CHAPTER 2
Elements of Decision Problems

Notes
This chapter is intended to start the reader thinking about decision problems in decision-analysis terms. Thus, we talk about decisions to make, uncertain events, and valuing consequences. To make sure that the richness of the terrain is understood, we introduce the concepts of dynamic decision making, a planning horizon, and trade-offs.

In our definition of terms, we refer to a decision maker's *objectives* where the term *values* is used to refer to the decision maker's set of objectives and their structure. The terms *decision* and *alternative* are adopted, and are used throughout the book rather than similar terms such as "choice" and "option." Likewise, we have adopted the term *uncertain event* (and sometimes *chance event*), which then has *outcomes*. Finally, and perhaps most significant, we have adopted Savage's term *consequence* to refer to what the decision maker experiences as a result of a combination of alternative(s) chosen and chance outcome(s). Another term that we use that comes from Keeney's value-focused thinking is the notion of *decision context*. This term is discussed in the text and still more thoroughly in Keeney's book. Briefly, it refers to the specific identification of the problem (from which we might suspect that when one solves the wrong problem, one has used the wrong decision context). It also can be used as a way to identify the class of alternatives that one is willing to consider; a broader context (safety in auto travel as compared to specific traffic laws, for example) leads a decision maker to consider a broader class of alternatives.

The time value of money appears in Chapter 2 and may seem out of place in some ways. It is here because it is a fundamental way that streams of cash flows are valued, and because it provides a nice example of a basic trade-off. Also, we have found that since most students have already been exposed to discounting, we have been able to incorporate NPV calculations into problems and case studies throughout the book. For the few students who have not encountered the topic, the early introduction to discounting in Chapter 2 provides enough information for them to proceed. Of course, the section on NPV may be skipped and used as a reference later for problems that require discounting or for the discussion of trade-offs in Chapter 15.

Topical cross-reference for problems

Requisite models	2.13
"Secretary" problem	2.6
Sequential decisions	2.2, 2.6, Early Bird, Inc.
Time value of money	2.9-2.12, The Value of Patience

Solutions
2.1. a. Some objectives might be to minimize cost, maximize safety, maximize comfort, maximize reliability, maximize cargo capacity (for shopping or vacationing), maximize maneuverability (in city traffic). Students will undoubtedly come up with others as well.

b. In this new context, appropriate objectives might be minimize travel time, maximize exercise, minimize total transportation cost, minimize use of fossil fuels, maximize ease (suitably defined) of visiting friends and shopping. New alternatives to consider include using a bicycle or public transportation, walking, rollerblading, skateboarding, motorcycle or scooter, renting a car, such as Zipcar. One might even consider moving in order to live in a more convenient location.

2.2. Future alternatives can affect the eventual value of the consequence. For example, a university faculty member, when accepting a position at a different institution, may not immediately resign his or her position at the first university. Instead, a leave of absence may be taken. The leave of absence provides the opportunity to decide in the future whether to stay at the new institution or return to the old one. A faculty member would most likely think about the two different situations — resigning the current position immediately versus taking a leave and postponing a permanent decision — in very different ways.

Another good example is purchasing a house. For many people in our mobile society, it is important to think about the potential for selling the house in the future. Many purchasers might buy an unusual house that suits them fine. However, if the house is too unusual, would-be purchasers might be afraid that, *if they decide to sell the house in the near future*, it may be difficult to find a buyer and the sales price might be lower than it would be for a more conventional house.

Finally, the current choice might eliminate a future valuable option. For example, our policy of powering cars with fossil fuels reduces our options for using oil for potentially more valuable and less destructive future activities.

2.3. In the first case, the planning horizon may be tied directly to the solution of the specific problem at hand. If the problem is an isolated one not expected to repeat, this is a reasonable horizon. If more similar problems are anticipated, the planning horizon might change to look forward in time far enough to anticipate future such situations. If the firm is considering hiring a permanent employee or training existing employees, then a planning horizon should be long enough to accommodate employee-related issues (training, reviews, career advancement, and so on). In this broader context, the firm must consider objectives related to hiring a new person (or training), which might include maximizing the welfare of current employees, minimizing long-term costs of dealing with the class of problems, satisfying affirmative-action requirements, or equity in treatment of employees.

2.4. In making any decision, it is important to 1) use all currently available information and 2) think carefully about future uncertainty. Thus it is necessary to keep track of exactly what information is available at each point in time. If information is lost or forgotten, then it will either be treated as an uncertainty or simply not used when deciding. Clearly, the farmer would want to keep up to date on the weather and incorporate any change to the forecast.

2.5. Some possibilities: insurance, hire another firm to manage the protection operation, press for regulatory decisions and evaluations (i.e., get public policy makers to do the necessary analysis), do nothing, develop a "cleanup cooperative" with other firms, or design and develop equipment that can serve a day-to-day purpose but be converted easily to cleanup equipment. Students may come up with a wide variety of ideas.

2.6. The employer should think about qualifications of the applicants. The qualifications that he seeks should be intimately related to what the employer wants to accomplish (objectives — e.g., increase market share) and hence to the way the successful applicant will be evaluated (attributes — e.g., sales). The planning horizon may be critical. Is the employer interested in long-term or short-term performance? The uncertainty that the employer faces, of course, is the uncertainty regarding the applicant's future performance on the specified attributes.

If the decision maker must decide whether to make a job offer at the end of each interview, then the problem becomes a dynamic one. That is, after each interview the decision maker must decide whether to make the offer (and end the search) or to continue the search for at least one more interview, at which time the same decision arises. In this version of the problem, the decision maker faces an added uncertainty: the qualifications of the applicants still to come. (This dynamic problem is sometimes known as the "Secretary Problem," and has been analyzed extensively and in many different forms in the operations-research literature. For example, see DeGroot (2004) *Optimal Statistical Decisions*, Hoboken, NJ: Wiley & Sons. P. 325.)

2.7. Decisions to make: How to invest current funds. Possible alternatives include do nothing, purchase specific properties, purchase options, etc. Other decisions might include how to finance the purchase, when to resell, how much rent to charge, and so on. Note that the situation is a dynamic one if we consider future investment opportunities that may be limited by current investments.

Uncertain events: Future market conditions (for resale or renting), occupancy rates, costs (management, maintenance, insurance), and rental income.

Possible outcomes: Most likely such an investor will be interested in future cash flows. Important trade-offs include time value of money and current versus future investment opportunities.

2.8. Answers depend on personal experience and will vary widely. Be sure to consider current and future decisions and uncertain events, the planning horizon, and important trade-offs.

2.9. NPV $= \frac{-2500}{1.13^0} + \frac{1500}{1.13^1} + \frac{1700}{1.13^2}$

$= -2500 + 1327.43 + 1331.35$

$= \$158.78.$

Or use Excel's function NPV:

=-2500+NPV(0.13,1500,1700) = $158.78

The Excel file, "Problem 2.9.xls" has the equation set-up as a reference to cells that contain the cash flows.

2.10. NPV $= \frac{-12000}{1.12} + \frac{5000}{1.12^2} + \frac{5000}{1.12^3} + \frac{-2000}{1.12^4} + \frac{6000}{1.12^5} + \frac{6000}{1.12^6}$

$= -10{,}714.29 + 3985.97 + 3558.90 - 1271.04 + 3404.56 + 3039.79$

$= \$2003.90$

Using Excel's NPV function:

=NPV(0.12,-12000,5000, 5000,-2000,6000,6000)
= $2,003.90

The internal rate of return (IRR) for this cash flow is approximately 19.2%.

The Excel file, "Problem 2.10.xls" has the equation set-up as a reference to cells that contain the cash flows.

2.11. If the annual rate = 10%, then the monthly (periodic) rate r = 10% / 12 = 0.83%.

NPV(0.83%) $= -1000 + \frac{90}{1.0083} + \frac{90}{1.0083^2} + \ldots + \frac{90}{1.0083^{12}}$

$= \$23.71.$

Or use Excel's NPV function, assume the 12 payments of $90 appear in cells B13:B24:

=-1000+NPV(0.1/12,B13:B24)= $23.71

(As shown in the Excel file "Problem 2.11.xls")

If the annual rate = 20%, then the monthly (periodic) rate r = 20% / 12 = 1.67%.

NPV(1.67%) $= -1000 + \frac{90}{1.0167} + \frac{90}{1.0167^2} + \ldots + \frac{90}{1.0167^{12}} = \$-28.44.$

Or use Excel's NPV function, assume the 12 payments of $90 appear in cells B13:B24:

=-1000+NPV(0.2/12,B13:B24)= $-28.44

(As shown in the Excel file "Problem 2.11.xls")

The annual interest rate (IRR) that gives NPV=0 is approximately 14.45%. You can verify this result by substituting 14.45% / 12 = 1.20% for *r* in the calculations above.

Or with Excel's IRR function, IRR(Values, Guess), assume the series of payments (the initial $1000 payment and the series of 12 payments of $90) are in cells B12:B24:
=IRR(B12:B24,0) = 1.20%

(As shown in the Excel file "Problem 2.11.xls")

2.12. a. If the annual rate = 10%, then the monthly rate r = 10%/12 = 0.83%. Always match the periodicity of the rate to that of the payments or cash flows.

$$\text{NPV(Terry)} = 600 + \frac{-55}{1.0083} + \frac{-55}{1.0083^2} + \ldots + \frac{-55}{1.0083^{12}}$$

= $-25.60.

Be sure to get the orientation correct. For Terry, the loan is a positive cash flow, and the payments are negative cash flows (outflows). Thus, the NPV is negative. Because of the negative NPV, Terry should know that this deal is not in his favor and that the actual interest rate being charged is not 10% annually. If it were, then NPV should equal zero. The actual annual interest being charged must be greater than 10% as NPV is less than zero.

Or with Excel's NPV function, assume the series of 12 payments of $55 are in cells B12:B23.

=NPV(0.1/12,B12:B23)+600

= -$25.60

These calculations and those associated with the remaining parts of the question are shown in the Excel file "Problem 2.12.xls".

b. For the manager, the $600 loan is a negative cash flow, and the payments are positive cash flows. Hence,

$$\text{NPV(Mgr)} = -600 + \frac{55}{1.0083} + \frac{55}{1.0083^2} + \ldots + \frac{55}{1.0083^{12}} = \$25.60.$$

Or with Excel's NPV function, assume the series of 12 receipts of $55 are in cells B12:B23.

=NPV(0.1/12,B12:B23)-600

= $25.60

c. If the annual rate is 18%, then NPV is about $-0.08. In other words, the actual rate on this loan (the internal rate of return or IRR) is just under 18%.

Using Excel's IRR function, and assuming the cash flows are in cells B11:B23:

=IRR(B11:B23,0)*12

= 17.97% annually

2.13. Should future decisions ever be treated as uncertain events? Under some circumstances, this may not be unreasonable.

If the node for selling the car is included at all, then possible consequences must be considered. For example, the consequence would be the price obtained if he decides to sell, whereas if he keeps the car, the consequence would be the length of the car's life and cost to maintain and repair it.

If the node is a decision node, the requisite model would have to identify the essential events and information prior to the decision. If the node is a chance event, this amounts to collapsing the model, and hence may be useful in a first-cut analysis of a complicated problem. It would be necessary to think about scenarios that would lead to selling the car or not, and to evaluate the uncertainty surrounding each scenario.

2.14. Vijay's objectives include maximizing profit, minimizing unsavory behavior, minimizing legal costs, and maximizing Rising Moon's appeal. Students will think of other objectives. Vijay's decision is to apply for a liquor license, and if granted, then he could decide on how to manage drinking at Rising Moon. For example, he might be able to create a separate area of his place, such as a beer garden, where drinking alcohol is allowed. Vijay could also decide to broaden his menu in other ways than serving alcohol. The uncertainties include future sales and profit for Rising Moon, market reaction to offering alcohol, amount of disruption occurring from serving alcohol, and legal liabilities. Consequence measures for sales, profit, and legal costs are clear. He could simply count the number of disruptions to the business due to alcohol or he could try to associate a cost figure to the unsavory behavior. Rising Moon's appeal could be measured by the change in sales volume due to introducing alcohol.

Vijay will certainly, as law requires, hedge by carrying insurance, and he will want to think carefully about the level of insurance. As mentioned, he might be able to have a designated area for drinking alcohol. He could gather information now via surveys or speaking to other local merchants. And he can always change his mind later and stop serving alcohol.

Case Study: The Value of Patience
The Excel solution for this case is provided in the file "Value of Patience case.xlsx".

1. NPV $= -385{,}000 + \frac{100{,}000}{1.18} + \frac{100{,}000}{1.18^2} + \ldots + \frac{100{,}000}{1.18^7} = \$-3847.$

Thus, Union should not accept the project because the NPV is negative.

Using Excel's NPV function and assuming the series of 7 payments of $100,000 are in cells B12:B18:
=-385000+NPV(0.18,B12:B18)
= -$3847

2. NPV $= -231{,}000 + \frac{50{,}000}{1.10} + \frac{50{,}000}{1.10^2} + \ldots + \frac{50{,}000}{1.10^7} = \$12{,}421.$

This portion of the project is acceptable to Briggs because it has a positive NPV.

Using Excel's NPV function and assuming the series of 7 payments of $50,000 are in cells E12:E18:

= -231,000+NPV(0.1,E12:E18)
= $12,421

3. $\text{NPV} = -154{,}000 + \frac{50{,}000}{1.18} + \frac{50{,}000}{1.18^2} + \ldots + \frac{50{,}000}{1.18^7} = \$36{,}576.$

Thus, this portion of the project is profitable to Union.

Using Excel's NPV function and assuming the series of 7 payments of $50,000 are in cells H12:H18:

= -154,000+NPV(0.18,H12:H18)

= $36,576

Some students will want to consider the other $231,000 that Union was considering investing as part of the entire project. Note, however, that if Union invests this money at their 18% rate, the NPV for that particular investment would be zero. Thus the NPV for the entire $385,000 would be the sum of the two NPVs, or $36,576.

4. Patience usually refers to a willingness to wait. Briggs, with the lower interest rate, is willing to wait longer than Union to be paid back. The higher interest rate for Union can be thought of as an indication of impatience; Union needs to be paid back sooner than Briggs.

The uneven split they have engineered exploits this difference between the two parties. For Briggs, a payment of $50,000 per year is adequate for the initial investment of $231,000. On the other hand, the less patient Union invests less ($154,000) and so the $50,000 per year is satisfactory.

As an alternative arrangement, suppose that the two parties arrange to split the annual payments in such a way that Union gets more money early, and Briggs gets more later. For example, suppose each invests half, or $192,500. Union gets $100,000 per year for years 1-3, and Briggs gets $100,000 per year for years 4-7. This arrangement provides a positive NPV for each side: NPV(Union) = $24,927, NPV(Briggs) = $45,657. Briggs really is more patient than Union!

Case Study: Early Bird, Inc.

1. The stated objective is to gain market share by the end of this time. Other objectives might be to maximize profit (perhaps appropriate in a broader strategic context) or to enhance its public image.

2. Early Bird's planning horizon must be at least through the end of the current promotion. In a first-cut analysis, the planning horizon might be set at the end of the promotion plus two months (to evaluate how sales, profits, and market share stabilize after the promotion is over). If another promotion is being planned, it may be appropriate to consider how the outcome of the current situation could affect the next promotion decision.

3, 4.