Section 1.2: Mathematical Models: A Catalog of Essential Functions

1. Classify the function $f(x) = \dfrac{x^2 + \pi}{x}$.

a. Power function
b. Root function
c. Polynomial function
d. Rational function
e. Algebraic function
f. Trigonometric function
g. Exponential function
h. Logarithmic function

ANS: D PTS: 1

2. Classify the function $f(x) = \dfrac{\pi^2 + x^2}{e}$.

a. Power function
b. Root function
c. Polynomial function
d. Rational function
e. Algebraic function
f. Trigonometric function
g. Exponential function
h. Logarithmic function

ANS: C PTS: 1

3. Classify the function $f(x) = \sin(5)\, x^2 + \sin(3)\, x$.

a. Power function
b. Root function
c. Polynomial function
d. Rational function
e. Algebraic function
f. Trigonometric function
g. Exponential function
h. Logarithmic function

ANS: C PTS: 1

4. The following time-of-day and temperature (F°) were gathered during a gorgeous midsummer day in Fargo, North Dakota:

Time of Day	Temperature
18	74
17	73
16	73
15	72
14	70
13	70
12	68
11	66
10	63
9	62
8	59
7	58

Source: National Weather Service; www.weather.gov

(a) Make a scatter plot of these data.

(b) Fit a linear model to the data.

(c) Fit an exponential model to the data.

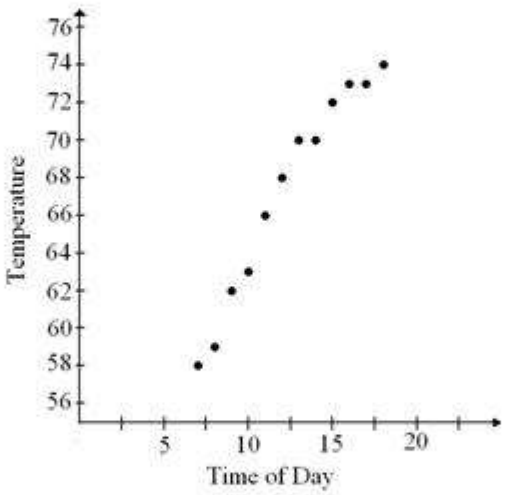
(d) Fit a quadratic model to the data.

(e) Use your equations to make a table showing the predicted temperature for each model, rounded to the nearest degree.

(f) The actual temperature at 8:00 p.m. (20 hours) was 70° F. Which model was closest? Which model was second-closest?

(g) All of the models give values that are too high for each of the times after 6:00 PM. What is one possible explanation for this?

ANS:
(a)



(b) $y = 1.561x + 47.68$
(c) $y = 49.89802\ e^{1.023831}$
(d) $y = -0.09263x^2 + 3.902611x + 33.934$
(e) Linear: 79
Exponential: 80
Quadratic: 75
(f) Closest: quadratic. Second-closest: linear
(g) Answers may vary, but only one explanation is that the data only reflect the part of the day when the air is warming and do not take into account cooling that takes place later in the day into evening. The only model that begins to reflect this is the quadratic model.

PTS: 1

5. Consider the data below:

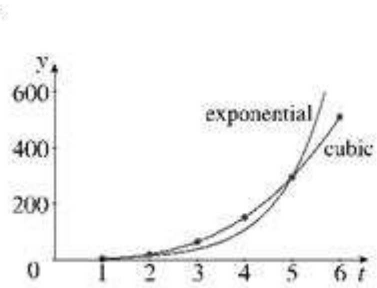
t	1	2	3	4	5	6
y	2.4	19	64	152	295	510

(a) Fit both an exponential curve and a third-degree polynomial to the data.

(b) Which of the models appears to be a better fit? Defend your choice.

ANS:

(a)



(b) A third degree polynomial, for example, $y = 2.40t^3$, appears to be a better fit.

PTS: 1

6. The following table contains United States population data for the years 1981–1990, as well as estimates based on the 1990 census.

Year	U.S. Population (millions)
1981	229.5
1982	231.6
1983	233.8
1984	235.8
1985	237.9
1986	240.1
1987	242.3
1988	244.4
1989	246.8
1990	249.5

Year	U.S. Population (millions)
1991	252.2
1992	255.0
1993	257.8
1994	260.3
1995	262.8
1996	265.2
1997	267.8
1998	270.2
1999	272.7
2000	275.1

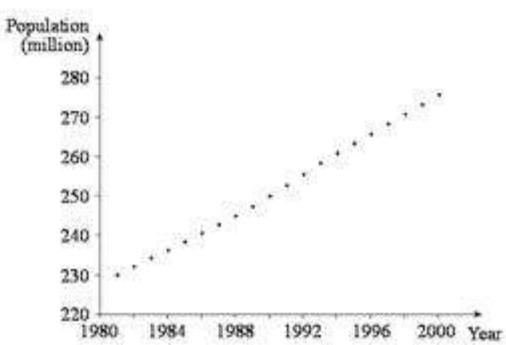
Source: U.S. Census Bureau website

(a) Make a scatter plot for the data and use your scatter plot to determine a mathematical model of the U.S. population.

(b) Use your model to predict the U.S. population in 2003.

ANS:

(a)



A linear model seems appropriate. Taking $t = 0$ in 1981, we obtain the model $P(t) = 2.4455t + 228.5$.

(b) $P(22) \approx 282.3$

PTS: 1

7. The following table contains United States population data for the years 1790–2000 at intervals of 10 years.

Year	Years since 1790	U.S. population (millions)
1790	0	3.9
1800	10	5.2
1810	20	7.2
1820	30	9.6
1830	40	12.9
1840	50	17.1
1850	60	23.2
1860	70	31.4
1870	80	39.8
1880	90	50.2
1890	100	62.9

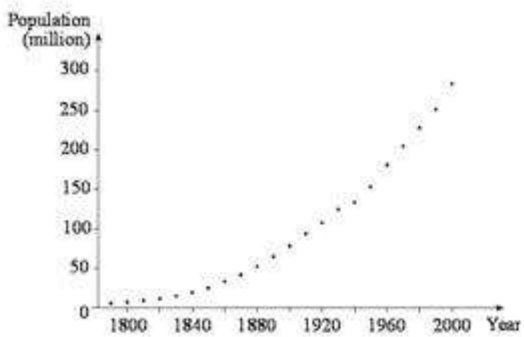
Year	Years since 1790	U.S. population (millions)
1900	110	76.0
1910	120	92.0
1920	130	105.7
1930	140	122.8
1940	150	131.7
1950	160	150.7
1960	170	178.5
1970	180	202.5
1980	190	225.5
1990	200	248.7
2000	210	281.4

(a) Make a scatter plot for the data and use your scatter plot to determine a mathematical model for the U.S. population.

(b) Use your model to predict the U.S. population in 2005.

ANS:

(a)



Answers will vary, but a quadratic or cubic model is most appropriate.
Linear model: $P(t) = 1.28545t - 40.47668$;
quadratic model: $P(t) = 0.006666t^2 - 0.1144t + 5.9$;
cubic model: $P(t) = (6.6365 \times 10^{-6})\,t^3 + 0.004575t^2 + 0.057155t + 3.7$;
exponential model: $P(t) = 6.04852453 \times 1.020407795^t$

(b) Linear model: $P(215) \approx 235.9$;
quadratic model: $P(215) \approx 289.4$;
cubic model: $P(215) \approx 293.4$;
exponential model: $P(215) \approx 465.6$

PTS: 1

8. Refer to your models from Problems 6 and 7 above. Why do the two data sets produce such

different models?

ANS:
Problem 6 covers a much shorter time span, so its data exhibit local linearity, while Problem 7 shows nonlinear population growth over a longer time span.

PTS: 1

9. The following are the winning times for the Olympic Men's 110 Meter Hurdles:

Year	**Time**	**Year**	**Time**	**Year**	**Time**
1896	17.6	1932	14.6	1976	13.3
1900	15.4	1936	14.2	1980	13.39
1904	16	1948	13.9	1984	13.2
1906	16.2	1952	13.7	1988	12.98
1908	15	1956	13.5	1992	13.12
1912	15.1	1960	13.8	1996	12.95
1920	14.8	1964	13.6	2000	13
1924	15	1968	13.3	2004	12.91
1928	14.8	1972	13.24		

(a) Make a scatter plot of these data.

(b) Fit a linear model to the data.

(c) Fit an exponential model to the data

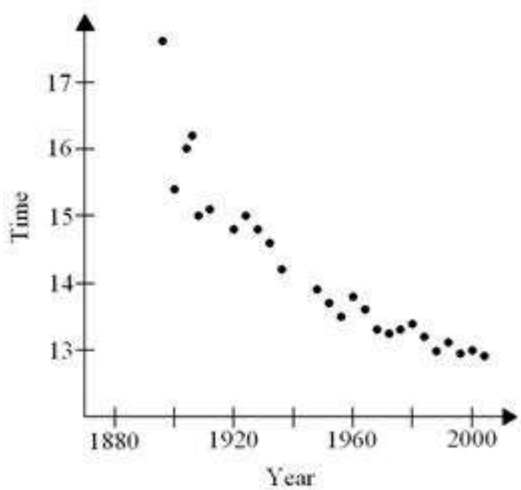
(d) Fit a quadratic model to the data.

(e) Use your equations to make a table showing the predicted winning time for each model for the 2008 Olympics, rounded to the nearest hundredth of a second.

(f) The actual time for the 2008 Olympics was 12.93 seconds. Which model was closest? Which model was second-closest?

ANS:

(a)



(b) $y = -0.0320057x + 76.595$

(c) $y = 1053.09176(0.997791842^x)$

(d) $y = 0.000322x^2 - 1.2872778x + 1299.573$

(e) Linear: 12.33

Exponential: 12.44

Quadratic: 13.04

(f) Closest: quadratic. Second-closest: exponential

PTS: 1