a truth table and a few words explaining how the truth table supports your answer.

5. State precisely (but concisely) what it means for two statement forms to be logically equivalent.

6. Write the following two statements in symbolic form and determine whether they are logically equivalent. Include a truth table and a few words explaining how the truth table supports your answer.

   If Sam bought it at Crown Books, then Sam didn't pay full price.

   Sam bought it at Crown Books or Sam paid full price.

7. Write the following two statements in symbolic form and determine whether they are logically equivalent. Include a truth table and a few words explaining how the truth table supports your answer.

   If Sam is out of Schlitz, then Sam is out of beer.

   Sam is not out of beer or Sam is not out of Schlitz.

8. Write the converse, inverse, and contrapositive of "If Ann is Jan's mother, then Jose is Jan's cousin."

9. Write the converse, inverse, and contrapositive of "If Ed is Sue's father, then Liu is Sue's cousin."

10. Write the converse, inverse, and contrapositive of "If Al is Tom's cousin, then Jim is Tom's grandfather."

11. Rewrite the following statement in if-then form without using the word "necessary": Getting an answer of 10 for problem 16 is a necessary condition for solving problem 16 correctly.

12. State precisely (but concisely) what it means for a form of argument to be valid.

13. Consider the argument form:

$$\begin{array}{rl} & p \rightarrow \sim q \\ & q \rightarrow \sim p \\ \therefore & p \vee q \end{array}$$

Use the truth table below to determine whether this form of argument is valid or invalid. Include a truth table and a few words explaining how the truth table supports your answer.

p	q	$\sim p$	$\sim q$	$p \rightarrow \sim q$	$q \rightarrow \sim p$	$p \vee q$
T	T	F	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	T	T	F

14. Consider the argument form:

$$\begin{array}{rl} & p \wedge \sim q \rightarrow r \\ & p \vee q \\ & q \rightarrow p \\ \text{Therefore} & r. \end{array}$$

Use the truth table below to determine whether this argument form is valid or invalid. Annotate the table (as appropriate) and include a few words explaining how the truth table supports your answer.

p	q	r	$\sim q$	$p \wedge \sim q$	$p \wedge \sim q \rightarrow r$	$p \vee q$	$q \rightarrow p$	r
T	T	T	F	F	T	T	T	T
T	T	F	F	F	T	T	T	F
T	F	T	T	T	T	T	T	T
T	F	F	T	T	F	T	T	F
F	T	T	F	F	T	T	F	T
F	T	F	F	F	T	T	F	F
F	F	T	T	F	T	F	T	T
F	F	F	T	F	T	F	T	F

15. Determine whether the following argument is valid or invalid. Include a truth table and a few words explaining why the truth table shows validity or invalidity.

    If Hugo is a physics major or if Hugo is a math major, then he needs to take calculus.

    Hugo needs to take calculus or Hugo is a math major.

    Therefore, Hugo is a physics major or Hugo is a math major.

16. Determine whether the following argument is valid or invalid. Include a truth table and a few words explaining why the truth table shows validity or invalidity.

    If 12 divides 709,438 then 3 divides 709,438.
    If the sum of the digits of 709,438 is divisible by 9 then 3 divides 709,438.
    The sum of the digits of 709,438 is not divisible by 9.
    Therefore, 12 does not divide 709,438.

17. Write the form of the following argument. Is the argument valid or invalid? Justify your answer.

    If 54,587 is a prime number, then 17 is not a divisor of 54,587.
    17 is a divisor of 54,587.
    Therefore, 54,587 is not a prime number.

18. Write the form of the following argument. Is the argument valid or invalid? Justify your answer.

    If Ann has the flu, then Ann has a fever.
    Ann has a fever.
    Therefore, Ann has the flu.

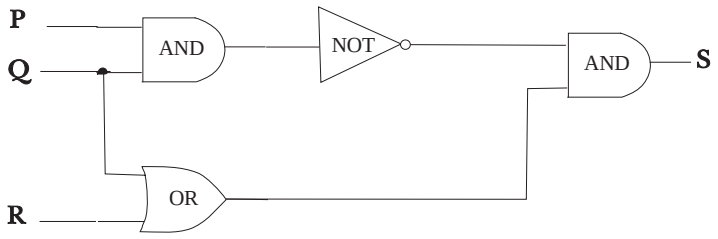
19. On the island of knights and knaves, you meet three natives, A, B, and C, who address you as follows:

    A: At least one of us is a knave.

    B: At most two of us are knaves.

    What are A, B, and C?

20. Consider the following circuit.



    (a) Find the output of the circuit corresponding to the input $P = 1$, $Q = 0$, and $R = 1$.

    (b) Write the Boolean expression corresponding to the circuit.

21. Write 110101_2 in decimal form.

22. Write 75 in binary notation.

23. Draw the circuit that corresponds to the following Boolean expression: $(P \wedge Q) \vee (\sim P \wedge \sim Q)$.(*Note for students who have studied some circuit design*: Do not simplify the circuit; just draw the one that exactly corresponds to the expression.)

24. Find a circuit with the following input/output table.

P	Q	R	S
1	1	1	0
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

25. Find $10111_2 + 1011_2$.

26. Write 100110_2 in decimal form.

27. Write the 8-bit two's complement for 49.